Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/198-7.4.1-Inverse-hyperbolic-cotangent-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [300]. This is test number [198].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (300)	0.00 (0)
Mathematica	98.00 (294)	2.00 (6)
Maple	91.00 (273)	9.00 (27)
Maxima	82.00 (246)	18.00 (54)
Fricas	74.33 (223)	25.67 (77)
Mupad	51.00 (153)	49.00 (147)
Giac	50.67 (152)	49.33 (148)
Sympy	36.33 (109)	63.67 (191)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

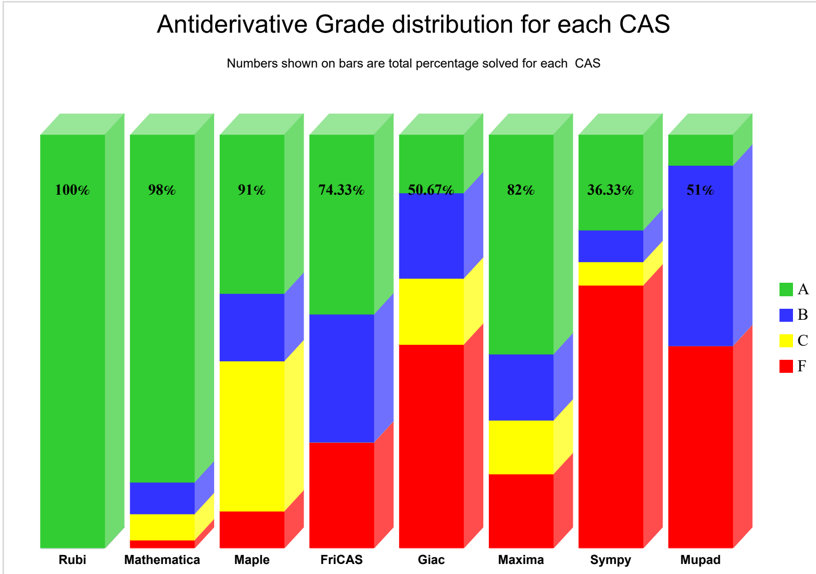
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

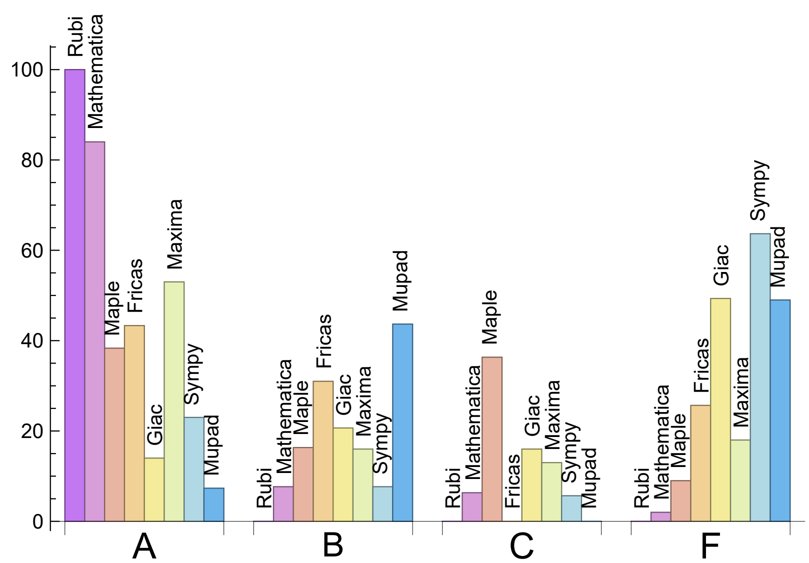
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	84.00	7.67	6.33	2.00
Maxima	53.00	16.00	13.00	18.00
Fricas	43.33	31.00	0.00	25.67
Maple	38.33	16.33	36.33	9.00
Sympy	23.00	7.67	5.67	63.67
Giac	14.00	20.67	16.00	49.33
Mupad	N/A	43.67	0.00	49.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	27	100.00 %	0.00 %	0.00 %
Fricas	77	100.00 %	0.00 %	0.00 %
Giac	148	97.97 %	0.00 %	2.03 %
Maxima	54	98.15 %	0.00 %	1.85 %
Sympy	191	92.15 %	7.33 %	0.52 %
Mupad	147	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

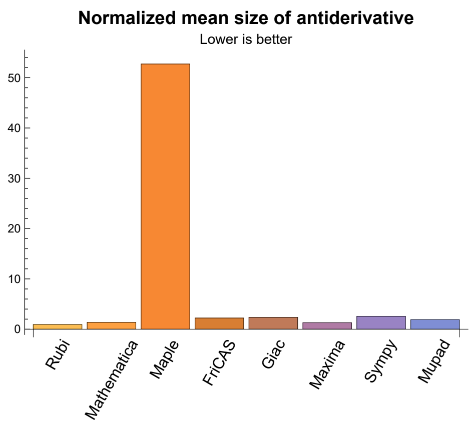
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

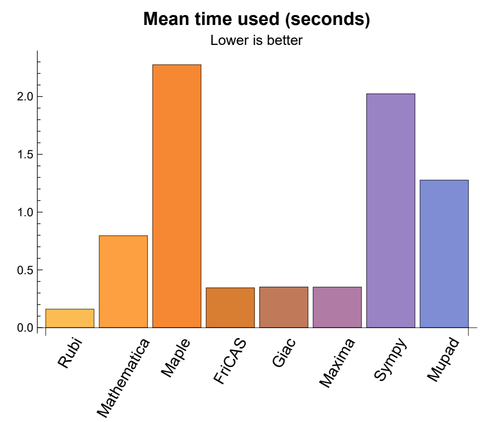
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	126.13	0.93	79.00	1.00
Mathematica	0.80	213.06	1.36	79.00	0.93
Maple	2.28	5640.69	52.72	334.00	2.45
Maxima	0.35	128.72	1.29	78.50	1.10
Fricas	0.34	276.11	2.22	92.00	1.42
Sympy	2.02	273.73	2.56	42.00	1.11
Giac	0.35	164.66	2.36	77.00	1.73
Mupad	1.28	145.89	1.89	48.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{41, 70, 73, 75, 78, 79, 82, 106, 109, 112, 113, 114, 118, 238, 255, 282\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

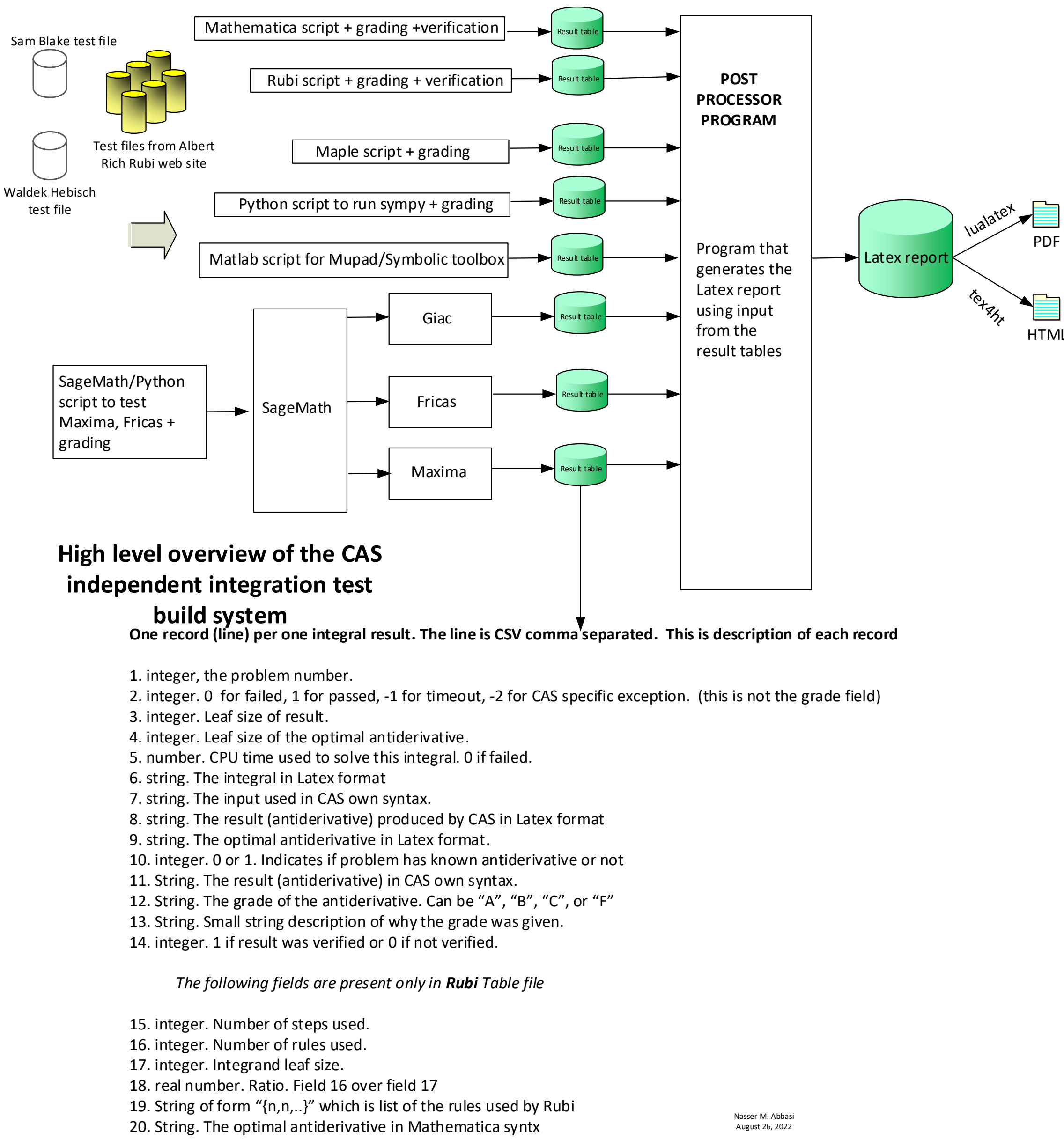
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 76, 77, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 110, 111, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 211, 212, 213, 214, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 270, 271, 272, 273, 274, 278, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { 41, 70, 95, 98, 100, 101, 109, 139, 150, 205, 210, 215, 219, 224, 229, 231, 238, 248, 255, 275, 276, 279, 281 }

C grade: { 24, 26, 34, 66, 73, 74, 75, 78, 79, 82, 106, 112, 113, 114, 115, 116, 118, 265, 282 }

F grade: { 117, 119, 123, 124, 269, 277 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 35, 36, 37, 38, 42, 43, 48, 49, 50, 51, 52, 53, 54, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 72, 74, 75, 77, 78, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 111, 113, 120, 121, 122, 125, 126, 127, 129, 130, 133, 134, 135, 140, 151, 162, 171, 180, 189, 194, 195, 197, 198, 199, 200, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 282, 283, 284, 285, 286, 289, 292, 293, 298 }

B grade: { 14, 16, 19, 20, 21, 22, 28, 39, 40, 41, 56, 59, 61, 69, 70, 71, 76, 82, 86, 102, 103, 104, 109, 110, 116, 123, 124, 131, 185, 196, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 259, 263, 287, 288, 290, 291, 294 }

C grade: { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 55, 73, 79, 93, 112, 114, 115, 117, 118, 128, 132, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 163, 167, 168, 169, 170, 173, 176, 177, 178, 179, 181, 186, 187, 188, 193, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 278, 279, 295, 296, 297, 299, 300 }

F grade: { 44, 45, 46, 47, 119, 147, 148, 149, 158, 164, 165, 166, 172, 174, 175, 182, 183, 184, 190, 191, 192, 270, 271, 275, 276, 277, 281 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 23, 35, 36, 37, 38, 39, 40, 42, 43, 50, 51, 52, 53, 54, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 103, 104, 105, 107, 108, 110, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 239, 243, 247, 252, 256, 260, 264, 268, 280, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299 }

B grade: { 7, 16, 17, 19, 22, 25, 27, 31, 33, 41, 44, 45, 46, 47, 55, 56, 59, 61, 86, 93, 95, 98, 100, 101, 102, 109, 140, 151, 234, 238, 240, 241, 242, 244, 245, 246, 251, 255, 257, 258, 259, 261, 262, 263, 283, 286, 295, 300 }

C grade: { 76, 79, 141, 152, 153, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 266, 267, 269, 272, 273, 274 }

F grade: { 18, 24, 26, 28, 29, 30, 32, 34, 48, 49, 73, 80, 81, 82, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 231, 232, 233, 236, 237, 248, 249, 250, 253, 254, 265, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 50, 51, 52, 55, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 104, 105, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 163, 193, 194, 195, 196, 197, 198, 199, 202, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 272, 273, 274, 280, 289, 291, 292, 293, 296, 297, 298, 299 }

B grade: { 44, 45, 46, 47, 53, 54, 57, 95, 102, 103, 107, 108, 151, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 200, 201, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 265, 283, 284, 285, 286, 287, 288, 290, 294, 295, 300 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

2.1.6 Sympy

A grade: { 8, 9, 10, 11, 12, 14, 16, 20, 22, 42, 43, 53, 54, 57, 58, 62, 63, 64, 65, 90, 94, 96, 105, 120, 122, 126, 127, 129, 131, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 155, 157, 162, 170, 171, 178, 179, 180, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 292, 293 }

B grade: { 60, 67, 68, 87, 88, 89, 91, 92, 97, 99, 102, 103, 104, 107, 108, 130, 156, 189, 194, 195, 196, 198, 199 }

C grade: { 1, 2, 3, 4, 5, 6, 35, 36, 37, 38, 266, 267, 268, 272, 273, 274, 297 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 59, 61, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 123, 124, 125, 128, 132, 136, 141, 142, 146, 152, 153, 154, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 197, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 298, 299, 300 }

2.1.7 Giac

A grade: { 42, 43, 44, 45, 46, 47, 50, 51, 52, 54, 55, 120, 121, 122, 126, 127, 189, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 295, 297, 298, 300 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 53, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 128, 129, 130, 131, 133, 134, 135, 193, 292, 293, 294, 296, 299 }

C grade: { 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 268, 272, 273, 274 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 61, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 136, 146, 158, 166, 175, 184, 185, 186, 187, 188, 190, 191, 192, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 266, 267, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291 }

2.1.8 Mupad

A grade: { 42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 91, 92, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 198, 199, 266, 267, 268, 272, 273, 274, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 90, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	67	54	61	51	49	245	41
N.S.	1	1.00	1.31	1.06	1.20	1.00	0.96	4.80	0.80
time (sec)	N/A	0.019	0.008	0.079	0.253	0.380	0.368	0.400	1.313

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	50	46	55	54	255	43
N.S.	1	1.00	1.00	1.00	0.92	1.10	1.08	5.10	0.86
time (sec)	N/A	0.026	0.007	0.059	0.252	0.352	0.334	0.392	1.267

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	46	52	43	41	195	33
N.S.	1	1.00	1.39	1.12	1.27	1.05	1.00	4.76	0.80
time (sec)	N/A	0.016	0.007	0.035	0.258	0.342	0.236	0.395	1.219

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	42	35	44	46	206	35
N.S.	1	1.00	1.00	1.05	0.88	1.10	1.15	5.15	0.88
time (sec)	N/A	0.020	0.007	0.029	0.272	0.415	0.200	0.393	1.225

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	47	38	41	34	32	144	26
N.S.	1	1.00	1.52	1.23	1.32	1.10	1.03	4.65	0.84
time (sec)	N/A	0.010	0.006	0.030	0.253	0.343	0.163	0.385	1.180

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	25	33	27	153	22
N.S.	1	1.00	1.00	1.00	1.00	1.32	1.08	6.12	0.88
time (sec)	N/A	0.005	0.003	0.028	0.266	0.354	0.117	0.398	1.149

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	37	86	0	0	0	-1
N.S.	1	1.00	0.93	1.32	3.07	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.007	0.039	0.257	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	36	30	39	26	143	27
N.S.	1	1.00	1.00	1.20	1.00	1.30	0.87	4.77	0.90
time (sec)	N/A	0.015	0.007	0.033	0.257	0.360	0.144	0.392	1.163

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	47	42	36	35	24	140	40
N.S.	1	1.00	1.52	1.35	1.16	1.13	0.77	4.52	1.29
time (sec)	N/A	0.012	0.007	0.036	0.249	0.348	0.242	0.414	1.190

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	48	40	50	46	209	39
N.S.	1	1.00	1.00	1.02	0.85	1.06	0.98	4.45	0.83
time (sec)	N/A	0.021	0.008	0.037	0.263	0.360	0.291	0.398	1.183

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	50	51	43	32	205	60
N.S.	1	1.00	1.39	1.22	1.24	1.05	0.78	5.00	1.46
time (sec)	N/A	0.014	0.008	0.040	0.254	0.355	0.316	0.394	1.606

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	168	135	98	114	534	85
N.S.	1	1.00	0.76	1.60	1.29	0.93	1.09	5.09	0.81
time (sec)	N/A	0.165	0.017	0.230	0.258	0.367	0.475	0.387	1.365

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	87	165	155	0	0	0	-1
N.S.	1	1.00	0.69	1.30	1.22	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.316	0.280	0.258	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	148	118	81	90	335	65
N.S.	1	1.00	0.77	1.83	1.46	1.00	1.11	4.14	0.80
time (sec)	N/A	0.112	0.014	0.135	0.261	0.388	0.352	0.404	1.265

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	66	145	134	0	0	0	-1
N.S.	1	1.00	0.64	1.41	1.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.181	0.244	0.264	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	127	97	62	60	154	44
N.S.	1	1.00	0.80	2.35	1.80	1.15	1.11	2.85	0.81
time (sec)	N/A	0.053	0.009	0.112	0.254	0.373	0.193	0.398	1.209

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	116	135	0	0	0	-1
N.S.	1	1.00	0.79	2.00	2.33	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.060	0.286	0.255	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	114	487	0	0	0	0	-1
N.S.	1	1.00	1.18	5.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.044	1.996	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	145	146	0	0	0	-1
N.S.	1	1.00	0.89	2.64	2.65	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.076	0.068	0.263	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	136	96	79	56	137	145
N.S.	1	1.00	0.93	2.23	1.57	1.30	0.92	2.25	2.38
time (sec)	N/A	0.072	0.012	0.079	0.253	0.386	0.248	0.389	1.447

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	185	176	0	0	0	-1
N.S.	1	1.00	0.84	1.80	1.71	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.158	0.079	0.259	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	158	154	97	90	319	196
N.S.	1	1.00	0.91	1.76	1.71	1.08	1.00	3.54	2.18
time (sec)	N/A	0.121	0.015	0.082	0.253	0.373	0.326	0.385	1.550

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	117	2135	289	0	0	0	-1
N.S.	1	1.00	0.63	11.48	1.55	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.487	0.370	5.717	0.272	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	175	737	0	0	0	0	-1
N.S.	1	1.00	0.89	3.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.390	0.405	4.967	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	88	871	262	0	0	0	-1
N.S.	1	1.00	0.63	6.27	1.88	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.221	3.950	0.265	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	140	683	0	0	0	0	-1
N.S.	1	1.00	0.94	4.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.262	3.451	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	68	2916	215	0	0	0	-1
N.S.	1	1.00	0.72	30.69	2.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.099	2.866	0.262	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	168	0	0	0	0	-1
N.S.	1	1.00	0.93	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.072	0.327	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	156	564	0	0	0	0	-1
N.S.	1	1.00	1.04	3.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.051	2.207	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	719	0	0	0	0	-1
N.S.	1	1.00	0.91	9.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.095	3.214	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	3502	252	0	0	0	-1
N.S.	1	1.00	0.83	36.86	2.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.114	2.632	0.266	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	142	840	0	0	0	0	-1
N.S.	1	1.00	0.92	5.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.160	5.096	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	659	342	0	0	0	-1
N.S.	1	1.00	0.84	4.67	2.43	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.164	3.607	0.278	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	741	897	0	0	0	0	-1
N.S.	1	1.00	4.52	5.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	6.744	3.944	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	213	301	276	247	427	1473	296
N.S.	1	1.00	0.87	1.23	1.13	1.01	1.74	6.01	1.21
time (sec)	N/A	0.127	0.056	0.094	0.256	0.355	1.043	0.437	1.509

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	211	198	177	282	932	190
N.S.	1	1.00	0.89	1.25	1.17	1.05	1.67	5.51	1.12
time (sec)	N/A	0.089	0.045	0.088	0.259	0.360	0.594	0.406	1.519

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	137	131	118	182	529	115
N.S.	1	1.00	0.89	1.25	1.19	1.07	1.65	4.81	1.05
time (sec)	N/A	0.095	0.031	0.144	0.255	0.340	0.417	0.409	1.370

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	74	65	64	87	268	60
N.S.	1	1.00	1.21	1.30	1.14	1.12	1.53	4.70	1.05
time (sec)	N/A	0.047	0.012	0.047	0.261	0.388	0.226	0.397	1.274

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	671	954	406	0	0	0	-1
N.S.	1	1.00	1.72	2.45	1.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	0.928	0.661	0.558	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	755	2071	550	0	0	0	-1
N.S.	1	1.00	1.28	3.51	0.93	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	5.568	0.794	0.525	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	1559	3791	1087	0	0	0	-1
N.S.	1	1.00	2.37	5.77	1.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.696	9.601	1.101	0.579	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.014	3.560	0.417	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	3.466	0.371	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	354	0	79	-1
N.S.	1	1.00	1.92	0.00	2.47	5.71	0.00	1.27	-0.02
time (sec)	N/A	0.067	0.076	0.356	0.277	0.364	0.000	0.405	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	223	728	0	143	-1
N.S.	1	1.00	1.77	0.00	1.74	5.69	0.00	1.12	-0.01
time (sec)	N/A	0.236	0.205	0.355	0.469	0.388	0.000	0.419	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	401	1278	0	226	-1
N.S.	1	1.00	1.64	0.00	2.00	6.39	0.00	1.13	-0.00
time (sec)	N/A	0.720	0.379	0.359	0.482	0.426	0.000	0.433	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	639	2004	0	357	-1
N.S.	1	1.00	1.52	0.00	2.26	7.08	0.00	1.26	-0.00
time (sec)	N/A	0.896	0.875	0.355	0.481	0.520	0.000	0.429	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	125	199	0	0	0	0	-1
N.S.	1	1.00	0.67	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.878	0.448	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	77	190	0	0	0	0	-1
N.S.	1	1.00	0.53	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.080	0.409	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	63	41	0	58	-1
N.S.	1	1.00	0.81	1.41	1.70	1.11	0.00	1.57	-0.03
time (sec)	N/A	0.017	0.031	0.372	0.468	0.365	0.000	0.408	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	67	61	0	90	-1
N.S.	1	1.00	0.54	1.35	0.81	0.73	0.00	1.08	-0.01
time (sec)	N/A	0.037	0.036	0.395	0.270	0.385	0.000	0.417	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	99	81	0	122	-1
N.S.	1	1.00	0.44	1.42	0.80	0.65	0.00	0.98	-0.01
time (sec)	N/A	0.056	0.042	0.414	0.262	0.364	0.000	0.418	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	12	3
N.S.	1	1.00	1.00	1.33	1.00	3.67	1.00	4.00	1.00
time (sec)	N/A	0.016	0.021	0.088	0.250	0.343	0.115	0.393	0.304

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	62	15	22	22
N.S.	1	1.00	1.00	1.08	1.00	5.17	1.25	1.83	1.83
time (sec)	N/A	0.019	0.008	0.084	0.283	0.349	1.104	0.397	1.363

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	707	171	63	0	53	201
N.S.	1	1.00	0.98	11.40	2.76	1.02	0.00	0.85	3.24
time (sec)	N/A	0.034	0.029	1.238	0.254	0.376	0.000	0.432	2.680

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	75	76	0	0	0	-1
N.S.	1	1.00	0.92	2.03	2.05	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.042	0.037	0.094	0.253	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	6	14	5	14	21
N.S.	1	1.00	1.00	1.62	0.75	1.75	0.62	1.75	2.62
time (sec)	N/A	0.011	0.004	0.085	0.260	0.354	0.451	0.388	1.203

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	44	39	34	29	31	101	21
N.S.	1	1.00	1.22	1.08	0.94	0.81	0.86	2.81	0.58
time (sec)	N/A	0.022	0.013	0.079	0.253	0.387	0.240	0.377	1.155

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	99	76	42	0	80	81
N.S.	1	1.00	0.74	2.61	2.00	1.11	0.00	2.11	2.13
time (sec)	N/A	0.012	0.013	0.108	0.251	0.359	0.000	0.421	1.216

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	53	47	47	88	154	34
N.S.	1	1.00	1.00	1.06	0.94	0.94	1.76	3.08	0.68
time (sec)	N/A	0.024	0.025	0.081	0.250	0.380	0.338	0.389	1.210

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	131	118	66	0	0	112
N.S.	1	1.00	0.64	1.96	1.76	0.99	0.00	0.00	1.67
time (sec)	N/A	0.025	0.021	0.115	0.263	0.349	0.000	0.000	1.318

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	81	172	106	112	153	512	134
N.S.	1	1.00	0.80	1.70	1.05	1.11	1.51	5.07	1.33
time (sec)	N/A	0.094	0.028	0.059	0.251	0.359	0.534	0.410	1.473

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	92	124	79	84	117	360	98
N.S.	1	1.00	1.18	1.59	1.01	1.08	1.50	4.62	1.26
time (sec)	N/A	0.075	0.018	0.054	0.255	0.382	0.384	0.413	1.364

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	70	61	66	76	259	62
N.S.	1	1.00	0.86	1.08	0.94	1.02	1.17	3.98	0.95
time (sec)	N/A	0.053	0.016	0.050	0.250	0.378	0.275	0.397	2.004

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	30	31	48	41	197	42
N.S.	1	1.00	1.23	0.86	0.89	1.37	1.17	5.63	1.20
time (sec)	N/A	0.013	0.010	0.042	0.258	0.333	0.195	0.397	1.707

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	259	104	128	0	0	0	-1
N.S.	1	1.00	2.82	1.13	1.39	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.108	0.283	0.257	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	66	54	68	144	259	62
N.S.	1	1.00	0.86	1.03	0.84	1.06	2.25	4.05	0.97
time (sec)	N/A	0.038	0.037	0.051	0.264	0.397	0.622	0.400	1.732

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	83	85	111	410	360	247
N.S.	1	1.00	0.84	0.92	0.94	1.23	4.56	4.00	2.74
time (sec)	N/A	0.074	0.079	0.073	0.259	0.417	0.984	0.406	1.951

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	203	887	320	0	0	0	-1
N.S.	1	1.00	0.77	3.37	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	1.070	0.249	0.265	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	607	666	259	0	0	0	-1
N.S.	1	1.00	2.98	3.26	1.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	3.205	0.240	0.267	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	106	312	202	0	0	0	-1
N.S.	1	1.00	0.78	2.29	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.181	0.223	0.271	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	133	139	0	0	0	-1
N.S.	1	1.00	0.68	1.64	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.050	0.285	0.260	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	714	900	0	0	0	0	-1
N.S.	1	1.00	4.82	6.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	2.638	2.924	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	206	356	244	0	0	0	-1
N.S.	1	1.00	0.82	1.42	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	0.761	0.426	0.261	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	291	449	360	0	0	0	-1
N.S.	1	1.00	0.79	1.21	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	1.400	0.435	0.270	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	529	1219	591	0	0	0	-1
N.S.	1	1.00	0.79	1.81	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	0.404	0.821	0.580	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	185	185	192	0	0	0	-1
N.S.	1	1.00	1.54	1.54	1.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.046	0.902	0.273	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	360	502	306	192	0	0	0	-1
N.S.	1	1.23	1.72	1.05	0.66	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	2.571	0.887	0.270	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	5056	9136	651	0	0	0	-1
N.S.	1	1.00	6.85	12.38	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.138	30.437	4.776	0.571	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	619	575	773	0	0	0	0	-1
N.S.	1	1.00	0.93	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.605	0.455	0.321	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	719	1001	0	0	0	0	-1
N.S.	1	1.00	0.97	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.707	0.470	0.328	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	1239	2090	0	0	0	0	-1
N.S.	1	1.00	3.70	6.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	24.718	1.223	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	42	41	38	0	164	31
N.S.	1	1.00	1.16	0.82	0.80	0.75	0.00	3.22	0.61
time (sec)	N/A	0.010	0.017	0.031	0.253	0.358	0.000	0.391	1.305

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	52	37	36	31	0	114	26
N.S.	1	1.00	1.24	0.88	0.86	0.74	0.00	2.71	0.62
time (sec)	N/A	0.008	0.010	0.033	0.256	0.361	0.000	0.391	1.263

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	27	26	24	0	65	16
N.S.	1	1.00	1.00	1.23	1.18	1.09	0.00	2.95	0.73
time (sec)	N/A	0.005	0.017	0.032	0.251	0.371	0.000	0.388	1.237

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	33	66	0	0	0	-1
N.S.	1	1.00	1.00	1.74	3.47	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	0.008	0.052	0.254	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	32	31	30	92	65	18
N.S.	1	1.00	1.80	1.28	1.24	1.20	3.68	2.60	0.72
time (sec)	N/A	0.009	0.016	0.040	0.249	0.342	0.592	0.388	1.272

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	37	36	38	160	114	45
N.S.	1	1.00	1.38	0.88	0.86	0.90	3.81	2.71	1.07
time (sec)	N/A	0.009	0.015	0.037	0.256	0.333	1.279	0.391	1.487

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	24	35	121	168	24
N.S.	1	1.00	0.82	0.92	0.63	0.92	3.18	4.42	0.63
time (sec)	N/A	0.012	0.012	0.034	0.252	0.354	1.061	0.399	1.259

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	19	30	39	119	-1
N.S.	1	1.00	0.81	0.97	0.61	0.97	1.26	3.84	-0.03
time (sec)	N/A	0.009	0.009	0.033	0.261	0.339	0.564	0.393	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	16	24	87	70	14
N.S.	1	1.00	1.00	0.75	0.80	1.20	4.35	3.50	0.70
time (sec)	N/A	0.006	0.007	0.032	0.250	0.390	0.225	0.398	1.289

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	36	126	70	22
N.S.	1	1.00	1.00	1.21	0.75	1.50	5.25	2.92	0.92
time (sec)	N/A	0.007	0.013	0.037	0.254	0.369	0.512	0.396	1.252

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	95	104	0	0	0	-1
N.S.	1	1.00	0.93	3.39	3.71	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.011	0.076	0.251	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	30	15	23	15	104	26
N.S.	1	1.00	1.00	1.58	0.79	1.21	0.79	5.47	1.37
time (sec)	N/A	0.005	0.002	0.137	0.257	0.388	0.075	0.386	1.135

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	53	147	128	0	0	-1
N.S.	1	1.00	2.55	1.39	3.87	3.37	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.035	0.178	0.316	0.346	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	66	46	62	44	56	188	50
N.S.	1	1.00	1.69	1.18	1.59	1.13	1.44	4.82	1.28
time (sec)	N/A	0.016	0.032	0.053	0.262	0.363	0.248	0.398	2.024

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	48	81	86	97	255	114
N.S.	1	1.00	0.78	0.89	1.50	1.59	1.80	4.72	2.11
time (sec)	N/A	0.030	0.019	0.076	0.249	0.373	0.361	0.400	1.542

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	286	51	112	0	0	0	-1
N.S.	1	1.00	8.17	1.46	3.20	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.018	0.017	0.112	0.286	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	45	53	67	136	198	93
N.S.	1	1.00	0.90	0.94	1.10	1.40	2.83	4.12	1.94
time (sec)	N/A	0.026	0.022	0.092	0.264	0.369	0.592	0.397	1.406

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	117	34	58	0	0	0	-1
N.S.	1	1.00	4.68	1.36	2.32	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.015	0.011	0.087	0.275	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	312	61	132	0	0	0	-1
N.S.	1	1.00	8.91	1.74	3.77	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.020	0.121	0.276	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	270	982	331	817	644	2333	742
N.S.	1	1.00	1.61	5.85	1.97	4.86	3.83	13.89	4.42
time (sec)	N/A	0.242	0.166	0.165	0.276	0.412	1.954	0.448	2.173

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	174	590	207	420	369	973	386
N.S.	1	1.00	1.45	4.92	1.72	3.50	3.08	8.11	3.22
time (sec)	N/A	0.145	0.096	0.148	0.282	0.417	1.327	0.416	1.926

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	138	185	111	176	173	338	136
N.S.	1	1.00	1.42	1.91	1.14	1.81	1.78	3.48	1.40
time (sec)	N/A	0.116	0.036	0.096	0.278	0.342	0.795	0.419	2.404

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	42	36	60	46	202	48
N.S.	1	1.00	1.20	1.05	0.90	1.50	1.15	5.05	1.20
time (sec)	N/A	0.017	0.012	0.075	0.272	0.360	0.227	0.395	1.754

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	352	220	0	0	0	0	-1
N.S.	1	1.00	2.71	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.193	0.960	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	170	128	516	1658	472	175
N.S.	1	1.00	1.09	1.48	1.11	4.49	14.42	4.10	1.52
time (sec)	N/A	0.117	0.122	0.158	0.283	0.470	4.389	0.413	2.080

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	174	266	316	2442	19912	2562	422
N.S.	1	1.00	1.04	1.59	1.89	14.62	119.23	15.34	2.53
time (sec)	N/A	0.173	0.214	0.208	0.275	1.086	10.403	0.462	3.296

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	1054	2890	843	0	0	0	-1
N.S.	1	1.00	2.82	7.73	2.25	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	6.816	0.408	0.447	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	295	812	422	0	0	0	-1
N.S.	1	1.00	1.33	3.67	1.91	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.346	0.334	0.450	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	186	0	0	0	0	-1
N.S.	1	1.00	1.14	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.098	0.367	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	1376	1766	0	0	0	0	-1
N.S.	1	1.00	6.43	8.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.104	12.516	5.340	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	485	470	857	0	0	0	0	-1
N.S.	1	1.01	0.98	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.196	5.486	1.572	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	2594	10601	0	0	0	0	-1
N.S.	1	1.00	4.75	19.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.707	8.950	22.133	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	325	600	12037	0	0	0	0	-1
N.S.	1	1.00	1.84	36.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.849	4.312	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	208	408	0	0	0	0	-1
N.S.	1	1.00	1.58	3.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.202	0.652	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	3624	0	0	0	0	-1
N.S.	1	1.00	0.00	11.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	20.001	6.148	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1094	1818	4599	0	0	0	0	-1
N.S.	1	1.00	1.67	4.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.943	13.448	6.292	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.747	0.306	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	1.811	0.345	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.164	0.365	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.065	0.477	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	0	1488	0	0	0	0	-1
N.S.	1	1.00	0.00	3.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.187	1.027	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	694	0	0	0	0	-1
N.S.	1	1.00	0.00	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.269	0.674	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	98	119	0	0	0	0	-1
N.S.	1	1.00	1.10	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.265	0.421	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.064	0.477	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.525	0.479	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	90	96
N.S.	1	1.00	0.92	18.27	1.03	0.89	0.00	2.43	2.59
time (sec)	N/A	0.020	0.045	0.095	0.268	0.351	0.000	0.397	1.610

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	19	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	0.83	3.09	0.83
time (sec)	N/A	0.007	0.013	0.346	0.294	0.358	0.115	0.384	0.090

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	39	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	1.70	3.09	0.83
time (sec)	N/A	0.005	0.011	0.337	0.304	0.334	0.096	0.403	1.127

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	16	10	19	69	16
N.S.	1	1.00	1.12	2.00	1.00	0.62	1.19	4.31	1.00
time (sec)	N/A	0.002	0.006	0.240	0.298	0.336	0.060	0.397	1.121

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	354	34	8	0	15	59
N.S.	1	1.00	0.90	16.86	1.62	0.38	0.00	0.71	2.81
time (sec)	N/A	0.031	0.011	0.246	0.262	0.380	0.000	0.399	0.180

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	21	17	13	14	70	17
N.S.	1	1.00	1.06	1.24	1.00	0.76	0.82	4.12	1.00
time (sec)	N/A	0.006	0.012	0.328	0.304	0.351	0.079	0.385	0.086

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	19	71	16
N.S.	1	1.00	0.78	0.87	0.83	0.48	0.83	3.09	0.70
time (sec)	N/A	0.006	0.011	0.310	0.302	0.363	0.191	0.402	1.135

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	20	71	19
N.S.	1	1.00	0.87	0.87	0.83	0.57	0.87	3.09	0.83
time (sec)	N/A	0.006	0.011	0.313	0.311	0.344	0.248	0.398	1.123

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	9175	73	101	0	0	203
N.S.	1	1.00	0.87	129.23	1.03	1.42	0.00	0.00	2.86
time (sec)	N/A	0.025	0.088	0.912	0.302	0.363	0.000	0.000	1.325

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	36	30	37	41	36
N.S.	1	1.00	0.88	81.38	0.86	0.71	0.88	0.98	0.86
time (sec)	N/A	0.016	0.022	41.903	0.333	0.346	0.257	0.413	1.175

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	36	30	37	41	36
N.S.	1	1.00	0.88	81.38	0.86	0.71	0.88	0.98	0.86
time (sec)	N/A	0.017	0.037	34.225	0.326	0.343	0.168	0.411	1.182

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	74	3418	36	30	41	41	36
N.S.	1	1.00	2.18	100.53	1.06	0.88	1.21	1.21	1.06
time (sec)	N/A	0.016	0.041	33.895	0.341	0.326	0.145	0.395	1.154

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	27	20	39	33
N.S.	1	1.00	1.00	0.94	2.06	1.69	1.25	2.44	2.06
time (sec)	N/A	0.004	0.007	0.500	0.325	0.344	0.084	0.404	1.121

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	3774	38	27	0	37	183
N.S.	1	1.00	1.08	77.02	0.78	0.55	0.00	0.76	3.73
time (sec)	N/A	0.020	0.034	0.244	0.644	0.362	0.000	0.398	0.292

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	1095	54	29	0	36	207
N.S.	1	1.00	0.95	28.08	1.38	0.74	0.00	0.92	5.31
time (sec)	N/A	0.018	0.034	0.165	0.317	0.350	0.000	0.396	1.247

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	3213	34	29	32	37	34
N.S.	1	1.00	1.17	89.25	0.94	0.81	0.89	1.03	0.94
time (sec)	N/A	0.016	0.025	0.349	0.340	0.357	0.187	0.418	1.158

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	3217	36	29	37	38	32
N.S.	1	1.00	1.10	103.77	1.16	0.94	1.19	1.23	1.03
time (sec)	N/A	0.010	0.030	0.330	0.339	0.415	0.252	0.400	1.119

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	37	3217	36	29	39	38	36
N.S.	1	1.00	0.58	50.27	0.56	0.45	0.61	0.59	0.56
time (sec)	N/A	0.023	0.021	0.343	0.344	0.345	0.344	0.406	1.169

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	63382	109	209	0	0	332
N.S.	1	1.00	0.88	576.20	0.99	1.90	0.00	0.00	3.02
time (sec)	N/A	0.043	0.094	10.115	0.351	0.360	0.000	0.000	1.452

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	0	54	52	56	77	53
N.S.	1	1.00	0.89	0.00	0.89	0.85	0.92	1.26	0.87
time (sec)	N/A	0.031	0.022	180.000	0.372	0.473	0.623	0.419	1.251

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	0	54	52	58	77	53
N.S.	1	1.00	0.89	0.00	0.89	0.85	0.95	1.26	0.87
time (sec)	N/A	0.027	0.024	180.000	0.372	0.465	0.404	0.406	1.232

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	54	52	56	77	53
N.S.	1	1.00	1.02	0.00	1.02	0.98	1.06	1.45	1.00
time (sec)	N/A	0.021	0.017	180.000	0.368	0.393	0.263	0.430	1.207

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	99	18111	54	52	41	77	53
N.S.	1	1.00	2.91	532.68	1.59	1.53	1.21	2.26	1.56
time (sec)	N/A	0.014	0.042	34.578	0.379	0.386	0.248	0.415	0.117

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	49	20	75	47
N.S.	1	1.00	1.00	0.94	3.19	3.06	1.25	4.69	2.94
time (sec)	N/A	0.004	0.006	36.294	0.382	0.397	0.127	0.390	1.181

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	104	21848	74	49	0	74	306
N.S.	1	1.00	1.35	283.74	0.96	0.64	0.00	0.96	3.97
time (sec)	N/A	0.062	0.044	0.827	0.671	0.363	0.000	0.420	0.144

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	7683	124	51	0	74	372
N.S.	1	1.00	0.91	112.99	1.82	0.75	0.00	1.09	5.47
time (sec)	N/A	0.030	0.027	0.388	0.586	0.415	0.000	0.419	1.203

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	7366	72	51	0	71	383
N.S.	1	1.00	1.10	122.77	1.20	0.85	0.00	1.18	6.38
time (sec)	N/A	0.028	0.027	0.401	0.354	0.378	0.000	0.418	1.325

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	17237	52	51	51	73	51
N.S.	1	1.00	1.09	313.40	0.95	0.93	0.93	1.33	0.93
time (sec)	N/A	0.027	0.018	1.687	0.398	0.344	0.279	0.413	1.180

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	17235	53	49	56	74	48
N.S.	1	1.00	1.61	555.97	1.71	1.58	1.81	2.39	1.55
time (sec)	N/A	0.011	0.017	1.596	0.373	0.359	0.366	0.390	1.193

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	17234	54	49	60	74	53
N.S.	1	1.00	0.84	269.28	0.84	0.77	0.94	1.16	0.83
time (sec)	N/A	0.024	0.024	1.450	0.392	0.377	0.524	0.411	0.118

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.063	0.039	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	130774	85	127	0	81	354
N.S.	1	1.00	0.98	1614.49	1.05	1.57	0.00	1.00	4.37
time (sec)	N/A	0.043	0.033	5.275	0.532	0.368	0.000	0.399	0.129

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	28786	51	97	0	50	234
N.S.	1	1.00	0.98	514.04	0.91	1.73	0.00	0.89	4.18
time (sec)	N/A	0.025	0.025	1.187	0.539	0.385	0.000	0.405	1.328

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	30	79	0	28	108
N.S.	1	1.00	1.00	138.81	0.97	2.55	0.00	0.90	3.48
time (sec)	N/A	0.011	0.018	0.286	0.547	0.350	0.000	0.404	0.144

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	28	17	14	12
N.S.	1	1.00	1.00	1.08	1.33	2.33	1.42	1.17	1.00
time (sec)	N/A	0.003	0.035	0.199	0.458	0.374	11.144	0.402	1.178

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	37	87	0	33	113
N.S.	1	1.00	0.66	22.09	0.84	1.98	0.00	0.75	2.57
time (sec)	N/A	0.021	0.015	8.689	0.532	0.363	0.000	0.406	2.906

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	0	65	137	0	64	220
N.S.	1	1.00	0.69	0.00	1.00	2.11	0.00	0.98	3.38
time (sec)	N/A	0.026	0.019	180.000	0.531	0.420	0.000	0.404	3.119

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	106	199	0	109	300
N.S.	1	1.00	0.72	0.00	1.15	2.16	0.00	1.18	3.26
time (sec)	N/A	0.049	0.028	0.062	0.533	0.392	0.000	0.410	3.595

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.374	0.032	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	131085	179	326	0	135	669
N.S.	1	1.00	1.08	1337.60	1.83	3.33	0.00	1.38	6.83
time (sec)	N/A	0.056	0.062	6.396	0.673	0.423	0.000	0.400	1.286

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	83	29109	123	244	0	97	490
N.S.	1	1.00	1.11	388.12	1.64	3.25	0.00	1.29	6.53
time (sec)	N/A	0.039	0.038	1.462	0.658	0.421	0.000	0.408	0.171

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	4626	81	189	0	66	302
N.S.	1	1.00	1.12	92.52	1.62	3.78	0.00	1.32	6.04
time (sec)	N/A	0.022	0.048	0.357	0.646	0.376	0.000	0.411	1.293

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	625	46	97	36	47	28
N.S.	1	1.00	0.96	22.32	1.64	3.46	1.29	1.68	1.00
time (sec)	N/A	0.010	0.027	0.156	0.669	0.416	21.719	0.416	0.088

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	18	36	20	19	14
N.S.	1	1.00	1.00	1.07	1.29	2.57	1.43	1.36	1.00
time (sec)	N/A	0.004	0.008	0.207	0.469	0.419	21.865	0.396	1.136

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	78	306	0	78	421
N.S.	1	1.00	0.76	0.00	1.11	4.37	0.00	1.11	6.01
time (sec)	N/A	0.034	0.054	180.000	0.643	0.404	0.000	0.402	4.025

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	27548	135	480	0	140	453
N.S.	1	1.00	0.69	270.08	1.32	4.71	0.00	1.37	4.44
time (sec)	N/A	0.045	0.042	0.480	0.654	0.365	0.000	0.415	3.799

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	0	190	644	0	203	689
N.S.	1	1.00	0.64	0.00	1.33	4.50	0.00	1.42	4.82
time (sec)	N/A	0.063	0.032	180.000	0.655	0.351	0.000	0.403	4.905

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.439	0.033	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	29456	198	493	0	163	867
N.S.	1	1.00	1.24	320.17	2.15	5.36	0.00	1.77	9.42
time (sec)	N/A	0.053	0.031	1.451	0.875	0.369	0.000	0.423	1.378

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	4977	146	418	0	123	620
N.S.	1	1.00	1.21	70.10	2.06	5.89	0.00	1.73	8.73
time (sec)	N/A	0.033	0.049	0.362	0.862	0.374	0.000	0.437	1.432

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	952	96	250	54	91	46
N.S.	1	1.00	1.04	20.26	2.04	5.32	1.15	1.94	0.98
time (sec)	N/A	0.021	0.029	0.167	0.862	0.378	33.103	0.413	1.219

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	634	63	124	42	61	25
N.S.	1	1.00	0.79	18.65	1.85	3.65	1.24	1.79	0.74
time (sec)	N/A	0.010	0.029	0.126	0.869	0.375	33.619	0.426	0.088

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	28	107	24	44	14
N.S.	1	1.00	1.00	0.94	1.75	6.69	1.50	2.75	0.88
time (sec)	N/A	0.004	0.007	0.192	0.472	0.363	22.441	0.383	0.066

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	27876	171	811	0	173	902
N.S.	1	1.00	0.76	287.38	1.76	8.36	0.00	1.78	9.30
time (sec)	N/A	0.048	0.075	0.470	0.876	0.385	0.000	0.422	6.493

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	93	0	245	1078	0	262	1074
N.S.	1	1.00	0.71	0.00	1.87	8.23	0.00	2.00	8.20
time (sec)	N/A	0.060	0.038	180.000	0.888	0.388	0.000	0.423	5.262

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	C	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	0	331	1316	0	343	1251
N.S.	1	1.00	0.63	0.00	1.95	7.74	0.00	2.02	7.36
time (sec)	N/A	0.087	0.032	180.000	0.883	0.454	0.000	0.434	8.121

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.099	0.032	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	504228	380	583	0	0	546
N.S.	1	1.00	0.88	3055.93	2.30	3.53	0.00	0.00	3.31
time (sec)	N/A	0.095	0.073	20.530	0.548	0.403	0.000	0.000	2.187

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	129477	257	411	0	0	418
N.S.	1	1.00	0.88	1070.06	2.12	3.40	0.00	0.00	3.45
time (sec)	N/A	0.060	0.056	11.509	0.555	0.397	0.000	0.000	1.500

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	25561	166	287	0	0	304
N.S.	1	1.00	0.87	311.72	2.02	3.50	0.00	0.00	3.71
time (sec)	N/A	0.036	0.044	9.273	0.547	0.350	0.000	0.000	1.361

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	480	102	210	0	0	205
N.S.	1	1.00	0.85	10.00	2.12	4.38	0.00	0.00	4.27
time (sec)	N/A	0.014	0.032	9.040	0.535	0.407	0.000	0.000	1.307

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	65	164	51	27	121
N.S.	1	1.00	1.00	1.05	3.25	8.20	2.55	1.35	6.05
time (sec)	N/A	0.005	0.013	9.024	0.532	0.344	0.195	0.397	1.268

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.062	0.031	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.029	0.032	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.032	0.031	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	90	96
N.S.	1	1.00	0.92	18.27	1.03	0.89	0.00	2.43	2.59
time (sec)	N/A	0.008	0.024	0.053	0.272	0.376	0.000	0.405	0.003

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	13	13	76	13	19
N.S.	1	1.00	0.87	0.87	0.57	0.57	3.30	0.57	0.83
time (sec)	N/A	0.015	0.018	0.218	0.254	0.360	7.062	0.381	0.079

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	13	13	180	13	19
N.S.	1	1.00	0.87	0.87	0.57	0.57	7.83	0.57	0.83
time (sec)	N/A	0.005	0.013	0.208	0.252	0.290	4.106	0.386	0.056

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	10	10	80	10	16
N.S.	1	1.00	1.12	2.00	0.62	0.62	5.00	0.62	1.00
time (sec)	N/A	0.002	0.004	0.175	0.274	0.300	2.078	0.377	1.207

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	8	8	0	9	58
N.S.	1	1.00	0.90	1.29	0.38	0.38	0.00	0.43	2.76
time (sec)	N/A	0.024	0.013	0.166	0.251	0.344	0.000	0.393	0.545

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	21	11	13	68	12	17
N.S.	1	1.00	1.06	1.24	0.65	0.76	4.00	0.71	1.00
time (sec)	N/A	0.006	0.014	0.206	0.258	0.354	3.785	0.387	0.074

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	11	11	80	11	16
N.S.	1	1.00	0.78	0.87	0.48	0.48	3.48	0.48	0.70
time (sec)	N/A	0.007	0.012	0.207	0.262	0.360	6.620	0.390	1.143

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	33	57	0	0	-1
N.S.	1	1.00	1.74	0.78	1.22	2.11	0.00	0.00	-0.04
time (sec)	N/A	0.021	0.013	0.201	0.294	0.345	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	449	56	87	0	0	-1
N.S.	1	1.00	1.59	8.80	1.10	1.71	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.012	0.175	0.298	0.371	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	471	78	117	0	0	-1
N.S.	1	1.00	1.42	6.12	1.01	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.022	0.177	0.311	0.369	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	271	5294	281	899	0	0	-1
N.S.	1	1.00	0.88	17.24	0.92	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.248	5.433	0.477	0.408	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	203	4990	215	745	0	0	-1
N.S.	1	1.00	0.88	21.60	0.93	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.085	1.372	0.484	0.381	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	366	361	142	551	0	0	-1
N.S.	1	1.00	2.44	2.41	0.95	3.67	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.917	1.523	0.482	0.369	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.117	4.466	0.111	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1726	149	450	0	0	-1
N.S.	1	1.00	0.93	11.14	0.96	2.90	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.130	1.715	0.691	0.387	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1667	125	381	0	0	-1
N.S.	1	1.00	0.92	13.02	0.98	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.078	1.519	0.687	0.363	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1584	101	322	0	0	-1
N.S.	1	1.00	0.90	15.68	1.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.066	1.496	0.699	0.356	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	265	72	238	0	0	-1
N.S.	1	1.00	2.91	3.84	1.04	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.487	0.886	0.688	0.361	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.065	2.676	0.177	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1802	146	423	0	0	-1
N.S.	1	1.00	0.86	10.73	0.87	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.134	1.793	0.703	0.389	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1745	123	359	0	0	-1
N.S.	1	1.00	0.86	12.55	0.88	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.074	1.702	0.707	0.362	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1664	100	305	0	0	-1
N.S.	1	1.00	0.85	15.13	0.91	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.064	1.567	0.689	0.358	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	200	281	73	227	0	0	-1
N.S.	1	1.00	2.63	3.70	0.96	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.479	1.000	0.714	0.356	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.048	2.236	0.190	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	265	5222	277	879	0	0	-1
N.S.	1	1.00	0.87	17.23	0.91	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.230	5.569	0.491	0.433	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	199	4918	213	729	0	0	-1
N.S.	1	1.00	0.87	21.48	0.93	3.18	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.092	1.241	0.484	0.376	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	369	361	142	539	0	0	-1
N.S.	1	1.00	2.46	2.41	0.95	3.59	0.00	0.00	-0.01
time (sec)	N/A	0.170	1.247	1.313	0.490	0.408	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.118	4.092	0.194	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1698	146	423	0	0	-1
N.S.	1	1.00	0.93	11.17	0.96	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.157	1.708	0.701	0.374	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1641	123	359	0	0	-1
N.S.	1	1.00	0.92	13.02	0.98	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.129	1.698	0.706	0.383	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1560	100	305	0	0	-1
N.S.	1	1.00	0.90	15.60	1.00	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.066	1.453	0.699	0.385	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	197	265	72	226	0	0	-1
N.S.	1	1.00	2.86	3.84	1.04	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.684	0.947	0.695	0.429	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.066	2.454	0.223	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1830	149	450	0	0	-1
N.S.	1	1.00	0.89	11.09	0.90	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.131	1.720	0.723	0.396	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1771	125	381	0	0	-1
N.S.	1	1.00	0.88	12.93	0.91	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.076	1.526	0.694	0.385	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1688	101	322	0	0	-1
N.S.	1	1.00	0.86	15.49	0.93	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.068	1.302	0.727	0.346	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	208	281	73	239	0	0	-1
N.S.	1	1.00	2.74	3.70	0.96	3.14	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.430	0.980	0.694	0.407	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.064	2.935	0.235	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	3040	0	0	-1
N.S.	1	1.00	2.17	24.60	0.00	10.07	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.183	19.098	0.000	0.470	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1787	0	0	-1
N.S.	1	1.00	1.75	23.69	0.00	7.64	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.121	13.249	0.000	0.430	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	295	2543	0	947	0	0	-1
N.S.	1	1.00	1.82	15.70	0.00	5.85	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.202	1.346	0.000	0.433	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	169	182	498	0	0	-1
N.S.	1	1.00	1.61	2.14	2.30	6.30	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.044	0.628	0.506	0.382	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.030	0.084	0.200	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6874	0	2164	0	0	-1
N.S.	1	1.00	0.88	17.40	0.00	5.48	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.323	15.415	0.000	0.487	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6500	0	1688	0	0	-1
N.S.	1	1.00	0.87	22.03	0.00	5.72	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.163	1.759	0.000	0.451	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	562	372	1184	0	0	-1
N.S.	1	1.00	23.99	2.90	1.92	6.10	0.00	0.00	-0.01
time (sec)	N/A	0.183	12.833	1.051	0.526	0.501	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.112	0.282	0.155	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2339	343	344	0	0	-1
N.S.	1	1.00	0.91	13.76	2.02	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.143	1.696	0.291	0.379	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	248	292	0	0	-1
N.S.	1	1.00	0.89	16.91	1.86	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.078	1.387	0.290	0.392	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	321	262	217	0	0	-1
N.S.	1	1.00	0.90	3.45	2.82	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.420	0.909	0.474	0.395	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	0.522	0.188	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2449	342	344	0	0	-1
N.S.	1	1.00	0.91	14.32	2.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.134	1.688	0.292	0.386	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2351	247	292	0	0	-1
N.S.	1	1.00	0.90	17.54	1.84	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.075	1.431	0.270	0.370	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	334	260	218	0	0	-1
N.S.	1	1.00	0.90	3.55	2.77	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.475	0.936	0.480	0.381	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.524	0.144	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	2798	0	0	-1
N.S.	1	1.00	2.17	24.60	0.00	9.26	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.171	17.467	0.000	0.511	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1589	0	0	-1
N.S.	1	1.00	1.75	23.69	0.00	6.79	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.117	15.790	0.000	0.469	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	295	2543	0	793	0	0	-1
N.S.	1	1.00	1.82	15.70	0.00	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.180	1.494	0.000	0.469	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	127	188	184	388	0	0	-1
N.S.	1	1.00	1.61	2.38	2.33	4.91	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.042	0.775	0.488	0.407	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.028	0.079	0.263	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6698	0	1798	0	0	-1
N.S.	1	1.00	0.87	17.13	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.230	18.155	0.000	0.559	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6348	0	1462	0	0	-1
N.S.	1	1.00	0.86	21.67	0.00	4.99	0.00	0.00	-0.00
time (sec)	N/A	0.287	0.094	1.865	0.000	0.538	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	570	392	1098	0	0	-1
N.S.	1	1.00	23.01	2.94	2.02	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.179	12.624	1.233	0.540	0.550	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.072	0.292	0.259	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2449	344	179	0	0	-1
N.S.	1	1.00	0.92	14.58	2.05	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.132	2.233	0.278	0.358	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2351	249	156	0	0	-1
N.S.	1	1.00	0.90	17.81	1.89	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.077	1.804	0.283	0.387	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	321	286	121	0	0	-1
N.S.	1	1.00	0.89	3.45	3.08	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.333	1.248	0.468	0.357	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	0.506	0.246	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2339	345	179	0	0	-1
N.S.	1	1.00	0.92	13.84	2.04	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.161	2.279	0.282	0.362	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	250	156	0	0	-1
N.S.	1	1.00	0.89	16.91	1.88	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.113	1.642	0.283	0.336	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	334	288	121	0	0	-1
N.S.	1	1.00	0.89	3.55	3.06	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.541	1.042	0.476	0.343	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	0.679	0.251	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	131	920	0	460	0	0	-1
N.S.	1	1.00	0.82	5.75	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.409	0.212	0.214	0.000	0.405	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	236	4034	334	389	362	0	510
N.S.	1	1.00	0.79	13.58	1.12	1.31	1.22	0.00	1.72
time (sec)	N/A	0.275	0.097	7.362	0.264	0.349	3.586	0.000	2.288

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3320	274	305	286	0	414
N.S.	1	1.00	0.85	14.76	1.22	1.36	1.27	0.00	1.84
time (sec)	N/A	0.182	0.080	5.302	0.263	0.348	1.555	0.000	2.280

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2616	174	214	209	241	329
N.S.	1	1.00	0.92	18.69	1.24	1.53	1.49	1.72	2.35
time (sec)	N/A	0.085	0.062	1.539	0.268	0.362	0.674	0.510	2.048

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	0	864	173	0	0	0	-1
N.S.	1	1.00	0.00	2.27	0.45	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	0.151	3.796	0.370	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	161	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.098	2.537	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	307	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.082	5.943	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4194	320	388	345	359	497
N.S.	1	1.00	0.75	13.31	1.02	1.23	1.10	1.14	1.58
time (sec)	N/A	0.507	0.082	2.564	0.270	0.346	2.320	0.628	2.339

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3514	255	304	265	282	414
N.S.	1	1.00	0.74	14.23	1.03	1.23	1.07	1.14	1.68
time (sec)	N/A	0.429	0.067	1.943	0.261	0.370	1.062	0.508	2.328

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2210	181	187	155	198	315
N.S.	1	1.00	1.38	21.25	1.74	1.80	1.49	1.90	3.03
time (sec)	N/A	0.142	0.017	1.010	0.255	0.339	0.510	0.470	2.089

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	332	0	0	0	0	0	-1
N.S.	1	1.00	3.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.133	1.076	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	457	0	0	0	0	0	-1
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.235	2.855	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.200	5.240	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	677	8491	0	0	0	0	-1
N.S.	1	1.00	1.32	16.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	3.667	1.585	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	1287	3508	0	0	0	0	-1
N.S.	1	1.00	2.36	6.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	2.436	4.224	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.159	0.331	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1236	0	0	0	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.792	2.918	0.320	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	1318	936	0	0	0	0	-1
N.S.	1	1.00	1.85	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.749	4.479	4.636	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	46	31	58	64	0	0	-1
N.S.	1	1.00	1.84	1.24	2.32	2.56	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.038	0.069	0.255	0.341	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	71	62	59	94	0	0	-1
N.S.	1	1.00	1.39	1.22	1.16	1.84	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.016	0.046	0.260	0.341	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	79	76	119	0	0	-1
N.S.	1	1.00	1.33	1.13	1.09	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.018	0.050	0.257	0.364	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	69	59	107	137	0	0	-1
N.S.	1	1.00	1.68	1.44	2.61	3.34	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.055	0.079	0.254	0.346	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	113	179	108	198	0	0	-1
N.S.	1	1.00	1.36	2.16	1.30	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.023	0.156	0.269	0.344	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	149	325	142	247	0	0	-1
N.S.	1	1.00	1.25	2.73	1.19	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.028	0.163	0.265	0.351	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	108	160	202	283	0	0	-1
N.S.	1	1.00	0.64	0.95	1.20	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.053	0.523	0.260	0.359	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	177	590	194	395	0	0	-1
N.S.	1	1.00	0.82	2.73	0.90	1.83	0.00	0.00	-0.00
time (sec)	N/A	1.889	0.072	0.069	0.298	0.348	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	235	666	254	479	0	0	-1
N.S.	1	1.00	0.87	2.48	0.94	1.78	0.00	0.00	-0.00
time (sec)	N/A	1.740	0.050	0.073	0.290	0.392	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	21	21	14	44	15
N.S.	1	1.00	1.00	1.12	1.24	1.24	0.82	2.59	0.88
time (sec)	N/A	0.029	0.041	0.097	0.266	0.331	0.231	0.407	1.395

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	37	37	58	60	225	107
N.S.	1	1.00	0.89	0.84	0.84	1.32	1.36	5.11	2.43
time (sec)	N/A	0.036	0.016	0.052	0.269	0.353	0.950	0.419	1.561

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	118	40	108	0	119	58
N.S.	1	1.00	0.89	2.51	0.85	2.30	0.00	2.53	1.23
time (sec)	N/A	0.038	0.027	0.050	0.257	0.357	0.000	0.422	2.510

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	153	794	184	233	0	167	187
N.S.	1	1.00	1.43	7.42	1.72	2.18	0.00	1.56	1.75
time (sec)	N/A	0.125	0.093	0.204	0.467	0.390	0.000	0.556	1.672

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	824	64	92	0	98	119
N.S.	1	1.00	1.22	16.82	1.31	1.88	0.00	2.00	2.43
time (sec)	N/A	0.052	0.045	0.168	0.260	0.371	0.000	0.481	1.830

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	349	43	25	63	40	28
N.S.	1	1.00	1.02	7.76	0.96	0.56	1.40	0.89	0.62
time (sec)	N/A	0.040	0.042	0.109	0.260	0.361	0.757	0.401	0.103

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	42	25	0	35	28
N.S.	1	1.00	1.02	1.51	0.93	0.56	0.00	0.78	0.62
time (sec)	N/A	0.041	0.064	0.097	0.252	0.324	0.000	0.418	1.205

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	939	64	93	0	147	111
N.S.	1	1.00	1.20	19.16	1.31	1.90	0.00	3.00	2.27
time (sec)	N/A	0.048	0.130	0.150	0.276	0.348	0.000	0.473	1.437

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	150	920	184	234	0	157	179
N.S.	1	1.00	1.40	8.60	1.72	2.19	0.00	1.47	1.67
time (sec)	N/A	0.118	0.148	0.236	0.465	0.371	0.000	0.544	1.604

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [123] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	2	2	1.00	4	0.500
7	A	1	1	1.00	8	0.125
8	A	5	5	1.00	8	0.625
9	A	3	3	1.00	8	0.375
10	A	4	3	1.00	8	0.375
11	A	4	3	1.00	8	0.375
12	A	15	7	1.00	10	0.700
13	A	14	9	1.00	10	0.900
14	A	10	7	1.00	10	0.700
15	A	9	8	1.00	10	0.800
16	A	5	5	1.00	8	0.625
17	A	5	5	1.00	6	0.833
18	A	6	5	1.00	10	0.500
19	A	4	4	1.00	10	0.400
20	A	8	7	1.00	10	0.700
21	A	8	7	1.00	10	0.700
22	A	13	8	1.00	10	0.800
23	A	33	11	1.00	10	1.100
24	A	22	11	1.00	10	1.100
25	A	18	10	1.00	10	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	11	9	1.00	10	0.900
27	A	8	8	1.00	8	1.000
28	A	5	6	1.00	6	1.000
29	A	8	6	1.00	10	0.600
30	A	5	6	1.00	10	0.600
31	A	7	6	1.00	10	0.600
32	A	14	11	1.00	10	1.100
33	A	16	8	1.00	10	0.800
34	A	1	1	1.00	14	0.071
35	A	4	4	1.00	14	0.286
36	A	4	4	1.00	14	0.286
37	A	5	5	1.00	14	0.357
38	A	5	4	1.00	12	0.333
39	A	27	13	1.00	14	0.929
40	A	25	13	1.00	14	0.929
41	A	23	11	1.00	14	0.786
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	5	6	1.00	16	0.375
45	A	7	9	1.00	16	0.562
46	A	8	9	1.00	16	0.562
47	A	8	9	1.00	16	0.562
48	A	3	3	1.00	15	0.200
49	A	2	2	1.00	15	0.133
50	A	1	1	1.00	15	0.067
51	A	2	2	1.00	15	0.133
52	A	3	2	1.00	15	0.133
53	A	1	1	1.00	14	0.071
54	A	1	1	1.00	14	0.071
55	A	4	4	1.00	14	0.286
56	A	4	4	1.00	13	0.308
57	A	1	1	1.00	12	0.083
58	A	3	3	1.00	13	0.231
59	A	2	2	1.00	12	0.167
60	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	12	0.250
62	A	7	5	1.00	10	0.500
63	A	7	5	1.00	10	0.500
64	A	7	5	1.00	8	0.625
65	A	3	3	1.00	6	0.500
66	A	5	5	1.00	10	0.500
67	A	7	5	1.00	10	0.500
68	A	5	4	1.00	10	0.400
69	A	19	15	1.00	12	1.250
70	A	15	13	1.00	12	1.083
71	A	12	10	1.00	10	1.000
72	A	6	6	1.00	8	0.750
73	A	2	2	1.00	12	0.167
74	A	17	15	1.00	12	1.250
75	A	21	16	1.00	12	1.333
76	A	15	5	1.00	16	0.312
77	A	5	5	1.00	14	0.357
78	A	37	10	1.23	16	0.625
79	A	57	11	1.00	16	0.688
80	A	55	16	1.00	18	0.889
81	A	65	19	1.00	18	1.056
82	A	12	8	1.00	19	0.421
83	A	6	4	1.00	10	0.400
84	A	5	4	1.00	8	0.500
85	A	4	4	1.00	6	0.667
86	A	2	2	1.00	10	0.200
87	A	4	4	1.00	10	0.400
88	A	5	4	1.00	10	0.400
89	A	3	2	1.00	12	0.167
90	A	3	2	1.00	12	0.167
91	A	2	2	1.00	12	0.167
92	A	4	4	1.00	12	0.333
93	A	2	2	1.00	10	0.200
94	A	3	3	1.00	4	0.750
95	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	12	0.333
97	A	5	4	1.00	14	0.286
98	A	2	2	1.00	14	0.143
99	A	6	6	1.00	14	0.429
100	A	3	3	1.00	12	0.250
101	A	3	3	1.00	19	0.158
102	A	7	5	1.00	18	0.278
103	A	7	5	1.00	18	0.278
104	A	7	5	1.00	16	0.312
105	A	4	3	1.00	10	0.300
106	A	5	5	1.00	18	0.278
107	A	7	5	1.00	18	0.278
108	A	5	4	1.00	18	0.222
109	A	16	13	1.00	20	0.650
110	A	13	10	1.00	18	0.556
111	A	6	6	1.00	12	0.500
112	A	2	2	1.00	20	0.100
113	A	21	19	1.01	20	0.950
114	A	21	14	1.00	20	0.700
115	A	15	11	1.00	18	0.611
116	A	6	7	1.00	12	0.583
117	A	2	2	1.00	20	0.100
118	A	30	18	1.00	20	0.900
119	A	6	4	1.00	18	0.222
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	9	7	1.00	40	0.175
124	A	7	6	1.00	40	0.150
125	A	2	3	1.00	38	0.079
126	A	0	0	0.00	0	0.000
127	A	0	0	0.00	0	0.000
128	A	2	2	1.00	11	0.182
129	A	2	2	1.00	11	0.182
130	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	7	0.286
132	A	2	2	1.00	11	0.182
133	A	2	2	1.00	11	0.182
134	A	2	2	1.00	11	0.182
135	A	2	2	1.00	11	0.182
136	A	3	2	1.00	13	0.154
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	3	1.00	11	0.273
140	A	2	2	1.00	9	0.222
141	A	3	3	1.00	13	0.231
142	A	3	3	1.00	13	0.231
143	A	3	2	1.00	13	0.154
144	A	1	1	1.00	13	0.077
145	A	2	2	1.00	13	0.154
146	A	4	2	1.00	13	0.154
147	A	4	2	1.00	13	0.154
148	A	4	2	1.00	13	0.154
149	A	4	3	1.00	13	0.231
150	A	3	3	1.00	11	0.273
151	A	2	2	1.00	9	0.222
152	A	4	3	1.00	13	0.231
153	A	4	4	1.00	13	0.308
154	A	4	3	1.00	13	0.231
155	A	4	2	1.00	13	0.154
156	A	1	1	1.00	13	0.077
157	A	2	2	1.00	13	0.154
158	A	1	1	1.00	13	0.077
159	A	5	4	1.00	13	0.308
160	A	4	4	1.00	13	0.308
161	A	3	3	1.00	11	0.273
162	A	2	2	1.00	9	0.222
163	A	4	3	1.00	13	0.231
164	A	5	4	1.00	13	0.308
165	A	6	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	13	0.154
167	A	6	5	1.00	13	0.385
168	A	5	5	1.00	13	0.385
169	A	4	4	1.00	13	0.308
170	A	3	3	1.00	11	0.273
171	A	2	2	1.00	9	0.222
172	A	5	4	1.00	13	0.308
173	A	6	5	1.00	13	0.385
174	A	7	5	1.00	13	0.385
175	A	3	2	1.00	13	0.154
176	A	6	5	1.00	13	0.385
177	A	5	4	1.00	13	0.308
178	A	4	3	1.00	13	0.231
179	A	3	3	1.00	11	0.273
180	A	2	2	1.00	9	0.222
181	A	6	4	1.00	13	0.308
182	A	7	5	1.00	13	0.385
183	A	8	5	1.00	13	0.385
184	A	1	1	1.00	13	0.077
185	A	6	3	1.00	13	0.231
186	A	5	3	1.00	13	0.231
187	A	4	3	1.00	13	0.231
188	A	3	3	1.00	11	0.273
189	A	2	2	1.00	9	0.222
190	A	1	1	1.00	13	0.077
191	A	2	2	1.00	13	0.154
192	A	3	2	1.00	13	0.154
193	A	2	2	1.00	11	0.182
194	A	2	2	1.00	11	0.182
195	A	2	2	1.00	9	0.222
196	A	2	2	1.00	7	0.286
197	A	2	2	1.00	11	0.182
198	A	2	2	1.00	11	0.182
199	A	2	2	1.00	11	0.182
200	A	6	4	1.00	3	1.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
201	A	8	5	1.00	5	1.000
202	A	10	6	1.00	7	0.857
203	A	11	6	1.00	15	0.400
204	A	9	5	1.00	13	0.385
205	A	7	4	1.00	11	0.364
206	A	0	0	0.00	0	0.000
207	A	8	7	1.00	16	0.438
208	A	7	7	1.00	16	0.438
209	A	6	6	1.00	14	0.429
210	A	5	5	1.00	12	0.417
211	A	0	0	0.00	0	0.000
212	A	8	7	1.00	19	0.368
213	A	7	7	1.00	19	0.368
214	A	6	6	1.00	17	0.353
215	A	5	5	1.00	15	0.333
216	A	0	0	0.00	0	0.000
217	A	11	6	1.00	15	0.400
218	A	9	5	1.00	13	0.385
219	A	7	4	1.00	11	0.364
220	A	0	0	0.00	0	0.000
221	A	8	7	1.00	16	0.438
222	A	7	7	1.00	16	0.438
223	A	6	6	1.00	14	0.429
224	A	5	5	1.00	12	0.417
225	A	0	0	0.00	0	0.000
226	A	8	7	1.00	19	0.368
227	A	7	7	1.00	19	0.368
228	A	6	6	1.00	17	0.353
229	A	5	5	1.00	15	0.333
230	A	0	0	0.00	0	0.000
231	A	12	6	1.00	15	0.400
232	A	10	6	1.00	15	0.400
233	A	8	5	1.00	13	0.385
234	A	6	4	1.00	7	0.571
235	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	11	6	1.00	15	0.400
237	A	9	5	1.00	13	0.385
238	A	7	4	1.00	11	0.364
239	A	0	0	0.00	0	0.000
240	A	7	7	1.00	20	0.350
241	A	6	6	1.00	18	0.333
242	A	5	5	1.00	16	0.312
243	A	0	0	0.00	0	0.000
244	A	7	7	1.00	21	0.333
245	A	6	6	1.00	19	0.316
246	A	5	5	1.00	17	0.294
247	A	0	0	0.00	0	0.000
248	A	12	6	1.00	15	0.400
249	A	10	6	1.00	15	0.400
250	A	8	5	1.00	13	0.385
251	A	6	4	1.00	7	0.571
252	A	0	0	0.00	0	0.000
253	A	11	6	1.00	15	0.400
254	A	9	5	1.00	13	0.385
255	A	7	4	1.00	11	0.364
256	A	0	0	0.00	0	0.000
257	A	7	7	1.00	20	0.350
258	A	6	6	1.00	18	0.333
259	A	5	5	1.00	16	0.312
260	A	0	0	0.00	0	0.000
261	A	7	7	1.00	21	0.333
262	A	6	6	1.00	19	0.316
263	A	5	5	1.00	17	0.294
264	A	0	0	0.00	0	0.000
265	A	11	8	1.00	24	0.333
266	A	23	11	1.00	27	0.407
267	A	14	11	1.00	27	0.407
268	A	7	8	1.00	25	0.320
269	A	21	11	1.00	27	0.407
270	A	13	13	1.00	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	17	14	1.00	27	0.518
272	A	26	15	1.00	27	0.556
273	A	21	15	1.00	27	0.556
274	A	9	8	1.00	24	0.333
275	A	6	6	1.00	27	0.222
276	A	15	14	1.00	27	0.518
277	A	24	16	1.00	27	0.593
278	A	22	17	1.00	22	0.773
279	A	38	20	1.00	21	0.952
280	A	0	0	0.00	0	0.000
281	A	38	22	1.00	24	0.917
282	A	32	17	1.00	24	0.708
283	A	2	2	1.00	4	0.500
284	A	7	4	1.00	6	0.667
285	A	9	5	1.00	8	0.625
286	A	2	2	1.00	8	0.250
287	A	7	4	1.00	10	0.400
288	A	9	5	1.00	12	0.417
289	A	6	6	1.00	12	0.500
290	A	25	8	1.00	14	0.571
291	A	29	9	1.00	16	0.562
292	A	1	1	1.00	20	0.050
293	A	4	4	1.00	12	0.333
294	A	4	4	1.00	14	0.286
295	A	8	7	1.00	20	0.350
296	A	5	5	1.00	20	0.250
297	A	3	2	1.00	20	0.100
298	A	3	2	1.00	20	0.100
299	A	5	5	1.00	20	0.250
300	A	8	7	1.00	20	0.350

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=51

$$\frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{6a^6}$$

```
[Out] 1/6*x/a^5+1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*arccoth(a*x)-1/6*arctanh(a*x)/a^6
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6038, 308, 212}

$$-\frac{\tanh^{-1}(ax)}{6a^6} + \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{1}{6}x^6\coth^{-1}(ax) + \frac{x^5}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x],x]
```

```
[Out] x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]
/(6*a^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5 \coth^{-1}(ax)\,dx &= \frac{1}{6}x^6\coth^{-1}(ax) - \frac{1}{6}a\int \frac{x^6}{1-a^2x^2}\,dx\\ &= \frac{1}{6}x^6\coth^{-1}(ax) - \frac{1}{6}a\int \left(-\frac{1}{a^6} - \frac{x^2}{a^4} - \frac{x^4}{a^2} + \frac{1}{a^6(1-a^2x^2)}\right)dx\\ &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) - \frac{\int \frac{1}{1-a^2x^2}\,dx}{6a^5}\\ &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{6a^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.31

$$\frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6\coth^{-1}(ax) + \frac{\log(1-ax)}{12a^6} - \frac{\log(1+ax)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCoth[a*x],x]
```

```
[Out] x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 + Log[1 - a*x]
/(12*a^6) - Log[1 + a*x]/(12*a^6)
```

Maple [A]
time = 0.08, size = 54, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccoth}(ax)}{6}+\frac{a^5x^5}{30}+\frac{a^3x^3}{18}+\frac{ax}{6}+\frac{\ln(ax-1)}{12}-\frac{\ln(ax+1)}{12}}{a^6}$	54
default	$\frac{\frac{a^6x^6\operatorname{arccoth}(ax)}{6}+\frac{a^5x^5}{30}+\frac{a^3x^3}{18}+\frac{ax}{6}+\frac{\ln(ax-1)}{12}-\frac{\ln(ax+1)}{12}}{a^6}$	54
risch	$\frac{x^6\ln(ax+1)}{12}-\frac{x^6\ln(ax-1)}{12}+\frac{x^5}{30a}+\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\ln(ax+1)}{12a^6}+\frac{\ln(-ax+1)}{12a^6}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arccoth(a*x)+1/30*a^5*x^5+1/18*a^3*x^3+1/6*a*x+1/12*ln(a
*x-1)-1/12*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 61, normalized size = 1.20

$$\frac{1}{6}x^6\operatorname{arcoth}(ax) + \frac{1}{180}a\left(\frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15\log(ax+1)}{a^7} + \frac{15\log(ax-1)}{a^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccoth(a*x) + 1/180*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*l
og(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)
```

Fricas [A]
time = 0.38, size = 51, normalized size = 1.00

$$\frac{6\,a^5x^5+10\,a^3x^3+30\,ax+15\,(a^6x^6-1)\log\left(\frac{ax+1}{ax-1}\right)}{180\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/180*(6*a^5*x^5 + 10*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*
x - 1)))/a^6
```

Sympy [C] Result contains complex when optimal does not.
time = 0.37, size = 49, normalized size = 0.96

$$\begin{cases} \frac{x^6\operatorname{acoth}(ax)}{6}+\frac{x^5}{30a}+\frac{x^3}{18a^3}+\frac{x}{6a^5}-\frac{\operatorname{acoth}(ax)}{6a^6} & \text{for}\ a\neq 0 \\ \frac{i\pi x^6}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x),x)
```

```
[Out] Piecewise((x**6*acoth(a*x)/6 + x**5/(30*a) + x**3/(18*a**3) + x/(6*a**5) -
acoth(a*x)/(6*a**6), Ne(a, 0)), (I*pi*x**6/12, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(41) = 82.
time = 0.40, size = 245, normalized size = 4.80

$$\frac{1}{45}\,a\left(\frac{\frac{45\,(ax+1)^4}{(ax-1)^4}-\frac{90\,(ax+1)^3}{(ax-1)^3}+\frac{140\,(ax+1)^2}{(ax-1)^2}-\frac{70\,(ax+1)}{ax-1}+23}{a^7\left(\frac{ax+1}{ax-1}-1\right)^5}+\frac{15\left(\frac{3\,(ax+1)^5}{(ax-1)^5}+\frac{10\,(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)}{ax-1}\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^7\left(\frac{ax+1}{ax-1}-1\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/45*a*((45*(a*x + 1)^4/(a*x - 1)^4 - 90*(a*x + 1)^3/(a*x - 1)^3 + 140*(a*x
 + 1)^2/(a*x - 1)^2 - 70*(a*x + 1)/(a*x - 1) + 23)/(a^7*((a*x + 1)/(a*x - 1
```

```
) - 1)^5) + 15*(3*(a*x + 1)^5/(a*x - 1)^5 + 10*(a*x + 1)^3/(a*x - 1)^3 + 3*
(a*x + 1)/(a*x - 1))*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x -
 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) -
 1))/(a^7*((a*x + 1)/(a*x - 1) - 1)^6))
```

Mupad [B]
time = 1.31, size = 41, normalized size = 0.80

$$\frac{\frac{a\,x}{6}-\frac{\mathrm{acoth}(a\,x)}{6}+\frac{a^3\,x^3}{18}+\frac{a^5\,x^5}{30}}{a^6}+\frac{x^6\,\mathrm{acoth}(a\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acoth(a*x),x)
[Out] ((a*x)/6 - acoth(a*x)/6 + (a^3*x^3)/18 + (a^5*x^5)/30)/a^6 + (x^6*acoth(a*x
))/6
```

3.2 $\int x^4 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$\frac{x^2}{10a^3}+\frac{x^4}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)+\frac{\log(1-a^2x^2)}{10a^5}$$

```
[Out] 1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*arccoth(a*x)+1/10*ln(-a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6038, 272, 45}

$$\frac{x^2}{10a^3}+\frac{\log(1-a^2x^2)}{10a^5}+\frac{1}{5}x^5\coth^{-1}(ax)+\frac{x^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x],x]
```

```
[Out] x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^5
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
 + 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4 \coth^{-1}(ax)\,dx &= \frac{1}{5}x^5\coth^{-1}(ax) - \frac{1}{5}a\int \frac{x^5}{1-a^2x^2}\,dx \\ &= \frac{1}{5}x^5\coth^{-1}(ax) - \frac{1}{10}a\text{Subst}\left(\int \frac{x^2}{1-a^2x}\,dx, x, x^2\right) \\ &= \frac{1}{5}x^5\coth^{-1}(ax) - \frac{1}{10}a\text{Subst}\left(\int \left(-\frac{1}{a^4} - \frac{x}{a^2} - \frac{1}{a^4(-1+a^2x)}\right)\,dx, x, x^2\right) \\ &= \frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{10a^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5\coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{10a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[a*x],x]
[Out] x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + Log[1 - a^2*x^2]/(10*a^5
)
```

Maple [A]
time = 0.06, size = 50, normalized size = 1.00

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\,\mathrm{arccoth}(ax)}{5}+\frac{a^4x^4}{20}+\frac{a^2x^2}{10}+\frac{\ln(ax-1)}{10}+\frac{\ln(ax+1)}{10}}{a^5}$	50
default	$\frac{\frac{a^5x^5\,\mathrm{arccoth}(ax)}{5}+\frac{a^4x^4}{20}+\frac{a^2x^2}{10}+\frac{\ln(ax-1)}{10}+\frac{\ln(ax+1)}{10}}{a^5}$	50
risch	$\frac{x^5\ln(ax+1)}{10}-\frac{x^5\ln(ax-1)}{10}+\frac{x^4}{20a}+\frac{x^2}{10a^3}+\frac{\ln(a^2x^2-1)}{10a^5}+\frac{1}{20a^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(1/5*a^5*x^5*arccoth(a*x)+1/20*a^4*x^4+1/10*a^2*x^2+1/10*ln(a*x-1)+1/
10*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 46, normalized size = 0.92

$$\frac{1}{5}\,x^5\,\mathrm{arcoth}\,(ax) + \frac{1}{20}\,a\left(\frac{a^2x^4+2\,x^2}{a^4} + \frac{2\,\log\,(a^2x^2-1)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccoth(a*x) + 1/20*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/a
^6)
```

Fricas [A]
time = 0.35, size = 55, normalized size = 1.10

$$\frac{2\,a^5x^5\log\left(\frac{ax+1}{ax-1}\right)+a^4x^4+2\,a^2x^2+2\,\log\left(a^2x^2-1\right)}{20\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/20*(2*a^5*x^5*log((a*x + 1)/(a*x - 1)) + a^4*x^4 + 2*a^2*x^2 + 2*log(a^2*
x^2 - 1))/a^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 54, normalized size = 1.08

$$\begin{cases}\frac{x^5\operatorname{acoth}(ax)}{5}+\frac{x^4}{20a}+\frac{x^2}{10a^3}+\frac{\log(ax+1)}{5a^5}-\frac{\operatorname{acoth}(ax)}{5a^5} & \text{for } a\neq 0\\ \frac{i\pi x^5}{10} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x),x)
```

```
[Out] Piecewise((x**5*acoth(a*x)/5 + x**4/(20*a) + x**2/(10*a**3) + log(a*x + 1)/
(5*a**5) - acoth(a*x)/(5*a**5), Ne(a, 0)), (I*pi*x**5/10, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(42) = 84.
time = 0.39, size = 255, normalized size = 5.10

$$\frac{1}{5}\,a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^6}-\frac{\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^6}+\frac{4\left(\frac{(ax+1)^3}{(ax-1)^3}-\frac{(ax+1)^2}{(ax-1)^2}+\frac{ax+1}{ax-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^4}+\frac{\left(\frac{5\,(ax+1)^4}{(ax-1)^4}+\frac{10\,(ax+1)^2}{(ax-1)^2}+1\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/5*a*(log(abs(a*x + 1)/abs(a*x - 1))/a^6 - log(abs((a*x + 1)/(a*x - 1) - 1
))/a^6 + 4*((a*x + 1)^3/(a*x - 1)^3 - (a*x + 1)^2/(a*x - 1)^2 + (a*x + 1)/(
```

```
a*x - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^4) + (5*(a*x + 1)^4/(a*x - 1)^4 +
10*(a*x + 1)^2/(a*x - 1)^2 + 1)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x
+ 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x -
 1) + 1)) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B]
time = 1.27, size = 43, normalized size = 0.86

$$\frac{\frac{\ln(a^2\,x^2-1)}{10}+\frac{a^2\,x^2}{10}+\frac{a^4\,x^4}{20}}{a^5}+\frac{x^5\,\mathrm{acoth}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(a*x),x)
```

```
[Out] (log(a^2*x^2 - 1)/10 + (a^2*x^2)/10 + (a^4*x^4)/20)/a^5 + (x^5*acoth(a*x))/
5
```

3.3 $\int x^3 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=41

$$\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4a^4}$$

```
[Out] 1/4*x/a^3+1/12*x^3/a+1/4*x^4*arccoth(a*x)-1/4*arctanh(a*x)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = $\{6038, 308, 212\}$

$$-\frac{\tanh^{-1}(ax)}{4a^4}+\frac{x}{4a^3}+\frac{1}{4}x^4\coth^{-1}(ax)+\frac{x^3}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x],x]
```

```
[Out] x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(ax)\,dx &= \frac{1}{4}x^4\coth^{-1}(ax) - \frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,dx\\ &= \frac{1}{4}x^4\coth^{-1}(ax) - \frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}+\frac{1}{a^4(1-a^2x^2)}\right)dx\\ &= \frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)-\frac{\int\frac{1}{1-a^2x^2}\,dx}{4a^3}\\ &= \frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{4a^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.39

$$\frac{x}{4a^3}+\frac{x^3}{12a}+\frac{1}{4}x^4\coth^{-1}(ax)+\frac{\log(1-ax)}{8a^4}-\frac{\log(1+ax)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a*x],x]
```

```
[Out] x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 + Log[1 - a*x]/(8*a^4) - Log[
1 + a*x]/(8*a^4)
```

Maple [A]
time = 0.04, size = 46, normalized size = 1.12

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)}{4}+\frac{a^3x^3}{12}+\frac{ax}{4}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}}{a^4}$	46
default	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)}{4}+\frac{a^3x^3}{12}+\frac{ax}{4}+\frac{\ln(ax-1)}{8}-\frac{\ln(ax+1)}{8}}{a^4}$	46
risch	$\frac{x^4\ln(ax+1)}{8}-\frac{x^4\ln(ax-1)}{8}+\frac{x^3}{12a}+\frac{x}{4a^3}+\frac{\ln(-ax+1)}{8a^4}-\frac{\ln(ax+1)}{8a^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccoth(a*x)+1/12*a^3*x^3+1/4*a*x+1/8*ln(a*x-1)-1/8*ln(a
*x+1))
```

Maxima [A]
time = 0.26, size = 52, normalized size = 1.27

$$\frac{1}{4}\,x^4\operatorname{arcoth}(ax)+\frac{1}{24}\,a\left(\frac{2\,(a^2x^3+3\,x)}{a^4}-\frac{3\,\log(ax+1)}{a^5}+\frac{3\,\log(ax-1)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x) + 1/24*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 +
 3*log(a*x - 1)/a^5)
```

Fricas [A]
time = 0.34, size = 43, normalized size = 1.05

$$\frac{2\,a^3x^3 + 6\,ax + 3\,(a^4x^4 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/24*(2*a^3*x^3 + 6*a*x + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/a^4
```

Sympy [C] Result contains complex when optimal does not.
time = 0.24, size = 41, normalized size = 1.00

$$\begin{cases} \frac{x^4\operatorname{acoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} - \frac{\operatorname{acoth}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x),x)
```

```
[Out] Piecewise((x**4*acoth(a*x)/4 + x**3/(12*a) + x/(4*a**3) - acoth(a*x)/(4*a**
4), Ne(a, 0)), (I*pi*x**4/8, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(33) = 66.
time = 0.39, size = 195, normalized size = 4.76

$$\frac{1}{3}\,a\left(\frac{\frac{3\,(ax+1)^2}{(ax-1)^2} - \frac{3\,(ax+1)}{ax-1} + 2}{a^5\left(\frac{ax+1}{ax-1} - 1\right)^3} + \frac{3\left(\frac{(ax+1)^3}{(ax-1)^3} + \frac{ax+1}{ax-1}\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1} - a}{a\left(\frac{ax+1}{ax-1}+1\right)} + 1}{\frac{\frac{(ax+1)a}{ax-1} - a}{a\left(\frac{ax+1}{ax-1}+1\right)} - 1}\right)}{a^5\left(\frac{ax+1}{ax-1} - 1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*(a*x + 1)^2/(a*x - 1)^2 - 3*(a*x + 1)/(a*x - 1) + 2)/(a^5*((a*x +
 1)/(a*x - 1) - 1)^3) + 3*((a*x + 1)^3/(a*x - 1)^3 + (a*x + 1)/(a*x - 1))*l
```

```
og(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x
+ 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^5*((a*x + 1)/(
a*x - 1) - 1)^4))
```

Mupad [B]
time = 1.22, size = 33, normalized size = 0.80

$$\frac{\frac{a\,x}{4}-\frac{\mathrm{acoth}(a\,x)}{4}+\frac{a^3\,x^3}{12}}{a^4}+\frac{x^4\,\mathrm{acoth}(a\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(a*x),x)
```

```
[Out] ((a*x)/4 - acoth(a*x)/4 + (a^3*x^3)/12)/a^4 + (x^4*acoth(a*x))/4
```

3.4 $\int x^2 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=40

$$\frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{6a^3}$$

```
[Out] 1/6*x^2/a+1/3*x^3*arccoth(a*x)+1/6*ln(-a^2*x^2+1)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6038, 272, 45}

$$\frac{\log\left(1-a^2x^2\right)}{6a^3}+\frac{1}{3}x^3\coth^{-1}(ax)+\frac{x^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x],x]
```

```
[Out] x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(ax)\,dx &= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{3}a\int\frac{x^3}{1-a^2x^2}\,dx\\ &= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\text{Subst}\left(\int\frac{x}{1-a^2x}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\coth^{-1}(ax)-\frac{1}{6}a\text{Subst}\left(\int\left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x)}\right)\,dx,x,x^2\right)\\ &= \frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)+\frac{\log(1-a^2x^2)}{6a^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^2}{6a}+\frac{1}{3}x^3\coth^{-1}(ax)+\frac{\log(1-a^2x^2)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a*x],x]
[Out] x^2/(6*a) + (x^3*ArcCoth[a*x])/3 + Log[1 - a^2*x^2]/(6*a^3)
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\frac{a^3x^3\,\mathrm{arccoth}(ax)}{3}+\frac{a^2x^2}{6}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^3}$	42
default	$\frac{\frac{a^3x^3\,\mathrm{arccoth}(ax)}{3}+\frac{a^2x^2}{6}+\frac{\ln(ax-1)}{6}+\frac{\ln(ax+1)}{6}}{a^3}$	42
risch	$\frac{x^3\ln(ax+1)}{6}-\frac{x^3\ln(ax-1)}{6}+\frac{x^2}{6a}+\frac{\ln(a^2x^2-1)}{6a^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/3*a^3*x^3*arccoth(a*x)+1/6*a^2*x^2+1/6*ln(a*x-1)+1/6*ln(a*x+1))
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.88

$$\frac{1}{3}x^3\,\mathrm{arcoth}\,(ax)+\frac{1}{6}a\left(\frac{x^2}{a^2}+\frac{\log(a^2x^2-1)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(a*x) + 1/6*a*(x^2/a^2 + log(a^2*x^2 - 1)/a^4)
```

Fricas [**A**]
time = 0.41, size = 44, normalized size = 1.10

$$\frac{a^3x^3\log\left(\frac{ax+1}{ax-1}\right)+a^2x^2+\log\left(a^2x^2-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3*log((a*x + 1)/(a*x - 1)) + a^2*x^2 + log(a^2*x^2 - 1))/a^3
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.20, size = 46, normalized size = 1.15

$$\begin{cases}\frac{x^3\operatorname{acoth}(ax)}{3}+\frac{x^2}{6a}+\frac{\log(ax+1)}{3a^3}-\frac{\operatorname{acoth}(ax)}{3a^3} & \text{for } a\neq 0\\ \frac{i\pi x^3}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x),x)
```

```
[Out] Piecewise((x**3*acoth(a*x)/3 + x**2/(6*a) + log(a*x + 1)/(3*a**3) - acoth(a
*x)/(3*a**3), Ne(a, 0)), (I*pi*x**3/6, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(34) = 68.
time = 0.39, size = 206, normalized size = 5.15

$$\frac{1}{3}\,a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^4}-\frac{\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^4}+\frac{\left(\frac{3\,(ax+1)^2}{(ax-1)^2}+1\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^4\left(\frac{ax+1}{ax-1}-1\right)^3}+\frac{2\,(ax+1)}{(ax-1)a^4\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/3*a*(log(abs(a*x + 1)/abs(a*x - 1))/a^4 - log(abs((a*x + 1)/(a*x - 1) - 1
))/a^4 + (3*(a*x + 1)^2/(a*x - 1)^2 + 1)*log(-(((a*x + 1)*a/(a*x - 1) - a)/
(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x +
1)/(a*x - 1) + 1)) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3) + 2*(a*x + 1)/((
a*x - 1)*a^4*((a*x + 1)/(a*x - 1) - 1)^2))
```

Mupad [B]
time = 1.22, size = 35, normalized size = 0.88

$$\frac{\frac{\ln(a^2\,x^2-1)}{6}+\frac{a^2\,x^2}{6}}{a^3}+\frac{x^3\,\mathrm{acoth}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a*x),x)
[Out] (log(a^2*x^2 - 1)/6 + (a^2*x^2)/6)/a^3 + (x^3*acoth(a*x))/3
```

3.5 $\int x \coth^{-1}(ax)\,dx$

Optimal. Leaf size=31

$$\frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2a^2}$$

```
[Out] 1/2*x/a+1/2*x^2*arccoth(a*x)-1/2*arctanh(a*x)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6038, 327, 212}

$$-\frac{\tanh^{-1}(ax)}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(ax)\,dx &= \frac{1}{2}x^2\coth^{-1}(ax)-\frac{1}{2}a\int \frac{x^2}{1-a^2x^2}\,dx\\ &= \frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)-\frac{\int \frac{1}{1-a^2x^2}\,dx}{2a}\\ &= \frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.52

$$\frac{x}{2a}+\frac{1}{2}x^2\coth^{-1}(ax)+\frac{\log(1-ax)}{4a^2}-\frac{\log(1+ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x],x]
```

```
[Out] x/(2*a) + (x^2*ArcCoth[a*x])/2 + Log[1 - a*x]/(4*a^2) - Log[1 + a*x]/(4*a^2
)
```

Maple [A]
time = 0.03, size = 38, normalized size = 1.23

method	result	size
derivativedivides	$\frac{\frac{\operatorname{arccoth}(ax)a^2x^2}{2}+\frac{ax}{2}+\frac{\ln(ax-1)}{4}-\frac{\ln(ax+1)}{4}}{a^2}$	38
default	$\frac{\frac{\operatorname{arccoth}(ax)a^2x^2}{2}+\frac{ax}{2}+\frac{\ln(ax-1)}{4}-\frac{\ln(ax+1)}{4}}{a^2}$	38
risch	$\frac{x^2\ln(ax+1)}{4}-\frac{x^2\ln(ax-1)}{4}+\frac{x}{2a}-\frac{\ln(ax+1)}{4a^2}+\frac{\ln(-ax+1)}{4a^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arccoth(a*x)*a^2*x^2+1/2*a*x+1/4*ln(a*x-1)-1/4*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 41, normalized size = 1.32

$$\frac{1}{2}\,x^2\operatorname{arcoth}\left(ax\right)+\frac{1}{4}\,a\left(\frac{2\,x}{a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x) + 1/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3
)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 1.10

$$\frac{2\,ax+\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*x + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.16, size = 32, normalized size = 1.03

$$\begin{cases} \frac{x^2\operatorname{acoth}(ax)}{2}+\frac{x}{2a}-\frac{\operatorname{acoth}(ax)}{2a^2} & \text{for } a\neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x),x)
```

```
[Out] Piecewise((x**2*acoth(a*x)/2 + x/(2*a) - acoth(a*x)/(2*a**2), Ne(a, 0)), (I
*pi*x**2/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(25) = 50.
time = 0.39, size = 144, normalized size = 4.65

$$a\left(\frac{1}{a^3\left(\frac{ax+1}{ax-1}-1\right)}+\frac{(ax+1)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{(ax-1)a^3\left(\frac{ax+1}{ax-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x),x, algorithm="giac")
```

```
[Out] a*(1/(a^3*((a*x + 1)/(a*x - 1) - 1)) + (a*x + 1)*log(-(((a*x + 1)*a/(a*x -
1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*
((a*x + 1)/(a*x - 1) + 1)) - 1))/((a*x - 1)*a^3*((a*x + 1)/(a*x - 1) - 1)^2
))
```

Mupad [B]
time = 1.18, size = 26, normalized size = 0.84

$$\frac{x^2\,\mathrm{acoth}(a\,x)}{2}-\frac{\frac{\mathrm{acoth}(a\,x)}{2}-\frac{a\,x}{2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a*x),x)
[Out] (x^2*acoth(a*x))/2 - (acoth(a*x)/2 - (a*x)/2)/a^2
```

3.6 $\int \coth^{-1}(ax)\,dx$

Optimal. Leaf size=25

$$x\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{2a}$$

```
[Out] x*arccoth(a*x)+1/2*ln(-a^2*x^2+1)/a
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6022, 266}

$$\frac{\log\left(1-a^2x^2\right)}{2a}+x\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x],x]
```

```
[Out] x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(ax)\,dx&=x\coth^{-1}(ax)-a\int \frac{x}{1-a^2x^2}\,dx\\&=x\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$x\coth^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x],x]
```

```
[Out] x*ArcCoth[a*x] + Log[1 - a^2*x^2]/(2*a)
```

Maple [A]
time = 0.03, size = 25, normalized size = 1.00

method	result	size
derivativedivides	$\frac{ax\,\mathrm{arccoth}(ax)+\frac{\ln\left(a^{2}x^{2}-1\right)}{2}}{a}$	25
default	$\frac{ax\,\mathrm{arccoth}(ax)+\frac{\ln\left(a^{2}x^{2}-1\right)}{2}}{a}$	25
risch	$\frac{x\ln(ax+1)}{2}-\frac{x\ln(ax-1)}{2}+\frac{\ln(a^{2}x^{2}-1)}{2a}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x*arccoth(a*x)+1/2*ln(a^2*x^2-1))
```

Maxima [A]
time = 0.27, size = 25, normalized size = 1.00

$$\frac{2\,ax\,\mathrm{arcoth}\left(ax\right)+\log\left(-a^{2}x^{2}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*x*arccoth(a*x) + log(-a^2*x^2 + 1))/a
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.32

$$\frac{ax\log\left(\frac{ax+1}{ax-1}\right)+\log\left(a^{2}x^{2}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x*log((a*x + 1)/(a*x - 1)) + log(a^2*x^2 - 1))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.12, size = 27, normalized size = 1.08

$$\begin{cases} x\,\mathrm{acoth}\left(ax\right)+\frac{\log\left(ax+1\right)}{a}-\frac{\mathrm{acoth}\left(ax\right)}{a} & \text{for}\ a\neq 0\\ \frac{i\pi x}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x),x)
```

```
[Out] Piecewise((x*acoth(a*x) + log(a*x + 1)/a - acoth(a*x)/a, Ne(a, 0)), (I*pi*x
/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(23) = 46.
time = 0.40, size = 153, normalized size = 6.12

$$a\left(\frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^2}-\frac{\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^2}+\frac{\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^2\left(\frac{ax+1}{ax-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x),x, algorithm="giac")
```

```
[Out] a*(log(abs(a*x + 1)/abs(a*x - 1))/a^2 - log(abs((a*x + 1)/(a*x - 1) - 1))/a
^2 + log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(
((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^2*((a*x
+ 1)/(a*x - 1) - 1)))
```

Mupad [B]
time = 1.15, size = 22, normalized size = 0.88

$$x\,\mathrm{acoth}(a\,x)+\frac{\ln\left(a^2\,x^2-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x),x)
```

```
[Out] x*acoth(a*x) + log(a^2*x^2 - 1)/(2*a)
```

3.7 $\int \frac{\coth^{-1}(ax)}{x} \, dx$

Optimal. Leaf size=28

$$\frac{1}{2}\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{1}{ax}\right)$$

```
[Out] 1/2*polylog(2,-1/a/x)-1/2*polylog(2,1/a/x)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6032}

$$\frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right)-\frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x))]/2 - PolyLog[2, 1/(a*x)]/2
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x} \, dx = \frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right)-\frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{1}{ax}\right)-\text{PolyLog}\left(2,\frac{1}{ax}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x,x]
```

```
[Out] (PolyLog[2, -(1/(a*x))] - PolyLog[2, 1/(a*x)])/2
```

Maple [A]
time = 0.04, size = 37, normalized size = 1.32

method	result	size
risch	$-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\operatorname{dilog}(ax)}{2}-\frac{\ln(ax-1)\ln(ax)}{2}$	28
derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}$	37
default	$\ln(ax)\operatorname{arccoth}(ax)-\frac{\operatorname{dilog}(ax)}{2}-\frac{\operatorname{dilog}(ax+1)}{2}-\frac{\ln(ax)\ln(ax+1)}{2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccoth(a*x)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(22) = 44.
time = 0.26, size = 86, normalized size = 3.07

$$-\frac{1}{2}a\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\log(x)-\frac{1}{2}a\left(\frac{\log(ax-1)\log(ax)+\operatorname{Li}_2(-ax+1)}{a}-\frac{\log(ax+1)\log(-ax)+\operatorname{Li}_2(ax+1)}{a}\right)+\operatorname{arcoth}(ax)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x + 1)/a - log(a*x - 1)/a)*log(x) - 1/2*a*((log(a*x - 1)*log(
a*x) + dilog(-a*x + 1))/a - (log(a*x + 1)*log(-a*x) + dilog(a*x + 1))/a) +
arccoth(a*x)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x,x)
```

```
[Out] Integral(acoth(a*x)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acoth}(a\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x,x)
```

```
[Out] int(acoth(a*x)/x, x)
```

3.8 $\int \frac{\coth^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=30

$$-\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1 - a^2x^2\right)$$

```
[Out] -arccoth(a*x)/x+a*ln(x)-1/2*a*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6038, 272, 36, 29, 31}

$$-\frac{1}{2}a\log\left(1 - a^2x^2\right) + a\log(x) - \frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^2,x]
```

```
[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^2}\,dx &= -\frac{\coth^{-1}(ax)}{x} + a\int \frac{1}{x(1-a^2x^2)}\,dx \\
&= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x(1-a^2x)}\,dx, x, x^2\right) \\
&= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{1-a^2x}\,dx, x, x^2\right) \\
&= -\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1-a^2x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$-\frac{\coth^{-1}(ax)}{x} + a\log(x) - \frac{1}{2}a\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^2,x]
[Out] -(ArcCoth[a*x]/x) + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Maple [A]
time = 0.03, size = 36, normalized size = 1.20

method	result	size
derivativedivides	$a\left(-\frac{\text{arccoth}(ax)}{ax} - \frac{\ln(ax-1)}{2} + \ln(ax) - \frac{\ln(ax+1)}{2}\right)$	36
default	$a\left(-\frac{\text{arccoth}(ax)}{ax} - \frac{\ln(ax-1)}{2} + \ln(ax) - \frac{\ln(ax+1)}{2}\right)$	36
risch	$-\frac{\ln(ax+1)}{2x} + \frac{2a\ln(x)x - a\ln(a^2x^2-1)x + \ln(ax-1)}{2x}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] a*(-1/a/x*arccoth(a*x)-1/2*ln(a*x-1)+ln(a*x)-1/2*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 1.00

$$-\frac{1}{2}\,a\left(\log\left(a^2x^2-1\right)-\log\left(x^2\right)\right)-\frac{\operatorname{arcoth}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a^2*x^2 - 1) - log(x^2)) - arccoth(a*x)/x
```

Fricas [A]
time = 0.36, size = 39, normalized size = 1.30

$$-\frac{ax\log\left(a^2x^2-1\right)-2\,ax\log\left(x\right)+\log\left(\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + log((a*x + 1)/(a*x - 1)))/x
```

Sympy [A]
time = 0.14, size = 26, normalized size = 0.87

$$a\log\left(x\right)-a\log\left(ax+1\right)+a\operatorname{acoth}\left(ax\right)-\frac{\operatorname{acoth}\left(ax\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**2,x)
```

```
[Out] a*log(x) - a*log(a*x + 1) + a*acoth(a*x) - acoth(a*x)/x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.
time = 0.39, size = 143, normalized size = 4.77

$$a\left(\frac{\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{\frac{ax+1}{ax-1}+1}-\log\left(\frac{|ax+1|}{|ax-1|}\right)+\log\left(\left|\frac{ax+1}{ax-1}+1\right|\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^2,x, algorithm="giac")
```

```
[Out] a*(log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((
a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/((a*x + 1)/(a
*x - 1) + 1) - log(abs(a*x + 1)/abs(a*x - 1)) + log(abs((a*x + 1)/(a*x - 1)
 + 1)))
```

Mupad [B]
time = 1.16, size = 27, normalized size = 0.90

$$a\,\ln\left(x\right)-\frac{a\,\ln\left(a^2\,x^2-1\right)}{2}-\frac{\mathrm{acoth}(a\,x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x^2,x)
```

```
[Out] a*log(x) - (a*log(a^2*x^2 - 1))/2 - acoth(a*x)/x
```

3.9 $\int \frac{\coth^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=31

$$-\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \tanh^{-1}(ax)$$

```
[Out] -1/2*a/x-1/2*arccoth(a*x)/x^2+1/2*a^2*arctanh(a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6038, 331, 212}

$$\frac{1}{2}a^2 \tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^3,x]
```

```
[Out] -1/2*a/x - ArcCoth[a*x]/(2*x^2) + (a^2*ArcTanh[a*x])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x^3}\,dx = -\frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx$$
$$= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx$$
$$= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tanh^{-1}(ax)$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.52

$$-\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\log(1-ax) + \frac{1}{4}a^2\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^3,x]
```

```
[Out] -1/2*a/x - ArcCoth[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x])
/4
```

Maple [A]
time = 0.04, size = 42, normalized size = 1.35

method	result	size
derivativedivides	$a^2\left(-\frac{\operatorname{arccoth}(ax)}{2a^2x^2} - \frac{\ln(ax-1)}{4} + \frac{\ln(ax+1)}{4} - \frac{1}{2ax}\right)$	42
default	$a^2\left(-\frac{\operatorname{arccoth}(ax)}{2a^2x^2} - \frac{\ln(ax-1)}{4} + \frac{\ln(ax+1)}{4} - \frac{1}{2ax}\right)$	42
risch	$-\frac{\ln(ax+1)}{4x^2} + \frac{\ln(-ax-1)a^2x^2 - \ln(-ax+1)a^2x^2 - 2ax + \ln(ax-1)}{4x^2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/a^2/x^2*arccoth(a*x)-1/4*ln(a*x-1)+1/4*ln(a*x+1)-1/2/a/x)
```

Maxima [A]
time = 0.25, size = 36, normalized size = 1.16

$$\frac{1}{4}\left(a\log(ax+1) - a\log(ax-1) - \frac{2}{x}\right)a - \frac{\operatorname{arcoth}(ax)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*arccoth(a*x)/x^2
```

Fricas [A]
time = 0.35, size = 35, normalized size = 1.13

$$-\frac{2\,ax-\left(a^{2}x^{2}-1\right)\log\left(\frac{ax+1}{ax-1}\right)}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="fricas")
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)))/x^2
```

Sympy [A]
time = 0.24, size = 24, normalized size = 0.77

$$\frac{a^{2}\operatorname{acoth}\left(ax\right)}{2}-\frac{a}{2x}-\frac{\operatorname{acoth}\left(ax\right)}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**3,x)
[Out] a**2*acoth(a*x)/2 - a/(2*x) - acoth(a*x)/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(25) = 50.
time = 0.41, size = 140, normalized size = 4.52

$$a\left(\frac{a}{\frac{ax+1}{ax-1}+1}+\frac{(ax+1)a\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^3,x, algorithm="giac")
[Out] a*(a/((a*x + 1)/(a*x - 1) + 1) + (a*x + 1)*a*log(-(((a*x + 1)*a/(a*x - 1) -
 a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*
x + 1)/(a*x - 1) + 1)) - 1))/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2))
```

Mupad [B]
time = 1.19, size = 40, normalized size = 1.29

$$\frac{a\,\mathrm{atan}\left(\frac{a^{2}\,x}{\sqrt{-a^{2}}}\right)\,\sqrt{-a^{2}}}{2}-\frac{\frac{\mathrm{acoth}\left(a\,x\right)}{2}+\frac{a\,x}{2}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x^3,x)
```

```
[Out] (a*atan((a^2*x)/(-a^2)^(1/2))*(-a^2)^(1/2))/2 - (acoth(a*x)/2 + (a*x)/2)/x^
2
```

3.10 $\int \frac{\coth^{-1}(ax)}{x^4} \, dx$

Optimal. Leaf size=47

$$-\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3\log(x) - \frac{1}{6}a^3\log\left(1 - a^2x^2\right)$$

```
[Out] -1/6*a/x^2-1/3*arccoth(a*x)/x^3+1/3*a^3*ln(x)-1/6*a^3*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6038, 272, 46}

$$\frac{1}{3}a^3\log(x) - \frac{1}{6}a^3\log\left(1 - a^2x^2\right) - \frac{\coth^{-1}(ax)}{3x^3} - \frac{a}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2])
/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x^4} \, dx = -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3} a \int \frac{1}{x^3 (1 - a^2 x^2)} \, dx$$
$$= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6} a \text{Subst}\left(\int \frac{1}{x^2 (1 - a^2 x)} \, dx, x, x^2\right)$$
$$= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6} a \text{Subst}\left(\int \left(\frac{1}{x^2} + \frac{a^2}{x} - \frac{a^4}{-1 + a^2 x}\right) \, dx, x, x^2\right)$$
$$= -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3} a^3 \log(x) - \frac{1}{6} a^3 \log\left(1 - a^2 x^2\right)$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$-\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3} a^3 \log(x) - \frac{1}{6} a^3 \log\left(1 - a^2 x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^4,x]
```

```
[Out] -1/6*a/x^2 - ArcCoth[a*x]/(3*x^3) + (a^3*Log[x])/3 - (a^3*Log[1 - a^2*x^2])
/6
```

Maple [A]
time = 0.04, size = 48, normalized size = 1.02

method	result	size
derivativedivides	$a^3 \left(-\frac{\operatorname{arccoth}(ax)}{3a^3x^3} - \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{1}{6a^2x^2} + \frac{\ln(ax)}{3}\right)$	48
default	$a^3 \left(-\frac{\operatorname{arccoth}(ax)}{3a^3x^3} - \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{1}{6a^2x^2} + \frac{\ln(ax)}{3}\right)$	48
risch	$-\frac{\ln(ax+1)}{6x^3} + \frac{2\ln(x)a^3x^3 - \ln(a^2x^2-1)a^3x^3 - ax + \ln(ax-1)}{6x^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arccoth(a*x)-1/6*ln(a*x-1)-1/6*ln(a*x+1)-1/6/a^2/x^2+1/3*
ln(a*x))
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.85

$$-\frac{1}{6}\left(a^2 \log\left(a^2 x^2 - 1\right) - a^2 \log\left(x^2\right) + \frac{1}{x^2}\right) a - \frac{\operatorname{arcoth}(ax)}{3\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(a^2*log(a^2*x^2 - 1) - a^2*log(x^2) + 1/x^2)*a - 1/3*arccoth(a*x)/x^3
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.06

$$-\frac{a^3x^3\log\left(a^2x^2-1\right)-2\,a^3x^3\log\left(x\right)+ax+\log\left(\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log(a^2*x^2 - 1) - 2*a^3*x^3*log(x) + a*x + log((a*x + 1)/(a*
x - 1)))/x^3
```

Sympy [A]
time = 0.29, size = 46, normalized size = 0.98

$$\frac{a^3\log\left(x\right)}{3}-\frac{a^3\log\left(ax+1\right)}{3}+\frac{a^3\operatorname{acoth}\left(ax\right)}{3}-\frac{a}{6x^2}-\frac{\operatorname{acoth}\left(ax\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**4,x)
```

```
[Out] a**3*log(x)/3 - a**3*log(a*x + 1)/3 + a**3*acoth(a*x)/3 - a/(6*x**2) - acot
h(a*x)/(3*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(39) = 78.
time = 0.40, size = 209, normalized size = 4.45

$$-\frac{1}{3}\left(a^2\log\left(\frac{|ax+1|}{|ax-1|}\right)-a^2\log\left(\left|\frac{ax+1}{ax-1}+1\right|\right)-\frac{2\,(ax+1)a^2}{(ax-1)\left(\frac{ax+1}{ax-1}+1\right)^2}-\frac{\left(\frac{3\,(ax+1)^2a^2}{(ax-1)^2}+a^2\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(a^2*log(abs(a*x + 1)/abs(a*x - 1)) - a^2*log(abs((a*x + 1)/(a*x - 1)
+ 1)) - 2*(a*x + 1)*a^2/((a*x - 1)*((a*x + 1)/(a*x - 1) + 1)^2) - (3*(a*x +
 1)^2*a^2/(a*x - 1)^2 + a^2)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1
```

```
)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1)
 + 1)) - 1))/((a*x + 1)/(a*x - 1) + 1)^3)*a
```

Mupad [B]
time = 1.18, size = 39, normalized size = 0.83

$$\frac{a^3\,\ln\left(x\right)}{3}-\frac{\frac{\mathrm{acoth}\left(a\,x\right)}{3}+\frac{a\,x}{6}}{x^3}-\frac{a^3\,\ln\left(a^2\,x^2-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x^4,x)
[Out] (a^3*log(x))/3 - (acoth(a*x)/3 + (a*x)/6)/x^3 - (a^3*log(a^2*x^2 - 1))/6
```

3.11 $\int \frac{\coth^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{12x^3}-\frac{a^3}{4x}-\frac{\coth^{-1}(ax)}{4x^4}+\frac{1}{4}a^4\tanh^{-1}(ax)$$

```
[Out] -1/12*a/x^3-1/4*a^3/x-1/4*arccoth(a*x)/x^4+1/4*a^4*arctanh(a*x)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6038, 331, 212}

$$\frac{1}{4}a^4\tanh^{-1}(ax)-\frac{a^3}{4x}-\frac{\coth^{-1}(ax)}{4x^4}-\frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/x^5,x]
```

```
[Out] -1/12*a/x^3 - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) + (a^4*ArcTanh[a*x])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x^5}\,dx = -\frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a\int \frac{1}{x^4\,(1-a^2x^2)}\,dx$$
$$= -\frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^3\int \frac{1}{x^2\,(1-a^2x^2)}\,dx$$
$$= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^5\int \frac{1}{1-a^2x^2}\,dx$$
$$= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^4\tanh^{-1}(ax)$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.39

$$-\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} - \frac{1}{8}a^4\log(1-ax) + \frac{1}{8}a^4\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/x^5,x]
```

```
[Out] -1/12*a/x^3 - a^3/(4*x) - ArcCoth[a*x]/(4*x^4) - (a^4*Log[1 - a*x])/8 + (a^
4*Log[1 + a*x])/8
```

Maple [A]
time = 0.04, size = 50, normalized size = 1.22

method	result	size
derivativedivides	$a^4\left(-\frac{\mathrm{arccoth}(ax)}{4a^4x^4} - \frac{\ln(ax-1)}{8} - \frac{1}{12a^3x^3} - \frac{1}{4ax} + \frac{\ln(ax+1)}{8}\right)$	50
default	$a^4\left(-\frac{\mathrm{arccoth}(ax)}{4a^4x^4} - \frac{\ln(ax-1)}{8} - \frac{1}{12a^3x^3} - \frac{1}{4ax} + \frac{\ln(ax+1)}{8}\right)$	50
risch	$-\frac{\ln(ax+1)}{8x^4} + \frac{3\ln(-ax-1)a^4x^4 - 3\ln(-ax+1)a^4x^4 - 6a^3x^3 - 2ax + 3\ln(ax-1)}{24x^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4/a^4/x^4*arccoth(a*x)-1/8*ln(a*x-1)-1/12/a^3/x^3-1/4/a/x+1/8*ln(a*
x+1))
```

Maxima [A]
time = 0.25, size = 51, normalized size = 1.24

$$\frac{1}{24}\left(3\,a^3\log\left(ax+1\right) - 3\,a^3\log\left(ax-1\right) - \frac{2\,(3\,a^2x^2+1)}{x^3}\right)a - \frac{\mathrm{arcoth}\left(ax\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a -
1/4*arccoth(a*x)/x^4
```

Fricas [A]
time = 0.35, size = 43, normalized size = 1.05

$$-\frac{6\,a^3x^3+2\,ax-3\,(a^4x^4-1)\log\left(\frac{ax+1}{ax-1}\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^3*x^3 + 2*a*x - 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1)))/x^4
```

Sympy [A]
time = 0.32, size = 32, normalized size = 0.78

$$\frac{a^4\operatorname{acoth}(ax)}{4}-\frac{a^3}{4x}-\frac{a}{12x^3}-\frac{\operatorname{acoth}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x**5,x)
```

```
[Out] a**4*acoth(a*x)/4 - a**3/(4*x) - a/(12*x**3) - acoth(a*x)/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(33) = 66.
time = 0.39, size = 205, normalized size = 5.00

$$\frac{1}{3}a\left(\frac{\frac{3\,(ax+1)^2a^3}{(ax-1)^2}+\frac{3\,(ax+1)a^3}{ax-1}+2\,a^3}{\left(\frac{ax+1}{ax-1}+1\right)^3}+\frac{3\left(\frac{(ax+1)^3a^3}{(ax-1)^3}+\frac{(ax+1)a^3}{ax-1}\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{\left(\frac{ax+1}{ax-1}+1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/x^5,x, algorithm="giac")
```

```
[Out] 1/3*a*((3*(a*x + 1)^2*a^3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1) + 2*a^3)/
((a*x + 1)/(a*x - 1) + 1)^3 + 3*((a*x + 1)^3*a^3/(a*x - 1)^3 + (a*x + 1)*a^
3/(a*x - 1))*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)
```

```
) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/((a
*x + 1)/(a*x - 1) + 1)^4)
```

Mupad [B]
time = 1.61, size = 60, normalized size = 1.46

$$\frac{\ln\left(1-\frac{1}{a\,x}\right)}{8\,x^4}-\frac{\ln\left(\frac{1}{a\,x}+1\right)}{8\,x^4}-\frac{a^3\,x^2+\frac{a}{3}}{4\,x^3}-\frac{a^4\,\mathrm{atan}(a\,x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x^5,x)
```

```
[Out] log(1 - 1/(a*x))/(8*x^4) - (a^4*atan(a*x*1i)*1i)/4 - log(1/(a*x) + 1)/(8*x^
4) - (a/3 + a^3*x^2)/(4*x^3)
```

3.12 $\int x^5 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=105

$$\frac{4x^2}{45a^4}+\frac{x^4}{60a^2}+\frac{x\coth^{-1}(ax)}{3a^5}+\frac{x^3\coth^{-1}(ax)}{9a^3}+\frac{x^5\coth^{-1}(ax)}{15a}-\frac{\coth^{-1}(ax)^2}{6a^6}+\frac{1}{6}x^6\coth^{-1}(ax)^2+\frac{23\log\left(1-a^2x\right.}{90a^6}$$

```
[Out] 4/45*x^2/a^4+1/60*x^4/a^2+1/3*x*arccoth(a*x)/a^5+1/9*x^3*arccoth(a*x)/a^3+1
/15*x^5*arccoth(a*x)/a-1/6*arccoth(a*x)^2/a^6+1/6*x^6*arccoth(a*x)^2+23/90*
ln(-a^2*x^2+1)/a^6
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6038, 6128, 272, 45, 6022, 266, 6096}

$$-\frac{\coth^{-1}(ax)^2}{6a^6}+\frac{x\coth^{-1}(ax)}{3a^5}+\frac{4x^2}{45a^4}+\frac{x^3\coth^{-1}(ax)}{9a^3}+\frac{x^4}{60a^2}+\frac{23\log\left(1-a^2x^2\right)}{90a^6}+\frac{1}{6}x^6\coth^{-1}(ax)^2+\frac{x^5\coth^{-1}(ax)}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x]^2,x]
```

```
[Out] (4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*ArcCoth[a*x])/(3*a^5) + (x^3*ArcCoth[a
*x])/(9*a^3) + (x^5*ArcCoth[a*x])/(15*a) - ArcCoth[a*x]^2/(6*a^6) + (x^6*Ar
cCoth[a*x]^2)/6 + (23*Log[1 - a^2*x^2])/(90*a^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\int x^5 \coth^{-1}(ax)^2\,dx = \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{3}a\int \frac{x^6\coth^{-1}(ax)}{1-a^2x^2}\,dx$$

$$= \frac{1}{6}x^6\coth^{-1}(ax)^2 + \frac{\int x^4\coth^{-1}(ax)\,dx}{3a} - \frac{\int \frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx}{3a}$$

$$= \frac{x^5\coth^{-1}(ax)}{15a} + \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{15}\int \frac{x^5}{1-a^2x^2}\,dx + \frac{\int x^2\coth^{-1}(ax)\,dx}{3a^3} - \frac{\int x^{\cdots}}{\cdots}$$

$$= \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} + \frac{1}{6}x^6\coth^{-1}(ax)^2 - \frac{1}{30}\text{Subst}\left(\int \frac{x^2}{1-a^2x}\,dx, x, x^{\cdots}\right.$$

$$= \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6\coth^{-1}(ax)^2 \cdots$$

$$= \frac{x^2}{30a^4} + \frac{x^4}{60a^2} + \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}\cdots$$

$$= \frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x\coth^{-1}(ax)}{3a^5} + \frac{x^3\coth^{-1}(ax)}{9a^3} + \frac{x^5\coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}\cdots$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 0.76

$$\frac{16a^2x^2 + 3a^4x^4 + 4ax(15 + 5a^2x^2 + 3a^4x^4)\coth^{-1}(ax) + 30(-1 + a^6x^6)\coth^{-1}(ax)^2 + 46\log(1 - a^2x^2)}{180a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCoth[a*x]^2,x]
```

```
[Out] (16*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 + 5*a^2*x^2 + 3*a^4*x^4)*ArcCoth[a*x] +
 30*(-1 + a^6*x^6)*ArcCoth[a*x]^2 + 46*Log[1 - a^2*x^2])/(180*a^6)
```

Maple [A]
time = 0.23, size = 168, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{a^6x^6\operatorname{arccoth}(ax)^2}{6} + \frac{a^5x^5\operatorname{arccoth}(ax)}{15} + \frac{a^3x^3\operatorname{arccoth}(ax)}{9} + \frac{ax\operatorname{arccoth}(ax)}{3} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{6} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{6} + \frac{\ln(ax-1)}{24}}{a^6}$
default	$\frac{\frac{a^6x^6\operatorname{arccoth}(ax)^2}{6} + \frac{a^5x^5\operatorname{arccoth}(ax)}{15} + \frac{a^3x^3\operatorname{arccoth}(ax)}{9} + \frac{ax\operatorname{arccoth}(ax)}{3} + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{6} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{6} + \frac{\ln(ax-1)}{24}}{a^6}$
risch	$\frac{(a^6x^6-1)\ln(ax+1)^2}{24a^6} - \frac{(15x^6\ln(ax-1)a^6-6a^5x^5-10a^3x^3-30ax-15\ln(ax-1))\ln(ax+1)}{180a^6} + \frac{x^6\ln(ax-1)^2}{24} - \frac{x^5\ln(}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arccoth(a*x)^2+1/15*a^5*x^5*arccoth(a*x)+1/9*a^3*x^3*arc
coth(a*x)+1/3*a*x*arccoth(a*x)+1/6*arccoth(a*x)*ln(a*x-1)-1/6*arccoth(a*x)*
ln(a*x+1)+1/24*ln(a*x-1)^2-1/12*ln(a*x-1)*ln(1/2*a*x+1/2)-1/12*(ln(a*x+1)-l
n(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)+1/24*ln(a*x+1)^2+1/60*a^4*x^4+4/45*a^2*x^2
+23/90*ln(a*x-1)+23/90*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 135, normalized size = 1.29

$$\frac{1}{6}x^6\operatorname{arcoth}(ax)^2 + \frac{1}{90}a\left(\frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15\log(ax+1)}{a^7} + \frac{15\log(ax-1)}{a^7}\right)\operatorname{arcoth}(ax) + \frac{6a^4x^4 + 32a^2x^2 - 2(15\log(ax-1) - 46)\log(ax+1) + 15\log(ax+1)^2 + 15\log(ax-1)^2 + 92\log(ax-1)}{360a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccoth(a*x)^2 + 1/90*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*
log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x) + 1/360*(6*a^4*x^4 + 3
2*a^2*x^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 + 15*
log(a*x - 1)^2 + 92*log(a*x - 1))/a^6
```

Fricas [A]
time = 0.37, size = 98, normalized size = 0.93

$$\frac{6\,a^4x^4+32\,a^2x^2+15\,(a^6x^6-1)\log\left(\frac{ax+1}{ax-1}\right)^2+4\,(3\,a^5x^5+5\,a^3x^3+15\,ax)\log\left(\frac{ax+1}{ax-1}\right)+92\,\log\left(a^2x^2-1\right)}{360\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(6*a^4*x^4 + 32*a^2*x^2 + 15*(a^6*x^6 - 1)*log((a*x + 1)/(a*x - 1))^2
 + 4*(3*a^5*x^5 + 5*a^3*x^3 + 15*a*x)*log((a*x + 1)/(a*x - 1)) + 92*log(a^2
*x^2 - 1))/a^6
```

Sympy [A]
time = 0.48, size = 114, normalized size = 1.09

$$\begin{cases} \frac{x^6\operatorname{acoth}^2(ax)}{6}+\frac{x^5\operatorname{acoth}(ax)}{15a}+\frac{x^4}{60a^2}+\frac{x^3\operatorname{acoth}(ax)}{9a^3}+\frac{4x^2}{45a^4}+\frac{x\operatorname{acoth}(ax)}{3a^5}+\frac{23\log(ax+1)}{45a^6}-\frac{\operatorname{acoth}^2(ax)}{6a^6}-\frac{23\operatorname{acoth}(ax)}{45a^6} & \text{for } a\neq 0 \\ -\frac{\pi^2x^6}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**6*acoth(a*x)**2/6 + x**5*acoth(a*x)/(15*a) + x**4/(60*a**2) +
 x**3*acoth(a*x)/(9*a**3) + 4*x**2/(45*a**4) + x*acoth(a*x)/(3*a**5) + 23*l
og(a*x + 1)/(45*a**6) - acoth(a*x)**2/(6*a**6) - 23*acoth(a*x)/(45*a**6), N
e(a, 0)), (-pi**2*x**6/24, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(89) = 178.
time = 0.39, size = 534, normalized size = 5.09

$$\frac{1}{90}\left(\frac{15\left(\frac{3(ax+1)^5}{(ax-1)^5}+\frac{10(ax+1)^3}{(ax-1)^3}+\frac{3(ax+1)}{ax-1}\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^6a^7}{(ax-1)^6}-\frac{6(ax+1)^5a^7}{(ax-1)^5}+\frac{15(ax+1)^4a^7}{(ax-1)^4}-\frac{20(ax+1)^3a^7}{(ax-1)^3}+\frac{15(ax+1)^2a^7}{(ax-1)^2}-\frac{6(ax+1)a^7}{ax-1}+a^7}+\frac{2\left(\frac{45(ax+1)^4}{(ax-1)^4}-\frac{90(ax+1)^3}{(ax-1)^3}+\frac{140(ax+1)^2}{(ax-1)^2}-\frac{70(ax+1)}{ax-1}+23\right)\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^5a^7}{(ax-1)^5}-\frac{5(ax+1)^4a^7}{(ax-1)^4}+\frac{10(ax+1)^3a^7}{(ax-1)^3}-\frac{10(ax+1)^2a^7}{(ax-1)^2}+\frac{5(ax+1)a^7}{ax-1}-a^7}+\frac{4\left(\frac{11(ax+1)^3}{(ax-1)^3}-\frac{16(ax+1)^2}{(ax-1)^2}+\frac{11(ax+1)}{ax-1}\right)}{\frac{(ax+1)^4a^7}{(ax-1)^4}-\frac{4(ax+1)^3a^7}{(ax-1)^3}+\frac{6(ax+1)^2a^7}{(ax-1)^2}-\frac{4(ax+1)a^7}{ax-1}+a^7}-\frac{46\log\left(\frac{ax+1}{ax-1}-1\right)}{a^7}+\frac{46\log\left(\frac{ax+1}{ax-1}\right)}{a^7}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/90*(15*(3*(a*x + 1)^5/(a*x - 1)^5 + 10*(a*x + 1)^3/(a*x - 1)^3 + 3*(a*x +
 1)/(a*x - 1))*log((a*x + 1)/(a*x - 1))^2/((a*x + 1)^6*a^7/(a*x - 1)^6 - 6*
(a*x + 1)^5*a^7/(a*x - 1)^5 + 15*(a*x + 1)^4*a^7/(a*x - 1)^4 - 20*(a*x + 1)
^3*a^7/(a*x - 1)^3 + 15*(a*x + 1)^2*a^7/(a*x - 1)^2 - 6*(a*x + 1)*a^7/(a*x
- 1) + a^7) + 2*(45*(a*x + 1)^4/(a*x - 1)^4 - 90*(a*x + 1)^3/(a*x - 1)^3 +
140*(a*x + 1)^2/(a*x - 1)^2 - 70*(a*x + 1)/(a*x - 1) + 23)*log((a*x + 1)/(a
*x - 1))/((a*x + 1)^5*a^7/(a*x - 1)^5 - 5*(a*x + 1)^4*a^7/(a*x - 1)^4 + 10*
(a*x + 1)^3*a^7/(a*x - 1)^3 - 10*(a*x + 1)^2*a^7/(a*x - 1)^2 + 5*(a*x + 1)*
a^7/(a*x - 1) - a^7) + 4*(11*(a*x + 1)^3/(a*x - 1)^3 - 16*(a*x + 1)^2/(a*x
```

```
- 1)^2 + 11*(a*x + 1)/(a*x - 1))/((a*x + 1)^4*a^7/(a*x - 1)^4 - 4*(a*x + 1)
^3*a^7/(a*x - 1)^3 + 6*(a*x + 1)^2*a^7/(a*x - 1)^2 - 4*(a*x + 1)*a^7/(a*x -
 1) + a^7) - 46*log((a*x + 1)/(a*x - 1) - 1)/a^7 + 46*log((a*x + 1)/(a*x -
1))/a^7)*a
```

Mupad [B]
time = 1.36, size = 85, normalized size = 0.81

$$\frac{x^6\,\mathrm{acoth}(a\,x)^2}{6}+\frac{\frac{23\,\ln(a^2\,x^2-1)}{90}+\frac{4\,a^2\,x^2}{45}+\frac{a^4\,x^4}{60}-\frac{\mathrm{acoth}(a\,x)^2}{6}+\frac{a^3\,x^3\,\mathrm{acoth}(a\,x)}{9}+\frac{a^5\,x^5\,\mathrm{acoth}(a\,x)}{15}+\frac{a\,x\,\mathrm{acoth}(a\,x)}{3}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acoth(a*x)^2,x)
```

```
[Out] (x^6*acoth(a*x)^2)/6 + ((23*log(a^2*x^2 - 1))/90 + (4*a^2*x^2)/45 + (a^4*x^
4)/60 - acoth(a*x)^2/6 + (a^3*x^3*acoth(a*x))/9 + (a^5*x^5*acoth(a*x))/15 +
 (a*x*acoth(a*x))/3)/a^6
```

3.13 $\int x^4 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=127

$$\frac{3x}{10a^4}+\frac{x^3}{30a^2}+\frac{x^2\coth^{-1}(ax)}{5a^3}+\frac{x^4\coth^{-1}(ax)}{10a}+\frac{\coth^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\coth^{-1}(ax)^2-\frac{3\tanh^{-1}(ax)}{10a^5}-\frac{2\coth^{-1}(ax)}{5a^5}$$

```
[Out] 3/10*x/a^4+1/30*x^3/a^2+1/5*x^2*arccoth(a*x)/a^3+1/10*x^4*arccoth(a*x)/a+1/
5*arccoth(a*x)^2/a^5+1/5*x^5*arccoth(a*x)^2-3/10*arctanh(a*x)/a^5-2/5*arccoth(a*x)*ln(2/(-a*x+1))/a^5-1/5*polylog(2,1-2/(-a*x+1))/a^5
```

Rubi [A]
time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6038, 6128, 308, 212, 327, 6132, 6056, 2449, 2352}

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{5a^5}-\frac{3\tanh^{-1}(ax)}{10a^5}+\frac{\coth^{-1}(ax)^2}{5a^5}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{5a^5}+\frac{3x}{10a^4}+\frac{x^2\coth^{-1}(ax)}{5a^3}+\frac{x^3}{30a^2}+\frac{1}{5}x^5\coth^{-1}(ax)^2+\frac{x^4\coth^{-1}(ax)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^2,x]
```

```
[Out] (3*x)/(10*a^4) + x^3/(30*a^2) + (x^2*ArcCoth[a*x])/(5*a^3) + (x^4*ArcCoth[a*x])/(10*a) + ArcCoth[a*x]^2/(5*a^5) + (x^5*ArcCoth[a*x]^2)/5 - (3*ArcTanh[a*x])/(10*a^5) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(5*a^5) - PolyLog[2, 1 - 2/(1 - a*x)]/(5*a^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \coth^{-1}(ax)^2 \, dx = \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \coth^{-1}(ax)}{1-a^2x^2} \, dx$$
$$= \frac{1}{5}x^5 \coth^{-1}(ax)^2 + \frac{2\int x^3 \coth^{-1}(ax) \, dx}{5a} - \frac{2\int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2} \, dx}{5a}$$
$$= \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10}\int \frac{x^4}{1-a^2x^2} \, dx + \frac{2\int x \coth^{-1}(ax) \, dx}{5a^3} - \frac{2\int$$
$$= \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10}\int \left(-\frac{1}{a^4} -\right.$$
$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 -$$
$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 -$$
$$= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 -$$

Mathematica [A]
time = 0.32, size = 87, normalized size = 0.69

$$\frac{ax(9+a^2x^2) + 6(-1+a^5x^5)\coth^{-1}(ax)^2 + 3\coth^{-1}(ax)\left(-3+2a^2x^2+a^4x^4-4\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right) + 6\text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right)}{30a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[a*x]^2,x]
```

```
[Out] (a*x*(9 + a^2*x^2) + 6*(-1 + a^5*x^5)*ArcCoth[a*x]^2 + 3*ArcCoth[a*x]*(-3 +
 2*a^2*x^2 + a^4*x^4 - 4*Log[1 - E^(-2*ArcCoth[a*x])]) + 6*PolyLog[2, E^(-2
*ArcCoth[a*x])])/(30*a^5)
```

Maple [A]
time = 0.28, size = 165, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{a^5x^5\operatorname{arccoth}(ax)^2}{5}+\frac{a^4x^4\operatorname{arccoth}(ax)}{10}+\frac{\operatorname{arccoth}(ax)a^2x^2}{5}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{5}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{5}+\frac{a^3x^3}{30}+\frac{3ax}{10}+\frac{3\ln(ax-1}{20}}{a^5}$
default	$\frac{\frac{a^5x^5\operatorname{arccoth}(ax)^2}{5}+\frac{a^4x^4\operatorname{arccoth}(ax)}{10}+\frac{\operatorname{arccoth}(ax)a^2x^2}{5}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{5}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{5}+\frac{a^3x^3}{30}+\frac{3ax}{10}+\frac{3\ln(ax-1}{20}}{a^5}$
risch	$\frac{3x}{10a^4}+\frac{x^3}{30a^2}+\frac{413}{2250a^5}-\frac{x^5\ln(ax+1)}{50}-\frac{\ln(ax-1)x^2}{20a^3}-\frac{\ln(ax-1)x}{10a^4}-\frac{\ln(ax-1)x^4}{40a}-\frac{\ln(ax-1)x^3}{30a^2}-\frac{x^5\ln(}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccoth(a*x)^2+1/10*a^4*x^4*arccoth(a*x)+1/5*arccoth(a*x
)*a^2*x^2+1/5*arccoth(a*x)*ln(a*x-1)+1/5*arccoth(a*x)*ln(a*x+1)+1/30*a^3*x^
3+3/10*a*x+3/20*ln(a*x-1)-3/20*ln(a*x+1)+1/20*ln(a*x-1)^2-1/5*dilog(1/2*a*x
+1/2)-1/10*ln(a*x-1)*ln(1/2*a*x+1/2)+1/10*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1
/2*a*x+1/2)-1/20*ln(a*x+1)^2)
```

Maxima [A]
time = 0.26, size = 155, normalized size = 1.22

$$\frac{1}{5}\,x^5\,\operatorname{arcoth}\left(ax\right)^2+\frac{1}{60}\,a^2\left(\frac{2\,a^3x^3+18\,ax-3\,\log\left(ax+1\right)^2+6\,\log\left(ax+1\right)\log\left(ax-1\right)+3\,\log\left(ax-1\right)^2+9\,\log\left(ax-1\right)}{a^7}-\frac{12\,\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a^7}-\frac{9\,\log\left(ax+1\right)}{a^7}\right)+\frac{1}{10}\,a\left(\frac{a^2x^4+2\,x^2}{a^4}+\frac{2\,\log\left(a^2x^2-1\right)}{a^6}\right)\operatorname{arcoth}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccoth(a*x)^2 + 1/60*a^2*((2*a^3*x^3 + 18*a*x - 3*log(a*x + 1)^2 +
 6*log(a*x + 1)*log(a*x - 1) + 3*log(a*x - 1)^2 + 9*log(a*x - 1))/a^7 - 12*
(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 - 9*log(a*x +
 1)/a^7) + 1/10*a*((a^2*x^4 + 2*x^2)/a^4 + 2*log(a^2*x^2 - 1)/a^6)*arccoth(
a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*arccoth(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\operatorname{acoth}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**2,x)
```

```
[Out] Integral(x**4*acoth(a*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(a*x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\mathrm{acoth}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(a*x)^2,x)
```

```
[Out] int(x^4*acoth(a*x)^2, x)
```

3.14 $\int x^3 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=81

$$\frac{x^2}{12a^2} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{3a^4}$$

```
[Out] 1/12*x^2/a^2+1/2*x*arccoth(a*x)/a^3+1/6*x^3*arccoth(a*x)/a-1/4*arccoth(a*x)
^2/a^4+1/4*x^4*arccoth(a*x)^2+1/3*ln(-a^2*x^2+1)/a^4
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6038, 6128, 272, 45, 6022, 266, 6096}

$$-\frac{\coth^{-1}(ax)^2}{4a^4} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^2}{12a^2} + \frac{\log(1-a^2x^2)}{3a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{x^3 \coth^{-1}(ax)}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x]^2,x]
```

```
[Out] x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCot
h[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(ax)^2\,dx &= \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{2}a\int \frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\int x^2\coth^{-1}(ax)\,dx}{2a} - \frac{\int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx}{2a} \\
&= \frac{x^3\coth^{-1}(ax)}{6a} + \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{6}\int \frac{x^3}{1-a^2x^2}\,dx + \frac{\int \coth^{-1}(ax)\,dx}{2a^3} - \frac{\int \frac{\coth^{-1}}{1-a^2}}{2a} \\
&= \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 - \frac{1}{12}\text{Subst}\bigg(\int \frac{}{1} \\
&= \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{4a^4} \\
&= \frac{x^2}{12a^2} + \frac{x\coth^{-1}(ax)}{2a^3} + \frac{x^3\coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^2 + \frac{\log(1-}{3a}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 62, normalized size = 0.77

$$\frac{a^2x^2 + 2ax(3 + a^2x^2)\coth^{-1}(ax) + 3(-1 + a^4x^4)\coth^{-1}(ax)^2 + 4\log(1 - a^2x^2)}{12a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a*x]^2,x]
```

```
[Out] (a^2*x^2 + 2*a*x*(3 + a^2*x^2)*ArcCoth[a*x] + 3*(-1 + a^4*x^4)*ArcCoth[a*x]
^2 + 4*Log[1 - a^2*x^2])/(12*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(69) = 138.
time = 0.14, size = 148, normalized size = 1.83

method	result
risch	$\frac{(a^4x^4-1)\ln(ax+1)^2}{16a^4} - \frac{(3x^4\ln(ax-1)a^4-2a^3x^3-6ax-3\ln(ax-1))\ln(ax+1)}{24a^4} + \frac{x^4\ln(ax-1)^2}{16} - \frac{x^3\ln(ax-1)}{12a} +$
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccoth}(ax)}{6}+\frac{ax\operatorname{arccoth}(ax)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}+\frac{\ln(ax-1)^2}{16}-}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arccoth}(ax)^2}{4}+\frac{a^3x^3\operatorname{arccoth}(ax)}{6}+\frac{ax\operatorname{arccoth}(ax)}{2}+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}+\frac{\ln(ax-1)^2}{16}-}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccoth(a*x)^2+1/6*a^3*x^3*arccoth(a*x)+1/2*a*x*arccoth(
a*x)+1/4*arccoth(a*x)*ln(a*x-1)-1/4*arccoth(a*x)*ln(a*x+1)+1/16*ln(a*x-1)^2
-1/8*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+
1/2)+1/16*ln(a*x+1)^2+1/12*a^2*x^2+1/3*ln(a*x-1)+1/3*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 118, normalized size = 1.46

$$\frac{1}{4}x^4\operatorname{arcoth}(ax)^2 + \frac{1}{12}a\left(\frac{2(a^2x^3+3x)}{a^4} - \frac{3\log(ax+1)}{a^5} + \frac{3\log(ax-1)}{a^5}\right)\operatorname{arcoth}(ax) + \frac{4a^2x^2 - 2(3\log(ax-1)-8)\log(ax+1) + 3\log(ax+1)^2 + 3\log(ax-1)^2 + 16\log(ax-1)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x)^2 + 1/12*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
 + 3*log(a*x - 1)/a^5)*arccoth(a*x) + 1/48*(4*a^2*x^2 - 2*(3*log(a*x - 1) -
 8)*log(a*x + 1) + 3*log(a*x + 1)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1))/a
^4
```

Fricas [A]
time = 0.39, size = 81, normalized size = 1.00

$$\frac{4a^2x^2 + 3(a^4x^4-1)\log\left(\frac{ax+1}{ax-1}\right)^2 + 4(a^3x^3+3ax)\log\left(\frac{ax+1}{ax-1}\right) + 16\log(a^2x^2-1)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(a^3*x^3 +
 3*a*x)*log((a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^4
```

Sympy [A]
time = 0.35, size = 90, normalized size = 1.11

$$\begin{cases} \frac{x^4 \operatorname{acoth}^2(ax)}{4} + \frac{x^3 \operatorname{acoth}(ax)}{6a} + \frac{x^2}{12a^2} + \frac{x \operatorname{acoth}(ax)}{2a^3} + \frac{2\log(ax+1)}{3a^4} - \frac{\operatorname{acoth}^2(ax)}{4a^4} - \frac{2\operatorname{acoth}(ax)}{3a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acoth(a*x)**2/4 + x**3*acoth(a*x)/(6*a) + x**2/(12*a**2) +
x*acoth(a*x)/(2*a**3) + 2*log(a*x + 1)/(3*a**4) - acoth(a*x)**2/(4*a**4) -
2*acoth(a*x)/(3*a**4), Ne(a, 0)), (-pi**2*x**4/16, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(69) = 138.
time = 0.40, size = 335, normalized size = 4.14

$$\frac{1}{6}\left(\frac{3\left(\frac{(ax+1)^3}{(ax-1)^3}+\frac{ax+1}{ax-1}\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^4a^5}{(ax-1)^4}-\frac{4(ax+1)^3a^5}{(ax-1)^3}+\frac{6(ax+1)^2a^5}{(ax-1)^2}-\frac{4(ax+1)a^5}{ax-1}+a^5}+\frac{2\left(\frac{3(ax+1)^2}{(ax-1)^2}-\frac{3(ax+1)}{ax-1}+2\right)\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3a^5}{(ax-1)^3}-\frac{3(ax+1)^2a^5}{(ax-1)^2}+\frac{3(ax+1)a^5}{ax-1}-a^5}+\frac{2(ax+1)}{\left(\frac{(ax+1)^2a^5}{(ax-1)^2}-\frac{2(ax+1)a^5}{ax-1}+a^5\right)(ax-1)}-\frac{4\log\left(\frac{ax+1}{ax-1}-1\right)}{a^5}+\frac{4\log\left(\frac{ax+1}{ax-1}\right)}{a^5}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*((a*x + 1)^3/(a*x - 1)^3 + (a*x + 1)/(a*x - 1))*log((a*x + 1)/(a*x -
 1))^2/((a*x + 1)^4*a^5/(a*x - 1)^4 - 4*(a*x + 1)^3*a^5/(a*x - 1)^3 + 6*(a*
x + 1)^2*a^5/(a*x - 1)^2 - 4*(a*x + 1)*a^5/(a*x - 1) + a^5) + 2*(3*(a*x + 1
)^2/(a*x - 1)^2 - 3*(a*x + 1)/(a*x - 1) + 2)*log((a*x + 1)/(a*x - 1))/((a*x
 + 1)^3*a^5/(a*x - 1)^3 - 3*(a*x + 1)^2*a^5/(a*x - 1)^2 + 3*(a*x + 1)*a^5/(
a*x - 1) - a^5) + 2*(a*x + 1)/(((a*x + 1)^2*a^5/(a*x - 1)^2 - 2*(a*x + 1)*a
^5/(a*x - 1) + a^5)*(a*x - 1)) - 4*log((a*x + 1)/(a*x - 1) - 1)/a^5 + 4*log
((a*x + 1)/(a*x - 1))/a^5)*a
```

Mupad [B]
time = 1.26, size = 65, normalized size = 0.80

$$\frac{x^4\,\mathrm{acoth}(a\,x)^2}{4}+\frac{\frac{\ln\left(a^2\,x^2-1\right)}{3}+\frac{a^2\,x^2}{12}-\frac{\mathrm{acoth}(a\,x)^2}{4}+\frac{a^3\,x^3\,\mathrm{acoth}(a\,x)}{6}+\frac{a\,x\,\mathrm{acoth}(a\,x)}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(a*x)^2,x)
```

```
[Out] (x^4*acoth(a*x)^2)/4 + (log(a^2*x^2 - 1)/3 + (a^2*x^2)/12 - acoth(a*x)^2/4
+ (a^3*x^3*acoth(a*x))/6 + (a*x*acoth(a*x))/2)/a^4
```

3.15 $\int x^2 \coth^{-1}(ax)^2 \, dx$

Optimal. Leaf size=103

$$\frac{x}{3a^2}+\frac{x^2\coth^{-1}(ax)}{3a}+\frac{\coth^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\coth^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{3a^3}-\frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{3a^3}-\frac{\text{PolyLog}(2,1-}{3a^3}$$

```
[Out] 1/3*x/a^2+1/3*x^2*arccoth(a*x)/a+1/3*arccoth(a*x)^2/a^3+1/3*x^3*arccoth(a*x
)^2-1/3*arctanh(a*x)/a^3-2/3*arccoth(a*x)*ln(2/(-a*x+1))/a^3-1/3*polylog(2,
1-2/(-a*x+1))/a^3
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6038, 6128, 327, 212, 6132, 6056, 2449, 2352}

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{3a^3}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{\coth^{-1}(ax)^2}{3a^3}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{3a^3}+\frac{x}{3a^2}+\frac{1}{3}x^3\coth^{-1}(ax)^2+\frac{x^2\coth^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x]^2,x]
```

```
[Out] x/(3*a^2) + (x^2*ArcCoth[a*x])/(3*a) + ArcCoth[a*x]^2/(3*a^3) + (x^3*ArcCot
h[a*x]^2)/3 - ArcTanh[a*x]/(3*a^3) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/(3*a
^3) - PolyLog[2, 1 - 2/(1 - a*x)]/(3*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(ax)^2 \, dx = \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2} \, dx$$
$$= \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{2\int x \coth^{-1}(ax) \, dx}{3a} - \frac{2\int \frac{x \coth^{-1}(ax)}{1-a^2x^2} \, dx}{3a}$$
$$= \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3}\int \frac{x^2}{1-a^2x^2} \, dx - \frac{2\int \frac{\coth^{-1}(ax)}{1-ax}}{3a^2}$$
$$= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{3a^3} -$$
$$= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2\coth^{-1}(ax}{3}$$
$$= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2\coth^{-1}(ax}{3}$$

Mathematica [A]
time = 0.18, size = 66, normalized size = 0.64

$$\frac{ax + (-1+a^3x^3)\coth^{-1}(ax)^2 + \coth^{-1}(ax)\left(-1+a^2x^2 - 2\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right) + \text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a*x]^2,x]
[Out] (a*x + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + ArcCoth[a*x]*(-1 + a^2*x^2 - 2*Log[1
 - E^(-2*ArcCoth[a*x])]) + PolyLog[2, E^(-2*ArcCoth[a*x])])/(3*a^3)
```

Maple [A]
time = 0.24, size = 145, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{a^3x^3\text{arccoth}(ax)^2}{3} + \frac{\text{arccoth}(ax)a^2x^2}{3} + \frac{\text{arccoth}(ax)\ln(ax-1)}{3} + \frac{\text{arccoth}(ax)\ln(ax+1)}{3} + \frac{ax}{3} + \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3} -}{a^3}$
default	$\frac{\frac{a^3x^3\text{arccoth}(ax)^2}{3} + \frac{\text{arccoth}(ax)a^2x^2}{3} + \frac{\text{arccoth}(ax)\ln(ax-1)}{3} + \frac{\text{arccoth}(ax)\ln(ax+1)}{3} + \frac{ax}{3} + \frac{\ln(ax-1)}{6} - \frac{\ln(ax+1)}{6} - \frac{\text{dilog}\left(\frac{ax}{2}+\frac{1}{2}\right)}{3} -}{a^3}$
risch	$\frac{\ln(ax-1)^2x^3}{12} - \frac{\ln(ax-1)^2}{12a^3} - \frac{\ln(ax-1)x^3}{18} - \frac{\ln(ax-1)x^2}{12a} - \frac{\ln(ax-1)x}{6a^2} + \frac{11\ln(ax-1)}{36a^3} + \frac{x}{3a^2} + \frac{\ln(ax+1)^2x^3}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccoth(a*x)^2+1/3*arccoth(a*x)*a^2*x^2+1/3*arccoth(a*x)
*ln(a*x-1)+1/3*arccoth(a*x)*ln(a*x+1)+1/3*a*x+1/6*ln(a*x-1)-1/6*ln(a*x+1)-1
/3*dilog(1/2*a*x+1/2)-1/6*ln(a*x-1)*ln(1/2*a*x+1/2)+1/12*ln(a*x-1)^2-1/12*l
n(a*x+1)^2+1/6*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2))
```

Maxima [A]
time = 0.26, size = 134, normalized size = 1.30

$$\frac{1}{3}x^3\operatorname{arcoth}(ax)^2+\frac{1}{12}a^2\left(\frac{4ax-\log(ax+1)^2+2\log(ax+1)\log(ax-1)+\log(ax-1)^2+2\log(ax-1)}{a^5}-\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^5}-\frac{2\log(ax+1)}{a^5}\right)+\frac{1}{3}a\left(\frac{x^2}{a^2}+\frac{\log(a^2x^2-1)}{a^4}\right)\operatorname{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(a*x)^2 + 1/12*a^2*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)
*log(a*x - 1) + log(a*x - 1)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(
1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) + 1/3*a*(
x^2/a^2 + log(a^2*x^2 - 1)/a^4)*arccoth(a*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{acoth}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x)**2,x)
```

```
[Out] Integral(x**2*acoth(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a*x)^2,x)
```

```
[Out] int(x^2*acoth(a*x)^2, x)
```

3.16 $\int x \coth^{-1}(ax)^2\,dx$

Optimal. Leaf size=54

$$\frac{x\coth^{-1}(ax)}{a}-\frac{\coth^{-1}(ax)^2}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^2+\frac{\log(1-a^2x^2)}{2a^2}$$

```
[Out] x*arccoth(a*x)/a-1/2*arccoth(a*x)^2/a^2+1/2*x^2*arccoth(a*x)^2+1/2*ln(-a^2*
x^2+1)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6038, 6128, 6022, 266, 6096}

$$\frac{\log(1-a^2x^2)}{2a^2}-\frac{\coth^{-1}(ax)^2}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^2+\frac{x\coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x]^2,x]
```

```
[Out] (x*ArcCoth[a*x])/a - ArcCoth[a*x]^2/(2*a^2) + (x^2*ArcCoth[a*x]^2)/2 + Log[
1 - a^2*x^2]/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x\coth^{-1}(ax)^2\,dx &= \frac{1}{2}x^2\coth^{-1}(ax)^2 - a\int \frac{x^2\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= \frac{1}{2}x^2\coth^{-1}(ax)^2 + \frac{\int \coth^{-1}(ax)\,dx}{a} - \frac{\int \frac{\coth^{-1}(ax)}{1-a^2x^2}\,dx}{a} \\
&= \frac{x\coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^2 - \int \frac{x}{1-a^2x^2}\,dx \\
&= \frac{x\coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^2 + \frac{\log(1-a^2x^2)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.80

$$\frac{2ax\coth^{-1}(ax) + (-1+a^2x^2)\coth^{-1}(ax)^2 + \log(1-a^2x^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x]^2,x]
```

```
[Out] (2*a*x*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 + Log[1 - a^2*x^2])/(2*
a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(48) = 96.
time = 0.11, size = 127, normalized size = 2.35

method	result
risch	$\frac{(a^2x^2-1)\ln(ax+1)^2}{8a^2} - \frac{(x^2\ln(ax-1)a^2-2ax-\ln(ax-1))\ln(ax+1)}{4a^2} + \frac{x^2\ln(ax-1)^2}{8} - \frac{x\ln(ax-1)}{2a} - \frac{\ln(ax-1)^2}{8a^2} +$
derivativedivides	$\frac{\frac{a^2x^2\operatorname{arccoth}(ax)^2}{2} + ax\operatorname{arccoth}(ax) + \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2} - \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2} - \frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4} + \frac{\ln(ax-1)^2}{8} + \frac{\ln(ax-1)}{2}}{a^2}$

default	$\frac{\frac{a^2x^2\operatorname{arccoth}(ax)^2}{2}+ax\operatorname{arccoth}(ax)+\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2}-\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2}-\frac{\ln(ax-1)\ln\left(\frac{ax}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(ax-1)^2}{8}+\frac{\ln(ax-1}{2}}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccoth(a*x)^2+a*x*arccoth(a*x)+1/2*arccoth(a*x)*ln(a*x-
1)-1/2*arccoth(a*x)*ln(a*x+1)-1/4*ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*ln(a*x-1)^2
+1/2*ln(a*x-1)+1/2*ln(a*x+1)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2*a*x+1/2)
)*ln(-1/2*a*x+1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(48) = 96$.
time = 0.25, size = 97, normalized size = 1.80

$$\frac{1}{2}x^2\operatorname{arcoth}(ax)^2+\frac{1}{2}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{arcoth}(ax)-\frac{2(\log(ax-1)-2)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x)^2 + 1/2*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a
^3)*arccoth(a*x) - 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2
- log(a*x - 1)^2 - 4*log(a*x - 1))/a^2
```

Fricas [A]
time = 0.37, size = 62, normalized size = 1.15

$$\frac{4\,ax\log\left(\frac{ax+1}{ax-1}\right)+(a^2x^2-1)\log\left(\frac{ax+1}{ax-1}\right)^2+4\log\left(a^2x^2-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*x*log((a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1)
)^2 + 4*log(a^2*x^2 - 1))/a^2
```

Sympy [A]
time = 0.19, size = 60, normalized size = 1.11

$$\begin{cases} \frac{x^2\operatorname{acoth}^2(ax)}{2}+\frac{x\operatorname{acoth}(ax)}{a}+\frac{\log(ax+1)}{a^2}-\frac{\operatorname{acoth}^2(ax)}{2a^2}-\frac{\operatorname{acoth}(ax)}{a^2} & \text{for}\ a\neq 0 \\ -\frac{\pi^2x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acoth(a*x)**2/2 + x*acoth(a*x)/a + log(a*x + 1)/a**2 - acot
h(a*x)**2/(2*a**2) - acoth(a*x)/a**2, Ne(a, 0)), (-pi**2*x**2/8, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(48) = 96.
time = 0.40, size = 154, normalized size = 2.85

$$\frac{1}{2}a\left(\frac{(ax+1)\log\left(\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^2a^3}{(ax-1)^2}-\frac{2(ax+1)a^3}{ax-1}+a^3\right)(ax-1)}+\frac{2\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)a^3}{ax-1}-a^3}-\frac{2\log\left(\frac{ax+1}{ax-1}-1\right)}{a^3}+\frac{2\log\left(\frac{ax+1}{ax-1}\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*a*((a*x + 1)*log((a*x + 1)/(a*x - 1))^2/(((a*x + 1)^2*a^3/(a*x - 1)^2 -
 2*(a*x + 1)*a^3/(a*x - 1) + a^3)*(a*x - 1)) + 2*log((a*x + 1)/(a*x - 1))/(
(a*x + 1)*a^3/(a*x - 1) - a^3) - 2*log((a*x + 1)/(a*x - 1) - 1)/a^3 + 2*log
((a*x + 1)/(a*x - 1))/a^3)
```

Mupad [B]
time = 1.21, size = 44, normalized size = 0.81

$$\frac{x^2\,\mathrm{acoth}(a\,x)^2}{2}+\frac{-\frac{\mathrm{acoth}(a\,x)^2}{2}+a\,x\,\mathrm{acoth}(a\,x)+\frac{\ln(a^2\,x^2-1)}{2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a*x)^2,x)
```

```
[Out] (x^2*acoth(a*x)^2)/2 + (log(a^2*x^2 - 1)/2 - acoth(a*x)^2/2 + a*x*acoth(a*x
))/a^2
```

3.17 $\int \coth^{-1}(ax)^2\,dx$

Optimal. Leaf size=58

$$\frac{\coth^{-1}(ax)^2}{a}+x\coth^{-1}(ax)^2-\frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}$$

```
[Out] arccoth(a*x)^2/a+x*arccoth(a*x)^2-2*arccoth(a*x)*ln(2/(-a*x+1))/a-polylog(2
,1-2/(-a*x+1))/a
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6022, 6132, 6056, 2449, 2352}

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a}+x\coth^{-1}(ax)^2+\frac{\coth^{-1}(ax)^2}{a}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2,x]
```

```
[Out] ArcCoth[a*x]^2/a + x*ArcCoth[a*x]^2 - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a -
 PolyLog[2, 1 - 2/(1 - a*x)]/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \coth^{-1}(ax)^2\,dx &= x\coth^{-1}(ax)^2 - (2a)\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= \frac{\coth^{-1}(ax)^2}{a} + x\coth^{-1}(ax)^2 - 2\int \frac{\coth^{-1}(ax)}{1-ax}\,dx \\
&= \frac{\coth^{-1}(ax)^2}{a} + x\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a} + 2\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx \\
&= \frac{\coth^{-1}(ax)^2}{a} + x\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a} - \frac{2\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-ax}\right)}{a} \\
&= \frac{\coth^{-1}(ax)^2}{a} + x\coth^{-1}(ax)^2 - \frac{2\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a} - \frac{\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.79

$$
\frac{\coth^{-1}(ax)\left((-1+ax)\coth^{-1}(ax) - 2\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right) + \mathrm{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2,x]
```

```
[Out] (ArcCoth[a*x]*((-1 + a*x)*ArcCoth[a*x] - 2*Log[1 - E^(-2*ArcCoth[a*x])]) +
PolyLog[2, E^(-2*ArcCoth[a*x])])/a
```

Maple [A]
time = 0.29, size = 116, normalized size = 2.00

method	result

derivativedivides	$\frac{\operatorname{arccoth}(ax)^2(ax-1)+2\operatorname{arccoth}(ax)^2-2\operatorname{arccoth}(ax)\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{arccoth}(ax)\ln}{a}$
default	$\frac{\operatorname{arccoth}(ax)^2(ax-1)+2\operatorname{arccoth}(ax)^2-2\operatorname{arccoth}(ax)\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-2\operatorname{arccoth}(ax)\ln}{a}$
risch	$\frac{\ln(ax-1)^2x}{4}-\frac{x\ln(ax-1)}{2}-\frac{\ln(ax-1)^2}{4a}+\frac{\ln(ax-1)}{2a}+\frac{\ln(ax+1)^2x}{4}-\frac{x\ln(ax+1)}{2}+\frac{\ln(ax+1)^2}{4a}+\frac{\ln(ax+1)}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)^2*(a*x-1)+2*arccoth(a*x)^2-2*arccoth(a*x)*ln(1-1/((a*x-1)
/(a*x+1))^(1/2))-2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)*ln(1
+1/((a*x-1)/(a*x+1))^(1/2))-2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.25, size = 135, normalized size = 2.33

$$x \operatorname{arcoth}(ax)^2 + \frac{1}{4}\left(a\left(\frac{\log(ax+1)^2 + 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a^3} - \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^3}\right) - \frac{2\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log\left(a^2x^2-1\right)}{a}\right)a + \frac{\operatorname{arcoth}(ax)\log\left(a^2x^2-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arccoth(a*x)^2 + 1/4*(a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) -
log(a*x - 1)^2)/a^3 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x +
 1/2))/a^3) - 2*(log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a)*a + a
rccoth(a*x)*log(a^2*x^2 - 1)/a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2,x)
```

```
[Out] Integral(acoth(a*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acoth}(a\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2,x)
```

```
[Out] int(acoth(a*x)^2, x)
```

3.18 $\int \frac{\coth^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=97

$$2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)-\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2a}{1+}\right.$$

```
[Out] 2*arccoth(a*x)^2*arccoth(1-2/(-a*x+1))+arccoth(a*x)*polylog(2,1-2/(a*x+1))-
arccoth(a*x)*polylog(2,1-2*a*x/(a*x+1))+1/2*polylog(3,1-2/(a*x+1))-1/2*poly
log(3,1-2*a*x/(a*x+1))
```

Rubi [A]

time = 0.16, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6034, 6200, 6096, 6204, 6745}

$$\frac{1}{2}\text{Li}_3\left(1-\frac{2}{ax+1}\right)-\frac{1}{2}\text{Li}_3\left(1-\frac{2ax}{ax+1}\right)+\text{Li}_2\left(1-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\text{Li}_2\left(1-\frac{2ax}{ax+1}\right)\coth^{-1}(ax)+2\coth^{-1}\left(1-\frac{2}{1-ax}\right)\coth^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x,x]
```

```
[Out] 2*ArcCoth[a*x]^2*ArcCoth[1 - 2/(1 - a*x)] + ArcCoth[a*x]*PolyLog[2, 1 - 2/(
1 + a*x)] - ArcCoth[a*x]*PolyLog[2, 1 - (2*a*x)/(1 + a*x)] + PolyLog[3, 1 -
 2/(1 + a*x)]/2 - PolyLog[3, 1 - (2*a*x)/(1 + a*x)]/2
```

Rule 6034

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6200

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^2}{x}\,dx &= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)-(4a)\int \frac{\coth^{-1}(ax)\coth^{-1}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx\\ &= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+(2a)\int \frac{\coth^{-1}(ax)\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx-(2a)\int \frac{\coth^{-1}\\ &= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)-\coth^{-1}(ax)\mathrm{Li}_2\Bigg(\\ &= 2\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)-\coth^{-1}(ax)\mathrm{Li}_2\Bigg(\end{aligned}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 1.18

$$\frac{2}{3}\coth^{-1}(ax)^3+\coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)-\coth^{-1}(ax)^2\log\left(1-e^{2\coth^{-1}(ax)}\right)-\coth^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-\coth^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)-\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x,x]
```

```
[Out] (2*ArcCoth[a*x]^3)/3 + ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*x])] - ArcCot
h[a*x]^2*Log[1 - E^(2*ArcCoth[a*x])] - ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCo
th[a*x])] - ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] - PolyLog[3, -E^(-2
*ArcCoth[a*x])]/2 + PolyLog[3, E^(2*ArcCoth[a*x])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.00, size = 487, normalized size = 5.02

method	result

derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)^2 + \frac{i\pi \operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\operatorname{arccoth}(ax)^2}{2} - \frac{i\pi \operatorname{csgn}\left(\right.}{}$
default	$\ln(ax)\operatorname{arccoth}(ax)^2 + \frac{i\pi \operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right)\operatorname{csgn}\left(i\left(1+\frac{ax+1}{ax-1}\right)\right)\operatorname{csgn}\left(\frac{i\left(1+\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right)\operatorname{arccoth}(ax)^2}{2} - \frac{i\pi \operatorname{csgn}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccoth(a*x)^2+1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I*(1+1/(
a*x-1)*(a*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))*arccot
h(a*x)^2-1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(1/(a*x-1)*(a*x+1)-1
)*(1+1/(a*x-1)*(a*x+1)))^2*arccoth(a*x)^2-1/2*I*Pi*csgn(I*(1+1/(a*x-1)*(a*x
+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^2*arccoth(a*x)^2+
1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^3*arccoth(a*x)
^2+arccoth(a*x)^2*ln(1/(a*x-1)*(a*x+1)-1)-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a
*x+1))^(1/2))-2*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylo
g(3,-1/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/
2))-2*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x
-1)/(a*x+1))^(1/2))+arccoth(a*x)*polylog(2,-1/(a*x-1)*(a*x+1))-1/2*polylog(
3,-1/(a*x-1)*(a*x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x,x)
```

```
[Out] Integral(acoth(a*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2/x,x)
```

```
[Out] int(acoth(a*x)^2/x, x)
```

3.19 $\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=55

$$a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{x}+2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-a\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

```
[Out] a*arccoth(a*x)^2-arccoth(a*x)^2/x+2*a*arccoth(a*x)*ln(2-2/(a*x+1))-a*polylo
g(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6038, 6136, 6080, 2497}

$$-a\text{Li}_2\left(\frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^2,x]
```

```
[Out] a*ArcCoth[a*x]^2 - ArcCoth[a*x]^2/x + 2*a*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx &= -\frac{\coth^{-1}(ax)^2}{x} + (2a)\int \frac{\coth^{-1}(ax)}{x\,(1-a^2x^2)}\,dx\\ &= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + (2a)\int \frac{\coth^{-1}(ax)}{x(1+ax)}\,dx\\ &= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - (2a^2)\int \frac{\log\left(2-\frac{2}{1+}\right.}{1-a^2x}\\ &= a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\text{Li}_2\left(-1+\frac{2}{1+ax}\right.\end{aligned}$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 0.89

$$\frac{(-1+ax)\coth^{-1}(ax)^2}{x} + 2a\coth^{-1}(ax)\log\left(1+e^{-2\coth^{-1}(ax)}\right) - a\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^2,x]
```

```
[Out] ((-1 + a*x)*ArcCoth[a*x]^2)/x + 2*a*ArcCoth[a*x]*Log[1 + E^(-2*ArcCoth[a*x]
)] - a*PolyLog[2, -E^(-2*ArcCoth[a*x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(55) = 110.
time = 0.07, size = 145, normalized size = 2.64

method	result
derivativedivides	$a\left(-\frac{\operatorname{arccoth}(ax)^2}{ax} - \operatorname{arccoth}(ax)\ln(ax+1) - \operatorname{arccoth}(ax)\ln(ax-1) + 2\ln(ax)\operatorname{arccoth}(a\right.$
default	$a\left(-\frac{\operatorname{arccoth}(ax)^2}{ax} - \operatorname{arccoth}(ax)\ln(ax+1) - \operatorname{arccoth}(ax)\ln(ax-1) + 2\ln(ax)\operatorname{arccoth}(a\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/a/x*arccoth(a*x)^2-arccoth(a*x)*ln(a*x+1)-arccoth(a*x)*ln(a*x-1)+2*ln
(a*x)*arccoth(a*x)+dilog(1/2*a*x+1/2)+1/2*ln(a*x-1)*ln(1/2*a*x+1/2)-1/4*ln(
a*x-1)^2+1/4*ln(a*x+1)^2-1/2*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1/2)-d
ilog(a*x+1)-ln(a*x)*ln(a*x+1)-dilog(a*x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(54) = 108$.
time = 0.26, size = 146, normalized size = 2.65

$$\frac{1}{4}a^2\left(\frac{\log(ax+1)^2 - 2\log(ax+1)\log(ax-1) - \log(ax-1)^2}{a} + \frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right) + \mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a} - \frac{4\left(\log(ax+1)\log(x) + \mathrm{Li}_2(-ax)\right)}{a} + \frac{4\left(\log(-ax+1)\log(x) + \mathrm{Li}_2(ax)\right)}{a}\right) - a\left(\log\left(a^2x^2-1\right) - \log\left(x^2\right)\right)\operatorname{arcoth}(ax) - \frac{\operatorname{arcoth}(ax)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
+ 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*
x + 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a) -
 a*(log(a^2*x^2 - 1) - log(x^2))*arccoth(a*x) - arccoth(a*x)^2/x
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^2/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(ax)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**2,x)
```

```
[Out] Integral(acoth(a*x)**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^2/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acoth}(a\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2/x^2,x)
```

```
[Out] int(acoth(a*x)^2/x^2, x)
```

3.20 $\int \frac{\coth^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=61

$$-\frac{a\coth^{-1}(ax)}{x}+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)$$

```
[Out] -a*arccoth(a*x)/x+1/2*a^2*arccoth(a*x)^2-1/2*arccoth(a*x)^2/x^2+a^2*ln(x)-1
/2*a^2*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {6038, 6130, 272, 36, 29, 31, 6096}

$$-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}-\frac{a\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^3,x]
```

```
[Out] -((a*ArcCoth[a*x])/x) + (a^2*ArcCoth[a*x]^2)/2 - ArcCoth[a*x]^2/(2*x^2) + a
^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx &= -\frac{\coth^{-1}(ax)^2}{2x^2}+a\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx \\
&= -\frac{\coth^{-1}(ax)^2}{2x^2}+a\int \frac{\coth^{-1}(ax)}{x^2}\,dx+a^3\int \frac{\coth^{-1}(ax)}{1-a^2x^2}\,dx \\
&= -\frac{a\coth^{-1}(ax)}{x}+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}+a^2\int \frac{1}{x\left(1-a^2x^2\right)}\,dx \\
&= -\frac{a\coth^{-1}(ax)}{x}+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx,x,x^2\right. \\
&= -\frac{a\coth^{-1}(ax)}{x}+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^4\text{S} \\
&= -\frac{a\coth^{-1}(ax)}{x}+\frac{1}{2}a^2\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 0.93

$$-\frac{a\coth^{-1}(ax)}{x}+\frac{\left(-1+a^2x^2\right)\coth^{-1}(ax)^2}{2x^2}+a^2\log(x)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^3,x]
```

```
[Out] -((a*ArcCoth[a*x])/x) + ((-1 + a^2*x^2)*ArcCoth[a*x]^2)/(2*x^2) + a^2*Log[x
] - (a^2*Log[1 - a^2*x^2])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.08, size = 136, normalized size = 2.23

method	result
risch	$\frac{(a^2x^2-1)\ln(ax+1)^2}{8x^2} - \frac{(x^2\ln(ax-1)a^2+2ax-\ln(ax-1))\ln(ax+1)}{4x^2} + \frac{a^2x^2\ln(ax-1)^2+8a^2\ln(x)x^2-4a^2\ln(a^2x^2-}{8x^2}$
derivativedivides	$a^2\left(-\frac{\operatorname{arccoth}(ax)^2}{2a^2x^2} - \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arccoth}(ax)}{ax} + \frac{\ln(ax-1)\ln(\frac{ax}{2}+\frac{1}{2})}{4} -\right.$
default	$a^2\left(-\frac{\operatorname{arccoth}(ax)^2}{2a^2x^2} - \frac{\operatorname{arccoth}(ax)\ln(ax-1)}{2} + \frac{\operatorname{arccoth}(ax)\ln(ax+1)}{2} - \frac{\operatorname{arccoth}(ax)}{ax} + \frac{\ln(ax-1)\ln(\frac{ax}{2}+\frac{1}{2})}{4} -\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2/a^2/x^2*arccoth(a*x)^2-1/2*arccoth(a*x)*ln(a*x-1)+1/2*arccoth(a*x
)*ln(a*x+1)-1/a/x*arccoth(a*x)+1/4*ln(a*x-1)*ln(1/2*a*x+1/2)-1/8*ln(a*x-1)^
2-1/2*ln(a*x-1)-1/2*ln(a*x+1)+ln(a*x)-1/8*ln(a*x+1)^2+1/4*(ln(a*x+1)-ln(1/2
*a*x+1/2))*ln(-1/2*a*x+1/2))
```

Maxima [A]
time = 0.25, size = 96, normalized size = 1.57

$$\frac{1}{8}\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2-\log\left(ax-1\right)^2-4\,\log\left(ax-1\right)+8\,\log\left(x\right)\right)a^2+\frac{1}{2}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)a\,\operatorname{arcoth}\left(ax\right)-\frac{\operatorname{arcoth}\left(ax\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
4*log(a*x - 1) + 8*log(x))*a^2 + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x
)*a*arccoth(a*x) - 1/2*arccoth(a*x)^2/x^2
```

Fricas [A]
time = 0.39, size = 79, normalized size = 1.30

$$-\frac{4\,a^2x^2\log\left(a^2x^2-1\right)-8\,a^2x^2\log\left(x\right)+4\,ax\log\left(\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^2*x^2*log(a^2*x^2 - 1) - 8*a^2*x^2*log(x) + 4*a*x*log((a*x + 1)/(
a*x - 1)) - (a^2*x^2 - 1)*log((a*x + 1)/(a*x - 1))^2)/x^2
```

Sympy [A]
time = 0.25, size = 56, normalized size = 0.92

$$a^2\log(x)-a^2\log(ax+1)+\frac{a^2\operatorname{acoth}^2(ax)}{2}+a^2\operatorname{acoth}(ax)-\frac{a\operatorname{acoth}(ax)}{x}-\frac{\operatorname{acoth}^2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**3,x)
```

```
[Out] a**2*log(x) - a**2*log(a*x + 1) + a**2*acoth(a*x)**2/2 + a**2*acoth(a*x) -
a*acoth(a*x)/x - acoth(a*x)**2/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(55) = 110.
time = 0.39, size = 137, normalized size = 2.25

$$\frac{1}{2}\left(2a\log\left(\frac{ax+1}{ax-1}+1\right)-2a\log\left(\frac{ax+1}{ax-1}\right)+\frac{(ax+1)a\log\left(\frac{ax+1}{ax-1}\right)^2}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2}+\frac{2(ax+1)}{ax-1}+1\right)}+\frac{2a\log\left(\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log((a*x + 1)/(a*x - 1) + 1) - 2*a*log((a*x + 1)/(a*x - 1)) + (a*x
 + 1)*a*log((a*x + 1)/(a*x - 1))^2/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 + 2*
(a*x + 1)/(a*x - 1) + 1)) + 2*a*log((a*x + 1)/(a*x - 1))/((a*x + 1)/(a*x -
1) + 1))*a
```

Mupad [B]
time = 1.45, size = 145, normalized size = 2.38

$$a^2\ln(x)+\ln\left(\frac{1}{a\,x}+1\right)^2\left(\frac{a^2}{8}-\frac{1}{8\,x^2}\right)+\ln\left(1-\frac{1}{a\,x}\right)^2\left(\frac{a^2}{8}-\frac{1}{8\,x^2}\right)-\frac{a^2\ln(a^2\,x^2-1)}{2}+\ln\left(1-\frac{1}{a\,x}\right)\left(\frac{4\,a\,x-2}{16\,x^2}+\frac{4\,a\,x+2}{16\,x^2}-\ln\left(\frac{1}{a\,x}+1\right)\left(\frac{a^2}{4}-\frac{1}{4\,x^2}\right)\right)-\frac{a\ln\left(\frac{1}{a\,x}+1\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2/x^3,x)
```

```
[Out] a^2*log(x) + log(1/(a*x) + 1)^2*(a^2/8 - 1/(8*x^2)) + log(1 - 1/(a*x))^2*(a
^2/8 - 1/(8*x^2)) - (a^2*log(a^2*x^2 - 1))/2 + log(1 - 1/(a*x))*((4*a*x - 2
)/(16*x^2) + (4*a*x + 2)/(16*x^2) - log(1/(a*x) + 1)*(a^2/4 - 1/(4*x^2))) -
 (a*log(1/(a*x) + 1))/(2*x)
```

3.21 $\int \frac{\coth^{-1}(ax)^2}{x^4} \, dx$

Optimal. Leaf size=103

$$-\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{2}{3}a^3\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-$$

```
[Out] -1/3*a^2/x-1/3*a*arccoth(a*x)/x^2+1/3*a^3*arccoth(a*x)^2-1/3*arccoth(a*x)^2
/x^3+1/3*a^3*arctanh(a*x)+2/3*a^3*arccoth(a*x)*ln(2-2/(a*x+1))-1/3*a^3*poly
log(2,-1+2/(a*x+1))
```

Rubi [A]
time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6038, 6130, 331, 212, 6136, 6080, 2497}

$$-\frac{1}{3}a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{1}{3}a^3\coth^{-1}(ax)^2+\frac{2}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{a^2}{3x}-\frac{\coth^{-1}(ax)^2}{3x^3}-\frac{a\coth^{-1}(ax)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^4,x]
```

```
[Out] -1/3*a^2/x - (a*ArcCoth[a*x])/(3*x^2) + (a^3*ArcCoth[a*x]^2)/3 - ArcCoth[a*
x]^2/(3*x^3) + (a^3*ArcTanh[a*x])/3 + (2*a^3*ArcCoth[a*x]*Log[2 - 2/(1 + a*
x)])/3 - (a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx &= -\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\coth^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\coth^{-1}(ax)}{x^3}\,dx+\frac{1}{3}\left(2a^3\right)\int \frac{\coth^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^2\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx+\frac{1}{3}\left(2a^3\right)\\
&= -\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\coth^{-1}(ax)\log\Bigg(2-\frac{2}{1+\ldots}\\
&= -\frac{a^2}{3x}-\frac{a\coth^{-1}(ax)}{3x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{3x^3}+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{2}{3}a^3\coth^{-1}\ldots
\end{aligned}
$$

Mathematica [**A**]
time = 0.16, size = 87, normalized size = 0.84

$$\frac{-a^2x^2+(-1+a^3x^3)\coth^{-1}(ax)^2+ax\coth^{-1}(ax)\left(-1+a^2x^2+2a^2x^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)\right)-a^3x^3\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^4,x]
```

```
[Out] (-(a^2*x^2) + (-1 + a^3*x^3)*ArcCoth[a*x]^2 + a*x*ArcCoth[a*x]*(-1 + a^2*x^
2 + 2*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]) - a^3*x^3*PolyLog[2, -E^(-2*Arc
Coth[a*x])])/(3*x^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(89) = 178.
time = 0.08, size = 185, normalized size = 1.80

method	result
derivativedivides	$a^3\left(-\frac{\mathrm{arccoth}(ax)^2}{3a^3x^3}-\frac{\mathrm{arccoth}(ax)}{3a^2x^2}+\frac{2\ln(ax)\mathrm{arccoth}(ax)}{3}-\frac{\mathrm{arccoth}(ax)\ln(ax-1)}{3}-\frac{\mathrm{arccoth}(ax)\ln(ax+1)}{3}-\right.$
default	$a^3\left(-\frac{\mathrm{arccoth}(ax)^2}{3a^3x^3}-\frac{\mathrm{arccoth}(ax)}{3a^2x^2}+\frac{2\ln(ax)\mathrm{arccoth}(ax)}{3}-\frac{\mathrm{arccoth}(ax)\ln(ax-1)}{3}-\frac{\mathrm{arccoth}(ax)\ln(ax+1)}{3}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arccoth(a*x)^2-1/3*arccoth(a*x)/a^2/x^2+2/3*ln(a*x)*arcco
th(a*x)-1/3*arccoth(a*x)*ln(a*x-1)-1/3*arccoth(a*x)*ln(a*x+1)-1/3/a/x-1/6*l
n(a*x-1)+1/6*ln(a*x+1)+1/3*dilog(1/2*a*x+1/2)+1/6*ln(a*x-1)*ln(1/2*a*x+1/2)
-1/12*ln(a*x-1)^2+1/12*ln(a*x+1)^2-1/6*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*
a*x+1/2)-1/3*dilog(a*x+1)-1/3*ln(a*x)*ln(a*x+1)-1/3*dilog(a*x))
```

Maxima [**A**]
time = 0.26, size = 176, normalized size = 1.71

$$\frac{1}{12}\left(4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-4\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a+4\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)a+2a\log(ax+1)-2a\log(ax-1)+\frac{ax\log(ax+1)^2-2ax\log(ax+1)\log(ax-1)-ax\log(ax-1)^2-4}{x}\right)a^2-\frac{1}{3}\left(a^2\log(a^2x^2-1)-a^2\log(x^2)+\frac{1}{x^2}\right)a\,\mathrm{arcoth}(ax)-\frac{\mathrm{arcoth}(ax)^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*
a + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a
*x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a^2 - 1/3*(a^2*log(a^2*x^
2 - 1) - a^2*log(x^2) + 1/x^2)*a*arccoth(a*x) - 1/3*arccoth(a*x)^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="fricas")
[Out] integral(arccoth(a*x)^2/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**4,x)
[Out] Integral(acoth(a*x)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^4,x, algorithm="giac")
[Out] integrate(arccoth(a*x)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2/x^4,x)
[Out] int(acoth(a*x)^2/x^4, x)
```

3.22 $\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2}{12x^2}-\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{2}{3}a^4\log(x)-\frac{1}{3}a^4\log\left(1-a^2x^2\right)$$

```
[Out] -1/12*a^2/x^2-1/6*a*arccoth(a*x)/x^3-1/2*a^3*arccoth(a*x)/x+1/4*a^4*arccoth
(a*x)^2-1/4*arccoth(a*x)^2/x^4+2/3*a^4*ln(x)-1/3*a^4*ln(-a^2*x^2+1)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6038, 6130, 272, 46, 36, 29, 31, 6096}

$$\frac{2}{3}a^4\log(x)+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{a^3\coth^{-1}(ax)}{2x}-\frac{a^2}{12x^2}-\frac{1}{3}a^4\log\left(1-a^2x^2\right)-\frac{\coth^{-1}(ax)^2}{4x^4}-\frac{a\coth^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*
ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1
- a^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^5}\,dx &= -\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\coth^{-1}(ax)}{x^4\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int \frac{\coth^{-1}(ax)}{x^4}\,dx+\frac{1}{2}a^3\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^2\int \frac{1}{x^3\left(1-a^2x^2\right)}\,dx+\frac{1}{2}a^3\int \frac{\coth^{-1}(ax)}{x^2}\,dx+\frac{1}{2}a\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\text{Subst}\Bigg(\int \frac{}{x^3}\\
&= -\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\text{Subst}\Bigg(\int \Bigg(\\
&= -\frac{a^2}{12x^2}-\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^4\log(a\\
&= -\frac{a^2}{12x^2}-\frac{a\coth^{-1}(ax)}{6x^3}-\frac{a^3\coth^{-1}(ax)}{2x}+\frac{1}{4}a^4\coth^{-1}(ax)^2-\frac{\coth^{-1}(ax)^2}{4x^4}+\frac{2}{3}a^4\log(x
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 82, normalized size = 0.91

$$-\frac{a^2}{12x^2}-\frac{a(1+3a^2x^2)\coth^{-1}(ax)}{6x^3}+\frac{(-1+a^4x^4)\coth^{-1}(ax)^2}{4x^4}+\frac{2}{3}a^4\log(x)-\frac{1}{3}a^4\log\left(1-a^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*(1 + 3*a^2*x^2)*ArcCoth[a*x])/(6*x^3) + ((-1 + a^4*x^4)*
ArcCoth[a*x]^2)/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(76) = 152.
time = 0.08, size = 158, normalized size = 1.76

method	result
derivativedivides	$a^4\Bigg(-\frac{\operatorname{arccoth}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arccoth}(ax)}{6a^3x^3}-\frac{\operatorname{arccoth}(ax)}{2ax}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\ln(ax-}{16}$
default	$a^4\Bigg(-\frac{\operatorname{arccoth}(ax)^2}{4a^4x^4}+\frac{\operatorname{arccoth}(ax)\ln(ax+1)}{4}-\frac{\operatorname{arccoth}(ax)}{6a^3x^3}-\frac{\operatorname{arccoth}(ax)}{2ax}-\frac{\operatorname{arccoth}(ax)\ln(ax-1)}{4}-\frac{\ln(ax-}{16}$
risch	$\frac{(a^4x^4-1)\ln(ax+1)^2}{16x^4}-\frac{(3x^4\ln(ax-1)a^4+6a^3x^3+2ax-3\ln(ax-1))\ln(ax+1)}{24x^4}+\frac{3a^4x^4\ln(ax-1)^2+32\ln(x)a^4x^4-1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4/a^4/x^4*arccoth(a*x)^2+1/4*arccoth(a*x)*ln(a*x+1)-1/6/a^3/x^3*arc
coth(a*x)-1/2/a/x*arccoth(a*x)-1/4*arccoth(a*x)*ln(a*x-1)-1/16*ln(a*x-1)^2+
1/8*ln(a*x-1)*ln(1/2*a*x+1/2)+1/8*(ln(a*x+1)-ln(1/2*a*x+1/2))*ln(-1/2*a*x+1
/2)-1/16*ln(a*x+1)^2-1/3*ln(a*x+1)-1/12/a^2/x^2+2/3*ln(a*x)-1/3*ln(a*x-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(76) = 152.
time = 0.25, size = 154, normalized size = 1.71

$$\frac{1}{48}\left(32\,a^2\log(x)-\frac{3\,a^2x^2\log(ax+1)^2+3\,a^2x^2\log(ax-1)^2+16\,a^2x^2\log(ax-1)-2\,(3\,a^2x^2\log(ax-1)-8\,a^2x^2)\log(ax+1)+4}{x^2}\right)a^2+\frac{1}{12}\left(3\,a^3\log(ax+1)-3\,a^3\log(ax-1)-\frac{2\,(3\,a^2x^2+1)}{x^3}\right)a\operatorname{arcoth}(ax)-\frac{\operatorname{arcoth}(ax)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
+ 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 8*a^2*x^2)*log(a*x
+ 1) + 4)/x^2)*a^2 + 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a
^2*x^2 + 1)/x^3)*a*arccoth(a*x) - 1/4*arccoth(a*x)^2/x^4
```

Fricas [A]
time = 0.37, size = 97, normalized size = 1.08

$$-\frac{16\,a^4x^4\log\left(a^2x^2-1\right)-32\,a^4x^4\log\left(x\right)+4\,a^2x^2-3\left(a^4x^4-1\right)\log\left(\frac{ax+1}{ax-1}\right)^2+4\left(3\,a^3x^3+ax\right)\log\left(\frac{ax+1}{ax-1}\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a^4*x^4*log(a^2*x^2 - 1) - 32*a^4*x^4*log(x) + 4*a^2*x^2 - 3*(a^4
*x^4 - 1)*log((a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 + a*x)*log((a*x + 1)/(a
*x - 1)))/x^4
```

Sympy [A]
time = 0.33, size = 90, normalized size = 1.00

$$\frac{2a^4\log\left(x\right)}{3}-\frac{2a^4\log\left(ax+1\right)}{3}+\frac{a^4\operatorname{acoth}^2\left(ax\right)}{4}+\frac{2a^4\operatorname{acoth}\left(ax\right)}{3}-\frac{a^3\operatorname{acoth}\left(ax\right)}{2x}-\frac{a^2}{12x^2}-\frac{a\operatorname{acoth}\left(ax\right)}{6x^3}-\frac{\operatorname{acoth}^2\left(ax\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**2/x**5,x)
```

```
[Out] 2*a**4*log(x)/3 - 2*a**4*log(a*x + 1)/3 + a**4*acoth(a*x)**2/4 + 2*a**4*aco
th(a*x)/3 - a**3*acoth(a*x)/(2*x) - a**2/(12*x**2) - a*acoth(a*x)/(6*x**3)
- acoth(a*x)**2/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(76) = 152.
time = 0.39, size = 319, normalized size = 3.54

$$\frac{1}{6}\left(4\,a^3\log\left(\frac{ax+1}{ax-1}+1\right)-4\,a^3\log\left(\frac{ax+1}{ax-1}\right)+\frac{2\,(ax+1)a^3}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2}+\frac{2\,(ax+1)}{ax-1}+1\right)}+\frac{3\left(\frac{(ax+1)^3a^3}{(ax-1)^3}+\frac{(ax+1)a^3}{ax-1}\right)\log\left(\frac{ax+1}{ax-1}\right)^2}{\frac{(ax+1)^4}{(ax-1)^4}+\frac{4\,(ax+1)^3}{(ax-1)^3}+\frac{6\,(ax+1)^2}{(ax-1)^2}+\frac{4\,(ax+1)}{ax-1}+1}+\frac{2\left(\frac{3\,(ax+1)^2a^3}{(ax-1)^2}+\frac{3\,(ax+1)a^3}{ax-1}+2\,a^3\right)\log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)^3}{(ax-1)^3}+\frac{3\,(ax+1)^2}{(ax-1)^2}+\frac{3\,(ax+1)}{ax-1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*(4*a^3*log((a*x + 1)/(a*x - 1) + 1) - 4*a^3*log((a*x + 1)/(a*x - 1)) +
2*(a*x + 1)*a^3/((a*x - 1)*((a*x + 1)^2/(a*x - 1)^2 + 2*(a*x + 1)/(a*x - 1)
 + 1)) + 3*((a*x + 1)^3*a^3/(a*x - 1)^3 + (a*x + 1)*a^3/(a*x - 1))*log((a*x
 + 1)/(a*x - 1))^2/((a*x + 1)^4/(a*x - 1)^4 + 4*(a*x + 1)^3/(a*x - 1)^3 + 6
*(a*x + 1)^2/(a*x - 1)^2 + 4*(a*x + 1)/(a*x - 1) + 1) + 2*(3*(a*x + 1)^2*a^
3/(a*x - 1)^2 + 3*(a*x + 1)*a^3/(a*x - 1) + 2*a^3)*log((a*x + 1)/(a*x - 1))
/((a*x + 1)^3/(a*x - 1)^3 + 3*(a*x + 1)^2/(a*x - 1)^2 + 3*(a*x + 1)/(a*x -
1) + 1))*a
```

Mupad [B]
time = 1.55, size = 196, normalized size = 2.18

$$\frac{2\,a^4\,\ln\left(x\right)}{3}+\ln\left(\frac{1}{a\,x}+1\right)^2\,\left(\frac{a^4}{16}-\frac{1}{16\,x^4}\right)+\ln\left(1-\frac{1}{a\,x}\right)^2\,\left(\frac{a^4}{16}-\frac{1}{16\,x^4}\right)+\ln\left(1-\frac{1}{a\,x}\right)\,\left(\frac{24\,a^3\,x^3-12\,a^2\,x^2+8\,a\,x-6}{192\,x^4}+\frac{24\,a^3\,x^3+12\,a^2\,x^2+8\,a\,x+6}{192\,x^4}-\ln\left(\frac{1}{a\,x}+1\right)\,\left(\frac{a^4}{8}-\frac{1}{8\,x^4}\right)\right)-\frac{a^4\,\ln\left(a^2\,x^2-1\right)}{3}-\frac{a^2}{12\,x^2}-\frac{a\,\ln\left(\frac{1}{a\,x}+1\right)\,\left(\frac{a^2\,x^2}{4}+\frac{1}{12}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^2/x^5,x)
```

```
[Out] (2*a^4*log(x))/3 + log(1/(a*x) + 1)^2*(a^4/16 - 1/(16*x^4)) + log(1 - 1/(a*
x))^2*(a^4/16 - 1/(16*x^4)) + log(1 - 1/(a*x))*((8*a*x - 12*a^2*x^2 + 24*a^
3*x^3 - 6)/(192*x^4) + (8*a*x + 12*a^2*x^2 + 24*a^3*x^3 + 6)/(192*x^4) - lo
g(1/(a*x) + 1)*(a^4/8 - 1/(8*x^4))) - (a^4*log(a^2*x^2 - 1))/3 - a^2/(12*x^
2) - (a*log(1/(a*x) + 1)*((a^2*x^2)/4 + 1/12))/x^3
```

3.23 $\int x^5 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=186

$$\frac{19x}{60a^5}+\frac{x^3}{60a^3}+\frac{4x^2\coth^{-1}(ax)}{15a^4}+\frac{x^4\coth^{-1}(ax)}{20a^2}+\frac{23\coth^{-1}(ax)^2}{30a^6}+\frac{x\coth^{-1}(ax)^2}{2a^5}+\frac{x^3\coth^{-1}(ax)^2}{6a^3}+\frac{x^5\coth^{-1}(a}{10a}$$

```
[Out] 19/60*x/a^5+1/60*x^3/a^3+4/15*x^2*arccoth(a*x)/a^4+1/20*x^4*arccoth(a*x)/a^
2+23/30*arccoth(a*x)^2/a^6+1/2*x*arccoth(a*x)^2/a^5+1/6*x^3*arccoth(a*x)^2/
a^3+1/10*x^5*arccoth(a*x)^2/a-1/6*arccoth(a*x)^3/a^6+1/6*x^6*arccoth(a*x)^3
-19/60*arctanh(a*x)/a^6-23/15*arccoth(a*x)*ln(2/(-a*x+1))/a^6-23/30*polylog
(2,1-2/(-a*x+1))/a^6
```

Rubi [A]
time = 0.49, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6038, 6128, 308, 212, 327, 6132, 6056, 2449, 2352, 6022, 6096}

$$-\frac{23\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{30a^6}-\frac{19\tanh^{-1}(ax)}{60a^6}-\frac{\coth^{-1}(ax)^3}{6a^6}+\frac{23\coth^{-1}(ax)^2}{30a^6}-\frac{23\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{15a^6}+\frac{19x}{60a^5}+\frac{x\coth^{-1}(ax)^2}{2a^5}+\frac{4x^2\coth^{-1}(ax)}{15a^4}+\frac{x^3}{60a^3}+\frac{x^3\coth^{-1}(ax)^2}{6a^3}+\frac{x^4\coth^{-1}(ax)}{20a^2}+\frac{1}{6}x^6\coth^{-1}(ax)^3+\frac{x^5\coth^{-1}(ax)^2}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcCoth[a*x]^3,x]
```

```
[Out] (19*x)/(60*a^5) + x^3/(60*a^3) + (4*x^2*ArcCoth[a*x])/(15*a^4) + (x^4*ArcCo
th[a*x])/(20*a^2) + (23*ArcCoth[a*x]^2)/(30*a^6) + (x*ArcCoth[a*x]^2)/(2*a^
5) + (x^3*ArcCoth[a*x]^2)/(6*a^3) + (x^5*ArcCoth[a*x]^2)/(10*a) - ArcCoth[a
*x]^3/(6*a^6) + (x^6*ArcCoth[a*x]^3)/6 - (19*ArcTanh[a*x])/(60*a^6) - (23*A
rcCoth[a*x]*Log[2/(1 - a*x)])/(15*a^6) - (23*PolyLog[2, 1 - 2/(1 - a*x)])/(
30*a^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
```

```
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5 \coth^{-1}(ax)^3\,dx &= \frac{1}{6}x^6\coth^{-1}(ax)^3 - \frac{1}{2}a\int \frac{x^6\coth^{-1}(ax)^2}{1-a^2x^2}\,dx \\
&= \frac{1}{6}x^6\coth^{-1}(ax)^3 + \frac{\int x^4\coth^{-1}(ax)^2\,dx}{2a} - \frac{\int \frac{x^4\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a} \\
&= \frac{x^5\coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\coth^{-1}(ax)^3 - \frac{1}{5}\int \frac{x^5\coth^{-1}(ax)}{1-a^2x^2}\,dx + \frac{\int x^2\coth^{-1}(ax)^2\,dx}{2a^3} - \\
&= \frac{x^3\coth^{-1}(ax)^2}{6a^3} + \frac{x^5\coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6\coth^{-1}(ax)^3 + \frac{\int \coth^{-1}(ax)^2\,dx}{2a^5} - \frac{\int \frac{\coth^{-1}(}{1-a^2}}{2a^5} \\
&= \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}(ax)^2}{6a^3} + \frac{x^5\coth^{-1}(ax)^2}{10a} - \frac{\coth^{-1}(ax)^3}{6a^6} + \\
&= \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} + \frac{x^3\coth^{-1}(ax)}{6a^3} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2\coth^{-1}(ax)}{15a^4} + \frac{x^4\coth^{-1}(ax)}{20a^2} + \frac{23\coth^{-1}(ax)^2}{30a^6} + \frac{x\coth^{-1}(ax)^2}{2a^5}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 117, normalized size = 0.63

$$\frac{ax(19+a^2x^2)+2(-23+15ax+5a^3x^3+3a^5x^5)\coth^{-1}(ax)^2+10(-1+a^6x^6)\coth^{-1}(ax)^3+\coth^{-1}(ax)\left(-19+16a^2x^2+3a^4x^4-92\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right)+46\text{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)}{60a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcCoth[a*x]^3,x]
```

```
[Out] (a*x*(19 + a^2*x^2) + 2*(-23 + 15*a*x + 5*a^3*x^3 + 3*a^5*x^5)*ArcCoth[a*x]
^2 + 10*(-1 + a^6*x^6)*ArcCoth[a*x]^3 + ArcCoth[a*x]*(-19 + 16*a^2*x^2 + 3*
a^4*x^4 - 92*Log[1 - E^(-2*ArcCoth[a*x])]) + 46*PolyLog[2, E^(-2*ArcCoth[a*
x])])/(60*a^6)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.72, size = 2135, normalized size = 11.48

method	result	size
derivativedivides	Expression too large to display	2135
default	Expression too large to display	2135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^3*x^3*arccoth(a*x)^2+1/10*a^5*x^5*arccoth(a*x)^2-23/15*arccoth
(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-1/6*arccoth(a*x)^3+23/30*arccoth(a*x)
^2-1/20*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2-2*a^2*x^2-((a*x-1)/(a*x+1))^(1/2
)+2*a*x)*arccoth(a*x)*(a*x+1)^2+1/80*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+2*(
(a*x-1)/(a*x+1))^(1/2)*a*x-2*a^2*x^2+1)*arccoth(a*x)*(a*x+1)^2-1/8*I*arccot
h(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))+1/8*I
*arccoth(a*x)^2*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/
(a*x-1)*(a*x+1)-1))^2-23/15*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+23/15*dilog(
1/((a*x-1)/(a*x+1))^(1/2))-47/240*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*
x+1))^(1/2)+a*x)*arccoth(a*x)*(a*x+1)-1/120*(a*x-1)*((a*x-1)/(a*x+1))^(1/2)
/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-2*a*x+1)-3/40*(2*((
a*x-1)/(a*x+1))^(1/2)*a^2*x^2+((a*x-1)/(a*x+1))^(1/2)*a*x+2*a^2*x^2-((a*x-1
)/(a*x+1))^(1/2)-a*x-1)*arccoth(a*x)*(a*x+1)+47/240*(((a*x-1)/(a*x+1))^(1/2
)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)*arccoth(a*x)*(a*x+1)+1/4*arccoth(a*x)^2*
ln(a*x-1)-1/4*arccoth(a*x)^2*ln(a*x+1)+1/80*(a*x-1)/(((a*x-1)/(a*x+1))^(1/2
)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x)-1/80*(a*x-1)/(((a*x-1)/(a*x+1))^(1/2)*a*
x+((a*x-1)/(a*x+1))^(1/2)-a*x)+1/8*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+
1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2+1/4*I*arccoth(a*x)^2*Pi
*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2+3/40*(2*((a*x-1)
/(a*x+1))^(1/2)*a^2*x^2+2*((a*x-1)/(a*x+1))^(1/2)*a*x-2*a^2*x^2+1)*arccoth(
a*x)*(a*x-1)*(a*x+1)+1/20*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+2*a^2*x^2-((a*
x-1)/(a*x+1))^(1/2)-2*a*x)*(a*x-1)*arccoth(a*x)*(a*x+1)-1/8*I*arccoth(a*x)^
2*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3-41/240*(((a*x-1)/(a*x+
1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x-1)*arccoth(a*x)-1/4*arccoth(a*x)^
2*ln((a*x-1)/(a*x+1))-41/120/(((a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)/(a*x+1))^
(1/2)-3/40*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+2*((a*x-1)/(a*x+1))^(1/2)*a*x
+2*a^2*x^2-1)*arccoth(a*x)*(a*x-1)*(a*x+1)-1/8*I*arccoth(a*x)^2*Pi*csgn(I/(
1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*
```

```
x-1)*(a*x+1)-1))-1/8*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1))^3-1/20*(2*
((a*x-1)/(a*x+1))^(1/2)*a^2*x^2-2*a^2*x^2-((a*x-1)/(a*x+1))^(1/2)+2*a*x)*(a
*x-1)*arccoth(a*x)*(a*x+1)+3/40*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+((a*x-1)
/(a*x+1))^(1/2)*a*x-2*a^2*x^2-((a*x-1)/(a*x+1))^(1/2)+a*x+1)*arccoth(a*x)*(
a*x+1)+1/2*arccoth(a*x)^2*a*x+1/6*a^6*x^6*arccoth(a*x)^3-1/40*(2*((a*x-1)/(
a*x+1))^(1/2)*a^2*x^2+3*((a*x-1)/(a*x+1))^(1/2)*a*x-2*a^2*x^2+((a*x-1)/(a*x
+1))^(1/2)-a*x+1)*arccoth(a*x)*(a*x+1)-47/120*(((a*x-1)/(a*x+1))^(1/2)*a*x-
a*x+1)*arccoth(a*x)*(a*x+1)+1/20*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+2*a^2*x
^2-((a*x-1)/(a*x+1))^(1/2)-2*a*x)*arccoth(a*x)*(a*x+1)^2-1/80*(2*((a*x-1)/(
a*x+1))^(1/2)*a^2*x^2+2*((a*x-1)/(a*x+1))^(1/2)*a*x+2*a^2*x^2-1)*arccoth(a*
x)*(a*x+1)^2-3/40*(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+3*((a*x-1)/(a*x+1))^(1
/2)*a*x-2*a^2*x^2+((a*x-1)/(a*x+1))^(1/2)-a*x+1)*arccoth(a*x)*(a*x-1)+3/40*
(2*((a*x-1)/(a*x+1))^(1/2)*a^2*x^2+3*((a*x-1)/(a*x+1))^(1/2)*a*x+2*a^2*x^2+
((a*x-1)/(a*x+1))^(1/2)+a*x-1)*arccoth(a*x)*(a*x-1)-41/120/(((a*x-1)/(a*x+1
))^(1/2)-1)*((a*x-1)/(a*x+1))^(1/2)+41/240*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a
*x-1)/(a*x+1))^(1/2)+a*x+1)*arccoth(a*x)+1/40*(2*((a*x-1)/(a*x+1))^(1/2)*a^
2*x^2+3*((a*x-1)/(a*x+1))^(1/2)*a*x+2*a^2*x^2+((a*x-1)/(a*x+1))^(1/2)+a*x-1
)*arccoth(a*x)*(a*x+1)-1/120*(a*x-1)*((a*x-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a
*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+2*a*x-1)+47/120*(((a*x-1)/(a*x+1))
^(1/2)*a*x+a*x-1)*arccoth(a*x)*(a*x+1))
```

Maxima [A]
time = 0.27, size = 289, normalized size = 1.55

$$\frac{1}{6}x^6\operatorname{arcoth}(ax)^3+\frac{1}{60}a\left(\frac{2\,(3\,a^4x^5+5\,a^2x^3+15\,x)}{a^6}-\frac{15\,\log(ax+1)}{a^7}+\frac{15\,\log(ax-1)}{a^7}\right)\operatorname{arcoth}(ax)^2+\frac{1}{240}a\left(\frac{\frac{4\,a^3x^3+(15\,\log(ax-1)-46)\log(ax+1)^2-5\,\log(ax+1)^3+5\,\log(ax-1)^3+76\,ax-\left(15\,\log(ax-1)^2-92\,\log(ax-1)\right)\log(ax+1)+46\,\log(ax-1)^2+38\,\log(ax-1)}{a}-\frac{184\,\left(\log(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a}-\frac{38\,\log(ax+1)}{a}}{a^6}+\frac{2\left(6\,a^4x^4+32\,a^2x^2-2\,(15\,\log(ax-1)-46)\log(ax+1)+15\,\log(ax+1)^2+15\,\log(ax-1)^2+92\,\log(ax-1)\right)\operatorname{arcoth}(ax)}{a^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arccoth(a*x)^3 + 1/60*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*
log(a*x + 1)/a^7 + 15*log(a*x - 1)/a^7)*arccoth(a*x)^2 + 1/240*a*(((4*a^3*x
^3 + (15*log(a*x - 1) - 46)*log(a*x + 1)^2 - 5*log(a*x + 1)^3 + 5*log(a*x -
 1)^3 + 76*a*x - (15*log(a*x - 1)^2 - 92*log(a*x - 1))*log(a*x + 1) + 46*lo
g(a*x - 1)^2 + 38*log(a*x - 1))/a - 184*(log(a*x - 1)*log(1/2*a*x + 1/2) +
dilog(-1/2*a*x + 1/2))/a - 38*log(a*x + 1)/a)/a^6 + 2*(6*a^4*x^4 + 32*a^2*x
^2 - 2*(15*log(a*x - 1) - 46)*log(a*x + 1) + 15*log(a*x + 1)^2 + 15*log(a*x
 - 1)^2 + 92*log(a*x - 1))*arccoth(a*x)/a^7)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^5*arccoth(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\operatorname{acoth}^{3}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*acoth(a*x)**3,x)
[Out] Integral(x**5*acoth(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="giac")
[Out] integrate(x^5*arccoth(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\mathrm{acoth}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*acoth(a*x)^3,x)
[Out] int(x^5*acoth(a*x)^3, x)
```

3.24 $\int x^4 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=196

$$\frac{x^2}{20a^3}+\frac{9x\coth^{-1}(ax)}{10a^4}+\frac{x^3\coth^{-1}(ax)}{10a^2}-\frac{9\coth^{-1}(ax)^2}{20a^5}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{3x^4\coth^{-1}(ax)^2}{20a}+\frac{\coth^{-1}(ax)^3}{5a^5}+\frac{1}{5}$$

```
[Out] 1/20*x^2/a^3+9/10*x*arccoth(a*x)/a^4+1/10*x^3*arccoth(a*x)/a^2-9/20*arccoth
(a*x)^2/a^5+3/10*x^2*arccoth(a*x)^2/a^3+3/20*x^4*arccoth(a*x)^2/a+1/5*arcco
th(a*x)^3/a^5+1/5*x^5*arccoth(a*x)^3-3/5*arccoth(a*x)^2*ln(2/(-a*x+1))/a^5+
1/2*ln(-a^2*x^2+1)/a^5-3/5*arccoth(a*x)*polylog(2,1-2/(-a*x+1))/a^5+3/10*po
lylog(3,1-2/(-a*x+1))/a^5
```

Rubi [A]
time = 0.39, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {6038, 6128, 272, 45, 6022, 266, 6096, 6132, 6056, 6206, 6745}

$$\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{10a^5}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{5a^5}+\frac{\coth^{-1}(ax)^3}{5a^5}-\frac{9\coth^{-1}(ax)^2}{20a^5}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{5a^5}+\frac{9x\coth^{-1}(ax)}{10a^4}+\frac{x^2}{20a^3}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{x^3\coth^{-1}(ax)}{10a^2}+\frac{\log\left(1-a^2x^2\right)}{2a^5}+\frac{1}{5}x^5\coth^{-1}(ax)^3+\frac{3x^4\coth^{-1}(ax)^2}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[a*x]^3,x]
```

```
[Out] x^2/(20*a^3) + (9*x*ArcCoth[a*x])/(10*a^4) + (x^3*ArcCoth[a*x])/(10*a^2) -
(9*ArcCoth[a*x]^2)/(20*a^5) + (3*x^2*ArcCoth[a*x]^2)/(10*a^3) + (3*x^4*ArcC
oth[a*x]^2)/(20*a) + ArcCoth[a*x]^3/(5*a^5) + (x^5*ArcCoth[a*x]^3)/5 - (3*A
rcCoth[a*x]^2*Log[2/(1 - a*x)])/(5*a^5) + Log[1 - a^2*x^2]/(2*a^5) - (3*Arc
Coth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*PolyLog[3, 1 - 2/(1 - a
*x)])/(10*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4\coth^{-1}(ax)^3\,dx=\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{1}{5}(3a)\int\frac{x^5\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$

$$=\frac{1}{5}x^5\coth^{-1}(ax)^3+\frac{3\int x^3\coth^{-1}(ax)^2\,dx}{5a}-\frac{3\int\frac{x^3\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{5a}$$

$$=\frac{3x^4\coth^{-1}(ax)^2}{20a}+\frac{1}{5}x^5\coth^{-1}(ax)^3-\frac{3}{10}\int\frac{x^4\coth^{-1}(ax)}{1-a^2x^2}\,dx+\frac{3\int x\coth^{-1}(ax)^2\,dx}{5a^3}$$

$$=\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{3x^4\coth^{-1}(ax)^2}{20a}+\frac{\coth^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\coth^{-1}(ax)^3-$$

$$=\frac{x^3\coth^{-1}(ax)}{10a^2}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{3x^4\coth^{-1}(ax)^2}{20a}+\frac{\coth^{-1}(ax)^3}{5a^5}+\frac{1}{5}x^5\coth^{-1}($$

$$=\frac{9x\coth^{-1}(ax)}{10a^4}+\frac{x^3\coth^{-1}(ax)}{10a^2}-\frac{9\coth^{-1}(ax)^2}{20a^5}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{3x^4\coth^{-1}(a}{20a}$$

$$=\frac{9x\coth^{-1}(ax)}{10a^4}+\frac{x^3\coth^{-1}(ax)}{10a^2}-\frac{9\coth^{-1}(ax)^2}{20a^5}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+\frac{3x^4\coth^{-1}(a}{20a}$$

$$=\frac{x^2}{20a^3}+\frac{9x\coth^{-1}(ax)}{10a^4}+\frac{x^3\coth^{-1}(ax)}{10a^2}-\frac{9\coth^{-1}(ax)^2}{20a^5}+\frac{3x^2\coth^{-1}(ax)^2}{10a^3}+$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.41, size = 175, normalized size = 0.89

$$\frac{-2-i\pi^3+2a^2x^2+36ax\coth^{-1}(ax)+4a^3x^3\coth^{-1}(ax)-18\coth^{-1}(ax)^2+12a^2x^2\coth^{-1}(ax)^2+6a^4x^4\coth^{-1}(ax)^2+8\coth^{-1}(ax)^3+8a^5x^5\coth^{-1}(ax)^3-24\coth^{-1}(ax)^2\log\left(1-e^{2\coth^{-1}(ax)}\right)-40\log\left(\frac{1}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\right)-24\coth^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)+12\mathrm{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)}{40a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[a*x]^3,x]
```

```
[Out] (-2 - I*Pi^3 + 2*a^2*x^2 + 36*a*x*ArcCoth[a*x] + 4*a^3*x^3*ArcCoth[a*x] - 1
8*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[a*x]^2 + 6*a^4*x^4*ArcCoth[a*x]^2 + 8
*ArcCoth[a*x]^3 + 8*a^5*x^5*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*Log[1 - E^(2
*ArcCoth[a*x])] - 40*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] - 24*ArcCoth[a*x]*P
olyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2*ArcCoth[a*x])])/(40*a^5)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.97, size = 737, normalized size = 3.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(9/10*a*x*arccoth(a*x)+3/20*arccoth(a*x)^2*a^4*x^4+3/10*a^2*x^2*arcco
th(a*x)^2+1/10*a^3*x^3*arccoth(a*x)+3/20*I*arccoth(a*x)^2*csgn(I/(1/(a*x-1)
*(a*x+1)-1))*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1
)*(a*x+1))-3/5*arccoth(a*x)^2*ln(2)+1/20*a^2*x^2-1/20-ln(1+1/((a*x-1)/(a*x+
1))^(1/2))+1/5*arccoth(a*x)^3-9/20*arccoth(a*x)^2+arccoth(a*x)-3/20*I*arcco
th(a*x)^2*csgn(I/(1/(a*x-1)*(a*x+1)-1))*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1
)*(a*x+1)-1))^2-3/20*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*
(a*x+1)-1))^2*csgn(I/(a*x-1)*(a*x+1))-3/10*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-
1)*(a*x+1))^2*csgn(I/((a*x-1)/(a*x+1))^(1/2))+3/20*I*arccoth(a*x)^2*Pi*csgn
(I/(a*x-1)*(a*x+1))*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2+1/5*a^5*x^5*arccoth(a
*x)^3+3/5*arccoth(a*x)^2*ln(1/(a*x-1)*(a*x+1)-1)-3/5*arccoth(a*x)^2*ln(1-1/
((a*x-1)/(a*x+1))^(1/2))+3/20*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1))^3
+3/20*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3+3
/10*arccoth(a*x)^2*ln(a*x-1)+3/10*arccoth(a*x)^2*ln(a*x+1)+6/5*polylog(3,1/
((a*x-1)/(a*x+1))^(1/2))-ln(1/((a*x-1)/(a*x+1))^(1/2)-1)-6/5*arccoth(a*x)*p
olylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/5*arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x
+1))^(1/2))-6/5*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6/5*poly
log(3,-1/((a*x-1)/(a*x+1))^(1/2))+3/10*arccoth(a*x)^2*ln((a*x-1)/(a*x+1)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(2*(a^5*x^5 + 1)*log(a*x + 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 2*(a^5*x^5
- 1)*log(a*x - 1))*log(a*x + 1)^2)/a^5 + 1/8*integrate(-1/5*(5*(a^5*x^5 + a
^4*x^4)*log(a*x - 1)^3 + 3*(a^4*x^4 + 2*a^2*x^2 - 5*(a^5*x^5 + a^4*x^4)*log
(a*x - 1)^2 - 2*(a^5*x^5 - 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x + a^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="fricas")
[Out] integral(x^4*arccoth(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**3,x)
[Out] Integral(x**4*acoth(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(a*x)^3,x, algorithm="giac")
[Out] integrate(x^4*arccoth(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acoth}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(a*x)^3,x)
[Out] int(x^4*acoth(a*x)^3, x)
```

3.25 $\int x^3 \coth^{-1}(ax)^3\,dx$

Optimal. Leaf size=139

$$\frac{x}{4a^3}+\frac{x^2\coth^{-1}(ax)}{4a^2}+\frac{\coth^{-1}(ax)^2}{a^4}+\frac{3x\coth^{-1}(ax)^2}{4a^3}+\frac{x^3\coth^{-1}(ax)^2}{4a}-\frac{\coth^{-1}(ax)^3}{4a^4}+\frac{1}{4}x^4\coth^{-1}(ax)^3-\text{tan}$$

```
[Out] 1/4*x/a^3+1/4*x^2*arccoth(a*x)/a^2+arccoth(a*x)^2/a^4+3/4*x*arccoth(a*x)^2/
a^3+1/4*x^3*arccoth(a*x)^2/a-1/4*arccoth(a*x)^3/a^4+1/4*x^4*arccoth(a*x)^3-
1/4*arctanh(a*x)/a^4-2*arccoth(a*x)*ln(2/(-a*x+1))/a^4-polylog(2,1-2/(-a*x+
1))/a^4
```

Rubi [A]
time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6038, 6128, 327, 212, 6132, 6056, 2449, 2352, 6022, 6096}

$$-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\coth^{-1}(ax)^3}{4a^4}+\frac{\coth^{-1}(ax)^2}{a^4}-\frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^4}+\frac{x}{4a^3}+\frac{3x\coth^{-1}(ax)^2}{4a^3}+\frac{x^2\coth^{-1}(ax)}{4a^2}+\frac{1}{4}x^4\coth^{-1}(ax)^3+\frac{x^3\coth^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a*x]^3,x]
```

```
[Out] x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[
a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^
4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x
)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3 \coth^{-1}(ax)^3\,dx = \frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$
$$= \frac{1}{4}x^4\coth^{-1}(ax)^3 + \frac{3\int x^2\coth^{-1}(ax)^2\,dx}{4a} - \frac{3\int \frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{4a}$$
$$= \frac{x^3\coth^{-1}(ax)^2}{4a} + \frac{1}{4}x^4\coth^{-1}(ax)^3 - \frac{1}{2}\int \frac{x^3\coth^{-1}(ax)}{1-a^2x^2}\,dx + \frac{3\int \coth^{-1}(ax)^2\,dx}{4a^3} -$$
$$= \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4\coth^{-1}(ax)^3 + \frac{\int x\coth^{-1}(}{2a^2}$$
$$= \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} +$$
$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(}{4a^4}$$
$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(}{4a^4}$$
$$= \frac{x}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^3\coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(}{4a^4}$$

Mathematica [A]
time = 0.22, size = 88, normalized size = 0.63

$$\frac{ax + (-4+3ax+a^3x^3)\coth^{-1}(ax)^2 + (-1+a^4x^4)\coth^{-1}(ax)^3 + \coth^{-1}(ax)\left(-1+a^2x^2-8\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right) + 4\text{PolyLog}\left(2, e^{-2\coth^{-1}(ax)}\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a*x]^3,x]
```

```
[Out] (a*x + (-4 + 3*a*x + a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*x^4)*ArcCoth[a*x]^
3 + ArcCoth[a*x]*(-1 + a^2*x^2 - 8*Log[1 - E^(-2*ArcCoth[a*x])]) + 4*PolyLo
g[2, E^(-2*ArcCoth[a*x])])/(4*a^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.95, size = 871, normalized size = 6.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^3*x^3*arccoth(a*x)^2-3/16*I*arccoth(a*x)^2*Pi*csgn(I/(1/(a*x-1
)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*
x+1)-1))-2*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-1/4*arccoth(a*x)^3+
arccoth(a*x)^2-3/16*I*arccoth(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*c
sgn(I/(a*x-1)*(a*x+1))+3/16*I*arccoth(a*x)^2*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1
))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2+3/16*I*arccoth(a*x)^2*Pi
*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2+3/
8*I*arccoth(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1
))^2-2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+2*dilog(1/((a*x-1)/(a*x+1))^(1/2)
)-1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x)*arccoth(a*x
)*(a*x+1)+1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)*arc
coth(a*x)*(a*x+1)+3/8*arccoth(a*x)^2*ln(a*x-1)-3/8*arccoth(a*x)^2*ln(a*x+1)
+1/4*a^4*x^4*arccoth(a*x)^3-1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+
1))^(1/2)-a*x-1)*arccoth(a*x)-3/8*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))-1/4/((
(a*x-1)/(a*x+1))^(1/2)+1)*((a*x-1)/(a*x+1))^(1/2)+3/4*arccoth(a*x)^2*a*x-1/
4*(((a*x-1)/(a*x+1))^(1/2)*a*x-a*x+1)*arccoth(a*x)*(a*x+1)-3/16*I*arccoth(a
*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1))^3-3/16*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*
(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3-1/4/(((a*x-1)/(a*x+1))^(1/2)-1)*((a*x-1)/(
a*x+1))^(1/2)+1/8*(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x+
1)*arccoth(a*x)+1/4*(((a*x-1)/(a*x+1))^(1/2)*a*x+a*x-1)*arccoth(a*x)*(a*x+1
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(122) = 244$.
time = 0.27, size = 262, normalized size = 1.88

$$\frac{1}{4}\,x^4\operatorname{arcoth}\left(ax\right)^3+\frac{1}{8}\,a\left(\frac{2\left(a^2x^3+3\,x\right)}{a^4}-\frac{3\,\log\left(ax+1\right)}{a^5}+\frac{3\,\log\left(ax-1\right)}{a^5}\right)\operatorname{arcoth}\left(ax\right)^2+\frac{1}{32}\,a\left(\frac{\frac{(3\,\log(ax-1)-8)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3+8\,ax-\left(3\,\log(ax-1)^2-16\,\log(ax-1)\right)\log(ax+1)+8\,\log(ax-1)^2+4\,\log(ax-1)}{a}-\frac{32\left(\log(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a}-\frac{4\,\log(ax+1)}{a}}{a^4}+\frac{2\left(4\,a^2x^2-2\left(3\,\log\left(ax-1\right)-8\right)\log\left(ax+1\right)+3\,\log\left(ax+1\right)^2+3\,\log\left(ax-1\right)^2+16\,\log\left(ax-1\right)\right)\operatorname{arcoth}\left(ax\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(a*x)^3 + 1/8*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5
+ 3*log(a*x - 1)/a^5)*arccoth(a*x)^2 + 1/32*a*((((3*log(a*x - 1) - 8)*log(a
*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 + 8*a*x - (3*log(a*x - 1)^2 - 1
6*log(a*x - 1))*log(a*x + 1) + 8*log(a*x - 1)^2 + 4*log(a*x - 1))/a - 32*(l
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*log(a*x + 1)/
a)/a^4 + 2*(4*a^2*x^2 - 2*(3*log(a*x - 1) - 8)*log(a*x + 1) + 3*log(a*x + 1
)^2 + 3*log(a*x - 1)^2 + 16*log(a*x - 1))*arccoth(a*x)/a^5)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccoth(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \operatorname{acoth}^{3}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(a*x)**3,x)
[Out] Integral(x**3*acoth(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(a*x)^3,x, algorithm="giac")
[Out] integrate(x^3*arccoth(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{acoth}(a\,x)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(a*x)^3,x)
[Out] int(x^3*acoth(a*x)^3, x)
```

3.26 $\int x^2 \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=149

$$\frac{x\coth^{-1}(ax)}{a^2}-\frac{\coth^{-1}(ax)^2}{2a^3}+\frac{x^2\coth^{-1}(ax)^2}{2a}+\frac{\coth^{-1}(ax)^3}{3a^3}+\frac{1}{3}x^3\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\log($$

```
[Out] x*arccoth(a*x)/a^2-1/2*arccoth(a*x)^2/a^3+1/2*x^2*arccoth(a*x)^2/a+1/3*arcc
oth(a*x)^3/a^3+1/3*x^3*arccoth(a*x)^3-arccoth(a*x)^2*ln(2/(-a*x+1))/a^3+1/2
*ln(-a^2*x^2+1)/a^3-arccoth(a*x)*polylog(2,1-2/(-a*x+1))/a^3+1/2*polylog(3,
1-2/(-a*x+1))/a^3
```

Rubi [A]
time = 0.23, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6038, 6128, 6022, 266, 6096, 6132, 6056, 6206, 6745}

$$\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^3}-\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^3}+\frac{\coth^{-1}(ax)^3}{3a^3}-\frac{\coth^{-1}(ax)^2}{2a^3}-\frac{\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a^3}+\frac{x\coth^{-1}(ax)}{a^2}+\frac{\log\left(1-a^2x^2\right)}{2a^3}+\frac{1}{3}x^3\coth^{-1}(ax)^3+\frac{x^2\coth^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a*x]^3,x]
```

```
[Out] (x*ArcCoth[a*x])/a^2 - ArcCoth[a*x]^2/(2*a^3) + (x^2*ArcCoth[a*x]^2)/(2*a)
+ ArcCoth[a*x]^3/(3*a^3) + (x^3*ArcCoth[a*x]^3)/3 - (ArcCoth[a*x]^2*Log[2/(
1 - a*x)])/a^3 + Log[1 - a^2*x^2]/(2*a^3) - (ArcCoth[a*x]*PolyLog[2, 1 - 2/
(1 - a*x)])/a^3 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \coth^{-1}(ax)^3 \, dx &= \frac{1}{3}x^3 \coth^{-1}(ax)^3 - a \int \frac{x^3 \coth^{-1}(ax)^2}{1-a^2x^2} \, dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax)^3 + \frac{\int x \coth^{-1}(ax)^2 \, dx}{a} - \frac{\int \frac{x \coth^{-1}(ax)^2}{1-a^2x^2} \, dx}{a} \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\int \frac{\coth^{-1}(ax)^2}{1-ax} \, dx}{a^2} - \int \frac{x^2 \coth^{-1}(\ldots}{1-a^2x^2 \ldots} \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\int \coth \ldots}{\ldots} \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \ldots \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \ldots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.26, size = 140, normalized size = 0.94

$$
\frac{-i\pi^3 + 24ax \coth^{-1}(ax) - 12\coth^{-1}(ax)^2 + 12a^2x^2\coth^{-1}(ax)^2 + 8\coth^{-1}(ax)^3 + 8a^3x^3\coth^{-1}(ax)^3 - 24\coth^{-1}(ax)^2 \log\left(1 - e^{2\coth^{-1}(ax)}\right) - 24\log\left(\frac{1}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\right) - 24\coth^{-1}(ax)\text{PolyLog}\left(2, e^{2\coth^{-1}(ax)}\right) + 12\text{PolyLog}\left(3, e^{2\coth^{-1}(ax)}\right)}{24a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a*x]^3,x]
```

```
[Out] ((-I)*Pi^3 + 24*a*x*ArcCoth[a*x] - 12*ArcCoth[a*x]^2 + 12*a^2*x^2*ArcCoth[a
*x]^2 + 8*ArcCoth[a*x]^3 + 8*a^3*x^3*ArcCoth[a*x]^3 - 24*ArcCoth[a*x]^2*Log
[1 - E^(2*ArcCoth[a*x])] - 24*Log[1/(a*Sqrt[1 - 1/(a^2*x^2)]*x)] - 24*ArcCo
th[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])] + 12*PolyLog[3, E^(2*ArcCoth[a*x])])
/(24*a^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.45, size = 683, normalized size = 4.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccoth(a*x)^3+1/2*a^2*x^2*arccoth(a*x)^2+1/2*arccoth(a*
x)^2*ln(a*x-1)+1/2*arccoth(a*x)^2*ln(a*x+1)+1/2*arccoth(a*x)^2*ln((a*x-1)/(
a*x+1))+arccoth(a*x)^2*ln(1/(a*x-1)*(a*x+1)-1)+1/12*arccoth(a*x)*(-3*I*arcc
oth(a*x)*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a
*x+1)-1))^2*Pi+3*I*arccoth(a*x)*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1
)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*Pi+3*I*arccoth(a*x
```

```
)*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*Pi-3*I*arccoth(a*x)*csgn(
I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*csgn(I/(a*x-1)*(a*x+1))*Pi+3*I*a
rccoth(a*x)*csgn(I/(a*x-1)*(a*x+1))^3*Pi-6*I*arccoth(a*x)*csgn(I/(a*x-1)*(a
*x+1))^2*csgn(I/((a*x-1)/(a*x+1))^(1/2))*Pi+3*I*arccoth(a*x)*csgn(I/(a*x-1)
*(a*x+1))*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*Pi+4*arccoth(a*x)^2-12*arccoth(
a*x)*ln(2)-6*arccoth(a*x)+12*a*x+12)-ln(1/((a*x-1)/(a*x+1))^(1/2)-1)-ln(1+1
/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x)^2*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-2*
arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,1/((a*x-1)/(a
*x+1))^(1/2))-arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-2*arccoth(a*x)
*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+2*polylog(3,-1/((a*x-1)/(a*x+1))^(1/
2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*((a^3*x^3 + 1)*log(a*x + 1)^3 + 3*(a^2*x^2 - (a^3*x^3 - 1)*log(a*x - 1
))*log(a*x + 1)^2)/a^3 + 1/8*integrate(-((a^3*x^3 + a^2*x^2)*log(a*x - 1)^3
 + (2*a^2*x^2 - 3*(a^3*x^3 + a^2*x^2)*log(a*x - 1)^2 - 2*(a^3*x^3 - 1)*log(
a*x - 1))*log(a*x + 1))/(a^3*x + a^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(a*x)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}^3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a*x)**3,x)
```

```
[Out] Integral(x**2*acoth(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a*x)^3,x)
```

```
[Out] int(x^2*acoth(a*x)^3, x)
```

3.27 $\int x \coth^{-1}(ax)^3 \, dx$

Optimal. Leaf size=95

$$\frac{3\coth^{-1}(ax)^2}{2a^2}+\frac{3x\coth^{-1}(ax)^2}{2a}-\frac{\coth^{-1}(ax)^3}{2a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{3\text{PolyLog}(2,1-}{2a^2}$$

```
[Out] 3/2*arccoth(a*x)^2/a^2+3/2*x*arccoth(a*x)^2/a-1/2*arccoth(a*x)^3/a^2+1/2*x^
2*arccoth(a*x)^3-3*arccoth(a*x)*ln(2/(-a*x+1))/a^2-3/2*polylog(2,1-2/(-a*x+
1))/a^2
```

Rubi [A]
time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6038, 6128, 6022, 6132, 6056, 2449, 2352, 6096}

$$-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\coth^{-1}(ax)^3}{2a^2}+\frac{3\coth^{-1}(ax)^2}{2a^2}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2}+\frac{1}{2}x^2\coth^{-1}(ax)^3+\frac{3x\coth^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a*x]^3,x]
```

```
[Out] (3*ArcCoth[a*x]^2)/(2*a^2) + (3*x*ArcCoth[a*x]^2)/(2*a) - ArcCoth[a*x]^3/(2
*a^2) + (x^2*ArcCoth[a*x]^3)/2 - (3*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^2 - (3
*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
```

```
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\coth^{-1}(ax)^3\,dx = \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{1}{2}(3a)\int \frac{x^2\coth^{-1}(ax)^2}{1-a^2x^2}\,dx$$
$$= \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3\int \coth^{-1}(ax)^2\,dx}{2a} - \frac{3\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}$$
$$= \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - 3\int \frac{x\coth^{-1}(ax)}{1-a^2x^2}\,dx$$
$$= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\int \frac{\coth^{-1}(ax)}{1-ax}\,d}{a}$$
$$= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\,\mathrm{l}}{a^2}$$
$$= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\,\mathrm{l}}{a^2}$$
$$= \frac{3\coth^{-1}(ax)^2}{2a^2} + \frac{3x\coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 - \frac{3\coth^{-1}(ax)\,\mathrm{l}}{a^2}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.72

$$\frac{\coth^{-1}(ax)\left(3(-1+ax)\coth^{-1}(ax)+(-1+a^2x^2)\coth^{-1}(ax)^2-6\log\left(1-e^{-2\coth^{-1}(ax)}\right)\right)+3\mathrm{PolyLog}\left(2,e^{-2\coth^{-1}(ax)}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a*x]^3,x]
```

```
[Out] (ArcCoth[a*x]*(3*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]^2 -
6*Log[1 - E^(-2*ArcCoth[a*x])]) + 3*PolyLog[2, E^(-2*ArcCoth[a*x])])/(2*a^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.87, size = 2916, normalized size = 30.69

method	result	size
derivativedivides	Expression too large to display	2916
default	Expression too large to display	2916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*p
olylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3*arc
```

```
coth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1
)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1+1/((a*x-1)/(a
*x+1))^(1/2))-3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)
/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/
(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arcc
oth(a*x)^2-3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1
/(a*x-1)*(a*x+1)-1))^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I
/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x
+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x
-1)*(a*x+1)-1))^2*arccoth(a*x)^2-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a
*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))
-3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(
a*x+1)-1))^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/((a*x-1)
/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(
1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*
x-1)*(a*x+1)-1))*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-
1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arccoth(a*x)*ln
(1-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I
/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*polylog(2,-
1/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/
(a*x-1)*(a*x+1))^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*cs
gn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/
(1/(a*x-1)*(a*x+1)-1))*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(
1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*
x-1)*(a*x+1)-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(1/(a*x-1
)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arccoth(a*x)*
ln(1-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*
csgn(I/(a*x-1)*(a*x+1))*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/2*po
lylog(2,1/((a*x-1)/(a*x+1))^(1/2))-3/2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1)
)^(1/2))-3*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-1/2*arccoth(a*x)^3+
3/2*arccoth(a*x)^2-3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a
*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*arccoth(a*x)^2+1/2*a^2
*x^2*arccoth(a*x)^3+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(
a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*arccoth(a*x)*ln(1-1/(
(a*x-1)/(a*x+1))^(1/2))-3/2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/2*dilog(1/
((a*x-1)/(a*x+1))^(1/2))+3/4*arccoth(a*x)^2*ln(a*x-1)-3/4*arccoth(a*x)^2*ln
(a*x+1)-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(
a*x-1)*(a*x+1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x
-1)*(a*x+1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1
)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(a*x-1)*(
a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*
Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*polylog(2,-1/((a*x-1)/(a
*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*dilo
g(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(
a*x+1)-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+
```

```
1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3
*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/4*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))+3/
2*arccoth(a*x)^2*a*x-3/2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csg
n(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2*polylog(2,1/((a*x-1)
/(a*x+1))^(1/2))-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a
*x+1))^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/((a*x-1)/(a*
x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))
-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)
-1))^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/((a*x-1)/(a*x+
1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8
*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*dilog(1/((a
*x-1)/(a*x+1))^(1/2))+3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-
1)*(a*x+1))^2*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1
)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/8*I
*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2
*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*
csgn(I/(a*x-1)*(a*x+1))^2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csgn(
I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2*dilog(1/((a*x-1)/(a*x+
1))^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(82) = 164$.
time = 0.26, size = 215, normalized size = 2.26

$$\frac{1}{2}x^2\operatorname{arcoth}(ax)^3+\frac{3}{4}a\left(\frac{2x}{a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{arcoth}(ax)^2+\frac{1}{16}a\left(\frac{\frac{3(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\left(\log(ax-1)^2-4\log(ax-1)\right)\log(ax+1)+6\log(ax-1)^2}{a}-\frac{24\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}}{a^2}-\frac{6\left(2\left(\log(ax-1)-2\right)\log(ax+1)-\log(ax+1)^2-\log(ax-1)^2-4\log(ax-1)\right)\operatorname{arcoth}(ax)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(a*x)^3 + 3/4*a*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a
^3)*arccoth(a*x)^2 + 1/16*a*(((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*
x + 1)^3 + log(a*x - 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1
) + 6*log(a*x - 1)^2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*
a*x + 1/2))/a)/a^2 - 6*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2
- log(a*x - 1)^2 - 4*log(a*x - 1))*arccoth(a*x)/a^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(a*x)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x)**3,x)
```

```
[Out] Integral(x*acoth(a*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(a*x)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a*x)^3,x)
```

```
[Out] int(x*acoth(a*x)^3, x)
```

3.28 $\int \coth^{-1}(ax)^3\,dx$

Optimal. Leaf size=85

$$\frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-\frac{3\coth^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}+\frac{3\text{PolyLog}(3,1}{2a}$$

```
[Out] arccoth(a*x)^3/a+x*arccoth(a*x)^3-3*arccoth(a*x)^2*ln(2/(-a*x+1))/a-3*arcco
th(a*x)*polylog(2,1-2/(-a*x+1))/a+3/2*polylog(3,1-2/(-a*x+1))/a
```

Rubi [A]
time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6022, 6132, 6056, 6096, 6206, 6745}

$$\frac{3\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a}-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a}+x\coth^{-1}(ax)^3+\frac{\coth^{-1}(ax)^3}{a}-\frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3,x]
```

```
[Out] ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[2/(1 - a*x)])/a
 - (3*ArcCoth[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + (3*PolyLog[3, 1 - 2/(1
- a*x)])/(2*a)
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(ax)^3\,dx &= x\coth^{-1}(ax)^3-(3a)\int \frac{x\coth^{-1}(ax)^2}{1-a^2x^2}\,dx\\ &= \frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-3\int \frac{\coth^{-1}(ax)^2}{1-ax}\,dx\\ &= \frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}+6\int \frac{\coth^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,d\\ &= \frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-\frac{3\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a}\\ &= \frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a}-\frac{3\coth^{-1}(ax)\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 79, normalized size = 0.93

$$\frac{\coth^{-1}(ax)^3}{a}+x\coth^{-1}(ax)^3-\frac{3\coth^{-1}(ax)^2\log\left(1-e^{2\coth^{-1}(ax)}\right)}{a}-\frac{3\coth^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)}{a}+\frac{3\mathrm{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3,x]
```

```
[Out] ArcCoth[a*x]^3/a + x*ArcCoth[a*x]^3 - (3*ArcCoth[a*x]^2*Log[1 - E^(2*ArcCot
h[a*x])])/a - (3*ArcCoth[a*x]*PolyLog[2, E^(2*ArcCoth[a*x])])/a + (3*PolyLo
g[3, E^(2*ArcCoth[a*x])])/(2*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(83) = 166$.
time = 0.33, size = 168, normalized size = 1.98

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)^3(ax-1)+2\operatorname{arccoth}(ax)^3-3\operatorname{arccoth}(ax)^2\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-6\operatorname{arccoth}(ax)\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)+6\,\mathrm{p}}{}$
default	$\frac{\operatorname{arccoth}(ax)^3(ax-1)+2\operatorname{arccoth}(ax)^3-3\operatorname{arccoth}(ax)^2\ln\left(1-\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)-6\operatorname{arccoth}(ax)\operatorname{polylog}\left(2,\frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)+6\,\mathrm{p}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)^3*(a*x-1)+2*arccoth(a*x)^3-3*arccoth(a*x)^2*ln(1-1/((a*x-
1)/(a*x+1))^(1/2))-6*arccoth(a*x)*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+6*po
lylog(3,1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*ln(1+1/((a*x-1)/(a*x+1)
)^(1/2))-6*arccoth(a*x)*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+6*polylog(3,-
1/((a*x-1)/(a*x+1))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((a*x + 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(a*x - 1))/a
+ 1/8*integrate(-((a*x + 1)*log(a*x - 1)^3 - 3*((a*x + 1)*log(a*x - 1)^2 +
2*(a*x - 1)*log(a*x - 1))*log(a*x + 1))/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3,x)
```

```
[Out] Integral(acoth(a*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(a\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3,x)
```

```
[Out] int(acoth(a*x)^3, x)
```

3.29 $\int \frac{\coth^{-1}(ax)^3}{x} \, dx$

Optimal. Leaf size=150

$$2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\Bigg(2,1$$

```
[Out] 2*arccoth(a*x)^3*arccoth(1-2/(-a*x+1))+3/2*arccoth(a*x)^2*polylog(2,1-2/(a*
x+1))-3/2*arccoth(a*x)^2*polylog(2,1-2*a*x/(a*x+1))+3/2*arccoth(a*x)*polylo
g(3,1-2/(a*x+1))-3/2*arccoth(a*x)*polylog(3,1-2*a*x/(a*x+1))+3/4*polylog(4,
1-2/(a*x+1))-3/4*polylog(4,1-2*a*x/(a*x+1))
```

Rubi [A]
time = 0.24, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6034, 6200, 6096, 6204, 6208, 6745}

$$\frac{3}{4}\text{Li}_4\left(1-\frac{2}{ax+1}\right)-\frac{3}{4}\text{Li}_4\left(1-\frac{2ax}{ax+1}\right)+\frac{3}{2}\text{Li}_2\left(1-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2-\frac{3}{2}\text{Li}_2\left(1-\frac{2ax}{ax+1}\right)\coth^{-1}(ax)^2+\frac{3}{2}\text{Li}_3\left(1-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{3}{2}\text{Li}_3\left(1-\frac{2ax}{ax+1}\right)\coth^{-1}(ax)+2\coth^{-1}\left(1-\frac{2}{1-ax}\right)\coth^{-1}(ax)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x,x]
```

```
[Out] 2*ArcCoth[a*x]^3*ArcCoth[1 - 2/(1 - a*x)] + (3*ArcCoth[a*x]^2*PolyLog[2, 1
- 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]^2*PolyLog[2, 1 - (2*a*x)/(1 + a*x)])/2
+ (3*ArcCoth[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/2 - (3*ArcCoth[a*x]*PolyLog[
3, 1 - (2*a*x)/(1 + a*x)])/2 + (3*PolyLog[4, 1 - 2/(1 + a*x)])/4 - (3*PolyL
og[4, 1 - (2*a*x)/(1 + a*x)])/4
```

Rule 6034

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6200

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegra
```

```
nd[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6208

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x}\,dx &= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)-(6a)\int\frac{\coth^{-1}(ax)^2\coth^{-1}\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx \\
&= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+(3a)\int\frac{\coth^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx-(3a)\int\frac{\coth
\\
&= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)
\\
&= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)
\\
&= 2\coth^{-1}(ax)^3\coth^{-1}\left(1-\frac{2}{1-ax}\right)+\frac{3}{2}\coth^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)-\frac{3}{2}\coth^{-1}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 156, normalized size = 1.04

$$\frac{1}{64}\left(-\pi^4+32\coth^{-1}(ax)^4+64\coth^{-1}(ax)^3\log\left(1+e^{-2\coth^{-1}(ax)}\right)-64\coth^{-1}(ax)^3\log\left(1-e^{2\coth^{-1}(ax)}\right)-96\coth^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-96\coth^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\coth^{-1}(ax)}\right)-96\coth^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)+96\coth^{-1}(ax)\text{PolyLog}\left(3,e^{2\coth^{-1}(ax)}\right)-48\text{PolyLog}\left(4,-e^{-2\coth^{-1}(ax)}\right)-48\text{PolyLog}\left(4,e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3/x,x]
```

```
[Out] (-Pi^4 + 32*ArcCoth[a*x]^4 + 64*ArcCoth[a*x]^3*Log[1 + E^(-2*ArcCoth[a*x])]
 - 64*ArcCoth[a*x]^3*Log[1 - E^(2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*PolyLo
g[2, -E^(-2*ArcCoth[a*x])] - 96*ArcCoth[a*x]^2*PolyLog[2, E^(2*ArcCoth[a*x]
)] - 96*ArcCoth[a*x]*PolyLog[3, -E^(-2*ArcCoth[a*x])] + 96*ArcCoth[a*x]*Pol
yLog[3, E^(2*ArcCoth[a*x])] - 48*PolyLog[4, -E^(-2*ArcCoth[a*x])] - 48*Poly
Log[4, E^(2*ArcCoth[a*x])])/64
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.21, size = 564, normalized size = 3.76

method	result
derivativedivides	$\ln(ax)\operatorname{arccoth}(ax)^3+\operatorname{arccoth}(ax)^3\ln\left(\frac{ax+1}{ax-1}-1\right)+\frac{3\operatorname{arccoth}(ax)^2\operatorname{polylog}\left(2,-\frac{ax+1}{ax-1}\right)}{2}-\frac{3\operatorname{arccot}}{}$
default	$\ln(ax)\operatorname{arccoth}(ax)^3+\operatorname{arccoth}(ax)^3\ln\left(\frac{ax+1}{ax-1}-1\right)+\frac{3\operatorname{arccoth}(ax)^2\operatorname{polylog}\left(2,-\frac{ax+1}{ax-1}\right)}{2}-\frac{3\operatorname{arccot}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a*x)*arccoth(a*x)^3+arccoth(a*x)^3*ln(1/(a*x-1)*(a*x+1)-1)+3/2*arccoth(a
*x)^2*polylog(2,-1/(a*x-1)*(a*x+1))-3/2*arccoth(a*x)*polylog(3,-1/(a*x-1)*(
a*x+1))+3/4*polylog(4,-1/(a*x-1)*(a*x+1))+1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1
)-1))*csgn(I*(1+1/(a*x-1)*(a*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-
1)*(a*x+1)))*arccoth(a*x)^3-1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(
1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^2*arccoth(a*x)^3-1/2*I*Pi*csgn(
I*(1+1/(a*x-1)*(a*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1))
)^2*arccoth(a*x)^3+1/2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+
1)))^3*arccoth(a*x)^3-arccoth(a*x)^3*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3*arcc
oth(a*x)^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3,1/
((a*x-1)/(a*x+1))^(1/2))-6*polylog(4,1/((a*x-1)/(a*x+1))^(1/2))-arccoth(a*x
)^3*ln(1+1/((a*x-1)/(a*x+1))^(1/2))-3*arccoth(a*x)^2*polylog(2,-1/((a*x-1)/
(a*x+1))^(1/2))+6*arccoth(a*x)*polylog(3,-1/((a*x-1)/(a*x+1))^(1/2))-6*poly
log(4,-1/((a*x-1)/(a*x+1))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)^3/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^{3}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x,x)
```

```
[Out] Integral(acoth(a*x)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3/x,x)
```

```
[Out] int(acoth(a*x)^3/x, x)
```

3.30 $\int \frac{\coth^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=79

$$a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\coth^{-1}(ax)\text{PolyLog}\left(2,-1+\frac{2}{1+ax}\right)$$

```
[Out] a*arccoth(a*x)^3-arccoth(a*x)^3/x+3*a*arccoth(a*x)^2*ln(2-2/(a*x+1))-3*a*ar
ccoth(a*x)*polylog(2,-1+2/(a*x+1))-3/2*a*polylog(3,-1+2/(a*x+1))
```

Rubi [A]
time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6038, 6136, 6080, 6096, 6204, 6745}

$$-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\coth^{-1}(ax)+a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^2,x]
```

```
[Out] a*ArcCoth[a*x]^3 - ArcCoth[a*x]^3/x + 3*a*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x
)] - 3*a*ArcCoth[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2
/(1 + a*x)])/2
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^3}{x^2}\,dx &= -\frac{\coth^{-1}(ax)^3}{x}+(3a)\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\\ &= a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+(3a)\int \frac{\coth^{-1}(ax)^2}{x(1+ax)}\,dx\\ &= a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-\left(6a^2\right)\int \frac{\coth^{-1}(a}{1}\\ &= a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\coth^{-1}(ax)\mathrm{Li}_2\Big(\\ &= a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)-3a\coth^{-1}(ax)\mathrm{Li}_2\Big(\end{aligned}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.91

$$\frac{(-1+ax)\coth^{-1}(ax)^3}{x}+3a\coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)-3a\coth^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)-\frac{3}{2}a\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3/x^2,x]
```

[Out] ((-1 + a*x)*ArcCoth[a*x]^3)/x + 3*a*ArcCoth[a*x]^2*Log[1 + E^(-2*ArcCoth[a*x])] - 3*a*ArcCoth[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - (3*a*PolyLog[3, -E^(-2*ArcCoth[a*x])])/2

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.21, size = 719, normalized size = 9.10

method	result
derivativedivides	$a\left(-\frac{\operatorname{arccoth}(ax)^3}{ax}-\frac{3\operatorname{arccoth}(ax)^2\ln(ax-1)}{2}+3\operatorname{arccoth}(ax)^2\ln(ax)-\frac{3\operatorname{arccoth}(ax)^2\ln(ax+1)}{2}-\frac{3\mathrm{ar}}{}\right.$
default	$a\left(-\frac{\operatorname{arccoth}(ax)^3}{ax}-\frac{3\operatorname{arccoth}(ax)^2\ln(ax-1)}{2}+3\operatorname{arccoth}(ax)^2\ln(ax)-\frac{3\operatorname{arccoth}(ax)^2\ln(ax+1)}{2}-\frac{3\mathrm{ar}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a*x)^3/x^2,x,method=_RETURNVERBOSE)

[Out] a*(-1/a/x*arccoth(a*x)^3-3/2*arccoth(a*x)^2*ln(a*x-1)+3*arccoth(a*x)^2*ln(a*x)-3/2*arccoth(a*x)^2*ln(a*x+1)-3/2*arccoth(a*x)^2*ln((a*x-1)/(a*x+1))-arccoth(a*x)^3+3/4*(2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^3-2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^2-2*I*Pi*csgn(I*(1+1/(a*x-1)*(a*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^2+2*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I*(1+1/(a*x-1)*(a*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))-I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))+I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2-I*Pi*csgn(I/(a*x-1)*(a*x+1))^3+2*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2+I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2-I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))-I*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3+4*ln(2))*arccoth(a*x)^2+3*arccoth(a*x)*polylog(2,-1/(a*x-1)*(a*x+1))-3/2*polylog(3,-1/(a*x-1)*(a*x+1)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*a*(log(a*x + 1) - log(x))*log(a)^3 + 3/8*a*integrate(x*log(a*x - 1)/(a*
x^3 + x^2), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^3 + x^2), x)*log(a)
^2 - 1/8*(a*log(a*x + 1) - a*log(x) - 1/x)*log(a)^3 + 3/4*a^2*integrate(x^2
*log(a*x + 1)*log(a*x - 1)/(a*x^3 + x^2), x) - 3/2*a^2*integrate(x^2*log(a*
x + 1)*log(x)/(a*x^3 + x^2), x) + 3/4*a*integrate(x*log(a*x - 1)*log(x)/(a*
x^3 + x^2), x)*log(a) - 3/8*a*integrate(x*log(x)^2/(a*x^3 + x^2), x)*log(a)
 + 3/8*integrate(log(a*x - 1)/(a*x^3 + x^2), x)*log(a)^2 - 3/8*integrate(lo
g(x)/(a*x^3 + x^2), x)*log(a)^2 + 3/8*a*integrate(x*log(a*x + 1)*log(a*x -
1)^2/(a*x^3 + x^2), x) - 3/8*a*integrate(x*log(a*x - 1)^2*log(x)/(a*x^3 + x
^2), x) + 3/8*a*integrate(x*log(a*x - 1)*log(x)^2/(a*x^3 + x^2), x) - 1/8*a
*integrate(x*log(x)^3/(a*x^3 + x^2), x) - 3/4*a*integrate(x*log(a*x + 1)*lo
g(a*x - 1)/(a*x^3 + x^2), x) - 3/8*integrate(a*x*log(a*x - 1)^2/(a*x^3 + x^
2), x)*log(a) - 3/8*integrate(log(a*x - 1)^2/(a*x^3 + x^2), x)*log(a) + 3/4
*integrate(log(a*x - 1)*log(x)/(a*x^3 + x^2), x)*log(a) - 3/8*integrate(log
(x)^2/(a*x^3 + x^2), x)*log(a) - 3/8*(a^2*log(a*x - 1) - a^2*log(x) + a/x)*
log(-1/(a*x) + 1)^2/a + 1/8*log(-1/(a*x) + 1)^3/x - 1/8*((a*x + 1)*log(a*x
+ 1)^3 - 3*(2*a*x*log(x) - (a*x - 1)*log(a*x - 1))*log(a*x + 1)^2)/x + 1/8*
(3*(a^3*x*log(a*x - 1)^2 + a^3*x*log(x)^2 - 2*a^3*x*log(x) + 2*a^2 - 2*(a^3
*x*log(x) - a^3*x)*log(a*x - 1))*log(-1/(a*x) + 1)/(a*x) - (a^4*x*log(a*x -
 1)^3 - a^4*x*log(x)^3 + 3*a^4*x*log(x)^2 - 6*a^4*x*log(x) + 6*a^3 - 3*(a^4
*x*log(x) - a^4*x)*log(a*x - 1)^2 + 3*(a^4*x*log(x)^2 - 2*a^4*x*log(x) + 2*
a^4*x)*log(a*x - 1))/(a^2*x))/a + 3/8*integrate(log(a*x + 1)*log(a*x - 1)^2
/(a*x^3 + x^2), x) - 3/8*integrate(log(a*x - 1)^2*log(x)/(a*x^3 + x^2), x)
+ 3/8*integrate(log(a*x - 1)*log(x)^2/(a*x^3 + x^2), x) - 1/8*integrate(log
(x)^3/(a*x^3 + x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**2,x)
```

```
[Out] Integral(acoth(a*x)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acoth}(a\,x)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3/x^2,x)
```

```
[Out] int(acoth(a*x)^3/x^2, x)
```

3.31 $\int \frac{\coth^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=95

$$\frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\frac{3}{2}a^2\mathrm{P}$$

[Out] 3/2*a^2*arccoth(a*x)^2-3/2*a*arccoth(a*x)^2/x+1/2*a^2*arccoth(a*x)^3-1/2*ar
ccoth(a*x)^3/x^2+3*a^2*arccoth(a*x)*ln(2-2/(a*x+1))-3/2*a^2*polylog(2,-1+2/
(a*x+1))

Rubi [A]
time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6038, 6130, 6136, 6080, 2497, 6096}

$$-\frac{3}{2}a^2\mathrm{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\coth^{-1}(ax)^3+\frac{3}{2}a^2\coth^{-1}(ax)^2+3a^2\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{\coth^{-1}(ax)^3}{2x^2}-\frac{3a\coth^{-1}(ax)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^3,x]
```

```
[Out] (3*a^2*ArcCoth[a*x]^2)/2 - (3*a*ArcCoth[a*x]^2)/(2*x) + (a^2*ArcCoth[a*x]^3
)/2 - ArcCoth[a*x]^3/(2*x^2) + 3*a^2*ArcCoth[a*x]*Log[2 - 2/(1 + a*x)] - (3
*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax)^3}{x^3}\,dx &= -\frac{\coth^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx\\
&= -\frac{\coth^{-1}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\coth^{-1}(ax)^2}{x^2}\,dx+\frac{1}{2}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2}\,dx\\
&= -\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)}\,dx\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\coth}{x(1}\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax\\
&= \frac{3}{2}a^2\coth^{-1}(ax)^2-\frac{3a\coth^{-1}(ax)^2}{2x}+\frac{1}{2}a^2\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{2x^2}+3a^2\coth^{-1}(ax\end{aligned}$$

Mathematica [A]

time = 0.11, size = 79, normalized size = 0.83

$$\frac{1}{2}\left(\frac{\coth^{-1}(ax)\left(3ax(-1+ax)\coth^{-1}(ax)+(-1+a^2x^2)\coth^{-1}(ax)^2+6a^2x^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)\right)}{x^2}-3a^2\text{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3/x^3,x]
```

```
[Out] ((ArcCoth[a*x]*(3*a*x*(-1 + a*x)*ArcCoth[a*x] + (-1 + a^2*x^2)*ArcCoth[a*x]
^2 + 6*a^2*x^2*Log[1 + E^(-2*ArcCoth[a*x])]))/x^2 - 3*a^2*PolyLog[2, -E^(-2
*ArcCoth[a*x])])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.63, size = 3502, normalized size = 36.86

method	result	size
derivativedivides	Expression too large to display	3502
default	Expression too large to display	3502

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*dilog(1-I/((a
*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3*dilog(1+I/((a*x-1)
/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3*dilog(1-I/((a*x-1)/(a*x
+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))^3*arccoth(a*x)^2-3/16*I*Pi*csg
n(I/(a*x-1)*(a*x+1))^3*polylog(2,-1/(a*x-1)*(a*x+1))+3/8*I*Pi*csgn(I/(a*x-1
)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*arccoth(a*x)^2-3/16*I*Pi*csgn(I/(a*x-1)*
(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*polylog(2,-1/(a*x-1)*(a*x+1))+3/8*I*Pi*csg
n(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*dilog(1+I/((a*x-1)/(a*x+1))^(1
/2))-3/2/a/x*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)
*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))-3/8*
I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(
a*x+1)/(1/(a*x-1)*(a*x+1)-1))*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))+1/2*arcc
oth(a*x)^3-3/2*arccoth(a*x)^2+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I
/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*arccoth(a*x
)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*
csgn(I/(a*x-1)*(a*x+1))^2*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))-3/8*I*Pi*csg
n(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*
arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/((a*x-1)/(a*x+
1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^
(1/2))+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*
x-1)*(a*x+1)-1))^2*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))-3/16*I*Pi*csgn(I/(1
/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x
-1)*(a*x+1)-1))*polylog(2,-1/(a*x-1)*(a*x+1))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+
1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arccoth(a*x)*ln(1+I/((a
*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*
x-1)*(a*x+1))*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))+3/8*I*Pi*csgn(I/(1/(a*x-
1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a
```

```
*x+1)-1))*arccoth(a*x)^2+3/16*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(
a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*polylog(2,-1/(a*x-1)*(a*x+1))-3/8*I*Pi*csgn
(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1
+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)
*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*
Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*dilog(1+I/((a*
x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*
(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*dilog(1+I/((a*x-1)/(
a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(
a*x-1)*(a*x+1)-1))^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*
csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1
)/(1/(a*x-1)*(a*x+1)-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I
/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arc
coth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))
^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/
2))-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2*arcc
oth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^
(1/2))*csgn(I/(a*x-1)*(a*x+1))^2*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2
))+3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a
*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))*arccoth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^
(1/2))-1/2/a^2/x^2*arccoth(a*x)^3-3/4*arccoth(a*x)^2*ln(a*x-1)+3/4*arccoth(
a*x)^2*ln(a*x+1)-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a
*x+1))^2*arccoth(a*x)^2+3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*
x-1)*(a*x+1))^2*polylog(2,-1/(a*x-1)*(a*x+1))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+
1))^3*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1)/
(1/(a*x-1)*(a*x+1)-1))^3*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))+3/8*I*Pi*csgn
(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*arccoth(a*x)*ln(1+I/((a*x-1)/(a
*x+1))^(1/2))+3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^3*arcc
oth(a*x)*ln(1-I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(a*x-1)*(a*x+1))*c
sgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*arccoth(a*x)^2-3/8*I*Pi*csgn
(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*d
ilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*cs
gn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*dilog(1-I/((a*x-1)/(a*x+1))^(
1/2))+3/8*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+1))*di
log(1-I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*c
sgn(I/(a*x-1)*(a*x+1))^2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*Pi*csgn(I
/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2*dilog(1-I/((a*x-1)/(a*x
+1))^(1/2))+3/2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/4*polylog(2,-1/(a*x-1)
*(a*x+1))+3/2*arccoth(a*x)*ln(1+1/(a*x-1)*(a*x+1))+3/2*arccoth(a*x)*ln(1+I/
((a*x-1)/(a*x+1))^(1/2))+3/2*arccoth(a*x)*ln(1-...
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(84) = 168$.
time = 0.27, size = 252, normalized size = 2.65

$$\frac{3}{4}\left(a\log\left(ax+1\right)-a\log\left(ax-1\right)-\frac{2}{x}\right)a\operatorname{arcoth}\left(ax\right)^{2}-\frac{1}{16}\left(a^{2}\left(\frac{3\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)^{2}-\log\left(ax+1\right)^{3}+\log\left(ax-1\right)^{3}-3\left(\log\left(ax-1\right)^{2}-4\log\left(ax-1\right)\right)\log\left(ax+1\right)+6\log\left(ax-1\right)^{2}}{a}-\frac{24\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\operatorname{Li}_{2}\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a}+\frac{24\left(\log\left(ax+1\right)\log\left(x\right)+\operatorname{Li}_{2}\left(-ax\right)\right)}{a}-\frac{24\left(\log\left(-ax+1\right)\log\left(x\right)+\operatorname{Li}_{2}\left(ax\right)\right)}{a}\right)-6\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^{2}-\log\left(ax-1\right)^{2}-4\log\left(ax-1\right)+8\log\left(x\right)\right)a\operatorname{arcoth}\left(ax\right)\right)a-\frac{\operatorname{arcoth}\left(ax\right)^{3}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a*arccoth(a*x)^2 - 1/16*(a^2*((
3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3*(
log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*(l
og(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*(log(a*x + 1
)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 6*(
2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log
(a*x - 1) + 8*log(x))*a*arccoth(a*x))*a - 1/2*arccoth(a*x)^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^{3}(ax)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**3,x)
```

```
[Out] Integral(acoth(a*x)**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{acoth}\left(a\,x\right)}^3}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3/x^3,x)
```

```
[Out] int(acoth(a*x)^3/x^3, x)
```

3.32 $\int \frac{\coth^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=154

$$-\frac{a^2\coth^{-1}(ax)}{x}+\frac{1}{2}a^3\coth^{-1}(ax)^2-\frac{a\coth^{-1}(ax)^2}{2x^2}+\frac{1}{3}a^3\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{3x^3}+a^3\log(x)-\frac{1}{2}a^3\log(1-$$

```
[Out] -a^2*arccoth(a*x)/x+1/2*a^3*arccoth(a*x)^2-1/2*a*arccoth(a*x)^2/x^2+1/3*a^3
*arccoth(a*x)^3-1/3*arccoth(a*x)^3/x^3+a^3*ln(x)-1/2*a^3*ln(-a^2*x^2+1)+a^3
*arccoth(a*x)^2*ln(2-2/(a*x+1))-a^3*arccoth(a*x)*polylog(2,-1+2/(a*x+1))-1/
2*a^3*polylog(3,-1+2/(a*x+1))
```

Rubi [A]
time = 0.25, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = $\{6038, 6130, 272, 36, 29, 31, 6096, 6136, 6080, 6204, 6745\}$

$$-\frac{1}{2}a^3\text{Li}_3\left(\frac{2}{ax+1}-1\right)-a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\coth^{-1}(ax)+a^3\log(x)+\frac{1}{3}a^3\coth^{-1}(ax)^3+\frac{1}{2}a^3\coth^{-1}(ax)^2+a^3\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)^2-\frac{a^2\coth^{-1}(ax)}{x}-\frac{1}{2}a^3\log\left(1-a^2x^2\right)-\frac{\coth^{-1}(ax)^3}{3x^3}-\frac{a\coth^{-1}(ax)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcCoth[a*x])/x) + (a^3*ArcCoth[a*x]^2)/2 - (a*ArcCoth[a*x]^2)/(2*x^
2) + (a^3*ArcCoth[a*x]^3)/3 - ArcCoth[a*x]^3/(3*x^3) + a^3*Log[x] - (a^3*Lo
g[1 - a^2*x^2])/2 + a^3*ArcCoth[a*x]^2*Log[2 - 2/(1 + a*x)] - a^3*ArcCoth[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (a^3*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)^3}{x^4}\,dx = -\frac{\coth^{-1}(ax)^3}{3x^3} + a\int \frac{\coth^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(ax)^3}{3x^3} + a\int \frac{\coth^{-1}(ax)^2}{x^3}\,dx + a^3\int \frac{\coth^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx$$

$$= -\frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2\int \frac{\coth^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + a^3\int \frac{\mathrm{co}}{}$$

$$= -\frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3\coth^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right.$$

$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax}{3x^3}$$

$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax}{3x^3}$$

$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax}{3x^3}$$

$$= -\frac{a^2\coth^{-1}(ax)}{x} + \frac{1}{2}a^3\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax}{3x^3}$$

Mathematica [A]
time = 0.16, size = 142, normalized size = 0.92

$$\frac{1}{6}\left(-\frac{6a^2\coth^{-1}(ax)}{x} + 3a^3\coth^{-1}(ax)^2 - \frac{3a\coth^{-1}(ax)^2}{x^2} + 2a^3\coth^{-1}(ax)^3 - \frac{2\coth^{-1}(ax)^3}{x^3} + 6a^3\coth^{-1}(ax)^2\log\left(1+e^{-2\coth^{-1}(ax)}\right) + 6a^3\log\left(\frac{1}{\sqrt{1-\frac{1}{a^2x^2}}}\right) - 6a^3\coth^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right) - 3a^3\mathrm{PolyLog}\left(3,-e^{-2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3/x^4,x]
```

```
[Out] ((-6*a^2*ArcCoth[a*x])/x + 3*a^3*ArcCoth[a*x]^2 - (3*a*ArcCoth[a*x]^2)/x^2
+ 2*a^3*ArcCoth[a*x]^3 - (2*ArcCoth[a*x]^3)/x^3 + 6*a^3*ArcCoth[a*x]^2*Log[
1 + E^(-2*ArcCoth[a*x])] + 6*a^3*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 6*a^3*ArcCo
th[a*x]*PolyLog[2, -E^(-2*ArcCoth[a*x])] - 3*a^3*PolyLog[3, -E^(-2*ArcCoth[
a*x])])/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.10, size = 840, normalized size = 5.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arccoth(a*x)^3-1/2*arccoth(a*x)^2*ln(a*x+1)-1/2/a^2/x^2*a
rccoth(a*x)^2+arccoth(a*x)^2*ln(a*x)-1/2*arccoth(a*x)^2*ln(a*x-1)-1/2*arcco
th(a*x)^2*ln((a*x-1)/(a*x+1))+arccoth(a*x)*polylog(2,-1/(a*x-1)*(a*x+1))-1/
2*polylog(3,-1/(a*x-1)*(a*x+1))-1/12*arccoth(a*x)*(6*I*arccoth(a*x)*Pi*csgn
(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)
))^2*a*x+3*I*arccoth(a*x)*Pi*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^
3*a*x-3*I*arccoth(a*x)*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x
+1)/(1/(a*x-1)*(a*x+1)-1))^2*a*x-6*I*arccoth(a*x)*Pi*csgn(I*(1+1/(a*x-1)*(a
*x+1)))*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*
x-1)*(a*x+1)))*a*x+6*I*arccoth(a*x)*Pi*csgn(I*(1+1/(a*x-1)*(a*x+1)))*csgn(I
/(1/(a*x-1)*(a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^2*a*x+3*I*arccoth(a*x)*Pi*csg
n(I/(a*x-1)*(a*x+1))^3*a*x-3*I*arccoth(a*x)*Pi*csgn(I/(a*x-1)*(a*x+1))*csgn
(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2*a*x+3*I*arccoth(a*x)*Pi*csgn(I/
(a*x-1)*(a*x+1))*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*a*x+3*I*arccoth(a*x)*Pi*
csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(a*x+1
)/(1/(a*x-1)*(a*x+1)-1))*a*x-6*I*arccoth(a*x)*Pi*csgn(I/(a*x-1)*(a*x+1))^2*
csgn(I/((a*x-1)/(a*x+1))^(1/2))*a*x-6*I*arccoth(a*x)*Pi*csgn(I/(1/(a*x-1)*(
a*x+1)-1)*(1+1/(a*x-1)*(a*x+1)))^3*a*x+4*arccoth(a*x)^2*a*x-12*arccoth(a*x)
*ln(2)*a*x-6*a*x*arccoth(a*x)+12*a*x+12)/a/x+ln(1+1/(a*x-1)*(a*x+1)))
```

__Maxima__ **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*integrate(x^4*log(a*x + 1)*log(a*x - 1)/(a*x^5 + x^4), x) - 1/2*a^4
*integrate(x^4*log(a*x + 1)*log(x)/(a*x^5 + x^4), x) + 1/16*(2*a^2*log(a*x
+ 1) - 2*a^2*log(x) - (2*a*x - 1)/x^2)*a*log(a)^3 + 3/8*a*integrate(x*log(a
*x - 1)/(a*x^5 + x^4), x)*log(a)^2 - 3/8*a*integrate(x*log(x)/(a*x^5 + x^4)
, x)*log(a)^2 - 1/48*(6*a^3*log(a*x + 1) - 6*a^3*log(x) - (6*a^2*x^2 - 3*a*
x + 2)/x^3)*log(a)^3 + 1/4*a^2*integrate(x^2*log(a*x + 1)/(a*x^5 + x^4), x)
 + 3/4*a*integrate(x*log(a*x - 1)*log(x)/(a*x^5 + x^4), x)*log(a) - 3/8*a*i
ntegrate(x*log(x)^2/(a*x^5 + x^4), x)*log(a) + 3/8*integrate(log(a*x - 1)/(
a*x^5 + x^4), x)*log(a)^2 - 3/8*integrate(log(x)/(a*x^5 + x^4), x)*log(a)^2
 + 3/8*a*integrate(x*log(a*x + 1)*log(a*x - 1)^2/(a*x^5 + x^4), x) - 3/8*a*
integrate(x*log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*a*integrate(x*log
(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*a*integrate(x*log(x)^3/(a*x^5 +
x^4), x) - 1/4*a*integrate(x*log(a*x + 1)*log(a*x - 1)/(a*x^5 + x^4), x) -
3/8*integrate(a*x*log(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) - 3/8*integrate(l
```

```
og(a*x - 1)^2/(a*x^5 + x^4), x)*log(a) + 3/4*integrate(log(a*x - 1)*log(x)/
(a*x^5 + x^4), x)*log(a) - 3/8*integrate(log(x)^2/(a*x^5 + x^4), x)*log(a)
- 1/48*(6*a^4*log(a*x - 1) - 6*a^4*log(x) + (6*a^3*x^2 + 3*a^2*x + 2*a)/x^3
)*log(-1/(a*x) + 1)^2/a + 1/864*(6*(18*a^5*x^3*log(a*x - 1)^2 + 18*a^5*x^3*
log(x)^2 - 66*a^5*x^3*log(x) + 66*a^4*x^2 + 15*a^3*x + 4*a^2 - 6*(6*a^5*x^3
*log(x) - 11*a^5*x^3)*log(a*x - 1))*log(-1/(a*x) + 1)/(a*x^3) - (36*a^6*x^3
*log(a*x - 1)^3 - 36*a^6*x^3*log(x)^3 + 198*a^6*x^3*log(x)^2 - 510*a^6*x^3*
log(x) + 510*a^5*x^2 + 57*a^4*x + 8*a^3 - 18*(6*a^6*x^3*log(x) - 11*a^6*x^3
)*log(a*x - 1)^2 + 6*(18*a^6*x^3*log(x)^2 - 66*a^6*x^3*log(x) + 85*a^6*x^3)
*log(a*x - 1))/(a^2*x^3))/a + 1/24*log(-1/(a*x) + 1)^3/x^3 - 1/24*((a^3*x^3
 + 1)*log(a*x + 1)^3 - 3*(2*a^3*x^3*log(x) - a*x - (a^3*x^3 - 1)*log(a*x -
1))*log(a*x + 1)^2)/x^3 + 3/8*integrate(log(a*x + 1)*log(a*x - 1)^2/(a*x^5
+ x^4), x) - 3/8*integrate(log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*in
tegrate(log(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*integrate(log(x)^3/(a
*x^5 + x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**4,x)
```

```
[Out] Integral(acoth(a*x)**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a\,x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3/x^4,x)
```

```
[Out] int(acoth(a*x)^3/x^4, x)
```

3.33 $\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx$

Optimal. Leaf size=141

$$-\frac{a^3}{4x}-\frac{a^2\coth^{-1}(ax)}{4x^2}+a^4\coth^{-1}(ax)^2-\frac{a\coth^{-1}(ax)^2}{4x^3}-\frac{3a^3\coth^{-1}(ax)^2}{4x}+\frac{1}{4}a^4\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{4x^4}+\frac{1}{4}$$

```
[Out] -1/4*a^3/x-1/4*a^2*arccoth(a*x)/x^2+a^4*arccoth(a*x)^2-1/4*a*arccoth(a*x)^2
/x^3-3/4*a^3*arccoth(a*x)^2/x+1/4*a^4*arccoth(a*x)^3-1/4*arccoth(a*x)^3/x^4
+1/4*a^4*arctanh(a*x)+2*a^4*arccoth(a*x)*ln(2-2/(a*x+1))-a^4*polylog(2,-1+2
/(a*x+1))
```

Rubi [A]
time = 0.31, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6038, 6130, 331, 212, 6136, 6080, 2497, 6096}

$$-a^4\text{Li}_2\left(\frac{2}{ax+1}-1\right)+\frac{1}{4}a^4\tanh^{-1}(ax)+\frac{1}{4}a^4\coth^{-1}(ax)^3+a^4\coth^{-1}(ax)^2+2a^4\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)-\frac{a^3}{4x}-\frac{3a^3\coth^{-1}(ax)^2}{4x}-\frac{a^2\coth^{-1}(ax)}{4x^2}-\frac{\coth^{-1}(ax)^3}{4x^4}-\frac{a\coth^{-1}(ax)^2}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]^3/x^5,x]
```

```
[Out] -1/4*a^3/x - (a^2*ArcCoth[a*x])/(4*x^2) + a^4*ArcCoth[a*x]^2 - (a*ArcCoth[a
*x]^2)/(4*x^3) - (3*a^3*ArcCoth[a*x]^2)/(4*x) + (a^4*ArcCoth[a*x]^3)/4 - Ar
cCoth[a*x]^3/(4*x^4) + (a^4*ArcTanh[a*x])/4 + 2*a^4*ArcCoth[a*x]*Log[2 - 2/
(1 + a*x)] - a^4*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^5}\,dx &= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\coth^{-1}(ax)^2}{x^4\left(1-a^2x^2\right)}\,dx \\
&= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a)\int \frac{\coth^{-1}(ax)^2}{x^4}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx \\
&= -\frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\coth^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx + \frac{1}{4}\left(3a^3\right)\int \frac{\coth^{-1}(ax)^2}{x^2}\,d \\
&= -\frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\coth^{-}}{x} \\
&= -\frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\coth^{-1}(a \\
&= -\frac{a^3}{4x} - \frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\mathrm{co} \\
&= -\frac{a^3}{4x} - \frac{a^2\coth^{-1}(ax)}{4x^2} + a^4\coth^{-1}(ax)^2 - \frac{a\coth^{-1}(ax)^2}{4x^3} - \frac{3a^3\coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4\mathrm{co}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 118, normalized size = 0.84

$$
\frac{-a^3x^3 + ax\left(-1-3a^2x^2+4a^3x^3\right)\coth^{-1}(ax)^2 + \left(-1+a^4x^4\right)\coth^{-1}(ax)^3 + a^2x^2\coth^{-1}(ax)\left(-1+a^2x^2+8a^2x^2\log\left(1+e^{-2\coth^{-1}(ax)}\right)\right) - 4a^4x^4\,\mathrm{PolyLog}\left(2,-e^{-2\coth^{-1}(ax)}\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]^3/x^5,x]
```

```
[Out] (-(a^3*x^3) + a*x*(-1 - 3*a^2*x^2 + 4*a^3*x^3)*ArcCoth[a*x]^2 + (-1 + a^4*x
^4)*ArcCoth[a*x]^3 + a^2*x^2*ArcCoth[a*x]*(-1 + a^2*x^2 + 8*a^2*x^2*Log[1 +
 E^(-2*ArcCoth[a*x])]) - 4*a^4*x^4*PolyLog[2, -E^(-2*ArcCoth[a*x])])/(4*x^4
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.61, size = 659, normalized size = 4.67 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4/a^4/x^4*arccoth(a*x)^3+3/8*arccoth(a*x)^2*ln(a*x+1)-1/4/a^3/x^3*a
rccoth(a*x)^2-3/4/a/x*arccoth(a*x)^2-3/8*arccoth(a*x)^2*ln(a*x-1)+2*dilog(1
+I/((a*x-1)/(a*x+1))^(1/2))+2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+1/4*(a*x-1
)/a/x-1/2*arccoth(a*x)*(a*x+1)/a/x-arccoth(a*x)^2+1/4*arccoth(a*x)^3-3/8*I*
arccoth(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I/(a*x-1)*(a*x+1))^2
+2*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+2*arccoth(a*x)*ln(1-I/((a*x
```

```
-1)/(a*x+1))^(1/2))+3/16*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+1))^3+3/16
*I*arccoth(a*x)^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I/(a*x-1)*(a*x+
1))+1/4*arccoth(a*x)*(a*x+1)^2/a^2/x^2+1/2*arccoth(a*x)*(a*x-1)*(a*x+1)/a^2
/x^2-3/16*I*arccoth(a*x)^2*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*
(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2-3/16*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a
*x+1))*csgn(I/(a*x-1)*(a*x+1)/(1/(a*x-1)*(a*x+1)-1))^2+3/16*I*arccoth(a*x)^
2*Pi*csgn(I/(1/(a*x-1)*(a*x+1)-1))*csgn(I/(a*x-1)*(a*x+1))*csgn(I/(a*x-1)*(
a*x+1)/(1/(a*x-1)*(a*x+1)-1))+3/16*I*arccoth(a*x)^2*Pi*csgn(I/(a*x-1)*(a*x+
1)/(1/(a*x-1)*(a*x+1)-1))^3+3/8*arccoth(a*x)^2*ln((a*x-1)/(a*x+1)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(126) = 252$.
time = 0.28, size = 342, normalized size = 2.43

$$\frac{1}{8}\left(3a^3\log(ax+1)-3a^3\log(ax-1)-\frac{2(3a^2x^2+1)}{x^3}\right)a\,\mathrm{arcoth}(ax)^2+\frac{1}{32}\left(\left(32\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-32(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax))a+32(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax))a+4a\log(ax+1)-4a\log(ax-1)+\frac{ax\log(ax+1)^3-ax\log(ax-1)^3-8ax\log(ax-1)^2-(3ax\log(ax-1)-8ax)\log(ax+1)^2+(3ax\log(ax-1)^2-16ax\log(ax-1))\log(ax+1)-8}{x}\right)a^2+2\left(32a^2\log(x)-\frac{3a^2x^2\log(ax+1)^2+3a^2x^2\log(ax-1)^2+16a^2x^2\log(ax-1)-2(3a^2x^2\log(ax-1)-8a^2x^2)\log(ax+1)+4}{x^2}\right)a\,\mathrm{arcoth}(ax)\right)a-\frac{\mathrm{arcoth}(ax)^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 + 1)/x^3)*a*arc
coth(a*x)^2 + 1/32*((32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x +
 1/2))*a - 32*(log(a*x + 1)*log(x) + dilog(-a*x))*a + 32*(log(-a*x + 1)*log
(x) + dilog(a*x))*a + 4*a*log(a*x + 1) - 4*a*log(a*x - 1) + (a*x*log(a*x +
1)^3 - a*x*log(a*x - 1)^3 - 8*a*x*log(a*x - 1)^2 - (3*a*x*log(a*x - 1) - 8*
a*x)*log(a*x + 1)^2 + (3*a*x*log(a*x - 1)^2 - 16*a*x*log(a*x - 1))*log(a*x
+ 1) - 8)/x)*a^2 + 2*(32*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2
*log(a*x - 1)^2 + 16*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 8*a
^2*x^2)*log(a*x + 1) + 4)/x^2)*a*arccoth(a*x))*a - 1/4*arccoth(a*x)^3/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)^3/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{acoth}^3(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)**3/x**5,x)
```

```
[Out] Integral(acoth(a*x)**3/x**5, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)^3/x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acoth}(a\,x)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)^3/x^5,x)
```

```
[Out] int(acoth(a*x)^3/x^5, x)
```

3.34 $\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx$

Optimal. Leaf size=164

$$-\frac{\coth^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{\coth^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2}{1+cx}\right)}{e}-\frac{\coth^{-1}(cx)\text{Poly}}{}$$

```
[Out] -arccoth(c*x)^2*ln(2/(c*x+1))/e+arccoth(c*x)^2*ln(2*c*(e*x+d)/(c*d+e)/(c*x+
1))/e+arccoth(c*x)*polylog(2,1-2/(c*x+1))/e-arccoth(c*x)*polylog(2,1-2*c*(e
*x+d)/(c*d+e)/(c*x+1))/e+1/2*polylog(3,1-2/(c*x+1))/e-1/2*polylog(3,1-2*c*(
e*x+d)/(c*d+e)/(c*x+1))/e
```

Rubi [A]
time = 0.02, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{6060\}$

$$-\frac{\text{Li}_3\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e}-\frac{\coth^{-1}(cx)\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{\text{Li}_3\left(1-\frac{2}{cx+1}\right)}{2e}+\frac{\text{Li}_2\left(1-\frac{2}{cx+1}\right)\coth^{-1}(cx)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\coth^{-1}(cx)^2}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c*x]^2/(d + e*x),x]
```

```
[Out] -((ArcCoth[c*x]^2*Log[2/(1 + c*x)])/e) + (ArcCoth[c*x]^2*Log[(2*c*(d + e*x)
)/((c*d + e)*(1 + c*x))])/e + (ArcCoth[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/e
- (ArcCoth[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e +
PolyLog[3, 1 - 2/(1 + c*x)]/(2*e) - PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))]/(2*e)
```

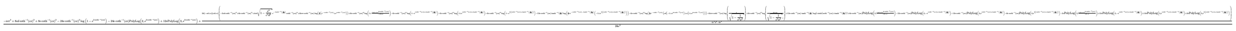
Rule 6060

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(cx)^2}{d+ex}\,dx=-\frac{\coth^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\coth^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{\coth^{-1}(cx)\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{e}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.74, size = 741, normalized size = 4.52



Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c*x]^2/(d + e*x),x]
```

```
[Out] ((-I)*e*Pi^3 + 8*c*d*ArcCoth[c*x]^3 + 8*e*ArcCoth[c*x]^3 - 24*e*ArcCoth[c*x
]^2*Log[1 - E^(2*ArcCoth[c*x])] - 24*e*ArcCoth[c*x]*PolyLog[2, E^(2*ArcCoth
[c*x])] + 12*e*PolyLog[3, E^(2*ArcCoth[c*x])] + (24*(-(c*d) + e)*(c*d + e)*
(-2*c*d*ArcCoth[c*x]^3 + 6*e*ArcCoth[c*x]^3 + (4*c*d*Sqrt[1 - e^2/(c^2*d^2)
]*ArcCoth[c*x]^3)/E^ArcTanh[e/(c*d)] + (6*I)*e*Pi*ArcCoth[c*x]*Log[(E^(-Arc
Coth[c*x]) + E^ArcCoth[c*x])/2] + 6*e*ArcCoth[c*x]^2*Log[1 + ((c*d + e)*E^(
2*ArcCoth[c*x]))/(-(c*d) + e)] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(ArcCoth[c*x]
 + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 + E^(ArcCoth[c*x] + ArcTan
h[e/(c*d)])] - 6*e*ArcCoth[c*x]^2*Log[1 - E^(2*(ArcCoth[c*x] + ArcTanh[e/(c
*d)]))] - 12*e*ArcCoth[c*x]*ArcTanh[e/(c*d)]*Log[(I/2)*E^(-ArcCoth[c*x] - A
rcTanh[e/(c*d)])*(-1 + E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)])))] - 6*e*ArcC
oth[c*x]^2*Log[(c*d*(-1 + E^(2*ArcCoth[c*x])) + e*(1 + E^(2*ArcCoth[c*x])))
/(2*E^ArcCoth[c*x])] - (6*I)*e*Pi*ArcCoth[c*x]*Log[1/Sqrt[1 - 1/(c^2*x^2)]]
 + 6*e*ArcCoth[c*x]^2*Log[(d + e*x)/(Sqrt[1 - 1/(c^2*x^2)]*x)] + 12*e*ArcCo
th[c*x]*ArcTanh[e/(c*d)]*Log[I*Sinh[ArcCoth[c*x] + ArcTanh[e/(c*d)]]] + 6*e
*ArcCoth[c*x]*PolyLog[2, ((c*d + e)*E^(2*ArcCoth[c*x]))/(c*d - e)] - 12*e*A
rcCoth[c*x]*PolyLog[2, -E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] - 12*e*ArcCoth
[c*x]*PolyLog[2, E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] - 6*e*ArcCoth[c*x]*Po
lyLog[2, E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)]))] - 3*e*PolyLog[3, ((c*d +
e)*E^(2*ArcCoth[c*x]))/(c*d - e)] + 12*e*PolyLog[3, -E^(ArcCoth[c*x] + ArcT
anh[e/(c*d)])] + 12*e*PolyLog[3, E^(ArcCoth[c*x] + ArcTanh[e/(c*d)])] + 3*e
*PolyLog[3, E^(2*(ArcCoth[c*x] + ArcTanh[e/(c*d)]))]))/(6*c^2*d^2 - 6*e^2))
/(24*e^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.94, size = 897, normalized size = 5.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c*x)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*ln(c*e*x+c*d)/e*arccoth(c*x)^2+2*c/e*(-1/2*arccoth(c*x)^2*ln(d*c*(1/
(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))+1/4*I*Pi*arccoth(c*x)^2*csgn(I/
(1/(c*x-1)*(c*x+1)-1))*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+
1)+1)))*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(c*x-
1)*(c*x+1)-1))-1/4*I*Pi*arccoth(c*x)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1))*csgn(I
*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(c*x-1)*(c*x+1)-1))
^2-1/4*I*Pi*arccoth(c*x)^2*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(
c*x+1)+1)))*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(
```

```
c*x-1)*(c*x+1)-1))^2+1/4*I*Pi*arccoth(c*x)^2*csgn(I*(d*c*(1/(c*x-1)*(c*x+1)
-1)+e*(1/(c*x-1)*(c*x+1)+1))/(1/(c*x-1)*(c*x+1)-1))^3+1/2*arccoth(c*x)^2*ln
(1/(c*x-1)*(c*x+1)-1)-1/2*arccoth(c*x)^2*ln(1+1/((c*x-1)/(c*x+1))^(1/2))-ar
ccoth(c*x)*polylog(2,-1/((c*x-1)/(c*x+1))^(1/2))+polylog(3,-1/((c*x-1)/(c*x
+1))^(1/2))-1/2*arccoth(c*x)^2*ln(1-1/((c*x-1)/(c*x+1))^(1/2))-arccoth(c*x)
*polylog(2,1/((c*x-1)/(c*x+1))^(1/2))+polylog(3,1/((c*x-1)/(c*x+1))^(1/2))+
1/2*e/(c*d+e)*arccoth(c*x)^2*ln(1-(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))+1/2*e/(c
*d+e)*arccoth(c*x)*polylog(2,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))-1/4*e/(c*d+e)
*polylog(3,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))+1/2*d*c/(c*d+e)*arccoth(c*x)^2*
ln(1-(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))+1/2*d*c/(c*d+e)*arccoth(c*x)*polylog(
2,(c*d+e)/(c*x-1)*(c*x+1)/(c*d-e))-1/4*d*c/(c*d+e)*polylog(3,(c*d+e)/(c*x-1
)*(c*x+1)/(c*d-e))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(arccoth(c*x)^2/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="fricas")
[Out] integral(arccoth(c*x)^2/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c*x)**2/(e*x+d),x)
[Out] Integral(acoth(c*x)**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccoth(c*x)^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acoth}(c\,x)^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c*x)^2/(d + e*x),x)
```

```
[Out] int(acoth(c*x)^2/(d + e*x), x)
```

3.35 $\int (c + dx^2)^4 \coth^{-1}(ax)\, dx$

Optimal. Leaf size=245

$$\frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)\, x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)\, x^4}{1260a^5} + \frac{d^3(36a^2c + 7d)\, x^6}{378a^3} + \frac{d^4x^8}{72a} + \cdots$$

```
[Out] 1/630*d*(420*a^6*c^3+378*a^4*c^2*d+180*a^2*c*d^2+35*d^3)*x^2/a^7+1/1260*d^2
*(378*a^4*c^2+180*a^2*c*d+35*d^2)*x^4/a^5+1/378*d^3*(36*a^2*c+7*d)*x^6/a^3+
1/72*d^4*x^8/a+c^4*x*arccoth(a*x)+4/3*c^3*d*x^3*arccoth(a*x)+6/5*c^2*d^2*x^
5*arccoth(a*x)+4/7*c*d^3*x^7*arccoth(a*x)+1/9*d^4*x^9*arccoth(a*x)+1/630*(3
15*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*ln(-a^2*x^2+
1)/a^9
```

Rubi [A]
time = 0.13, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 6124, 1824, 266}

$$\frac{d^3x^6(36a^2c + 7d)}{378a^3} + \frac{d^2x^4(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} + \frac{dx^2(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)}{630a^7} + \frac{(315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4)\log(1 - a^2x^2)}{630a^9} + c^4x\coth^{-1}(ax) + \frac{4}{3}c^3dx^3\coth^{-1}(ax) + \frac{6}{5}c^2d^2x^5\coth^{-1}(ax) + \frac{4}{7}cd^3x^7\coth^{-1}(ax) + \frac{1}{9}d^4x^9\coth^{-1}(ax) + \frac{d^4x^8}{72a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCoth[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcCoth[a*x] + (4*c^3*d*x^
3*ArcCoth[a*x])/3 + (6*c^2*d^2*x^5*ArcCoth[a*x])/5 + (4*c*d^3*x^7*ArcCoth[a
*x])/7 + (d^4*x^9*ArcCoth[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx^2)^4 \coth^{-1}(ax)\,dx &= c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}\\ &= c^4x\coth^{-1}(ax)+\frac{4}{3}c^3dx^3\coth^{-1}(ax)+\frac{6}{5}c^2d^2x^5\coth^{-1}(ax)+\frac{4}{7}cd^3x^7\coth^{-1}\\ &= \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d}{1260a^5}\\ &= \frac{d(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3)\,x^2}{630a^7}+\frac{d^2(378a^4c^2+180a^2cd+35d}{1260a^5}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 213, normalized size = 0.87

$$\frac{a^2dx^2(420d^3+30a^2d^2(72c+7dx^2)+4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+240cd^2x^4+35d^3x^6))+24a^9x(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8)\coth^{-1}(ax)+12(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\log(1-a^2x^2)}{7560a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270
*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*
d^3*x^6 + 35*d^4*x^8)*ArcCoth[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*
a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

Maple [A]
time = 0.09, size = 301, normalized size = 1.23

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^4ax+\frac{4a\operatorname{arccoth}(ax)c^3dx^3}{3}+\frac{6a\operatorname{arccoth}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccoth}(ax)cd^3x^7}{7}+\frac{a\operatorname{arccoth}(ax)d^4x^9}{9}+-\frac{(-315a^8c^4-420}{}}{}$
default	$\frac{\operatorname{arccoth}(ax)c^4ax+\frac{4a\operatorname{arccoth}(ax)c^3dx^3}{3}+\frac{6a\operatorname{arccoth}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccoth}(ax)cd^3x^7}{7}+\frac{a\operatorname{arccoth}(ax)d^4x^9}{9}+-\frac{(-315a^8c^4-420}{}}{}$

risch	$\left(\frac{1}{18}d^4x^9+\frac{2}{7}d^3c\,x^7+\frac{3}{5}c^2d^2x^5+\frac{2}{3}c^3d\,x^3+\frac{1}{2}c^4x\right)\ln(ax+1)-\frac{d^4x^9\ln(ax-1)}{18}-\frac{2c\,d^3x^7\ln(ax-1)}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)*c^4*a*x+4/3*a*arccoth(a*x)*c^3*d*x^3+6/5*a*arccoth(a*x)*c
^2*d^2*x^5+4/7*a*arccoth(a*x)*c*d^3*x^7+1/9*a*arccoth(a*x)*d^4*x^9+1/315/a^
8*(-1/2*(-315*a^8*c^4-420*a^6*c^3*d-378*a^4*c^2*d^2-180*a^2*c*d^3-35*d^4)*l
n(a*x+1)+1/2*(315*a^8*c^4+420*a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^
4)*ln(a*x-1)+35/6*d^4*a^6*x^6+35/2*d^4*a^2*x^2+45*a^6*c*d^3*x^4+90*a^4*c*d^
3*x^2+189*c^2*a^6*d^2*x^2+30*c*a^8*d^3*x^6+189/2*c^2*a^8*d^2*x^4+210*c^3*a^
8*d*x^2+35/4*d^4*a^4*x^4+35/8*d^4*a^8*x^8))
```

Maxima [A]
time = 0.26, size = 276, normalized size = 1.13

$$\frac{1}{7560}a\left(\frac{105\,a^6d^4x^8+20\,(36\,a^6cd^3+7\,a^4d^4)x^6+6\,(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^4+12\,(420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)x^2}{a^8}+\frac{12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log(ax+1)}{a^{10}}+\frac{12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log(ax-1)}{a^{10}}\right)+\frac{1}{315}\,(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x)\,\mathrm{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8
*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x -
1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*
x^3 + 315*c^4*x)*arccoth(a*x)
```

Fricas [A]
time = 0.35, size = 247, normalized size = 1.01

$$\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3+7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2+180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d+378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4)x^2+12\,(315\,a^8c^4+420\,a^6c^3d+378\,a^4c^2d^2+180\,a^2cd^3+35\,d^4)\log(a^2x^2-1)+12\,(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x)\log\left(\frac{ax+1}{ax-1}\right)}{7560\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 37
8*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d^4*x
^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*
c^4*x)*log((a*x + 1)/(a*x - 1)))/a^9
```

Sympy [C] Result contains complex when optimal does not.
time = 1.04, size = 427, normalized size = 1.74

$$\begin{cases} c^4x\operatorname{acoth}(ax) + \frac{4c^3dx^3\operatorname{acoth}(ax)}{3} + \frac{6c^2d^2x^5\operatorname{acoth}(ax)}{5} + \frac{4cd^3x^7\operatorname{acoth}(ax)}{7} + \frac{d^4x^9\operatorname{acoth}(ax)}{9} + \frac{c^4\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^4\operatorname{acoth}(ax)}{a} + \frac{2c^3dx^2}{3a} + \frac{3c^2d^2x^4}{10a} + \frac{2cd^3x^6}{21a} + \frac{d^4x^8}{72a} + \frac{4c^3d\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{4c^3d\operatorname{acoth}(ax)}{3a^3} + \frac{3c^2d^2x^2}{5a^3} + \frac{cd^3x^4}{7a^3} + \frac{d^4x^6}{54a^3} + \frac{6c^2d^2\log\left(x-\frac{1}{a}\right)}{5a^5} + \frac{6c^2d^2\operatorname{acoth}(ax)}{5a^5} + \frac{2cd^3x^2}{7a^5} + \frac{d^4x^4}{36a^5} + \frac{4cd^3\log\left(x-\frac{1}{a}\right)}{7a^7} + \frac{4cd^3\operatorname{acoth}(ax)}{7a^7} + \frac{d^4x^2}{18a^7} + \frac{d^4\log\left(x-\frac{1}{a}\right)}{9a^9} + \frac{d^4\operatorname{acoth}(ax)}{9a^9} & \text{for}\: a \neq 0 \\ \frac{i\pi\left(c^4x+\frac{4c^3dx^3}{3}+\frac{6c^2d^2x^5}{5}+\frac{4cd^3x^7}{7}+\frac{d^4x^9}{9}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acoth(a*x),x)
```

```
[Out] Piecewise((c**4*x*acoth(a*x) + 4*c**3*d*x**3*acoth(a*x)/3 + 6*c**2*d**2*x**
5*acoth(a*x)/5 + 4*c*d**3*x**7*acoth(a*x)/7 + d**4*x**9*acoth(a*x)/9 + c**4
*log(x - 1/a)/a + c**4*acoth(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**
4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/
(3*a**3) + 4*c**3*d*acoth(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**
3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
 6*c**2*d**2*acoth(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*acoth(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*acoth(a*x)/(9*a**9), Ne
(a, 0)), (I*pi*(c**4*x + 4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x*
*7/7 + d**4*x**9/9)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(227) = 454$.
time = 0.44, size = 1473, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/945*a*(3*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 +
 35*d^4)*log(abs(a*x + 1)/abs(a*x - 1))/a^10 - 3*(315*a^8*c^4 + 420*a^6*c^3
*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(abs((a*x + 1)/(a*x - 1)
- 1))/a^10 + 8*(3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d^4
)*(a*x + 1)^7/(a*x - 1)^7 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^
3 + 7*d^4)*(a*x + 1)^6/(a*x - 1)^6 + (4725*a^6*c^3*d + 6237*a^4*c^2*d^2 + 3
555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^5/(a*x - 1)^5 - 2*(3150*a^6*c^3*d + 3969
*a^4*c^2*d^2 + 2340*a^2*c*d^3 + 455*d^4)*(a*x + 1)^4/(a*x - 1)^4 + (4725*a^
6*c^3*d + 6237*a^4*c^2*d^2 + 3555*a^2*c*d^3 + 875*d^4)*(a*x + 1)^3/(a*x - 1
)^3 - 45*(42*a^6*c^3*d + 63*a^4*c^2*d^2 + 36*a^2*c*d^3 + 7*d^4)*(a*x + 1)^2
/(a*x - 1)^2 + 3*(105*a^6*c^3*d + 189*a^4*c^2*d^2 + 135*a^2*c*d^3 + 35*d^4)
*(a*x + 1)/(a*x - 1))/(a^10*((a*x + 1)/(a*x - 1) - 1)^8) + 3*(315*(a*x + 1)
^8*a^8*c^4/(a*x - 1)^8 - 2520*(a*x + 1)^7*a^8*c^4/(a*x - 1)^7 + 8820*(a*x +
 1)^6*a^8*c^4/(a*x - 1)^6 - 17640*(a*x + 1)^5*a^8*c^4/(a*x - 1)^5 + 22050*(
a*x + 1)^4*a^8*c^4/(a*x - 1)^4 - 17640*(a*x + 1)^3*a^8*c^4/(a*x - 1)^3 + 88
20*(a*x + 1)^2*a^8*c^4/(a*x - 1)^2 - 2520*(a*x + 1)*a^8*c^4/(a*x - 1) + 315
```

```
*a^8*c^4 + 1260*(a*x + 1)^8*a^6*c^3*d/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^6*c^
3*d/(a*x - 1)^7 + 19320*(a*x + 1)^6*a^6*c^3*d/(a*x - 1)^6 - 27720*(a*x + 1)
^5*a^6*c^3*d/(a*x - 1)^5 + 25200*(a*x + 1)^4*a^6*c^3*d/(a*x - 1)^4 - 15960*
(a*x + 1)^3*a^6*c^3*d/(a*x - 1)^3 + 7560*(a*x + 1)^2*a^6*c^3*d/(a*x - 1)^2
- 2520*(a*x + 1)*a^6*c^3*d/(a*x - 1) + 420*a^6*c^3*d + 1890*(a*x + 1)^8*a^4
*c^2*d^2/(a*x - 1)^8 - 7560*(a*x + 1)^7*a^4*c^2*d^2/(a*x - 1)^7 + 15120*(a*
x + 1)^6*a^4*c^2*d^2/(a*x - 1)^6 - 22680*(a*x + 1)^5*a^4*c^2*d^2/(a*x - 1)^
5 + 24948*(a*x + 1)^4*a^4*c^2*d^2/(a*x - 1)^4 - 16632*(a*x + 1)^3*a^4*c^2*d
^2/(a*x - 1)^3 + 6048*(a*x + 1)^2*a^4*c^2*d^2/(a*x - 1)^2 - 1512*(a*x + 1)*
a^4*c^2*d^2/(a*x - 1) + 378*a^4*c^2*d^2 + 1260*(a*x + 1)^8*a^2*c*d^3/(a*x -
 1)^8 - 2520*(a*x + 1)^7*a^2*c*d^3/(a*x - 1)^7 + 7560*(a*x + 1)^6*a^2*c*d^3
/(a*x - 1)^6 - 12600*(a*x + 1)^5*a^2*c*d^3/(a*x - 1)^5 + 10080*(a*x + 1)^4*
a^2*c*d^3/(a*x - 1)^4 - 7560*(a*x + 1)^3*a^2*c*d^3/(a*x - 1)^3 + 3960*(a*x
+ 1)^2*a^2*c*d^3/(a*x - 1)^2 - 360*(a*x + 1)*a^2*c*d^3/(a*x - 1) + 180*a^2*
c*d^3 + 315*(a*x + 1)^8*d^4/(a*x - 1)^8 + 2940*(a*x + 1)^6*d^4/(a*x - 1)^6
+ 4410*(a*x + 1)^4*d^4/(a*x - 1)^4 + 1260*(a*x + 1)^2*d^4/(a*x - 1)^2 + 35*
d^4)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(
((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^10*((a*x
 + 1)/(a*x - 1) - 1)^9))
```

Mupad [B]
time = 1.51, size = 296, normalized size = 1.21

$$\ln\left(\frac{1}{a\,x}+1\right)\,\left(\frac{c^4\,x}{2}+\frac{2\,c^3\,d\,x^3}{3}+\frac{3\,c^2\,d^2\,x^5}{5}+\frac{2\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{18}\right)-\ln\left(1-\frac{1}{a\,x}\right)\,\left(\frac{c^4\,x}{2}+\frac{2\,c^3\,d\,x^3}{3}+\frac{3\,c^2\,d^2\,x^5}{5}+\frac{2\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{18}\right)+x^2\,\left(\frac{\frac{\frac{d^4}{9\,a^3}+\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}+\frac{2\,c^3\,d}{3\,a}\right)+x^6\,\left(\frac{d^4}{54\,a^3}+\frac{2\,c\,d^3}{21\,a}\right)+x^4\,\left(\frac{\frac{d^4}{9\,a^3}+\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)+\frac{\ln\left(a^2\,x^2-1\right)\,\left(315\,a^8\,c^4+420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2+180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}+\frac{d^4\,x^8}{72\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)*(c + d*x^2)^4,x)
```

```
[Out] log(1/(a*x) + 1)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7
)/7 + (3*c^2*d^2*x^5)/5) - log(1 - 1/(a*x))*((c^4*x)/2 + (d^4*x^9)/18 + (2*
c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/5) + x^2*(((d^4/(9*a^3) +
(4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (2*c^3*d)/(3*a)) + x^6*
(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a))/(4
*a^2) + (3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4 + 180
*a^2*c*d^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

3.36 $\int (c+dx^2)^3 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=169

$$\frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5}+\frac{d^2(21a^2c+5d)\,x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}$$

```
[Out] 1/70*d*(35*a^4*c^2+21*a^2*c*d+5*d^2)*x^2/a^5+1/140*d^2*(21*a^2*c+5*d)*x^4/a
^3+1/42*d^3*x^6/a+c^3*x*arccoth(a*x)+c^2*d*x^3*arccoth(a*x)+3/5*c*d^2*x^5*a
rccoth(a*x)+1/7*d^3*x^7*arccoth(a*x)+1/70*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c
*d^2+5*d^3)*ln(-a^2*x^2+1)/a^7
```

Rubi [A]
time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {200, 6124, 1824, 266}

$$\frac{d^2x^4(21a^2c+5d)}{140a^3}+\frac{dx^2(35a^4c^2+21a^2cd+5d^2)}{70a^5}+\frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log(1-a^2x^2)}{70a^7}+c^3x\coth^{-1}(ax)+c^2dx^3\coth^{-1}(ax)+\frac{3}{5}cd^2x^5\coth^{-1}(ax)+\frac{1}{7}d^3x^7\coth^{-1}(ax)+\frac{d^3x^6}{42a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCoth[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcCoth[a*x] + c^2*d*x^3*ArcCoth[
a*x] + (3*c*d^2*x^5*ArcCoth[a*x])/5 + (d^3*x^7*ArcCoth[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \coth^{-1}(ax)\,dx = c^3x\coth^{-1}(ax) + c^2dx^3\coth^{-1}(ax) + \frac{3}{5}cd^2x^5\coth^{-1}(ax) + \frac{1}{7}d^3x^7\coth^{-1}(ax)$$
$$= c^3x\coth^{-1}(ax) + c^2dx^3\coth^{-1}(ax) + \frac{3}{5}cd^2x^5\coth^{-1}(ax) + \frac{1}{7}d^3x^7\coth^{-1}(ax)$$
$$= \frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5} + \frac{d^2(21a^2c+5d)\,x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\coth^{-1}(ax) +$$
$$= \frac{d(35a^4c^2+21a^2cd+5d^2)\,x^2}{70a^5} + \frac{d^2(21a^2c+5d)\,x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x\coth^{-1}(ax) +$$

Mathematica [A]
time = 0.05, size = 150, normalized size = 0.89

$$\frac{a^2dx^2(30d^2+3a^2d(42c+5dx^2)+a^4(210c^2+63cdx^2+10d^2x^4))+12a^7x(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6)\coth^{-1}(ax)+6(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\log\left(1-a^2x^2\right)}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcCoth[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

Maple [A]
time = 0.09, size = 211, normalized size = 1.25

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^3ax+a\operatorname{arccoth}(ax)c^2dx^3+\frac{3a\operatorname{arccoth}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccoth}(ax)d^3x^7}{7}+\frac{-\frac{\left(-35a^6c^3-35a^4c^2d-21a^2c\,d^2-5d^3\right)\ln(a}{2}}{}}{a}$
default	$\frac{\operatorname{arccoth}(ax)c^3ax+a\operatorname{arccoth}(ax)c^2dx^3+\frac{3a\operatorname{arccoth}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccoth}(ax)d^3x^7}{7}+\frac{-\frac{\left(-35a^6c^3-35a^4c^2d-21a^2c\,d^2-5d^3\right)\ln(a}{2}}{}}{a}$
risch	$\left(\frac{1}{14}d^3x^7+\frac{3}{10}c\,d^2x^5+\frac{1}{2}c^2d\,x^3+\frac{1}{2}c^3x\right)\ln\left(ax+1\right)-\frac{d^3x^7\ln(ax-1)}{14}-\frac{3c\,d^2x^5\ln(ax-1)}{10}+\frac{d^3x^6}{42a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)*c^3*a*x+a*arccoth(a*x)*c^2*d*x^3+3/5*a*arccoth(a*x)*c*d^2
*x^5+1/7*a*arccoth(a*x)*d^3*x^7+1/35/a^6*(-1/2*(-35*a^6*c^3-35*a^4*c^2*d-21
```

```
*a^2*c*d^2-5*d^3)*ln(a*x+1)+1/2*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c*d^2+5*d^3
)*ln(a*x-1)+5/4*d^3*a^4*x^4+5/2*d^3*a^2*x^2+21/2*c*a^4*d^2*x^2+21/4*c*a^6*d
^2*x^4+35/2*c^2*a^6*d*x^2+5/6*d^3*a^6*x^6))
```

Maxima [A]
time = 0.26, size = 198, normalized size = 1.17

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\,(21\,a^4cd^2+5\,a^2d^3)x^4+6\,(35\,a^4c^2d+21\,a^2cd^2+5\,d^3)x^2}{a^6}+\frac{6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log\left(ax+1\right)}{a^8}+\frac{6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log\left(ax-1\right)}{a^8}\right)+\frac{1}{35}\left(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x\right)\operatorname{arcoth}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*
d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*
x^3 + 35*c^3*x)*arccoth(a*x)
```

Fricas [A]
time = 0.36, size = 177, normalized size = 1.05

$$\frac{10\,a^6d^3x^6+3\,(21\,a^6cd^2+5\,a^4d^3)x^4+6\,(35\,a^6c^2d+21\,a^4cd^2+5\,a^2d^3)x^2+6\,(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\log\left(a^2x^2-1\right)+6\,(5\,a^7d^3x^7+21\,a^7cd^2x^5+35\,a^7c^2dx^3+35\,a^7c^3x)\log\left(\frac{ax+1}{ax-1}\right)}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7
*c^2*d*x^3 + 35*a^7*c^3*x)*log((a*x + 1)/(a*x - 1)))/a^7
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 282, normalized size = 1.67

$$\begin{cases} c^3x\operatorname{acoth}\left(ax\right)+c^2dx^3\operatorname{acoth}\left(ax\right)+\frac{3cd^2x^5\operatorname{acoth}\left(ax\right)}{5}+\frac{d^3x^7\operatorname{acoth}\left(ax\right)}{7}+\frac{c^3\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^3\operatorname{acoth}\left(ax\right)}{a}+\frac{c^2dx^2}{2a}+\frac{3cd^2x^4}{20a}+\frac{d^3x^6}{42a}+\frac{c^2d\log\left(x-\frac{1}{a}\right)}{a^3}+\frac{c^2d\operatorname{acoth}\left(ax\right)}{a^3}+\frac{3cd^2x^2}{10a^3}+\frac{d^3x^4}{28a^3}+\frac{3cd^2\log\left(x-\frac{1}{a}\right)}{5a^5}+\frac{3cd^2\operatorname{acoth}\left(ax\right)}{5a^5}+\frac{d^3x^2}{14a^5}+\frac{d^3\log\left(x-\frac{1}{a}\right)}{7a^7}+\frac{d^3\operatorname{acoth}\left(ax\right)}{7a^7} & \text{for}\ a\neq 0 \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acoth(a*x),x)
```

```
[Out] Piecewise((c**3*x*acoth(a*x) + c**2*d*x**3*acoth(a*x) + 3*c*d**2*x**5*acoth
(a*x)/5 + d**3*x**7*acoth(a*x)/7 + c**3*log(x - 1/a)/a + c**3*acoth(a*x)/a
+ c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(
x - 1/a)/a**3 + c**2*d*acoth(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**
4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*acoth(a*x)/(5*a**5)
 + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*acoth(a*x)/(7*a*
```

```
*7), Ne(a, 0)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7
)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(157) = 314.
time = 0.41, size = 932, normalized size = 5.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(a*x +
 1)/abs(a*x - 1))/a^8 - 3*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3
)*log(abs((a*x + 1)/(a*x - 1) - 1))/a^8 + 2*(3*(35*a^4*c^2*d + 42*a^2*c*d^2
 + 15*d^3)*(a*x + 1)^5/(a*x - 1)^5 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^
3)*(a*x + 1)^4/(a*x - 1)^4 + 2*(315*a^4*c^2*d + 252*a^2*c*d^2 + 85*d^3)*(a*
x + 1)^3/(a*x - 1)^3 - 6*(70*a^4*c^2*d + 63*a^2*c*d^2 + 15*d^3)*(a*x + 1)^2
/(a*x - 1)^2 + 3*(35*a^4*c^2*d + 42*a^2*c*d^2 + 15*d^3)*(a*x + 1)/(a*x - 1)
)/(a^8*((a*x + 1)/(a*x - 1) - 1)^6) + 3*(35*(a*x + 1)^6*a^6*c^3/(a*x - 1)^6
 - 210*(a*x + 1)^5*a^6*c^3/(a*x - 1)^5 + 525*(a*x + 1)^4*a^6*c^3/(a*x - 1)^
4 - 700*(a*x + 1)^3*a^6*c^3/(a*x - 1)^3 + 525*(a*x + 1)^2*a^6*c^3/(a*x - 1)
^2 - 210*(a*x + 1)*a^6*c^3/(a*x - 1) + 35*a^6*c^3 + 105*(a*x + 1)^6*a^4*c^2
*d/(a*x - 1)^6 - 420*(a*x + 1)^5*a^4*c^2*d/(a*x - 1)^5 + 665*(a*x + 1)^4*a^
4*c^2*d/(a*x - 1)^4 - 560*(a*x + 1)^3*a^4*c^2*d/(a*x - 1)^3 + 315*(a*x + 1)
^2*a^4*c^2*d/(a*x - 1)^2 - 140*(a*x + 1)*a^4*c^2*d/(a*x - 1) + 35*a^4*c^2*d
 + 105*(a*x + 1)^6*a^2*c*d^2/(a*x - 1)^6 - 210*(a*x + 1)^5*a^2*c*d^2/(a*x -
 1)^5 + 315*(a*x + 1)^4*a^2*c*d^2/(a*x - 1)^4 - 420*(a*x + 1)^3*a^2*c*d^2/(
a*x - 1)^3 + 231*(a*x + 1)^2*a^2*c*d^2/(a*x - 1)^2 - 42*(a*x + 1)*a^2*c*d^2
/(a*x - 1) + 21*a^2*c*d^2 + 35*(a*x + 1)^6*d^3/(a*x - 1)^6 + 175*(a*x + 1)^
4*d^3/(a*x - 1)^4 + 105*(a*x + 1)^2*d^3/(a*x - 1)^2 + 5*d^3)*log(-(((a*x +
1)*a/(a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x -
 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^8*((a*x + 1)/(a*x - 1) - 1)
^7))
```

Mupad [B]
time = 1.52, size = 190, normalized size = 1.12

$$c^3\,x\,\mathrm{acoth}(a\,x)+\frac{d^3\,x^7\,\mathrm{acoth}(a\,x)}{7}+\frac{c^3\,\ln\left(a^2\,x^2-1\right)}{2\,a}+\frac{d^3\,\ln\left(a^2\,x^2-1\right)}{14\,a^7}+\frac{d^3\,x^6}{42\,a}+\frac{d^3\,x^4}{28\,a^3}+\frac{d^3\,x^2}{14\,a^5}+\frac{c^2\,d\,\ln\left(a^2\,x^2-1\right)}{2\,a^3}+\frac{3\,c\,d^2\,\ln\left(a^2\,x^2-1\right)}{10\,a^5}+\frac{c^2\,d\,x^2}{2\,a}+\frac{3\,c\,d^2\,x^4}{20\,a}+\frac{3\,c\,d^2\,x^2}{10\,a^3}+c^2\,d\,x^3\,\mathrm{acoth}(a\,x)+\frac{3\,c\,d^2\,x^5\,\mathrm{acoth}(a\,x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)*(c + d*x^2)^3,x)
```

```
[Out] c^3*x*acoth(a*x) + (d^3*x^7*acoth(a*x))/7 + (c^3*log(a^2*x^2 - 1))/(2*a) +
(d^3*log(a^2*x^2 - 1))/(14*a^7) + (d^3*x^6)/(42*a) + (d^3*x^4)/(28*a^3) + (
d^3*x^2)/(14*a^5) + (c^2*d*log(a^2*x^2 - 1))/(2*a^3) + (3*c*d^2*log(a^2*x^2
 - 1))/(10*a^5) + (c^2*d*x^2)/(2*a) + (3*c*d^2*x^4)/(20*a) + (3*c*d^2*x^2)/
(10*a^3) + c^2*d*x^3*acoth(a*x) + (3*c*d^2*x^5*acoth(a*x))/5
```

3.37 $\int (c+dx^2)^2 \coth^{-1}(ax)\,dx$

Optimal. Leaf size=110

$$\frac{d(10a^2c+3d)\,x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)+\frac{(15a^4c^2+10a^2cd+3d^2)\,l}{30a^5}$$

```
[Out] 1/30*d*(10*a^2*c+3*d)*x^2/a^3+1/20*d^2*x^4/a+c^2*x*arccoth(a*x)+2/3*c*d*x^3
*arccoth(a*x)+1/5*d^2*x^5*arccoth(a*x)+1/30*(15*a^4*c^2+10*a^2*c*d+3*d^2)*l
n(-a^2*x^2+1)/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {200, 6124, 1608, 1261, 712}

$$\frac{dx^2(10a^2c+3d)}{30a^3}+\frac{(15a^4c^2+10a^2cd+3d^2)\log(1-a^2x^2)}{30a^5}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)+\frac{d^2x^4}{20a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCoth[a*x],x]
```

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcCoth[a*x] +
 (2*c*d*x^3*ArcCoth[a*x])/3 + (d^2*x^5*ArcCoth[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^2\right)^2\coth^{-1}(ax)\,dx &= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-a\int\frac{c^2x+\frac{2}{3}cdx^3+}{1-a^2x^2}\\ &= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-a\int\frac{x\left(c^2+\frac{2}{3}cdx^2\right.}{1-a^2x}\\ &= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int\frac{c^2+}{1}\right.\\ &= c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int\left(-\frac{d}{}\right.\right.\\ &= \frac{d(10a^2c+3d)\,x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\coth^{-1}(ax)+\frac{2}{3}cdx^3\coth^{-1}(ax)+\frac{1}{5}d^2x^5\coth\end{aligned}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.89

$$\frac{a^2dx^2(6d+a^2(20c+3dx^2))+4a^5x(15c^2+10cdx^2+3d^2x^4)\coth^{-1}(ax)+(30a^4c^2+20a^2cd+6d^2)\log\left(1-a^2x^2\right)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCoth[a*x],x]
```

```
[Out] (a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3*
d^2*x^4)*ArcCoth[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2])
/(60*a^5)
```

Maple [A]
time = 0.14, size = 137, normalized size = 1.25

method	result
derivativedivides	$\frac{\operatorname{arccoth}(ax)c^2ax+\frac{2a\operatorname{arccoth}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccoth}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}+\frac{3d^2a^2x^2}{2}+\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\ln(ax-1)}{2}}{15a^4}}{a}$
default	$\frac{\operatorname{arccoth}(ax)c^2ax+\frac{2a\operatorname{arccoth}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccoth}(ax)d^2x^5}{5}+\frac{5c\,a^4d\,x^2+\frac{3d^2a^4x^4}{4}+\frac{3d^2a^2x^2}{2}+\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\ln(ax-1)}{2}}{15a^4}}{a}$
risch	$\left(\frac{1}{10}d^2x^5+\frac{1}{3}cd\,x^3+\frac{1}{2}c^2x\right)\ln(ax+1)-\frac{d^2x^5\ln(ax-1)}{10}-\frac{cd\,x^3\ln(ax-1)}{3}+\frac{d^2x^4}{20a}-\frac{c^2x\ln(ax-1)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)*c^2*a*x+2/3*a*arccoth(a*x)*c*d*x^3+1/5*a*arccoth(a*x)*d^2
*x^5+1/15/a^4*(5*c*a^4*d*x^2+3/4*d^2*a^4*x^4+3/2*d^2*a^2*x^2+1/2*(15*a^4*c^
2+10*a^2*c*d+3*d^2)*ln(a*x-1)-1/2*(-15*a^4*c^2-10*a^2*c*d-3*d^2)*ln(a*x+1))
)
```

Maxima [A]
time = 0.26, size = 131, normalized size = 1.19

$$\frac{1}{60}a\left(\frac{3a^2d^2x^4+2(10a^2cd+3d^2)x^2}{a^4}+\frac{2(15a^4c^2+10a^2cd+3d^2)\log(ax+1)}{a^6}+\frac{2(15a^4c^2+10a^2cd+3d^2)\log(ax-1)}{a^6}\right)+\frac{1}{15}(3d^2x^5+10cdx^3+15c^2x)\operatorname{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*
log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arccoth(a*x)
```

Fricas [A]
time = 0.34, size = 118, normalized size = 1.07

$$\frac{3a^4d^2x^4+2(10a^4cd+3a^2d^2)x^2+2(15a^4c^2+10a^2cd+3d^2)\log(a^2x^2-1)+2(3a^5d^2x^5+10a^5cdx^3+15a^5c^2x)\log\left(\frac{ax+1}{ax-1}\right)}{60a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a
^5*c^2*x)*log((a*x + 1)/(a*x - 1)))/a^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 182, normalized size = 1.65

$$\begin{cases} c^2x\operatorname{acoth}(ax)+\frac{2cdx^3\operatorname{acoth}(ax)}{3}+\frac{d^2x^5\operatorname{acoth}(ax)}{5}+\frac{c^2\log\left(x-\frac{1}{a}\right)}{a}+\frac{c^2\operatorname{acoth}(ax)}{a}+\frac{cdx^2}{3a}+\frac{d^2x^4}{20a}+\frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3}+\frac{2cd\operatorname{acoth}(ax)}{3a^3}+\frac{d^2x^2}{10a^3}+\frac{d^2\log\left(x-\frac{1}{a}\right)}{5a^5}+\frac{d^2\operatorname{acoth}(ax)}{5a^5} & \text{for}\ a\neq 0\\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acoth(a*x),x)
```

```
[Out] Piecewise((c**2*x*acoth(a*x) + 2*c*d*x**3*acoth(a*x)/3 + d**2*x**5*acoth(a*
x)/5 + c**2*log(x - 1/a)/a + c**2*acoth(a*x)/a + c*d*x**2/(3*a) + d**2*x**4
/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*acoth(a*x)/(3*a**3) + d**2*x*
*2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*acoth(a*x)/(5*a**5), Ne(a,
 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(100) = 200$.
time $= 0.41$, size $= 529$, normalized size $= 4.81$

$$\frac{1}{15}\,a\left(\frac{(15\,a^4c^2+10\,a^2cd+3\,d^2)\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^6}-\frac{(15\,a^4c^2+10\,a^2cd+3\,d^2)\log\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}{a^6}+\frac{4\left(\frac{(5\,a^2cd+3\,d^2)(ax+1)^3}{(ax-1)^3}-\frac{(10\,a^2cd+3\,d^2)(ax+1)^2}{(ax-1)^2}+\frac{(5\,a^2cd+3\,d^2)(ax+1)}{ax-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^4}+\frac{\left(\frac{15\,(ax+1)^4a^4c^2}{(ax-1)^4}-\frac{60\,(ax+1)^3a^4c^2}{(ax-1)^3}+\frac{90\,(ax+1)^2a^4c^2}{(ax-1)^2}-\frac{60\,(ax+1)a^4c^2}{ax-1}+15\,a^4c^2+\frac{30\,(ax+1)^4a^2cd}{(ax-1)^4}-\frac{60\,(ax+1)^3a^2cd}{(ax-1)^3}+\frac{40\,(ax+1)^2a^2cd}{(ax-1)^2}-\frac{20\,(ax+1)a^2cd}{ax-1}+10\,a^2cd+\frac{15\,(ax+1)^4d^2}{(ax-1)^4}+\frac{30\,(ax+1)^2d^2}{(ax-1)^2}+3\,d^2\right)\log\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}{a^6\left(\frac{ax+1}{ax-1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs(a*x + 1)/abs(a*x - 1))/a^
6 - (15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*log(abs((a*x + 1)/(a*x - 1) - 1))/a^6
 + 4*((5*a^2*c*d + 3*d^2)*(a*x + 1)^3/(a*x - 1)^3 - (10*a^2*c*d + 3*d^2)*(a
*x + 1)^2/(a*x - 1)^2 + (5*a^2*c*d + 3*d^2)*(a*x + 1)/(a*x - 1))/(a^6*((a*x
 + 1)/(a*x - 1) - 1)^4) + (15*(a*x + 1)^4*a^4*c^2/(a*x - 1)^4 - 60*(a*x + 1
)^3*a^4*c^2/(a*x - 1)^3 + 90*(a*x + 1)^2*a^4*c^2/(a*x - 1)^2 - 60*(a*x + 1)
*a^4*c^2/(a*x - 1) + 15*a^4*c^2 + 30*(a*x + 1)^4*a^2*c*d/(a*x - 1)^4 - 60*(
a*x + 1)^3*a^2*c*d/(a*x - 1)^3 + 40*(a*x + 1)^2*a^2*c*d/(a*x - 1)^2 - 20*(a
*x + 1)*a^2*c*d/(a*x - 1) + 10*a^2*c*d + 15*(a*x + 1)^4*d^2/(a*x - 1)^4 + 3
0*(a*x + 1)^2*d^2/(a*x - 1)^2 + 3*d^2)*log(-(((a*x + 1)*a/(a*x - 1) - a)/(a
*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) - a)/(a*((a*x + 1)
/(a*x - 1) + 1)) - 1))/(a^6*((a*x + 1)/(a*x - 1) - 1)^5))
```

Mupad [B]
time $= 1.37$, size $= 115$, normalized size $= 1.05$

$$\frac{a^4\left(\frac{c^2\,\ln\left(a^2\,x^2-1\right)}{2}+\frac{d^2\,x^4}{20}+\frac{c\,d\,x^2}{3}\right)+a^2\left(\frac{d^2\,x^2}{10}+\frac{c\,d\,\ln\left(a^2\,x^2-1\right)}{3}\right)+\frac{d^2\,\ln\left(a^2\,x^2-1\right)}{10}}{a^5}+c^2\,x\,\mathrm{acoth}(a\,x)+\frac{d^2\,x^5\,\mathrm{acoth}(a\,x)}{5}+\frac{2\,c\,d\,x^3\,\mathrm{acoth}(a\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)*(c + d*x^2)^2,x)
```

```
[Out] (a^4*((c^2*log(a^2*x^2 - 1))/2 + (d^2*x^4)/20 + (c*d*x^2)/3) + a^2*((d^2*x^
2)/10 + (c*d*log(a^2*x^2 - 1))/3) + (d^2*log(a^2*x^2 - 1))/10)/a^5 + c^2*x*
acoth(a*x) + (d^2*x^5*acoth(a*x))/5 + (2*c*d*x^3*acoth(a*x))/3
```

3.38 $\int (c+dx^2)\coth^{-1}(ax)\,dx$

Optimal. Leaf size=57

$$\frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}$$

```
[Out] 1/6*d*x^2/a+c*x*arccoth(a*x)+1/3*d*x^3*arccoth(a*x)+1/6*(3*a^2*c+d)*ln(-a^2
*x^2+1)/a^3
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6124, 1607, 455, 45}

$$\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{dx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCoth[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
```

```
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx^2)\coth^{-1}(ax)\,dx &= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-a\int\frac{cx+\frac{dx^3}{3}}{1-a^2x^2}\,dx\\
&= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1-a^2x^2}\,dx\\
&= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int\frac{c+\frac{dx}{3}}{1-a^2x}\,dx,x,x^2\right)\\
&= cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)-\frac{1}{2}a\text{Subst}\left(\int\left(-\frac{d}{3a^2}+\frac{-3a^2c-d}{3a^2(-1+a^2x)}\right)\right.\\
&= \frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{(3a^2c+d)\log(1-a^2x^2)}{6a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.21

$$\frac{dx^2}{6a}+cx\coth^{-1}(ax)+\frac{1}{3}dx^3\coth^{-1}(ax)+\frac{c\log(1-a^2x^2)}{2a}+\frac{d\log(1-a^2x^2)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCoth[a*x],x]
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + (c*Log[1 - a^2*
x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

Maple [A]
time = 0.05, size = 74, normalized size = 1.30

method	result	siz
derivativedivides	$\frac{\mathrm{arccoth}(ax)cax+\frac{a\,\mathrm{arccoth}(ax)d\,x^3}{3}+\frac{\frac{a^2d\,x^2}{2}+\frac{(3a^2c+d)\ln(ax-1)}{2}-\frac{(-3a^2c-d)\ln(ax+1)}{2}}{3a^2}}{a}$	74
default	$\frac{\mathrm{arccoth}(ax)cax+\frac{a\,\mathrm{arccoth}(ax)d\,x^3}{3}+\frac{\frac{a^2d\,x^2}{2}+\frac{(3a^2c+d)\ln(ax-1)}{2}-\frac{(-3a^2c-d)\ln(ax+1)}{2}}{3a^2}}{a}$	74
risch	$\left(\frac{1}{6}d\,x^3+\frac{1}{2}cx\right)\ln(ax+1)-\frac{d\,x^3\ln(ax-1)}{6}-\frac{cx\ln(ax-1)}{2}+\frac{d\,x^2}{6a}+\frac{\ln(a^2x^2-1)c}{2a}+\frac{\ln(a^2x^2-1)d}{6a^3}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccoth(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccoth(a*x)*c*a*x+1/3*a*arccoth(a*x)*d*x^3+1/3/a^2*(1/2*a^2*d*x^2+1/2
*(3*a^2*c+d)*ln(a*x-1)-1/2*(-3*a^2*c-d)*ln(a*x+1)))
```

Maxima [A]
time = 0.26, size = 65, normalized size = 1.14

$$\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{(3\,a^2c+d)\log{(ax+1)}}{a^4}+\frac{(3\,a^2c+d)\log{(ax-1)}}{a^4}\right)+\frac{1}{3}\,(dx^3+3\,cx)\operatorname{arcoth}{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x -
 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arccoth(a*x)
```

Fricas [A]
time = 0.39, size = 64, normalized size = 1.12

$$\frac{a^2dx^2+(3\,a^2c+d)\log{(a^2x^2-1)}+(a^3dx^3+3\,a^3cx)\log{\left(\frac{ax+1}{ax-1}\right)}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*l
og((a*x + 1)/(a*x - 1)))/a^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.23, size = 87, normalized size = 1.53

$$\begin{cases} cx\operatorname{acoth}{(ax)}+\frac{dx^3\operatorname{acoth}{(ax)}}{3}+\frac{c\log{(x-\frac{1}{a})}}{a}+\frac{c\operatorname{acoth}{(ax)}}{a}+\frac{dx^2}{6a}+\frac{d\log{(x-\frac{1}{a})}}{3a^3}+\frac{d\operatorname{acoth}{(ax)}}{3a^3} & \text{for } a\neq 0 \\ \frac{i\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acoth(a*x),x)
```

```
[Out] Piecewise((c*x*acoth(a*x) + d*x**3*acoth(a*x)/3 + c*log(x - 1/a)/a + c*acot
h(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*acoth(a*x)/(3*a**3),
Ne(a, 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(51) = 102.
time = 0.40, size = 268, normalized size = 4.70

$$\frac{1}{3}\,a\left(\frac{(3\,a^2c+d)\log{\left(\frac{|ax+1|}{|ax-1|}\right)}}{a^4}-\frac{(3\,a^2c+d)\log{\left(\left|\frac{ax+1}{ax-1}-1\right|\right)}}{a^4}+\frac{2\,(ax+1)d}{(ax-1)a^4\left(\frac{ax+1}{ax-1}-1\right)^2}+\frac{\left(\frac{3\,(ax+1)^2a^2c}{(ax-1)^2}-\frac{6\,(ax+1)a^2c}{ax-1}+3\,a^2c+\frac{3\,(ax+1)^2d}{(ax-1)^2}+d\right)\log{\left(-\frac{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}+1}{\frac{\frac{(ax+1)a}{ax-1}-a}{a\left(\frac{ax+1}{ax-1}+1\right)}-1}\right)}}{a^4\left(\frac{ax+1}{ax-1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + d)*log(abs(a*x + 1)/abs(a*x - 1))/a^4 - (3*a^2*c + d)*log
(abs((a*x + 1)/(a*x - 1) - 1))/a^4 + 2*(a*x + 1)*d/((a*x - 1)*a^4*((a*x + 1
)/(a*x - 1) - 1)^2) + (3*(a*x + 1)^2*a^2*c/(a*x - 1)^2 - 6*(a*x + 1)*a^2*c/
(a*x - 1) + 3*a^2*c + 3*(a*x + 1)^2*d/(a*x - 1)^2 + d)*log(-(((a*x + 1)*a/(
a*x - 1) - a)/(a*((a*x + 1)/(a*x - 1) + 1)) + 1)/(((a*x + 1)*a/(a*x - 1) -
a)/(a*((a*x + 1)/(a*x - 1) + 1)) - 1))/(a^4*((a*x + 1)/(a*x - 1) - 1)^3))
```

Mupad [B]
time = 1.27, size = 60, normalized size = 1.05

$$\frac{\frac{d\,\ln\left(a^2\,x^2-1\right)}{6}+a^2\,\left(\frac{c\,\ln\left(a^2\,x^2-1\right)}{2}+\frac{d\,x^2}{6}\right)}{a^3}+\frac{d\,x^3\,\mathrm{acoth}\left(a\,x\right)}{3}+c\,x\,\mathrm{acoth}\left(a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)*(c + d*x^2),x)
```

```
[Out] ((d*log(a^2*x^2 - 1))/6 + a^2*((c*log(a^2*x^2 - 1))/2 + (d*x^2)/6))/a^3 + (
d*x^3*acoth(a*x))/3 + c*x*acoth(a*x)
```

3.39 $\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=390

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\,\sqrt{d}\,(1-}{\left(ia\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}\right.}\right.}{2\sqrt{c}\,\sqrt{d}}$$

```
[Out] -1/2*arctan(x*d^(1/2)/c^(1/2))*ln(1-1/a/x)/c^(1/2)/d^(1/2)+1/2*arctan(x*d^(
1/2)/c^(1/2))*ln(1+1/a/x)/c^(1/2)/d^(1/2)+1/2*arctan(x*d^(1/2)/c^(1/2))*ln(
-2*(-a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^
(1/2)/d^(1/2)-1/2*arctan(x*d^(1/2)/c^(1/2))*ln(2*(a*x+1)*c^(1/2)*d^(1/2)/(I
*a*c^(1/2)+d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)-1/4*I*polylog(2,
1+2*(-a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1/2)-d^(1/2))/(c^(1/2)-I*x*d^(1/2)))/c
^(1/2)/d^(1/2)+1/4*I*polylog(2,1-2*(a*x+1)*c^(1/2)*d^(1/2)/(I*a*c^(1/2)+d^(
1/2))/(c^(1/2)-I*x*d^(1/2)))/c^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6120, 211, 2520, 12, 266, 6820, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$-\frac{\log\left(1-\frac{1}{ax}\right)\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\log\left(\frac{1}{ax}+1\right)\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\,\sqrt{d}\,(1-ax)}{\left(-\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}\,\sqrt{d}\,(ax+1)}{\left(\sqrt{d}+ia\sqrt{c}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{2\sqrt{c}\,\sqrt{d}}-\frac{i\operatorname{Li}_2\left(\frac{2\sqrt{c}\,\sqrt{d}\,(1-ax)}{\left(ia\sqrt{c}-\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}+1\right)}{4\sqrt{c}\,\sqrt{d}}+\frac{i\operatorname{Li}_2\left(1-\frac{2\sqrt{c}\,\sqrt{d}\,(ax+1)}{\left(i\sqrt{c}\,a+\sqrt{d}\right)\left(\sqrt{c}-i\sqrt{d}\,x\right)}\right)}{4\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2),x]
```

```
[Out] -1/2*(ArcTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 - 1/(a*x)])/(Sqrt[c]*Sqrt[d]) + (Ar
cTan[(Sqrt[d]*x)/Sqrt[c]]*Log[1 + 1/(a*x)])/(2*Sqrt[c]*Sqrt[d]) + (ArcTan[(
Sqrt[d]*x)/Sqrt[c]]*Log[(-2*Sqrt[c]*Sqrt[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt
[d])*(Sqrt[c] - I*Sqrt[d]*x))]])/(2*Sqrt[c]*Sqrt[d]) - (ArcTan[(Sqrt[d]*x)/S
qrt[c]]*Log[(2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sqrt[c] + Sqrt[d])*(Sqrt[c]
 - I*Sqrt[d]*x))]])/(2*Sqrt[c]*Sqrt[d]) - ((I/4)*PolyLog[2, 1 + (2*Sqrt[c]*S
qrt[d]*(1 - a*x))/((I*a*Sqrt[c] - Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]])/(Sqrt
[c]*Sqrt[d]) + ((I/4)*PolyLog[2, 1 - (2*Sqrt[c]*Sqrt[d]*(1 + a*x))/((I*a*Sq
rt[c] + Sqrt[d])*(Sqrt[c] - I*Sqrt[d]*x))]])/(Sqrt[c]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6120

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(1-\frac{1}{ax}\right)}{c+dx^2}\,dx\right)+\frac{1}{2}\int \frac{\log\left(1+\frac{1}{ax}\right)}{c+dx^2}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}\,\left(1-\frac{1}{ax}\right)x^2}\,dx}{2a}+$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\left(1-\frac{1}{ax}\right)x^2}\,dx}{2a\sqrt{c}\,\sqrt{d}}+$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\int \frac{a\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x(-1+ax)}\,dx}{2a\sqrt{c}\,\sqrt{d}}+$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x(-1+ax)}\,dx}{2\sqrt{c}\,\sqrt{d}}+$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{x}+\right.}{2\sqrt{c}\,\sqrt{d}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{-1+ax}\,dx}{2\sqrt{c}\,\sqrt{d}}-$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\right.}{2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1-\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(1+\frac{1}{ax}\right)}{2\sqrt{c}\,\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\right.}{2}$$

Mathematica [A]

time = 0.93, size = 671, normalized size = 1.72

$$\frac{a\left(-2i\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+4\coth^{-1}(ax)\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)-\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)+2\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)\right)\log\left(\frac{2d\left(a^2c-i\sqrt{a^2cd}\right)(-1+ax)}{(a^2c+d)\left(i\sqrt{a^2cd}+adx\right)}\right)-\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)-2\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)\right)\log\left(\frac{2d\left(a^2c+i\sqrt{a^2cd}\right)(1+ax)}{(a^2c+d)\left(i\sqrt{a^2cd}+adx\right)}\right)+\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)+2\left(\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}\,e^{-\coth^{-1}(ax)}}{\sqrt{a^2c+d}\sqrt{-a^2c+d+(a^2c+d)\cosh\left(2\coth^{-1}(ax)\right)}}\right)+\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)-2\left(\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}\,e^{\coth^{-1}(ax)}}{\sqrt{a^2c+d}\sqrt{-a^2c+d+(a^2c+d)\cosh\left(2\coth^{-1}(ax)\right)}}\right)+i\left(-\text{PolyLog}\left(2,\frac{\left(a^2c-d-2i\sqrt{a^2cd}\right)\left(\sqrt{a^2cd}+iadx\right)}{(a^2c+d)\left(\sqrt{a^2cd}-iadx\right)}\right)+\text{PolyLog}\left(2,\frac{\left(a^2c-d+2i\sqrt{a^2cd}\right)\left(\sqrt{a^2cd}+iadx\right)}{(a^2c+d)\left(\sqrt{a^2cd}-iadx\right)}\right)\right)\right)}{4\sqrt{a^2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2),x]
```

```
[Out] (a*((-2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]
+ 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^2*c
 + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2*c*
d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos[(a^2*c -
 d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c + I*S
qrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos
[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*
d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcCot
h[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + (ArcCos[(a
^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x
)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*c +
 d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I*(-PolyLog[2
, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d
)*(Sqrt[a^2*c*d] - I*a*d*x))] + PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c*d
])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))])))/(
4*Sqrt[a^2*c*d])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(280) = 560$.
time = 0.66, size = 954, normalized size = 2.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^2/d/(a^4*
c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)
-d))*arccoth(a*x)-1/2*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d
)*a^2/d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2+1/4*(-(-a^2*c*d)^(1/2)*a^2*c
+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^2/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2
*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))-(a^2*c+2*(-a^2*c*d)^(1/
2)-d)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(
a*x)*a^2/(a^4*c^2+2*a^2*c*d+d^2)-1/2*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a
^2*c*d)^(1/2)*d)/c/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(
a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)+(a^2*c+2*(-a^2*c*d)^(1/2)-d)*arcc
oth(a*x)^2*a^2/(a^4*c^2+2*a^2*c*d+d^2)+1/2*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c
*d+(-a^2*c*d)^(1/2)*d)/c/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2-1/2*(a^2*c+
2*(-a^2*c*d)^(1/2)-d)*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*
d)^(1/2)-d))*a^2/(a^4*c^2+2*a^2*c*d+d^2)-1/4*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2
*c*d+(-a^2*c*d)^(1/2)*d)/c/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)/(a*x
```

```
-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))+1/2*(-a^2*c*d)^(1/2)/c/d*arccoth(
a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1/2*(-a^2
*c*d)^(1/2)/c/d*arccoth(a*x)^2+1/4*(-a^2*c*d)^(1/2)/c/d*polylog(2,(a^2*c+d)
/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))
```

Maxima [A]
time = 0.56, size = 406, normalized size = 1.04

$$\frac{\operatorname{arcoth}(ax)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{\left(\arctan\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-\arctan\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right)-\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2\,adx+d}{a^2c+d}\right)+\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2-2\,adx+d}{a^2c+d}\right)-i\,\mathrm{Li}_2\left(\frac{a^2c+adx-(i\,a^2x-i\,a)\sqrt{c}\sqrt{d}}{a^2c+2i\,a\sqrt{c}\sqrt{d}-d}\right)-i\,\mathrm{Li}_2\left(\frac{a^2c-adx+(i\,a^2x+i\,a)\sqrt{c}\sqrt{d}}{a^2c+2i\,a\sqrt{c}\sqrt{d}-d}\right)+i\,\mathrm{Li}_2\left(\frac{a^2c+adx+(i\,a^2x-i\,a)\sqrt{c}\sqrt{d}}{a^2c-2i\,a\sqrt{c}\sqrt{d}-d}\right)+i\,\mathrm{Li}_2\left(\frac{a^2c-adx-(i\,a^2x+i\,a)\sqrt{c}\sqrt{d}}{a^2c-2i\,a\sqrt{c}\sqrt{d}-d}\right)}{4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arccoth(a*x)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + 1/4*((arctan2((a^2*x + a)*sq
rt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - arctan2((a^2*x - a)*s
qrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c) - arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + arctan(
sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*dilog((a^
2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(
d) - d)) - I*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c
 + 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + I*dilog((a^2*c - a*d
*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)))
/sqrt(c*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(arccoth(a*x)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acoth}(a\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2),x)
[Out] int(acoth(a*x)/(c + d*x^2), x)
```

3.40 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=590

$$\frac{x\coth^{-1}(ax)}{2c\,(c+dx^2)}+\frac{\coth^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(-\frac{\sqrt{d}\,(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\text{lo}}{8c^{3/2}\sqrt{d}}$$

```
[Out] 1/2*x*arccoth(a*x)/c/(d*x^2+c)+1/4*a*ln(-a^2*x^2+1)/c/(a^2*c+d)-1/4*a*ln(d*
x^2+c)/c/(a^2*c+d)+1/2*arccoth(a*x)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/
2)-1/8*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2
))/c^(3/2)/d^(1/2)+1/8*I*ln((-a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1-I*
x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)-1/8*I*ln(-(-a*x+1)*d^(1/2)/(I*a*c^(1/2)-
d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*ln((a*x+1)*d^(1/2
)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(3/2)/d^(1/2)+1/8*I*po
lylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1/2)-1/8*
I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/d^(1/2)+
1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)-I*d^(1/2)))/c^(3/2)/d^(1
/2)-1/8*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a*c^(1/2)+I*d^(1/2)))/c^(3/2)/
d^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {205, 211, 6124, 6857, 531, 455, 36, 31, 5028, 2456, 2441, 2440, 2438}

$$\frac{a\log(1-a^2x^2)}{4c(a^2c+d)}-\frac{a\log(c+dx^2)}{4c(a^2c+d)}+\frac{\coth^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(1-ax)}{\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(ax+1)}{-\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(1-ax)}{-\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(ax+1)}{\sqrt{d}+ia\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}+\frac{x\coth^{-1}(ax)}{2c\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcCoth[a*x])/(2*c*(c + d*x^2)) + (ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/
```

```
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2
)*Sqrt[d])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2}\,dx = \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a\int \frac{\frac{x}{2c(c+dx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a\int \left(-\frac{x}{2c(-1+ax)(1+ax)(c+dx^2)}\right.$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+ax)(1+ax)(c+dx^2)}\,dx}{2c} + \frac{a\int \frac{\tan^{-1}\left(\ldots\right)}{-1+a^2\ldots}}{2c^{3/2}\sqrt{\ldots}}$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+a^2x^2)(c+dx^2)}\,dx}{2c} + \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{\ldots}}{\ldots}\right)}{-1+a^2\ldots}}{4c^{3/2}\sqrt{d}\ldots}$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \frac{1}{(-1+a^2x)(c+dx)}\,dx, x, x^2\right)}{4c} + \frac{(ia\ldots}{\ldots}$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{1-ax}\,dx}{8c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{\ldots}}{\ldots}\right)}{1+a\ldots}}{8c^{3/2}\sqrt{\ldots}}$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \ldots$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \ldots$$

$$= \frac{x\coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \ldots$$

Mathematica [A]
time = 5.57, size = 755, normalized size = 1.28

$$-\frac{a\left(\frac{2\log\left(1-\frac{(a^2c+d)\cosh(2\coth^{-1}(ax))}{a^2c-d}\right)}{a^2c+d}+\frac{2i\,\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)-4\coth^{-1}(ax)\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)+\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)+2\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)\right)\log\left(\frac{2d\left(a^2c-i\sqrt{a^2cd}\right)(-1+ax)}{(a^2c+d)\left(i\sqrt{a^2cd}+adx\right)}\right)+\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)-2\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)\right)\log\left(\frac{2d\left(a^2c+i\sqrt{a^2cd}\right)(1+ax)}{(a^2c+d)\left(i\sqrt{a^2cd}+adx\right)}\right)-\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)+2\left(\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}\,e^{-\coth^{-1}(ax)}}{\sqrt{a^2c+d}\sqrt{-a^2c+d+(a^2c+d)\cosh(2\coth^{-1}(ax))}}\right)-\left(\text{ArcCos}\left(\frac{a^2c-d}{a^2c+d}\right)-2\left(\text{ArcTan}\left(\frac{ac}{\sqrt{a^2cd}\,x}\right)+\text{ArcTan}\left(\frac{adx}{\sqrt{a^2cd}}\right)\right)\right)\log\left(\frac{\sqrt{2}\sqrt{a^2cd}\,e^{\coth^{-1}(ax)}}{\sqrt{a^2c+d}\sqrt{-a^2c+d+(a^2c+d)\cosh(2\coth^{-1}(ax))}}\right)+i\left(\text{PolyLog}\left(2,\frac{\left(a^2c-d-2i\sqrt{a^2cd}\right)\left(\sqrt{a^2cd}+iadx\right)}{(a^2c+d)\left(\sqrt{a^2cd}-iadx\right)}\right)-\text{PolyLog}\left(2,\frac{\left(a^2c-d+2i\sqrt{a^2cd}\right)\left(\sqrt{a^2cd}+iadx\right)}{(a^2c+d)\left(\sqrt{a^2cd}-iadx\right)}\right)\right)}{\sqrt{a^2cd}}-\frac{4\coth^{-1}(ax)\sinh(2\coth^{-1}(ax))}{-a^2c+d+(a^2c+d)\cosh(2\coth^{-1}(ax))}\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -1/8*(a*((2*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)])/(a^2*c
 + d) + ((2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*
x)] - 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + (ArcCos[(a^2*c - d)/(a
^2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^
2*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos[(a^2
*c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c +
 I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (Ar
cCos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan
[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^Ar
cCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] - (ArcCo
s[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a
*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2
*c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I*(PolyLo
g[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c
+ d)*(Sqrt[a^2*c*d] - I*a*d*x))] - PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*
c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))]))
/Sqrt[a^2*c*d] - (4*ArcCoth[a*x]*Sinh[2*ArcCoth[a*x]])/(-(a^2*c) + d + (a^2
*c + d)*Cosh[2*ArcCoth[a*x]])))/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2070 vs. 2(430) = 860.
time = 0.79, size = 2071, normalized size = 3.51

method	result	size
risch	Expression too large to display	1926
derivativedivides	Expression too large to display	2071
default	Expression too large to display	2071

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/8*(-a^2*c*d)^(1/2)/c*a^2/d/(a^2*c+d)*polylog(2,(a^2*c+d)/(a*x-1)*(a*
x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))+1/4*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(
-a^2*c*d)^(1/2)*d)/(a^2*c+d)/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2-1
/8*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^2*c+d)/c^2*d/(
a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c
```

```
*d)^(1/2)-d))-1/4*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^
4/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2-1/4*(c*d)^(1/2)/c/d*a^
3*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^2*
c+d)-1/2*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^4*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a
^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)+
1/8*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^4/d/(a^2*c+d)/
(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*
c*d)^(1/2)-d))-1/2*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^2*d*ln(1-(a^2*c+d)/(a*x-1
)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)/c/(a^2*c+d)/(a^4*c^2+2
*a^2*c*d+d^2)-1/4/(a^2*c+d)^2/c*a^2*d*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c/
(a*x-1)*(a*x+1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*d/(a*x-1)*(a*x+1)+d)-1/4*(c*d
)^(1/2)/c^2*d*a*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d
)^(1/2))/(a^2*c+d)^2-1/4*(-a^2*c*d)^(1/2)/c*a^2/d/(a^2*c+d)*arccoth(a*x)^2+
1/4*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^4/d/(a^2*c+d)/
(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^
(1/2)-d))*arccoth(a*x)-1/4*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^2*d*polylog(2,(a^
2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))/c/(a^2*c+d)/(a^4*c^2+2
*a^2*c*d+d^2)+1/2*a^2*arccoth(a*x)*(a^2*c-a*d*x)*(a*x-1)/c/(a^2*c+d)/(a^2*d
*x^2+a^2*c)+1/4*(-a^2*c*d)^(1/2)/c*a^2/d/(a^2*c+d)*arccoth(a*x)*ln(1-(a^2*c
+d)/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1/4*(-(-a^2*c*d)^(1/2)*a^
2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^2*c+d)/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)*l
n(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)+1/
2*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^2*d*arccoth(a*x)^2/c/(a^2*c+d)/(a^4*c^2+2*
a^2*c*d+d^2)-1/2/(a^2*c+d)^2/c*a^2*d*ln((a*x-1)/(a*x+1))-1/4*(a^2*c+2*(-a^2
*c*d)^(1/2)-d)*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2
)-d))*a^4/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)+1/4*(c*d)^(1/2)/c^2*a*arctan(1/
4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^2*c+d)+1/4*(-
a^2*c*d)^(1/2)/(a^2*c+d)/c^2*arccoth(a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a
^2*c+2*(-a^2*c*d)^(1/2)-d))+1/4*(c*d)^(1/2)/d*a^5*arctan(1/4*(2*(a^2*c+d)/(
a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^2*c+d)^2+1/2*(a^2*c+2*(-a^2*c
*d)^(1/2)-d)*arccoth(a*x)^2*a^4/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)-1/2/(a^2*
c+d)^2*a^4*ln((a*x-1)/(a*x+1))-1/4/(a^2*c+d)^2*a^4*ln(a^2*c/(a*x-1)^2*(a*x+
1)^2-2*a^2*c/(a*x-1)*(a*x+1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*d/(a*x-1)*(a*x+1
)+d)+1/8*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)
/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-1/4*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*arccoth(a
*x)^2)
```

Maxima [A]
time = 0.53, size = 550, normalized size = 0.93

$$\frac{1}{2}\left(\frac{x}{cdx^2+c^2}+\frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}\right)\operatorname{arcoth}(ax)-\frac{\left(2\,acd\log\left(dx^2+c\right)-2\,acd\log\left(ax+1\right)-2\,acd\log\left(ax-1\right)+\left(\left(a^2c+d\right)\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2+2\,adx+d}{a^2c+d}\right)-\left(a^2c+d\right)\arctan\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{a^2dx^2-2\,adx+d}{a^2c+d}\right)+\left(i\,a^2c+i\,d\right)\mathrm{Li}_2\left(\frac{a^2c+adx-\left(i\,a^2x-i\,a\right)\sqrt{c}\,\sqrt{d}}{a^2c+2i\,a\sqrt{c}\,\sqrt{d}-d}\right)+\left(i\,a^2c+i\,d\right)\mathrm{Li}_2\left(\frac{a^2c-adx+\left(i\,a^2x+i\,a\right)\sqrt{c}\,\sqrt{d}}{a^2c+2i\,a\sqrt{c}\,\sqrt{d}-d}\right)+\left(-i\,a^2c-i\,d\right)\mathrm{Li}_2\left(\frac{a^2c+adx+\left(i\,a^2x-i\,a\right)\sqrt{c}\,\sqrt{d}}{a^2c-2i\,a\sqrt{c}\,\sqrt{d}-d}\right)+\left(-i\,a^2c-i\,d\right)\mathrm{Li}_2\left(\frac{a^2c-adx-\left(i\,a^2x+i\,a\right)\sqrt{c}\,\sqrt{d}}{a^2c-2i\,a\sqrt{c}\,\sqrt{d}-d}\right)-\left(\left(a^2c+d\right)\arctan\left(\frac{\left(a^2x+a\right)\sqrt{c}\,\sqrt{d}}{a^2c+d},\frac{adx+d}{a^2c+d}\right)-\left(a^2c+d\right)\arctan\left(\frac{\left(a^2x-a\right)\sqrt{c}\,\sqrt{d}}{a^2c+d},-\frac{adx-d}{a^2c+d}\right)\right)\log\left(dx^2+c\right)\right)\sqrt{c}\,\sqrt{d}\right)a}{8\left(a^3c^3d+ac^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccoth(a*x)
- 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x - 1)
 + ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^
2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x
+ d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*
sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*d)*dil
og((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*sq
rt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilo
g((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*
sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d
), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*sqrt(c)*sqrt(
d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d))
*a/(a^3*c^3*d + a*c^2*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a\,x)}{{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^2,x)
[Out] int(acoth(a*x)/(c + d*x^2)^2, x)
```

3.41 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=657

$$\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\log\left(\frac{\sqrt{d}\,(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\cdots}{32c^{5/2}\sqrt{\cdots}}$$

```
[Out] 1/8*a/c/(a^2*c+d)/(d*x^2+c)+1/4*x*arccoth(a*x)/c/(d*x^2+c)^2+3/8*x*arccoth(
a*x)/c^2/(d*x^2+c)+1/16*a*(5*a^2*c+3*d)*ln(-a^2*x^2+1)/c^2/(a^2*c+d)^2-1/16
*a*(5*a^2*c+3*d)*ln(d*x^2+c)/c^2/(a^2*c+d)^2+3/8*arccoth(a*x)*arctan(x*d^(1
/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-(a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2
)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32*I*ln((-a*x+1)*d^(1/2)/(I
*a*c^(1/2)+d^(1/2)))*ln(1-I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32*I*ln(-(
-a*x+1)*d^(1/2)/(I*a*c^(1/2)-d^(1/2)))*ln(1+I*x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(1/2)+3/32*I*ln((a*x+1)*d^(1/2)/(I*a*c^(1/2)+d^(1/2)))*ln(1+I*x*d^(1/2)/c^(
1/2))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1/2)-I
*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)-I*x*d^(1/2))/(a*c^(1
/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)+3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2))/(a
*c^(1/2)-I*d^(1/2)))/c^(5/2)/d^(1/2)-3/32*I*polylog(2,a*(c^(1/2)+I*x*d^(1/2
))/(a*c^(1/2)+I*d^(1/2)))/c^(5/2)/d^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {205, 211, 6124, 6857, 585, 78, 5028, 2456, 2441, 2440, 2438}

$$\frac{a(5a^2c+3d)\log\left(1-a^2x^2\right)}{16c^2\left(a^2c+d\right)^2}-\frac{a(5a^2c+3d)\log\left(c+dx^2\right)}{16c^2\left(a^2c+d\right)^2}+\frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)}+\frac{3\coth^{-1}(ax)\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}+\frac{3i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{Li}_2\left(\frac{a\left(\sqrt{c}-i\sqrt{d}\,x\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{Li}_2\left(\frac{a\left(i\sqrt{d}\,x+\sqrt{c}\right)}{\sqrt{c}\,a+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(1-ax)}{\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\log\left(1-\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(ax+1)}{-\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(-\frac{\sqrt{d}\,(1-ax)}{-\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\log\left(1+\frac{i\sqrt{d}\,x}{\sqrt{c}}\right)\log\left(\frac{\sqrt{d}\,(ax+1)}{\sqrt{d}+ia\sqrt{c}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3x\coth^{-1}(ax)}{8c^2\left(c+dx^2\right)}+\frac{x\coth^{-1}(ax)}{4c\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcCoth[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcCoth[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcCoth[a*x]*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
```

```
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 5028

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3}\,dx = \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a\int \frac{\frac{x}{4c(c+dx^2)^2} + \frac{3x}{8c^2(c+}}{1}$$

$$= \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a\int \left(-\frac{x(5c}{8c^2\,(-1+a^2}\right.$$

$$= \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a\int \frac{x(5c+3dx^2)}{(-1+a^2x^2)(c+dx^2)^2}\,d}{8c^2}$$

$$= \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \frac{5c+3dx}{(-1+a^2x)(c+}\right.}{16c^2}$$

$$= \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a\text{Subst}\left(\int \left(\frac{a^2(5a^2c+}{(a^2c+d)^2(-}\right.\right.}{}$$

$$= \frac{a}{8c\,(a^2c+d)\,(c+dx^2)} + \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\,(a^2c+d)\,(c+dx^2)} + \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\,(a^2c+d)\,(c+dx^2)} + \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}$$

$$= \frac{a}{8c\,(a^2c+d)\,(c+dx^2)} + \frac{x\coth^{-1}(ax)}{4c\,(c+dx^2)^2} + \frac{3x\coth^{-1}(ax)}{8c^2\,(c+dx^2)} + \frac{3\coth^{-1}(ax)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1559 vs. $2(657) = 1314$.
time = 9.60, size = 1559, normalized size = 2.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^3,x]
```

```
[Out] -1/32*(a*(10*a^2*c*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)]
+ 6*d*Log[1 - ((a^2*c + d)*Cosh[2*ArcCoth[a*x]])/(a^2*c - d)] - (3*d*(a^2*c
 + d)*((-2*I)*ArcCos[(a^2*c - d)/(a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x
)] + 4*ArcCoth[a*x]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^
2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2
*c*d])*(-1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos[(a^2*
c - d)/(a^2*c + d)] - 2*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c +
I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (Arc
Cos[(a^2*c - d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[
(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^Arc
Coth[a*x]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + (ArcCos
[(a^2*c - d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*
d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*
c + d]*Sqrt[-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]]])] + I*(-PolyLo
g[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c
+ d)*(Sqrt[a^2*c*d] - I*a*d*x))] + PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*
c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))]))
)/Sqrt[a^2*c*d] - (3*Sqrt[a^2*c*d]*(a^2*c + d)*((-2*I)*ArcCos[(a^2*c - d)/(
a^2*c + d)]*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + 4*ArcCoth[a*x]*ArcTan[(a*d*x)
/Sqrt[a^2*c*d]] - (ArcCos[(a^2*c - d)/(a^2*c + d)] + 2*ArcTan[(a*c)/(Sqrt[a
^2*c*d]*x)])*Log[(2*d*(a^2*c - I*Sqrt[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(I
*Sqrt[a^2*c*d] + a*d*x))] - (ArcCos[(a^2*c - d)/(a^2*c + d)] - 2*ArcTan[(a*
c)/(Sqrt[a^2*c*d]*x)])*Log[(2*d*(a^2*c + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*
c + d)*(I*Sqrt[a^2*c*d] + a*d*x))] + (ArcCos[(a^2*c - d)/(a^2*c + d)] + 2*(
ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt
[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcCoth[a*x]*Sqrt[-(a^2*c) + d + (a^2
*c + d)*Cosh[2*ArcCoth[a*x]]])] + (ArcCos[(a^2*c - d)/(a^2*c + d)] - 2*(Arc
Tan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]
*Sqrt[a^2*c*d]*E^ArcCoth[a*x])/(Sqrt[a^2*c + d]*Sqrt[-(a^2*c) + d + (a^2*c
+ d)*Cosh[2*ArcCoth[a*x]]])] + I*(-PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[a^2*
c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))] +
 PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c*d])*(Sqrt[a^2*c*d] + I*a*d*x))/(
(a^2*c + d)*(Sqrt[a^2*c*d] - I*a*d*x))])))/d + (16*a^2*c*d*(a^2*c + d)*ArcC
oth[a*x]*Sinh[2*ArcCoth[a*x]])/(-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a
*x]])^2 + (8*a^2*c*d - 4*(5*a^4*c^2 + 8*a^2*c*d + 3*d^2)*ArcCoth[a*x]*Sinh[
```

```
2*ArcCoth[a*x]])/(-(a^2*c) + d + (a^2*c + d)*Cosh[2*ArcCoth[a*x]])))/(c^2*(
a^2*c + d)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3790 vs. $2(493) = 986$.
time = 1.10, size = 3791, normalized size = 5.77

method	result	size
derivativedivides	Expression too large to display	3791
default	Expression too large to display	3791
risch	Expression too large to display	4508

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(3/16*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^2/c^2*d/
(a^4*c^2+2*a^2*c*d+d^2)^2*arccoth(a*x)^2-3/16*(-a^2*c*d)^(1/2)/(a^4*c^2+2*a
^2*c*d+d^2)/c*a^4/d*arccoth(a*x)^2-3/16*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+
(-a^2*c*d)^(1/2)*d)/c^3*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)/(a*x-1
)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)-1/2/c/(a^4*c^2+2*a^2*c
*d+d^2)*a^4*d/(a^2*c+d)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c/(a*x-1)*(a*x+1
)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*d/(a*x-1)*(a*x+1)+d)-3/16/c^2/(a^4*c^2+2*a^
2*c*d+d^2)*a^2*d^2/(a^2*c+d)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c/(a*x-1)*(
a*x+1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*d/(a*x-1)*(a*x+1)+d)-3/8/c^2/(a^4*c^2+
2*a^2*c*d+d^2)*a^2*d^2/(a^2*c+d)*ln((a*x-1)/(a*x+1))-1/c/(a^4*c^2+2*a^2*c*d
+d^2)*a^4*d/(a^2*c+d)*ln((a*x-1)/(a*x+1))-3/32*(-(-a^2*c*d)^(1/2)*a^2*c+2*a
^2*c*d+(-a^2*c*d)^(1/2)*d)*a^2/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a
^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))-3/16*(a^2*c+2*(-a^2*c
*d)^(1/2)-d)*a^2*d^2*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d
)^(1/2)-d))/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2+3/16*(c*d)^(1/2)/c*a^5*arctan(1/4
*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*
d+d^2)/(a^2*c+d)+1/8*a^2*(5*a^6*c^3*arccoth(a*x)+3*arccoth(a*x)*a^6*c^2*d*x
^2-7*arccoth(a*x)*a^5*c^2*d*x-5*arccoth(a*x)*a^5*c*d^2*x^3+3*a^4*c^2*d*arcc
oth(a*x)+arccoth(a*x)*a^4*c*d^2*x^2-a^5*c^2*d*x-a^5*c*d^2*x^3-5*arccoth(a*x
)*a^3*c*d^2*x-3*arccoth(a*x)*d^3*a^3*x^3-a^4*c^2*d-c*a^4*d^2*x^2)*(a*x-1)/(
a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2-3/32*(-(-a^2*c*d)^(1/2)*a^2*
c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/c^3*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2
,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))-3/16*(-(-a^2*c*d)^
(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^4*c^2+2*a^2*c*d+d^2)^2*a^6/d*a
rccoth(a*x)^2-3/8*(-a^2*c*d)^(1/2)*a^2/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(
a*x)^2-3/16*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^4*c^2
+2*a^2*c*d+d^2)^2/c*a^4*arccoth(a*x)^2+3/16*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*
c*d+(-a^2*c*d)^(1/2)*d)/c^3*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*arccoth(a*x)^2+1/
8*(c*d)^(1/2)/c^2*a^3*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/
a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^2)+3/32*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*
```

```
c*d+(-a^2*c*d)^(1/2)*d)/(a^4*c^2+2*a^2*c*d+d^2)^2*a^6/d*polylog(2,(a^2*c+d)
/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))+3/32*(-(-a^2*c*d)^(1/2)*a^2*
c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^4*c^2+2*a^2*c*d+d^2)^2/c*a^4*polylog(2,(
a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))+3/16*(-a^2*c*d)^(1/2
)*a^2/c^2/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*
c+2*(-a^2*c*d)^(1/2)-d))-3/8*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*ln(1-(a^2*c+d)/(a
*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)*a^6/(a^4*c^2+2*a^2
*c*d+d^2)^2+3/32*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(
a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/16*(-a^2*c*d)^(1/2
)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)^2-5/8/(a^4*c^2+2*a^2*c*d+d^2)*
a^6/(a^2*c+d)*ln((a*x-1)/(a*x+1))-5/16/(a^4*c^2+2*a^2*c*d+d^2)*a^6/(a^2*c+d
)*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c/(a*x-1)*(a*x+1)+d/(a*x-1)^2*(a*x+1)^
2+a^2*c+2*d/(a*x-1)*(a*x+1)+d)+3/8*(-a^2*c*d)^(1/2)*a^2/c^2/(a^4*c^2+2*a^2*
c*d+d^2)*arccoth(a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c+2*(-a^2*c*d)^(1
/2)-d))-3/16*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^6*polylog(2,(a^2*c+d)/(a*x-1)*(
a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))/(a^4*c^2+2*a^2*c*d+d^2)^2+3/8*(a^2*c+2
*(-a^2*c*d)^(1/2)-d)*a^6*arccoth(a*x)^2/(a^4*c^2+2*a^2*c*d+d^2)^2-3/16*(-(-
a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)*a^2/c^2*d/(a^4*c^2+2*a^2
*c*d+d^2)^2*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*ar
ccoth(a*x)-5/16*(c*d)^(1/2)/c^2*d*a^3*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+
1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)-3/4*(a^2*c
+2*(-a^2*c*d)^(1/2)-d)*a^4*d*ln(1-(a^2*c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*
c*d)^(1/2)-d))*arccoth(a*x)/c/(a^4*c^2+2*a^2*c*d+d^2)^2+3/16*(-a^2*c*d)^(1/
2)/(a^4*c^2+2*a^2*c*d+d^2)/c*a^4/d*arccoth(a*x)*ln(1-(a^2*c+d)/(a*x-1)*(a*x
+1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))-3/16*(c*d)^(1/2)/c^3*d^2*a*arctan(1/4*(2*
(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^
2)/(a^2*c+d)-3/8*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^2*d^2*ln(1-(a^2*c+d)/(a*x-1
)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)/c^2/(a^4*c^2+2*a^2*c*d
+d^2)^2+5/16*(c*d)^(1/2)/d*a^7*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^
2*c+2*d)/a/(c*d)^(1/2))/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)+3/16*(c*d)^(1/2)/
c^3*d*a*arctan(1/4*(2*(a^2*c+d)/(a*x-1)*(a*x+1)-2*a^2*c+2*d)/a/(c*d)^(1/2))
/(a^4*c^2+2*a^2*c*d+d^2)+3/4*(a^2*c+2*(-a^2*c*d)^(1/2)-d)*a^4*d*arccoth(a*x
)^2/c/(a^4*c^2+2*a^2*c*d+d^2)^2+3/16*(-(-a^2*c*d)^(1/2)*a^2*c+2*a^2*c*d+(-a
^2*c*d)^(1/2)*d)/(a^4*c^2+2*a^2*c*d+d^2)^2*a^6/d*ln(1-(a^2*c+d)/(a*x-1)*(a*
x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arccoth(a*x)+3/16*(-(-a^2*c*d)^(1/2)*a^2
*c+2*a^2*c*d+(-a^2*c*d)^(1/2)*d)/(a^4*c^2+2*a^2*c*d+d^2)^2/c*a^4*ln(1-(a^2*
c+d)/(a*x-1)*(a*x+1)/(a^2*c-2*(-a^2*c*d)^(1/2)-...
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. 2(463) = 926.
time = 0.58, size = 1087, normalized size = 1.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sqr
t(c*d))/(sqrt(c*d)*c^2))*arccoth(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 - 3
*((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arc
tan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - (a^4*c^
3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(sqrt(
d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - (-I*a^4*c^3 - 2*
I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^2*c*d^2 - I*d^3)*x^2)*dilog((
a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqr
t(d) - d)) - (-I*a^4*c^3 - 2*I*a^2*c^2*d - I*c*d^2 + (-I*a^4*c^2*d - 2*I*a^
2*c*d^2 - I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d
))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4*c^3 + 2*I*a^2*c^2*d + I*c*
d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)*dilog((a^2*c + a*d*x + (I*
a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - (I*a^4
*c^3 + 2*I*a^2*c^2*d + I*c*d^2 + (I*a^4*c^2*d + 2*I*a^2*c*d^2 + I*d^3)*x^2)
*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqr
t(c)*sqrt(d) - d)) - ((a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c
*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d), (a*d*x +
d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2
 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/
(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d) - 2*(5*a^3*c^3*d + 3*a*c^2*d^
2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(d*x^2 + c) + 2*(5*a^3*c^3*d + 3*a*
c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(a*x + 1) + 2*(5*a^3*c^3*d +
3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*log(a*x - 1))*a/(a^5*c^6*d +
 2*a^3*c^5*d^2 + a*c^4*d^3 + (a^5*c^5*d^2 + 2*a^3*c^4*d^3 + a*c^3*d^4)*x^2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/(d*x^2 + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a\,x)}{{\left(d\,x^2+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^3,x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^3, x)
```

3.42 $\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\sqrt{c+dx^2} \coth^{-1}(ax), x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$$

Mathematica [A]
time = 3.56, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2} \coth^{-1}(ax) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x],x]
[Out] Integrate[Sqrt[c + d*x^2]*ArcCoth[a*x], x]
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c}\, \operatorname{arccoth}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccoth(a*x),x)
[Out] int((d*x^2+c)^(1/2)*arccoth(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2}\,\operatorname{acoth}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acoth(a*x),x)
[Out] Integral(sqrt(c + d*x**2)*acoth(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccoth(a*x),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*arccoth(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{acoth}(a\,x)\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)*(c + d*x^2)^(1/2),x)
[Out] int(acoth(a*x)*(c + d*x^2)^(1/2), x)
```

3.43 $\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx=\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [A]
time = 3.47, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcCoth[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(acoth(a*x)/sqrt(c + d*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^(1/2),x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^(1/2), x)
```

3.44 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

```
[Out] -arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c/(a^2*c+d)^(1/2)+x*arccoth(a*x
)/c/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {197, 6124, 12, 455, 65, 214}

$$\frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCoth[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx &= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\,(1-a^2x^2)\sqrt{c+dx^2}}\,dx \\
&= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{(1-a^2x^2)\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\mathrm{Subst}\left(\int \frac{1}{(1-a^2x)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c} \\
&= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd} \\
&= \frac{x\coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 119, normalized size = 1.92

$$\frac{\frac{2x\coth^{-1}(ax)}{\sqrt{c+dx^2}} + \frac{\log(1-ax)+\log(1+ax)-\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)-\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{\sqrt{a^2c+d}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcCoth[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a*
c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c + d
]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.
time = 0.28, size = 153, normalized size = 2.47

$$\frac{a^2\left(\frac{\operatorname{arsinh}\left(-\frac{2\,a^2c}{\sqrt{cd}\,|2\,a^2x+2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^2x+2\,a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}-\frac{\operatorname{arsinh}\left(\frac{2\,a^2c}{\sqrt{cd}\,|2\,a^2x-2\,a|}+\frac{2\,adx}{\sqrt{cd}\,|2\,a^2x-2\,a|}\right)}{a^3\sqrt{c+\frac{d}{a^2}}}\right)}{2\,c}+\frac{x\operatorname{arcoth}\left(ax\right)}{\sqrt{dx^2+c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c*
d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d)*
abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(c +
 d/a^2)))/c + x*arccoth(a*x)/(sqrt(d*x^2 + c)*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(54) = 108.
time = 0.36, size = 354, normalized size = 5.71

$$\left[\frac{2\,(a^2c+d)\sqrt{dx^2+c}\,x\log\left(\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}\,(dx^2+c)\log\left(\frac{a^4d^2x^4+8\,a^4c^2+8\,a^2cd+2\,(4\,a^4cd+3\,a^2d^2)x^2-4\,(a^3dx^2+2\,a^3c+ad)\sqrt{a^2c+d}\,\sqrt{dx^2+c}+d^2}{a^4x^4-2\,a^2x^2+1}\right)}{4\,(a^2c^3+c^2d+(a^2c^2d+cd^2)x^2)},\ \frac{(a^2c+d)\sqrt{dx^2+c}\,x\log\left(\frac{ax+1}{ax-1}\right)+\sqrt{-a^2c-d}\,(dx^2+c)\arctan\left(\frac{(a^2dx^2+2\,a^2c+d)\sqrt{-a^2c-d}\,\sqrt{dx^2+c}}{2\,(a^3c^2+acd+(a^3cd+ad^2)x^2)}\right)}{2\,(a^2c^3+c^2d+(a^2c^2d+cd^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1)) + sqrt(a^2*c
 + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d +
```

```
 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2
+ c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d + c*d
^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log((a*x + 1)/(a*x - 1)) + sqr
t(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c
- d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^2*c^3 +
 c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}{\left(ax \right)}}{\left(c + dx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [A]
time = 0.41, size = 79, normalized size = 1.27

$$\frac{x \log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{2\sqrt{dx^2+c}\,c} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d
*x^2 + c)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acoth}(a\,x)}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^(3/2), x)
```

3.45 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}-\frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}$$

```
[Out] 1/3*x*arccoth(a*x)/c/(d*x^2+c)^(3/2)-1/3*(3*a^2*c+2*d)*arctanh(a*(d*x^2+c)^
(1/2)/(a^2*c+d)^(1/2))/c^2/(a^2*c+d)^(3/2)+1/3*a/c/(a^2*c+d)/(d*x^2+c)^(1/2
)+2/3*x*arccoth(a*x)/c^2/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 6124, 6820, 12, 585, 79, 65, 214}

$$-\frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}+\frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcCoth[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x(3c+2dx^2)}{3c^2\,(1-a^2x^2)\,(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x(3c+2dx^2)}{(1-a^2x^2)(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{3c+2dx}{(1-a^2x)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d))\,\text{Subst}\Big(\int}{6c^2\,(a}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d))\,\text{Subst}\Big(\int}{3c^2d}$$

$$= \frac{a}{3c\,(a^2c+d)\,\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d)\tanh^{-1}\Big(\frac{a\sqrt{}}{\sqrt{}}}{3c^2\,(a^2c+d)^{3/}}$$

Mathematica [A]
time = 0.20, size = 226, normalized size = 1.77

$$\frac{\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} + \frac{2x(3c+2dx^2)\coth^{-1}(ax)}{(c+dx^2)^{3/2}} + \frac{(3a^2c+2d)\log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d)\log(1+ax)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d)\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d)\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{(a^2c+d)^{3/2}}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcCoth[a*x])
/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + ((3
*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c -
d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d
)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))/(6*c
^2)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(108) = 216$.
time = 0.47, size = 223, normalized size = 1.74

$$\frac{1}{6}a\left(\frac{\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^2c^2+cd)\sqrt{a^2c+d}}+\frac{2d}{(a^2c^2+cd)\sqrt{dx^2+c}}}{d}+\frac{2\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^2}\right)+\frac{1}{3}\left(\frac{2x}{\sqrt{dx^2+c}\,c^2}+\frac{x}{(dx^2+c)^{\frac{3}{2}}c}\right)\operatorname{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*
a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^2
 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*
a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2)) + 1/
3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arccoth(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(108) = 216$.
time = 0.39, size = 728, normalized size = 5.69

$$\left[\frac{(3a^2c^3+(3a^2cd^2+2d^3)x^4+2c^2d+2(3a^2c^2d+2cd^2)x^2)\sqrt{a^2c+d}\log\left(\frac{a^4d^2x^4+8a^4c^2+8a^2cd+2(4a^4cd+3a^2d^2)x^2-4(a^3dx^2+2a^3c+ad)\sqrt{a^2c+d}\sqrt{dx^2+c}+d^2}{a^4x^4-2a^2x^2+1}\right)+2(2a^3c^3+2ac^2d+2(a^3c^2d+acd^2)x^2+(2(a^4c^2d+2a^2cd^2+d^3)x^3+3(a^4c^3+2a^2c^2d+cd^2)x)\log\left(\frac{ax+1}{ax-1}\right))\sqrt{dx^2+c}}{12(a^4c^6+2a^2c^5d+c^4d^2+(a^4c^4d^2+2a^2c^3d^3+c^2d^4)x^4+2(a^4c^5d+2a^2c^4d^2+c^3d^3)x^2)}, \frac{(3a^2c^3+(3a^2cd^2+2d^3)x^4+2c^2d+2(3a^2c^2d+2cd^2)x^2)\sqrt{-a^2c-d}\arctan\left(\frac{(a^2dx^2+2a^2c+d)\sqrt{-a^2c-d}\sqrt{dx^2+c}}{2(a^3c^2+acd+(a^3cd+ad^2)x^2)}\right)+(2a^3c^3+2ac^2d+2(a^3c^2d+acd^2)x^2+(2(a^4c^2d+2a^2cd^2+d^3)x^3+3(a^4c^3+2a^2c^2d+cd^2)x)\log\left(\frac{ax+1}{ax-1}\right))\sqrt{dx^2+c}}{6(a^4c^6+2a^2c^5d+c^4d^2+(a^4c^4d^2+2a^2c^3d^3+c^2d^4)x^4+2(a^4c^5d+2a^2c^4d^2+c^3d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*
(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2
*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 +
 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2
+ c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2
*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3 +
 (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt(
-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^
2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d
+ 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3*
(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c
```

```
))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*d^
4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [**A**]
time = 0.42, size = 143, normalized size = 1.12

$$\frac{1}{3}a\left(\frac{(3a^2c+2d)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^2c^3+c^2d)\sqrt{-a^2c-d}\,a}+\frac{1}{(a^2c^2+cd)\sqrt{dx^2+c}}\right)+\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{6\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^3
 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/6*
x*(2*d*x^2/c^2 + 3/c)*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(d*x^2 + c)^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a\,x)}{{(d\,x^2+c)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^(5/2), x)
```

3.46 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}+\frac{a(7a^2c+4d)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}-$$

```
[Out] 1/15*a/c/(a^2*c+d)/(d*x^2+c)^(3/2)+1/5*x*arccoth(a*x)/c/(d*x^2+c)^(5/2)+4/1
5*x*arccoth(a*x)/c^2/(d*x^2+c)^(3/2)-1/15*(15*a^4*c^2+20*a^2*c*d+8*d^2)*arc
tanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))/c^3/(a^2*c+d)^(5/2)+1/15*a*(7*a^2*c
+4*d)/c^2/(a^2*c+d)^2/(d*x^2+c)^(1/2)+8/15*x*arccoth(a*x)/c^3/(d*x^2+c)^(1/
2)
```

Rubi [A]
time = 0.72, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 6124, 6820, 12, 6847, 911, 1275, 214}

$$\frac{a(7a^2c+4d)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}}-\frac{\left(15a^4c^2+20a^2cd+8d^2\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3\left(a^2c+d\right)^{5/2}}+\frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\coth^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\coth^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcCoth[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCoth[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}} +}{1-a^2x^2}$$

$$= \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3\,(1-a^2x^2)\,(c+dx^2)^{5/}}$$

$$= \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1-a^2x^2)(c+dx^2)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1-a^2x)(c+dx)^{5/2}}\,dx, x\right.}{30c^3}$$

$$= \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)}\,dx, x,\right.}{15c^3d}$$

$$= \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\text{Subst}\left(\int \left(\frac{3c^2d}{(a^2c+d)x^4} + \frac{cd(7a^2c+4}{(a^2c+d)^2}\right.\right.}{}$$

$$= \frac{a}{15c\,(a^2c+d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2\,(a^2c+d)^2\,\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(}{15c^2\,(c+dx}$$

$$= \frac{a}{15c\,(a^2c+d)\,(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2\,(a^2c+d)^2\,\sqrt{c+dx^2}} + \frac{x\coth^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\coth^{-1}(}{15c^2\,(c+dx}$$

Mathematica [A]
time = 0.38, size = 329, normalized size = 1.64

$$\frac{2ac\sqrt{a^2c+d}\,(c+dx^2)\,(d(5c+4dx^2)+a^2c(8c+7dx^2)) + 2(a^2c+d)^{5/2}x(15c^2+20cdx^2+8d^2x^4)\coth^{-1}(ax) + (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log(1-ax) + (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log(1+ax) - (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right) - (15a^4c^2+20a^2cd+8d^2)\,(c+dx^2)^{5/2}\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{30c^3\,(a^2c+d)^{5/2}\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^
2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcCoth[a*x]
+ (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15*a^
4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^2 +
20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sqrt[
c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c +
 d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(c + d*x
^2)^(5/2))
```

Maple [**F**]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(172) = 344$.
time = 0.48, size = 401, normalized size = 2.00

$$\frac{1}{30}\,a\left(\frac{\frac{3\,a^{3}d\log\left(\frac{\sqrt{dx^{2}+c}\,a^{2}-\sqrt{a^{2}c+d}\,a}{\sqrt{dx^{2}+c}\,a^{2}+\sqrt{a^{2}c+d}\,a}\right)}{(a^{4}c^{3}+2\,a^{2}c^{2}d+cd^{2})\sqrt{a^{2}c+d}}+\frac{2\,(3\,(dx^{2}+c)a^{2}d+a^{2}cd+d^{2})}{(a^{4}c^{3}+2\,a^{2}c^{2}d+cd^{2})(dx^{2}+c)^{\frac{3}{2}}}}{d}+\frac{4\left(\frac{ad\log\left(\frac{\sqrt{dx^{2}+c}\,a^{2}-\sqrt{a^{2}c+d}\,a}{\sqrt{dx^{2}+c}\,a^{2}+\sqrt{a^{2}c+d}\,a}\right)}{(a^{2}c^{3}+c^{2}d)\sqrt{a^{2}c+d}}+\frac{2\,d}{(a^{2}c^{3}+c^{2}d)\sqrt{dx^{2}+c}}\right)}{d}+\frac{8\log\left(\frac{\sqrt{dx^{2}+c}\,a^{2}-\sqrt{a^{2}c+d}\,a}{\sqrt{dx^{2}+c}\,a^{2}+\sqrt{a^{2}c+d}\,a}\right)}{\sqrt{a^{2}c+d}\,ac^{3}}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{dx^{2}+c}\,c^{3}}+\frac{4\,x}{(dx^{2}+c)^{\frac{3}{2}}c^{2}}+\frac{3\,x}{(dx^{2}+c)^{\frac{5}{2}}c}\right)\operatorname{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/30*a*((3*a^3*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2
+ c)*a^2 + sqrt(a^2*c + d)*a))/((a^4*c^3 + 2*a^2*c^2*d + c*d^2)*sqrt(a^2*c
+ d)) + 2*(3*(d*x^2 + c)*a^2*d + a^2*c*d + d^2)/((a^4*c^3 + 2*a^2*c^2*d + c
*d^2)*(d*x^2 + c)^(3/2)))/d + 4*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c
+ d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^3 + c^2*d)*sqrt(
a^2*c + d)) + 2*d/((a^2*c^3 + c^2*d)*sqrt(d*x^2 + c)))/d + 8*log((sqrt(d*x^
2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/
(sqrt(a^2*c + d)*a*c^3)) + 1/15*(8*x/(sqrt(d*x^2 + c)*c^3) + 4*x/((d*x^2 +
c)^(3/2)*c^2) + 3*x/((d*x^2 + c)^(5/2)*c))*arccoth(a*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(172) = 344$.
time = 0.43, size = 1278, normalized size = 6.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
```

```
 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2
+ c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a
^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
+ 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d
^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20
*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2
*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
+ 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*
a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*
d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^
3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*lo
g((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7
*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^
6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*
c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(7/2), x)
```

Giac [**A**]
time = 0.43, size = 226, normalized size = 1.13

$$\frac{1}{15}\,a\left(\frac{(15\,a^4c^2+20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^4c^5+2\,a^2c^4d+c^3d^2)\sqrt{-a^2c-d}\,a}+\frac{7\,(dx^2+c)a^2c+a^2c^2+4\,(dx^2+c)d+cd}{(a^4c^4+2\,a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{30\,(dx^2+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*x^
```

```
2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*log(-(1/(a*x) + 1)/(1/(a*x) - 1))/(d*x^2 + c)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a\,x)}{{\left(d\,x^2+c\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^(7/2), x)
```

3.47 $\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=283

$$\frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}}+\frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}}+\frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}+\frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}+\frac{6x\,c\cdots}{35c^2\,(\cdots}$$

```
[Out] 1/35*a/c/(a^2*c+d)/(d*x^2+c)^(5/2)+1/105*a*(11*a^2*c+6*d)/c^2/(a^2*c+d)^2/(
d*x^2+c)^(3/2)+1/7*x*arccoth(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccoth(a*x)/c^2
/(d*x^2+c)^(5/2)+8/35*x*arccoth(a*x)/c^3/(d*x^2+c)^(3/2)-1/35*(35*a^6*c^3+7
0*a^4*c^2*d+56*a^2*c*d^2+16*d^3)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c+d)^(1/2))
/c^4/(a^2*c+d)^(7/2)+1/35*a*(19*a^4*c^2+22*a^2*c*d+8*d^2)/c^3/(a^2*c+d)^3/(
d*x^2+c)^(1/2)+16/35*x*arccoth(a*x)/c^4/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {198, 197, 6124, 6820, 12, 6847, 1633, 65, 214}

$$\frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}}+\frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}}+\frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}}-\frac{(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\,(a^2c+d)^{7/2}}+\frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}}+\frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcCoth[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcCoth[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCoth[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcCoth[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1633

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 6124

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x}{7c(c+dx \cdots}$$

$$= \frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x(35c^3 \cdots}{\cdots}$$

$$= \frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x(35c^3+ \cdots}{(\cdots}}{\cdots}$$

$$= \frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= \frac{x\coth^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\coth^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\coth^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}} \cdots$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}} \cdots$$

$$= \frac{a}{35c\,(a^2c+d)\,(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2\,(a^2c+d)^2\,(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3\,(a^2c+d)^3\,\sqrt{c+dx^2}} \cdots$$

Mathematica [**A**]
time = 0.87, size = 431, normalized size = 1.52

$$\frac{2ac\sqrt{a^2c+d}\,(c+dx^2)\left(3c^2(a^2c+d)^2+c(a^2c+d)\,(11a^2c+6d)\,(c+dx^2)+3(19a^4c^2+22a^2cd+8d^2)\,(c+dx^2)^2\right)+6(a^2c+d)^{7/2}x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\coth^{-1}(ax)+3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log(1-ax)+3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log(1+ax)-3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log\left(ac-dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)-3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3)\,(c+dx^2)^{7/2}\log\left(ac+dx+\sqrt{a^2c+d}\,\sqrt{c+dx^2}\right)}{210c^4\,(a^2c+d)^{7/2}\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(11
*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^2)
^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*
x^6)*ArcCoth[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(
c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2
 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^2*d +
56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sq
```

```
rt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(c +
 d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(210*c^4*(a
^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\left(d\,x^{2}+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccoth(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(247) = 494.
time = 0.48, size = 639, normalized size = 2.26

$$\frac{1}{210}\,a\left(\frac{\frac{15\,a^5 d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)\sqrt{a^2c+d}}+\frac{2\left(15\,(dx^2+c)^2a^4d+3\,a^4c^2d+6\,a^2cd^2+3\,d^3+5\,(a^4cd+a^2d^2)\,(dx^2+c)\right)}{(a^6c^4+3\,a^4c^3d+3\,a^2c^2d^2+cd^3)(dx^2+c)^{\frac{5}{2}}}}{d}+\frac{6\left(\frac{3\,a^3d\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^4c^4+2\,a^2c^3d+c^2d^2)\sqrt{a^2c+d}}+\frac{2\,(3\,(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^4+2\,a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)}{d}+\frac{24\left(\frac{ad\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{(a^2c^4+c^3d)\sqrt{a^2c+d}}+\frac{2\,d}{(a^2c^4+c^3d)\sqrt{dx^2+c}}\right)}{d}+\frac{48\,\log\left(\frac{\sqrt{dx^2+c}\,a^2-\sqrt{a^2c+d}\,a}{\sqrt{dx^2+c}\,a^2+\sqrt{a^2c+d}\,a}\right)}{\sqrt{a^2c+d}\,ac^4}\right)+\frac{1}{35}\left(\frac{16\,x}{\sqrt{dx^2+c}\,c^4}+\frac{8\,x}{(dx^2+c)^{\frac{3}{2}}c^3}+\frac{6\,x}{(dx^2+c)^{\frac{5}{2}}c^2}+\frac{5\,x}{(dx^2+c)^{\frac{7}{2}}c}\right)\operatorname{arcoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/210*a*((15*a^5*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^
2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^6*c^4 + 3*a^4*c^3*d + 3*a^2*c^2*d^2 +
c*d^3)*sqrt(a^2*c + d)) + 2*(15*(d*x^2 + c)^2*a^4*d + 3*a^4*c^2*d + 6*a^2*c
*d^2 + 3*d^3 + 5*(a^4*c*d + a^2*d^2)*(d*x^2 + c))/((a^6*c^4 + 3*a^4*c^3*d +
 3*a^2*c^2*d^2 + c*d^3)*(d*x^2 + c)^(5/2)))/d + 6*(3*a^3*d*log((sqrt(d*x^2
+ c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((
a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*sqrt(a^2*c + d)) + 2*(3*(d*x^2 + c)*a^2*d
+ a^2*c*d + d^2)/((a^4*c^4 + 2*a^2*c^3*d + c^2*d^2)*(d*x^2 + c)^(3/2)))/d +
 24*(a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2
 + sqrt(a^2*c + d)*a))/((a^2*c^4 + c^3*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^4
+ c^3*d)*sqrt(d*x^2 + c)))/d + 48*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d
)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^4)) +
1/35*(16*x/(sqrt(d*x^2 + c)*c^4) + 8*x/((d*x^2 + c)^(3/2)*c^3) + 6*x/((d*x^
2 + c)^(5/2)*c^2) + 5*x/((d*x^2 + c)^(7/2)*c))*arccoth(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(247) = 494.
time = 0.52, size = 2004, normalized size = 7.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2
+ 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6
*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(18
2*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2*(
196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3*(
16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7 +
56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*x^
5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5
)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4
)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d +
6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 + 6
*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6*
c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4*
a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35*
a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
+ 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3*
d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3 +
 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*a
^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^
2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d
^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 +
 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(
182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2
*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3
*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
+ 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*
x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d
^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d
^4)*x)*log((a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d
+ 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*
d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^
6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d +
4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**(9/2), x)
```

Giac [A]
time = 0.43, size = 357, normalized size = 1.26

$$\frac{1}{105}\,a\left(\frac{3\,(35\,a^6c^3+70\,a^4c^2d+56\,a^2cd^2+16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}\,a}{\sqrt{-a^2c-d}}\right)}{(a^6c^7+3\,a^4c^6d+3\,a^2c^5d^2+c^4d^3)\sqrt{-a^2c-d}\,a}+\frac{57\,(dx^2+c)^2a^4c^2+11\,(dx^2+c)a^4c^3+3\,a^4c^4+66\,(dx^2+c)^2a^2cd+17\,(dx^2+c)a^2c^2d+6\,a^2c^3d+24\,(dx^2+c)^2d^2+6\,(dx^2+c)cd^2+3\,c^2d^2}{(a^6c^6+3\,a^4c^5d+3\,a^2c^4d^2+c^3d^3)(dx^2+c)^{\frac{5}{2}}}\right)+\frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{70\,(dx^2+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + c^
4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d + 6*
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/70*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(1/(a*x) +
 1)/(1/(a*x) - 1))/(d*x^2 + c)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a\,x)}{{\left(d\,x^2+c\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^(9/2), x)
```

3.48 $\int \sqrt{a-ax^2} \coth^{-1}(x)\,dx$

Optimal. Leaf size=186

$$\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)-\frac{a\sqrt{1-x^2}\coth^{-1}(x)\mathrm{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\,\mathrm{PolyLog}\left(2,-\right.}{2\sqrt{a-ax^2}}$$

```
[Out] -a*arccoth(x)*arctan((1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-1/2*I*a*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^
(1/2)+1/2*I*a*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a
)^(1/2)+1/2*(-a*x^2+a)^(1/2)+1/2*x*arccoth(x)*(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6090, 6102, 6098}

$$-\frac{a\sqrt{1-x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\mathrm{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*x^2]*ArcCoth[x],x]
```

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcCoth[x])/2 - (a*Sqrt[1 - x^2]*Arc
Coth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rule 6090

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[b*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Dist[2*d*(q/(2*q + 1
)), Int[(d + e*x^2)^(q - 1)*(a + b*ArcCoth[c*x]), x], x] + Simp[x*(d + e*x^
2)^q*((a + b*ArcCoth[c*x])/(2*q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcCoth[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6102

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCoth[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-ax^2}\coth^{-1}(x)\,dx &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)+\frac{1}{2}a\int\frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx\\ &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)+\frac{\left(a\sqrt{1-x^2}\right)\int\frac{\coth^{-1}(x)}{\sqrt{1-x^2}}\,dx}{2\sqrt{a-ax^2}}\\ &= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\coth^{-1}(x)-\frac{a\sqrt{1-x^2}\coth^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A]
time = 0.88, size = 125, normalized size = 0.67

$$-\frac{\sqrt{a-ax^2}\left(-2\coth\left(\frac{1}{2}\coth^{-1}(x)\right)-\coth^{-1}(x)\text{csch}^2\left(\frac{1}{2}\coth^{-1}(x)\right)-4\coth^{-1}(x)\log\left(1-e^{-\coth^{-1}(x)}\right)+4\coth^{-1}(x)\log\left(1+e^{-\coth^{-1}(x)}\right)-4\text{PolyLog}\left(2,-e^{-\coth^{-1}(x)}\right)+4\text{PolyLog}\left(2,e^{-\coth^{-1}(x)}\right)-\coth^{-1}(x)\text{sech}^2\left(\frac{1}{2}\coth^{-1}(x)\right)+2\tanh\left(\frac{1}{2}\coth^{-1}(x)\right)\right)}{8\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*x^2]*ArcCoth[x],x]
```

```
[Out] -1/8*(Sqrt[a - a*x^2]*(-2*Coth[ArcCoth[x]/2] - ArcCoth[x]*Csch[ArcCoth[x]/2
]^2 - 4*ArcCoth[x]*Log[1 - E^(-ArcCoth[x])] + 4*ArcCoth[x]*Log[1 + E^(-ArcC
oth[x])] - 4*PolyLog[2, -E^(-ArcCoth[x])] + 4*PolyLog[2, E^(-ArcCoth[x])] -
 ArcCoth[x]*Sech[ArcCoth[x]/2]^2 + 2*Tanh[ArcCoth[x]/2]))/(Sqrt[1 - x^(-2)]
*x)
```

Maple [A]
time = 0.45, size = 199, normalized size = 1.07

method	result
default	$\frac{(\text{arccoth}(x)x+1)\sqrt{-a(1+x)(-1+x)}}{2}-\frac{\sqrt{-a(1+x)(-1+x)}\sqrt{\frac{-1+x}{1+x}}\,\text{arccoth}(x)\ln\left(\frac{1}{\sqrt{\frac{-1+x}{1+x}}}+1\right)}{2(-1+x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+a)^(1/2)*arccoth(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arccoth(x)*x+1)*(-a*(1+x)*(-1+x))^(1/2)-1/2*(-a*(1+x)*(-1+x))^(1/2)*((
-1+x)/(1+x))^(1/2)/(-1+x)*arccoth(x)*ln(1/((-1+x)/(1+x))^(1/2)+1)-1/2*(-a*(
1+x)*(-1+x))^(1/2)*((-1+x)/(1+x))^(1/2)/(-1+x)*polylog(2,-1/((-1+x)/(1+x))^
(1/2))+1/2*(-a*(1+x)*(-1+x))^(1/2)*((-1+x)/(1+x))^(1/2)/(-1+x)*arccoth(x)*l
n(1-1/((-1+x)/(1+x))^(1/2))+1/2*(-a*(1+x)*(-1+x))^(1/2)*((-1+x)/(1+x))^(1/2
)/(-1+x)*polylog(2,1/((-1+x)/(1+x))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + a)*arccoth(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*x^2 + a)*arccoth(x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a(x-1)(x+1)}\,\operatorname{acoth}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+a)**(1/2)*acoth(x),x)
```

```
[Out] Integral(sqrt(-a*(x - 1)*(x + 1))*acoth(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arccoth(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + a)*arccoth(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(x)\,\sqrt{a-a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)*(a - a*x^2)^(1/2),x)
[Out] int(acoth(x)*(a - a*x^2)^(1/2), x)
```

3.49 $\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}} \, dx$

Optimal. Leaf size=144

$$-\frac{2\sqrt{1-x^2}\coth^{-1}(x)\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{}}{\sqrt{}}\right.}{\sqrt{a-ax^2}}$$

```
[Out] -2*arccoth(x)*arctan((1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/
2)-I*polylog(2,-I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)+
I*polylog(2,I*(1-x)^(1/2)/(1+x)^(1/2))*(-x^2+1)^(1/2)/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6102, 6098}

$$-\frac{2\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\coth^{-1}(x)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2]*ArcCoth[x]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[-2*(a + b*ArcCoth[c*x])*(ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]]/(c*S
qrt[d])), x] + (-Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(
c*Sqrt[d])), x] + Simp[I*b*(PolyLog[2, I*(Sqrt[1 - c*x]/Sqrt[1 + c*x])]/(c*
Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 6102

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCoth[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}}\,dx = \frac{\sqrt{1-x^2}\int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}}\,dx}{\sqrt{a-ax^2}}$$

$$= -\frac{2\sqrt{1-x^2}\coth^{-1}(x)\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2}\,\mathrm{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2}}{\sqrt{a}}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.53

$$\frac{\sqrt{a-ax^2}\left(\coth^{-1}(x)\left(\log\left(1-e^{-\coth^{-1}(x)}\right)-\log\left(1+e^{-\coth^{-1}(x)}\right)\right)+\mathrm{PolyLog}\left(2,-e^{-\coth^{-1}(x)}\right)-\mathrm{PolyLog}\left(2,e^{-\coth^{-1}(x)}\right)\right)}{a\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (Sqrt[a - a*x^2]*(ArcCoth[x]*(Log[1 - E^(-ArcCoth[x])] - Log[1 + E^(-ArcCot
h[x])]) + PolyLog[2, -E^(-ArcCoth[x])] - PolyLog[2, E^(-ArcCoth[x])]))/(a*S
qrt[1 - x^(-2)]*x)
```

Maple [A]
time = 0.41, size = 190, normalized size = 1.32

method	result
default	$-\frac{\ln\left(\frac{1}{\sqrt{\frac{-1+x}{1+x}}}+1\right)\operatorname{arccoth}(x)\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-a\,(1+x)\,(-1+x)}}{(-1+x)a} - \frac{\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-a\,(}}{(-1+x)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1/((-1+x)/(1+x))^(1/2)+1)*arccoth(x)*((-1+x)/(1+x))^(1/2)*(-a*(1+x)*(-1
+x))^(1/2)/(-1+x)/a-polylog(2,-1/((-1+x)/(1+x))^(1/2))*((-1+x)/(1+x))^(1/2)
*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a+ln(1-1/((-1+x)/(1+x))^(1/2))*arccoth(x)*(
(-1+x)/(1+x))^(1/2)*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a+polylog(2,1/((-1+x)/(1
+x))^(1/2))*((-1+x)/(1+x))^(1/2)*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(x)/sqrt(-a*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arccoth(x)/(a*x^2 - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(1/2),x)
```

```
[Out] Integral(acoth(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccoth(x)/sqrt(-a*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{a-a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)/(a - a*x^2)^(1/2),x)
```

```
[Out] int(acoth(x)/(a - a*x^2)^(1/2), x)
```

3.50 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

```
[Out] -1/a/(-a*x^2+a)^(1/2)+x*arccoth(x)/a/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6106}

$$\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(3/2),x]
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcCoth[x])/(a*Sqrt[a - a*x^2])
```

Rule 6106

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx=-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}\,\left(1-x\coth^{-1}(x)\right)}{a^2\left(-1+x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(3/2),x]
[Out] (Sqrt[a - a*x^2]*(1 - x*ArcCoth[x]))/(a^2*(-1 + x^2))
```

Maple [A]
time = 0.37, size = 52, normalized size = 1.41

method	result	size
risch	$\frac{x\ln(1+x)}{2a\sqrt{-a\left(x^{2}-1\right)}}-\frac{\ln(-1+x)x+2}{2a\sqrt{-a\left(x^{2}-1\right)}}$	45
default	$-\frac{(-1+\operatorname{arccoth}(x))\sqrt{-a\left(1+x\right)\left(-1+x\right)}}{2(-1+x)a^{2}}-\frac{(1+\operatorname{arccoth}(x))\sqrt{-a\left(1+x\right)\left(-1+x\right)}}{2(1+x)a^{2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-1+arccoth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a^2-1/2*(1+arccoth(x))*
(-a*(1+x)*(-1+x))^(1/2)/(1+x)/a^2
```

Maxima [A]
time = 0.47, size = 63, normalized size = 1.70

$$\frac{x\operatorname{arcoth}\left(x\right)}{\sqrt{-ax^{2}+a}\;a}-\frac{\frac{\sqrt{-ax^{2}+a}}{ax+a}-\frac{\sqrt{-ax^{2}+a}}{ax-a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arccoth(x)/(sqrt(-a*x^2 + a)*a) - 1/2*(sqrt(-a*x^2 + a)/(a*x + a) - sqrt(
-a*x^2 + a)/(a*x - a))/a
```

Fricas [A]
time = 0.37, size = 41, normalized size = 1.11

$$-\frac{\sqrt{-ax^{2}+a}\left(x\log\left(\frac{x+1}{x-1}\right)-2\right)}{2\left(a^{2}x^{2}-a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*(x*log((x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}\left(x\right)}{\left(-a\left(x-1\right)\left(x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(3/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.57

$$-\frac{\sqrt{-ax^2+a}\,x\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{2\,(ax^2-a)a}-\frac{1}{\sqrt{-ax^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*x*log(-(1/x + 1)/(1/x - 1))/((a*x^2 - a)*a) - 1/(sqrt
(-a*x^2 + a)*a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acoth}(x)}{(a-a\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)/(a - a*x^2)^(3/2),x)
```

```
[Out] int(acoth(x)/(a - a*x^2)^(3/2), x)
```

3.51 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}$$

```
[Out] -1/9/a/(-a*x^2+a)^(3/2)+1/3*x*arccoth(x)/a/(-a*x^2+a)^(3/2)-2/3/a^2/(-a*x^2
+a)^(1/2)+2/3*x*arccoth(x)/a^2/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6108, 6106}

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/9*1/(a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcCoth[x])/(
3*a*(a - a*x^2)^(3/2)) + (2*x*ArcCoth[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rule 6106

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.54

$$-\frac{\sqrt{a-ax^2}\,(7-6x^2+(-9x+6x^3)\coth^{-1}(x))}{9a^3\,(-1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/9*(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcCoth[x]))/(a^3*(-1 + x
^2)^2)
```

Maple [A]
time = 0.40, size = 112, normalized size = 1.35

method	result
risch	$\frac{x(2x^2-3)\ln(1+x)}{6a^2(x^2-1)\sqrt{-a\,(x^2-1)}}-\frac{6x^3\ln(-1+x)+12x^2-9\ln(-1+x)x-14}{18a^2(x^2-1)\sqrt{-a\,(x^2-1)}}$
default	$\frac{(1+x)(-1+3\,\mathrm{arccoth}(x))\sqrt{-a\,(1+x)\,(-1+x)}}{72(-1+x)^2a^3}-\frac{3(-1+\mathrm{arccoth}(x))\sqrt{-a\,(1+x)\,(-1+x)}}{8(-1+x)a^3}-\frac{3(1+\mathrm{arccoth}($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(1+x)*(-1+3*arccoth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)^2/a^3-3/8*(-1+a
rccoth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a^3-3/8*(1+arccoth(x))*(-a*(1+x)*
(-1+x))^(1/2)/(1+x)/a^3+1/72*(1+3*arccoth(x))*(-1+x)*(-a*(1+x)*(-1+x))^(1/2
)/(1+x)^2/a^3
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.81

$$\frac{1}{3}\left(\frac{2\,x}{\sqrt{-ax^2+a}\,a^2}+\frac{x}{(-ax^2+a)^{\frac{3}{2}}a}\right)\mathrm{arcoth}\,(x)-\frac{2}{3\sqrt{-ax^2+a}\,a^2}-\frac{1}{9\,(-ax^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arccoth(x) - 2/
3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

Fricas [A]
time = 0.39, size = 61, normalized size = 0.73

$$\frac{\sqrt{-ax^2+a}\,\left(12\,x^2-3\,(2\,x^3-3\,x)\log\left(\frac{x+1}{x-1}\right)-14\right)}{18\,(a^3x^4-2\,a^3x^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log((x + 1)/(x - 1)) - 14)/
(a^3*x^4 - 2*a^3*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{\left(-a(x-1)(x+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

Giac [A]
time = 0.42, size = 90, normalized size = 1.08

$$-\frac{\sqrt{-ax^2+a}\,x\left(\frac{2x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{6\left(ax^2-a\right)^2}-\frac{6\,ax^2-7\,a}{9\left(ax^2-a\right)\sqrt{-ax^2+a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(1/x + 1)/(1/x - 1))/(a*x^2 -
a)^2 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(x)}{\left(a-a\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)/(a - a*x^2)^(5/2),x)
```

```
[Out] int(acoth(x)/(a - a*x^2)^(5/2), x)
```

3.52 $\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}} \, dx$

Optimal. Leaf size=124

$$-\frac{1}{25a(a-ax^2)^{5/2}}-\frac{4}{45a^2(a-ax^2)^{3/2}}-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}+\frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}}+\frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}}$$

```
[Out] -1/25/a/(-a*x^2+a)^(5/2)-4/45/a^2/(-a*x^2+a)^(3/2)+1/5*x*arccoth(x)/a/(-a*x
^2+a)^(5/2)+4/15*x*arccoth(x)/a^2/(-a*x^2+a)^(3/2)-8/15/a^3/(-a*x^2+a)^(1/2
)+8/15*x*arccoth(x)/a^3/(-a*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6108, 6106}

$$-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2(a-ax^2)^{3/2}}+\frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}}-\frac{1}{25a(a-ax^2)^{5/2}}+\frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] -1/25*1/(a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sq
rt[a - a*x^2]) + (x*ArcCoth[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcCoth[x])/
(15*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcCoth[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rule 6106

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[-b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcCoth[c*x])/(d*
Sqrt[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}}\,dx = -\frac{1}{25a\,(a-ax^2)^{5/2}} + \frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}} + \frac{4\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}}\,dx}{5a}$$

$$= -\frac{1}{25a\,(a-ax^2)^{5/2}} - \frac{4}{45a^2\,(a-ax^2)^{3/2}} + \frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}} + \frac{4x\coth^{-1}(x)}{15a^2\,(a-ax^2)^{3/2}} + \frac{8\int \frac{\text{co}}{(a-}}{}$$

$$= -\frac{1}{25a\,(a-ax^2)^{5/2}} - \frac{4}{45a^2\,(a-ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a-ax^2}} + \frac{x\coth^{-1}(x)}{5a\,(a-ax^2)^{5/2}} + \frac{4x\,\text{cot}}{15a^2\,(a}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2}\,\left(149-260x^2+120x^4-15x(15-20x^2+8x^4)\coth^{-1}(x)\right)}{225a^4\,(-1+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(a - a*x^2)^(7/2),x]
[Out] (Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*ArcC
oth[x]))/(225*a^4*(-1 + x^2)^3)
```

Maple [A]
time = 0.41, size = 176, normalized size = 1.42

method	result
risch	$\frac{x(8x^4-20x^2+15)\ln(1+x)}{30a^3(x^2-1)^2\sqrt{-a\,(x^2-1)}} - \frac{120x^5\ln(-1+x)+240x^4-300x^3\ln(-1+x)-520x^2+225\ln(-1+x)x+298}{450a^3(x^2-1)^2\sqrt{-a\,(x^2-1)}}$
default	$-\frac{(1+x)^2(-1+5\,\mathrm{arccoth}(x))\sqrt{-a\,(1+x)\,(-1+x)}}{800(-1+x)^3a^4} + \frac{5(1+x)(-1+3\,\mathrm{arccoth}(x))\sqrt{-a\,(1+x)\,(-1+x)}}{288(-1+x)^2a^4} - \frac{5(-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-a*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/800*(1+x)^2*(-1+5*arccoth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)^3/a^4+5/288
*(1+x)*(-1+3*arccoth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)^2/a^4-5/16*(-1+arcc
oth(x))*(-a*(1+x)*(-1+x))^(1/2)/(-1+x)/a^4-5/16*(1+arccoth(x))*(-a*(1+x)*(-
1+x))^(1/2)/(1+x)/a^4+5/288*(1+3*arccoth(x))*(-1+x)*(-a*(1+x)*(-1+x))^(1/2)
/(1+x)^2/a^4-1/800*(1+5*arccoth(x))*(-1+x)^2*(-a*(1+x)*(-1+x))^(1/2)/(1+x)^
3/a^4
```

Maxima [A]
time = 0.26, size = 99, normalized size = 0.80

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-ax^2+a}\,a^3} + \frac{4\,x}{(-ax^2+a)^{\frac{3}{2}}a^2} + \frac{3\,x}{(-ax^2+a)^{\frac{5}{2}}a}\right)\mathrm{arcoth}\,(x) - \frac{8}{15\,\sqrt{-ax^2+a}\,a^3} - \frac{4}{45\,(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25\,(-ax^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a*
x^2 + a)^(5/2)*a))*arccoth(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x^2
 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

Fricas [A]
time = 0.36, size = 81, normalized size = 0.65

$$\frac{\left(240\,x^{4}-520\,x^{2}-15\left(8\,x^{5}-20\,x^{3}+15\,x\right)\log\left(\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^{2}+a}}{450\left(a^{4}x^{6}-3\,a^{4}x^{4}+3\,a^{4}x^{2}-a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log((x + 1)/(x - 1))
+ 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}\left(x\right)}{\left(-a\left(x-1\right)\left(x+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(7/2), x)
```

Giac [A]
time = 0.42, size = 122, normalized size = 0.98

$$-\frac{\sqrt{-ax^{2}+a}\left(4\,x^{2}\left(\frac{2\,x^{2}}{a}-\frac{5}{a}\right)+\frac{15}{a}\right)x\log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{30\left(ax^{2}-a\right)^{3}}-\frac{120\left(ax^{2}-a\right)^{2}-20\left(ax^{2}-a\right)a+9\,a^{2}}{225\left(ax^{2}-a\right)^{2}\sqrt{-ax^{2}+a}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(1/x + 1)/(1/x
 - 1))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^2)
/((a*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{acoth}(x)}{{\left(a-a\,x^{2}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)/(a - a*x^2)^(7/2),x)
[Out] int(acoth(x)/(a - a*x^2)^(7/2), x)
```

3.53 $\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\log\left(\coth^{-1}(x)\right)$$

```
[Out] ln(arccoth(x))
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6094}

$$\log\left(\coth^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)*ArcCoth[x]),x]
[Out] Log[ArcCoth[x]]
```

Rule 6094

```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)}\,dx = \log\left(\coth^{-1}(x)\right)$$

Mathematica [A]
time = 0.02, size = 3, normalized size = 1.00

$$\log\left(\coth^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^2)*ArcCoth[x]),x]
[Out] Log[ArcCoth[x]]
```

Maple [A]
time = 0.09, size = 4, normalized size = 1.33

method	result	size
default	$\ln(\operatorname{arccoth}(x))$	4
risch	$\ln(\ln(1+x)-\ln(-1+x))$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/arccoth(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(arccoth(x))
```

Maxima [A]
time = 0.25, size = 3, normalized size = 1.00

$$\log(\operatorname{arcoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="maxima")
```

```
[Out] log(arccoth(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(3) = 6$.
time = 0.34, size = 11, normalized size = 3.67

$$\log\left(\log\left(\frac{x+1}{x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="fricas")
```

```
[Out] log(log((x + 1)/(x - 1)))
```

Sympy [A]
time = 0.12, size = 3, normalized size = 1.00

$$\log(\operatorname{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/acoth(x),x)
```

```
[Out] log(acoth(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(3) = 6$.
time = 0.39, size = 12, normalized size = 4.00

$$\log\left(\left|\log\left(\frac{x+1}{x-1}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="giac")
[Out] log(abs(log((x + 1)/(x - 1))))
```

Mupad [B]
time = 0.30, size = 3, normalized size = 1.00

$$\ln(\mathrm{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(acoth(x)*(x^2 - 1)),x)
[Out] log(acoth(x))
```

3.54 $\int \frac{\coth^{-1}(x)^n}{1-x^2} dx$

Optimal. Leaf size=12

$$\frac{\coth^{-1}(x)^{1+n}}{1+n}$$

```
[Out] arccoth(x)^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6096}

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]^n/(1 - x^2),x]
```

```
[Out] ArcCoth[x]^(1 + n)/(1 + n)
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{\coth^{-1}(x)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]^n/(1 - x^2),x]
```

```
[Out] ArcCoth[x]^(1 + n)/(1 + n)
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
default	$\frac{\operatorname{arccoth}(x)^{1+n}}{1+n}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)^n/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arccoth(x)^(1+n)/(1+n)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 1.00

$$\frac{\operatorname{arcoth}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="maxima")
```

```
[Out] arccoth(x)^(n + 1)/(n + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(12) = 24$.
time = 0.35, size = 62, normalized size = 5.17

$$\frac{\cosh\left(n\log\left(\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)\right)\right)\log\left(\frac{x+1}{x-1}\right)+\log\left(\frac{x+1}{x-1}\right)\sinh\left(n\log\left(\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)\right)\right)}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(n*log(1/2*log((x + 1)/(x - 1))))*log((x + 1)/(x - 1)) + log((x +
1)/(x - 1))*sinh(n*log(1/2*log((x + 1)/(x - 1)))))/(n + 1)
```

Sympy [A]
time = 1.10, size = 15, normalized size = 1.25

$$\begin{cases}\frac{\operatorname{acoth}^{n+1}(x)}{n+1} & \text{for}\ n\neq -1\\ \log\left(\operatorname{acoth}(x)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)**n/(-x**2+1),x)
```

```
[Out] Piecewise((acoth(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acoth(x)), True))
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.83

$$\frac{\left(\frac{1}{2}\,\log\left(\frac{x+1}{x-1}\right)\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="giac")
```

```
[Out] (1/2*log((x + 1)/(x - 1)))^(n + 1)/(n + 1)
```

Mupad [B]
time = 1.36, size = 22, normalized size = 1.83

$$\begin{cases} \ln\left(\operatorname{acoth}(x)\right) & \text{if } n=-1\\ \frac{\operatorname{acoth}(x)^{n+1}}{n+1} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(x)^n/(x^2 - 1),x)
```

```
[Out] piecewise(n == -1, log(acoth(x)), n ~= -1, acoth(x)^(n + 1)/(n + 1))
```

3.55 $\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx$

Optimal. Leaf size=62

$$\frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x\coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6}\coth^{-1}(x)^3 + \frac{1}{4}\tanh^{-1}(x)$$

```
[Out] 1/4*x/(-x^2+1)-1/2*arccoth(x)/(-x^2+1)+1/2*x*arccoth(x)^2/(-x^2+1)+1/6*arcc
oth(x)^3+1/4*arctanh(x)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6104, 6142, 205, 212}

$$\frac{x}{4(1-x^2)} + \frac{x\coth^{-1}(x)^2}{2(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4}\tanh^{-1}(x) + \frac{1}{6}\coth^{-1}(x)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]^2/(1 - x^2)^2,x]
```

```
[Out] x/(4*(1 - x^2)) - ArcCoth[x]/(2*(1 - x^2)) + (x*ArcCoth[x]^2)/(2*(1 - x^2))
 + ArcCoth[x]^3/6 + ArcTanh[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6142

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2}\,dx &= \frac{x\coth^{-1}(x)^2}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3-\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}\,dx\\ &= -\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3+\frac{1}{2}\int \frac{1}{(1-x^2)^2}\,dx\\ &= \frac{x}{4(1-x^2)}-\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3+\frac{1}{4}\int \frac{1}{1-x^2}\,dx\\ &= \frac{x}{4(1-x^2)}-\frac{\coth^{-1}(x)}{2(1-x^2)}+\frac{x\coth^{-1}(x)^2}{2(1-x^2)}+\frac{1}{6}\coth^{-1}(x)^3+\frac{1}{4}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.98

$$\frac{-6x+12\coth^{-1}(x)-12x\coth^{-1}(x)^2+4(-1+x^2)\coth^{-1}(x)^3-3(-1+x^2)\log(1-x)+3(-1+x^2)\log(1+x)}{24(-1+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]^2/(1 - x^2)^2,x]
[Out] (-6*x + 12*ArcCoth[x] - 12*x*ArcCoth[x]^2 + 4*(-1 + x^2)*ArcCoth[x]^3 - 3*(
-1 + x^2)*Log[1 - x] + 3*(-1 + x^2)*Log[1 + x])/(24*(-1 + x^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.24, size = 707, normalized size = 11.40

method	result
risch	$\frac{\ln(1+x)^3}{48}-\frac{(\ln(-1+x)x^2+2x-\ln(-1+x))\ln(1+x)^2}{16(x^2-1)}+\frac{\left(x^2\ln(-1+x)^2+4\ln(-1+x)x-\ln(-1+x)^2+4\right)\ln(1+x)}{16(-1+x)(1+x)}+\frac{-x^2\ln(-1+x)}{}$
default	$-\frac{\operatorname{arccoth}(x)^2}{4(1+x)}+\frac{\operatorname{arccoth}(x)^2\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)^2}{4(-1+x)}-\frac{\operatorname{arccoth}(x)^2\ln(-1+x)}{4}+\frac{\operatorname{arccoth}(x)^2\ln\left(\frac{-1+x}{1+x}\right)}{4}+\frac{3i\operatorname{arccoth}(x)^2\pi\operatorname{csg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)^2/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arccoth(x)^2/(1+x)+1/4*arccoth(x)^2*ln(1+x)-1/4*arccoth(x)^2/(-1+x)-1/
4*arccoth(x)^2*ln(-1+x)+1/4*arccoth(x)^2*ln((-1+x)/(1+x))+1/24*(3*I*arccoth
(x)^2*Pi*csgn(I*(1+x)/(-1+x))^3*x^2-3*I*csgn(I*(1+x)/(-1+x))*csgn(I/((-1+x)
/(1+x))^(1/2))^2*arccoth(x)^2*Pi-3*I*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))*
csgn(I*(1+x)/(-1+x))*csgn(I/((1+x)/(-1+x)-1))*arccoth(x)^2*Pi+6*I*csgn(I*(1
+x)/(-1+x))^2*csgn(I/((-1+x)/(1+x))^(1/2))*arccoth(x)^2*Pi+3*I*arccoth(x)^2
*Pi*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3*x^2-3*I*csgn(I*(1+x)/(-1+x))^3*
arccoth(x)^2*Pi+3*I*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*csgn(I*(1+x)/(-
1+x))*arccoth(x)^2*Pi-6*I*arccoth(x)^2*Pi*csgn(I*(1+x)/(-1+x))^2*csgn(I/((-
1+x)/(1+x))^(1/2))*x^2+3*I*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*csgn(I/(
(1+x)/(-1+x)-1))*arccoth(x)^2*Pi+3*I*arccoth(x)^2*Pi*csgn(I*(1+x)/(-1+x))*c
sgn(I/((-1+x)/(1+x))^(1/2))^2*x^2-3*I*arccoth(x)^2*Pi*csgn(I*(1+x)/(-1+x))*
csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2*x^2-3*I*csgn(I*(1+x)/(-1+x)/((1+x)/
(-1+x)-1))^3*arccoth(x)^2*Pi-3*I*arccoth(x)^2*Pi*csgn(I*(1+x)/(-1+x)/((1+x)
/(-1+x)-1))^2*csgn(I/((1+x)/(-1+x)-1))*x^2+3*I*arccoth(x)^2*Pi*csgn(I*(1+x)
/(-1+x))*csgn(I*(1+x)/(-1+x)/((1+x)/(-1+x)-1))*csgn(I/((1+x)/(-1+x)-1))*x^2
+4*arccoth(x)^3*x^2-4*arccoth(x)^3+6*arccoth(x)*x^2+6*arccoth(x)-6*x)/(-1+x
)/(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(46) = 92$.
time = 0.25, size = 171, normalized size = 2.76

$$-\frac{1}{4}\left(\frac{2x}{x^2-1}-\log(x+1)+\log(x-1)\right)\operatorname{arcoth}(x)^2-\frac{\left((x^2-1)\log(x+1)^2-2(x^2-1)\log(x+1)\log(x-1)+(x^2-1)\log(x-1)^2-4\right)\operatorname{arcoth}(x)}{8(x^2-1)}+\frac{(x^2-1)\log(x+1)^3-3(x^2-1)\log(x+1)^2\log(x-1)-(x^2-1)\log(x-1)^3+3\left((x^2-1)\log(x-1)^2+2x^2-2\right)\log(x+1)-6(x^2-1)\log(x-1)-12x}{48(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x)^2 - 1/8*((x^2 - 1
)*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)^2
 - 4)*arccoth(x)/(x^2 - 1) + 1/48*((x^2 - 1)*log(x + 1)^3 - 3*(x^2 - 1)*log
(x + 1)^2*log(x - 1) - (x^2 - 1)*log(x - 1)^3 + 3*((x^2 - 1)*log(x - 1)^2 +
 2*x^2 - 2)*log(x + 1) - 6*(x^2 - 1)*log(x - 1) - 12*x)/(x^2 - 1)
```

Fricas [A]
time = 0.38, size = 63, normalized size = 1.02

$$\frac{(x^2-1)\log\left(\frac{x+1}{x-1}\right)^3-6x\log\left(\frac{x+1}{x-1}\right)^2+6(x^2+1)\log\left(\frac{x+1}{x-1}\right)-12x}{48(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/48*((x^2 - 1)*log((x + 1)/(x - 1))^3 - 6*x*log((x + 1)/(x - 1))^2 + 6*(x^
2 + 1)*log((x + 1)/(x - 1)) - 12*x)/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(x)}{(x-1)^2(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)**2/(-x**2+1)**2,x)
```

```
[Out] Integral(acoth(x)**2/((x - 1)**2*(x + 1)**2), x)
```

Giac [A]
time = 0.43, size = 53, normalized size = 0.85

$$-\frac{(x-1)\log\left(\frac{x+1}{x-1}\right)^2}{16(x+1)}-\frac{(x-1)\log\left(\frac{x+1}{x-1}\right)}{8(x+1)}-\frac{x-1}{8(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/16*(x - 1)*log((x + 1)/(x - 1))^2/(x + 1) - 1/8*(x - 1)*log((x + 1)/(x -
 1))/(x + 1) - 1/8*(x - 1)/(x + 1)
```

Mupad [B]
time = 2.68, size = 201, normalized size = 3.24

$$\frac{\ln\left(\frac{1}{x}+1\right)^3}{48}-\frac{\ln\left(1-\frac{1}{x}\right)^3}{48}-\frac{x}{4\,(x^2-1)}+\ln\left(1-\frac{1}{x}\right)\left(\frac{\frac{3\,x}{32}-\frac{1}{8}}{x^2-1}-\frac{\frac{x}{8}+\frac{1}{8}}{x^2-1}-\frac{\ln\left(\frac{1}{x}+1\right)^2}{16}+\frac{x}{32\,(x^2-1)}+\ln\left(\frac{1}{x}+1\right)\left(\frac{\frac{x}{4}+\frac{1}{16}}{x^2-1}-\frac{1}{16\,(x^2-1)}\right)\right)+\ln\left(1-\frac{1}{x}\right)^2\left(\frac{\ln\left(\frac{1}{x}+1\right)}{16}-\frac{x}{8\,(x^2-1)}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{4\,(x^2-1)}-\frac{x\ln\left(\frac{1}{x}+1\right)^2}{8\,(x^2-1)}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)^2/(x^2 - 1)^2,x)
```

```
[Out] log(1/x + 1)^3/48 - (atan(x*1i)*1i)/4 - log(1 - 1/x)^3/48 - x/(4*(x^2 - 1))
 + log(1 - 1/x)*(((3*x)/32 - 1/8)/(x^2 - 1) - (x/8 + 1/8)/(x^2 - 1) - log(1
/x + 1)^2/16 + x/(32*(x^2 - 1)) + log(1/x + 1)*((x/4 + 1/16)/(x^2 - 1) - 1/
(16*(x^2 - 1)))) + log(1 - 1/x)^2*(log(1/x + 1)/16 - x/(8*(x^2 - 1))) + log
(1/x + 1)/(4*(x^2 - 1)) - (x*log(1/x + 1)^2)/(8*(x^2 - 1))
```

3.56 $\int \frac{x \coth^{-1}(x)}{1-x^2} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\coth^{-1}(x)^2+\coth^{-1}(x)\log\left(\frac{2}{1-x}\right)+\frac{1}{2}\text{PolyLog}\left(2,\frac{1+x}{-1+x}\right)$$

```
[Out] -1/2*arccoth(x)^2+arccoth(x)*ln(2/(1-x))+1/2*polylog(2,(1+x)/(-1+x))
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6132, 6056, 2449, 2352}

$$\frac{1}{2}\text{Li}_2\left(\frac{x+1}{x-1}\right)-\frac{1}{2}\coth^{-1}(x)^2+\log\left(\frac{2}{1-x}\right)\coth^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2),x]
```

```
[Out] -1/2*ArcCoth[x]^2 + ArcCoth[x]*Log[2/(1 - x)] + PolyLog[2, (1 + x)/(-1 + x)
]/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx &= -\frac{1}{2}\coth^{-1}(x)^2+\int \frac{\coth^{-1}(x)}{1-x}\,dx\\ &= -\frac{1}{2}\coth^{-1}(x)^2+\coth^{-1}(x)\log\left(\frac{2}{1-x}\right)-\int \frac{\log\left(\frac{2}{1-x}\right)}{1-x^2}\,dx\\ &= -\frac{1}{2}\coth^{-1}(x)^2+\coth^{-1}(x)\log\left(\frac{2}{1-x}\right)+\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-x}\right)\\ &= -\frac{1}{2}\coth^{-1}(x)^2+\coth^{-1}(x)\log\left(\frac{2}{1-x}\right)+\frac{1}{2}\text{Li}_2\left(\frac{1+x}{-1+x}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 34, normalized size = 0.92

$$\frac{1}{2}\left(\coth^{-1}(x)\left(\coth^{-1}(x)+2\log\left(1-e^{-2\coth^{-1}(x)}\right)\right)-\text{PolyLog}\left(2,e^{-2\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2),x]
```

```
[Out] (ArcCoth[x]*(ArcCoth[x] + 2*Log[1 - E^(-2*ArcCoth[x])]) - PolyLog[2, E^(-2*
ArcCoth[x])])/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.
time = 0.09, size = 75, normalized size = 2.03

method	result
risch	$\frac{\text{dilog}\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln(-1+x)\ln\left(\frac{1}{2}+\frac{x}{2}\right)}{4}+\frac{\ln(-1+x)^2}{8}-\frac{\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{x}{2}\right)}{4}-\frac{\ln(1+x)^2}{8}$
default	$-\frac{\text{arccoth}(x)\ln(-1+x)}{2}-\frac{\text{arccoth}(x)\ln(1+x)}{2}+\frac{\text{dilog}\left(\frac{1}{2}+\frac{x}{2}\right)}{2}+\frac{\ln(-1+x)\ln\left(\frac{1}{2}+\frac{x}{2}\right)}{4}-\frac{\ln(-1+x)^2}{8}-\frac{\left(\ln(1+x)-\ln\left(\frac{1}{2}+\frac{x}{2}\right)\right)\ln}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccoth(x)*ln(-1+x)-1/2*arccoth(x)*ln(1+x)+1/2*dilog(1/2+1/2*x)+1/4*ln
(-1+x)*ln(1/2+1/2*x)-1/8*ln(-1+x)^2-1/4*(ln(1+x)-ln(1/2+1/2*x))*ln(1/2-1/2*
x)+1/8*ln(1+x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.
time = 0.25, size = 76, normalized size = 2.05

$$\frac{1}{4}(\log(x+1)-\log(x-1))\log(x^2-1)-\frac{1}{2}\,\text{arcoth}(x)\log(x^2-1)-\frac{1}{8}\log(x+1)^2-\frac{1}{4}\log(x+1)\log(x-1)+\frac{1}{8}\log(x-1)^2+\frac{1}{2}\log(x-1)\log\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{2}\text{Li}_2\left(-\frac{1}{2}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="maxima")
[Out] 1/4*(log(x + 1) - log(x - 1))*log(x^2 - 1) - 1/2*arccoth(x)*log(x^2 - 1) -
1/8*log(x + 1)^2 - 1/4*log(x + 1)*log(x - 1) + 1/8*log(x - 1)^2 + 1/2*log(x
 - 1)*log(1/2*x + 1/2) + 1/2*dilog(-1/2*x + 1/2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="fricas")
[Out] integral(-x*arccoth(x)/(x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\,\text{acoth}\,(x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1),x)
[Out] -Integral(x*acoth(x)/(x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1),x, algorithm="giac")
[Out] integrate(-x*arccoth(x)/(x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x\,\text{acoth}(x)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*acoth(x))/(x^2 - 1),x)
[Out] -int((x*acoth(x))/(x^2 - 1), x)
```

3.57 $\int \frac{\coth^{-1}(x)}{1-x^2} dx$

Optimal. Leaf size=8

$$\frac{1}{2}\coth^{-1}(x)^2$$

```
[Out] 1/2*arccoth(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6096}

$$\frac{1}{2}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2),x]
[Out] ArcCoth[x]^2/2
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(x)}{1-x^2} \, dx = \frac{1}{2}\coth^{-1}(x)^2$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2),x]
[Out] ArcCoth[x]^2/2
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.62

method	result	size
default	$\text{arctanh}\left(x\right)\text{arccoth}(x)-\frac{\text{arctanh}(x)^{2}}{2}$	13
risch	$\frac{\ln(1+x)^{2}}{8}-\frac{\ln(-1+x)\ln(1+x)}{4}+\frac{\ln(-1+x)^{2}}{8}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] arctanh(x)*arccoth(x)-1/2*arctanh(x)^2
```

Maxima [A]
time = 0.26, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\text{arcoth}\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="maxima")
[Out] 1/2*arccoth(x)^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.35, size = 14, normalized size = 1.75

$$\frac{1}{8}\,\log\left(\frac{x+1}{x-1}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="fricas")
[Out] 1/8*log((x + 1)/(x - 1))^2
```

Sympy [A]
time = 0.45, size = 5, normalized size = 0.62

$$\frac{\text{acoth}^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1),x)
[Out] acoth(x)**2/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.39, size = 14, normalized size = 1.75

$$\frac{1}{8}\,\log\left(\frac{x+1}{x-1}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*log((x + 1)/(x - 1))^2
```

Mupad [B]
time = 1.20, size = 21, normalized size = 2.62

$$\frac{\left(\ln\left(1-\frac{1}{x}\right)-\ln\left(\frac{1}{x}+1\right)\right)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(x)/(x^2 - 1),x)
```

```
[Out] (log(1 - 1/x) - log(1/x + 1))^2/8
```

3.58 $\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$

Optimal. Leaf size=36

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/4*x/(-x^2+1)+1/2*arccoth(x)/(-x^2+1)-1/4*arctanh(x)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6142, 205, 212}

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

```
[Out] -1/4*x/(1 - x^2) + ArcCoth[x]/(2*(1 - x^2)) - ArcTanh[x]/4
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6142

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x\coth^{-1}(x)}{(1-x^2)^2}\,dx = \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{(1-x^2)^2}\,dx$$
$$= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4}\int \frac{1}{1-x^2}\,dx$$
$$= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4}\tanh^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.22

$$\frac{x}{4(-1+x^2)} - \frac{\coth^{-1}(x)}{2(-1+x^2)} + \frac{1}{8}\log(1-x) - \frac{1}{8}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2)^2,x]
```

```
[Out] x/(4*(-1 + x^2)) - ArcCoth[x]/(2*(-1 + x^2)) + Log[1 - x]/8 - Log[1 + x]/8
```

Maple [A]
time = 0.08, size = 39, normalized size = 1.08

method	result	size
default	$-\frac{\mathrm{arccoth}(x)}{2(x^2-1)} + \frac{1}{8+8x} - \frac{\ln(1+x)}{8} + \frac{1}{-8+8x} + \frac{\ln(-1+x)}{8}$	39
risch	$-\frac{\ln(1+x)}{4(x^2-1)} + \frac{\ln(-1+x)x^2 - \ln(1+x)x^2 + \ln(-1+x) + \ln(1+x) + 2x}{8(-1+x)(1+x)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccoth(x)/(x^2-1)+1/8/(1+x)-1/8*ln(1+x)+1/8/(-1+x)+1/8*ln(-1+x)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 0.94

$$\frac{x}{4(x^2-1)} - \frac{\mathrm{arcoth}\,(x)}{2(x^2-1)} - \frac{1}{8}\log(x+1) + \frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(x^2 - 1) - 1/2*arccoth(x)/(x^2 - 1) - 1/8*log(x + 1) + 1/8*log(x - 1
)
```

Fricas [A]
time = 0.39, size = 29, normalized size = 0.81

$$-\frac{(x^2+1)\log\left(\frac{x+1}{x-1}\right)-2\,x}{8\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((x^2 + 1)*log((x + 1)/(x - 1)) - 2*x)/(x^2 - 1)
```

Sympy [A]
time = 0.24, size = 31, normalized size = 0.86

$$-\frac{x^2\operatorname{acoth}(x)}{4x^2-4}+\frac{x}{4x^2-4}-\frac{\operatorname{acoth}(x)}{4x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1)**2,x)
```

```
[Out] -x**2*acoth(x)/(4*x**2 - 4) + x/(4*x**2 - 4) - acoth(x)/(4*x**2 - 4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(26) = 52.
time = 0.38, size = 101, normalized size = 2.81

$$-\frac{1}{16}\left(\frac{x+1}{x-1}+\frac{x-1}{x+1}\right)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)+\frac{x+1}{16\,(x-1)}-\frac{x-1}{16\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/16*((x + 1)/(x - 1) + (x - 1)/(x + 1))*log(-(((x + 1)/(x - 1) - 1)/((x +
 1)/(x - 1) + 1) + 1)/(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1)) +
1/16*(x + 1)/(x - 1) - 1/16*(x - 1)/(x + 1)
```

Mupad [B]
time = 1.16, size = 21, normalized size = 0.58

$$\frac{\frac{x}{4}-\frac{\operatorname{acoth}(x)}{2}}{x^2-1}-\frac{\operatorname{acoth}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acoth(x))/(x^2 - 1)^2,x)
```

```
[Out] (x/4 - acoth(x)/2)/(x^2 - 1) - acoth(x)/4
```

3.59 $\int \frac{\coth^{-1}(x)}{(1-x^2)^2} \, dx$

Optimal. Leaf size=38

$$-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

```
[Out] -1/4/(-x^2+1)+1/2*x*arccoth(x)/(-x^2+1)+1/4*arccoth(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6104, 267}

$$-\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2)^2,x]
```

```
[Out] -1/4*1/(1 - x^2) + (x*ArcCoth[x])/(2*(1 - x^2)) + ArcCoth[x]^2/4
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx &= \frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2-\frac{1}{2}\int \frac{x}{(1-x^2)^2}\,dx\\ &= -\frac{1}{4(1-x^2)}+\frac{x\coth^{-1}(x)}{2(1-x^2)}+\frac{1}{4}\coth^{-1}(x)^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.74

$$\frac{1-2x\coth^{-1}(x)+(-1+x^2)\coth^{-1}(x)^2}{4(-1+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2)^2,x]
```

```
[Out] (1 - 2*x*ArcCoth[x] + (-1 + x^2)*ArcCoth[x]^2)/(4*(-1 + x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.
time = 0.11, size = 99, normalized size = 2.61

method	result
risch	$\frac{\ln(1+x)^2}{16}-\frac{(\ln(-1+x)x^2+2x-\ln(-1+x))\ln(1+x)}{8(x^2-1)}+\frac{x^2\ln(-1+x)^2+4\ln(-1+x)x-\ln(-1+x)^2+4}{16(-1+x)(1+x)}$
default	$-\frac{\operatorname{arccoth}(x)}{4(1+x)}+\frac{\operatorname{arccoth}(x)\ln(1+x)}{4}-\frac{\operatorname{arccoth}(x)}{4(-1+x)}-\frac{\operatorname{arccoth}(x)\ln(-1+x)}{4}+\frac{(\ln(1+x)-\ln(\frac{1}{2}+\frac{x}{2}))\ln(\frac{1}{2}-\frac{x}{2})}{8}-\frac{\ln(1+x)^2}{16}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arccoth(x)/(1+x)+1/4*arccoth(x)*ln(1+x)-1/4*arccoth(x)/(-1+x)-1/4*arcc
oth(x)*ln(-1+x)+1/8*(ln(1+x)-ln(1/2+1/2*x))*ln(1/2-1/2*x)-1/16*ln(1+x)^2+1/
8*ln(-1+x)*ln(1/2+1/2*x)-1/16*ln(-1+x)^2-1/8/(1+x)+1/8/(-1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.
time = 0.25, size = 76, normalized size = 2.00

$$-\frac{1}{4}\left(\frac{2x}{x^2-1}-\log(x+1)+\log(x-1)\right)\operatorname{arcoth}(x)-\frac{(x^2-1)\log(x+1)^2-2(x^2-1)\log(x+1)\log(x-1)+(x^2-1)\log(x-1)^2-4}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(x^2 - 1) - log(x + 1) + log(x - 1))*arccoth(x) - 1/16*((x^2 - 1)
*log(x + 1)^2 - 2*(x^2 - 1)*log(x + 1)*log(x - 1) + (x^2 - 1)*log(x - 1)^2
- 4)/(x^2 - 1)
```

Fricas [A]
time = 0.36, size = 42, normalized size = 1.11

$$\frac{(x^2-1)\log\left(\frac{x+1}{x-1}\right)^2-4x\log\left(\frac{x+1}{x-1}\right)+4}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((x^2 - 1)*log((x + 1)/(x - 1))^2 - 4*x*log((x + 1)/(x - 1)) + 4)/(x^2
 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{(x-1)^2 (x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1)**2,x)
```

```
[Out] Integral(acoth(x)/((x - 1)**2*(x + 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.
time = 0.42, size = 80, normalized size = 2.11

$$-\frac{(x-1)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)}{8(x+1)}-\frac{x-1}{8(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/8*(x - 1)*log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/(((x +
1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1))/(x + 1) - 1/8*(x - 1)/(x + 1)
```

Mupad [B]
time = 1.22, size = 81, normalized size = 2.13

$$\frac{\ln\left(\frac{1}{x}+1\right)^2}{16}-\ln\left(1-\frac{1}{x}\right)\left(\frac{\ln\left(\frac{1}{x}+1\right)}{8}-\frac{x}{4\,(x^2-1)}\right)+\frac{\ln\left(1-\frac{1}{x}\right)^2}{16}+\frac{1}{4\,(x^2-1)}-\frac{x\,\ln\left(\frac{1}{x}+1\right)}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x)/(x^2 - 1)^2,x)
```

```
[Out] log(1/x + 1)^2/16 - log(1 - 1/x)*(log(1/x + 1)/8 - x/(4*(x^2 - 1))) + log(1
 - 1/x)^2/16 + 1/(4*(x^2 - 1)) - (x*log(1/x + 1))/(4*(x^2 - 1))
```

3.60 $\int \frac{x \coth^{-1}(x)}{(1-x^2)^3} \, dx$

Optimal. Leaf size=50

$$-\frac{x}{16\left(1-x^2\right)^2}-\frac{3x}{32\left(1-x^2\right)}+\frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2}-\frac{3}{32}\tanh^{-1}(x)$$

```
[Out] -1/16*x/(-x^2+1)^2-3/32*x/(-x^2+1)+1/4*arccoth(x)/(-x^2+1)^2-3/32*arctanh(x
)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6142, 205, 212}

$$-\frac{3x}{32\left(1-x^2\right)}-\frac{x}{16\left(1-x^2\right)^2}+\frac{\coth^{-1}(x)}{4\left(1-x^2\right)^2}-\frac{3}{32}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

```
[Out] -1/16*x/(1 - x^2)^2 - (3*x)/(32*(1 - x^2)) + ArcCoth[x]/(4*(1 - x^2)^2) - (
3*ArcTanh[x])/32
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6142

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])^p/(2*e*(q
+ 1))), x] + Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\coth^{-1}(x)}{(1-x^2)^3}\,dx &= \frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{1}{4}\int \frac{1}{(1-x^2)^3}\,dx\\ &= -\frac{x}{16(1-x^2)^2}+\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{3}{16}\int \frac{1}{(1-x^2)^2}\,dx\\ &= -\frac{x}{16(1-x^2)^2}-\frac{3x}{32(1-x^2)}+\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{3}{32}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{x}{16(1-x^2)^2}-\frac{3x}{32(1-x^2)}+\frac{\coth^{-1}(x)}{4(1-x^2)^2}-\frac{3}{32}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.00

$$\frac{1}{64}\left(-\frac{4x}{(-1+x^2)^2}+\frac{6x}{-1+x^2}+\frac{16\coth^{-1}(x)}{(-1+x^2)^2}+3\log(1-x)-3\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCoth[x])/(1 - x^2)^3,x]
```

```
[Out] ((-4*x)/(-1 + x^2)^2 + (6*x)/(-1 + x^2) + (16*ArcCoth[x])/(-1 + x^2)^2 + 3*
Log[1 - x] - 3*Log[1 + x])/64
```

Maple [A]
time = 0.08, size = 53, normalized size = 1.06

method	result	size
default	$\frac{\mathrm{arccoth}(x)}{4(x^2-1)^2}+\frac{1}{64(1+x)^2}+\frac{3}{64(1+x)}-\frac{3\ln(1+x)}{64}-\frac{1}{64(-1+x)^2}+\frac{3}{64(-1+x)}+\frac{3\ln(-1+x)}{64}$	53
risch	$\frac{\ln(1+x)}{8(x^2-1)^2}+\frac{3\ln(-1+x)x^4-3\ln(1+x)x^4-6\ln(-1+x)x^2+6\ln(1+x)x^2+6x^3-5\ln(-1+x)-3\ln(1+x)-10x}{64(1+x)(-1+x)(x^2-1)}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x)/(-x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arccoth(x)/(x^2-1)^2+1/64/(1+x)^2+3/64/(1+x)-3/64*ln(1+x)-1/64/(-1+x)^2
+3/64/(-1+x)+3/64*ln(-1+x)
```

Maxima [A]
time = 0.25, size = 47, normalized size = 0.94

$$\frac{3x^3-5x}{32(x^4-2x^2+1)}+\frac{\mathrm{arcoth}\left(x\right)}{4(x^2-1)^2}-\frac{3}{64}\log\left(x+1\right)+\frac{3}{64}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) + 1/4*arccoth(x)/(x^2 - 1)^2 - 3/64*lo
g(x + 1) + 3/64*log(x - 1)
```

Fricas [A]
time = 0.38, size = 47, normalized size = 0.94

$$\frac{6\,x^3-(3\,x^4-6\,x^2-5)\log\left(\frac{x+1}{x-1}\right)-10\,x}{64\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(6*x^3 - (3*x^4 - 6*x^2 - 5)*log((x + 1)/(x - 1)) - 10*x)/(x^4 - 2*x^2
 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(37) = 74$.
time = 0.34, size = 88, normalized size = 1.76

$$-\frac{3x^4\operatorname{acoth}(x)}{32x^4-64x^2+32}+\frac{3x^3}{32x^4-64x^2+32}+\frac{6x^2\operatorname{acoth}(x)}{32x^4-64x^2+32}-\frac{5x}{32x^4-64x^2+32}+\frac{5\operatorname{acoth}(x)}{32x^4-64x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1)**3,x)
```

```
[Out] -3*x**4*acoth(x)/(32*x**4 - 64*x**2 + 32) + 3*x**3/(32*x**4 - 64*x**2 + 32)
 + 6*x**2*acoth(x)/(32*x**4 - 64*x**2 + 32) - 5*x/(32*x**4 - 64*x**2 + 32)
+ 5*acoth(x)/(32*x**4 - 64*x**2 + 32)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(36) = 72$.
time = 0.39, size = 154, normalized size = 3.08

$$-\frac{1}{128}\left(\frac{(x-1)^2\left(\frac{4(x+1)}{x-1}-1\right)}{(x+1)^2}-\frac{(x+1)^2}{(x-1)^2}+\frac{4(x+1)}{x-1}\right)\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)-\frac{(x-1)^2\left(\frac{8(x+1)}{x-1}-1\right)}{256(x+1)^2}-\frac{(x+1)^2}{256(x-1)^2}+\frac{x+1}{32(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/128*((x - 1)^2*(4*(x + 1)/(x - 1) - 1)/(x + 1)^2 - (x + 1)^2/(x - 1)^2 +
 4*(x + 1)/(x - 1))*log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/
(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) - 1)) - 1/256*(x - 1)^2*(8*(x
```

```
+ 1)/(x - 1) - 1)/(x + 1)^2 - 1/256*(x + 1)^2/(x - 1)^2 + 1/32*(x + 1)/(x -
 1)
```

Mupad [B]
time = 1.21, size = 34, normalized size = 0.68

$$\frac{3\,\ln\left(x-1\right)}{64}-\frac{3\,\ln\left(x+1\right)}{64}+\frac{\frac{\mathrm{acoth}\left(x\right)}{4}-\frac{5\,x}{32}+\frac{3\,x^3}{32}}{{\left(x^2-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*acoth(x))/(x^2 - 1)^3,x)
```

```
[Out] (3*log(x - 1))/64 - (3*log(x + 1))/64 + (acoth(x)/4 - (5*x)/32 + (3*x^3)/32
)/(x^2 - 1)^2
```

3.61 $\int \frac{\coth^{-1}(x)}{(1-x^2)^3} dx$

Optimal. Leaf size=67

$$-\frac{1}{16(1-x^2)^2}-\frac{3}{16(1-x^2)}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}+\frac{3x\coth^{-1}(x)}{8(1-x^2)}+\frac{3}{16}\coth^{-1}(x)^2$$

```
[Out] -1/16/(-x^2+1)^2-3/16/(-x^2+1)+1/4*x*arccoth(x)/(-x^2+1)^2+3/8*x*arccoth(x)
/(-x^2+1)+3/16*arccoth(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6108, 6104, 267}

$$-\frac{3}{16(1-x^2)}-\frac{1}{16(1-x^2)^2}+\frac{3x\coth^{-1}(x)}{8(1-x^2)}+\frac{x\coth^{-1}(x)}{4(1-x^2)^2}+\frac{3}{16}\coth^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x]/(1 - x^2)^3,x]
```

```
[Out] -1/16*1/(1 - x^2)^2 - 3/(16*(1 - x^2)) + (x*ArcCoth[x])/(4*(1 - x^2)^2) + (
3*x*ArcCoth[x])/(8*(1 - x^2)) + (3*ArcCoth[x]^2)/16
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6104

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[x*((a + b*ArcCoth[c*x])^p/(2*d*(d + e*x^2))), x] + (-Dist[b*c
*(p/2), Int[x*((a + b*ArcCoth[c*x])^(p - 1)/(d + e*x^2)^2), x], x] + Simp[(
a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (Dist[(2*q +
3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]), x], x] - Si
mp[x*(d + e*x^2)^(q + 1)*((a + b*ArcCoth[c*x])/(2*d*(q + 1))), x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(x)}{(1-x^2)^3}\,dx &= -\frac{1}{16\,(1-x^2)^2}+\frac{x\coth^{-1}(x)}{4\,(1-x^2)^2}+\frac{3}{4}\int \frac{\coth^{-1}(x)}{(1-x^2)^2}\,dx\\
&= -\frac{1}{16\,(1-x^2)^2}+\frac{x\coth^{-1}(x)}{4\,(1-x^2)^2}+\frac{3x\coth^{-1}(x)}{8\,(1-x^2)}+\frac{3}{16}\coth^{-1}(x)^2-\frac{3}{8}\int \frac{x}{(1-x^2)^2}\,dx\\
&= -\frac{1}{16\,(1-x^2)^2}-\frac{3}{16\,(1-x^2)}+\frac{x\coth^{-1}(x)}{4\,(1-x^2)^2}+\frac{3x\coth^{-1}(x)}{8\,(1-x^2)}+\frac{3}{16}\coth^{-1}(x)^2
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.64

$$-\frac{4-3x^2+2x(-5+3x^2)\coth^{-1}(x)-3(-1+x^2)^2\coth^{-1}(x)^2}{16\,(-1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x]/(1 - x^2)^3,x]
```

```
[Out] -1/16*(4 - 3*x^2 + 2*x*(-5 + 3*x^2)*ArcCoth[x] - 3*(-1 + x^2)^2*ArcCoth[x]^
2)/(-1 + x^2)^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(57) = 114.
time = 0.12, size = 131, normalized size = 1.96

method	result
risch	$\frac{3\ln(1+x)^2}{64}-\frac{(3\ln(-1+x)x^4+6x^3-6\ln(-1+x)x^2-10x+3\ln(-1+x))\ln(1+x)}{32(x^2-1)^2}+\frac{3x^4\ln(-1+x)^2+12x^3\ln(-1+x)-6x^2\ln(-1+x}{64(1+x)(-1+}$
default	$-\frac{\operatorname{arccoth}(x)}{16(1+x)^2}-\frac{3\operatorname{arccoth}(x)}{16(1+x)}+\frac{3\operatorname{arccoth}(x)\ln(1+x)}{16}+\frac{\operatorname{arccoth}(x)}{16(-1+x)^2}-\frac{3\operatorname{arccoth}(x)}{16(-1+x)}-\frac{3\operatorname{arccoth}(x)\ln(-1+x)}{16}+\frac{3\ln(-1+x}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x)/(-x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*arccoth(x)/(1+x)^2-3/16*arccoth(x)/(1+x)+3/16*arccoth(x)*ln(1+x)+1/16
*arccoth(x)/(-1+x)^2-3/16*arccoth(x)/(-1+x)-3/16*arccoth(x)*ln(-1+x)+3/32*l
n(-1+x)*ln(1/2+1/2*x)-3/64*ln(-1+x)^2+3/32*(ln(1+x)-ln(1/2+1/2*x))*ln(1/2-1
/2*x)-3/64*ln(1+x)^2-1/64/(-1+x)^2+7/64/(-1+x)-1/64/(1+x)^2-7/64/(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(49) = 98.
time = 0.26, size = 118, normalized size = 1.76

$$-\frac{1}{16}\left(\frac{2\,(3\,x^3-5\,x)}{x^4-2\,x^2+1}-3\,\log\left(x+1\right)+3\,\log\left(x-1\right)\right)\operatorname{arcoth}\left(x\right)-\frac{3\,(x^4-2\,x^2+1)\log\left(x+1\right)^2-6\,(x^4-2\,x^2+1)\log\left(x+1\right)\log\left(x-1\right)+3\,(x^4-2\,x^2+1)\log\left(x-1\right)^2-12\,x^2+16}{64\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*x^3 - 5*x)/(x^4 - 2*x^2 + 1) - 3*log(x + 1) + 3*log(x - 1))*arc
coth(x) - 1/64*(3*(x^4 - 2*x^2 + 1)*log(x + 1)^2 - 6*(x^4 - 2*x^2 + 1)*log(
x + 1)*log(x - 1) + 3*(x^4 - 2*x^2 + 1)*log(x - 1)^2 - 12*x^2 + 16)/(x^4 -
2*x^2 + 1)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 0.99

$$\frac{3\,(x^4-2\,x^2+1)\log\left(\frac{x+1}{x-1}\right)^2+12\,x^2-4\,(3\,x^3-5\,x)\log\left(\frac{x+1}{x-1}\right)-16}{64\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(x^4 - 2*x^2 + 1)*log((x + 1)/(x - 1))^2 + 12*x^2 - 4*(3*x^3 - 5*x)
*log((x + 1)/(x - 1)) - 16)/(x^4 - 2*x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{acoth}(x)}{x^6-3x^4+3x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1)**3,x)
```

```
[Out] -Integral(acoth(x)/(x**6 - 3*x**4 + 3*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccoth(x)/(x^2 - 1)^3, x)
```

Mupad [B]
time = 1.32, size = 112, normalized size = 1.67

$$\frac{3\ln\left(\frac{1}{x}+1\right)^2}{64}-\ln\left(1-\frac{1}{x}\right)\left(\frac{3\ln\left(\frac{1}{x}+1\right)}{32}+\frac{\frac{5\,x}{16}-\frac{3\,x^3}{16}}{x^4-2\,x^2+1}\right)+\frac{3\ln\left(1-\frac{1}{x}\right)^2}{64}+\frac{\frac{3\,x^2}{16}-\frac{1}{4}}{x^4-2\,x^2+1}+\frac{\ln\left(\frac{1}{x}+1\right)\left(\frac{5\,x}{16}-\frac{3\,x^3}{16}\right)}{x^4-2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(x)/(x^2 - 1)^3,x)
```

```
[Out] (3*log(1/x + 1)^2)/64 - log(1 - 1/x)*((3*log(1/x + 1))/32 + ((5*x)/16 - (3*
x^3)/16)/(x^4 - 2*x^2 + 1)) + (3*log(1 - 1/x)^2)/64 + ((3*x^2)/16 - 1/4)/(x
^4 - 2*x^2 + 1) + (log(1/x + 1)*((5*x)/16 - (3*x^3)/16))/(x^4 - 2*x^2 + 1)
```

3.62 $\int x^3 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=101

$$\frac{(1+6a^2)\,x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)+\frac{(1-a)^4\log(1-a-bx)}{8b^4}-\frac{(1+a)^4\log(1+a+bx)}{8b^4}$$

```
[Out] 1/4*(6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*arccoth(
b*x+a)+1/8*(1-a)^4*ln(-b*x-a+1)/b^4-1/8*(1+a)^4*ln(b*x+a+1)/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(6a^2+1)\,x}{4b^3}+\frac{(a+bx)^3}{12b^4}-\frac{a(a+bx)^2}{2b^4}+\frac{(1-a)^4\log(-a-bx+1)}{8b^4}-\frac{(a+1)^4\log(a+bx+1)}{8b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x],x]
```

```
[Out] ((1 + 6*a^2)*x)/(4*b^3) - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) +
(x^4*ArcCoth[a + b*x])/4 + ((1 - a)^4*Log[1 - a - b*x])/(8*b^4) - ((1 + a)^
4*Log[1 + a + b*x])/(8*b^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^3\coth^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{1}{4}x^4\coth^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{1-x^2}\,dx,x,a+bx\right)\\ &= \frac{1}{4}x^4\coth^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\left(-\frac{1+6a^2}{b^4}+\frac{4ax}{b^4}-\frac{x^2}{b^4}+\frac{1+6a^2+a^4-4a(}{b^4\left(1-x^2\right)}\right.\right.\\ &= \frac{(1+6a^2)\,x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{1+6a^2+a^4}{1}\right.}{}\\ &= \frac{(1+6a^2)\,x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)-\frac{(1-a)^4\text{Subst}\left(\int\right.}{8}\\ &= \frac{(1+6a^2)\,x}{4b^3}-\frac{a(a+bx)^2}{2b^4}+\frac{(a+bx)^3}{12b^4}+\frac{1}{4}x^4\coth^{-1}(a+bx)+\frac{(1-a)^4\log(1-}{8b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 0.80

$$\frac{6(1+3a^2)\,bx-6ab^2x^2+2b^3x^3+6b^4x^4\coth^{-1}(a+bx)+3(-1+a)^4\log(1-a-bx)-3(1+a)^4\log(1+a+bx)}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x],x]
[Out] (6*(1 + 3*a^2)*b*x - 6*a*b^2*x^2 + 2*b^3*x^3 + 6*b^4*x^4*ArcCoth[a + b*x] +
 3*(-1 + a)^4*Log[1 - a - b*x] - 3*(1 + a)^4*Log[1 + a + b*x])/(24*b^4)
```

Maple [A]
time = 0.06, size = 172, normalized size = 1.70

method	result

derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)a^4}{4}-\operatorname{arccoth}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccoth}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)a(bx+a)^3+\frac{\operatorname{arccoth}(bx+a)(bx+a)^4}{4}+3}{b^4}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)a^4}{4}-\operatorname{arccoth}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccoth}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)a(bx+a)^3+\frac{\operatorname{arccoth}(bx+a)(bx+a)^4}{4}+3}{b^4}$
risch	$\frac{x^4\ln(bx+a+1)}{8}-\frac{x^4\ln(bx+a-1)}{8}+\frac{x^3}{12b}+\frac{\ln(-bx-a+1)a^4}{8b^4}-\frac{\ln(bx+a+1)a^4}{8b^4}-\frac{a\,x^2}{4b^2}-\frac{\ln(-bx-a+1)a^3}{2b^4}-\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arccoth(b*x+a)*a^4-arccoth(b*x+a)*a^3*(b*x+a)+3/2*arccoth(b*x+a)
*a^2*(b*x+a)^2-arccoth(b*x+a)*a*(b*x+a)^3+1/4*arccoth(b*x+a)*(b*x+a)^4+3/2*
a^2*(b*x+a)-1/2*(b*x+a)^2*a+1/12*(b*x+a)^3+1/4*b*x+1/4*a+1/8*(a^4-4*a^3+6*a
^2-4*a+1)*ln(b*x+a-1)-1/8*(a^4+4*a^3+6*a^2+4*a+1)*ln(b*x+a+1))
```

Maxima [A]
time = 0.25, size = 106, normalized size = 1.05

$$\frac{1}{4}x^4\operatorname{arcoth}(bx+a)+\frac{1}{24}b\left(\frac{2(b^2x^3-3abx^2+3(3a^2+1)x)}{b^4}-\frac{3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)}{b^5}+\frac{3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(b*x + a) + 1/24*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x
)/b^4 - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4
*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1)/b^5)
```

Fricas [A]
time = 0.36, size = 112, normalized size = 1.11

$$\frac{3b^4x^4\log\left(\frac{bx+a+1}{bx+a-1}\right)+2b^3x^3-6ab^2x^2+6(3a^2+1)bx-3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)+3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*x^4*log((b*x + a + 1)/(b*x + a - 1)) + 2*b^3*x^3 - 6*a*b^2*x^2
+ 6*(3*a^2 + 1)*b*x - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1) +
3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b*x + a - 1))/b^4
```

Sympy [A]
time = 0.53, size = 153, normalized size = 1.51

$$\begin{cases}-\frac{a^4\operatorname{acoth}(a+bx)}{4b^4}-\frac{a^3\log(a+bx+1)}{b^4}+\frac{a^3\operatorname{acoth}(a+bx)}{b^4}+\frac{3a^2x}{4b^3}-\frac{3a^2\operatorname{acoth}(a+bx)}{2b^4}-\frac{ax^2}{4b^2}-\frac{a\log(a+bx+1)}{b^4}+\frac{a\operatorname{acoth}(a+bx)}{b^4}+\frac{x^4\operatorname{acoth}(a+bx)}{4}+\frac{x^3}{12b}+\frac{x}{4b^3}-\frac{\operatorname{acoth}(a+bx)}{4b^4} & \text{for } b\neq 0\\ \frac{x^4\operatorname{acoth}(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acoth(a + b*x)/(4*b**4) - a**3*log(a + b*x + 1)/b**4 + a**
3*acoth(a + b*x)/b**4 + 3*a**2*x/(4*b**3) - 3*a**2*acoth(a + b*x)/(2*b**4)
- a*x**2/(4*b**2) - a*log(a + b*x + 1)/b**4 + a*acoth(a + b*x)/b**4 + x**4*
acoth(a + b*x)/4 + x**3/(12*b) + x/(4*b**3) - acoth(a + b*x)/(4*b**4), Ne(b
, 0)), (x**4*acoth(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(87) = 174$.
time = 0.41, size = 512, normalized size = 5.07

$$-\frac{1}{6}\,((a+1)b-(a-1)b)\left(\frac{3\,(a^3+a)\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^5}-\frac{3\,(a^3+a)\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^5}-\frac{9\,a^2+\frac{3\,(3\,a^2-2\,a+1)(bx+a+1)^2}{(bx+a-1)^2}-\frac{3\,(6\,a^2-2\,a+1)(bx+a+1)}{bx+a-1}+2}{b^5\left(\frac{bx+a+1}{bx+a-1}-1\right)^3}+\frac{3\left(\frac{(bx+a+1)^3a^3}{(bx+a-1)^3}-\frac{3\,(bx+a+1)^2a^3}{(bx+a-1)^2}+\frac{3\,(bx+a+1)a^3}{bx+a-1}-a^3-\frac{3\,(bx+a+1)^3a^2}{(bx+a-1)^3}+\frac{6\,(bx+a+1)^2a^2}{(bx+a-1)^2}-\frac{3\,(bx+a+1)a^2}{bx+a-1}+\frac{3\,(bx+a+1)^3a}{(bx+a-1)^3}-\frac{3\,(bx+a+1)^2a}{(bx+a-1)^2}+\frac{(bx+a+1)a}{bx+a-1}-a-\frac{(bx+a+1)^3}{(bx+a-1)^3}-\frac{bx+a+1}{bx+a-1}\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^5\left(\frac{bx+a+1}{bx+a-1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*((a + 1)*b - (a - 1)*b)*(3*(a^3 + a)*log(abs(b*x + a + 1)/abs(b*x + a
- 1))/b^5 - 3*(a^3 + a)*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^5 - (9*
a^2 + 3*(3*a^2 - 2*a + 1)*(b*x + a + 1)^2/(b*x + a - 1)^2 - 3*(6*a^2 - 2*a
+ 1)*(b*x + a + 1)/(b*x + a - 1) + 2)/(b^5*((b*x + a + 1)/(b*x + a - 1) - 1
)^3) + 3*((b*x + a + 1)^3*a^3/(b*x + a - 1)^3 - 3*(b*x + a + 1)^2*a^3/(b*x
+ a - 1)^2 + 3*(b*x + a + 1)*a^3/(b*x + a - 1) - a^3 - 3*(b*x + a + 1)^3*a^
2/(b*x + a - 1)^3 + 6*(b*x + a + 1)^2*a^2/(b*x + a - 1)^2 - 3*(b*x + a + 1)
*a^2/(b*x + a - 1) + 3*(b*x + a + 1)^3*a/(b*x + a - 1)^3 - 3*(b*x + a + 1)^
2*a/(b*x + a - 1)^2 + (b*x + a + 1)*a/(b*x + a - 1) - a - (b*x + a + 1)^3/(
b*x + a - 1)^3 - (b*x + a + 1)/(b*x + a - 1))*log(-(1/(a - ((b*x + a + 1)*(
a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(
1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*
x + a - 1) - b)) - 1))/(b^5*((b*x + a + 1)/(b*x + a - 1) - 1)^4))
```

Mupad [B]
time = 1.47, size = 134, normalized size = 1.33

$$\frac{x^4\,\ln\left(\frac{1}{a+b\,x}+1\right)}{8}-x\,\left(\frac{4\,a^2-4}{16\,b^3}-\frac{a^2}{b^3}\right)-\frac{x^4\,\ln\left(1-\frac{1}{a+b\,x}\right)}{8}+\frac{x^3}{12\,b}-\frac{a\,x^2}{4\,b^2}+\frac{\ln\left(a+b\,x-1\right)\,\left(a^4-4\,a^3+6\,a^2-4\,a+1\right)}{8\,b^4}-\frac{\ln\left(a+b\,x+1\right)\,\left(a^4+4\,a^3+6\,a^2+4\,a+1\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(a + b*x),x)
```

```
[Out] (x^4*log(1/(a + b*x) + 1))/8 - x*((4*a^2 - 4)/(16*b^3) - a^2/b^3) - (x^4*lo
g(1 - 1/(a + b*x)))/8 + x^3/(12*b) - (a*x^2)/(4*b^2) + (log(a + b*x - 1)*(6
*a^2 - 4*a - 4*a^3 + a^4 + 1))/(8*b^4) - (log(a + b*x + 1)*(4*a + 6*a^2 + 4
*a^3 + a^4 + 1))/(8*b^4)
```

3.63 $\int x^2 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=78

$$-\frac{ax}{b^2}+\frac{(a+bx)^2}{6b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)+\frac{(1-a)^3\log(1-a-bx)}{6b^3}+\frac{(1+a)^3\log(1+a+bx)}{6b^3}$$

```
[Out] -a*x/b^2+1/6*(b*x+a)^2/b^3+1/3*x^3*arccoth(b*x+a)+1/6*(1-a)^3*ln(-b*x-a+1)/
b^3+1/6*(1+a)^3*ln(b*x+a+1)/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(a+bx)^2}{6b^3}+\frac{(1-a)^3\log(-a-bx+1)}{6b^3}+\frac{(a+1)^3\log(a+bx+1)}{6b^3}-\frac{ax}{b^2}+\frac{1}{3}x^3\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*x],x]
```

```
[Out] -((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*ArcCoth[a + b*x])/3 + ((1 - a)^3*
Log[1 - a - b*x])/(6*b^3) + ((1 + a)^3*Log[1 + a + b*x])/(6*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \coth^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{1}{3}x^3\coth^{-1}(a+bx) - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{1-x^2}\,dx, x, a+bx\right)$$
$$= \frac{1}{3}x^3\coth^{-1}(a+bx) - \frac{1}{3}\text{Subst}\left(\int \left(\frac{3a}{b^3} - \frac{x}{b^3} - \frac{a(3+a^2)-(1+3a^2)\,x}{b^3\,(1-x^2)}\right)\,dx, x, a$$
$$= -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx) + \frac{\text{Subst}\left(\int \frac{a(3+a^2)-(1+3a^2)x}{1-x^2}\,dx, x, a+bx\right.}{3b^3}$$
$$= -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx) - \frac{(1-a)^3\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, a+bx\right)}{6b^3} -$$
$$= -\frac{ax}{b^2} + \frac{(a+bx)^2}{6b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx) + \frac{(1-a)^3\log(1-a-bx)}{6b^3} + \frac{(1+a)^3\,\text{lo}}{}$$

Mathematica [A]
time = 0.02, size = 92, normalized size = 1.18

$$-\frac{2ax}{3b^2} + \frac{x^2}{6b} + \frac{1}{3}x^3\coth^{-1}(a+bx) + \frac{(1-3a+3a^2-a^3)\log(1-a-bx)}{6b^3} + \frac{(1+3a+3a^2+a^3)\log(1+a+bx)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a + b*x],x]
```

```
[Out] (-2*a*x)/(3*b^2) + x^2/(6*b) + (x^3*ArcCoth[a + b*x])/3 + ((1 - 3*a + 3*a^2
 - a^3)*Log[1 - a - b*x])/(6*b^3) + ((1 + 3*a + 3*a^2 + a^3)*Log[1 + a + b*
x])/(6*b^3)
```

Maple [A]
time = 0.05, size = 124, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(bx+a)a^3}{3}+\operatorname{arccoth}(bx+a)a^2(bx+a)-\operatorname{arccoth}(bx+a)a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(a^3-\cdots}{}}{b^3}$
default	$\frac{-\frac{\operatorname{arccoth}(bx+a)a^3}{3}+\operatorname{arccoth}(bx+a)a^2(bx+a)-\operatorname{arccoth}(bx+a)a(bx+a)^2+\frac{\operatorname{arccoth}(bx+a)(bx+a)^3}{3}-(bx+a)a+\frac{(bx+a)^2}{6}-\frac{(a^3-\cdots}{}}{b^3}$
risch	$\frac{x^3\ln(bx+a+1)}{6}-\frac{x^3\ln(bx+a-1)}{6}+\frac{\ln(-bx-a-1)a^3}{6b^3}-\frac{\ln(bx+a-1)a^3}{6b^3}+\frac{x^2}{6b}+\frac{\ln(-bx-a-1)a^2}{2b^3}+\frac{\ln(bx+a-1)\cdots}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arccoth(b*x+a)*a^3+arccoth(b*x+a)*a^2*(b*x+a)-arccoth(b*x+a)*a*
(b*x+a)^2+1/3*arccoth(b*x+a)*(b*x+a)^3-(b*x+a)*a+1/6*(b*x+a)^2-1/6*(a^3-3*a
^2+3*a-1)*ln(b*x+a-1)+1/6*(a^3+3*a^2+3*a+1)*ln(b*x+a+1))
```

Maxima [A]
time = 0.25, size = 79, normalized size = 1.01

$$\frac{1}{3}\,x^3\operatorname{arcoth}\left(bx+a\right)+\frac{1}{6}\,b\left(\frac{bx^2-4\,ax}{b^3}+\frac{(a^3+3\,a^2+3\,a+1)\log\left(bx+a+1\right)}{b^4}-\frac{(a^3-3\,a^2+3\,a-1)\log\left(bx+a-1\right)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*x + a) + 1/6*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a
+ 1)*log(b*x + a + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 1.08

$$\frac{b^3x^3\log\left(\frac{bx+a+1}{bx+a-1}\right)+b^2x^2-4\,abx+(a^3+3\,a^2+3\,a+1)\log\left(bx+a+1\right)-(a^3-3\,a^2+3\,a-1)\log\left(bx+a-1\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((b*x + a + 1)/(b*x + a - 1)) + b^2*x^2 - 4*a*b*x + (a^3 +
3*a^2 + 3*a + 1)*log(b*x + a + 1) - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1
))/b^3
```

Sympy [A]
time = 0.38, size = 117, normalized size = 1.50

$$\begin{cases} \frac{a^3\operatorname{acoth}\left(a+bx\right)}{3b^3}+\frac{a^2\log\left(a+bx+1\right)}{b^3}-\frac{a^2\operatorname{acoth}\left(a+bx\right)}{b^3}-\frac{2ax}{3b^2}+\frac{a\operatorname{acoth}\left(a+bx\right)}{b^3}+\frac{x^3\operatorname{acoth}\left(a+bx\right)}{3}+\frac{x^2}{6b}+\frac{\log\left(a+bx+1\right)}{3b^3}-\frac{\operatorname{acoth}\left(a+bx\right)}{3b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\operatorname{acoth}\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**3*acoth(a + b*x)/(3*b**3) + a**2*log(a + b*x + 1)/b**3 - a**2
*acoth(a + b*x)/b**3 - 2*a*x/(3*b**2) + a*acoth(a + b*x)/b**3 + x**3*acoth(
a + b*x)/3 + x**2/(6*b) + log(a + b*x + 1)/(3*b**3) - acoth(a + b*x)/(3*b**
3), Ne(b, 0)), (x**3*acoth(a)/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(68) = 136.
time = 0.41, size = 360, normalized size = 4.62

$$\frac{1}{6}\left((a+1)b-(a-1)b\right)\left(\frac{(3a^2+1)\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^4}-\frac{(3a^2+1)\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^4}-\frac{2\left(\frac{(bx+a+1)(3a-1)}{bx+a-1}-3a\right)}{b^4\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}+\frac{\left(\frac{3(bx+a+1)^2a^2}{(bx+a-1)^2}-\frac{6(bx+a+1)a^2}{bx+a-1}+3a^2-\frac{6(bx+a+1)^2a}{(bx+a-1)^2}+\frac{6(bx+a+1)a}{bx+a-1}+\frac{3(bx+a+1)^2}{(bx+a-1)^2}+1\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^4\left(\frac{bx+a+1}{bx+a-1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*((a + 1)*b - (a - 1)*b)*((3*a^2 + 1)*log(abs(b*x + a + 1)/abs(b*x + a -
 1))/b^4 - (3*a^2 + 1)*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^4 - 2*((
b*x + a + 1)*(3*a - 1)/(b*x + a - 1) - 3*a)/(b^4*((b*x + a + 1)/(b*x + a -
1) - 1)^2) + (3*(b*x + a + 1)^2*a^2/(b*x + a - 1)^2 - 6*(b*x + a + 1)*a^2/(
b*x + a - 1) + 3*a^2 - 6*(b*x + a + 1)^2*a/(b*x + a - 1)^2 + 6*(b*x + a + 1
)*a/(b*x + a - 1) + 3*(b*x + a + 1)^2/(b*x + a - 1)^2 + 1)*log(-(1/(a - ((b
*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1)
 - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x +
 a + 1)*b/(b*x + a - 1) - b)) - 1))/(b^4*((b*x + a + 1)/(b*x + a - 1) - 1)^
3))
```

Mupad [B]
time = 1.36, size = 98, normalized size = 1.26

$$\frac{x^3\ln\left(\frac{1}{a+b\,x}+1\right)}{6}-\frac{x^3\ln\left(1-\frac{1}{a+b\,x}\right)}{6}+\frac{x^2}{6\,b}-\frac{\ln\left(a+b\,x-1\right)\,\left(a^3-3\,a^2+3\,a-1\right)}{6\,b^3}+\frac{\ln\left(a+b\,x+1\right)\,\left(a^3+3\,a^2+3\,a+1\right)}{6\,b^3}-\frac{2\,a\,x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a + b*x),x)
```

```
[Out] (x^3*log(1/(a + b*x) + 1))/6 - (x^3*log(1 - 1/(a + b*x)))/6 + x^2/(6*b) - (
log(a + b*x - 1)*(3*a - 3*a^2 + a^3 - 1))/(6*b^3) + (log(a + b*x + 1)*(3*a
+ 3*a^2 + a^3 + 1))/(6*b^3) - (2*a*x)/(3*b^2)
```

3.64 $\int x \coth^{-1}(a+bx)\, dx$

Optimal. Leaf size=65

$$\frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)+\frac{(1-a)^2\log(1-a-bx)}{4b^2}-\frac{(1+a)^2\log(1+a+bx)}{4b^2}$$

```
[Out] 1/2*x/b+1/2*x^2*arccoth(b*x+a)+1/4*(1-a)^2*ln(-b*x-a+1)/b^2-1/4*(1+a)^2*ln(
b*x+a+1)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(1-a)^2\log(-a-bx+1)}{4b^2}-\frac{(a+1)^2\log(a+bx+1)}{4b^2}+\frac{1}{2}x^2\coth^{-1}(a+bx)+\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*x],x]
```

```
[Out] x/(2*b) + (x^2*ArcCoth[a + b*x])/2 + ((1 - a)^2*Log[1 - a - b*x])/(4*b^2) -
 ((1 + a)^2*Log[1 + a + b*x])/(4*b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\coth^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{1-x^2}\,dx,x,a+bx\right)\\ &= \frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\left(-\frac{1}{b^2}+\frac{1+a^2-2ax}{b^2(1-x^2)}\right)dx,x,a+bx\right)\\ &= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{\text{Subst}\left(\int\frac{1+a^2-2ax}{1-x^2}\,dx,x,a+bx\right)}{2b^2}\\ &= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)-\frac{(1-a)^2\text{Subst}\left(\int\frac{1}{1-x}\,dx,x,a+bx\right)}{4b^2}+\frac{(1+a)^2\text{Subst}(}{}\\ &= \frac{x}{2b}+\frac{1}{2}x^2\coth^{-1}(a+bx)+\frac{(1-a)^2\log(1-a-bx)}{4b^2}-\frac{(1+a)^2\log(1+a+bx)}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.86

$$\frac{2bx+2b^2x^2\coth^{-1}(a+bx)+(-1+a)^2\log(1-a-bx)-(1+a)^2\log(1+a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a + b*x],x]
[Out] (2*b*x + 2*b^2*x^2*ArcCoth[a + b*x] + (-1 + a)^2*Log[1 - a - b*x] - (1 + a)
^2*Log[1 + a + b*x])/(4*b^2)
```

Maple [A]
time = 0.05, size = 70, normalized size = 1.08

method	result

derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}-\operatorname{arccoth}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{(2a-1)\ln(bx+a-1)}{4}+\frac{(-2a-1)\ln(bx+a+1)}{4}}{b^2}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}-\operatorname{arccoth}(bx+a)a(bx+a)+\frac{bx}{2}+\frac{a}{2}-\frac{(2a-1)\ln(bx+a-1)}{4}+\frac{(-2a-1)\ln(bx+a+1)}{4}}{b^2}$
risch	$\frac{x^2\ln(bx+a+1)}{4}-\frac{x^2\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)a^2}{4b^2}+\frac{\ln(-bx-a+1)a^2}{4b^2}-\frac{\ln(bx+a+1)a}{2b^2}-\frac{\ln(-bx-a+1)a}{2b^2}+\frac{x}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*(b*x+a)^2*arccoth(b*x+a)-arccoth(b*x+a)*a*(b*x+a)+1/2*b*x+1/2*a-
1/4*(2*a-1)*ln(b*x+a-1)+1/4*(-2*a-1)*ln(b*x+a+1))
```

Maxima [A]
time = 0.25, size = 61, normalized size = 0.94

$$\frac{1}{2}x^2\operatorname{arcoth}(bx+a)+\frac{1}{4}b\left(\frac{2x}{b^2}-\frac{(a^2+2a+1)\log(bx+a+1)}{b^3}+\frac{(a^2-2a+1)\log(bx+a-1)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(b*x + a) + 1/4*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1
)/b^3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)
```

Fricas [A]
time = 0.38, size = 66, normalized size = 1.02

$$\frac{b^2x^2\log\left(\frac{bx+a+1}{bx+a-1}\right)+2bx-(a^2+2a+1)\log(bx+a+1)+(a^2-2a+1)\log(bx+a-1)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((b*x + a + 1)/(b*x + a - 1)) + 2*b*x - (a^2 + 2*a + 1)*log
(b*x + a + 1) + (a^2 - 2*a + 1)*log(b*x + a - 1))/b^2
```

Sympy [A]
time = 0.27, size = 76, normalized size = 1.17

$$\begin{cases} -\frac{a^2\operatorname{acoth}(a+bx)}{2b^2}-\frac{a\log(a+bx+1)}{b^2}+\frac{a\operatorname{acoth}(a+bx)}{b^2}+\frac{x^2\operatorname{acoth}(a+bx)}{2}+\frac{x}{2b}-\frac{\operatorname{acoth}(a+bx)}{2b^2} & \text{for } b\neq 0 \\ \frac{x^2\operatorname{acoth}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acoth(a + b*x)/(2*b**2) - a*log(a + b*x + 1)/b**2 + a*acot
h(a + b*x)/b**2 + x**2*acoth(a + b*x)/2 + x/(2*b) - acoth(a + b*x)/(2*b**2)
, Ne(b, 0)), (x**2*acoth(a)/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(55) = 110$.

time = 0.40, size = 259, normalized size = 3.98

$$-\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{a\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^3}-\frac{a\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^3}+\frac{\left(\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^3\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}-\frac{1}{b^3\left(\frac{bx+a+1}{bx+a-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*((a + 1)*b - (a - 1)*b)*(a*log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^3
- a*log(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^3 + ((b*x + a + 1)*a/(b*x +
 a - 1) - a - (b*x + a + 1)/(b*x + a - 1))*log(-(1/(a - ((b*x + a + 1)*(a -
 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(
a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x +
 a - 1) - b)) - 1))/(b^3*((b*x + a + 1)/(b*x + a - 1) - 1)^2) - 1/(b^3*((b*
x + a + 1)/(b*x + a - 1) - 1)))
```

Mupad [B]

time = 2.00, size = 62, normalized size = 0.95

$$\frac{x^2\,\mathrm{acoth}(a+b\,x)}{2}-\frac{\frac{\mathrm{acoth}(a+b\,x)}{2}-\frac{b\,x}{2}+\frac{a^2\,\mathrm{acoth}(a+b\,x)}{2}+\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a + b*x),x)
```

```
[Out] (x^2*acoth(a + b*x))/2 - (acoth(a + b*x)/2 - (b*x)/2 + (a^2*acoth(a + b*x))
/2 + (a*log(a^2 + b^2*x^2 + 2*a*b*x - 1))/2)/b^2
```

3.65 $\int \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)\coth^{-1}(a+bx)}{b}+\frac{\log\left(1-(a+bx)^2\right)}{2b}$$

```
[Out] (b*x+a)*arccoth(b*x+a)/b+1/2*ln(1-(b*x+a)^2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6239, 6022, 266}

$$\frac{\log\left(1-(a+bx)^2\right)}{2b}+\frac{(a+bx)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x],x]
```

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b + Log[1 - (a + b*x)^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \coth^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \coth^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b} + \frac{\log\left(1-(a+bx)^2\right)}{2b}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.23

$$x\coth^{-1}(a+bx) + \frac{-((-1+a)\log(1-a-bx)) + (1+a)\log(1+a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x],x]
```

```
[Out] x*ArcCoth[a + b*x] + (-((-1 + a)*Log[1 - a - b*x]) + (1 + a)*Log[1 + a + b*
x])/(2*b)
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arccoth}(bx+a)+\frac{\ln\left((bx+a)^2-1\right)}{2}}{b}$	30
default	$\frac{(bx+a)\operatorname{arccoth}(bx+a)+\frac{\ln\left((bx+a)^2-1\right)}{2}}{b}$	30
risch	$\frac{x\ln(bx+a+1)}{2} - \frac{x\ln(bx+a-1)}{2} - \frac{\ln(bx+a-1)a}{2b} + \frac{\ln(-bx-a-1)a}{2b} + \frac{\ln(bx+a-1)}{2b} + \frac{\ln(-bx-a-1)}{2b}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arccoth(b*x+a)+1/2*ln((b*x+a)^2-1))
```

Maxima [A]
time = 0.26, size = 31, normalized size = 0.89

$$\frac{2\,(bx+a)\operatorname{arcoth}\left(bx+a\right) + \log\left(-(bx+a)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x + a)*arccoth(b*x + a) + log(-(b*x + a)^2 + 1))/b
```

Fricas [A]
time = 0.33, size = 48, normalized size = 1.37

$$\frac{bx\log\left(\frac{bx+a+1}{bx+a-1}\right)+(a+1)\log\left(bx+a+1\right)-(a-1)\log\left(bx+a-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((b*x + a + 1)/(b*x + a - 1)) + (a + 1)*log(b*x + a + 1) - (a -
 1)*log(b*x + a - 1))/b
```

Sympy [A]
time = 0.19, size = 41, normalized size = 1.17

$$\begin{cases}\frac{a\operatorname{acoth}\left(a+bx\right)}{b}+x\operatorname{acoth}\left(a+bx\right)+\frac{\log\left(a+bx+1\right)}{b}-\frac{\operatorname{acoth}\left(a+bx\right)}{b} & \text{for}\ b\neq 0\\ x\operatorname{acoth}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a),x)
```

```
[Out] Piecewise((a*acoth(a + b*x)/b + x*acoth(a + b*x) + log(a + b*x + 1)/b - aco
th(a + b*x)/b, Ne(b, 0)), (x*acoth(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(33) = 66.
time = 0.40, size = 197, normalized size = 5.63

$$\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^2}+\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 - l
og(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^2 + log(-(1/(a - ((b*x + a + 1)*
```

```
(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/
(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b
*x + a - 1) - b)) - 1))/(b^2*((b*x + a + 1)/(b*x + a - 1) - 1)))
```

Mupad [B]
time = 1.71, size = 42, normalized size = 1.20

$$\frac{\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2}+a\,\mathrm{acoth}(a+b\,x)}{b}+x\,\mathrm{acoth}(a+b\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x),x)
[Out] (log(a^2 + b^2*x^2 + 2*a*b*x - 1)/2 + a*acoth(a + b*x))/b + x*acoth(a + b*x
)
```

3.66 $\int \frac{\coth^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=92

$$-\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right.$$

```
[Out] -arccoth(b*x+a)*ln(2/(b*x+a+1))+arccoth(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))+1/
2*polylog(2,1-2/(b*x+a+1))-1/2*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6247, 6058, 2449, 2352, 2497}

$$\frac{1}{2}\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\log\left(\frac{2}{a+bx+1}\right)\left(-\coth^{-1}(a+bx)\right)+\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x,x]
```

```
[Out] -(ArcCoth[a + b*x]*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]*Log[(2*b*x)/((1
 - a)*(1 + a + b*x))] + PolyLog[2, 1 - 2/(1 + a + b*x)]/2 - PolyLog[2, 1 -
(2*b*x)/((1 - a)*(1 + a + b*x))]/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)-$$

$$= -\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 259, normalized size = 2.82

$$(\coth^{-1}(a+bx)-\tanh^{-1}(a+bx))\log(x)+\tanh^{-1}(a+bx)\left(-\log\left(\frac{1}{\sqrt{1-(a+bx)^2}}\right)+\log\left(-i\sinh\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\right)\right)+\frac{1}{8}\left(4\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)^2-\left(\pi-2i\tanh^{-1}(a+bx)\right)^2-8\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\log\left(1-e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)-4i\left(\pi-2i\tanh^{-1}(a+bx)\right)\log\left(1+e^{2\tanh^{-1}(a+bx)}\right)+4\left(i\pi+2\tanh^{-1}(a+bx)\right)\log\left(\frac{2}{\sqrt{1-(a+bx)^2}}\right)+8\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\log\left(-2i\sinh\left(\tanh^{-1}(a)-\tanh^{-1}(a+bx)\right)\right)-4\text{PolyLog}\left(2,e^{2\tanh^{-1}(a)-2\tanh^{-1}(a+bx)}\right)-4\text{PolyLog}\left(2,-e^{2\tanh^{-1}(a+bx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x,x]
```

```
[Out] (ArcCoth[a + b*x] - ArcTanh[a + b*x])*Log[x] + ArcTanh[a + b*x]*(-Log[1/Sqr
t[1 - (a + b*x)^2]] + Log[(-I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]]) + (4*(
ArcTanh[a] - ArcTanh[a + b*x])^2 - (Pi - (2*I)*ArcTanh[a + b*x])^2 - 8*(Arc
Tanh[a] - ArcTanh[a + b*x])*Log[1 - E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])]
- (4*I)*(Pi - (2*I)*ArcTanh[a + b*x])*Log[1 + E^(2*ArcTanh[a + b*x])] + 4*(
I*Pi + 2*ArcTanh[a + b*x])*Log[2/Sqrt[1 - (a + b*x)^2]] + 8*(ArcTanh[a] - A
rcTanh[a + b*x])*Log[(-2*I)*Sinh[ArcTanh[a] - ArcTanh[a + b*x]]] - 4*PolyLo
g[2, E^(2*ArcTanh[a] - 2*ArcTanh[a + b*x])] - 4*PolyLog[2, -E^(2*ArcTanh[a
+ b*x])])/8
```

Maple [A]
time = 0.28, size = 104, normalized size = 1.13

method	result
risch	$\frac{\operatorname{dilog}\left(\frac{bx}{-a-1}\right)}{2}+\frac{\ln(bx+a+1)\ln\left(\frac{bx}{-a-1}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{bx}{1-a}\right)}{2}-\frac{\ln(bx+a-1)\ln\left(\frac{bx}{1-a}\right)}{2}$
derivativedivides	$\ln(-bx)\operatorname{arccoth}(bx+a)+\frac{\operatorname{dilog}\left(\frac{-bx-a+1}{1-a}\right)}{2}+\frac{\ln(-bx)\ln\left(\frac{-bx-a+1}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-bx-a-1}{-a-1}\right)}{2}-\ln(-bx)\ln($
default	$\ln(-bx)\operatorname{arccoth}(bx+a)+\frac{\operatorname{dilog}\left(\frac{-bx-a+1}{1-a}\right)}{2}+\frac{\ln(-bx)\ln\left(\frac{-bx-a+1}{1-a}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{-bx-a-1}{-a-1}\right)}{2}-\ln(-bx)\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-b*x)*arccoth(b*x+a)+1/2*dilog((-b*x-a+1)/(1-a))+1/2*ln(-b*x)*ln((-b*x-a
+1)/(1-a))-1/2*dilog((-b*x-a-1)/(-a-1))-1/2*ln(-b*x)*ln((-b*x-a-1)/(-a-1))
```

Maxima [A]
time = 0.26, size = 128, normalized size = 1.39

$$-\frac{1}{2}\,b\left(\frac{\log\left(bx+a+1\right)}{b}-\frac{\log\left(bx+a-1\right)}{b}\right)\log\left(x\right)+\frac{1}{2}\,b\left(\frac{\log\left(bx+a+1\right)\log\left(-\frac{bx+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(\frac{bx+a+1}{a+1}\right)}{b}-\frac{\log\left(bx+a-1\right)\log\left(-\frac{bx+a-1}{a-1}+1\right)+\mathrm{Li}_2\left(\frac{bx+a-1}{a-1}\right)}{b}\right)+\operatorname{arcoth}\left(bx+a\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(x) + 1/2*b*((log(b*x +
 a + 1)*log(-(b*x + a + 1)/(a + 1) + 1) + dilog((b*x + a + 1)/(a + 1)))/b -
 (log(b*x + a - 1)*log(-(b*x + a - 1)/(a - 1) + 1) + dilog((b*x + a - 1)/(a
 - 1)))/b) + arccoth(b*x + a)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(a+bx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x,x)
```

```
[Out] Integral(acoth(a + b*x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/x,x)
```

```
[Out] int(acoth(a + b*x)/x, x)
```

3.67 $\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\coth^{-1}(a+bx)}{x}+\frac{b\log(x)}{1-a^2}-\frac{b\log(1-a-bx)}{2(1-a)}-\frac{b\log(1+a+bx)}{2(1+a)}$$

```
[Out] -arccoth(b*x+a)/x+b*ln(x)/(-a^2+1)-1/2*b*ln(-b*x-a+1)/(1-a)-1/2*b*ln(b*x+a+
1)/(1+a)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6245, 378, 720, 31, 647}

$$\frac{b\log(x)}{1-a^2}-\frac{b\log(-a-bx+1)}{2(1-a)}-\frac{b\log(a+bx+1)}{2(a+1)}-\frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/x) + (b*Log[x])/(1 - a^2) - (b*Log[1 - a - b*x])/(2*(1 -
 a)) - (b*Log[1 + a + b*x])/(2*(1 + a))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
```

```
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{x^2}\,dx &= -\frac{\coth^{-1}(a+bx)}{x}+b\int \frac{1}{x\left(1-(a+bx)^2\right)}\,dx\\
&= -\frac{\coth^{-1}(a+bx)}{x}+b\text{Subst}\left(\int \frac{1}{(-a+x)\left(1-x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{1-a^2}+\frac{b\text{Subst}\left(\int \frac{a+x}{1-x^2}\,dx,x,a+bx\right)}{1-a^2}\\
&= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\log(x)}{1-a^2}+\frac{b\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,a+bx\right)}{2(1-a)}+\frac{b\text{Subst}\left(\int \frac{1}{-1-x}\,dx,x\right.}{2(1+a)}\\
&= -\frac{\coth^{-1}(a+bx)}{x}+\frac{b\log(x)}{1-a^2}-\frac{b\log(1-a-bx)}{2(1-a)}-\frac{b\log(1+a+bx)}{2(1+a)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.86

$$-\frac{\coth^{-1}(a+bx)}{x}+\frac{b(-2\log(x)+(1+a)\log(1-a-bx)-(-1+a)\log(1+a+bx))}{2\left(-1+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/x) + (b*(-2*Log[x] + (1 + a)*Log[1 - a - b*x] - (-1 + a)
*Log[1 + a + b*x]))/(2*(-1 + a^2))
```

Maple [A]
time = 0.05, size = 66, normalized size = 1.03

method	result

derivativedivides	$b\left(-\frac{\operatorname{arccoth}(bx+a)}{bx}-\frac{\ln(-bx)}{(-1+a)(1+a)}+\frac{\ln(bx+a-1)}{-2+2a}-\frac{\ln(bx+a+1)}{2+2a}\right)$
default	$b\left(-\frac{\operatorname{arccoth}(bx+a)}{bx}-\frac{\ln(-bx)}{(-1+a)(1+a)}+\frac{\ln(bx+a-1)}{-2+2a}-\frac{\ln(bx+a+1)}{2+2a}\right)$
risch	$-\frac{\ln(bx+a+1)}{2x}-\frac{\ln(bx+a+1)abx-\ln(-bx-a+1)abx-\ln(bx+a+1)bx+2\ln(-x)bx-\ln(-bx-a+1)bx-\ln(bx+a-1)a^2+\mathrm{l}}{2x(-1+a)(1+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arccoth(b*x+a)-1/(-1+a)/(1+a)*ln(-b*x)+1/(-2+2*a)*ln(b*x+a-1)-1/(
2+2*a)*ln(b*x+a+1))
```

Maxima [A]
time = 0.26, size = 54, normalized size = 0.84

$$-\frac{1}{2}b\left(\frac{\log(bx+a+1)}{a+1}-\frac{\log(bx+a-1)}{a-1}+\frac{2\log(x)}{a^2-1}\right)-\frac{\operatorname{arcoth}(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(b*x + a + 1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2
 - 1)) - arccoth(b*x + a)/x
```

Fricas [A]
time = 0.40, size = 68, normalized size = 1.06

$$-\frac{(a-1)bx\log(bx+a+1)-(a+1)bx\log(bx+a-1)+2\,bx\log(x)+(a^2-1)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\,(a^2-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a - 1)*b*x*log(b*x + a + 1) - (a + 1)*b*x*log(b*x + a - 1) + 2*b*x*l
og(x) + (a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1)))/((a^2 - 1)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(48)=96$.
time = 0.62, size = 144, normalized size = 2.25

$$\begin{cases}\frac{b\operatorname{acoth}(bx-1)}{2}-\frac{\operatorname{acoth}(bx-1)}{x}-\frac{1}{2x} & \text{for}\ a=-1\\ -\frac{b\operatorname{acoth}(bx+1)}{2}-\frac{\operatorname{acoth}(bx+1)}{x}+\frac{1}{2x} & \text{for}\ a=1\\ -\frac{a^2\operatorname{acoth}(a+bx)}{a^2x-x}-\frac{abx\operatorname{acoth}(a+bx)}{a^2x-x}-\frac{bx\log(x)}{a^2x-x}+\frac{bx\log(a+bx+1)}{a^2x-x}-\frac{bx\operatorname{acoth}(a+bx)}{a^2x-x}+\frac{\operatorname{acoth}(a+bx)}{a^2x-x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x**2,x)
```

```
[Out] Piecewise((b*acoth(b*x - 1)/2 - acoth(b*x - 1)/x - 1/(2*x), Eq(a, -1)), (-b
*acoth(b*x + 1)/2 - acoth(b*x + 1)/x + 1/(2*x), Eq(a, 1)), (-a**2*acoth(a +
 b*x)/(a**2*x - x) - a*b*x*acoth(a + b*x)/(a**2*x - x) - b*x*log(x)/(a**2*x
 - x) + b*x*log(a + b*x + 1)/(a**2*x - x) - b*x*acoth(a + b*x)/(a**2*x - x)
 + acoth(a + b*x)/(a**2*x - x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(57) = 114$.
time = 0.40, size = 259, normalized size = 4.05

$$-\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{(a-1)\log\left(\left|\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right|\right)}{a^3-a^2-a+1}-\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{a^2-1}-\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{\left(\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right)(a-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*((a + 1)*b - (a - 1)*b)*((a - 1)*log(abs((b*x + a + 1)*a/(b*x + a - 1)
 - a - (b*x + a + 1)/(b*x + a - 1) - 1))/(a^3 - a^2 - a + 1) - log(abs(b*x
+ a + 1)/abs(b*x + a - 1))/(a^2 - 1) - log(-(1/(a - ((b*x + a + 1)*(a - 1)/
(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a -
((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a -
 1) - b)) - 1))/(((b*x + a + 1)*a/(b*x + a - 1) - a - (b*x + a + 1)/(b*x +
a - 1) - 1)*(a - 1)))
```

Mupad [B]
time = 1.73, size = 62, normalized size = 0.97

$$-\frac{\mathrm{acoth}(a+b\,x)}{x}-\frac{b\,x\,\ln\left(x\right)-\frac{b\,x\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2}+a\,b\,x\,\mathrm{acoth}(a+b\,x)}{x\,\left(a^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/x^2,x)
```

```
[Out] - acoth(a + b*x)/x - (b*x*log(x) - (b*x*log(a^2 + b^2*x^2 + 2*a*b*x - 1))/2
 + a*b*x*acoth(a + b*x))/(x*(a^2 - 1))
```

3.68 $\int \frac{\coth^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=90

$$-\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b^2\log(1-a-bx)}{4(1-a)^2}+\frac{b^2\log(1+a+bx)}{4(1+a)^2}$$

```
[Out] -1/2*b/(-a^2+1)/x-1/2*arccoth(b*x+a)/x^2+a*b^2*ln(x)/(-a^2+1)^2-1/4*b^2*ln(
-b*x-a+1)/(1-a)^2+1/4*b^2*ln(b*x+a+1)/(1+a)^2
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6245, 378, 724, 815}

$$\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b}{2\left(1-a^2\right)x}-\frac{b^2\log(-a-bx+1)}{4(1-a)^2}+\frac{b^2\log(a+bx+1)}{4(a+1)^2}-\frac{\coth^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/x^3,x]
```

```
[Out] -1/2*b/((1 - a^2)*x) - ArcCoth[a + b*x]/(2*x^2) + (a*b^2*Log[x])/(1 - a^2)^
2 - (b^2*Log[1 - a - b*x])/(4*(1 - a)^2) + (b^2*Log[1 + a + b*x])/(4*(1 + a
)^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{x^3}\,dx &= -\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2\left(1-(a+bx)^2\right)}\,dx\\ &= -\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{1}{2}b^2\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(1-x^2\right)}\,dx,x,a+bx\right)\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}-\frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)(1-x^2)}\,dx,x,a+bx\right)}{2\left(1-a^2\right)}\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}-\frac{b^2\text{Subst}\left(\int \left(-\frac{2a}{(-1+a^2)(a-x)}+\frac{-1-a}{2(-1+a)(-1+x)}+\frac{}{2(1}\right.\right.}{2\left(1-a^2\right)}\\ &= -\frac{b}{2\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)}{2x^2}+\frac{ab^2\log(x)}{\left(1-a^2\right)^2}-\frac{b^2\log(1-a-bx)}{4(1-a)^2}+\frac{b^2\log(1+a-}{4(1+a)^2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.84

$$\frac{1}{4}\left(-\frac{2\coth^{-1}(a+bx)}{x^2}+b\left(\frac{2}{\left(-1+a^2\right)x}+\frac{4ab\log(x)}{\left(-1+a^2\right)^2}-\frac{b\log(1-a-bx)}{(-1+a)^2}+\frac{b\log(1+a+bx)}{(1+a)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^3,x]
[Out] ((-2*ArcCoth[a + b*x])/x^2 + b*(2/((-1 + a^2)*x) + (4*a*b*Log[x])/(-1 + a^2
)^2 - (b*Log[1 - a - b*x])/(-1 + a)^2 + (b*Log[1 + a + b*x])/(1 + a)^2))/4
```

Maple [A]
time = 0.07, size = 83, normalized size = 0.92

method	result
derivativedivides	$b^2\left(-\frac{\mathrm{arccoth}(bx+a)}{2b^2x^2}+\frac{1}{2(-1+a)(1+a)bx}+\frac{a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\ln(bx+a+1)}{4(1+a)^2}-\frac{\ln(bx+a-1)}{4(-1+a)^2}\right)$
default	$b^2\left(-\frac{\mathrm{arccoth}(bx+a)}{2b^2x^2}+\frac{1}{2(-1+a)(1+a)bx}+\frac{a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\ln(bx+a+1)}{4(1+a)^2}-\frac{\ln(bx+a-1)}{4(-1+a)^2}\right)$

risch	$-\frac{\ln(bx+a+1)}{4x^2}+\frac{\ln(-bx-a-1)a^2b^2x^2-\ln(-bx-a+1)a^2b^2x^2+4\ln(x)ab^2x^2-2\ln(-bx-a-1)ab^2x^2-2\ln(-bx-a+1)a}{4x^2(a^2-2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arccoth(b*x+a)+1/2/(-1+a)/(1+a)/b/x+a/(-1+a)^2/(1+a)^2*ln
(-b*x)+1/4/(1+a)^2*ln(b*x+a+1)-1/4/(-1+a)^2*ln(b*x+a-1))
```

Maxima [A]
time = 0.26, size = 85, normalized size = 0.94

$$\frac{1}{4}\left(\frac{4\,ab\log\left(x\right)}{a^4-2\,a^2+1}+\frac{b\log\left(bx+a+1\right)}{a^2+2\,a+1}-\frac{b\log\left(bx+a-1\right)}{a^2-2\,a+1}+\frac{2}{(a^2-1)x}\right)b-\frac{\operatorname{arcoth}\left(bx+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) -
b*log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b - 1/2*arccoth(b*x +
 a)/x^2
```

Fricas [A]
time = 0.42, size = 111, normalized size = 1.23

$$\frac{(a^2-2\,a+1)b^2x^2\log\left(bx+a+1\right)-(a^2+2\,a+1)b^2x^2\log\left(bx+a-1\right)+4\,ab^2x^2\log\left(x\right)+2\,(a^2-1)bx-(a^4-2\,a^2+1)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{4\,(a^4-2\,a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 2*a + 1)*b^2*x^2*log(b*x + a + 1) - (a^2 + 2*a + 1)*b^2*x^2*log
(b*x + a - 1) + 4*a*b^2*x^2*log(x) + 2*(a^2 - 1)*b*x - (a^4 - 2*a^2 + 1)*lo
g((b*x + a + 1)/(b*x + a - 1)))/((a^4 - 2*a^2 + 1)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(73) = 146.
time = 0.98, size = 410, normalized size = 4.56

$$\begin{cases} \frac{b^2\operatorname{acoth}(bx-1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx-1)}{2x^2}-\frac{1}{8x^2} & \text{for } a=-1 \\ \frac{b^2\operatorname{acoth}(bx+1)}{8}-\frac{b}{8x}-\frac{\operatorname{acoth}(bx+1)}{2x^2}+\frac{1}{8x^2} & \text{for } a=1 \\ -\frac{a^4\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2b^2x^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{a^2bx}{2a^4x^2-4a^2x^2+2x^2}+\frac{2a^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{2ab^2x^2\log(x)}{2a^4x^2-4a^2x^2+2x^2}-\frac{2ab^2x^2\log(a+bx+1)}{2a^4x^2-4a^2x^2+2x^2}+\frac{2ab^2x^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}+\frac{b^2x^2\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2}-\frac{bx}{2a^4x^2-4a^2x^2+2x^2}-\frac{\operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/x**3,x)
```

```
[Out] Piecewise((b**2*acoth(b*x - 1)/8 - b/(8*x) - acoth(b*x - 1)/(2*x**2) - 1/(8
*x**2), Eq(a, -1)), (b**2*acoth(b*x + 1)/8 - b/(8*x) - acoth(b*x + 1)/(2*x*
```

```
*2) + 1/(8*x**2), Eq(a, 1)), (-a**4*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x*
*2 + 2*x**2) + a**2*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2
*x**2) + a**2*b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a**2*acoth(a + b
*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*log(x)/(2*a**4*x**
2 - 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*log(a + b*x + 1)/(2*a**4*x**2 - 4
*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x
**2 + 2*x**2) + b**2*x**2*acoth(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**
2) - b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - acoth(a + b*x)/(2*a**4*x**2
 - 4*a**2*x**2 + 2*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(76) = 152$.
time = 0.41, size = 360, normalized size = 4.00

$$-\frac{1}{2}((a+1)b-(a-1)b)\left(\frac{ab\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{a^4-2a^2+1}-\frac{ab\log\left(\left|\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right|\right)}{a^4-2a^2+1}+\frac{\left(\frac{(bx+a+1)ab}{bx+a-1}-ab-\frac{(bx+a+1)b}{bx+a-1}\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{(a^2-2a+1)\left(\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right)^2}+\frac{ab+b}{\left(\frac{(bx+a+1)a}{bx+a-1}-a-\frac{bx+a+1}{bx+a-1}-1\right)(a+1)^2(a-1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((a + 1)*b - (a - 1)*b)*(a*b*log(abs(b*x + a + 1)/abs(b*x + a - 1))/(a
^4 - 2*a^2 + 1) - a*b*log(abs((b*x + a + 1)*a/(b*x + a - 1) - a - (b*x + a
+ 1)/(b*x + a - 1) - 1))/(a^4 - 2*a^2 + 1) + ((b*x + a + 1)*a*b/(b*x + a -
1) - a*b - (b*x + a + 1)*b/(b*x + a - 1))*log(-(1/(a - ((b*x + a + 1)*(a -
1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a
 - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x +
a - 1) - b)) - 1))/((a^2 - 2*a + 1)*((b*x + a + 1)*a/(b*x + a - 1) - a - (b
*x + a + 1)/(b*x + a - 1) - 1)^2) + (a*b + b)/(((b*x + a + 1)*a/(b*x + a -
1) - a - (b*x + a + 1)/(b*x + a - 1) - 1)*(a + 1)^2*(a - 1)^2))
```

Mupad [B]
time = 1.95, size = 247, normalized size = 2.74

$$\ln(x)\left(\frac{b^2}{4(a-1)^2}-\frac{b^2}{4(a+1)^2}\right)-\ln\left(a^2+2abx+b^2x^2-1\right)\left(\frac{b^2}{8(a-1)^2}-\frac{b^2}{8(a+1)^2}\right)-\frac{\mathrm{acoth}(a+bx)\left(\frac{a^2}{2}-\frac{1}{2}\right)-\frac{bx}{2}+\frac{b^2x^2\,\mathrm{acoth}(a+bx)}{2}+\frac{x^3(3a^2b^3+b^3)}{2(a^2-1)^2}+\frac{ab^4x^4}{(a^2-1)^2}+abx\,\mathrm{acoth}(a+bx)}{a^2x^2+2abx^3+b^2x^4-x^2}-\frac{\mathrm{atan}\left(\frac{2xb^2+2ab}{2\sqrt{b^2(a^2-1)-a^2b^2}}\right)(a^2b^3+b^3)}{\sqrt{-b^2}\,(2a^4-4a^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/x^3,x)
```

```
[Out] log(x)*(b^2/(4*(a - 1)^2) - b^2/(4*(a + 1)^2)) - log(a^2 + b^2*x^2 + 2*a*b*
x - 1)*(b^2/(8*(a - 1)^2) - b^2/(8*(a + 1)^2)) - (acoth(a + b*x)*(a^2/2 - 1
/2) - (b*x)/2 + (b^2*x^2*acoth(a + b*x))/2 + (x^3*(b^3 + 3*a^2*b^3))/(2*(a^
2 - 1)^2) + (a*b^4*x^4)/(a^2 - 1)^2 + a*b*x*acoth(a + b*x))/(a^2*x^2 - x^2
+ b^2*x^4 + 2*a*b*x^3) - (atan((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 - 1) - a^2*b^
2)^(1/2)))*(b^3 + a^2*b^3))/((-b^2)^(1/2)*(2*a^4 - 4*a^2 + 2))
```

3.69 $\int x^3 \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=263

$$-\frac{ax}{b^3}+\frac{(a+bx)^2}{12b^4}+\frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4}-\frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4}-$$

```
[Out] -a*x/b^3+1/12*(b*x+a)^2/b^4+1/2*(6*a^2+1)*(b*x+a)*arccoth(b*x+a)/b^4-a*(b*x
+a)^2*arccoth(b*x+a)/b^4+1/6*(b*x+a)^3*arccoth(b*x+a)/b^4-a*(a^2+1)*arccoth
(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*arccoth(b*x+a)^2/b^4+1/4*x^4*arccoth(b*x+a)
^2+a*arctanh(b*x+a)/b^4+2*a*(a^2+1)*arccoth(b*x+a)*ln(2/(-b*x-a+1))/b^4+1/1
2*ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*polylog(2
,(-b*x-a-1)/(-b*x-a+1))/b^4
```

Rubi [A]
time = 0.25, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6247, 6066, 6022, 266, 6038, 327, 212, 272, 45, 6196, 6096, 6132, 6056, 2449, 2352}

$$\frac{a(a^2+1)\operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^4}+\frac{(6a^2+1)\log\left(1-(a+bx)^2\right)}{4b^4}-\frac{a(a^2+1)\coth^{-1}(a+bx)^2}{b^4}+\frac{(6a^2+1)(a+bx)\coth^{-1}(a+bx)}{2b^4}+\frac{2a(a^2+1)\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b^4}-\frac{(a^4+6a^2+1)\coth^{-1}(a+bx)^2}{4b^4}+\frac{(a+bx)^2}{12b^4}+\frac{\log\left(1-(a+bx)^2\right)}{12b^4}+\frac{a\tanh^{-1}(a+bx)}{b^4}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4}-\frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4}-\frac{ax}{b^3}+\frac{1}{4}x^4\coth^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x]^2,x]
```

```
[Out] -((a*x)/b^3) + (a + b*x)^2/(12*b^4) + ((1 + 6*a^2)*(a + b*x)*ArcCoth[a + b*
x])/(2*b^4) - (a*(a + b*x)^2*ArcCoth[a + b*x])/b^4 + ((a + b*x)^3*ArcCoth[a
 + b*x])/(6*b^4) - (a*(1 + a^2)*ArcCoth[a + b*x]^2)/b^4 - ((1 + 6*a^2 + a^4
)*ArcCoth[a + b*x]^2)/(4*b^4) + (x^4*ArcCoth[a + b*x]^2)/4 + (a*ArcTanh[a +
 b*x])/b^4 + (2*a*(1 + a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/b^4 + Lo
g[1 - (a + b*x)^2]/(12*b^4) + ((1 + 6*a^2)*Log[1 - (a + b*x)^2])/(4*b^4) +
(a*(1 + a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{1}{4}x^4 \coth^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{(1+6a^2)\coth^{-1}(x)}{b^4} + \frac{4ax\coth^{-1}(x)}{b^4} - \right.\right. \\
&= \frac{1}{4}x^4 \coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int x^2\coth^{-1}(x)\,dx, x, a+bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{(1+6a^2+a}{}\right.}{} \\
&= \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\text{cot}}{6b} \\
&= -\frac{ax}{b^3} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+}{} \\
&= -\frac{ax}{b^3} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+}{} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+}{b^4} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+}{b^4} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+}{b^4}
\end{aligned}$$

Mathematica [A]
time = 1.07, size = 203, normalized size = 0.77

$$-\frac{1+11a^2+10abx-b^2x^2+3(1-4a+6a^2-4a^3+a^4-b^4x^4)\coth^{-1}(a+bx)^2-2\coth^{-1}(a+bx)\left(9a+13a^3+3bx+9a^2bx-3ab^2x^2+b^3x^3+12(a+a^3)\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right)+8\log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right)+36a^2\log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right)+12(a+a^3)\,\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)}\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x]^2,x]
```

```
[Out] -1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 -
 b^4*x^4)*ArcCoth[a + b*x]^2 - 2*ArcCoth[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9
*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 - E^(-2*ArcCoth[a + b
*x])]) + 8*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 36*a^2*Log[1/((a +
 b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 12*(a + a^3)*PolyLog[2, E^(-2*ArcCoth[a
+ b*x])])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(251) = 502$.
time = 0.25, size = 887, normalized size = 3.37

method	result
risch	$-\frac{5ax}{6b^3}-\frac{1}{12b^4}+\frac{x^2}{12b^2}+\frac{25\ln(bx+a-1)}{96b^4}-\frac{\ln(bx+a-1)^2}{16b^4}+\frac{\ln(bx+a-1)^2x^4}{16}+\frac{a}{b^4}-\frac{11a^2}{12b^4}-\frac{\ln(bx+a-1)(bx+}{2b^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(3*arccoth(b*x+a)*a^2*(b*x+a)-arccoth(b*x+a)^2*a^3*(b*x+a)-arccoth(b*
x+a)*a*(b*x+a)^2+3/2*arccoth(b*x+a)^2*a^2*(b*x+a)^2-arccoth(b*x+a)^2*a*(b*x
+a)^3-3/2*arccoth(b*x+a)*ln(b*x+a+1)*a^2+1/2*(b*x+a)*arccoth(b*x+a)-arccoth
(b*x+a)*ln(b*x+a+1)*a+1/6*arccoth(b*x+a)*(b*x+a)^3-(b*x+a)*a+1/12*(b*x+a)^2
+1/8*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)+1/16*ln(b*x+a+1)^2*a^4+1/
4*ln(b*x+a+1)^2*a^3+3/8*ln(b*x+a+1)^2*a^2+1/4*ln(b*x+a+1)^2*a+1/16*ln(b*x+a
-1)^2*a^4-1/4*ln(b*x+a-1)^2*a^3+3/8*ln(b*x+a-1)^2*a^2-1/4*ln(b*x+a-1)^2*a+d
ilog(1/2*b*x+1/2*a+1/2)*a^3+dilog(1/2*b*x+1/2*a+1/2)*a-1/8*ln(b*x+a-1)*ln(1
/2*b*x+1/2*a+1/2)+1/4*arccoth(b*x+a)*ln(b*x+a-1)-1/4*arccoth(b*x+a)*ln(b*x+
a+1)+1/4*arccoth(b*x+a)^2*a^4+1/4*arccoth(b*x+a)^2*(b*x+a)^4-1/8*ln(-1/2*b*
x-1/2*a+1/2)*ln(b*x+a+1)+1/16*ln(b*x+a-1)^2+1/16*ln(b*x+a+1)^2+3/2*ln(b*x+a
-1)*a^2-1/2*ln(b*x+a-1)*a+3/2*ln(b*x+a+1)*a^2+1/2*ln(b*x+a+1)*a-1/4*arccoth
(b*x+a)*ln(b*x+a+1)*a^4-arccoth(b*x+a)*ln(b*x+a+1)*a^3+1/4*arccoth(b*x+a)*l
n(b*x+a-1)*a^4-arccoth(b*x+a)*ln(b*x+a-1)*a^3+3/2*arccoth(b*x+a)*ln(b*x+a-1
)*a^2-arccoth(b*x+a)*ln(b*x+a-1)*a-1/8*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^
4+1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^3-3/4*ln(b*x+a-1)*ln(1/2*b*x+1/2*
a+1/2)*a^2+1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a-1/8*ln(b*x+a+1)*ln(-1/2*
b*x-1/2*a+1/2)*a^4-1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^3-3/4*ln(b*x+a+
1)*ln(-1/2*b*x-1/2*a+1/2)*a^2-1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a+1/8*
ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a^4+1/2*ln(1/2*b*x+1/2*a+1/2)*
ln(-1/2*b*x-1/2*a+1/2)*a^3+3/4*ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)
*a^2+1/2*ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a+1/3*ln(b*x+a+1)+1/3
*ln(b*x+a-1))
```

Maxima [A]

time = 0.26, size = 320, normalized size = 1.22

$$\frac{1}{4}x^4\operatorname{arcoth}(bx+a)^2+\frac{1}{48}b^2\left(\frac{48(a^3+a)\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^6}+\frac{4(13a^3+18a^2+9a+4)\log(bx+a+1)}{b^6}+\frac{4b^2x^2-40abx+3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)^2-6(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)\log(bx+a-1)+3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)^2-4(13a^3-18a^2+9a-4)\log(bx+a-1)}{b^6}\right)+\frac{1}{12}b\left(\frac{2(b^2x^3-3abx^2+3(3a^2+1)x)}{b^4}-\frac{3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1)}{b^5}+\frac{3(a^4-4a^3+6a^2-4a+1)\log(bx+a-1)}{b^5}\right)\operatorname{arcoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(b*x + a)^2 + 1/48*b^2*(48*(a^3 + a)*(log(b*x + a - 1)*log(1
/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^6 + 4*(13*a^3 + 18
```

```
*a^2 + 9*a + 4)*log(b*x + a + 1)/b^6 + (4*b^2*x^2 - 40*a*b*x + 3*(a^4 + 4*a
^3 + 6*a^2 + 4*a + 1)*log(b*x + a + 1)^2 - 6*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1
)*log(b*x + a + 1)*log(b*x + a - 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log
(b*x + a - 1)^2 - 4*(13*a^3 - 18*a^2 + 9*a - 4)*log(b*x + a - 1))/b^6) + 1/
12*b*(2*(b^2*x^3 - 3*a*b*x^2 + 3*(3*a^2 + 1)*x)/b^4 - 3*(a^4 + 4*a^3 + 6*a^
2 + 4*a + 1)*log(b*x + a + 1)/b^5 + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*log(b
*x + a - 1)/b^5)*arccoth(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^3*arccoth(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a)**2,x)
[Out] Integral(x**3*acoth(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*arccoth(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acoth}(a + b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(a + b*x)^2,x)
[Out] int(x^3*acoth(a + b*x)^2, x)
```

3.70 $\int x^2 \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=204

$$\frac{x}{3b^2}-\frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3}+\frac{a(3+a^2)\coth^{-1}(a+bx)^2}{3b^3}+\frac{(1+3a^2)\coth^{-1}(a\cdot}{3b^3}$$

```
[Out] 1/3*x/b^2-2*a*(b*x+a)*arccoth(b*x+a)/b^3+1/3*(b*x+a)^2*arccoth(b*x+a)/b^3+1
/3*a*(a^2+3)*arccoth(b*x+a)^2/b^3+1/3*(3*a^2+1)*arccoth(b*x+a)^2/b^3+1/3*x^
3*arccoth(b*x+a)^2-1/3*arctanh(b*x+a)/b^3-2/3*(3*a^2+1)*arccoth(b*x+a)*ln(2
/(-b*x-a+1))/b^3-a*ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*polylog(2,(-b*x-a-1)/(
-b*x-a+1))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$,
Rules used = {6247, 6066, 6022, 266, 6038, 327, 212, 6196, 6096, 6132, 6056, 2449, 2352}

$$-\frac{(3a^2+1)\,\mathrm{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3}+\frac{a(a^2+3)\coth^{-1}(a+bx)^2}{3b^3}+\frac{(3a^2+1)\coth^{-1}(a+bx)^2}{3b^3}-\frac{2(3a^2+1)\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{3b^3}-\frac{a\log\left(1-(a+bx)^2\right)}{b^3}-\frac{\tanh^{-1}(a+bx)}{3b^3}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3}-\frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3}+\frac{1}{3}x^3\coth^{-1}(a+bx)^2+\frac{x}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*x]^2,x]
```

```
[Out] x/(3*b^2) - (2*a*(a + b*x)*ArcCoth[a + b*x])/b^3 + ((a + b*x)^2*ArcCoth[a +
 b*x])/(3*b^3) + (a*(3 + a^2)*ArcCoth[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*Ar
cCoth[a + b*x]^2)/(3*b^3) + (x^3*ArcCoth[a + b*x]^2)/3 - ArcTanh[a + b*x]/(
3*b^3) - (2*(1 + 3*a^2)*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(3*b^3) - (a
*Log[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*PolyLog[2, -((1 + a + b*x)/(1 - a
 - b*x))])/(3*b^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\coth^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \left(\frac{3a\coth^{-1}(x)}{b^3} - \frac{x\coth^{-1}(x)}{b^3} - \frac{(a(3+a^2)\right.\right.$$

$$= \frac{1}{3}x^3\coth^{-1}(a+bx)^2 + \frac{2\text{Subst}\left(\int x\coth^{-1}(x)\,dx, x, a+bx\right)}{3b^3} + \frac{2\text{Subst}\left(\int \frac{(a(3+a^2}{}\right.}{}$$

$$= -\frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\coth^{-1}(a+bx)^2$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{1}{3}x^3\coth^{-1}(a+$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a(3+a^2)\coth^{-}}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a(3+a^2)\coth^{-}}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a(3+a^2)\coth^{-}}{3b^3}$$

$$= \frac{x}{3b^2} - \frac{2a(a+bx)\coth^{-1}(a+bx)}{b^3} + \frac{(a+bx)^2\coth^{-1}(a+bx)}{3b^3} + \frac{a(3+a^2)\coth^{-}}{3b^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 607 vs. 2(204) = 408.
time = 3.20, size = 607, normalized size = 2.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCoth[a + b*x]^2,x]
```

```
[Out] -1/12*((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*(1 - (a + b*x)^2)*((4*ArcCoth[a +
 b*x])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (3*ArcCoth[a + b*x]^2)/((a +
b*x)*Sqrt[1 - (a + b*x)^(-2)]) - (12*a*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[
1 - (a + b*x)^(-2)]) + (9*a^2*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[1 - (a +
b*x)^(-2)]) + (-1 + 6*a*ArcCoth[a + b*x] - 3*(-1 + a^2)*ArcCoth[a + b*x]^2)
/Sqrt[1 - (a + b*x)^(-2)] + Cosh[3*ArcCoth[a + b*x]] - 6*a*ArcCoth[a + b*x]
*Cosh[3*ArcCoth[a + b*x]] + ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + 3
*a^2*ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + (6*ArcCoth[a + b*x]*Log[
```

```
1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (18*a^
2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)]) - (18*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])])/((a + b
*x)*Sqrt[1 - (a + b*x)^(-2)]) + (4*(1 + 3*a^2)*PolyLog[2, E^(-2*ArcCoth[a +
 b*x])])/((a + b*x)^3*(1 - (a + b*x)^(-2))^(3/2)) - ArcCoth[a + b*x]^2*Sinh
[3*ArcCoth[a + b*x]] - 3*a^2*ArcCoth[a + b*x]^2*Sinh[3*ArcCoth[a + b*x]] -
2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a + b*x]
] - 6*a^2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[
a + b*x]] + 6*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])]*Sinh[3*ArcCoth[
a + b*x]]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(190) = 380$.
time = 0.24, size = 666, normalized size = 3.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*a-2*arccoth(b*x+a)*a*(b*x+a)+arccoth(b*x+a)^2*a^2*(b*x+a)-arccot
h(b*x+a)^2*a*(b*x+a)^2+arccoth(b*x+a)*ln(b*x+a+1)*a^2+1/3*(b*x+a)^2*arccoth
(b*x+a)+arccoth(b*x+a)*ln(b*x+a+1)*a+1/3*b*x-1/6*ln(-1/2*b*x-1/2*a+1/2)*ln(
1/2*b*x+1/2*a+1/2)-1/12*ln(b*x+a+1)^2*a^3-1/4*ln(b*x+a+1)^2*a^2-1/4*ln(b*x+
a+1)^2*a-1/12*ln(b*x+a-1)^2*a^3+1/4*ln(b*x+a-1)^2*a^2-1/4*ln(b*x+a-1)^2*a-1
/6*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)+1/3*arccoth(b*x+a)*ln(b*x+a-1)+1/3*arc
coth(b*x+a)*ln(b*x+a+1)+1/6*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)-1/3*arccoth(
b*x+a)^2*a^3+1/3*arccoth(b*x+a)^2*(b*x+a)^3-dilog(1/2*b*x+1/2*a+1/2)*a^2+1/
12*ln(b*x+a-1)^2-1/3*dilog(1/2*b*x+1/2*a+1/2)-1/12*ln(b*x+a+1)^2-ln(b*x+a-1
)*a-ln(b*x+a+1)*a+1/3*arccoth(b*x+a)*ln(b*x+a+1)*a^3-1/3*arccoth(b*x+a)*ln(
b*x+a-1)*a^3+arccoth(b*x+a)*ln(b*x+a-1)*a^2-arccoth(b*x+a)*ln(b*x+a-1)*a+1/
6*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a^3-1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/
2)*a^2+1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)*a+1/6*ln(b*x+a+1)*ln(-1/2*b*x-
1/2*a+1/2)*a^3+1/2*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^2+1/2*ln(b*x+a+1)*l
n(-1/2*b*x-1/2*a+1/2)*a-1/6*ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a^
3-1/2*ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a^2-1/2*ln(1/2*b*x+1/2*a
+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a-1/6*ln(b*x+a+1)+1/6*ln(b*x+a-1))
```

Maxima [A]
time = 0.27, size = 259, normalized size = 1.27

$$\frac{1}{3}x^3\operatorname{arcoth}(bx+a)^2-\frac{1}{12}b^2\left(\frac{4(3a^2+1)\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^5}+\frac{2(5a^2+6a+1)\log(bx+a+1)}{b^5}+\frac{(a^3+3a^2+3a+1)\log(bx+a+1)^2-2(a^3+3a^2+3a+1)\log(bx+a+1)\log(bx+a-1)+(a^3-3a^2+3a-1)\log(bx+a-1)^2-4bx-2(5a^2-6a+1)\log(bx+a-1)}{b^5}\right)+\frac{1}{3}b\left(\frac{bx^2-4ax}{b^3}+\frac{(a^3+3a^2+3a+1)\log(bx+a+1)}{b^4}-\frac{(a^3-3a^2+3a-1)\log(bx+a-1)}{b^4}\right)\operatorname{arcoth}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 + 6*
```

```
a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^2 -
 2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3*a^2
 + 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x + a -
1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b*x + a
 + 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arccoth(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x**2*acoth(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(a + b\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a + b*x)^2,x)
```

```
[Out] int(x^2*acoth(a + b*x)^2, x)
```

3.71 $\int x \coth^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=136

$$\frac{(a+bx)\coth^{-1}(a+bx)}{b^2}-\frac{a\coth^{-1}(a+bx)^2}{b^2}-\frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2}+\frac{1}{2}x^2\coth^{-1}(a+bx)^2+\frac{2a\coth^{-1}(a+$$

```
[Out] (b*x+a)*arccoth(b*x+a)/b^2-a*arccoth(b*x+a)^2/b^2-1/2*(a^2+1)*arccoth(b*x+a
)^2/b^2+1/2*x^2*arccoth(b*x+a)^2+2*a*arccoth(b*x+a)*ln(2/(-b*x-a+1))/b^2+1/
2*ln(1-(b*x+a)^2)/b^2+a*polylog(2,(-b*x-a-1)/(-b*x-a+1))/b^2
```

Rubi [A]

time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6247, 6066, 6022, 266, 6196, 6096, 6132, 6056, 2449, 2352}

$$-\frac{(a^2+1)\coth^{-1}(a+bx)^2}{2b^2}+\frac{a\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^2}+\frac{\log\left(1-(a+bx)^2\right)}{2b^2}-\frac{a\coth^{-1}(a+bx)^2}{b^2}+\frac{(a+bx)\coth^{-1}(a+bx)}{b^2}+\frac{2a\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b^2}+\frac{1}{2}x^2\coth^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*ArcCoth[a + b*x])/b^2 - (a*ArcCoth[a + b*x]^2)/b^2 - ((1 + a^2)*
ArcCoth[a + b*x]^2)/(2*b^2) + (x^2*ArcCoth[a + b*x]^2)/2 + (2*a*ArcCoth[a +
 b*x]*Log[2/(1 - a - b*x)])/b^2 + Log[1 - (a + b*x)^2]/(2*b^2) + (a*PolyLog
[2, -((1 + a + b*x)/(1 - a - b*x))])/b^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

```
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int x\coth^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\coth^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)^2 - \text{Subst}\left(\int\left(-\frac{\coth^{-1}(x)}{b^2}+\frac{(1+a^2-2ax)\coth^{-1}(x)}{b^2(1-x^2)}\right)dx,\right.$$
$$= \frac{1}{2}x^2\coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int\coth^{-1}(x)\,dx,x,a+bx\right)}{b^2} - \frac{\text{Subst}\left(\int\frac{(1+a^2-2ax)\coth}{1-x^2}\right.}{b^2}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int\frac{x}{1-x^2}\,dx,x,a+bx\right)}{b^2} -$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} + \frac{1}{2}x^2\coth^{-1}(a+bx)^2 + \frac{\log\left(1-(a+bx)^2\right)}{2b^2} + \frac{(2a)\text{Subs}}{}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\text{co}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\text{co}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\text{co}$$
$$= \frac{(a+bx)\coth^{-1}(a+bx)}{b^2} - \frac{a\coth^{-1}(a+bx)^2}{b^2} - \frac{(1+a^2)\coth^{-1}(a+bx)^2}{2b^2} + \frac{1}{2}x^2\text{co}$$

Mathematica [A]
time = 0.18, size = 106, normalized size = 0.78

$$\frac{(-1+2a-a^2+b^2x^2)\coth^{-1}(a+bx)^2+2\coth^{-1}(a+bx)\left(a+bx+2a\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right)-2\log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right)-2a\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a + b*x]^2,x]
```

```
[Out] ((-1 + 2*a - a^2 + b^2*x^2)*ArcCoth[a + b*x]^2 + 2*ArcCoth[a + b*x]*(a + b*
x + 2*a*Log[1 - E^(-2*ArcCoth[a + b*x])]) - 2*Log[1/((a + b*x)*Sqrt[1 - (a
+ b*x)^(-2)])] - 2*a*PolyLog[2, E^(-2*ArcCoth[a + b*x])])/(2*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(132) = 264$.
time = 0.22, size = 312, normalized size = 2.29

method	result
derivativedivides	$\frac{\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)^2a(bx+a)+(bx+a)\operatorname{arccoth}(bx+a)-\operatorname{arccoth}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arccoth}(bx+a)\,\mathrm{l}}{2}}{}$
default	$\frac{\frac{\operatorname{arccoth}(bx+a)^2(bx+a)^2}{2}-\operatorname{arccoth}(bx+a)^2a(bx+a)+(bx+a)\operatorname{arccoth}(bx+a)-\operatorname{arccoth}(bx+a)\ln(bx+a-1)a+\frac{\operatorname{arccoth}(bx+a)\,\mathrm{l}}{2}}{}$
risch	$-\frac{(-b^2x^2+a^2+2a+1)\ln(bx+a+1)^2}{8b^2}+\left(-\frac{x^2\ln(bx+a-1)}{4}+\frac{\ln(bx+a-1)a^2-2\ln(bx+a-1)a+2bx+\ln(bx+a-1)}{4b^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arccoth(b*x+a)^2*(b*x+a)^2-arccoth(b*x+a)^2*a*(b*x+a)+(b*x+a)*ar
ccoth(b*x+a)-arccoth(b*x+a)*ln(b*x+a-1)*a+1/2*arccoth(b*x+a)*ln(b*x+a-1)-ar
ccoth(b*x+a)*ln(b*x+a+1)*a-1/2*arccoth(b*x+a)*ln(b*x+a+1)+1/2*ln(b*x+a-1)+1
/2*ln(b*x+a+1)+dilog(1/2*b*x+1/2*a+1/2)*a+1/2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+
1/2)*a-1/4*ln(b*x+a-1)^2*a-1/4*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/2)+1/8*ln(b*x
+a-1)^2+1/2*ln(1/2*b*x+1/2*a+1/2)*ln(-1/2*b*x-1/2*a+1/2)*a-1/2*ln(b*x+a+1)*
ln(-1/2*b*x-1/2*a+1/2)*a+1/4*ln(b*x+a+1)^2*a+1/4*ln(-1/2*b*x-1/2*a+1/2)*ln(
1/2*b*x+1/2*a+1/2)-1/4*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/8*ln(b*x+a+1)^2
)
```

Maxima [A]
time = 0.27, size = 202, normalized size = 1.49

$$\frac{1}{2}\,x^2\operatorname{arcoth}\left(bx+a\right)^2+\frac{1}{8}\,b^2\left(\frac{8\left(\log\left(bx+a-1\right)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)a}{b^4}+\frac{4\left(a+1\right)\log\left(bx+a+1\right)}{b^4}+\frac{\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)^2-2\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)\log\left(bx+a-1\right)+\left(a^2-2\,a+1\right)\log\left(bx+a-1\right)^2-4\left(a-1\right)\log\left(bx+a-1\right)}{b^4}\right)+\frac{1}{2}\,b\left(\frac{2\,x}{b^2}-\frac{\left(a^2+2\,a+1\right)\log\left(bx+a+1\right)}{b^3}+\frac{\left(a^2-2\,a+1\right)\log\left(bx+a-1\right)}{b^3}\right)\operatorname{arcoth}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(b*x + a)^2 + 1/8*b^2*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2
*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))*a/b^4 + 4*(a + 1)*log(b*x + a +
1)/b^4 + ((a^2 + 2*a + 1)*log(b*x + a + 1)^2 - 2*(a^2 + 2*a + 1)*log(b*x +
a + 1)*log(b*x + a - 1) + (a^2 - 2*a + 1)*log(b*x + a - 1)^2 - 4*(a - 1)*lo
g(b*x + a - 1))/b^4) + 1/2*b*(2*x/b^2 - (a^2 + 2*a + 1)*log(b*x + a + 1)/b^
3 + (a^2 - 2*a + 1)*log(b*x + a - 1)/b^3)*arccoth(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x*arccoth(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acoth}^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x*acoth(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a + b*x)^2,x)
```

```
[Out] int(x*acoth(a + b*x)^2, x)
```

3.72 $\int \coth^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=81

$$\frac{\coth^{-1}(a+bx)^2}{b}+\frac{(a+bx)\coth^{-1}(a+bx)^2}{b}-\frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b}-\frac{\text{PolyLog}\left(2,-\frac{1+a+bx}{1-a-bx}\right)}{b}$$

```
[Out] arccoth(b*x+a)^2/b+(b*x+a)*arccoth(b*x+a)^2/b-2*arccoth(b*x+a)*ln(2/(-b*x-a
+1))/b-polylog(2,(-b*x-a-1)/(-b*x-a+1))/b
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6239, 6022, 6132, 6056, 2449, 2352}

$$-\frac{\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b}+\frac{(a+bx)\coth^{-1}(a+bx)^2}{b}+\frac{\coth^{-1}(a+bx)^2}{b}-\frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2,x]
```

```
[Out] ArcCoth[a + b*x]^2/b + ((a + b*x)*ArcCoth[a + b*x]^2)/b - (2*ArcCoth[a + b*
x]*Log[2/(1 - a - b*x)])/b - PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))]/b
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x)^2\,dx, x, a+bx\right)}{b}\\
&= \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{x\coth^{-1}(x)}{1-x^2}\,dx, x, a+bx\right)}{b}\\
&= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1-x}\,dx, x, a+bx\right)}{b}\\
&= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2\text{Su}}{}\\
&= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2\text{Su}}{}\\
&= \frac{\coth^{-1}(a+bx)^2}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} - \frac{2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}_2}{}
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 55, normalized size = 0.68

$$\frac{\coth^{-1}(a+bx)\left((-1+a+bx)\coth^{-1}(a+bx) - 2\log\left(1-e^{-2\coth^{-1}(a+bx)}\right)\right) + \text{PolyLog}\left(2, e^{-2\coth^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]^2,x]
```

```
[Out] (ArcCoth[a + b*x]*((-1 + a + b*x)*ArcCoth[a + b*x] - 2*Log[1 - E^(-2*ArcCot
h[a + b*x])]) + PolyLog[2, E^(-2*ArcCoth[a + b*x])])/b
```

Maple [A]
time = 0.28, size = 133, normalized size = 1.64

method	result
derivativedivides	$\frac{\operatorname{arccoth}(bx+a)^2(bx+a-1)+2\operatorname{arccoth}(bx+a)^2-2\operatorname{arccoth}(bx+a)\ln\left(1+\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right.}{b}$
default	$\frac{\operatorname{arccoth}(bx+a)^2(bx+a-1)+2\operatorname{arccoth}(bx+a)^2-2\operatorname{arccoth}(bx+a)\ln\left(1+\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)-2\operatorname{polylog}\left(2,-\frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right.}{b}$
risch	$\frac{(bx+a+1)\ln(bx+a+1)^2}{4b}+\left(-\frac{x\ln(bx+a-1)}{2}+\frac{-\ln(bx+a-1)a+\ln(bx+a-1)}{2b}\right)\ln(bx+a+1)+\frac{x\ln(bx+a}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccoth(b*x+a)^2*(b*x+a-1)+2*arccoth(b*x+a)^2-2*arccoth(b*x+a)*ln(1+1/
((b*x+a-1)/(b*x+a+1))^(1/2))-2*polylog(2,-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*
arccoth(b*x+a)*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*polylog(2,1/((b*x+a-1)
/(b*x+a+1))^(1/2)))
```

Maxima [A]
time = 0.26, size = 139, normalized size = 1.72

$$-\frac{1}{4}b^2\left(\frac{(a+1)\log(bx+a+1)^2-2(a+1)\log(bx+a+1)\log(bx+a-1)+(a-1)\log(bx+a-1)^2}{b^3}+\frac{4\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)}{b^3}\right)+b\left(\frac{(a+1)\log(bx+a+1)}{b^2}-\frac{(a-1)\log(bx+a-1)}{b^2}\right)\operatorname{arcoth}(bx+a)+x\operatorname{arcoth}(bx+a)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x
+ a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*b*
x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(b*x
 + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arccoth(b*x + a) + x*arccoth(
b*x + a)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)^2,x)
```

```
[Out] int(acoth(a + b*x)^2, x)
```

3.73 $\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=148

$$-\coth^{-1}(a+bx)^2\log\left(\frac{2}{1+a+bx}\right)+\coth^{-1}(a+bx)^2\log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)+\coth^{-1}(a+bx)\text{PolyLog}\left(2\right.$$

```
[Out] -arccoth(b*x+a)^2*ln(2/(b*x+a+1))+arccoth(b*x+a)^2*ln(2*b*x/(1-a)/(b*x+a+1)
)+arccoth(b*x+a)*polylog(2,1-2/(b*x+a+1))-arccoth(b*x+a)*polylog(2,1-2*b*x/
(1-a)/(b*x+a+1))+1/2*polylog(3,1-2/(b*x+a+1))-1/2*polylog(3,1-2*b*x/(1-a)/(
b*x+a+1))
```

Rubi [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6247, 6060}

$$\frac{1}{2}\text{Li}_3\left(1-\frac{2}{a+bx+1}\right)-\frac{1}{2}\text{Li}_3\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)+\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)-\text{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)-\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)^2+\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x,x]
```

```
[Out] -(ArcCoth[a + b*x]^2*Log[2/(1 + a + b*x)]) + ArcCoth[a + b*x]^2*Log[(2*b*x)
/((1 - a)*(1 + a + b*x))] + ArcCoth[a + b*x]*PolyLog[2, 1 - 2/(1 + a + b*x)
] - ArcCoth[a + b*x]*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + Poly
Log[3, 1 - 2/(1 + a + b*x)]/2 - PolyLog[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*
x))]/2
```

Rule 6060

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$
$$= -\coth^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.64, size = 714, normalized size = 4.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2/x,x]
```

```
[Out] (-1/24*I)*Pi^3 - (2*ArcCoth[a + b*x]^3)/3 - (2*a*ArcCoth[a + b*x]^3)/3 + (2
*Sqrt[1 - a^(-2)]*a*E^ArcTanh[a^(-1)]*ArcCoth[a + b*x]^3)/3 - I*Pi*ArcCoth[
a + b*x]*Log[(E^(-ArcCoth[a + b*x]) + E^ArcCoth[a + b*x])/2] + ArcCoth[a +
b*x]^2*Log[1 - Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] + ArcCoth[a + b*x
]^2*Log[1 + Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]^2
*Log[1 - E^(2*ArcCoth[a + b*x])] - ArcCoth[a + b*x]^2*Log[1 - ((-1 + a)*E^(
2*ArcCoth[a + b*x]))/(1 + a)] + ArcCoth[a + b*x]^2*Log[1 - E^(2*ArcCoth[a +
 b*x] - 2*ArcTanh[a^(-1)])] - 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[(I/2)*
(E^(ArcCoth[a + b*x] - ArcTanh[a^(-1)]) - E^(-ArcCoth[a + b*x] + ArcTanh[a^
(-1)]))] + ArcCoth[a + b*x]^2*Log[(-1 - E^(2*ArcCoth[a + b*x]) + a*(-1 + E^
(2*ArcCoth[a + b*x])))/(2*E^ArcCoth[a + b*x])] + I*Pi*ArcCoth[a + b*x]*Log[
1/Sqrt[1 - (a + b*x)^(-2)]] - ArcCoth[a + b*x]^2*Log[-((b*x)/((a + b*x)*Sqr
t[1 - (a + b*x)^(-2)]))] + 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[I*Sinh[Ar
cCoth[a + b*x] - ArcTanh[a^(-1)]]] + 2*ArcCoth[a + b*x]*PolyLog[2, -(Sqrt[(
-1 + a)/(1 + a)]*E^ArcCoth[a + b*x])] + 2*ArcCoth[a + b*x]*PolyLog[2, Sqrt[
(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]*PolyLog[2, E^(2*Ar
cCoth[a + b*x])] - ArcCoth[a + b*x]*PolyLog[2, ((-1 + a)*E^(2*ArcCoth[a + b
*x]))/(1 + a)] + ArcCoth[a + b*x]*PolyLog[2, E^(2*ArcCoth[a + b*x] - 2*ArcT
anh[a^(-1)])] - 2*PolyLog[3, -(Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x])]
- 2*PolyLog[3, Sqrt[(-1 + a)/(1 + a)]*E^ArcCoth[a + b*x]] + PolyLog[3, E^(2
*ArcCoth[a + b*x])]/2 + PolyLog[3, ((-1 + a)*E^(2*ArcCoth[a + b*x]))/(1 + a
)]/2 - PolyLog[3, E^(2*ArcCoth[a + b*x] - 2*ArcTanh[a^(-1)])]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.92, size = 900, normalized size = 6.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(-b*x)*arccoth(b*x+a)^2-arccoth(b*x+a)^2*ln(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1
/(b*x+a-1)*(b*x+a+1)-1)+1/2*I*Pi*arccoth(b*x+a)^2*csgn(I/(1/(b*x+a-1)*(b*x+
a+1)-1))*csgn(I*(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1/(b*x+a-1)*(b*x+a+1)-1))*csgn
(I*(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1/(b*x+a-1)*(b*x+a+1)-1)/(1/(b*x+a-1)*(b*x+
a+1)-1))-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1/(b
*x+a-1)*(b*x+a+1)-1))*csgn(I*(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1/(b*x+a-1)*(b*x+
a+1)-1)/(1/(b*x+a-1)*(b*x+a+1)-1))^2-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I/(1/(b
*x+a-1)*(b*x+a+1)-1))*csgn(I*(a*(1/(b*x+a-1)*(b*x+a+1)-1)-1/(b*x+a-1)*(b*x+
a+1)-1)/(1/(b*x+a-1)*(b*x+a+1)-1))^2+1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*(1
/(b*x+a-1)*(b*x+a+1)-1)-1/(b*x+a-1)*(b*x+a+1)-1)/(1/(b*x+a-1)*(b*x+a+1)-1))
^3+arccoth(b*x+a)^2*ln(1/(b*x+a-1)*(b*x+a+1)-1)-arccoth(b*x+a)^2*ln(1-1/((b
*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,1/((b*x+a-1)/(b*x+a+1)
)^(1/2))+2*polylog(3,1/((b*x+a-1)/(b*x+a+1))^(1/2))-arccoth(b*x+a)^2*ln(1+1
/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,-1/((b*x+a-1)/(b*x
+a+1))^(1/2))+2*polylog(3,-1/((b*x+a-1)/(b*x+a+1))^(1/2))+a/(-1+a)*arccoth(
b*x+a)^2*ln(1-(-1+a)/(b*x+a-1)*(b*x+a+1)/(1+a))+a/(-1+a)*arccoth(b*x+a)*pol
ylog(2,(-1+a)/(b*x+a-1)*(b*x+a+1)/(1+a))-1/2*a/(-1+a)*polylog(3,(-1+a)/(b*x
+a-1)*(b*x+a+1)/(1+a))-1/(-1+a)*arccoth(b*x+a)^2*ln(1-(-1+a)/(b*x+a-1)*(b*x
+a+1)/(1+a))-1/(-1+a)*arccoth(b*x+a)*polylog(2,(-1+a)/(b*x+a-1)*(b*x+a+1)/(
1+a))+1/2/(-1+a)*polylog(3,(-1+a)/(b*x+a-1)*(b*x+a+1)/(1+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(b*x + a)^2/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acoth}(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)^2/x,x)
```

```
[Out] int(acoth(a + b*x)^2/x, x)
```

3.74 $\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=251

$$-\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a}+\frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}-\frac{2b\coth^{-1}(a+bx)\log\left(\frac{}{1\cdots}\right.}{1-a^2}$$

```
[Out] -arccoth(b*x+a)^2/x+b*arccoth(b*x+a)*ln(2/(-b*x-a+1))/(1-a)+b*arccoth(b*x+a
)*ln(2/(b*x+a+1))/(1+a)-2*b*arccoth(b*x+a)*ln(2/(b*x+a+1))/(-a^2+1)+2*b*arc
coth(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)+1/2*b*polylog(2,(-b*x-a-1)/(
-b*x-a+1))/(1-a)-1/2*b*polylog(2,1-2/(b*x+a+1))/(1+a)+b*polylog(2,1-2/(b*x+
a+1))/(-a^2+1)-b*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)
```

Rubi [A]

time = 0.51, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {6245, 378, 720, 31, 647, 6873, 6257, 6820, 12, 6857, 6058, 2449, 2352, 2497, 6056}

$$\frac{b\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{1-a^2}-\frac{b\text{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2}-\frac{2b\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{1-a^2}+\frac{2b\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)}{1-a^2}+\frac{b\text{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)}-\frac{b\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{2(a+1)}-\frac{\coth^{-1}(a+bx)^2}{x}+\frac{b\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{1-a}+\frac{b\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{a+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcCoth[a + b*x]^2/x) + (b*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(1 - a)
 + (b*ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/(1 + a) - (2*b*ArcCoth[a + b*x
]*Log[2/(1 + a + b*x)])/(1 - a^2) + (2*b*ArcCoth[a + b*x]*Log[(2*b*x)/((1 -
 a)*(1 + a + b*x))])/(1 - a^2) + (b*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x
))])/(2*(1 - a)) - (b*PolyLog[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*Pol
yLog[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*PolyLog[2, 1 - (2*b*x)/((1 - a
)*(1 + a + b*x))])/(1 - a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
```

```
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6257

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x^2}\,dx = -\frac{\coth^{-1}(a+bx)^2}{x} + (2b)\int \frac{\coth^{-1}(a+bx)}{x\left(1-(a+bx)^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b)\int \frac{\coth^{-1}(a+bx)}{x\left(1-a^2-2abx-b^2x^2\right)}\,dx$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)(1-x^2)}\,dx, x, a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{b\coth^{-1}(x)}{(-a+x)(1-x^2)}\,dx, x, a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \frac{\coth^{-1}(x)}{(-a+x)(1-x^2)}\,dx, x, a+bx\right)$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b)\text{Subst}\left(\int \left(\frac{\coth^{-1}(x)}{(-1+a^2)(a-x)} + \frac{\coth^{-1}(x)}{2(-1+a)(-1+x)} - \frac{}{2(}\right.\right.$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} - \frac{b\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-1+x}\,dx, x, a+bx\right)}{1-a} - \frac{b\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1+x}\,dx, x, a\right.}{1+a}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}$$

$$= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{1+a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.76, size = 206, normalized size = 0.82

$$-\frac{\left(\left(-1+a^2+\sqrt{1-\frac{1}{a^2}}\,abe^{\tanh^{-1}\left(\frac{1}{a}\right)}x\right)\coth^{-1}(a+bx)^2\right)+bx\coth^{-1}(a+bx)\left(-i\pi+2\tanh^{-1}\left(\frac{1}{a}\right)-2\log\left(1-e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)\right)+bx\left(i\pi\left(\log\left(1+e^{2\coth^{-1}(a+bx)}\right)-\log\left(\frac{1}{\sqrt{1-\frac{1}{(a+bx)^2}}}\right)\right)+2\tanh^{-1}\left(\frac{1}{a}\right)\left(\log\left(1-e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)-\log\left(i\sinh\left(\coth^{-1}(a+bx)-\tanh^{-1}\left(\frac{1}{a}\right)\right)\right)\right)\right)+bx\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)}{(-1+a^2)\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]^2/x^2,x]
```

```
[Out] (-((-1 + a^2 + Sqrt[1 - a^(-2)]*a*b*E^ArcTanh[a^(-1)]*x)*ArcCoth[a + b*x]^2
) + b*x*ArcCoth[a + b*x]*((-I)*Pi + 2*ArcTanh[a^(-1)] - 2*Log[1 - E^(-2*Arc
Coth[a + b*x] + 2*ArcTanh[a^(-1)])]) + b*x*(I*Pi*(Log[1 + E^(2*ArcCoth[a +
b*x])] - Log[1/Sqrt[1 - (a + b*x)^(-2)]]) + 2*ArcTanh[a^(-1)]*(Log[1 - E^(-
2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x] - Ar
cTanh[a^(-1)]]])) + b*x*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1
)])])/((-1 + a^2)*x)
```

Maple [A]
time = 0.43, size = 356, normalized size = 1.42

method	result
derivativedivides	$b\left(-\frac{\operatorname{arccoth}(bx+a)^2}{bx}+\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a-1)}{-2+2a}-\frac{2\operatorname{arccoth}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2+2a}\right.$
default	$b\left(-\frac{\operatorname{arccoth}(bx+a)^2}{bx}+\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a-1)}{-2+2a}-\frac{2\operatorname{arccoth}(bx+a)\ln(-bx)}{(-1+a)(1+a)}-\frac{2\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2+2a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arccoth(b*x+a)^2+2*arccoth(b*x+a)/(-2+2*a)*ln(b*x+a-1)-2*arccoth(
b*x+a)/(-1+a)/(1+a)*ln(-b*x)-2*arccoth(b*x+a)/(2+2*a)*ln(b*x+a+1)+1/4/(-1+a
)*ln(b*x+a-1)^2-1/2/(-1+a)*dilog(1/2*b*x+1/2*a+1/2)-1/2/(-1+a)*ln(b*x+a-1)*
ln(1/2*b*x+1/2*a+1/2)+1/2/(1+a)*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2
)-1/2/(1+a)*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/2/(1+a)*dilog(1/2*b*x+1/2*
a+1/2)+1/4/(1+a)*ln(b*x+a+1)^2+1/(-1+a)/(1+a)*dilog((-b*x-a-1)/(-a-1))+1/(-
1+a)/(1+a)*ln(-b*x)*ln((-b*x-a-1)/(-a-1))-1/(-1+a)/(1+a)*dilog((-b*x-a+1)/(
1-a))-1/(-1+a)/(1+a)*ln(-b*x)*ln((-b*x-a+1)/(1-a)))
```

Maxima [A]
time = 0.26, size = 244, normalized size = 0.97

$$\frac{1}{4}\,b^2\left(\frac{(a-1)\log(bx+a+1)^2-2\,(a-1)\log(bx+a+1)\log(bx+a-1)+(a+1)\log(bx+a-1)^2}{a^2b-b}-\frac{4\left(\log(bx+a-1)\log\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a+\frac{1}{2}\right)\right)}{a^2b-b}+\frac{4\left(\log\left(\frac{bx}{a+1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a+1}\right)\right)}{a^2b-b}-\frac{4\left(\log\left(\frac{bx}{a-1}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a-1}\right)\right)}{a^2b-b}\right)-b\left(\frac{\log(bx+a+1)}{a+1}-\frac{\log(bx+a-1)}{a-1}+\frac{2\,\log(x)}{a^2-1}\right)\operatorname{arcoth}(bx+a)-\frac{\operatorname{arcoth}(bx+a)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(((a - 1)*log(b*x + a + 1)^2 - 2*(a - 1)*log(b*x + a + 1)*log(b*x +
 a - 1) + (a + 1)*log(b*x + a - 1)^2)/(a^2*b - b) - 4*(log(b*x + a - 1)*log
(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/(a^2*b - b) + 4*(l
og(b*x/(a + 1) + 1)*log(x) + dilog(-b*x/(a + 1)))/(a^2*b - b) - 4*(log(b*x/
(a - 1) + 1)*log(x) + dilog(-b*x/(a - 1)))/(a^2*b - b)) - b*(log(b*x + a +
1)/(a + 1) - log(b*x + a - 1)/(a - 1) + 2*log(x)/(a^2 - 1))*arccoth(b*x + a
) - arccoth(b*x + a)^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)^2/x^2,x)
```

```
[Out] int(acoth(a + b*x)^2/x^2, x)
```

3.75 $\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=370

$$-\frac{b\coth^{-1}(a+bx)}{(1-a^2)\,x}-\frac{\coth^{-1}(a+bx)^2}{2x^2}+\frac{b^2\log(x)}{(1-a^2)^2}+\frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2}-\frac{b^2\log(1-a-bx)}{2(1-a)^2(1+a)}-\frac{b^2\,\mathrm{co}}{}$$

```
[Out] -b*arccoth(b*x+a)/(-a^2+1)/x-1/2*arccoth(b*x+a)^2/x^2+b^2*ln(x)/(-a^2+1)^2+
1/2*b^2*arccoth(b*x+a)*ln(2/(-b*x-a+1))/(1-a)^2-1/2*b^2*ln(-b*x-a+1)/(1-a)^
2/(1+a)-1/2*b^2*arccoth(b*x+a)*ln(2/(b*x+a+1))/(1+a)^2-2*a*b^2*arccoth(b*x+
a)*ln(2/(b*x+a+1))/(-a^2+1)^2+2*a*b^2*arccoth(b*x+a)*ln(2*b*x/(1-a)/(b*x+a+
1))/(-a^2+1)^2-1/2*b^2*ln(b*x+a+1)/(1-a)/(1+a)^2+1/4*b^2*polylog(2,(-b*x-a-
1)/(-b*x-a+1))/(1-a)^2+1/4*b^2*polylog(2,1-2/(b*x+a+1))/(1+a)^2+a*b^2*polyl
og(2,1-2/(b*x+a+1))/(-a^2+1)^2-a*b^2*polylog(2,1-2*b*x/(1-a)/(b*x+a+1))/(-a
^2+1)^2
```

Rubi [A]
time = 0.58, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6245, 378, 724, 815, 6873, 6257, 6857, 6064, 720, 31, 647, 6058, 2449, 2352, 2497, 6056}

$$\frac{ab^2\mathrm{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{(1-a^2)^2}-\frac{ab^2\mathrm{Li}_2\left(1-\frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2}+\frac{b^2\log(x)}{(1-a^2)^2}-\frac{2ab^2\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{(1-a^2)^2}+\frac{2ab^2\log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)\coth^{-1}(a+bx)}{(1-a^2)^2}-\frac{b\coth^{-1}(a+bx)}{(1-a^2)\,x}+\frac{b^2\mathrm{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2}+\frac{b^2\mathrm{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{4(a+1)^2}-\frac{b^2\log(-a-bx+1)}{2(1-a)^2(a+1)}-\frac{b^2\log(a+bx+1)}{2(1-a)(a+1)^2}+\frac{b^2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{2(1-a)^2}-\frac{b^2\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{2(a+1)^2}-\frac{\coth^{-1}(a+bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]^2/x^3,x]
```

```
[Out] -((b*ArcCoth[a + b*x])/((1 - a^2)*x)) - ArcCoth[a + b*x]^2/(2*x^2) + (b^2*L
og[x])/(1 - a^2)^2 + (b^2*ArcCoth[a + b*x]*Log[2/(1 - a - b*x)])/(2*(1 - a)
^2) - (b^2*Log[1 - a - b*x])/(2*(1 - a)^2*(1 + a)) - (b^2*ArcCoth[a + b*x]*
Log[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2*ArcCoth[a + b*x]*Log[2/(1 +
a + b*x)])/(1 - a^2)^2 + (2*a*b^2*ArcCoth[a + b*x]*Log[(2*b*x)/((1 - a)*(1
+ a + b*x))])/(1 - a^2)^2 - (b^2*Log[1 + a + b*x])/(2*(1 - a)*(1 + a)^2) +
(b^2*PolyLog[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2*PolyL
og[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2*PolyLog[2, 1 - 2/(1 + a
+ b*x)])/(1 - a^2)^2 - (a*b^2*PolyLog[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x)
)])/(1 - a^2)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6257

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(a+bx)^2}{x^3}\,dx &= -\frac{\coth^{-1}(a+bx)^2}{2x^2}+b\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-(a+bx)^2\right)}\,dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2}+b\int \frac{\coth^{-1}(a+bx)}{x^2\left(1-a^2-2abx-b^2x^2\right)}\,dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2}+\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\left(1-x^2\right)}\,dx,x,a+bx\right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2}+\text{Subst}\left(\int \left(-\frac{b^2\coth^{-1}(x)}{\left(-1+a^2\right)(a-x)^2}-\frac{2ab^2\coth^{-1}(x)}{\left(-1+a^2\right)^2(a-x)}-\frac{\cdots}{2(-\cdots}\right.\right.\cdots \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2}-\frac{b^2\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-1+x}\,dx,x,a+bx\right)}{2(1-a)^2}+\frac{b^2\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1+x}\,dx,x,\cdots\right.}{2(1+a)^2}\cdots \\
&= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)^2}{2x^2}+\frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2}-\frac{b^2\coth^{-1}\cdots}{\cdots} \\
&= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)^2}{2x^2}+\frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2}-\frac{b^2\coth^{-1}\cdots}{\cdots} \\
&= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)^2}{2x^2}+\frac{b^2\log(x)}{\left(1-a^2\right)^2}+\frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right.\cdots}{2(1-a)^2}\cdots \\
&= -\frac{b\coth^{-1}(a+bx)}{\left(1-a^2\right)x}-\frac{\coth^{-1}(a+bx)^2}{2x^2}+\frac{b^2\log(x)}{\left(1-a^2\right)^2}+\frac{b^2\coth^{-1}(a+bx)\log\left(\frac{2}{1-a-bx}\right.\cdots}{2(1-a)^2}\cdots
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.40, size = 291, normalized size = 0.79

$$
\frac{\left(-1-a^4+b^2x^2+a^2\left(2+b^2\left(-1+2\sqrt{1-\frac{1}{a^2}}\,e^{\tanh^{-1}\left(\frac{1}{a}\right)}\right)x^2\right)\right)\coth^{-1}(a+bx)^2+2bx\coth^{-1}(a+bx)\left(-1+a^2+abx+iab\pi x-2abx\tanh^{-1}\left(\frac{1}{a}\right)+2abx\log\left(1-e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)\right)+2b^2x^2\left(-ia\pi\log\left(1+e^{2\coth^{-1}(a+bx)}\right)+ia\pi\log\left(\frac{1}{\sqrt{1-\frac{1}{(a+bx)^2}}}\right)+\log\left(-\frac{bx}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right)-2a\tanh^{-1}\left(\frac{1}{a}\right)\left(\log\left(1-e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)-\log\left(i\sinh\left(\coth^{-1}(a+bx)-\tanh^{-1}\left(\frac{1}{a}\right)\right)\right)\right)\right)-2ab^2x^2\text{PolyLog}\left(2,e^{-2\coth^{-1}(a+bx)+2\tanh^{-1}\left(\frac{1}{a}\right)}\right)}{2\left(-1+a^2\right)^2x^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]^2/x^3,x]
```

```
[Out] ((-1 - a^4 + b^2*x^2 + a^2*(2 + b^2*(-1 + 2*Sqrt[1 - a^(-2)]*E^ArcTanh[a^(-
1)])*x^2))*ArcCoth[a + b*x]^2 + 2*b*x*ArcCoth[a + b*x]*(-1 + a^2 + a*b*x +
I*a*b*Pi*x - 2*a*b*x*ArcTanh[a^(-1)] + 2*a*b*x*Log[1 - E^(-2*ArcCoth[a + b*
x] + 2*ArcTanh[a^(-1)])]) + 2*b^2*x^2*((-I)*a*Pi*Log[1 + E^(2*ArcCoth[a + b
*x])] + I*a*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] + Log[-((b*x)/((a + b*x)*Sqr
t[1 - (a + b*x)^(-2)]))] - 2*a*ArcTanh[a^(-1)]*(Log[1 - E^(-2*ArcCoth[a + b
*x] + 2*ArcTanh[a^(-1)])] - Log[I*Sinh[ArcCoth[a + b*x] - ArcTanh[a^(-1)]]]
)) - 2*a*b^2*x^2*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])])/(
2*(-1 + a^2)^2*x^2)
```

Maple [A]
time = 0.44, size = 449, normalized size = 1.21

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arccoth}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arccoth}(bx+a)}{(-1+a)(1+a)bx}+\frac{2\operatorname{arccoth}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2}-\right.$ arccotl
default	$b^2\left(-\frac{\operatorname{arccoth}(bx+a)^2}{2b^2x^2}+\frac{\operatorname{arccoth}(bx+a)}{(-1+a)(1+a)bx}+\frac{2\operatorname{arccoth}(bx+a)a\ln(-bx)}{(-1+a)^2(1+a)^2}+\frac{\operatorname{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2}-\right.$ arccotl

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arccoth(b*x+a)^2+arccoth(b*x+a)/(-1+a)/(1+a)/b/x+2*arccot
h(b*x+a)*a/(-1+a)^2/(1+a)^2*ln(-b*x)+1/2*arccoth(b*x+a)/(1+a)^2*ln(b*x+a+1)
-1/2*arccoth(b*x+a)/(-1+a)^2*ln(b*x+a-1)-1/8/(-1+a)^2*ln(b*x+a-1)^2+1/4/(-1
+a)^2*dilog(1/2*b*x+1/2*a+1/2)+1/4/(-1+a)^2*ln(b*x+a-1)*ln(1/2*b*x+1/2*a+1/
2)-1/8/(1+a)^2*ln(b*x+a+1)^2+1/4/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)
-1/4/(1+a)^2*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2*b*x+1/2*a+1/2)-1/4/(1+a)^2*dilog
(1/2*b*x+1/2*a+1/2)+1/(-1+a)^2/(1+a)^2*ln(-b*x)+1/(-1+a)/(1+a)/(2+2*a)*ln(b
*x+a+1)-1/(-1+a)/(1+a)/(-2+2*a)*ln(b*x+a-1)+a/(-1+a)^2/(1+a)^2*dilog((-b*x-
a+1)/(1-a))+a/(-1+a)^2/(1+a)^2*ln(-b*x)*ln((-b*x-a+1)/(1-a))-a/(-1+a)^2/(1+
a)^2*dilog((-b*x-a-1)/(-a-1))-a/(-1+a)^2/(1+a)^2*ln(-b*x)*ln((-b*x-a-1)/(-a
-1)))
```

Maxima [A]
time = 0.27, size = 360, normalized size = 0.97

$$\frac{1}{8}\left(\frac{8\left(\log(bx+a-1)\log\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{1}{2}\right)+\operatorname{Li}_2\left(-\frac{1}{2}bx-\frac{1}{2}a+\frac{1}{2}\right)\right)a}{a^4-2a^2+1}-\frac{8\left(\log\left(\frac{bx}{a+1}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{bx}{a+1}\right)\right)a}{a^4-2a^2+1}+\frac{8\left(\log\left(\frac{bx}{a-1}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{bx}{a-1}\right)\right)a}{a^4-2a^2+1}-\frac{(a^2-2a+1)\log(bx+a+1)^2-2(a^2-2a+1)\log(bx+a+1)\log(bx+a-1)+(a^2+2a+1)\log(bx+a-1)^2}{a^4-2a^2+1}+\frac{4\log(bx+a+1)}{a^3+a^2-a-1}-\frac{4\log(bx+a-1)}{a^3-a^2-a+1}+\frac{8\log(x)}{a^4-2a^2+1}\right)b^2+\frac{1}{2}\left(\frac{4ab\log(x)}{a^4-2a^2+1}+\frac{b\log(bx+a+1)}{a^2+2a+1}-\frac{b\log(bx+a-1)}{a^2-2a+1}+\frac{2}{(a^2-1)x}\right)b\operatorname{arcoth}(bx+a)-\frac{\operatorname{arcoth}(bx+a)^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*
a + 1/2))*a/(a^4 - 2*a^2 + 1) - 8*(log(b*x/(a + 1) + 1)*log(x) + dilog(-b*x
/(a + 1)))*a/(a^4 - 2*a^2 + 1) + 8*(log(b*x/(a - 1) + 1)*log(x) + dilog(-b*
```

```
x/(a - 1)))*a/(a^4 - 2*a^2 + 1) - ((a^2 - 2*a + 1)*log(b*x + a + 1)^2 - 2*(
a^2 - 2*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^2 + 2*a + 1)*log(b*x
+ a - 1)^2)/(a^4 - 2*a^2 + 1) + 4*log(b*x + a + 1)/(a^3 + a^2 - a - 1) - 4*
log(b*x + a - 1)/(a^3 - a^2 - a + 1) + 8*log(x)/(a^4 - 2*a^2 + 1))*b^2 + 1/
2*(4*a*b*log(x)/(a^4 - 2*a^2 + 1) + b*log(b*x + a + 1)/(a^2 + 2*a + 1) - b*
log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b*arccoth(b*x + a) - 1/
2*arccoth(b*x + a)^2/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="fricas")
[Out] integral(arccoth(b*x + a)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**3,x)
[Out] Integral(acoth(a + b*x)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^3,x, algorithm="giac")
[Out] integrate(arccoth(b*x + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)^2/x^3,x)
[Out] int(acoth(a + b*x)^2/x^3, x)
```

3.76 $\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=673

$$\frac{\log\left(-\frac{1-a-bx}{a+bx}\right)\log\left(1+\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c-b\sqrt{-c}\,\sqrt{d}-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log\left(-\frac{1-a-bx}{a+bx}\right)\log\left(1+\frac{(b^2c+a^2d)(1-a-bx)}{\left(b^2c+b\sqrt{-c}\,\sqrt{d}-(1-a)ad\right)}\right.}{4\sqrt{-c}\,\sqrt{d}}$$

```
[Out] 1/4*ln((b*x+a-1)/(b*x+a))*ln(1+(a^2*d+b^2*c)*(-b*x-a+1)/(b*x+a)/(b^2*c-(1-a
)*a*d-b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*ln((b*x+a+1)/(b*x+a))*l
n(1-(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d-b*(-c)^(1/2)*d^(1/2)))
/(-c)^(1/2)/d^(1/2)-1/4*ln((b*x+a-1)/(b*x+a))*ln(1+(a^2*d+b^2*c)*(-b*x-a+1)
/(b*x+a)/(b^2*c-(1-a)*a*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*ln(
(b*x+a+1)/(b*x+a))*ln(1-(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d+b*
(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4*polylog(2,-(a^2*d+b^2*c)*(-b*x-
a+1)/(b*x+a)/(b^2*c-(1-a)*a*d-b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)+1/4
*polylog(2,(a^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d-b*(-c)^(1/2)*d^
(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,-(a^2*d+b^2*c)*(-b*x-a+1)/(b*x+a)/
(b^2*c-(1-a)*a*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^(1/2)/d^(1/2)-1/4*polylog(2,(a
^2*d+b^2*c)*(b*x+a+1)/(b*x+a)/(b^2*c+a*(1+a)*d+b*(-c)^(1/2)*d^(1/2)))/(-c)^
(1/2)/d^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6251, 2576, 2404, 2354, 2438}

$$\frac{\text{Li}_2\left(-\frac{(da^2+b^2c)(-a-bx+1)}{\left(cb^2-\sqrt{-c}\,\sqrt{d}\,b-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{(da^2+b^2c)(-a-bx+1)}{\left(cb^2+\sqrt{-c}\,\sqrt{d}\,b-(1-a)ad\right)(a+bx)}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2-\sqrt{-c}\,\sqrt{d}\,b+a(a+1)d\right)(a+bx)}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\text{Li}_2\left(\frac{(da^2+b^2c)(a+bx+1)}{\left(cb^2+\sqrt{-c}\,\sqrt{d}\,b+a(a+1)d\right)(a+bx)}\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(\frac{(-a-bx+1)(a^2d+b^2c)}{(a+bx)\left(-(1-a)ad+b^2c-b\sqrt{-c}\,\sqrt{d}\right)}+1\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(\frac{(-a-bx+1)(a^2d+b^2c)}{(a+bx)\left(-(1-a)ad+b^2c+b\sqrt{-c}\,\sqrt{d}\right)}+1\right)}{4\sqrt{-c}\,\sqrt{d}}+\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(1-\frac{(a+bx+1)(a^2d+b^2c)}{(a+bx)\left(a(a+1)d+b^2c-b\sqrt{-c}\,\sqrt{d}\right)}\right)}{4\sqrt{-c}\,\sqrt{d}}-\frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(1-\frac{(a+bx+1)(a^2d+b^2c)}{(a+bx)\left(a(a+1)d+b^2c+b\sqrt{-c}\,\sqrt{d}\right)}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))/((
b^2*c - b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))]])/(4*Sqrt[-c]*Sqrt[d])
 - (Log[-((1 - a - b*x)/(a + b*x))]*Log[1 + ((b^2*c + a^2*d)*(1 - a - b*x))
/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a + b*x))]])/(4*Sqrt[-c]*Sqrt[
d]) + (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))
/((b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))]])/(4*Sqrt[-c]*Sqrt[
d]) - (Log[(1 + a + b*x)/(a + b*x)]*Log[1 - ((b^2*c + a^2*d)*(1 + a + b*x))
/((b^2*c + b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))]])/(4*Sqrt[-c]*Sqrt[
d]) + PolyLog[2, -(((b^2*c + a^2*d)*(1 - a - b*x))/((b^2*c - b*Sqrt[-c]*Sqr
t[d] - (1 - a)*a*d)*(a + b*x)))]/(4*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -(((b^2*
c + a^2*d)*(1 - a - b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d] - (1 - a)*a*d)*(a +
```

```
b*x)))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/(
(b^2*c - b*Sqrt[-c]*Sqrt[d] + a*(1 + a)*d)*(a + b*x))]/(4*Sqrt[-c]*Sqrt[d])
 - PolyLog[2, ((b^2*c + a^2*d)*(1 + a + b*x))/((b^2*c + b*Sqrt[-c]*Sqrt[d]
+ a*(1 + a)*d)*(a + b*x))]/(4*Sqrt[-c]*Sqrt[d])
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2576

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Dist[b*c - a*d, Subst[Int[(b^2*f - a*b
*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^2*h
)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/(c
+ d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] && N
eQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

Rule 6251

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+dx^2}\,dx\right) + \frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+dx^2}\,dx\right) + \frac{1}{2}\int \frac{\log(1+a+bx)}{c+dx^2}\,dx - \frac{1}{2}\Big(-\log(-1+a+bx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\tan^{-1}\Big(\frac{\sqrt{d}}{\sqrt{c}}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\tan^{-1}\Big(\frac{\sqrt{d}}{\sqrt{c}}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\tan^{-1}\Big(\frac{\sqrt{d}}{\sqrt{c}}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\tan^{-1}\Big(\frac{\sqrt{d}}{\sqrt{c}}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\,\sqrt{d}} + \frac{\tan^{-1}\Big(\frac{\sqrt{d}}{\sqrt{c}}}{}$$

<u>Mathematica</u> [A]
time = 0.40, size = 529, normalized size = 0.79

$$\frac{\log\left(\frac{\sqrt{d}(-1+a+bx)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right) - \log\left(\frac{-1+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right) - \log\left(\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right) + \log\left(\frac{1+a+bx}{a+bx}\right)\log\left(\sqrt{-c}-\sqrt{d}\,x\right) - \log\left(-\frac{\sqrt{d}(-1+a+bx)}{b\sqrt{-c}-(-1+a)\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right) + \log\left(\frac{-1+a+bx}{a+bx}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right) + \log\left(-\frac{\sqrt{d}(1+a+bx)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right) - \log\left(\frac{1+a+bx}{a+bx}\right)\log\left(\sqrt{-c}+\sqrt{d}\,x\right) + \text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(-1+a)\sqrt{d}}\right) - \text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}-\sqrt{d}\,x\right)}{b\sqrt{-c}+(1+a)\sqrt{d}}\right) - \text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(-1+a)\sqrt{d}}\right) + \text{PolyLog}\left(2,\frac{b\left(\sqrt{-c}+\sqrt{d}\,x\right)}{b\sqrt{-c}-(1+a)\sqrt{d}}\right)}{4\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Log[(Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])]*Log[Sqrt[-c]
 - Sqrt[d]*x] - Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - L
og[(Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])]*Log[Sqrt[-c] - S
qrt[d]*x] + Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] - Sqrt[d]*x] - Log[-(
(Sqrt[d]*(-1 + a + b*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d]))]*Log[Sqrt[-c] + S
```

```
qrt[d]*x] + Log[(-1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + Log[-
((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d]))]*Log[Sqrt[-c] + Sq
rt[d]*x] - Log[(1 + a + b*x)/(a + b*x)]*Log[Sqrt[-c] + Sqrt[d]*x] + PolyLog
[2, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (-1 + a)*Sqrt[d])] - PolyLog[2
, (b*(Sqrt[-c] - Sqrt[d]*x))/(b*Sqrt[-c] + (1 + a)*Sqrt[d])] - PolyLog[2, (
b*(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (-1 + a)*Sqrt[d])] + PolyLog[2, (b*
(Sqrt[-c] + Sqrt[d]*x))/(b*Sqrt[-c] - (1 + a)*Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1218 vs. $2(585) = 1170$.
time = 0.82, size = 1219, normalized size = 1.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*(-b^2*c*d)^(1/2)/c/d*arccoth(b*x+a)*ln(1-(a^2*d+b^2*c-2*a*d+d)/(b*
x+a-1)*(b*x+a+1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))-1/2*(-b^2*c*d)^(1/2)/c
/d*arccoth(b*x+a)^2+1/4*(-b^2*c*d)^(1/2)/c/d*polylog(2,(a^2*d+b^2*c-2*a*d+d
)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))-1/2*(-b^2*c*d)^(1
/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c
*d)^(1/2)-d))*a^2*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*(-b
^2*c*d)^(1/2)*b^2/d*ln(1-(a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b
^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-
d)+1/2*(-b^2*c*d)^(1/2)/c*a^2*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1
/2)-d)+1/2*(-b^2*c*d)^(1/2)*b^2/d*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d
)^(1/2)-d)-1/4*(-b^2*c*d)^(1/2)/c*polylog(2,(a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)
*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*a^2/(a^2*d+b^2*c-2*(-b^2*c*d
)^(1/2)-d)-1/4*(-b^2*c*d)^(1/2)*b^2/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)/(b*x+
a-1)*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))/(a^2*d+b^2*c-2*(-b^2*c*d
)^(1/2)-d)-b^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*ln(1-(a^2*d+b^2*c-2*a*d+d
)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)+b^
2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*arccoth(b*x+a)^2+1/2*(-b^2*c*d)^(1/2)/
c*ln(1-(a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^
(1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*(-b^2*c*d)^
(1/2)/c*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*b^2/(a^2*d+
b^2*c-2*(-b^2*c*d)^(1/2)-d)*polylog(2,(a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)*(b*x+
a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))+1/4*(-b^2*c*d)^(1/2)/c*polylog(2,(
a^2*d+b^2*c-2*a*d+d)/(b*x+a-1)*(b*x+a+1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)
)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.58, size = 591, normalized size = 0.88

$$\frac{\operatorname{arcoth}(bx+a)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{\left(\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d},\frac{(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right)-\arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d},\frac{(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)\right)\log(dx^2+c)-\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{b^2dx^2+2(a+1)bdx+(a^2+2a+1)d}{b^2c+(a^2+2a+1)d}\right)+\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\log\left(\frac{b^2dx^2+2(a-1)bdx+(a^2-2a+1)d}{b^2c+(a^2-2a+1)d}\right)-i\,\mathrm{Li}_2\left(\frac{(a-1)bdx+b^2c+(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{b^2c+2(-ia+i)b\sqrt{c}\sqrt{d}-(a^2-2a+1)d}\right)+i\,\mathrm{Li}_2\left(\frac{(a-1)bdx+b^2c-(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{b^2c-2(-ia+i)b\sqrt{c}\sqrt{d}-(a^2-2a+1)d}\right)+i\,\mathrm{Li}_2\left(\frac{(a+1)bdx+b^2c+(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{b^2c+2(-ia-i)b\sqrt{c}\sqrt{d}-(a^2+2a+1)d}\right)-i\,\mathrm{Li}_2\left(\frac{(a+1)bdx+b^2c-(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{b^2c-2(-ia-i)b\sqrt{c}\sqrt{d}-(a^2+2a+1)d}\right)}{4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arccoth(b*x + a)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + 1/4*((arctan2((b^2*x + (
a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (a^
2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*b)*
sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*a +
1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/sqrt(
c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2
*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b*d*x +
(a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*b*d*x +
b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a + I)*b*s
qrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b^2*c - (I*
b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a + I)*b*sqrt(c)*sqrt
(d) - (a^2 - 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I*b^2*x + (-I
*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (a^2
+ 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a - I)*b)*
sqrt(c)*sqrt(d))/(b^2*c - 2*(-I*a - I)*b*sqrt(c)*sqrt(d) - (a^2 + 2*a + 1)*
d)))/sqrt(c*d)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/(d*x^2 + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d*x^2),x)
```

```
[Out] int(acoth(a + b*x)/(c + d*x^2), x)
```

3.77 $\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=120

$$-\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1+a+bx}\right)}{2d}-\frac{\text{PolyLog}(}{}$$

```
[Out] -arccoth(b*x+a)*ln(2/(b*x+a+1))/d+arccoth(b*x+a)*ln(2*b*(d*x+c)/(-a*d+b*c+d
)/(b*x+a+1))/d+1/2*polylog(2,1-2/(b*x+a+1))/d-1/2*polylog(2,1-2*b*(d*x+c)/(
-a*d+b*c+d)/(b*x+a+1))/d
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6247, 6058, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(1-\frac{2b(c+dx)}{(bc-ad+d)(a+bx+1)}\right)}{2d}+\frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d}+\frac{\text{Li}_2\left(1-\frac{2}{a+bx+1}\right)}{2d}-\frac{\log\left(\frac{2}{a+bx+1}\right)\coth^{-1}(a+bx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d*x),x]
```

```
[Out] -((ArcCoth[a + b*x]*Log[2/(1 + a + b*x)])/d) + (ArcCoth[a + b*x]*Log[(2*b*(
c + d*x))/((b*c + d - a*d)*(1 + a + b*x))])/d + PolyLog[2, 1 - 2/(1 + a + b
*x)]/(2*d) - PolyLog[2, 1 - (2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))
]/(2*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+dx}\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{bc-ad}{b}+\frac{dx}{b}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1-\frac{}{(b}\right.}{}$$

$$= -\frac{\coth^{-1}(a+bx)\log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx)\log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2\left(1-\frac{}{1-}\right.}{2d}$$

Mathematica [A]
time = 0.05, size = 185, normalized size = 1.54

$$\frac{\log\left(\frac{d(1-a-bx)}{bc+d-ad}\right)\log(c+dx)}{2d} - \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)\log(c+dx)}{2d} - \frac{\log\left(-\frac{d(1+a+bx)}{bc-d-ad}\right)\log(c+dx)}{2d} + \frac{\log\left(\frac{1+a+bx}{a+bx}\right)\log(c+dx)}{2d} - \frac{\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-d-ad}\right)}{2d} + \frac{\text{PolyLog}\left(2, \frac{b(c+dx)}{bc+d-ad}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*x),x]
```

```
[Out] (Log[(d*(1 - a - b*x))/(b*c + d - a*d)]*Log[c + d*x])/(2*d) - (Log[(-1 + a
+ b*x)/(a + b*x)]*Log[c + d*x])/(2*d) - (Log[-((d*(1 + a + b*x))/(b*c - d -
 a*d))]*Log[c + d*x])/(2*d) + (Log[(1 + a + b*x)/(a + b*x)]*Log[c + d*x])/(
2*d) - PolyLog[2, (b*(c + d*x))/(b*c - d - a*d)]/(2*d) + PolyLog[2, (b*(c +
 d*x))/(b*c + d - a*d)]/(2*d)
```

Maple [A]

time = 0.90, size = 185, normalized size = 1.54

method	result
risch	$-\frac{\operatorname{dilog}\left(\frac{(bx+a-1)d-ad+cb+d}{-ad+cb+d}\right)}{2d}-\frac{\ln(bx+a-1)\ln\left(\frac{(bx+a-1)d-ad+cb+d}{-ad+cb+d}\right)}{2d}+\frac{\operatorname{dilog}\left(\frac{(bx+a+1)d-ad+cb-d}{-ad+cb-d}\right)}{2d}+\frac{\ln(bx+a$
derivativedivides	$\frac{\frac{b\ln(ad-cb-d(bx+a))\operatorname{arccoth}(bx+a)}{d}-\frac{b\left(\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+cb-d}\right)+\ln(ad-cb-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+cb-d}\right)\right)d}{2}-\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)+d}{-ad+cb+d}\right.\right.}{}\right.}{d^2}}{b}$
default	$\frac{\frac{b\ln(ad-cb-d(bx+a))\operatorname{arccoth}(bx+a)}{d}-\frac{b\left(\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)-d}{-ad+cb-d}\right)+\ln(ad-cb-d(bx+a))\ln\left(\frac{-d(bx+a)-d}{-ad+cb-d}\right)\right)d}{2}-\frac{\left(\operatorname{dilog}\left(\frac{-d(bx+a)+d}{-ad+cb+d}\right.\right.}{}\right.}{d^2}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b*ln(a*d-c*b-d*(b*x+a))/d*arccoth(b*x+a)-b/d^2*(1/2*(dilog((-d*(b*x+a)
-d)/(-a*d+b*c-d))+ln(a*d-c*b-d*(b*x+a))*ln((-d*(b*x+a)-d)/(-a*d+b*c-d)))*d-
1/2*(dilog((-d*(b*x+a)+d)/(-a*d+b*c+d))+ln(a*d-c*b-d*(b*x+a))*ln((-d*(b*x+a
)+d)/(-a*d+b*c+d)))*d))
```

Maxima [A]

time = 0.27, size = 192, normalized size = 1.60

$$-\frac{1}{2}b\left(\frac{\log(bx+a-1)\log\left(\frac{bdx+ad-d}{bc-ad+d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd}-\frac{\log(bx+a+1)\log\left(\frac{bdx+ad+d}{bc-ad-d}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd}\right)-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(dx+c)}{2d}+\frac{\operatorname{arcoth}(bx+a)\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a - 1)*log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + dilo
g(-(b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (log(b*x + a + 1)*log((b*d*x
 + a*d + d)/(b*c - a*d - d) + 1) + dilog(-(b*d*x + a*d + d)/(b*c - a*d - d)
))/(b*d)) - 1/2*b*(log(b*x + a + 1)/b - log(b*x + a - 1)/b)*log(d*x + c)/d
+ arccoth(b*x + a)*log(d*x + c)/d
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)/(d*x + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(acoth(a + b*x)/(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d*x),x)
```

```
[Out] int(acoth(a + b*x)/(c + d*x), x)
```

3.78 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=292

$$\frac{(1-a-bx)\log\left(-\frac{1-a-bx}{a+bx}\right)}{2bc}+\frac{\log(a+bx)}{2bc}+\frac{\log(1+a+bx)}{2bc}+\frac{(a+bx)\log\left(\frac{1+a+bx}{a+bx}\right)}{2bc}-\frac{d\log\left(\frac{c(1-a-bx)}{c-ac+bd}\right)\log(d\cdots}{2c^2}$$

```
[Out] 1/2*(-b*x-a+1)*ln((b*x+a-1)/(b*x+a))/b/c+1/2*ln(b*x+a)/b/c+1/2*ln(b*x+a+1)/
b/c+1/2*(b*x+a)*ln((b*x+a+1)/(b*x+a))/b/c-1/2*d*ln(c*(-b*x-a+1)/(-a*c+b*d+c
))*ln(c*x+d)/c^2+1/2*d*ln((b*x+a-1)/(b*x+a))*ln(c*x+d)/c^2+1/2*d*ln(c*(b*x+
a+1)/(a*c-b*d+c))*ln(c*x+d)/c^2-1/2*d*ln((b*x+a+1)/(b*x+a))*ln(c*x+d)/c^2+1
/2*d*polylog(2,-b*(c*x+d)/(a*c-b*d+c))/c^2-1/2*d*polylog(2,b*(c*x+d)/(-a*c+
b*d+c))/c^2
```

Rubi [A]

time = 0.36, antiderivative size = 360, normalized size of antiderivative = 1.23, number of steps used = 37, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6251, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 45}

$$\frac{d\,\text{Li}_2\left(\frac{c(-a-bx+1)}{-ac+c+bd}\right)}{2c^2}-\frac{d\,\text{Li}_2\left(\frac{c(a+bx+1)}{ac+c-bd}\right)}{2c^2}+\frac{d\log(a+bx-1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2}-\frac{d\left(\log(a+bx-1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\log(cx+d)}{2c^2}-\frac{d\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\log(cx+d)}{2c^2}-\frac{d\log(a+bx+1)\log\left(-\frac{b(cx+d)}{ac-bd+c}\right)}{2c^2}+\frac{(-a-bx+1)\log(a+bx-1)}{2bc}+\frac{x\left(\log(a+bx-1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}+\frac{x\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/x),x]
```

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((
1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) + ((1 + a + b*x)*Log[1 + a
+ b*x])/(2*b*c) + (x*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(
a + b*x)]))/(2*c) - (d*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))]
 - Log[a + b*x])*Log[d + c*x])/(2*c^2) - (d*(Log[a + b*x] - Log[1 + a + b*x
] + Log[(1 + a + b*x)/(a + b*x)])*Log[d + c*x])/(2*c^2) - (d*Log[1 + a + b*
x]*Log[-((b*(d + c*x))/(c + a*c - b*d))])/(2*c^2) + (d*Log[-1 + a + b*x]*Lo
g[(b*(d + c*x))/(c - a*c + b*d)])/(2*c^2) + (d*PolyLog[2, (c*(1 - a - b*x))
/(c - a*c + b*d)])/(2*c^2) - (d*PolyLog[2, (c*(1 + a + b*x))/(c + a*c - b*d
)])/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 6251

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}}\,dx &= -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x}}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+\frac{d}{x}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x}}\,dx-\frac{1}{2}\Big(-\log(-1+a+bx\\
&= -\left(\frac{1}{2}\int \left(\frac{\log(-1+a+bx)}{c}-\frac{d\log(-1+a+bx)}{c(d+cx)}\right)dx\right)+\frac{1}{2}\int \Big(\frac{\log(1+a+bx)}{c}\\
&= -\frac{\int \log(-1+a+bx)\,dx}{2c}+\frac{\int \log(1+a+bx)\,dx}{2c}+\frac{d\int \frac{\log(-1+a+bx)}{d+cx}\,dx}{2c}-\frac{d\int \frac{\log(1+a}{d+c}}{2c}\\
&= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{x\big(\log(a+bx)-\log(1+a}{2c}\\
&= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\big(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b}{2c}\\
&= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\big(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b}{2c}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.57, size = 502, normalized size = 1.72

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/x),x]
```

```
[Out] (2*a*c^2*ArcCoth[a + b*x] - I*b*c*d*Pi*ArcCoth[a + b*x] + 2*b*c^2*x*ArcCoth
[a + b*x] + b*c*d*ArcCoth[a + b*x]^2 + a*b*c*d*ArcCoth[a + b*x]^2 - b^2*d^2
*ArcCoth[a + b*x]^2 - a*b*c*d*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcTanh[c/(a*c
- b*d)]*ArcCoth[a + b*x]^2 + b^2*d^2*Sqrt[1 - c^2/(a*c - b*d)^2]*E^ArcTanh[
c/(a*c - b*d)]*ArcCoth[a + b*x]^2 + 2*b*c*d*ArcCoth[a + b*x]*ArcTanh[c/(a*c
 - b*d)] + 2*b*c*d*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])] + I*b*
```

```
c*d*Pi*Log[1 + E^(2*ArcCoth[a + b*x])] - 2*b*c*d*ArcCoth[a + b*x]*Log[1 - E
^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])] + 2*b*c*d*ArcTanh[c/(a*c
 - b*d)]*Log[1 - E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])] - I*b*
c*d*Pi*Log[1/Sqrt[1 - (a + b*x)^(-2)]] - 2*c^2*Log[1/((a + b*x)*Sqrt[1 - (a
 + b*x)^(-2)])] - 2*b*c*d*ArcTanh[c/(a*c - b*d)]*Log[I*Sinh[ArcCoth[a + b*x
] - ArcTanh[c/(a*c - b*d)]]] - b*c*d*PolyLog[2, E^(-2*ArcCoth[a + b*x])] +
b*c*d*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[c/(a*c - b*d)])])/(2*b*
c^3)
```

Maple [A]
time = 0.89, size = 306, normalized size = 1.05

method	result
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2cb}-\frac{1}{bc}-\frac{d\,\mathrm{dilog}\left(\frac{(bx+a+1)c-ac+bd-c}{-ac+bd-c}\right)}{2c^2}-\frac{d\ln(bx+a+1)\ln\left(\frac{(bx+a+1)}{-ac}\right.}{2c^2}$
derivativedivides	$\frac{\frac{\mathrm{arccoth}(bx+a)(bx+a)}{c}-\frac{\mathrm{arccoth}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$
default	$\frac{\frac{\mathrm{arccoth}(bx+a)(bx+a)}{c}-\frac{\mathrm{arccoth}(bx+a)db\ln(ac-bd-c(bx+a))}{c^2}-\frac{-\frac{\ln\left(a^2c^2-2abcd+b^2d^2-2ac(ac-bd-c(bx+a))+2bd(ac-bd-c(bx+a))\right.}{2}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arccoth(b*x+a)/c*(b*x+a)-arccoth(b*x+a)*d*b/c^2*ln(a*c-b*d-c*(b*x+a))-
1/c*(-1/2*ln(a^2*c^2-2*a*b*c*d+b^2*d^2-2*a*c*(a*c-b*d-c*(b*x+a))+2*b*d*(a*c
-b*d-c*(b*x+a))-c^2+(a*c-b*d-c*(b*x+a))^2)-1/2*b*d/c*ln(a*c-b*d-c*(b*x+a))*
ln((-c*(b*x+a)-c)/(-a*c+b*d-c))-1/2*b*d/c*dilog((-c*(b*x+a)-c)/(-a*c+b*d-c)
)+1/2*b*d/c*ln(a*c-b*d-c*(b*x+a))*ln((-c*(b*x+a)+c)/(-a*c+b*d+c))+1/2*b*d/c
*dilog((-c*(b*x+a)+c)/(-a*c+b*d+c))))
```

Maxima [A]
time = 0.27, size = 192, normalized size = 0.66

$$\frac{1}{2}b\left(\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd+c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right)\right)d}{bc^2}-\frac{\left(\log\left(cx+d\right)\log\left(\frac{bcx+bd}{ac-bd-c}+1\right)+\mathrm{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right)\right)d}{bc^2}+\frac{(a+1)\log\left(bx+a+1\right)}{b^2c}-\frac{(a-1)\log\left(bx+a-1\right)}{b^2c}\right)+\left(\frac{x}{c}-\frac{d\log\left(cx+d\right)}{c^2}\right)\mathrm{arcoth}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*b*((log(c*x + d)*log((b*c*x + b*d)/(a*c - b*d + c) + 1) + dilog(-(b*c*x
 + b*d)/(a*c - b*d + c)))*d/(b*c^2) - (log(c*x + d)*log((b*c*x + b*d)/(a*c
- b*d - c) + 1) + dilog(-(b*c*x + b*d)/(a*c - b*d - c)))*d/(b*c^2) + (a + 1
)*log(b*x + a + 1)/(b^2*c) - (a - 1)*log(b*x + a - 1)/(b^2*c)) + (x/c - d*l
og(c*x + d)/c^2)*arccoth(b*x + a)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="fricas")
[Out] integral(x*arccoth(b*x + a)/(c*x + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{acoth}\left(a+bx\right)}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x),x)
[Out] Integral(x*acoth(a + b*x)/(c*x + d), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x),x, algorithm="giac")
[Out] integrate(arccoth(b*x + a)/(c + d/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acoth}(a+b\,x)}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d/x),x)
[Out] int(acoth(a + b*x)/(c + d/x), x)
```

3.79 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}} \, dx$

Optimal. Leaf size=738

$$\frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{}$$

```
[Out] 1/2*(-b*x-a+1)*ln(b*x+a-1)/b/c+1/2*x*(ln(b*x+a-1)-ln((b*x+a-1)/(b*x+a))-ln(
b*x+a))/c+1/2*(b*x+a+1)*ln(b*x+a+1)/b/c+1/2*x*(ln(b*x+a)-ln(b*x+a+1)+ln((b*
x+a+1)/(b*x+a)))/c-1/2*arctan(x*c^(1/2)/d^(1/2))*(ln(b*x+a-1)-ln((b*x+a-1)/
(b*x+a))-ln(b*x+a))*d^(1/2)/c^(3/2)-1/2*arctan(x*c^(1/2)/d^(1/2))*(ln(b*x+a
)-ln(b*x+a+1)+ln((b*x+a+1)/(b*x+a)))*d^(1/2)/c^(3/2)+1/4*ln(b*x+a-1)*ln(-b*
(-x*(-c)^(1/2)+d^(1/2))/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/
4*ln(b*x+a+1)*ln(-b*(x*(-c)^(1/2)+d^(1/2))/((1+a)*(-c)^(1/2)-b*d^(1/2)))*d^
(1/2)/(-c)^(3/2)-1/4*ln(b*x+a-1)*ln(b*(x*(-c)^(1/2)+d^(1/2))/((1-a)*(-c)^(1
/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/4*ln(b*x+a+1)*ln(b*(-x*(-c)^(1/2)+d^(1
/2))/((1+a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*polylog(2,(-b*x-a
+1)*(-c)^(1/2)/((1-a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)+1/4*polylog
(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)-b*d^(1/2)))*d^(1/2)/(-c)^(3/2)-1/
4*polylog(2,(-b*x-a+1)*(-c)^(1/2)/((1-a)*(-c)^(1/2)+b*d^(1/2)))*d^(1/2)/(-c
)^(3/2)-1/4*polylog(2,(b*x+a+1)*(-c)^(1/2)/((1+a)*(-c)^(1/2)+b*d^(1/2)))*d^
(1/2)/(-c)^(3/2)
```

Rubi [A]

time = 1.14, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6251, 2593, 2456, 2436, 2332, 2441, 2440, 2438, 199, 327, 211}

$$-\frac{\sqrt{d}\left(\log(a+bx-1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{3/2}}-\frac{\sqrt{d}\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{3/2}}+\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}(-a-bx+1)}{\sqrt{-c}(1-a)+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\,\text{Li}_2\left(\frac{\sqrt{-c}(a+bx+1)}{\sqrt{-c}(a+1)+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx-1)\log\left(-\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\log(a+bx-1)\log\left(\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}-\frac{\sqrt{d}\log(a+bx+1)\log\left(\frac{b\left(\sqrt{d}-\sqrt{-c}\,x\right)}{(a+1)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{\sqrt{d}\log(a+bx+1)\log\left(-\frac{b\left(\sqrt{-c}\,x+\sqrt{d}\right)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}}+\frac{(-a-bx+1)\log(a+bx-1)}{2bc}+\frac{x\left(\log(a+bx-1)-\log\left(-\frac{-a-bx+1}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{(a+bx+1)\log(a+bx+1)}{2bc}+\frac{x\left(\log(a+bx)-\log(a+bx+1)+\log\left(\frac{a+bx+1}{a+bx}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d/x^2),x]
```

```
[Out] ((1 - a - b*x)*Log[-1 + a + b*x])/(2*b*c) + (x*(Log[-1 + a + b*x] - Log[-((
1 - a - b*x)/(a + b*x))] - Log[a + b*x]))/(2*c) - (Sqrt[d]*ArcTan[(Sqrt[c]*
x)/Sqrt[d]]*(Log[-1 + a + b*x] - Log[-((1 - a - b*x)/(a + b*x))] - Log[a +
b*x]))/(2*c^(3/2)) + ((1 + a + b*x)*Log[1 + a + b*x])/(2*b*c) + (x*(Log[a +
 b*x] - Log[1 + a + b*x] + Log[(1 + a + b*x)/(a + b*x)]))/(2*c) - (Sqrt[d]*
ArcTan[(Sqrt[c]*x)/Sqrt[d]]*(Log[a + b*x] - Log[1 + a + b*x] + Log[(1 + a +
 b*x)/(a + b*x)]))/(2*c^(3/2)) + (Sqrt[d]*Log[-1 + a + b*x]*Log[-((b*(Sqrt[
d] - Sqrt[-c]*x))/((1 - a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[
d]*Log[1 + a + b*x]*Log[(b*(Sqrt[d] - Sqrt[-c]*x))/((1 + a)*Sqrt[-c] + b*Sq
```

```
rt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*Log[1 + a + b*x]*Log[-((b*(Sqrt[d] + Sqr
t[-c]*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d]))])/(4*(-c)^(3/2)) - (Sqrt[d]*Log[-
1 + a + b*x]*Log[(b*(Sqrt[d] + Sqrt[-c]*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])]
)/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a - b*x))/((1 - a)*Sq
rt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 - a
 - b*x))/((1 - a)*Sqrt[-c] + b*Sqrt[d])])/(4*(-c)^(3/2)) + (Sqrt[d]*PolyLog
[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] - b*Sqrt[d])])/(4*(-c)^(3/2)
) - (Sqrt[d]*PolyLog[2, (Sqrt[-c]*(1 + a + b*x))/((1 + a)*Sqrt[-c] + b*Sqrt
[d])])/(4*(-c)^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2593

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_.), x_Symbol] :> Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r], Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] &&  !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 6251

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\log(-1+a+bx)}{c+\frac{d}{x^2}}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}}\,dx-\frac{1}{2}\Big(-\log(-1+a+bx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\log(-1+a+bx)}{c}-\frac{d\log(-1+a+bx)}{c\,(d+cx^2)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{c}\right.$$

$$= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}+\frac{x\left(\log(a+bx)-\log(1+a\right.}{2c}$$

$$= \frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+bx)\right)}{2c}-\frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)(\log(-}{}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b\right.}{2c}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b\right.}{2c}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b\right.}{2c}$$

$$= \frac{(1-a-bx)\log(-1+a+bx)}{2bc}+\frac{x\left(\log(-1+a+bx)-\log\left(-\frac{1-a-bx}{a+bx}\right)-\log(a+b\right.}{2c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.44, size = 5056, normalized size = 6.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/x^2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.78, size = 9136, normalized size = 12.38

method	result
risch	$\frac{\ln(bx+a+1)x}{2c}+\frac{\ln(bx+a+1)a}{2bc}+\frac{\ln(bx+a+1)}{2cb}-\frac{1}{bc}-\frac{d\ln(bx+a+1)\ln\left(\frac{b\sqrt{-cd}-(bx+a+1)c+ac+c}{b\sqrt{-cd}+ac+c}\right)}{4c\sqrt{-cd}}+\frac{d\ln(bx-}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 651, normalized size = 0.88

$$-\left(\frac{d\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}-\frac{x}{c}\right)\operatorname{arcoth}(bx+a)+\frac{2(a+1)c\log(bx+a+1)-2(a-1)c\log(bx+a-1)+\left(b\arctan\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2(a+1)bcx+(a^2+2a+1)c}{b^2d+(a^2+2a+1)c}\right)-b\arctan\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\log\left(\frac{b^2cx^2+2(a-1)bcx+(a^2-2a+1)c}{b^2d+(a^2-2a+1)c}\right)+ib\operatorname{Li}_2\left(\frac{(a-1)bcx+b^2d+(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{2(-ia+i)b\sqrt{c}\sqrt{d}+b^2d-(a^2-2a+1)c}\right)-ib\operatorname{Li}_2\left(-\frac{(a-1)bcx+b^2d-(ib^2x+(-ia+i)b)\sqrt{c}\sqrt{d}}{2(-ia+i)b\sqrt{c}\sqrt{d}-b^2d+(a^2-2a+1)c}\right)-ib\operatorname{Li}_2\left(\frac{(a+1)bcx+b^2d+(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{2(-ia-i)b\sqrt{c}\sqrt{d}+b^2d-(a^2+2a+1)c}\right)+ib\operatorname{Li}_2\left(-\frac{(a+1)bcx+b^2d-(ib^2x+(-ia-i)b)\sqrt{c}\sqrt{d}}{2(-ia-i)b\sqrt{c}\sqrt{d}-b^2d+(a^2+2a+1)c}\right)-\left(b\arctan\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2d+(a^2+2a+1)c},\frac{(a+1)bcx+(a^2+2a+1)c}{b^2d+(a^2+2a+1)c}\right)-b\arctan\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2d+(a^2-2a+1)c},\frac{(a-1)bcx+(a^2-2a+1)c}{b^2d+(a^2-2a+1)c}\right)\right)\log(cx^2+d)\right)\sqrt{c}\sqrt{d}}{4bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] -(d*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c) - x/c)*arccoth(b*x + a) + 1/4*(2*(a
 + 1)*c*log(b*x + a + 1) - 2*(a - 1)*c*log(b*x + a - 1) + (b*arctan(sqrt(c)
*x/sqrt(d))*log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d +
(a^2 + 2*a + 1)*c)) - b*arctan(sqrt(c)*x/sqrt(d))*log((b^2*c*x^2 + 2*(a - 1
)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*dilog(((a -
 1)*b*c*x + b^2*d + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)
*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 - 2*a + 1)*c)) - I*b*dilog(-((a - 1)*b*c*
x + b^2*d - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a + I)*b*sqrt(
c)*sqrt(d) - b^2*d + (a^2 - 2*a + 1)*c)) - I*b*dilog(((a + 1)*b*c*x + b^2*d
 + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sqrt(d
) + b^2*d - (a^2 + 2*a + 1)*c)) + I*b*dilog(-((a + 1)*b*c*x + b^2*d - (I*b^
2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(2*(-I*a - I)*b*sqrt(c)*sqrt(d) - b^2*
d + (a^2 + 2*a + 1)*c)) - (b*arctan2((b^2*x + (a + 1)*b)*sqrt(c)*sqrt(d)/(b
^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d + (a^
2 + 2*a + 1)*c)) - b*arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqrt(d)/(b^2*d + (
a^2 - 2*a + 1)*c), ((a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a
+ 1)*c)))*log(c*x^2 + d))*sqrt(c)*sqrt(d))/(b*c^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="fricas")
[Out] integral(x^2*arccoth(b*x + a)/(c*x^2 + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x**2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^2),x, algorithm="giac")
[Out] integrate(arccoth(b*x + a)/(c + d/x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acoth}(a+b\,x)}{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d/x^2),x)
[Out] int(acoth(a + b*x)/(c + d/x^2), x)
```

3.80 $\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=619

$$\frac{2\sqrt{1+a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{\right.}}{d^2}$$

```
[Out] c*ln((b*x+a-1)/(b*x+a))*ln(c+d*x^(1/2))/d^2-c*ln((b*x+a+1)/(b*x+a))*ln(c+d*
x^(1/2))/d^2+c*ln(c+d*x^(1/2))*ln(d*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(d*(-1-a
)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(d*((1-a)^(1/2)-b^(1/2)*x^(1/2)
)/(d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*ln(c+d*x^(1/2))*ln(-d*((-1-a)^(1/2)+b^(1
/2)*x^(1/2))/(-d*(-1-a)^(1/2)+c*b^(1/2)))/d^2-c*ln(c+d*x^(1/2))*ln(-d*((1-a
)^(1/2)+b^(1/2)*x^(1/2))/(-d*(1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2
)*(c+d*x^(1/2))/(-d*(-1-a)^(1/2)+c*b^(1/2)))/d^2+c*polylog(2,b^(1/2)*(c+d*x
^(1/2))/(d*(-1-a)^(1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(
-d*(1-a)^(1/2)+c*b^(1/2)))/d^2-c*polylog(2,b^(1/2)*(c+d*x^(1/2))/(d*(1-a)^(
1/2)+c*b^(1/2)))/d^2-2*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/d/b
^(1/2)+2*arctan(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/d/b^(1/2)-ln((b*x+
a-1)/(b*x+a))*x^(1/2)/d+ln((b*x+a+1)/(b*x+a))*x^(1/2)/d
```

Rubi [A]
time = 1.61, antiderivative size = 619, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6251, 196, 45, 2608, 2603, 12, 492, 211, 2604, 2465, 266, 2463, 2441, 2440, 2438, 214}

$$\frac{2\sqrt{a+1}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}\,d}+\frac{c\,\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-1}\,d}\right)}{d^2}+\frac{c\,\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-a-1}\,d}\right)}{d^2}-\frac{c\,\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)}{d^2}-\frac{c\,\mathrm{Li}_2\left(\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{-a-1}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{-a-1}\,d+\sqrt{b}\,c}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\,\sqrt{x}\right)}{\sqrt{1-a}\,d+\sqrt{b}\,c}\right)}{d^2}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{-a-1}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-a-1}\,d}\right)}{d^2}-\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{d\left(\sqrt{1-a}+\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)}{d^2}+\frac{c\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{c\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(c+d\sqrt{x}\right)}{d^2}-\frac{\sqrt{x}\,\log\left(-\frac{-a-bx+1}{a+bx}\right)}{d}+\frac{\sqrt{x}\,\log\left(\frac{a+bx+1}{a+bx}\right)}{d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[1 + a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*d) - (2*Sqrt
[1 - a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*d) + (c*Log[(d*(Sq
rt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt
[x]])/d^2 - (c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1
- a]*d)]*Log[c + d*Sqrt[x]])/d^2 + (c*Log[-((d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt
[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d))]*Log[c + d*Sqrt[x]])/d^2 - (c*Log[-((d*
(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d))]*Log[c + d*Sq
rt[x]])/d^2 - (Sqrt[x]*Log[-((1 - a - b*x)/(a + b*x))])/d + (c*Log[c + d*Sq
rt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/d^2 + (Sqrt[x]*Log[(1 + a + b*x)/(a
 + b*x)])/d - (c*Log[c + d*Sqrt[x]]*Log[(1 + a + b*x)/(a + b*x)])/d^2 + (c*
PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[-1 - a]*d)])/d^2 + (
```

```
c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)])/d^2 -
 (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1 - a]*d)])/d^2
- (c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6251

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx\right)+\frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+d\sqrt{x}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{x\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c+dx}\,dx, x, \sqrt{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d}-\frac{c\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right)\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \left(\frac{\log}{}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d}+\frac{\text{Subst}\left(\int \log\left(\frac{1+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{d}+\frac{c\text{Su}}{}$$

$$= -\frac{\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2}+\frac{\sqrt{x}\,\log\left(\frac{1+a+bx}{a+bx}\right)}{d}-\frac{c\log}{}$$

$$= -\frac{\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2}+\frac{\sqrt{x}\,\log\left(\frac{1+a+bx}{a+bx}\right)}{d}-\frac{c\log}{}$$

$$= -\frac{\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}+\frac{c\log\left(c+d\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2}+\frac{\sqrt{x}\,\log\left(\frac{1+a+bx}{a+bx}\right)}{d}-\frac{c\log}{}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}-\frac{\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{d}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

$$= \frac{2\sqrt{1+a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,d}-\frac{2\sqrt{1-a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,d}+\frac{c\log\left(\frac{d\left(\sqrt{-1-a}\right.}{\sqrt{b}\,c+\sqrt{-}}\right.}{}$$

Mathematica [A]
time = 0.46, size = 575, normalized size = 0.93

$$\frac{\frac{2\sqrt{1+a}\,d\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}}-\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}}+c\log\left(\frac{d\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)+c\log\left(\frac{d\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-c\log\left(\frac{d\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{-\sqrt{b}\,c+\sqrt{1-a}\,d}\right)\log\left(c+d\sqrt{x}\right)-d\sqrt{x}\log\left(\frac{-1+a+bx}{a+bx}\right)+c\log\left(c+d\sqrt{x}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)+d\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)-c\log\left(c+d\sqrt{x}\right)\log\left(\frac{1+a+bx}{a+bx}\right)+c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{-1-a}\,d}\right)+c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{-1-a}\,d}\right)-c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c-\sqrt{1-a}\,d}\right)-c\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}\left(c+d\sqrt{x}\right)}{\sqrt{b}\,c+\sqrt{1-a}\,d}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d*Sqrt[x]),x]
```

```
[Out] ((2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/Sqrt[b] - (2*Sqrt[
1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/Sqrt[b] + c*Log[(d*(Sqrt[-
1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]]
 - c*Log[(d*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]*d)]*L
og[c + d*Sqrt[x]] + c*Log[(d*(Sqrt[-1 - a] + Sqrt[b]*Sqrt[x]))/(-(Sqrt[b]*c
) + Sqrt[-1 - a]*d)]*Log[c + d*Sqrt[x]] - c*Log[(d*(Sqrt[1 - a] + Sqrt[b]*S
qrt[x]))/(-(Sqrt[b]*c) + Sqrt[1 - a]*d)]*Log[c + d*Sqrt[x]] - d*Sqrt[x]*Log
[(-1 + a + b*x)/(a + b*x)] + c*Log[c + d*Sqrt[x]]*Log[(-1 + a + b*x)/(a + b
*x)] + d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)] - c*Log[c + d*Sqrt[x]]*Log[(1
 + a + b*x)/(a + b*x)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c
- Sqrt[-1 - a]*d)] + c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sq
rt[-1 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c - Sqrt[1
 - a]*d)] - c*PolyLog[2, (Sqrt[b]*(c + d*Sqrt[x]))/(Sqrt[b]*c + Sqrt[1 - a]
*d)])/d^2
```

Maple [A]
time = 0.32, size = 773, normalized size = 1.25

method	result
derivativedivides	$\frac{2\,\mathrm{arccoth}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccoth}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}+\frac{4b\left(\frac{d^2\arctan\left(\frac{-2cb+2b\left(c+d\sqrt{x}\right)}{2\sqrt{ab\,d^2-b\,d^2}}\right)}{2b\sqrt{ab\,d^2-b\,d^2}}-\frac{d^2\arctan\left(\frac{-2cb}{2\sqrt{a}}\right.}{2b\sqrt{ab}}\right.}{}$ …
default	$\frac{2\,\mathrm{arccoth}(bx+a)\sqrt{x}}{d}-\frac{2\,\mathrm{arccoth}(bx+a)c\ln\left(c+d\sqrt{x}\right)}{d^2}+\frac{4b\left(\frac{d^2\arctan\left(\frac{-2cb+2b\left(c+d\sqrt{x}\right)}{2\sqrt{ab\,d^2-b\,d^2}}\right)}{2b\sqrt{ab\,d^2-b\,d^2}}-\frac{d^2\arctan\left(\frac{-2cb}{2\sqrt{a}}\right.}{2b\sqrt{ab}}\right.}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arccoth(b*x+a)/d*x^(1/2)-2*arccoth(b*x+a)*c/d^2*ln(c+d*x^(1/2))+4*b/d^2*(
1/2*d^2/b/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(-2*c*b+2*b*(c+d*x^(1/2)))/(a*b*
```

```
d^2-b*d^2)^(1/2))-1/2*d^2/b/(a*b*d^2-b*d^2)^(1/2)*arctan(1/2*(-2*c*b+2*b*(c
+d*x^(1/2)))/(a*b*d^2-b*d^2)^(1/2))*a+1/2*d^2/b/(a*b*d^2+b*d^2)^(1/2)*arcta
n(1/2*(-2*c*b+2*b*(c+d*x^(1/2)))/(a*b*d^2+b*d^2)^(1/2))+1/2*d^2/b/(a*b*d^2+
b*d^2)^(1/2)*arctan(1/2*(-2*c*b+2*b*(c+d*x^(1/2)))/(a*b*d^2+b*d^2)^(1/2))*a
-1/4*c/b*ln(c+d*x^(1/2))*ln((c*b-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(c
*b+(-a*b*d^2+b*d^2)^(1/2)))-1/4*c/b*ln(c+d*x^(1/2))*ln((-c*b+b*(c+d*x^(1/2)
)+(-a*b*d^2+b*d^2)^(1/2))/(-c*b+(-a*b*d^2+b*d^2)^(1/2)))-1/4*c/b*dilog((c*b
-b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(c*b+(-a*b*d^2+b*d^2)^(1/2)))-1/4*
c/b*dilog((-c*b+b*(c+d*x^(1/2))+(-a*b*d^2+b*d^2)^(1/2))/(-c*b+(-a*b*d^2+b*d
^2)^(1/2)))+1/4*c/b*ln(c+d*x^(1/2))*ln((c*b-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2
)^(1/2))/(c*b+(-a*b*d^2-b*d^2)^(1/2)))+1/4*c/b*ln(c+d*x^(1/2))*ln((-c*b+b*(
c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(-c*b+(-a*b*d^2-b*d^2)^(1/2)))+1/4*c/b
*dilog((c*b-b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(c*b+(-a*b*d^2-b*d^2)^(
1/2)))+1/4*c/b*dilog((-c*b+b*(c+d*x^(1/2))+(-a*b*d^2-b*d^2)^(1/2))/(-c*b+(-
a*b*d^2-b*d^2)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((d*sqrt(x)*arccoth(b*x + a) - c*arccoth(b*x + a))/(d^2*x - c^2), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d*x^(1/2)),x, algorithm="giac")
[Out] integrate(arccoth(b*x + a)/(d*sqrt(x) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a+b\,x)}{c+d\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d*x^(1/2)),x)
[Out] int(acoth(a + b*x)/(c + d*x^(1/2)), x)
```

3.81 $\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx$

Optimal. Leaf size=738

$$-\frac{2\sqrt{1+a}\,d\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)\log\left(d\right.}{c^3}$$

[Out] 1/2*(1-a)*ln(-b*x-a+1)/b/c-1/2*x*ln((b*x+a-1)/(b*x+a))/c+1/2*(1+a)*ln(b*x+a
+1)/b/c+1/2*x*ln((b*x+a+1)/(b*x+a))/c-d^2*ln((b*x+a-1)/(b*x+a))*ln(d+c*x^(1
/2))/c^3+d^2*ln((b*x+a+1)/(b*x+a))*ln(d+c*x^(1/2))/c^3-d^2*ln(d+c*x^(1/2))*
ln(c*((-1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(-1-a)^(1/2)+d*b^(1/2)))/c^3+d^2*ln(
d+c*x^(1/2))*ln(c*((1-a)^(1/2)-b^(1/2)*x^(1/2))/(c*(1-a)^(1/2)+d*b^(1/2)))/
c^3-d^2*ln(d+c*x^(1/2))*ln(c*((-1-a)^(1/2)+b^(1/2)*x^(1/2))/(c*(-1-a)^(1/2)
-d*b^(1/2)))/c^3+d^2*ln(d+c*x^(1/2))*ln(c*((1-a)^(1/2)+b^(1/2)*x^(1/2))/(c*
(1-a)^(1/2)-d*b^(1/2)))/c^3-d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(-1-a)^
(1/2)-d*b^(1/2)))/c^3+d^2*polylog(2,-b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1/2)-d
*b^(1/2)))/c^3-d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(-1-a)^(1/2)+d*b^(1/2
)))/c^3+d^2*polylog(2,b^(1/2)*(d+c*x^(1/2))/(c*(1-a)^(1/2)+d*b^(1/2)))/c^3+
2*d*arctanh(b^(1/2)*x^(1/2)/(1-a)^(1/2))*(1-a)^(1/2)/c^2/b^(1/2)-2*d*arctan
(b^(1/2)*x^(1/2)/(1+a)^(1/2))*(1+a)^(1/2)/c^2/b^(1/2)+d*ln((b*x+a-1)/(b*x+a
))*x^(1/2)/c^2-d*ln((b*x+a+1)/(b*x+a))*x^(1/2)/c^2

Rubi [A]
time = 1.71, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 65, number of rules used = 19, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$, Rules used = {6251, 196, 46, 2608, 2603, 12, 492, 211, 2605, 457, 78, 2604, 2465, 266, 2463, 2441, 2440, 2438, 214}

$$-\frac{2\sqrt{a+1}\,d\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+1}}\right)}{\sqrt{b}\,c^2}-\frac{d^2\mathrm{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{-a-1}\,c-\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\mathrm{Li}_2\left(-\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\mathrm{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{-a-1}\,c+\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\mathrm{Li}_2\left(\frac{\sqrt{b}\left(\sqrt{x}\,c+d\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}\,c+\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{-a-1}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-a-1}\,c-\sqrt{b}\,d}\right)}{c^3}+\frac{d^2\log\left(c\sqrt{x}+d\right)\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)}{c^3}-\frac{d^2\log\left(-\frac{-a-bx+1}{a+bx}\right)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d^2\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(c\sqrt{x}+d\right)}{c^3}+\frac{d\sqrt{x}\log\left(-\frac{-a-bx+1}{a+bx}\right)}{c^2}-\frac{d\sqrt{x}\log\left(\frac{a+bx+1}{a+bx}\right)}{c^2}+\frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2}+\frac{(1-a)\log(-a-bx+1)}{2bc}-\frac{x\log\left(-\frac{-a-bx+1}{a+bx}\right)}{2c}+\frac{(a+1)\log(a+bx+1)}{2bc}+\frac{x\log\left(\frac{a+bx+1}{a+bx}\right)}{2c}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]

[Out] (-2*Sqrt[1 + a]*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]])/(Sqrt[b]*c^2) + (2
*Sqrt[1 - a]*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]])/(Sqrt[b]*c^2) - (d^2
Log[(c(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log
[d + c*Sqrt[x]])/c^3 + (d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1
 - a]*c + Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 - (d^2*Log[(c*(Sqrt[-1 - a] +
 Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]])/c^3 +
(d^2*Log[(c*(Sqrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*L
og[d + c*Sqrt[x]])/c^3 + ((1 - a)*Log[1 - a - b*x])/(2*b*c) + (d*Sqrt[x]*Lo

```
g[-((1 - a - b*x)/(a + b*x))])/c^2 - (x*Log[-((1 - a - b*x)/(a + b*x))])/(2
*c) - (d^2*Log[d + c*Sqrt[x]]*Log[-((1 - a - b*x)/(a + b*x))])/c^3 + ((1 +
a)*Log[1 + a + b*x])/(2*b*c) - (d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)])/c^2
 + (x*Log[(1 + a + b*x)/(a + b*x)])/(2*c) + (d^2*Log[d + c*Sqrt[x]]*Log[(1
+ a + b*x)/(a + b*x)])/c^3 - (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[x]))/(S
qrt[-1 - a]*c - Sqrt[b]*d))])/c^3 + (d^2*PolyLog[2, -((Sqrt[b]*(d + c*Sqrt[
x]))/(Sqrt[1 - a]*c - Sqrt[b]*d))])/c^3 - (d^2*PolyLog[2, (Sqrt[b]*(d + c*S
qrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)])/c^3 + (d^2*PolyLog[2, (Sqrt[b]*(d +
 c*Sqrt[x]))/(Sqrt[1 - a]*c + Sqrt[b]*d)])/c^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6251

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
 && RationalQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}}\,dx = -\left(\frac{1}{2}\int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx\right) + \frac{1}{2}\int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}}\,dx$$

$$= -\text{Subst}\left(\int \frac{x^2\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x^2\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d+cx}\,dx, x, \sqrt{x}\right.$$

$$= -\text{Subst}\left(\int \left(-\frac{d\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right)\,dx, x,\right.$$

$$= -\frac{\text{Subst}\left(\int x\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} + \frac{\text{Subst}\left(\int x\log\left(\frac{1+a+bx^2}{a+bx^2}\right)\,dx, x, \sqrt{x}\right)}{c} +$$

$$= \frac{d\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x}}{}$$

$$= \frac{d\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x}}{}$$

$$= \frac{d\sqrt{x}\,\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x\log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2\log\left(d+c\sqrt{x}\right)\log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x}}{}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2} + \frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2} + \frac{d\sqrt{x}\,\log\left(-\frac{1-}{a-}\right.}{c^2}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2} + \frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2} + \frac{(1-a)\log(1-}{2bc}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2} + \frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2} - \frac{d^2\log\left(\frac{c\left(\sqrt{-1}\right.}{\sqrt{-}}\right.}{}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2} + \frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2} - \frac{d^2\log\left(\frac{c\left(\sqrt{-1}\right.}{\sqrt{-}}\right.}{}$$

$$= -\frac{2\sqrt{1+a}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}\,c^2} + \frac{2\sqrt{1-a}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}\,c^2} - \frac{d^2\log\left(\frac{c\left(\sqrt{-1}\right.}{\sqrt{-}}\right.}{}$$

Mathematica [A]
time = 0.47, size = 719, normalized size = 0.97

$$\frac{-4\sqrt{1+a}\sqrt{b}\,cd\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)+4\sqrt{1-a}\sqrt{b}\,cd\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)-2bd^2\log\left(\frac{c\left(\sqrt{-1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)+2bd^2\log\left(\frac{c\left(\sqrt{1-a}-\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)-2bd^2\log\left(\frac{c\left(\sqrt{-1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{-1-a}\,c-\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)+2bd^2\log\left(\frac{c\left(\sqrt{1-a}+\sqrt{b}\sqrt{x}\right)}{\sqrt{1-a}\,c-\sqrt{b}\,d}\right)\log\left(d+c\sqrt{x}\right)+c^2\log(1-a-bx)-ac^2\log(1-a-bx)+2bcd\sqrt{x}\log\left(\frac{-1+a+bx}{a+bx}\right)-bc^2x\log\left(\frac{-1+a+bx}{a+bx}\right)-2bd^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)+c^2\log(1+a+bx)+ac^2\log(1+a+bx)-2bcd\sqrt{x}\log\left(\frac{1+a+bx}{a+bx}\right)+bc^2x\log\left(\frac{1+a+bx}{a+bx}\right)+2bd^2\log\left(d+c\sqrt{x}\right)\log\left(\frac{1+a+bx}{a+bx}\right)-2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)-2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{-1-a}\,c+\sqrt{b}\,d}\right)+2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{-\sqrt{1-a}\,c+\sqrt{b}\,d}\right)+2bd^2\text{PolyLog}\left(2,\frac{\sqrt{b}\left(d+c\sqrt{x}\right)}{\sqrt{1-a}\,c+\sqrt{b}\,d}\right)}{2bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(c + d/Sqrt[x]),x]
```

```
[Out] (-4*Sqrt[1 + a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[1 + a]] + 4*Sqrt[
1 - a]*Sqrt[b]*c*d*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[1 - a]] - 2*b*d^2*Log[(c*
(Sqrt[-1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[-1 - a]*c + Sqrt[b]*d)]*Log[d + c*S
qrt[x]] + 2*b*d^2*Log[(c*(Sqrt[1 - a] - Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c +
Sqrt[b]*d)]*Log[d + c*Sqrt[x]] - 2*b*d^2*Log[(c*(Sqrt[-1 - a] + Sqrt[b]*Sqr
t[x]))/(Sqrt[-1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[x]] + 2*b*d^2*Log[(c*(S
qrt[1 - a] + Sqrt[b]*Sqrt[x]))/(Sqrt[1 - a]*c - Sqrt[b]*d)]*Log[d + c*Sqrt[
x]] + c^2*Log[1 - a - b*x] - a*c^2*Log[1 - a - b*x] + 2*b*c*d*Sqrt[x]*Log[(
-1 + a + b*x)/(a + b*x)] - b*c^2*x*Log[(-1 + a + b*x)/(a + b*x)] - 2*b*d^2*
Log[d + c*Sqrt[x]]*Log[(-1 + a + b*x)/(a + b*x)] + c^2*Log[1 + a + b*x] + a
*c^2*Log[1 + a + b*x] - 2*b*c*d*Sqrt[x]*Log[(1 + a + b*x)/(a + b*x)] + b*c^
2*x*Log[(1 + a + b*x)/(a + b*x)] + 2*b*d^2*Log[d + c*Sqrt[x]]*Log[(1 + a +
b*x)/(a + b*x)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sqrt[-1 -
 a]*c) + Sqrt[b]*d)] - 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(Sqrt[-
1 - a]*c + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(-(Sq
rt[1 - a]*c) + Sqrt[b]*d)] + 2*b*d^2*PolyLog[2, (Sqrt[b]*(d + c*Sqrt[x]))/(
Sqrt[1 - a]*c + Sqrt[b]*d)])/(2*b*c^3)
```

Maple [A]
time = 0.33, size = 1001, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(c+d/x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arccoth(b*x+a)/c*x-2*arccoth(b*x+a)/c^2*d*x^(1/2)+2*arccoth(b*x+a)*d^2/c^3*
ln(d+c*x^(1/2))+4*b/c^2*(1/8*c/b^2*ln(a*c^2+b*d^2-2*b*d*(d+c*x^(1/2))+b*(d+
c*x^(1/2))^2+c^2)-1/2*c/b*d/(a*b*c^2+b*c^2)^(1/2)*arctan(1/2*(-2*b*d+2*b*(d
+c*x^(1/2)))/(a*b*c^2+b*c^2)^(1/2))+1/8*c/b^2*a*ln(a*c^2+b*d^2-2*b*d*(d+c*x
^(1/2))+b*(d+c*x^(1/2))^2+c^2)-1/2*c/b*a*d/(a*b*c^2+b*c^2)^(1/2)*arctan(1/2
*(-2*b*d+2*b*(d+c*x^(1/2)))/(a*b*c^2+b*c^2)^(1/2))+1/8*c/b^2*ln(a*c^2+b*d^2
-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2-c^2)-1/2*c/b*d/(a*b*c^2-b*c^2)^(1/2)
*arctan(1/2*(-2*b*d+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2))-1/8*c/b^2*a*l
n(a*c^2+b*d^2-2*b*d*(d+c*x^(1/2))+b*(d+c*x^(1/2))^2-c^2)+1/2*c/b*a*d/(a*b*c
^2-b*c^2)^(1/2)*arctan(1/2*(-2*b*d+2*b*(d+c*x^(1/2)))/(a*b*c^2-b*c^2)^(1/2)
)-1/4/c*d^2/b*ln(d+c*x^(1/2))*ln((b*d-b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2
))/(b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/4/c*d^2/b*ln(d+c*x^(1/2))*ln((-b*d+b*(d+
c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-b*d+(-a*b*c^2-b*c^2)^(1/2)))-1/4/c*d^2
/b*dilog((b*d-b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(b*d+(-a*b*c^2-b*c^2)
```

```
^(1/2))-1/4/c*d^2/b*dilog((-b*d+b*(d+c*x^(1/2))+(-a*b*c^2-b*c^2)^(1/2))/(-
b*d+(-a*b*c^2-b*c^2)^(1/2)))+1/4/c*d^2/b*ln(d+c*x^(1/2))*ln((b*d-b*(d+c*x^(
1/2))+(-a*b*c^2+b*c^2)^(1/2))/(b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/4/c*d^2/b*ln(
d+c*x^(1/2))*ln((-b*d+b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(1/2))/(-b*d+(-a*b*c
^2+b*c^2)^(1/2)))+1/4/c*d^2/b*dilog((b*d-b*(d+c*x^(1/2))+(-a*b*c^2+b*c^2)^(
1/2))/(b*d+(-a*b*c^2+b*c^2)^(1/2)))+1/4/c*d^2/b*dilog((-b*d+b*(d+c*x^(1/2))
+(-a*b*c^2+b*c^2)^(1/2))/(-b*d+(-a*b*c^2+b*c^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*((b*x + a + 1)*log(b*x + a + 1) - (b*x + a - 1)*log(b*x + a - 1))/(b*c)
 - 1/2*integrate((d*log(b*x + a + 1) - d*log(b*x + a - 1))/(c^2*sqrt(x) + c
*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral((c*x*arccoth(b*x + a) - d*sqrt(x)*arccoth(b*x + a))/(c^2*x - d^2),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d/x**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(c+d/x^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(c + d/sqrt(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a+b\,x)}{c+\frac{d}{\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d/x^(1/2)),x)
```

```
[Out] int(acoth(a + b*x)/(c + d/x^(1/2)), x)
```

3.82 $\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx$

Optimal. Leaf size=335

$$\frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b-\sqrt{b^2-4ac}\right)e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}}-\frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(1\right.}}{\sqrt{b^2-4ac}}$$

```
[Out] arccoth(e*x+d)*ln(2*e*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(
b-(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)-arccoth(e*x+d)*ln(2*e*(b+2*c*x+(
-4*a*c+b^2)^(1/2))/(e*x+d+1)/(2*c*(1-d)+e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+
b^2)^(1/2)-1/2*polylog(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b-(-4*a*c+b^2)^(1/2)))/(
e*x+d+1)/(2*c-2*c*d+b*e-e*(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+1/2*polyl
og(2,1+2*(2*c*d-2*c*(e*x+d)-e*(b+(-4*a*c+b^2)^(1/2)))/(e*x+d+1)/(2*c*(1-d)+
e*(b+(-4*a*c+b^2)^(1/2))))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {632, 212, 6860, 6247, 6058, 2449, 2352, 2497}

$$-\frac{\text{Li}_2\left(\frac{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(-2dc+2c+be-\sqrt{b^2-4ac}\,e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\text{Li}_2\left(\frac{2\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e-2c(d+ex)\right)}{\left(2c(1-d)+\left(b+\sqrt{b^2-4ac}\right)e\right)(d+ex+1)}+1\right)}{2\sqrt{b^2-4ac}}+\frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(d+ex+1)\left(e\left(b-\sqrt{b^2-4ac}\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}-\frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(\sqrt{b^2-4ac}+b+2cx\right)}{(d+ex+1)\left(e\left(\sqrt{b^2-4ac}+b\right)+2c(1-d)\right)}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] (ArcCoth[d + e*x]*Log[(2*e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) +
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - (ArcCoth[d
 + e*x]*Log[(2*e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*(1 - d) + (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + d + e*x))])/Sqrt[b^2 - 4*a*c] - PolyLog[2, 1 + (2*(2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c - 2*c*d + b*e - Sq
rt[b^2 - 4*a*c]*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 + (
2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e - 2*c*(d + e*x)))/((2*c*(1 - d) + (b +
 Sqrt[b^2 - 4*a*c])*e)*(1 + d + e*x))]/(2*Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\coth^{-1}(d+ex)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)$$

$$= \frac{(2c)\int \frac{\coth^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\coth^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}{e}+\frac{2cx}{e}}\,dx, x, d+ex\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2c)\text{Subst}\left(\int \frac{\coth\ldots}{\frac{-2cd+\left(b+\sqrt{b^2\ldots}\right.}{e}}\right.}{\sqrt{b^2\ldots}}$$

$$= \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}\,e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex)\log\left(\frac{\ldots}{\left(2c(1\ldots\right.}\right.}{\sqrt{b\ldots}}$$

$$= \frac{\coth^{-1}(d+ex)\log\left(\frac{2e\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(2c-2cd+be-\sqrt{b^2-4ac}\,e\right)(1+d+ex)}\right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex)\log\left(\frac{\ldots}{\left(2c(1\ldots\right.}\right.}{\sqrt{b\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 24.72, size = 1239, normalized size = 3.70

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[d + e*x]/(a + b*x + c*x^2),x]
```

```
[Out] ((1 - (d + e*x)^2)*(-2*Sqrt[b^2 - 4*a*c]*e*E^(-ArcTanh[(2*c*(-1 + d) - b*e)
/(Sqrt[b^2 - 4*a*c]*e)] - ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)
])*(b*e*(Sqrt[-((c*(c*(1 + d)^2 + e*(-(b*(1 + d)) + a*e)))/((b^2 - 4*a*c)*e
```

```
^2))]*E^ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] - Sqrt[-((c*(c*
(-1 + d)^2 + e*(b - b*d + a*e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c*(1 +
d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]) - 2*c*((-1 + d)*Sqrt[-((c*(c*(1 + d)^2 +
e*(-(b*(1 + d)) + a*e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c*(-1 + d) - b*
e)/(Sqrt[b^2 - 4*a*c]*e)] - (1 + d)*Sqrt[-((c*(c*(-1 + d)^2 + e*(b - b*d +
a*e)))/((b^2 - 4*a*c)*e^2))]*E^ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*
c]*e)] + E^(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(
2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]]^2 + 2*(4*c^2*(-1 + d^2) - 4*b*c*d*e + b^2*e^2)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]]*(2*ArcCoth[d + e*x] + ArcTanh[(2*c*(-1 + d) - b*e)/(Sqr
t[b^2 - 4*a*c]*e)] - ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + L
og[1 - E^(-2*(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh
[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] - Log[1 - E^(-2*(ArcTanh[(2*c*(1 + d) -
b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))]) +
2*(4*c^2*(-1 + d^2) - 4*b*c*d*e + b^2*e^2)*(ArcTanh[(2*c*(-1 + d) - b*e)/(S
qrt[b^2 - 4*a*c]*e)]*(Log[1 - E^(-2*(ArcTanh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2
 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] - Log[I*Sinh[ArcT
anh[(2*c*(-1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]]]]) + ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)]*(-Log
[1 - E^(-2*(ArcTanh[(2*c*(1 + d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] + Log[I*Sinh[ArcTanh[(2*c*(1 + d) - b*e)/(S
qrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]]])) - (4*c^2*
(-1 + d^2) - 4*b*c*d*e + b^2*e^2)*PolyLog[2, E^(-2*(ArcTanh[(2*c*(-1 + d) -
 b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]))] +
(4*c^2*(-1 + d^2) - 4*b*c*d*e + b^2*e^2)*PolyLog[2, E^(-2*(ArcTanh[(2*c*(1
+ d) - b*e)/(Sqrt[b^2 - 4*a*c]*e)] + ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
))]))/(2*Sqrt[b^2 - 4*a*c]*(-2*c*(-1 + d) + b*e)*(-2*c*(1 + d) + b*e)*(d +
e*x)^2*(1 - (d + e*x)^(-2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2089 vs. $2(307) = 614$.
time = 1.22, size = 2090, normalized size = 6.24

method	result
risch	$-\frac{e\ln(ex+d-1)\ln\left(\frac{-2(ex+d-1)c-be+2cd+\sqrt{-4ac\,e^2+b^2e^2}-2c}{-be+2cd-2c+\sqrt{-4ac\,e^2+b^2e^2}}\right)}{2\sqrt{-4ac\,e^2+b^2e^2}}+\frac{e\ln(ex+d-1)\ln\left(\frac{2(ex+d-1)c+be-2cd+\sqrt{}}{be-2cd+\sqrt{-4a}}\right.}{2\sqrt{-4ac\,e^2+b}}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*((e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)*ln(1-(a*e^2-b*d*e+
c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2)
```

```
)^(1/2)-c))-(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2+1/2*(e^2*
(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(
e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2+(-e^2*(4*a*c-b^2))^(1/2)-c))-(e^2*(-4
*a*c+b^2))^(1/2)*e^2/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+
d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*a*arccoth(e*
x+d)/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2))^(1/2
)*e/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a
*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*b*d*arccoth(e*x+d)/(a*e^2-b*e
*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*l
n(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-
(-e^2*(4*a*c-b^2))^(1/2)-c))*c*d^2*arccoth(e*x+d)/(a*e^2-b*e*d+c*d^2-(e^2*(
-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2))^(1/2)*e^2/(4*a*c-b^2)*a*arccoth(e*
x+d)^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1
/2)*e/(4*a*c-b^2)*b*d*arccoth(e*x+d)^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2)
)^(1/2)-c)+(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*c*d^2*arccoth(e*x+d)^2/(a*e
^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/2*(e^2*(-4*a*c+b^2))^(1/2)*e^2
/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/
(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*a/(a*e^2-b*e*d+c*d^2-(e^2*(
-4*a*c+b^2))^(1/2)-c)+1/2*(e^2*(-4*a*c+b^2))^(1/2)*e/(4*a*c-b^2)*polylog(2,
(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^
2*(4*a*c-b^2))^(1/2)-c))*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)
-1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-
2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c)
)*c*d^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-e^2/(a*e^2-b*e*d+c*d
^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-
1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)
+e^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*arccoth(e*x+d)^2+(e^2*(
-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-
1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*c*arccoth(e*x+
d)/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b^2))^(1/2)/
(4*a*c-b^2)*c*arccoth(e*x+d)^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-
c)-1/2*e^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*polylog(2,(a*e^2-
b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*
c-b^2))^(1/2)-c))+1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2
-b*d*e+c*d^2+b*e-2*c*d+c)/(e*x+d-1)*(e*x+d+1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a
*c-b^2))^(1/2)-c))*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccoth(x*e + d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acoth}(d+e\,x)}{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d + e*x)/(a + b*x + c*x^2),x)
```

```
[Out] int(acoth(d + e*x)/(a + b*x + c*x^2), x)
```

3.83 $\int x^2 \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}+\frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/9*x^(3/2)+1/15*x^(5/2)+1/3*x^3*arccoth(x^(1/2))-1/3*arctanh(x^(1/2))+1/3*
x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {6038, 52, 65, 212}

$$\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}+\frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{3}-\frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqr
t[x]]/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1-x} dx \\
&= \frac{x^{5/2}}{15} + \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{3/2}}{1-x} dx \\
&= \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{\sqrt{x}}{1-x} dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{1}{(1-x)\sqrt{x}} dx \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
&= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3\coth^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 59, normalized size = 1.16

$$\frac{1}{90}\left(30\sqrt{x} + 10x^{3/2} + 6x^{5/2} + 30x^3\coth^{-1}\left(\sqrt{x}\right) + 15\log\left(1-\sqrt{x}\right) - 15\log\left(1+\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Sqrt[x]],x]
[Out] (30*Sqrt[x] + 10*x^(3/2) + 6*x^(5/2) + 30*x^3*ArcCoth[Sqrt[x]] + 15*Log[1 -
 Sqrt[x]] - 15*Log[1 + Sqrt[x]])/90
```

Maple [**A**]
time = 0.03, size = 42, normalized size = 0.82

method	result	size
derivativedivides	$\frac{x^3 \operatorname{arccoth}\left(\sqrt{x}\right)}{3} + \frac{x^{\frac{5}{2}}}{15} + \frac{x^{\frac{3}{2}}}{9} + \frac{\sqrt{x}}{3} + \frac{\ln\left(\sqrt{x}-1\right)}{6} - \frac{\ln\left(\sqrt{x}+1\right)}{6}$	42

default	$\frac{x^3 \operatorname{arccoth}(\sqrt{x})}{3}+\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}+\frac{\sqrt{x}}{3}+\frac{\ln(\sqrt{x}-1)}{6}-\frac{\ln(\sqrt{x}+1)}{6}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccoth(x^(1/2))+1/15*x^(5/2)+1/9*x^(3/2)+1/3*x^(1/2)+1/6*ln(x^(1/2
)-1)-1/6*ln(x^(1/2)+1)
```

Maxima [A]
time = 0.25, size = 41, normalized size = 0.80

$$\frac{1}{3}\,x^3\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{15}\,x^{\frac{5}{2}}+\frac{1}{9}\,x^{\frac{3}{2}}+\frac{1}{3}\,\sqrt{x}-\frac{1}{6}\,\log\left(\sqrt{x}+1\right)+\frac{1}{6}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(sqrt(x)) + 1/15*x^(5/2) + 1/9*x^(3/2) + 1/3*sqrt(x) - 1/6*l
og(sqrt(x) + 1) + 1/6*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.36, size = 38, normalized size = 0.75

$$\frac{1}{6}\left(x^3-1\right)\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{45}\left(3\,x^2+5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(x^3 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/45*(3*x^2 + 5*x + 15)*sq
rt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{acoth}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(x**(1/2)),x)
```

```
[Out] Integral(x**2*acoth(sqrt(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(31) = 62.

time = 0.39, size = 164, normalized size = 3.22

$$\frac{2\left(\frac{45\left(\sqrt{x}+1\right)^{4}}{\left(\sqrt{x}-1\right)^{4}}-\frac{90\left(\sqrt{x}+1\right)^{3}}{\left(\sqrt{x}-1\right)^{3}}+\frac{140\left(\sqrt{x}+1\right)^{2}}{\left(\sqrt{x}-1\right)^{2}}-\frac{70\left(\sqrt{x}+1\right)}{\sqrt{x}-1}+23\right)}{45\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^{5}}+\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^{5}}{\left(\sqrt{x}-1\right)^{5}}+\frac{10\left(\sqrt{x}+1\right)^{3}}{\left(\sqrt{x}-1\right)^{3}}+\frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/45*(45*(sqrt(x) + 1)^4/(sqrt(x) - 1)^4 - 90*(sqrt(x) + 1)^3/(sqrt(x) - 1)
^3 + 140*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 - 70*(sqrt(x) + 1)/(sqrt(x) - 1) +
 23)/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/3*(3*(sqrt(x) + 1)^5/(sqrt(x)
- 1)^5 + 10*(sqrt(x) + 1)^3/(sqrt(x) - 1)^3 + 3*(sqrt(x) + 1)/(sqrt(x) - 1)
)*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^6
```

Mupad [B]
time = 1.30, size = 31, normalized size = 0.61

$$\frac{x^3\,\mathrm{acoth}\left(\sqrt{x}\right)}{3}-\frac{\mathrm{acoth}\left(\sqrt{x}\right)}{3}+\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}+\frac{x^{5/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(x^(1/2)),x)
```

```
[Out] (x^3*acoth(x^(1/2)))/3 - acoth(x^(1/2))/3 + x^(1/2)/3 + x^(3/2)/9 + x^(5/2)
/15
```

3.84 $\int x \coth^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=42

$$\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 1/6*x^(3/2)+1/2*x^2*arccoth(x^(1/2))-1/2*arctanh(x^(1/2))+1/2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6038, 52, 65, 212}

$$\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x \coth^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{1-x}\,dx\\
&= \frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{\sqrt{x}}{1-x}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{(1-x)\sqrt{x}}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}+\frac{1}{2}x^2\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.24

$$\frac{1}{12}\left(6\sqrt{x}+2x^{3/2}+6x^2\coth^{-1}\left(\sqrt{x}\right)+3\log\left(1-\sqrt{x}\right)-3\log\left(1+\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Sqrt[x]],x]
```

```
[Out] (6*Sqrt[x] + 2*x^(3/2) + 6*x^2*ArcCoth[Sqrt[x]] + 3*Log[1 - Sqrt[x]] - 3*Lo
g[1 + Sqrt[x]])/12
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.88

method	result	size
derivativedivides	$\frac{x^2\operatorname{arccoth}\left(\sqrt{x}\right)}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{\ln\left(\sqrt{x}-1\right)}{4}-\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37
default	$\frac{x^2\operatorname{arccoth}\left(\sqrt{x}\right)}{2}+\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{\ln\left(\sqrt{x}-1\right)}{4}-\frac{\ln\left(\sqrt{x}+1\right)}{4}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccoth(x^(1/2))+1/6*x^(3/2)+1/2*x^(1/2)+1/4*ln(x^(1/2)-1)-1/4*ln(x
^(1/2)+1)
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.86

$$\frac{1}{2}\,x^2\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{6}\,x^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x}-\frac{1}{4}\,\log\left(\sqrt{x}+1\right)+\frac{1}{4}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(sqrt(x)) + 1/6*x^(3/2) + 1/2*sqrt(x) - 1/4*log(sqrt(x) + 1)
 + 1/4*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.74

$$\frac{1}{4}\left(x^2-1\right)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{6}\left(x+3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/6*(x + 3)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acoth}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x**(1/2)),x)
```

```
[Out] Integral(x*acoth(sqrt(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.
time = 0.39, size = 114, normalized size = 2.71

$$\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}-\frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1}+2\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3}+\frac{2\left(\frac{\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 - 3*(sqrt(x) + 1)/(sqrt(x) - 1) + 2)
/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 2*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 +
 (sqrt(x) + 1)/(sqrt(x) - 1))*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) +
1)/(sqrt(x) - 1) - 1)^4
```

Mupad [**B**]
time = 1.26, size = 26, normalized size = 0.62

$$\frac{x^2\,\mathrm{acoth}\left(\sqrt{x}\right)}{2}-\frac{\mathrm{acoth}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}+\frac{x^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(x^(1/2)),x)
```

```
[Out] (x^2*acoth(x^(1/2)))/2 - acoth(x^(1/2))/2 + x^(1/2)/2 + x^(3/2)/6
```

3.85 $\int \coth^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$\sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] x*arccoth(x^(1/2))-arctanh(x^(1/2))+x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.667$, Rules used = {6022, 52, 65, 212}

$$\sqrt{x}-\tanh^{-1}\left(\sqrt{x}\right)+x\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]],x]
```

```
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(\sqrt{x}\right)\,dx &= x\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1-x}\,dx\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x}}\,dx\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\sqrt{x}+x\coth^{-1}\left(\sqrt{x}\right)-\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]],x]
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

Maple [A]
time = 0.03, size = 27, normalized size = 1.23

method	result	size
derivativedivides	$x\,\text{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{\ln\left(\sqrt{x}-1\right)}{2}-\frac{\ln\left(\sqrt{x}+1\right)}{2}$	27
default	$x\,\text{arccoth}\left(\sqrt{x}\right)+\sqrt{x}+\frac{\ln\left(\sqrt{x}-1\right)}{2}-\frac{\ln\left(\sqrt{x}+1\right)}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccoth(x^(1/2))+x^(1/2)+1/2*ln(x^(1/2)-1)-1/2*ln(x^(1/2)+1)
```

Maxima [A]
time = 0.25, size = 26, normalized size = 1.18

$$x\,\text{arcoth}\left(\sqrt{x}\right)+\sqrt{x}-\frac{1}{2}\log\left(\sqrt{x}+1\right)+\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="maxima")
[Out] x*arccoth(sqrt(x)) + sqrt(x) - 1/2*log(sqrt(x) + 1) + 1/2*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 1.09

$$\frac{1}{2}(x-1)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2)),x)
[Out] Integral(acoth(sqrt(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(16) = 32$.
time = 0.39, size = 65, normalized size = 2.95

$$\frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1}+\frac{2\left(\sqrt{x}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)),x, algorithm="giac")
[Out] 2/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*(sqrt(x) + 1)*log((sqrt(x) + 1)/(sq
rt(x) - 1))/((sqrt(x) - 1)*((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2)
```

Mupad [B]
time = 1.24, size = 16, normalized size = 0.73

$$x\operatorname{acoth}\left(\sqrt{x}\right)-\operatorname{acoth}\left(\sqrt{x}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2)),x)
[Out] x*acoth(x^(1/2)) - acoth(x^(1/2)) + x^(1/2)
```

3.86 $\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=19

$$\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)$$

```
[Out] polylog(2,-1/x^(1/2))-polylog(2,1/x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6036, 6032}

$$\text{Li}_2\left(-\frac{1}{\sqrt{x}}\right)-\text{Li}_2\left(\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x,x]
```

```
[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6036

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,\sqrt{x}\right)\\ &= \text{Li}_2\left(-\frac{1}{\sqrt{x}}\right)-\text{Li}_2\left(\frac{1}{\sqrt{x}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\text{PolyLog}\left(2,-\frac{1}{\sqrt{x}}\right)-\text{PolyLog}\left(2,\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x,x]
```

```
[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 0.05, size = 33, normalized size = 1.74

method	result	size
derivativedivides	$\ln(x)\,\text{arccoth}\left(\sqrt{x}\right)-\text{dilog}\left(\sqrt{x}+1\right)-\frac{\ln(x)\ln\left(\sqrt{x}+1\right)}{2}-\text{dilog}\left(\sqrt{x}\right)$	33
default	$\ln(x)\,\text{arccoth}\left(\sqrt{x}\right)-\text{dilog}\left(\sqrt{x}+1\right)-\frac{\ln(x)\ln\left(\sqrt{x}+1\right)}{2}-\text{dilog}\left(\sqrt{x}\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arccoth(x^(1/2))-dilog(x^(1/2)+1)-1/2*ln(x)*ln(x^(1/2)+1)-dilog(x^(1/
2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(13) = 26$.
time = 0.25, size = 66, normalized size = 3.47

$$-\frac{1}{2}\left(\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}-1\right)\right)\log\left(x\right)+\text{arcoth}\left(\sqrt{x}\right)\log\left(x\right)+\log\left(-\sqrt{x}\right)\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(x\right)\log\left(\sqrt{x}-1\right)+\text{Li}_2\left(\sqrt{x}+1\right)-\text{Li}_2\left(-\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(log(sqrt(x) + 1) - log(sqrt(x) - 1))*log(x) + arccoth(sqrt(x))*log(x)
 + log(-sqrt(x))*log(sqrt(x) + 1) - 1/2*log(x)*log(sqrt(x) - 1) + dilog(sqr
t(x) + 1) - dilog(-sqrt(x) + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x,x)
```

```
[Out] Integral(acoth(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2))/x,x)
```

```
[Out] int(acoth(x^(1/2))/x, x)
```

3.87 $\int \frac{\coth^{-1}(\sqrt{x})}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arccoth(x^(1/2))/x+arctanh(x^(1/2))-1/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6038, 53, 65, 212}

$$-\frac{1}{\sqrt{x}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^2,x]
```

```
[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x + ArcTanh[Sqrt[x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{(1-x)x^{3/2}}\,dx\\ &= -\frac{1}{\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x}}\,dx\\ &= -\frac{1}{\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.80

$$-\frac{1}{\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{x}-\frac{1}{2}\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log\left(1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^2,x]
[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2
```

Maple [A]
time = 0.04, size = 32, normalized size = 1.28

method	result	size
derivativedivides	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}$	32
default	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}-\frac{\ln\left(\sqrt{x}-1\right)}{2}+\frac{\ln\left(\sqrt{x}+1\right)}{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arccoth(x^(1/2))/x-1/x^(1/2)-1/2*ln(x^(1/2)-1)+1/2*ln(x^(1/2)+1)
```

Maxima [A]
time = 0.25, size = 31, normalized size = 1.24

$$-\frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -arccoth(sqrt(x))/x - 1/sqrt(x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) -
1)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 1.20

$$\frac{(x-1)\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)-2\sqrt{x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="fricas")
[Out] 1/2*((x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*sqrt(x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(20) = 40$.
time = 0.59, size = 92, normalized size = 3.68

$$\frac{x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{x^{2}}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{x}{x^{\frac{5}{2}}-x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**2,x)
[Out] x**(5/2)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - 2*x**(3/2)*acoth(sqrt(x))/(
x**(5/2) - x**(3/2)) + sqrt(x)*acoth(sqrt(x))/(x**(5/2) - x**(3/2)) - x**2/
(x**(5/2) - x**(3/2)) + x/(x**(5/2) - x**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(19) = 38$.
time = 0.39, size = 65, normalized size = 2.60

$$\frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1}+\frac{2\left(\sqrt{x}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] 2/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) + 2*(sqrt(x) + 1)*log((sqrt(x) + 1)/(sq
rt(x) - 1))/((sqrt(x) - 1)*((sqrt(x) + 1)/(sqrt(x) - 1) + 1)^2)
```

Mupad [B]
time = 1.27, size = 18, normalized size = 0.72

$$\mathrm{atanh}\left(\sqrt{x}\right)-\frac{\mathrm{acoth}\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2))/x^2,x)
```

```
[Out] atanh(x^(1/2)) - (acoth(x^(1/2)) + x^(1/2))/x
```

3.88 $\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/6/x^(3/2)-1/2*arccoth(x^(1/2))/x^2+1/2*arctanh(x^(1/2))-1/2/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6038, 53, 65, 212}

$$-\frac{1}{6x^{3/2}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{2\sqrt{x}}+\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^3,x]
```

```
[Out] -1/6*1/x^(3/2) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) + ArcTanh[Sqrt[x]
]/2
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{(1-x)x^{5/2}}\,dx \\
&= -\frac{1}{6x^{3/2}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{(1-x)x^{3/2}}\,dx \\
&= -\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{(1-x)\sqrt{x}}\,dx \\
&= -\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.38

$$-\frac{1}{6x^{3/2}}-\frac{1}{2\sqrt{x}}-\frac{\coth^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\log\left(1-\sqrt{x}\right)+\frac{1}{4}\log\left(1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^3,x]
[Out] -1/6*1/x^(3/2) - 1/(2*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2*x^2) - Log[1 - Sqrt[x]
]/4 + Log[1 + Sqrt[x]]/4
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{2x^2}-\frac{\ln\left(\sqrt{x}-1\right)}{4}+\frac{\ln\left(\sqrt{x}+1\right)}{4}-\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}$	37

default	$-\frac{\operatorname{arccoth}\left(\sqrt{x}\right)}{2x^{2}}-\frac{\ln\left(\sqrt{x}-1\right)}{4}+\frac{\ln\left(\sqrt{x}+1\right)}{4}-\frac{1}{6x^{\frac{3}{2}}}-\frac{1}{2\sqrt{x}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccoth(x^(1/2))/x^2-1/4*ln(x^(1/2)-1)+1/4*ln(x^(1/2)+1)-1/6/x^(3/2)-1
/2/x^(1/2)
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.86

$$-\frac{3\,x+1}{6\,x^{\frac{3}{2}}}-\frac{\operatorname{arcoth}\left(\sqrt{x}\right)}{2\,x^{2}}+\frac{1}{4}\,\log\left(\sqrt{x}+1\right)-\frac{1}{4}\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*x + 1)/x^(3/2) - 1/2*arccoth(sqrt(x))/x^2 + 1/4*log(sqrt(x) + 1) -
1/4*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.33, size = 38, normalized size = 0.90

$$\frac{3\,(x^{2}-1)\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)-2\,(3\,x+1)\sqrt{x}}{12\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) - 2*(3*x + 1)*sqrt(x))/x
^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(36) = 72$.
time = 1.28, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3\sqrt{x}\operatorname{acoth}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3x^{3}}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2x^{2}}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{x}{6x^{\frac{7}{2}}-6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(5/2)*acoth(sqrt
(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**(3/2)*acoth(sqrt(x))/(6*x**(7/2) - 6*
```

```
x**(5/2)) + 3*sqrt(x)*acoth(sqrt(x))/(6*x**(7/2) - 6*x**(5/2)) - 3*x**3/(6*
x**(7/2) - 6*x**(5/2)) + 2*x**2/(6*x**(7/2) - 6*x**(5/2)) + x/(6*x**(7/2) -
 6*x**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(26) = 52.
time = 0.39, size = 114, normalized size = 2.71

$$\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^{2}}{\left(\sqrt{x}-1\right)^{2}}+\frac{3\left(\sqrt{x}+1\right)}{\sqrt{x}-1}+2\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^{3}}+\frac{2\left(\frac{\left(\sqrt{x}+1\right)^{3}}{\left(\sqrt{x}-1\right)^{3}}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 3*(sqrt(x) + 1)/(sqrt(x) - 1) + 2)
/((sqrt(x) + 1)/(sqrt(x) - 1) + 1)^3 + 2*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 +
 (sqrt(x) + 1)/(sqrt(x) - 1))*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) +
1)/(sqrt(x) - 1) + 1)^4
```

Mupad [B]
time = 1.49, size = 45, normalized size = 1.07

$$\frac{\ln\left(1-\frac{1}{\sqrt{x}}\right)}{4\,x^2}-\frac{\frac{x}{2}+\frac{1}{6}}{x^{3/2}}-\frac{\ln\left(\frac{1}{\sqrt{x}}+1\right)}{4\,x^2}-\frac{\mathrm{atan}\left(\sqrt{x}\;1\mathrm{i}\right)\;1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2))/x^3,x)
```

```
[Out] log(1 - 1/x^(1/2))/(4*x^2) - (atan(x^(1/2)*1i)*1i)/2 - (x/2 + 1/6)/x^(3/2)
- log(1/x^(1/2) + 1)/(4*x^2)
```

3.89 $\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=38

$$\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)$$

```
[Out] 1/5*x+1/10*x^2+2/5*x^(5/2)*arccoth(x^(1/2))+1/5*ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6038, 45}

$$\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x^2}{10}+\frac{x}{5}+\frac{1}{5}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

```
[Out] x/5 + x^2/10 + (2*x^(5/2)*ArcCoth[Sqrt[x]])/5 + Log[1 - x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\coth^{-1}\left(\sqrt{x}\right)\,dx&=\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1-x}\,dx\\&=\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+\frac{1}{1-x}-x\right)dx\\&=\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.82

$$\frac{1}{10}\left(x(2+x)+4x^{5/2}\coth^{-1}\left(\sqrt{x}\right)+2\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcCoth[Sqrt[x]],x]
```

```
[Out] (x*(2 + x) + 4*x^(5/2)*ArcCoth[Sqrt[x]] + 2*Log[1 - x])/10
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.92

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35
default	$\frac{2x^{\frac{5}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}+\frac{x}{5}+\frac{\ln\left(\sqrt{x}-1\right)}{5}+\frac{\ln\left(\sqrt{x}+1\right)}{5}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arccoth(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)*arccoth(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(x^(1/2)-1)+1/5*ln(x^(1/2
)+1)
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.63

$$\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*arccoth(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.92

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{10}\,x^2+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*x^(5/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(x
 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.
time = 1.06, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}-\frac{4x^{\frac{5}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}+\frac{x^3}{10x-10}+\frac{x^2}{10x-10}+\frac{4x\log\left(\sqrt{x}+1\right)}{10x-10}-\frac{4x\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}-\frac{4\log\left(\sqrt{x}+1\right)}{10x-10}+\frac{4\operatorname{acoth}\left(\sqrt{x}\right)}{10x-10}-\frac{2}{10x-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*acoth(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*acoth(sqrt(x))/(10*x - 10) - 4*x**(5/2)*acoth(sqrt(x))/(10*x - 1
0) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 10)
 - 4*x*acoth(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*acot
h(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(26) = 52$.
time = 0.40, size = 168, normalized size = 4.42

$$\frac{8\left(\frac{\left(\sqrt{x}+1\right)^3}{\left(\sqrt{x}-1\right)^3}-\frac{\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^4}+\frac{2\left(\frac{5\left(\sqrt{x}+1\right)^4}{\left(\sqrt{x}-1\right)^4}+\frac{10\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^5}+\frac{2}{5}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-\frac{2}{5}\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="giac")
```

```
[Out] 8/5*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 - (sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + (s
qrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^4 + 2/5*(5*(sq
rt(x) + 1)^4/(sqrt(x) - 1)^4 + 10*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(
(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/5*log(
(sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2/5*log(abs((sqrt(x) + 1)/(sqrt(x) - 1) -
 1))
```

Mupad [B]
time = 1.26, size = 24, normalized size = 0.63

$$\frac{x}{5}+\frac{\ln\left(x-1\right)}{5}+\frac{2\,x^{5/2}\,\operatorname{acoth}\left(\sqrt{x}\right)}{5}+\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*acoth(x^(1/2)),x)
```

```
[Out] x/5 + log(x - 1)/5 + (2*x^(5/2)*acoth(x^(1/2)))/5 + x^2/10
```

3.90 $\int \sqrt{x} \coth^{-1}\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=31

$$\frac{x}{3}+\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)$$

```
[Out] 1/3*x+2/3*x^(3/2)*arccoth(x^(1/2))+1/3*ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6038, 45}

$$\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{x}{3}+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

```
[Out] x/3 + (2*x^(3/2)*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\coth^{-1}\left(\sqrt{x}\right)\,dx &= \frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x}{1-x}\,dx\\
&= \frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\left(-1+\frac{1}{1-x}\right)\,dx\\
&= \frac{x}{3}+\frac{2}{3}x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.81

$$\frac{1}{3}\left(x+2x^{3/2}\coth^{-1}\left(\sqrt{x}\right)+\log(1-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcCoth[Sqrt[x]],x]
```

```
[Out] (x + 2*x^(3/2)*ArcCoth[Sqrt[x]] + Log[1 - x])/3
```

Maple [A]
time = 0.03, size = 30, normalized size = 0.97

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{3}+\frac{\ln\left(\sqrt{x}+1\right)}{3}$	30
default	$\frac{2x^{\frac{3}{2}}\operatorname{arccoth}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(\sqrt{x}-1\right)}{3}+\frac{\ln\left(\sqrt{x}+1\right)}{3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)*arccoth(x^(1/2))+1/3*x+1/3*ln(x^(1/2)-1)+1/3*ln(x^(1/2)+1)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.61

$$\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{arcoth}\left(\sqrt{x}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arccoth(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 0.97

$$\frac{1}{3}\,x^{\frac{3}{2}}\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*x^(3/2)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

Sympy [A]
time = 0.56, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}-\frac{2\operatorname{acoth}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*acoth(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*acoth(sqrt(x))
/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(21) = 42.
time = 0.39, size = 119, normalized size = 3.84

$$\frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3}+\frac{4\left(\sqrt{x}+1\right)}{3\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2}+\frac{2}{3}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-\frac{2}{3}\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log((sqrt(x) + 1)/(sqrt(x) - 1)
)/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 4/3*(sqrt(x) + 1)/((sqrt(x) - 1)*((
sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2) + 2/3*log((sqrt(x) + 1)/abs(sqrt(x) - 1)
) - 2/3*log(abs((sqrt(x) + 1)/(sqrt(x) - 1) - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{x}\,\operatorname{acoth}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*acoth(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*acoth(x^(1/2)), x)
```

3.91 $\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}\,\coth^{-1}\left(\sqrt{x}\right)+\log(1-x)$$

```
[Out] ln(1-x)+2*arccoth(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6038, 31}

$$\log(1-x)+2\sqrt{x}\,\coth^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\,\coth^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1-x}\,dx\\ &= 2\sqrt{x}\,\coth^{-1}\left(\sqrt{x}\right)+\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$2\sqrt{x}\,\coth^{-1}\left(\sqrt{x}\right)+\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sqrt[x]*ArcCoth[Sqrt[x]] + Log[1 - x]
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.75

method	result	size
derivativedivides	$2 \operatorname{arccoth}\left(\sqrt{x}\right) \sqrt{x}+\ln \left(-1+x\right)$	15
default	$2 \operatorname{arccoth}\left(\sqrt{x}\right) \sqrt{x}+\ln \left(-1+x\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arccoth(x^(1/2))*x^(1/2)+ln(-1+x)
```

Maxima [A]
time = 0.25, size = 16, normalized size = 0.80

$$2 \sqrt{x} \operatorname{arcoth}\left(\sqrt{x}\right)+\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arccoth(sqrt(x)) + log(-x + 1)
```

Fricas [A]
time = 0.39, size = 24, normalized size = 1.20

$$\sqrt{x} \log\left(\frac{x+2 \sqrt{x}+1}{x-1}\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.
time = 0.22, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}} \operatorname{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x} \operatorname{acoth}\left(\sqrt{x}\right)}{x-1}+\frac{2x \log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x \operatorname{acoth}\left(\sqrt{x}\right)}{x-1}-\frac{2 \log\left(\sqrt{x}+1\right)}{x-1}+\frac{2 \operatorname{acoth}\left(\sqrt{x}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*x**(3/2)*acoth(sqrt(x))/(x - 1) - 2*sqrt(x)*acoth(sqrt(x))/(x - 1) + 2*x*
log(sqrt(x) + 1)/(x - 1) - 2*x*acoth(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/
(x - 1) + 2*acoth(sqrt(x))/(x - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(16) = 32$.
time = 0.40, size = 70, normalized size = 3.50

$$\frac{2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1}+2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)-2\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*lo
g((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2*log(abs((sqrt(x) + 1)/(sqrt(x) - 1) -
 1))
```

Mupad [B]
time = 1.29, size = 14, normalized size = 0.70

$$\ln(x-1)+2\sqrt{x}\,\mathrm{acoth}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2))/x^(1/2),x)
```

```
[Out] log(x - 1) + 2*x^(1/2)*acoth(x^(1/2))
```

3.92 $\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)$$

```
[Out] -ln(1-x)+ln(x)-2*arccoth(x^(1/2))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6038, 36, 31, 29}

$$-\log(1-x)+\log(x)-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{(1-x)x}\,dx\\ &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{1-x}\,dx+\int \frac{1}{x}\,dx\\ &= -\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\coth^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}-\log(1-x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

Maple [A]
time = 0.04, size = 29, normalized size = 1.21

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arccoth}\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)$	29
default	$-\frac{2\,\mathrm{arccoth}\left(\sqrt{x}\right)}{\sqrt{x}}+\ln\left(x\right)-\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arccoth(x^(1/2))/x^(1/2)+ln(x)-ln(x^(1/2)-1)-ln(x^(1/2)+1)
```

Maxima [A]
time = 0.25, size = 18, normalized size = 0.75

$$-\frac{2\,\mathrm{arcoth}\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arccoth(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

Fricas [A]
time = 0.37, size = 36, normalized size = 1.50

$$-\frac{x\log\left(x-1\right)-x\log\left(x\right)+\sqrt{x}\,\log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -(x*log(x - 1) - x*log(x) + sqrt(x)*log((x + 2*sqrt(x) + 1)/(x - 1)))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(20) = 40.
time = 0.51, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}}\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{2\sqrt{x}\,\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}+\frac{x^2\log\left(x\right)}{x^2-x}-\frac{2x^2\log\left(\sqrt{x}+1\right)}{x^2-x}+\frac{2x^2\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}-\frac{x\log\left(x\right)}{x^2-x}+\frac{2x\log\left(\sqrt{x}+1\right)}{x^2-x}-\frac{2x\operatorname{acoth}\left(\sqrt{x}\right)}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x**(1/2))/x**(3/2),x)
[Out] -2*x**(3/2)*acoth(sqrt(x))/(x**2 - x) + 2*sqrt(x)*acoth(sqrt(x))/(x**2 - x)
 + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*aco
th(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 -
 x) - 2*x*acoth(sqrt(x))/(x**2 - x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(20) = 40.
time = 0.40, size = 70, normalized size = 2.92

$$\frac{2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1}-2\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right)+2\log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] 2*log((sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) - 2*lo
g((sqrt(x) + 1)/abs(sqrt(x) - 1)) + 2*log(abs((sqrt(x) + 1)/(sqrt(x) - 1) +
 1))
```

Mupad [B]
time = 1.25, size = 22, normalized size = 0.92

$$2\ln\left(\sqrt{x}\right)-\ln\left(x-1\right)-\frac{2\operatorname{acoth}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x^(1/2))/x^(3/2),x)
[Out] 2*log(x^(1/2)) - log(x - 1) - (2*acoth(x^(1/2)))/x^(1/2)
```

3.93 $\int \frac{\coth^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{10}\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\frac{1}{10}\text{PolyLog}\left(2,\frac{1}{ax^5}\right)$$

```
[Out] 1/10*polylog(2,-1/a/x^5)-1/10*polylog(2,1/a/x^5)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6036, 6032}

$$\frac{1}{10}\text{Li}_2\left(-\frac{1}{ax^5}\right)-\frac{1}{10}\text{Li}_2\left(\frac{1}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x^5]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x^5))]/10 - PolyLog[2, 1/(a*x^5)]/10
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6036

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax^5)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{\coth^{-1}(ax)}{x}\,dx,x,x^5\right)\\ &= \frac{1}{10}\text{Li}_2\left(-\frac{1}{ax^5}\right)-\frac{1}{10}\text{Li}_2\left(\frac{1}{ax^5}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{10}\left(\text{PolyLog}\left(2,-\frac{1}{ax^5}\right)-\text{PolyLog}\left(2,\frac{1}{ax^5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x^5]/x,x]
```

```
[Out] (PolyLog[2, -(1/(a*x^5))] - PolyLog[2, 1/(a*x^5)])/10
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 95, normalized size = 3.39

method	result
default	$\ln(x)\operatorname{arccoth}(a\,x^5)+5a\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)}{10a}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5-\right.}\right.$
risch	$\frac{\ln(x)\ln\left(a\,x^5+1\right)}{2}-\frac{\left(\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{\ln(x)\ln\left(a\,x^5-1\right)}{2}+\frac{\left(\sum_{\_R1=\mathrm{RootO}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arccoth(a*x^5)+5*a*(-1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_
R1),_R1=RootOf(_Z^5*a+1))+1/10/a*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R
1),_R1=RootOf(_Z^5*a-1)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(22) = 44.
time = 0.25, size = 104, normalized size = 3.71

$$-\frac{1}{2}a\left(\frac{\log\left(ax^5+1\right)}{a}-\frac{\log\left(ax^5-1\right)}{a}\right)\log(x)-\frac{1}{10}a\left(\frac{\log\left(ax^5-1\right)\log\left(ax^5\right)+\mathrm{Li}_2\left(-ax^5+1\right)}{a}-\frac{\log\left(ax^5+1\right)\log\left(-ax^5\right)+\mathrm{Li}_2\left(ax^5+1\right)}{a}\right)+\operatorname{arcoth}\left(ax^5\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilog(
a*x^5 + 1))/a) + arccoth(a*x^5)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(a*x^5)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(ax^{5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x**5)/x,x)
[Out] Integral(acoth(a*x**5)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arccoth(a*x^5)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acoth}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x^5)/x,x)
[Out] int(acoth(a*x^5)/x, x)
```

3.94 $\int \coth^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=19

$$x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] x*arccoth(1/x)+1/2*ln(-x^2+1)
```

Rubi [A]

time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6022, 269, 266}

$$\frac{1}{2}\log\left(1-x^2\right)+x\coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[x^(-1)],x]
```

```
[Out] x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}\left(\frac{1}{x}\right)\,dx &= x\coth^{-1}\left(\frac{1}{x}\right)+\int \frac{1}{\left(1-\frac{1}{x^2}\right)x}\,dx\\
&= x\coth^{-1}\left(\frac{1}{x}\right)+\int \frac{x}{-1+x^2}\,dx\\
&= x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$x\coth^{-1}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[x^(-1)],x]
```

```
[Out] x*ArcCoth[x^(-1)] + Log[1 - x^2]/2
```

Maple [A]
time = 0.14, size = 30, normalized size = 1.58

method	result
derivativedivides	$x\,\mathrm{arccoth}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}$
default	$x\,\mathrm{arccoth}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}$
risch	$\frac{x\ln(1+x)}{2}-\frac{\ln(-1+x)x}{2}+\frac{i\pi\mathrm{csgn}\left(\frac{i(-1+x)}{x}\right)^2x}{2}+\frac{i\pi\,\mathrm{csgn}(i(1+x))\mathrm{csgn}\left(\frac{i(1+x)}{x}\right)^2x}{4}+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}(i(-1+x))}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1/x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccoth(1/x)+1/2*ln(1/x-1)-ln(1/x)+1/2*ln(1/x+1)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.79

$$x\,\mathrm{arcoth}\left(\frac{1}{x}\right)+\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="maxima")
```

```
[Out] x*arccoth(1/x) + 1/2*log(x^2 - 1)
```

Fricas [A]
time = 0.39, size = 23, normalized size = 1.21

$$\frac{1}{2}\,x\log\left(-\frac{x+1}{x-1}\right)+\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

Sympy [A]
time = 0.07, size = 15, normalized size = 0.79

$$x\operatorname{acoth}\left(\frac{1}{x}\right)+\log\left(x+1\right)-\operatorname{acoth}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1/x),x)
```

```
[Out] x*acoth(1/x) + log(x + 1) - acoth(1/x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(17) = 34.
time = 0.39, size = 104, normalized size = 5.47

$$\frac{\log\left(-\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}-1}+\log\left(\frac{|-x-1|}{|x-1|}\right)-\log\left(\left|-\frac{x+1}{x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1/x),x, algorithm="giac")
```

```
[Out] log(-(((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1) +
1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) - 1) + log(abs(-x - 1)/abs(
x - 1)) - log(abs(-(x + 1)/(x - 1) + 1))
```

Mupad [B]
time = 1.13, size = 26, normalized size = 1.37

$$\frac{\ln\left(x^2-1\right)}{2}+x\left(\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(1-x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(1/x),x)
```

```
[Out] log(x^2 - 1)/2 + x*(log(x + 1)/2 - log(1 - x)/2)
```

3.95 $\int \frac{\coth^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=38

$$\frac{\text{PolyLog}\left(2,-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{PolyLog}\left(2,\frac{x^{-n}}{a}\right)}{2n}$$

```
[Out] 1/2*polylog(2,-1/a/(x^n))/n-1/2*polylog(2,1/a/(x^n))/n
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6036, 6032}

$$\frac{\text{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a*x^n]/x,x]
```

```
[Out] PolyLog[2, -(1/(a*x^n))]/(2*n) - PolyLog[2, 1/(a*x^n)]/(2*n)
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6036

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(ax^n)}{x} \, dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(ax)}{x} \, dx, x, x^n\right)}{n}\\ &= \frac{\text{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n}-\frac{\text{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(38) = 76.
time = 0.04, size = 97, normalized size = 2.55

$$\frac{2n\coth^{-1}(ax^n)\log(x)+n\log(x)\log(-1+ax^n)-\log(ax^n)\log(-1+ax^n)-n\log(x)\log(1+ax^n)+\log(-ax^n)\log(1+ax^n)-\text{PolyLog}(2,1-ax^n)+\text{PolyLog}(2,1+ax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a*x^n]/x,x]
```

```
[Out] (2*n*ArcCoth[a*x^n]*Log[x] + n*Log[x]*Log[-1 + a*x^n] - Log[a*x^n]*Log[-1 +
 a*x^n] - n*Log[x]*Log[1 + a*x^n] + Log[-(a*x^n)]*Log[1 + a*x^n] - PolyLog[
2, 1 - a*x^n] + PolyLog[2, 1 + a*x^n])/(2*n)
```

Maple [A]
time = 0.18, size = 53, normalized size = 1.39

method	result	size
risch	$-\frac{\ln(a\,x^n-1)\ln(a\,x^n)}{2n}-\frac{\mathrm{dilog}(a\,x^n)}{2n}-\frac{\mathrm{dilog}(a\,x^n+1)}{2n}$	45
derivativedivides	$\frac{\ln(a\,x^n)\mathrm{arccoth}(a\,x^n)-\frac{\mathrm{dilog}(a\,x^n)}{2}-\frac{\mathrm{dilog}(a\,x^n+1)}{2}-\frac{\ln(a\,x^n)\ln(a\,x^n+1)}{2}}{n}$	53
default	$\frac{\ln(a\,x^n)\mathrm{arccoth}(a\,x^n)-\frac{\mathrm{dilog}(a\,x^n)}{2}-\frac{\mathrm{dilog}(a\,x^n+1)}{2}-\frac{\ln(a\,x^n)\ln(a\,x^n+1)}{2}}{n}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(a*x^n)*arccoth(a*x^n)-1/2*dilog(a*x^n)-1/2*dilog(a*x^n+1)-1/2*ln(a*
x^n)*ln(a*x^n+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(32) = 64.
time = 0.32, size = 147, normalized size = 3.87

$$-\frac{1}{2}an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an}-\frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x)+\frac{1}{2}an\left(\frac{\log(ax^n+1)\log(x)-\log(ax^n-1)\log(x)}{an}-\frac{n\log(ax^n+1)\log(x)+\mathrm{Li}_2(-ax^n)}{an^2}+\frac{n\log(-ax^n+1)\log(x)+\mathrm{Li}_2(ax^n)}{an^2}\right)+\mathrm{arccoth}(ax^n)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/2
*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^n
+ 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog(a*
x^n))/(a*n^2)) + arccoth(a*x^n)*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(32) = 64.
time = 0.35, size = 128, normalized size = 3.37

$$-\frac{n\log(a\cosh(n\log(x))+a\sinh(n\log(x))+1)\log(x)-n\log(-a\cosh(n\log(x))-a\sinh(n\log(x))+1)\log(x)-n\log(x)\log\left(\frac{a\cosh(n\log(x))+a\sinh(n\log(x))+1}{a\cosh(n\log(x))+a\sinh(n\log(x))-1}\right)-\mathrm{Li}_2(a\cosh(n\log(x))+a\sinh(n\log(x)))+\mathrm{Li}_2(-a\cosh(n\log(x))-a\sinh(n\log(x)))}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cosh
(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log((a*cosh(n*log(x))
+ a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - dilog(
a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sinh(n*l
og(x))))/n
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x**n)/x,x)
```

```
[Out] Integral(acoth(a*x**n)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(a*x^n)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x^n)/x,x)
```

```
[Out] int(acoth(a*x^n)/x, x)
```

3.96 $\int (a+bx)\coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{x}{2}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}-\frac{\tanh^{-1}(a+bx)}{2b}$$

[Out] 1/2*x+1/2*(b*x+a)^2*arccoth(b*x+a)/b-1/2*arctanh(b*x+a)/b

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6243, 6038, 327, 212}

$$-\frac{\tanh^{-1}(a+bx)}{2b}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*ArcCoth[a + b*x],x]

[Out] x/2 + ((a + b*x)^2*ArcCoth[a + b*x])/(2*b) - ArcTanh[a + b*x]/(2*b)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
```

```
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\coth^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\coth^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,a+bx\right)}{2b}\\ &= \frac{x}{2}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,a+bx\right)}{2b}\\ &= \frac{x}{2}+\frac{(a+bx)^2\coth^{-1}(a+bx)}{2b}-\frac{\tanh^{-1}(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.69

$$\frac{2bx+2bx(2a+bx)\coth^{-1}(a+bx)-(-1+a^2)\log(1-a-bx)-\log(1+a+bx)+a^2\log(1+a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*ArcCoth[a + b*x],x]
[Out] (2*b*x + 2*b*x*(2*a + b*x)*ArcCoth[a + b*x] - (-1 + a^2)*Log[1 - a - b*x] -
 Log[1 + a + b*x] + a^2*Log[1 + a + b*x])/(4*b)
```

Maple [A]
time = 0.05, size = 46, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)}{4}}{b}$
default	$\frac{\frac{(bx+a)^2\operatorname{arccoth}(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}+\frac{\ln(bx+a-1)}{4}-\frac{\ln(bx+a+1)}{4}}{b}$
risch	$\left(\frac{1}{4}b\,x^2+\frac{1}{2}ax\right)\ln\left(bx+a+1\right)-\frac{b\,x^2\ln(bx+a-1)}{4}-\frac{ax\ln(bx+a-1)}{2}-\frac{\ln(bx+a-1)a^2}{4b}+\frac{\ln(-bx-a-1)}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*arccoth(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*(b*x+a)^2*arccoth(b*x+a)+1/2*b*x+1/2*a+1/4*ln(b*x+a-1)-1/4*ln(b*x+
a+1))
```

Maxima [A]
time = 0.26, size = 62, normalized size = 1.59

$$\frac{1}{4}\,b\left(\frac{2\,x}{b}+\frac{(a^2-1)\log\left(bx+a+1\right)}{b^2}-\frac{(a^2-1)\log\left(bx+a-1\right)}{b^2}\right)+\frac{1}{2}\left(bx^2+2\,ax\right)\operatorname{arcoth}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*x/b + (a^2 - 1)*log(b*x + a + 1)/b^2 - (a^2 - 1)*log(b*x + a - 1)/
b^2) + 1/2*(b*x^2 + 2*a*x)*arccoth(b*x + a)
```

Fricas [A]
time = 0.36, size = 44, normalized size = 1.13

$$\frac{2\,bx+\left(b^2x^2+2\,abx+a^2-1\right)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x + (b^2*x^2 + 2*a*b*x + a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1))
)/b
```

Sympy [A]
time = 0.25, size = 56, normalized size = 1.44

$$\begin{cases}\frac{a^2\operatorname{acoth}\left(a+bx\right)}{2b}+ax\operatorname{acoth}\left(a+bx\right)+\frac{bx^2\operatorname{acoth}\left(a+bx\right)}{2}+\frac{x}{2}-\frac{\operatorname{acoth}\left(a+bx\right)}{2b} & \text{for } b\neq 0\\ ax\operatorname{acoth}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*acoth(b*x+a),x)
```

```
[Out] Piecewise((a**2*acoth(a + b*x)/(2*b) + a*x*acoth(a + b*x) + b*x**2*acoth(a
+ b*x)/2 + x/2 - acoth(a + b*x)/(2*b), Ne(b, 0)), (a*x*acoth(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(33) = 66.
time = 0.40, size = 188, normalized size = 4.82

$$\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{1}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)}+\frac{(bx+a+1)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{(bx+a-1)b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - (a - 1)*b)*(1/(b^2*((b*x + a + 1)/(b*x + a - 1) - 1)) + (b
*x + a + 1)*log(-(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((
b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x
 + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) - 1))/((b*x + a -
 1)*b^2*((b*x + a + 1)/(b*x + a - 1) - 1)^2))
```

Mupad [B]
time = 2.02, size = 50, normalized size = 1.28

$$\frac{x}{2}-\frac{\frac{\mathrm{acoth}(a+b\,x)}{2}-\frac{a^2\,\mathrm{acoth}(a+b\,x)}{2}}{b}+a\,x\,\mathrm{acoth}(a+b\,x)+\frac{b\,x^2\,\mathrm{acoth}(a+b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)*(a + b*x),x)
```

```
[Out] x/2 - (acoth(a + b*x)/2 - (a^2*acoth(a + b*x))/2)/b + a*x*acoth(a + b*x) +
(b*x^2*acoth(a + b*x))/2
```

3.97 $\int (a+bx)^2 \coth^{-1}(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{(a+bx)^2}{6b}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}+\frac{\log\left(1-(a+bx)^2\right)}{6b}$$

```
[Out] 1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*arccoth(b*x+a)/b+1/6*ln(1-(b*x+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6243, 6038, 272, 45}

$$\frac{(a+bx)^2}{6b}+\frac{\log\left(1-(a+bx)^2\right)}{6b}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*ArcCoth[a + b*x],x]
```

```
[Out] (a + b*x)^2/(6*b) + ((a + b*x)^3*ArcCoth[a + b*x])/(3*b) + Log[1 - (a + b*x
)^2]/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
```

```
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\coth^{-1}(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\coth^{-1}(x)\,dx,x,a+bx\right)}{b}\\ &= \frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{x^3}{1-x^2}\,dx,x,a+bx\right)}{3b}\\ &= \frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}-\frac{\text{Subst}\left(\int \frac{x}{1-x}\,dx,x,(a+bx)^2\right)}{6b}\\ &= \frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}-\frac{\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)\,dx,x,(a+bx)^2\right)}{6b}\\ &= \frac{(a+bx)^2}{6b}+\frac{(a+bx)^3\coth^{-1}(a+bx)}{3b}+\frac{\log\left(1-(a+bx)^2\right)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.78

$$\frac{(a+bx)^2+2(a+bx)^3\coth^{-1}(a+bx)+\log\left(1-(a+bx)^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*ArcCoth[a + b*x],x]
[Out] ((a + b*x)^2 + 2*(a + b*x)^3*ArcCoth[a + b*x] + Log[1 - (a + b*x)^2])/(6*b)
```

Maple [A]
time = 0.08, size = 48, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\mathrm{arccoth}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}+\frac{\ln(bx+a-1)}{6}+\frac{\ln(bx+a+1)}{6}}{b}$
default	$\frac{\frac{\mathrm{arccoth}(bx+a)(bx+a)^3}{3}+\frac{(bx+a)^2}{6}+\frac{\ln(bx+a-1)}{6}+\frac{\ln(bx+a+1)}{6}}{b}$
risch	$\frac{(bx+a)^3\ln(bx+a+1)}{6b}-\frac{b^2x^3\ln(bx+a-1)}{6}-\frac{ba\,x^2\ln(bx+a-1)}{2}-\frac{a^2x\ln(bx+a-1)}{2}-\frac{\ln(bx+a-1)a^3}{6b}+\frac{b\,x^2}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*arccoth(b*x+a),x,method=_RETURNVERBOSE)
```

[Out] 1/b*(1/3*arccoth(b*x+a)*(b*x+a)^3+1/6*(b*x+a)^2+1/6*ln(b*x+a-1)+1/6*ln(b*x+a+1))

Maxima [A]

time = 0.25, size = 81, normalized size = 1.50

$$\frac{1}{6}\,b\left(\frac{bx^2+2\,ax}{b}+\frac{(a^3+1)\log\left(bx+a+1\right)}{b^2}-\frac{(a^3-1)\log\left(bx+a-1\right)}{b^2}\right)+\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\operatorname{arcoth}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="maxima")

[Out] 1/6*b*((b*x^2 + 2*a*x)/b + (a^3 + 1)*log(b*x + a + 1)/b^2 - (a^3 - 1)*log(b*x + a - 1)/b^2) + 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*arccoth(b*x + a)

Fricas [A]

time = 0.37, size = 86, normalized size = 1.59

$$\frac{b^2x^2+2\,abx+(a^3+1)\log\left(bx+a+1\right)-(a^3-1)\log\left(bx+a-1\right)+\left(b^3x^3+3\,ab^2x^2+3\,a^2bx\right)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="fricas")

[Out] 1/6*(b^2*x^2 + 2*a*b*x + (a^3 + 1)*log(b*x + a + 1) - (a^3 - 1)*log(b*x + a - 1) + (b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)*log((b*x + a + 1)/(b*x + a - 1)))/b

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

time = 0.36, size = 97, normalized size = 1.80

$$\begin{cases}\frac{a^3\operatorname{acoth}(a+bx)}{3b}+a^2x\operatorname{acoth}\left(a+bx\right)+abx^2\operatorname{acoth}\left(a+bx\right)+\frac{ax}{3}+\frac{b^2x^3\operatorname{acoth}(a+bx)}{3}+\frac{bx^2}{6}+\frac{\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{3b}-\frac{\operatorname{acoth}(a+bx)}{3b} & \text{for}\ b\neq 0\\ a^2x\operatorname{acoth}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2*acoth(b*x+a),x)

[Out] Piecewise((a**3*acoth(a + b*x)/(3*b) + a**2*x*acoth(a + b*x) + a*b*x**2*acoth(a + b*x) + a*x/3 + b**2*x**3*acoth(a + b*x)/3 + b*x**2/6 + log(a/b + x + 1/b)/(3*b) - acoth(a + b*x)/(3*b), Ne(b, 0)), (a**2*x*acoth(a), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(48) = 96.

time = 0.40, size = 255, normalized size = 4.72

$$\frac{1}{6}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}-1\right|\right)}{b^2}+\frac{\left(\frac{3\,(bx+a+1)^2}{(bx+a-1)^2}+1\right)\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)^3}+\frac{2\,(bx+a+1)}{(bx+a-1)b^2\left(\frac{bx+a+1}{bx+a-1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 - l
og(abs((b*x + a + 1)/(b*x + a - 1) - 1))/b^2 + (3*(b*x + a + 1)^2/(b*x + a
- 1)^2 + 1)*log(-(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((
b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x
 + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) - 1))/(b^2*((b*x
+ a + 1)/(b*x + a - 1) - 1)^3) + 2*(b*x + a + 1)/((b*x + a - 1)*b^2*((b*x +
 a + 1)/(b*x + a - 1) - 1)^2))
```

Mupad [B]
time = 1.54, size = 114, normalized size = 2.11

$$\frac{a\,x}{3}+\ln\left(\frac{1}{a+b\,x}+1\right)\,\left(\frac{a^2\,x}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^3}{6}\right)+\frac{b\,x^2}{6}-\ln\left(1-\frac{1}{a+b\,x}\right)\,\left(\frac{a^2\,x}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^3}{6}\right)-\frac{\ln\left(a+b\,x-1\right)\,\left(a^3-1\right)}{6\,b}+\frac{\ln\left(a+b\,x+1\right)\,\left(a^3+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a*x)/3 + log(1/(a + b*x) + 1)*((a^2*x)/2 + (b^2*x^3)/6 + (a*b*x^2)/2) + (b
*x^2)/6 - log(1 - 1/(a + b*x))*((a^2*x)/2 + (b^2*x^3)/6 + (a*b*x^2)/2) - (l
og(a + b*x - 1)*(a^3 - 1))/(6*b) + (log(a + b*x + 1)*(a^3 + 1))/(6*b)
```

3.98 $\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2b}$$

```
[Out] 1/2*polylog(2,-1/(b*x+a))/b-1/2*polylog(2,1/(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6243, 6032}

$$\frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2b}-\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(a + b*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*b) - PolyLog[2, (a + b*x)^(-1)]/(2*b)
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(a+bx)}{a+bx}\,dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,a+bx\right)}{b}\\ &= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2b}-\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 286 vs. $2(35) = 70$.
time = 0.02, size = 286, normalized size = 8.17

$$-\frac{\log\left(\frac{b(-1+a+bx)}{(-1+a)b-ab}\right)\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{2b}-\frac{\log^2\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{4b}+\frac{\log\left(\frac{b(-1-a-bx)}{(-1-a)b+ab}\right)\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{2b}+\frac{\log^2\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{4b}+\frac{\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)}{2b}-\frac{\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)\log\left(\frac{1+a+bx}{a+bx}\right)}{2b}-\frac{\text{PolyLog}(2,-a-bx)}{2b}+\frac{\text{PolyLog}(2,a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(a + b*x),x]
```

```
[Out] -1/2*(Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*b)/
(b*(a + b*x))])/b - Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(4*b) + (Log
[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(a + b*x))
])/(2*b) + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(4*b) + (Log[(-((-1 + a)*
b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/(2*b) - (Log[(a*b -
 (1 + a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/(2*b) - PolyLog[2,
 -a - b*x]/(2*b) + PolyLog[2, a + b*x]/(2*b)
```

Maple [A]
time = 0.11, size = 51, normalized size = 1.46

method	result	size
risch	$-\frac{\ln(bx+a-1)\ln(bx+a)}{2b}-\frac{\operatorname{dilog}(bx+a)}{2b}-\frac{\operatorname{dilog}(bx+a+1)}{2b}$	43
derivativedivides	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{b}$	51
default	$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)-\frac{\operatorname{dilog}(bx+a)}{2}-\frac{\operatorname{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}}{b}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(b*x+a)*arccoth(b*x+a)-1/2*dilog(b*x+a)-1/2*dilog(b*x+a+1)-1/2*ln(b*
x+a)*ln(b*x+a+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(29) = 58$.
time = 0.29, size = 112, normalized size = 3.20

$$-\frac{1}{2}b\left(\frac{\log(bx+a)\log(bx+a-1)+\mathrm{Li}_2(-bx-a+1)}{b^2}-\frac{\log(bx+a+1)\log(-bx-a)+\mathrm{Li}_2(bx+a+1)}{b^2}\right)-\frac{1}{2}\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log(bx+a)+\frac{\operatorname{arcoth}(bx+a)\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/b^2 - (log(b*
x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/b^2) - 1/2*(log(b*x + a + 1)
/b - log(b*x + a - 1)/b)*log(b*x + a) + arccoth(b*x + a)*log(b*x + a)/b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="fricas")
[Out] integral(arccoth(b*x + a)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}{\left(a + bx \right)}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(b*x+a),x)
[Out] Integral(acoth(a + b*x)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="giac")
[Out] integrate(arccoth(b*x + a)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{acoth}(a+b\,x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(a + b*x),x)
[Out] int(acoth(a + b*x)/(a + b*x), x)
```

3.99 $\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx$

Optimal. Leaf size=48

$$-\frac{\coth^{-1}(a+bx)}{b(a+bx)}+\frac{\log(a+bx)}{b}-\frac{\log\left(1-(a+bx)^2\right)}{2b}$$

```
[Out] -arccoth(b*x+a)/b/(b*x+a)+ln(b*x+a)/b-1/2*ln(1-(b*x+a)^2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6243, 6038, 272, 36, 31, 29}

$$\frac{\log(a+bx)}{b}-\frac{\log\left(1-(a+bx)^2\right)}{2b}-\frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(ArcCoth[a + b*x]/(b*(a + b*x))) + Log[a + b*x]/b - Log[1 - (a + b*x)^2]/(
2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, (a+bx)^2\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (a+bx)^2\right)}{2b}\\
&= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\log(a+bx)}{b} - \frac{\log\left(1-(a+bx)^2\right)}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.90

$$-\frac{\frac{2\coth^{-1}(a+bx)}{a+bx} - 2\log(a+bx) + \log\left(1-(a+bx)^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -1/2*((2*ArcCoth[a + b*x])/(a + b*x) - 2*Log[a + b*x] + Log[1 - (a + b*x)^2
])/b
```

Maple [A]
time = 0.09, size = 45, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccoth}(bx+a)}{bx+a}+\ln(bx+a)-\frac{\ln(bx+a-1)}{2}-\frac{\ln(bx+a+1)}{2}}{b}$
default	$\frac{-\frac{\operatorname{arccoth}(bx+a)}{bx+a}+\ln(bx+a)-\frac{\ln(bx+a-1)}{2}-\frac{\ln(bx+a+1)}{2}}{b}$
risch	$-\frac{\ln(bx+a+1)}{2b(bx+a)}+\frac{2\ln(-bx-a)bx-\ln(b^2x^2+2bxa+a^2-1)bx+2\ln(-bx-a)a-\ln(b^2x^2+2bxa+a^2-1)a+\ln(bx+a-1)}{2b(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/b*(-arccoth(b*x+a)/(b*x+a)+ln(b*x+a)-1/2*ln(b*x+a-1)-1/2*ln(b*x+a+1))

Maxima [A]
time = 0.26, size = 53, normalized size = 1.10

$$-\frac{\log(bx+a+1)}{2\,b}+\frac{\log(bx+a)}{b}-\frac{\log(bx+a-1)}{2\,b}-\frac{\operatorname{arcoth}(bx+a)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*log(b*x + a + 1)/b + log(b*x + a)/b - 1/2*log(b*x + a - 1)/b - arccoth(b*x + a)/((b*x + a)*b)

Fricas [A]
time = 0.37, size = 67, normalized size = 1.40

$$-\frac{(bx+a)\log(b^2x^2+2\,abx+a^2-1)-2\,(bx+a)\log(bx+a)+\log\left(\frac{bx+a+1}{bx+a-1}\right)}{2\,(b^2x+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="fricas")

[Out] -1/2*((b*x + a)*log(b^2*x^2 + 2*a*b*x + a^2 - 1) - 2*(b*x + a)*log(b*x + a) + log((b*x + a + 1)/(b*x + a - 1)))/(b^2*x + a*b)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(34) = 68.
time = 0.59, size = 136, normalized size = 2.83

$$\begin{cases}\frac{a\log\left(\frac{a}{b}+x\right)}{ab+b^2x}-\frac{a\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x}+\frac{a\operatorname{acoth}(a+bx)}{ab+b^2x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab+b^2x}-\frac{bx\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{ab+b^2x}+\frac{bx\operatorname{acoth}(a+bx)}{ab+b^2x}-\frac{\operatorname{acoth}(a+bx)}{ab+b^2x} & \text{for } b\neq 0\\ \frac{x\operatorname{acoth}(a)}{a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(b*x+a)**2,x)
```

```
[Out] Piecewise((a*log(a/b + x)/(a*b + b**2*x) - a*log(a/b + x + 1/b)/(a*b + b**2
*x) + a*acoth(a + b*x)/(a*b + b**2*x) + b*x*log(a/b + x)/(a*b + b**2*x) - b
*x*log(a/b + x + 1/b)/(a*b + b**2*x) + b*x*acoth(a + b*x)/(a*b + b**2*x) -
acoth(a + b*x)/(a*b + b**2*x), Ne(b, 0)), (x*acoth(a)/a**2, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(46) = 92.
time = 0.40, size = 198, normalized size = 4.12

$$-\frac{1}{2}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}+1\right|\right)}{b^2}-\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{(bx+a+1)(a-1)}{bx+a-1}-a-1\right)b}{\frac{(bx+a+1)b}{bx+a-1}-b}}-1}\right)}{b^2\left(\frac{bx+a+1}{bx+a-1}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x + a + 1)/abs(b*x + a - 1))/b^2 -
log(abs((b*x + a + 1)/(b*x + a - 1) + 1))/b^2 - log(-(1/(a - ((b*x + a + 1)
*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(b*x + a - 1) - b)) + 1)
/(1/(a - ((b*x + a + 1)*(a - 1)/(b*x + a - 1) - a - 1)*b/((b*x + a + 1)*b/(
b*x + a - 1) - b)) - 1))/(b^2*((b*x + a + 1)/(b*x + a - 1) + 1)))
```

Mupad [**B**]
time = 1.41, size = 93, normalized size = 1.94

$$\frac{\ln\left(a+b\,x\right)}{b}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2-1\right)}{2\,b}-\frac{\ln\left(\frac{a+b\,x+1}{a+b\,x}\right)}{2\,\left(x\,b^2+a\,b\right)}+\frac{\ln\left(\frac{a+b\,x-1}{a+b\,x}\right)}{2\,x\,b^2+2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(a + b*x)^2,x)
```

```
[Out] log(a + b*x)/b - log(a^2 + b^2*x^2 + 2*a*b*x - 1)/(2*b) - log((a + b*x + 1)
/(a + b*x))/(2*(a*b + b^2*x)) + log((a + b*x - 1)/(a + b*x))/(2*a*b + 2*b^2
*x)
```

3.100 $\int \frac{\coth^{-1}(1+x)}{2+2x} \, dx$

Optimal. Leaf size=25

$$\frac{1}{4}\text{PolyLog}\left(2,-\frac{1}{1+x}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1}{1+x}\right)$$

```
[Out] 1/4*polylog(2,-1/(1+x))-1/4*polylog(2,1/(1+x))
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6243, 12, 6032}

$$\frac{1}{4}\text{Li}_2\left(-\frac{1}{x+1}\right)-\frac{1}{4}\text{Li}_2\left(\frac{1}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + x]/(2 + 2*x),x]
```

```
[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(1+x)}{2+2x}\,dx &= \text{Subst}\left(\int \frac{\coth^{-1}(x)}{2x}\,dx, x, 1+x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx, x, 1+x\right)\\ &= \frac{1}{4}\text{Li}_2\left(-\frac{1}{1+x}\right) - \frac{1}{4}\text{Li}_2\left(\frac{1}{1+x}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 117 vs. $2(25) = 50$.
time = 0.01, size = 117, normalized size = 4.68

$$\frac{1}{8}\log^2\left(-\frac{1}{1+x}\right) - \frac{1}{4}\log(-x)\log\left(\frac{1}{1+x}\right) - \frac{1}{8}\log^2\left(\frac{1}{1+x}\right) + \frac{1}{4}\log\left(\frac{1}{1+x}\right)\log\left(\frac{x}{1+x}\right) + \frac{1}{4}\log\left(-\frac{1}{1+x}\right)\log(2+x) - \frac{1}{4}\log\left(-\frac{1}{1+x}\right)\log\left(\frac{2+x}{1+x}\right) - \frac{1}{4}\text{PolyLog}(2,-1-x) + \frac{1}{4}\text{PolyLog}(2,1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 + x]/(2 + 2*x),x]
```

```
[Out] Log[-(1 + x)^(-1)]^2/8 - (Log[-x]*Log[(1 + x)^(-1)])/4 - Log[(1 + x)^(-1)]^
2/8 + (Log[(1 + x)^(-1)]*Log[x/(1 + x)])/4 + (Log[-(1 + x)^(-1)]*Log[2 + x]
)/4 - (Log[-(1 + x)^(-1)]*Log[(2 + x)/(1 + x)])/4 - PolyLog[2, -1 - x]/4 +
PolyLog[2, 1 + x]/4
```

Maple [A]
time = 0.09, size = 34, normalized size = 1.36

method	result	size
risch	$-\frac{\text{dilog}(1+x)}{4} - \frac{\ln(x)\ln(1+x)}{4} - \frac{\text{dilog}(x+2)}{4}$	22
derivativedivides	$\frac{\ln(1+x)\text{arccoth}(1+x)}{2} - \frac{\text{dilog}(1+x)}{4} - \frac{\text{dilog}(x+2)}{4} - \frac{\ln(1+x)\ln(x+2)}{4}$	34
default	$\frac{\ln(1+x)\text{arccoth}(1+x)}{2} - \frac{\text{dilog}(1+x)}{4} - \frac{\text{dilog}(x+2)}{4} - \frac{\ln(1+x)\ln(x+2)}{4}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+x)/(2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1+x)*arccoth(1+x)-1/4*dilog(1+x)-1/4*dilog(x+2)-1/4*ln(1+x)*ln(x+2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(19) = 38$.
time = 0.27, size = 58, normalized size = 2.32

$$-\frac{1}{4}\left(\log\left(x+2\right) - \log\left(x\right)\right)\log\left(x+1\right) + \frac{1}{2}\,\text{arcoth}\left(x+1\right)\log\left(x+1\right) - \frac{1}{4}\,\log\left(x+1\right)\log\left(x\right) + \frac{1}{4}\,\log\left(x+2\right)\log\left(-x-1\right) - \frac{1}{4}\,\text{Li}_2(-x) + \frac{1}{4}\,\text{Li}_2(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(log(x + 2) - log(x))*log(x + 1) + 1/2*arccoth(x + 1)*log(x + 1) - 1/4
*log(x + 1)*log(x) + 1/4*log(x + 2)*log(-x - 1) - 1/4*dilog(-x) + 1/4*dilog
(x + 2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="fricas")
```

```
[Out] integral(1/2*arccoth(x + 1)/(x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acoth}(x+1)}{x+1}\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+x)/(2+2*x),x)
```

```
[Out] Integral(acoth(x + 1)/(x + 1), x)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+x)/(2+2*x),x, algorithm="giac")
```

```
[Out] integrate(1/2*arccoth(x + 1)/(x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(x+1)}{2\,x+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(x + 1)/(2*x + 2),x)
```

```
[Out] int(acoth(x + 1)/(2*x + 2), x)
```

3.101 $\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(2,-\frac{1}{a+bx}\right)}{2d}-\frac{\text{PolyLog}\left(2,\frac{1}{a+bx}\right)}{2d}$$

```
[Out] 1/2*polylog(2,-1/(b*x+a))/d-1/2*polylog(2,1/(b*x+a))/d
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6243, 12, 6032}

$$\frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2d}-\frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] PolyLog[2, -(a + b*x)^(-1)]/(2*d) - PolyLog[2, (a + b*x)^(-1)]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6243

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(f*(x/d))^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\coth^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2d} - \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 312 vs. $2(35) = 70$.

time = 0.02, size = 312, normalized size = 8.91

$$b\left(-\frac{\log\left(\frac{b(-1+a+bx)}{(-1+a)b-ab}\right)\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{2bd}-\frac{\log^2\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)}{4bd}+\frac{\log\left(\frac{b(-1-a-bx)}{(-1-a)b+ab}\right)\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{2bd}+\frac{\log^2\left(\frac{ab-(1+a)b}{b(a+bx)}\right)}{4bd}+\frac{\log\left(\frac{-((-1+a)b)+ab}{b(a+bx)}\right)\log\left(\frac{-1+a+bx}{a+bx}\right)}{2bd}-\frac{\log\left(\frac{ab-(1+a)b}{b(a+bx)}\right)\log\left(\frac{1+a+bx}{a+bx}\right)}{2bd}-\frac{\text{PolyLog}(2,-a-bx)}{2bd}+\frac{\text{PolyLog}(2,a+bx)}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] b*(-1/2*(Log[(b*(-1 + a + b*x))/((-1 + a)*b - a*b)]*Log[(-((-1 + a)*b) + a*
b)/(b*(a + b*x))])/(b*d) - Log[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]^2/(4*b*
d) + (Log[(b*(-1 - a - b*x))/((-1 - a)*b + a*b)]*Log[(a*b - (1 + a)*b)/(b*(
a + b*x))])/(2*b*d) + Log[(a*b - (1 + a)*b)/(b*(a + b*x))]^2/(4*b*d) + (Log
[(-((-1 + a)*b) + a*b)/(b*(a + b*x))]*Log[(-1 + a + b*x)/(a + b*x)])/(2*b*d
) - (Log[(a*b - (1 + a)*b)/(b*(a + b*x))]*Log[(1 + a + b*x)/(a + b*x)])/(2*
b*d) - PolyLog[2, -a - b*x]/(2*b*d) + PolyLog[2, a + b*x]/(2*b*d))
```

Maple [**A**]

time = 0.12, size = 61, normalized size = 1.74

method	result	size
risch	$-\frac{\text{dilog}(bx+a+1)}{2d}-\frac{\ln(bx+a-1)\ln(bx+a)}{2d}-\frac{\text{dilog}(bx+a)}{2d}$	43
derivativedivides	$\frac{\frac{b\ln(bx+a)\text{arccoth}(bx+a)}{d}+\frac{b\left(-\frac{\text{dilog}(bx+a)}{2}-\frac{\text{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}\right)}{d}}{b}$	61
default	$\frac{\frac{b\ln(bx+a)\text{arccoth}(bx+a)}{d}+\frac{b\left(-\frac{\text{dilog}(bx+a)}{2}-\frac{\text{dilog}(bx+a+1)}{2}-\frac{\ln(bx+a)\ln(bx+a+1)}{2}\right)}{d}}{b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(b/d*ln(b*x+a)*arccoth(b*x+a)+b/d*(-1/2*dilog(b*x+a)-1/2*dilog(b*x+a+1)
-1/2*ln(b*x+a)*ln(b*x+a+1)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(29) = 58$.
time = 0.28, size = 132, normalized size = 3.77

$$-\frac{1}{2}b\left(\frac{\log\left(bx+a\right)\log\left(bx+a-1\right)+\mathrm{Li}_2(-bx-a+1)}{bd}-\frac{\log\left(bx+a+1\right)\log\left(-bx-a\right)+\mathrm{Li}_2(bx+a+1)}{bd}\right)-\frac{b\left(\frac{\log(bx+a+1)}{b}-\frac{\log(bx+a-1)}{b}\right)\log\left(dx+\frac{ad}{b}\right)}{2\,d}+\frac{\operatorname{arcoth}\left(bx+a\right)\log\left(dx+\frac{ad}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] -1/2*b*((log(b*x + a)*log(b*x + a - 1) + dilog(-b*x - a + 1))/(b*d) - (log(
b*x + a + 1)*log(-b*x - a) + dilog(b*x + a + 1))/(b*d)) - 1/2*b*(log(b*x +
a + 1)/b - log(b*x + a - 1)/b)*log(d*x + a*d/b)/d + arccoth(b*x + a)*log(d*
x + a*d/b)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccoth(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{acoth}\left(a+bx\right)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acoth(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(d*x + (a*d)/b),x)
[Out] int(acoth(a + b*x)/(d*x + (a*d)/b), x)
```

3.102 $\int (e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=168

$$\frac{bf(6d^2e^2-12cdef+(1+6c^2)f^2)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+\frac{bf^3(c+dx)^3}{12d^4}+\frac{(e+fx)^4 (a+b\coth^{-1}(c+dx}{4f}$$

```
[Out] 1/4*b*f*(6*d^2*e^2-12*c*d*e*f+(6*c^2+1)*f^2)*x/d^3+1/2*b*f^2*(-c*f+d*e)*(d*
x+c)^2/d^4+1/12*b*f^3*(d*x+c)^3/d^4+1/4*(f*x+e)^4*(a+b*arccoth(d*x+c))/f+1/
8*b*(-c*f+d*e+f)^4*ln(-d*x-c+1)/d^4/f-1/8*b*(-c*f+d*e-f)^4*ln(d*x+c+1)/d^4/
f
```

Rubi [A]
time = 0.24, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(e+fx)^4 (a+b\coth^{-1}(c+dx))}{4f}+\frac{bfx((6c^2+1)f^2-12cdef+6d^2e^2)}{4d^3}+\frac{bf^2(c+dx)^2(de-cf)}{2d^4}-\frac{b(-cf+de-f)^4\log(c+dx+1)}{8d^4f}+\frac{b(-cf+de+f)^4\log(-c-dx+1)}{8d^4f}+\frac{bf^3(c+dx)^3}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e -
c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a
+ b*ArcCoth[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*Log[1 - c - d*x])/(8*d^
4*f) - (b*(d*e - f - c*f)^4*Log[1 + c + d*x])/(8*d^4*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3\left(a+b\coth^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}-\frac{b\text{Subst}\left(\int\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^4}{1-x^2}\,dx,x,c\right.}{4f}$$

$$=\frac{(e+fx)^4\left(a+b\coth^{-1}(c+dx)\right)}{4f}-\frac{b\text{Subst}\left(\int\left(-\frac{f^2(6d^2e^2-12cdef+(}{d^4}\right.\right.}{}$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+$$

$$=\frac{bf\left(6d^2e^2-12cdef+\left(1+6c^2\right)f^2\right)x}{4d^3}+\frac{bf^2(de-cf)(c+dx)^2}{2d^4}+$$

Mathematica [A]
time = 0.17, size = 270, normalized size = 1.61

$$\frac{6d\left(4ad^3e^3+bf\left(6d^2e^2-8cdef+\left(1+3c^2\right)f^2\right)\right)x+6d^2f\left(6ad^2e^2+bf(2de-cf)\right)x^2+2d^3f^2(12ade+bf)x^3+6ad^4f^3x^4+6bd^4x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\coth^{-1}(c+dx)-3b(-1+c)\left(4d^3e^3-6(-1+c)d^2e^2f+4(-1+c)^2def^2-(-1+c)^3f^3\right)\log(1-c-dx)-3b(1+c)\left(-4d^3e^3+6(1+c)d^2e^2f-4(1+c)^2def^2+(1+c)^3f^3\right)\log(1+c+dx)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (6*d*(4*a*d^3*e^3 + b*f*(6*d^2*e^2 - 8*c*d*e*f + (1 + 3*c^2)*f^2))*x + 6*d^
2*f*(6*a*d^2*e^2 + b*f*(2*d*e - c*f))*x^2 + 2*d^3*f^2*(12*a*d*e + b*f)*x^3
```

```
+ 6*a*d^4*f^3*x^4 + 6*b*d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*A
rcCoth[c + d*x] - 3*b*(-1 + c)*(4*d^3*e^3 - 6*(-1 + c)*d^2*e^2*f + 4*(-1 +
c)^2*d*e*f^2 - (-1 + c)^3*f^3)*Log[1 - c - d*x] - 3*b*(1 + c)*(-4*d^3*e^3 +
 6*(1 + c)*d^2*e^2*f - 4*(1 + c)^2*d*e*f^2 + (1 + c)^3*f^3)*Log[1 + c + d*x
])/(24*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(156) = 312$.
time = 0.16, size = 982, normalized size = 5.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b*ln(d*x+c+1)*e^3+1/2*b*ln(d*x+c-1)*e^3-1/2*b/d^2*f^2*ln(d*x+c-1)*
c^3*e+3/2*b/d^2*f^2*ln(d*x+c+1)*c*e-3*b/d^2*f^2*c*e*(d*x+c)+3/2*b/d^3*f^3*a
rccoth(d*x+c)*c^2*(d*x+c)^2+3/4*b/d*f*ln(d*x+c-1)*c^2*e^2+3/2*b/d^2*f^2*ln(
d*x+c-1)*c^2*e-3/2*b/d*f*ln(d*x+c-1)*c*e^2+3/2*b/d*f*arccoth(d*x+c)*c^2*e^2
-b/d^2*f^2*arccoth(d*x+c)*c^3*e-b/d^3*f^3*arccoth(d*x+c)*c*(d*x+c)^3-3/2*b/
d^2*f^2*ln(d*x+c-1)*c*e+3/2*b/d*f*arccoth(d*x+c)*e^2*(d*x+c)^2+b/d^2*f^2*ar
ccoth(d*x+c)*e*(d*x+c)^3-b/d^3*f^3*arccoth(d*x+c)*c^3*(d*x+c)+1/2*b/d^2*f^2
*ln(d*x+c+1)*c^3*e-3/4*b/d*f*ln(d*x+c+1)*c^2*e^2+3/2*b/d^2*f^2*ln(d*x+c+1)*
c^2*e-3/2*b/d*f*ln(d*x+c+1)*c*e^2-3*b/d^2*f^2*arccoth(d*x+c)*c*e*(d*x+c)^2+
3*b/d^2*f^2*arccoth(d*x+c)*c^2*e*(d*x+c)-3*b/d*f*arccoth(d*x+c)*c*e^2*(d*x+
c)+1/4*(c*f-d*e-f*(d*x+c))^4*a/d^3/f+1/12*b/d^3*f^3*(d*x+c)^3-1/2*b*ln(d*x+
c-1)*c*e^3-1/8*b/d^3*f^3*ln(d*x+c+1)+1/8*b/d^3*f^3*ln(d*x+c-1)-b*arccoth(d*
x+c)*c*e^3+b*arccoth(d*x+c)*e^3*(d*x+c)+1/2*b*ln(d*x+c+1)*c*e^3+1/4*b/d^3*f
^3*(d*x+c)-1/2*b/d^3*f^3*c*(d*x+c)^2+1/2*b/d^2*f^2*e*(d*x+c)^2+3/2*b/d*f*e^
2*(d*x+c)-1/2*b/d^3*f^3*ln(d*x+c-1)*c-3/4*b/d^3*f^3*ln(d*x+c+1)*c^2-1/2*b/d
^3*f^3*ln(d*x+c+1)*c-1/8*b/d^3*f^3*ln(d*x+c+1)*c^4-1/2*b/d^3*f^3*ln(d*x+c+1
)*c^3+1/8*b*d/f*ln(d*x+c-1)*e^4-1/8*b*d/f*ln(d*x+c+1)*e^4-3/4*b/d*f*ln(d*x+
c+1)*e^2+1/4*b*d/f*arccoth(d*x+c)*e^4+1/2*b/d^2*f^2*ln(d*x+c+1)*e+3/4*b/d*f
*ln(d*x+c-1)*e^2+1/2*b/d^2*f^2*ln(d*x+c-1)*e+1/4*b/d^3*f^3*arccoth(d*x+c)*c
^4+1/4*b/d^3*f^3*arccoth(d*x+c)*(d*x+c)^4+1/8*b/d^3*f^3*ln(d*x+c-1)*c^4-1/2
*b/d^3*f^3*ln(d*x+c-1)*c^3+3/2*b/d^3*f^3*c^2*(d*x+c)+3/4*b/d^3*f^3*ln(d*x+c
-1)*c^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(161) = 322$.
time = 0.28, size = 331, normalized size = 1.97

$$\frac{1}{4}\,af^3x^4 + af^2x^3e + \frac{1}{24}\left(6\,x^4\,\mathrm{arcoth}\left(dx+c\right) + d\left(\frac{2\left(d^2x^3 - 3\,cdx^2 + 3\left(3\,c^2+1\right)x\right)}{d^4} - \frac{3\left(c^4+4\,c^3+6\,c^2+4\,c+1\right)\log\left(dx+c+1\right)}{d^5} + \frac{3\left(c^4-4\,c^3+6\,c^2-4\,c+1\right)\log\left(dx+c-1\right)}{d^5}\right)\right)bf^3 + \frac{3}{2}\,afx^2e^2 + \frac{1}{2}\left(2\,x^3\,\mathrm{arcoth}\left(dx+c\right) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{\left(c^3+3\,c^2+3\,c+1\right)\log\left(dx+c+1\right)}{d^4} - \frac{\left(c^3-3\,c^2+3\,c-1\right)\log\left(dx+c-1\right)}{d^4}\right)\right)bf^2e + \frac{3}{4}\left(2\,x^2\,\mathrm{arcoth}\left(dx+c\right) + d\left(\frac{2\,x}{d^2} - \frac{\left(c^2+2\,c+1\right)\log\left(dx+c+1\right)}{d^3} + \frac{\left(c^2-2\,c+1\right)\log\left(dx+c-1\right)}{d^3}\right)\right)bfe^2 + axe^3 + \frac{\left(2\left(dx+c\right)\mathrm{arcoth}\left(dx+c\right) + \log\left(-\left(dx+c\right)^2+1\right)\right)be^3}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*f^3*x^4 + a*f^2*x^3*e + 1/24*(6*x^4*arccoth(d*x + c) + d*(2*(d^2*x^3
- 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*log(
```

```
d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*log(d*x + c - 1)/d^5))
*b*f^3 + 3/2*a*f*x^2*e^2 + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)
/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c -
1)*log(d*x + c - 1)/d^4))*b*f^2*e + 3/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^
2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)
/d^3))*b*f*e^2 + a*x*e^3 + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x +
c)^2 + 1))*b*e^3/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(161) = 322$.
time = 0.41, size = 817, normalized size = 4.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*d^4*f^3*x^4 + 2*b*d^3*f^3*x^3 - 6*b*c*d^2*f^3*x^2 + 24*a*d^4*x*co
sh(1)^3 + 24*a*d^4*x*sinh(1)^3 + 6*(3*b*c^2 + b)*d*f^3*x + 36*(a*d^4*f*x^2
+ b*d^3*f*x)*cosh(1)^2 + 36*(a*d^4*f*x^2 + 2*a*d^4*x*cosh(1) + b*d^3*f*x)*s
inh(1)^2 + 12*(2*a*d^4*f^2*x^3 + b*d^3*f^2*x^2 - 4*b*c*d^2*f^2*x)*cosh(1) +
 3*(4*(b*c + b)*d^3*cosh(1)^3 + 4*(b*c + b)*d^3*sinh(1)^3 - 6*(b*c^2 + 2*b*
c + b)*d^2*f*cosh(1)^2 + 4*(b*c^3 + 3*b*c^2 + 3*b*c + b)*d*f^2*cosh(1) - (b
*c^4 + 4*b*c^3 + 6*b*c^2 + 4*b*c + b)*f^3 + 6*(2*(b*c + b)*d^3*cosh(1) - (b
*c^2 + 2*b*c + b)*d^2*f)*sinh(1)^2 + 4*(3*(b*c + b)*d^3*cosh(1)^2 - 3*(b*c^
2 + 2*b*c + b)*d^2*f*cosh(1) + (b*c^3 + 3*b*c^2 + 3*b*c + b)*d*f^2)*sinh(1)
)*log(d*x + c + 1) - 3*(4*(b*c - b)*d^3*cosh(1)^3 + 4*(b*c - b)*d^3*sinh(1)
^3 - 6*(b*c^2 - 2*b*c + b)*d^2*f*cosh(1)^2 + 4*(b*c^3 - 3*b*c^2 + 3*b*c - b
)*d*f^2*cosh(1) - (b*c^4 - 4*b*c^3 + 6*b*c^2 - 4*b*c + b)*f^3 + 6*(2*(b*c -
 b)*d^3*cosh(1) - (b*c^2 - 2*b*c + b)*d^2*f)*sinh(1)^2 + 4*(3*(b*c - b)*d^3
*cosh(1)^2 - 3*(b*c^2 - 2*b*c + b)*d^2*f*cosh(1) + (b*c^3 - 3*b*c^2 + 3*b*c
 - b)*d*f^2)*sinh(1))*log(d*x + c - 1) + 3*(b*d^4*f^3*x^4 + 4*b*d^4*f^2*x^3
*cosh(1) + 6*b*d^4*f*x^2*cosh(1)^2 + 4*b*d^4*x*cosh(1)^3 + 4*b*d^4*x*sinh(1
)^3 + 6*(b*d^4*f*x^2 + 2*b*d^4*x*cosh(1))*sinh(1)^2 + 4*(b*d^4*f^2*x^3 + 3*
b*d^4*f*x^2*cosh(1) + 3*b*d^4*x*cosh(1)^2)*sinh(1))*log((d*x + c + 1)/(d*x
+ c - 1)) + 12*(2*a*d^4*f^2*x^3 + b*d^3*f^2*x^2 + 6*a*d^4*x*cosh(1)^2 - 4*b
*c*d^2*f^2*x + 6*(a*d^4*f*x^2 + b*d^3*f*x)*cosh(1))*sinh(1))/d^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(151) = 302$.
time = 1.95, size = 644, normalized size = 3.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b
*c**4*f**3*acoth(c + d*x)/(4*d**4) + b*c**3*e*f**2*acoth(c + d*x)/d**3 - b*
c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*acoth(c + d*x)/d**4 - 3*b*c
**2*e**2*f*acoth(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)/d**
3 - 3*b*c**2*e*f**2*acoth(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3*b*c*
*2*f**3*acoth(c + d*x)/(2*d**4) + b*c*e**3*acoth(c + d*x)/d - 3*b*c*e**2*f*
log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*acoth(c + d*x)/d**2 - 2*b*c*e*f**2*x
/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*acoth(c + d*x)/d**3 - b*c*f**
3*log(c/d + x + 1/d)/d**4 + b*c*f**3*acoth(c + d*x)/d**4 + b*e**3*x*acoth(c
 + d*x) + 3*b*e**2*f*x**2*acoth(c + d*x)/2 + b*e*f**2*x**3*acoth(c + d*x) +
 b*f**3*x**4*acoth(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*e**3*acoth(
c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/(12*d)
- 3*b*e**2*f*acoth(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x + 1/d)/d**3 - b
*e*f**2*acoth(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*acoth(c + d*x)/(4*
d**4), Ne(d, 0)), ((a + b*acoth(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3
 + f**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2333 vs. $2(156) = 312$.
time = 0.45, size = 2333, normalized size = 13.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*((c + 1)*d - (c - 1)*d)*(3*((d*x + c + 1)^3*b*d^3*e^3/(d*x + c - 1)^3 -
 3*(d*x + c + 1)^2*b*d^3*e^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*d^3*e^3/(d
*x + c - 1) - b*d^3*e^3 - 3*(d*x + c + 1)^3*b*c*d^2*e^2*f/(d*x + c - 1)^3 +
 9*(d*x + c + 1)^2*b*c*d^2*e^2*f/(d*x + c - 1)^2 - 9*(d*x + c + 1)*b*c*d^2*
e^2*f/(d*x + c - 1) + 3*b*c*d^2*e^2*f + 3*(d*x + c + 1)^3*b*c^2*d*e*f^2/(d*
x + c - 1)^3 - 9*(d*x + c + 1)^2*b*c^2*d*e*f^2/(d*x + c - 1)^2 + 9*(d*x + c
 + 1)*b*c^2*d*e*f^2/(d*x + c - 1) - 3*b*c^2*d*e*f^2 - (d*x + c + 1)^3*b*c^3
*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c^3*f^3/(d*x + c - 1)^2 - 3*(d*x
 + c + 1)*b*c^3*f^3/(d*x + c - 1) + b*c^3*f^3 + 3*(d*x + c + 1)^3*b*d^2*e^2
*f/(d*x + c - 1)^3 - 6*(d*x + c + 1)^2*b*d^2*e^2*f/(d*x + c - 1)^2 + 3*(d*x
 + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 6*(d*x + c + 1)^3*b*c*d*e*f^2/(d*x +
c - 1)^3 + 12*(d*x + c + 1)^2*b*c*d*e*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)
*b*c*d*e*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*c^2*f^3/(d*x + c - 1)^3 -
6*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c - 1)^2 + 3*(d*x + c + 1)*b*c^2*f^3/(d*
x + c - 1) + 3*(d*x + c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 3*(d*x + c + 1)^
2*b*d*e*f^2/(d*x + c - 1)^2 + (d*x + c + 1)*b*d*e*f^2/(d*x + c - 1) - b*d*e
*f^2 - 3*(d*x + c + 1)^3*b*c*f^3/(d*x + c - 1)^3 + 3*(d*x + c + 1)^2*b*c*f^
3/(d*x + c - 1)^2 - (d*x + c + 1)*b*c*f^3/(d*x + c - 1) + b*c*f^3 + (d*x +
c + 1)^3*b*f^3/(d*x + c - 1)^3 + (d*x + c + 1)*b*f^3/(d*x + c - 1))*log((d*
```

x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^4*d^5/(d*x + c - 1)^4 - 4*(d*x + c
 + 1)^3*d^5/(d*x + c - 1)^3 + 6*(d*x + c + 1)^2*d^5/(d*x + c - 1)^2 - 4*(d*
x + c + 1)*d^5/(d*x + c - 1) + d^5) + (6*(d*x + c + 1)^3*a*d^3*e^3/(d*x + c
 - 1)^3 - 18*(d*x + c + 1)^2*a*d^3*e^3/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a
*d^3*e^3/(d*x + c - 1) - 6*a*d^3*e^3 - 18*(d*x + c + 1)^3*a*c*d^2*e^2*f/(d*
x + c - 1)^3 + 54*(d*x + c + 1)^2*a*c*d^2*e^2*f/(d*x + c - 1)^2 - 54*(d*x +
 c + 1)*a*c*d^2*e^2*f/(d*x + c - 1) + 18*a*c*d^2*e^2*f + 18*(d*x + c + 1)^3
*a*c^2*d*e*f^2/(d*x + c - 1)^3 - 54*(d*x + c + 1)^2*a*c^2*d*e*f^2/(d*x + c
- 1)^2 + 54*(d*x + c + 1)*a*c^2*d*e*f^2/(d*x + c - 1) - 18*a*c^2*d*e*f^2 -
6*(d*x + c + 1)^3*a*c^3*f^3/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*a*c^3*f^3/
(d*x + c - 1)^2 - 18*(d*x + c + 1)*a*c^3*f^3/(d*x + c - 1) + 6*a*c^3*f^3 +
18*(d*x + c + 1)^3*a*d^2*e^2*f/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*d^2*e
^2*f/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a*d^2*e^2*f/(d*x + c - 1) + 9*(d*x
+ c + 1)^3*b*d^2*e^2*f/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*d^2*e^2*f/(d*
x + c - 1)^2 + 27*(d*x + c + 1)*b*d^2*e^2*f/(d*x + c - 1) - 9*b*d^2*e^2*f -
 36*(d*x + c + 1)^3*a*c*d*e*f^2/(d*x + c - 1)^3 + 72*(d*x + c + 1)^2*a*c*d*
e*f^2/(d*x + c - 1)^2 - 36*(d*x + c + 1)*a*c*d*e*f^2/(d*x + c - 1) - 18*(d*
x + c + 1)^3*b*c*d*e*f^2/(d*x + c - 1)^3 + 54*(d*x + c + 1)^2*b*c*d*e*f^2/(
d*x + c - 1)^2 - 54*(d*x + c + 1)*b*c*d*e*f^2/(d*x + c - 1) + 18*b*c*d*e*f^
2 + 18*(d*x + c + 1)^3*a*c^2*f^3/(d*x + c - 1)^3 - 36*(d*x + c + 1)^2*a*c^2
*f^3/(d*x + c - 1)^2 + 18*(d*x + c + 1)*a*c^2*f^3/(d*x + c - 1) + 9*(d*x +
c + 1)^3*b*c^2*f^3/(d*x + c - 1)^3 - 27*(d*x + c + 1)^2*b*c^2*f^3/(d*x + c
- 1)^2 + 27*(d*x + c + 1)*b*c^2*f^3/(d*x + c - 1) - 9*b*c^2*f^3 + 18*(d*x +
 c + 1)^3*a*d*e*f^2/(d*x + c - 1)^3 - 18*(d*x + c + 1)^2*a*d*e*f^2/(d*x + c
 - 1)^2 + 6*(d*x + c + 1)*a*d*e*f^2/(d*x + c - 1) - 6*a*d*e*f^2 + 6*(d*x +
c + 1)^3*b*d*e*f^2/(d*x + c - 1)^3 - 12*(d*x + c + 1)^2*b*d*e*f^2/(d*x + c
- 1)^2 + 6*(d*x + c + 1)*b*d*e*f^2/(d*x + c - 1) - 18*(d*x + c + 1)^3*a*c*f
^3/(d*x + c - 1)^3 + 18*(d*x + c + 1)^2*a*c*f^3/(d*x + c - 1)^2 - 6*(d*x +
c + 1)*a*c*f^3/(d*x + c - 1) + 6*a*c*f^3 - 6*(d*x + c + 1)^3*b*c*f^3/(d*x +
 c - 1)^3 + 12*(d*x + c + 1)^2*b*c*f^3/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*
c*f^3/(d*x + c - 1) + 6*(d*x + c + 1)^3*a*f^3/(d*x + c - 1)^3 + 6*(d*x + c
+ 1)*a*f^3/(d*x + c - 1) + 3*(d*x + c + 1)^3*b*f^3/(d*x + c - 1)^3 - 6*(d*x
 + c + 1)^2*b*f^3/(d*x + c - 1)^2 + 5*(d*x + c + 1)*b*f^3/(d*x + c - 1) - 2
*b*f^3)/((d*x + c + 1)^4*d^5/(d*x + c - 1)^4 - 4*(d*x + c + 1)^3*d^5/(d*x +
 c - 1)^3 + 6*(d*x + c + 1)^2*d^5/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^5/(d*
x + c - 1) + d^5) - 3*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^
3*f^3 + b*d*e*f^2 - b*c*f^3)*log((d*x + c + 1)/(d*x + c - 1) - 1)/d^5 + 3*(
b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 + b*d*e*f^2 - b*c
*f^3)*log((d*x + c + 1)/(d*x + c - 1))/d^5)

Mupad [B]
time = 2.17, size = 742, normalized size = 4.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a + b*acoth(c + d*x)),x)

[Out] x*((e*(6*a*c^2*f^2 - 6*a*f^2 + 2*a*d^2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f))/(2*
d^2) - ((4*c^2 - 4)*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d
))/(4*d^2) + (2*c*((2*c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^
3)/d))/d - (4*a*c^2*f^3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d
*e*f^2)/(4*d^2) + (a*f^3*(4*c^2 - 4))/(4*d^2)))/d) - log(1 - 1/(c + d*x))*(
(b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*b*e^2*f*x^2)/4 + (b*e*f^2*x^3)/2) - x^2*((
c*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(4*d) - (2*a*c*f^3)/d))/d - (4*a*c^2*f^
3 - 4*a*f^3 + 4*b*d*e*f^2 + 12*a*d^2*e^2*f + 24*a*c*d*e*f^2)/(8*d^2) + (a*f
^3*(4*c^2 - 4))/(8*d^2)) + x^3*((f^2*(b*f + 8*a*c*f + 12*a*d*e))/(12*d) - (
2*a*c*f^3)/(3*d)) + log(1/(c + d*x) + 1)*((b*f^3*x^4)/8 + (b*e^3*x)/2 + (3*
b*e^2*f*x^2)/4 + (b*e*f^2*x^3)/2) + (a*f^3*x^4)/4 + (log(c + d*x - 1)*(b*f^
3 + 6*b*c^2*f^3 - 4*b*c^3*f^3 + 4*b*d^3*e^3 + b*c^4*f^3 - 4*b*c*f^3 + 4*b*d
*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f - 12*b*c*d^2*e^2*f + 12*b*c^2*d*e*f^
2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))/(8*d^4) - (log(c
 + d*x + 1)*(b*f^3 + 6*b*c^2*f^3 + 4*b*c^3*f^3 - 4*b*d^3*e^3 + b*c^4*f^3 +
4*b*c*f^3 - 4*b*d*e*f^2 - 4*b*c*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*c*d^2*e^2*f
- 12*b*c^2*d*e*f^2 - 4*b*c^3*d*e*f^2 + 6*b*c^2*d^2*e^2*f - 12*b*c*d*e*f^2))
/(8*d^4)
```

3.103 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) dx$

Optimal. Leaf size=120

$$\frac{bf(de-cf)x}{d^2}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}+\frac{b(de+f-cf)^3\log(1-c-dx)}{6d^3f}-\frac{b(de-$$

```
[Out] b*f*(-c*f+d*e)*x/d^2+1/6*b*f^2*(d*x+c)^2/d^3+1/3*(f*x+e)^3*(a+b*arccoth(d*x
+c))/f+1/6*b*(-c*f+d*e+f)^3*ln(-d*x-c+1)/d^3/f-1/6*b*(d*e-(1+c)*f)^3*ln(d*x
+c+1)/d^3/f
```

Rubi [A]
time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)}{3f}+\frac{b(-cf+de+f)^3\log(-c-dx+1)}{6d^3f}-\frac{b(de-(c+1)f)^3\log(c+dx+1)}{6d^3f}+\frac{bf^2(c+dx)^2}{6d^3}+\frac{bfx(de-cf)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*(d*e - c*f)*x)/d^2 + (b*f^2*(c + d*x)^2)/(6*d^3) + ((e + f*x)^3*(a + b
*ArcCoth[c + d*x]))/(3*f) + (b*(d*e + f - c*f)^3*Log[1 - c - d*x])/(6*d^3*f
) - (b*(d*e - (1 + c)*f)^3*Log[1 + c + d*x])/(6*d^3*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\coth^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^3}{1-x^2} dx, x, c \cdots\right.}{3f}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} - \frac{b\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)}{d^3} - \frac{f^3x}{d^3} + \cdots\right.\right.}{}$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} \cdots$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} \cdots$$

$$= \frac{bf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2}{6d^3} + \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)}{3f} \cdots$$

Mathematica [A]
time = 0.10, size = 174, normalized size = 1.45

$$\frac{2d(3ad^2e^2+bf(3de-2cf))x+d^2f(6ade+bf)x^2+2ad^3f^2x^3+2bd^3x(3e^2+3efx+f^2x^2)\coth^{-1}(c+dx)-b(-1+c)\left(3d^2e^2-3(-1+c)def+(-1+c)^2f^2\right)\log(1-c-dx)+b(1+c)\left(3d^2e^2-3(1+c)def+(1+c)^2f^2\right)\log(1+c+dx)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (2*d*(3*a*d^2*e^2 + b*f*(3*d*e - 2*c*f))*x + d^2*f*(6*a*d*e + b*f)*x^2 + 2*
a*d^3*f^2*x^3 + 2*b*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c + d*x] - b*
(-1 + c)*(3*d^2*e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] +
 b*(1 + c)*(3*d^2*e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/
(6*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(112) = 224$.

time = 0.15, size = 590, normalized size = 4.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b*ln(d*x+c-1)*e^2+1/2*b*ln(d*x+c+1)*e^2-b/d*f*ln(d*x+c-1)*c*e-1/2*
b/d*f*ln(d*x+c+1)*c^2*e-b/d*f*ln(d*x+c+1)*c*e+b/d*f*arccoth(d*x+c)*c^2*e+b/
d*f*arccoth(d*x+c)*e*(d*x+c)^2-b/d^2*f^2*arccoth(d*x+c)*c*(d*x+c)^2+b/d^2*f
^2*arccoth(d*x+c)*c^2*(d*x+c)+1/2*b/d*f*ln(d*x+c-1)*c^2*e+1/6*b/d^2*f^2*ln(
d*x+c+1)+1/6*b/d^2*f^2*ln(d*x+c-1)-b*arccoth(d*x+c)*c*e^2+b*arccoth(d*x+c)*
e^2*(d*x+c)-1/2*b*ln(d*x+c-1)*c*e^2+1/2*b*ln(d*x+c+1)*c*e^2-1/3*(c*f-d*e-f*
(d*x+c))^3*a/d^2/f+1/6*b/d^2*f^2*(d*x+c)^2-2*b/d*f*arccoth(d*x+c)*c*e*(d*x+
c)+b/d*f*e*(d*x+c)-b/d^2*f^2*c*(d*x+c)-1/3*b/d^2*f^2*arccoth(d*x+c)*c^3+1/3
*b/d^2*f^2*arccoth(d*x+c)*(d*x+c)^3+1/6*b*d/f*ln(d*x+c-1)*e^3+1/2*b/d*f*ln(
d*x+c-1)*e-1/6*b*d/f*ln(d*x+c+1)*e^3-1/2*b/d*f*ln(d*x+c+1)*e+1/6*b/d^2*f^2*
ln(d*x+c+1)*c^3+1/2*b/d^2*f^2*ln(d*x+c+1)*c^2+1/2*b/d^2*f^2*ln(d*x+c+1)*c-1
/6*b/d^2*f^2*ln(d*x+c-1)*c^3+1/2*b/d^2*f^2*ln(d*x+c-1)*c^2-1/2*b/d^2*f^2*ln
(d*x+c-1)*c+1/3*b*d/f*arccoth(d*x+c)*e^3)
```

Maxima [A]

time = 0.28, size = 207, normalized size = 1.72

$$\frac{1}{3}\,af^2x^3 + afx^2e + \frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\left(dx+c\right) + d\left(\frac{dx^2-4\,cx}{d^3} + \frac{\left(c^3+3\,c^2+3\,c+1\right)\log\left(dx+c+1\right)}{d^4} - \frac{\left(c^3-3\,c^2+3\,c-1\right)\log\left(dx+c-1\right)}{d^4}\right)\right)bf^2 + \frac{1}{2}\left(2\,x^2\,\mathrm{arcoth}\left(dx+c\right) + d\left(\frac{2\,x}{d^2} - \frac{\left(c^2+2\,c+1\right)\log\left(dx+c+1\right)}{d^3} + \frac{\left(c^2-2\,c+1\right)\log\left(dx+c-1\right)}{d^3}\right)\right)bfe + axe^2 + \frac{\left(2\left(dx+c\right)\mathrm{arcoth}\left(dx+c\right) + \log\left(-\left(dx+c\right)^2+1\right)\right)be^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*f^2*x^3 + a*f*x^2*e + 1/6*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x
)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c -
 1)*log(d*x + c - 1)/d^4))*b*f^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2
 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/
d^3))*b*f*e + a*x*e^2 + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^
2 + 1))*b*e^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(119) = 238$.

time = 0.42, size = 420, normalized size = 3.50

$$\frac{2\,ad^3f^2x^3 + bd^2f^2x^2 + 6\,ad^3x\cosh\left(1\right)^2 + 6\,ad^3x\sinh\left(1\right)^2 - 4\,bcdf^2x + 6\left(ad^3fx^2 + bd^2fx\right)\cosh\left(1\right) + \left(3\left(bc+b\right)d^2\cosh\left(1\right)^2 + 3\left(bc+b\right)d^2\sinh\left(1\right)^2 - 3\left(bc^2+2\,bc+b\right)df\cosh\left(1\right) + \left(bc^3+3\,bc^2+3\,bc+b\right)f^2 + 3\left(2\left(bc+b\right)d^2\cosh\left(1\right) - \left(bc^2+2\,bc+b\right)df\right)\sinh\left(1\right)\right)\log\left(dx+c+1\right) - \left(3\left(bc-b\right)d^2\cosh\left(1\right)^2 + 3\left(bc-b\right)d^2\sinh\left(1\right)^2 - 3\left(bc^2-2\,bc+b\right)df\cosh\left(1\right) + \left(bc^3-3\,bc^2+3\,bc-b\right)f^2 + 3\left(2\left(bc-b\right)d^2\cosh\left(1\right) - \left(bc^2-2\,bc+b\right)df\right)\sinh\left(1\right)\right)\log\left(dx+c-1\right) + \left(bd^3f^2x^3 + 3\,bd^3fx^2\cosh\left(1\right) + 3\,bd^3x\cosh\left(1\right)^2 + 3\,bd^3x\sinh\left(1\right)^2 + 3\left(bd^3fx^2 + 2\,bd^3x\cosh\left(1\right)\right)\sinh\left(1\right)\right)\log\left(\frac{dx+c+1}{dx+c-1}\right) + 6\left(ad^3fx^2 + 2\,ad^3x\cosh\left(1\right) + bd^2fx\right)\sinh\left(1\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*f^2*x^3 + b*d^2*f^2*x^2 + 6*a*d^3*x*cosh(1)^2 + 6*a*d^3*x*sinh
(1)^2 - 4*b*c*d*f^2*x + 6*(a*d^3*f*x^2 + b*d^2*f*x)*cosh(1) + (3*(b*c + b)*
```

```
d^2*cosh(1)^2 + 3*(b*c + b)*d^2*sinh(1)^2 - 3*(b*c^2 + 2*b*c + b)*d*f*cosh(
1) + (b*c^3 + 3*b*c^2 + 3*b*c + b)*f^2 + 3*(2*(b*c + b)*d^2*cosh(1) - (b*c^
2 + 2*b*c + b)*d*f)*sinh(1))*log(d*x + c + 1) - (3*(b*c - b)*d^2*cosh(1)^2
+ 3*(b*c - b)*d^2*sinh(1)^2 - 3*(b*c^2 - 2*b*c + b)*d*f*cosh(1) + (b*c^3 -
3*b*c^2 + 3*b*c - b)*f^2 + 3*(2*(b*c - b)*d^2*cosh(1) - (b*c^2 - 2*b*c + b)
*d*f)*sinh(1))*log(d*x + c - 1) + (b*d^3*f^2*x^3 + 3*b*d^3*f*x^2*cosh(1) +
3*b*d^3*x*cosh(1)^2 + 3*b*d^3*x*sinh(1)^2 + 3*(b*d^3*f*x^2 + 2*b*d^3*x*cosh
(1))*sinh(1))*log((d*x + c + 1)/(d*x + c - 1)) + 6*(a*d^3*f*x^2 + 2*a*d^3*x
*cosh(1) + b*d^2*f*x)*sinh(1))/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(105) = 210.
time = 1.33, size = 369, normalized size = 3.08

$$\begin{cases} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2\operatorname{acoth}(c+dx)}{3d^3} - \frac{bc^2ef\operatorname{acoth}(c+dx)}{d^2} + \frac{bc^2f^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^3} - \frac{bc^2f^2\operatorname{acoth}(c+dx)}{d^3} + \frac{bce^2\operatorname{acoth}(c+dx)}{d} - \frac{2bcef\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{2bcef\operatorname{acoth}(c+dx)}{d^2} - \frac{2bcf^2x}{3d^2} + \frac{bcf^2\operatorname{acoth}(c+dx)}{d^3} + be^2x\operatorname{acoth}(c+dx) + befx^2\operatorname{acoth}(c+dx) + \frac{bf^2x^3\operatorname{acoth}(c+dx)}{3} + \frac{be^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{be^2\operatorname{acoth}(c+dx)}{d} + \frac{befx}{d} + \frac{bf^2x^2}{6d} - \frac{bef\operatorname{acoth}(c+dx)}{d^2} + \frac{bf^2\log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{3d^3} - \frac{bf^2\operatorname{acoth}(c+dx)}{3d^3} & \text{for } d \neq 0 \\ (a + b\operatorname{acoth}(c))\left(e^2x + efx^2 + \frac{f^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e**2*x + a*e*f*x**2 + a*f**2*x**3/3 + b*c**3*f**2*acoth(c + d*
x)/(3*d**3) - b*c**2*e*f*acoth(c + d*x)/d**2 + b*c**2*f**2*log(c/d + x + 1/
d)/d**3 - b*c**2*f**2*acoth(c + d*x)/d**3 + b*c*e**2*acoth(c + d*x)/d - 2*b
*c*e*f*log(c/d + x + 1/d)/d**2 + 2*b*c*e*f*acoth(c + d*x)/d**2 - 2*b*c*f**2
*x/(3*d**2) + b*c*f**2*acoth(c + d*x)/d**3 + b*e**2*x*acoth(c + d*x) + b*e*
f*x**2*acoth(c + d*x) + b*f**2*x**3*acoth(c + d*x)/3 + b*e**2*log(c/d + x +
 1/d)/d - b*e**2*acoth(c + d*x)/d + b*e*f*x/d + b*f**2*x**2/(6*d) - b*e*f*a
coth(c + d*x)/d**2 + b*f**2*log(c/d + x + 1/d)/(3*d**3) - b*f**2*acoth(c +
d*x)/(3*d**3), Ne(d, 0)), ((a + b*acoth(c))*(e**2*x + e*f*x**2 + f**2*x**3/
3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(112) = 224.
time = 0.42, size = 973, normalized size = 8.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*((c + 1)*d - (c - 1)*d)*((3*(d*x + c + 1)^2*b*d^2*e^2/(d*x + c - 1)^2 -
 6*(d*x + c + 1)*b*d^2*e^2/(d*x + c - 1) + 3*b*d^2*e^2 - 6*(d*x + c + 1)^2*
b*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*b*c*d*e*f/(d*x + c - 1) - 6*b*
c*d*e*f + 3*(d*x + c + 1)^2*b*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*c
^2*f^2/(d*x + c - 1) + 3*b*c^2*f^2 + 6*(d*x + c + 1)^2*b*d*e*f/(d*x + c - 1
)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) - 6*(d*x + c + 1)^2*b*c*f^2/(d*
x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^2/(d*x + c - 1) + 3*(d*x + c + 1)^2*b*
```

```
f^2/(d*x + c - 1)^2 + b*f^2)*log((d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1
)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c - 1)^2 + 3*(d*x +
c + 1)*d^4/(d*x + c - 1) - d^4) + 2*(3*(d*x + c + 1)^2*a*d^2*e^2/(d*x + c -
 1)^2 - 6*(d*x + c + 1)*a*d^2*e^2/(d*x + c - 1) + 3*a*d^2*e^2 - 6*(d*x + c
+ 1)^2*a*c*d*e*f/(d*x + c - 1)^2 + 12*(d*x + c + 1)*a*c*d*e*f/(d*x + c - 1)
 - 6*a*c*d*e*f + 3*(d*x + c + 1)^2*a*c^2*f^2/(d*x + c - 1)^2 - 6*(d*x + c +
 1)*a*c^2*f^2/(d*x + c - 1) + 3*a*c^2*f^2 + 6*(d*x + c + 1)^2*a*d*e*f/(d*x
+ c - 1)^2 - 6*(d*x + c + 1)*a*d*e*f/(d*x + c - 1) + 3*(d*x + c + 1)^2*b*d*
e*f/(d*x + c - 1)^2 - 6*(d*x + c + 1)*b*d*e*f/(d*x + c - 1) + 3*b*d*e*f - 6
*(d*x + c + 1)^2*a*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*a*c*f^2/(d*x + c
 - 1) - 3*(d*x + c + 1)^2*b*c*f^2/(d*x + c - 1)^2 + 6*(d*x + c + 1)*b*c*f^2
/(d*x + c - 1) - 3*b*c*f^2 + 3*(d*x + c + 1)^2*a*f^2/(d*x + c - 1)^2 + a*f^
2 + (d*x + c + 1)^2*b*f^2/(d*x + c - 1)^2 - (d*x + c + 1)*b*f^2/(d*x + c -
1))/((d*x + c + 1)^3*d^4/(d*x + c - 1)^3 - 3*(d*x + c + 1)^2*d^4/(d*x + c -
 1)^2 + 3*(d*x + c + 1)*d^4/(d*x + c - 1) - d^4) - (3*b*d^2*e^2 - 6*b*c*d*e
*f + 3*b*c^2*f^2 + b*f^2)*log((d*x + c + 1)/(d*x + c - 1) - 1)/d^4 + (3*b*d
^2*e^2 - 6*b*c*d*e*f + 3*b*c^2*f^2 + b*f^2)*log((d*x + c + 1)/(d*x + c - 1)
)/d^4)
```

Mupad [B]
time = 1.93, size = 386, normalized size = 3.22

$$x^2\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{6\,d}-\frac{a\,c\,f^2}{d}\right)-\ln\left(1-\frac{1}{c+d\,x}\right)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)-x\left(\frac{2\,c\left(\frac{f\,(b\,f+6\,a\,c\,f+6\,a\,d\,e)}{3\,d}-\frac{2\,a\,c\,f^2}{d}\right)}{d}-\frac{3\,a\,c^2\,f^2+12\,a\,c\,d\,e\,f+3\,a\,d^2\,e^2+3\,b\,d\,e\,f-3\,a\,f^2}{3\,d^2}+\frac{a\,f^2\,(3\,c^2-3)}{3\,d^2}\right)+\ln\left(\frac{1}{c+d\,x}+1\right)\left(\frac{b\,e^2\,x}{2}+\frac{b\,e\,f\,x^2}{2}+\frac{b\,f^2\,x^3}{6}\right)+\frac{a\,f^2\,x^3}{3}+\frac{\ln(c+d\,x-1)\left(\frac{b\,f^2}{6}+d\left(\frac{b\,e\,f\,c^2}{2}-b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}-\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}-\frac{b\,c^3\,f^2}{6}-\frac{b\,c\,f^2}{2}\right)}{d^3}+\frac{\ln(c+d\,x+1)\left(\frac{b\,f^2}{6}-d\left(\frac{b\,e\,f\,c^2}{2}+b\,e\,f\,c+\frac{b\,e\,f}{2}\right)+d^2\left(\frac{b\,e^2}{2}+\frac{b\,c\,e^2}{2}\right)+\frac{b\,c^2\,f^2}{2}+\frac{b\,c^3\,f^2}{6}+\frac{b\,c\,f^2}{2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acoth(c + d*x)),x)
```

```
[Out] x^2*((f*(b*f + 6*a*c*f + 6*a*d*e))/(6*d) - (a*c*f^2)/d) - log(1 - 1/(c + d*
x))*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) - x*((2*c*((f*(b*f + 6*a*
c*f + 6*a*d*e))/(3*d) - (2*a*c*f^2)/d))/d - (3*a*c^2*f^2 - 3*a*f^2 + 3*a*d^
2*e^2 + 3*b*d*e*f + 12*a*c*d*e*f)/(3*d^2) + (a*f^2*(3*c^2 - 3))/(3*d^2)) +
log(1/(c + d*x) + 1)*((b*f^2*x^3)/6 + (b*e^2*x)/2 + (b*e*f*x^2)/2) + (a*f^2
*x^3)/3 + (log(c + d*x - 1)*((b*f^2)/6 + d*((b*e*f)/2 + (b*c^2*e*f)/2 - b*c
*e*f) + d^2*((b*e^2)/2 - (b*c*e^2)/2) + (b*c^2*f^2)/2 - (b*c^3*f^2)/6 - (b*
c*f^2)/2))/d^3 + (log(c + d*x + 1)*((b*f^2)/6 - d*((b*e*f)/2 + (b*c^2*e*f)/
2 + b*c*e*f) + d^2*((b*e^2)/2 + (b*c*e^2)/2) + (b*c^2*f^2)/2 + (b*c^3*f^2)/
6 + (b*c*f^2)/2))/d^3
```

3.104 $\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$\frac{bfx}{2d}+\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}+\frac{b(de+f-cf)^2\log(1-c-dx)}{4d^2f}-\frac{b(de-(1+c)f)^2\log(1+c+dx)}{4d^2f}$$

```
[Out] 1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*arccoth(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*ln(-d
*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*ln(d*x+c+1)/d^2/f
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6247, 6064, 716, 647, 31}

$$\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f}+\frac{b(-cf+de+f)^2\log(-c-dx+1)}{4d^2f}-\frac{b(de-(c+1)f)^2\log(c+dx+1)}{4d^2f}+\frac{bfx}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] (b*f*x)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x]))/(2*f) + (b*(d*e + f
- c*f)^2*Log[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*Log[1 + c + d
*x])/(4*d^2*f)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2}{1-x^2}\,dx,x,c}{2f}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \left(-\frac{f^2}{d^2}+\frac{d^2e^2-2cdef+(}{d}\right.\right.}{}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{b\text{Subst}\left(\int \frac{d^2e^2-2cdef+(1+}{1}\right.}{}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} - \frac{(b(de+f-cf)^2)\,\text{Subs}}{4d^2}$$

$$= \frac{bfx}{2d} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)}{2f} + \frac{b(de+f-cf)^2\log(1-}{4d^2f}$$

Mathematica [A]
time = 0.04, size = 138, normalized size = 1.42

$$aex+\frac{bfx}{2d}+\frac{1}{2}afx^2+bex\coth^{-1}(c+dx)+\frac{1}{2}bfx^2\coth^{-1}(c+dx)+\frac{b(1-2c+c^2)f\log(1-c-dx)}{4d^2}+\frac{b(-1-2c-c^2)f\log(1+c+dx)}{4d^2}+\frac{be(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] a*e*x + (b*f*x)/(2*d) + (a*f*x^2)/2 + b*e*x*ArcCoth[c + d*x] + (b*f*x^2*Arc
Coth[c + d*x])/2 + (b*(1 - 2*c + c^2)*f*Log[1 - c - d*x])/(4*d^2) + (b*(-1
- 2*c - c^2)*f*Log[1 + c + d*x])/(4*d^2) + (b*e*(-((-1 + c)*Log[1 - c - d*x
]) + (1 + c)*Log[1 + c + d*x]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(89) = 178$.

time = 0.10, size = 185, normalized size = 1.91

method	result
derivativedivides	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arccoth}(dx+c)fc(dx+c)}{d}+b\,\mathrm{arccoth}(dx+c)e(dx+c)+\frac{b\,\mathrm{arccoth}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx+\cdots}{2d}}{d}$
default	$\frac{-\frac{a\left(fc(dx+c)-ed(dx+c)-\frac{f(dx+c)^2}{2}\right)}{d}-\frac{b\,\mathrm{arccoth}(dx+c)fc(dx+c)}{d}+b\,\mathrm{arccoth}(dx+c)e(dx+c)+\frac{b\,\mathrm{arccoth}(dx+c)f(dx+c)^2}{2d}+\frac{bf(dx+\cdots}{2d}}{d}$
risch	$\frac{bx(fx+2e)\ln(dx+c+1)}{4}-\frac{bf\,x^2\ln(dx+c-1)}{4}-\frac{bex\ln(dx+c-1)}{2}+\frac{af\,x^2}{2}-\frac{\ln(dx+c+1)b\,c^2f}{4d^2}+\frac{\ln(dx+c+1)bce}{2d}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)-b/d*arccoth(d*x+c)*f*c*
(d*x+c)+b*arccoth(d*x+c)*e*(d*x+c)+1/2*b/d*arccoth(d*x+c)*f*(d*x+c)^2+1/2*b
/d*f*(d*x+c)-1/2*b/d*ln(d*x+c-1)*f*c+1/2*b*ln(d*x+c-1)*e+1/4*b/d*ln(d*x+c-1
)*f-1/2*b/d*ln(d*x+c+1)*f*c+1/2*b*ln(d*x+c+1)*e-1/4*b/d*ln(d*x+c+1)*f)
```

Maxima [A]

time = 0.28, size = 111, normalized size = 1.14

$$\frac{1}{2}afx^2+\frac{1}{4}\left(2x^2\,\mathrm{arcoth}\,(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)bf+axe+\frac{(2(dx+c)\,\mathrm{arcoth}\,(dx+c)+\log(-(dx+c)^2+1))be}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*f*x^2 + 1/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*lo
g(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*b*f + a*x*e + 1
/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b*e/d
```

Fricas [A]

time = 0.34, size = 176, normalized size = 1.81

$$\frac{2\,ad^2fx^2+4\,ad^2x\cosh(1)+4\,ad^2x\sinh(1)+2\,bdfx+(2\,(bc+b)d\cosh(1)+2\,(bc+b)d\sinh(1)-(bc^2+2\,bc+b)f)\log(dx+c+1)-(2\,(bc-b)d\cosh(1)+2\,(bc-b)d\sinh(1)-(bc^2-2\,bc+b)f)\log(dx+c-1)+(bd^2fx^2+2\,bd^2x\cosh(1)+2\,bd^2x\sinh(1))\log\left(\frac{dx+c+1}{dx+c-1}\right)}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*f*x^2 + 4*a*d^2*x*cosh(1) + 4*a*d^2*x*sinh(1) + 2*b*d*f*x + (2
*(b*c + b)*d*cosh(1) + 2*(b*c + b)*d*sinh(1) - (b*c^2 + 2*b*c + b)*f)*log(d
*x + c + 1) - (2*(b*c - b)*d*cosh(1) + 2*(b*c - b)*d*sinh(1) - (b*c^2 - 2*b
*c + b)*f)*log(d*x + c - 1) + (b*d^2*f*x^2 + 2*b*d^2*x*cosh(1) + 2*b*d^2*x*
sinh(1))*log((d*x + c + 1)/(d*x + c - 1)))/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.

time = 0.80, size = 173, normalized size = 1.78

$$\begin{cases} aex + \frac{afx^2}{2} - \frac{bc^2 f \operatorname{acoth}(c+dx)}{2d^2} + \frac{bce \operatorname{acoth}(c+dx)}{d} - \frac{bcf \log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d^2} + \frac{bcf \operatorname{acoth}(c+dx)}{d^2} + bex \operatorname{acoth}(c+dx) + \frac{bfx^2 \operatorname{acoth}(c+dx)}{2} + \frac{be \log\left(\frac{c}{d}+x+\frac{1}{d}\right)}{d} - \frac{be \operatorname{acoth}(c+dx)}{d} + \frac{bfx}{2d} - \frac{bf \operatorname{acoth}(c+dx)}{2d^2} & \text{for } d \neq 0 \\ (a + b \operatorname{acoth}(c)) \left(ex + \frac{fx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c)),x)
```

```
[Out] Piecewise((a*e*x + a*f*x**2/2 - b*c**2*f*acoth(c + d*x)/(2*d**2) + b*c*e*ac
oth(c + d*x)/d - b*c*f*log(c/d + x + 1/d)/d**2 + b*c*f*acoth(c + d*x)/d**2
+ b*e*x*acoth(c + d*x) + b*f*x**2*acoth(c + d*x)/2 + b*e*log(c/d + x + 1/d)
/d - b*e*acoth(c + d*x)/d + b*f*x/(2*d) - b*f*acoth(c + d*x)/(2*d**2), Ne(d
, 0)), ((a + b*acoth(c))*(e*x + f*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(89) = 178$.

time = 0.42, size = 338, normalized size = 3.48

$$\frac{1}{2}\left((c+1)d - (c-1)d\right)\left(\frac{\left(\frac{(dx+c+1)bde}{dx+c-1} - bde - \frac{(dx+c+1)bcf}{dx+c-1} + bcf + \frac{(dx+c+1)bf}{dx+c-1}\right)\log\left(\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2 d^3}{(dx+c-1)^2} - \frac{2(dx+c+1)d^3}{dx+c-1} + d^3} + \frac{\frac{2(dx+c+1)ade}{dx+c-1} - 2ade - \frac{2(dx+c+1)acf}{dx+c-1} + 2acf + \frac{2(dx+c+1)af}{dx+c-1} + \frac{(dx+c+1)bf}{dx+c-1} - bf}{\frac{(dx+c+1)^2 d^3}{(dx+c-1)^2} - \frac{2(dx+c+1)d^3}{dx+c-1} + d^3} - \frac{(bde - bcf)\log\left(\frac{dx+c+1}{dx+c-1} - 1\right)}{d^3} + \frac{(bde - bcf)\log\left(\frac{dx+c+1}{dx+c-1}\right)}{d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*(((d*x + c + 1)*b*d*e/(d*x + c - 1) - b*d*e - (
d*x + c + 1)*b*c*f/(d*x + c - 1) + b*c*f + (d*x + c + 1)*b*f/(d*x + c - 1))
*log((d*x + c + 1)/(d*x + c - 1))/((d*x + c + 1)^2*d^3/(d*x + c - 1)^2 - 2*
(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) + (2*(d*x + c + 1)*a*d*e/(d*x + c -
1) - 2*a*d*e - 2*(d*x + c + 1)*a*c*f/(d*x + c - 1) + 2*a*c*f + 2*(d*x + c +
 1)*a*f/(d*x + c - 1) + (d*x + c + 1)*b*f/(d*x + c - 1) - b*f)/((d*x + c +
1)^2*d^3/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^3/(d*x + c - 1) + d^3) - (b*d*
e - b*c*f)*log((d*x + c + 1)/(d*x + c - 1) - 1)/d^3 + (b*d*e - b*c*f)*log((
d*x + c + 1)/(d*x + c - 1))/d^3)
```

Mupad [B]

time = 2.40, size = 136, normalized size = 1.40

$$a\,e\,x + \frac{a\,f\,x^2}{2} + \frac{b\,e\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 - 1\right)}{2\,d} - \frac{b\,f\,\operatorname{acoth}(c+d\,x)}{2\,d^2} + \frac{b\,f\,x^2\,\operatorname{acoth}(c+d\,x)}{2} + \frac{b\,f\,x}{2\,d} + b\,e\,x\,\operatorname{acoth}(c+d\,x) - \frac{b\,c^2\,f\,\operatorname{acoth}(c+d\,x)}{2\,d^2} - \frac{b\,c\,f\,\ln\left(c^2 + 2\,c\,d\,x + d^2\,x^2 - 1\right)}{2\,d^2} + \frac{b\,c\,e\,\operatorname{acoth}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acoth(c + d*x)),x)
```

```
[Out] a*e*x + (a*f*x^2)/2 + (b*e*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/(2*d) - (b*f*a
coth(c + d*x))/(2*d^2) + (b*f*x^2*acoth(c + d*x))/2 + (b*f*x)/(2*d) + b*e*x
*acoth(c + d*x) - (b*c^2*f*acoth(c + d*x))/(2*d^2) - (b*c*f*log(c^2 + d^2*x
^2 + 2*c*d*x - 1))/(2*d^2) + (b*c*e*acoth(c + d*x))/d
```

3.105 $\int \left(a+b\coth^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}$$

```
[Out] a*x+b*(d*x+c)*arccoth(d*x+c)/d+1/2*b*ln(1-(d*x+c)^2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {6239, 6022, 266}

$$ax+\frac{b\log\left(1-(c+dx)^2\right)}{2d}+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCoth[c + d*x],x]
```

```
[Out] a*x + (b*(c + d*x)*ArcCoth[c + d*x])/d + (b*Log[1 - (c + d*x)^2])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+b\coth^{-1}(c+dx))\,dx = ax+b\int \coth^{-1}(c+dx)\,dx$$
$$= ax+\frac{b\text{Subst}\left(\int \coth^{-1}(x)\,dx, x, c+dx\right)}{d}$$
$$= ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, c+dx\right)}{d}$$
$$= ax+\frac{b(c+dx)\coth^{-1}(c+dx)}{d}+\frac{b\log\left(1-(c+dx)^2\right)}{2d}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.20

$$ax+bx\coth^{-1}(c+dx)+\frac{b(-((-1+c)\log(1-c-dx))+(1+c)\log(1+c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCoth[c + d*x],x]
[Out] a*x + b*x*ArcCoth[c + d*x] + (b*(-((-1 + c)*Log[1 - c - d*x]) + (1 + c)*Log
[1 + c + d*x]))/(2*d)
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.05

method	result
derivativedivides	$\frac{(dx+c)a+b(dx+c)\operatorname{arccoth}(dx+c)+\frac{b\ln\left((dx+c)^2-1\right)}{2}}{d}$
default	$ax+b\operatorname{arccoth}(dx+c)\,x+\frac{b\operatorname{arccoth}(dx+c)c}{d}+\frac{b\ln\left((dx+c)^2-1\right)}{2d}$
risch	$ax+\frac{b\ln(dx+c+1)x}{2}-\frac{bx\ln(dx+c-1)}{2}-\frac{b\ln(dx+c-1)c}{2d}+\frac{b\ln(-dx-c-1)c}{2d}+\frac{b\ln(dx+c-1)}{2d}+\frac{b\ln(-dx-c-}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccoth(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b*arccoth(d*x+c)*x+b/d*arccoth(d*x+c)*c+1/2*b/d*ln((d*x+c)^2-1)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.90

$$ax+\frac{\left(2\,(dx+c)\operatorname{arcoth}(dx+c)+\log\left(-(dx+c)^2+1\right)\right)b}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b/d
```

Fricas [A]
time = 0.36, size = 60, normalized size = 1.50

$$\frac{bdx\log\left(\frac{dx+c+1}{dx+c-1}\right)+2\,adx+(bc+b)\log\left(dx+c+1\right)-(bc-b)\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x*log((d*x + c + 1)/(d*x + c - 1)) + 2*a*d*x + (b*c + b)*log(d*x +
 c + 1) - (b*c - b)*log(d*x + c - 1))/d
```

Sympy [A]
time = 0.23, size = 46, normalized size = 1.15

$$ax+b\left(\begin{cases}\frac{c\operatorname{acoth}\left(c+dx\right)}{d}+x\operatorname{acoth}\left(c+dx\right)+\frac{\log\left(c+dx+1\right)}{d}-\frac{\operatorname{acoth}\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\operatorname{acoth}\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acoth(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acoth(c + d*x)/d + x*acoth(c + d*x) + log(c + d*x + 1)
/d - acoth(c + d*x)/d, Ne(d, 0)), (x*acoth(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(38) = 76.
time = 0.39, size = 202, normalized size = 5.05

$$\frac{1}{2}\left((c+1)d-(c-1)d\right)b\left(\frac{\log\left(\frac{|dx+c+1|}{|dx+c-1|}\right)}{d^2}-\frac{\log\left(\left|\frac{dx+c+1}{dx+c-1}-1\right|\right)}{d^2}+\frac{\log\left(-\frac{\frac{1}{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}}+1}{\frac{1}{c-\frac{\left(\frac{(dx+c+1)(c-1)}{dx+c-1}-c-1\right)d}{\frac{(dx+c+1)d}{dx+c-1}-d}}-1}\right)}{d^2\left(\frac{dx+c+1}{dx+c-1}-1\right)}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccoth(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*((c + 1)*d - (c - 1)*d)*b*(log(abs(d*x + c + 1)/abs(d*x + c - 1))/d^2 -
 log(abs((d*x + c + 1)/(d*x + c - 1) - 1))/d^2 + log(-(1/(c - ((d*x + c + 1
)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/(d*x + c - 1) - d)) + 1
)/(1/(c - ((d*x + c + 1)*(c - 1)/(d*x + c - 1) - c - 1)*d/((d*x + c + 1)*d/
(d*x + c - 1) - d)) - 1))/(d^2*((d*x + c + 1)/(d*x + c - 1) - 1))) + a*x
```

Mupad [B]
time = 1.75, size = 48, normalized size = 1.20

$$a\,x+\frac{\frac{b\,\ln\left(c^2+2\,c\,d\,x+d^2\,x^2-1\right)}{2}+b\,c\,\mathrm{acoth}\left(c+d\,x\right)}{d}+b\,x\,\mathrm{acoth}\left(c+d\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acoth(c + d*x),x)
```

```
[Out] a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*acoth(c + d*x))/d + b*x
*acoth(c + d*x)
```

3.106 $\int \frac{a+b \coth^{-1}(c+dx)}{e+fx} \, dx$

Optimal. Leaf size=130

$$-\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{1+c+dx}\right)}{2f}$$

```
[Out] -(a+b*arccoth(d*x+c))*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))*ln(2*d*(f*x+e)
/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*polylog(2,1-2/(d*x+c+1))/f-1/2*b*polylog(2
,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6247, 6058, 2449, 2352, 2497}

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\frac{b\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c +
d*x])*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*PolyLog[
2, 1 - 2/(1 + c + d*x)])/(2*f) - (b*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e +
f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}(c+dx)}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e-\ldots}{(de+f-cf\ldots}\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e-\ldots}{(de+f-cf\ldots}\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2d(e-\ldots}{(de+f-cf\ldots}\right.}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 352, normalized size = 2.71

$$\frac{a\log(e+fx)+b\left(\coth^{-1}(c+dx)-\tanh^{-1}(c+dx)\right)\log(e+fx)+b\tanh^{-1}(c+dx)\left(-\log\left(\frac{1}{\sqrt{1-(c+dx)^2}}\right)+\log\left(i\sinh\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\right)\right)-\frac{1}{2}ib\left(-\frac{1}{4}i\left(\pi-2i\tanh^{-1}(c+dx)\right)^2+i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)^2+\left(\pi-2i\tanh^{-1}(c+dx)\right)\log\left(1+e^{2\tanh^{-1}(c+dx)}\right)+2i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\log\left(1-e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)-\left(\pi-2i\tanh^{-1}(c+dx)\right)\log\left(\frac{2}{\sqrt{1-(c+dx)^2}}\right)-2i\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\log\left(2i\sinh\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)\right)-i\,\text{PolyLog}\left(2,-e^{2\tanh^{-1}(c+dx)}\right)-i\,\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{de-cf}{f}\right)+\tanh^{-1}(c+dx)\right)}\right)\right)}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x),x]
```

```
[Out] (a*Log[e + f*x] + b*(ArcCoth[c + d*x] - ArcTanh[c + d*x])*Log[e + f*x] + b*
ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I*Sinh[ArcTanh[(d*e -
 c*f)/f] + ArcTanh[c + d*x]]]) - (I/2)*b*((-1/4*I)*(Pi - (2*I)*ArcTanh[c +
d*x])^2 + I*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])^2 + (Pi - (2*I)*Arc
```

```
Tanh[c + d*x])*Log[1 + E^(2*ArcTanh[c + d*x])] + (2*I)*(ArcTanh[(d*e - c*f)
/f] + ArcTanh[c + d*x])*Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c +
 d*x]))] - (Pi - (2*I)*ArcTanh[c + d*x])*Log[2/Sqrt[1 - (c + d*x)^2]] - (2*
I)*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e
- c*f)/f] + ArcTanh[c + d*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c + d*x])] - I*
PolyLog[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))]))/f
```

Maple [A]

time = 0.96, size = 220, normalized size = 1.69

method	result
risch	$\frac{a\ln((dx+c-1)f-cf+de+f)}{f}-\frac{b\,\mathrm{dilog}\left(\frac{(dx+c-1)f-cf+de+f}{-cf+de+f}\right)}{2f}-\frac{b\ln(dx+c-1)\ln\left(\frac{(dx+c-1)f-cf+de+f}{-cf+de+f}\right)}{2f}+\frac{b\,\mathrm{dilog}(\cdots}{}$
derivativedivides	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\mathrm{arccoth}(dx+c)}{f}+\frac{bd\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)}{2f}+\frac{bd\,\mathrm{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)}{2f}\cdots}{d}$
default	$\frac{\frac{ad\ln(cf-de-f(dx+c))}{f}+\frac{bd\ln(cf-de-f(dx+c))\mathrm{arccoth}(dx+c)}{f}+\frac{bd\ln(cf-de-f(dx+c))\ln\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)}{2f}+\frac{bd\,\mathrm{dilog}\left(\frac{-f(dx+c)+f}{-cf+de+f}\right)}{2f}\cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d*ln(c*f-d*e-f*(d*x+c))/f+b*d*ln(c*f-d*e-f*(d*x+c))/f*arccoth(d*x+c)
+1/2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))+1/2*b*d/f*
dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))-1/2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(
d*x+c)-f)/(-c*f+d*e-f))-1/2*b*d/f*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1))/(f*x + e), x
) + a*log(f*x + e)/f
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acoth}{(c + dx)}}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e),x)
[Out] Integral((a + b*acoth(c + d*x))/(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arccoth(d*x + c) + a)/(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acoth}(c + d\,x)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))/(e + f*x),x)
[Out] int((a + b*acoth(c + d*x))/(e + f*x), x)
```

3.107 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(1-c-dx)}{2f(de+f-cf)}+\frac{bd\log(1+c+dx)}{2f(de-f-cf)}-\frac{bd\log(e+fx)}{(de+f-cf)(de-(1+c)f)}$$

```
[Out] (-a-b*arccoth(d*x+c))/f/(f*x+e)-1/2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*d
*ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6245, 2007, 719, 31, 646}

$$-\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(-c-dx+1)}{2f(-cf+de+f)}+\frac{bd\log(c+dx+1)}{2f(-cf+de-f)}-\frac{bd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])/(f*(e + f*x))) - (b*d*Log[1 - c - d*x])/(2*f*(d*
e + f - c*f)) + (b*d*Log[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*Log[e +
 f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^2}\,dx = -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)(1-(c+dx)^2)}\,dx}{f}$$
$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{1}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$
$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{(bd)\int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2}\,dx}{f\left(-d^2e^2+2cdef+(1-c^2)f^2\right)}+\frac{(bdf)\int \frac{}{e}}{-d^2e^2+2cdef}$$
$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(e+fx)}{(de-f-cf)(de+f-cf)}-\frac{(bd^3)\int \frac{1}{-d-cd-d^2x}\,dx}{2f(de-f-cf)}$$
$$= -\frac{a+b\coth^{-1}(c+dx)}{f(e+fx)}-\frac{bd\log(1-c-dx)}{2f(de+f-cf)}+\frac{bd\log(1+c+dx)}{2f(de-f-cf)}-\frac{b}{(de-f}$$

Mathematica [A]
time = 0.12, size = 125, normalized size = 1.09

$$\frac{1}{2}\left(-\frac{2a}{f(e+fx)}-\frac{2b\coth^{-1}(c+dx)}{f(e+fx)}+\frac{bd\log(1-c-dx)}{f(-de+(-1+c)f)}-\frac{bd\log(1+c+dx)}{f(-de+f+cf)}-\frac{2bd\log(e+fx)}{d^2e^2-2cdef+(-1+c^2)f^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^2,x]
[Out] ((-2*a)/(f*(e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (b*d*Log[1 -
 c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*Log[1 + c + d*x])/(f*(-(d*e) +
f + c*f)) - (2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2
```

Maple [A]
time = 0.16, size = 170, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\operatorname{arccoth}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(dx+c+1)}{f(2cf-2de+2f)}+\frac{b\,d^2\ln(dx+c-1)}{f(2cf-2de-2f)}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}}{d}$
default	$\frac{\frac{a\,d^2}{(cf-de-f(dx+c))f}+\frac{b\,d^2\operatorname{arccoth}(dx+c)}{(cf-de-f(dx+c))f}-\frac{b\,d^2\ln(dx+c+1)}{f(2cf-2de+2f)}+\frac{b\,d^2\ln(dx+c-1)}{f(2cf-2de-2f)}-\frac{b\,d^2\ln(cf-de-f(dx+c))}{(cf-de-f)(cf-de+f)}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{2f(fx+e)}-\frac{\ln(dx+c+1)bcd\,f^2x-\ln(dx+c+1)b\,d^2efx-\ln(-dx-c+1)bcd\,f^2x+\ln(-dx-c+1)b\,d^2efx+\ln(dx+c+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*d^2/(c*f-d*e-f*(d*x+c))/f+b*d^2/(c*f-d*e-f*(d*x+c))/f*arccoth(d*x+c)
-b*d^2/f/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)+b*d^2/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1
)-b*d^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c)))
```

Maxima [A]
time = 0.28, size = 128, normalized size = 1.11

$$-\frac{1}{2}\left(d\left(\frac{\log(dx+c+1)}{(c+1)f^2-dfe}-\frac{\log(dx+c-1)}{(c-1)f^2-dfe}-\frac{2\log(fx+e)}{2\,cdfe-(c^2-1)f^2-d^2e^2}\right)+\frac{2\operatorname{arcoth}(dx+c)}{f^2x+fe}\right)b-\frac{a}{f^2x+fe}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)
*f^2 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*a
rccoth(d*x + c)/(f^2*x + f*e))*b - a/(f^2*x + f*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(118) = 236$.
time = 0.47, size = 516, normalized size = 4.49

$$\frac{4\,acdf\cosh(1)-2\,ad^2\cosh(1)^2-2\,ad^2\sinh(1)^2-2\,(ac^2-a)f^2-((bc-b)df^2x-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bd^2fx-(bc-b)df)\cosh(1)-(bd^2fx+2\,bd^2\cosh(1)-(bc-b)df)\sinh(1))\log(dx+c+1)+((bc+b)df^2x-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bd^2fx-(bc+b)df)\cosh(1)-(bd^2fx+2\,bd^2\cosh(1)-(bc+b)df)\sinh(1))\log(dx+c-1)-2\,(bdf^2x+bdf\cosh(1)+bdf\sinh(1))\log(fx+\cosh(1)+\sinh(1))+(2\,bcdf\cosh(1)-bd^2\cosh(1)^2-bd^2\sinh(1)^2-(bc^2-b)f^2+2\,(bcdf-bd^2\cosh(1))\sinh(1))\log\left(\frac{dx+c+1}{dx+c-1}\right)+4\,(acdf-ad^2\cosh(1))\sinh(1)}{2\,((c^2-1)f^4x+d^2f\cosh(1)^3+d^2f\sinh(1)^3+(d^2f^2x-2\,cdf^2)\cosh(1)^2+(d^2f^2x-2\,cdf^2+3\,d^2f\cosh(1))\sinh(1)^2-(2\,cdf^3x-(c^2-1)f^3)\cosh(1)-(2\,cdf^3x-3\,d^2f\cosh(1)^2-(c^2-1)f^3-2\,(d^2f^2x-2\,cdf^2)\cosh(1))\sinh(1))}$$

Verification of antiderivative is not currently implemented for this CAS.

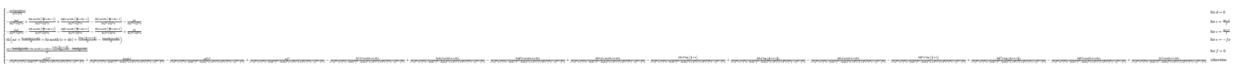
```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*c*d*f*cosh(1) - 2*a*d^2*cosh(1)^2 - 2*a*d^2*sinh(1)^2 - 2*(a*c^2 -
 a)*f^2 - ((b*c - b)*d*f^2*x - b*d^2*cosh(1)^2 - b*d^2*sinh(1)^2 - (b*d^2*f
*x - (b*c - b)*d*f)*cosh(1) - (b*d^2*f*x + 2*b*d^2*cosh(1) - (b*c - b)*d*f)
*sinh(1))*log(d*x + c + 1) + ((b*c + b)*d*f^2*x - b*d^2*cosh(1)^2 - b*d^2*s
inh(1)^2 - (b*d^2*f*x - (b*c + b)*d*f)*cosh(1) - (b*d^2*f*x + 2*b*d^2*cosh(
1) - (b*c + b)*d*f)*sinh(1))*log(d*x + c - 1) - 2*(b*d*f^2*x + b*d*f*cosh(1
) + b*d*f*sinh(1))*log(f*x + cosh(1) + sinh(1)) + (2*b*c*d*f*cosh(1) - b*d^
2*cosh(1)^2 - b*d^2*sinh(1)^2 - (b*c^2 - b)*f^2 + 2*(b*c*d*f - b*d^2*cosh(1
```

```
))*sinh(1))*log((d*x + c + 1)/(d*x + c - 1)) + 4*(a*c*d*f - a*d^2*cosh(1))*
sinh(1))/((c^2 - 1)*f^4*x + d^2*f*cosh(1)^3 + d^2*f*sinh(1)^3 + (d^2*f^2*x
- 2*c*d*f^2)*cosh(1)^2 + (d^2*f^2*x - 2*c*d*f^2 + 3*d^2*f*cosh(1))*sinh(1)^
2 - (2*c*d*f^3*x - (c^2 - 1)*f^3)*cosh(1) - (2*c*d*f^3*x - 3*d^2*f*cosh(1)^
2 - (c^2 - 1)*f^3 - 2*(d^2*f^2*x - 2*c*d*f^2)*cosh(1))*sinh(1))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(92) = 184$.

time = 4.39, size = 1658, normalized size = 14.42



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e)**2,x)
```

```
[Out] Piecewise((-(a + b*acoth(c))/(e*f + f**2*x), Eq(d, 0)), (-2*a*f/(2*e*f**2 +
 2*f**3*x) + b*d*e*acoth(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) + b*d*f*x*a
coth(d*e/f + d*x - 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*acoth(d*e/f + d*x - 1)/
(2*e*f**2 + 2*f**3*x) - b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d*e - f)/f)), (-2
*a*f/(2*e*f**2 + 2*f**3*x) - b*d*e*acoth(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**
3*x) - b*d*f*x*acoth(d*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) - 2*b*f*acoth(d
*e/f + d*x + 1)/(2*e*f**2 + 2*f**3*x) + b*f/(2*e*f**2 + 2*f**3*x), Eq(c, (d
*e + f)/f)), (zoo*(a*x + b*c*acoth(c + d*x)/d + b*x*acoth(c + d*x) + b*log(
c/d + x + 1/d)/d - b*acoth(c + d*x)/d), Eq(e, -f*x)), ((a*x + b*c*acoth(c +
 d*x)/d + b*x*acoth(c + d*x) + b*log(c/d + x + 1/d)/d - b*acoth(c + d*x)/d)
/e**2, Eq(f, 0)), (-a*c**2*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**
2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*
a*c*d*e*f/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d
**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3
 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) + a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2
*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x)
- b*c**2*f**2*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 -
 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d
*e*f*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*
f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*a
coth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x
 + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d**2*e*f*x*acoth(c
 + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**
2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*e*f*log(e/f + x)/(c**2
*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d*
*2*e**2*f**2*x - e*f**3 - f**4*x) + b*d*e*f*log(c/d + x + 1/d)/(c**2*e*f**3
 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) - b*d*e*f*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**
4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e
```

```
*f**3 - f**4*x) - b*d*f**2*x*log(e/f + x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*
d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f*
*4*x) + b*d*f**2*x*log(c/d + x + 1/d)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e*
*2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x
) - b*d*f**2*x*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2
- 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*f*
*2*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f*
*3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(112) = 224$.
time = 0.41, size = 472, normalized size = 4.10

$$-\frac{1}{2}\left((c+1)d-(c-1)d\right)\left(\frac{b\log\left(-\frac{(dx+c+1)de}{dx+c-1}+de+\frac{(dx+c+1)cf}{dx+c-1}-cf-\frac{(dx+c+1)f}{dx+c-1}-f\right)}{d^2e^2-2\,cdef+c^2f^2-f^2}-\frac{b\log\left(\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d^2e^2-\frac{2\,(dx+c+1)cdef}{dx+c-1}+2\,cdef+\frac{(dx+c+1)c^2f^2}{dx+c-1}-c^2f^2+\frac{2\,(dx+c+1)def}{dx+c-1}-\frac{2\,(dx+c+1)cf^2}{dx+c-1}+\frac{(dx+c+1)f^2}{dx+c-1}+f^2}-\frac{b\log\left(\frac{dx+c+1}{dx+c-1}\right)}{d^2e^2-2\,cdef+c^2f^2-f^2}-\frac{2\,a}{\frac{(dx+c+1)d^2e^2}{dx+c-1}-d^2e^2-\frac{2\,(dx+c+1)cdef}{dx+c-1}+2\,cdef+\frac{(dx+c+1)c^2f^2}{dx+c-1}-c^2f^2+\frac{2\,(dx+c+1)def}{dx+c-1}-\frac{2\,(dx+c+1)cf^2}{dx+c-1}+\frac{(dx+c+1)f^2}{dx+c-1}+f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*((c + 1)*d - (c - 1)*d)*(b*log(-(d*x + c + 1)*d*e/(d*x + c - 1) + d*e
+ (d*x + c + 1)*c*f/(d*x + c - 1) - c*f - (d*x + c + 1)*f/(d*x + c - 1) - f
)/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - b*log((d*x + c + 1)/(d*x + c - 1)
)/((d*x + c + 1)*d^2*e^2/(d*x + c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f/
(d*x + c - 1) + 2*c*d*e*f + (d*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2 +
 2*(d*x + c + 1)*d*e*f/(d*x + c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1)
+ (d*x + c + 1)*f^2/(d*x + c - 1) + f^2) - b*log((d*x + c + 1)/(d*x + c - 1
))/(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - f^2) - 2*a/((d*x + c + 1)*d^2*e^2/(d*x
+ c - 1) - d^2*e^2 - 2*(d*x + c + 1)*c*d*e*f/(d*x + c - 1) + 2*c*d*e*f + (d
*x + c + 1)*c^2*f^2/(d*x + c - 1) - c^2*f^2 + 2*(d*x + c + 1)*d*e*f/(d*x +
c - 1) - 2*(d*x + c + 1)*c*f^2/(d*x + c - 1) + (d*x + c + 1)*f^2/(d*x + c -
 1) + f^2))
```

Mupad [**B**]
time = 2.08, size = 175, normalized size = 1.52

$$\ln\left(e+f\,x\right)\,\left(\frac{b\,\left(c-1\right)}{2\,e\,\left(d\,e-f\,\left(c-1\right)\right)}-\frac{b\,\left(c+1\right)}{2\,e\,\left(d\,e-f\,\left(c+1\right)\right)}\right)-\frac{a}{x\,f^2+e\,f}-\frac{b\,\ln\left(\frac{1}{c+d\,x}+1\right)}{2\,f\,\left(e+f\,x\right)}-\frac{b\,d\,\ln\left(c+d\,x-1\right)}{2\,f^2-2\,c\,f^2+2\,d\,e\,f}-\frac{b\,d\,\ln\left(c+d\,x+1\right)}{2\,c\,f^2+2\,f^2-2\,d\,e\,f}+\frac{b\,\ln\left(1-\frac{1}{c+d\,x}\right)}{f\,\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))/(e + f*x)^2,x)
```

```
[Out] log(e + f*x)*((b*(c - 1))/(2*e*(d*e - f*(c - 1))) - (b*(c + 1))/(2*e*(d*e -
 f*(c + 1)))) - a/(e*f + f^2*x) - (b*log(1/(c + d*x) + 1))/(2*f*(e + f*x))
- (b*d*log(c + d*x - 1))/(2*f^2 - 2*c*f^2 + 2*d*e*f) - (b*d*log(c + d*x + 1
))/(2*c*f^2 + 2*f^2 - 2*d*e*f) + (b*log(1 - 1/(c + d*x)))/(f*(2*e + 2*f*x))
```

3.108 $\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx$

Optimal. Leaf size=167

$$\frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(1-c-dx)}{4f(de+f-cf)^2}+\frac{bd^2\log(1+c+dx)}{4f(de-f-cf)^2}-$$

```
[Out] 1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*arccoth(d*x+c))/f/(f*x+
e)^2-1/4*b*d^2*ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*ln(d*x+c+1)/f/(-c*f+
d*e-f)^2-b*d^2*(-c*f+d*e)*ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6245, 2007, 723, 814}

$$-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(-c-dx+1)}{4f(-cf+de+f)^2}+\frac{bd^2\log(c+dx+1)}{4f(-cf+de-f)^2}-\frac{bd^2(de-cf)\log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2}+\frac{bd}{2(e+fx)(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

```
[Out] (b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*ArcCoth[c +
d*x])/(2*f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x])/(4*f*(d*e + f - c*f)^2)
+ (b*d^2*Log[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*Log
[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
```

```
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)^3}\,dx &= -\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2(1-(c+dx)^2)}\,dx}{2f}\\ &= -\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{1}{(e+fx)^2(1-c^2-2cdx-d^2x^2)}\,dx}{2f}\\ &= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int \frac{-}{(e+f}}{2f\left(-d^2e^2+\right.}\\ &= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}+\frac{(bd)\int\left(\frac{d^2}{2(de+}\right.}{}\\ &= \frac{bd}{2(de+f-cf)(de-(1+c)f)(e+fx)}-\frac{a+b\coth^{-1}(c+dx)}{2f(e+fx)^2}-\frac{bd^2\log(1-}{4f(de+f}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 174, normalized size = 1.04

$$\frac{1}{4}\left(-\frac{2a}{f(e+fx)^2}+\frac{2bd}{\left(d^2e^2-2cdef+(-1+c^2)f^2\right)(e+fx)}-\frac{2b\coth^{-1}(c+dx)}{f(e+fx)^2}-\frac{bd^2\log(1-c-dx)}{f(de+f-cf)^2}+\frac{bd^2\log(1+c+dx)}{f(-de+f+cf)^2}-\frac{4bd^2(de-cf)\log(e+fx)}{\left(d^2e^2-2cdef+(-1+c^2)f^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^3,x]
```

```
[Out] ((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(
e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x
])/(f*(d*e + f - c*f)^2) + (b*d^2*Log[1 + c + d*x])/(f*(-(d*e) + f + c*f)^2
) - (4*b*d^2*(d*e - c*f)*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^
2)^2)/4
```

Maple [A]
time = 0.21, size = 266, normalized size = 1.59

method	result
derivativedivides	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2f}-\frac{b\,d^3\operatorname{arccoth}(dx+c)}{2(cf-de-f(dx+c))^2f}-\frac{b\,d^3\ln(dx+c-1)}{4f(cf-de-f)^2}+\frac{b\,d^3\ln(dx+c+1)}{4f(cf-de+f)^2}-\frac{b\,d^3}{2(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}}{d}$
default	$\frac{-\frac{a\,d^3}{2(cf-de-f(dx+c))^2f}-\frac{b\,d^3\operatorname{arccoth}(dx+c)}{2(cf-de-f(dx+c))^2f}-\frac{b\,d^3\ln(dx+c-1)}{4f(cf-de-f)^2}+\frac{b\,d^3\ln(dx+c+1)}{4f(cf-de+f)^2}-\frac{b\,d^3}{2(cf-de-f)(cf-de+f)(cf-de-f(dx+c))}}{d}$
risch	$-\frac{b\ln(dx+c+1)}{4f(fx+e)^2}+\frac{8a\,c^3de\,f^3-12a\,c^2d^2e^2f^2+8ac\,d^3e^3f+2b\,c^2de\,f^3-8acde\,f^3-4bc\,d^2e^2f^2+6b\,c^2d^2e^2f^2\ln(dx+c-1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a*d^3/(c*f-d*e-f*(d*x+c))^2/f-1/2*b*d^3/(c*f-d*e-f*(d*x+c))^2/f*a
rccoth(d*x+c)-1/4*b*d^3/f/(c*f-d*e-f)^2*ln(d*x+c-1)+1/4*b*d^3/f/(c*f-d*e+f)
^2*ln(d*x+c+1)-1/2*b*d^3/(c*f-d*e-f)/(c*f-d*e+f)/(c*f-d*e-f*(d*x+c))+b*d^3*
f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*c-b*d^4/(c*f-d*e-f)^2/(
c*f-d*e+f)^2*ln(c*f-d*e-f*(d*x+c))*e)
```

Maxima [A]
time = 0.28, size = 316, normalized size = 1.89

$$-\frac{1}{4}\left(d\left(\frac{d\log\left(dx+c+1\right)}{2\left(ce+e\right)df^2-\left(c^2+2\,c+1\right)f^3-d^2fe^2}-\frac{d\log\left(dx+c-1\right)}{2\left(ce-e\right)df^2-\left(c^2-2\,c+1\right)f^3-d^2fe^2}+\frac{4\left(cdf-d^2e\right)\log\left(fx+e\right)}{4\,cd^3fe^3-2\left(3\,c^2e^2-e^2\right)d^2f^2+4\left(c^3e-ce\right)df^3-\left(c^4-2\,c^2+1\right)f^4-d^4e^4}+\frac{2}{2\,cdfe^2-\left(c^2e-e\right)f^2-d^2e^3+\left(2\,cdf^2e-\left(c^2-1\right)f^3-d^2fe^2\right)x}\right)+\frac{2\,\operatorname{arcoth}\left(dx+c\right)}{f^3x^2+2\,f^2xe+fe^2}\right)b-\frac{a}{2\left(f^3x^2+2\,f^2xe+fe^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(d*(d*log(d*x + c + 1)/(2*(c*e + e)*d*f^2 - (c^2 + 2*c + 1)*f^3 - d^2*
f*e^2) - d*log(d*x + c - 1)/(2*(c*e - e)*d*f^2 - (c^2 - 2*c + 1)*f^3 - d^2*
f*e^2) + 4*(c*d*f - d^2*e)*log(f*x + e)/(4*c*d^3*f*e^3 - 2*(3*c^2*e^2 - e^2
)*d^2*f^2 + 4*(c^3*e - c*e)*d*f^3 - (c^4 - 2*c^2 + 1)*f^4 - d^4*e^4) + 2/(2
*c*d*f*e^2 - (c^2*e - e)*f^2 - d^2*e^3 + (2*c*d*f^2*e - (c^2 - 1)*f^3 - d^2
*f*e^2)*x)) + 2*arccoth(d*x + c)/(f^3*x^2 + 2*f^2*x*e + f*e^2))*b - 1/2*a/(
f^3*x^2 + 2*f^2*x*e + f*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2442 vs. $2(171) = 342$.
time = 1.09, size = 2442, normalized size = 14.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^4*cosh(1)^4 + 2*a*d^4*sinh(1)^4 - 2*(4*a*c + b)*d^3*f*cosh(1)^3
 - 2*(b*c^2 - b)*d*f^4*x + 2*(a*c^4 - 2*a*c^2 + a)*f^4 + 2*(4*a*d^4*cosh(1)
```

```
 - (4*a*c + b)*d^3*f)*sinh(1)^3 - 2*(b*d^3*f^2*x - 2*(3*a*c^2 + b*c - a)*d^
2*f^2)*cosh(1)^2 - 2*(b*d^3*f^2*x - 6*a*d^4*cosh(1)^2 + 3*(4*a*c + b)*d^3*f
*cosh(1) - 2*(3*a*c^2 + b*c - a)*d^2*f^2)*sinh(1)^2 + 2*(2*b*c*d^2*f^3*x -
(4*a*c^3 + b*c^2 - 4*a*c - b)*d*f^3)*cosh(1) - ((b*c^2 - 2*b*c + b)*d^2*f^4
*x^2 + b*d^4*cosh(1)^4 + b*d^4*sinh(1)^4 + 2*(b*d^4*f*x - (b*c - b)*d^3*f)*
cosh(1)^3 + 2*(b*d^4*f*x + 2*b*d^4*cosh(1) - (b*c - b)*d^3*f)*sinh(1)^3 + (
b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f^2*x + (b*c^2 - 2*b*c + b)*d^2*f^2)*cosh(1
)^2 + (b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f^2*x + 6*b*d^4*cosh(1)^2 + (b*c^2 -
 2*b*c + b)*d^2*f^2 + 6*(b*d^4*f*x - (b*c - b)*d^3*f)*cosh(1))*sinh(1)^2 -
2*((b*c - b)*d^3*f^3*x^2 - (b*c^2 - 2*b*c + b)*d^2*f^3*x)*cosh(1) - 2*((b*c
 - b)*d^3*f^3*x^2 - 2*b*d^4*cosh(1)^3 - (b*c^2 - 2*b*c + b)*d^2*f^3*x - 3*(
b*d^4*f*x - (b*c - b)*d^3*f)*cosh(1)^2 - (b*d^4*f^2*x^2 - 4*(b*c - b)*d^3*f
^2*x + (b*c^2 - 2*b*c + b)*d^2*f^2)*cosh(1))*sinh(1))*log(d*x + c + 1) + ((
b*c^2 + 2*b*c + b)*d^2*f^4*x^2 + b*d^4*cosh(1)^4 + b*d^4*sinh(1)^4 + 2*(b*d
^4*f*x - (b*c + b)*d^3*f)*cosh(1)^3 + 2*(b*d^4*f*x + 2*b*d^4*cosh(1) - (b*c
 + b)*d^3*f)*sinh(1)^3 + (b*d^4*f^2*x^2 - 4*(b*c + b)*d^3*f^2*x + (b*c^2 +
2*b*c + b)*d^2*f^2)*cosh(1)^2 + (b*d^4*f^2*x^2 - 4*(b*c + b)*d^3*f^2*x + 6*
b*d^4*cosh(1)^2 + (b*c^2 + 2*b*c + b)*d^2*f^2 + 6*(b*d^4*f*x - (b*c + b)*d^
3*f)*cosh(1))*sinh(1)^2 - 2*((b*c + b)*d^3*f^3*x^2 - (b*c^2 + 2*b*c + b)*d^
2*f^3*x)*cosh(1) - 2*((b*c + b)*d^3*f^3*x^2 - 2*b*d^4*cosh(1)^3 - (b*c^2 +
2*b*c + b)*d^2*f^3*x - 3*(b*d^4*f*x - (b*c + b)*d^3*f)*cosh(1)^2 - (b*d^4*f
^2*x^2 - 4*(b*c + b)*d^3*f^2*x + (b*c^2 + 2*b*c + b)*d^2*f^2)*cosh(1))*sinh
(1))*log(d*x + c - 1) - 4*(b*c*d^2*f^4*x^2 - b*d^3*f*cosh(1)^3 - b*d^3*f*si
nh(1)^3 - (2*b*d^3*f^2*x - b*c*d^2*f^2)*cosh(1)^2 - (2*b*d^3*f^2*x - b*c*d^
2*f^2 + 3*b*d^3*f*cosh(1))*sinh(1)^2 - (b*d^3*f^3*x^2 - 2*b*c*d^2*f^3*x)*co
sh(1) - (b*d^3*f^3*x^2 - 2*b*c*d^2*f^3*x + 3*b*d^3*f*cosh(1)^2 + 2*(2*b*d^3
*f^2*x - b*c*d^2*f^2)*cosh(1))*sinh(1))*log(f*x + cosh(1) + sinh(1)) - (4*b
*c*d^3*f*cosh(1)^3 - b*d^4*cosh(1)^4 - b*d^4*sinh(1)^4 - 2*(3*b*c^2 - b)*d^
2*f^2*cosh(1)^2 + 4*(b*c^3 - b*c)*d*f^3*cosh(1) - (b*c^4 - 2*b*c^2 + b)*f^4
 + 4*(b*c*d^3*f - b*d^4*cosh(1))*sinh(1)^3 + 2*(6*b*c*d^3*f*cosh(1) - 3*b*d
^4*cosh(1)^2 - (3*b*c^2 - b)*d^2*f^2)*sinh(1)^2 + 4*(3*b*c*d^3*f*cosh(1)^2
 - b*d^4*cosh(1)^3 - (3*b*c^2 - b)*d^2*f^2*cosh(1) + (b*c^3 - b*c)*d*f^3)*si
nh(1))*log((d*x + c + 1)/(d*x + c - 1)) + 2*(2*b*c*d^2*f^3*x + 4*a*d^4*cosh
(1)^3 - 3*(4*a*c + b)*d^3*f*cosh(1)^2 - (4*a*c^3 + b*c^2 - 4*a*c - b)*d*f^3
 - 2*(b*d^3*f^2*x - 2*(3*a*c^2 + b*c - a)*d^2*f^2)*cosh(1))*sinh(1))/(d^4*f
*cosh(1)^6 + d^4*f*sinh(1)^6 + (c^4 - 2*c^2 + 1)*f^7*x^2 + 2*(d^4*f^2*x - 2
*c*d^3*f^2)*cosh(1)^5 + 2*(d^4*f^2*x - 2*c*d^3*f^2 + 3*d^4*f*cosh(1))*sinh(
1)^5 + (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cosh(1)^4 + (d
^4*f^3*x^2 - 8*c*d^3*f^3*x + 15*d^4*f*cosh(1)^2 + 2*(3*c^2 - 1)*d^2*f^3 + 1
0*(d^4*f^2*x - 2*c*d^3*f^2)*cosh(1))*sinh(1)^4 - 4*(c*d^3*f^4*x^2 - (3*c^2
 - 1)*d^2*f^4*x + (c^3 - c)*d*f^4)*cosh(1)^3 - 4*(c*d^3*f^4*x^2 - (3*c^2 - 1
)*d^2*f^4*x - 5*d^4*f*cosh(1)^3 + (c^3 - c)*d*f^4 - 5*(d^4*f^2*x - 2*c*d^3*
f^2)*cosh(1)^2 - (d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cosh
(1))*sinh(1)^3 + (2*(3*c^2 - 1)*d^2*f^5*x^2 - 8*(c^3 - c)*d*f^5*x + (c^4 -
2*c^2 + 1)*f^5)*cosh(1)^2 + (2*(3*c^2 - 1)*d^2*f^5*x^2 + 15*d^4*f*cosh(1)^4
```

```
 - 8*(c^3 - c)*d*f^5*x + (c^4 - 2*c^2 + 1)*f^5 + 20*(d^4*f^2*x - 2*c*d^3*f^
2)*cosh(1)^3 + 6*(d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*(3*c^2 - 1)*d^2*f^3)*cosh
(1)^2 - 12*(c*d^3*f^4*x^2 - (3*c^2 - 1)*d^2*f^4*x + (c^3 - c)*d*f^4)*cosh(1
))*sinh(1)^2 - 2*(2*(c^3 - c)*d*f^6*x^2 - (c^4 - 2*c^2 + 1)*f^6*x)*cosh(1)
 - 2*(2*(c^3 - c)*d*f^6*x^2 - 3*d^4*f*cosh(1)^5 - (c^4 - 2*c^2 + 1)*f^6*x -
5*(d^4*f^2*x - 2*c*d^3*f^2)*cosh(1)^4 - 2*(d^4*f^3*x^2 - 8*c*d^3*f^3*x + 2*
(3*c^2 - 1)*d^2*f^3)*cosh(1)^3 + 6*(c*d^3*f^4*x^2 - (3*c^2 - 1)*d^2*f^4*x +
 (c^3 - c)*d*f^4)*cosh(1)^2 - (2*(3*c^2 - 1)*d^2*f^5*x^2 - 8*(c^3 - c)*d*f^
5*x + (c^4 - 2*c^2 + 1)*f^5)*cosh(1))*sinh(1))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19912 vs. $2(143) = 286$.
time = 10.40, size = 19912, normalized size = 119.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))/(f*x+e)**3,x)
```

```
[Out] Piecewise(((a*x + b*c*acoth(c + d*x)/d + b*x*acoth(c + d*x) + b*log(c/d + x
 + 1/d)/d - b*acoth(c + d*x)/d)/e**3, Eq(f, 0)), (-(a + b*acoth(c))/(2*e**2
*f + 4*e*f**2*x + 2*f**3*x**2), Eq(d, 0)), (-4*a*f**2/(8*e**2*f**3 + 16*e*f
**4*x + 8*f**5*x**2) + b*d**2*e**2*acoth(d*e/f + d*x - 1)/(8*e**2*f**3 + 16
*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*acoth(d*e/f + d*x - 1)/(8*e**2*f*
*3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*x**2*acoth(d*e/f + d*x - 1)/(
8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*e*f/(8*e**2*f**3 + 16*e*f**4
*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) -
4*b*f**2*acoth(d*e/f + d*x - 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) -
 b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2), Eq(c, (d*e - f)/f)), (-4
*a*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*e**2*acoth(d*e/f
 + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + 2*b*d**2*e*f*x*acot
h(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) + b*d**2*f**2*
x**2*acoth(d*e/f + d*x + 1)/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d
*e*f/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x**2) - b*d*f**2*x/(8*e**2*f**3 +
16*e*f**4*x + 8*f**5*x**2) - 4*b*f**2*acoth(d*e/f + d*x + 1)/(8*e**2*f**3 +
 16*e*f**4*x + 8*f**5*x**2) + b*f**2/(8*e**2*f**3 + 16*e*f**4*x + 8*f**5*x*
*2), Eq(c, (d*e + f)/f)), (zoo*(a*x + b*c*acoth(c + d*x)/d + b*x*acoth(c +
d*x) + b*log(c/d + x + 1/d)/d - b*acoth(c + d*x)/d), Eq(e, -f*x)), (-a*c**4
*f**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**
3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f
**3 + 24*c**2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*
f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*
e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*
x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f*
*3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2
```

```
*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2) + 4*a*c**3*d*e*f**3/(2*c**4*e**2*f**
5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**
2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**
3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x
 - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3
*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2
+ 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*
f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*
x + 2*f**7*x**2) - 6*a*c**2*d**2*e**2*f**2/(2*c**4*e**2*f**5 + 4*c**4*e*f**
6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**
3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*c*
*2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x
**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2
+ 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f
+ 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e*
*3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2)
 + 2*a*c**2*f**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8
*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*
d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4
*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 -
 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d
*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d
**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f*
*5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**2) + 4*a*c*d**3*e**3*f/(2*c*
*4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16
*c**3*d*e**2*f**5*x - 8*c**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c*
*2*d**2*e**3*f**4*x + 12*c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c*
*2*e*f**6*x - 4*c**2*f**7*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x
 - 8*c*d**3*e**3*f**4*x**2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e
*f**6*x**2 + 2*d**4*e**6*f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4
*d**2*e**4*f**3 - 8*d**2*e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5
+ 4*e*f**6*x + 2*f**7*x**2) - 4*a*c*d*e*f**3/(2*c**4*e**2*f**5 + 4*c**4*e*f
**6*x + 2*c**4*f**7*x**2 - 8*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c
**3*d*e*f**6*x**2 + 12*c**2*d**2*e**4*f**3 + 24*c**2*d**2*e**3*f**4*x + 12*
c**2*d**2*e**2*f**5*x**2 - 4*c**2*e**2*f**5 - 8*c**2*e*f**6*x - 4*c**2*f**7
*x**2 - 8*c*d**3*e**5*f**2 - 16*c*d**3*e**4*f**3*x - 8*c*d**3*e**3*f**4*x**
2 + 8*c*d*e**3*f**4 + 16*c*d*e**2*f**5*x + 8*c*d*e*f**6*x**2 + 2*d**4*e**6*
f + 4*d**4*e**5*f**2*x + 2*d**4*e**4*f**3*x**2 - 4*d**2*e**4*f**3 - 8*d**2*
e**3*f**4*x - 4*d**2*e**2*f**5*x**2 + 2*e**2*f**5 + 4*e*f**6*x + 2*f**7*x**
2) - a*d**4*e**4/(2*c**4*e**2*f**5 + 4*c**4*e*f**6*x + 2*c**4*f**7*x**2 - 8
*c**3*d*e**3*f**4 - 16*c**3*d*e**2*f**5*x - 8*c...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. $2(160) = 320$.

time = 0.46, size = 2562, normalized size = 15.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

```
[Out] -1/2*((c + 1)*d - (c - 1)*d)*((b*d^2*e - b*c*d*f)*log((d*x + c + 1)*d*e/(d*
x + c - 1) - d*e - (d*x + c + 1)*c*f/(d*x + c - 1) + c*f + (d*x + c + 1)*f/
(d*x + c - 1) + f)/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3*d*e
*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - ((d*x + c
 + 1)*b*d^2*e/(d*x + c - 1) - b*d^2*e - (d*x + c + 1)*b*c*d*f/(d*x + c - 1)
 + b*c*d*f + (d*x + c + 1)*b*d*f/(d*x + c - 1))*log((d*x + c + 1)/(d*x + c
- 1))/((d*x + c + 1)^2*d^4*e^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^4*e^4/(d
*x + c - 1) + d^4*e^4 - 4*(d*x + c + 1)^2*c*d^3*e^3*f/(d*x + c - 1)^2 + 8*(
d*x + c + 1)*c*d^3*e^3*f/(d*x + c - 1) - 4*c*d^3*e^3*f + 6*(d*x + c + 1)^2*
c^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*f^2/(d*x + c
 - 1) + 6*c^2*d^2*e^2*f^2 - 4*(d*x + c + 1)^2*c^3*d*e*f^3/(d*x + c - 1)^2 +
 8*(d*x + c + 1)*c^3*d*e*f^3/(d*x + c - 1) - 4*c^3*d*e*f^3 + (d*x + c + 1)^
2*c^4*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^4/(d*x + c - 1) + c^4*f^4
 + 4*(d*x + c + 1)^2*d^3*e^3*f/(d*x + c - 1)^2 - 4*(d*x + c + 1)*d^3*e^3*f/
(d*x + c - 1) - 12*(d*x + c + 1)^2*c*d^2*e^2*f^2/(d*x + c - 1)^2 + 12*(d*x
+ c + 1)*c*d^2*e^2*f^2/(d*x + c - 1) + 12*(d*x + c + 1)^2*c^2*d*e*f^3/(d*x
+ c - 1)^2 - 12*(d*x + c + 1)*c^2*d*e*f^3/(d*x + c - 1) - 4*(d*x + c + 1)^2
*c^3*f^4/(d*x + c - 1)^2 + 4*(d*x + c + 1)*c^3*f^4/(d*x + c - 1) + 6*(d*x +
 c + 1)^2*d^2*e^2*f^2/(d*x + c - 1)^2 - 2*d^2*e^2*f^2 - 12*(d*x + c + 1)^2*
c*d*e*f^3/(d*x + c - 1)^2 + 4*c*d*e*f^3 + 6*(d*x + c + 1)^2*c^2*f^4/(d*x +
c - 1)^2 - 2*c^2*f^4 + 4*(d*x + c + 1)^2*d*e*f^3/(d*x + c - 1)^2 + 4*(d*x +
 c + 1)*d*e*f^3/(d*x + c - 1) - 4*(d*x + c + 1)^2*c*f^4/(d*x + c - 1)^2 - 4
*(d*x + c + 1)*c*f^4/(d*x + c - 1) + (d*x + c + 1)^2*f^4/(d*x + c - 1)^2 +
2*(d*x + c + 1)*f^4/(d*x + c - 1) + f^4) - (b*d^2*e - b*c*d*f)*log((d*x + c
 + 1)/(d*x + c - 1))/(d^4*e^4 - 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 - 4*c^3*d
*e*f^3 + c^4*f^4 - 2*d^2*e^2*f^2 + 4*c*d*e*f^3 - 2*c^2*f^4 + f^4) - (2*(d*x
 + c + 1)*a*d^3*e^2/(d*x + c - 1) - 2*a*d^3*e^2 - 4*(d*x + c + 1)*a*c*d^2*e
*f/(d*x + c - 1) + 4*a*c*d^2*e*f + 2*(d*x + c + 1)*a*c^2*d*f^2/(d*x + c - 1
) - 2*a*c^2*d*f^2 + 2*a*d^2*e*f - (d*x + c + 1)*b*d^2*e*f/(d*x + c - 1) + b
*d^2*e*f - 2*a*c*d*f^2 + (d*x + c + 1)*b*c*d*f^2/(d*x + c - 1) - b*c*d*f^2
- 2*(d*x + c + 1)*a*d*f^2/(d*x + c - 1) - (d*x + c + 1)*b*d*f^2/(d*x + c -
1) - b*d*f^2)/((d*x + c + 1)^2*d^5*e^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^
5*e^5/(d*x + c - 1) + d^5*e^5 - 5*(d*x + c + 1)^2*c*d^4*e^4*f/(d*x + c - 1)
^2 + 10*(d*x + c + 1)*c*d^4*e^4*f/(d*x + c - 1) - 5*c*d^4*e^4*f + 10*(d*x +
 c + 1)^2*c^2*d^3*e^3*f^2/(d*x + c - 1)^2 - 20*(d*x + c + 1)*c^2*d^3*e^3*f^
2/(d*x + c - 1) + 10*c^2*d^3*e^3*f^2 - 10*(d*x + c + 1)^2*c^3*d^2*e^2*f^3/(
d*x + c - 1)^2 + 20*(d*x + c + 1)*c^3*d^2*e^2*f^3/(d*x + c - 1) - 10*c^3*d^
2*e^2*f^3 + 5*(d*x + c + 1)^2*c^4*d*e*f^4/(d*x + c - 1)^2 - 10*(d*x + c + 1
```

```
)*c^4*d*e*f^4/(d*x + c - 1) + 5*c^4*d*e*f^4 - (d*x + c + 1)^2*c^5*f^5/(d*x
+ c - 1)^2 + 2*(d*x + c + 1)*c^5*f^5/(d*x + c - 1) - c^5*f^5 + 3*(d*x + c +
 1)^2*d^4*e^4*f/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d^4*e^4*f/(d*x + c - 1) -
 d^4*e^4*f - 12*(d*x + c + 1)^2*c*d^3*e^3*f^2/(d*x + c - 1)^2 + 8*(d*x + c
+ 1)*c*d^3*e^3*f^2/(d*x + c - 1) + 4*c*d^3*e^3*f^2 + 18*(d*x + c + 1)^2*c^2
*d^2*e^2*f^3/(d*x + c - 1)^2 - 12*(d*x + c + 1)*c^2*d^2*e^2*f^3/(d*x + c -
1) - 6*c^2*d^2*e^2*f^3 - 12*(d*x + c + 1)^2*c^3*d*e*f^4/(d*x + c - 1)^2 + 8
*(d*x + c + 1)*c^3*d*e*f^4/(d*x + c - 1) + 4*c^3*d*e*f^4 + 3*(d*x + c + 1)^
2*c^4*f^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)*c^4*f^5/(d*x + c - 1) - c^4*f^5
 + 2*(d*x + c + 1)^2*d^3*e^3*f^2/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^3*e^3*
f^2/(d*x + c - 1) - 2*d^3*e^3*f^2 - 6*(d*x + c + 1)^2*c*d^2*e^2*f^3/(d*x +
c - 1)^2 - 12*(d*x + c + 1)*c*d^2*e^2*f^3/(d*x + c - 1) + 6*c*d^2*e^2*f^3 +
 6*(d*x + c + 1)^2*c^2*d*e*f^4/(d*x + c - 1)^2 + 12*(d*x + c + 1)*c^2*d*e*f
^4/(d*x + c - 1) - 6*c^2*d*e*f^4 - 2*(d*x + c + 1)^2*c^3*f^5/(d*x + c - 1)^
2 - 4*(d*x + c + 1)*c^3*f^5/(d*x + c - 1) + 2*c^3*f^5 - 2*(d*x + c + 1)^2*d
^2*e^2*f^3/(d*x + c - 1)^2 + 4*(d*x + c + 1)*d^2*e^2*f^3/(d*x + c - 1) + 2*
d^2*e^2*f^3 + 4*(d*x + c + 1)^2*c*d*e*f^4/(d*x + c - 1)^2 - 8*(d*x + c + 1)
*c*d*e*f^4/(d*x + c - 1) - 4*c*d*e*f^4 - 2*(d*x + c + 1)^2*c^2*f^5/(d*x + c
 - 1)^2 + 4*(d*x + c + 1)*c^2*f^5/(d*x + c - 1) + 2*c^2*f^5 - 3*(d*x + c +
1)^2*d*e*f^4/(d*x + c - 1)^2 - 2*(d*x + c + 1)*d*e*f^4/(d*x + c - 1) + d*e*
f^4 + 3*(d*x + c + 1)^2*c*f^5/(d*x + c - 1)^2 + 2*(d*x + c + 1)*c*f^5/(d*x
+ c - 1) - c*f^5 - (d*x + c + 1)^2*f^5/(d*x + c - 1)^2 - 2*(d*x + c + 1)*f^
5/(d*x + c - 1) - f^5))
```

Mupad [B]
time = 3.30, size = 422, normalized size = 2.53

$$\frac{b\,\ln\left(1-\frac{1}{c+d\,x}\right)}{2\,f\,\left(2\,e^2+4\,e\,f\,x+2\,f^2\,x^2\right)}-\frac{\ln\left(e+f\,x\right)\,\left(b\,d^3\,e-b\,c\,d^2\,f\right)}{c^4\,f^4-4\,c^3\,d\,e\,f^3+6\,c^2\,d^2\,e^2\,f^2-2\,c^2\,f^4-4\,c\,d^3\,e^3\,f+4\,c\,d\,e\,f^3+d^4\,e^4-2\,d^2\,e^2\,f^2+f^4}-\frac{\frac{-a\,c^2\,f^2+2\,a\,c\,d\,e\,f-a\,d^2\,e^2+b\,d\,e\,f+a\,f^2}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}+\frac{b\,d\,f^2\,x}{-c^2\,f^2+2\,c\,d\,e\,f-d^2\,e^2+f^2}}{2\,e^2\,f+4\,e\,f^2\,x+2\,f^3\,x^2}-\frac{b\,d^2\,\ln\left(c+d\,x-1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2-8\,c\,f^3+4\,d^2\,e^2\,f+8\,d\,e\,f^2+4\,f^3}+\frac{b\,d^2\,\ln\left(c+d\,x+1\right)}{4\,c^2\,f^3-8\,c\,d\,e\,f^2+8\,c\,f^3+4\,d^2\,e^2\,f-8\,d\,e\,f^2+4\,f^3}-\frac{b\,\ln\left(\frac{1}{c+d\,x}+1\right)}{4\,f\,\left(e^2+2\,e\,f\,x+f^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))/(e + f*x)^3,x)
```

```
[Out] (b*log(1 - 1/(c + d*x)))/(2*f*(2*e^2 + 2*f^2*x^2 + 4*e*f*x)) - (log(e + f*x
)*(b*d^3*e - b*c*d^2*f))/(f^4 - 2*c^2*f^4 + c^4*f^4 + d^4*e^4 - 2*d^2*e^2*f
^2 + 4*c*d*e*f^3 + 6*c^2*d^2*e^2*f^2 - 4*c*d^3*e^3*f - 4*c^3*d*e*f^3) - ((a
*f^2 - a*c^2*f^2 - a*d^2*e^2 + b*d*e*f + 2*a*c*d*e*f)/(f^2 - c^2*f^2 - d^2*
e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f))/(2*e^
2*f + 2*f^3*x^2 + 4*e*f^2*x) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*f^3 +
4*c^2*f^3 + 4*d^2*e^2*f + 8*d*e*f^2 - 8*c*d*e*f^2) + (b*d^2*log(c + d*x + 1
))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*e*f^2 - 8*c*d*e*f^2) -
(b*log(1/(c + d*x) + 1))/(4*f*(e^2 + f^2*x^2 + 2*e*f*x))
```

3.109 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=374

$$\frac{b^2f^2x}{3d^2}+\frac{2abf(de-cf)x}{d^2}+\frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}+\frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+dx)\right)}{3d^3}-\frac{(de$$

```
[Out] 1/3*b^2*f^2*x/d^2+2*a*b*f*(-c*f+d*e)*x/d^2+2*b^2*f*(-c*f+d*e)*(d*x+c)*arcco
th(d*x+c)/d^3+1/3*b*f^2*(d*x+c)^2*(a+b*arccoth(d*x+c))/d^3-1/3*(-c*f+d*e)*(
d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arccoth(d*x+c))^2/d^3/f+1/3*(3*d^2*e^2-
6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))^2/d^3+1/3*(f*x+e)^3*(a+b*arcc
oth(d*x+c))^2/f-1/3*b^2*f^2*arctanh(d*x+c)/d^3-2/3*b*(3*d^2*e^2-6*c*d*e*f+(
3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))*ln(2/(-d*x-c+1))/d^3+b^2*f*(-c*f+d*e)*ln
(1-(d*x+c)^2)/d^3-1/3*b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(2,(-d
*x-c-1)/(-d*x-c+1))/d^3
```

Rubi [A]
time = 0.42, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {6247, 6066, 6022, 266, 6038, 327, 212, 6196, 6096, 6132, 6056, 2449, 2352}

$$-\frac{(de-cf)\left(\left(c^2+3\right)f^2-2cdef+d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{3d^3f}+\frac{\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{3d^3}-\frac{2b\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{3d^3}+\frac{bf^2(c+dx)^2\left(a+b\coth^{-1}(c+dx)\right)}{3d^3}+\frac{2abfx(de-cf)}{d^2}+\frac{(e+fx)^3\left(a+b\coth^{-1}(c+dx)\right)^2}{3f}-\frac{b^2\left(\left(3c^2+1\right)f^2-6cdef+3d^2e^2\right)\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{3d^3}+\frac{b^2f(de-cf)\log\left(1-(c+dx)^2\right)}{d^3}+\frac{2b^2f(c+dx)(de-cf)\coth^{-1}(c+dx)}{d^3}-\frac{b^2f^2\tanh^{-1}(c+dx)}{3d^3}+\frac{b^2f^2x}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (b^2*f^2*x)/(3*d^2) + (2*a*b*f*(d*e - c*f)*x)/d^2 + (2*b^2*f*(d*e - c*f)*(c
 + d*x)*ArcCoth[c + d*x])/d^3 + (b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])
)/(3*d^3) - ((d*e - c*f)*(d^2*e^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCo
th[c + d*x])^2)/(3*d^3*f) + ((3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a +
 b*ArcCoth[c + d*x])^2)/(3*d^3) + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^2)/
(3*f) - (b^2*f^2*ArcTanh[c + d*x])/(3*d^3) - (2*b*(3*d^2*e^2 - 6*c*d*e*f +
(1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/(3*d^3) + (
b^2*f*(d*e - c*f)*Log[1 - (c + d*x)^2])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f +
 (1 + 3*c^2)*f^2)*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(3*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
```

```
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\coth^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+\right.\right.}{d^3}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^2}{3f} - \frac{(2b)\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cde}{}\right.}{}$$

$$= \frac{2abf(de-cf)x}{d^2} + \frac{bf^2(c+dx)^2 \left(a+b\coth^{-1}(c+dx)\right)}{3d^3} + \frac{(e+fx}{}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}$$

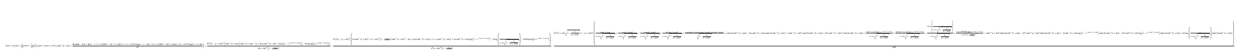
$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}$$

$$= \frac{b^2f^2x}{3d^2} + \frac{2abf(de-cf)x}{d^2} + \frac{2b^2f(de-cf)(c+dx)\coth^{-1}(c+dx)}{d^3}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1054 vs. $2(374) = 748$.

time = 6.82, size = 1054, normalized size = 2.82



Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] a^2*e^2*x + a^2*e*f*x^2 + (a^2*f^2*x^3)/3 + (a*b*(2*x*(3*e^2 + 3*e*f*x + f^
2*x^2)*ArcCoth[c + d*x] + (d*f*x*(6*d*e - 4*c*f + d*f*x) - (-1 + c)*(3*d^2*
e^2 - 3*(-1 + c)*d*e*f + (-1 + c)^2*f^2)*Log[1 - c - d*x] + (1 + c)*(3*d^2*
e^2 - 3*(1 + c)*d*e*f + (1 + c)^2*f^2)*Log[1 + c + d*x])/d^3))/3 + (b^2*e^2
*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d*x)*ArcCoth[
c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])]) - PolyLog[2, E^(-2*ArcCoth[c
```

```
 + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (b^2*e*f*(1 - (c + d*x)^
2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2))*ArcCoth[c + d
*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*x]) + 4*c*ArcCot
h[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - 2*Log[1/((c + d*x)*Sqrt[1 - (
c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])]))/(d^2*(c + d*x)
^2*(1 - (c + d*x)^(-2))) - (b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 -
 (c + d*x)^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) +
(3*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]
^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*A
rcCoth[c + d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Co
sh[3*ArcCoth[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + Ar
cCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3
*ArcCoth[c + d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/
((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(
-2*ArcCoth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/(
(c + d*x)*Sqrt[1 - (c + d*x)^(-2)])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
+ (4*(1 + 3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c
+ d*x)^(-2))^(3/2)) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*A
rcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(
-2*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log
[1 - E^(-2*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*
x)*Sqrt[1 - (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]]))/(12*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2889 vs. $2(360) = 720$.
time = 0.41, size = 2890, normalized size = 7.73

method	result	size
risch	Expression too large to display	2100
derivativedivides	Expression too large to display	2890
default	Expression too large to display	2890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*b/d*f*ln(d*x+c+1)*e-4*a*b/d*f*arccoth(d*x+c)*c*e*(d*x+c)-1/4*b^2*ln
(d*x+c+1)^2*e^2+1/4*b^2*ln(d*x+c-1)^2*e^2-b^2*dilog(1/2*d*x+1/2*c+1/2)*e^2-
1/12*b^2/d^2*f^2*ln(d*x+c+1)^2-1/3*b^2/d^2*f^2*dilog(1/2*d*x+1/2*c+1/2)+1/1
2*b^2/d^2*f^2*ln(d*x+c-1)^2+1/6*b^2/d^2*f^2*ln(d*x+c-1)-1/6*b^2/d^2*f^2*ln(
d*x+c+1)-1/4*b^2*ln(d*x+c-1)^2*c*e^2+b^2*arccoth(d*x+c)*ln(d*x+c+1)*e^2-b^2
*arccoth(d*x+c)^2*c*e^2+b^2*arccoth(d*x+c)^2*e^2*(d*x+c)-1/3*(c*f-d*e-f*(d*
x+c))^3*a^2/d^2/f+a*b*ln(d*x+c-1)*e^2+a*b*ln(d*x+c+1)*e^2-1/4*b^2*ln(d*x+c+
1)^2*c*e^2+b^2*arccoth(d*x+c)*ln(d*x+c-1)*e^2-1/2*b^2*ln(-1/2*d*x-1/2*c+1/2
)*ln(1/2*d*x+1/2*c+1/2)*e^2+1/2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*e^2-
```

1/2*b^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*e^2+1/3*b^2/d^2*f^2*(d*x+c)-1/4*b
^2/d^2*f^2*ln(d*x+c+1)^2*c^2-1/4*b^2/d^2*f^2*ln(d*x+c+1)^2*c+1/6*b^2/d^2*f^
2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)-1/6*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)
*ln(1/2*d*x+1/2*c+1/2)-b^2/d^2*f^2*dilog(1/2*d*x+1/2*c+1/2)*c^2-1/6*b^2/d^2
*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-1/12*b^2/d^2*f^2*ln(d*x+c-1)^2*c^3+1
/4*b^2/d^2*f^2*ln(d*x+c-1)^2*c^2+1/4*b^2/d*f*ln(d*x+c+1)^2*e+1/4*b^2/d*f*ln
(d*x+c-1)^2*e+b^2/d*f*ln(d*x+c+1)*e+b^2/d*f*ln(d*x+c-1)*e+1/12*b^2*d/f*ln(d
*x+c-1)^2*e^3+1/12*b^2*d/f*ln(d*x+c+1)^2*e^3+1/3*a*b/d^2*f^2*(d*x+c)^2-2*a*
b*arccoth(d*x+c)*c*e^2+2*a*b*arccoth(d*x+c)*e^2*(d*x+c)-a*b*ln(d*x+c-1)*c*e
^2+a*b*ln(d*x+c+1)*c*e^2+1/3*a*b/d^2*f^2*ln(d*x+c+1)+1/3*a*b/d^2*f^2*ln(d*x
+c-1)-1/3*b^2/d^2*f^2*arccoth(d*x+c)^2*c^3+b^2*arccoth(d*x+c)*ln(d*x+c+1)*c
*e^2-b^2*arccoth(d*x+c)*ln(d*x+c-1)*c*e^2-1/2*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/
2*d*x+1/2*c+1/2)*c^2+1/2*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*c+2*
a*b/d*f*e*(d*x+c)+a*b/d*f*ln(d*x+c-1)*e-a*b/d^2*f^2*ln(d*x+c-1)*c+1/3*a*b*d
/f*ln(d*x+c-1)*e^3-1/3*a*b*d/f*ln(d*x+c+1)*e^3+2/3*a*b*d/f*arccoth(d*x+c)*e
^3-1/3*a*b/d^2*f^2*ln(d*x+c-1)*c^3+a*b/d^2*f^2*ln(d*x+c-1)*c^2+1/3*a*b/d^2*
f^2*ln(d*x+c+1)*c^3+a*b/d^2*f^2*ln(d*x+c+1)*c^2-2/3*a*b/d^2*f^2*arccoth(d*x
+c)*c^3+2/3*a*b/d^2*f^2*arccoth(d*x+c)*(d*x+c)^3+a*b/d^2*f^2*ln(d*x+c+1)*c+
1/3*b^2/d^2*f^2*arccoth(d*x+c)*ln(d*x+c+1)*c^3+1/2*b^2/d*f*ln(d*x+c+1)^2*c*
e-1/2*b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*e+1/3*b^2*d/f*arccoth(d*x+
c)*ln(d*x+c-1)*e^3-1/3*b^2*d/f*arccoth(d*x+c)*ln(d*x+c+1)*e^3+2*b^2/d*f*dil
og(1/2*d*x+1/2*c+1/2)*c*e-1/2*b^2/d*f*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*e+1
/4*b^2/d*f*ln(d*x+c+1)^2*c^2*e-2*a*b/d^2*f^2*c*(d*x+c)+b^2/d*f*arccoth(d*x+
c)^2*c^2*e+b^2/d*f*arccoth(d*x+c)^2*e*(d*x+c)^2+b^2/d^2*f^2*arccoth(d*x+c)^
2*c^2*(d*x+c)-b^2/d^2*f^2*arccoth(d*x+c)^2*c*(d*x+c)^2-1/6*b^2*d/f*ln(d*x+c
-1)*ln(1/2*d*x+1/2*c+1/2)*e^3+2*b^2/d*f*arccoth(d*x+c)*e*(d*x+c)+b^2/d*f*ar
ccoth(d*x+c)*ln(d*x+c-1)*e-b^2/d*f*arccoth(d*x+c)*ln(d*x+c+1)*e+1/4*b^2/d*f
*ln(d*x+c-1)^2*c^2*e-1/2*b^2/d*f*ln(d*x+c-1)^2*c*e-1/6*b^2*d/f*ln(-1/2*d*x-
1/2*c+1/2)*ln(d*x+c+1)*e^3+1/6*b^2*d/f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/
2*c+1/2)*e^3+1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c^2+1/2*b^2
/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c-1/6*b^2/d^2*f^2*ln(-1/2*d*x-1
/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c^3-1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*
ln(1/2*d*x+1/2*c+1/2)*c^2-1/2*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x
+1/2*c+1/2)*c+1/6*b^2/d^2*f^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c^3-2*b^2/
d^2*f^2*arccoth(d*x+c)*c*(d*x+c)+1/6*b^2/d^2*f^2*ln(d*x+c-1)*ln(1/2*d*x+1/2
*c+1/2)*c^3+b^2/d^2*f^2*arccoth(d*x+c)*ln(d*x+c+1)*c^2+b^2/d^2*f^2*arccoth(
d*x+c)*ln(d*x+c+1)*c-1/3*b^2/d^2*f^2*arccoth(d*x+c)*ln(d*x+c-1)*c^3+b^2/d^2
*f^2*arccoth(d*x+c)*ln(d*x+c-1)*c^2-b^2/d^2*f^2*arccoth(d*x+c)*ln(d*x+c-1)*
c+1/2*b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*e+1/3*b^2/d^2*f^
2*arccoth(d*x+c)^2*(d*x+c)^3+1/3*b^2*d/f*arccoth(d*x+c)^2*e^3-1/2*b^2*ln(-1
/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)*c*e^2+1/2*b^2*ln(d*x+c-1)*ln(1/2*d*
x+1/2*c+1/2)*c*e^2+1/2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c*e^2-1/4*b^2
/d^2*f^2*ln(d*x+c-1)^2*c-b^2/d^2*f^2*ln(d*x+c+1)*c+1/3*b^2/d^2*f^2*arccoth(
d*x+c)*(d*x+c)^2-b^2/d^2*f^2*ln(d*x+c-1)*c+1/3*b^2/d^2*f^2*arccoth(d*x+c)*l
n(d*x+c-1)+1/3*b^2/d^2*f^2*arccoth(d*x+c)*ln(d*x+c+1)-1/12*b^2/d^2*f^2*ln(d

```
*x+c+1)^2*c^3-b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)*c*e+b^2/d*f*arccot
h(d*x+c)*ln(d*x+c-1)*c^2*e+a*b/d*f*ln(d*x+c-1)*c^2*e-2*a*b/d*f*ln(d*x+c-1)*
c*e-a*b/d*f*ln(d*x+c+1)*c^2*e-2*a*b/d*f*ln(d*x+c+1)*c*e+2*a*b/d^2*f^2*arcco
th(d*x+c)*c^2*(d*x+c)-2*a*b/d^2*f^2*arccoth(d*x+c)*c*(d*x+c)^2+2*a*b/d*f*ar
ccoth(d*x+c)*c^2*e+2*a*b/d*f*arccoth(d*x+c)*e*(d*x+c)^2-1/2*b^2/d*f*ln(-1/2
*d*x-1/2*c+1/2)*ln(d*x+c+1)*c^2*e+b^2/d*f*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)
*c*e-2*b^2/d*f*arccoth(d*x+c)*ln(d*x+c+1)*c*e+1/2*b^2/d*f*ln(-1/2*d*x-1/2*c
+1/2)*ln(1/2*d*x+1/2*c+1/2)*c^2*e+b^2/d*f*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x
+1/2*c+1/2)*c*e-1/2*b^2/d*f*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)*c^2*e-2*b^2/d
*f*arccoth(d*x+c)*ln(d*x+c-1)*c*e-b^2/d*f*arccoth(d*x+c)*ln(d*x+c+1)*c^2*e-
2*b^2/d*f*arccoth(d*x+c)^2*c*e*(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(361) = 722$.
time = 0.45, size = 843, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*f^2*x^3 + a^2*f*x^2*e + 1/3*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3
*c - 1)*log(d*x + c - 1)/d^4))*a*b*f^2 + (2*x^2*arccoth(d*x + c) + d*(2*x/d
^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1
)/d^3))*a*b*f*e + a^2*x*e^2 + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c
)^2 + 1))*a*b*e^2/d + 1/3*(6*b^2*c*d*f*e - 3*b^2*d^2*e^2 - (3*c^2*f^2 + f^2
)*b^2)*(log(d*x + c - 1)*log(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x - 1/2*
c + 1/2))/d^3 + 1/6*(6*(c*d*f + d*f)*b^2*e - (5*c^2*f^2 + 6*c*f^2 + f^2)*b^
2)*log(d*x + c + 1)/d^3 + 1/12*(4*b^2*d*f^2*x + (b^2*d^3*f^2*x^3 + 3*b^2*d^
3*f*x^2*e + 3*b^2*d^3*x*e^2 + 3*(c*d^2 + d^2)*b^2*e^2 - 3*(c^2*d*f + 2*c*d*
f + d*f)*b^2*e + (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^2)*log(d*x + c + 1
)^2 + (b^2*d^3*f^2*x^3 + 3*b^2*d^3*f*x^2*e + 3*b^2*d^3*x*e^2 + 3*(c*d^2 - d
^2)*b^2*e^2 - 3*(c^2*d*f - 2*c*d*f + d*f)*b^2*e + (c^3*f^2 - 3*c^2*f^2 + 3*
c*f^2 - f^2)*b^2)*log(d*x + c - 1)^2 + 2*(b^2*d^2*f^2*x^2 - 2*(2*b^2*c*d*f^
2 - 3*b^2*d^2*f*e)*x - (b^2*d^3*f^2*x^3 + 3*b^2*d^3*f*x^2*e + 3*b^2*d^3*x*e
^2 + 3*(c*d^2 - d^2)*b^2*e^2 - 3*(c^2*d*f - 2*c*d*f + d*f)*b^2*e + (c^3*f^2
 - 3*c^2*f^2 + 3*c*f^2 - f^2)*b^2)*log(d*x + c - 1))*log(d*x + c + 1) - 2*(
b^2*d^2*f^2*x^2 + 6*(c*d*f - d*f)*b^2*e - (5*c^2*f^2 - 6*c*f^2 + f^2)*b^2 -
 2*(2*b^2*c*d*f^2 - 3*b^2*d^2*f*e)*x)*log(d*x + c - 1))/d^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f^2*x^2 + 2*a^2*f*x*e + (b^2*f^2*x^2 + 2*b^2*f*x*e + b^2*e^2)*
arccoth(d*x + c)^2 + a^2*e^2 + 2*(a*b*f^2*x^2 + 2*a*b*f*x*e + a*b*e^2)*arcc
oth(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)^2\,(a + b\,\mathrm{acoth}(c + d\,x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acoth(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^2*(a + b*acoth(c + d*x))^2, x)
```

3.110 $\int (e+fx) \left(a+b \coth^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=221

$$\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2}-\frac{(d^2e^2-2cdef+(1+c^2)\,f^2)\left(a+\right.}{2d^2f}$$

```
[Out] a*b*f*x/d+b^2*f*(d*x+c)*arccoth(d*x+c)/d^2+(-c*f+d*e)*(a+b*arccoth(d*x+c))^
2/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*arccoth(d*x+c))^2/d^2/f+1/2*
(f*x+e)^2*(a+b*arccoth(d*x+c))^2/f-2*b*(-c*f+d*e)*(a+b*arccoth(d*x+c))*ln(2
/(-d*x-c+1))/d^2+1/2*b^2*f*ln(1-(d*x+c)^2)/d^2-b^2*(-c*f+d*e)*polylog(2,(-d
*x-c-1)/(-d*x-c+1))/d^2
```

Rubi [A]

time = 0.31, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6247, 6066, 6022, 266, 6196, 6096, 6132, 6056, 2449, 2352}

$$\frac{\left(-\frac{(c^2+1)f}{d}+2ce-\frac{de^2}{f}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d}+\frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2}-\frac{2b(de-cf)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^2}+\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}+\frac{abfx}{d}-\frac{b^2(de-cf)\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{d^2}+\frac{b^2f\log\left(1-(c+dx)^2\right)}{2d^2}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (a*b*f*x)/d + (b^2*f*(c + d*x)*ArcCoth[c + d*x])/d^2 + ((d*e - c*f)*(a + b*
ArcCoth[c + d*x])^2)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*Ar
cCoth[c + d*x])^2)/(2*d) + ((e + f*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*f) -
 (2*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^2 + (b^2
*f*Log[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*PolyLog[2, -((1 + c + d
*x)/(1 - c - d*x))])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\left(-\frac{f^2(a+b\coth^{-1}(x))}{d^2}\right.\right.}{}$$

$$=\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\frac{(d^2e^2-2cdef+(1+c^2)f^2}{}\right.}{}$$

$$=\frac{abfx}{d}+\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{b\text{Subst}\left(\int\left(\frac{d^2e^2\left(1+\right.}{}\right.\right.}{}$$

$$=\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(e+fx)^2\left(a+b\coth^{-1}(c\right.}{2f}$$

$$=\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c\right.}{d^2}$$

$$=\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c\right.}{d^2}$$

$$=\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c\right.}{d^2}$$

$$=\frac{abfx}{d}+\frac{b^2f(c+dx)\coth^{-1}(c+dx)}{d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c\right.}{d^2}$$

Mathematica [A]
time = 0.35, size = 295, normalized size = 1.33

$$\frac{2a^2cde+2abcf-a^2c^2f+2a^2d^2ex+2abdfx+a^2d^2fx^2+b^2(-1+c+dx)(2de+f-cf+dfx)\coth^{-1}(c+dx)^2+2b\coth^{-1}(c+dx)\left(-((c+dx)(-bf+acf-ad(2e+fx)))-2b(de-cf)\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)+abf\log(1-c-dx)-abf\log(1+c+dx)-4abde\log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)-2b^2f\log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)+4abcf\log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)+2b^2(de-cf)\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^
2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcCoth[c + d*x]^2 +
 2*b*ArcCoth[c + d*x]*(-((c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*
b*(d*e - c*f)*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*b*f*Log[1 - c - d*x] -
a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)
])] - 2*b^2*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1
/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 2*b^2*(d*e - c*f)*PolyLog[2, E^(-2
*ArcCoth[c + d*x])])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(217) = 434$.
time = 0.33, size = 812, normalized size = 3.67 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/d*(f*c*(d*x+c)-e*d*(d*x+c)-1/2*f*(d*x+c)^2)+a*b/d*arccoth(d*x+c)*
f*(d*x+c)^2-a*b/d*ln(d*x+c-1)*c*f-a*b/d*ln(d*x+c+1)*c*f+1/2*b^2/d*ln(1/2*d*
x+1/2*c+1/2)*ln(-1/2*d*x-1/2*c+1/2)*c*f-1/2*b^2/d*ln(d*x+c+1)*ln(-1/2*d*x-1
/2*c+1/2)*c*f+1/2*b^2/d*ln(1/2*d*x+1/2*c+1/2)*ln(d*x+c-1)*c*f-b^2/d*arccoth
(d*x+c)*ln(d*x+c-1)*c*f-b^2/d*arccoth(d*x+c)*ln(d*x+c+1)*c*f-b^2/d*arccoth(
d*x+c)^2*f*c*(d*x+c)-2*a*b/d*arccoth(d*x+c)*f*c*(d*x+c)+a*b/d*f*(d*x+c)+2*a
*b*arccoth(d*x+c)*e*(d*x+c)+b^2/d*dilog(1/2*d*x+1/2*c+1/2)*c*f-1/4*b^2/d*ln
(1/2*d*x+1/2*c+1/2)*ln(d*x+c-1)*f+b^2/d*arccoth(d*x+c)*f*(d*x+c)+1/2*b^2/d*
arccoth(d*x+c)*ln(d*x+c-1)*f-1/2*b^2/d*arccoth(d*x+c)*ln(d*x+c+1)*f+1/4*b^2
/d*ln(d*x+c+1)^2*c*f+1/4*b^2/d*ln(1/2*d*x+1/2*c+1/2)*ln(-1/2*d*x-1/2*c+1/2)
*f-1/4*b^2/d*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*f-1/4*b^2/d*ln(d*x+c-1)^2*c
*f+1/2*b^2/d*arccoth(d*x+c)^2*f*(d*x+c)^2+a*b*ln(d*x+c-1)*e+a*b*ln(d*x+c+1)
*e+1/8*b^2/d*ln(d*x+c+1)^2*f+1/8*b^2/d*ln(d*x+c-1)^2*f+1/2*b^2/d*ln(d*x+c-1
)*f+1/2*b^2/d*ln(d*x+c+1)*f+b^2*arccoth(d*x+c)^2*e*(d*x+c)+b^2*arccoth(d*x+
c)*ln(d*x+c-1)*e+b^2*arccoth(d*x+c)*ln(d*x+c+1)*e-1/2*b^2*ln(1/2*d*x+1/2*c+
1/2)*ln(-1/2*d*x-1/2*c+1/2)*e+1/2*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*e-
1/2*b^2*ln(1/2*d*x+1/2*c+1/2)*ln(d*x+c-1)*e-b^2*dilog(1/2*d*x+1/2*c+1/2)*e-
1/4*b^2*ln(d*x+c+1)^2*e+1/4*b^2*ln(d*x+c-1)^2*e+1/2*a*b/d*ln(d*x+c-1)*f-1/2
*a*b/d*ln(d*x+c+1)*f)
```

Maxima [A]
time = 0.45, size = 422, normalized size = 1.91

$$\frac{1}{2}\,a^{2}fx^{2}+\frac{1}{2}\left(2\,x^{2}\operatorname{arcoth}\left(dx+c\right)+d\left(\frac{2\,x}{d^{2}}-\frac{(c^{2}+2\,c+1)\log\left(dx+c+1\right)}{d^{3}}+\frac{(c^{2}-2\,c+1)\log\left(dx+c-1\right)}{d^{3}}\right)\right)abf+a^{2}xe+\frac{(2\,(dx+c)\operatorname{arcoth}\left(dx+c\right)+\log\left(-(dx+c)^{2}+1\right))abe}{d}+\frac{(cf+f)b^{2}\log\left(dx+c+1\right)}{2\,d^{2}}+\frac{(b^{2}cf-b^{2}de)\left(\log\left(dx+c-1\right)\log\left(\frac{1}{2}\,dx+\frac{1}{2}\,c+\frac{1}{2}\right)+\mathrm{Li}_{2}\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c+\frac{1}{2}\right)\right)}{d^{2}}+\frac{(b^{2}d^{2}fx^{2}+2\,b^{2}d^{2}xe+2\,(cd+d)b^{2}e-(c^{2}f+2\,cf+f)b^{2})\log\left(dx+c+1\right)^{2}+(b^{2}d^{2}fx^{2}+2\,b^{2}d^{2}xe+2\,(cd-d)b^{2}e-(c^{2}f-2\,cf+f)b^{2})\log\left(dx+c-1\right)^{2}+2\,(2\,b^{2}dfx-(b^{2}d^{2}fx^{2}+2\,b^{2}d^{2}xe+2\,(cd-d)b^{2}e-(c^{2}f-2\,cf+f)b^{2})\log\left(dx+c-1\right))\log\left(dx+c+1\right)-4\,(b^{2}dfx+(cf-f)b^{2})\log\left(dx+c-1\right)}{8\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*f*x^2 + 1/2*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a*b*f + a^2*x
```

```
*e + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b*e/d + 1/2*(
c*f + f)*b^2*log(d*x + c + 1)/d^2 + (b^2*c*f - b^2*d*e)*(log(d*x + c - 1)*l
og(1/2*d*x + 1/2*c + 1/2) + dilog(-1/2*d*x - 1/2*c + 1/2))/d^2 + 1/8*((b^2*
d^2*f*x^2 + 2*b^2*d^2*x*e + 2*(c*d + d)*b^2*e - (c^2*f + 2*c*f + f)*b^2)*lo
g(d*x + c + 1)^2 + (b^2*d^2*f*x^2 + 2*b^2*d^2*x*e + 2*(c*d - d)*b^2*e - (c^
2*f - 2*c*f + f)*b^2)*log(d*x + c - 1)^2 + 2*(2*b^2*d*f*x - (b^2*d^2*f*x^2
+ 2*b^2*d^2*x*e + 2*(c*d - d)*b^2*e - (c^2*f - 2*c*f + f)*b^2)*log(d*x + c
- 1))*log(d*x + c + 1) - 4*(b^2*d*f*x + (c*f - f)*b^2)*log(d*x + c - 1))/d^
2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
[Out] integral(a^2*f*x + (b^2*f*x + b^2*e)*arccoth(d*x + c)^2 + a^2*e + 2*(a*b*f*
x + a*b*e)*arccoth(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**2,x)
[Out] Integral((a + b*acoth(c + d*x))**2*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\,\left(a + b\,\mathrm{acoth}(c + d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acoth(c + d*x))^2,x)
[Out] int((e + f*x)*(a + b*acoth(c + d*x))^2, x)
```

3.111 $\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}-\frac{2b\left(a+b\coth^{-1}(c+dx)\right)\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{b^2\text{PolyL}}{}$$

```
[Out] (a+b*arccoth(d*x+c))^2/d+(d*x+c)*(a+b*arccoth(d*x+c))^2/d-2*b*(a+b*arccoth(
d*x+c))*ln(2/(-d*x-c+1))/d-b^2*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6239, 6022, 6132, 6056, 2449, 2352}

$$\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d}-\frac{2b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d}-\frac{b^2\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (a + b*ArcCoth[c + d*x])^2/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d - (
2*b*(a + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d - (b^2*PolyLog[2, -((1
 + c + d*x)/(1 - c - d*x))])/d
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
```

```
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\coth^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x(a+b\coth^{-1}(x))}{1-x^2}\,dx,x,c+d\right.}{d}$$

$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\right.}{}$$

$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\,\text{co}\right.}{}$$

$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\,\text{co}\right.}{}$$

$$= \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d} - \frac{2b\left(a+b\,\text{co}\right.}{}$$

Mathematica [A]
time = 0.10, size = 111, normalized size = 1.14

$$\frac{b^2(-1+c+dx)\coth^{-1}(c+dx)^2+2b\coth^{-1}(c+dx)\left(ac+adx-b\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)+a\left(ac+adx-2b\log\left(\frac{1}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)\right)+b^2\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] (b^2*(-1 + c + d*x)*ArcCoth[c + d*x]^2 + 2*b*ArcCoth[c + d*x]*(a*c + a*d*x
- b*Log[1 - E^(-2*ArcCoth[c + d*x])]) + a*(a*c + a*d*x - 2*b*Log[1/((c + d*
x)*Sqrt[1 - (c + d*x)^(-2)])]) + b^2*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/d
```

Maple [A]
time = 0.37, size = 186, normalized size = 1.92

method	result
derivativedivides	$\frac{(dx+c)a^2+\operatorname{arccoth}(dx+c)^2b^2(dx+c)+b^2\operatorname{arccoth}(dx+c)^2-2\operatorname{arccoth}(dx+c)\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)b^2-2\operatorname{arccoth}(dx+c)\ln$
default	$\frac{(dx+c)a^2+\operatorname{arccoth}(dx+c)^2b^2(dx+c)+b^2\operatorname{arccoth}(dx+c)^2-2\operatorname{arccoth}(dx+c)\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)b^2-2\operatorname{arccoth}(dx+c)\ln$
risch	$\frac{b^2(dx+c+1)\ln(dx+c+1)^2}{4d}+\left(-\frac{b^2x\ln(dx+c-1)}{2}+\frac{b(2adx-b\ln(dx+c-1)c+b\ln(dx+c-1))}{2d}\right)\ln(dx+c+1)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2+arccoth(d*x+c)^2*b^2*(d*x+c)+b^2*arccoth(d*x+c)^2-2*arccot
h(d*x+c)*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2-2*arccoth(d*x+c)*ln(1-1/((
d*x+c-1)/(d*x+c+1))^(1/2))*b^2-2*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*
b^2-2*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^2+2*a*b*(d*x+c)*arccoth(d*
x+c)+a*b*ln((d*x+c)^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*b^2*((d*x*log(d*x + c - 1)^2 + (d*x + c + 1)*log(d*x + c + 1)^2
 - 2*(d*x + c - 1)*log(d*x + c + 1)*log(d*x + c - 1))/d + integrate(2*(c^2
+ (c*d - 3*d)*x - 2*c + 1)*log(d*x + c - 1)/(d^2*x^2 + 2*c*d*x + c^2 - 1),
x)) + (2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a*b/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
[Out] integral(b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acoth}\,(c + dx))^2\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2,x)
[Out] Integral((a + b*acoth(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arccoth(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acoth}(c+d\,x)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^2,x)
[Out] int((a + b*acoth(c + d*x))^2, x)
```

3.112 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx$

Optimal. Leaf size=214

$$-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{b\left(a+b\coth^{-1}(c+\right.}{}$$

```
[Out] -(a+b*arccoth(d*x+c))^2*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))^2*ln(2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+b*(a+b*arccoth(d*x+c))*polylog(2,1-2/(d*x+c+
1))/f-b*(a+b*arccoth(d*x+c))*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1)
)/f+1/2*b^2*polylog(3,1-2/(d*x+c+1))/f-1/2*b^2*polylog(3,1-2*d*(f*x+e)/(-c*
f+d*e+f)/(d*x+c+1))/f
```

Rubi [A]
time = 0.10, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6247, 6060}

$$-\frac{b\left(a+b\coth^{-1}(c+dx)\right)\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{b\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{f}-\frac{b^2\text{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{b^2\text{Li}_3\left(1-\frac{2}{c+dx+1}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^2*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
 + d*x])^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (b*(a +
 b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 + c + d*x)])/f - (b*(a + b*ArcCoth
[c + d*x])*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])
/f + (b^2*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (b^2*PolyLog[3, 1 - (2*d
*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f)
```

Rule 6060

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{}{(de-}\right.}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.52, size = 1376, normalized size = 6.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x),x]
```

```
[Out] (24*a^2*f*Log[e + f*x] + 48*a*b*f*((ArcCoth[c + d*x] - ArcTanh[c + d*x])*Lo
g[e + f*x] + ArcTanh[c + d*x]*(-Log[1/Sqrt[1 - (c + d*x)^2]] + Log[I*Sinh[A
rcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]]]) + (-(Pi - (2*I)*ArcTanh[c + d*x
])^2 + 4*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])^2 - (4*I)*(Pi - (2*I)*
ArcTanh[c + d*x])*Log[1 + E^(2*ArcTanh[c + d*x])] + 8*(ArcTanh[(d*e - c*f)/
f] + ArcTanh[c + d*x])*Log[1 - E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c +
d*x]))] + 4*(I*Pi + 2*ArcTanh[c + d*x])*Log[2/Sqrt[1 - (c + d*x)^2]] - 8*(A
rcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x])*Log[(2*I)*Sinh[ArcTanh[(d*e - c*f
)/f] + ArcTanh[c + d*x]]] - 4*PolyLog[2, -E^(2*ArcTanh[c + d*x])] - 4*PolyL
og[2, E^(-2*(ArcTanh[(d*e - c*f)/f] + ArcTanh[c + d*x]))])/8) - b^2*(I*f*Pi
^3 - 16*d*e*ArcCoth[c + d*x]^3 + 16*f*ArcCoth[c + d*x]^3 + 16*c*f*ArcCoth[c
 + d*x]^3 + (16*d*e*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)
^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] - (16*c*f*Sqrt[(d^2*e^2 -
2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/
(d*e - c*f)] - (24*I)*f*Pi*ArcCoth[c + d*x]*Log[2] + 4*f*ArcCoth[c + d*x]^2
*Log[64] + (24*I)*f*Pi*ArcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x]) + E^ArcCo
th[c + d*x]] + 24*f*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 24
*f*ArcCoth[c + d*x]^2*Log[1 - E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])
] - 24*f*ArcCoth[c + d*x]^2*Log[1 + E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e -
c*f)])] - 24*f*ArcCoth[c + d*x]^2*Log[1 - E^(2*(ArcCoth[c + d*x] + ArcTanh[
f/(d*e - c*f)]))] - 48*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[(I/2)*
E^(-ArcCoth[c + d*x] - ArcTanh[f/(d*e - c*f)])*(-1 + E^(2*(ArcCoth[c + d*x]
```

```
 + ArcTanh[f/(d*e - c*f)]))] - 24*f*ArcCoth[c + d*x]^2*Log[-((d*e*(-1 + E^
(2*ArcCoth[c + d*x])) + (1 + c + E^(2*ArcCoth[c + d*x]) - c*E^(2*ArcCoth[c
+ d*x]))*f)/E^ArcCoth[c + d*x])] + 24*f*ArcCoth[c + d*x]^2*Log[(-(d*e*(-1 +
 E^(2*ArcCoth[c + d*x]))) + (-1 - E^(2*ArcCoth[c + d*x]) + c*(-1 + E^(2*Arc
Coth[c + d*x])))*f)/(d*e - (1 + c)*f)] - (24*I)*f*Pi*ArcCoth[c + d*x]*Log[1
/Sqrt[1 - (c + d*x)^(-2)]] + 24*f*ArcCoth[c + d*x]^2*Log[-((d*(e + f*x))/((
c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))] + 48*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e
 - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] + 24*f*ArcC
oth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] - 48*f*ArcCoth[c + d*x]*Pol
yLog[2, -E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] - 48*f*ArcCoth[c +
d*x]*PolyLog[2, E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] - 24*f*ArcCo
th[c + d*x]*PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] +
 24*f*ArcCoth[c + d*x]*PolyLog[2, (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/
(d*e - (1 + c)*f)] - 12*f*PolyLog[3, E^(2*ArcCoth[c + d*x])] + 48*f*PolyLog
[3, -E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] + 48*f*PolyLog[3, E^(Ar
cCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] + 12*f*PolyLog[3, E^(2*(ArcCoth[c
 + d*x] + ArcTanh[f/(d*e - c*f)]))] - 12*f*PolyLog[3, (E^(2*ArcCoth[c + d*x
])*(d*e + f - c*f))/(d*e - (1 + c)*f)]))/(24*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 5.34, size = 1766, normalized size = 8.25

method	result	size
derivativedivides	Expression too large to display	1766
default	Expression too large to display	1766

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a*b*d*ln(c*f-d*e-f*(d*x+c))/f*arccoth(d*x+c)-1/2*I*b^2*d/f*Pi*arccot
h(d*x+c)^2*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)
*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(
-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d
*x+c+1)-1))^2+a*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))
-a*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))-1/2*I*b^2*d/
f*Pi*arccoth(d*x+c)^2*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I*(f*c*(1/(d*x
+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1
)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))^2+1/2*I*b^2*d/f*Pi*arccoth(d*x+c)^2*csgn(I/
(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c
-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f))*csgn(I*(f*c*(1/(d*x+c-1
)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)
/(1/(d*x+c-1)*(d*x+c+1)-1))-b^2*d^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(
c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-b^2*d^2/f*e/(c*f-d*e-f)*arccoth
(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+1/2*I*b^2*d/
f*Pi*arccoth(d*x+c)^2*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(
```

```
d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))^3+
b^2*d*c/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(
c*f-d*e+f))+b^2*d*c/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c
-1)*(d*x+c+1)/(c*f-d*e+f))+1/2*b^2*d^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f
)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arccoth(d*
x+c)^2-b^2*d/f*arccoth(d*x+c)^2*ln(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c
-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)+b^2*d/f*arccoth(d*x+c)^2
*ln(1/(d*x+c-1)*(d*x+c+1)-1)-b^2*d/f*arccoth(d*x+c)^2*ln(1-1/((d*x+c-1)/(d*
x+c+1))^(1/2))-2*b^2*d/f*arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(
1/2))-b^2*d/f*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))-2*b^2*d/
f*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-1/2*b^2*d*c/(c*f
-d*e-f)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-b^2*d/(c*f-d
*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-b^
2*d/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c
*f-d*e+f))+a*b*d/f*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))-a*b*d/f*dilog((-f*(d*
x+c)-f)/(-c*f+d*e-f))+a^2*d*ln(c*f-d*e-f*(d*x+c))/f+2*b^2*d/f*polylog(3,1/(
(d*x+c-1)/(d*x+c+1))^(1/2))+2*b^2*d/f*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1
/2))+1/2*b^2*d/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d
*e+f)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^2*log(f*x + e)/f + integrate(1/4*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x
+ c) + 1))^2/(f*x + e) + a*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1))
/(f*x + e), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f*x + e),
 x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(c + dx))^2}{e + fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2/(f*x+e),x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2/(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2/(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acoth}(c + d\,x))^2}{e + f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^2/(e + f*x),x)
```

```
[Out] int((a + b*acoth(c + d*x))^2/(e + f*x), x)
```

3.113 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx$

Optimal. Leaf size=480

$$-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}-\frac{abd\log(1-c-dx)}{f(de+f-cf)}-\frac{b^2d\coth^{-1}(c+dx)\log\left(\right.}{f(de-f-cf)}$$

```
[Out] -(a+b*arccoth(d*x+c))^2/f/(f*x+e)+b^2*d*arccoth(d*x+c)*ln(2/(-d*x-c+1))/f/(
-c*f+d*e+f)-a*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*arccoth(d*x+c)*ln(2/(d*
x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*arccoth(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-f)/
(-c*f+d*e+f)+a*b*d*ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*ln(f*x+e)/(f^2-(-c*f+
d*e)^2)-2*b^2*d*arccoth(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f
+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e
+f)+1/2*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*polylog(2,1-2/(
d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e
+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 1.20, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {6245, 2007, 719, 31, 646, 6873, 6257, 720, 647, 6820, 12, 6857, 84, 6874, 6056, 2449, 2352, 6058, 2497}

$$-\frac{abd\log(-c-dx+1)}{f(-cf+de+f)}+\frac{abd\log(c+dx+1)}{f(-cf+de-f)}-\frac{2abd\log(e+fx)}{(-cf+de+f)(de-(c+1)f)}-\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)}+\frac{b^2d\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)}+\frac{b^2d\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{2f(-cf+de-f)}-\frac{b^2d\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{(-cf+de+f)(de-(c+1)f)}+\frac{b^2d\log\left(\frac{2}{-c-dx+1}\right)\coth^{-1}(c+dx)}{f(-cf+de+f)}-\frac{b^2d\log\left(\frac{2}{c+dx+1}\right)\coth^{-1}(c+dx)}{f(-cf+de-f)}+\frac{2b^2d\log\left(\frac{2}{c+dx+1}\right)\coth^{-1}(c+dx)}{(-cf+de+f)(de-(c+1)f)}-\frac{2b^2d\coth^{-1}(c+dx)\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])^2/(f*(e + f*x))) + (b^2*d*ArcCoth[c + d*x]*Log[2
/(1 - c - d*x)])/(f*(d*e + f - c*f)) - (a*b*d*Log[1 - c - d*x])/(f*(d*e + f
 - c*f)) - (b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d*e - f - c*f)
) + (2*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e -
 (1 + c)*f)) + (a*b*d*Log[1 + c + d*x])/(f*(d*e - f - c*f)) - (2*a*b*d*Log[
e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (2*b^2*d*ArcCoth[c + d*x]*L
og[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e
- (1 + c)*f)) + (b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*f*(d*
e + f - c*f)) + (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f
)) - (b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) + (b^2*d*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x
))])/((d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2007

```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6257

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
```

```
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)^2}\,dx = -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)(1-(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\int \frac{a+b\coth^{-1}(c+dx)}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2b)\text{Subst}\left(\int \frac{d(a+b\coth^{-1}(x))}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{(de-cf+fx)(1-x^2)}\,dx, x, c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{(2bd)\text{Subst}\left(\int \left(-\frac{a}{(-1+x)(1+x)(de-cf+fx)} - \frac{}{(-1+}\right.\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx, x, c+\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{(2abd)\text{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+}\right.\right.}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} - \frac{abd\log(1-c-dx)}{f(de+f-cf)} + \frac{abd\log(1+c+dx)}{f(de-f-cf)} -$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{f(e+fx)} + \frac{b^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} - \frac{abd\log(1-}{f(de+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.49, size = 470, normalized size = 0.98

$$\frac{-\frac{a^2}{f} + \frac{2ab\left((f-c^2f+d^2ex+cd(e-fx))\coth^{-1}(c+dx)-d(e+fx)\log\left(-\frac{d(e+fx)}{(c+dx)\sqrt{1-\frac{1}{(c+dx)^2}}}\right)\right)}{(de+f-cf)(de-(1+c)f)} + \frac{b^2d(e+fx)(1-(c+dx)^2)\left(\frac{e^{\tanh^{-1}\left(\frac{f}{-de+cf}\right)}\coth^{-1}(c+dx)^2}{(-de+cf)\sqrt{1-\frac{f^2}{(de-cf)^2}}} + \frac{\coth^{-1}(c+dx)^2}{de+dfx} + \frac{f\left(-i\pi\log\left(1+e^{2\coth^{-1}(c+dx)}\right)-2\tanh^{-1}\left(\frac{f}{-de+cf}\right)\log\left(1-e^{-2\left(\coth^{-1}(c+dx)+\tanh^{-1}\left(\frac{f}{de-cf}\right)\right)}\right)+\coth^{-1}(c+dx)\left(i\pi+2\tanh^{-1}\left(\frac{f}{de-cf}\right)+2\log\left(1-e^{-2\left(\coth^{-1}(c+dx)+\tanh^{-1}\left(\frac{f}{de-cf}\right)\right)}\right)\right)+i\pi\log\left(\frac{1}{\sqrt{1-\frac{1}{(c+dx)^2}}}\right)+2\tanh^{-1}\left(\frac{f}{-de+cf}\right)\log\left(i\sinh\left(\coth^{-1}(c+dx)+\tanh^{-1}\left(\frac{f}{de-cf}\right)\right)\right)-\text{PolyLog}\left(2,e^{-2\left(\coth^{-1}(c+dx)+\tanh^{-1}\left(\frac{f}{de-cf}\right)\right)}\right)\right)}{d^2e^2-2cdef+(-1+c^2)f^2}\right)}{(c+dx)^2\left(f-\frac{f}{(c+dx)^2}\right)}}{e+fx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^2/(e + f*x)^2,x]
```

```
[Out] (-(a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*ArcCoth[c + d*x]
- d*(e + f*x)*Log[-((d*(e + f*x))/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))]))/
((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(1 - (c + d*x)^2)*((
E^ArcTanh[f/(-(d*e) + c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1 - f^
2/(d*e - c*f)^2]) + ArcCoth[c + d*x]^2/(d*e + d*f*x) + (f*((-I)*Pi*Log[1 +
E^(2*ArcCoth[c + d*x])] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(ArcCot
h[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + ArcCoth[c + d*x]*(I*Pi + 2*ArcTanh
[f/(d*e - c*f)] + 2*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)
]))]) + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*
Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[2, E^(-2*(
ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 +
 c^2)*f^2)))/((c + d*x)^2*(f - f/(c + d*x)^2)))/(e + f*x)
```

Maple [A]
time = 1.57, size = 857, normalized size = 1.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*d^2/(c*f-d*e-f*(d*x+c))/f+b^2*d^2/(c*f-d*e-f*(d*x+c))/f*arccoth(d*
x+c)^2+2*b^2*d^2/f*arccoth(d*x+c)/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*b^2*d^2*a
rccoth(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))-2*b^2*d^2/f*arc
coth(d*x+c)/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)-1/2*b^2*d^2/f/(c*f-d*e-f)*dilog(1
/2*d*x+1/2*c+1/2)-1/2*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/
2)+1/4*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2+1/4*b^2*d^2/f/(c*f-d*e+f)*ln(d*x
+c+1)^2-1/2*b^2*d^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+1/2*b^
2*d^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+1/2*b^2*d^
2/f/(c*f-d*e+f)*dilog(1/2*d*x+1/2*c+1/2)+b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)*ln
(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))+b^2*d^2/(c*f-d*e-f)/(c*
f-d*e+f)*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))-b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)
*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))-b^2*d^2/(c*f-d*e-f)/
(c*f-d*e+f)*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))+2*a*b*d^2/(c*f-d*e-f*(d*x+c)
)/f*arccoth(d*x+c)+2*a*b*d^2/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-2*a*b*d^2/(c*f
-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))-2*a*b*d^2/f/(2*c*f-2*d*e+2*f)*ln(
d*x+c+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)*f^2
 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*arcco
th(d*x + c)/(f^2*x + f*e))*a*b - 1/4*b^2*(log(d*x + c + 1)^2/(f^2*x + f*e)
+ integrate(-((d*f*x + c*f + f)*log(d*x + c - 1)^2 + 2*(d*f*x + d*e - (d*f*
x + c*f + f)*log(d*x + c - 1))*log(d*x + c + 1))/(d*f^3*x^3 + (c*f^3 + 2*d*
f^2*e + f^3)*x^2 + (d*f*e^2 + 2*(c*f^2 + f^2)*e)*x + (c*f + f)*e^2), x)) -
a^2/(f^2*x + f*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="fricas")
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)/(f^2*x^2 +
 2*f*x*e + e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(c + dx))^2}{(e + fx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**2/(f*x+e)**2,x)
[Out] Integral((a + b*acoth(c + d*x))**2/(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^2/(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*arccoth(d*x + c) + a)^2/(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acoth}(c + d\,x))^2}{(e + f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^2/(e + f*x)^2,x)
[Out] int((a + b*acoth(c + d*x))^2/(e + f*x)^2, x)
```

3.114 $\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=546

$$\frac{ab^2f^2x}{d^2}+\frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3}-\frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3}+\frac{3bf(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3}+$$

```
[Out] a*b^2*f^2*x/d^2+b^3*f^2*(d*x+c)*arccoth(d*x+c)/d^3-1/2*b*f^2*(a+b*arccoth(d
*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(a+b*arccoth(d*x+c))^2/d^3+3*b*f*(-c*f+d*e)*(
d*x+c)*(a+b*arccoth(d*x+c))^2/d^3+1/2*b*f^2*(d*x+c)^2*(a+b*arccoth(d*x+c))^
2/d^3-1/3*(-c*f+d*e)*(d^2*e^2-2*c*d*e*f+(c^2+3)*f^2)*(a+b*arccoth(d*x+c))^3
/d^3/f+1/3*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arccoth(d*x+c))^3/d^3+1
/3*(f*x+e)^3*(a+b*arccoth(d*x+c))^3/f-6*b^2*f*(-c*f+d*e)*(a+b*arccoth(d*x+c
))*ln(2/(-d*x-c+1))/d^3-b*(3*d^2*e^2-6*c*d*e*f+(3*c^2+1)*f^2)*(a+b*arccoth(
d*x+c))^2*ln(2/(-d*x-c+1))/d^3+1/2*b^3*f^2*ln(1-(d*x+c)^2)/d^3-3*b^3*f*(-c*
f+d*e)*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^3-b^2*(3*d^2*e^2-6*c*d*e*f+(3*c^2
+1)*f^2)*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^3+1/2*b^3*(3*d^2*
e^2-6*c*d*e*f+(3*c^2+1)*f^2)*polylog(3,1-2/(-d*x-c+1))/d^3
```

Rubi [A]
time = 0.71, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6247, 6066, 6022, 6132, 6056, 2449, 2352, 6038, 6128, 266, 6096, 6196, 6206, 6745}

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a*b^2*f^2*x)/d^2 + (b^3*f^2*(c + d*x)*ArcCoth[c + d*x])/d^3 - (b*f^2*(a +
b*ArcCoth[c + d*x])^2)/(2*d^3) + (3*b*f*(d*e - c*f)*(a + b*ArcCoth[c + d*x]
)^2)/d^3 + (3*b*f*(d*e - c*f)*(c + d*x)*(a + b*ArcCoth[c + d*x])^2)/d^3 + (
b*f^2*(c + d*x)^2*(a + b*ArcCoth[c + d*x])^2)/(2*d^3) - ((d*e - c*f)*(d^2*e
^2 - 2*c*d*e*f + (3 + c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3*f) + ((3
*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^3)/(3*d^3)
 + ((e + f*x)^3*(a + b*ArcCoth[c + d*x])^3)/(3*f) - (6*b^2*f*(d*e - c*f)*(a
 + b*ArcCoth[c + d*x])*Log[2/(1 - c - d*x)])/d^3 - (b*(3*d^2*e^2 - 6*c*d*e*
f + (1 + 3*c^2)*f^2)*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d^3 +
 (b^3*f^2*Log[1 - (c + d*x)^2])/(2*d^3) - (3*b^3*f*(d*e - c*f)*PolyLog[2, -
((1 + c + d*x)/(1 - c - d*x))])/d^3 - (b^2*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*
c^2)*f^2)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d^3 + (
b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x
)])/(2*d^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)^2 \left(a+b\coth^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^2 \left(a+b\coth^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+bc}{d^3}\right.\right.}{}$$

$$= \frac{(e+fx)^3 \left(a+b\coth^{-1}(c+dx)\right)^3}{3f} - \frac{b\text{Subst}\left(\int \frac{((de-cf)(d^2e^2-2cdef}{}\right.}{}$$

$$= \frac{3bf(de-cf)(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} + \frac{bf^2(c+dx)^2 (a}{}$$

$$= \frac{3bf(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^3} + \frac{3bf(de-cf)(c+dx) (a}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} - \frac{bf^2\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^3} + \frac{3bf(de-cf)\left(a+b\coth\right.}{d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+d\right.}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+d\right.}{2d^3}$$

$$= \frac{ab^2f^2x}{d^2} + \frac{b^3f^2(c+dx)\coth^{-1}(c+dx)}{d^3} - \frac{bf^2\left(a+b\coth^{-1}(c+d\right.}{2d^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.95, size = 2594, normalized size = 4.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)^2*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c
+ d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b*c*
d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 - a
^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*c*d^2
```

```
*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^2 + 3*
a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*d^3) +
(3*a*b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d
*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x])]) - PolyLog[2, E^(
-2*ArcCoth[c + d*x])]))/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (3*a*b^2*e*f
*(1 - (c + d*x)^2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2
))*ArcCoth[c + d*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*
x]) + 4*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] - 2*Log[1/((c +
 d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])])
)/(d^2*(c + d*x)^2*(1 - (c + d*x)^(-2))) + (b^3*e^2*(1 - (c + d*x)^2)*((I/8
)*Pi^3 - ArcCoth[c + d*x]^3 - (c + d*x)*ArcCoth[c + d*x]^3 + 3*ArcCoth[c +
d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 3*ArcCoth[c + d*x]*PolyLog[2, E^(2
*ArcCoth[c + d*x])] - (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/2))/(d*(c + d*
x)^2*(1 - (c + d*x)^(-2))) - (b^3*e*f*(1 - (c + d*x)^2)*(I*c*Pi^3 - 12*ArcC
oth[c + d*x]^2 + 12*(c + d*x)*ArcCoth[c + d*x]^2 - 8*c*ArcCoth[c + d*x]^3 -
 8*c*(c + d*x)*ArcCoth[c + d*x]^3 + 4*(c + d*x)^2*(1 - (c + d*x)^(-2))*ArcC
oth[c + d*x]^3 - 24*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])] + 24*
c*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] + 12*PolyLog[2, E^(-2*
ArcCoth[c + d*x])] + 24*c*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x]
)] - 12*c*PolyLog[3, E^(2*ArcCoth[c + d*x])]))/(4*d^2*(c + d*x)^2*(1 - (c +
 d*x)^(-2))) - (a*b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 - (c + d*x)
^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (3*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth[c + d*x]^
2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]^2)/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*ArcCoth[c +
 d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Cosh[3*ArcCo
th[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + ArcCoth[c +
d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c
 + d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/((c + d*x)
*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/((c + d*x)*
Sqrt[1 - (c + d*x)^(-2)])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (4*(1 +
3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c + d*x)^(-2
))^(3/2)) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*ArcCoth[c +
 d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2
*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*x)*Sqrt[1
- (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]]))/(4*d^3) + (b^3*f^2*(1 - (c +
 d*x)^2)*(3*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])] + ((c + d*x)^3*(1 - (c +
d*x)^(-2))^(3/2)*(((-3*I)*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - ((9*
I)*c^2*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (24*ArcCoth[c + d*x])/S
qrt[1 - (c + d*x)^(-2)] - (72*c*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)
] - (48*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (216*c*A
rcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (24*ArcCoth[c + d
*x]^3)/Sqrt[1 - (c + d*x)^(-2)] + (24*c^2*ArcCoth[c + d*x]^3)/Sqrt[1 - (c +
```

```
 d*x)^(-2)] + (24*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
+ (96*c*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (72*c^2*
ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - 24*ArcCoth[c + d
*x]*Cosh[3*ArcCoth[c + d*x]] + 72*c*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d
*x]] - 8*ArcCoth[c + d*x]^3*Cosh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d
*x]^3*Cosh[3*ArcCoth[c + d*x]] + (432*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcC
oth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (72*ArcCoth[c + d*x]
^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) -
(216*c^2*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqr
t[1 - (c + d*x)^(-2)]) - (72*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])])/(
(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (96*(1 + 3*c^2)*ArcCoth[c + d*x]*Poly
Log[2, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c + d*x)^(-2))^(3/2)) -
(48*(1 + 3*c^2)*PolyLog[3, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c +
d*x)^(-2))^(3/2)) + I*Pi^3*Sinh[3*ArcCoth[c + d*x]] + (3*I)*c^2*Pi^3*Sinh[3
*ArcCoth[c + d*x]] - 72*c*ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 8*A
rcCoth[c + d*x]^3*Sinh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d*x]^3*Sinh
[3*ArcCoth[c + d*x]] - 144*c*ArcCoth[c + d*x]*L...
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 22.13, size = 10601, normalized size = 19.42

method	result	size
derivativedivides	Expression too large to display	10601
default	Expression too large to display	10601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*f^2*x^3 + a^3*f*x^2*e + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4
*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3
*c - 1)*log(d*x + c - 1)/d^4))*a^2*b*f^2 + 3/2*(2*x^2*arccoth(d*x + c) + d*
(2*x/d^2 - (c^2 + 2*c + 1)*log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x +
 c - 1)/d^3))*a^2*b*f*e + a^3*x*e^2 + 3/2*(2*(d*x + c)*arccoth(d*x + c) + l
og(-(d*x + c)^2 + 1))*a^2*b*e^2/d + 1/24*((b^3*d^3*f^2*x^3 + 3*b^3*d^3*f*x^
2*e + 3*b^3*d^3*x*e^2 + 3*(c*d^2 + d^2)*b^3*e^2 - 3*(c^2*d*f + 2*c*d*f + d*
```

```
f)*b^3*e + (c^3*f^2 + 3*c^2*f^2 + 3*c*f^2 + f^2)*b^3)*log(d*x + c + 1)^3 +
3*(2*a*b^2*d^3*f^2*x^3 + (6*a*b^2*d^3*f*e + b^3*d^2*f^2)*x^2 - 2*(2*b^3*c*d
*f^2 - 3*a*b^2*d^3*e^2 - 3*b^3*d^2*f*e)*x - (b^3*d^3*f^2*x^3 + 3*b^3*d^3*f*
x^2*e + 3*b^3*d^3*x*e^2 + 3*(c*d^2 - d^2)*b^3*e^2 - 3*(c^2*d*f - 2*c*d*f +
d*f)*b^3*e + (c^3*f^2 - 3*c^2*f^2 + 3*c*f^2 - f^2)*b^3)*log(d*x + c - 1))*l
og(d*x + c + 1)^2)/d^3 + integrate(-1/8*((b^3*d^3*f^2*x^3 + (c*d^2 + d^2)*b
^3*e^2 + (2*b^3*d^3*f*e + (c*d^2*f^2 + d^2*f^2)*b^3)*x^2 + (b^3*d^3*e^2 + 2
*(c*d^2*f + d^2*f)*b^3*e)*x)*log(d*x + c - 1)^3 - 6*(a*b^2*d^3*f^2*x^3 + (c
*d^2 + d^2)*a*b^2*e^2 + (2*a*b^2*d^3*f*e + (c*d^2*f^2 + d^2*f^2)*a*b^2)*x^2
 + (a*b^2*d^3*e^2 + 2*(c*d^2*f + d^2*f)*a*b^2*e)*x)*log(d*x + c - 1)^2 + (4
*a*b^2*d^3*f^2*x^3 + 2*(6*a*b^2*d^3*f*e + b^3*d^2*f^2)*x^2 - 3*(b^3*d^3*f^2
*x^3 + (c*d^2 + d^2)*b^3*e^2 + (2*b^3*d^3*f*e + (c*d^2*f^2 + d^2*f^2)*b^3)*
x^2 + (b^3*d^3*e^2 + 2*(c*d^2*f + d^2*f)*b^3*e)*x)*log(d*x + c - 1)^2 - 4*(
2*b^3*c*d*f^2 - 3*a*b^2*d^3*e^2 - 3*b^3*d^2*f*e)*x + 2*(3*(c^2*d*f - 2*c*d*
f + d*f)*b^3*e - (c^3*f^2 - 3*c^2*f^2 + 3*c*f^2 - f^2)*b^3 + (6*a*b^2*d^3*f
^2 - b^3*d^3*f^2)*x^3 + 3*(2*(c*d^2*f^2 + d^2*f^2)*a*b^2 + (4*a*b^2*d^3*f -
 b^3*d^3*f)*e)*x^2 + 3*(4*(c*d^2*f + d^2*f)*a*b^2*e + (2*a*b^2*d^3 - b^3*d^
3)*e^2)*x + 3*(2*(c*d^2 + d^2)*a*b^2 - (c*d^2 - d^2)*b^3)*e^2)*log(d*x + c
- 1))*log(d*x + c + 1))/(d^3*x + c*d^2 + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*f*x*e + (b^3*f^2*x^2 + 2*b^3*f*x*e + b^3*e^2)*
arccoth(d*x + c)^3 + a^3*e^2 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*f*x*e + a*b^2*e^2
)*arccoth(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*f*x*e + a^2*b*e^2)*arccot
h(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acoth}(c + dx))^3 (e + fx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**3*(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*(b*arccoth(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^2\,(a+b\,\mathrm{acoth}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a + b*acoth(c + d*x))^3,x)
```

```
[Out] int((e + f*x)^2*(a + b*acoth(c + d*x))^3, x)
```

3.115 $\int (e+fx) \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=326

$$\frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2}+\frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^3}{d^2}-\frac{(d^2e^{\cdots}}{}$$

```
[Out] 3/2*b*f*(a+b*arccoth(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*arccoth(d*x+c))^2/d
^2+(-c*f+d*e)*(a+b*arccoth(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2
)*(a+b*arccoth(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*arccoth(d*x+c))^3/f-3*b^2
*f*(a+b*arccoth(d*x+c))*ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*arccoth(d*
x+c))^2*ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*polylog(2,(-d*x-c-1)/(-d*x-c+1))/d^2
-3*b^2*(-c*f+d*e)*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d*x-c+1))/d^2+3/2*b^
3*(-c*f+d*e)*polylog(3,1-2/(-d*x-c+1))/d^2
```

Rubi [A]
time = 0.49, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$,
Rules used = {6247, 6066, 6022, 6132, 6056, 2449, 2352, 6196, 6096, 6206, 6745}

$$-\frac{3b^2(de-cf)\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^2}-\frac{3b^2f\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d^2}+\frac{\left(-\frac{(c^2+1)f}{d}+2ce-\frac{de^2}{f}\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{2d}+\frac{(de-cf)\left(a+b\coth^{-1}(c+dx)\right)^3}{d^2}-\frac{3b(de-cf)\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{d^2}+\frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2}+\frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2}+\frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f}+\frac{3b^3(de-cf)\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d^2}-\frac{3b^3f\text{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (3*b*f*(a + b*ArcCoth[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*ArcCot
h[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*ArcCoth[c + d*x])^3)/d^2 + ((2
*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*ArcCoth[c + d*x])^3)/(2*d) + ((e
 + f*x)^2*(a + b*ArcCoth[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*ArcCoth[c + d
*x])*Log[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*ArcCoth[c + d*x])^
2*Log[2/(1 - c - d*x)])/d^2 - (3*b^3*f*PolyLog[2, -((1 + c + d*x)/(1 - c -
d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*ArcCoth[c + d*x])*PolyLog[2, 1
- 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*PolyLog[3, 1 - 2/(1 - c - d*x)
])/(2*d^2)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[e^2*f + d^2*g, 0]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6066

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])^p/(e*(q + 1))), x] -
 Dist[b*c*(p/(e*(q + 1))), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6196

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
```

```
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (e+fx)\left(a+b\coth^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)\left(a+b\coth^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \left(-\frac{f^2(a+b\coth^{-1}(}{d^2}\right.\right.}{}$$

$$= \frac{(e+fx)^2\left(a+b\coth^{-1}(c+dx)\right)^3}{2f} - \frac{(3b)\text{Subst}\left(\int \frac{(d^2e^2-2cdef+(1+c}{}\right.}{}$$

$$= \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{(e+fx)^2\left(a+b\coth^{-1}(c\right.}{2f}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{2d^2}$$

$$= \frac{3bf\left(a+b\coth^{-1}(c+dx)\right)^2}{2d^2} + \frac{3bf(c+dx)\left(a+b\coth^{-1}(c+dx)\right)}{2d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.85, size = 600, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (2*a^2*(2*a*d*e + 3*b*f - 2*a*c*f)*(c + d*x) + 2*a^3*f*(c + d*x)^2 - 6*a^2*
b*(c + d*x)*(c*f - d*(2*e + f*x))*ArcCoth[c + d*x] + 3*a^2*b*(2*d*e + f - 2
*c*f)*Log[1 - c - d*x] + 3*a^2*b*(2*d*e - (1 + 2*c)*f)*Log[1 + c + d*x] + 1
2*a*b^2*f*((c + d*x)*ArcCoth[c + d*x] + ((-1 + (c + d*x)^2)*ArcCoth[c + d*x
```

```
]^2)/2 - Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])]) + 12*a*b^2*d*e*(ArcCo
th[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCoth[c +
d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) - 12*a*b^2*c*f*(ArcCoth[c +
d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*Log[1 - E^(-2*ArcCoth[c + d*x])])
 + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) + 2*b^3*f*(ArcCoth[c + d*x]*(3*(-1
+ c + d*x)*ArcCoth[c + d*x] + (-1 + c^2 + 2*c*d*x + d^2*x^2)*ArcCoth[c + d*
x]^2 - 6*Log[1 - E^(-2*ArcCoth[c + d*x])]) + 3*PolyLog[2, E^(-2*ArcCoth[c +
 d*x])]) + 4*b^3*d*e*((-1/8*I)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*ArcCot
h[c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 3*Arc
Coth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*ArcCo
th[c + d*x])])/2) - 4*b^3*c*f*((-1/8*I)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*
x)*ArcCoth[c + d*x]^3 - 3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])
] - 3*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E
^(2*ArcCoth[c + d*x])])/2))/(4*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.31, size = 12037, normalized size = 36.92

method	result	size
derivativedivides	Expression too large to display	12037
default	Expression too large to display	12037

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*a^3*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c) + d*(2*x/d^2 - (c^2 + 2*c + 1)*
log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*log(d*x + c - 1)/d^3))*a^2*b*f + a^3
*x*e + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b*e/d
 + 1/16*((b^3*d^2*f*x^2 + 2*b^3*d^2*x*e + 2*(c*d + d)*b^3*e - (c^2*f + 2*c*
f + f)*b^3)*log(d*x + c + 1)^3 + 3*(2*a*b^2*d^2*f*x^2 + 2*(2*a*b^2*d^2*e +
b^3*d*f)*x - (b^3*d^2*f*x^2 + 2*b^3*d^2*x*e + 2*(c*d - d)*b^3*e - (c^2*f -
2*c*f + f)*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2)/d^2 + integrate(-1/8*
((b^3*d^2*f*x^2 + (c*d + d)*b^3*e + (b^3*d^2*e + (c*d*f + d*f)*b^3)*x)*log(
d*x + c - 1)^3 - 6*(a*b^2*d^2*f*x^2 + (c*d + d)*a*b^2*e + (a*b^2*d^2*e + (c
*d*f + d*f)*a*b^2)*x)*log(d*x + c - 1)^2 + 3*(2*a*b^2*d^2*f*x^2 - (b^3*d^2*
```

```
f*x^2 + (c*d + d)*b^3*e + (b^3*d^2*e + (c*d*f + d*f)*b^3)*x)*log(d*x + c -
1)^2 + 2*(2*a*b^2*d^2*e + b^3*d*f)*x + ((c^2*f - 2*c*f + f)*b^3 + (4*a*b^2*
d^2*f - b^3*d^2*f)*x^2 + 2*(2*(c*d*f + d*f)*a*b^2 + (2*a*b^2*d^2 - b^3*d^2)
*e)*x + 2*(2*(c*d + d)*a*b^2 - (c*d - d)*b^3)*e)*log(d*x + c - 1))*log(d*x
+ c + 1))/(d^2*x + c*d + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
[Out] integral(a^3*f*x + (b^3*f*x + b^3*e)*arccoth(d*x + c)^3 + a^3*e + 3*(a*b^2*
f*x + a*b^2*e)*arccoth(d*x + c)^2 + 3*(a^2*b*f*x + a^2*b*e)*arccoth(d*x + c
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}{(c + dx)})^3 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**3,x)
[Out] Integral((a + b*acoth(c + d*x))**3*(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f\,x)\,\left(a + b\,\mathrm{acoth}(c + d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a + b*acoth(c + d*x))^3,x)
[Out] int((e + f*x)*(a + b*acoth(c + d*x))^3, x)
```

3.116 $\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=132

$$\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2\log\left(\frac{2}{1-c-dx}\right)}{d}-\frac{3b^2(a-$$

```
[Out] (a+b*arccoth(d*x+c))^3/d+(d*x+c)*(a+b*arccoth(d*x+c))^3/d-3*b*(a+b*arccoth(
d*x+c))^2*ln(2/(-d*x-c+1))/d-3*b^2*(a+b*arccoth(d*x+c))*polylog(2,1-2/(-d*x
-c+1))/d+3/2*b^3*polylog(3,1-2/(-d*x-c+1))/d
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6239, 6022, 6132, 6056, 6096, 6206, 6745}

$$-\frac{3b^2\text{Li}_2\left(1-\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\log\left(\frac{2}{-c-dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{d}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{-c-dx+1}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (a + b*ArcCoth[c + d*x])^3/d + ((c + d*x)*(a + b*ArcCoth[c + d*x])^3)/d - (
3*b*(a + b*ArcCoth[c + d*x])^2*Log[2/(1 - c - d*x)])/d - (3*b^2*(a + b*ArcC
oth[c + d*x])*PolyLog[2, 1 - 2/(1 - c - d*x)])/d + (3*b^3*PolyLog[3, 1 - 2/
(1 - c - d*x)])/(2*d)
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6132

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(c+dx)\right)^3\,dx=\frac{\mathrm{Subst}\left(\int\left(a+b\coth^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\mathrm{Subst}\left(\int\frac{x\left(a+b\coth^{-1}(x)\right)^2}{1-x^2}\,dx,x,c+\right.}{d}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\mathrm{Subst}\left(\right.}{}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\mathrm{co}\right.}{}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\mathrm{co}\right.}{}$$

$$=\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\coth^{-1}(c+dx)\right)^3}{d}-\frac{3b\left(a+b\,\mathrm{co}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 208, normalized size = 1.58

$$\frac{2a^3(c+dx)+6a^2b(c+dx)\coth^{-1}(c+dx)+3a^2b\log\left(1-(c+dx)^2\right)+6ab^2\left(\coth^{-1}(c+dx)\left((-1+c+dx)\coth^{-1}(c+dx)-2\log\left(1-e^{-2\coth^{-1}(c+dx)}\right)\right)+\text{PolyLog}\left(2,e^{-2\coth^{-1}(c+dx)}\right)\right)+2b^3\left(-\frac{i\pi^3}{8}+\coth^{-1}(c+dx)^3+(c+dx)\coth^{-1}(c+dx)^3-3\coth^{-1}(c+dx)^2\log\left(1-e^{2\coth^{-1}(c+dx)}\right)-3\coth^{-1}(c+dx)\text{PolyLog}\left(2,e^{2\coth^{-1}(c+dx)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2\coth^{-1}(c+dx)}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] (2*a^3*(c + d*x) + 6*a^2*b*(c + d*x)*ArcCoth[c + d*x] + 3*a^2*b*Log[1 - (c
+ d*x)^2] + 6*a*b^2*(ArcCoth[c + d*x]*((-1 + c + d*x)*ArcCoth[c + d*x] - 2*
Log[1 - E^(-2*ArcCoth[c + d*x])]) + PolyLog[2, E^(-2*ArcCoth[c + d*x])]) +
2*b^3*((-1/8*I)*Pi^3 + ArcCoth[c + d*x]^3 + (c + d*x)*ArcCoth[c + d*x]^3 -
3*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])] - 3*ArcCoth[c + d*x]*P
olyLog[2, E^(2*ArcCoth[c + d*x])] + (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/
2))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(130) = 260.
time = 0.65, size = 408, normalized size = 3.09

method	result
derivativedivides	$\frac{(dx+c)a^3+\operatorname{arccoth}(dx+c)^3b^3(dx+c)+b^3\operatorname{arccoth}(dx+c)^3-3\operatorname{arccoth}(dx+c)^2\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)b^3-3\operatorname{arccoth}(dx+c)^2}{}$
default	$\frac{(dx+c)a^3+\operatorname{arccoth}(dx+c)^3b^3(dx+c)+b^3\operatorname{arccoth}(dx+c)^3-3\operatorname{arccoth}(dx+c)^2\ln\left(1+\frac{1}{\sqrt{\frac{dx+c-1}{dx+c+1}}}\right)b^3-3\operatorname{arccoth}(dx+c)^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+arccoth(d*x+c)^3*b^3*(d*x+c)+b^3*arccoth(d*x+c)^3-3*arccot
h(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3-3*arccoth(d*x+c)^2*ln(1-
1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3-6*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)
/(d*x+c+1))^(1/2))*b^3-6*arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(
1/2))*b^3+6*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))*b^3+6*polylog(3,1/((d
*x+c-1)/(d*x+c+1))^(1/2))*b^3+3*arccoth(d*x+c)^2*a*b^2*(d*x+c)+3*a*b^2*arcc
oth(d*x+c)^2-6*arccoth(d*x+c)*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2))*a*b^2-6*a
rccoth(d*x+c)*ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2))*a*b^2-6*polylog(2,-1/((d*
x+c-1)/(d*x+c+1))^(1/2))*a*b^2-6*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))*a
*b^2+3*a^2*b*(d*x+c)*arccoth(d*x+c)+3/2*a^2*b*ln((d*x+c)^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*a^2*b/d
+ 1/8*((b^3*d*x + b^3*(c + 1))*log(d*x + c + 1)^3 + 3*(2*a*b^2*d*x - (b^3*d
*x + b^3*(c - 1))*log(d*x + c - 1))*log(d*x + c + 1)^2)/d + integrate(-1/8*
((b^3*d*x + b^3*(c + 1))*log(d*x + c - 1)^3 - 6*(a*b^2*d*x + a*b^2*(c + 1))
*log(d*x + c - 1)^2 + 3*(4*a*b^2*d*x - (b^3*d*x + b^3*(c + 1))*log(d*x + c
- 1)^2 + 2*(2*a*b^2*(c + 1) - b^3*(c - 1) + (2*a*b^2*d - b^3*d)*x)*log(d*x
+ c - 1))*log(d*x + c + 1))/(d*x + c + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arcc
oth(d*x + c) + a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acoth}(c + dx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acoth}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^3,x)
```

```
[Out] int((a + b*acoth(c + d*x))^3, x)
```

3.117 $\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$

Optimal. Leaf size=308

$$-\frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2}{1+c+dx}\right)}{f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}+\frac{3b\left(a+b\coth^{-1}(c\right.}{}$$

```
[Out] -(a+b*arccoth(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))^3*ln(2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2/(
d*x+c+1))/f-3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+
f)/(d*x+c+1))/f+3/2*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2/(d*x+c+1))/f-3/2
*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(-c*f+d
*e+f)/(d*x+c+1))/f
```

Rubi [A]
time = 0.13, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6247, 6062}

$$-\frac{3b^2\left(a+b\coth^{-1}(c+dx)\right)\text{Li}_3\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{3b^2\text{Li}_3\left(1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)}{2f}-\frac{3b\left(a+b\coth^{-1}(c+dx)\right)^2\text{Li}_2\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}+\frac{\left(a+b\coth^{-1}(c+dx)\right)^3\log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}+\frac{3b\text{Li}_2\left(1-\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^2}{2f}-\frac{\log\left(\frac{2}{c+dx+1}\right)\left(a+b\coth^{-1}(c+dx)\right)^3}{f}-\frac{3b^3\text{Li}_4\left(1-\frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{4f}+\frac{3b^3\text{Li}_4\left(1-\frac{2}{c+dx+1}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c
+ d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a
 + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a +
 b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 +
 c + d*x))])/(2*f) + (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - 2/(1 +
c + d*x)])/(2*f) - (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - (2*d*(e +
 f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*f) + (3*b^3*PolyLog[4, 1 - 2/(1
 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f
)*(1 + c + d*x))])/(4*f)
```

Rule 6062

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^3/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^3)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^3*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[3*b*
(a + b*ArcCoth[c*x])^2*(PolyLog[2, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[3*b*(
a + b*ArcCoth[c*x])^2*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))
]/(2*e)), x] + Simp[3*b^2*(a + b*ArcCoth[c*x])*(PolyLog[3, 1 - 2/(1 + c*x)]
/(2*e)), x] - Simp[3*b^2*(a + b*ArcCoth[c*x])*(PolyLog[3, 1 - 2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(2*e)), x] + Simp[3*b^3*(PolyLog[4, 1 - 2/(1 + c*
```

```
x)]/(4*e)), x] - Simp[3*b^3*(PolyLog[4, 1 - 2*c*((d + e*x)/((c*d + e)*(1 +
c*x)))]/(4*e)), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^3}{\frac{de-cf}{d}+\frac{fx}{d}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3 \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{\left(a+b\coth^{-1}(c+dx)\right)^3 \log\left(\frac{\cdots}{(de+f\cdots}\right.}{f}$$

Mathematica [F]
time = 20.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x),x]
```

```
[Out] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.15, size = 3624, normalized size = 11.77

method	result	size
derivativedivides	Expression too large to display	3624
default	Expression too large to display	3624

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3/(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/2*a^2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))+3
/2*a^2*b*d/f*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(-c*f+d*e+f))-3/2*b^3*
```

d*c/(c*f-d*e-f)*arccoth(d*x+c)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c
*f-d*e+f))-3/4*b^3*d^2/f*e/(c*f-d*e-f)*polylog(4,(c*f-d*e-f)/(d*x+c-1)*(d*x
+c+1)/(c*f-d*e+f))+b^3*d*c/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1-(c*f-d*e-f)/(d
x+c-1)(d*x+c+1)/(c*f-d*e+f))+3/2*b^3*d*c/(c*f-d*e-f)*arccoth(d*x+c)^2*pol
ylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3*a^2*b*d*ln(c*f-d*e-f*
(d*x+c))/f*arccoth(d*x+c)+3*a*b^2*d*ln(c*f-d*e-f*(d*x+c))/f*arccoth(d*x+c)^
2-3*a*b^2*d/f*arccoth(d*x+c)^2*ln(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-
1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)+3*a*b^2*d/f*arccoth(d*x+c
)^2*ln(1/(d*x+c-1)*(d*x+c+1)-1)-3*a*b^2*d/f*arccoth(d*x+c)^2*ln(1-1/((d*x+c
-1)/(d*x+c+1))^(1/2))-6*a*b^2*d/f*arccoth(d*x+c)*polylog(2,1/((d*x+c-1)/(d*
x+c+1))^(1/2))-3*a*b^2*d/f*arccoth(d*x+c)^2*ln(1+1/((d*x+c-1)/(d*x+c+1))^(1
/2))-6*a*b^2*d/f*arccoth(d*x+c)*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-3
/2*a*b^2*d*c/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e
+f))-3*a*b^2*d/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x
+c+1)/(c*f-d*e+f))-3*a*b^2*d/(c*f-d*e-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-
f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3*a*b^2*d*c/(c*f-d*e-f)*arccoth(d*x+c)^
2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3*a*b^2*d*c/(c*f-d*e-f)
*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3/2*
a*b^2*d^2/f*e/(c*f-d*e-f)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*
e+f))-b^3*d^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^3*ln(1-(c*f-d*e-f)/(d*x+c-1)*(
d*x+c+1)/(c*f-d*e+f))-3/2*b^3*d^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*polylog(
2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3/2*b^3*d^2/f*e/(c*f-d*e-f)*
arccoth(d*x+c)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+1/2*I
*b^3*d/f*Pi*arccoth(d*x+c)^3*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x
+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)
-1))^3+1/2*I*b^3*d/f*Pi*arccoth(d*x+c)^3*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*
csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d
x+c-1)(d*x+c+1)-1)*f))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1
)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))
-3/2*I*a*b^2*d/f*Pi*arccoth(d*x+c)^2*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+
(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f))*csgn(I*(f*c*(
1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+
c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))^2-3/2*I*a*b^2*d/f*Pi*arccoth(d*x+c)^2
*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-
1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*
x+c+1)-1))^2-1/2*I*b^3*d/f*Pi*arccoth(d*x+c)^3*csgn(I*(f*c*(1/(d*x+c-1)*(d*
x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f))*csg
n(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+
c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))^2-1/2*I*b^3*d/f*Pi*arccoth(
d*x+c)^3*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+
1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+
c-1)*(d*x+c+1)-1))^2-3*a*b^2*d^2/f*e/(c*f-d*e-f)*arccoth(d*x+c)^2*ln(1-(c*f
-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3*a*b^2*d^2/f*e/(c*f-d*e-f)*arccot
h(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3/2*I*a*b^2
*d/f*Pi*arccoth(d*x+c)^2*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1

```
)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))
^3+3/2*I*a*b^2*d/f*Pi*arccoth(d*x+c)^2*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*cs
gn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x
+c-1)*(d*x+c+1)-1)*f))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*
(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))+6
*a*b^2*d/f*polylog(3,1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*a*b^2*d/f*polylog(3,-
1/((d*x+c-1)/(d*x+c+1))^(1/2))+3/2*a*b^2*d/(c*f-d*e-f)*polylog(3,(c*f-d*e-f
)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-6*b^3*d/f*polylog(4,1/((d*x+c-1)/(d*x+c+
1))^(1/2))-6*b^3*d/f*polylog(4,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-3/4*b^3*d/(c
*f-d*e-f)*polylog(4,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3/2*a^2*b*
d/f*dilog((-f*(d*x+c)-f)/(-c*f+d*e-f))+3/2*a^2*b*d/f*dilog((-f*(d*x+c)+f)/(
-c*f+d*e+f))+3/2*b^3*d/(c*f-d*e-f)*arccoth(d*x+c)*polylog(3,(c*f-d*e-f)/(d*
x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3/2*b^3*d/(c*f-d*e-f)*arccoth(d*x+c)^2*polylo
g(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-b^3*d/(c*f-d*e-f)*arccoth(
d*x+c)^3*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3/4*b^3*d*c/(c*f
-d*e-f)*polylog(4,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3*b^3*d/f*ar
ccoth(d*x+c)^2*polylog(2,-1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*b^3*d/f*arccoth(
d*x+c)*polylog(3,-1/((d*x+c-1)/(d*x+c+1))^(1/2))-3*b^3*d/f*arccoth(d*x+c)^2
*polylog(2,1/((d*x+c-1)/(d*x+c+1))^(1/2))+6*b^3*d/f*arccoth(d*x+c)*polylog(
3,1/((d*x+c-1)/(d*x+c+1))^(1/2))+b^3*d/f*arccot...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="maxima")
```

```
[Out] a^3*log(f*x + e)/f + integrate(1/8*b^3*(log(1/(d*x + c) + 1) - log(-1/(d*x
+ c) + 1))^3/(f*x + e) + 3/4*a*b^2*(log(1/(d*x + c) + 1) - log(-1/(d*x + c)
 + 1))^2/(f*x + e) + 3/2*a^2*b*(log(1/(d*x + c) + 1) - log(-1/(d*x + c) + 1
))/(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)/(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(c + dx))^3}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3/(f*x+e),x)
[Out] Integral((a + b*acoth(c + d*x))**3/(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="giac")
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\operatorname{acoth}(c + d\,x))^3}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^3/(e + f*x),x)
[Out] int((a + b*acoth(c + d*x))^3/(e + f*x), x)
```

3.118 $\int \frac{(a+b\coth^{-1}(c+dx))^3}{(e+fx)^2}\,dx$

Optimal. Leaf size=1089

$$-\frac{(a+b\coth^{-1}(c+dx))^3}{f(e+fx)}+\frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)}+\frac{3b^3d\coth^{-1}(c+dx)^2\log\left(\frac{2}{1-c-dx}\right)}{2f(de+f-cf)}-\frac{3a^2bd\log}{2f(de}$$

```
[Out] -(a+b*arccoth(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*arccoth(d*x+c)*ln(2/(-d*x-c+1))
/f/(-c*f+d*e+f)+3/2*b^3*d*arccoth(d*x+c)^2*ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-
3/2*a^2*b*d*ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*arccoth(d*x+c)*ln(2/(d*x+
c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*arccoth(d*x+c)*ln(2/(d*x+c+1))/(-c*f+d*e-f)/
(-c*f+d*e+f)-3/2*b^3*d*arccoth(d*x+c)^2*ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+3*b^
3*d*arccoth(d*x+c)^2*ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^2*b*d*
ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*a*b^2*d
*arccoth(d*x+c)*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d
*e+f)-3*b^3*d*arccoth(d*x+c)^2*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f
+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*polylog(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d
*e+f)+3/2*b^3*d*arccoth(d*x+c)*polylog(2,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2
*a*b^2*d*polylog(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*polylog(2,1-2/(d
*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*arccoth(d*x+c)*polylog(2,1-2/(
d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*arccoth(d*x+c)*polylog(2,1-2/(d*x+c+1))/(-
c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x
+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*arccoth(d*x+c)*polylog(2,1-2*d*(f*
x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4*b^3*d*polylog(3,
1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*polylog(3,1-2/(d*x+c+1))/f/(-c*f+d
*e-f)-3/2*b^3*d*polylog(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*
d*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)
```

Rubi [A]
time = 1.94, antiderivative size = 1094, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {6245, 6873, 6257, 6820, 12, 6857, 84, 6874, 6056, 2449, 2352, 6058, 2497, 6096, 6206, 6745, 6204, 6060}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] -((a + b*ArcCoth[c + d*x])^3/(f*(e + f*x))) + (3*a*b^2*d*ArcCoth[c + d*x]*L
og[2/(1 - c - d*x)])/(f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[
2/(1 - c - d*x)])/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f
*(d*e + f - c*f)) - (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d
```

```
*e - f - c*f)) + (6*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e +
f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*
x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x
)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f
*(d*e - f - c*f)) - (3*a^2*b*d*Log[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (6*a*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1
 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d
*x]^2*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f
)*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x))
])/(2*f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 -
c - d*x)])/(2*f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x
)])/(2*f*(d*e - f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d
*e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 -
 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcCoth[c + d*x]*PolyLo
g[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d
*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f
 - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - (2*d*
(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((d*e + f - c*f)*(d*e - (1 + c
)*f)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + (
3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*P
olyLog[3, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*
b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/(2*(
d*e + f - c*f)*(d*e - (1 + c)*f))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6056

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6060

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 Simp[(-(a + b*ArcCoth[c*x])^2)*(Log[2/(1 + c*x)]/e), x] + (Simp[(a + b*Arc
Coth[c*x])^2*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + Simp[b*(a
 + b*ArcCoth[c*x])*(PolyLog[2, 1 - 2/(1 + c*x)]/e), x] - Simp[b*(a + b*ArcC
oth[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x] + S
imp[b^2*(PolyLog[3, 1 - 2/(1 + c*x)]/(2*e)), x] - Simp[b^2*(PolyLog[3, 1 -
2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/(2*e)), x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6206

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
+ e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6245

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcCoth[c + d*x])^p/(f*(m
+ 1))), x] - Dist[b*d*(p/(f*(m + 1))), Int[(e + f*x)^(m + 1)*((a + b*ArcCot
h[c + d*x])^(p - 1)/(1 - (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6257

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcCoth[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{(e+fx)^2}\,dx = -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)(1-(c+dx)^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\int \frac{\left(a+b\coth^{-1}(c+dx)\right)^2}{(e+fx)(1-c^2-2cdx-d^2x^2)}\,dx}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)(1-x^2)}\,dx,x,c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3b)\text{Subst}\left(\int \frac{d\left(a+b\coth^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx,x,c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \frac{\left(a+b\coth^{-1}(x)\right)^2}{(de-cf+fx)(1-x^2)}\,dx,x,c+dx\right)}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{(3bd)\text{Subst}\left(\int \left(-\frac{a^2}{(-1+x)(1+x)(de-cf+fx)} - \frac{2}{(-1+x)}\right.\right.}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)(de-cf+fx)}\,dx,x,c+\right.}{f}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{(3a^2bd)\,\text{Subst}\left(\int \left(\frac{1}{2(de+f-cf)(-1+x)} + \frac{1}{2(-de+(1+c}\right.\right.}{}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} - \frac{3a^2bd\log(1-c-dx)}{2f(de+f-cf)} + \frac{3a^2bd\log(1+c+dx)}{2f(de-f-cf)}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth}{2}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth}{2}$$

$$= -\frac{\left(a+b\coth^{-1}(c+dx)\right)^3}{f(e+fx)} + \frac{3ab^2d\coth^{-1}(c+dx)\log\left(\frac{2}{1-c-dx}\right)}{f(de+f-cf)} + \frac{3b^3d\coth}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.45, size = 1818, normalized size = 1.67

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])^3/(e + f*x)^2,x]
```

```
[Out] -(a^3/(f*(e + f*x))) - (3*a^2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (3*a^2*b*
d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])
/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f -
 f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] +
(d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((E^ArcTanh[f/(-(d*e) +
 c*f)]*ArcCoth[c + d*x]^2)/((-(d*e) + c*f)*Sqrt[1 - f^2/(d*e - c*f)^2]) + A
rcCoth[c + d*x]^2/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(f/Sqrt[1 - (c + d*x)
^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) + (f*(I*Pi*ArcC
oth[c + d*x] + 2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)] - I*Pi*Log[1 + E^(
2*ArcCoth[c + d*x])] + 2*ArcCoth[c + d*x]*Log[1 - E^(-2*(ArcCoth[c + d*x] +
 ArcTanh[f/(d*e - c*f)]))] - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(Arc
Coth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(
-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f
/(d*e - c*f)]]] - PolyLog[2, E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f
)]))]))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*f*(e + f*x)^2) - (b^3*(
1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[
1 - (c + d*x)^(-2)]))^2*((d*ArcCoth[c + d*x]^3)/(f*(c + d*x)*Sqrt[1 - (c +
d*x)^(-2)]*(-(f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c +
d*x)^(-2)]) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) - (d*(2*d*e*ArcC
oth[c + d*x]^3 - 6*f*ArcCoth[c + d*x]^3 - 2*c*f*ArcCoth[c + d*x]^3 - (4*d*e
*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x
]^3)/E^ArcTanh[f/(d*e - c*f)] + (4*c*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^
2)*f^2)/(d*e - c*f)^2]*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)
*f*Pi*ArcCoth[c + d*x]*Log[2] - f*ArcCoth[c + d*x]^2*Log[64] - (6*I)*f*Pi*A
rcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x]) + E^ArcCoth[c + d*x]] + 6*f*ArcCo
th[c + d*x]^2*Log[1 - E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] + 6*f*
ArcCoth[c + d*x]^2*Log[1 + E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] +
 6*f*ArcCoth[c + d*x]^2*Log[1 - E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c
*f)]))] + 12*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[(I/2)*E^(-ArcCot
h[c + d*x] - ArcTanh[f/(d*e - c*f)])*(-1 + E^(2*(ArcCoth[c + d*x] + ArcTanh
[f/(d*e - c*f)])))] + 6*f*ArcCoth[c + d*x]^2*Log[-((d*e*(-1 + E^(2*ArcCoth[
c + d*x])) + (1 + c + E^(2*ArcCoth[c + d*x]) - c*E^(2*ArcCoth[c + d*x]))*f)
/E^ArcCoth[c + d*x])] - 6*f*ArcCoth[c + d*x]^2*Log[(-(d*e*(-1 + E^(2*ArcCot
h[c + d*x]))) + (-1 - E^(2*ArcCoth[c + d*x]) + c*(-1 + E^(2*ArcCoth[c + d*x
])))*f)/(d*e - (1 + c)*f)] + (6*I)*f*Pi*ArcCoth[c + d*x]*Log[1/Sqrt[1 - (c
+ d*x)^(-2)]] - 6*f*ArcCoth[c + d*x]^2*Log[-(f/Sqrt[1 - (c + d*x)^(-2)]) -
(d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (c*f)/((c + d*x)*Sqrt[1 - (c +
```

```
 d*x)^(-2)])] - 12*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[I*Sinh[Arc
Coth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] + 12*f*ArcCoth[c + d*x]*PolyLog[2,
 -E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] + 12*f*ArcCoth[c + d*x]*Po
lyLog[2, E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] + 6*f*ArcCoth[c + d
*x]*PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 6*f*Arc
Coth[c + d*x]*PolyLog[2, (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1
 + c)*f)] - 12*f*PolyLog[3, -E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]
 - 12*f*PolyLog[3, E^(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])] - 3*f*Pol
yLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 3*f*PolyLog[3,
 (E^(2*ArcCoth[c + d*x])*(d*e + f - c*f))/(d*e - (1 + c)*f)]))/(2*f*(d*e +
f - c*f)*(d*e - (1 + c)*f))))/(d^2*(e + f*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.29, size = 4599, normalized size = 4.22

method	result	size
derivativedivides	Expression too large to display	4599
default	Expression too large to display	4599

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*d^2/(c*f-d*e-f*(d*x+c))/f+3*a*b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)*dilo
g((-f*(d*x+c)-f)/(-c*f+d*e-f))+3/4*I*b^3*d^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arcc
oth(d*x+c)^2*Pi*e*csgn(I/(d*x+c-1)*(d*x+c+1))^3+3/4*I*b^3*d^3/f/(c*f-d*e-f)
/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I/(d*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*
(d*x+c+1)-1))^3+3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c
*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I/(d*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*
(d*x+c+1)-1))^2-3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c
*csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))^2*csgn(I/(d*x+c-1)*(d*x+c+1))+3/2*I*b^
3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I/((d*x+c-1)/(d*x+
c+1))^(1/2))*csgn(I/(d*x+c-1)*(d*x+c+1))^2+3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d
*e+f)*arccoth(d*x+c)^2*Pi*c*csgn(I/(d*x+c-1)*(d*x+c+1))*csgn(I/(d*x+c-1)*(d
*x+c+1)/(1/(d*x+c-1)*(d*x+c+1)-1))^2+3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*
arccoth(d*x+c)^2*Pi*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I/(d*x+c-1)*(d*x
+c+1))*csgn(I/(d*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*(d*x+c+1)-1))-3/2*I*b^3*d^2/
(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I/(1/(d*x+c-1)*(d*x+c+1)-1
))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1
/(d*x+c-1)*(d*x+c+1)-1)*f))*csgn(I*(f*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+
c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-
1))+3/2*b^3*d^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)*c*polylog(3,(c*f-d*e-f)/(d*x+c-
1)*(d*x+c+1)/(c*f-d*e+f))+3*b^3*d^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)*arccoth(d*x
+c)^2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3*b^3*d^2*f/(c*f-d*
e-f)^2/(c*f-d*e+f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)
/(c*f-d*e+f))+3*b^3*d^3/(c*f-d*e-f)^2/(c*f-d*e+f)*e*arccoth(d*x+c)^2*ln(1-(
```

```
c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3*b^3*d^3/(c*f-d*e-f)^2/(c*f-d*
e+f)*e*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f)
)+3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I/(d*x+c-1
)*(d*x+c+1))^3+3/4*a*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)^2+3/4*a*b^2*d^2/f/(c
*f-d*e+f)*ln(d*x+c+1)^2+3/2*a*b^2*d^2/f/(c*f-d*e+f)*dilog(1/2*d*x+1/2*c+1/2
)+3*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*ln(f*c*(1/(d*x+c-1)*(d
*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(d*x+c+1)-1)*f)-3*b
^3*d^2*arccoth(d*x+c)^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))-3*b^3
*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*ln(2)-3/2*b^3*d^3/(c*f-d*e-f)
^2/(c*f-d*e+f)*e*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3*b
^3*d^2/f*arccoth(d*x+c)^2/(2*c*f-2*d*e+2*f)*ln(d*x+c+1)+3*b^3*d^2/f*arccoth
(d*x+c)^2/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-3/2*b^3*d^2*f/(c*f-d*e-f)^2/(c*f-d*
e+f)*polylog(3,(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))-3/2*b^3*d^2/f*a
rccoth(d*x+c)^2/(c*f-d*e+f)*ln((d*x+c-1)/(d*x+c+1))-3*b^3*d^2*f/(c*f-d*e-f)
^2/(c*f-d*e+f)*c*arccoth(d*x+c)^2*ln(1-(c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f
-d*e+f))-3*b^3*d^2*f/(c*f-d*e-f)^2/(c*f-d*e+f)*c*arccoth(d*x+c)*polylog(2,(
c*f-d*e-f)/(d*x+c-1)*(d*x+c+1)/(c*f-d*e+f))+3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-
d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I/(d*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*(d*x+c+1
)-1))^3-3/2*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*csgn(I*(f
*c*(1/(d*x+c-1)*(d*x+c+1)-1)+(-1/(d*x+c-1)*(d*x+c+1)+1)*e*d+(-1/(d*x+c-1)*(
d*x+c+1)-1)*f)/(1/(d*x+c-1)*(d*x+c+1)-1))^3-6*a*b^2*d^2/f*arccoth(d*x+c)/(2
*c*f-2*d*e+2*f)*ln(d*x+c+1)+6*a*b^2*d^2/f*arccoth(d*x+c)/(2*c*f-2*d*e-2*f)*
ln(d*x+c-1)-6*a*b^2*d^2*arccoth(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f
*(d*x+c))-3/2*a*b^2*d^2/f/(c*f-d*e-f)*ln(d*x+c-1)*ln(1/2*d*x+1/2*c+1/2)-3/2
*a*b^2*d^2/f/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(d*x+c+1)+3/2*a*b^2*d^2/f
/(c*f-d*e+f)*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2*d*x+1/2*c+1/2)+3*a*b^2*d^2/(c*f-
d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)-f)/(-c*f+d*e-f))-3*
a*b^2*d^2/(c*f-d*e-f)/(c*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))*ln((-f*(d*x+c)+f)/(
-c*f+d*e+f))+3/4*I*b^3*d^3/f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*
csgn(I/((d*x+c-1)/(d*x+c+1))^(1/2))^2*csgn(I/(d*x+c-1)*(d*x+c+1))-3*a*b^2*d
^2/(c*f-d*e-f)/(c*f-d*e+f)*dilog((-f*(d*x+c)+f)/(-c*f+d*e+f))+3*a^2*b*d^2/(
c*f-d*e-f*(d*x+c))/f*arccoth(d*x+c)-3*a^2*b*d^2/f/(2*c*f-2*d*e+2*f)*ln(d*x+
c+1)+3*a^2*b*d^2/f/(2*c*f-2*d*e-2*f)*ln(d*x+c-1)-3*a^2*b*d^2/(c*f-d*e-f)/(c
*f-d*e+f)*ln(c*f-d*e-f*(d*x+c))+3*a*b^2*d^2/(c*f-d*e-f*(d*x+c))/f*arccoth(d
*x+c)^2-3/2*a*b^2*d^2/f/(c*f-d*e-f)*dilog(1/2*d*x+1/2*c+1/2)+3/4*I*b^3*d^3/
f/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I/(1/(d*x+c-1)*(d*x+c+
1)-1))*csgn(I/(d*x+c-1)*(d*x+c+1))*csgn(I/(d*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*
(d*x+c+1)-1))-3/4*I*b^3*d^2/(c*f-d*e-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*c*c
sgn(I/(1/(d*x+c-1)*(d*x+c+1)-1))*csgn(I/(d*x+c-1)*(d*x+c+1))*csgn(I/(d*x+c-
1)*(d*x+c+1)/(1/(d*x+c-1)*(d*x+c+1)-1))+b^3*d^2/(c*f-d*e-f*(d*x+c))/f*arcco
th(d*x+c)^3-b^3*d^2/f*arccoth(d*x+c)^3/(c*f-d*e+f)-3/4*I*b^3*d^3/f/(c*f-d*e
-f)/(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I/(d*x+c-1)*(d*x+c+1))*csgn(I/(d
*x+c-1)*(d*x+c+1)/(1/(d*x+c-1)*(d*x+c+1)-1))^2-3/4*I*b^3*d^3/f/(c*f-d*e-f)/
(c*f-d*e+f)*arccoth(d*x+c)^2*Pi*e*csgn(I/(1/(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -3/2*(d*(log(d*x + c + 1)/((c + 1)*f^2 - d*f*e) - log(d*x + c - 1)/((c - 1)
*f^2 - d*f*e) - 2*log(f*x + e)/(2*c*d*f*e - (c^2 - 1)*f^2 - d^2*e^2)) + 2*a
rccoth(d*x + c)/(f^2*x + f*e))*a^2*b - a^3/(f^2*x + f*e) - 1/8*(((c*d*f + d
*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*d*f^2 - d*f^2)*b^3)*x)*
log(d*x + c + 1)^3 + 3*(4*a*b^2*c*d*f*e - 2*a*b^2*d^2*e^2 - 2*(c^2*f^2 - f^
2)*a*b^2 - ((c*d*f - d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*d
*f^2 + d*f^2)*b^3)*x)*log(d*x + c - 1))*log(d*x + c + 1)^2)/(2*c*d*f^2*e^2
- d^2*f*e^3 - (c^2*f^4 - 2*c*d*f^3*e + d^2*f^2*e^2 - f^4)*x - (c^2*f^3 - f^
3)*e) + integrate(1/8*(((c*d*f + d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^
2*f*e - (c*d*f^2 - d*f^2)*b^3)*x)*log(d*x + c - 1)^3 - 6*((c*d*f + d*f)*a*b
^2*e - (c^2*f^2 - f^2)*a*b^2 + (a*b^2*d^2*f*e - (c*d*f^2 - d*f^2)*a*b^2)*x)
*log(d*x + c - 1)^2 - 3*(4*a*b^2*d^2*e^2 - 4*(c*d*f - d*f)*a*b^2*e + ((c*d*
f + d*f)*b^3*e - (c^2*f^2 - f^2)*b^3 + (b^3*d^2*f*e - (c*d*f^2 - d*f^2)*b^3
)*x)*log(d*x + c - 1)^2 + 4*(a*b^2*d^2*f*e - (c*d*f^2 - d*f^2)*a*b^2)*x + 2
*(b^3*d^2*f^2*x^2 + 2*(c^2*f^2 - f^2)*a*b^2 + (2*(c*d*f^2 - d*f^2)*a*b^2 +
(c*d*f^2 - d*f^2)*b^3 - (2*a*b^2*d^2*f - b^3*d^2*f)*e)*x - (2*(c*d*f + d*f)
*a*b^2 - (c*d*f - d*f)*b^3)*e)*log(d*x + c - 1))*log(d*x + c + 1))/((c*d*f^
4 - d^2*f^3*e - d*f^4)*x^3 + (c^2*f^4 - 2*d^2*f^2*e^2 - f^4 + (c*d*f^3 - 3*
d*f^3)*e)*x^2 - (d^2*f*e^3 + (c*d*f^2 + 3*d*f^2)*e^2 - 2*(c^2*f^3 - f^3)*e)
*x - (c*d*f + d*f)*e^3 + (c^2*f^2 - f^2)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)/(f^2*x^2 + 2*f*x*e + e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acoth}\left(c + dx\right)\right)^{3}}{\left(e + fx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(d*x+c))**3/(f*x+e)**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**3/(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(d*x+c))^3/(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^3/(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acoth}(c + d\,x))^3}{(e + f\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c + d*x))^3/(e + f*x)^2,x)
```

```
[Out] int((a + b*acoth(c + d*x))^3/(e + f*x)^2, x)
```

3.119 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right) dx$

Optimal. Leaf size=162

$$\frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)}+\frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{d(e+fx)}{de-f-cf}\right)}{2f(de-(1+c)f)(1+m)(2+m)}-\frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2\right.}{2f(de+f-cf)}$$

```
[Out] (f*x+e)^(1+m)*(a+b*arccoth(d*x+c))/f/(1+m)+1/2*b*d*(f*x+e)^(2+m)*hypergeom(
[1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e-f))/f/(d*e-(1+c)*f)/(1+m)/(2+m)-1/2*b*d*
(f*x+e)^(2+m)*hypergeom([1, 2+m],[3+m],d*(f*x+e)/(-c*f+d*e+f))/f/(-c*f+d*e+
f)/(1+m)/(2+m)
```

Rubi [A]
time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6247, 6064, 726, 70}

$$\frac{(e+fx)^{m+1}\left(a+b\coth^{-1}(c+dx)\right)}{f(m+1)}+\frac{bd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf-f}\right)}{2f(m+1)(m+2)(de-(c+1)f)}-\frac{bd(e+fx)^{m+2}\,{}_2F_1\left(1,m+2;m+3;\frac{d(e+fx)}{de-cf+f}\right)}{2f(m+1)(m+2)(-cf+de+f)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
```

```
[Out] ((e + f*x)^(1 + m)*(a + b*ArcCoth[c + d*x]))/(f*(1 + m)) + (b*d*(e + f*x)^(
2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e - f - c*f)])/(
2*f*(d*e - (1 + c)*f)*(1 + m)*(2 + m)) - (b*d*(e + f*x)^(2 + m)*Hypergeomet
ric2F1[1, 2 + m, 3 + m, (d*(e + f*x))/(d*e + f - c*f)])/(2*f*(d*e + f - c*f
)*(1 + m)*(2 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 6064

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcCoth[c*x])/(e*(q + 1))), x] - Dist[b
```

```
*(c/(e*(q + 1))), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx = \frac{\mathrm{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\mathrm{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x^2}\, d\right.}{f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\mathrm{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{2(1-x)}\right.\right.}{}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} - \frac{b\mathrm{Subst}\left(\int \frac{\left(\frac{de-cf}{d}+\frac{fx}{d}\right)^{1+m}}{1-x}\, d\right.}{2f(1+m)}$$

$$= \frac{(e+fx)^{1+m}\left(a+b\coth^{-1}(c+dx)\right)}{f(1+m)} + \frac{bd(e+fx)^{2+m}\,{}_2F_1\left(1,2+\right.}{2f(de-(1+c)f)(1}$$

Mathematica [**F**]
time = 1.75, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]),x]
[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]), x]
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arccoth}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)
```

```
[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*b*((f*x + e)*(f*x + e)^m*log(d*x + c + 1)/(f*(m + 1)) - integrate((d*f*
x + d*e + (d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1))*log(d*x + c - 1))*(f*x
+ e)^m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)) + (f*x + e)^(m + 1)*a
/(f*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}{(c + dx)}) (e + fx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c)),x)
```

```
[Out] Integral((a + b*acoth(c + d*x))*(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)*(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e+f\,x)^m\,(a+b\,\mathrm{acoth}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acoth(c + d*x)),x)
[Out] int((e + f*x)^m*(a + b*acoth(c + d*x)), x)
```

3.120 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2,x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^2, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^2, x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int (fx+e)^m \left(a+b\,\mathrm{arccoth}(dx+c)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*f*x + b^2*e)*(f*x + e)^m*log(d*x + c + 1)^2/(f*(m + 1)) + (f*x + e
)^(m + 1)*a^2/(f*(m + 1)) - integrate(-1/4*((b^2*d*f*(m + 1)*x + (c*f*(m +
1) + f*(m + 1))*b^2)*log(d*x + c - 1)^2 - 2*(b^2*d*e - 2*(c*f*(m + 1) + f*(
m + 1))*a*b - (2*a*b*d*f*(m + 1) - b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c*f*(
m + 1) + f*(m + 1))*b^2)*log(d*x + c - 1))*log(d*x + c + 1) - 4*(a*b*d*f*(m
 + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b)*log(d*x + c - 1))*(f*x + e)^m/(d*f
*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccoth(d*x + c)^2 + 2*a*b*arccoth(d*x + c) + a^2)*(f*x + e)^
m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2*(e + f*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(d*x + c) + a)^2*(f*x + e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acoth(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*acoth(c + d*x))^2, x)
```

3.121 $\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3, x\right)$$

```
[Out] Unintegrable((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] Defer[Subst][Defer[Int][((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^3, x
], x, c + d*x]/d
```

Rubi steps

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d}+\frac{fx}{d}\right)^m \left(a+b\coth^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int (e+fx)^m \left(a+b\coth^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3,x]
```

```
[Out] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x])^3, x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int (fx+e)^m (a+b\,\text{arccoth}(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

```
[Out] int((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b^3*f*x + b^3*e)*(f*x + e)^m*log(d*x + c + 1)^3/(f*(m + 1)) + (f*x + e
)^(m + 1)*a^3/(f*(m + 1)) - integrate(1/8*((b^3*d*f*(m + 1)*x + (c*f*(m + 1
) + f*(m + 1))*b^3)*log(d*x + c - 1)^3 + 3*(b^3*d*e - 2*(c*f*(m + 1) + f*(m
 + 1))*a*b^2 - (2*a*b^2*d*f*(m + 1) - b^3*d*f)*x + (b^3*d*f*(m + 1)*x + (c*
f*(m + 1) + f*(m + 1))*b^3)*log(d*x + c - 1))*log(d*x + c + 1)^2 - 6*(a*b^2
*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b^2)*log(d*x + c - 1)^2 - 3*(4
*a^2*b*d*f*(m + 1)*x + 4*(c*f*(m + 1) + f*(m + 1))*a^2*b + (b^3*d*f*(m + 1)
*x + (c*f*(m + 1) + f*(m + 1))*b^3)*log(d*x + c - 1)^2 - 4*(a*b^2*d*f*(m +
1)*x + (c*f*(m + 1) + f*(m + 1))*a*b^2)*log(d*x + c - 1))*log(d*x + c + 1)
+ 12*(a^2*b*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a^2*b)*log(d*x + c -
1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccoth(d*x + c)^3 + 3*a*b^2*arccoth(d*x + c)^2 + 3*a^2*b*arc
coth(d*x + c) + a^3)*(f*x + e)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^m*(a+b*arccoth(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*arccoth(d*x + c) + a)^3*(f*x + e)^m, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e+f\,x)^m\,\left(a+b\,\mathrm{acoth}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acoth(c + d*x))^3,x)
[Out] int((e + f*x)^m*(a + b*acoth(c + d*x))^3, x)
```

3.122 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.123 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=460

$$-\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}-\frac{3b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\Bigg(}{2c}$$

```
[Out] -2*arccoth(1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arccoth((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^3/c-3/2*b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*p
olylog(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b*(a+b*arccoth((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+1)/(c*x+1)^(1/2))/c-3/2*b^2*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(
1/2)))*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+3/2*b^2*(a+b*arcco
th((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+1)/(c*x+1)^(1/2))/c-3/4*b^3*polylog(4,1-2/((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1))/c+3/4*b^3*polylog(4,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(
c*x+1)^(1/2)+1)/(c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.39, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 6034, 6200, 6096, 6204, 6208, 6745}

$$-\frac{3b^2\text{Li}_3\left(1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{Li}_3\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b\text{Li}_2\left(1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}+\frac{3b\text{Li}_2\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}-\frac{3b^3\text{Li}_4\left(1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{4c}+\frac{3b^3\text{Li}_4\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (3*b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b*(a
 + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, 1 - (2*Sqrt[1 - c*x
])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c) - (3*b^2*(a +
b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1 + Sqrt[1 - c*x]
/Sqrt[1 + c*x])])/(2*c) + (3*b^2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[3, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[
1 + c*x]))])/(2*c) - (3*b^3*PolyLog[4, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*
x])])/(4*c) + (3*b^3*PolyLog[4, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 + c*x]*(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x]))])/(4*c)
```

Rule 6034

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6200

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6208

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[k + 1, u]/
(2*c*d)), x] + Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[k +
1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
 EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
```

```
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\coth^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(3}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b}{}$$

Mathematica [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1487 vs. $2(388) = 776$.
time = 1.03, size = 1488, normalized size = 3.23

method	result	size
default	Expression too large to display	1488

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^3*ln(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)+1)-3/2*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+3/2*b^3/c
*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)
^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-3/4*b^3/c*polylog(4,-1/((-c*x+1
)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+b^3/c*arccoth((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)
^(1/2))^2*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c
*x+1)^(1/2)+1))^(1/2))-6*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylo
g(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^
(1/2))+6*b^3/c*polylog(4,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+1))^(1/2))+b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*l
n(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(
1/2))+3*b^3/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,1/(((-c*x+1
)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*b^3/c*a
rccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(
1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+6*b^3/c*polylog(4,1/(((-c*
x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-3*a*b^
2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1/((-c*x+1)^(1/2)/(c*x+1)^(1
/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)+1)-3*a*b^2/c*arccoth((-c*x+1)^(1/2)
/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+1))+3/2*a*b^2/c*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)
-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+3*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x
+1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+
1)^(1/2)+1))^(1/2))+6*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog
(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(
1/2))-6*a*b^2/c*polylog(3,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+1))^(1/2))+3*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+
1))^(1/2))+6*a*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,1/(((-
```

```
c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*a*
b^2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)
^(1/2)+1))^(1/2))+3/4*a^2*b*(4*dilog(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1))-dilog(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^2/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(
sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*
x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c
*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
 + 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) +
 sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sq
rt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1
)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt
(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x
 + 1) + sqrt(-c*x + 1)))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/((c^2*x^2 -
1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccoth(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3 \operatorname{acoth}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acoth(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acoth(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acoth(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.124 $\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=302

$$-\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c}-\frac{b\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2\right.}{c}$$

```
[Out] -2*arccoth(1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arccoth((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^2/c-b*(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog
(2,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c+b*(a+b*arccoth((-c*x+1)^(1/2)/(c
*x+1)^(1/2)))*polylog(2,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
/(c*x+1)^(1/2))/c-1/2*b^2*polylog(3,1-2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))/c
+1/2*b^2*polylog(3,1-2*(-c*x+1)^(1/2)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(c*x
+1)^(1/2))/c
```

Rubi [A]

time = 0.25, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6813, 6034, 6200, 6096, 6204, 6745}

$$-\frac{b\text{Li}_2\left(1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\text{Li}_2\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}-\frac{b^2\text{Li}_3\left(1-\frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{2c}+\frac{b^2\text{Li}_3\left(1-\frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcCoth[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcCo
th[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, 1 - (2*Sqrt[1 - c*x])/(Sqrt[1 +
 c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/c - (b^2*PolyLog[3, 1 - 2/(1 + S
qrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (b^2*PolyLog[3, 1 - (2*Sqrt[1 - c*x])
/(Sqrt[1 + c*x]*(1 + Sqrt[1 - c*x]/Sqrt[1 + c*x]))])/(2*c)
```

Rule 6034

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcCoth[c*x])^p*ArcCoth[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcCoth[c*x])^(p - 1)*(ArcCoth[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6200

```
Int[(ArcCoth[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[Log[SimplifyIntegrand[1 + 1/u, x]]*((a +
 b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] - Dist[1/2, Int[Log[SimplifyIntegra
nd[1 - 1/u, x]]*((a + b*ArcCoth[c*x])^p/(d + e*x^2)), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]
```

Rule 6204

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*(p/2), Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\coth^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(2}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b(}{}$$

$$= -\frac{2\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\coth^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b(}{}$$

Mathematica [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(258) = 516.
time = 0.67, size = 694, normalized size = 2.30

method	result

default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-\frac{b^2\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}+1\right)}{c}-\frac{b^2\mathrm{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\mathrm{poly}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2*ln(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)+1)-b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/((-c*x+
1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+1/2*b^2/c*polyl
og(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))+
b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+1/(((-c*x+1)^(1/2)/(c*x+
1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+2*b^2/c*arccoth((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c
*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-2*b^2/c*polylog(3,-1/(((-c*x+1)^(1/2)/
(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+b^2/c*arccoth((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+2*b^2/c*arccoth((-c*x+1)^(1/2)/(c*x+1)^
(1/2))*polylog(2,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1))^(1/2))-2*b^2/c*polylog(3,1/(((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((
-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+1/2*a*b*(4*dilog(((-c*x+1)^(1/2)/(c*
x+1)^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-dilog(((-c*x+1)^(1/2)/(c*x+
1)^(1/2)-1)^2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(s
qrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^
```

```
2 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*
x + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + s
qrt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*lo
g(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c
*x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1
)))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccoth(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acoth(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acoth(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
[Out] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.125 $\int \frac{a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2} dx$

Optimal. Leaf size=89

$$-\frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*b*polylog(2,-(c*x+1)^(1/2)/(-c*x+
1)^(1/2))/c+1/2*b*polylog(2,(c*x+1)^(1/2)/(-c*x+1)^(1/2))/c
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {212, 6813, 6032}

$$-\frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}-\frac{b\text{Li}_2\left(-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}+\frac{b\text{Li}_2\left(\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) - (b*PolyLog[2, -(Sqrt[1 + c*x]/S
qrt[1 - c*x])])/(2*c) + (b*PolyLog[2, Sqrt[1 + c*x]/Sqrt[1 - c*x]])/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\coth^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{b\text{Li}_2\left(-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} + \frac{b\text{Li}_2\left(\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

Mathematica [A]
time = 0.26, size = 98, normalized size = 1.10

$$\frac{a\tanh^{-1}(cx)}{c} + \frac{b\left(\tanh^{-1}(cx)\left(2\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right) + \log\left(1-e^{-\tanh^{-1}(cx)}\right) - \log\left(1+e^{-\tanh^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{-\tanh^{-1}(cx)}\right) - \text{PolyLog}\left(2,e^{-\tanh^{-1}(cx)}\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c + (b*(ArcTanh[c*x]*(2*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x
]] + Log[1 - E^(-ArcTanh[c*x])] - Log[1 + E^(-ArcTanh[c*x])]) + PolyLog[2,
-E^(-ArcTanh[c*x])] - PolyLog[2, E^(-ArcTanh[c*x])]))/(2*c)
```

Maple [A]
time = 0.42, size = 119, normalized size = 1.34

method	result	size
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{b\left(4\,\text{dilog}\left(\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}\right)-\text{dilog}\left(\frac{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1\right)^2}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1\right)^2}\right)\right)}{4c}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/4*b*(4*dilog(((-c*x+1)^(1/2)/(c*x+1)
^(1/2)-1)/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-dilog(((-c*x+1)^(1/2)/(c*x+1)^(
1/2)-1)^2/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2))/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] 1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
- (log(c*x + 1) - log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/c -
2*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1
)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sqrt(
c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2
*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*acoth(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.126 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccoth}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a c^{2} x^{2} - a + b c^{2} x^{2} \operatorname{acoth}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)} - b \operatorname{acoth}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acoth(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.127 $\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}dx = \int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCoth[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccoth}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x + 1
)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)
) + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 - b^
2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b^2*
c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(-sqrt(c*x + 1) + sqrt(-c*x
+ 1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccoth(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccoth(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2}c^{2}x^{2} - a^{2} + 2abc^{2}x^{2} \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^{2}c^{2}x^{2} \operatorname{acoth}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^{2} \operatorname{acoth}^{2}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*acoth(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*acoth(sqrt(-c*x + 1)/sqrt(c*x
 + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccoth((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arccoth(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{acoth}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.128 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\coth^{-1}(\tanh(a+bx))\,dx&=\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 0.92

$$x^m\left(\frac{bx^2}{2+m}+\frac{x(-bx+\coth^{-1}(\tanh(a+bx)))}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 676, normalized size = 18.27

method	result
risch	$\frac{x\,x^m\ln\left(\mathrm{e}^{bx+a}\right)}{1+m}-\frac{x\left(i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)m-2i\pi\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^2m+2i\pi\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^3m+4bx-2i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))*m-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^3*m+4*b*x-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*m-2*I*Pi*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+I*Pi*csgn(I*exp(2*b*x+2*a))^3*
m+2*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+4*I*Pi+4*I*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^3-4*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-I*Pi
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2
*I*Pi*m-2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*m+2*I*Pi*csgn(I*
exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+I*
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*m+2*I*Pi*csgn(I*exp(2*b*x+2*
a))^3+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1)))/(1+m)/(2+m)*x^m
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.89

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log(x)-\frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for}\ m=-2 \\ \int\frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for}\ m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2}+\frac{2xx^m\operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.40, size = 90, normalized size = 2.43

$$\frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)}-\frac{bx^{m+2}}{(m+2)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(2
*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m + 2)
*(m + 1))
```

Mupad [B]
time = 1.61, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,\left(m+1\right)}{2\,m^2+6\,m+4}-\frac{x\,x^m\,\left(m+2\right)\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))/(6*m + 2*m^2 + 4)
```

3.129 $\int x^2 \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCoth[Tanh[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(\tanh(a+bx))\,dx&=\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))-\frac{1}{3}b\int x^3\,dx\\&=-\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcCoth[Tanh[a + b*x]]))
```

Maple [A]
time = 0.35, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^4}{12}+\frac{x^3 \operatorname{arccoth}(\tanh(bx+a))}{3}$
risch	$\frac{x^3\ln\left(e^{bx+a}\right)}{3}-\frac{b\,x^4}{12}+\frac{i\pi\,x^3 \operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2}{12}-\frac{i\pi\,x^3\operatorname{csgn}\left(i e^{2bx+2a}\right)^3}{12}+\frac{i\pi\,x^3\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+}}{e^{2bx+2a}}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arccoth(tanh(b*x+a))
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.83

$$-\frac{1}{12}\,bx^4+\frac{1}{3}\,x^3\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/12*b*x^4 + 1/3*x^3*arccoth(tanh(b*x + a))
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.11, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12}+\frac{x^3\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a)),x)
```

```
[Out] -b*x**4/12 + x**3*acoth(tanh(a + b*x))/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.38, size = 71, normalized size = 3.09

$$-\frac{1}{12}\,bx^4+\frac{1}{6}\,x^3\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 0.09, size = 19, normalized size = 0.83

$$\frac{x^3\operatorname{acoth}(\tanh(a+b\,x))}{3}-\frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(tanh(a + b*x)),x)
```

```
[Out] (x^3*acoth(tanh(a + b*x)))/3 - (b*x^4)/12
```

3.130 $\int x \coth^{-1}(\tanh(a+bx)) \, dx$

Optimal. Leaf size=23

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6375, 30}

$$\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCoth[Tanh[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(\tanh(a+bx))\,dx&=\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]],x]
[Out] -1/6*(x^2*(b*x - 3*ArcCoth[Tanh[a + b*x]]))
```

Maple [A]
time = 0.34, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^3}{6}+\frac{x^2\operatorname{arccoth}(\tanh(bx+a))}{2}$
risch	$\frac{x^2\ln(\mathrm{e}^{bx+a})}{2}-\frac{b\,x^3}{6}+\frac{i\pi\,x^2\operatorname{csgn}(i\mathrm{e}^{2bx+2a})\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2}{8}-\frac{i\pi\,x^2\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^3}{4}-\frac{i\pi\,x^2\operatorname{csgn}(i\mathrm{e}^{2bx+2a})^3}{8}-\frac{i\pi\,x^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*x^3+1/2*x^2*arccoth(tanh(b*x+a))
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{1}{6}\,bx^3+\frac{1}{2}\,x^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/6*b*x^3 + 1/2*x^2*arccoth(tanh(b*x + a))
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.
time = 0.10, size = 39, normalized size = 1.70

$$\begin{cases}\frac{x\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{2b}-\frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{6b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{acoth}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((x*acoth(tanh(a + b*x))**2/(2*b) - acoth(tanh(a + b*x))**3/(6*b**
2), Ne(b, 0)), (x**2*acoth(tanh(a))/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.
time = 0.40, size = 71, normalized size = 3.09

$$-\frac{1}{6}\,bx^3 + \frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} + 1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 1.13, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{acoth}(\tanh(a + b\,x))}{2} - \frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(tanh(a + b*x)),x)
```

```
[Out] (x^2*acoth(tanh(a + b*x)))/2 - (b*x^3)/6
```

3.131 $\int \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

```
[Out] 1/2*arccoth(tanh(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2} + x\coth^{-1}(\tanh(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCoth[Tanh[a + b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.24, size = 32, normalized size = 2.00

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccoth}(\tanh(bx+a))-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2}}{b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))\operatorname{arccoth}(\tanh(bx+a))-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2}}{b}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi x\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3}{4}+\frac{i\pi\operatorname{csgn}\left(i\mathrm{e}^{bx+a}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2 x}{2}+\frac{i\pi\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(tanh(b*x+a))*arccoth(tanh(b*x+a))-1/2*arctanh(tanh(b*x+a))^2)
```

Maxima [A]
time = 0.30, size = 16, normalized size = 1.00

$$-\frac{1}{2}\,bx^2+x\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arccoth(tanh(b*x + a))
```

Fricas [A]
time = 0.34, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.06, size = 19, normalized size = 1.19

$$\begin{cases}\frac{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{acoth}\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.
time = 0.40, size = 69, normalized size = 4.31

$$-\frac{1}{2}bx^2 + \frac{1}{2}x\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/(
(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

Mupad [B]
time = 1.12, size = 16, normalized size = 1.00

$$x\,\mathrm{acoth}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x)),x)
```

```
[Out] x*acoth(tanh(a + b*x)) - (b*x^2)/2
```

3.132 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] b*x-(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$bx-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x} \, dx &= bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{x} \, dx\\ &= bx-\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$bx+\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b*x]]/x,x]

[Out] b*x + (-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 354, normalized size = 16.86

method	result
risch	$\ln(x)\ln\left(\mathrm{e}^{bx+a}\right)-\ln(x)\,xb+bx-\frac{i\pi\ln(x)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{3}}{4}+\frac{i\pi\ln(x)\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b*x+a))/x,x,method=_RETURNVERBOSE)

[Out] ln(x)*ln(exp(b*x+a))-ln(x)*x*b+b*x-1/4*I*Pi*ln(x)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/4*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2+1/4*I*Pi*ln(x)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi*ln(x)*csgn(I*exp(2*b*x+2*a))^3-1/2*I*Pi*ln(x)+1/2*I*Pi*ln(x)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2*I*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3-1/4*I*Pi*ln(x)*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))

Maxima [A]
time = 0.26, size = 34, normalized size = 1.62

$$-b\left(x+\frac{a}{b}\right)\log(x)+b\left(x+\frac{a\log(x)}{b}\right)+\operatorname{arcoth}(\tanh(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="maxima")

[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arccoth(tanh(b*x + a))*log(x)

Fricas [A]
time = 0.38, size = 8, normalized size = 0.38

$$bx+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="fricas")

[Out] b*x + a*log(x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))/x, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 15, normalized size = 0.71

$$bx + \frac{1}{2}\,(i\,\pi + 2\,a)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + 1/2*(I*pi + 2*a)*log(x)
```

Mupad [B]
time = 0.18, size = 59, normalized size = 2.81

$$b\,x - \ln\left(x\right)\,\left(\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2} - \frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2} + b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))/x,x)
```

```
[Out] b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)
```

3.133 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx$

Optimal. Leaf size=17

$$-\frac{\coth^{-1}(\tanh(a+bx))}{x}+b\log(x)$$

```
[Out] -arccoth(tanh(b*x+a))/x+b*ln(x)
```

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 29}

$$b\log(x)-\frac{\coth^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]/x) + b*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{x}+b\int \frac{1}{x}\,dx\\ &= -\frac{\coth^{-1}(\tanh(a+bx))}{x}+b\log(x)\end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.06

$$b-\frac{\coth^{-1}(\tanh(a+bx))}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^2,x]
```

```
[Out] b - ArcCoth[Tanh[a + b*x]]/x + b*Log[x]
```

Maple [A]
time = 0.33, size = 21, normalized size = 1.24

method	result
default	$-\frac{\operatorname{arccoth}(\tanh(bx+a))}{x}+b\ln\left(-bx\right)$
risch	$-\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{x}+\frac{-i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-2i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}-2i\pi\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)^{2}+i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccoth(tanh(b*x+a))/x+b*ln(-b*x)
```

Maxima [A]
time = 0.30, size = 17, normalized size = 1.00

$$b\log\left(x\right)-\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arccoth(tanh(b*x + a))/x
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.76

$$\frac{bx\log\left(x\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [A]
time = 0.08, size = 14, normalized size = 0.82

$$b\log\left(x\right)-\frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**2,x)
```

```
[Out] b*log(x) - acoth(tanh(a + b*x))/x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(17) = 34$.
time = 0.38, size = 70, normalized size = 4.12

$$b\log\left(|x|\right) - \frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

Mupad [B]
time = 0.09, size = 17, normalized size = 1.00

$$b\,\ln\left(x\right) - \frac{\mathrm{acoth}(\tanh(a+b\,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))/x^2,x)
```

```
[Out] b*log(x) - acoth(tanh(a + b*x))/x
```

3.134 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{b}{2x}-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}$$

```
[Out] -1/2*b/x-1/2*arccoth(tanh(b*x+a))/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -1/2*b/x - ArcCoth[Tanh[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x^3}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\ &= -\frac{b}{2x}-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$-\frac{bx+\coth^{-1}(\tanh(a+bx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -1/2*(b*x + ArcCoth[Tanh[a + b*x]])/x^2
```

Maple [A]
time = 0.31, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{2x^2}$
risch	$-\frac{\ln(\mathrm{e}^{bx+a})}{2x^2}-\frac{4bx+i\pi\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2+2i\pi\operatorname{csgn}(i\mathrm{e}^{bx+a})\operatorname{csgn}(i\mathrm{e}^{2bx+2a})^2-i\pi\operatorname{csgn}(i\mathrm{e}^{bx+a})^2\operatorname{csgn}(i\mathrm{e}^{2bx+2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/x-1/2*arccoth(tanh(b*x+a))/x^2
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.83

$$-\frac{b}{2\,x}-\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arccoth(tanh(b*x + a))/x^2
```

Fricas [A]
time = 0.36, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A]
time = 0.19, size = 19, normalized size = 0.83

$$-\frac{b}{2x}-\frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**3,x)
```

```
[Out] -b/(2*x) - acoth(tanh(a + b*x))/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.
time = 0.40, size = 71, normalized size = 3.09

$$-\frac{b}{2\,x}-\frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

Mupad [B]
time = 1.14, size = 16, normalized size = 0.70

$$-\frac{\mathrm{acoth}(\tanh(a+b\,x))+b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))/x^3,x)
```

```
[Out] -(acoth(tanh(a + b*x)) + b*x)/(2*x^2)
```

3.135 $\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$

Optimal. Leaf size=23

$$-\frac{b}{6x^2}-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}$$

[Out] -1/6*b/x^2-1/3*arccoth(tanh(b*x+a))/x^3

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b*x]]/x^4,x]

[Out] -1/6*b/x^2 - ArcCoth[Tanh[a + b*x]]/(3*x^3)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))}{x^4}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}+\frac{1}{3}b\int \frac{1}{x^3}\,dx\\ &= -\frac{b}{6x^2}-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{bx+2\coth^{-1}(\tanh(a+bx))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]/x^4,x]
[Out] -1/6*(b*x + 2*ArcCoth[Tanh[a + b*x]])/x^3
```

Maple [A]
time = 0.31, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{6x^2}-\frac{\operatorname{arccoth}(\tanh(bx+a))}{3x^3}$
risch	$-\frac{\ln(e^{bx+a})}{3x^3}-\frac{2bx+i\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+2i\pi\,\mathrm{csgn}\left(ie^{bx+a}\right)\mathrm{csgn}\left(ie^{2bx+2a}\right)^2-i\pi\mathrm{csgn}\left(ie^{bx+a}\right)^2\mathrm{csgn}\left(ie^{2bx+2a}\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/6*b/x^2-1/3*arccoth(tanh(b*x+a))/x^3
```

Maxima [A]
time = 0.31, size = 19, normalized size = 0.83

$$-\frac{b}{6\,x^2}-\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="maxima")
[Out] -1/6*b/x^2 - 1/3*arccoth(tanh(b*x + a))/x^3
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.57

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A]
time = 0.25, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2}-\frac{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))/x**4,x)
```

```
[Out] -b/(6*x**2) - acoth(tanh(a + b*x))/(3*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.
time = 0.40, size = 71, normalized size = 3.09

$$-\frac{b}{6\,x^2}-\frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b/x^2 - 1/6*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e
^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^3
```

Mupad [B]
time = 1.12, size = 19, normalized size = 0.83

$$-\frac{\operatorname{acoth}(\tanh(a+b\,x))}{3\,x^3}-\frac{b}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))/x^4,x)
```

```
[Out] - acoth(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

3.136 $\int x^m \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=71

$$\frac{2b^2x^{3+m}}{6+11m+6m^2+m^3}-\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}$$

```
[Out] 2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arccoth(tanh(b*x+a))/(m^2+3*m+
2)+x^(1+m)*arccoth(tanh(b*x+a))^2/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{2bx^{m+2}\coth^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcCoth[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^2)/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\coth^{-1}(\tanh(a+bx))^2\,dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}-\frac{(2b)\int x^{1+m}\coth^{-1}(\tanh(a+bx))\,dx}{1+m}\\ &= -\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^2}{1+m}+\frac{(2b^2)\int}{2+3m}\\ &= \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3}-\frac{2bx^{2+m}\coth^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tan}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 0.87

$$\frac{x^{1+m}\left(2b^2x^2-2b(3+m)x\coth^{-1}(\tanh(a+bx))+(6+5m+m^2)\coth^{-1}(\tanh(a+bx))^2\right)}{(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcCoth[Tanh[a + b*x]] + (6 + 5*m + m
^2)*ArcCoth[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.91, size = 9175, normalized size = 129.23

method	result	size
risch	Expression too large to display	9175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 73, normalized size = 1.03

$$\frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)}-\frac{2\,bx^2x^m\,\mathrm{arcoth}\,(\tanh\,(bx+a))}{(m+2)(m+1)}+\frac{x^{m+1}\,\mathrm{arcoth}\,(\tanh\,(bx+a))^2}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arccoth(tanh(b*x + a)
)/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))^2/(m + 1)
```

Fricas [A]
time = 0.36, size = 101, normalized size = 1.42

$$\frac{(4\,(b^2m^2+3\,b^2m+2\,b^2)x^3+8\,(abm^2+4\,abm+3\,ab)x^2+(4\,a^2m^2-\pi^2(m^2+5\,m+6)+20\,a^2m+24\,a^2)x)x^m}{4\,(m^3+6\,m^2+11\,m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 8*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2
+ (4*a^2*m^2 - pi^2*(m^2 + 5*m + 6) + 20*a^2*m + 24*a^2)*x)*x^m/(m^3 + 6*m^
2 + 11*m + 6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2} & \text{for } m = -3 \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x^2}\, dx & \text{for } m = -2 \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x}\, dx & \text{for } m = -1 \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m \operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m \operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m \operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m \operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{6xx^m \operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2
/(2*x**2), Eq(m, -3)), (Integral(acoth(tanh(a + b*x))**2/x**2, x), Eq(m, -2
)), (Integral(acoth(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x**m/
(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6
*m**2 + 11*m + 6) - 6*b*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6*m**2 + 11*
m + 6) + m**2*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 5
*m*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x**m*aco
th(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^2, x)
```

Mupad [B]
time = 1.32, size = 203, normalized size = 2.86

$$\frac{4\,b^2\,x^m\,x^3\,\left(m^2+3\,m+2\right)}{4\,m^3+24\,m^2+44\,m+24}+\frac{x\,x^m\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2\,\left(m^2+5\,m+6\right)}{4\,m^3+24\,m^2+44\,m+24}-\frac{4\,b\,x^m\,x^2\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(m^2+4\,m+3\right)}{4\,m^3+24\,m^2+44\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x))^2,x)
```

```
[Out] (4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log(
-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) - 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (4*b
*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4*m^3
+ 24)
```

3.137 $\int x^3 \coth^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$\frac{b^2x^6}{60} - \frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2$$

```
[Out] 1/60*b^2*x^6-1/10*b*x^5*arccoth(tanh(b*x+a))+1/4*x^4*arccoth(tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$-\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2 + \frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcCoth[Tanh[a + b*x]])/10 + (x^4*ArcCoth[Tanh[a + b*
x]]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2 - \frac{1}{2}b\int x^4\coth^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{10}b^2\int x^5\\ &= \frac{b^2x^6}{60} - \frac{1}{10}bx^5\coth^{-1}(\tanh(a+bx)) + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4\left(b^2x^2-6bx\coth^{-1}(\tanh(a+bx))+15\coth^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^4*(b^2*x^2 - 6*b*x*ArcCoth[Tanh[a + b*x]] + 15*ArcCoth[Tanh[a + b*x]]^2)
)/60
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 41.90, size = 3418, normalized size = 81.38

method	result	size
risch	Expression too large to display	3418

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(exp(b*x+a))^2-1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*ex
p(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2-1/40*I*Pi*b*x^5*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/40*I*Pi*b*x^
5*csgn(I*exp(2*b*x+2*a))^3+1/40*I*Pi*b*x^5*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^3+1/60*b^2*x^6+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/32*Pi^2*x^4*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))+1/32*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x
+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/40*I*Pi*b*x^5*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))+1/40*I*Pi*b*x^5*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/20
*I*Pi*b*x^5*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/40*I*Pi*b*x^5*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/32*Pi^2*x
^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2-1/16*Pi^2*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/32*Pi^2*x^4*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/32*Pi^2*x^4*csgn(I
```

```
*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/32*Pi^2*x^
4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/64*P
i^2*x^4*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+1/16*Pi^2*x^4*csgn(I*
exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-3/32*Pi^2*x^4*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))^4+1/16*Pi^2*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^5+1/32*Pi^2*x^4*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-1/64*Pi^2*x^4*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^4-1/64*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/16*Pi^2*x^4*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/16*Pi^2*x^4
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^4-1/32*Pi^2*x^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/64*Pi^2*x^4*csgn(I/(exp(2*
b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2-1/64*Pi^2*x^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-1/64*Pi^
2*x^4*csgn(I*exp(2*b*x+2*a))^6-1/16*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2*x^4-1
/8*csgn(I/(exp(2*b*x+2*a)+1))^3*Pi^2*x^4-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*
a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn(I/(
exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*Pi^2*
x^4*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2+1/8*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2
*b*x+2*a))^2-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^6+(-1/10*b*x^5-1/8*I*
Pi*x^4*csgn(I*exp(2*b*x+2*a))^3-1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+1/4*I*Pi*x^4*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/8*
I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2-1/8*I*Pi*x^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*I*Pi*x^4-
1/8*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/8*I*Pi*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))^
3+1/4*I*Pi*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2)*ln(exp(b*x+a))+1/8*Pi^2*x^4*cs
gn(I/(exp(2*b*x+2*a)+1))^5-1/20*I*Pi*b*x^5*csgn(I/(exp(2*b*x+2*a)+1))^2+1/2
0*I*Pi*b*x^5*csgn(I/(exp(2*b*x+2*a)+1))^3-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/
8*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^2-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*Pi^2*x^4*csgn(I/(exp(2*b*x
+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)-1/16*Pi^2*x^4*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/16*Pi^2*x^4*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^3-1/16*csgn(I*exp(2*b*x+2*a))^
3*Pi^2*x^4+1/8*csgn(I/(exp(2*b*x+2*a)+1))^2*Pi^2*x^4+1/16*Pi^2*x^4*csgn(I*e
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*
x+2*a)+1))^3+1/20*I*Pi*b*x^5-1/16*Pi^2*x^4*csgn...
```

Maxima [A]

time = 0.33, size = 36, normalized size = 0.86

$$\frac{1}{60}\,b^2x^6-\frac{1}{10}\,bx^5\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)+\frac{1}{4}\,x^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/60*b^2*x^6 - 1/10*b*x^5*arccoth(tanh(b*x + a)) + 1/4*x^4*arccoth(tanh(b*x
 + a))^2
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.71

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5-\frac{1}{16}\,\left(\pi^2-4\,a^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 - 1/16*(pi^2 - 4*a^2)*x^4
```

Sympy [A]
time = 0.26, size = 37, normalized size = 0.88

$$\frac{b^2x^6}{60}-\frac{bx^5\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{10}+\frac{x^4\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**6/60 - b*x**5*acoth(tanh(a + b*x))/10 + x**4*acoth(tanh(a + b*x))**
2/4
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 41, normalized size = 0.98

$$\frac{1}{6}\,b^2x^6-\frac{1}{5}\,(-i\,\pi b-2\,ab)x^5-\frac{1}{16}\,\left(\pi^2-4i\,\pi a-4\,a^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 - 1/5*(-I*pi*b - 2*a*b)*x^5 - 1/16*(pi^2 - 4*I*pi*a - 4*a^2)*x^
4
```

Mupad [B]
time = 1.18, size = 36, normalized size = 0.86

$$\frac{b^2\,x^6}{60}-\frac{b\,x^5\,\mathrm{acoth}(\tanh(a+b\,x))}{10}+\frac{x^4\,\mathrm{acoth}(\tanh(a+b\,x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(tanh(a + b*x))^2,x)
```

```
[Out] (x^4*acoth(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*acoth(tanh(a + b*x))
)/10
```

3.138 $\int x^2 \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=42

$$\frac{b^2x^5}{30}-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2$$

```
[Out] 1/30*b^2*x^5-1/6*b*x^4*arccoth(tanh(b*x+a))+1/3*x^3*arccoth(tanh(b*x+a))^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2199, 30}

$$-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2+\frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcCoth[Tanh[a + b*x]])/6 + (x^3*ArcCoth[Tanh[a + b*x
]]^2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(\tanh(a+bx))^2\,dx&=\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2-\frac{1}{3}(2b)\int x^3\coth^{-1}(\tanh(a+bx))\,dx\\&=-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2+\frac{1}{6}b^2\int x^4\,dx\\&=\frac{b^2x^5}{30}-\frac{1}{6}bx^4\coth^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\coth^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3\left(b^2x^2-5bx\coth^{-1}(\tanh(a+bx))+10\coth^{-1}(\tanh(a+bx))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^3*(b^2*x^2 - 5*b*x*ArcCoth[Tanh[a + b*x]] + 10*ArcCoth[Tanh[a + b*x]]^2)
)/30
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 34.22, size = 3418, normalized size = 81.38

method	result	size
risch	Expression too large to display	3418

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*Pi*b*x^4-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a
)+1))^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/
12*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))^3+1/30*b^2*x^5+1/3*x^3*ln(exp(b*x+
a))^2+1/24*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/24*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/24*I*Pi*b*x^4*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/12*I*Pi*b*x^4*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2-1/24*I*Pi*b*x^4*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2-1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))-1/24*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^3+1/24*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^5+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/12*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(b*x+
a))^2*Pi^2*x^3+1/6*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(b*x+a))*Pi^2*x^3+1/6
*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(exp(2*b*x+2*a
)+1))^3+1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^3+1/6*Pi^2*x^3*csgn(I/(exp(2*b*x+2
*a)+1))^5-1/6*csgn(I/(exp(2*b*x+2*a)+1))^3*Pi^2*x^3-1/12*Pi^2*x^3*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*csgn(I/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x
^3*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3*csgn
(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*P
```

```
i^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+1/6*csgn(I/(exp(2*b*x+2*a)+1))^2*Pi^2*x^3-1/12*I*Pi*b*x^4*csgn(I/(exp(2
*b*x+2*a)+1))^2-1/12*csgn(I*exp(2*b*x+2*a))^3*Pi^2*x^3-1/24*Pi^2*x^3*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))-1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn
(I/(exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^6+(-1/6*b
*x^4+1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2-1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a))^3+1/6*I*Pi*x^3*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*I*Pi*x^3*csgn(
I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/3*I*Pi*x^3*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2-1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3-1/6*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2-
1/3*I*Pi*x^3-1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^3)*ln(exp(b*x+a))+1/24
*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/12*Pi^2
*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))-1/48*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^6+1/12*Pi^2*x^3*
csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/
6*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^2-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3-1/12*csgn(I/(exp(
2*b*x+2*a)+1))^4*Pi^2*x^3+1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3*csgn
(I/(exp(2*b*x+2*a)+1))^3+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+
2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/24*Pi^2*x^3*csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2+1/24*I*Pi*b*x^4*csgn(I*exp(2*b*x+2*a))^3-1/12*Pi^2*x^3*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I
*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*Pi^2*x^3*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/48*Pi^2*x^3*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^6+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*ex
p(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/48*Pi^2*x^3*csgn(
I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^4+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/24*P
i^2*x^3*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b...
```

Maxima [**A**]

time = 0.33, size = 36, normalized size = 0.86

$$\frac{1}{30}\,b^2x^5-\frac{1}{6}\,bx^4\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)+\frac{1}{3}\,x^3\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/30*b^2*x^5 - 1/6*b*x^4*arccoth(tanh(b*x + a)) + 1/3*x^3*arccoth(tanh(b*x
+ a))^2
```

Fricas [A]
time = 0.34, size = 30, normalized size = 0.71

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4-\frac{1}{12}\,\left(\pi^2-4\,a^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 - 1/12*(pi^2 - 4*a^2)*x^3
```

Sympy [A]
time = 0.17, size = 37, normalized size = 0.88

$$\frac{b^2x^5}{30}-\frac{bx^4\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{6}+\frac{x^3\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**2,x)
```

```
[Out] b**2*x**5/30 - b*x**4*acoth(tanh(a + b*x))/6 + x**3*acoth(tanh(a + b*x))**2
/3
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 41, normalized size = 0.98

$$\frac{1}{5}\,b^2x^5-\frac{1}{4}\,(-i\,\pi b-2\,ab)x^4-\frac{1}{12}\,\left(\pi^2-4i\,\pi a-4\,a^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 - 1/4*(-I*pi*b - 2*a*b)*x^4 - 1/12*(pi^2 - 4*I*pi*a - 4*a^2)*x^
3
```

Mupad [B]
time = 1.18, size = 36, normalized size = 0.86

$$\frac{b^2\,x^5}{30}-\frac{b\,x^4\,\mathrm{acoth}(\tanh(a+b\,x))}{6}+\frac{x^3\,\mathrm{acoth}(\tanh(a+b\,x))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(tanh(a + b*x))^2,x)
```

```
[Out] (x^3*acoth(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*acoth(tanh(a + b*x))
)/6
```

3.139 $\int x \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=34

$$\frac{x\coth^{-1}(\tanh(a+bx))^3}{3b}-\frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

```
[Out] 1/3*x*arccoth(tanh(b*x+a))^3/b-1/12*arccoth(tanh(b*x+a))^4/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2199, 2188, 30}

$$\frac{x\coth^{-1}(\tanh(a+bx))^3}{3b}-\frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x\coth^{-1}(\tanh(a+bx))^2\,dx = \frac{x\coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a+bx))^3\,dx}{3b}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\text{Subst}\left(\int x^3\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{3b^2}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a+bx))^4}{12b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(34) = 68.
time = 0.04, size = 74, normalized size = 2.18

$$\frac{(a+bx)\left(-((3a-bx)(a+bx)^2)+4(2a^2+abx-b^2x^2)\coth^{-1}(\tanh(a+bx))-6(a-bx)\coth^{-1}(\tanh(a+bx))^2\right)}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcCot
h[Tanh[a + b*x]] - 6*(a - b*x)*ArcCoth[Tanh[a + b*x]]^2))/(12*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 33.90, size = 3418, normalized size = 100.53

method	result	size
risch	Expression too large to display	3418

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*Pi*b*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*x^2*ln(exp(b*x+a))^2+1/12*b^2*x^4-1/8*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(b*x+a))^2*Pi^2*x^2+1/4*csgn(I*exp(2*b*x+2
*a))^2*csgn(I*exp(b*x+a))*Pi^2*x^2+1/4*csgn(I/(exp(2*b*x+2*a)+1))^2*Pi^2*x^
2-1/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b
*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*Pi^2*x^2*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b
*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn
(I/(exp(2*b*x+2*a)+1))^2+(-1/3*b*x^3+1/4*I*Pi*x^2*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*x^2*csgn(I/(exp(2*b*x+2*
a)+1))^3-1/2*I*Pi*x^2+1/2*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2-1/4*I*Pi*x^
2*csgn(I*exp(2*b*x+2*a))^3-1/4*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
```

```
xp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*I*Pi*x^2*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/4*I*Pi*x^2*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/4*I*Pi*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))+1/4*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2)*ln(exp(b*x+a))-1/32*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^6+1/16*Pi^
2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^3+1/12*I*Pi*b*x^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^3-1/4*csgn(I/(exp(2*b*x+2*a)+1))^3*Pi^2*x^2+1/4*Pi^2*x^2*csgn(I/(
exp(2*b*x+2*a)+1))^5-3/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))^4+1/8*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/16*Pi^2*x^
2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16
*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^3-1/32*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^4+1/16*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^5-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^6-1/8*Pi^2*x^2*cs
gn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*
Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/32*Pi^2*x^2*csgn(I/(exp
(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2-1/8*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2*x^2+1/6*I*Pi*b*x^3+1/8*Pi
^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+1/8*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I/(exp(2*b*x+2*a)+1))^2-1/8*c
sgn(I*exp(2*b*x+2*a))^3*Pi^2*x^2-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))^4*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*Pi^2*x
^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*x^2-1/8*Pi^2*x^2*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^4+1/16*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^
2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*Pi^2*x^2-1/8*Pi^2*x^2*csgn(I/(exp
(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))^3-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2
*a)+1))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*Pi^2*x^2*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2+1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a
))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/12*I*Pi*b*x^3*csgn(I*exp
(2*b*x+2*a))^3-1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/8*Pi^2*x^2*csgn(I*exp(b*x
+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1
/8*Pi^2*x^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2
*a)+1))^3+1/4*Pi^2*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I/(
exp(2*b*x+2*a)+1))^3+1/8*Pi^2*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2*csgn(I/(exp(2*b*x+2*a)+1))^3-1/32*Pi^2*x^2*csgn(I
/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/16*Pi^
2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
```

```
5-1/32*Pi^2*x^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+1/8*Pi^2*x^2*
csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/8*Pi^2*x^2*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
-1/16*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2
*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/16*Pi^2*x^2*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))-1/8*Pi^2*x^2*csgn(I/(exp(2*b*x+2*a)+1...
```

Maxima [A]
time = 0.34, size = 36, normalized size = 1.06

$$\frac{1}{12}\,b^2x^4-\frac{1}{3}\,bx^3\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)+\frac{1}{2}\,x^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^4 - 1/3*b*x^3*arccoth(tanh(b*x + a)) + 1/2*x^2*arccoth(tanh(b*x
+ a))^2
```

Fricas [A]
time = 0.33, size = 30, normalized size = 0.88

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3-\frac{1}{8}\left(\pi^2-4\,a^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 - 1/8*(pi^2 - 4*a^2)*x^2
```

Sympy [A]
time = 0.14, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x\operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}{3b}-\frac{\operatorname{acoth}^{4}\left(\tanh\left(a+bx\right)\right)}{12b^2} & \text{for}\ b\neq 0\\ \frac{x^2\operatorname{acoth}^{2}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x*acoth(tanh(a + b*x))**3/(3*b) - acoth(tanh(a + b*x))**4/(12*b*
*2), Ne(b, 0)), (x**2*acoth(tanh(a))**2/2, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 41, normalized size = 1.21

$$\frac{1}{4}b^2x^4 - \frac{1}{3}(-i\pi b - 2ab)x^3 - \frac{1}{8}(\pi^2 - 4i\pi a - 4a^2)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 - 1/3*(-I*pi*b - 2*a*b)*x^3 - 1/8*(pi^2 - 4*I*pi*a - 4*a^2)*x^2
```

Mupad [B]
time = 1.15, size = 36, normalized size = 1.06

$$\frac{b^2x^4}{12} - \frac{bx^3 \operatorname{acoth}(\tanh(a+bx))}{3} + \frac{x^2 \operatorname{acoth}(\tanh(a+bx))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(tanh(a + b*x))^2,x)
```

```
[Out] (x^2*acoth(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*acoth(tanh(a + b*x))
)/3
```

3.140 $\int \coth^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

```
[Out] 1/3*arccoth(tanh(b*x+a))^3/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = $\{2188, 30\}$

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^2\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^3}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*b)
```

Maple [A]
time = 0.50, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^3}{3b}$	15
risch	Expression too large to display	14844

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arccoth(tanh(b*x+a))^3/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.
time = 0.33, size = 33, normalized size = 2.06

$$\frac{1}{3}\,b^2x^3 - bx^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right) + x\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 - b*x^2*arccoth(tanh(b*x + a)) + x*arccoth(tanh(b*x + a))^2
```

Fricas [A]
time = 0.34, size = 27, normalized size = 1.69

$$\frac{1}{3}\,b^2x^3 + abx^2 - \frac{1}{4}\,\left(\pi^2 - 4\,a^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 - 1/4*(pi^2 - 4*a^2)*x
```

Sympy [A]
time = 0.08, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{3b} & \text{for}\ b \neq 0 \\ x\operatorname{acoth}^2\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2,
True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 39, normalized size = 2.44

$$\frac{1}{3}b^2x^3 - \frac{1}{2}(-i\pi b - 2ab)x^2 - \frac{1}{4}(\pi^2 - 4i\pi a - 4a^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 - 1/2*(-I*pi*b - 2*a*b)*x^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x
```

Mupad [B]
time = 1.12, size = 33, normalized size = 2.06

$$\frac{b^2x^3}{3} - bx^2\,\mathrm{acoth}(\tanh(a+bx)) + x\,\mathrm{acoth}(\tanh(a+bx))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2,x)
```

```
[Out] x*acoth(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*acoth(tanh(a + b*x))
```

3.141 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} \, dx$

Optimal. Leaf size=49

$$-bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2+\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log(x)$$

```
[Out] -b*x*(b*x-arccoth(tanh(b*x+a)))+1/2*arccoth(tanh(b*x+a))^2+(b*x-arccoth(tan
h(b*x+a)))^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2190, 2189, 29}

$$-bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

```
[Out] -(b*x*(b*x - ArcCoth[Tanh[a + b*x]])) + ArcCoth[Tanh[a + b*x]]^2/2 + (b*x -
 ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx = \frac{1}{2}\coth^{-1}(\tanh(a+bx))^2 - \left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a+$$
$$= -bx\left(bx-\coth^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\coth^{-1}(\tanh(a+bx))^2 - \left(\left(bx-\coth^{-1}\right.\right.$$
$$= -bx\left(bx-\coth^{-1}(\tanh(a+bx))\right) + \frac{1}{2}\coth^{-1}(\tanh(a+bx))^2 + \left(bx-\coth^{-1}\right.$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2 - (a+bx)\left(a+2bx-2\coth^{-1}(\tanh(a+bx))\right) + \left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x,x]
```

```
[Out] (a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcCoth[Tanh[a + b*x]]) + (-(b*x)
+ ArcCoth[Tanh[a + b*x]])^2*Log[b*x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 3774, normalized size = 77.02

method	result	size
risch	Expression too large to display	3774

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))^3-1/2*I*Pi*b*x*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*Pi^2*ln(x)*csgn(I/
(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2*I*ln(
x)*Pi*x*b*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*ln(x)*Pi*x*b*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a
)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2-I*ln(exp(b*x+a))*Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3-I*Pi*x*b*csgn(I/(e
xp(2*b*x+2*a)+1))^3+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*ln(x)*Pi^2*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*ln(x)*Pi^2*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(
```

2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^2+1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csg
n(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*I*Pi*b*
x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/
4*ln(x)*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^3-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(
I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2*I*ln(x)*Pi*x*b*csgn(I/(exp(2*b*x
+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-
1/8*ln(x)*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3-1/2*I*ln(x)*ln(exp(b*x+a))*Pi*csgn(I*exp(2*b*x+
2*a))^3-1/2*I*ln(x)*ln(exp(b*x+a))*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3-1/4*ln(x)*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/4*
Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+ln(x)*ln(
exp(b*x+a))^2+b^2*x^2*ln(x)+2*b*ln(exp(b*x+a))*x+1/8*ln(x)*Pi^2*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2-1/2*I*ln(x)*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^2-1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*ln(x)*ln(exp(b*x+a))*Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)-1/16*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a))^6+I*Pi*b*x*csgn(I*exp(b*x+a))*csgn
(I*exp(2*b*x+2*a))^2+I*ln(x)*ln(exp(b*x+a))*Pi*csgn(I*exp(b*x+a))*csgn(I*ex
p(2*b*x+2*a))^2+1/2*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^2-1/2*I*ln(x)*ln(exp(b*x+a))*Pi*csgn(I*exp(b*x+a))^2*csg
n(I*exp(2*b*x+2*a))+I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2+I*ln(exp(b*x+a))*
Pi*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2-1/4*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2*b*x+2*a))^3+1/2*I*ln(x)*l
n(exp(b*x+a))*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2-1/16*ln(x)*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-2*
b*ln(x)*ln(exp(b*x+a))*x-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I
*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*Pi^2*ln(x)*c
sgn(I/(exp(2*b*x+2*a)+1))^6-1/4*Pi^2*ln(x)*csgn(I*exp(2*b*x+2*a))^3-1/4*Pi^
2*ln(x)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/2*csgn(I/(exp(2*b*x+2
*a)+1))^2*Pi^2*ln(x)-1/2*csgn(I/(exp(2*b*x+2*a)+1))^3*Pi^2*ln(x)-1/4*Pi^2*l
n(x)-1/2*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^2-I*Pi*ln(x)*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2+I*Pi*ln(x)*x*
b*csgn(I/(exp(2*b*x+2*a)+1))^3-1/2*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*csgn(I/(ex
p(2*b*x+2*a)+1))^4*Pi^2*ln(x)+1/2*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^5-3
/2*b^2*x^2+1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))^2

```
*csgn(I*exp(2*b*x+2*a))-1/2*I*Pi*b*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))-1/16*ln(x)*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2-1/16*ln
(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^4+1/8*ln(x)*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^5+1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I*ln(x)*Pi*x*b*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*ln(exp(b*x+a)...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.64, size = 38, normalized size = 0.78

$$\frac{1}{2}\,b^2x^2-(i\,\pi b-2\,ab)x-\frac{1}{4}\left(\pi^2+4i\,\pi a-4\,a^2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 - (I*pi*b - 2*a*b)*x - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(x)
```

Fricas [A]
time = 0.36, size = 27, normalized size = 0.55

$$\frac{1}{2}\,b^2x^2+2\,abx-\frac{1}{4}\left(\pi^2-4\,a^2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="fricas")
[Out] 1/2*b^2*x^2 + 2*a*b*x - 1/4*(pi^2 - 4*a^2)*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x,x)
[Out] Integral(acoth(tanh(a + b*x))**2/x, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 37, normalized size = 0.76

$$\frac{1}{2}\,b^2x^2+(i\,\pi b+2\,ab)x-\frac{1}{4}\left(\pi^2-4i\,\pi a-4\,a^2\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + (I*pi*b + 2*a*b)*x - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(x)
```

Mupad [B]
time = 0.29, size = 183, normalized size = 3.73

$$\ln(x)\left(\frac{\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)^2}{4}-a\left(2a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)+a^2\right)+\frac{b^2x^2}{2}-bx\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2/x,x)
```

```
[Out] log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log
(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*
b*x) + a^2) + (b^2*x^2)/2 - b*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
```

3.142 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx$

Optimal. Leaf size=39

$$2b^2x-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-2b\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] 2*b^2*x-arccoth(tanh(b*x+a))^2/x-2*b*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2199, 2189, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] 2*b^2*x - ArcCoth[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcCoth[Tanh[a + b*x]])*L
og[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^2}{x} + (2b)\int \frac{\coth^{-1}(\tanh(a+bx))}{x}\,dx$$
$$= 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - \left(2b\left(bx - \coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{x}\,dx$$
$$= 2b^2x - \frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b\left(bx - \coth^{-1}(\tanh(a+bx))\right)\log(x)$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 0.95

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b^2x\log(x) + 2b\coth^{-1}(\tanh(a+bx))(1+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcCoth[Tanh[a + b*x]]
*(1 + Log[x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 1095, normalized size = 28.08

method	result	size
risch	Expression too large to display	1095

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln(exp(b*x+a))^2+2*b^2*x-1/2*I*Pi*ln(x)*b*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+I*Pi*ln(x)*
b*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/2*I*Pi*ln(x)*b*csgn(I*exp(b
*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2*I*Pi*ln(x)*b*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I*Pi/x*ln(exp(b*x+a))*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*Pi/x*ln(exp(b*x+a))*csgn(I*exp(b*x+a)
)*csgn(I*exp(2*b*x+2*a))^2+1/16*Pi^2*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*csgn(I/(exp(2*b*x+2*a)
+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x
+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))
^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2)^2/x+1/2*I*Pi*ln(x)*b*csgn(I/(exp(
```

```
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I*Pi/x*ln(ex
p(b*x+a))*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))-2*ln(x)*x*b^2+2*ln(x)*ln(exp(b*x+a))*b+I*Pi*ln(e
xp(b*x+a))/x+1/2*I*Pi/x*ln(exp(b*x+a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))^3-1/2*I*Pi*ln(x)*b*csgn(I*exp(2*b*x+2*a))^3-1/2*I*Pi*ln(x)*b*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*ln(exp(b*x+a))/x*csgn(I/(exp(2*b*x
+2*a)+1))^2+I*Pi*b*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2+I*Pi*ln(exp(b*x+a))/x
*csgn(I/(exp(2*b*x+2*a)+1))^3-I*Pi*b*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^3-1/2
*I*Pi/x*ln(exp(b*x+a))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-1/2*I*Pi/x*ln(exp(b*x+a))*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/2*I*Pi/x*ln(exp(b*x+a))*csgn(I*exp(2
*b*x+2*a))^3-I*Pi*b*ln(x)
```

Maxima [A]
time = 0.32, size = 54, normalized size = 1.38

$$2\,b\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right)-2\,\left(b\left(x+\frac{a}{b}\right)\log\left(x\right)-b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b-\frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*arccoth(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b - arccoth(tanh(b*x + a))^2/x
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.74

$$\frac{4\,b^{2}x^{2}+8\,abx\log\left(x\right)+\pi^{2}-4\,a^{2}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 + 8*a*b*x*log(x) + pi^2 - 4*a^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**2/x**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 36, normalized size = 0.92

$$b^2x + (i\,\pi b + 2\,ab)\log\left(x\right) + \frac{\pi^2 - 4i\,\pi a - 4\,a^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + (I*pi*b + 2*a*b)*log(x) + 1/4*(pi^2 - 4*I*pi*a - 4*a^2)/x
```

Mupad [B]
time = 1.25, size = 207, normalized size = 5.31

$$b\,\ln\left(\frac{{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\frac{{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}^2}{4\,x}-b\,\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\frac{{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}^2}{4\,x}+b\,\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\ln\left(x\right)-b\,\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\ln\left(x\right)+\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2\,x}-2\,b^2\,x\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2/x^2,x)
```

```
[Out] b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) - 1)) - lo
g(-2/(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) + b*log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1))*log(x) - b*log(-2/(exp(2*a)*exp(2*b*x) - 1))*log(x)
 + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)
*exp(2*b*x) - 1)))/(2*x) - 2*b^2*x*log(x)
```

3.143 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx$

Optimal. Leaf size=36

$$-\frac{b\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\log(x)$$

```
[Out] -b*arccoth(tanh(b*x+a))/x-1/2*arccoth(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2199, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\coth^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -((b*ArcCoth[Tanh[a + b*x]])/x) - ArcCoth[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b\int \frac{\coth^{-1}(\tanh(a+bx))}{x^2}\,dx\\
&= -\frac{b\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\int \frac{1}{x}\,dx\\
&= -\frac{b\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\log(x)
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 42, normalized size = 1.17

$$-\frac{2bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2-b^2x^2(3+2\log(x))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -1/2*(2*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 - b^2*x^2*(3
+ 2*Log[x]))/x^2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.35, size = 3213, normalized size = 89.25

method	result	size
risch	Expression too large to display	3213

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln(exp(b*x+a))^2-1/4*(4*b*x+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+I*Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*exp(2*b*x+2*a))^3-2*I*Pi-2*I*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+2*I*Pi*csgn(I/(exp(2*
b*x+2*a)+1))^2)/x^2*ln(exp(b*x+a))+1/32*(-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
^2*csgn(I*exp(2*b*x+2*a))^3+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2+4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*
exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+8*I*Pi
*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))
-8*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+32*b^2*x^2*ln(x)+8*I*Pi
*b*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+16*I*Pi*x*b-4*Pi^2*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi^2*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2
*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+2*Pi^2*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-
```

```
2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(b
*x+a))^3*csgn(I*exp(2*b*x+2*a))^3+6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))^4-4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5-2*Pi^2*csgn(
I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi^2*csgn(I
*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2-8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-16*I*Pi*b*x*csgn(I*exp(b*x+a
))*csgn(I*exp(2*b*x+2*a))^2-8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-2*Pi^2*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I*exp(b*x+a))*csgn(
I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+Pi^2*csgn(I
/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2-2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+Pi^2*csgn(I*exp(2*b*x+2*a))^6-
2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^5+2*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^3-2*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^5+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi^2*csgn(I*exp(2*b*x+2*a
))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+Pi^2*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^6+Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^4+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^6+4*csgn(I/(exp
(2*b*x+2*a)+1))^4*Pi^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^5-4*Pi^2*csgn(I/(e
xp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-16*I*Pi*x*b*csgn(I/(exp(2*b*x+2*
a)+1))^2+4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csgn(I/(
exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)
+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+8
*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3+8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(e
xp(2*b*x+2*a)+1))^2+8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*
csgn(I*exp(2*b*x+2*a))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-4*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*csgn(I/(exp(2*b*x
+2*a)+1))^2*Pi^2+16*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))^3+8*I*Pi*b*x*csgn(I
*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a...
```

<u>Maxima</u> [A]

time = 0.34, size = 34, normalized size = 0.94

$$b^2 \log(x) - \frac{b \operatorname{arcoth}(\tanh(bx+a))}{x} - \frac{\operatorname{arcoth}(\tanh(bx+a))^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
[Out] b^2*log(x) - b*arccoth(tanh(b*x + a))/x - 1/2*arccoth(tanh(b*x + a))^2/x^2
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.81

$$\frac{8b^2x^2\log(x) - 16abx + \pi^2 - 4a^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
[Out] 1/8*(8*b^2*x^2*log(x) - 16*a*b*x + pi^2 - 4*a^2)/x^2
```

Sympy [A]
time = 0.19, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**3,x)
[Out] b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2/(2*x**2)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 37, normalized size = 1.03

$$b^2 \log(x) - \frac{8i\pi bx + 16abx - \pi^2 + 4i\pi a + 4a^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="giac")
[Out] b^2*log(x) - 1/8*(8*I*pi*b*x + 16*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^2
```

Mupad [B]
time = 1.16, size = 34, normalized size = 0.94

$$b^2 \ln(x) - \frac{\frac{\operatorname{acoth}(\tanh(a+b\,x))^2}{2} + b\,x\operatorname{acoth}(\tanh(a+b\,x))}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2/x^3,x)
[Out] b^2*log(x) - (acoth(tanh(a + b*x))^2/2 + b*x*acoth(tanh(a + b*x)))/x^2
```

3.144 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} \, dx$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/3*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2198}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 1.10

$$-\frac{b^2x^2+bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] -1/3*(b^2*x^2 + b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2)/x^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 3217, normalized size = 103.77

method	result	size
risch	Expression too large to display	3217

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*ln(exp(b*x+a))^2-1/6*(2*b*x+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2-I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+I*Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-I*Pi*csgn(I*exp(2*b*x+2*a))^3-2*I*Pi+2*I*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-2*I*Pi*csgn(I/(exp(2*
b*x+2*a)+1))^3)/x^3*ln(exp(b*x+a))-1/48*(4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^
2*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+8*I*Pi*b*x*csgn(I*exp(b*x+a))*csgn(I*exp(2*
b*x+2*a))^2-4*Pi^2-4*I*Pi*b*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-4
*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))^3-4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2
*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-8*I*Pi*x*b*
csgn(I/(exp(2*b*x+2*a)+1))^3+8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))^2+4*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*P
i^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+8*Pi^2*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^2+4*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*I*Pi*x*b+4*Pi^2*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-4*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-2*Pi^2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*ex
p(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-2*Pi^2*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*Pi^2*csg
n(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-6*Pi^2*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))^4+4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+2*P
i^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-Pi
^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2-4*Pi^2*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+
```

```
2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2-2*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*exp(b*x+a))*csgn(I*
exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+4*Pi^2*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3-Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+16*b^2*x^2+4
*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2-Pi^2*csgn(I*exp(2*b*x+2*a))^6+2*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-2*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^2-Pi^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^4-Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-Pi^2*csgn(I/(exp(2*b
*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-4*Pi^2*csgn(I/(ex
p(2*b*x+2*a)+1))^6-4*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2+8*Pi^2*csgn(I/(exp(2
*b*x+2*a)+1))^5+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+8*Pi^
2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-
4*Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I/(exp(2*b*x
+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))^3+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*Pi^2*csg
n(I/(exp(2*b*x+2*a)+1))^3-4*I*Pi*b*x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^3+4*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+
2*a)+1))^2-8*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(b*x+a))*csgn(I*ex
p(2*b*x+2*a))^2-4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+8*csgn(I/(exp(2*b*x+2*a)+1))
^2*Pi^2+4*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csg...
```

Maxima [A]
time = 0.34, size = 36, normalized size = 1.16

$$-\frac{b^2}{3\,x}-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{3\,x^2}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/x - 1/3*b*arccoth(tanh(b*x + a))/x^2 - 1/3*arccoth(tanh(b*x + a))^
2/x^3
```

Fricas [A]

time = 0.42, size = 29, normalized size = 0.94

$$-\frac{12\,b^2x^2+12\,abx-\pi^2+4\,a^2}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^2*x^2 + 12*a*b*x - pi^2 + 4*a^2)/x^3
```

Sympy [A]
time = 0.25, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x}-\frac{b\,\mathrm{acoth}\,(\tanh\,(a+bx))}{3x^2}-\frac{\mathrm{acoth}^2\,(\tanh\,(a+bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**4,x)
```

```
[Out] -b**2/(3*x) - b*acoth(tanh(a + b*x))/(3*x**2) - acoth(tanh(a + b*x))**2/(3*
x**3)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 38, normalized size = 1.23

$$-\frac{12\,b^2x^2+6i\,\pi bx+12\,abx-\pi^2+4i\,\pi a+4\,a^2}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(12*b^2*x^2 + 6*I*pi*b*x + 12*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)/x^3
```

Mupad [B]
time = 1.12, size = 32, normalized size = 1.03

$$-\frac{b^2\,x^2+b\,x\,\mathrm{acoth}(\tanh(a+b\,x))+\mathrm{acoth}(\tanh(a+b\,x))^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2/x^4,x)
```

```
[Out] -(acoth(tanh(a + b*x))^2 + b^2*x^2 + b*x*acoth(tanh(a + b*x)))/(3*x^3)
```

3.145 $\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx$

Optimal. Leaf size=64

$$\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/12*b*arccoth(tanh(b*x+a))^3/x^3/(b*x-arccoth(tanh(b*x+a)))^2+1/4*arccoth(
tanh(b*x+a))^3/x^4/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2202, 2198}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5}\,dx &= \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4}\,dx}{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\coth^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.58

$$-\frac{b^2x^2 + 2bx\coth^{-1}(\tanh(a+bx)) + 3\coth^{-1}(\tanh(a+bx))^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] -1/12*(b^2*x^2 + 2*b*x*ArcCoth[Tanh[a + b*x]] + 3*ArcCoth[Tanh[a + b*x]]^2)
/x^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 3217, normalized size = 50.27

method	result	size
risch	Expression too large to display	3217

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*ln(exp(b*x+a))^2-1/24*(4*b*x+3*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-6*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^
3+6*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-3*I*Pi*csgn(I*exp(b*x+
a))^2*csgn(I*exp(2*b*x+2*a))+3*I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2+6*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-3*I*Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3*I*Pi*csgn(I*exp(2*b*x+2*a))^3-6*I
*Pi-3*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1)))/x^4*ln(exp(b*x+a))-1/192*(12*Pi^2*csgn(I/(exp(
2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a))^3-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1)
)^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+16*I*Pi*b*x*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2+16*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))^2-12*Pi
^2-12*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*
a)+1))^3-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-16*I*Pi*x*b*csgn(I/(exp(2*b*x+2*a)+1))^3+
12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2-12*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+24*Pi^2*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2+12*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-6*Pi^2*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*e
xp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x
+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-16*I*Pi*x*b-6*Pi^2*csgn(
```

```
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+12*Pi
^2*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-18*Pi^2*csgn(I*exp(b*x+a))
^2*csgn(I*exp(2*b*x+2*a))^4+12*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a
))^5+6*Pi^2*csgn(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+
1))^2-3*Pi^2*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2-12*Pi^2*csgn(I/(
exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))^4+6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-6*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-12*Pi^2*csgn(I*exp(b*x
+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1
2*Pi^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^3-3*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*
a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+6*Pi^2*csgn(I/(exp(2*b*x+
2*a)+1))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^3+16*b^2*x^2+8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))^2+8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^2-8*I*Pi*b*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-8*I*Pi*b*x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-3*Pi^2*csgn(I*ex
p(2*b*x+2*a))^6+6*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^5-6*Pi^2*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^3+6*Pi^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^5-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*cs
gn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3*Pi^2*c
sgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-3*Pi^2*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^6-3*Pi^2*csgn(I/(exp(2*b*x+2*a)+1
))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4-12*Pi^2*csgn(I/(exp(2*b*x+
2*a)+1))^6-12*csgn(I/(exp(2*b*x+2*a)+1))^4*Pi^2+24*Pi^2*csgn(I/(exp(2*b*x+2
*a)+1))^5+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+24*Pi^2*c
sgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-12*
Pi^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-12*Pi^2*csgn(I/(exp(2*b*x+
2*a)+1))^3*csgn(I*exp(2*b*x+2*a))^3+12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^3*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-24*Pi^2*cs
gn(I/(exp(2*b*x+2*a)+1))^3+12*Pi^2*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))*csgn(I/(exp(2*b*x+2*a)+1))^2-24*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-12*Pi^2*csgn(I/(exp(2*b*x+2*a)+1))^4
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+12*Pi^2*c
sgn(I/(exp(2*b*x+2*a)+1))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+24*
csgn(I/(exp(2*b*x+2*a)+1))^2*Pi^2-8*I*Pi*b*x*cs...
```

Maxima [A]

time = 0.34, size = 36, normalized size = 0.56

$$-\frac{b^{2}}{12\,x^{2}}-\frac{b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{6\,x^{3}}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*b^2/x^2 - 1/6*b*arccoth(tanh(b*x + a))/x^3 - 1/4*arccoth(tanh(b*x + a
))^2/x^4
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.45

$$-\frac{24\,b^{2}x^{2}+32\,abx-3\,\pi^{2}+12\,a^{2}}{48\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*b^2*x^2 + 32*a*b*x - 3*pi^2 + 12*a^2)/x^4
```

Sympy [A]
time = 0.34, size = 39, normalized size = 0.61

$$-\frac{b^{2}}{12x^{2}}-\frac{b\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{6x^{3}}-\frac{\mathrm{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**2/x**5,x)
```

```
[Out] -b**2/(12*x**2) - b*acoth(tanh(a + b*x))/(6*x**3) - acoth(tanh(a + b*x))**2
/(4*x**4)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 38, normalized size = 0.59

$$-\frac{24\,b^{2}x^{2}+16i\,\pi bx+32\,abx-3\,\pi^{2}+12i\,\pi a+12\,a^{2}}{48\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(24*b^2*x^2 + 16*I*pi*b*x + 32*a*b*x - 3*pi^2 + 12*I*pi*a + 12*a^2)/x
^4
```

Mupad [B]
time = 1.17, size = 36, normalized size = 0.56

$$-\frac{\operatorname{acoth}(\tanh(a+b\,x))^2}{4\,x^4}-\frac{b^2}{12\,x^2}-\frac{b\operatorname{acoth}(\tanh(a+b\,x))}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^2/x^5,x)
[Out] - acoth(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*acoth(tanh(a + b*x)))/
(6*x^3)
```

3.146 $\int x^m \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=110

$$-\frac{6b^3x^{4+m}}{(1+m)\left(24+26m+9m^2+m^3\right)}+\frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}}{}$$

```
[Out] -6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arccoth(tanh(b*x+a))
/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arccoth(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m)*
arccoth(tanh(b*x+a))^3/(1+m)
```

Rubi [A]

time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{6b^2x^{m+3}\coth^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\coth^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b^3x^{m+4}}{(m+1)\left(m^3+9m^2+26m+24\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcCoth[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcCoth[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^m \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m} - \frac{(3b)\int x^{1+m}\coth^{-1}(\tanh(a+bx))^2\,dx}{1+m}$$
$$= -\frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))^3}{1+m} + \frac{(6b^2)\int$$
$$= \frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx^{2+m}\coth^{-1}(\tanh(a+bx))^2}{2+3m+m^2} + \frac{x^{1+m}\,\mathrm{c}$$
$$= -\frac{6b^3x^{4+m}}{(4+m)(6+11m+6m^2+m^3)} + \frac{6b^2x^{3+m}\coth^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3} - \frac{3bx$$

Mathematica [A]
time = 0.09, size = 97, normalized size = 0.88

$$\frac{x^{1+m}\left(-6b^3x^3+6b^2(4+m)x^2\coth^{-1}(\tanh(a+bx))-3b(12+7m+m^2)\,x\coth^{-1}(\tanh(a+bx))^2+(24+26m+9m^2+m^3)\coth^{-1}(\tanh(a+bx))^3\right)}{(1+m)(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*(12
 + 7*m + m^2)*x*ArcCoth[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcCot
h[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 10.12, size = 63382, normalized size = 576.20

method	result	size
risch	Expression too large to display	63382

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.35, size = 109, normalized size = 0.99

$$-\frac{3\,bx^2x^m\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{(m+2)(m+1)} + \frac{x^{m+1}\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{m+1} - \frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)} - \frac{bx^3x^m\,\mathrm{arcoth}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3*b*x^2*x^m*arccoth(tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^(m + 1)*arccoth
(tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*x^
3*x^m*arccoth(tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)
```

Fricas [A]
time = 0.36, size = 209, normalized size = 1.90

$$\frac{(4\,(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+12\,(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2)x^3+3\,(4\,a^2bm^3+32\,a^2bm^2+76\,a^2bm-\pi^2(bm^3+8\,bm^2+19\,bm+12\,b)+48\,a^2b)x^2+(4\,a^3m^3+36\,a^3m^2+104\,a^3m-3\,\pi^2(am^3+9\,am^2+26\,am+24\,a)+96\,a^3)x)x^m}{4\,(m^4+10\,m^3+35\,m^2+50\,m+24)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 12*(a*b^2*m^3 + 7*a*b
^2*m^2 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(4*a^2*b*m^3 + 32*a^2*b*m^2 + 76*a^2
*b*m - pi^2*(b*m^3 + 8*b*m^2 + 19*b*m + 12*b) + 48*a^2*b)*x^2 + (4*a^3*m^3
+ 36*a^3*m^2 + 104*a^3*m - 3*pi^2*(a*m^3 + 9*a*m^2 + 26*a*m + 24*a) + 96*a^
3)*x)*x^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^3\log(x) - \frac{b^2\operatorname{acoth}(\tanh(a+bx))}{x} - \frac{b\operatorname{acoth}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3x^3} & \text{for}\ m=-4 \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3}\,dx & \text{for}\ m=-3 \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^2}\,dx & \text{for}\ m=-2 \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x}\,dx & \text{for}\ m=-1 \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m\operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m\operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m\operatorname{acoth}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{21bmx^2x^m\operatorname{acoth}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{36bx^2x^m\operatorname{acoth}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{m^3xx^m\operatorname{acoth}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{9m^2xx^m\operatorname{acoth}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{26mxx^m\operatorname{acoth}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24xx^m\operatorname{acoth}^3(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*x
))**2/(2*x**2) - acoth(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(ac
oth(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(acoth(tanh(a + b*x))*
*3/x**2, x), Eq(m, -2)), (Integral(acoth(tanh(a + b*x))**3/x, x), Eq(m, -1)
), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*m*x**
3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*b**
2*x**3*x**m*acoth(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 3
*b*m**2*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2 + 50*m
+ 24) - 21*b*m*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
+ 50*m + 24) - 36*b*x**2*x**m*acoth(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + m**3*x*x**m*acoth(tanh(a + b*x))**3/(m**4 + 10*m**3 + 3
5*m**2 + 50*m + 24) + 9*m**2*x*x**m*acoth(tanh(a + b*x))**3/(m**4 + 10*m**3
 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*acoth(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24) + 24*x*x**m*acoth(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^3, x)
```

Mupad [B]
time = 1.45, size = 332, normalized size = 3.02

$$\frac{8\,b^3\,x^m\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}-\frac{x\,x^m\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^3\,\left(m^3+9\,m^2+26\,m+24\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}-\frac{12\,b^2\,x^m\,x^3\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(m^3+7\,m^2+14\,m+8\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}+\frac{6\,b\,x^m\,x^2\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2\,\left(m^3+8\,m^2+19\,m+12\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x))^3,x)
```

```
[Out] (8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
+ 192) - (x*x^m*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(400
*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(-2/(exp(2*a)*ex
p(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) +
(6*b*x^m*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))/(400*m
+ 280*m^2 + 80*m^3 + 8*m^4 + 192)
```

3.147 $\int x^4 \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=61

$$-\frac{1}{280}b^3x^8+\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a+bx))-\frac{1}{10}bx^6 \coth^{-1}(\tanh(a+bx))^2+\frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3$$

```
[Out] -1/280*b^3*x^8+1/35*b^2*x^7*arccoth(tanh(b*x+a))-1/10*b*x^6*arccoth(tanh(b*
x+a))^2+1/5*x^5*arccoth(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3 - \frac{1}{280}b^3x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/280*(b^3*x^8) + (b^2*x^7*ArcCoth[Tanh[a + b*x]])/35 - (b*x^6*ArcCoth[Tan
h[a + b*x]]^2)/10 + (x^5*ArcCoth[Tanh[a + b*x]]^3)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3 - \frac{1}{5}(3b)\int x^5\coth^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5\coth^{-1}(\tanh(a+bx))^3 + \frac{1}{5}b^2\int x^6 \mathrm{c}$$
$$= \frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5\coth^{-}$$
$$= -\frac{1}{280}b^3x^8 + \frac{1}{35}b^2x^7\coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6\coth^{-1}(\tanh(a+bx))^2$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.89

$$-\frac{1}{280}x^5\left(b^3x^3 - 8b^2x^2\coth^{-1}(\tanh(a+bx)) + 28bx\coth^{-1}(\tanh(a+bx))^2 - 56\coth^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[Tanh[a + b*x]]^3,x]
[Out] -1/280*(x^5*(b^3*x^3 - 8*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 28*b*x*ArcCoth[Ta
nh[a + b*x]]^2 - 56*ArcCoth[Tanh[a + b*x]]^3))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^4 \mathrm{arccoth}(\tanh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(tanh(b*x+a))^3,x)
[Out] int(x^4*arccoth(tanh(b*x+a))^3,x)
```

Maxima [A]
time = 0.37, size = 54, normalized size = 0.89

$$-\frac{1}{10}bx^6\,\mathrm{arcoth}(\tanh(bx+a))^2 + \frac{1}{5}x^5\,\mathrm{arcoth}(\tanh(bx+a))^3 - \frac{1}{280}\left(b^2x^8 - 8bx^7\,\mathrm{arcoth}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] -1/10*b*x^6*arccoth(tanh(b*x + a))^2 + 1/5*x^5*arccoth(tanh(b*x + a))^3 - 1
/280*(b^2*x^8 - 8*b*x^7*arccoth(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.47, size = 52, normalized size = 0.85

$$\frac{1}{8}\,b^{3}x^{8}+\frac{3}{7}\,ab^{2}x^{7}-\frac{1}{8}\,\left(\pi^{2}b-4\,a^{2}b\right)x^{6}-\frac{1}{20}\,\left(3\,\pi^{2}a-4\,a^{3}\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 - 1/8*(pi^2*b - 4*a^2*b)*x^6 - 1/20*(3*pi^2*a -
 4*a^3)*x^5
```

Sympy [A]
time = 0.62, size = 56, normalized size = 0.92

$$-\frac{b^{3}x^{8}}{280}+\frac{b^{2}x^{7}\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{35}-\frac{bx^{6}\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{10}+\frac{x^{5}\operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**8/280 + b**2*x**7*acoth(tanh(a + b*x))/35 - b*x**6*acoth(tanh(a +
b*x))**2/10 + x**5*acoth(tanh(a + b*x))**3/5
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 77, normalized size = 1.26

$$\frac{1}{8}\,b^{3}x^{8}-\frac{3}{14}\,\left(-i\,\pi b^{2}-2\,ab^{2}\right)x^{7}-\frac{1}{8}\,\left(\pi^{2}b-4i\,\pi ab-4\,a^{2}b\right)x^{6}-\frac{1}{40}\,\left(i\,\pi^{3}+6\,\pi^{2}a-12i\,\pi a^{2}-8\,a^{3}\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8 - 3/14*(-I*pi*b^2 - 2*a*b^2)*x^7 - 1/8*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^6 - 1/40*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^5
```

Mupad [B]
time = 1.25, size = 53, normalized size = 0.87

$$-\frac{b^3\,x^8}{280}+\frac{b^2\,x^7\,\mathrm{acoth}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}{35}-\frac{b\,x^6\,{\mathrm{acoth}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^2}{10}+\frac{x^5\,{\mathrm{acoth}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(tanh(a + b*x))^3,x)
```

```
[Out] (x^5*acoth(tanh(a + b*x))^3)/5 - (b^3*x^8)/280 - (b*x^6*acoth(tanh(a + b*x)
)^2)/10 + (b^2*x^7*acoth(tanh(a + b*x)))/35
```

3.148 $\int x^3 \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=61

$$-\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3$$

```
[Out] -1/140*b^3*x^7+1/20*b^2*x^6*arccoth(tanh(b*x+a))-3/20*b*x^5*arccoth(tanh(b*
x+a))^2+1/4*x^4*arccoth(tanh(b*x+a))^3
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 30}

$$\frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/140*(b^3*x^7) + (b^2*x^6*ArcCoth[Tanh[a + b*x]])/20 - (3*b*x^5*ArcCoth[T
anh[a + b*x]]^2)/20 + (x^4*ArcCoth[Tanh[a + b*x]]^3)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3 - \frac{1}{4}(3b)\int x^4\coth^{-1}(\tanh(a+bx))^2\,dx$$
$$= -\frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\coth^{-1}(\tanh(a+bx))^3 + \frac{1}{10}(3b^2)$$
$$= \frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4\coth$$
$$= -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6\coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5\coth^{-1}(\tanh(a+bx))$$

Mathematica [**A**]
time = 0.02, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4\left(b^3x^3 - 7b^2x^2\coth^{-1}(\tanh(a+bx)) + 21bx\coth^{-1}(\tanh(a+bx))^2 - 35\coth^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^3,x]
[Out] -1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 21*b*x*ArcCoth[Ta
nh[a + b*x]]^2 - 35*ArcCoth[Tanh[a + b*x]]^3))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{arccoth}(\tanh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^3,x)
[Out] int(x^3*arccoth(tanh(b*x+a))^3,x)
```

Maxima [**A**]
time = 0.37, size = 54, normalized size = 0.89

$$-\frac{3}{20}bx^5\operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{4}x^4\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{140}\left(b^2x^7 - 7bx^6\operatorname{arcoth}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] -3/20*b*x^5*arccoth(tanh(b*x + a))^2 + 1/4*x^4*arccoth(tanh(b*x + a))^3 - 1
/140*(b^2*x^7 - 7*b*x^6*arccoth(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.47, size = 52, normalized size = 0.85

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6-\frac{3}{20}\left(\pi^2b-4\,a^2b\right)x^5-\frac{1}{16}\left(3\,\pi^2a-4\,a^3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 - 3/20*(pi^2*b - 4*a^2*b)*x^5 - 1/16*(3*pi^2*a
- 4*a^3)*x^4
```

Sympy [A]
time = 0.40, size = 58, normalized size = 0.95

$$-\frac{b^3x^7}{140}+\frac{b^2x^6\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}{20}-\frac{3bx^5\,\mathrm{acoth}^2\left(\tanh\left(a+bx\right)\right)}{20}+\frac{x^4\,\mathrm{acoth}^3\left(\tanh\left(a+bx\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**7/140 + b**2*x**6*acoth(tanh(a + b*x))/20 - 3*b*x**5*acoth(tanh(a
+ b*x))**2/20 + x**4*acoth(tanh(a + b*x))**3/4
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 77, normalized size = 1.26

$$\frac{1}{7}\,b^3x^7-\frac{1}{4}\left(-i\,\pi b^2-2\,ab^2\right)x^6-\frac{3}{20}\left(\pi^2b-4i\,\pi ab-4\,a^2b\right)x^5-\frac{1}{32}\left(i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 - 1/4*(-I*pi*b^2 - 2*a*b^2)*x^6 - 3/20*(pi^2*b - 4*I*pi*a*b - 4
*a^2*b)*x^5 - 1/32*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^4
```

Mupad [B]
time = 1.23, size = 53, normalized size = 0.87

$$-\frac{b^3\,x^7}{140}+\frac{b^2\,x^6\,\mathrm{acoth}(\tanh(a+b\,x))}{20}-\frac{3\,b\,x^5\,\mathrm{acoth}(\tanh(a+b\,x))^2}{20}+\frac{x^4\,\mathrm{acoth}(\tanh(a+b\,x))^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(tanh(a + b*x))^3,x)
```

```
[Out] (x^4*acoth(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*acoth(tanh(a + b*
x))^2)/20 + (b^2*x^6*acoth(tanh(a + b*x)))/20
```

3.149 $\int x^2 \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=53

$$\frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3}$$

```
[Out] 1/4*x^2*arccoth(tanh(b*x+a))^4/b-1/10*x*arccoth(tanh(b*x+a))^5/b^2+1/60*arc
coth(tanh(b*x+a))^6/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^6}{60b^3} - \frac{x \coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{x^2 \coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - (x*ArcCoth[Tanh[a + b*x]]^5)/(10*b^2
) + ArcCoth[Tanh[a + b*x]]^6/(60*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^2 \coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int x\coth^{-1}(\tanh(a+bx))^4\,dx}{2b}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\int \coth^{-1}(\tanh(a -}{10b^2}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\text{Subst}\left(\int x^5\,dx, x, \right.}{}$$
$$= \frac{x^2\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{x\coth^{-1}(\tanh(a+bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a+}{60b^3}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3\left(b^3x^3 - 6b^2x^2\coth^{-1}(\tanh(a+bx)) + 15bx\coth^{-1}(\tanh(a+bx))^2 - 20\coth^{-1}(\tanh(a+bx))^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 15*b*x*ArcCoth[Tan
h[a + b*x]]^2 - 20*ArcCoth[Tanh[a + b*x]]^3))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\tanh(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(x^2*arccoth(tanh(b*x+a))^3,x)
```

Maxima [A]
time = 0.37, size = 54, normalized size = 1.02

$$-\frac{1}{4}bx^4\operatorname{arcoth}(\tanh(bx+a))^2 + \frac{1}{3}x^3\operatorname{arcoth}(\tanh(bx+a))^3 - \frac{1}{60}\left(b^2x^6 - 6bx^5\operatorname{arcoth}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4*arccoth(tanh(b*x + a))^2 + 1/3*x^3*arccoth(tanh(b*x + a))^3 - 1/
60*(b^2*x^6 - 6*b*x^5*arccoth(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.39, size = 52, normalized size = 0.98

$$\frac{1}{6}\,b^{3}x^{6}+\frac{3}{5}\,ab^{2}x^{5}-\frac{3}{16}\,\left(\pi^{2}b-4\,a^{2}b\right)x^{4}-\frac{1}{12}\,\left(3\,\pi^{2}a-4\,a^{3}\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 - 3/16*(pi^2*b - 4*a^2*b)*x^4 - 1/12*(3*pi^2*a
- 4*a^3)*x^3
```

Sympy [A]
time = 0.26, size = 56, normalized size = 1.06

$$-\frac{b^{3}x^{6}}{60}+\frac{b^{2}x^{5}\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{10}-\frac{bx^{4}\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}{4}+\frac{x^{3}\operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**3,x)
```

```
[Out] -b**3*x**6/60 + b**2*x**5*acoth(tanh(a + b*x))/10 - b*x**4*acoth(tanh(a + b
*x))**2/4 + x**3*acoth(tanh(a + b*x))**3/3
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 77, normalized size = 1.45

$$\frac{1}{6}\,b^{3}x^{6}-\frac{3}{10}\,\left(-i\,\pi b^{2}-2\,ab^{2}\right)x^{5}-\frac{3}{16}\,\left(\pi^{2}b-4i\,\pi ab-4\,a^{2}b\right)x^{4}-\frac{1}{24}\,\left(i\,\pi^{3}+6\,\pi^{2}a-12i\,\pi a^{2}-8\,a^{3}\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 - 3/10*(-I*pi*b^2 - 2*a*b^2)*x^5 - 3/16*(pi^2*b - 4*I*pi*a*b -
4*a^2*b)*x^4 - 1/24*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^3
```

Mupad [B]
time = 1.21, size = 53, normalized size = 1.00

$$-\frac{b^{3}\,x^{6}}{60}+\frac{b^{2}\,x^{5}\,\mathrm{acoth}(\tanh(a+b\,x))}{10}-\frac{b\,x^{4}\,{\mathrm{acoth}(\tanh(a+b\,x))}^{2}}{4}+\frac{x^{3}\,{\mathrm{acoth}(\tanh(a+b\,x))}^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(tanh(a + b*x))^3,x)
```

```
[Out] (x^3*acoth(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*acoth(tanh(a + b*x))
^2)/4 + (b^2*x^5*acoth(tanh(a + b*x)))/10
```

3.150 $\int x \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=34

$$\frac{x\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

```
[Out] 1/4*x*arccoth(tanh(b*x+a))^4/b-1/20*arccoth(tanh(b*x+a))^5/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x\coth^{-1}(\tanh(a+bx))^4}{4b}-\frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^4)/(4*b) - ArcCoth[Tanh[a + b*x]]^5/(20*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x\coth^{-1}(\tanh(a+bx))^3\,dx = \frac{x\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a+bx))^4\,dx}{4b}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{4b^2}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a+bx))^5}{20b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(34) = 68.
time = 0.04, size = 99, normalized size = 2.91

$$\frac{(a+bx)\left((4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\coth^{-1}(\tanh(a+bx))+10(2a^2+abx-b^2x^2)\coth^{-1}(\tanh(a+bx))^2-10(a-bx)\coth^{-1}(\tanh(a+bx))^3\right)}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcCoth[Tan
h[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcCoth[Tanh[a + b*x]]^2 - 10*(a
 - b*x)*ArcCoth[Tanh[a + b*x]]^3))/(20*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 34.58, size = 18111, normalized size = 532.68

method	result	size
risch	Expression too large to display	18111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.38, size = 54, normalized size = 1.59

$$-\frac{1}{2}\,bx^3\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{2}\,x^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{20}\,\left(b^2x^5-5\,bx^4\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^3*arccoth(tanh(b*x + a))^2 + 1/2*x^2*arccoth(tanh(b*x + a))^3 - 1/
20*(b^2*x^5 - 5*b*x^4*arccoth(tanh(b*x + a)))*b
```

Fricas [A]
time = 0.39, size = 52, normalized size = 1.53

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4-\frac{1}{4}\,(\pi^2b-4\,a^2b)x^3-\frac{1}{8}\,(3\,\pi^2a-4\,a^3)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 - 1/4*(pi^2*b - 4*a^2*b)*x^3 - 1/8*(3*pi^2*a -
4*a^3)*x^2
```

Sympy [A]
time = 0.25, size = 41, normalized size = 1.21

$$\begin{cases}\frac{x\operatorname{acoth}^4(\tanh(a+bx))}{4b}-\frac{\operatorname{acoth}^5(\tanh(a+bx))}{20b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{acoth}^3(\tanh(a))}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x*acoth(tanh(a + b*x))**4/(4*b) - acoth(tanh(a + b*x))**5/(20*b*
*2), Ne(b, 0)), (x**2*acoth(tanh(a))**3/2, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 77, normalized size = 2.26

$$\frac{1}{5}\,b^3x^5-\frac{3}{8}\,(-i\,\pi b^2-2\,ab^2)x^4-\frac{1}{4}\,(\pi^2b-4i\,\pi ab-4\,a^2b)x^3-\frac{1}{16}\,(i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 - 3/8*(-I*pi*b^2 - 2*a*b^2)*x^4 - 1/4*(pi^2*b - 4*I*pi*a*b - 4*
a^2*b)*x^3 - 1/16*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x^2
```

Mupad [B]
time = 0.12, size = 53, normalized size = 1.56

$$-\frac{b^3\,x^5}{20}+\frac{b^2\,x^4\,\mathrm{acoth}(\tanh(a+b\,x))}{4}-\frac{b\,x^3\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2}+\frac{x^2\,\mathrm{acoth}(\tanh(a+b\,x))^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(tanh(a + b*x))^3,x)
```

```
[Out] (x^2*acoth(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*acoth(tanh(a + b*x))
^2)/2 + (b^2*x^4*acoth(tanh(a + b*x)))/4
```

3.151 $\int \coth^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] 1/4*arccoth(tanh(b*x+a))^4/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^3 \, dx &= \frac{\text{Subst}\left(\int x^3 \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*b)
```

Maple [A]
time = 36.29, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$	15
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^4}{4b}$	15
risch	Expression too large to display	14682

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arccoth(tanh(b*x+a))^4/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.
time = 0.38, size = 51, normalized size = 3.19

$$-\frac{3}{2}\,bx^2\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2+x\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{4}\left(b^2x^4-4\,bx^3\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*x^2*arccoth(tanh(b*x + a))^2 + x*arccoth(tanh(b*x + a))^3 - 1/4*(b^2
*x^4 - 4*b*x^3*arccoth(tanh(b*x + a)))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.
time = 0.40, size = 49, normalized size = 3.06

$$\frac{1}{4}\,b^3x^4+ab^2x^3-\frac{3}{8}\left(\pi^2b-4\,a^2b\right)x^2-\frac{1}{4}\left(3\,\pi^2a-4\,a^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 - 3/8*(pi^2*b - 4*a^2*b)*x^2 - 1/4*(3*pi^2*a - 4*a^
3)*x
```

Sympy [A]
time = 0.13, size = 20, normalized size = 1.25

$$\begin{cases}\frac{\operatorname{acoth}^4\left(\tanh\left(a+bx\right)\right)}{4b} & \text{for}\ b\neq 0\\ x\operatorname{acoth}^3\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3,
True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 75, normalized size = 4.69

$$\frac{1}{4}b^3x^4 - \frac{1}{2}\left(-i\pi b^2 - 2ab^2\right)x^3 - \frac{3}{8}\left(\pi^2 b - 4i\pi ab - 4a^2b\right)x^2 - \frac{1}{8}\left(i\pi^3 + 6\pi^2 a - 12i\pi a^2 - 8a^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 - 1/2*(-I*pi*b^2 - 2*a*b^2)*x^3 - 3/8*(pi^2*b - 4*I*pi*a*b - 4*
a^2*b)*x^2 - 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*x
```

Mupad [B]
time = 1.18, size = 47, normalized size = 2.94

$$\frac{x\left(2\operatorname{acoth}(\tanh(a+bx)) - bx\right)\left(b^2x^2 - 2bx\operatorname{acoth}(\tanh(a+bx)) + 2\operatorname{acoth}(\tanh(a+bx))^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3,x)
```

```
[Out] (x*(2*acoth(tanh(a + b*x)) - b*x)*(2*acoth(tanh(a + b*x))^2 + b^2*x^2 - 2*b
*x*acoth(tanh(a + b*x))))/4
```

3.152 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} \, dx$

Optimal. Leaf size=77

$$bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2+\frac{1}{3}\coth^{-1}(\tanh(a+bx$$

```
[Out] b*x*(b*x-arccoth(tanh(b*x+a)))^2-1/2*(b*x-arccoth(tanh(b*x+a)))*arccoth(tan
h(b*x+a))^2+1/3*arccoth(tanh(b*x+a))^3-(b*x-arccoth(tanh(b*x+a)))^3*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2190, 2189, 29}

$$bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\coth^{-1}(\tanh(a+bx))^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\coth^{-1}(\tanh(a+bx))^3-\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

```
[Out] b*x*(b*x - ArcCoth[Tanh[a + b*x]])^2 - ((b*x - ArcCoth[Tanh[a + b*x]])*ArcC
oth[Tanh[a + b*x]]^2)/2 + ArcCoth[Tanh[a + b*x]]^3/3 - (b*x - ArcCoth[Tanh[
a + b*x]])^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x}\,dx &= \frac{1}{3}\coth^{-1}(\tanh(a+bx))^3 - \left(bx-\coth^{-1}(\tanh(a+bx))\right)\int \frac{\coth^{-1}(\tanh(a\\ &= -\frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2 + \frac{1}{3}\coth^{-1}(\tanh(a\\ &= bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(t\\ &= bx\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 - \frac{1}{2}\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(t\end{aligned}$$

Mathematica [A]
time = 0.04, size = 104, normalized size = 1.35

$$\frac{1}{3}(a+bx)^3+(a+bx)\left(a^2-3a(a+bx-\coth^{-1}(\tanh(a+bx)))+3(a+bx-\coth^{-1}(\tanh(a+bx)))^2\right)-\frac{1}{2}(a+bx)^2\left(2a+3bx-3\coth^{-1}(\tanh(a+bx))\right)+\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x,x]
```

```
[Out] (a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcCoth[Tanh[a + b*x]]) + 3
*(a + b*x - ArcCoth[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*ArcC
oth[Tanh[a + b*x]]))/2 + (-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[b*x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.83, size = 21848, normalized size = 283.74

method	result	size
risch	Expression too large to display	21848

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.67, size = 74, normalized size = 0.96

$$\frac{1}{3}\,b^3x^3-\frac{3}{4}\left(i\,\pi b^2-2\,ab^2\right)x^2-\frac{3}{4}\left(\pi^2b+4i\,\pi ab-4\,a^2b\right)x+\frac{1}{8}\left(i\,\pi^3-6\,\pi^2a-12i\,\pi a^2+8\,a^3\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 - 3/4*(I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b + 4*I*pi*a*b - 4*a
^2*b)*x + 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(x)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 0.64

$$\frac{1}{3}\,b^{3}x^{3}+\frac{3}{2}\,ab^{2}x^{2}-\frac{3}{4}\,\left(\pi^{2}b-4\,a^{2}b\right)x-\frac{1}{4}\,\left(3\,\pi^{2}a-4\,a^{3}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 - 3/4*(pi^2*b - 4*a^2*b)*x - 1/4*(3*pi^2*a - 4*
a^3)*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 74, normalized size = 0.96

$$\frac{1}{3}\,b^{3}x^{3}-\frac{3}{4}\,\left(-i\,\pi b^{2}-2\,ab^{2}\right)x^{2}-\frac{3}{4}\,\left(\pi^{2}b-4i\,\pi ab-4\,a^{2}b\right)x+\frac{1}{8}\,\left(-i\,\pi^{3}-6\,\pi^{2}a+12i\,\pi a^{2}+8\,a^{3}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 - 3/4*(-I*pi*b^2 - 2*a*b^2)*x^2 - 3/4*(pi^2*b - 4*I*pi*a*b - 4*
a^2*b)*x + 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)*log(x)
```

Mupad [B]
time = 0.14, size = 306, normalized size = 3.97

$$\frac{b^3\,x^3}{3}-\ln\left(x\right)\,\left(\frac{{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^3}{8}-a^3-\frac{3\,a\,{\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}{4}+\frac{3\,a^2\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2}\right)-\frac{3\,b^2\,x^2\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{4}+\frac{3\,b\,x\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x,x)
```

```
[Out] (b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2*a
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a
)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/
2) - (3*b^2*x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/4 + (3*b*x*(log(-2/(exp(2*a)*exp
(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*
b*x)^2)/4
```

3.153 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx$

Optimal. Leaf size=68

$$-3b^2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)+\frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}+3b\left(bx-\coth^{-1}(\tan\right.$$

```
[Out] -3*b^2*x*(b*x-arccoth(tanh(b*x+a)))+3/2*b*arccoth(tanh(b*x+a))^2-arccoth(ta
nh(b*x+a))^3/x+3*b*(b*x-arccoth(tanh(b*x+a)))^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2199, 2190, 2189, 29}

$$-3b^2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -3*b^2*x*(b*x - ArcCoth[Tanh[a + b*x]]) + (3*b*ArcCoth[Tanh[a + b*x]]^2)/2
- ArcCoth[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2}\,dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{x} + (3b)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x}\,dx \\
&= \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} - \big(3b\big(bx - \coth^{-1}(\tanh \\
&= -3b^2x\big(bx - \coth^{-1}(\tanh(a+bx))\big) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\mathrm{ta}}{} \\
&= -3b^2x\big(bx - \coth^{-1}(\tanh(a+bx))\big) + \frac{3}{2}b\coth^{-1}(\tanh(a+bx))^2 - \frac{\coth^{-1}(\mathrm{ta}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.91

$$
-\frac{\coth^{-1}(\tanh(a+bx))^3}{x} - 6b^2x\coth^{-1}(\tanh(a+bx))\log(x) + 3b\coth^{-1}(\tanh(a+bx))^2(1+\log(x)) + \frac{3}{2}b^3x^2(-1+2\log(x))
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcCoth[Tanh[a + b*x]]*Log[x] + 3*b
*ArcCoth[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 7683, normalized size = 112.99

method	result	size
risch	Expression too large to display	7683

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.59, size = 124, normalized size = 1.82

$$
3\,b\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \frac{3}{2}\left(2\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right) - \left(bx^2 - 2\left(-i\,\pi - 2\,a\right)x + 2\left(-\frac{i\,\pi(bx+a)}{b} - \frac{(bx+a)^2}{b}\right)\log\left(x\right) + \frac{2\,\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^2\log\left(x\right)}{b} + \frac{2\left(i\,\pi a + a^2\right)\log\left(x\right)}{b}\right)b\right)b - \frac{\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*b*arccoth(tanh(b*x + a))^2*log(x) - 3/2*(2*arccoth(tanh(b*x + a))^2*log(x
) - (b*x^2 - 2*(-I*pi - 2*a)*x + 2*(-I*pi*(b*x + a)/b - (b*x + a)^2/b)*log(
x) + 2*arccoth(tanh(b*x + a))^2*log(x)/b + 2*(I*pi*a + a^2)*log(x)/b)*b)*b
- arccoth(tanh(b*x + a))^3/x
```

Fricas [A]
time = 0.41, size = 51, normalized size = 0.75

$$\frac{2\,b^3x^3+12\,ab^2x^2+3\,\pi^2a-4\,a^3-3\,(\pi^2b-4\,a^2b)x\log{(x)}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^3 + 12*a*b^2*x^2 + 3*pi^2*a - 4*a^3 - 3*(pi^2*b - 4*a^2*b)*x*l
og(x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}^3{(\tanh{(a+bx)})}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 74, normalized size = 1.09

$$\frac{1}{2}\,b^3x^2-\frac{3}{2}\,(-i\,\pi b^2-2\,ab^2)x-\frac{3}{4}\,(\pi^2b-4i\,\pi ab-4\,a^2b)\log{(x)}-\frac{-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 - 3/2*(-I*pi*b^2 - 2*a*b^2)*x - 3/4*(pi^2*b - 4*I*pi*a*b - 4*a^
2*b)*log(x) - 1/8*(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x
```

Mupad [B]
time = 1.20, size = 372, normalized size = 5.47

$$\ln(x)\left(3\,a^2\,b+\frac{3\,b\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4}-3\,a\,b\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)+\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3-8\,a^3-6\,a\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2+12\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{8\,x}+\frac{b^3\,x^2}{2}-\frac{3\,b^2\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x^2,x)
```

```
[Out] log(x)*(3*a^2*b + (3*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 - 3*a*b*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*e
xp(2*b*x) - 1)) + 2*b*x)) + ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*
a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
))/(8*x) + (b^3*x^2)/2 - (3*b^2*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/2
```

3.154 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx$

Optimal. Leaf size=60

$$3b^3x-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-3b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

```
[Out] 3*b^3*x-3/2*b*arccoth(tanh(b*x+a))^2/x-1/2*arccoth(tanh(b*x+a))^3/x^2-3*b^2
*(b*x-arccoth(tanh(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2189, 29}

$$-3b^2\log(x)\left(bx-\coth^{-1}(\tanh(a+bx))\right)-\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] 3*b^3*x - (3*b*ArcCoth[Tanh[a + b*x]]^2)/(2*x) - ArcCoth[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcCoth[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + \frac{1}{2}(3b)\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2}\,dx$$
$$= -\frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + (3b^2)\int \frac{\coth^{-1}(\tanh}{x}$$
$$= 3b^3x - \frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - \left(3b^2\left(bx - \coth\right.\right.$$
$$= 3b^3x - \frac{3b\coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2\left(bx - \coth^{-}\right.$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.10

$$b^3x - \frac{3b\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{x} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{2x^2} + 3b^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] b^3*x - (3*b*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcCoth[Tan
h[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 7366, normalized size = 122.77

method	result	size
risch	Expression too large to display	7366

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.35, size = 72, normalized size = 1.20

$$3\left(b\,\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)\log\left(x\right) - \left(b\left(x+\frac{a}{b}\right)\log\left(x\right) - b\left(x+\frac{a\log\left(x\right)}{b}\right)\right)b\right)b - \frac{3\,b\,\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)^2}{2\,x} - \frac{\mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3*(b*arccoth(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)/
b))*b)*b - 3/2*b*arccoth(tanh(b*x + a))^2/x - 1/2*arccoth(tanh(b*x + a))^3/
x^2
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.85

$$\frac{8\,b^3x^3+24\,ab^2x^2\log\left(x\right)+3\,\pi^2a-4\,a^3+6\,(\pi^2b-4\,a^2b)x}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*x^3 + 24*a*b^2*x^2*log(x) + 3*pi^2*a - 4*a^3 + 6*(pi^2*b - 4*a^2
*b)*x)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 71, normalized size = 1.18

$$b^3x+\frac{3}{2}\left(i\,\pi b^2+2\,ab^2\right)\log\left(x\right)+\frac{12\,\pi^2bx-48i\,\pi abx-48\,a^2bx+i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

```
[Out] b^3*x + 3/2*(I*pi*b^2 + 2*a*b^2)*log(x) + 1/16*(12*pi^2*b*x - 48*I*pi*a*b*x
 - 48*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/x^2
```

Mupad [B]
time = 1.33, size = 383, normalized size = 6.38

$$\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^3}{16\,x^2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^3}{16\,x^2}+\frac{9\,b^2\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{4}-\frac{9\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{4}-\frac{3\,b^3\,x}{2}-\frac{3\,b\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{8\,x}+\frac{3\,b^2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\ln\left(x\right)}{2}-3\,b^3\,x\,\ln\left(x\right)-\frac{3\,b\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{8\,x}-\frac{3\,b^2\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\ln\left(x\right)}{2}-\frac{3\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2}{16\,x^2}+\frac{3\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)^2\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{16\,x^2}+\frac{3\,b\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x^3,x)
```

```
[Out] log(-2/(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))^3/(16*x^2) + (9*b^2*log(exp(2*b*x)/(exp(2*a)*exp
(2*b*x) - 1)))/4 - (9*b^2*log(1/(exp(2*a)*exp(2*b*x) - 1)))/4 - (3*b^3*x)/2
 - (3*b*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x) + (
```

```
3*b^2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2 - 3*
b^3*x*log(x) - (3*b*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(8*x) - (3*b^2*log
(-2/(exp(2*a)*exp(2*b*x) - 1))*log(x))/2 - (3*log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1))^2)/(16*x^2) + (
3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2*log(-2/(exp(2*a)
*exp(2*b*x) - 1)))/(16*x^2) + (3*b*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1)))/(4*x)
```

3.155 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx$

Optimal. Leaf size=55

$$-\frac{b^2\coth^{-1}(\tanh(a+bx))}{x}-\frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

```
[Out] -b^2*arccoth(tanh(b*x+a))/x-1/2*b*arccoth(tanh(b*x+a))^2/x^2-1/3*arccoth(ta
nh(b*x+a))^3/x^3+b^3*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 29}

$$-\frac{b^2\coth^{-1}(\tanh(a+bx))}{x}-\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] -((b^2*ArcCoth[Tanh[a + b*x]])/x) - (b*ArcCoth[Tanh[a + b*x]]^2)/(2*x^2) -
ArcCoth[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3}\,dx$$
$$= -\frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^2\int \frac{\coth^{-1}(\tanh(a+$$
$$= -\frac{b^2\coth^{-1}(\tanh(a+bx))}{x} - \frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx}{3x^3}$$
$$= -\frac{b^2\coth^{-1}(\tanh(a+bx))}{x} - \frac{b\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx}{3x^3}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2\coth^{-1}(\tanh(a+bx)) - 3bx\coth^{-1}(\tanh(a+bx))^2 - 2\coth^{-1}(\tanh(a+bx))^3 + b^3x^3(11+6\log(x))}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] (-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*Arc
Coth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.69, size = 17237, normalized size = 313.40

method	result	size
risch	Expression too large to display	17237

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.40, size = 52, normalized size = 0.95

$$\left(b^2\log(x) - \frac{b\,\mathrm{arcoth}(\tanh(bx+a))}{x}\right)b - \frac{b\,\mathrm{arcoth}(\tanh(bx+a))^2}{2\,x^2} - \frac{\mathrm{arcoth}(\tanh(bx+a))^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*log(x) - b*arccoth(tanh(b*x + a))/x)*b - 1/2*b*arccoth(tanh(b*x + a))^
2/x^2 - 1/3*arccoth(tanh(b*x + a))^3/x^3
```

Fricas [A]
time = 0.34, size = 51, normalized size = 0.93

$$\frac{24\,b^3x^3\log\left(x\right)-72\,ab^2x^2+6\,\pi^2a-8\,a^3+9\,(\pi^2b-4\,a^2b)x}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^3*x^3*log(x) - 72*a*b^2*x^2 + 6*pi^2*a - 8*a^3 + 9*(pi^2*b - 4*a
^2*b)*x)/x^3
```

Sympy [A]
time = 0.28, size = 51, normalized size = 0.93

$$b^3\log\left(x\right)-\frac{b^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}{x}-\frac{b\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}{2x^2}-\frac{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**4,x)
```

```
[Out] b**3*log(x) - b**2*acoth(tanh(a + b*x))/x - b*acoth(tanh(a + b*x))**2/(2*x*
*2) - acoth(tanh(a + b*x))**3/(3*x**3)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 73, normalized size = 1.33

$$b^3\log\left(x\right)-\frac{36i\,\pi b^2x^2+72\,ab^2x^2-9\,\pi^2bx+36i\,\pi abx+36\,a^2bx-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(x) - 1/24*(36*I*pi*b^2*x^2 + 72*a*b^2*x^2 - 9*pi^2*b*x + 36*I*pi*a*
b*x + 36*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^3
```

Mupad [B]
time = 1.18, size = 51, normalized size = 0.93

$$b^3\,\ln\left(x\right)-\frac{b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x))+\frac{b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2}{2}+\frac{\mathrm{acoth}(\tanh(a+b\,x))^3}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x^4,x)
```

```
[Out] b^3*log(x) - (acoth(tanh(a + b*x))^3/3 + (b*x*acoth(tanh(a + b*x))^2)/2 + b
^2*x^2*acoth(tanh(a + b*x)))/x^3
```

3.156 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/4*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2198\}$

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.61

$$-\frac{b^3x^3+b^2x^2\coth^{-1}(\tanh(a+bx))+bx\coth^{-1}(\tanh(a+bx))^2+\coth^{-1}(\tanh(a+bx))^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] -1/4*(b^3*x^3 + b^2*x^2*ArcCoth[Tanh[a + b*x]] + b*x*ArcCoth[Tanh[a + b*x]]
^2 + ArcCoth[Tanh[a + b*x]]^3)/x^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.60, size = 17235, normalized size = 555.97

method	result	size
risch	Expression too large to display	17235

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.37, size = 53, normalized size = 1.71

$$-\frac{1}{4}b\left(\frac{b^2}{x}+\frac{b\operatorname{arcoth}(\tanh(bx+a))}{x^2}\right)-\frac{b\operatorname{arcoth}(\tanh(bx+a))^2}{4x^3}-\frac{\operatorname{arcoth}(\tanh(bx+a))^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*(b^2/x + b*arccoth(tanh(b*x + a))/x^2) - 1/4*b*arccoth(tanh(b*x + a)
)^2/x^3 - 1/4*arccoth(tanh(b*x + a))^3/x^4
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.58

$$-\frac{16\,b^3x^3+24\,ab^2x^2-3\,\pi^2a+4\,a^3-4\,(\pi^2b-4\,a^2b)x}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^3*x^3 + 24*a*b^2*x^2 - 3*pi^2*a + 4*a^3 - 4*(pi^2*b - 4*a^2*b)*
x)/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.
time = 0.37, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x}-\frac{b^2\operatorname{acoth}(\tanh(a+bx))}{4x^2}-\frac{b\operatorname{acoth}^2(\tanh(a+bx))}{4x^3}-\frac{\operatorname{acoth}^3(\tanh(a+bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**5,x)
```

```
[Out] -b**3/(4*x) - b**2*acoth(tanh(a + b*x))/(4*x**2) - b*acoth(tanh(a + b*x))**
2/(4*x**3) - acoth(tanh(a + b*x))**3/(4*x**4)
```

Giac [C] Result contains complex when optimal does not.
time = 0.39, size = 74, normalized size = 2.39

$$-\frac{32\,b^3x^3+24i\,\pi b^2x^2+48\,ab^2x^2-8\,\pi^2bx+32i\,\pi abx+32\,a^2bx-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/32*(32*b^3*x^3 + 24*I*pi*b^2*x^2 + 48*a*b^2*x^2 - 8*pi^2*b*x + 32*I*pi*a
*b*x + 32*a^2*b*x - I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)/x^4
```

Mupad [B]
time = 1.19, size = 48, normalized size = 1.55

$$-\frac{b^3\,x^3+b^2\,x^2\,\mathrm{acoth}(\tanh(a+b\,x))+b\,x\,\mathrm{acoth}(\tanh(a+b\,x))^2+\mathrm{acoth}(\tanh(a+b\,x))^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x^5,x)
```

```
[Out] -(acoth(tanh(a + b*x))^3 + b^3*x^3 + b*x*acoth(tanh(a + b*x))^2 + b^2*x^2*a
coth(tanh(a + b*x)))/(4*x^4)
```

3.157 $\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx$

Optimal. Leaf size=64

$$\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/20*b*arccoth(tanh(b*x+a))^4/x^4/(b*x-arccoth(tanh(b*x+a)))^2+1/5*arccoth(
tanh(b*x+a))^4/x^5/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2202, 2198}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] (b*ArcCoth[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcCoth[Tanh[a + b*x]])^2) + Ar
cCoth[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2198

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] /;
NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[
m + n + 2, 0] && NeQ[m, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6}\,dx &= \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5}\,dx}{5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\coth^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\coth^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.84

$$-\frac{b^3x^3+2b^2x^2\coth^{-1}(\tanh(a+bx))+3bx\coth^{-1}(\tanh(a+bx))^2+4\coth^{-1}(\tanh(a+bx))^3}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] -1/20*(b^3*x^3 + 2*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 3*b*x*ArcCoth[Tanh[a +
b*x]]^2 + 4*ArcCoth[Tanh[a + b*x]]^3)/x^5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.45, size = 17234, normalized size = 269.28

method	result	size
risch	Expression too large to display	17234

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.39, size = 54, normalized size = 0.84

$$-\frac{1}{20}\,b\left(\frac{b^2}{x^2}+\frac{2\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)}{x^3}\right)-\frac{3\,b\,\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4}-\frac{\mathrm{arcoth}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*b*(b^2/x^2 + 2*b*arccoth(tanh(b*x + a))/x^3) - 3/20*b*arccoth(tanh(b*
x + a))^2/x^4 - 1/5*arccoth(tanh(b*x + a))^3/x^5
```

Fricas [A]
time = 0.38, size = 49, normalized size = 0.77

$$-\frac{40\,b^3x^3+80\,ab^2x^2-12\,\pi^2a+16\,a^3-15\,(\pi^2b-4\,a^2b)x}{80\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/80*(40*b^3*x^3 + 80*a*b^2*x^2 - 12*pi^2*a + 16*a^3 - 15*(pi^2*b - 4*a^2*
b)*x)/x^5
```

Sympy [A]
time = 0.52, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2} - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{10x^3} - \frac{3b \operatorname{acoth}^2(\tanh(a+bx))}{20x^4} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**6,x)
```

```
[Out] -b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a +
b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 74, normalized size = 1.16

$$-\frac{40\,b^3x^3 + 40i\,\pi b^2x^2 + 80\,ab^2x^2 - 15\,\pi^2bx + 60i\,\pi abx + 60\,a^2bx - 2i\,\pi^3 - 12\,\pi^2a + 24i\,\pi a^2 + 16\,a^3}{80\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

```
[Out] -1/80*(40*b^3*x^3 + 40*I*pi*b^2*x^2 + 80*a*b^2*x^2 - 15*pi^2*b*x + 60*I*pi*
a*b*x + 60*a^2*b*x - 2*I*pi^3 - 12*pi^2*a + 24*I*pi*a^2 + 16*a^3)/x^5
```

Mupad [B]
time = 0.12, size = 53, normalized size = 0.83

$$-\frac{\operatorname{acoth}(\tanh(a+b\,x))^3}{5\,x^5} - \frac{b^3}{20\,x^2} - \frac{b^2\operatorname{acoth}(\tanh(a+b\,x))}{10\,x^3} - \frac{3\,b\operatorname{acoth}(\tanh(a+b\,x))^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^3/x^6,x)
```

```
[Out] - acoth(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*acoth(tanh(a + b*x))
)/(10*x^3) - (3*b*acoth(tanh(a + b*x))^2)/(20*x^4)
```

3.158 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=53

$$-\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arccoth(tanh(b*x+a))))/(1+m)/(b*
x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2195}

$$-\frac{x^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcCoth[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcCoth[Tanh[a + b*x]])))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))}\,dx = -\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.96

$$\frac{x^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]]))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx+a))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a)),x)
```

```
[Out] int(x^m/arccoth(tanh(b*x+a)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(x^m/arccoth(tanh(b*x + a)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a+bx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**m/acoth(tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x^m/arccoth(tanh(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/acoth(tanh(a + b*x)),x)
[Out] int(x^m/acoth(tanh(a + b*x)), x)
```

3.159 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=81

$$\frac{x^3}{3b}+\frac{x^2(bx-\coth^{-1}(\tanh(a+bx)))}{2b^2}+\frac{x(bx-\coth^{-1}(\tanh(a+bx)))^2}{b^3}+\frac{(bx-\coth^{-1}(\tanh(a+bx)))^3\log(}{b^4}$$

```
[Out] 1/3*x^3/b+1/2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^2+x*(b*x-arccoth(tanh(b*x+a)
))^2/b^3+(b*x-arccoth(tanh(b*x+a)))^3*ln(arccoth(tanh(b*x+a)))/b^4
```

Rubi [A]

time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2190, 2189, 2188, 29}

$$\frac{(bx-\coth^{-1}(\tanh(a+bx)))^3\log(\coth^{-1}(\tanh(a+bx)))}{b^4}+\frac{x(bx-\coth^{-1}(\tanh(a+bx)))^2}{b^3}+\frac{x^2(bx-\coth^{-1}(\tanh(a+bx)))}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcCot
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Ta
nh[a + b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \int \frac{\ldots}{\coth\ldots}}{b^2}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-\ldots}{\ldots}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-\ldots}{\ldots}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{(b\ldots}{\ldots}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.98

$$\frac{x^3}{3b} - \frac{x^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
ArcCoth[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[A
rcCoth[Tanh[a + b*x]]])/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.28, size = 130774, normalized size = 1614.49

method	result	size
risch	Expression too large to display	130774

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 85, normalized size = 1.05

$$\frac{4\,b^2x^3 - 3\,(-i\,\pi b + 2\,ab)x^2 - 3\,(\pi^2 + 4i\,\pi a - 4\,a^2)x}{12\,b^3} - \frac{(i\,\pi^3 - 6\,\pi^2 a - 12i\,\pi a^2 + 8\,a^3)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^2*x^3 - 3*(-I*pi*b + 2*a*b)*x^2 - 3*(pi^2 + 4*I*pi*a - 4*a^2)*x)/
b^3 - 1/8*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(-I*pi + 2*b*x + 2*a
)/b^4
```

Fricas [A]
time = 0.37, size = 127, normalized size = 1.57

$$\frac{8\,b^3x^3 - 12\,ab^2x^2 - 6\,(\pi^2b - 4\,a^2b)x - 6\,(\pi^3 - 12\,\pi a^2)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) + 3\,(3\,\pi^2a - 4\,a^3)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*b^3*x^3 - 12*a*b^2*x^2 - 6*(pi^2*b - 4*a^2*b)*x - 6*(pi^3 - 12*pi*a
^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) +
3*(3*pi^2*a - 4*a^3)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 81, normalized size = 1.00

$$\frac{x^3}{3\,b} - \frac{(i\,\pi + 2\,a)x^2}{4\,b^2} - \frac{(\pi^2 - 4i\,\pi a - 4\,a^2)x}{4\,b^3} + \frac{(i\,\pi^3 + 6\,\pi^2a - 12i\,\pi a^2 - 8\,a^3)\log\left(\pi - 2i\,bx - 2i\,a\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*x^3/b - 1/4*(I*pi + 2*a)*x^2/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*x/b^3
+ 1/8*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(pi - 2*I*b*x - 2*I*a)/b
^4
```

Mupad [B]
time = 0.13, size = 354, normalized size = 4.37

$$\frac{x^3}{3\,b} + \frac{x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{4\,b^2} + \frac{x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2}{4\,b^3} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^3 - 8\,a^3 - 6\,a\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 + 12\,a^2\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)\right)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/acoth(tanh(a + b*x)),x)
```

```
[Out] x^3/(3*b) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b^2) + (x*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x)))/(8*b^4)
```

3.160 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=56

$$\frac{x^2}{2b}+\frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

```
[Out] 1/2*x^2/b+x*(b*x-arccoth(tanh(b*x+a)))/b^2+(b*x-arccoth(tanh(b*x+a)))^2*ln(
arccoth(tanh(b*x+a)))/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2190, 2189, 2188, 29}

$$\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) + (x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^2 + ((b*x - ArcCoth[Tanh[a
 + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\coth}}{b^2}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \text{Subst}}{b^3}$$

$$= \frac{x^2}{2b} + \frac{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\text{cot}\right.}{b^3}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.98

$$\frac{x^2}{2b} - \frac{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2 \log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth[
Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.19, size = 28786, normalized size = 514.04

method	result	size
risch	Expression too large to display	28786

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 51, normalized size = 0.91

$$\frac{bx^2 + (i\,\pi - 2\,a)x}{2\,b^2} - \frac{(\pi^2 + 4i\,\pi a - 4\,a^2)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + (I*pi - 2*a)*x)/b^2 - 1/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi
+ 2*b*x + 2*a)/b^3
```

Fricas [A]
time = 0.38, size = 97, normalized size = 1.73

$$\frac{4\,b^2x^2-8\,abx-16\,\pi a\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-(\pi^2-4\,a^2)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*x^2 - 8*a*b*x - 16*pi*a*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 +
8*a*b*x + pi^2 + 4*a^2))/pi) - (pi^2 - 4*a^2)*log(4*b^2*x^2 + 8*a*b*x + pi^
2 + 4*a^2))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 50, normalized size = 0.89

$$\frac{x^2}{2\,b}-\frac{(i\,\pi+2\,a)x}{2\,b^2}-\frac{(\pi^2-4i\,\pi a-4\,a^2)\log\left(\pi-2i\,bx-2i\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/2*(I*pi + 2*a)*x/b^2 - 1/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(pi -
 2*I*b*x - 2*I*a)/b^3
```

Mupad [B]
time = 1.33, size = 234, normalized size = 4.18

$$\frac{x^2}{2\,b}+\frac{x\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b^2}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)\,\left({\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)+4\,a^2\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/acoth(tanh(a + b*x)),x)
```

```
[Out] x^2/(2*b) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp
(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4
*a^2))/(4*b^3)
```

3.161 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=31

$$\frac{x}{b}+\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

```
[Out] x/b+(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2189, 2188, 29}

$$\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x/b + ((b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx &= \frac{x}{b}-\frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= \frac{x}{b}-\frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,\coth^{-1}(\tanh(a+bx))\right)}{b^2}\\ &= \frac{x}{b}+\frac{\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{x}{b} - \frac{\left(-bx + \coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x/b - ((-(b*x) + ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 4303, normalized size = 138.81

method	result	size
risch	Expression too large to display	4303

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(ex
p(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn
(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a
+2*Pi)*x-1/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi
*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)
)-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a)
)^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+P
i*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b
*x-a)+4*I*a+2*Pi)*ln(exp(b*x+a))+1/4*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(
I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x
+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a)
)^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4
*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*I/b^2
*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
```

```
/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2
*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*
Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2-1/2*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-
2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^
3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x
-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+1/2*I/b^2*ln(Pi*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I
*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp
(2*b*x+2*a)+1))^3+1/4*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a
)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*
a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*e
xp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*e
xp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(
exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a
))-1/2*I/b^2*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-
2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^
3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x
-a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/4*I/b^2*ln
(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*
x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp...
```

Maxima [C] Result contains complex when optimal does not.

time = 0.55, size = 30, normalized size = 0.97

$$\frac{x}{b} - \frac{(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x/b - 1/2*(-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(31) = 62.
time = 0.35, size = 79, normalized size = 2.55

$$\frac{2\,bx + 2\,\pi\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right) - a\log\left(4\,b^2x^2 + 8\,abx + \pi^2 + 4\,a^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x + 2*pi*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 +
4*a^2))/pi) - a*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acoth}\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x/acoth(tanh(a + b*x)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 28, normalized size = 0.90

$$\frac{x}{b} - \frac{(i\,\pi + 2\,a)\log\left(\pi - 2i\,bx - 2i\,a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] x/b - 1/2*(I*pi + 2*a)*log(pi - 2*I*b*x - 2*I*a)/b^2
```

Mupad [B]
time = 0.14, size = 108, normalized size = 3.48

$$\frac{x}{b}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/acoth(tanh(a + b*x)),x)
```

```
[Out] x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/
(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^2)
```

3.162 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=12

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

```
[Out] ln(arccoth(tanh(b*x+a)))/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcCoth[Tanh[a + b*x]]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 1.00

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcCoth[Tanh[a + b*x]]]/b
```

Maple [A]
time = 0.20, size = 13, normalized size = 1.08

method	result
derivativedivides	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
default	$\frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{b}$
risch	$\ln\left(\ln(\mathrm{e}^{bx+a})-\frac{i\pi\left(\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)-\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^{2}-2\operatorname{csgn}\left(\frac{}{\mathrm{e}^{2b}}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(arccoth(tanh(b*x+a)))/b
```

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 16, normalized size = 1.33

$$\frac{\log\left(-\frac{1}{2}i\,\pi-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] log(-1/2*I*pi - b*x - a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.37, size = 28, normalized size = 2.33

$$\frac{\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/b
```

Sympy [A]
time = 11.14, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b} & \text{for}\ b \neq 0 \\ \frac{x}{\operatorname{acoth}\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((log(acoth(tanh(a + b*x)))/b, Ne(b, 0)), (x/acoth(tanh(a)), True)
)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 14, normalized size = 1.17

$$\frac{\log\left(\pi - 2i\,bx - 2i\,a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] log(pi - 2*I*b*x - 2*I*a)/b
```

Mupad [B]
time = 1.18, size = 12, normalized size = 1.00

$$\frac{\ln\left(\operatorname{acoth}(\tanh(a + b\,x))\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acoth(tanh(a + b*x)),x)
```

```
[Out] log(acoth(tanh(a + b*x)))/b
```

3.163 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=44

$$-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -ln(x)/(b*x-arccoth(tanh(b*x+a)))+ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tan
h(b*x+a)))
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2191, 2188, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] -(Log[x]/(b*x - ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/(b*x
 - ArcCoth[Tanh[a + b*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x \coth^{-1}(\tanh(a+bx))}\,dx = -\frac{\int \frac{1}{x}\,dx}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{b\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$= -\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$= -\frac{\log(x)}{bx-\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.66

$$\frac{-\log(x)+\log\left(\coth^{-1}(\tanh(a+bx))\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] (-Log[x] + Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.69, size = 972, normalized size = 22.09

method	result
risch	$-\frac{4i\ln\left(\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(i e^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2+2\pi\mathrm{cs}\right.}{\pi\mathrm{csgn}\left(i e^{2bx+2a}\right)^3-2\pi\,\mathrm{csgn}\left(i e^{bx+a}\right)\mathrm{csgn}\left(i e^{2bx+2a}\right)^2-\pi\,\mathrm{csgn}\left(i e^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2+\pi\,\mathrm{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\mathrm{csgn}\left(i e\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/(Pi*csgn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+
2*a))^2-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-
4*I*b*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*ln(Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)
+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a
))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x
```

```
+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)+4*I/(
Pi*csgn(I*exp(2*b*x+2*a))^3-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^
2-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-4*I*b*
x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+4*I*ln(exp(b*x+a))+2*Pi)*ln(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 37, normalized size = 0.84

$$\frac{2 \log(-i\pi + 2bx + 2a)}{i\pi - 2a} - \frac{2 \log(x)}{i\pi - 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="maxima")
[Out] 2*log(-I*pi + 2*b*x + 2*a)/(I*pi - 2*a) - 2*log(x)/(I*pi - 2*a)
```

Fricas [A]
time = 0.36, size = 87, normalized size = 1.98

$$-\frac{2\left(2\pi\arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)+a\log\left(4b^2x^2+8abx+\pi^2+4a^2\right)-2a\log(x)\right)}{\pi^2+4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="fricas")
[Out] -2*(2*pi*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/p
i) + a*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) - 2*a*log(x))/(pi^2 + 4*a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a)),x)
[Out] Integral(1/(x*acoth(tanh(a + b*x))), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 33, normalized size = 0.75

$$\frac{2 \log(\pi - 2ibx - 2ia)}{-i\pi - 2a} - \frac{2i \log(x)}{\pi - 2ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*log(pi - 2*I*b*x - 2*I*a)/(-I*pi - 2*a) - 2*I*log(x)/(pi - 2*I*a)
```

Mupad [B]
time = 2.91, size = 113, normalized size = 2.57

$$-\frac{4\,\mathrm{atanh}\left(\frac{4\,b\,x}{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x}-1\right)}{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acoth(tanh(a + b*x))),x)
```

```
[Out] -(4*atanh((4*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - 1))/(log(-2/(exp(2*a)*exp(2*b
*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
```

3.164 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=65

$$\frac{1}{x \left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{b \log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{b \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/x/(b*x-arccoth(tanh(b*x+a)))-b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^2+b*ln(ar
ccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2194, 2191, 2188, 29}

$$\frac{1}{x \left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{b \log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{b \log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] 1/(x*(b*x - ArcCoth[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcCoth[Tanh[a + b
*x]])^2 + (b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))}\,dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{x\coth^{-1}(\tanh(a+bx))}\,dx}{bx-\coth^{-1}(\tanh(a+bx))}$$
$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x}\,dx}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b^2 \int}{(bx-c}$$
$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\text{Subst}}{(b}$$
$$= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log(c}{(bx-c}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.69

$$\frac{-\coth^{-1}(\tanh(a+bx)) + bx\left(1-\log(x)+\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]),x]
[Out] (-ArcCoth[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcCoth[Tanh[a + b*x]]]))/
(x*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a)),x)
[Out] int(1/x^2/arccoth(tanh(b*x+a)),x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 65, normalized size = 1.00

$$-\frac{4\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{4\,b\log\left(x\right)}{\pi^2+4i\,\pi a-4\,a^2} + \frac{2}{(i\,\pi-2\,a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4*b*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) + 4*b*log(x)/(pi^2
+ 4*I*pi*a - 4*a^2) + 2/((I*pi - 2*a)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.
time = 0.42, size = 137, normalized size = 2.11

$$\frac{2\left(16\,\pi abx\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-2\,\pi^2a-8\,a^3-(\pi^2b-4\,a^2b)x\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)+2\,(\pi^2b-4\,a^2b)x\log\left(x\right)\right)}{(\pi^4+8\,\pi^2a^2+16\,a^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2*(16*pi*a*b*x*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a
^2))/pi) - 2*pi^2*a - 8*a^3 - (pi^2*b - 4*a^2*b)*x*log(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2) + 2*(pi^2*b - 4*a^2*b)*x*log(x))/((pi^4 + 8*pi^2*a^2 + 16*a
^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 64, normalized size = 0.98

$$-\frac{4i\,b\log\left(\pi-2i\,bx-2i\,a\right)}{i\,\pi^2+4\,\pi a-4i\,a^2}+\frac{4\,b\log\left(x\right)}{\pi^2-4i\,\pi a-4\,a^2}+\frac{2}{-i\,\pi x-2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -4*I*b*log(pi - 2*I*b*x - 2*I*a)/(I*pi^2 + 4*pi*a - 4*I*a^2) + 4*b*log(x)/(
pi^2 - 4*I*pi*a - 4*a^2) + 2/(-I*pi*x - 2*a*x)
```

Mupad [B]
time = 3.12, size = 220, normalized size = 3.38

$$\frac{2\,\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-2\,\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+4\,b\,x+b\,x\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\,1{}\mathrm{i}+b\,x\,2{}\mathrm{i}}{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x}\right)\,8{}\mathrm{i}}{x\,{\left(\ln\left(-\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acoth(tanh(a + b*x))),x)
```

```
[Out] (2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1)) + 4*b*x + b*x*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log
(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x))*8i)/(x*(log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)
```

3.165 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=92

$$\frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2\log}{(bx-}$$

```
[Out] b/x/(b*x-arccoth(tanh(b*x+a)))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a)))-b^2*ln(
x)/(b*x-arccoth(tanh(b*x+a)))^3+b^2*ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(t
anh(b*x+a)))^3
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$-\frac{b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}+\frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
```

```
[Out] b/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcCoth[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcCoth[Tanh[a + b*x]])^3 + (b^2*Log[ArcCoth[T
anh[a + b*x]]])/(b*x - ArcCoth[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))}\,dx}{-bx+\coth^{-1}(\tanh(a+bx))}$$

$$= \frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{}{(bx}$$

$$= \frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} + \frac{}{(bx}$$

$$= \frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{}{(bx}$$

$$= \frac{b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)} - \frac{}{(bx}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.72

$$\frac{-4bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2+b^2x^2\left(3-2\log(x)+2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]),x]
[Out] (-4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2*
Log[x] + 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x
]])^3)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a)),x)
[Out] int(1/x^3/arccoth(tanh(b*x+a)),x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 106, normalized size = 1.15

$$\frac{8\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}-\frac{8\,b^2\log\left(x\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}-\frac{i\,\pi+4\,bx-2\,a}{(\pi^2+4i\,\pi a-4\,a^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 8*b^2*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) -
 8*b^2*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - (I*pi + 4*b*x -
2*a)/((pi^2 + 4*I*pi*a - 4*a^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(90) = 180.
time = 0.39, size = 199, normalized size = 2.16

$$-\frac{2\left(\pi^4a+8\,\pi^2a^3+16\,a^5-8\,(\pi^3b^2-12\,\pi a^2b^2)x^2\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-4\,(3\,\pi^2ab^2-4\,a^3b^2)x^2\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)+8\,(3\,\pi^2ab^2-4\,a^3b^2)x^2\log\left(x\right)+2\,(\pi^4b-16\,a^4b)x\right)}{(\pi^6+12\,\pi^4a^2+48\,\pi^2a^4+64\,a^6)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -2*(pi^4*a + 8*pi^2*a^3 + 16*a^5 - 8*(pi^3*b^2 - 12*pi*a^2*b^2)*x^2*arctan(
-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - 4*(3*pi^2*a
*b^2 - 4*a^3*b^2)*x^2*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 8*(3*pi^2*a
*b^2 - 4*a^3*b^2)*x^2*log(x) + 2*(pi^4*b - 16*a^4*b)*x)/((pi^6 + 12*pi^4*a^
2 + 48*pi^2*a^4 + 64*a^6)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\,\mathrm{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 109, normalized size = 1.18

$$-\frac{8\,b^2\log\left(\pi-2i\,bx-2i\,a\right)}{-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}+\frac{8i\,b^2\log\left(x\right)}{\pi^3-6i\,\pi^2a-12\,\pi a^2+8i\,a^3}-\frac{-i\,\pi+4\,bx-2\,a}{\pi^2x^2-4i\,\pi ax^2-4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -8*b^2*log(pi - 2*I*b*x - 2*I*a)/(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3)
 + 8*I*b^2*log(x)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) - (-I*pi + 4*b*
x - 2*a)/(pi^2*x^2 - 4*I*pi*a*x^2 - 4*a^2*x^2)
```

Mupad [B]
time = 3.60, size = 300, normalized size = 3.26

$$\frac{\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)^2-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)\,\left(2\,\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)+8\,b\,x\right)+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)^2+12\,b^2\,x^2+8\,b\,x\,\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)+b^2\,x^2\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)\,1\mathrm{i}-\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x}\right)\,16\mathrm{i}}{x^2\,\left(\ln\left(-\frac{1}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*acoth(tanh(a + b*x))),x)
```

```
[Out] (log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) - 1))*(2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + 8*b*x) + log((exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 12*b^2*x^2 + b^2*x^2*atan((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)
*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*16i + 8*b*x*log(-
1/(exp(2*a)*exp(2*b*x) - 1)))/(x^2*(log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log
((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

3.166 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=65

$$-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}-\frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -x^m/b/arccoth(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arccoth(tan
h(b*x+a))))/b/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$-\frac{x^m\,{}_2F_1\left(1,m;m+1;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x^m/(b*ArcCoth[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcCoth[Tanh[a + b*x]])])/(b*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))} + \frac{m\int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= -\frac{x^m}{b\coth^{-1}(\tanh(a+bx))} - \frac{x^m\,{}_2F_1\left(1,m;1+m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.37, size = 51, normalized size = 0.78

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]]^2,x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a))^2,x)
[Out] int(x^m/arccoth(tanh(b*x+a))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arccoth(tanh(b*x + a))^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**m/acoth(tanh(a + b*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/acoth(tanh(a + b*x))^2,x)
```

```
[Out] int(x^m/acoth(tanh(a + b*x))^2, x)
```

3.167 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=98

$$\frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}+\frac{4\left(bx-\right.}{}$$

[Out] 4/3*x^3/b^2+2*x^2*(b*x-arccoth(tanh(b*x+a)))/b^3+4*x*(b*x-arccoth(tanh(b*x+a)))^2/b^4-x^4/b/arccoth(tanh(b*x+a))+4*(b*x-arccoth(tanh(b*x+a)))^3*ln(arccoth(tanh(b*x+a)))/b^5

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{4\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}+\frac{4x^3}{3b^2}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCoth[Tanh[a + b*x]]^2,x]

[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 + (4*x*(b*x - ArcCoth[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcCoth[Tanh[a + b*x]]) + (4*(b*x - ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth[Tanh[a + b*x]]])/b^5

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}+\frac{4\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= \frac{4x^3}{3b^2}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}-\frac{\left(4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\coth^{-1}}}{b^2}\\ &= \frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^4}{b\coth^{-1}(\tanh(a+bx))}+\frac{\left(4(-\right.}{}\\ &= \frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}\\ &= \frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}\\ &= \frac{4x^3}{3b^2}+\frac{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 1.08

$$\frac{x^3}{3b^2}-\frac{x^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}{b^5\coth^{-1}(\tanh(a+bx))}-\frac{4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] x^3/(3*b^2) - (x^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
ArcCoth[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcCoth[Tanh[a + b*x]])^4/(b^5*A
```

```
rcCoth[Tanh[a + b*x]]) - (4*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3*Log[ArcCoth
[Tanh[a + b*x]]])/b^5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 6.40, size = 131085, normalized size = 1337.60

method	result	size
risch	Expression too large to display	131085

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.67, size = 179, normalized size = 1.83

$$\frac{16\,b^4x^4-3\,\pi^4-24i\,\pi^3a+72\,\pi^2a^2+96i\,\pi a^3-48\,a^4-16\,(-i\,\pi b^3+2\,ab^3)x^3-24\,(\pi^2b^2+4i\,\pi ab^2-4\,a^2b^2)x^2-18\,(-i\,\pi^3b+6\,\pi^2ab+12i\,\pi a^2b-8\,a^3b)x}{24\,(2\,b^6x-i\,\pi b^5+2\,ab^5)}-\frac{(i\,\pi^3-6\,\pi^2a-12i\,\pi a^2+8\,a^3)\log\left(-i\,\pi+2\,bx+2\,a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(16*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 + 96*I*pi*a^3 - 48*a^
4 - 16*(-I*pi*b^3 + 2*a*b^3)*x^3 - 24*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^2)
*x^2 - 18*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(2*b^6*x -
I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3)*log(-I*
pi + 2*b*x + 2*a)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(96) = 192.
time = 0.42, size = 326, normalized size = 3.33

$$\frac{16\,b^5x^5-16\,ab^4x^4+9\,\pi^4a+24\,\pi^2a^3-48\,a^5-32\,(\pi^2b^3-2\,a^2b^3)x^3-12\,(7\,\pi^2ab^2-20\,a^3b^2)x^2-12\,(\pi^4b-6\,\pi^2a^2b-8\,a^4b)x-12\,(\pi^5-8\,\pi^3a^2-48\,\pi a^4+4\,(\pi^3b^2-12\,\pi a^2b^2)x^2+8\,(\pi^3ab-12\,\pi a^3b)x)\arctan\left(-\frac{2bx+2a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)+6\,(3\,\pi^4a+8\,\pi^2a^3-16\,a^5+4\,(3\,\pi^2ab^2-4\,a^3b^2)x^2+8\,(3\,\pi^2a^2b-4\,a^4b)x)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{12\,(4\,b^7x^2+8\,ab^6x+\pi^2b^5+4\,a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(16*b^5*x^5 - 16*a*b^4*x^4 + 9*pi^4*a + 24*pi^2*a^3 - 48*a^5 - 32*(pi^
2*b^3 - 2*a^2*b^3)*x^3 - 12*(7*pi^2*a*b^2 - 20*a^3*b^2)*x^2 - 12*(pi^4*b -
6*pi^2*a^2*b - 8*a^4*b)*x - 12*(pi^5 - 8*pi^3*a^2 - 48*pi*a^4 + 4*(pi^3*b^2
 - 12*pi*a^2*b^2)*x^2 + 8*(pi^3*a*b - 12*pi*a^3*b)*x)*arctan(-(2*b*x + 2*a
- sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) + 6*(3*pi^4*a + 8*pi^2*a^3
- 16*a^5 + 4*(3*pi^2*a*b^2 - 4*a^3*b^2)*x^2 + 8*(3*pi^2*a^2*b - 4*a^4*b)*x)
*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^7*x^2 + 8*a*b^6*x + pi^2*b^5
 + 4*a^2*b^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**4/acoth(tanh(a + b*x))**2, x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.40, size = 135, normalized size = 1.38

$$\frac{x^3}{3\,b^2} - \frac{\pi^4 - 8 i\,\pi^3 a - 24\,\pi^2 a^2 + 32 i\,\pi a^3 + 16\,a^4}{8\,(2\,b^6 x + i\,\pi b^5 + 2\,a b^5)} - \frac{(i\,\pi + 2\,a)x^2}{2\,b^3} - \frac{3\,(\pi^2 - 4 i\,\pi a - 4\,a^2)x}{4\,b^4} + \frac{(i\,\pi^3 + 6\,\pi^2 a - 12 i\,\pi a^2 - 8\,a^3)\log(i\,\pi + 2\,bx + 2\,a)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3/b^2 - 1/8*(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4)/
(2*b^6*x + I*pi*b^5 + 2*a*b^5) - 1/2*(I*pi + 2*a)*x^2/b^3 - 3/4*(pi^2 - 4*I
*pi*a - 4*a^2)*x/b^4 + 1/2*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)*log(I*
pi + 2*b*x + 2*a)/b^5
```

Mupad [**B**]
time = 1.29, size = 669, normalized size = 6.83

$$\frac{x^3}{3\,b^2} - \frac{\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^4 + 24\,a^2\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 + 16\,a^4 - 8\,a\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^3 - 32\,a^3\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b\left(8\,a\,b^4 + 8\,b^5\,x - 4\,b^4\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)\right)} + \frac{x^2\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{2\,b^3} + \frac{3\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2}{4\,b^4} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^3 - 8\,a^3 - 6\,a\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 + 12\,a^2\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/acoth(tanh(a + b*x))^2,x)
```

```
[Out] x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x)
- 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp
(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x))) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^3) + (3*x*(l
og(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2*a - log((2*e
```

```
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(ex
p(2*a)*exp(2*b*x) - 1)) + 2*b*x)))/(2*b^5)
```

3.168 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{3x^2}{2b^2}+\frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-}\right.}{b^4}$$

```
[Out] 3/2*x^2/b^2+3*x*(b*x-arccoth(tanh(b*x+a)))/b^3-x^3/b/arccoth(tanh(b*x+a))+3
*(b*x-arccoth(tanh(b*x+a)))^2*ln(arccoth(tanh(b*x+a)))/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\coth^{-1}(\tanh(a+bx))}+\frac{3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcCoth
[Tanh[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a +
 b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{3x^2}{2b^2} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{\cdots}{\coth^{-1}(\cdots}}{b^2}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{\left(3(-bx \cdots\right.}{\cdots}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{\left(3(-bx \cdots\right.}{\cdots}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \cdots}{\cdots}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 1.11

$$\frac{x^2}{2b^2} - \frac{2x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^4\coth^{-1}(\tanh(a+bx))} + \frac{3\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] x^2/(2*b^2) - (2*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcCo
th[Tanh[a + b*x]])^3/(b^4*ArcCoth[Tanh[a + b*x]]) + (3*(-(b*x) + ArcCoth[Ta
nh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.46, size = 29109, normalized size = 388.12

method	result	size
risch	Expression too large to display	29109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.66, size = 123, normalized size = 1.64

$$\frac{4\,b^3x^3+i\,\pi^3-6\,\pi^2a-12i\,\pi a^2+8\,a^3-6\,(-i\,\pi b^2+2\,ab^2)x^2+4\,(\pi^2b+4i\,\pi ab-4\,a^2b)x}{4\,(2\,b^5x-i\,\pi b^4+2\,ab^4)}-\frac{3\,(\pi^2+4i\,\pi a-4\,a^2)\log\left(-i\,\pi+2\,bx+2\,a\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 6*(-I*pi*b^2 + 2
*a*b^2)*x^2 + 4*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(2*b^5*x - I*pi*b^4 + 2*
a*b^4) - 3/4*(pi^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(73) = 146.
time = 0.42, size = 244, normalized size = 3.25

$$\frac{16\,b^4x^4-32\,ab^3x^3-2\,\pi^4+32\,a^4+4\,(\pi^2b^2-28\,a^2b^2)x^2-8\,(5\,\pi^2ab+4\,a^3b)x-48\,(4\,\pi ab^2x^2+8\,\pi a^2bx+\pi^3a+4\,\pi a^3)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-3\,(\pi^4-16\,a^4+4\,(\pi^2b^2-4\,a^2b^2)x^2+8\,(\pi^2ab-4\,a^3b)x)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{8\,(4\,b^6x^2+8\,ab^5x+\pi^2b^4+4\,a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(16*b^4*x^4 - 32*a*b^3*x^3 - 2*pi^4 + 32*a^4 + 4*(pi^2*b^2 - 28*a^2*b^2
)*x^2 - 8*(5*pi^2*a*b + 4*a^3*b)*x - 48*(4*pi*a*b^2*x^2 + 8*pi*a^2*b*x + pi
^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4
*a^2))/pi) - 3*(pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b
 - 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^6*x^2 + 8*a*b^5
*x + pi^2*b^4 + 4*a^2*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x))**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 97, normalized size = 1.29

$$-\frac{i\pi^3+6\pi^2a-12i\pi a^2-8a^3}{4\left(2b^5x+i\pi b^4+2ab^4\right)}+\frac{x^2}{2b^2}+\frac{(-i\pi-2a)x}{b^3}-\frac{3\left(\pi^2-4i\pi a-4a^2\right)\log\left(i\pi+2bx+2a\right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -1/4*(I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3)/(2*b^5*x + I*pi*b^4 + 2*a*b^
4) + 1/2*x^2/b^2 + (-I*pi - 2*a)*x/b^3 - 3/4*(pi^2 - 4*I*pi*a - 4*a^2)*log(
I*pi + 2*b*x + 2*a)/b^4
```

Mupad [B]
time = 0.17, size = 490, normalized size = 6.53

$$\frac{x^2}{2b^2}+\frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right)\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2-12a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)+12a^2\right)}{4b^4}-\frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^3-8a^3-6a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2+12a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)}{4b\left(2ab^3+2b^4x-b^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)\right)}+\frac{x\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/acoth(tanh(a + b*x))^2,x)
```

```
[Out] x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) -
 log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 -
12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2
/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x)
 - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp
(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*
b*x) - 1)) + 2*b*x))) + (x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

3.169 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=50

$$\frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2(bx - \coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

```
[Out] 2*x/b^2-x^2/b/arccoth(tanh(b*x+a))+2*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(
tanh(b*x+a)))/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2189, 2188, 29}

$$\frac{2(bx - \coth^{-1}(\tanh(a+bx)))\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x)/b^2 - x^2/(b*ArcCoth[Tanh[a + b*x]]) + (2*(b*x - ArcCoth[Tanh[a + b*x
]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\int \frac{x}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\coth^{-1}(\text{ta}}}{b^2}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)\right)\text{Subst}\left(\int\right.}{b^3}$$
$$= \frac{2x}{b^2} - \frac{x^2}{b\coth^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\text{ta}\right.}{b^3}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 1.12

$$\frac{bx - \frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}{\coth^{-1}(\tanh(a+bx))} + 2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (b*x - (-(b*x) + ArcCoth[Tanh[a + b*x]])^2/ArcCoth[Tanh[a + b*x]] + 2*(b*x
- ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]])/b^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 4626, normalized size = 92.52

method	result	size
risch	Expression too large to display	4626

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*x^2/b/(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(
I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi
*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi
```

```
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+2/b^2*ln(P
i*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b
*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*x
-2/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/
(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*
I*a+2*Pi)*ln(exp(b*x+a))+1/2*I/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*
b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2
*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*c
sgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*c
sgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4
*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*I/b^3*ln(Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a
)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a
)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I/b^3
*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2
*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*
Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+I/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I
/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+
a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))
```

```
^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*
I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^
3+1/2*I/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2
*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3
-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-
a)+4*I*a+2*Pi)*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I/b^3*ln(Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a
)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*
x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.65, size = 81, normalized size = 1.62

$$\frac{4 b^2 x^2 + \pi^2 + 4 i \pi a - 4 a^2 - 2 (i \pi b - 2 ab)x}{2 (2 b^4 x - i \pi b^3 + 2 ab^3)} - \frac{(-i \pi + 2 a) \log(-i \pi + 2 bx + 2 a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b - 2*a*b)*x)/(2*b^4*x -
 I*pi*b^3 + 2*a*b^3) - (-I*pi + 2*a)*log(-I*pi + 2*b*x + 2*a)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(50) = 100.
time = 0.38, size = 189, normalized size = 3.78

$$\frac{4 b^3 x^3 + 8 ab^2 x^2 + 2 \pi^2 bx - \pi^2 a - 4 a^3 + 2 (4 \pi b^2 x^2 + 8 \pi abx + \pi^3 + 4 \pi a^2) \arctan\left(-\frac{2 bx + 2 a - \sqrt{4 b^2 x^2 + 8 abx + \pi^2 + 4 a^2}}{\pi}\right) - (4 ab^2 x^2 + 8 a^2 bx + \pi^2 a + 4 a^3) \log(4 b^2 x^2 + 8 abx + \pi^2 + 4 a^2)}{4 b^5 x^2 + 8 ab^4 x + \pi^2 b^3 + 4 a^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (4*b^3*x^3 + 8*a*b^2*x^2 + 2*pi^2*b*x - pi^2*a - 4*a^3 + 2*(4*pi*b^2*x^2 +
8*pi*a*b*x + pi^3 + 4*pi*a^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b
*x + pi^2 + 4*a^2))/pi) - (4*a*b^2*x^2 + 8*a^2*b*x + pi^2*a + 4*a^3)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^5*x^2 + 8*a*b^4*x + pi^2*b^3 + 4*a^
2*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**2/acoth(tanh(a + b*x))**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 66, normalized size = 1.32

$$\frac{\pi^2 - 4i\,\pi a - 4\,a^2}{2\,(2\,b^4x + i\,\pi b^3 + 2\,ab^3)} + \frac{x}{b^2} - \frac{(i\,\pi + 2\,a)\log(i\,\pi + 2\,bx + 2\,a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi^2 - 4*I*pi*a - 4*a^2)/(2*b^4*x + I*pi*b^3 + 2*a*b^3) + x/b^2 - (I*p
i + 2*a)*log(I*pi + 2*b*x + 2*a)/b^3
```

Mupad [B]
time = 1.29, size = 302, normalized size = 6.04

$$\frac{x}{b^2} - \frac{\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)^2 - 4\,a\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right) + 4\,a^2}{2\,b\left(2\,a\,b^2 + 2\,b^3\,x - b^2\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)\right)} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right) + 2\,b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/acoth(tanh(a + b*x))^2,x)
```

```
[Out] x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
))) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/
(exp(2*a)*exp(2*b*x) - 1)))*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/b^3
```

3.170 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=28

$$-\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

```
[Out] -x/b/arccoth(tanh(b*x+a))+ln(arccoth(tanh(b*x+a)))/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x/(b*ArcCoth[Tanh[a + b*x]])) + Log[ArcCoth[Tanh[a + b*x]]]/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx &= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2}\\ &= -\frac{x}{b\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.96

$$\frac{1-\frac{bx}{\coth^{-1}(\tanh(a+bx))}+\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]]^2,x]
```

```
[Out] (1 - (b*x)/ArcCoth[Tanh[a + b*x]] + Log[ArcCoth[Tanh[a + b*x]]])/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 625, normalized size = 22.32

method	result
risch	$-\frac{}{b\left(\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(i e^{2bx+2a}\right)\text{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2+2\pi\text{csg}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*x/b/(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/
(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*c
sgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+1/b^2*ln(ln(
exp(b*x+a))-1/4*I*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*csgn(I/(exp(2*b*x+2*a)+1))^2+2*csgn(I/
```

```
(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^3+2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.67, size = 46, normalized size = 1.64

$$\frac{-i\,\pi+2\,a}{2\,b^3x-i\,\pi b^2+2\,ab^2}+\frac{\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] (-I*pi + 2*a)/(2*b^3*x - I*pi*b^2 + 2*a*b^2) + log(-I*pi + 2*b*x + 2*a)/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(28) = 56.
time = 0.42, size = 97, normalized size = 3.46

$$\frac{8\,abx+2\,\pi^2+8\,a^2+\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{2\left(4\,b^4x^2+8\,ab^3x+\pi^2b^2+4\,a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(8*a*b*x + 2*pi^2 + 8*a^2 + (4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^4*x^2 + 8*a*b^3*x + pi^2*b^2 + 4*a^
2*b^2)
```

Sympy [A]
time = 21.72, size = 36, normalized size = 1.29

$$\begin{cases} -\frac{x}{b\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}+\frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b^2} & \text{for}\ b\neq 0\\ \frac{x^2}{2\operatorname{acoth}^2\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-x/(b*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**2, Ne
(b, 0)), (x**2/(2*acoth(tanh(a))**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 47, normalized size = 1.68

$$-\frac{-i\,\pi-2\,a}{2\,b^3x+i\,\pi b^2+2\,ab^2}+\frac{\log\left(i\,\pi+2\,bx+2\,a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -(-I*pi - 2*a)/(2*b^3*x + I*pi*b^2 + 2*a*b^2) + log(I*pi + 2*b*x + 2*a)/b^2
```

Mupad [B]
time = 0.09, size = 28, normalized size = 1.00

$$\frac{\ln\left(\mathrm{acoth}(\tanh(a+b\,x))\right)}{b^2}-\frac{x}{b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/acoth(tanh(a + b*x))^2,x)
```

```
[Out] log(acoth(tanh(a + b*x)))/b^2 - x/(b*acoth(tanh(a + b*x)))
```

3.171 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{b \coth^{-1}(\tanh(a+bx))}$$

```
[Out] -1/b/arccoth(tanh(b*x+a))
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$-\frac{1}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcCoth[Tanh[a + b*x]]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{1}{b \coth^{-1}(\tanh(a+bx))} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-2),x]
[Out] -(1/(b*ArcCoth[Tanh[a + b*x]]))
```

Maple [A]
time = 0.21, size = 15, normalized size = 1.07

method	result
derivativedivides	$-\frac{1}{b\operatorname{arccoth}(\tanh(bx+a))}$
default	$-\frac{1}{b\operatorname{arccoth}(\tanh(bx+a))}$
risch	$-\frac{}{b\left(\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^{2}-2\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
[Out] -1/b/arccoth(tanh(b*x+a))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 18, normalized size = 1.29

$$\frac{4}{-2\left(i\pi+2bx+2a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 4/((-2*I*pi - 4*b*x - 4*a)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.
time = 0.42, size = 36, normalized size = 2.57

$$-\frac{4\left(bx+a\right)}{4\,b^{3}x^{2}+8\,ab^{2}x+\pi^{2}b+4\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
[Out] -4*(b*x + a)/(4*b^3*x^2 + 8*a*b^2*x + pi^2*b + 4*a^2*b)
```

Sympy [A]
time = 21.87, size = 20, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)} & \text{for}\ b\neq 0\\ \frac{x}{\operatorname{acoth}^{2}\left(\tanh\left(a\right)\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((-1/(b*acoth(tanh(a + b*x))), Ne(b, 0)), (x/acoth(tanh(a))**2, Tr
ue))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 19, normalized size = 1.36

$$-\frac{2}{2\,b^2x+i\,\pi b+2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -2/(2*b^2*x + I*pi*b + 2*a*b)
```

Mupad [B]
time = 1.14, size = 14, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acoth(tanh(a + b*x))^2,x)
```

```
[Out] -1/(b*acoth(tanh(a + b*x)))
```

3.172 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=70

$$-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}+\frac{\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^2}-\frac{\log\left(\coth^{-1}(\tanh(\right.}{(bx-\coth^{-1}(\tanh(}$$

```
[Out] -1/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))+ln(x)/(b*x-arccoth(tanh(
b*x+a)))^2-ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^2
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$-\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}+\frac{\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^2}-\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{(bx-\coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] -(1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcCoth[Tanh[a + b*x]])^2 - Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Ta
nh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{x\coth^{-1}(\tanh(a\text{-}}}{-bx+\coth^{-1}(\tan}$$

$$= -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} - \frac{}{(bx-\coth^{-1}(\tanh}$$

$$= -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx-\coth^{-1}(\tanh}$$

$$= -\frac{1}{(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))} + \frac{\log(x)}{(bx-\coth^{-1}(\tanh}$$

Mathematica [**A**]
time = 0.05, size = 53, normalized size = 0.76

$$\frac{-bx+\coth^{-1}(\tanh(a+bx))\left(1+\log(bx)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{\coth^{-1}(\tanh(a+bx))\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b*x) + ArcCoth[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcCoth[Tanh[a + b*x]]
]))/(ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.64, size = 78, normalized size = 1.11

$$\frac{4\log\left(-i\pi+2bx+2a\right)}{\pi^2+4i\pi a-4a^2} - \frac{4\log\left(x\right)}{\pi^2+4i\pi a-4a^2} - \frac{4}{\pi^2+4i\pi a-4a^2-2\left(-i\pi b+2ab\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*I
*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 - 2*(-I*pi*b + 2*a*b)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(70) = 140$.
time = 0.40, size = 306, normalized size = 4.37

$$-\frac{2\left(2\pi^4-32a^4-8(\pi^2ab+4a^3b)x+16(4\pi ab^2x^2+8\pi a^2bx+\pi^3a+4\pi a^3)\arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)-(\pi^4-16a^4+4(\pi^2b^2-4a^2b^2)x^2+8(\pi^2ab-4a^3b)x)\log(4b^2x^2+8abx+\pi^2+4a^2)+2(\pi^4-16a^4+4(\pi^2b^2-4a^2b^2)x^2+8(\pi^2ab-4a^3b)x)\log(x)\right)}{\pi^6+12\pi^4a^2+48\pi^2a^4+64a^6+4(\pi^4b^2+8\pi^2a^2b^2+16a^4b^2)x^2+8(\pi^4ab+8\pi^2a^3b+16a^5b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*pi^4 - 32*a^4 - 8*(pi^2*a*b + 4*a^3*b)*x + 16*(4*pi*a*b^2*x^2 + 8*pi*
a^2*b*x + pi^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*
x + pi^2 + 4*a^2))/pi) - (pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*
(pi^2*a*b - 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(pi^4 -
 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b - 4*a^3*b)*x)*log(x))/
(pi^6 + 12*pi^4*a^2 + 48*pi^2*a^4 + 64*a^6 + 4*(pi^4*b^2 + 8*pi^2*a^2*b^2 +
 16*a^4*b^2)*x^2 + 8*(pi^4*a*b + 8*pi^2*a^3*b + 16*a^5*b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 78, normalized size = 1.11

$$\frac{4\log(i\pi+2bx+2a)}{\pi^2-4i\pi a-4a^2}-\frac{4\log(x)}{\pi^2-4i\pi a-4a^2}+\frac{4}{2i\pi bx+4abx-\pi^2+4i\pi a+4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 4*log(I*pi + 2*b*x + 2*a)/(pi^2 - 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 - 4*I*
pi*a - 4*a^2) + 4/(2*I*pi*b*x + 4*a*b*x - pi^2 + 4*I*pi*a + 4*a^2)
```

Mupad [B]
time = 4.03, size = 421, normalized size = 6.01

$$-\frac{4\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)-4\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+8bx+\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\mathrm{atan}\left(\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx}\right)8\mathrm{i}-\mathrm{atan}\left(\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx}\right)\left(\ln(2)+\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)\right)8\mathrm{i}}{\left(\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)-\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)\right)\left(\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acoth(tanh(a + b*x))^2),x)
```

```
[Out] -(4*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 4*log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 8*b*x + log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i -
 log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x
) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*
8i - atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(
-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*(log(
2) + log(-1/(exp(2*a)*exp(2*b*x) - 1)))*8i)/((log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2
*a)*exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x)^2)
```

3.173 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=102

$$-\frac{2b}{(bx-\coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))}+\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx$$

```
[Out] -2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/x/(b*x-arccoth(tan
h(b*x+a)))/arccoth(tanh(b*x+a))+2*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^3-2*b*
ln(arccoth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$-\frac{2b}{(bx-\coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))}+\frac{1}{x\,(bx-\coth^{-1}(\tanh(a+bx)))\coth^{-1}(\tanh(a+bx))}+\frac{2b\log(x)}{(bx-\coth^{-1}(\tanh(a+bx)))^3}-\frac{2b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{(bx-\coth^{-1}(\tanh(a+bx)))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-2*b)/((b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^3 - (2*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth
[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^2\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} - \frac{(2b)\int \frac{1}{x\coth^{-1}(\tanh}}{-bx+\coth^{-1}(\tan}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\coth^{-1}(\mathrm{t}\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\coth^{-1}(\mathrm{t}\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\coth^{-1}(\mathrm{t}\right.}$$

$$= -\frac{2b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\coth^{-1}(\mathrm{t}\right.}$$

Mathematica [**A**]
time = 0.04, size = 70, normalized size = 0.69

$$\frac{-b^2x^2+\coth^{-1}(\tanh(a+bx))^2+2bx\coth^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b^2*x^2) + ArcCoth[Tanh[a + b*x]]^2 + 2*b*x*ArcCoth[Tanh[a + b*x]]*(Log[
x] - Log[ArcCoth[Tanh[a + b*x]]]))/(x*(b*x - ArcCoth[Tanh[a + b*x]])^3*ArcC
oth[Tanh[a + b*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.48, size = 27548, normalized size = 270.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.65, size = 135, normalized size = 1.32

$$-\frac{16\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^{3}+6\,\pi^{2}a+12i\,\pi a^{2}-8\,a^{3}}+\frac{16\,b\log\left(x\right)}{-i\,\pi^{3}+6\,\pi^{2}a+12i\,\pi a^{2}-8\,a^{3}}-\frac{4\,(i\,\pi-4\,bx-2\,a)}{2\,(\pi^{2}b+4i\,\pi ab-4\,a^{2}b)x^{2}-(i\,\pi^{3}-6\,\pi^{2}a-12i\,\pi a^{2}+8\,a^{3})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -16*b*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) +
 16*b*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 4*(I*pi - 4*b*x -
 2*a)/(2*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x^2 - (I*pi^3 - 6*pi^2*a - 12*I*pi
*a^2 + 8*a^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(102) = 204$.
time = 0.36, size = 480, normalized size = 4.71

$$\frac{4\left(\pi^{6}+4\,\pi^{4}a^{2}-16\,\pi^{2}a^{4}-64\,a^{6}+8\,(\pi^{4}b^{2}-16\,a^{4}b^{2})x^{2}+4\,(5\,\pi^{4}ab+8\,\pi^{2}a^{3}b-48\,a^{5}b)x-8\,(4\,(\pi^{3}b^{3}-12\,\pi a^{2}b^{3})x^{3}+8\,(\pi^{3}ab^{2}-12\,\pi a^{3}b^{2})x^{2}+(\pi^{5}b-8\,\pi^{3}a^{2}b-48\,\pi a^{4}b)x)\arctan\left(-\frac{2bx+2a-\sqrt{4\,b^{2}x^{2}+8\,abx+\pi^{2}+4\,a^{2}}}{\pi}\right)-4\,(4\,(3\,\pi^{2}ab^{3}-4\,a^{3}b^{3})x^{3}+8\,(3\,\pi^{2}a^{2}b^{2}-4\,a^{4}b^{2})x^{2}+(3\,\pi^{4}ab+8\,\pi^{2}a^{3}b-16\,a^{5}b)x)\log\left(4\,b^{2}x^{2}+8\,abx+\pi^{2}+4\,a^{2}\right)+8\,(4\,(3\,\pi^{2}ab^{3}-4\,a^{3}b^{3})x^{3}+8\,(3\,\pi^{2}a^{2}b^{2}-4\,a^{4}b^{2})x^{2}+(3\,\pi^{4}ab+8\,\pi^{2}a^{3}b-16\,a^{5}b)x)\log\left(x\right)\right)}{4\,(\pi^{6}b^{2}+12\,\pi^{4}a^{2}b^{2}+48\,\pi^{2}a^{4}b^{2}+64\,a^{6}b^{2})x^{3}+8\,(\pi^{6}ab+12\,\pi^{4}a^{3}b+48\,\pi^{2}a^{5}b+64\,a^{7}b)x^{2}+(\pi^{8}+16\,\pi^{6}a^{2}+96\,\pi^{4}a^{4}+256\,\pi^{2}a^{6}+256\,a^{8})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 4*(pi^6 + 4*pi^4*a^2 - 16*pi^2*a^4 - 64*a^6 + 8*(pi^4*b^2 - 16*a^4*b^2)*x^2
 + 4*(5*pi^4*a*b + 8*pi^2*a^3*b - 48*a^5*b)*x - 8*(4*(pi^3*b^3 - 12*pi*a^2*
b^3)*x^3 + 8*(pi^3*a*b^2 - 12*pi*a^3*b^2)*x^2 + (pi^5*b - 8*pi^3*a^2*b - 48
*pi*a^4*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^
2))/pi) - 4*(4*(3*pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(3*pi^2*a^2*b^2 - 4*a^4*b
^2)*x^2 + (3*pi^4*a*b + 8*pi^2*a^3*b - 16*a^5*b)*x)*log(4*b^2*x^2 + 8*a*b*x
 + pi^2 + 4*a^2) + 8*(4*(3*pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(3*pi^2*a^2*b^2
- 4*a^4*b^2)*x^2 + (3*pi^4*a*b + 8*pi^2*a^3*b - 16*a^5*b)*x)*log(x))/(4*(pi
^6*b^2 + 12*pi^4*a^2*b^2 + 48*pi^2*a^4*b^2 + 64*a^6*b^2)*x^3 + 8*(pi^6*a*b
+ 12*pi^4*a^3*b + 48*pi^2*a^5*b + 64*a^7*b)*x^2 + (pi^8 + 16*pi^6*a^2 + 96*
pi^4*a^4 + 256*pi^2*a^6 + 256*a^8)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\operatorname{acoth}^{2}\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 140, normalized size = 1.37

$$\frac{16\,b\log\left(i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}-\frac{16\,b\log\left(x\right)}{-i\,\pi^3-6\,\pi^2a+12i\,\pi a^2+8\,a^3}+\frac{8\,b}{2\,\pi^2bx-8i\,\pi abx-8\,a^2bx+i\,\pi^3+6\,\pi^2a-12i\,\pi a^2-8\,a^3}+\frac{4}{\pi^2x-4i\,\pi ax-4\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 16*b*log(I*pi + 2*b*x + 2*a)/(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3) - 1
6*b*log(x)/(-I*pi^3 - 6*pi^2*a + 12*I*pi*a^2 + 8*a^3) + 8*b/(2*pi^2*b*x - 8
*I*pi*a*b*x - 8*a^2*b*x + I*pi^3 + 6*pi^2*a - 12*I*pi*a^2 - 8*a^3) + 4/(pi^
2*x - 4*I*pi*a*x - 4*a^2*x)
```

Mupad [B]
time = 3.80, size = 453, normalized size = 4.44

$$\frac{4\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2-\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\left(8\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+bx\,\mathrm{atan}\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx}\right)32\mathrm{i}\right)+4\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2-16\,b^2x^2+bx\,\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\mathrm{atan}\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx}\right)32\mathrm{i}}{x\left(\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\right)\left(\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acoth(tanh(a + b*x))^2),x)
```

```
[Out] (4*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) - 1))*(8*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + b*x*atan((log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2
*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*32i) + 4*log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 - 16*b^2*x^2 + b*x*log(-1/(exp(2*a
)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))*32i)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - lo
g(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2*a)*exp(2*b*x) - 1)) - log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)
```

3.174 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=143

$$-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))}+\frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+$$

```
[Out] -3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))+3/2*b/x/(b*x-arcco
th(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))+1/2/x^2/(b*x-arccoth(tanh(b*x+a)))/
arccoth(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4-3*b^2*ln(arcc
oth(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))}+\frac{3b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))}+\frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcCoth[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - Ar
cCoth[Tanh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^3\coth^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))} + \frac{(3b)\int \frac{}{x^2\coth^{-1}}}{2\left(bx-\coth^{-1}(\right.}$$

$$= \frac{3b}{2x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))} + \frac{}{2x^2\left(bx-\coth^{-}\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\coth^{-1}(\right.}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.64

$$-\frac{2b^3x^3-6bx\coth^{-1}(\tanh(a+bx))^2+\coth^{-1}(\tanh(a+bx))^3-3b^2x^2\coth^{-1}(\tanh(a+bx))\left(-1+2\log(x)-2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2x^2\coth^{-1}(\tanh(a+bx))\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^2),x]
```

```
[Out] -1/2*(2*b^3*x^3 - 6*b*x*ArcCoth[Tanh[a + b*x]]^2 + ArcCoth[Tanh[a + b*x]]^3
 - 3*b^2*x^2*ArcCoth[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a +
 b*x]]]))/(x^2*ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x^3/arccoth(tanh(b*x+a))^2,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.65, size = 190, normalized size = 1.33

$$-\frac{48\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}+\frac{48\,b^2\log\left(x\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}+\frac{2\,(24\,b^2x^2+\pi^2+4i\,\pi a-4\,a^2-6\,(i\,\pi b-2\,ab)x)}{2\,(i\,\pi^3b-6\,\pi^2ab-12i\,\pi a^2b+8\,a^3b)x^3+(\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -48*b^2*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi
*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a
^3 + 16*a^4) + 2*(24*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 6*(I*pi*b - 2*a*b)
*x)/(2*(I*pi^3*b - 6*pi^2*a*b - 12*I*pi*a^2*b + 8*a^3*b)*x^3 + (pi^4 + 8*I*
pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 + 16*a^4)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(139) = 278$.
time = 0.35, size = 644, normalized size = 4.50

$$\frac{2\left(\pi^8+8\,\pi^6a^2-128\,\pi^2a^6-256\,a^8-96\,(3\,\pi^4ab^3+8\,\pi^2a^3b^3-16\,a^5b^3)x^3+12\,(\pi^6b^2-44\,\pi^4a^2b^2-144\,\pi^2a^4b^2+192\,a^6b^2)x^2-8\,(5\,\pi^6ab+36\,\pi^4a^3b+48\,\pi^2a^5b-64\,a^7b)x+384\,(4\,(\pi^3ab^4-4\,\pi a^3b^4)x^4+8\,(\pi^3a^2b^3-4\,\pi a^4b^3)x^3+(\pi^5ab^2-16\,\pi a^5b^2)x^2)\arctan\left(-\frac{2\,bx+2\,a-\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)-12\,(4\,(\pi^4b^4-24\,\pi^2a^2b^4+16\,a^4b^4)x^4+8\,(\pi^4ab^3-24\,\pi^2a^3b^3+16\,a^5b^3)x^3+(\pi^6b^2-20\,\pi^4a^2b^2-80\,\pi^2a^4b^2+64\,a^6b^2)x^2)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)+24\,(4\,(\pi^4b^4-24\,\pi^2a^2b^4+16\,a^4b^4)x^4+8\,(\pi^4ab^3-24\,\pi^2a^3b^3+16\,a^5b^3)x^3+(\pi^6b^2-20\,\pi^4a^2b^2-80\,\pi^2a^4b^2+64\,a^6b^2)x^2)\log\left(x\right)\right)}{4\,(\pi^8b^2+16\,\pi^6a^2b^2+96\,\pi^4a^4b^2+256\,\pi^2a^6b^2+256\,a^8b^2)x^4+8\,(\pi^8ab+16\,\pi^6a^3b+96\,\pi^4a^5b+256\,\pi^2a^7b+256\,a^9b)x^3+(\pi^{10}+20\,\pi^8a^2+160\,\pi^6a^4+640\,\pi^4a^6+1280\,\pi^2a^8+1024\,a^{10})x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2*(pi^8 + 8*pi^6*a^2 - 128*pi^2*a^6 - 256*a^8 - 96*(3*pi^4*a*b^3 + 8*pi^2*a
^3*b^3 - 16*a^5*b^3)*x^3 + 12*(pi^6*b^2 - 44*pi^4*a^2*b^2 - 144*pi^2*a^4*b^
2 + 192*a^6*b^2)*x^2 - 8*(5*pi^6*a*b + 36*pi^4*a^3*b + 48*pi^2*a^5*b - 64*a
^7*b)*x + 384*(4*(pi^3*a*b^4 - 4*pi*a^3*b^4)*x^4 + 8*(pi^3*a^2*b^3 - 4*pi*a
^4*b^3)*x^3 + (pi^5*a*b^2 - 16*pi*a^5*b^2)*x^2)*arctan(-(2*b*x + 2*a - sqrt
(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - 12*(4*(pi^4*b^4 - 24*pi^2*a^2*b
^4 + 16*a^4*b^4)*x^4 + 8*(pi^4*a*b^3 - 24*pi^2*a^3*b^3 + 16*a^5*b^3)*x^3 +
(pi^6*b^2 - 20*pi^4*a^2*b^2 - 80*pi^2*a^4*b^2 + 64*a^6*b^2)*x^2)*log(4*b^2*
x^2 + 8*a*b*x + pi^2 + 4*a^2) + 24*(4*(pi^4*b^4 - 24*pi^2*a^2*b^4 + 16*a^4*
b^4)*x^4 + 8*(pi^4*a*b^3 - 24*pi^2*a^3*b^3 + 16*a^5*b^3)*x^3 + (pi^6*b^2 -
20*pi^4*a^2*b^2 - 80*pi^2*a^4*b^2 + 64*a^6*b^2)*x^2)*log(x))/(4*(pi^8*b^2 +
```

16*pi^6*a^2*b^2 + 96*pi^4*a^4*b^2 + 256*pi^2*a^6*b^2 + 256*a^8*b^2)*x^4 + 8*(pi^8*a*b + 16*pi^6*a^3*b + 96*pi^4*a^5*b + 256*pi^2*a^7*b + 256*a^9*b)*x^3 + (pi^10 + 20*pi^8*a^2 + 160*pi^6*a^4 + 640*pi^4*a^6 + 1280*pi^2*a^8 + 1024*a^10)*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{acoth}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 203, normalized size = 1.42

$$-\frac{48\,b^2\log(i\pi+2\,bx+2\,a)}{\pi^4-8i\,\pi^3 a-24\,\pi^2 a^2+32i\,\pi a^3+16\,a^4}+\frac{48\,b^2\log(x)}{\pi^4-8i\,\pi^3 a-24\,\pi^2 a^2+32i\,\pi a^3+16\,a^4}+\frac{16\,b^2}{-2i\,\pi^3 bx-12\,\pi^2 abx+24i\,\pi a^2 bx+16\,a^3 bx+\pi^4-8i\,\pi^3 a-24\,\pi^2 a^2+32i\,\pi a^3+16\,a^4}-\frac{4\,(i\pi-8\,bx+2\,a)}{-2i\,\pi^3 x^2-12\,\pi^2 ax^2+24i\,\pi a^2 x^2+16\,a^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -48*b^2*log(I*pi + 2*b*x + 2*a)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4) + 16*b^2/(-2*I*pi^3*b*x - 12*pi^2*a*b*x + 24*I*pi*a^2*b*x + 16*a^3*b*x + pi^4 - 8*I*pi^3*a - 24*pi^2*a^2 + 32*I*pi*a^3 + 16*a^4) - 4*(I*pi - 8*b*x + 2*a)/(-2*I*pi^3*x^2 - 12*pi^2*a*x^2 + 24*I*pi*a^2*x^2 + 16*a^3*x^2)
```

Mupad [B]
time = 4.90, size = 689, normalized size = 4.82

$$\frac{2\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^3-2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^3-6\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2+6\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-32\,b^3x^3+24\,bx\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2+24\,bx\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)^2-24\,b^2x^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+24\,b^2x^2\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-b^2x^2\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\mathrm{atan}\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx}\right)96\mathrm{i}-48\,bx\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+b^2x^2\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\mathrm{atan}\left(\frac{\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}-\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)1\mathrm{i}+bx\,2\mathrm{i}}{\ln\left(-\frac{2}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx}\right)96\mathrm{i}}{x^2\left(\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)\right)\left(\ln\left(-\frac{1}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)-\ln\left(\frac{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}}{\mathrm{e}^{2a}\,\mathrm{e}^{2bx}-1}\right)+2\,bx\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*acoth(tanh(a + b*x))^2),x)
```

```
[Out] (2*log(-1/(exp(2*a)*exp(2*b*x) - 1))^3 - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^3 - 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 + 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 32*b^3*x^3 + 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 24*b*x*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 24*b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - b^2*
```

```
x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x)
 - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i - 48*b*x*log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + b
^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*
2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + 2*b*x))*96i)/(x^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2*a)*
exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2
*b*x)^4)
```

3.175 $\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=94

$$-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{mx^{-1+m}\,{}_2F_1\left(1,-1+m;m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -1/2*x^m/b/arccoth(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arccoth(tanh(b*x+a))-1
/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arccoth(tanh(b*x+a))))/b^2/(
b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^m/(b*ArcCoth[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcCoth[Tanh[
a + b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - Arc
Coth[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcCoth[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{m\int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{((1-m)m)\int \frac{}{\text{co}}}{2}$$
$$= -\frac{x^m}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m}\,{}_2F_1\Big(1,}{2b^2\,(bx-}$$

Mathematica [A]
time = 0.44, size = 51, normalized size = 0.54

$$\frac{x^{1+m}\,{}_2F_1\left(3,1+m;2+m;-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcCoth[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcCoth[Tanh[a + b*x]])^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(x^m/arccoth(tanh(b*x+a))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] integral(x^m/arccoth(tanh(b*x + a))^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/acoth(tanh(b*x+a))**3,x)
[Out] Integral(x**m/acoth(tanh(a + b*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
[Out] integrate(x^m/arccoth(tanh(b*x + a))^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/acoth(tanh(a + b*x))^3,x)
[Out] int(x^m/acoth(tanh(a + b*x))^3, x)
```

3.176 $\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{3x^2}{b^3}+\frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))}+\frac{6\left(bx-\coth^{-1}\right.}{}$$

```
[Out] 3*x^2/b^3+6*x*(b*x-arccoth(tanh(b*x+a)))/b^4-1/2*x^4/b/arccoth(tanh(b*x+a))
^2-2*x^3/b^2/arccoth(tanh(b*x+a))+6*(b*x-arccoth(tanh(b*x+a)))^2*ln(arccoth
(tanh(b*x+a)))/b^5
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2199, 2190, 2189, 2188, 29}

$$\frac{6\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{3x^2}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcCoth[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcCoth[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcCoth[Tanh[a + b*x]]) + (6*(b*x - ArcCoth
[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2190

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
```

```
1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{2\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))}+\frac{6\int \frac{x^2}{\coth^{-1}(\tanh(a+bx)}$$

$$= \frac{3x^2}{b^3}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{2x^3}{b^2\coth^{-1}(\tanh(a+bx))}-\frac{(6(-bx+\cot}{}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\coth}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\coth}$$

$$= \frac{3x^2}{b^3}+\frac{6x\left(bx-\coth^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^4}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{}{b^2\coth}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.24

$$\frac{x^2}{2b^3}-\frac{3x\left(-bx+\coth^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{4\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}{b^5\coth^{-1}(\tanh(a+bx))}-\frac{\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}{2b^5\coth^{-1}(\tanh(a+bx))^2}+\frac{6\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] x^2/(2*b^3) - (3*x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + Ar
cCoth[Tanh[a + b*x]])^3)/(b^5*ArcCoth[Tanh[a + b*x]]) - (-(b*x) + ArcCoth[T
```

```
anh[a + b*x]])^4/(2*b^5*ArcCoth[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcCoth[Ta
nh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.45, size = 29456, normalized size = 320.17

method	result	size
risch	Expression too large to display	29456

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.88, size = 198, normalized size = 2.15

$$\frac{16\,b^{4}x^{4}+7\,\pi^{4}+56 i\,\pi^{3}a-168\,\pi^{2}a^{2}-224 i\,\pi a^{3}+112\,a^{4}-32\,(-i\,\pi b^{3}+2\,ab^{3})x^{3}+44\,(\pi^{2}b^{2}+4 i\,\pi ab^{2}-4\,a^{2}b^{2})x^{2}-4\,(-i\,\pi^{3}b+6\,\pi^{2}ab+12 i\,\pi a^{2}b-8\,a^{3}b)x}{8\,(4\,b^{7}x^{2}-\pi^{2}b^{5}-4 i\,\pi ab^{5}+4\,a^{2}b^{5}-4\,(i\,\pi b^{6}-2\,ab^{6})x)}-\frac{3\,(\pi^{2}+4 i\,\pi a-4\,a^{2})\log(-i\,\pi+2\,bx+2\,a)}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*b^4*x^4 + 7*pi^4 + 56*I*pi^3*a - 168*pi^2*a^2 - 224*I*pi*a^3 + 112*
a^4 - 32*(-I*pi*b^3 + 2*a*b^3)*x^3 + 44*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^
2)*x^2 - 4*(-I*pi^3*b + 6*pi^2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x)/(4*b^7*x^2
 - pi^2*b^5 - 4*I*pi*a*b^5 + 4*a^2*b^5 - 4*(I*pi*b^6 - 2*a*b^6)*x) - 3/2*(p
i^2 + 4*I*pi*a - 4*a^2)*log(-I*pi + 2*b*x + 2*a)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(90) = 180.
time = 0.37, size = 493, normalized size = 5.36

$$\frac{64\,b^{6}x^{6}-128\,ab^{5}x^{5}-7\,\pi^{6}-28\,\pi^{4}a^{2}+112\,\pi^{2}a^{4}+448\,a^{6}+32\,(\pi^{2}b^{4}-36\,a^{2}b^{4})x^{4}-512\,(\pi^{2}ab^{3}+3\,a^{3}b^{3})x^{3}-32\,(\pi^{4}b^{2}+32\,\pi^{2}a^{2}b^{2})x^{2}-32\,(5\,\pi^{4}ab+12\,\pi^{2}a^{3}b-32\,a^{5}b)x-96\,(16\,\pi ab^{4}x^{4}+64\,\pi a^{2}b^{3}x^{3}+\pi^{5}a+8\,\pi^{3}a^{3}+16\,\pi a^{5}+8\,(\pi^{3}ab^{2}+12\,\pi a^{3}b^{2})x^{2}+16\,(\pi^{3}a^{2}b+4\,\pi a^{4}b)x)\arctan\left(-\frac{2bx+2a-\sqrt{4\,b^{2}x^{2}+8\,abx+\pi^{2}+4\,a^{2}}}{\pi}\right)-6\,(\pi^{6}+4\,\pi^{4}a^{2}-16\,\pi^{2}a^{4}-64\,a^{6}+16\,(\pi^{2}b^{4}-4\,a^{2}b^{4})x^{4}+64\,(\pi^{2}ab^{3}-4\,a^{3}b^{3})x^{3}+8\,(\pi^{4}b^{2}+8\,\pi^{2}a^{2}b^{2}-48\,a^{4}b^{2})x^{2}+16\,(\pi^{4}ab-16\,a^{5}b)x)\log(4\,b^{2}x^{2}+8\,abx+\pi^{2}+4\,a^{2})}{8\,(16\,b^{9}x^{4}+64\,ab^{8}x^{3}+\pi^{4}b^{5}+8\,\pi^{2}a^{2}b^{5}+16\,a^{4}b^{5}+8\,(\pi^{2}b^{7}+12\,a^{2}b^{7})x^{2}+16\,(\pi^{2}ab^{6}+4\,a^{3}b^{6})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(64*b^6*x^6 - 128*a*b^5*x^5 - 7*pi^6 - 28*pi^4*a^2 + 112*pi^2*a^4 + 448
*a^6 + 32*(pi^2*b^4 - 36*a^2*b^4)*x^4 - 512*(pi^2*a*b^3 + 3*a^3*b^3)*x^3 -
32*(pi^4*b^2 + 32*pi^2*a^2*b^2)*x^2 - 32*(5*pi^4*a*b + 12*pi^2*a^3*b - 32*a
^5*b)*x - 96*(16*pi*a*b^4*x^4 + 64*pi*a^2*b^3*x^3 + pi^5*a + 8*pi^3*a^3 + 1
6*pi*a^5 + 8*(pi^3*a*b^2 + 12*pi*a^3*b^2)*x^2 + 16*(pi^3*a^2*b + 4*pi*a^4*b
)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) -
 6*(pi^6 + 4*pi^4*a^2 - 16*pi^2*a^4 - 64*a^6 + 16*(pi^2*b^4 - 4*a^2*b^4)*x^
4 + 64*(pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(pi^4*b^2 + 8*pi^2*a^2*b^2 - 48*a^4
*b^2)*x^2 + 16*(pi^4*a*b - 16*a^5*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*
```

```
a^2))/(16*b^9*x^4 + 64*a*b^8*x^3 + pi^4*b^5 + 8*pi^2*a^2*b^5 + 16*a^4*b^5 +
 8*(pi^2*b^7 + 12*a^2*b^7)*x^2 + 16*(pi^2*a*b^6 + 4*a^3*b^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**4/acoth(tanh(a + b*x))**3, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 163, normalized size = 1.77

$$-\frac{16\,\pi^3 bx - 96i\,\pi^2 abx - 192\,\pi a^2 bx + 128i\,a^3 bx + 7i\,\pi^4 + 56\,\pi^3 a - 168i\,\pi^2 a^2 - 224\,\pi a^3 + 112i\,a^4}{-32i\,b^7 x^2 + 32\,\pi b^6 x - 64i\,ab^6 x + 8i\,\pi^2 b^5 + 32\,\pi ab^5 - 32i\,a^2 b^5} + \frac{x^2}{2\,b^3} - \frac{3\,(i\,\pi + 2\,a)x}{2\,b^4} - \frac{3\,(\pi^2 - 4i\,\pi a - 4\,a^2)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -(16*pi^3*b*x - 96*I*pi^2*a*b*x - 192*pi*a^2*b*x + 128*I*a^3*b*x + 7*I*pi^4
 + 56*pi^3*a - 168*I*pi^2*a^2 - 224*pi*a^3 + 112*I*a^4)/(-32*I*b^7*x^2 + 32
*pi*b^6*x - 64*I*a*b^6*x + 8*I*pi^2*b^5 + 32*pi*a*b^5 - 32*I*a^2*b^5) + 1/2
*x^2/b^3 - 3/2*(I*pi + 2*a)*x/b^4 - 3/2*(pi^2 - 4*I*pi*a - 4*a^2)*log(I*pi
+ 2*b*x + 2*a)/b^5
```

Mupad [B]
time = 1.38, size = 867, normalized size = 9.42

$$\frac{\frac{7\left(\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^4+24a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2+16a^4-8a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^3-32a^3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)\right)}{4b}-x\left(4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^3-32a^3-24a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2+48a^2\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)\right)}{2b^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2+x\left(16ab^5-8b^5\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)\right)+8a^2b^4+8b^6x^2-8ab^4\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)}+\frac{x^2}{2b^3}+\frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)-\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right)\left(3\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)^2-12a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)+2bx\right)+12a^2\right)}{2b^5}+\frac{3x\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)+2bx\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/acoth(tanh(a + b*x))^3,x)
```

```
[Out] ((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2
/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*
b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(
2*b*x) - 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 32
*a^3 - 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 48*a^2*(2*a - log((2*exp(2*
```

```
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(16*a*b^5 - 8*b^
5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 - 8*a*b^4*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*(2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*e
xp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 12*a^2)
)/(2*b^5) + (3*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^4)
```

3.177 $\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=71

$$\frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(\right.}{b^4}$$

```
[Out] 3*x/b^3-1/2*x^3/b/arccoth(tanh(b*x+a))^2-3/2*x^2/b^2/arccoth(tanh(b*x+a))+3
*(b*x-arccoth(tanh(b*x+a)))*ln(arccoth(tanh(b*x+a)))/b^4
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2199, 2189, 2188, 29}

$$\frac{3\left(bx-\coth^{-1}(\tanh(a+bx))\right)\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcCoth[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcCoth[Tan
h[a + b*x]]) + (3*(b*x - ArcCoth[Tanh[a + b*x]])*Log[ArcCoth[Tanh[a + b*x]]
])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$

$$= -\frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x}{\coth^{-1}(\tanh(a+}}{b^2}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{(3(-bx+\mathrm{co}}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} - \frac{(3(-bx+\mathrm{co}}{}$$

$$= \frac{3x}{b^3} - \frac{x^3}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\coth^{-1}(\tanh(a+bx))} + \frac{3(bx-\mathrm{coth}}{}$$

Mathematica [A]
time = 0.05, size = 86, normalized size = 1.21

$$-\frac{b^3x^3 + 3b^2x^2\coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^3\left(5+6\log\left(\coth^{-1}(\tanh(a+bx))\right)\right) - bx\coth^{-1}(\tanh(a+bx))^2\left(11+6\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2b^4\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^3
*(5 + 6*Log[ArcCoth[Tanh[a + b*x]]]) - b*x*ArcCoth[Tanh[a + b*x]]^2*(11 + 6
*Log[ArcCoth[Tanh[a + b*x]]]))/(b^4*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 4977, normalized size = 70.10

method	result	size
risch	Expression too large to display	4977

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*
```

```
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^2+
6*Pi*x^2*csgn(I/(exp(2*b*x+2*a)+1))^3+3*Pi*x^2*csgn(I*exp(b*x+a))^2*csgn(I*
exp(2*b*x+2*a))-6*Pi*x^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+3*Pi*x
^2*csgn(I*exp(2*b*x+2*a))^3-3*Pi*x^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*
x+2*a)/(exp(2*b*x+2*a)+1))^2+3*Pi*x^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3+4*I*x^3*b+6*Pi*x^2+12*I*x^2*ln(exp(b*x+a)))/b^2/(Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-
2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn
(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2+3/b^3*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I
/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+
a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))
^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*
I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*x-3/b^4*ln(Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-
2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn
(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(
2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*ln(exp(b*x+a))+3/4*
I/b^4*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(
exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I
*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)+1))-3/4*I/b^4*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))
^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+
Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b
*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-3/2*I/b^4*ln(Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-P
```

```
i*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I
*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*
b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))^3+4*I*b*x+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+
2*a)+1))^2+3/2*I/b^4*ln(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^
2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*
x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*
x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*b*x+4*I*(ln(exp(b*x
+a))-b*x-a)+4*I*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+3/4*I/b^4*ln(Pi*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+
1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a)
)*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+
2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.86, size = 146, normalized size = 2.06

$$\frac{16\,b^3x^3 - 5i\,\pi^3 + 30\,\pi^2a + 60i\,\pi a^2 - 40\,a^3 - 16\,(i\,\pi b^2 - 2\,ab^2)x^2 + 8\,(\pi^2b + 4i\,\pi ab - 4\,a^2b)x}{4\,(4\,b^6x^2 - \pi^2b^4 - 4i\,\pi ab^4 + 4\,a^2b^4 - 4\,(i\,\pi b^5 - 2\,ab^5)x)} - \frac{3\,(-i\,\pi + 2\,a)\log\left(-i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*b^3*x^3 - 5*I*pi^3 + 30*pi^2*a + 60*I*pi*a^2 - 40*a^3 - 16*(I*pi*b^
2 - 2*a*b^2)*x^2 + 8*(pi^2*b + 4*I*pi*a*b - 4*a^2*b)*x)/(4*b^6*x^2 - pi^2*b
^4 - 4*I*pi*a*b^4 + 4*a^2*b^4 - 4*(I*pi*b^5 - 2*a*b^5)*x) - 3/2*(-I*pi + 2*
a)*log(-I*pi + 2*b*x + 2*a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(67) = 134.
time = 0.37, size = 418, normalized size = 5.89

$$\frac{32\,b^5x^5 + 128\,ab^4x^4 - 5\,\pi^4a - 40\,\pi^2a^3 - 80\,a^5 + 8\,(5\,\pi^2b^3 + 12\,a^2b^3)x^3 + 4\,(11\,\pi^2ab^2 - 36\,a^3b^2)x^2 + 2\,(3\,\pi^4b - 16\,\pi^2a^2b - 112\,a^4b)x + 6\,(16\,\pi b^4x^4 + 64\,\pi ab^3x^3 + \pi^5 + 8\,\pi^3a^2 + 16\,\pi a^4 + 8\,(\pi^3b^2 + 12\,\pi a^2b^2)x^2 + 16\,(\pi^3ab + 4\,\pi a^3b)x)\arctan\left(-\frac{2\,bx + 2\,a - \sqrt{4\,b^2x^2 + 8\,abx + \pi^2 + 4\,a^2}}{\pi}\right) - 3\,(16\,ab^4x^4 + 64\,a^2b^3x^3 + \pi^4a + 8\,\pi^2a^3 + 16\,a^5 + 8\,(\pi^2ab^2 + 12\,a^3b^2)x^2 + 16\,(\pi^2a^2b + 4\,a^4b)x)\log\left(4\,b^2x^2 + 8\,abx + \pi^2 + 4\,a^2\right)}{2\,(16\,b^8x^4 + 64\,ab^7x^3 + \pi^4b^4 + 8\,\pi^2a^2b^4 + 16\,a^4b^4 + 8\,(\pi^2b^6 + 12\,a^2b^6)x^2 + 16\,(\pi^2ab^5 + 4\,a^3b^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(32*b^5*x^5 + 128*a*b^4*x^4 - 5*pi^4*a - 40*pi^2*a^3 - 80*a^5 + 8*(5*pi
^2*b^3 + 12*a^2*b^3)*x^3 + 4*(11*pi^2*a*b^2 - 36*a^3*b^2)*x^2 + 2*(3*pi^4*b
```

```
 - 16*pi^2*a^2*b - 112*a^4*b)*x + 6*(16*pi*b^4*x^4 + 64*pi*a*b^3*x^3 + pi^5
 + 8*pi^3*a^2 + 16*pi*a^4 + 8*(pi^3*b^2 + 12*pi*a^2*b^2)*x^2 + 16*(pi^3*a*b
 + 4*pi*a^3*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 +
4*a^2))/pi) - 3*(16*a*b^4*x^4 + 64*a^2*b^3*x^3 + pi^4*a + 8*pi^2*a^3 + 16*a
^5 + 8*(pi^2*a*b^2 + 12*a^3*b^2)*x^2 + 16*(pi^2*a^2*b + 4*a^4*b)*x)*log(4*b
^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(16*b^8*x^4 + 64*a*b^7*x^3 + pi^4*b^4 + 8
*pi^2*a^2*b^4 + 16*a^4*b^4 + 8*(pi^2*b^6 + 12*a^2*b^6)*x^2 + 16*(pi^2*a*b^5
 + 4*a^3*b^5)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x))**3, x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.44, size = 123, normalized size = 1.73

$$\frac{12\,\pi^2 bx - 48i\,\pi abx - 48\,a^2bx + 5i\,\pi^3 + 30\,\pi^2 a - 60i\,\pi a^2 - 40\,a^3}{4\,(4\,b^6x^2 + 4i\,\pi b^5x + 8\,ab^5x - \pi^2b^4 + 4i\,\pi ab^4 + 4\,a^2b^4)} + \frac{x}{b^3} + \frac{3\,(-i\,\pi - 2\,a)\log\left(i\,\pi + 2\,bx + 2\,a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*pi^2*b*x - 48*I*pi*a*b*x - 48*a^2*b*x + 5*I*pi^3 + 30*pi^2*a - 60*I
*pi*a^2 - 40*a^3)/(4*b^6*x^2 + 4*I*pi*b^5*x + 8*a*b^5*x - pi^2*b^4 + 4*I*pi
*a*b^4 + 4*a^2*b^4) + x/b^3 + 3/2*(-I*pi - 2*a)*log(I*pi + 2*b*x + 2*a)/b^4
```

Mupad [**B**]
time = 1.43, size = 620, normalized size = 8.73

$$\frac{x}{b^3}-\frac{x\,\left(3\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2-12\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)+12\,a^2\right)-\frac{5\,\left({\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}^3-8\,a^3-6\,a\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2+12\,a^2\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)}{4\,b}}{b^3\,{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2+x\,\left(8\,a\,b^4-4\,b^4\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)+4\,a^2\,b^3+4\,b^5\,x^2-4\,a\,b^3\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)\right)\,\left(3\,\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-3\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+6\,b\,x\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/acoth(tanh(a + b*x))^3,x)
```

```
[Out] x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a
```

```
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(ex
p(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
))/(4*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4*b^4*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)
*exp(2*b*x) - 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b
*x) - 1)) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*(3*log(-2/(exp(2*a)*exp(2*b*x)
- 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 6*b*x))/
(2*b^4)
```

3.178 $\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=47

$$-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{x}{b^2\coth^{-1}(\tanh(a+bx))}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}$$

```
[Out] -1/2*x^2/b/arccoth(tanh(b*x+a))^2-x/b^2/arccoth(tanh(b*x+a))+ln(arccoth(tan
h(b*x+a)))/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 29}

$$\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x}{b^2\coth^{-1}(\tanh(a+bx))}-\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^2/(b*ArcCoth[Tanh[a + b*x]]^2) - x/(b^2*ArcCoth[Tanh[a + b*x]]) + Lo
g[ArcCoth[Tanh[a + b*x]]]/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))}}{b^2}$$

$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, c\right.}{}$$

$$= -\frac{x^2}{2b\coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2\coth^{-1}(\tanh(a+bx))} + \frac{\log\left(\coth^{-1}(\tanh(\right.}{b^3}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 1.04

$$\frac{3 - \frac{b^2x^2}{\coth^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\coth^{-1}(\tanh(a+bx))} + 2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] (3 - (b^2*x^2)/ArcCoth[Tanh[a + b*x]]^2 - (2*b*x)/ArcCoth[Tanh[a + b*x]] +
2*Log[ArcCoth[Tanh[a + b*x]]])/(2*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 952, normalized size = 20.26

method	result
risch	$-\frac{4i\left(\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)x-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2x-2\pi\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2x+\right.}{b^2\left(\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi\text{csgn}\left(\frac{i}{e^{2bx+2a}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))*x-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^2*x-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*x+2*Pi*csgn(
I/(exp(2*b*x+2*a)+1))^3*x+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*x-
2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*x+Pi*csgn(I*exp(2*b*x+2*a)
)^3*x-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
*x+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*x+4*I*x*ln(exp(b*x+a))+2*
```

```
Pi*x+2*I*b*x^2)/b^2/(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2
*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2
*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2
*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^
2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2
+1/b^3*ln(ln(exp(b*x+a))-1/4*I*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*csgn(I/(exp(2*b*x+2*a)+1)
)^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-
csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.86, size = 96, normalized size = 2.04

$$-\frac{3\,\pi^2+12i\,\pi a-12\,a^2-8\,(-i\,\pi b+2\,ab)x}{2\,(4\,b^5x^2-\pi^2b^3-4i\,\pi ab^3+4\,a^2b^3-4\,(i\,\pi b^4-2\,ab^4)x)}+\frac{\log\left(-i\,\pi+2\,bx+2\,a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*pi^2 + 12*I*pi*a - 12*a^2 - 8*(-I*pi*b + 2*a*b)*x)/(4*b^5*x^2 - pi^
2*b^3 - 4*I*pi*a*b^3 + 4*a^2*b^3 - 4*(I*pi*b^4 - 2*a*b^4)*x) + log(-I*pi +
2*b*x + 2*a)/b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(45) = 90.
time = 0.38, size = 250, normalized size = 5.32

$$\frac{64\,ab^3x^3+3\,\pi^4+24\,\pi^2a^2+48\,a^4+4\,(5\,\pi^2b^2+44\,a^2b^2)x^2+40\,(\pi^2ab+4\,a^3b)x+(16\,b^4x^4+64\,ab^3x^3+\pi^4+8\,\pi^2a^2+16\,a^4+8\,(\pi^2b^2+12\,a^2b^2)x^2+16\,(\pi^2ab+4\,a^3b)x)\log\left(4\,b^2x^2+8\,abx+\pi^2+4\,a^2\right)}{2\,(16\,b^7x^4+64\,ab^6x^3+\pi^4b^3+8\,\pi^2a^2b^3+16\,a^4b^3+8\,(\pi^2b^5+12\,a^2b^5)x^2+16\,(\pi^2ab^4+4\,a^3b^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(64*a*b^3*x^3 + 3*pi^4 + 24*pi^2*a^2 + 48*a^4 + 4*(5*pi^2*b^2 + 44*a^2*
b^2)*x^2 + 40*(pi^2*a*b + 4*a^3*b)*x + (16*b^4*x^4 + 64*a*b^3*x^3 + pi^4 +
8*pi^2*a^2 + 16*a^4 + 8*(pi^2*b^2 + 12*a^2*b^2)*x^2 + 16*(pi^2*a*b + 4*a^3*
b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(16*b^7*x^4 + 64*a*b^6*x^3 +
 pi^4*b^3 + 8*pi^2*a^2*b^3 + 16*a^4*b^3 + 8*(pi^2*b^5 + 12*a^2*b^5)*x^2 + 1
6*(pi^2*a*b^4 + 4*a^3*b^4)*x)
```

Sympy [A]

time = 33.10, size = 54, normalized size = 1.15

$$\begin{cases} -\frac{x^2}{2b\,\mathrm{acoth}^2\,(\tanh\,(a+bx))} - \frac{x}{b^2\,\mathrm{acoth}\,(\tanh\,(a+bx))} + \frac{\log\,(\mathrm{acoth}\,(\tanh\,(a+bx)))}{b^3} & \text{for } b \neq 0 \\ \frac{x^3}{3\,\mathrm{acoth}^3\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*x
))) + log(acoth(tanh(a + b*x)))/b**3, Ne(b, 0)), (x**3/(3*acoth(tanh(a))**3
), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 91, normalized size = 1.94

$$\frac{8\,\pi bx - 16i\,abx + 3i\,\pi^2 + 12\,\pi a - 12i\,a^2}{-8i\,b^5x^2 + 8\,\pi b^4x - 16i\,ab^4x + 2i\,\pi^2b^3 + 8\,\pi ab^3 - 8i\,a^2b^3} + \frac{\log\,(i\,\pi + 2\,bx + 2\,a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] (8*pi*b*x - 16*I*a*b*x + 3*I*pi^2 + 12*pi*a - 12*I*a^2)/(-8*I*b^5*x^2 + 8*p
i*b^4*x - 16*I*a*b^4*x + 2*I*pi^2*b^3 + 8*pi*a*b^3 - 8*I*a^2*b^3) + log(I*p
i + 2*b*x + 2*a)/b^3
```

Mupad [B]
time = 1.22, size = 46, normalized size = 0.98

$$\frac{\ln\left(\mathrm{acoth}(\tanh(a+b\,x))\right)}{b^3} - \frac{\frac{b^2\,x^2}{2} + b\,x\,\mathrm{acoth}(\tanh(a+b\,x))}{b^3\,\mathrm{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/acoth(tanh(a + b*x))^3,x)
```

```
[Out] log(acoth(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*acoth(tanh(a + b*x)))/(b
^3*acoth(tanh(a + b*x))^2)
```

3.179 $\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=34

$$-\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -1/2*x/b/arccoth(tanh(b*x+a))^2-1/2/b^2/arccoth(tanh(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2199, 2188, 30}

$$-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}-\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcCoth[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x/(b*ArcCoth[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcCoth[Tanh[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\coth^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2}\,dx}{2b}\\ &= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}+\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx,x,\coth^{-1}(\tanh(a+bx))\right)}{2b^2}\\ &= -\frac{x}{2b\coth^{-1}(\tanh(a+bx))^2}-\frac{1}{2b^2\coth^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.79

$$-\frac{bx+\coth^{-1}(\tanh(a+bx))}{2b^2\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcCoth[Tanh[a + b*x]]^3,x]
[Out] -1/2*(b*x + ArcCoth[Tanh[a + b*x]])/(b^2*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 634, normalized size = 18.65

method	result
risch	$-\frac{2i\left(\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2+2\pi\text{cs}\ldots\right.}{b^2\left(\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(ie^{2bx+2a}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi\,\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi\text{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)^2+2\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
[Out] -2*I*(Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp
(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(
I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*b*x+4*I*ln(exp(b*x+a)))/b^2/(Pi*
csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))^2-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*
a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x
+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b
```

```
*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.87, size = 63, normalized size = 1.85

$$-\frac{-i\,\pi+4\,bx+2\,a}{4\,b^4x^2-\pi^2b^2-4i\,\pi ab^2+4\,a^2b^2-4\,(i\,\pi b^3-2\,ab^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -(-I*pi + 4*b*x + 2*a)/(4*b^4*x^2 - pi^2*b^2 - 4*I*pi*a*b^2 + 4*a^2*b^2 - 4
*(I*pi*b^3 - 2*a*b^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(30) = 60$.
time = 0.38, size = 124, normalized size = 3.65

$$-\frac{2\,(8\,b^3x^3+20\,ab^2x^2+16\,a^2bx+\pi^2a+4\,a^3)}{16\,b^6x^4+64\,ab^5x^3+\pi^4b^2+8\,\pi^2a^2b^2+16\,a^4b^2+8\,(\pi^2b^4+12\,a^2b^4)x^2+16\,(\pi^2ab^3+4\,a^3b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -2*(8*b^3*x^3 + 20*a*b^2*x^2 + 16*a^2*b*x + pi^2*a + 4*a^3)/(16*b^6*x^4 + 6
4*a*b^5*x^3 + pi^4*b^2 + 8*pi^2*a^2*b^2 + 16*a^4*b^2 + 8*(pi^2*b^4 + 12*a^2
*b^4)*x^2 + 16*(pi^2*a*b^3 + 4*a^3*b^3)*x)
```

Sympy [A]
time = 33.62, size = 42, normalized size = 1.24

$$\begin{cases} -\frac{x}{2b\operatorname{acoth}^2(\tanh(a+bx))}-\frac{1}{2b^2\operatorname{acoth}(\tanh(a+bx))} & \text{for}\ b\neq 0\\ \frac{x^2}{2\operatorname{acoth}^3(\tanh(a))} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-x/(2*b*acoth(tanh(a + b*x))**2) - 1/(2*b**2*acoth(tanh(a + b*x)
)), Ne(b, 0)), (x**2/(2*acoth(tanh(a))**3), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 61, normalized size = 1.79

$$-\frac{i\,\pi+4\,bx+2\,a}{4\,b^4x^2+4i\,\pi b^3x+8\,ab^3x-\pi^2b^2+4i\,\pi ab^2+4\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -(I*pi + 4*b*x + 2*a)/(4*b^4*x^2 + 4*I*pi*b^3*x + 8*a*b^3*x - pi^2*b^2 + 4*
I*pi*a*b^2 + 4*a^2*b^2)
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.74

$$-\frac{\operatorname{acoth}(\tanh(a+b\,x))+b\,x}{2\,b^2\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/acoth(tanh(a + b*x))^3,x)
```

```
[Out] -(acoth(tanh(a + b*x)) + b*x)/(2*b^2*acoth(tanh(a + b*x))^2)
```

3.180 $\int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2/b/arccoth(tanh(b*x+a))^2
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2188, 30}

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} \, dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{2b\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/2*1/(b*ArcCoth[Tanh[a + b*x]]^2)
```

Maple [A]
time = 0.19, size = 15, normalized size = 0.94

method	result
derivativedivides	$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$
default	$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$
risch	$\frac{}{b\left(\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(i e^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)-\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{i e^{2bx+2a}}{e^{2bx+2a}+1}\right)^2-2\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccoth(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/arccoth(tanh(b*x+a))^2
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 28, normalized size = 1.75

$$\frac{2}{\left(\pi^2 - 4i\,\pi(bx+a) - 4\,(bx+a)^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 2/((pi^2 - 4*I*pi*(b*x + a) - 4*(b*x + a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(14) = 28.
time = 0.36, size = 107, normalized size = 6.69

$$-\frac{2\,(4\,b^2x^2 + 8\,abx - \pi^2 + 4\,a^2)}{16\,b^5x^4 + 64\,ab^4x^3 + \pi^4b + 8\,\pi^2a^2b + 16\,a^4b + 8\,(\pi^2b^3 + 12\,a^2b^3)x^2 + 16\,(\pi^2ab^2 + 4\,a^3b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*b^2*x^2 + 8*a*b*x - pi^2 + 4*a^2)/(16*b^5*x^4 + 64*a*b^4*x^3 + pi^4*b
 + 8*pi^2*a^2*b + 16*a^4*b + 8*(pi^2*b^3 + 12*a^2*b^3)*x^2 + 16*(pi^2*a*b^2
 + 4*a^3*b^2)*x)
```

Sympy [A]
time = 22.44, size = 24, normalized size = 1.50

$$\begin{cases} -\frac{1}{2b\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)} & \text{for } b \neq 0 \\ \frac{x}{\operatorname{acoth}^3\left(\tanh\left(a\right)\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((-1/(2*b*acoth(tanh(a + b*x))**2), Ne(b, 0)), (x/acoth(tanh(a))**
3, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.38, size = 44, normalized size = 2.75

$$-\frac{2i}{4i\,b^3x^2 - 4\,\pi b^2x + 8i\,ab^2x - i\,\pi^2b - 4\,\pi ab + 4i\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -2*I/(4*I*b^3*x^2 - 4*pi*b^2*x + 8*I*a*b^2*x - I*pi^2*b - 4*pi*a*b + 4*I*a^
2*b)
```

Mupad [B]
time = 0.07, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\operatorname{acoth}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/acoth(tanh(a + b*x))^3,x)
```

```
[Out] -1/(2*b*acoth(tanh(a + b*x))^2)
```

3.181 $\int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=97

$$-\frac{1}{2 \left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}+\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+b$$

```
[Out] -1/2/(b*x-arccoth(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+1/(b*x-arccoth(tanh(
b*x+a)))^2/arccoth(tanh(b*x+a))-ln(x)/(b*x-arccoth(tanh(b*x+a)))^3+ln(arcco
th(tanh(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^3
```

Rubi [A]

time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2194, 2191, 2188, 29}

$$\frac{1}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))}-\frac{1}{2 \left(bx-\coth^{-1}(\tanh(a+bx))\right) \coth^{-1}(\tanh(a+bx))^2}-\frac{\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}+\frac{\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/2*1/((b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + 1/((b*x
- ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]) - Log[x]/(b*x - ArcCoth
[Tanh[a + b*x]])^3 + Log[ArcCoth[Tanh[a + b*x]]]/(b*x - ArcCoth[Tanh[a + b*
x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
```

```
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\coth^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{\int \frac{1}{x\coth^{-1}(\tanh(}}{bx-\coth^{-1}(\tan}\\ &= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{}{\left(bx-\coth^{-1}(\mathrm{ta}\right.}\\ &= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{}{\left(bx-\coth^{-1}(\mathrm{ta}\right.}\\ &= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{}{\left(bx-\coth^{-1}(\mathrm{ta}\right.}\\ &= -\frac{1}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{}{\left(bx-\coth^{-1}(\mathrm{ta}\right.}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 0.76

$$\frac{b^2x^2-4bx\coth^{-1}(\tanh(a+bx))+\coth^{-1}(\tanh(a+bx))^2\left(3+2\log(bx)-2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2\coth^{-1}(\tanh(a+bx))^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcCoth[Tanh[a + b*x]]^3),x]
[Out] (b^2*x^2 - 4*b*x*ArcCoth[Tanh[a + b*x]] + ArcCoth[Tanh[a + b*x]]^2*(3 + 2*L
og[b*x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(2*ArcCoth[Tanh[a + b*x]]^2*(-(b*
x) + ArcCoth[Tanh[a + b*x]])^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.47, size = 27876, normalized size = 287.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a))^3,x)
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.88, size = 171, normalized size = 1.76

$$\frac{4\,(-3i\,\pi+4\,bx+6\,a)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4-4\,(\pi^2b^2+4i\,\pi ab^2-4\,a^2b^2)x^2-4\,(-i\,\pi^3b+6\,\pi^2ab+12i\,\pi a^2b-8\,a^3b)x}+\frac{8\,\log\left(-i\,\pi+2\,bx+2\,a\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}-\frac{8\,\log\left(x\right)}{-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 4*(-3*I*pi + 4*b*x + 6*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 +
16*a^4 - 4*(pi^2*b^2 + 4*I*pi*a*b^2 - 4*a^2*b^2)*x^2 - 4*(-I*pi^3*b + 6*pi^
2*a*b + 12*I*pi*a^2*b - 8*a^3*b)*x) + 8*log(-I*pi + 2*b*x + 2*a)/(-I*pi^3 +
 6*pi^2*a + 12*I*pi*a^2 - 8*a^3) - 8*log(x)/(-I*pi^3 + 6*pi^2*a + 12*I*pi*a
^2 - 8*a^3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(95) = 190$.
time = 0.38, size = 811, normalized size = 8.36

$$-\frac{8\left(9\pi^6 a+60\pi^4 a^3+48\pi^2 a^5-192 a^7+8(\pi^4 b^3-16 a^4 b^3)x^3+4(9\pi^4 a b^2+8\pi^2 a^3 b^2-112 a^5 b^2)x^2+4(\pi^6 b+16\pi^4 a^2 b+16\pi^2 a^4 b-128 a^6 b)x-2(\pi^7-4\pi^5 a^2-80\pi^3 a^4-192\pi a^6+16(\pi^3 b^4-12\pi a^2 b^4)x^4+64(\pi^3 a b^3-12\pi a^3 b^3)x^3+8(\pi^5 b^2-144\pi a^4 b^2)x^2+16(\pi^5 a b-8\pi^3 a^3 b-48\pi a^5 b)x)\arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)-(3\pi^6 a+20\pi^4 a^3+16\pi^2 a^5-64 a^7+16(3\pi^2 a b^4-4 a^3 b^4)x^4+64(3\pi^2 a^2 b^3-4 a^4 b^3)x^3+8(3\pi^4 a b^2+32\pi^2 a^3 b^2-48 a^5 b^2)x^2+16(3\pi^4 a^2 b+8\pi^2 a^4 b-16 a^6 b)x)\log(4b^2x^2+8abx+\pi^2+4a^2)+2(3\pi^6 a+20\pi^4 a^3+16\pi^2 a^5-64 a^7+16(3\pi^2 a b^4-4 a^3 b^4)x^4+64(3\pi^2 a^2 b^3-4 a^4 b^3)x^3+8(3\pi^4 a b^2+32\pi^2 a^3 b^2-48 a^5 b^2)x^2+16(3\pi^4 a^2 b+8\pi^2 a^4 b-16 a^6 b)x)\log(x)\right)}{\pi^{10}+20\pi^8 a^2+160\pi^6 a^4+640\pi^4 a^6+1280\pi^2 a^8+1024 a^{10}+16(\pi^6 b^4+12\pi^4 a^2 b^4+48\pi^2 a^4 b^4+64 a^6 b^4)x^4+64(\pi^6 a b^3+12\pi^4 a^3 b^3+48\pi^2 a^5 b^3+64 a^7 b^3)x^3+8(\pi^8 b^2+24\pi^6 a^2 b^2+192\pi^4 a^4 b^2+640\pi^2 a^6 b^2+768 a^8 b^2)x^2+16(\pi^8 a b+16\pi^6 a^3 b+96\pi^4 a^5 b+256\pi^2 a^7 b+256 a^9 b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -8*(9*pi^6*a + 60*pi^4*a^3 + 48*pi^2*a^5 - 192*a^7 + 8*(pi^4*b^3 - 16*a^4*b
^3)*x^3 + 4*(9*pi^4*a*b^2 + 8*pi^2*a^3*b^2 - 112*a^5*b^2)*x^2 + 4*(pi^6*b +
 16*pi^4*a^2*b + 16*pi^2*a^4*b - 128*a^6*b)*x - 2*(pi^7 - 4*pi^5*a^2 - 80*p
i^3*a^4 - 192*pi*a^6 + 16*(pi^3*b^4 - 12*pi*a^2*b^4)*x^4 + 64*(pi^3*a*b^3 -
 12*pi*a^3*b^3)*x^3 + 8*(pi^5*b^2 - 144*pi*a^4*b^2)*x^2 + 16*(pi^5*a*b - 8*
pi^3*a^3*b - 48*pi*a^5*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*
x + pi^2 + 4*a^2))/pi) - (3*pi^6*a + 20*pi^4*a^3 + 16*pi^2*a^5 - 64*a^7 + 1
6*(3*pi^2*a*b^4 - 4*a^3*b^4)*x^4 + 64*(3*pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*
(3*pi^4*a*b^2 + 32*pi^2*a^3*b^2 - 48*a^5*b^2)*x^2 + 16*(3*pi^4*a^2*b + 8*pi
^2*a^4*b - 16*a^6*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(3*pi^6
*a + 20*pi^4*a^3 + 16*pi^2*a^5 - 64*a^7 + 16*(3*pi^2*a*b^4 - 4*a^3*b^4)*x^4
 + 64*(3*pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*(3*pi^4*a*b^2 + 32*pi^2*a^3*b^2
- 48*a^5*b^2)*x^2 + 16*(3*pi^4*a^2*b + 8*pi^2*a^4*b - 16*a^6*b)*x)*log(x))/
(pi^10 + 20*pi^8*a^2 + 160*pi^6*a^4 + 640*pi^4*a^6 + 1280*pi^2*a^8 + 1024*a
^10 + 16*(pi^6*b^4 + 12*pi^4*a^2*b^4 + 48*pi^2*a^4*b^4 + 64*a^6*b^4)*x^4 +
64*(pi^6*a*b^3 + 12*pi^4*a^3*b^3 + 48*pi^2*a^5*b^3 + 64*a^7*b^3)*x^3 + 8*(p
i^8*b^2 + 24*pi^6*a^2*b^2 + 192*pi^4*a^4*b^2 + 640*pi^2*a^6*b^2 + 768*a^8*b
^2)*x^2 + 16*(pi^8*a*b + 16*pi^6*a^3*b + 96*pi^4*a^5*b + 256*pi^2*a^7*b + 2
56*a^9*b)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acoth}^{3}\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))**3), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 173, normalized size = 1.78

$$\frac{4\,(-3i\,\pi-4\,bx-6\,a)}{4\,\pi^2b^2x^2-16i\,\pi ab^2x^2-16\,a^2b^2x^2+4i\,\pi^3bx+24\,\pi^2abx-48i\,\pi a^2bx-32\,a^3bx-\pi^4+8i\,\pi^3a+24\,\pi^2a^2-32i\,\pi a^3-16\,a^4}-\frac{8i\,\log\left(i\,\pi+2\,bx+2\,a\right)}{\pi^3-6i\,\pi^2a-12\,\pi a^2+8i\,a^3}+\frac{8i\,\log\left(x\right)}{\pi^3-6i\,\pi^2a-12\,\pi a^2+8i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 4*(-3*I*pi - 4*b*x - 6*a)/(4*pi^2*b^2*x^2 - 16*I*pi*a*b^2*x^2 - 16*a^2*b^2*
x^2 + 4*I*pi^3*b*x + 24*pi^2*a*b*x - 48*I*pi*a^2*b*x - 32*a^3*b*x - pi^4 +
8*I*pi^3*a + 24*pi^2*a^2 - 32*I*pi*a^3 - 16*a^4) - 8*I*log(I*pi + 2*b*x + 2
*a)/(pi^3 - 6*I*pi^2*a - 12*pi*a^2 + 8*I*a^3) + 8*I*log(x)/(pi^3 - 6*I*pi^2
*a - 12*pi*a^2 + 8*I*a^3)
```

Mupad [B]
time = 6.49, size = 902, normalized size = 9.30

$$-\frac{16\,\mathrm{atanh}\left(\frac{16\,\left(4\,b\,x-\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3-8\,a^3-6\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2+12\,a^2\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)+4\,a^2}\right)\,\left(\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{16}-\frac{a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{4}+\frac{a^2}{4}\right)}{\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}\right)}{\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}-\frac{\frac{12}{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x}-\frac{16\,b\,x}{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)+4\,a^2}}{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)+4\,a^2+x\,\left(8\,a\,b-4\,b\,\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\right)+4\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acoth(tanh(a + b*x))^3),x)
```

```
[Out] - (16*atanh((16*(4*b*x - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/
((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(ex
p(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) +
4*a^2))*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/16 - (a*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))/4 + a^2/4))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3))/(log(-2/(exp(2*a)*exp(2*
b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x
)^3 - (12/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) - (16*b*x)/((2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*
b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
+ log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))/((2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) -
```

```
1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2 + x*(8*a*b - 4
*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + 4*b^2*x^2)
```

3.182 $\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=131

$$-\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+b$$

```
[Out] -3/2*b/(b*x-arccoth(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/x/(b*x-arccoth
(tanh(b*x+a)))/arccoth(tanh(b*x+a))^2+3*b/(b*x-arccoth(tanh(b*x+a)))^3/arcc
oth(tanh(b*x+a))-3*b*ln(x)/(b*x-arccoth(tanh(b*x+a)))^4+3*b*ln(arccoth(tanh
(b*x+a)))/(b*x-arccoth(tanh(b*x+a)))^4
```

Rubi [A]
time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$\frac{3b}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2}+\frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}-\frac{3b\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}+\frac{3b\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b)/(2*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcC
oth[Tanh[a + b*x]])^4 + (3*b*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCoth[Ta
nh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
```

```
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3}\,dx &= \frac{1}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} - \frac{(3b)\int \frac{}{x\coth^{-1}(t}}{-bx+\coth^{-1}(} \\
&= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth\right.} \\
&= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth\right.} \\
&= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth\right.} \\
&= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth\right.} \\
&= -\frac{3b}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth\right.}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 0.71

$$-\frac{b^3x^3-6b^2x^2\coth^{-1}(\tanh(a+bx))+2\coth^{-1}(\tanh(a+bx))^3+3bx\coth^{-1}(\tanh(a+bx))^2\left(1+2\log(x)-2\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2x\coth^{-1}(\tanh(a+bx))^2\left(-bx+\coth^{-1}(\tanh(a+bx))\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/2*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]
^3 + 3*b*x*ArcCoth[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a +
b*x]]]))/(x*ArcCoth[Tanh[a + b*x]]^2*(-(b*x) + ArcCoth[Tanh[a + b*x]])^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\text{arccoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(1/x^2/arccoth(tanh(b*x+a))^3,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.89, size = 245, normalized size = 1.87

$$\frac{48\,b\log\left(-i\,\pi+2\,bx+2\,a\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}-\frac{48\,b\log\left(x\right)}{\pi^4+8i\,\pi^3a-24\,\pi^2a^2-32i\,\pi a^3+16\,a^4}-\frac{8\left(12\,b^2x^2-\pi^2-4i\,\pi a+4\,a^2-9\left(i\,\pi b-2\,ab\right)x\right)}{4\left(i\,\pi^3b^2-6\,\pi^2ab^2-12i\,\pi a^2b^2+8\,a^3b^2\right)x^3+4\left(\pi^4b+8i\,\pi^3ab-24\,\pi^2a^2b-32i\,\pi a^3b+16\,a^4b\right)x^2-\left(i\,\pi^5-10\,\pi^4a-40i\,\pi^3a^2+80\,\pi^2a^3+80i\,\pi a^4-32\,a^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 48*b*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^
3 + 16*a^4) - 48*b*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 +
16*a^4) - 8*(12*b^2*x^2 - pi^2 - 4*I*pi*a + 4*a^2 - 9*(I*pi*b - 2*a*b)*x)/(
4*(I*pi^3*b^2 - 6*pi^2*a*b^2 - 12*I*pi*a^2*b^2 + 8*a^3*b^2)*x^3 + 4*(pi^4*b
 + 8*I*pi^3*a*b - 24*pi^2*a^2*b - 32*I*pi*a^3*b + 16*a^4*b)*x^2 - (I*pi^5 -
 10*pi^4*a - 40*I*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(129) = 258.
time = 0.39, size = 1078, normalized size = 8.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 8*(6*pi^8*a + 64*pi^6*a^3 + 192*pi^4*a^5 - 512*a^9 + 96*(3*pi^4*a*b^4 + 8*p
i^2*a^3*b^4 - 16*a^5*b^4)*x^4 - 12*(pi^6*b^3 - 92*pi^4*a^2*b^3 - 272*pi^2*a
^4*b^3 + 448*a^6*b^3)*x^3 + 8*(11*pi^6*a*b^2 + 228*pi^4*a^3*b^2 + 528*pi^2*
a^5*b^2 - 832*a^7*b^2)*x^2 - (5*pi^8*b - 176*pi^6*a^2*b - 1440*pi^4*a^4*b -
 1792*pi^2*a^6*b + 3328*a^8*b)*x - 96*(16*(pi^3*a*b^5 - 4*pi*a^3*b^5)*x^5 +
 64*(pi^3*a^2*b^4 - 4*pi*a^4*b^4)*x^4 + 8*(pi^5*a*b^3 + 8*pi^3*a^3*b^3 - 48
*pi*a^5*b^3)*x^3 + 16*(pi^5*a^2*b^2 - 16*pi*a^6*b^2)*x^2 + (pi^7*a*b + 4*pi
^5*a^3*b - 16*pi^3*a^5*b - 64*pi*a^7*b)*x)*arctan(-(2*b*x + 2*a - sqrt(4*b^
2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) + 3*(16*(pi^4*b^5 - 24*pi^2*a^2*b^5 +
16*a^4*b^5)*x^5 + 64*(pi^4*a*b^4 - 24*pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(p
```

```
i^6*b^3 - 12*pi^4*a^2*b^3 - 272*pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(pi^6*
a*b^2 - 20*pi^4*a^3*b^2 - 80*pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (pi^8*b - 16*
pi^6*a^2*b - 160*pi^4*a^4*b - 256*pi^2*a^6*b + 256*a^8*b)*x)*log(4*b^2*x^2
+ 8*a*b*x + pi^2 + 4*a^2) - 6*(16*(pi^4*b^5 - 24*pi^2*a^2*b^5 + 16*a^4*b^5)
*x^5 + 64*(pi^4*a*b^4 - 24*pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(pi^6*b^3 - 1
2*pi^4*a^2*b^3 - 272*pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(pi^6*a*b^2 - 20*
pi^4*a^3*b^2 - 80*pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (pi^8*b - 16*pi^6*a^2*b
- 160*pi^4*a^4*b - 256*pi^2*a^6*b + 256*a^8*b)*x)*log(x))/(16*(pi^8*b^4 + 1
6*pi^6*a^2*b^4 + 96*pi^4*a^4*b^4 + 256*pi^2*a^6*b^4 + 256*a^8*b^4)*x^5 + 64
*(pi^8*a*b^3 + 16*pi^6*a^3*b^3 + 96*pi^4*a^5*b^3 + 256*pi^2*a^7*b^3 + 256*a
^9*b^3)*x^4 + 8*(pi^10*b^2 + 28*pi^8*a^2*b^2 + 288*pi^6*a^4*b^2 + 1408*pi^4
*a^6*b^2 + 3328*pi^2*a^8*b^2 + 3072*a^10*b^2)*x^3 + 16*(pi^10*a*b + 20*pi^8
*a^3*b + 160*pi^6*a^5*b + 640*pi^4*a^7*b + 1280*pi^2*a^9*b + 1024*a^11*b)*x
^2 + (pi^12 + 24*pi^10*a^2 + 240*pi^8*a^4 + 1280*pi^6*a^6 + 3840*pi^4*a^8 +
 6144*pi^2*a^10 + 4096*a^12)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acoth}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))**3), x)
```

Giac [**C**] Result contains complex when optimal does not.
time = 0.42, size = 262, normalized size = 2.00

$$\frac{48i\,b\log\left(i\,\pi+2\,bx+2\,a\right)}{i\,\pi^4+8\,\pi^3a-24i\,\pi^2a^2-32\,\pi a^3+16i\,a^4}-\frac{48i\,b\log\left(x\right)}{i\,\pi^4+8\,\pi^3a-24i\,\pi^2a^2-32\,\pi a^3+16i\,a^4}+\frac{16\left(8\,b^2x+5i\,\pi b+10\,ab\right)}{8i\,\pi^3b^2x^2+48\,\pi^2ab^2x^2-96i\,\pi a^2b^2x^2-64\,a^3b^2x^2-8\,\pi^4bx+64i\,\pi^3abx+192\,\pi^2a^2bx-256i\,\pi a^3bx-128\,a^4bx-2i\,\pi^5-20\,\pi^4a+80i\,\pi^3a^2+160\,\pi^2a^3-160i\,\pi a^4-64\,a^5}+\frac{8}{i\,\pi^3x+6\,\pi^2ax-12i\,\pi a^2x-8\,a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 48*I*b*log(I*pi + 2*b*x + 2*a)/(I*pi^4 + 8*pi^3*a - 24*I*pi^2*a^2 - 32*pi*a
^3 + 16*I*a^4) - 48*I*b*log(x)/(I*pi^4 + 8*pi^3*a - 24*I*pi^2*a^2 - 32*pi*a
^3 + 16*I*a^4) + 16*(8*b^2*x + 5*I*pi*b + 10*a*b)/(8*I*pi^3*b^2*x^2 + 48*pi
^2*a*b^2*x^2 - 96*I*pi*a^2*b^2*x^2 - 64*a^3*b^2*x^2 - 8*pi^4*b*x + 64*I*pi^
3*a*b*x + 192*pi^2*a^2*b*x - 256*I*pi*a^3*b*x - 128*a^4*b*x - 2*I*pi^5 - 20
*pi^4*a + 80*I*pi^3*a^2 + 160*pi^2*a^3 - 160*I*pi*a^4 - 64*a^5) + 8/(I*pi^3
*x + 6*pi^2*a*x - 12*I*pi*a^2*x - 8*a^3*x)
```

Mupad [**B**]
time = 5.26, size = 1074, normalized size = 8.20

$$\frac{\frac{8}{\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x}-\frac{72\,b\,x}{{\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)+4\,a^2}+\frac{96\,b^2\,x^2}{\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)\,\left({\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)+4\,a^2\right)}}{x\,\left({\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)+4\,a^2\right)+x^2\,\left(8\,a\,b-4\,b\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)\right)+4\,b^2\,x^3}+\frac{96\,b\,\mathrm{atanh}\left(\frac{{\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^4+24\,a^2\,{\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^2+16\,a^4-8\,a\,{\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^3-32\,a^3\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}{{\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^4}-\frac{4\,b\,x\,\left({\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^2-4\,a\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)+4\,a^2\right)}{{\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^3}\right)}{{\left(\ln\left(-\frac{2}{e^{2a}\,e^{2bx}-1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}-1}\right)+2\,b\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acoth(tanh(a + b*x))^3),x)
```

```
[Out] (8/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 2*b*x) - (72*b*x)/((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2
- 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + (96*b^2*x^2)/((log(-2/(exp
(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x) + 4*a^2)))/(x*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*ex
p(2*b*x) - 1)) + 2*b*x) + 4*a^2) + x^2*(8*a*b - 4*b*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1))
+ 2*b*x)) + 4*b^2*x^3) + (96*b*atanh(((2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 2
4*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(log(-2/(exp(2*a)*e
xp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^4 - (4*b*x*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x)
 - 1)) + 2*b*x) + 4*a^2))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3))/(log(-2/(exp(2*a)*e
xp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^4
```

3.183 $\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=170

$$-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2}+\frac{2b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+$$

```
[Out] -3*b^2/(b*x-arccoth(tanh(b*x+a)))^3/arccoth(tanh(b*x+a))^2+2*b/x/(b*x-arcco
th(tanh(b*x+a)))^2/arccoth(tanh(b*x+a))^2+1/2/x^2/(b*x-arccoth(tanh(b*x+a))
)/arccoth(tanh(b*x+a))^2+6*b^2/(b*x-arccoth(tanh(b*x+a)))^4/arccoth(tanh(b*
x+a))-6*b^2*ln(x)/(b*x-arccoth(tanh(b*x+a)))^5+6*b^2*ln(arccoth(tanh(b*x+a)
))/(b*x-arccoth(tanh(b*x+a)))^5
```

Rubi [A]
time = 0.09, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2202, 2194, 2191, 2188, 29}

$$\frac{6b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^4 \coth^{-1}(\tanh(a+bx))}-\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3 \coth^{-1}(\tanh(a+bx))^2}-\frac{6b^2\log(x)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5}+\frac{6b^2\log\left(\coth^{-1}(\tanh(a+bx))\right)}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5}+\frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2}+\frac{2b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2 \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b^2)/((b*x - ArcCoth[Tanh[a + b*x]])^3*ArcCoth[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcCoth[Tanh[a + b*x]])^2*ArcCoth[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcCoth[Tanh[a + b*x]])^4*ArcCoth[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcCoth[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcCoth[Tanh[a + b*x]]])/(b*x - ArcCo
th[Tanh[a + b*x]])^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2194

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[a*((n + 1)/((n +
 1)*(b*u - a*v))), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2202

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(-u^(m + 1))*(v^(n + 1)/((m + 1)*(b*u - a*v))), x] + D
ist[b*((m + n + 2)/((m + 1)*(b*u - a*v))), Int[u^(m + 1)*v^n, x], x] /; NeQ
[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^2} + \frac{(2b)\int \frac{}{x^2 \coth^{-1}(}}{bx-\coth^{-1}(t}$$

$$= \frac{2b}{x\left(bx-\coth^{-1}(\tanh(a+bx))\right)^2\coth^{-1}(\tanh(a+bx))^2} + \frac{}{2x^2\left(bx-\coth^{-}\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\right.}$$

$$= -\frac{3b^2}{\left(bx-\coth^{-1}(\tanh(a+bx))\right)^3\coth^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\coth^{-1}(\right.}$$

Mathematica [A]
time = 0.03, size = 107, normalized size = 0.63

$$\frac{-b^4x^4+8b^3x^3\coth^{-1}(\tanh(a+bx))-8bx\coth^{-1}(\tanh(a+bx))^3+\coth^{-1}(\tanh(a+bx))^4-12b^2x^2\coth^{-1}(\tanh(a+bx))^2\left(\log(x)-\log\left(\coth^{-1}(\tanh(a+bx))\right)\right)}{2x^2\left(bx-\coth^{-1}(\tanh(a+bx))\right)^5\coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcCoth[Tanh[a + b*x]]^3),x]
```

```
[Out] (-(b^4*x^4) + 8*b^3*x^3*ArcCoth[Tanh[a + b*x]] - 8*b*x*ArcCoth[Tanh[a + b*x
]]^3 + ArcCoth[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcCoth[Tanh[a + b*x]]^2*(Log[
x] - Log[ArcCoth[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcCoth[Tanh[a + b*x]])^5*
ArcCoth[Tanh[a + b*x]]^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \mathrm{arccoth}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

```
[Out] int(1/x^3/arccoth(tanh(b*x+a))^3,x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.88, size = 331, normalized size = 1.95

$$\frac{192\,b^2\log\left(-i\,\pi+2\,bx+2\,a\right)}{i\,\pi^5-10\,\pi^4a-40i\,\pi^3a^2+80\,\pi^2a^3+80i\,\pi a^4-32\,a^5}-\frac{192\,b^2\log\left(x\right)}{i\,\pi^5-10\,\pi^4a-40i\,\pi^3a^2+80\,\pi^2a^3+80i\,\pi a^4-32\,a^5}+\frac{4\,(96\,b^3x^3-i\,\pi^3+6\,\pi^2a+12i\,\pi a^2-8\,a^3-72\,(i\,\pi b^2-2\,ab^2)x^2-8\,(\pi^2b+4i\,\pi ab-4\,a^2b)x)}{4\,(\pi^4b^2+8i\,\pi^3ab^2-24\,\pi^2a^2b^2-32i\,\pi a^3b^2+16\,a^4b^2)x^4-4\,(i\,\pi^5b-10\,\pi^4ab-40i\,\pi^3a^2b+80\,\pi^2a^3b+80i\,\pi a^4b-32\,a^5b)x^3-(\pi^6+12i\,\pi^5a-60\,\pi^4a^2-160i\,\pi^3a^3+240\,\pi^2a^4+192i\,\pi a^5-64\,a^6)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 192*b^2*log(-I*pi + 2*b*x + 2*a)/(I*pi^5 - 10*pi^4*a - 40*I*pi^3*a^2 + 80*p
i^2*a^3 + 80*I*pi*a^4 - 32*a^5) - 192*b^2*log(x)/(I*pi^5 - 10*pi^4*a - 40*I
*pi^3*a^2 + 80*pi^2*a^3 + 80*I*pi*a^4 - 32*a^5) + 4*(96*b^3*x^3 - I*pi^3 +
6*pi^2*a + 12*I*pi*a^2 - 8*a^3 - 72*(I*pi*b^2 - 2*a*b^2)*x^2 - 8*(pi^2*b +
4*I*pi*a*b - 4*a^2*b)*x)/(4*(pi^4*b^2 + 8*I*pi^3*a*b^2 - 24*pi^2*a^2*b^2 -
32*I*pi*a^3*b^2 + 16*a^4*b^2)*x^4 - 4*(I*pi^5*b - 10*pi^4*a*b - 40*I*pi^3*a
^2*b + 80*pi^2*a^3*b + 80*I*pi*a^4*b - 32*a^5*b)*x^3 - (pi^6 + 12*I*pi^5*a
- 60*pi^4*a^2 - 160*I*pi^3*a^3 + 240*pi^2*a^4 + 192*I*pi*a^5 - 64*a^6)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. 2(168) = 336.
time = 0.45, size = 1316, normalized size = 7.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 8*(3*pi^10*a + 44*pi^8*a^3 + 224*pi^6*a^5 + 384*pi^4*a^7 - 256*pi^2*a^9 - 1
024*a^11 + 192*(pi^6*b^5 - 20*pi^4*a^2*b^5 - 80*pi^2*a^4*b^5 + 64*a^6*b^5)*
x^5 + 96*(11*pi^6*a*b^4 - 140*pi^4*a^3*b^4 - 624*pi^2*a^5*b^4 + 448*a^7*b^4
)*x^4 + 16*(5*pi^8*b^3 + 16*pi^6*a^2*b^3 - 1440*pi^4*a^4*b^3 - 4864*pi^2*a^
```

```
6*b^3 + 3328*a^8*b^3)*x^3 + 4*(65*pi^8*a*b^2 - 272*pi^6*a^3*b^2 - 4896*pi^4
*a^5*b^2 - 9472*pi^2*a^7*b^2 + 6400*a^9*b^2)*x^2 + 2*(3*pi^10*b - 12*pi^8*a
^2*b - 416*pi^6*a^4*b - 1920*pi^4*a^6*b - 2304*pi^2*a^8*b + 1024*a^10*b)*x
- 48*(16*(pi^5*b^6 - 40*pi^3*a^2*b^6 + 80*pi*a^4*b^6)*x^6 + 64*(pi^5*a*b^5
- 40*pi^3*a^3*b^5 + 80*pi*a^5*b^5)*x^5 + 8*(pi^7*b^4 - 28*pi^5*a^2*b^4 - 40
0*pi^3*a^4*b^4 + 960*pi*a^6*b^4)*x^4 + 16*(pi^7*a*b^3 - 36*pi^5*a^3*b^3 - 8
0*pi^3*a^5*b^3 + 320*pi*a^7*b^3)*x^3 + (pi^9*b^2 - 32*pi^7*a^2*b^2 - 224*pi
^5*a^4*b^2 + 1280*pi*a^8*b^2)*x^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 +
8*a*b*x + pi^2 + 4*a^2))/pi) - 24*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6 + 16*
a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5 + 8*(
5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 + 16*(
5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3 + (5*
pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 48*(16*(5*pi^4*a*b^6 - 40*pi^2*a^3*b^6
+ 16*a^5*b^6)*x^6 + 64*(5*pi^4*a^2*b^5 - 40*pi^2*a^4*b^5 + 16*a^6*b^5)*x^5
+ 8*(5*pi^6*a*b^4 + 20*pi^4*a^3*b^4 - 464*pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 +
 16*(5*pi^6*a^2*b^3 - 20*pi^4*a^4*b^3 - 144*pi^2*a^6*b^3 + 64*a^8*b^3)*x^3
+ (5*pi^8*a*b^2 - 224*pi^4*a^5*b^2 - 512*pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*l
og(x))/(16*(pi^10*b^4 + 20*pi^8*a^2*b^4 + 160*pi^6*a^4*b^4 + 640*pi^4*a^6*b
^4 + 1280*pi^2*a^8*b^4 + 1024*a^10*b^4)*x^6 + 64*(pi^10*a*b^3 + 20*pi^8*a^3
*b^3 + 160*pi^6*a^5*b^3 + 640*pi^4*a^7*b^3 + 1280*pi^2*a^9*b^3 + 1024*a^11*
b^3)*x^5 + 8*(pi^12*b^2 + 32*pi^10*a^2*b^2 + 400*pi^8*a^4*b^2 + 2560*pi^6*a
^6*b^2 + 8960*pi^4*a^8*b^2 + 16384*pi^2*a^10*b^2 + 12288*a^12*b^2)*x^4 + 16
*(pi^12*a*b + 24*pi^10*a^3*b + 240*pi^8*a^5*b + 1280*pi^6*a^7*b + 3840*pi^4
*a^9*b + 6144*pi^2*a^11*b + 4096*a^13*b)*x^3 + (pi^14 + 28*pi^12*a^2 + 336*
pi^10*a^4 + 2240*pi^8*a^6 + 8960*pi^6*a^8 + 21504*pi^4*a^10 + 28672*pi^2*a^
12 + 16384*a^14)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{acoth}^3(\tanh(a+bx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*acoth(tanh(a + b*x))**3), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 343, normalized size = 2.02

$$\frac{192i\,b^2\log(i\pi+2\,bx+2\,a)}{\pi^5-10i\,\pi^4a-40\,\pi^3a^2+80i\,\pi^2a^3+80\,\pi a^4-32i\,a^5}-\frac{192i\,b^2\log(x)}{\pi^5-10i\,\pi^4a-40\,\pi^3a^2+80i\,\pi^2a^3+80\,\pi a^4-32i\,a^5}-\frac{4\,(i\pi-12\,bx+2\,a)}{\pi^4x^2-8i\,\pi^3ax^2-24\,\pi^2a^2x^2+32i\,\pi a^3x^2+16\,a^4x^2}+\frac{16\,(12\,b^3x+7i\,\pi b^2+14\,ab^2)}{4\,\pi^4b^2x^2-32i\,\pi^3ab^2x^2-96\,\pi^2a^2b^2x^2+128i\,\pi a^3b^2x^2+64\,a^4b^2x^2+4i\,\pi^5bx+40\,\pi^4abx-160i\,\pi^3a^2bx-320\,\pi^2a^3bx+320i\,\pi a^4bx+128\,a^5bx-\pi^6+12i\,\pi^5a+60\,\pi^4a^2-160i\,\pi^3a^3-240\,\pi^2a^4+192i\,\pi a^5+64\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 192*I*b^2*log(I*pi + 2*b*x + 2*a)/(pi^5 - 10*I*pi^4*a - 40*pi^3*a^2 + 80*I*
pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 192*I*b^2*log(x)/(pi^5 - 10*I*pi^4*a - 4
0*pi^3*a^2 + 80*I*pi^2*a^3 + 80*pi*a^4 - 32*I*a^5) - 4*(I*pi - 12*b*x + 2*a
)/(pi^4*x^2 - 8*I*pi^3*a*x^2 - 24*pi^2*a^2*x^2 + 32*I*pi*a^3*x^2 + 16*a^4*x
^2) + 16*(12*b^3*x + 7*I*pi*b^2 + 14*a*b^2)/(4*pi^4*b^2*x^2 - 32*I*pi^3*a*b
^2*x^2 - 96*pi^2*a^2*b^2*x^2 + 128*I*pi*a^3*b^2*x^2 + 64*a^4*b^2*x^2 + 4*I*
pi^5*b*x + 40*pi^4*a*b*x - 160*I*pi^3*a^2*b*x - 320*pi^2*a^3*b*x + 320*I*pi
*a^4*b*x + 128*a^5*b*x - pi^6 + 12*I*pi^5*a + 60*pi^4*a^2 - 160*I*pi^3*a^3
- 240*pi^2*a^4 + 192*I*pi*a^5 + 64*a^6)
```

Mupad [B]
time = 8.12, size = 1251, normalized size = 7.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*acoth(tanh(a + b*x))^3),x)
```

```
[Out] (4/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 2*b*x) + (32*b*x)/(log(-2/(exp(2*a)*exp(2*b*x) - 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - (288*
b^2*x^2)/((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*
a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2)) + (384*b^3*x^3)/((log(-2/(exp(2
*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x) + 4*a^2)))/(x^3*(8*a*b - 4*b*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) +
 x^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*
x) + 4*a^2) + 4*b^2*x^4) - (384*b^2*atanh((4*b*x*((2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)
) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))/(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x)^3 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
+ log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) -
1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(
```

```
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/
(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)^5))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5
```

3.184 $\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=79

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{1+n} \, {}_2F_1\left(-m,1+n;2+n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(1+n)}$$

```
[Out] x^m*arccoth(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arccoth(tanh(b*x+
a))/(b*x-arccoth(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arccoth(tanh(b*x+a))))^m
)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2204}

$$\frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} \, {}_2F_1\left(-m,n+1;n+2;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^m*ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcCoth[Tanh[a + b*x]]))^m)
```

Rule 2204

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[u^m*(v^(n + 1)/(b*(n + 1)*(b*(u/(b*u - a*v)))^m))*Hype
rgeometric2F1[-m, n + 1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int x^m \coth^{-1}(\tanh(a+bx))^n \, dx = \frac{x^m \left(\frac{bx}{bx-\coth^{-1}(\tanh(a+bx))}\right)^{-m} \coth^{-1}(\tanh(a+bx))^{1+n} \, {}_2F_1\left(-m,1+n;2+\right.}{b(1+n)}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.90

$$\frac{x^{1+m} \coth^{-1}(\tanh(a+bx))^n \left(1+\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)^{-n} \, {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{-bx+\coth^{-1}(\tanh(a+bx))}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^(1 + m)*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((
b*x)/(-(b*x) + ArcCoth[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + Arc
Coth[Tanh[a + b*x]]))^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{m}\mathrm{arccoth}(\tanh(bx+a))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] int(x^m*arccoth(tanh(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral(x^m*arccoth(tanh(b*x + a))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m}\,\mathrm{acoth}^{n}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Integral(x**m*acoth(tanh(a + b*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
[Out] integrate(x^m*arccoth(tanh(b*x + a))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\mathrm{acoth}(\tanh(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x))^n,x)
[Out] int(x^m*acoth(tanh(a + b*x))^n, x)
```

3.185 $\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=165

$$\frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{12x^2 \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}-\frac{24x \coth^{-1}(\text{ta}}{b^4(1+n)(2+}$$

```
[Out] x^4*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arccoth(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+12*x^2*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-24*x*a
rccoth(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)+24*arccoth(tanh(b*x+a
))^(5+n)/b^5/(n^2+7*n+12)/(n^3+8*n^2+17*n+10)
```

Rubi [A]
time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$\frac{24 \coth^{-1}(\tanh(a+bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}-\frac{24x \coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{12x^2 \coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{4x^3 \coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^4 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcCoth[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcCoth[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcCoth[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int x^4 \coth^{-1}(\tanh(a+bx))^n \, dx &= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4 \int x^3 \coth^{-1}(\tanh(a+bx))^{1+n} \, dx}{b(1+n)} \\
&= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12 \int x^2 \mathrm{co}}{} \\
&= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \mathrm{coth}}{b^3(1+} \\
&= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \mathrm{coth}}{b^3(1+} \\
&= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \mathrm{coth}}{b^3(1+} \\
&= \frac{x^4 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \mathrm{coth}}{b^3(1+}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 146, normalized size = 0.88

$$
\frac{\coth^{-1}(\tanh(a+bx))^{1+n} \left(b^4(120+154n+71n^2+14n^3+n^4) x^4 - 4b^3(60+47n+12n^2+n^3) x^3 \coth^{-1}(\tanh(a+bx)) + 12b^2(20+9n+n^2) x^2 \coth^{-1}(\tanh(a+bx))^2 - 24b(5+n)x \coth^{-1}(\tanh(a+bx))^3 + 24 \coth^{-1}(\tanh(a+bx))^4\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*
x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcCoth[Tanh[a + b*x]] + 12*b^2*
(20 + 9*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcCoth[Tanh
[a + b*x]]^3 + 24*ArcCoth[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*(5 + n))
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 20.53, size = 504228, normalized size = 3055.93

method	result	size
risch	Expression too large to display	504228

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 380, normalized size = 2.30

$$\frac{\left(4\left(n^4+10n^3+35n^2+50n+24\right)b^5x^5-3i\pi^5+30\pi^4a+120i\pi^3a^2-240\pi^2a^3-240i\pi a^4+96a^5-2\left(i\pi\left(n^4+6n^3+11n^2+6n\right)b^4-2\left(n^4+6n^3+11n^2+6n\right)ab^4\right)x^4+4\left(\pi^2\left(n^3+3n^2+2n\right)b^3+4i\pi\left(n^3+3n^2+2n\right)ab^3-4\left(n^3+3n^2+2n\right)a^2b^3\right)x^3-6\left(-i\pi^3\left(n^2+n\right)b^2+6\pi^2\left(n^2+n\right)ab^2+12i\pi\left(n^2+n\right)a^2b^2-8\left(n^2+n\right)a^3b^2\right)x^2-6\left(\pi^4bn+8i\pi^3abn-24\pi^2a^2bn-32i\pi a^3bn+16a^4bn\right)x\right)\left(\cosh\left(-n\log\left(-i\pi+2bx+2a\right)\right)-\sinh\left(-n\log\left(-i\pi+2bx+2a\right)\right)\right)}{\left(2^{n+2}n^5+15\cdot 2^{n+2}n^4+85\cdot 2^{n+2}n^3+225\cdot 2^{n+2}n^2+137\cdot 2^{n+3}n+15\cdot 2^{n+5}\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*(n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 - 3*I*pi^5 + 30*pi^4*a + 120
*I*pi^3*a^2 - 240*pi^2*a^3 - 240*I*pi*a^4 + 96*a^5 - 2*(I*pi*(n^4 + 6*n^3 +
 11*n^2 + 6*n)*b^4 - 2*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4)*x^4 + 4*(pi^2*(n
^3 + 3*n^2 + 2*n)*b^3 + 4*I*pi*(n^3 + 3*n^2 + 2*n)*a*b^3 - 4*(n^3 + 3*n^2 +
 2*n)*a^2*b^3)*x^3 - 6*(-I*pi^3*(n^2 + n)*b^2 + 6*pi^2*(n^2 + n)*a*b^2 + 12
*I*pi*(n^2 + n)*a^2*b^2 - 8*(n^2 + n)*a^3*b^2)*x^2 - 6*(pi^4*b*n + 8*I*pi^3
*a*b*n - 24*pi^2*a^2*b*n - 32*I*pi*a^3*b*n + 16*a^4*b*n)*x)*(cosh(-n*log(-I
*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(n + 2)*n^5 +
15*2^(n + 2)*n^4 + 85*2^(n + 2)*n^3 + 225*2^(n + 2)*n^2 + 137*2^(n + 3)*n +
 15*2^(n + 5))*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(165) = 330.
time = 0.40, size = 583, normalized size = 3.53

$$\frac{2\left(2\left(b^5n^4+10b^5n^3+35b^5n^2+50b^5n+24b^5\right)x^5+15\pi^4a-120\pi^2a^3+48a^5+2\left(ab^4n^4+6ab^4n^3+11ab^4n^2+6ab^4n\right)x^4-2\left(4a^2b^3n^3+12a^2b^3n^2+8a^2b^3n-\pi^2\left(b^3n^3+3b^3n^2+2b^3n\right)\right)x^3+6\left(4a^3b^2n^2+4a^3b^2n-3\pi^2\left(ab^2n^2+ab^2n\right)\right)x^2-3\left(\pi^4bn-24\pi^2a^2bn+16a^4bn\right)x\right)\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right)^{\frac{1}{2}n}\cos\left(2n\arctan\left(-\frac{2bx}{\pi}-\frac{2a}{\pi}+\frac{\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)\right)-\left(2\pi\left(b^4n^4+6b^4n^3+11b^4n^2+6b^4n\right)x^4+3\pi^5-120\pi^3a^2+240\pi a^4-16\pi\left(ab^3n^3+3ab^3n^2+2ab^3n\right)x^3-6\left(\pi^3\left(b^2n^2+b^2n\right)-12\pi\left(a^2b^2n^2+a^2b^2n\right)\right)x^2+48\left(\pi^3abn-4\pi a^3bn\right)x\right)\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right)^{\frac{1}{2}n}\sin\left(2n\arctan\left(-\frac{2bx}{\pi}-\frac{2a}{\pi}+\frac{\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)\right)}{4\left(b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*(b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 + 15*
pi^4*a - 120*pi^2*a^3 + 48*a^5 + 2*(a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2
+ 6*a*b^4*n)*x^4 - 2*(4*a^2*b^3*n^3 + 12*a^2*b^3*n^2 + 8*a^2*b^3*n - pi^2*(
b^3*n^3 + 3*b^3*n^2 + 2*b^3*n))*x^3 + 6*(4*a^3*b^2*n^2 + 4*a^3*b^2*n - 3*pi
^2*(a*b^2*n^2 + a*b^2*n))*x^2 - 3*(pi^4*b*n - 24*pi^2*a^2*b*n + 16*a^4*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - (2*pi*(b^4*n^4 +
6*b^4*n^3 + 11*b^4*n^2 + 6*b^4*n)*x^4 + 3*pi^5 - 120*pi^3*a^2 + 240*pi*a^4
- 16*pi*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(pi^3*(b^2*n^2 + b^2*
n) - 12*pi*(a^2*b^2*n^2 + a^2*b^2*n))*x^2 + 48*(pi^3*a*b*n - 4*pi*a^3*b*n)*
x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi -
2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^5*n^5 + 15*b^5*n
^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^5\operatorname{acoth}^n(\tanh(a))}{5} & \text{for}\ b=0 \\ -\frac{x^4}{4b\operatorname{acoth}^4(\tanh(a+bx))}-\frac{x^3}{3b^2\operatorname{acoth}^3(\tanh(a+bx))}-\frac{x^2}{2b^3\operatorname{acoth}^2(\tanh(a+bx))}-\frac{x}{b^4\operatorname{acoth}(\tanh(a+bx))}+\frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^5} & \text{for}\ n=-5 \\ \int\frac{x^4}{\operatorname{acoth}^4(\tanh(a+bx))}\,dx & \text{for}\ n=-4 \\ \int\frac{x^4}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx & \text{for}\ n=-3 \\ \int\frac{x^4}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx & \text{for}\ n=-2 \\ \int\frac{x^4}{\operatorname{acoth}(\tanh(a+bx))}\,dx & \text{for}\ n=-1 \\ \frac{b^4n^4x^4\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{14b^4n^3x^4\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{71b^4n^2x^4\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{154b^4nx^4\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{120b^4x^4\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{4b^3n^3x^3\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{48b^3n^2x^3\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{188b^3nx^3\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{240b^3x^3\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{12b^2n^2x^2\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{108b^2nx^2\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{240b^2x^2\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{24bnx\operatorname{acoth}^4(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}-\frac{120bx\operatorname{acoth}^4(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}+\frac{24\operatorname{acoth}^5(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**5*acoth(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*acoth(tanh(a +
b*x))**4) - x**3/(3*b**2*acoth(tanh(a + b*x))**3) - x**2/(2*b**3*acoth(tanh
(a + b*x))**2) - x/(b**4*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/
b**5, Eq(n, -5)), (Integral(x**4/acoth(tanh(a + b*x))**4, x), Eq(n, -4)), (
Integral(x**4/acoth(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/acoth
(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/acoth(tanh(a + b*x)), x)
, Eq(n, -1)), (b**4*n**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120
*b**5) + 14*b**4*n**3*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
*5) + 71*b**4*n**2*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 154*b**4*n*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12
0*b**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*b**3*n*
*3*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 48*b**3*n*
*2*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 188*b**3*n
*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 240*b**3*x**
3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**2*n**2*x**
2*acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 108*b**2*n*x**2*
acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 240*b**2*x**2*acot
h(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*b*n*x*acoth(tanh(a
+ b*x))**4*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 120*b*x*acoth(tanh(a + b*x))**4
*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 24*acoth(tanh(a + b*x))**5*acoth(tanh(a +
 b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
*5*n + 120*b**5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^4*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B]
time = 2.19, size = 546, normalized size = 3.31

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^5}{4\,b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{3\,n\,x\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^4}{2\,b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^4\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\,\left(n^3+6\,n^2+11\,n+6\right)}{2\,b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{2\,b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{n\,x^3\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2*
a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5)/(4*b^5*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 + n^4
 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 - 1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (n*x^4*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
 + 2*b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (n*x
^3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*
n^3 + 15*n^4 + n^5 + 120)))
```

3.186 $\int x^3 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=121

$$\frac{x^3 \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{3x^2 \coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{6x \coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}-\frac{6 \coth^{-1}(\tanh}{b^4(1+n)(2+n}$$

```
[Out] x^3*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arccoth(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+6*x*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-6*arccoth
(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)
```

Rubi [A]
time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2199, 2188, 30}

$$-\frac{6 \coth^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{6x \coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{3x^2 \coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^3 \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^3*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcCoth[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcCoth[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcCoth[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^3 \coth^{-1}(\tanh(a+bx))^n\,dx &= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3\int x^2\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)} \\ &= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6\int x\coth^{-}}{b^2(} \\ &= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(}{b^3(1+n)} \\ &= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(}{b^3(1+n)} \\ &= \frac{x^3\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x\coth^{-1}(}{b^3(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a+bx))^{1+n}\left(b^3(24+26n+9n^2+n^3)\,x^3-3b^2(12+7n+n^2)\,x^2\coth^{-1}(\tanh(a+bx))+6b(4+n)x\coth^{-1}(\tanh(a+bx))^2-6\coth^{-1}(\tanh(a+bx))^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*
(12 + 7*n + n^2)*x^2*ArcCoth[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcCoth[Tanh[a
+ b*x]]^2 - 6*ArcCoth[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.51, size = 129477, normalized size = 1070.06

method	result	size
risch	Expression too large to display	129477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 257, normalized size = 2.12

$$\frac{(8\,(n^3+6\,n^2+11\,n+6)b^4x^4-3\,\pi^4-24i\,\pi^3a+72\,\pi^2a^2+96i\,\pi a^3-48\,a^4-4\,(i\,\pi(n^3+3\,n^2+2\,n)b^3-2\,(n^3+3\,n^2+2\,n)ab^3)x^3+6\,(\pi^2(n^2+n)b^2+4i\,\pi(n^2+n)ab^2-4\,(n^2+n)a^2b^2)x^2-6\,(-i\,\pi^3bn+6\,\pi^2abn+12i\,\pi a^2bn-8\,a^3bn)x)(\cosh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)-\sinh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right))}{(2^{n+3}n^4+5\cdot 2^{n+4}n^3+35\cdot 2^{n+3}n^2+25\cdot 2^{n+4}n+3\cdot 2^{n+6})b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (8*(n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 +
96*I*pi*a^3 - 48*a^4 - 4*(I*pi*(n^3 + 3*n^2 + 2*n)*b^3 - 2*(n^3 + 3*n^2 + 2
*n)*a*b^3)*x^3 + 6*(pi^2*(n^2 + n)*b^2 + 4*I*pi*(n^2 + n)*a*b^2 - 4*(n^2 +
n)*a^2*b^2)*x^2 - 6*(-I*pi^3*b*n + 6*pi^2*a*b*n + 12*I*pi*a^2*b*n - 8*a^3*b
*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a
)))/((2^(n + 3)*n^4 + 5*2^(n + 4)*n^3 + 35*2^(n + 3)*n^2 + 25*2^(n + 4)*n +
 3*2^(n + 6))*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(121) = 242$.
time = 0.40, size = 411, normalized size = 3.40

$$\frac{(8\,(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-3\,\pi^4+72\,\pi^2a^2-48\,a^4+8\,(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-6\,(4\,a^2b^2n^2+4\,a^2b^2n-\pi^2(b^2n^2+b^2n))x^2-12\,(3\,\pi^2abn-4\,a^3bn)x)(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2)^{\frac{1}{2}\,n}\cos\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)-2\,(2\,\pi(b^3n^3+3\,b^3n^2+2\,b^3n)x^3+12\,\pi^3a-48\,\pi a^3-12\,\pi(ab^2n^2+ab^2n)x^2-3\,(\pi^3bn-12\,\pi a^2bn)x)(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2)^{\frac{1}{2}\,n}\sin\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)}{8\,(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/8*((8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 3*pi^4 + 72*pi^2*a^2
 - 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(4*a^2*b^2*n^2
+ 4*a^2*b^2*n - pi^2*(b^2*n^2 + b^2*n))*x^2 - 12*(3*pi^2*a*b*n - 4*a^3*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - 2*(2*pi*(b^3*n^3
+ 3*b^3*n^2 + 2*b^3*n)*x^3 + 12*pi^3*a - 48*pi*a^3 - 12*pi*(a*b^2*n^2 + a*b
^2*n)*x^2 - 3*(pi^3*b*n - 12*pi*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 +
 a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2)/pi)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4\operatorname{acoth}^n(\tanh(a))}{4} & \text{for}\ b=0 \\ -\frac{x^3}{3b\operatorname{acoth}^3(\tanh(a+bx))}-\frac{x^2}{2b^2\operatorname{acoth}^2(\tanh(a+bx))}-\frac{x}{b^3\operatorname{acoth}(\tanh(a+bx))}+\frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^4} & \text{for}\ n=-4 \\ \int\frac{x^3}{\operatorname{acoth}^3(\tanh(a+bx))}\,dx & \text{for}\ n=-3 \\ \int\frac{x^3}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx & \text{for}\ n=-2 \\ \int\frac{x^3}{\operatorname{acoth}(\tanh(a+bx))}\,dx & \text{for}\ n=-1 \\ \frac{b^3n^3x^3\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{9b^3n^2x^3\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{26b^3nx^3\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{24b^3x^3\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{3b^2n^2x^2\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{21b^2nx^2\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{36b^2x^2\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{6bnx\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}+\frac{24bx\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4}-\frac{6\operatorname{acoth}^4(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**4*acoth(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*acoth(tanh(a +
b*x))**3) - x**2/(2*b**2*acoth(tanh(a + b*x))**2) - x/(b**3*acoth(tanh(a +
b*x))) + log(acoth(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/acoth(t
anh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/acoth(tanh(a + b*x))**2, x
), Eq(n, -2)), (Integral(x**3/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b**3*n
```

```
**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*acoth(tanh(a
+ b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) + 26*b**3*n*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b
*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2
4*b**3*x**3*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**2*acoth(tanh
(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*acoth(tanh(a + b*x))**2*acoth(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 36*b**2*x**2*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b*n*x*acoth(t
anh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**
4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*acoth(tanh(a + b*x))**3*acoth(tanh(a
 + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 - 6*acoth(tanh(a + b*x))**4*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n
**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B]
time = 1.50, size = 418, normalized size = 3.45

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^4}{8\,b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{3\,n\,x\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{n\,x^3\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)\left(n^2+3\,n+2\right)}{2\,b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,n\,x^2\,\left(n+1\right)\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)^2}{4\,b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2*
a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^4)/(8*b^4*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2
+ 10*n^3 + n^4 + 24) + (3*n*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3)/(4*b^3*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(3*n + n^2 +
 2))/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*(log(-2/(
exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.187 $\int x^2 \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=82

$$\frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{2\coth^{-1}(\tanh(a+bx))^{3+n}}{b^3(1+n)(2+n)(3+n)}$$

```
[Out] x^2*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arccoth(tanh(b*x+a))^(2+n)/b^2/(
1+n)/(2+n)+2*arccoth(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)
```

Rubi [A]

time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2199, 2188, 30}

$$\frac{2\coth^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{2x\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^2\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^2*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcCoth[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcCoth[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}(\tanh(a+bx))^n\,dx &= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2\int x\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)} \\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\int \coth^{-1}(\mathrm{t}}{b^2(1} \\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\mathrm{Subst}\left(\int x^2\right.}{} \\ &= \frac{x^2\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{2x\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2\coth^{-1}(\mathrm{tan}}{b^3(1+n)(2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.87

$$\frac{\coth^{-1}(\tanh(a+bx))^{1+n}\left(b^2(6+5n+n^2)\,x^2-2b(3+n)x\coth^{-1}(\tanh(a+bx))+2\coth^{-1}(\tanh(a+bx))^2\right)}{b^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*Ar
cCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*(3
 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 9.27, size = 25561, normalized size = 311.72

method	result	size
risch	Expression too large to display	25561

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [C] Result contains complex when optimal does not.
time = 0.55, size = 166, normalized size = 2.02

$$\frac{\left(4\,(n^2+3\,n+2)b^3x^3+i\,\pi^3-6\,\pi^2a-12i\,\pi a^2+8\,a^3-2\,(i\,\pi(n^2+n)b^2-2\,(n^2+n)ab^2)x^2+2\,(\pi^2bn+4i\,\pi abn-4\,a^2bn)x\right)\left(\cosh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)-\sinh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)\right)}{(2^{n+2}n^3+3\cdot 2^{n+3}n^2+11\cdot 2^{n+2}n+3\cdot 2^{n+3})b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*(n^2 + 3*n + 2)*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 - 2*(I
*pi*(n^2 + n)*b^2 - 2*(n^2 + n)*a*b^2)*x^2 + 2*(pi^2*b*n + 4*I*pi*a*b*n - 4
*a^2*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x
 + 2*a)))/((2^(n + 2)*n^3 + 3*2^(n + 3)*n^2 + 11*2^(n + 2)*n + 3*2^(n + 3))
*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(82) = 164$.
time = 0.35, size = 287, normalized size = 3.50

$$\frac{2\left(2\left(b^3n^2+3b^3n+2b^3\right)x^3-3\pi^2a+4a^3+2\left(ab^2n^2+ab^2n\right)x^2+\left(\pi^2bn-4a^2bn\right)x\right)\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right)^{\frac{1}{2}n}\cos\left(2n\arctan\left(-\frac{2bx}{\pi}-\frac{2a}{\pi}+\frac{\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)\right)+\left(8\pi abnx-2\pi(b^2n^2+b^2n)x^2+\pi^3-12\pi a^2\right)\left(b^2x^2+2abx+\frac{1}{4}\pi^2+a^2\right)^{\frac{1}{2}n}\sin\left(2n\arctan\left(-\frac{2bx}{\pi}-\frac{2a}{\pi}+\frac{\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)\right)}{4\left(b^3n^3+6b^3n^2+11b^3n+6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 3*pi^2*a + 4*a^3 + 2*(a*b^2*n^2
 + a*b^2*n)*x^2 + (pi^2*b*n - 4*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 +
 a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2)/pi)) + (8*pi*a*b*n*x - 2*pi*(b^2*n^2 + b^2*n)*x^2 + pi^3 -
12*pi*a^2)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b
*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^3*n^3 +
6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3\operatorname{acoth}^n(\tanh(a))}{3} & \text{for}\ b=0 \\ -\frac{x^2}{2b\operatorname{acoth}^2(\tanh(a+bx))}-\frac{x}{b^2\operatorname{acoth}(\tanh(a+bx))}+\frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} & \text{for}\ n=-3 \\ \int\frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))}\,dx & \text{for}\ n=-2 \\ \int\frac{x^2}{\operatorname{acoth}(\tanh(a+bx))}\,dx & \text{for}\ n=-1 \\ \frac{b^2n^2x^2\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5b^2nx^2\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6b^2x^2\operatorname{acoth}(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{2bnx\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}-\frac{6bx\operatorname{acoth}^2(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{2\operatorname{acoth}^3(\tanh(a+bx))\operatorname{acoth}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**3*acoth(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*acoth(tanh(a +
b*x))**2) - x/(b**2*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**3,
 Eq(n, -3)), (Integral(x**2/acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (Integ
ral(x**2/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*acoth(tanh(a
 + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) + 5*b**2*n*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*acoth(tanh(a + b*x))*ac
oth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*b*
n*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**
2 + 11*b**3*n + 6*b**3) - 6*b*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x)
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*acoth(tanh(a + b*x)
)**3*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3)
, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B]
time = 1.36, size = 304, normalized size = 3.71

$$-\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}\right)^n\,\left(\frac{{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^3}{4\,b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\,{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}{2\,b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{n\,x^2\,\left(n+1\right)\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2*
a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n^2
+ n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(log(-2
/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) - 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n + 1)*(lo
g(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) - 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

3.188 $\int x \coth^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=48

$$\frac{x \coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

```
[Out] x*arccoth(tanh(b*x+a))^(1+n)/b/(1+n)-arccoth(tanh(b*x+a))^(2+n)/b^2/(1+n)/(
2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2199, 2188, 30}

$$\frac{x \coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] (x*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcCoth[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x\coth^{-1}(\tanh(a+bx))^n\,dx = \frac{x\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\int\coth^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\text{Subst}\left(\int x^{1+n}\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2(1+n)}$$
$$= \frac{x\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.85

$$\frac{\left(b(2+n)x-\coth^{-1}(\tanh(a+bx))\right)\coth^{-1}(\tanh(a+bx))^{1+n}}{b^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcCoth[Tanh[a + b*x]])*ArcCoth[Tanh[a + b*x]]^(1 + n))/(b^
2*(1 + n)*(2 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 9.04, size = 480, normalized size = 10.00

method	result
risch	$\frac{\left(\frac{1}{2}\right)^n\left(2\ln\left(e^{bx+a}\right)-\frac{i\pi\,\text{csgn}\left(ie^{2bx+2a}\right)\left(-\text{csgn}\left(ie^{2bx+2a}\right)+\text{csgn}\left(ie^{bx+a}\right)\right)^2}{2}-\frac{i\pi\,\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)\left(-\text{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)+\text{csgn}\left(ie^{2bx+2a}\right)\right)(}{2}\right.}{2b(1+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(1/2)^n*(2*ln(exp(b*x+a))-1/2*I*Pi*csgn(I*exp(2*b*x+2*a))*(-csgn(I*ex
p(2*b*x+2*a))+csgn(I*exp(b*x+a)))^2-1/2*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))*(-csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I*exp(2*b*x+2*
a)))*(-csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I/(exp(2*b*x+2*a)+1))
)-I*Pi-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*(csgn(I/(exp(2*b*x+2*a)+1))-1))^(1
+n)/(1+n)*x-1/4/b^2*(1/2)^n/(1+n)*(2*ln(exp(b*x+a))-1/2*I*Pi*csgn(I*exp(2*b
*x+2*a))*(-csgn(I*exp(2*b*x+2*a))+csgn(I*exp(b*x+a)))^2-1/2*I*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*(-csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
+csgn(I*exp(2*b*x+2*a)))*(-csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+csgn(I
/(exp(2*b*x+2*a)+1)))-I*Pi-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2*(csgn(I/(exp(2
*b*x+2*a)+1))-1))^(2+n)/(2+n)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 102, normalized size = 2.12

$$\frac{\left(4\,b^2(n+1)x^2+\pi^2+4i\,\pi a-4\,a^2-2\,(i\,\pi bn-2\,abn)x\right)\left(\cosh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)-\sinh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)\right)}{\left(2^{n+2}n^2+3\cdot 2^{n+2}n+2^{n+3}\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (4*b^2*(n + 1)*x^2 + pi^2 + 4*I*pi*a - 4*a^2 - 2*(I*pi*b*n - 2*a*b*n)*x)*(c
osh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(
n + 2)*n^2 + 3*2^(n + 2)*n + 2^(n + 3))*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(48) = 96$.
time = 0.41, size = 210, normalized size = 4.38

$$\frac{\left(4\,abnx+4\,(b^2n+b^2)x^2+\pi^2-4\,a^2\right)\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\cos\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)-2\,(\pi bnx-2\,\pi a)\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\sin\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)}{4\,(b^2n^2+3\,b^2n+2\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/4*((4*a*b*n*x + 4*(b^2*n + b^2)*x^2 + pi^2 - 4*a^2)*(b^2*x^2 + 2*a*b*x +
1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2
+ 8*a*b*x + pi^2 + 4*a^2)/pi)) - 2*(pi*b*n*x - 2*pi*a)*(b^2*x^2 + 2*a*b*x +
 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2
 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2\operatorname{acoth}^n\left(\tanh\left(a\right)\right)}{2} & \text{for}\: b=0 \\ -\frac{x}{b\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}+\frac{\log\left(\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\right)}{b^2} & \text{for}\: n=-2 \\ \int\frac{x}{\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)}\,dx & \text{for}\: n=-1 \\ \frac{bnx\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2}+\frac{2bx\operatorname{acoth}\left(\tanh\left(a+bx\right)\right)\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2}-\frac{\operatorname{acoth}^2\left(\tanh\left(a+bx\right)\right)\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**2*acoth(tanh(a))**n/2, Eq(b, 0)), (-x/(b*acoth(tanh(a + b*x))
) + log(acoth(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/acoth(tanh(a +
b*x)), x), Eq(n, -1)), (b*n*x*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/
(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*acoth(tanh(a + b*x))*acoth(tanh(a +
 b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - acoth(tanh(a + b*x))**2*acoth(t
anh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
[Out] integrate(x*arccoth(tanh(b*x + a))^n, x)
```

Mupad [B]
time = 1.31, size = 205, normalized size = 4.27

$$-\left(\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}\right)^n\,\left(\frac{{\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}^2}{4\,b^2\,\left(n^2+3\,n+2\right)}-\frac{x^2\,\left(n+1\right)}{n^2+3\,n+2}+\frac{n\,x\,\left(\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,b\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(tanh(a + b*x))^n,x)
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2*
a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2 + 2
)) - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(
3*n + n^2 + 2)))
```

3.189 $\int \coth^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=20

$$\frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

```
[Out] arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2188, 30}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\tanh(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcCoth[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Maple [A]
time = 9.02, size = 21, normalized size = 1.05

method	result
derivativedivides	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
default	$\frac{\operatorname{arccoth}(\tanh(bx+a))^{1+n}}{b(1+n)}$
risch	$\frac{\left(\frac{1}{2}\right)^{n}\left(2\ln\left(\mathrm{e}^{bx+a}\right)-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\left(-\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\right)^{2}}{2}-\frac{i\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)\left(-\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)+\mathrm{csgn}\right.}{2b(1+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] arccoth(tanh(b*x+a))^(1+n)/b/(1+n)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.53, size = 65, normalized size = 3.25

$$\frac{(-i\,\pi+2\,bx+2\,a)(\cosh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right)-\sinh\left(-n\log\left(-i\,\pi+2\,bx+2\,a\right)\right))}{(2^{n+1}n+2^{n+1})b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (-I*pi + 2*b*x + 2*a)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*p
i + 2*b*x + 2*a)))/((2^(n + 1)*n + 2^(n + 1))*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(20) = 40$.
time = 0.34, size = 164, normalized size = 8.20

$$\frac{2\,(bx+a)\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\cos\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)-\pi\left(b^2x^2+2\,abx+\frac{1}{4}\,\pi^2+a^2\right)^{\frac{1}{2}\,n}\sin\left(2\,n\arctan\left(-\frac{2\,bx}{\pi}-\frac{2\,a}{\pi}+\frac{\sqrt{4\,b^2x^2+8\,abx+\pi^2+4\,a^2}}{\pi}\right)\right)}{2\,(bn+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*x + a)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arcta
n(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - pi*(
```

```
b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/
pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)))/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.
time = 0.19, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{acoth}\left(\tanh\left(a\right)\right)} & \text{for}\: b = 0 \wedge n = -1 \\ x \operatorname{acoth}^{n}\left(\tanh\left(a\right)\right) & \text{for}\: b = 0 \\ \frac{\log\left(\operatorname{acoth}\left(\tanh\left(a + bx\right)\right)\right)}{b} & \text{for}\: n = -1 \\ \frac{\operatorname{acoth}\left(\tanh\left(a + bx\right)\right) \operatorname{acoth}^{n}\left(\tanh\left(a + bx\right)\right)}{bn + b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*acoth(tanh(a))**n, E
q(b, 0)), (log(acoth(tanh(a + b*x)))/b, Eq(n, -1)), (acoth(tanh(a + b*x))*a
coth(tanh(a + b*x))**n/(b*n + b), True))
```

Giac [A]
time = 0.40, size = 27, normalized size = 1.35

$$\frac{\left(\frac{1}{2}\,\log\left(-e^{\left(2\,bx+2\,a\right)}\right)\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] (1/2*log(-e^(2*b*x + 2*a)))^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 1.27, size = 121, normalized size = 6.05

$$\left(\frac{1}{2}\right)^n\,\left(\frac{x}{n+1}-\frac{\frac{\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}-\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)}{2}+b\,x}{b\,\left(n+1\right)}\right)\,{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)-\ln\left(-\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}-1}\right)\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^n,x)
```

```
[Out] (1/2)^n*(x/(n + 1) - (log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) -
1)))^n
```

3.190 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^{1+n} \, {}_2F_1\left(1,1+n;2+n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] arccoth(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arccoth(tanh(b*x+a))/(
b*x-arccoth(tanh(b*x+a))))/(1+n)/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2195}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} \, {}_2F_1\left(1,n+1;n+2;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot
h[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcCoth[
Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} \, dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} \, {}_2F_1\left(1,1+n;2+n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} \, {}_2F_1\left(-n,-n;1-n;1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tanh
[a + b*x]]/(b*x)])/(n*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(tanh(b*x + a))^n/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^n(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**n/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x,x, algorithm="giac")
[Out] integrate(arccoth(tanh(b*x + a))^n/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acoth}(\tanh(a+b\,x))^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^n/x,x)
[Out] int(acoth(tanh(a + b*x))^n/x, x)
```

3.191 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\coth^{-1}(\tanh(a+bx))^n}{x}+\frac{b\coth^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}$$

```
[Out] -arccoth(tanh(b*x+a))^n/x+b*arccoth(tanh(b*x+a))^n*hypergeom([1, n],[1+n],-
arccoth(tanh(b*x+a))/(b*x-arccoth(tanh(b*x+a))))/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$\frac{b\coth^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;n+1;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{bx-\coth^{-1}(\tanh(a+bx))}-\frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^n/x) + (b*ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/
(b*x - ArcCoth[Tanh[a + b*x]])
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2}\,dx = -\frac{\coth^{-1}(\tanh(a+bx))^n}{x} + (bn)\int \frac{\coth^{-1}(\tanh(a+bx))^{-1+n}}{x}\,dx$$

$$= -\frac{\coth^{-1}(\tanh(a+bx))^n}{x} + \frac{b\coth^{-1}(\tanh(a+bx))^n\,{}_2F_1\left(1,n;1+n;-\frac{\coth^{-}}{bx-\coth}\right.}{bx-\coth^{-1}(\tanh(a+bx))}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a+bx))^n\left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n}{}_2F_1\left(1-n,-n;2-n;1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(-1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x^2,x]
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[T
anh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x^2,x)
[Out] int(arccoth(tanh(b*x+a))^n/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
[Out] integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(tanh(b*x + a))^n/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^n\left(\tanh\left(a+bx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**n/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{acoth}\left(\mathrm{tanh}\left(a+b\,x\right)\right)}^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^n/x^2,x)
```

```
[Out] int(acoth(tanh(a + b*x))^n/x^2, x)
```

3.192 $\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{bn\coth^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{b^2n\coth^{-1}(\tanh(a+bx))^{-1+n}\,{}_2F_1\left(1,-1+n;n;-\right.}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -1/2*b*n*arccoth(tanh(b*x+a))^(-1+n)/x-1/2*arccoth(tanh(b*x+a))^n/x^2+1/2*b
^2*n*arccoth(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arccoth(tanh(b*x+
a))/(b*x-arccoth(tanh(b*x+a))))/(b*x-arccoth(tanh(b*x+a)))
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2199, 2195}

$$\frac{b^2n\coth^{-1}(\tanh(a+bx))^{n-1}\,{}_2F_1\left(1,n-1;n;-\frac{\coth^{-1}(\tanh(a+bx))}{bx-\coth^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\coth^{-1}(\tanh(a+bx))\right)}-\frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{bn\coth^{-1}(\tanh(a+bx))^{n-1}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] -1/2*(b*n*ArcCoth[Tanh[a + b*x]]^(-1 + n))/x - ArcCoth[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcCoth[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcCoth[Tanh[a + b*x]]/(b*x - ArcCoth[Tanh[a + b*x]]))])/(2*(b*x - Arc
Coth[Tanh[a + b*x]]))
```

Rule 2195

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)/((n + 1)*(b*u - a*v)))*Hypergeometric2F1[1, n +
1, n + 2, (-a)*(v/(b*u - a*v))], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx = -\frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{1}{2}(bn) \int \frac{\coth^{-1}(\tanh(a+bx))^{-1+n}}{x^2} dx$$

$$= -\frac{bn \coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{1}{2}(b^2(1-n)n)$$

$$= -\frac{bn \coth^{-1}(\tanh(a+bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} + \frac{b^2 n \coth^{-1}(\tan$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.66

$$\frac{\coth^{-1}(\tanh(a+bx))^n \left(\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)^{-n} {}_2F_1\left(2-n,-n;3-n;1-\frac{\coth^{-1}(\tanh(a+bx))}{bx}\right)}{(-2+n)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] (ArcCoth[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[T
anh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcCoth[Tanh[a + b*x]]/(b*x))^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx+a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

```
[Out] int(arccoth(tanh(b*x+a))^n/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
[Out] integral(arccoth(tanh(b*x + a))^n/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**n/x**3,x)
[Out] Integral(acoth(tanh(a + b*x))**n/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^n/x^3,x, algorithm="giac")
[Out] integrate(arccoth(tanh(b*x + a))^n/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(\tanh(a+b\,x))^{n}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tanh(a + b*x))^n/x^3,x)
[Out] int(acoth(tanh(a + b*x))^n/x^3, x)
```

3.193 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))\,dx&=\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.92

$$x^m\left(\frac{bx^2}{2+m}+\frac{x(-bx+\coth^{-1}(\tanh(a+bx)))}{1+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 676, normalized size = 18.27

method	result
risch	$\frac{x\,x^m \ln(\mathrm{e}^{bx+a})}{1+m} - \frac{x\left(-2i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}+1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^2+2i\pi\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3+2i\pi\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}+1}\right)^3+4bx-i\pi\,\mathrm{csgn}\left(\frac{}{\mathrm{e}^{2bx}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*exp(2*b*x+2*a))^3+2*I
*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*b*x-I*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I/(exp
(2*b*x+2*a)+1))^3*m+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)
)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*I*Pi+I*Pi*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^3*m+I*Pi*csgn(I*exp(2*b*x+2*a))^3*m+I*Pi*csgn(I*exp(
b*x+a))^2*csgn(I*exp(2*b*x+2*a))*m-4*I*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b
*x+2*a))^2+4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-2*I*Pi*csgn(I*exp(b*x+a))*cs
gn(I*exp(2*b*x+2*a))^2*m+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*m-2*I*Pi*csgn(I/(exp(2*b*x+2
*a)+1))^2*m-4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*exp(2*b*x+2*a))
*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I*exp(b*x+a))^2*
csgn(I*exp(2*b*x+2*a))-2*I*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^2+2*I*Pi*m)/(1+m)/(2+m)*x^m
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\,\mathrm{arcoth}\,(\tanh\,(bx+a))}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

Fricas [A]
time = 0.38, size = 33, normalized size = 0.89

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log{(x)}-\frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x} & \text{for}\: m=-2 \\ \int\frac{\operatorname{acoth}{(\tanh{(a+bx)})}}{x}\,dx & \text{for}\: m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2}+\frac{2xx^m\operatorname{acoth}{(\tanh{(a+bx)})}}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.
time = 0.41, size = 90, normalized size = 2.43

$$\frac{x^{m+1}\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,(m+1)}-\frac{bx^{m+2}}{(m+2)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*x^(m + 1)*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(2
*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/(m + 1) - b*x^(m + 2)/((m + 2)
*(m + 1))
```

Mupad [B]
time = 0.00, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,(m+1)}{2\,m^2+6\,m+4}-\frac{x\,x^m\,(m+2)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))/(6*m + 2*m^2 + 4)
```

3.194 $\int x^2 \coth^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))$$

```
[Out] -1/12*b*x^4+1/3*x^3*arccoth(coth(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2199, 30}

$$\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Coth[a + b*x]],x]
```

```
[Out] -1/12*(b*x^4) + (x^3*ArcCoth[Coth[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(\coth(a+bx))\,dx&=\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))-\frac{1}{3}b\int x^3\,dx\\&=-\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\coth^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Coth[a + b*x]],x]
[Out] -1/12*(x^3*(b*x - 4*ArcCoth[Coth[a + b*x]]))
```

Maple [A]
time = 0.22, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^4}{12}+\frac{x^3\mathrm{arccoth}(\coth(bx+a))}{3}$
risch	$\frac{x^3\ln\left(\mathrm{e}^{bx+a}\right)}{3}-\frac{b\,x^4}{12}-\frac{i\pi\,x^3\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^3}{12}-\frac{i\pi\,x^3\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)}{12}+\frac{i\pi\,x^3\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^2}{6}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/12*b*x^4+1/3*x^3*arccoth(coth(b*x+a))
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(19) = 38$.
time = 7.06, size = 76, normalized size = 3.30

$$\begin{cases} \frac{x^3 \operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(-e^{-bx}\right) \\ \frac{x^3 \operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{3} & \text{for } a = \log\left(e^{-bx}\right) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((x**3*acoth(coth(b*x + log(-exp(-b*x))))/3, Eq(a, log(-exp(-b*x))
)), (x**3*acoth(coth(b*x + log(exp(-b*x))))/3, Eq(a, log(exp(-b*x)))), (-b*
x**4/12 + x**3*acoth(1/tanh(a + b*x))/3, True))
```

Giac [A]
time = 0.38, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.83

$$\frac{x^3\,\operatorname{acoth}(\coth(a+b\,x))}{3} - \frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(coth(a + b*x)),x)
```

```
[Out] (x^3*acoth(coth(a + b*x)))/3 - (b*x^4)/12
```

3.195 $\int x \coth^{-1}(\coth(a+bx)) \, dx$

Optimal. Leaf size=23

$$-\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))$$

```
[Out] -1/6*b*x^3+1/2*x^2*arccoth(coth(b*x+a))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6377, 30}

$$\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Coth[a + b*x]],x]
```

```
[Out] -1/6*(b*x^3) + (x^2*ArcCoth[Coth[a + b*x]])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(\coth(a+bx))\,dx&=\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\coth^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Coth[a + b*x]],x]
[Out] -1/6*(x^2*(b*x - 3*ArcCoth[Coth[a + b*x]]))
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arccoth}\left(\coth\left(bx+a\right)\right)}{2}$
risch	$\frac{x^{2}\ln\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{b\,x^{3}}{6}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)}{8}-\frac{i\pi\,x^{2}\operatorname{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^{3}}{8}+\frac{i\pi\,x^{2}\operatorname{csgn}\left(i\mathrm{e}^{2b\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*x^3+1/2*x^2*arccoth(coth(b*x+a))
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="maxima")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A]
time = 0.29, size = 13, normalized size = 0.57

$$\frac{1}{3}x^{3}b+\frac{1}{2}x^{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/3*x^3*b + 1/2*x^2*a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(19) = 38$.
time = 4.11, size = 180, normalized size = 7.83

$$\begin{cases} \frac{x^{2}\operatorname{acoth}{\left(\coth{\left(a \right)} \right)}}{2} & \text{for}\: b = 0 \\ - \frac{x \log{\left(- e^{- b x} \right)} \operatorname{acoth}{\left(\coth{\left(b x + \log{\left(- e^{- b x} \right)} \right)} \right)}}{b} - \frac{\log{\left(- e^{- b x} \right)}^{2} \operatorname{acoth}{\left(\coth{\left(b x + \log{\left(- e^{- b x} \right)} \right)} \right)}}{2 b^{2}} & \text{for}\: a = \log{\left(- e^{- b x} \right)} \\ - \frac{x \log{\left(e^{- b x} \right)} \operatorname{acoth}{\left(\coth{\left(b x + \log{\left(e^{- b x} \right)} \right)} \right)}}{b} - \frac{\log{\left(e^{- b x} \right)}^{2} \operatorname{acoth}{\left(\coth{\left(b x + \log{\left(e^{- b x} \right)} \right)} \right)}}{2 b^{2}} & \text{for}\: a = \log{\left(e^{- b x} \right)} \\ \frac{x \operatorname{acoth}^{2}{\left(\frac{1}{\tanh{\left(a + b x \right)}} \right)}}{2 b} - \frac{\operatorname{acoth}^{3}{\left(\frac{1}{\tanh{\left(a + b x \right)}} \right)}}{6 b^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((x**2*acoth(coth(a))/2, Eq(b, 0)), (-x*log(-exp(-b*x))*acoth(coth
(b*x + log(-exp(-b*x))))/b - log(-exp(-b*x))**2*acoth(coth(b*x + log(-exp(-
b*x))))/(2*b**2), Eq(a, log(-exp(-b*x)))), (-x*log(exp(-b*x))*acoth(coth(b*
x + log(exp(-b*x))))/b - log(exp(-b*x))**2*acoth(coth(b*x + log(exp(-b*x)))
)/(2*b**2), Eq(a, log(exp(-b*x)))), (x*acoth(1/tanh(a + b*x))**2/(2*b) - ac
oth(1/tanh(a + b*x))**3/(6*b**2), True))
```

Giac [**A**]
time = 0.39, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [**B**]
time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{acoth}(\coth(a+b\,x))}{2}-\frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(coth(a + b*x)),x)
```

```
[Out] (x^2*acoth(coth(a + b*x)))/2 - (b*x^3)/6
```

3.196 $\int \coth^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

```
[Out] 1/2*arccoth(coth(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2188, 30}

$$\frac{\coth^{-1}(\coth(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]],x]
```

```
[Out] ArcCoth[Coth[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \coth^{-1}(\coth(a+bx))\right)}{b}\\ &= \frac{\coth^{-1}(\coth(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.12

$$-\frac{bx^2}{2}+x\coth^{-1}(\coth(a+bx))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcCoth[Coth[a + b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.18, size = 32, normalized size = 2.00

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))\operatorname{arccoth}(\coth(bx+a))-\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2}}{b}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))\operatorname{arccoth}(\coth(bx+a))-\frac{\operatorname{arctanh}(\coth(bx+a))^2}{2}}{b}$
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)^2 x}{4}-\frac{i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctanh(coth(b*x+a))*arccoth(coth(b*x+a))-1/2*arctanh(coth(b*x+a))^2)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.62

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.30, size = 10, normalized size = 0.62

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(12) = 24$.

time = 2.08, size = 80, normalized size = 5.00

$$\begin{cases} -\frac{\log\left(-e^{-bx}\right)\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{b} & \text{for } a=\log\left(-e^{-bx}\right) \\ x\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right) & \text{for } a=\log\left(e^{-bx}\right) \\ x\operatorname{acoth}\left(\coth\left(a\right)\right) & \text{for } b=0 \\ \frac{\operatorname{acoth}^2\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a)),x)
```

```
[Out] Piecewise((-log(-exp(-b*x))*acoth(coth(b*x + log(-exp(-b*x))))/b, Eq(a, log
(-exp(-b*x)))), (x*acoth(coth(b*x + log(exp(-b*x)))), Eq(a, log(exp(-b*x)))
), (x*acoth(coth(a)), Eq(b, 0)), (acoth(1/tanh(a + b*x))**2/(2*b), True))
```

Giac [A]
time = 0.38, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 1.21, size = 16, normalized size = 1.00

$$x\operatorname{acoth}(\coth(a+b\,x))-\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(coth(a + b*x)),x)
```

```
[Out] x*acoth(coth(a + b*x)) - (b*x^2)/2
```

3.197 $\int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \log(x)$$

```
[Out] b*x-(b*x-arccoth(coth(b*x+a)))*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2189, 29}

$$bx - \log(x) \left(bx - \coth^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcCoth[Coth[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2189

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\coth(a+bx))}{x} \, dx &= bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \int \frac{1}{x} \, dx \\ &= bx - \left(bx - \coth^{-1}(\coth(a+bx))\right) \log(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$bx + \left(-bx + \coth^{-1}(\coth(a+bx))\right) \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x,x]
[Out] b*x + (-(b*x) + ArcCoth[Coth[a + b*x]])*Log[x]
```

Maple [A]
time = 0.17, size = 27, normalized size = 1.29

method	result
default	$bx+a\ln\left(x\right)+\ln\left(x\right)\left(\operatorname{arccoth}(\coth\left(bx+a\right))-bx-a\right)$
risch	$\ln\left(x\right)\ln\left(\mathrm{e}^{bx+a}\right)-\ln\left(x\right)xb+bx-\frac{i\ln(x)\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2bx+2a}}{\mathrm{e}^{2bx+2a}-1}\right)}{4}-\frac{i\ln(x)\pi\mathrm{csgn}\left(\frac{i\mathrm{e}^{2b}}{\mathrm{e}^{2bx-}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x,x,method=_RETURNVERBOSE)
[Out] b*x+a*ln(x)+ln(x)*(arccoth(coth(b*x+a))-b*x-a)
```

Maxima [A]
time = 0.25, size = 8, normalized size = 0.38

$$bx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="maxima")
[Out] b*x + a*log(x)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.38

$$bx+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="fricas")
[Out] b*x + a*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acoth}\left(\coth\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x,x)
[Out] Integral(acoth(coth(a + b*x))/x, x)
```

Giac [A]
time = 0.39, size = 9, normalized size = 0.43

$$bx + a\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

Mupad [B]
time = 0.55, size = 58, normalized size = 2.76

$$b\,x-\frac{\ln\left(\frac{1}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}+\frac{\ln\left(\frac{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\,\ln\left(x\right)}{2}-b\,x\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(coth(a + b*x))/x,x)
```

```
[Out] b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

3.198 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} \, dx$

Optimal. Leaf size=17

$$-\frac{\coth^{-1}(\coth(a+bx))}{x}+b\log(x)$$

```
[Out] -arccoth(coth(b*x+a))/x+b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2199, 29}

$$b\log(x)-\frac{\coth^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x^2,x]
```

```
[Out] -(ArcCoth[Coth[a + b*x]]/x) + b*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\coth(a+bx))}{x^2} \, dx &= -\frac{\coth^{-1}(\coth(a+bx))}{x}+b\int \frac{1}{x} \, dx\\ &= -\frac{\coth^{-1}(\coth(a+bx))}{x}+b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.06

$$b-\frac{\coth^{-1}(\coth(a+bx))}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x^2,x]
```

```
[Out] b - ArcCoth[Coth[a + b*x]]/x + b*Log[x]
```

Maple [A]
time = 0.21, size = 21, normalized size = 1.24

method	result
default	$-\frac{\operatorname{arccoth}(\coth(bx+a))}{x}+b\ln\left(-bx\right)$
risch	$-\frac{\ln\left(\mathrm{e}^{bx+a}\right)}{x}+\frac{-2i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)^{2}+i\pi\mathrm{csgn}\left(i\mathrm{e}^{bx+a}\right)^{2}\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)+i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2bx+2a}\right)\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2bx+2a}-1}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}}{\mathrm{e}^{2b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccoth(coth(b*x+a))/x+b*ln(-b*x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.65

$$b\log\left(x\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - a/x
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.76

$$\frac{bx\log\left(x\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 3.79, size = 68, normalized size = 4.00

$$\begin{cases} -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{x} & \text{for}\: a=\log\left(-e^{-bx}\right) \\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{x} & \text{for}\: a=\log\left(e^{-bx}\right) \\ b\log\left(x\right)-\frac{\operatorname{acoth}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x**2,x)
```

```
[Out] Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/x, Eq(a, log(-exp(-b*x)))),
(-acoth(coth(b*x + log(exp(-b*x))))/x, Eq(a, log(exp(-b*x)))), (b*log(x) -
acoth(1/tanh(a + b*x))/x, True))
```

Giac [A]
time = 0.39, size = 12, normalized size = 0.71

$$b\log\left(|x|\right) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

Mupad [B]
time = 0.07, size = 17, normalized size = 1.00

$$b\,\ln\left(x\right) - \frac{\mathrm{acoth}(\coth(a+b\,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(coth(a + b*x))/x^2,x)
```

```
[Out] b*log(x) - acoth(coth(a + b*x))/x
```

3.199 $\int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$

Optimal. Leaf size=23

$$-\frac{b}{2x}-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}$$

```
[Out] -1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2199, 30}

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Coth[a + b*x]]/x^3,x]
```

```
[Out] -1/2*b/x - ArcCoth[Coth[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2199

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Dist[b*(n/(a*(m + 1
))), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^{-1}(\coth(a+bx))}{x^3}\,dx &= -\frac{\coth^{-1}(\coth(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\ &= -\frac{b}{2x}-\frac{\coth^{-1}(\coth(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$-\frac{bx+\coth^{-1}(\coth(a+bx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Coth[a + b*x]]/x^3,x]
[Out] -1/2*(b*x + ArcCoth[Coth[a + b*x]])/x^2
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.87

method	result
default	$-\frac{b}{2x}-\frac{\operatorname{arccoth}(\coth(bx+a))}{2x^2}$
risch	$-\frac{\ln(e^{bx+a})}{2x^2}-\frac{4bx+2i\pi\operatorname{csgn}(ie^{bx+a})\operatorname{csgn}(ie^{2bx+2a})^2-i\pi\operatorname{csgn}(ie^{bx+a})^2\operatorname{csgn}(ie^{2bx+2a})-i\pi\operatorname{csgn}(ie^{2bx+2a})^3-i\pi\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(coth(b*x+a))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*b/x-1/2*arccoth(coth(b*x+a))/x^2
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x + a)/x^2
```

Fricas [A]
time = 0.36, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(19) = 38$.
time = 6.62, size = 80, normalized size = 3.48

$$\begin{cases} -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(-e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a = \log\left(-e^{-bx}\right) \\ -\frac{\operatorname{acoth}\left(\coth\left(bx+\log\left(e^{-bx}\right)\right)\right)}{2x^2} & \text{for } a = \log\left(e^{-bx}\right) \\ -\frac{b}{2x} - \frac{\operatorname{acoth}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(coth(b*x+a))/x**3,x)
```

```
[Out] Piecewise((-acoth(coth(b*x + log(-exp(-b*x))))/(2*x**2), Eq(a, log(-exp(-b*
x)))), (-acoth(coth(b*x + log(exp(-b*x))))/(2*x**2), Eq(a, log(exp(-b*x))))
, (-b/(2*x) - acoth(1/tanh(a + b*x))/(2*x**2), True))
```

Giac [A]
time = 0.39, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(coth(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Mupad [B]
time = 1.14, size = 16, normalized size = 0.70

$$-\frac{\operatorname{acoth}(\coth(a+b\,x))+b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(coth(a + b*x))/x^3,x)
```

```
[Out] -(acoth(coth(a + b*x)) + b*x)/(2*x^2)
```

3.200 $\int \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=27

$$x\coth^{-1}(\cosh(x)) - 2x\tanh^{-1}(e^x) - \text{PolyLog}(2, -e^x) + \text{PolyLog}(2, e^x)$$

```
[Out] x*arccoth(cosh(x))-2*x*arctanh(exp(x))-polylog(2,-exp(x))+polylog(2,exp(x))
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {6407, 4267, 2317, 2438}

$$-\text{Li}_2(-e^x) + \text{Li}_2(e^x) - 2x\tanh^{-1}(e^x) + x\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Cosh[x]],x]
```

```
[Out] x*ArcCoth[Cosh[x]] - 2*x*ArcTanh[E^x] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6407

```
Int[ArcCoth[u_], x_Symbol] :> Simp[x*ArcCoth[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \coth^{-1}(\cosh(x))\,dx = x\coth^{-1}(\cosh(x)) + \int x\text{csch}(x)\,dx$$
$$= x\coth^{-1}(\cosh(x)) - 2x\tanh^{-1}(e^x) - \int \log(1-e^x)\,dx + \int \log(1+e^x)\,dx$$
$$= x\coth^{-1}(\cosh(x)) - 2x\tanh^{-1}(e^x) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right) + \text{Subst}\left(\int \right.$$
$$= x\coth^{-1}(\cosh(x)) - 2x\tanh^{-1}(e^x) - \text{Li}_2(-e^x) + \text{Li}_2(e^x)$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.74

$$x\coth^{-1}(\cosh(x)) + x\left(\log\left(1-e^{-x}\right) - \log\left(1+e^{-x}\right)\right) + \text{PolyLog}\left(2,-e^{-x}\right) - \text{PolyLog}\left(2,e^{-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cosh[x]],x]
[Out] x*ArcCoth[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^
(-x)] - PolyLog[2, E^(-x)]
```

Maple [A]
time = 0.20, size = 21, normalized size = 0.78

method	result
default	$x\,\text{arccoth}(\cosh(x)) + 2\,\text{dilog}\left(e^{-x}\right) - \frac{\text{dilog}\left(e^{-2x}\right)}{2}$
risch	$-\frac{ix\pi\text{csgn}\left(i e^{-x}(e^x+1)^2\right)^3}{4} + \frac{ix\pi\,\text{csgn}(i(e^x+1))\text{csgn}\left(i(e^x+1)^2\right)^2}{2} + \frac{ix\pi\text{csgn}(i(e^x-1))^2\text{csgn}\left(i(e^x-1)^2\right)}{4} - \frac{ix\pi\text{csgn}(i(e^x+1))^2\text{cs}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cosh(x)),x,method=_RETURNVERBOSE)
[Out] x*arccoth(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 1.22

$$x\,\text{arcoth}(\cosh(x)) - x\log(e^x+1) + x\log(-e^x+1) - \text{Li}_2(-e^x) + \text{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="maxima")
[Out] x*arccoth(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog
(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(22) = 44$.
time = 0.34, size = 57, normalized size = 2.11

$$\frac{1}{2}x\log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - x\log(\cosh(x)+\sinh(x)+1) + x\log(-\cosh(x)-\sinh(x)+1) + \mathrm{Li}_2(\cosh(x)+\sinh(x)) - \mathrm{Li}_2(-\cosh(x)-\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*l
og(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - si
nh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cosh(x)),x)
```

```
[Out] Integral(acoth(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{acoth}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cosh(x)),x)
```

```
[Out] int(acoth(cosh(x)), x)
```

3.201 $\int x \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}\left(e^x\right)-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)$$

```
[Out] 1/2*x^2*arccoth(cosh(x))-x^2*arctanh(exp(x))-x*polylog(2,-exp(x))+x*polylog
(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6409, 4267, 2611, 2320, 6724}

$$-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Li}_3(-e^x)-\text{Li}_3(e^x)-x^2\tanh^{-1}\left(e^x\right)+\frac{1}{2}x^2\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^2*ArcCoth[Cosh[x]])/2 - x^2*ArcTanh[E^x] - x*PolyLog[2, -E^x] + x*PolyLo
g[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6409

```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(\cosh(x))\,dx &= \frac{1}{2}x^2\coth^{-1}(\cosh(x))+\frac{1}{2}\int x^2\text{csch}(x)\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-\int x\log(1-e^x)\,dx+\int x\log(1+e^x)\,d\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\int\text{Li}_2(-e^x)\,dx\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Subst}\bigg(\int\frac{\text{Li}_2(-}{x}\\
&= \frac{1}{2}x^2\coth^{-1}(\cosh(x))-x^2\tanh^{-1}(e^x)-x\text{Li}_2(-e^x)+x\text{Li}_2(e^x)+\text{Li}_3(-e^x)-\text{Li}_3(\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.59

$$\frac{1}{2}\left(x^2\coth^{-1}(\cosh(x))+x^2\log\left(1-e^{-x}\right)-x^2\log\left(1+e^{-x}\right)+2x\text{PolyLog}\left(2,-e^{-x}\right)-2x\text{PolyLog}\left(2,e^{-x}\right)+2\text{PolyLog}\left(3,-e^{-x}\right)-2\text{PolyLog}\left(3,e^{-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^2*ArcCoth[Cosh[x]] + x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*Pol
yLog[2, -E^(-x)] - 2*x*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyL
og[3, E^(-x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 449, normalized size = 8.80

method	result
risch	$-\frac{i\pi \operatorname{csgn}(i\mathrm{e}^{-x})\operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^{x}-1)^{2}\right)^{2} x^{2}}{8}+\frac{i\pi \operatorname{csgn}(i\mathrm{e}^{-x})\operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^{x}+1)^{2}\right)^{2} x^{2}}{8}+\frac{i\pi \operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^{x}-1)^{2}\right)^{3} x^{2}}{8}-\frac{i\pi \operatorname{csgn}(i(\mathrm{e}^{x}-1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2+1/8*I*Pi*csgn(
I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2+1/8*I*Pi*csgn(I*exp(-x)*(exp(
x)-1)^2)^3*x^2-1/4*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x^2+1/8*I
*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2)*x^2+x*polylog(2,exp(x))-x*pol
ylog(2,-exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))+1/2*x^2*ln(1-exp(x))-1
/2*x^2*ln(exp(x)-1)+1/4*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^2-
1/8*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*c
sgn(I*(exp(x)+1)^2)^3*x^2-1/8*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csg
n(I*exp(-x)*(exp(x)+1)^2)*x^2-1/8*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)+
1)^2)*x^2-1/8*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^2+1/8*I*Pi*csgn(I*(exp(
x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x^2+1/8*I*Pi*csgn(I*(
exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2+1/8*I*Pi*csgn(I*(exp(x)-1)^
2)^3*x^2
```

Maxima [A]
time = 0.30, size = 56, normalized size = 1.10

$$\frac{1}{2}\,x^2\operatorname{arcoth}\left(\cosh\left(x\right)\right)-\frac{1}{2}\,x^2\log\left(e^x+1\right)+\frac{1}{2}\,x^2\log\left(-e^x+1\right)-x\mathrm{Li}_2(-e^x)+x\mathrm{Li}_2(e^x)+\mathrm{Li}_3(-e^x)-\mathrm{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) - x
*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.
time = 0.37, size = 87, normalized size = 1.71

$$\frac{1}{4}\,x^2\log\left(\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-\frac{1}{2}\,x^2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{2}\,x^2\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+x\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-x\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))-\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x*d
```

```
ilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cosh
(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(cosh(x)),x)
[Out] Integral(x*acoth(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(cosh(x)),x, algorithm="giac")
[Out] integrate(x*arccoth(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acoth}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(cosh(x)),x)
[Out] int(x*acoth(cosh(x)), x)
```

3.202 $\int x^2 \coth^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=77

$$\frac{1}{3}x^3\coth^{-1}(\cosh(x))-\frac{2}{3}x^3\tanh^{-1}(e^x)-x^2\text{PolyLog}(2,-e^x)+x^2\text{PolyLog}(2,e^x)+2x\text{PolyLog}(3,-e^x)-2x\text{PolyL}$$

```
[Out] 1/3*x^3*arccoth(cosh(x))-2/3*x^3*arctanh(exp(x))-x^2*polylog(2,-exp(x))+x^2
*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog(4
,-exp(x))+2*polylog(4,exp(x))
```

Rubi [A]
time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6409, 4267, 2611, 6744, 2320, 6724}

$$-x^2\text{Li}_2(-e^x)+x^2\text{Li}_2(e^x)+2x\text{Li}_3(-e^x)-2x\text{Li}_3(e^x)-2\text{Li}_4(-e^x)+2\text{Li}_4(e^x)-\frac{2}{3}x^3\tanh^{-1}(e^x)+\frac{1}{3}x^3\coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[Cosh[x]],x]
```

```
[Out] (x^3*ArcCoth[Cosh[x]])/3 - (2*x^3*ArcTanh[E^x])/3 - x^2*PolyLog[2, -E^x] +
x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyLo
g[4, -E^x] + 2*PolyLog[4, E^x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6409

```
Int[((a_.) + ArcCoth[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCoth[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \coth^{-1}(\cosh(x))\,dx &= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) + \frac{1}{3}\int x^3\operatorname{csch}(x)\,dx \\
&= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\tanh^{-1}\left(e^x\right) - \int x^2\log\left(1-e^x\right)\,dx + \int x^2\log\left(1+\right. \\
&= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\tanh^{-1}\left(e^x\right) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2\int x\mathrm{Li}_2(- \\
&= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\tanh^{-1}\left(e^x\right) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) \\
&= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\tanh^{-1}\left(e^x\right) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x) \\
&= \frac{1}{3}x^3\coth^{-1}(\cosh(x)) - \frac{2}{3}x^3\tanh^{-1}\left(e^x\right) - x^2\mathrm{Li}_2(-e^x) + x^2\mathrm{Li}_2(e^x) + 2x\mathrm{Li}_3(-e^x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 109, normalized size = 1.42

$$
\frac{1}{24}\left(\pi^4 - 2x^4 + 8x^3\coth^{-1}(\cosh(x)) - 8x^3\log\left(1+e^{-x}\right) + 8x^3\log\left(1-e^x\right) + 24x^2\mathrm{PolyLog}(2,-e^{-x}) + 24x^2\mathrm{PolyLog}(2,e^x) + 48x\mathrm{PolyLog}(3,-e^{-x}) - 48x\mathrm{PolyLog}(3,e^x) + 48\mathrm{PolyLog}(4,-e^{-x}) + 48\mathrm{PolyLog}(4,e^x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[Cosh[x]],x]
```

```
[Out] (Pi^4 - 2*x^4 + 8*x^3*ArcCoth[Cosh[x]] - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[
1 - E^x] + 24*x^2*PolyLog[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyL
og[3, -E^(-x)] - 48*x*PolyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog
[4, E^x])/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 471, normalized size = 6.12

method	result
risch	$-\frac{i\pi \operatorname{csgn}\left(i(\mathrm{e}^{x}-1)^{2}\right)\operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^{x}-1)^{2}\right)^{2}x^{3}}{12}+\frac{i\pi \operatorname{csgn}\left(i(\mathrm{e}^{x}+1)^{2}\right)\operatorname{csgn}\left(i\mathrm{e}^{-x}(\mathrm{e}^{x}+1)^{2}\right)^{2}x^{3}}{12}+\frac{i\pi \operatorname{csgn}(i(\mathrm{e}^{x}+1))\operatorname{csgn}\left(i(\mathrm{e}^{x}+1)^{2}\right)^{2}x}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*P
i*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3+1/6*I*Pi*csgn(I*(
exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^3+2*x*polylog(3,-exp(x))-2*x*polylog(3,
exp(x))-2*polylog(4,-exp(x))+2*polylog(4,exp(x))+x^2*polylog(2,exp(x))-x^2*
polylog(2,-exp(x))+1/3*x^3*ln(1-exp(x))-1/3*x^3*ln(exp(x)-1)+1/12*I*Pi*csgn
(I*exp(-x)*(exp(x)-1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x
))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(
exp(x)+1)^2)*x^3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csg
n(I*(exp(x)-1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csgn
(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*exp(-x))*cs
gn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp
(x)-1)^2)*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)
*(exp(x)-1)^2)*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x^3
```

Maxima [A]
time = 0.31, size = 78, normalized size = 1.01

$$\frac{1}{3}\,x^{3}\operatorname{arcoth}\left(\cosh\left(x\right)\right)-\frac{1}{3}\,x^{3}\log\left(e^{x}+1\right)+\frac{1}{3}\,x^{3}\log\left(-e^{x}+1\right)-x^{2}\mathrm{Li}_{2}(-e^{x})+x^{2}\mathrm{Li}_{2}(e^{x})+2\,x\mathrm{Li}_{3}(-e^{x})-2\,x\mathrm{Li}_{3}(e^{x})-2\,\mathrm{Li}_{4}(-e^{x})+2\,\mathrm{Li}_{4}(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.52

$$\frac{1}{6}\,x^{3}\log\left(\frac{\cosh\left(x\right)+1}{\cosh\left(x\right)-1}\right)-\frac{1}{3}\,x^{3}\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{3}\,x^{3}\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+x^{2}\mathrm{Li}_{2}(\cosh\left(x\right)+\sinh\left(x\right))-x^{2}\mathrm{Li}_{2}(-\cosh\left(x\right)-\sinh\left(x\right))-2\,x\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+2\,x\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))+2\,\mathrm{polylog}(4,\cosh\left(x\right)+\sinh\left(x\right))-2\,\mathrm{polylog}(4,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) - x
^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*poly
log(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4,
 -cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(cosh(x)),x)
```

```
[Out] Integral(x**2*acoth(cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acoth}(\cosh(x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(cosh(x)),x)
```

```
[Out] int(x^2*acoth(cosh(x)), x)
```

3.203 $\int x^2 \coth^{-1}(c + d \tanh(a + bx)) \, dx$

Optimal. Leaf size=307

$$\frac{1}{3}x^3 \coth^{-1}(c+d\tanh(a+bx))+\frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x^2\text{Poly}}{}$$

```
[Out] 1/3*x^3*arccoth(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d)
)/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,-(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A]
time = 0.33, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6379, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{Li}_4\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\text{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x^2\text{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{6}x^3\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{6}x^3\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6379

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(
(1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c - d
 + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2b}}{1-c+d+(1-}$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) -$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) -$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) -$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) -$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) -$$

Mathematica [A]
time = 0.25, size = 271, normalized size = 0.88

$$\frac{1}{3}x^3\coth^{-1}(c+d\tanh(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 4b^3x^3\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) + 6b^2x^2\text{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) - 6bx\text{PolyLog}\left(3,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) + 6bx\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) + 3\text{PolyLog}\left(4,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 3\text{PolyLog}\left(4,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(-1 + c - d)] - 4*b^3*x^3*Log[1 + ((1 + c + d)*E^(2*(a + b*x)
))/(1 + c - d)] + 6*b^2*x^2*PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1
 + c - d))] - 6*b^2*x^2*PolyLog[2, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c -
 d))] - 6*b*x*PolyLog[3, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] +
6*b*x*PolyLog[3, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))] + 3*PolyLog[
4, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - 3*PolyLog[4, -(((1 + c
 + d)*E^(2*(a + b*x)))/(1 + c - d))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.43, size = 5294, normalized size = 17.24

method	result	size
risch	Expression too large to display	5294

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.48, size = 281, normalized size = 0.92

$$\frac{1}{3}x^{3}\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)-\frac{1}{18}bd\left(\frac{4b^{3}x^{3}\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^{2}x^{2}\mathrm{Li}_{2}\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_{3}(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1})+3\mathrm{Li}_{4}(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1})}{b^{4}d}-\frac{4b^{3}x^{3}\log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+6b^{2}x^{2}\mathrm{Li}_{2}\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)-6bx\mathrm{Li}_{3}(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1})+3\mathrm{Li}_{4}(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1})}{b^{4}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d +
 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4
*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog
(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^4*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(263) = 526$.
time = 0.41, size = 899, normalized size = 2.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*
x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d -
1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-(
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d
 + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c
+ d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1
)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(2
*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sq
rt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
+ d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6*
b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sin
h(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(co
```

```
sh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c -
d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-(
c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a
^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b
*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(co
sh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(c + d \tanh\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(c + d*tanh(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \tanh(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*acoth(c + d*tanh(a + b*x)), x)
```

3.204 $\int x \coth^{-1}(c + d\tanh(a + bx))\, dx$

Optimal. Leaf size=231

$$\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x\mathrm{Pol}}{}$$

```
[Out] 1/2*x^2*arccoth(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*ex
p(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-
1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

Rubi [A]
time = 0.27, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6379, 2221, 2611, 2320, 6724}

$$-\frac{\mathrm{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\mathrm{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\mathrm{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\mathrm{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{4}x^2\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6379

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + (Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^
(2*a + 2*b*x)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(
(1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c - d
 + (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\tanh(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{2}(b(1-c-d))\int\frac{e^{2a+2bx}}{1-c+d+(1-}$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-$$

Mathematica [A]

time = 0.09, size = 203, normalized size = 0.88

$$\frac{1}{2}x^2\coth^{-1}(c+d\tanh(a+bx))+\frac{2b^2x^2\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-2b^2x^2\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)+2bx\text{PolyLog}\left(2,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-2bx\text{PolyLog}\left(2,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)-\text{PolyLog}\left(3,-\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+\text{PolyLog}\left(3,-\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tanh[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(-1 + c - d)] - 2*b^2*x^2*Log[1 + ((1 + c + d)*E^(2*(a + b*x)
))/(1 + c - d)] + 2*b*x*PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c
 - d))] - 2*b*x*PolyLog[2, -(((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))] -
PolyLog[3, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] + PolyLog[3, -((
(1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(8*b^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 4990, normalized size = 21.60

method	result	size
risch	Expression too large to display	4990

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))-1/8/b^2*d/(c+d
-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))+1/2/b^2*a/(c+d-1)*dilog((-exp
(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-
1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+
d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/4/b^2/(c+d-1)*ln(1-(c+d
-1)*exp(2*b*x+2*a)/(1-c+d))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2
*a)/(1-c+d))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a+
1/2/b^2*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)
+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/(c+d-1)*ln((exp(b*x+a)*c
+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2)
)+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2+1/4*d/(c+d-1)*ln(1
-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x^2-1/2/b^2*c*a/(c+d-1)*dilog((exp(b*x+a)*
c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2
))-1/2/b^2*d*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))
^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((exp
(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-
1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c
+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/4/b^2*c/(c+d-1)*ln(1-(
c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*
b*x+2*a)/(1-c+d))*x+1/4/b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c
+d))*a+1/4/b^2*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a^2+1/4/b*d/(
c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x+1/4/b^2*d/(c+d-1)*polylo
g(2,(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*a-1/2/b/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+
2*a)/(1-c+d))*x*a+1/2/b*a/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*
(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x+1/2/b*a/(c+d-1)*ln((
exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c
```

```
+d-1))^(1/2))*x-1/2/b^2*c*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+(-(c-d
-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a^2/(c+d
-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(-(c
-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+
(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))+1/8/b^2/(1+c
+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))-1/4/(1+c+d)*ln(1-(1+c+d)*exp
(2*b*x+2*a)/(-1-c+d))*x^2+1/2/b*a*c/(1+c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+
(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/
(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/
(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*d*a/(1+c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*
d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))*x+1/2/b*d*
a/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a)
)/(-(1+c-d)*(1+c+d))^(1/2))*x-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(
-1-c+d))*x*a-1/2/b*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a+1/4/
b^2*c*a^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+c-d-1
)+1/4/b^2*d*a^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)
+c-d-1)+1/2/b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x*a+1/2/b*d/(c
+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(1-c+d))*x*a-1/2/b*c*a/(c+d-1)*ln((-exp(b
*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-1)
)^(1/2))*x-1/2/b*c*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d-1
))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((-exp
(b*x+a)*c-exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(-(c-d-1)*(c+d-
1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(c-d-1)*(c+d
-1))^(1/2)-exp(b*x+a))/(-(c-d-1)*(c+d-1))^(1/2))*x-1/4/b^2*c*a^2/(1+c+d)*ln
(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/4/b^2*d*a^2/(1+c
+d)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+c-d+1)-1/4*x^2*ln((
exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d-exp(2*b*x+2*a)-1)+1/4*x^2*ln((exp(
2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)-1/2/b/(1+c+d)*ln(1-(
1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a+1/2/b^2*a*c/(1+c+d)*dilog((exp(b*x+a)*c
+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2)
)+1/2/b^2*d*a/(1+c+d)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^
(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a/(1+c+d)*dilog((exp(
b*x+a)*c+exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d
))^(1/2))+1/2/b^2*a*c/(1+c+d)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+(-(1+c-d)*(
1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*c/(1+c+d)*l
n((-exp(b*x+a)*c-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-(1+c-d
)*(1+c+d))^(1/2))+1/2/b^2*a^2*c/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(-(1+
c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+
c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/(-
(1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d
+(-(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(-(1+c-d)...
```

Maxima [A]

time = 0.48, size = 215, normalized size = 0.93

$$-\frac{1}{8}bd\left(\frac{2b^2x^2\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+2bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-\mathrm{Li}_3\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^3d}-\frac{2b^2x^2\log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+2bx\mathrm{Li}_2\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)-\mathrm{Li}_3\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^3d}\right)+\frac{1}{2}x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*b
*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) +
1/2*x^2*arccoth(d*tanh(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(197) = 394$.
time = 0.38, size = 745, normalized size = 3.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*
x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x +
 a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d
+ 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a)
- 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cos
h(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/
(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*
x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*lo
g(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^
2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)/
(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d
+ 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c
+ d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt
(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acoth}\left(c+d\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*acoth(c + d*tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acoth}(c+d\,\tanh(a+b\,x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x*acoth(c + d*tanh(a + b*x)), x)
```

3.205 $\int \coth^{-1}(c+d\tanh(a+bx))\,dx$

Optimal. Leaf size=150

$$x\coth^{-1}(c+d\tanh(a+bx))+\frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{2}x\log\left(1+\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{\text{PolyLog}(}{}$$

```
[Out] x*arccoth(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*a)
/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A]
time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6371, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)+x\coth^{-1}(d\tanh(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6371

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[x*(E^(2*a + 2*b*x
)/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x))), x], x] - Dist[b*(1 + c + d),
Int[x*(E^(2*a + 2*b*x)/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x))), x], x])
/; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\tanh(a+bx))\,dx = x\coth^{-1}(c+d\tanh(a+bx)) + (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(1-c-d)}$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\,\mathrm{lo}$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\,\mathrm{lo}$$

$$= x\coth^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\,\mathrm{lo}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(150) = 300.
time = 0.92, size = 366, normalized size = 2.44

$$x\coth^{-1}(c+d\tanh(a+bx)) + \frac{(a+bx)\log\left(1-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) + (a+bx)\log\left(1+\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) - (a+bx)\log\left(1-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) - (a+bx)\log\left(1+\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) + a\log\left(1+c-d+e^{2(a+bx)}+ce^{2(a+bx)}+de^{2(a+bx)}\right) - a\log\left(1+d+e^{2(a+bx)}-de^{2(a+bx)}-c\left(1+e^{2(a+bx)}\right)\right) + \mathrm{PolyLog}\left(2,-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) + \mathrm{PolyLog}\left(2,\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{1-c+d}}\right) - \mathrm{PolyLog}\left(2,-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right) - \mathrm{PolyLog}\left(2,\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{-1-c+d}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tanh[a + b*x]] + ((a + b*x)*Log[1 - (Sqrt[-1 + c + d]*E^(a
+ b*x))/Sqrt[1 - c + d]] + (a + b*x)*Log[1 + (Sqrt[-1 + c + d]*E^(a + b*x))
/Sqrt[1 - c + d]] - (a + b*x)*Log[1 - (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1
 - c + d]] - (a + b*x)*Log[1 + (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1 - c +
d]] + a*Log[1 + c - d + E^(2*(a + b*x)) + c*E^(2*(a + b*x)) + d*E^(2*(a + b
*x))] - a*Log[1 + d + E^(2*(a + b*x)) - d*E^(2*(a + b*x)) - c*(1 + E^(2*(a
+ b*x)))] + PolyLog[2, -((Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[1 - c + d])] +
 PolyLog[2, (Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[1 - c + d]] - PolyLog[2, -(
(Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[-1 - c + d])] - PolyLog[2, (Sqrt[1 + c +
 d]*E^(a + b*x))/Sqrt[-1 - c + d]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(138) = 276.
time = 1.52, size = 361, normalized size = 2.41

method	result
derivativedivides	$-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)-}{1-c-d}\right.}{2d}\right.}{}$
default	$-\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\tanh(bx+a)-}{1-c-d}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arccoth(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*arccoth(c+d
*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*d^2*(1/2/d*dilog((-d*tanh(b*x+a)-c
+1)/(1-c-d))+1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-c+1)/(1-c-d))-1/
2/d*dilog((-d*tanh(b*x+a)-c-1)/(-1-c-d))-1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*
tanh(b*x+a)-c-1)/(-1-c-d))-1/2/d*dilog((-d*tanh(b*x+a)-c+1)/(1-c+d))-1/2/d*
ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c+1)/(1-c+d))+1/2/d*dilog((-d*tanh(
b*x+a)-c-1)/(-1-c+d))+1/2/d*ln(-d*tanh(b*x+a)-d)*ln((-d*tanh(b*x+a)-c-1)/(-
1-c+d))))
```

Maxima [A]
time = 0.48, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd\left(\frac{2bx\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^2d}-\frac{2bx\log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^2d}\right)+x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(-
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(128) = 256$.
time = 0.37, size = 551, normalized size = 3.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*
sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-
(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2*
(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqr
t(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-s
qrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(
sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sq
rt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt(
-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-(
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(c+d\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(c + d*tanh(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(c+d\tanh(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*tanh(a + b*x)),x)
```

```
[Out] int(acoth(c + d*tanh(a + b*x)), x)
```

3.206 $\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\operatorname{Int}\left(\frac{\coth^{-1}(c+d\tanh(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arccoth(c+d*tanh(b*x+a))/x,x)

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcCoth[c + d*Tanh[a + b*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d*Tanh[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 4.47, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x,x]

[Out] Integrate[ArcCoth[c + d*Tanh[a + b*x]]/x, x]

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c+d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(c + d\tanh\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(c + d*tanh(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c+d\tanh(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*tanh(a + b*x))/x,x)
[Out] int(acoth(c + d*tanh(a + b*x))/x, x)
```

3.207 $\int x^3 \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\frac{3x^2}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arccoth(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,-(1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6375, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left(-\left((d+1)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{8}x^4\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{4}x^4\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\coth^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))+\frac{1}{4}b\int\frac{x^4}{1+(1+d)e^{2a+2bx}}\,dx\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}(b(1+d))\int\frac{e}{1+(}\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\\
&= \frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1+d)\right.\end{aligned}$$

Mathematica [A]
time = 0.13, size = 144, normalized size = 0.93

$$\frac{1}{16}\left(4x^4\coth^{-1}(1+d+d\tanh(a+bx))-2x^4\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)+\frac{4x^3\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b}+\frac{6x^2\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2}+\frac{6x\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}+\frac{3\text{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 + d + d*Tanh[a + b*x]] - 2*x^4*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x^2*P
olyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((1 +
d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x))))])
/b^4)/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.72, size = 1726, normalized size = 11.14

method	result	size
risch	Expression too large to display	1726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b
*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/20*b*x^5-1/8/b^4*d*a^4/(1+d)*ln(exp(2*b*x+
2*a)*d+exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+
2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2
*b*x+2*a)+1))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp
(2*b*x+2*a)+1))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*I*x^4*Pi*csgn(I*d*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^3+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))+
1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-ex
p(b*x+a)*(-1-d)^(1/2))*x-1/2/b^3*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x-1/4
/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2*d/(1+d)*polylog(3,-
(1+d)*exp(2*b*x+2*a))*x^2-1/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2
*a)+1)+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1+(1+d)
*exp(2*b*x+2*a))*x^4-1/8*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4+1/2/b^4*a^3
/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a
)*(-1-d)^(1/2))+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*a
^4/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-
1-d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))-1/4/b^4*a^3/(1+d)*
polylog(2,-(1+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a
))*x^3+3/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3/(1+d)*pol
ylog(4,-(1+d)*exp(2*b*x+2*a))*x-3/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+
2*a))*x-1/4/b^4*d*a^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))-3/8/b^4*d*a^4/
(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(-1
-d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/16*I*x^4*
Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+
a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^
4*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*
csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(
2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(-1-d
)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+
d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-1-
d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2+1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/4*x^4*ln(
exp(b*x+a))-1/8*x^4*ln(d)
```

Maxima [A]

time = 0.69, size = 149, normalized size = 0.96

$$\frac{1}{4}\,x^{4}\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{40}\left(\frac{2\,x^{5}}{d}-\frac{5\,\left(2\,b^{4}x^{4}\log\left(\left(d+1\right)e^{\left(2\,bx+2\,a\right)}+1\right)+4\,b^{3}x^{3}\mathrm{Li}_{2}\left(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)}\right)-6\,b^{2}x^{2}\mathrm{Li}_{3}\left(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)}\right)+6\,bx\mathrm{Li}_{4}\left(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)}\right)-3\,\mathrm{Li}_{5}\left(-\left(d+1\right)e^{\left(2\,bx+2\,a\right)}\right)\right)}{b^{5}d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d + 1)
*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(135) = 270$.
time = 0.39, size = 450, normalized size = 2.90

$$\frac{2 b^5 x^5 + 5 b^4 x^4 \log\left(\frac{(d+2)\cosh(bx+a) + d\sinh(bx+a)}{d\cosh(bx+a) + d\sinh(bx+a)}\right) - 20 b^3 x^3 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 20 b^3 x^3 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 5a^4\log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) + \sqrt{-4d-4}\right) - 5a^4\log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) - \sqrt{-4d-4}\right) + 60 b^2x^2 \mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 60 b^2x^2 \mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 120 bx\, \mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 120 bx\, \mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 5\left(b^4x^4 - a^4\right)\log\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 5\left(b^4x^4 - a^4\right)\log\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) + 120\,\mathrm{polylog}\left(5, \frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 120\,\mathrm{polylog}\left(5, -\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{40 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1
)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(
d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(-
4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*sq
rt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqrt
(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*d
 - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}\left(d\tanh\left(a + bx\right) + d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**3*acoth(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acoth}(d + d\tanh(a + b\,x) + 1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x^3*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.208 $\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) \, dx$

Optimal. Leaf size=128

$$\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\frac{x\text{P}\cdots}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccoth(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6375, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{3}x^3\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{3}(b(1+d))\int \frac{e}{1+(}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)\right.$$

Mathematica [A]
time = 0.08, size = 118, normalized size = 0.92

$$\frac{1}{24}\left(8x^3\coth^{-1}(1+d+d\tanh(a+bx)) - 4x^3\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 + d + d*Tanh[a + b*x]] - 4*x^3*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x*Pol
yLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((1 + d)*E
^(2*(a + b*x))))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.52, size = 1667, normalized size = 13.02

method	result	size
risch	Expression too large to display	1667

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x
+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/3*x^3*ln(exp(b*x+a))+1/12
```

```
*b*x^4+1/3/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3+1/12*I*x^3*Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2
*a))*x*a^2-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^3*a^2/(
1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/12*I*x^3*Pi*csgn(I*d*exp(2*b*x+2*a)
/(exp(2*b*x+2*a)+1))^3-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x-
1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x+1/6/b^3*d*a^3/(1+d)*ln(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp
(2*b*x+2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b
^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*
x+a)*(-1-d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))*x-1/2/
b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1-exp(
b*x+a)*(-1-d)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))-1
/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2/(1+d)*ln(1+(1+d
)*exp(2*b*x+2*a))*x*a^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(ex
p(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn
(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3+1/3/b^3/(1+d)*ln(1+
(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+
1/4/b^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,
-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))-1/
2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))-1/8/b^3*d/(1+d)*polylog(4,-(1
+d)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/6/(1+d)*ln(1
+(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))+1
/6/b^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/6*I*x^3*Pi*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csg
n(I*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*P
i*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2-1/6*x^3*ln(d)+1/6*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)*x^3
```

Maxima [A]
time = 0.69, size = 125, normalized size = 0.98

$$\frac{1}{3}\,x^{3}\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3\,x^{4}}{d}-\frac{2\,\left(4\,b^{3}x^{3}\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\operatorname{Li}_2(-(d+1)e^{(2\,bx+2\,a)})-6\,bx\operatorname{Li}_3(-(d+1)e^{(2\,bx+2\,a)})+3\,\operatorname{Li}_4(-(d+1)e^{(2\,bx+2\,a)})\right)}{b^{4}d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
```

```
6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2*b*
x + 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.
time = 0.36, size = 381, normalized size = 2.98

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a)+2\,(d+1)\sinh(bx+a)+\sqrt{-4\,d-4}\right) + 2\,a^3\log\left(2\,(d+1)\cosh(bx+a)+2\,(d+1)\sinh(bx+a)-\sqrt{-4\,d-4}\right) + 12\,bx\,\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 12\,bx\,\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 2\,(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 2\,(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d -
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d + d \tanh(a + b x) + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d + d*tanh(a + b*x) + 1),x)
[Out] int(x^2*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.209 $\int x \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=101

$$\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}+\text{Poly}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arccoth(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*b*
x+2*a))/b^2
```

Rubi [A]
time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6375, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+d+d\tanh(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1+d)e^{2a+2bx}}\,dx$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}(b(1+d))\int\frac{}{1+(}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)\right.$$

Mathematica [A]

time = 0.07, size = 91, normalized size = 0.90

$$\frac{2b^2x^2\left(2\coth^{-1}(1+d+d\tanh(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[3,
 -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.50, size = 1584, normalized size = 15.68

method	result	size
risch	Expression too large to display	1584

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,-(1+d)*
exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*
a^2/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(
-1-d)^(1/2))-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp
(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*a/(1+
d)*dilog(1-exp(b*x+a)*(-1-d)^(1/2))-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*
x^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I
*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(
I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))^2-1/2/b*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a+1/2/b
*d*a/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1-exp(b*x+a)*
(-1-d)^(1/2))*x-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)+1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(
exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))^2-1/4/b^2*d*a^2/(1+d
)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*
(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1))+1/8*I*x^2*Pi*csgn(I*d*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^3-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2/(1
+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))
*x*a+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(-1-d)^(1/2))+1/2/b^2*d*a/(1+d)*d
ilog(1-exp(b*x+a)*(-1-d)^(1/2))-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*
```

```
a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a-1/4/b^2*a^2/(1+d)*
ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)-1/4/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+
2*a))*a^2+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b*a/(1+d)*ln(1-
exp(b*x+a)*(-1-d)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-1-d)^(1/2)
)+1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-1-d)^(1/2))-1/8*I*x^2*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*I*x^2*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*
x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)
*d+exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+
2*a))-1/4*x^2*ln(d)+1/4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)+1)*x^2
```

Maxima [A]

time = 0.70, size = 101, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4x^3}{d} - \frac{3\left(2b^2x^2\log\left((d+1)e^{(2bx+2a)}+1\right) + 2bx\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right) - \mathrm{Li}_3\left(-(d+1)e^{(2bx+2a)}\right)\right)}{b^3d}\right)bd + \frac{1}{2}x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d + 1/2*x^2*arccoth(d*tanh(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(87) = 174$.

time = 0.36, size = 322, normalized size = 3.19

$$\frac{2b^3x^3 + 3b^2x^2\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 6bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)+\sqrt{-4d-4}\right) - 3a^2\log\left(2(d+1)\cosh\left(bx+a\right)+2(d+1)\sinh\left(bx+a\right)-\sqrt{-4d-4}\right) - 3(b^2x^2-a^2)\log\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) - 3(b^2x^2-a^2)\log\left(-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) + 6\,\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) + 6\,\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{-4d-4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a)
 + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d -
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*acoth(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}(d+d\tanh(a+b\,x)+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x*acoth(d + d*tanh(a + b*x) + 1), x)
```

3.210 $\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=69

$$\frac{bx^2}{2}+x\coth^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,-\left((1+d)e^{2a+2bx}\right)\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccoth(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6367, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\coth^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6367

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \coth^{-1}(1+d+d\tanh(a+bx))\,dx &= x\coth^{-1}(1+d+d\tanh(a+bx)) + b\int \frac{x}{1+(1+d)e^{2a+2bx}}\,dx \\
&= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - (b(1+d))\int \frac{e^{2a+2bx}}{1+(1+d)e \ldots} \\
&= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2b \ldots}\right. \\
&= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2b \ldots}\right. \\
&= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2b \ldots}\right.
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. $2(69) = 138$.
time = 0.49, size = 201, normalized size = 2.91

$$x\coth^{-1}(1+d+d\tanh(a+bx)) + \frac{b^2x^2 + \log^2\left(e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-1-d}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{-1-d}\,e^{a+bx}\right) + 2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}+(1+d)e^{a+bx}\right) - 2bx\log((2+d)\cosh(a+bx)+d\sinh(a+bx)) - 2\mathrm{PolyLog}\left(2,-\sqrt{-1-d}\,e^{a+bx}\right) - 2\mathrm{PolyLog}\left(2,\sqrt{-1-d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*
E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
og[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)]
)/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(61) = 122$.
time = 0.89, size = 265, normalized size = 3.84

method	result
derivativedivides	$\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{d\tanh(bx+a)}{2}\right.}{2d}\right.}{}$
default	$\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(d\tanh(bx+a)+d)}{2}-\frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(-\frac{\operatorname{dilog}\left(\frac{d\tanh(bx+a)}{2}\right.}{2d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arccoth(1+d+d*tanh(b*x+a))*d*ln(d*tanh(b*x+a)+d)-1/2*arccoth(1+d
+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*d^2*(-1/2/d*dilog(1/2*d*tanh(b*x
+a)+1/2*d+1)-1/2/d*ln(d*tanh(b*x+a)+d)*ln(1/2*d*tanh(b*x+a)+1/2*d+1)+1/4/d*
ln(d*tanh(b*x+a)+d)^2-1/2/d*dilog(-1/2*(-d*tanh(b*x+a)-d)/d)-1/2/d*ln(-d*ta
nh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)/d)+1/2/d*dilog((-d*tanh(b*x+a)-d-2)
/(-2*d-2))+1/2/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d-2)/(-2*d-2))))
```

Maxima [A]
time = 0.69, size = 72, normalized size = 1.04

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\,\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*tanh(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(60) = 120.
time = 0.36, size = 238, normalized size = 3.45

$$\frac{b^2x^2+bx\log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+\sqrt{-4\,d-4}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-\sqrt{-4\,d-4}\right)-(bx+a)\log\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d-4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
```

```
h(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a
) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*
x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(acoth(d*tanh(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d+d\tanh(a+b\,x)+1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(acoth(d + d*tanh(a + b*x) + 1), x)
```

3.211 $\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.68, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1+d+d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+d+d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*tanh(a + b*x) + d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tanh(b*x + a) + d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(d+d\tanh(a+b\,x)+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d + d*tanh(a + b*x) + 1)/x,x)
```

```
[Out] int(acoth(d + d*tanh(a + b*x) + 1)/x, x)
```

3.212 $\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=168

$$\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{3x^2}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arccoth(1-d-d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,-(1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6375, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left(-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{8}x^4\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{4}x^4\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}(b(1-d))\int \frac{e}{1+(}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)\right.$$

Mathematica [A]
time = 0.13, size = 144, normalized size = 0.86

$$\frac{1}{16}\left(4x^4\coth^{-1}(1-d-d\tanh(a+bx)) - 2x^4\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3} + \frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 - d - d*Tanh[a + b*x]] - 2*x^4*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Po
lyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((-1 + d)*E
^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/b^4)/1
6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.79, size = 1802, normalized size = 10.73

method	result	size
risch	Expression too large to display	1802

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*b*x^5+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+
1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*
b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn
(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I/(exp
(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)+1))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3-3/16/b^4/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))+
1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(
2*b*x+2*a))*x^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1
/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^
4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp
(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2-1/2/b^3*d*a^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*d*a^3
/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(
d-1)^(1/2))*x+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d
/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*
(d-1)^(1/2))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*a^3
/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x-3/8/b^4*d*a^4/(d-1)*ln(1-(d-1)*exp(2*b*
x+2*a))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^4/(d-1
)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)
^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/16*I*x^4*Pi*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+3/8/b^4*a
^4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))+1/4/b^4*a^3/(d-1)*polylog(2,(d-1)*exp(2
*b*x+2*a))+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-3/8/b^2/(d-1)*po
lylog(3,(d-1)*exp(2*b*x+2*a))*x^2+3/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2
*a))*x+3/16/b^4*d/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^3/(d-1)*d
ilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(
1/2))-1/8*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4-1/2/b^4*a^4/(d-1)*ln(1-exp
(b*x+a)*(d-1)^(1/2))-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^4
*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3-1/16*I*x^4*Pi*csgn(I*d*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2
*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/8/b^4*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(
2*b*x+2*a)-1)-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*
a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b
*x+2*a)-1))+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/4*x^4*ln(exp(b*
x+a))-1/8*x^4*ln(d)+1/8*I*x^4*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a
)-1))^3-1/8/b^4*d*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)
```

Maxima [A]
time = 0.70, size = 146, normalized size = 0.87

$$-\frac{1}{4}\,x^4\,\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2((d-1)e^{(2\,bx+2\,a)})-6\,b^2x^2\mathrm{Li}_3((d-1)e^{(2\,bx+2\,a)})+6\,bx\mathrm{Li}_4((d-1)e^{(2\,bx+2\,a)})-3\,\mathrm{Li}_5((d-1)e^{(2\,bx+2\,a)})\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")

[Out] -1/4*x^4*arccoth(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a)) - 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d - 1)*e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(135) = 270$.
time = 0.39, size = 423, normalized size = 2.52

$$\frac{2 b^5 x^5 - 5 b^4 x^4 \log\left(\frac{d \cosh\left(bx+a\right) + d \sinh\left(bx+a\right)}{(d-2)\cosh\left(bx+a\right) + d \sinh\left(bx+a\right)}\right) - 20 b^3 x^3 \mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) - 20 b^3 x^3 \mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) - 5 a^4 \log\left(2 (d-1)\cosh\left(bx+a\right) + 2 (d-1)\sinh\left(bx+a\right) + 2\sqrt{d-1}\right) - 5 a^4 \log\left(2 (d-1)\cosh\left(bx+a\right) + 2 (d-1)\sinh\left(bx+a\right) - 2\sqrt{d-1}\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, \sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, -\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) - 120 bx \mathrm{polylog}\left(4, \sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) - 120 bx \mathrm{polylog}\left(4, -\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) - 5\left(b^4 x^4 - a^4\right)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right) + 1\right) - 5\left(b^4 x^4 - a^4\right)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right) + 1\right) + 120 \mathrm{polylog}\left(5, \sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right) + 120 \mathrm{polylog}\left(5, -\sqrt{d-1}\left(\cosh\left(bx+a\right) + \sinh\left(bx+a\right)\right)\right)}{40 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")

[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^3 \operatorname{acoth}\left(d \tanh\left(a+bx\right) + d - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*acoth(1-d-d*tanh(b*x+a)),x)

[Out] -Integral(x**3*acoth(d*tanh(a + b*x) + d - 1), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3 \operatorname{acoth}(d + d \tanh(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*acoth(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x^3*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.213 $\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=139

$$\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{x\text{P}}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccoth(1-d-d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6375, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{3}(b(1-d))\int \frac{e}{1+(}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)\right.$$

Mathematica [A]
time = 0.07, size = 119, normalized size = 0.86

$$\frac{1}{24}\left(8x^3\coth^{-1}(1-d-d\tanh(a+bx)) - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 - d - d*Tanh[a + b*x]] - 4*x^3*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x*Poly
Log[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((-1 + d)*E^(2*
(a + b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.70, size = 1745, normalized size = 12.55

method	result	size
risch	Expression too large to display	1745

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+exp(b
*x+a)*(d-1)^(1/2))-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(d-
```

```
1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-
1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/3/b^3/(d-1)*l
n(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x
^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(
3,(d-1)*exp(2*b*x+2*a))*x-1/3*x^3*ln(exp(b*x+a))+1/12*b*x^4+1/12*I*x^3*Pi*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/12*I*x^3*Pi*csgn(I*d*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)+1))^3-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2)
)*x-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/6*I*x^3*Pi*csgn(I/
(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/6/b^3*d*a^3/(d-
1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/6/(d-1)*ln(1-(d-1)*exp(2*b
*x+2*a))*x^3+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*
b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*cs
gn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/12*I*x^3*P
i*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d)*csgn(I*d*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)+1))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*I*x^3*Pi*csgn(I/
(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp
(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))-1/2/b^3*d*a^2/(d-1)*dil
og(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(
1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3-1/4/b*d/(d-1)*polylog(
2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))
*a^2+1/4/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(d-1)*ln(1-(
d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x
+1/6*I*x^3*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6/b^3*a^3/(d-
1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)+1/2/b^2*d/(d-1)*ln(1-(d-1)*exp(2*b
*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b^3*d*a
^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(
d-1)^(1/2))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2
*b*x+2*a)-1))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+
2*a))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*I*x^
3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn
(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*x^3*ln(d)+1/6*ln(ex
p(2*b*x+2*a)*d-exp(2*b*x+2*a)-1)*x^3
```

Maxima [A]
time = 0.71, size = 123, normalized size = 0.88

$$-\frac{1}{3}x^3\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d-1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3((d-1)e^{(2bx+2a)})+3\mathrm{Li}_4((d-1)e^{(2bx+2a)})\right)}{b^4d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arccoth(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*b*x
 + 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(112) = 224$.
time = 0.36, size = 359, normalized size = 2.58

$$\frac{b^4x^4 - 2b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6b^2x^2\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 6b^2x^2\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)+2\sqrt{d-1}\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)-2\sqrt{d-1}\right) + 12bx\,\mathrm{polylog}\left(3,\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 12bx\,\mathrm{polylog}\left(3,-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 2\left(b^3x^3+a^3\right)\log\left(\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 2\left(b^3x^3+a^3\right)\log\left(-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 12\,\mathrm{polylog}\left(4,\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 12\,\mathrm{polylog}\left(4,-\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*
x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2\,\mathrm{acoth}\left(d\tanh(a+bx)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x**2*acoth(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{acoth}(d + d \tanh(a + b x) - 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*acoth(d + d*tanh(a + b*x) - 1),x)
[Out] int(-x^2*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.214 $\int x \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=110

$$\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}+\frac{\text{Poly}}{}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arccoth(1-d-d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2*b*
x+2*a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6375, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6375

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tanh[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1-d)e^{2a+2bx}}\,d$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}(b(1-d))\int\frac{}{1+(}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d\right.$$

Mathematica [A]

time = 0.06, size = 93, normalized size = 0.85

$$\frac{2b^2x^2\left(2\coth^{-1}(1-d-d\tanh(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{-1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.57, size = 1664, normalized size = 15.13

method	result	size
risch	Expression too large to display	1664

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/8*I*x^2*P
i*csgn(I*exp(2*b*x+2*a))^3+1/2/b^2*d*a/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2)
)+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^2*d/(d-1)*ln(1-(d
-1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x-1/4
/b^2*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1-(d-1)*exp(2
*b*x+2*a))*x*a-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/8*I*x^2*Pi*cs
gn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*
b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/4/b^2*d*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp
(2*b*x+2*a)-1)+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/b*d/(d-1)
*ln(1-(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2)
)*x+1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/8/b^2/(d-1)*polylog(3,(d-1)*
exp(2*b*x+2*a))+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn
(I*d)*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*I*x^2*Pi*csgn(I*exp(b
*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*
x+2*a))*a-1/4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^2-1/2/b^2*a^2/(d-1)*ln(1
-exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8
*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a
)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))+1/4*I*
x^2*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d)
*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2/b*a/(d-1)*ln(1+exp(b*x+a
)*(d-1)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d
*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*
```

```
(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/8/b^2*d/(d-1
)*polylog(3,(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*
a^2+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x-1/4*I*x^2*Pi*csgn(I/(exp(
2*b*x+2*a)+1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*ex
p(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)
)^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(e
xp(2*b*x+2*a)*d-exp(2*b*x+2*a)-1))^3+1/4/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-
exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-
1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*
a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^2
*Pi*csgn(I*d*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/4*x^2*ln(d)+1/4*ln(exp(
2*b*x+2*a)*d-exp(2*b*x+2*a)-1)*x^2
```

Maxima [A]
time = 0.69, size = 100, normalized size = 0.91

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d - 1/2*x^2*arccoth(d*tanh(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(89) = 178.
time = 0.36, size = 305, normalized size = 2.77

$$\frac{2\,b^3x^3-3\,b^2x^2\log\left(\frac{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}{(d-2)\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}\right)-6\,bx\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6\,bx\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3\,a^2\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d-1}\right)-3\,a^2\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-2\,\sqrt{d-1}\right)-3\,(b^2x^2-a^2)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3\,(b^2x^2-a^2)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\,\mathrm{polylog}\left(3,\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\,\mathrm{polylog}\left(3,-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2
)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqrt
(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2*
sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*p
olylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x\,\mathrm{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-d-d*tanh(b*x+a)),x)
[Out] -Integral(x*acoth(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x\,\mathrm{acoth}(d+d\tanh(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*acoth(d + d*tanh(a + b*x) - 1),x)
[Out] int(-x*acoth(d + d*tanh(a + b*x) - 1), x)
```

3.215 $\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=76

$$\frac{bx^2}{2}+x\coth^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,-\left((1-d)e^{2a+2bx}\right)\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccoth(1-d-d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1/4
*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A]

time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6367, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\coth^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6367

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1-d-d\tanh(a+bx))\,dx = x\coth^{-1}(1-d-d\tanh(a+bx)) + b\int \frac{x}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - (b(1-d))\int \frac{e^{2a+2bx}}{1+(1-d)e}$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right.$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right.$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right.$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(76) = 152.
time = 0.48, size = 200, normalized size = 2.63

$$x\coth^{-1}(1-d-d\tanh(a+bx)) + \frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-1+d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{-1+d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(-1+(-1+d)e^{2(a+bx)}\right)\right)-2bx\log((-2+d)\cosh(a+bx)+d\sinh(a+bx))-2\text{PolyLog}\left(2,-\sqrt{-1+d}\,e^{a+bx}\right)-2\text{PolyLog}\left(2,\sqrt{-1+d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1
+ d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]
] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E
^(a + b*x)])/(4*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(68) = 136.
time = 1.00, size = 281, normalized size = 3.70

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\tan}{}\right.}{}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}-\frac{d^2\left(-\frac{\operatorname{dilog}\left(-\frac{d\tan}{}\right.}{}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(-1/2*arccoth(1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)+1/2*arccoth(
1-d-d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)-1/2*d^2*(-1/2/d*dilog(-1/2*d*tanh
(b*x+a)-1/2*d+1)-1/2/d*ln(-d*tanh(b*x+a)-d)*ln(-1/2*d*tanh(b*x+a)-1/2*d+1)+
1/4/d*ln(-d*tanh(b*x+a)-d)^2+1/2/d*dilog((-d*tanh(b*x+a)-d+2)/(-2*d+2))+1/2
/d*ln(-d*tanh(b*x+a)+d)*ln((-d*tanh(b*x+a)-d+2)/(-2*d+2))-1/2/d*dilog(-1/2*
(-d*tanh(b*x+a)-d)/d)-1/2/d*ln(-d*tanh(b*x+a)+d)*ln(-1/2*(-d*tanh(b*x+a)-d)
/d)))
```

Maxima [A]
time = 0.71, size = 73, normalized size = 0.96

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\operatorname{arcoth}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*tanh(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(63) = 126.
time = 0.36, size = 227, normalized size = 2.99

$$\frac{b^2x^2-bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+2\,\sqrt{d-1}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-2\,\sqrt{d-1}\right)-(bx+a)\log\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-(bx+a)\log\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-\mathrm{Li}_2\left(\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-\mathrm{Li}_2\left(-\sqrt{d-1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
```

```
(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(
d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*tanh(b*x+a)),x)
[Out] -Integral(acoth(d*tanh(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(-d*tanh(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d+d\tanh(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d + d*tanh(a + b*x) - 1),x)
[Out] int(-acoth(d + d*tanh(a + b*x) - 1), x)
```

3.216 $\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.24, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]]/x, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1-d-d\tanh(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-d-d*tanh(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -integrate(arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-arccoth(d*tanh(b*x + a) + d - 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{acoth}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*tanh(b*x+a))/x,x)
```

```
[Out] -Integral(acoth(d*tanh(a + b*x) + d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tanh(b*x + a) - d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{acoth}(d + d\tanh(a + b\,x) - 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d + d*tanh(a + b*x) - 1)/x,x)
```

```
[Out] int(-acoth(d + d*tanh(a + b*x) - 1)/x, x)
```

3.217 $\int x^2 \coth^{-1}(c + d\coth(a + bx))\,dx$

Optimal. Leaf size=303

$$\frac{1}{3}x^3 \coth^{-1}(c+d\coth(a+bx))+\frac{1}{6}x^3 \log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{6}x^3 \log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x^2\mathrm{Poly}}{}$$

```
[Out] 1/3*x^3*arccoth(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/
b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1-c+d)
)/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A]
time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6381, 2221, 2611, 6744, 2320, 6724}

$$\frac{\mathrm{Li}_4\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\mathrm{Li}_4\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\mathrm{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\mathrm{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\mathrm{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{6}x^3\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6381

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*
((1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(c + d\coth(a + bx))\, dx = \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) - \frac{1}{3}(b(1 - c - d)) \int \frac{e^{2a+2b}}{1 - c + d + (-1}$$

$$= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) -$$

$$= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) -$$

$$= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) -$$

$$= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) -$$

$$= \frac{1}{3}x^3 \coth^{-1}(c + d\coth(a + bx)) + \frac{1}{6}x^3 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) -$$

Mathematica [A]
time = 0.23, size = 265, normalized size = 0.87

$$\frac{1}{3}x^3\coth^{-1}(c+d\coth(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{1-c+d}\right) - 4b^3x^3\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{-1-c+d}\right) + 6b^2x^2\text{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 6b^2x^2\text{PolyLog}\left(2,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) - 6bx\text{PolyLog}\left(3,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) + 6bx\text{PolyLog}\left(3,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right) + 3\text{PolyLog}\left(4,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right) - 3\text{PolyLog}\left(4,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Coth[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(1 - c + d)] - 4*b^3*x^3*Log[1 + ((1 + c + d)*E^(2*(a + b*x))
)/(-1 - c + d)] + 6*b^2*x^2*PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 +
 c - d)] - 6*b^2*x^2*PolyLog[2, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]
- 6*b*x*PolyLog[3, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + 6*b*x*Pol
yLog[3, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)] + 3*PolyLog[4, ((-1 + c
+ d)*E^(2*(a + b*x)))/(-1 + c - d)] - 3*PolyLog[4, ((1 + c + d)*E^(2*(a + b
*x)))/(1 + c - d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.57, size = 5222, normalized size = 17.23

method	result	size
risch	Expression too large to display	5222

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.49, size = 277, normalized size = 0.91

$$\frac{1}{3}x^3\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)-\frac{1}{18}bd\left(\frac{4b^3x^3\log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)+3\mathrm{Li}_4\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d}-\frac{4b^3x^3\log\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}+1\right)+6b^2x^2\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)-6bx\mathrm{Li}_3\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)+3\mathrm{Li}_4\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d +
1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b
^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(
(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)))/(b^4*d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(259) = 518$.
time = 0.43, size = 879, normalized size = 2.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
 + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c - d
 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d + 1
)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d +
 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sin
h(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
+ d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1
)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*poly
log(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*b
*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)
)) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b
*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
```

```
+ sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cosh
(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt((
c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a
^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 - 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + s
inh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(c + d\coth\left(a + bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(c + d*coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acoth}\left(c+d\,\mathrm{coth}\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*acoth(c + d*coth(a + b*x)), x)
```

3.218 $\int x \coth^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=229

$$\frac{1}{2}x^2 \coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2 \log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{4}x^2 \log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{x\text{Pol}}{}$$

```
[Out] 1/2*x^2*arccoth(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*exp
(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-1/
8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))/b^2
```

Rubi [A]
time = 0.27, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6381, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{4}x^2\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6381

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + (-Dist[b*((1 - c - d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*
((1 + c + d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\coth^{-1}(c+d\coth(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))-\frac{1}{2}(b(1-c-d))\int\frac{e^{2a+2bx}}{1-c+d+(-1+\cdots}\\
&= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\cdots\\
&= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\cdots\\
&= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\cdots\\
&= \frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\cdots
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 199, normalized size = 0.87

$$\frac{1}{2}x^2\coth^{-1}(c+d\coth(a+bx))+\frac{2b^2x^2\log\left(1+\frac{(-1+c+d)e^{2(a+bx)}}{1-c+d}\right)-2b^2x^2\log\left(1+\frac{(1+c+d)e^{2(a+bx)}}{-1-c+d}\right)+2bx\,\text{PolyLog}\left(2,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)-2bx\,\text{PolyLog}\left(2,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)-\text{PolyLog}\left(3,\frac{(-1+c+d)e^{2(a+bx)}}{-1+c-d}\right)+\text{PolyLog}\left(3,\frac{(1+c+d)e^{2(a+bx)}}{1+c-d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Coth[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c + d)*E^(
2*(a + b*x)))/(1 - c + d)] - 2*b^2*x^2*Log[1 + ((1 + c + d)*E^(2*(a + b*x))
)/(-1 - c + d)] + 2*b*x*PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c -
 d)] - 2*b*x*PolyLog[2, ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)] - PolyLo
g[3, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[3, ((1 + c + d)
*E^(2*(a + b*x)))/(1 + c - d)])/(8*b^2)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.24, size = 4918, normalized size = 21.48

method	result	size
risch	Expression too large to display	4918

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a-1/4/b*c/(1+c+d)*pol
ylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x+1/2/b*a/(1+c+d)*ln((-exp(b*x+a)*c-
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x
+1/2/b*a/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(
b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/4/b*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*
b*x+2*a)/(1+c-d))*x+1/2/b^2*a^2*c/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1
+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+
c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1
+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a^2/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+(
(1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b^2*c/(1+c+
d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*c/(1+c+d)*polylog(2,(1+
c+d)*exp(2*b*x+2*a)/(1+c-d))*a-1/4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a
)/(1+c-d))*a^2-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*
a-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+
2*a)-1)/(exp(2*b*x+2*a)-1))^3+1/2/b^2*a^2/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+
a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/4/b/(1+
c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x-1/4/b^2*d*a^2/(1+c+d)*ln(e
xp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)-1/4/b^2*c*a^2/(1+c+d
)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-c+d-1)+1/2/b*d*a/(1+c
+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+
c-d)*(1+c+d))^(1/2))*x+1/2/b*d*a/(1+c+d)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((1+
c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/2/b*c/(1+c+d)*
ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a-1/2/b*d/(1+c+d)*ln(1-(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))*x*a+1/2/b*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))
*x*a+1/2/b*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a-1/2/b*c*a/(c+
d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c
```

```
-d-1)*(c+d-1))^(1/2))*x-1/2/b*c*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c
-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-
1)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d
-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c-d
-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x+1/4/b^2*c*a^2/(c+
d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/4/b^2*d*a
^2/(c+d-1)*ln(exp(2*b*x+2*a)*c+exp(2*b*x+2*a)*d-exp(2*b*x+2*a)-c+d+1)+1/2/b
^2*d*a/(1+c+d)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-ex
p(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*d*a/(1+c+d)*dilog((exp(b*x+a)*c+
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1
/2/b^2*a^2*c/(1+c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*c/(1+c+d)*dilog((-exp(b*x+a
)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2
))+1/2/b*a*c/(1+c+d)*ln((-exp(b*x+a)*c-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*c/(1+c+d)*ln((exp(b*x+a)*c+
exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x
+1/2/b^2*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2
)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((exp(b*x+a)*
c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))
-1/4/b^2/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c+d-1)*pol
ylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*
exp(2*b*x+2*a)/(c-d-1))*a+1/2/b^2*a^2/(c+d-1)*ln((-exp(b*x+a)*c-exp(b*x+a)*
d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/
(c+d-1)*ln((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/(
(c-d-1)*(c+d-1))^(1/2))-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/
(c-d-1))-1/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))+1/4*c/
(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)
*exp(2*b*x+2*a)/(c-d-1))*x^2-1/2/b^2*c*a/(c+d-1)*dilog((-exp(b*x+a)*c-exp(b
*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^
2*a*c/(1+c+d)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(
b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*
x+2*a)/(1+c-d))-1/4/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/2/b^
2*c*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(
b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((-exp(b*x+a)*c-e
xp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/
2/b^2*d*a/(c+d-1)*dilog((exp(b*x+a)*c+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-
exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/4/b^2*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b
*x+2*a)/(c-d-1))*a^2+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-
1))*x+1/4/b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a+1/4/b^2
*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*d/(c+d-1)*polylog
(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*d/...
```

Maxima [A]

time = 0.48, size = 213, normalized size = 0.93

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^3d}-\frac{2\,b^2x^2\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)-\mathrm{Li}_3\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^3d}\right)+\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) + 1/
2*x^2*arccoth(d*coth(b*x + a) + c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(195) = 390$.
time = 0.38, size = 729, normalized size = 3.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
 + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
+ 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))
) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(
c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
+ a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
- 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
- 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt((
c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 - a
^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c - d
 - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(c
 - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}\left(c + d \coth\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(x*acoth(c + d*coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \coth(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(c + d*coth(a + b*x)),x)
```

```
[Out] int(x*acoth(c + d*coth(a + b*x)), x)
```

3.219 $\int \coth^{-1}(c + d\coth(a + bx))\, dx$

Optimal. Leaf size=150

$$x\coth^{-1}(c+d\coth(a+bx))+\frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right)-\frac{1}{2}x\log\left(1-\frac{(1+c+d)e^{2a+2bx}}{1+c-d}\right)+\frac{\text{PolyLog}(}{}$$

```
[Out] x*arccoth(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a)/
(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A]
time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6373, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+x\coth^{-1}(d\coth(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6373

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + (-Dist[b*(1 - c - d), Int[x*(E^(2*a + 2*b*
x)/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x))), x], x] + Dist[b*(1 + c + d),
 Int[x*(E^(2*a + 2*b*x)/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x))), x], x])
 /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\coth(a+bx))\,dx = x\coth^{-1}(c+d\coth(a+bx)) - (b(1-c-d))\int \frac{e^{2a+2bx}x}{1-c+d+(-1+c+d}$$

$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log$$

$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log$$

$$= x\coth^{-1}(c+d\coth(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{2}x\log$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(150) = 300.
time = 1.25, size = 369, normalized size = 2.46

$$x\coth^{-1}(c+d\coth(a+bx)) - \frac{-\left((a+bx)\log\left(1-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right)\right) - (a+bx)\log\left(1+\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) + (a+bx)\log\left(1-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + (a+bx)\log\left(1+\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + a\log\left(1+d-e^{2(a+bx)}+de^{2(a+bx)}+c\left(-1+e^{2(a+bx)}\right)\right) - a\log\left(1+c-e^{2(a+bx)}-ce^{2(a+bx)}-d\left(1+e^{2(a+bx)}\right)\right) - \text{PolyLog}\left(2,-\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) - \text{PolyLog}\left(2,\frac{\sqrt{-1+c+d}\,e^{a+bx}}{\sqrt{-1+c-d}}\right) + \text{PolyLog}\left(2,-\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right) + \text{PolyLog}\left(2,\frac{\sqrt{1+c+d}\,e^{a+bx}}{\sqrt{1+c-d}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Coth[a + b*x]] - (-((a + b*x)*Log[1 - (Sqrt[-1 + c + d]*E^(
a + b*x))/Sqrt[-1 + c - d]]) - (a + b*x)*Log[1 + (Sqrt[-1 + c + d]*E^(a + b
*x))/Sqrt[-1 + c - d]] + (a + b*x)*Log[1 - (Sqrt[1 + c + d]*E^(a + b*x))/Sq
rt[1 + c - d]] + (a + b*x)*Log[1 + (Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[1 + c
 - d]] + a*Log[1 + d - E^(2*(a + b*x)) + d*E^(2*(a + b*x)) + c*(-1 + E^(2*(
a + b*x)))] - a*Log[1 + c - E^(2*(a + b*x)) - c*E^(2*(a + b*x)) - d*(1 + E^
(2*(a + b*x)))] - PolyLog[2, -((Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[-1 + c -
 d])] - PolyLog[2, (Sqrt[-1 + c + d]*E^(a + b*x))/Sqrt[-1 + c - d]] + PolyL
og[2, -((Sqrt[1 + c + d]*E^(a + b*x))/Sqrt[1 + c - d])] + PolyLog[2, (Sqrt[
1 + c + d]*E^(a + b*x))/Sqrt[1 + c - d]])/(2*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(138) = 276.
time = 1.31, size = 361, normalized size = 2.41

method	result
derivativedivides	$\frac{\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c+}{1-c-d}\right.}{2d}\right.}{}}{}$
default	$\frac{\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\operatorname{arccoth}(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{-d\coth(bx+a)-c+}{1-c-d}\right.}{2d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(1/2*arccoth(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*arccoth(c+d*
coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*d^2*(1/2/d*dilog((-d*coth(b*x+a)-c+
1)/(1-c-d))+1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-c+1)/(1-c-d))-1/2
/d*dilog((-d*coth(b*x+a)-c-1)/(-1-c-d))-1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*c
oth(b*x+a)-c-1)/(-1-c-d))+1/2/d*dilog((-d*coth(b*x+a)-c-1)/(-1-c+d))+1/2/d*
ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c-1)/(-1-c+d))-1/2/d*dilog((-d*coth
(b*x+a)-c+1)/(1-c+d))-1/2/d*ln(-d*coth(b*x+a)-d)*ln((-d*coth(b*x+a)-c+1)/(1
-c+d))))
```

Maxima [A]
time = 0.49, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right)+x\operatorname{arcoth}\left(d\coth\left(bx+a\right)+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d - 1)
*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(128) = 256$.
time = 0.41, size = 539, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) + (
c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*
sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(c +
 d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((c
 + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)
*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log(2*(c
+ d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((
c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt((c
+ d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt((c
+ d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt((c + d
 + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt((c + d - 1
)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt((c + d - 1)/(
c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(c + d \coth\left(a + bx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*coth(b*x+a)),x)
[Out] Integral(acoth(c + d*coth(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(d*coth(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}\left(c+d\,\mathrm{coth}\left(a+b\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*coth(a + b*x)),x)
[Out] int(acoth(c + d*coth(a + b*x)), x)
```

3.220 $\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(c+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(c+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[c + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[c + d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccoth}(c+d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*coth(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(c + d \coth\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(acoth(c + d*coth(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c + d \coth(a + b x))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*coth(a + b*x))/x,x)
[Out] int(acoth(c + d*coth(a + b*x))/x, x)
```

3.221 $\int x^3 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=152

$$\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x^3\mathrm{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{3x^2\mathrm{Poly}}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arccoth(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,(
1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6377, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left((d+1)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{4}x^4\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{4}b\int\frac{x^4}{1+(-1-d)e^{2a+2bx}}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{4}(b(1+d))\int\frac{}{1+(}$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

$$=\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1+d)e\right.$$

Mathematica [A]
time = 0.16, size = 141, normalized size = 0.93

$$\frac{1}{16}\left(4x^4\coth^{-1}(1+d+d\coth(a+bx))-2x^4\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)+\frac{4x^3\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{b}+\frac{6x^2\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2}+\frac{6x\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}+\frac{3\text{PolyLog}\left(5,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 + d + d*Coth[a + b*x]] - 2*x^4*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Poly
Log[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((1 + d)*E^(2*
(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/b^4)/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.71, size = 1698, normalized size = 11.17

method	result	size
risch	Expression too large to display	1698

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-3/8/b^4*d*a^4/(1+d)*ln(1-(1+d
)*exp(2*b*x+2*a))-1/4/b^4*d*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))+1/2/b
^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*
x+a)*(1+d)^(1/2))-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^2
*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2+1/20*b*x^5-1/16*I*x^4*Pi*csgn(
I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi
*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8
/b^4*d*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I
*exp(2*b*x+2*a))^3-3/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^
3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a
)*(1+d)^(1/2))*x+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^
3*a^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*d*a^3/(1+d)*ln(1-(1+d)*exp
(2*b*x+2*a))*x+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^3*d
*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^4*d*a^3/(1+d)*dilog(1-exp(b
*x+a)*(1+d)^(1/2))-1/8*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-3/8/b^3/(1+d)
*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^
(1/2))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+3/16/b^4*d/(1+d)*poly
log(5,(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2
))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-3/8/b^4*a^4/(1+d)*ln(1
-(1+d)*exp(2*b*x+2*a))-1/4/b^4*a^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))+3/
8/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-1/4/b/(1+d)*polylog(2,(1+d)
*exp(2*b*x+2*a))*x^3+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*
csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3+3/16/b^4/(1+d)*polylog(5,(1+d
)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^4-1/16*I*x^4*Pi*cs
gn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I
/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn
(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*
x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I
*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(e
xp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d+e
xp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a
)-1))^2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1
/8/b^4*a^4/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-
1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln
(d)
```

Maxima [A]
time = 0.70, size = 146, normalized size = 0.96

$$\frac{1}{4}\,x^4\,\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\left(2\,b^4x^4\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)+6\,bx\mathrm{Li}_4\left((d+1)e^{(2\,bx+2\,a)}\right)-3\,\mathrm{Li}_5\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccoth(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1)*e
^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(132) = 264$.
time = 0.37, size = 423, normalized size = 2.78

$$\frac{2 b^5 x^5 + 5 b^4 x^4 \log\left(\frac{d \cosh(bx+a) + (d+2)\sinh(bx+a)}{d\cosh(bx+a) + d\sinh(bx+a)}\right) - 20 b^3 x^3 \mathrm{Li}_2\left(\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) - 20 b^3 x^3 \mathrm{Li}_2\left(-\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) - 5 a^4 \log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) + 2\sqrt{d+1}\right) - 5 a^4 \log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) - 2\sqrt{d+1}\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, \sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, -\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) - 120 bx \,\mathrm{polylog}\left(4, \sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) - 120 bx \,\mathrm{polylog}\left(4, -\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) - 5(b^4x^4 - a^4)\log\left(\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a)) + 1\right) - 5(b^4x^4 - a^4)\log\left(-\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a)) + 1\right) + 120 \,\mathrm{polylog}\left(5, \sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right) + 120 \,\mathrm{polylog}\left(5, -\sqrt{d+1}\,(\cosh(bx+a) + \sinh(bx+a))\right)}{40 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(b*
x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a) + s
inh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x
 + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x + a) +
 sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)
)) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(
b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^
4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*po
lylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sqr
t(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}\left(d \coth\left(a + bx\right) + d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**3*acoth(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acoth}(d + d \coth(a + b x) + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acoth(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^3*acoth(d + d*coth(a + b*x) + 1), x)
```

3.222 $\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=126

$$\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x^2\mathrm{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{x\mathrm{PolyL}}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccoth(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp(2
*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6377, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\mathrm{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{x\mathrm{Li}_3\left((d+1)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\mathrm{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{3}x^3\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1-d)e^{2a+2bx}}\,d$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) + \frac{1}{3}(b(1+d))\int \frac{e}{1+(-}$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e\right.$$

$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+d)e\right.$$

Mathematica [A]
time = 0.13, size = 116, normalized size = 0.92

$$\frac{1}{24}\left(8x^3\coth^{-1}(1+d+d\coth(a+bx)) - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,\frac{e^{-2(a+bx)}}{1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 + d + d*Coth[a + b*x]] - 4*x^3*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x*PolyLo
g[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((1 + d)*E^(2*(a +
 b*x)))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.70, size = 1641, normalized size = 13.02

method	result	size
risch	Expression too large to display	1641

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b^3*d*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/4/b/(1+d)*polyl
og(2,(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a)
```

```
)*a^2+1/4/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(
1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/
3*x^3*ln(exp(b*x+a))+1/12*b*x^4-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a
))*x^2+1/4/b^3*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*
polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(
1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^3/(1+d
)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1
/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^2/(1+d
)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)
*exp(2*b*x+2*a))^3+1/3/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/12*I*x^
3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2
-1/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-
exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/2/b^3*
a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*
x+2*a))*a^3-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2
*a))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/12*I*x^3
*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12
*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+
1/12*I*x^3*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x
+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*
b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)
-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/6/b
^3*a^3/(1+d)*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)-1/6*I*x^3*Pi*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))
*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+
d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I
*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x
+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(
2*b*x+2*a)-1))-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polyl
og(4,(1+d)*exp(2*b*x+2*a))+1/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1
/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(
exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn
(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/6*x^3*ln
(d)+1/6*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)*x^3
```

Maxima [A]
time = 0.71, size = 123, normalized size = 0.98

$$\frac{1}{3}x^3\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d+1)e^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left((d+1)e^{(2bx+2a)}\right)+3\mathrm{Li}_4\left((d+1)e^{(2bx+2a)}\right)\right)}{b^4d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
```

```
6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(109) = 218$.
time = 0.38, size = 359, normalized size = 2.85

$$\frac{b^4x^4 + 2\,b^3x^3\log\left(\frac{d\cosh\left(bx+a\right)+\left(d+2\right)\sinh\left(bx+a\right)}{d\cosh\left(bx+a\right)+d\sinh\left(bx+a\right)}\right) - 6\,b^2x^2\mathrm{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 6\,b^2x^2\mathrm{Li}_2\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) + 2\,a^3\log\left(2\left(d+1\right)\cosh\left(bx+a\right)+2\left(d+1\right)\sinh\left(bx+a\right)+2\sqrt{d+1}\right) + 2\,a^3\log\left(2\left(d+1\right)\cosh\left(bx+a\right)+2\left(d+1\right)\sinh\left(bx+a\right)-2\sqrt{d+1}\right) + 12\,bx\mathrm{polylog}\left(3,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) + 12\,bx\mathrm{polylog}\left(3,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 2\left(b^3x^3+a^3\right)\log\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) - 2\left(b^3x^3+a^3\right)\log\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right) - 12\,\mathrm{polylog}\left(4,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 12\,\mathrm{polylog}\left(4,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a
) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*
x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\mathrm{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acoth}(d+d\,\mathrm{coth}(a+b\,x)+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^2*acoth(d + d*coth(a + b*x) + 1), x)
```

3.223 $\int x \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=100

$$\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}+\frac{\text{PolyLog}}{}$$

[Out] 1/6*b*x^3+1/2*x^2*arccoth(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+2*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*x+2*a))/b^2

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6377, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]

[Out] (b*x^3)/6 + (x^2*ArcCoth[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + PolyLog[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1-d)e^{2a+2bx}}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}(b(1+d))\int\frac{}{1+(}$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)\right.$$

$$= \frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)\right.$$

Mathematica [A]

time = 0.07, size = 90, normalized size = 0.90

$$\frac{2b^2x^2\left(2\coth^{-1}(1+d+d\coth(a+bx))-\log\left(1-\frac{e^{-2(a+bx)}}{1+d}\right)\right)+2bx\text{PolyLog}\left(2,\frac{e^{-2(a+bx)}}{1+d}\right)+\text{PolyLog}\left(3,\frac{e^{-2(a+bx)}}{1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3, 1/
((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.45, size = 1560, normalized size = 15.60

method	result	size
risch	Expression too large to display	1560

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/8*I*x^2*Pi*csgn(
I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*cs
gn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)-1))-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/4/b^2*d*a^2/(1+d)*ln(exp(
2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*I*
x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*
b*x+2*a)-1)*exp(2*b*x+2*a))+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2)
)-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*
x+2*a)-1)*exp(2*b*x+2*a))^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x
-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I
*x^2*Pi*csgn(I*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-
1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp
(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1))^2-1/4/b^2*a^2/(1+d)*ln(
exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+
d)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b/(1+d)*ln(
1-(1+d)*exp(2*b*x+2*a))*x*a-1/4/b^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))
*a+1/2/b*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+
a)*(1+d)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/4/b^2*
d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2+1/8/b^2/(1+d)*polylog(3,(1+d)*exp(2*
b*x+2*a))-1/4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2+1/2/b*d*a/(1+d)*ln(1-exp
(b*x+a)*(1+d)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b
*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/4/b^2/(1+d)*polylog(2,(1+d)*exp(2
*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2/(1+
```

```
d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2
*a))+1/2/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(
1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^2-1/4/
b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x-1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+
2*a))*x^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*
a)*d+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d+exp(2*b
*x+2*a)-1))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^3+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*x^2*ln(d)
+1/4*ln(exp(2*b*x+2*a)*d+exp(2*b*x+2*a)-1)*x^2
```

Maxima [A]
time = 0.70, size = 100, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left((d+1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)bd+\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arccoth(d*coth(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(86) = 172.
time = 0.39, size = 305, normalized size = 3.05

$$\frac{2\,b^3x^3+3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6\,bx\mathrm{Li}_2\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3\,a^2\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+2\,\sqrt{d+1}\right)-3\,a^2\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-2\,\sqrt{d+1}\right)-3\,(b^2x^2-a^2)\log\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3\,(b^2x^2-a^2)\log\left(-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\,\mathrm{polylog}\left(3,\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\,\mathrm{polylog}\left(3,-\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(
d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x + a
) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqrt
(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2*
sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
+ a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
 a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*p
olylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x*acoth(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}(d+d\coth(a+b\,x)+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x*acoth(d + d*coth(a + b*x) + 1), x)
```

3.224 $\int \coth^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=69

$$\frac{bx^2}{2}+x\coth^{-1}(1+d+d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,(1+d)e^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccoth(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6369, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\coth^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6369

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1+d+d\coth(a+bx))\,dx = x\coth^{-1}(1+d+d\coth(a+bx)) + b\int \frac{x}{1+(-1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) + (b(1+d))\int \frac{e^{2a+2bx}}{1+(-1-d)}$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right.$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right.$$

$$= \frac{bx^2}{2} + x\coth^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right.$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(69) = 138.
time = 0.68, size = 197, normalized size = 2.86

$$x\coth^{-1}(1+d+d\coth(a+bx)) + \frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1+d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{1+d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(-1+(1+d)e^{2(a+bx)}\right)\right)-2bx\log(d\cosh(a+bx)+(2+d)\sinh(a+bx))-2\mathrm{PolyLog}\left(2,-\sqrt{1+d}\,e^{a+bx}\right)-2\mathrm{PolyLog}\left(2,\sqrt{1+d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
2*PolyLog[2, -(Sqrt[1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 + d]*E^(a +
b*x)])/(4*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(61) = 122.
time = 0.95, size = 265, normalized size = 3.84

method	result
derivativedivides	$-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{\ln(d\coth(bx+a)+d)^2}{4d}\right.}{}$
default	$-\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{\operatorname{arccoth}(1+d+d\coth(bx+a))d\ln(d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{\ln(d\coth(bx+a)+d)^2}{4d}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*arccoth(1+d+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*arccoth(1
+d+d*coth(b*x+a))*d*ln(d*coth(b*x+a)+d)+1/2*d^2*(1/4/d*ln(d*coth(b*x+a)+d)^
2-1/2/d*dilog(1/2*d*coth(b*x+a)+1/2*d+1)-1/2/d*ln(d*coth(b*x+a)+d)*ln(1/2*d
*coth(b*x+a)+1/2*d+1)-1/2/d*dilog(-1/2*(-d*coth(b*x+a)-d)/d)-1/2/d*ln(-d*co
th(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/d)+1/2/d*dilog((-d*coth(b*x+a)-d-2)
/(-2*d-2))+1/2/d*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d-2)/(-2*d-2))))
```

Maxima [A]
time = 0.70, size = 72, normalized size = 1.04

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\operatorname{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\,\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arccoth(d*coth(b*x + a) + d + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(60) = 120.
time = 0.43, size = 226, normalized size = 3.28

$$\frac{b^2x^2+bx\log\left(\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)+2\,\sqrt{d+1}\right)+a\log\left(2\,(d+1)\cosh\left(bx+a\right)+2\,(d+1)\sinh\left(bx+a\right)-2\,\sqrt{d+1}\right)-(bx+a)\log\left(\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-(bx+a)\log\left(-\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+1\right)-\operatorname{Li}_2\left(\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-\operatorname{Li}_2\left(-\sqrt{d+1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b*
x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh
```

```
(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt(
d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*coth(b*x+a)),x)
[Out] Integral(acoth(d*coth(a + b*x) + d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d+d\coth(a+b\,x)+1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d + d*coth(a + b*x) + 1),x)
[Out] int(acoth(d + d*coth(a + b*x) + 1), x)
```

3.225 $\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\coth^{-1}(1+d+d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1+d+d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 + d + d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1+d+d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+d+d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+d+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*coth(a + b*x) + d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*coth(b*x + a) + d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(d + d\coth(a + b\,x) + 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d + d*coth(a + b*x) + 1)/x,x)
[Out] int(acoth(d + d*coth(a + b*x) + 1)/x, x)
```

3.226 $\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=165

$$\frac{bx^5}{20}+\frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x^3\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{3x^2\text{Poly}}{}$$

```
[Out] 1/20*b*x^5+1/4*x^4*arccoth(1-d-d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*exp
(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,(
1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A]
time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6377, 2215, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\text{Li}_5\left((1-d)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{4}x^4\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^5}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcCoth[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \coth^{-1}(1-d-d\coth(a+bx))\,dx &= \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1+d)e^{2a+2bx}} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}(b(1-d))\int \frac{}{1+(} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right. \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right. \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right. \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right. \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right. \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)\right.
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 147, normalized size = 0.89

$$
\frac{1}{16}\left(4x^4\coth^{-1}(1-d-d\coth(a+bx)) - 2x^4\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{4x^3\mathrm{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x^2\mathrm{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{6x\mathrm{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3} + \frac{3\mathrm{PolyLog}\left(5,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^4}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcCoth[1 - d - d*Coth[a + b*x]] - 2*x^4*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x^2
*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((-
1 + d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((-1 + d)*E^(2*(a + b*x)
)))])/b^4)/16
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.72, size = 1830, normalized size = 11.09

method	result	size
risch	Expression too large to display	1830

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)
)^3+1/16*I*x^4*Pi*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2
*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/20*b*x^5-1/16*I*x^4*P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/16*I
*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1)
)^2-1/8/b^4*d*a^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/2/b^3*d*a^3
/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+
2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/16*I*x^4*P
i*csgn(I*exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/2/b^4*a^3/(d-1)
*dilog(1-exp(b*x+a)*(1-d)^(1/2))+3/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2
*a))*x-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b/(d-1)*polyl
og(2,-(d-1)*exp(2*b*x+2*a))*x^3-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/
2))+3/16/b^4*d/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^3/(d-1)*dil
og(1+exp(b*x+a)*(1-d)^(1/2))+3/8/b^4*a^4/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))-1
/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4+1/4/b^4*a^3/(d-1)*polylog(2,-(d-1
)*exp(2*b*x+2*a))-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/8/(d-1)*
ln(1+(d-1)*exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylog(5,-(d-1)*exp(2*b*x+2*
a))+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^4*Pi*
csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2/b^3*a^3/(d-1)*ln(1+(d-1)*ex
p(2*b*x+2*a))*x+1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*
a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b
*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-3/8/b^4
*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(
2*b*x+2*a))*x^3-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^
2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polylog(4,-(
d-1)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2
/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2
*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16
*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8/b^4*a
^4/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*exp(2*b
*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1
))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I
*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/8*x^4*ln(exp(2*b*x+2*a)*d-exp(2*b*x
+2*a)+1)+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/16*I*
x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/8*I*x^4*Pi*csgn(I/(exp
(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/4*x^4*ln(exp(b*x+a)
)-1/8*x^4*ln(d)-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3
```

Maxima [A]

time = 0.72, size = 149, normalized size = 0.90

$$-\frac{1}{4}\,x^4\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2\,x^5}{d}-\frac{5\,\left(2\,b^4x^4\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+4\,b^3x^3\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-6\,b^2x^2\mathrm{Li}_3\left(-(d-1)e^{(2\,bx+2\,a)}\right)+6\,bx\mathrm{Li}_4\left(-(d-1)e^{(2\,bx+2\,a)}\right)-3\,\mathrm{Li}_5\left(-(d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^5d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arccoth(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d - 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(138) = 276$.
time = 0.40, size = 450, normalized size = 2.73

$$\frac{2 b^5 x^5 - 5 b^4 x^4 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2)\sinh(bx+a)}\right) - 20 b^3 x^3 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 20 b^3 x^3 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 5 a^4 \log\left(2(d-1)\cosh(bx+a) + 2(d-1)\sinh(bx+a) + \sqrt{-4d+4}\right) - 5 a^4 \log\left(2(d-1)\cosh(bx+a) + 2(d-1)\sinh(bx+a) - \sqrt{-4d+4}\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, \frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 60 b^2 x^2 \mathrm{polylog}\left(3, -\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 120 bx \mathrm{polylog}\left(4, \frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 120 bx \mathrm{polylog}\left(4, -\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 5\left(b^4x^4 - a^4\right)\log\left(\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) - 5\left(b^4x^4 - a^4\right)\log\left(-\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)+1\right) + 120 \mathrm{polylog}\left(5, \frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) + 120 \mathrm{polylog}\left(5, -\frac{1}{2}\sqrt{-4d+4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right)}{40 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh
(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1
)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(
d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(-
4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*sq
rt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqrt
(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*d
 + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^3 \operatorname{acoth}\left(d \coth\left(a+bx\right)+d-1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**3*acoth(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x^3*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3\,\mathrm{acoth}(d+d\coth(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*acoth(d + d*coth(a + b*x) - 1),x)
[Out] int(-x^3*acoth(d + d*coth(a + b*x) - 1), x)
```

3.227 $\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=137

$$\frac{bx^4}{12}+\frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{x\text{PolyL}}{}$$

```
[Out] 1/12*b*x^4+1/3*x^3*arccoth(1-d-d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*exp(2
*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3
```

Rubi [A]
time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6377, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{x\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcCoth[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(-1+d)e^{2a+2bx}}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) + \frac{1}{3}(b(1-d))\int \frac{}{1+(}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\coth^{-1}(1-d-d\coth(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-d)\right.$$

Mathematica [A]
time = 0.08, size = 121, normalized size = 0.88

$$\frac{1}{24}\left(8x^3\coth^{-1}(1-d-d\coth(a+bx)) - 4x^3\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right) + \frac{6x^2\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b} + \frac{6x\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^2} + \frac{3\text{PolyLog}\left(4,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{b^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcCoth[1 - d - d*Coth[a + b*x]] - 4*x^3*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x*P
olyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((-1 +
d)*E^(2*(a + b*x))))])/b^3)/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.53, size = 1771, normalized size = 12.93

method	result	size
risch	Expression too large to display	1771

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3*ln(exp(b*x+a))+1/12*b*x^4-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))
*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp
```

```
(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))
*a^3-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*pol
ylog(2,-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*
x+2*a))*x-1/2/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*
ln(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2
))*x-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*l
n(1-exp(b*x+a)*(1-d)^(1/2))+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*
b*x+2*a))^2-1/6/b^3*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)-1/6*I*x
^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/2/
b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/6/(d-1)*ln(1+(d-1)*exp(2*
b*x+2*a))*x^3+1/8/b^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12
*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*I*x^
3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/12*I*x^3*Pi*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I
*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)+1))^3-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-
1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*(exp
(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*
d-exp(2*b*x+2*a)+1))^2-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a
))^2-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,-(d
-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2-
1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1+exp
(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))+1/2/b^3
*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*
x+a)*(1-d)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/6*d/(d
-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a
))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/12*I*x^3*Pi*c
sgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x
+a)*(1-d)^(1/2))*x+1/6/b^3*d*a^3/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1
)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2
*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*
exp(2*b*x+2*a))^3-1/2/b^3*d*a^2/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/12*
I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(
2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(
exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/6*x^3*ln(d)+
1/6*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)*x^3
```

Maxima [A]
time = 0.69, size = 125, normalized size = 0.91

$$-\frac{1}{3}\,x^{3}\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{36}\left(\frac{3\,x^{4}}{d}-\frac{2\,\left(4\,b^{3}x^{3}\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+6\,b^{2}x^{2}\mathrm{Li}_{2}\left(-(d-1)e^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_{3}\left(-(d-1)e^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_{4}\left(-(d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^{4}d}\right)bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arccoth(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(114) = 228$.
time = 0.39, size = 381, normalized size = 2.78

$$\frac{b^4x^4 - 2b^3x^3\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 6b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)+\sqrt{-4d+4}\right) + 2a^3\log\left(2(d-1)\cosh(bx+a)+2(d-1)\sinh(bx+a)-\sqrt{-4d+4}\right) + 12bx\,\mathrm{polylog}\left(3,\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) + 12bx\,\mathrm{polylog}\left(3,-\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 2(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 2(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))+1\right) - 12\,\mathrm{polylog}\left(4,\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 12\,\mathrm{polylog}\left(4,-\frac{1}{2}\sqrt{-4d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d +
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2\,\mathrm{acoth}\,(d\coth(a+bx)+d-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**2*acoth(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{acoth}(d + d\coth(a + b\,x) - 1)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*acoth(d + d*coth(a + b*x) - 1),x)
[Out] int(-x^2*acoth(d + d*coth(a + b*x) - 1), x)
```

3.228 $\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=109

$$\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}+\frac{\text{PolyLog}}{}$$

[Out] 1/6*b*x^3+1/2*x^2*arccoth(1-d-d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x+2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b*x+2*a))/b^2

Rubi [A]
time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6377, 2215, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]

[Out] (b*x^3)/6 + (x^2*ArcCoth[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + PolyLog[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6377

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Coth[a + b*x]]/(f*(
m + 1))), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-d-d\coth(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1+d)e^{2a+2bx}}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))+\frac{1}{2}(b(1-d))\int\frac{}{1+(}$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)\right.$$

$$=\frac{bx^3}{6}+\frac{1}{2}x^2\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1-d)\right.$$

Mathematica [A]

time = 0.07, size = 94, normalized size = 0.86

$$\frac{2b^2x^2\left(2\coth^{-1}(1-d-d\coth(a+bx))-\log\left(1+\frac{e^{-2(a+bx)}}{-1+d}\right)\right)+2bx\text{PolyLog}\left(2,-\frac{e^{-2(a+bx)}}{-1+d}\right)+\text{PolyLog}\left(3,-\frac{e^{-2(a+bx)}}{-1+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcCoth[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 1688, normalized size = 15.49

method	result	size
risch	Expression too large to display	1688

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(
I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)-1))-1/2*x^2*ln(exp(b*x+a))+1/6*b*x^3-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp
(2*b*x+2*a))*a^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/4/b^2*d*a^2/(d-1)*
ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(e
xp(2*b*x+2*a)-1))*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/4
/b^2*a^2/(d-1)*ln(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)+1/4/b/(d-1)*polylog(2,
-(d-1)*exp(2*b*x+2*a))*x-1/4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/8*I*x
^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)
*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2
*b*x+2*a))^2-1/2/b*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*l
n(1+exp(b*x+a)*(1-d)^(1/2))*x+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*
x-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8*I*x^2*Pi*csg
n(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/
(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))+1/4/b^2/(d-1)*polyl
og(2,-(d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2)
)-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+ex
p(b*x+a)*(1-d)^(1/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/8/b
^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*
b*x+2*a))*a^2-1/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))+1/4/(d-1)*ln(1
+(d-1)*exp(2*b*x+2*a))*x^2-1/2/b*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/4
/b^2*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a+1/2/b^2*d*a/(d-1)*dilog(1-e
xp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/
2/b*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x
```

```
+a)*(1-d)^(1/2))-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-e
xp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)-1))^2+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1
/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x+1/2/b/(d-1)*ln(1+(d-1)*exp(
2*b*x+2*a))*x*a-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*
a)/(exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(ex
p(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*
(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2
*a)*d-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b
*x+2*a))^3+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/8*I
*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1))^3+1/
8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4*x^2*ln(d)+1/4*ln
(exp(2*b*x+2*a)*d-exp(2*b*x+2*a)+1)*x^2
```

Maxima [A]
time = 0.73, size = 101, normalized size = 0.93

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-(d-1)e^{(2\,bx+2\,a)}\right)\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d - 1/2*x^2*arccoth(d*coth(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(90) = 180$.
time = 0.35, size = 322, normalized size = 2.95

$$\frac{2\,b^3x^3-3\,b^2x^2\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)-6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-3\,a^2\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)-3\,a^2\log\left(2\left(d-1\right)\cosh\left(bx+a\right)+2\left(d-1\right)\sinh\left(bx+a\right)-\sqrt{-4\,d+4}\right)-3\left(b^2x^2-a^2\right)\log\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-3\left(b^2x^2-a^2\right)\log\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)+6\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)+6\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh
(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cosh(
b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a)
 + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x
 + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d +
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a))))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x \operatorname{acoth}\left(d \coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-d-d*coth(b*x+a)),x)
[Out] -Integral(x*acoth(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{acoth}(d+d \coth(a+b\,x)-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*acoth(d + d*coth(a + b*x) - 1),x)
[Out] int(-x*acoth(d + d*coth(a + b*x) - 1), x)
```

3.229 $\int \coth^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=76

$$\frac{bx^2}{2}+x\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}$$

```
[Out] 1/2*b*x^2+x*arccoth(1-d-d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1/4
*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6369, 2215, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\coth^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcCoth[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6369

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cCoth[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}(1-d-d\coth(a+bx))\,dx &= x\coth^{-1}(1-d-d\coth(a+bx))+b\int \frac{x}{1+(-1+d)e^{2a+2bx}}\,dx \\
&= \frac{bx^2}{2}+x\coth^{-1}(1-d-d\coth(a+bx))+(b(1-d))\int \frac{e^{2a+2bx}}{1+(-1+d)} \\
&= \frac{bx^2}{2}+x\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right. \\
&= \frac{bx^2}{2}+x\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right. \\
&= \frac{bx^2}{2}+x\coth^{-1}(1-d-d\coth(a+bx))-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right.
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(76) = 152.
time = 0.43, size = 208, normalized size = 2.74

$$x\coth^{-1}(1-d-d\coth(a+bx))+\frac{b^2x^2+\log^2\left(e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1-d}\,e^{a+bx}\right)-2\log\left(e^{a+bx}\right)\log\left(1+\sqrt{1-d}\,e^{a+bx}\right)+2\log\left(e^{a+bx}\right)\log\left(e^{-a-bx}\left(1+(-1+d)e^{2(a+bx)}\right)\right)-2bx\log(d\cosh(a+bx)+(-2+d)\sinh(a+bx))-2\text{PolyLog}\left(2,-\sqrt{1-d}\,e^{a+bx}\right)-2\text{PolyLog}\left(2,\sqrt{1-d}\,e^{a+bx}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
 2*PolyLog[2, -(Sqrt[1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 - d]*E^(a +
 b*x)])/(4*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(68) = 136.
time = 0.98, size = 281, normalized size = 3.70

method	result
derivativedivides	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\ln(-d\coth(bx+}{4d}\right.}{}}{}$
default	$-\frac{-\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}+\frac{\operatorname{arccoth}(1-d-d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}-\frac{d^2\left(\frac{\ln(-d\coth(bx+}{4d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(-1/2*arccoth(1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)+1/2*arccoth(
1-d-d*coth(b*x+a))*d*ln(-d*coth(b*x+a)+d)-1/2*d^2*(1/4/d*ln(-d*coth(b*x+a)-
d)^2-1/2/d*dilog(-1/2*d*coth(b*x+a)-1/2*d+1)-1/2/d*ln(-d*coth(b*x+a)-d)*ln(
-1/2*d*coth(b*x+a)-1/2*d+1)+1/2/d*dilog((-d*coth(b*x+a)-d+2)/(-2*d+2))+1/2/
d*ln(-d*coth(b*x+a)+d)*ln((-d*coth(b*x+a)-d+2)/(-2*d+2))-1/2/d*dilog(-1/2*(
-d*coth(b*x+a)-d)/d)-1/2/d*ln(-d*coth(b*x+a)+d)*ln(-1/2*(-d*coth(b*x+a)-d)/
d)))
```

Maxima [A]
time = 0.69, size = 73, normalized size = 0.96

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\,\operatorname{arcoth}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arccoth(d*coth(b*x + a) + d - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(63) = 126.
time = 0.41, size = 239, normalized size = 3.14

$$\frac{b^2x^2-bx\log\left(\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)+\sqrt{-4\,d+4}\right)+a\log\left(2\,(d-1)\cosh\left(bx+a\right)+2\,(d-1)\sinh\left(bx+a\right)-\sqrt{-4\,d+4}\right)-(bx+a)\log\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)+1\right)-\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4\,d+4}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a)
 + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh
```

```
(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b*x +
 a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a
) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*
x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{acoth}\left(d \coth\left(a+bx\right)+d-1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*coth(b*x+a)),x)
[Out] -Integral(acoth(d*coth(a + b*x) + d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(-d*coth(b*x + a) - d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d+d\coth(a+b\,x)-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d + d*coth(a + b*x) - 1),x)
[Out] int(-acoth(d + d*coth(a + b*x) - 1), x)
```

3.230 $\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\coth^{-1}(1-d-d\coth(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1-d-d*coth(b*x+a))/x,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Mathematica [A]
time = 2.94, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 - d - d*Coth[a + b*x]]/x, x]
```

Maple [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccoth}(1-d-d\coth(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-d-d*coth(b*x+a))/x,x)
[Out] int(arccoth(1-d-d*coth(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] -integrate(arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-arccoth(d*coth(b*x + a) + d - 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{acoth}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-d-d*coth(b*x+a))/x,x)
[Out] -Integral(acoth(d*coth(a + b*x) + d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-d-d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccoth(-d*coth(b*x + a) - d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\mathrm{acoth}(d+d\coth(a+b\,x)-1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d + d*coth(a + b*x) - 1)/x,x)
```

```
[Out] int(-acoth(d + d*coth(a + b*x) - 1)/x, x)
```

3.231 $\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}+\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/4*(f*x+e)^4*arccoth(tan(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A]
time = 0.17, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6387, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{3f^3\text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3\text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e+fx)^2\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^4\coth^{-1}(\tan(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCoth[Tan[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6387

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int (e+fx)^3 \coth^{-1}(\tan(a+bx))\,dx &= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f} \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{1}{2}\int (e \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+}{} \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+}{} \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+}{} \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+}{} \\
&= \frac{(e+fx)^4 \coth^{-1}(\tan(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+}{}
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.
time = 0.18, size = 654, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Tan[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.10, size = 7429, normalized size = 24.60

method	result	size
risch	Expression too large to display	7429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*f^2
*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*f^2*x^3*e
+ 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3040 vs. 2(244) = 488.
time = 0.47, size = 3040, normalized size = 10.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(
b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I
)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b*x +
 a) + I)/(tan(b*x + a)^2 + 1)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1
) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*
x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cos
```

h(1)^2)*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(
tan(b*x + a)^2 + 1) + 1) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*
I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^
3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2
)*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*
x + a)^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f
*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(
1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(
1))*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1) + 1) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cos
h(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*si
nh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*di
log(-(-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
) + 1) + 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x
+ a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a
^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^
2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(
b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*ta
n(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1
) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*
b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3
*a*b^3*cosh(1)^2)*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) +
 I - 1)/(tan(b*x + a)^2 + 1)) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^
2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*
b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(
1)^2)*sinh(1))*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan
(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3
+ 4*(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^
4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3
 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh
(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x +
a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^4*f^3*x^4 - a^4
f^3 + 4(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f
*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*
cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 +
 a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1
))*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x
 + a)^2 + 1)) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*
(b^4*x + a*b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*
x^2 - a^2*b^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a
^3*b*f^2)*cosh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^
2 + 3*(b^4*f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log((-(I - 1)*tan(b*x + a)^
2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^4*f^3 - 4*a^3*b*f^
2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 +
 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cos

```
h(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*
x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) +
6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*
f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b
^3*cosh(1)^2)*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I +
 1)/(tan(b*x + a)^2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*
f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 + 4*b^4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*
b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + 3*b^4*f*x^2*cosh(1) + 3*b^4*x*c
osh(1)^2)*sinh(1))*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + 6*(-I*b*f^3
*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, (I*tan(b*x + a)^2 + 2*ta
n(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(-I*b*f^3*x - I*b*f^2*cosh(1) - I
*b*f^2*sinh(1))*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x
 + a)^2 + 1)) + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4
, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(I*b*f
^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polyl...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{acoth}(\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*acoth(tan(b*x+a)),x)
[Out] Integral((e + f*x)**3*acoth(tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*arccoth(tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\tan(a+b\,x))\,{(e+f\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tan(a + b*x))*(e + f*x)^3,x)
[Out] int(acoth(tan(a + b*x))*(e + f*x)^3, x)
```

3.232 $\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}+\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/3*(f*x+e)^3*arccoth(tan(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6387, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^3\text{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{if^2\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^3\coth^{-1}(\tan(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Tan[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6387

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3 \sec(2a+2bx)\,dx}{3f}$$
$$= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{1}{2}\int (e$$
$$= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+}{}$$
$$= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+}{}$$
$$= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+}{}$$
$$= \frac{(e+fx)^3 \coth^{-1}(\tan(a+bx))}{3f} + \frac{i(e+fx)^3 \tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} - \frac{i(e+}{}$$

Mathematica [A]
time = 0.12, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right) - 12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right) - 4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right) + 12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right) + 12b^3efx^2\log\left(1+ie^{2i(a+bx)}\right) + 4b^3f^2x^3\log\left(1+ie^{2i(a+bx)}\right) - 6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right) + 6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right) + 6bef\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) + 6bf^2x\,\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right) - 6bef\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) - 6bf^2x\,\text{PolyLog}\left(3,ie^{2i(a+bx)}\right) + 3if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right) - 3if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.25, size = 5543, normalized size = 23.69

method	result	size
risch	Expression too large to display	5543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1787 vs. 2(186) = 372.
time = 0.43, size = 1787, normalized size = 7.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a
) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cos
h(1) - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh
(1))*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)
^2 + 1) + 1) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cosh(1)^2 -
I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(-((I + 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*f
^2*x^2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2
*f*x + b^2*cosh(1))*sinh(1))*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*f*x*cosh
(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1))*sinh(
1))*dilog(-(-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)
^2 + 1) + 1) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(
b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 -
```

```
 a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2
+ 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(a^3*f^2 - 3*a^2*b*f*co
sh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1
))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*
x + a)^2 + 1)) - 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b
^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(((I + 1)*tan(b*x
+ a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 +
 a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b
^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*co
sh(1))*sinh(1))*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(
b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 +
 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x
^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log((-(I - 1)*tan(b*x +
a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 + a^3
*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f
*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1
))*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x
 + a)^2 + 1)) + 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^
2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(((I - 1)*tan(b*x +
 a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(a^3*f^2 - 3*a^
2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^
2*cosh(1))*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1)) + 8*(b^3*f^2*x^3 + 3*b^3*f*x^2*cosh(1) + 3*b^3*x*cos
h(1)^2 + 3*b^3*x*sinh(1)^2 + 3*(b^3*f*x^2 + 2*b^3*x*cosh(1))*sinh(1))*log((
tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh
(1))*polylog(3, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1
)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, (I*tan(b*x + a)^2 -
 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b*f^2*x + b*f*cosh(1) + b*f
*sinh(1))*polylog(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)
^2 + 1)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, (-I*tan(b*x +
 a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^2 \operatorname{acoth}(\tan(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acoth(tan(b*x+a)),x)
[Out] Integral((e + f*x)**2*acoth(tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arccoth(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\tan(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tan(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(acoth(tan(a + b*x))*(e + f*x)^2, x)
```

3.233 $\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}+\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{P}}{}$$

```
[Out] 1/2*(f*x+e)^2*arccoth(tan(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6387, 4266, 2611, 2320, 6724}

$$\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{f\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCoth[Tan[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6387

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Tan[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\coth^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$
$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{1}{2}\int (\ldots$$
$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+\ldots}{}$$
$$= \frac{(e+fx)^2\coth^{-1}(\tan(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+\ldots}{}$$

Mathematica [A]
time = 0.20, size = 295, normalized size = 1.82

$$ex\coth^{-1}(\tan(a+bx)) + \frac{1}{2}fx^2\coth^{-1}(\tan(a+bx)) - \frac{e\left((-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)\right)}{8b} + \frac{f(4ib^2x^2\text{ArcTan}(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)))-2ibx\text{PolyLog}(2,-i\cos(2(a+bx))+\sin(2(a+bx)))-\text{PolyLog}(3,i\cos(2(a+bx))-\sin(2(a+bx)))+\text{PolyLog}(3,-i\cos(2(a+bx))+\sin(2(a+bx))))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcCoth[Tan[a + b*x]],x]
```

```
[Out] e*x*ArcCoth[Tan[a + b*x]] + (f*x^2*ArcCoth[Tan[a + b*x]])/2 - (e*((-4*a + P
i - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x))]
) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*
(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^2*x^
2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I*Co
s[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a + b
*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]
] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.35, size = 2543, normalized size = 15.70

method	result	size
risch	Expression too large to display	2543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a))
)/b^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-
I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn(I*(ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))+1))^2+1/2*I*f/b^2*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I
*f/b^2*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*(I*b*x+I*a)*ln(1+exp(
I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4)
)-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*(I*
b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*(I*b*x+I*a)*l
n(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*f/b^2*(I*b*x+I*a)^2*ln(1+I*ex
p(2*I*(b*x+a)))-1/4*f/b^2*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))-1/8*I*
Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x
*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(
I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn(I*(exp(2
*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4
*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*
(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+
a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))+1))*x^2+1/8*I*Pi*f*x^2+1/4*I*Pi*e*x-1/4*ln(exp(2*I*(b*x+a))-I)*f*x^
2-1/2*ln(exp(2*I*(b*x+a))-I)*e*x+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*cs
gn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
)*x^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)
)*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/4/b^2*f*a^2*ln(exp(2*
I*(b*x+a))+I)+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((
1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn(I*(ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(ex
```

```
p(2*I*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/4*f/b^2*a^2*
ln(-exp(2*I*(b*x+a))+I)-1/2*I*e/b*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*
e/b*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I*f/b^2*a*(I*b*x+I*a)*ln(1-exp(I
*(b*x+a))*(-1)^(3/4))-1/2*I*f/b^2*a*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a
)))/(-I)^(1/2))-1/2*I*f/b^2*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-
I)^(1/2))-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^
3*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*
I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/2*I/b*e*
dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+e
xp(I*(b*x+a)))/(-I)^(1/2))+1/4*f/b^2*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))
+1/4*f/b^2*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))+1/2/b*e*a*ln(exp(2*I*(
b*x+a))+I)-1/2*e/b*a*ln(-exp(2*I*(b*x+a))+I)-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*
x+a))+1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))+1))^3*x^2+1/2*I*f/b^2*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)
^(3/4))+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*
x^2+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/
4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*Pi*f*c
sgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/2*I/b^2*f*a*di
log(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*
(b*x+a))+I)+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+
a))+I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*cs
gn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I*(exp(2*
I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/4*
I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(
2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*c
sgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1
))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I
)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I/(exp(2
*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*x*e)*co
```

```
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(134) = 268$.
time = 0.43, size = 947, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(-((I + 1)*tan(b*x + a
)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(-I*b*f*x - I*b
*cosh(1) - I*b*sinh(1))*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
 + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*d
ilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 +
1) + 1) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*dilog(-(-(I - 1)*tan(b*x
+ a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*f*x^2 -
 a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1))*log(((I + 1)*ta
n(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(a^2*f - 2
*a*b*cosh(1) - 2*a*b*sinh(1))*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a
) + I - 1)/(tan(b*x + a)^2 + 1)) - 2*(a^2*f - 2*a*b*cosh(1) - 2*a*b*sinh(1)
)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
 1)) - 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sin
h(1))*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2
 + 1)) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*s
inh(1))*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a
)^2 + 1)) - 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b
)*sinh(1))*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) + 2*(a^2*f - 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(((I - 1)*tan(b
*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(a^2*f - 2*
a*b*cosh(1) - 2*a*b*sinh(1))*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a)
 + I + 1)/(tan(b*x + a)^2 + 1)) - 4*(b^2*f*x^2 + 2*b^2*x*cosh(1) + 2*b^2*x*
sinh(1))*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - f*polylog(3, (I*tan(b
*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (I*tan
(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (-I*
tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (
-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{acoth}(\tan(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acoth(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acoth(tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arccoth(tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(\tan(a+b\,x))\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tan(a + b*x))*(e + f*x),x)
```

```
[Out] int(acoth(tan(a + b*x))*(e + f*x), x)
```

3.234 $\int \coth^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=79

$$x\coth^{-1}(\tan(a+bx))+ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] x*arccoth(tan(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6383, 4266, 2317, 2438}

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+x\coth^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[Tan[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6383

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Tan[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \coth^{-1}(\tan(a+bx))\,dx = x\coth^{-1}(\tan(a+bx)) - b\int x\sec(2a+2bx)\,dx$$

$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)\,dx - \frac{1}{2}$$

$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2b}\right.}{4b}$$

$$= x\coth^{-1}(\tan(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+\right.}{4b}$$

Mathematica [A]
time = 0.04, size = 127, normalized size = 1.61

$$x\coth^{-1}(\tan(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[Tan[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x
))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]
] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a
+ b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.
time = 0.63, size = 169, normalized size = 2.14

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))\text{arccoth}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(bx}{1+\tan^2(bx}\right.}{2}}{b}$
default	$\frac{\arctan(\tan(bx+a))\text{arccoth}(\tan(bx+a))+\frac{\arctan(\tan(bx+a))\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2}-\frac{\arctan(\tan(bx+a))\ln\left(1-\frac{i(1+i\tan(bx}{1+\tan^2(bx}\right.}{2}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arctan(tan(b*x+a))*arccoth(tan(b*x+a))+1/2*arctan(tan(b*x+a))*ln(1+I*(
1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2*arctan(tan(b*x+a))*ln(1-I*(1+I*tan(
b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a
)^2))+1/4*I*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(57) = 114$.
time = 0.51, size = 182, normalized size = 2.30

$$\frac{4\,(bx+a)\,\mathrm{arcoth}\left(\tan\left(bx+a\right)\right) + \left(\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}, \frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right) - \arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)-\frac{1}{2}, -\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right)\right)\log\left(\tan\left(bx+a\right)^2+1\right) - (bx+a)\log\left(\frac{1}{2}\,\tan\left(bx+a\right)^2+\tan\left(bx+a\right)+\frac{1}{2}\right) + (bx+a)\log\left(\frac{1}{2}\,\tan\left(bx+a\right)^2-\tan\left(bx+a\right)+\frac{1}{2}\right) - i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan\left(bx+a\right)-\frac{1}{2}i+\frac{1}{2}\right) + i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan\left(bx+a\right)+\frac{1}{2}i+\frac{1}{2}\right) + i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\tan\left(bx+a\right)+\frac{1}{2}i+\frac{1}{2}\right) - i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\tan\left(bx+a\right)-\frac{1}{2}i+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1
/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a)
+ 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(b*
x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) - I*
dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*ta
n(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I + 1/
2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(57) = 114$.
time = 0.38, size = 498, normalized size = 6.30

$$\frac{4\,bx\log\left(\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right) - 2\,(bx+a)\log\left(\frac{(i+1)\,\tan(bx+a)^2+2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right) + 2\,a\log\left(\frac{(i+1)\,\tan(bx+a)^2+2i\,\tan(bx+a)+i-1}{\tan(bx+a)^2+1}\right) - 2\,a\log\left(\frac{(i+1)\,\tan(bx+a)^2-2i\,\tan(bx+a)+i-1}{\tan(bx+a)^2+1}\right) + 2\,(bx+a)\log\left(\frac{(i+1)\,\tan(bx+a)^2-2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right) - 2\,(bx+a)\log\left(\frac{-(i-1)\,\tan(bx+a)^2+2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right) + 2\,(bx+a)\log\left(\frac{-(i-1)\,\tan(bx+a)^2-2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right) + 2\,a\log\left(\frac{(i-1)\,\tan(bx+a)^2+2i\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right) - 2\,a\log\left(\frac{(i-1)\,\tan(bx+a)^2-2i\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}\right) + i\,\mathrm{Li}_2\left(-\frac{(i+1)\,\tan(bx+a)^2+2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}+1\right) + i\,\mathrm{Li}_2\left(-\frac{(i+1)\,\tan(bx+a)^2-2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}+1\right) - i\,\mathrm{Li}_2\left(-\frac{-(i-1)\,\tan(bx+a)^2+2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}+1\right) - i\,\mathrm{Li}_2\left(-\frac{-(i-1)\,\tan(bx+a)^2-2\,\tan(bx+a)+i+1}{\tan(bx+a)^2+1}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((I
 + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*
log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1
)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
+ 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2 + 2*t
an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*l
og(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)
) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a
)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/
(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x +
 a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
- 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tan(b*x+a)),x)
[Out] Integral(acoth(tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\tan(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tan(a + b*x)),x)
[Out] int(acoth(tan(a + b*x)), x)
```

3.235 $\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccoth(tan(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

```
[Out] Defer[Int][ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx = \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x),x]
```

```
[Out] Integrate[ArcCoth[Tan[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tan(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tan(b*x+a))/(f*x+e),x)
[Out] int(arccoth(tan(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccoth(tan(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(\tan\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tan(b*x+a))/(f*x+e),x)
[Out] Integral(acoth(tan(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] integrate(arccoth(tan(b*x + a))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acoth}(\tan(a+b\,x))}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(tan(a + b*x))/(e + f*x),x)
[Out] int(acoth(tan(a + b*x))/(e + f*x), x)
```

3.236 $\int x^2 \coth^{-1}(c + d\tan(a + bx))\, dx$

Optimal. Leaf size=395

$$\frac{1}{3}x^3 \coth^{-1}(c+d\tan(a+bx))+\frac{1}{6}x^3 \log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{6}x^3 \log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{ix}{}$$

```
[Out] 1/3*x^3*arccoth(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-(1+
c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2*I*a
+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c
+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b^3-1/8
*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

Rubi [A]
time = 0.35, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6403, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix^2\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{6}x^3\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6403

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 + c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 - c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(i(1-c)-d))\int \frac{e^{2ia\cdots}}{1-c-id+(1\cdots}$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)\cdots$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)\cdots$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)\cdots$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)\cdots$$

$$= \frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)\cdots$$

Mathematica [A]
time = 0.32, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3\coth^{-1}(c+d\tan(a+bx)) + \frac{4b^3x^3\log\left(1+\frac{(-1+c-id)e^{2i(a+bx)}}{-1+c+id}\right) - 4b^3x^3\log\left(1+\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) - 6ib^2x^2\text{PolyLog}\left(2,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) + 6ib^2x^2\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) + 6bx\text{PolyLog}\left(3,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) - 6bx\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right) + 3i\text{PolyLog}\left(4,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right) - 3i\text{PolyLog}\left(4,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*
E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^
((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)
*(a + b*x)))/(1 + c + I*d))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.42, size = 6874, normalized size = 17.40

method	result	size
risch	Expression too large to display	6874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)

[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")

[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*
a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^
2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c
^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
 + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
(c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4
*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^
4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (
c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*si
n(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2164 vs. 2(279) = 558.
time = 0.49, size = 2164, normalized size = 5.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")

[Out] 1/48*(8*b^3*x^3*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 6*
I*b^2*x^2*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d +
 (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 +
d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dil
og(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2
```

```
*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c +
 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(2*((I*(c
 - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d -
I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x +
 a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(2*((-I*(c - 1)*d - d^
2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I
*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2
 + d^2 - 2*c + 1) + 1) + 4*a^3*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^
2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(
b*x + a)^2 + 1)) + 4*a^3*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*
(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x +
a)^2 + 1)) - 4*a^3*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1
)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 +
 1)) - 4*a^3*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d +
(I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) -
 6*b*x*polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c
^2 - 2*I*(c + 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan
(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*
c + 1)) - 6*b*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x +
 a)^2 - c^2 + 2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c
+ I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 +
d^2 + 2*c + 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*t
an(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2
 + 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
 + c^2 + d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2
*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d
 - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*
x + a)^2 + c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(-2*((I*(c + 1)*d -
 d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2
*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(-2*((-I*(c + 1)*d - d^2)*tan(b
*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*
tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 +
 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(-2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 -
 c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a)
 + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) +
 4*(b^3*x^3 + a^3)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c
 - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a) + 2*c - 1
)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polyl
og(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c
+ 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) -
2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*
I*polylog(4, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 +
2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x +
 a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)
```

```
) - 3*I*polylog(4, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 -
c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*ta
n(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2
*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x +
a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
 I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d
^2 - 2*c + 1)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(c + d\tan\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*tan(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(c + d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{acoth}\left(c+d\,\tan\left(a+b\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*acoth(c + d*tan(a + b*x)), x)
```

3.237 $\int x \coth^{-1}(c + d\tan(a + bx))\, dx$

Optimal. Leaf size=295

$$\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{ix}{}$$

```
[Out] 1/2*x^2*arccoth(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*poly
log(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c-I*
d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2*I*b
*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b
^2
```

Rubi [A]
time = 0.28, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6403, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{4}x^2\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6403

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 + c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 - c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{2}(b(i(1-c)-d))\int\frac{e^{2ia+2i}}{1-c-id+(1-}$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-$$

Mathematica [A]
time = 0.16, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2\coth^{-1}(c+d\tan(a+bx))+\frac{2b^2x^2\log\left(1+\frac{(-1+c-id)e^{2i(a+bx)}}{-1+c+id}\right)-2b^2x^2\log\left(1+\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)-2ibx\text{PolyLog}\left(2,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right)+2ibx\text{PolyLog}\left(2,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)+\text{PolyLog}\left(3,\frac{(1-c+id)e^{2i(a+bx)}}{-1+c+id}\right)-\text{PolyLog}\left(3,-\frac{(1+c-id)e^{2i(a+bx)}}{1+c+id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Tan[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 2*b^2*x^2*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (2*I)*b*x*PolyLog[2, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] + (2*I)*b*x*PolyLog[2, -(((1 + c - I*d)*E^((
2*I)*(a + b*x)))/(1 + c + I*d))] + PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a +
b*x)))/(-1 + c + I*d)] - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(
1 + c + I*d))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.76, size = 6500, normalized size = 22.03

method	result	size
risch	Expression too large to display	6500

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*
x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 - d^2
 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2
+ (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d
^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2
+ 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*
```

```
d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*
x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(
c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2
*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*
c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4
*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^
2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(209) = 418$.
time = 0.45, size = 1688, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*
I*b*x*dilog(2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*
c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2
+ 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((-
I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1)
*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(
b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x*dilog(2*((I*(c - 1)*d - d^
2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*
c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2
+ d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog(2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^
2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x
+ a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1
) + 1) - 2*a^2*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d
+ (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1))
 - 2*a^2*log(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c
^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) + 2*a
^2*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I
*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log
(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 -
 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*x^2 - a^
2)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2
- 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*
c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*log(-2*((
-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1
)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan
(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-2*((I*(c - 1)*
```

```
d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2
- 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-2*((-I*(c - 1)*d - d^2)*ta
n(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c -
I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^
2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x +
 a)^2 - c^2 - 2*I*(c + 1)*d + d^2 - 2*(-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c
 - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 +
 d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*
x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 - 2*(I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I
*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2
 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(
b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 - 2*(-I*c^2 + 2*(c - 1)*d + I*d^2 +
2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 +
c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*t
an(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 - 2*(I*c^2 + 2*(c - 1)*d - I*d^2
- 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}\left(c + d \tan\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*tan(b*x+a)),x)
[Out] Integral(x*acoth(c + d*tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccoth(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \tan(a + b\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(c + d*tan(a + b*x)),x)
[Out] int(x*acoth(c + d*tan(a + b*x)), x)
```

3.238 $\int \coth^{-1}(c+d\tan(a+bx))\,dx$

Optimal. Leaf size=194

$$x\coth^{-1}(c+d\tan(a+bx))+\frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{2}x\log\left(1+\frac{(1+c-id)e^{2ia+2ibx}}{1+c+id}\right)-\frac{i\text{PolyLo}}{}$$

```
[Out] x*arccoth(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d)
)-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-c+
I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*a+2
*I*b*x)/(1+c+I*d))/b
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6395, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)+x\coth^{-1}(d\tan(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6395

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[x*(E^(2*I*a +
2*I*b*x)/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[
I*b*(1 - c + I*d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 - c - I*d + (1 - c + I*d)*
E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2
, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\tan(a+bx))\,dx = x\coth^{-1}(c+d\tan(a+bx)) + (b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x}{1-c-id+(1-c+$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,l$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,l$$

$$= x\coth^{-1}(c+d\tan(a+bx)) + \frac{1}{2}x\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{2}x\,l$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4654 vs. $2(194) = 388$.
time = 12.83, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
+ c*Cos[a + b*x] + d*Sin[a + b*x])] + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^
2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*
x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Log[(-d + Sqrt[1 - 2*
c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((-1 + c)*(I + Tan[(
a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
- 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + (a + b*x)*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(-I + T
an[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - I*Log[((-1 + c)*(I + Ta
```

n[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - (a + b*x)*Log[-((d + Sq
rt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - I*Log[((1 + c)*(-I
+ Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d +
 Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] + I*Log[((1 + c)*(
I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d
+ Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(
-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + I*Log
[((1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d
^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 +
c)] - I*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*d - I*Sqrt[1 + 2*
c + c^2 + d^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2])/(1 + c)] + I*PolyLog[2, (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan
[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d
 + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] +
 (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] + I*
PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I
 + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d + Sqrt[1 + 2*c +
 c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2
+ d^2])] + I*PolyLog[2, (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b
*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 +
 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c
)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])])*((-2*a)/(b*
(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a +
b*x)] + 2*c*d*Sin[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - Cos[2
*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a
+ b*x)]))))/(Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/
2]] + Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - Log
[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - Log[(-d +
 Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + (Log[(-d
+ Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b
x)/2]^2)/(2(1 - I*Tan[(a + b*x)/2])) - (Log[(-d + Sqrt[1 - 2*c + c^2 + d^
2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x
)/2])) + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(
1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + ((I/2)*Log[(d +
Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/
(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 +
 c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/
2)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a
 + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 +
 d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2
]) + ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/
2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2

```
]^2)/(2*((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2])) + (
(I/2)*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*
c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1
+ c) + Tan[(a + b*x)/2]) - ((I/2)*Log[-(((-1 + c)*(I + Tan[(a + b*x)/2]))/(
I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[
1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*
x)/2]^2)/(2*((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2])) +
 ((I/2)*Log[((-1 + c)*(-I + Tan[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c
+ c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c)
 + Tan[(a + b*x)/2]) - ((I/2)*Log[((-1 + c)*(I + Tan[(a + b*x)/2]))/(-I + I
*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(164) = 328$.
time = 1.05, size = 562, normalized size = 2.90

method	result
derivativedivides	$d\arctan(\tan(bx+a))\operatorname{arccoth}(c+d\tan(bx+a))+d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\arctan\left(\right.\right.$
default	$d\arctan(\tan(bx+a))\operatorname{arccoth}(c+d\tan(bx+a))+d^2\left(\frac{\arctan\left(-\frac{c+d\tan(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\arctan\left(\right.\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(d*arctan(tan(b*x+a))*arccoth(c+d*tan(b*x+a))+d^2*(1/2*arctan(-(c+d*t
an(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)-1/2*arctan(-(c+d*tan
(b*x+a))/d+c/d)/d*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)+1/4*I*ln(d*((c+d*tan(b
*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-ln((I*d+
d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I/d*dilog((I*d-d*((c+d*tan(b*
x+a))/d-c/d))/(I*d+c-1))-1/4*I/d*dilog((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-
c+I*d))-1/4*I*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d*tan(b*x+a
))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1)))/d-1/4
*I/d*dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/d*dilog((I*d+d
*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.

time = 0.53, size = 372, normalized size = 1.92

$$\frac{4\,(bx+a)\operatorname{arcoth}\left(d\tan\left(bx+a\right)+c\right)+\left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{d^2\tan(bx+a)+(c-1)d}{c^2+d^2-2\,c+1},\frac{(c-1)d\tan(bx+a)+c^2-2\,c+1}{c^2+d^2-2\,c+1}\right)\right)\log\left(\tan\left(bx+a\right)^2+1\right)-(bx+a)\log\left(\frac{d^2\tan(bx+a)^2+2\,(c+1)d\tan(bx+a)+c^2+2\,c+1}{c^2+d^2+2\,c+1}\right)+(bx+a)\log\left(\frac{d^2\tan(bx+a)^2+2\,(c-1)d\tan(bx+a)+c^2-2\,c+1}{c^2+d^2-2\,c+1}\right)-i\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d+i}\right)+i\,\mathrm{Li}_2\left(-\frac{i\,d\tan(bx+a)-d}{i\,c+d-i}\right)-i\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d+i}\right)+i\,\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d}{-i\,c+d-i}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
 (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

<u>Fricas</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(136) = 272.
time = 0.50, size = 1184, normalized size = 6.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
+ a)*log(-2*((I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^
2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 +
2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*log(-2*((-I*(
c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d
+ I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x
 + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-2*((I*(c - 1)*d - d^2)*t
an(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c +
I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^
2 - 2*c + 1)) + 2*(b*x + a)*log(-2*((-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c
^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*tan(b*x + a)
+ 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) +
2*a*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 +
I*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(
((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 +
2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c -
 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c +
I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d -
```

```
d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b
*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - I*dilog(2*((I*(c + 1)*d - d^2)*t
an(b*x + a)^2 - c^2 - I*(c + 1)*d + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c +
I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^
2 + 2*c + 1) + 1) + I*dilog(2*((-I*(c + 1)*d - d^2)*tan(b*x + a)^2 - c^2 +
I*(c + 1)*d + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*tan(b*x + a) - 2*c
 - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + I
*dilog(2*((I*(c - 1)*d - d^2)*tan(b*x + a)^2 - c^2 - I*(c - 1)*d + (I*c^2 -
 2*(c - 1)*d - I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c
 + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - I*dilog(2*((-I*(c - 1)*d
 - d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (-I*c^2 - 2*(c - 1)*d + I*d^2
+ 2*I*c - I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2
+ c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(c+d\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tan(b*x+a)),x)
[Out] Integral(acoth(c + d*tan(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(arccoth(d*tan(b*x + a) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(c+d\tan(a+b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*tan(a + b*x)),x)
[Out] int(acoth(c + d*tan(a + b*x)), x)
```

3.239 $\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x} dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(c+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(c+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Defer[Int][ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x} dx = \int \frac{\coth^{-1}(c+d\tan(a+bx))}{x} dx$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c+d\tan(a+bx))}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]]/x,x]
```

```
[Out] Integrate[ArcCoth[c + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c+d\tan(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*tan(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(c + d\tan\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(acoth(c + d*tan(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c+d\tan(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*tan(a + b*x))/x,x)
```

```
[Out] int(acoth(c + d*tan(a + b*x))/x, x)
```

3.240 $\int x^2 \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=170

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ib}\right.\right.}{4b}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6399, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\coth^{-1}(d\tan(a+bx)-id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6399

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1-id)e^{2ia+2}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{3}(b(i+d))\int \frac{}{1}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-\right.$$

Mathematica [A]
time = 0.14, size = 155, normalized size = 0.91

$$\frac{1}{3}x^3\coth^{-1}(1-id+d\tan(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6ib^2x^2\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right) + 6bx\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right) - 3i\text{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.70, size = 2339, normalized size = 13.76

method	result	size
risch	Expression too large to display	2339

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2*csgn(I*d)+
1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*
x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+
```

1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2
I(b*x+a))+1))^2+1/8/b^3/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))-1/6*d/(
I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(
b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I
+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+
d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b
*x+a))+1)*exp(2*I*(b*x+a)))^3+1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d+I)+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*cs
gn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*e
xp(2*I*(b*x+a)))*csgn(I*d)-1/4*I/b^3*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x
+a)))*a^2-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2
*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/4*I/b*d/(I+d)*polylog(2
,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)
))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/
b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+
d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b^3*a^2*d/(I+d)*dilog(1-
I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b
x+a))(-I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(
b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*
x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d+I)+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2
I(b*x+a))*d+I)+1/3/b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/12*I*
x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x
+a))+1)*exp(2*I*(b*x+a)))^2-1/4/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+
a)))*x-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*
a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn(I*d/
(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(
I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/6*x^3*ln(d)-1/6*I*x^3*Pi*csg
n(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*
(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a))
)-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*
Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*c
sgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/3*I/b
^3/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+
d)*exp(2*I*(b*x+a)))*x-1/6*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/12*
I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(
exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/
(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x
+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+
I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(

```
b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+
a))*(-I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^
(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*
csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/3*
x^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(118) = 236$.
time = 0.29, size = 343, normalized size = 2.02

$$\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arcoth}(d\tan(bx+a)-id+1)}{b^2}-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)+\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2\left((id-1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left(\left(d^2+1\right)\cos(2bx+2a)^2+\left(d^2+1\right)\sin(2bx+2a)^2+2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3\left((id-1)e^{(2ibx+2ia)}\right)+6i\operatorname{Li}_4\left((id-1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b*
x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*a
rctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(118) = 236$.
time = 0.38, size = 344, normalized size = 2.02

$$\frac{ib^4x^4+2b^3x^3\log\left(\frac{\left((d+i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)+6ib^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)+6ib^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-ia^4+2a^3\log\left(\frac{2(d+i)e^{(ibx+ia)}+i\sqrt{4id-4}}{2(d+i)}\right)+2a^3\log\left(\frac{2(d+i)e^{(ibx+ia)}-i\sqrt{4id-4}}{2(d+i)}\right)-12bx\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-12bx\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-2\left(b^3x^3+a^3\right)\log\left(\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}+1\right)-2\left(b^3x^3+a^3\right)\log\left(-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}+1\right)-12i\operatorname{polylog}\left(4,\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)-12i\operatorname{polylog}\left(4,-\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b
*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) + 6
*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log(
1/2*(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + 2*a^3*log(1/
2*(2*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 12*b*x*polylog
(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(4*I*
d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*
x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)
+ 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*polylog(
4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(d \tan(a + bx) - id + 1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
[Out] Integral(x**2*acoth(d*tan(a + b*x) - I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d \tan(a + b\,x) + 1 - d\,1\mathrm{i})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d*tan(a + b*x) - d*1i + 1),x)
[Out] int(x^2*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.241 $\int x \coth^{-1}(1 - id + d\tan(a + bx))\, dx$

Optimal. Leaf size=133

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(
3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6399, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d\tan(a+bx)-id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6399

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}(1-id+d\tan(a+bx))\,dx &= \frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1-id)e^{2ia+2ibx}}\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}(b(i+d))\int\frac{e}{1+(}\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id\right.\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id\right.\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id\right.\\ &= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id\right.\end{aligned}$$

Mathematica [A]

time = 0.08, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\coth^{-1}(1-id+d\tan(a+bx))-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right)+2ibx\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.39, size = 2249, normalized size = 16.91

method	result	size
risch	Expression too large to display	2249

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(ex
p(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*c
sgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/2*I/b^2*a*d/(I+d)*dilog(1+
I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+
a)))*x*a+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/
(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*
(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a
))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))-1/2/b*d/(I+d)*ln(1-I*(I+d)*e
xp(2*I*(b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)*x-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*
(b*x+a))+1))^2-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))
^2+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2*I/b*a/(I+d
)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*
(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d+I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2
*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I
*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x
+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a)
)+1))+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*
I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*
I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)
))+1/2/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a*d/
(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(I*d/(exp
(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-1/4*I/b^2/(I+d)*ln(1-I*(I+d)
```

```
*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))
^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*I/b
^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn(
I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(
2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*
csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(I*d)+1/2/b^2*a^2*d/(I+
d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp
(2*I*(b*x+a)))*a^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1
))^2*csgn(I/(exp(2*I*(b*x+a))+1))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/4/b^2
*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/2/b^2*a^2*d/(I+d
)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))
+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*exp(I*(b
*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/4*x^2*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b
*x+a))*d+I)-1/4*x^2*ln(d)-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)
))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4*d/(I+d)*l
n(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*
(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2
/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))+1))^3-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn
(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*(I*exp(2*
I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(
(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2
*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I/b^2/(I+d)*polylog
(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(93) = 186$.
time = 0.29, size = 248, normalized size = 1.86

$$\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-id+1)}{b}-\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((id-1)e^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)+\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((id-1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I*b
*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(93) = 186$.

time = 0.39, size = 292, normalized size = 2.20

$$\frac{2i\,b^3x^3+3\,b^2x^2\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)+2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)-3\,a^2\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}+i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)-3\,a^2\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}-i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)-3\,(b^2x^2-a^2)\log\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-3\,(b^2x^2-a^2)\log\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-6\,\mathrm{polylog}\left(3,\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)-6\,\mathrm{polylog}\left(3,-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I
*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1/2*
(2*(d + I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) - 3*a^2*log(1/2*(2
*(d + I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) - 3*(b^2*x^2 - a^2)*
log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*s
qrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d - 4)*e^(I
*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{acoth}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(x*acoth(d*tan(a + b*x) - I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(x*acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.242 $\int \coth^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}ibx^2+x\coth^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccoth(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6391, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\coth^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6391

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1-id+d\tan(a+bx))\,dx = x\coth^{-1}(1-id+d\tan(a+bx)) + (ib)\int \frac{x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id+d\tan(a+bx)) - (b(i+d))\int \frac{e^{2ia+}}{1+(1-i}$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia}\right.$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia}\right.$$
$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia}\right.$$

Mathematica [A]
time = 0.42, size = 84, normalized size = 0.90

$$x\coth^{-1}(1-id+d\tan(a+bx)) - \frac{2bx\log\left(1+\frac{ie^{-2i(a+bx)}}{i+d}\right) + i\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - I*d + d*Tan[a + b*x]] - (2*b*x*Log[1 + I/((I + d)*E^((2*I)*(a
 + b*x)))] + I*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(76) = 152.
time = 0.91, size = 321, normalized size = 3.45

method	result

derivativedivides	$-\frac{i\,\mathrm{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i\,\mathrm{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}+\frac{d^2\left(-\frac{i\ln(-id+d\tan(bx+\dots}{4d}\right.\dots}{\dots}$
default	$-\frac{i\,\mathrm{arccoth}(1-id+d\tan(bx+a))d\ln(-id+d\tan(bx+a))}{2}+\frac{i\,\mathrm{arccoth}(1-id+d\tan(bx+a))d\ln(id+d\tan(bx+a))}{2}+\frac{d^2\left(-\frac{i\ln(-id+d\tan(bx+\dots}{4d}\right.\dots}{\dots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*I*arccoth(1-I*d+d*tan(b*x+a))*d*ln(-I*d+d*tan(b*x+a))+1/2*I*arc
coth(1-I*d+d*tan(b*x+a))*d*ln(I*d+d*tan(b*x+a))+1/2*d^2*(-1/4*I/d*ln(-I*d+d
*tan(b*x+a))^2+1/2*I/d*dilog(1-1/2*I*d+1/2*d*tan(b*x+a))+1/2*I/d*ln(-I*d+d*
tan(b*x+a))*ln(1-1/2*I*d+1/2*d*tan(b*x+a))-1/2*I/d*dilog(I*(I*d+d*tan(b*x+a
)-I*(2*d+2*I))/(2*d+2*I))-1/2*I/d*ln(I*d+d*tan(b*x+a))*ln(I*(I*d+d*tan(b*x+
a)-I*(2*d+2*I))/(2*d+2*I))+1/2*I/d*dilog(1/2*I*(-I*d+d*tan(b*x+a))/d)+1/2*I
/d*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(65) = 130$.
time = 0.47, size = 262, normalized size = 2.82

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d+2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)-d\left(\frac{2i\left(\log(d\tan(bx+a)-i\,d+2)\log\left(-\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d+2i}{2\,(d+i)}\right)\right)}{d}-\frac{2i\,\log(d\tan(bx+a)-i\,d+2)\log(\tan(bx+a)-i)-i\,\log(\tan(bx+a)-i)^2}{d}+\frac{2i\left(\log\left(\frac{1}{2}\,d\tan(bx+a)-\frac{1}{2}i\,d+1\right)\log(\tan(bx+a)-i)+\mathrm{Li}_2\left(-\frac{1}{2}\,d\tan(bx+a)+\frac{1}{2}i\,d\right)\right)}{d}-\frac{2i\left(\log(\tan(bx+a)-i)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)-8\,(bx+a)\,\mathrm{arcoth}\,(d\tan\,(bx+a)-i\,d+1)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) - I
)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-1/2*(I*d*tan(b*x + a) + d
 + 2*I)/(d + I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d + 2*I)/(d + I)))/d -
 (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b*x +
 a) - I)^2)/d + 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x + a)
 - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d - 2*I*(log(tan(b*x + a) - I
)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8*
(b*x + a)*arccoth(d*tan(b*x + a) - I*d + 1))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(65) = 130$.
time = 0.39, size = 217, normalized size = 2.33

$$\frac{i\,b^2x^2+bx\log\left(\frac{((d+i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)+a\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}+i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)+a\log\left(\frac{2\,(d+i)e^{(i\,bx+i\,a)}-i\,\sqrt{4i\,d-4}}{2\,(d+i)}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + b*x*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*
I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) -
(b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(d +
 I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(d + I)) + a*log(1/2*(2*(d + I)*e^
(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(d + I)) + I*dilog(1/2*sqrt(4*I*d - 4)*e
^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(acoth(d*tan(a + b*x) - I*d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*tan(b*x + a) - I*d + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(acoth(d*tan(a + b*x) - d*1i + 1), x)
```

3.243 $\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\coth^{-1}(1-id+d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1-id+d\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-I*d+d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)/
x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(d\tan\left(a+bx\right)-id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*tan(a + b*x) - I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccoth(d*tan(b*x + a) - I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(d\tan(a+b\,x)+1-d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*tan(a + b*x) - d*1i + 1)/x,x)
[Out] int(acoth(d*tan(a + b*x) - d*1i + 1)/x, x)
```

3.244 $\int x^2 \coth^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=171

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ib\right.\right.}}{4b}$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d-d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Rubi [A]
time = 0.20, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6399, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6399

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\coth^{-1}(1+id-d\tan(a+bx))\,dx &= \frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(1+id)e^{2ia+2}}\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{3}(b(i-d))\int\frac{}{1}\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+\right.\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+\right.\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+\right.\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+\right.\\ &= \frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{6}x^3\log\left(1+(1+\right.\end{aligned}$$

Mathematica [A]
time = 0.13, size = 156, normalized size = 0.91

$$\frac{1}{3}x^3\coth^{-1}(1+id-d\tan(a+bx))-\frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)-3i\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a +
b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.69, size = 2449, normalized size = 14.32

method	result	size
risch	Expression too large to display	2449

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-
1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*
x+a))+1))^2+1/6*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/4/b/(I-d)*poly
```

log(2,I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog(2,I*(I-d)*exp(2*I
*(b*x+a)))*a^2-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))
-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/4/b^2*d/(I-
d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x+1/2/b^3*a^3*d/(I-d)*ln(1+I*exp(I*(
b*x+a))*(-I*(I-d))^(1/2))+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I*exp(2
I(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*
(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*cs
gn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(I*d)+1/2*I/b^2/(I-d)*ln(
1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I
*(b*x+a)))*x^2-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*
x-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*I/b^
3*a^2*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/4*I/b^3*d/(I-d)*
polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+
a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
I(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/6*I/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*
x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*(e
xp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
I(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*
x+a))*(-I*(I-d))^(1/2))*x+1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-
d))^(1/2))*x-1/2/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^2
*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/12*I*x^3*Pi*csgn(I*(
exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))-1/12*I*x
^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*
d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*
I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))
*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*exp(2*I
*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^2-1/6*x^3*ln(d)-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*
x+a)))^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/
8/b^3/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(2*
I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3
+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3
*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a)
)+1))-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))^2+1/2/b^3*a^3*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))
-1/3/b^3*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/6/b^3*a^3*d/(I-d)*ln(
I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))-I)-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2
))+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/8*I/b^
3*d/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I
*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/3*I/b^3/(I-d)*ln(1-I*

```
(I-d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+
a)))*x-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/12*I*x
^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
)*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/3*
x^3*ln(exp(I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(119) = 238.
time = 0.29, size = 342, normalized size = 2.00

$$-\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arccoth}(d\tan(bx+a)-id-1)}{b^2}+\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(-4i(bx+a)^3+9i(bx+a)^2a-9i(bx+a)a^2\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)+\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2\left((-id-1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3\left((-id-1)e^{(2ibx+2ia)}\right)+6i\operatorname{Li}_4\left((-id-1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2)*
arctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*
b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(119) = 238.
time = 0.39, size = 344, normalized size = 2.01

$$\frac{ib^4x^4-2b^3x^3\log\left(\frac{de^{(2ibx+2ia)}}{(d-i)e^{(2ibx+2ia)}-i}\right)+6ib^2x^2\operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)+6ib^2x^2\operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-ia^4+2a^3\log\left(\frac{2(d-i)e^{(ibx+ia)}+i\sqrt{-4id-4}}{2(d-i)}\right)+2a^3\log\left(\frac{2(d-i)e^{(ibx+ia)}-i\sqrt{-4id-4}}{2(d-i)}\right)-12bx\operatorname{polylog}\left(3,\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-12bx\operatorname{polylog}\left(3,-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-2(b^3x^3+a^3)\log\left(\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}+1\right)-2(b^3x^3+a^3)\log\left(-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}+1\right)-12i\operatorname{polylog}\left(4,\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)-12i\operatorname{polylog}\left(4,-\frac{1}{2}\sqrt{-4id-4}\,e^{(ibx+ia)}\right)}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x +
 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) + 6
*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log
(1/2*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + 2*a^3*log(
1/2*(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 12*b*x*poly
log(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(
-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d - 4)*e
^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x
+ I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*
polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d-d*tan(b*x+a)),x)
[Out] Integral(x**2*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d*1i - d*tan(a + b*x) + 1),x)
[Out] int(x^2*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.245 $\int x \coth^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=134

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d-d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(
3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6399, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6399

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Tan[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+id-d\tan(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1+id)e^{2ia+2ibx}}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}(b(i-d))\int\frac{e}{1+(}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id\right.$$

Mathematica [A]

time = 0.08, size = 120, normalized size = 0.90

$$\frac{1}{2}x^2\coth^{-1}(1+id-d\tan(a+bx))-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)
))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.43, size = 2351, normalized size = 17.54

method	result	size
risch	Expression too large to display	2351

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/4*ln(exp(2*I*(b*x+a))
*d-I*exp(2*I*(b*x+a))-I)*x^2-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x
+a)))*x-1/4*I/b^2*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a+1/4/b^2*d/(
I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/4/b^2*a^2*d/(I-d)*ln(I*exp(2*I*(b
*x+a))-exp(2*I*(b*x+a))*d+I)-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*
(I-d))^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))
/(exp(2*I*(b*x+a))+1))^2-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/
(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/2/b*d/(I-d)*ln(1-I*(I-
d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I
*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/2*I/b*a/(I-d)*ln
(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2*I/b^2*a*d/(I-d)*dilog(1+I*exp(I
*(b*x+a))*(-I*(I-d))^(1/2))-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(
2*I*(b*x+a)))^2*csgn(I*d)-1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-
d))^(1/2))+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))^2-1/4*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^
2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*I/b/(I-d)*ln(1-I*(I
-d)*exp(2*I*(b*x+a)))*x*a-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*
I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*cs
gn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-
I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1))+1/
2*I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/8*I*x^2*Pi*csgn(I
*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2
```

```
*I*(b*x+a)))*csgn(I*d)-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))+1))-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^
(1/2))*x-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2*I/b^
2*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2*I/b^2*a^2/(I-d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b
*x+a))-exp(2*I*(b*x+a))*d+I)-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*
(b*x+a))+1))^2*csgn(I/(exp(2*I*(b*x+a))+1))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*
x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))
^2+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/8*I*x^2
*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-
1/4*I/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/8*I/b^2/(I-d)*polylog(3,I*
(I-d)*exp(2*I*(b*x+a)))-1/4/b/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x-1
/4/b^2/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a+1/4*d/(I-d)*ln(1-I*(I-d)
*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x
+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*ex
p(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a)
)-I)/(exp(2*I*(b*x+a))+1))^3-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x
+a))/(exp(2*I*(b*x+a))+1))^3+1/8/b^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x
+a)))+1/2/b^2*a/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2/b^2*a/
(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/2*x^2*ln(exp(I*(b*x+a)))
+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3+1/8*I*x^2*P
i*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a)
)*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*(exp(
2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*csgn(I/(exp(2*
I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(94) = 188$.
time = 0.27, size = 247, normalized size = 1.84

$$-\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{arcoth}(d\tan(bx+a)-id-1)}{b}+\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2\left((-id-1)e^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a)a\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)+\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)+2\cos(2bx+2a)+1\right)+3\operatorname{Li}_3\left((-id-1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*tan(b*x + a) - I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2*I
*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(94) = 188$.

time = 0.37, size = 292, normalized size = 2.18

$$\frac{2i\,b^3x^3 - 3\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 3\,a^2\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right) - 3\,a^2\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right) - 3\,(b^2x^2-a^2)\log\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 3\,(b^2x^2-a^2)\log\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - 6\,\mathrm{polylog}\left(3, \frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - 6\,\mathrm{polylog}\left(3, -\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
 + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I
*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(1/2
*(2*(d - I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) - 3*a^2*log(1/2*
(2*(d - I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) - 3*(b^2*x^2 - a^
2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-1
/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*I*d - 4
)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Integral(x*acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(x*acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.246 $\int \coth^{-1}(1+id-d\tan(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}ibx^2+x\coth^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,-\left((1+id)e^{2ia+2ibx}\right)\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccoth(1+I*d-d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6391, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6391

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Coth[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1+id-d\tan(a+bx))\,dx = x\coth^{-1}(1+id-d\tan(a+bx)) + (ib)\int \frac{x}{1+(1+id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - (b(i-d))\int \frac{e^{2ia+}}{1+(1+i}$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia}\right.$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia}\right.$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia}\right.$$

Mathematica [A]
time = 0.48, size = 85, normalized size = 0.90

$$x\coth^{-1}(1+id-d\tan(a+bx)) - \frac{2bx\log\left(1-\frac{ie^{-2i(a+bx)}}{-i+d}\right) + i\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + I*d - d*Tan[a + b*x]] - (2*b*x*Log[1 - I/((-I + d)*E^((2*I)*(
a + b*x)))] + I*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(77) = 154.
time = 0.94, size = 334, normalized size = 3.55

method	result

derivativedivides	$-\frac{\frac{i\,\mathrm{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i\,\mathrm{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(\frac{i\,\mathrm{dilog}\left(1+\frac{id}{2}-\frac{d\tan}{}\right.}{2d}\right.}{}}{}$
default	$-\frac{\frac{i\,\mathrm{arccoth}(1+id-d\tan(bx+a))d\ln(id-d\tan(bx+a))}{2}-\frac{i\,\mathrm{arccoth}(1+id-d\tan(bx+a))d\ln(-id-d\tan(bx+a))}{2}-\frac{d^2\left(\frac{i\,\mathrm{dilog}\left(1+\frac{id}{2}-\frac{d\tan}{}\right.}{2d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d-d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(1/2*I*arccoth(1+I*d-d*tan(b*x+a))*d*ln(I*d-d*tan(b*x+a))-1/2*I*arcc
oth(1+I*d-d*tan(b*x+a))*d*ln(-I*d-d*tan(b*x+a))-1/2*d^2*(1/2*I/d*dilog(1+1/
2*I*d-1/2*d*tan(b*x+a))+1/2*I/d*ln(I*d-d*tan(b*x+a))*ln(1+1/2*I*d-1/2*d*tan
(b*x+a))-1/4*I/d*ln(I*d-d*tan(b*x+a))^2-1/2*I/d*dilog(I*(-I*d-d*tan(b*x+a)-
I*(2*I-2*d))/(2*I-2*d))-1/2*I/d*ln(-I*d-d*tan(b*x+a))*ln(I*(-I*d-d*tan(b*x+
a)-I*(2*I-2*d))/(2*I-2*d))+1/2*I/d*dilog(-1/2*I*(I*d-d*tan(b*x+a))/d)+1/2*I
/d*ln(-I*d-d*tan(b*x+a))*ln(-1/2*I*(I*d-d*tan(b*x+a))/d)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(66) = 132$.
time = 0.48, size = 260, normalized size = 2.77

$$-\frac{4\,(bx+a)d\left(\frac{\log(d\tan(bx+a)-i\,d-2)}{d}-\frac{\log(\tan(bx+a)-i)}{d}\right)+d\left(-\frac{2i\left(\log(d\tan(bx+a)-i\,d-2)\log\left(-\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}+1\right)+\mathrm{Li}_2\left(\frac{i\,d\tan(bx+a)+d-2i}{2\,(d-i)}\right)\right)}{d}+\frac{2i\,\log(d\tan(bx+a)-i\,d-2)\log(\tan(bx+a)-i)-i\,\log(\tan(bx+a)-i)^2}{d}-\frac{2i\,(\log(-\frac{1}{2}\,d\tan(bx+a)+\frac{1}{2}i\,d+1)\log(\tan(bx+a)-i)+\mathrm{Li}_2(\frac{1}{2}\,d\tan(bx+a)-\frac{1}{2}i\,d))}{d}+\frac{2i\,(\log(\tan(bx+a)-i)\log(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2})+\mathrm{Li}_2(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}))}{d}\right)+8\,(bx+a)\,\mathrm{arcoth}\,(d\tan\,(bx+a)-i\,d-1)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) - I
)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-1/2*(I*d*tan(b*x + a) +
d - 2*I)/(d - I) + 1) + dilog(1/2*(I*d*tan(b*x + a) + d - 2*I)/(d - I)))/d
+ (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b*x
+ a) - I)^2)/d - 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*x +
a) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d + 2*I*(log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8
*(b*x + a)*arccoth(d*tan(b*x + a) - I*d - 1))/b
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(66) = 132$.
time = 0.38, size = 218, normalized size = 2.32

$$\frac{i\,b^2x^2-bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)-i\,a^2-(bx+a)\log\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)-(bx+a)\log\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right)+a\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}+i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right)+a\log\left(\frac{2\,(d-i)e^{(i\,bx+i\,a)}-i\,\sqrt{-4i\,d-4}}{2\,(d-i)}\right)+i\,\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+i\,\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 - b*x*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a)
 - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) -
(b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(1/2*(2*(d
- I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(d - I)) + a*log(1/2*(2*(d - I)*
e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(d - I)) + I*dilog(1/2*sqrt(-4*I*d -
4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d-d*tan(b*x+a)),x)
```

```
[Out] Integral(acoth(-d*tan(a + b*x) + I*d + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*tan(b*x + a) + I*d + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(1-d\tan(a+b\,x)+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(acoth(d*1i - d*tan(a + b*x) + 1), x)
```

3.247 $\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\coth^{-1}(1+id-d\tan(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1+id-d\tan(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+I*d-d*tan(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)
 + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))/
x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(-d\tan\left(a+bx\right)+id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d-d*tan(b*x+a))/x,x)
```

```
[Out] Integral(acoth(-d*tan(a + b*x) + I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d-d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccoth(-d*tan(b*x + a) + I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{acoth}(1 - d\tan(a + b\,x) + d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*1i - d*tan(a + b*x) + 1)/x,x)
[Out] int(acoth(d*1i - d*tan(a + b*x) + 1)/x, x)
```

3.248 $\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=302

$$\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}+\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{i(e+fx)^3\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx}{}$$

```
[Out] 1/4*(f*x+e)^4*arccoth(cot(b*x+a))/f+1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A]

time = 0.17, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6389, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^4\text{ArcTan}\left(e^{2i(a+bx)}\right)}{4f}-\frac{3f^3\text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3\text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{3f(e+fx)^2\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)^3\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^3\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^4\coth^{-1}(\cot(a+bx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] ((e + f*x)^4*ArcCoth[Cot[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6389

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \coth^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f}$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{1}{2}\int (\ldots$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+\ldots}{}$$

$$= \frac{(e+fx)^4 \coth^{-1}(\cot(a+bx))}{4f} + \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} - \frac{i(e+\ldots}{}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 654 vs. $2(302) = 604$.
time = 0.17, size = 654, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcCoth[Cot[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 17.47, size = 7429, normalized size = 24.60

method	result	size
risch	Expression too large to display	7429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*f^2*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*f^2
*x^3*e + 6*f*x^2*e^2 + 4*x*e^3)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*f^2*x^3*e
+ 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*f^2*x^3*e + 6*b*f*x^2*e^2 + 4*b*x*e^3)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2798 vs. 2(244) = 488.
time = 0.51, size = 2798, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*poly
log(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*f^2*x^2*cosh(1) - 3*I*b^3*f*x*c
osh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3 - 3*I*(b^3*f*x + b^3*cosh(1))*
sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*
dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(-I*b^3*f^3*x^3 - 3*I*b^3*
```

f^2*x^2*cosh(1) - 3*I*b^3*f*x*cosh(1)^2 - I*b^3*cosh(1)^3 - I*b^3*sinh(1)^3
 - 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 - 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cos
h(1) + b^3*cosh(1)^2)*sinh(1))*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))
 + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1) + 3*I*b^3*f*x*cosh(1)^2 + I*b
^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*x + b^3*cosh(1))*sinh(1)^2 + 3*
I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cosh(1)^2)*sinh(1))*dilog(-I*cos(2
*b*x + 2*a) + sin(2*b*x + 2*a)) + 4*(I*b^3*f^3*x^3 + 3*I*b^3*f^2*x^2*cosh(1
) + 3*I*b^3*f*x*cosh(1)^2 + I*b^3*cosh(1)^3 + I*b^3*sinh(1)^3 + 3*I*(b^3*f*
x + b^3*cosh(1))*sinh(1)^2 + 3*I*(b^3*f^2*x^2 + 2*b^3*f*x*cosh(1) + b^3*cos
h(1)^2)*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3
*x^4 + 4*b^4*f^2*x^3*cosh(1) + 6*b^4*f*x^2*cosh(1)^2 + 4*b^4*x*cosh(1)^3 +
4*b^4*x*sinh(1)^3 + 6*(b^4*f*x^2 + 2*b^4*x*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*
x^3 + 3*b^4*f*x^2*cosh(1) + 3*b^4*x*cosh(1)^2)*sinh(1))*log((cos(2*b*x + 2*
a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 2*(
a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 -
 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f
^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(cos(2*b*x + 2*a)
 + I*sin(2*b*x + 2*a) + I) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*f
*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^3
*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^
2)*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4
 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)^3 + 6*
(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4*x + a
*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(b^4*f^2
*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b^2*f)*
cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4*f
^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*sinh(1)^
3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f + 2*(b^4
*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1) + 4*(b
^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 - a^2*b
^2*f)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*
(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*b^3)*si
nh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b^2*f +
2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*cosh(1)
+ 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*f*x^2 -
 a^2*b^2*f)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) +
1) - 2*(b^4*f^3*x^4 - a^4*f^3 + 4*(b^4*x + a*b^3)*cosh(1)^3 + 4*(b^4*x + a*
b^3)*sinh(1)^3 + 6*(b^4*f*x^2 - a^2*b^2*f)*cosh(1)^2 + 6*(b^4*f*x^2 - a^2*b
^2*f + 2*(b^4*x + a*b^3)*cosh(1))*sinh(1)^2 + 4*(b^4*f^2*x^3 + a^3*b*f^2)*c
osh(1) + 4*(b^4*f^2*x^3 + a^3*b*f^2 + 3*(b^4*x + a*b^3)*cosh(1)^2 + 3*(b^4*
f*x^2 - a^2*b^2*f)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) - sin(2*b*x +
2*a) + 1) + 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*
b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2*b^2*f - 2*a*b^3*cosh(1))*sinh(1)
^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) + 3*a*b^3*cosh(1)^2)*sinh(1))*log(-
cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^4*f^3 - 4*a^3*b*f^2*cosh(

```
1) + 6*a^2*b^2*f*cosh(1)^2 - 4*a*b^3*cosh(1)^3 - 4*a*b^3*sinh(1)^3 + 6*(a^2
*b^2*f - 2*a*b^3*cosh(1))*sinh(1)^2 - 4*(a^3*b*f^2 - 3*a^2*b^2*f*cosh(1) +
3*a*b^3*cosh(1)^2)*sinh(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I)
 + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh(1))*polylog(4, I*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a)) + 6*(I*b*f^3*x + I*b*f^2*cosh(1) + I*b*f^2*sinh
(1))*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 6*(-I*b*f^3*x - I*
b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*x
 + 2*a)) + 6*(-I*b*f^3*x - I*b*f^2*cosh(1) - I*b*f^2*sinh(1))*polylog(4, -I
*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*f^2*x*cosh(1
) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*(b^2*f^2*x + b^2*f*cosh(1))*sinh(
1))*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*
b^2*f^2*x*cosh(1) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*(b^2*f^2*x + b^2*
f*cosh(1))*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(
b^2*f^3*x^2 + 2*b^2*f^2*x*cosh(1) + b^2*f*cosh(1)^2 + b^2*f*sinh(1)^2 + 2*(
b^2*f^2*x + b^2*f*cosh(1))*sinh(1))*polylog(3, ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{acoth}(\cot(a+bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*acoth(cot(b*x+a)),x)
[Out] Integral((e + f*x)**3*acoth(cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(cot(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*arccoth(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acoth}(\cot(a+b\,x))\ (e+f\,x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x))*(e + f*x)^3,x)
[Out] int(acoth(cot(a + b*x))*(e + f*x)^3, x)
```

3.249 $\int (e+fx)^2 \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\mathrm{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+fx)^2\mathrm{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx}{}$$

```
[Out] 1/3*(f*x+e)^3*arccoth(cot(b*x+a))/f+1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6389, 4266, 2611, 6744, 2320, 6724}

$$\frac{i(e+fx)^3\mathrm{ArcTan}\left(e^{2i(a+bx)}\right)}{3f}+\frac{if^2\mathrm{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\mathrm{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\mathrm{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\mathrm{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\mathrm{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\mathrm{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] ((e + f*x)^3*ArcCoth[Cot[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6389

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (e+fx)^2\coth^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}-\frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}\\ &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f}+\frac{1}{2}\int (\\ &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+}{}\\ &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+}{}\\ &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+}{}\\ &= \frac{(e+fx)^3\coth^{-1}(\cot(a+bx))}{3f}+\frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f}-\frac{i(e+}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\coth^{-1}(\cot(a+bx))+\frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)-12b^3efx^2\log\left(1-ie^{2i(a+bx)}\right)-4b^3f^2x^3\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right)+12b^3efx^2\log\left(1+ie^{2i(a+bx)}\right)+4b^3f^2x^3\log\left(1+ie^{2i(a+bx)}\right)-6ib^2(e+fx)^2\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)+6ib^2(e+fx)^2\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)+6bef\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)+6bf^2x\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)-6bef\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)-6bf^2x\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)+3if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)-3if^2\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 15.79, size = 5543, normalized size = 23.69

method	result	size
risch	Expression too large to display	5543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*f*x^2*e + 3*x*e^2)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*f*x^2*e + 3*b*x*e^2)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(186) = 372.
time = 0.47, size = 1589, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*
polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1) - I*b^2*cos
h(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))*dilog(I*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(-I*b^2*f^2*x^2 - 2*I*b^2*f*x*cosh(1)
 - I*b^2*cosh(1)^2 - I*b^2*sinh(1)^2 - 2*I*(b^2*f*x + b^2*cosh(1))*sinh(1))
*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(I*b^2*f^2*x^2 + 2*I*b^2*
f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x + b^2*cosh(1
))*sinh(1))*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(I*b^2*f^2*x^
2 + 2*I*b^2*f*x*cosh(1) + I*b^2*cosh(1)^2 + I*b^2*sinh(1)^2 + 2*I*(b^2*f*x
+ b^2*cosh(1))*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(
b^3*f^2*x^3 + 3*b^3*f*x^2*cosh(1) + 3*b^3*x*cosh(1)^2 + 3*b^3*x*sinh(1)^2 +
 3*(b^3*f*x^2 + 2*b^3*x*cosh(1))*sinh(1))*log((cos(2*b*x + 2*a) + sin(2*b*x
 + 2*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 4*(a^3*f^2 - 3*a^
2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^
2*cosh(1))*sinh(1))*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(a^3
```

```
*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b
*f - 2*a*b^2*cosh(1))*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
I) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*x + a*
b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2*b*f +
 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2
*a) + 1) + 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3*(b^3*
x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2 - a^2
*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(I*cos(2*b*x + 2*a) - sin(2*b
*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1)^2 + 3
*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^3*f*x^2
 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-I*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + a^3*f^2 + 3*(b^3*x + a*b^2)*cosh(1
)^2 + 3*(b^3*x + a*b^2)*sinh(1)^2 + 3*(b^3*f*x^2 - a^2*b*f)*cosh(1) + 3*(b^
3*f*x^2 - a^2*b*f + 2*(b^3*x + a*b^2)*cosh(1))*sinh(1))*log(-I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a) + 1) + 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) + 3*a*b^2*cos
h(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh(1))*log(-co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(a^3*f^2 - 3*a^2*b*f*cosh(1) +
 3*a*b^2*cosh(1)^2 + 3*a*b^2*sinh(1)^2 - 3*(a^2*b*f - 2*a*b^2*cosh(1))*sinh
(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 6*(b*f^2*x + b*f*cos
h(1) + b*f*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 6*(
b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, I*cos(2*b*x + 2*a) - sin(2*
b*x + 2*a)) + 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*polylog(3, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 6*(b*f^2*x + b*f*cosh(1) + b*f*sinh(1))*pol
ylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^2 \operatorname{acoth}(\cot(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*acoth(cot(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arccoth(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\cot(a+bx))\,(e+fx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(acoth(cot(a + b*x))*(e + f*x)^2, x)
```

3.250 $\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}+\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)}{}$$

```
[Out] 1/2*(f*x+e)^2*arccoth(cot(b*x+a))/f+1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a))
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A]

time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6389, 4266, 2611, 2320, 6724}

$$\frac{i(e+fx)^2\text{ArcTan}\left(e^{2i(a+bx)}\right)}{2f}+\frac{f\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] ((e + f*x)^2*ArcCoth[Cot[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*ArcTan[E^((2
*I)*(a + b*x))])/f - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6389

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcCoth[Cot[a + b*x]]/(f*(m + 1))), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (e+fx)\coth^{-1}(\cot(a+bx))\,dx &= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f} \\
&= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{1}{2}\int (e \cdots \\
&= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+f \cdots}{} \\
&= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+f \cdots}{} \\
&= \frac{(e+fx)^2\coth^{-1}(\cot(a+bx))}{2f} + \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} - \frac{i(e+f \cdots}{}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 295, normalized size = 1.82

$$ex\coth^{-1}(\cot(a+bx))+\frac{1}{2}fx^2\coth^{-1}(\cot(a+bx))-\frac{e\left(\left(-4a+\pi-4bx\right)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-\left(-4a+\pi\right)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)\right)}{8b}+\frac{f(4ib^2x^2\text{ArcTan}(\cos(2(a+bx))+i\sin(2(a+bx)))+2ibx\text{PolyLog}(2,i\cos(2(a+bx))-\sin(2(a+bx)))-2ibx\text{PolyLog}(2,-i\cos(2(a+bx))+\sin(2(a+bx)))-\text{PolyLog}(3,i\cos(2(a+bx))-\sin(2(a+bx)))+\text{PolyLog}(3,-i\cos(2(a+bx))+\sin(2(a+bx))))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcCoth[Cot[a + b*x]] + (f*x^2*ArcCoth[Cot[a + b*x]])/2 - (e*((-4*a + P
i - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x))] - Log[1 + I/E^((2*I)*(a + b*x))]
) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*
(a + b*x))] - PolyLog[2, I/E^((2*I)*(a + b*x))])))/(8*b) + (f*((4*I)*b^2*x^
2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I*Co
s[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a + b
*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]
] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.49, size = 2543, normalized size = 15.70

method	result	size
risch	Expression too large to display	2543

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/8*
f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^
2+1/2*I*f/b^2*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))+1/2*I*f/b^2*a*dilog(1-ex
p(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/
4))-1/2*I*e/b*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b^2*f*a*dil
og(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*e*x-1/8*I*Pi*f*x^2+1/2*
I*e/b*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I*e/b*(I*b
*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*f/b^2*(I*b*x+I*a)^2*
ln(1+I*exp(2*I*(b*x+a)))-1/4*f/b^2*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)
))+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4
*ln(exp(2*I*(b*x+a))-I)*f*x^2-1/2*ln(exp(2*I*(b*x+a))-I)*e*x-1/4/b^2*f*a^2*
ln(exp(2*I*(b*x+a))+I)+1/4*f/b^2*a^2*ln(-exp(2*I*(b*x+a))+I)-1/2*I*e/b*dilo
g(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*e/b*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))
+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))-1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^
2*csgn(I*(exp(2*I*(b*x+a))-I))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(
2*I*(b*x+a))-1))^2*csgn(I*(exp(2*I*(b*x+a))-I))*x^2+1/2*I*f/b^2*a*(I*b*x+I*
a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/2*I*f/b^2*a*(I*b*x+I*a)*ln(((-I)^(1/2)
-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I*f/b^2*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I
*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/
2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/4*f/b^2*(I*b*
x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))+1/4*f/b^2*(I*b*x+I*a)*polylog(2,I*exp(2*I
*(b*x+a)))+1/2/b*e*a*ln(exp(2*I*(b*x+a))+I)-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x
+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))*x^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a
```

```
))+I)/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I))*x^2-1/4*I*Pi*x*e*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1
))*csgn(I*(exp(2*I*(b*x+a))+I))-1/2*e/b*a*ln(-exp(2*I*(b*x+a))+I)+1/2*I*f/b
^2*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/8*I*Pi*f*csgn(I*(exp(2*I
*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))-1))^2*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+
a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*
e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))-1))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a
)))/(-I)^(1/2))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))-1))^3+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))-1))^3-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(
2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))
-I)/(exp(2*I*(b*x+a))-1))^3*x^2+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b*x+a))+I)+
1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*
Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2+1/8*I*Pi*f
*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*csgn(I*(exp(2*I*(b*x+a
))+I))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2
*csgn(I*(exp(2*I*(b*x+a))+I))+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(
I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I))*x
^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(e
xp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I))-1/8*I*Pi*f*csgn(I*(exp(2*I
*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+
a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+
a))-1))^2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+
1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))-1))^2*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))-1))^3-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))
^3*x^2-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2
-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*x*e)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*x*e)*co
```

```
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*x*e)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(134) = 268$.
time = 0.47, size = 793, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(I*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a)) + 2*(-I*b*f*x - I*b*cosh(1) - I*b*sinh(1))*dilog(I*cos(2*
b*x + 2*a) - sin(2*b*x + 2*a)) + 2*(I*b*f*x + I*b*cosh(1) + I*b*sinh(1))*di
log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 2*(I*b*f*x + I*b*cosh(1) + I*
b*sinh(1))*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^2*f*x^2 + 2
*b^2*x*cosh(1) + 2*b^2*x*sinh(1))*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
+ 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 2*(a^2*f - 2*a*b*cosh(1)
- 2*a*b*sinh(1))*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^2*f
- 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)
+ I) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sin
h(1))*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 - a^2*f
 + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1))*log(I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1)
+ 2*(b^2*x + a*b)*sinh(1))*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)
- 2*(b^2*f*x^2 - a^2*f + 2*(b^2*x + a*b)*cosh(1) + 2*(b^2*x + a*b)*sinh(1))
*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 2*(a^2*f - 2*a*b*cosh(1)
 - 2*a*b*sinh(1))*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*(a^2*
f - 2*a*b*cosh(1) - 2*a*b*sinh(1))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*
a) + I) - f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + f*polylog(3
, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a)
 + sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))
)/b^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{acoth}(\cot(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acoth(cot(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arccoth(cot(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\cot(a+b\,x))\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x))*(e + f*x),x)
```

```
[Out] int(acoth(cot(a + b*x))*(e + f*x), x)
```

3.251 $\int \coth^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=79

$$x\coth^{-1}(\cot(a+bx))+ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}$$

```
[Out] x*arccoth(cot(b*x+a))+I*x*arctan(exp(2*I*(b*x+a)))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6385, 4266, 2317, 2438}

$$ix\text{ArcTan}\left(e^{2i(a+bx)}\right)-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+x\coth^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[Cot[a + b*x]] + I*x*ArcTan[E^((2*I)*(a + b*x))] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6385

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[Cot[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \coth^{-1}(\cot(a+bx))\,dx = x\coth^{-1}(\cot(a+bx)) - b\int x\sec(2a+2bx)\,dx$$

$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) + \frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)\,dx - \frac{1}{2}\int$$

$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i(2a+2bx)}\right)}{4b}$$

$$= x\coth^{-1}(\cot(a+bx)) + ix\tan^{-1}\left(e^{2i(a+bx)}\right) - \frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b} + \frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}$$

Mathematica [A]
time = 0.04, size = 127, normalized size = 1.61

$$x\coth^{-1}(\cot(a+bx)) - \frac{(-4a+\pi-4bx)\left(\log\left(1-ie^{-2i(a+bx)}\right)-\log\left(1+ie^{-2i(a+bx)}\right)\right)-(-4a+\pi)\log\left(\cot\left(a+\frac{\pi}{4}+bx\right)\right)+2i\left(\text{PolyLog}\left(2,-ie^{-2i(a+bx)}\right)-\text{PolyLog}\left(2,ie^{-2i(a+bx)}\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[Cot[a + b*x]] - ((-4*a + Pi - 4*b*x)*(Log[1 - I/E^((2*I)*(a + b*x
))] - Log[1 + I/E^((2*I)*(a + b*x))]) - (-4*a + Pi)*Log[Cot[a + Pi/4 + b*x]
] + (2*I)*(PolyLog[2, (-I)/E^((2*I)*(a + b*x))] - PolyLog[2, I/E^((2*I)*(a
+ b*x))]))/(8*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(64) = 128.
time = 0.78, size = 188, normalized size = 2.38

method	result
derivativedivides	$\frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccoth}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot^2(bx+a)+1}\right)}{2}+\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\ln}{2}}{b}$
default	$\frac{-\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\text{arccoth}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\ln\left(1+\frac{i(1+i\cot(bx+a))^2}{\cot^2(bx+a)+1}\right)}{2}+\frac{\left(\frac{\pi}{2}-\text{arccot}(\cot(bx+a))\right)\ln}{2}}{b}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-(1/2*Pi-arccot(cot(b*x+a)))*arccoth(cot(b*x+a))-1/2*(1/2*Pi-arccot(co
t(b*x+a)))*ln(1+I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))+1/2*(1/2*Pi-arccot(c
ot(b*x+a)))*ln(1-I*(1+I*cot(b*x+a))^2/(cot(b*x+a)^2+1))+1/4*I*dilog(1+I*(1+
I*cot(b*x+a))^2/(cot(b*x+a)^2+1))-1/4*I*dilog(1-I*(1+I*cot(b*x+a))^2/(cot(b
*x+a)^2+1)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(57) = 114$.
time = 0.49, size = 184, normalized size = 2.33

$$\frac{4\,(bx+a)\operatorname{arcoth}\left(\frac{1}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{1}{2}\tan(bx+a)+\frac{1}{2},\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\tan(bx+a)-\frac{1}{2},-\frac{1}{2}\tan(bx+a)+\frac{1}{2}\right)\right)\log\left(\tan(bx+a)^2+1\right)-(bx+a)\log\left(\frac{1}{2}\tan(bx+a)^2+\tan(bx+a)+\frac{1}{2}\right)+(bx+a)\log\left(\frac{1}{2}\tan(bx+a)^2-\tan(bx+a)+\frac{1}{2}\right)-i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\tan(bx+a)-\frac{1}{2}i+\frac{1}{2}\right)+i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\tan(bx+a)+\frac{1}{2}i+\frac{1}{2}\right)+i\,\mathrm{Li}_2\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\tan(bx+a)+\frac{1}{2}i+\frac{1}{2}\right)-i\,\mathrm{Li}_2\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\tan(bx+a)-\frac{1}{2}i+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(
b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) -
I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*
tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I +
1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(57) = 114$.
time = 0.41, size = 388, normalized size = 4.91

$$\frac{4\,bx\log\left(\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos(2\,bx+2\,a)+i\sin(2\,bx+2\,a)+i\right)-2\,a\log\left(\cos(2\,bx+2\,a)-i\sin(2\,bx+2\,a)+i\right)-2\,(bx+a)\log\left(i\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1\right)+2\,(bx+a)\log\left(i\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1\right)-2\,(bx+a)\log\left(-i\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1\right)+2\,(bx+a)\log\left(-i\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1\right)+2\,a\log\left(-\cos(2\,bx+2\,a)+i\sin(2\,bx+2\,a)+i\right)-2\,a\log\left(-\cos(2\,bx+2\,a)-i\sin(2\,bx+2\,a)+i\right)+i\,\mathrm{Li}_2\left(i\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)\right)+i\,\mathrm{Li}_2\left(i\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)\right)-i\,\mathrm{Li}_2\left(-i\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)\right)-i\,\mathrm{Li}_2\left(-i\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) +
I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(I
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x + 2
*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*b
*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1
) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b*
x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x
 + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos(
2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(\cot\left(a+bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cot(b*x+a)),x)
```

```
[Out] Integral(acoth(cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(cot(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(\cot(a + b\,x))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x)),x)
```

```
[Out] int(acoth(cot(a + b*x)), x)
```

3.252 $\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arccoth(cot(b*x+a))/(f*x+e),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Mathematica [A]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcCoth[Cot[a + b*x]]/(e + f*x), x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\cot(bx+a))}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(cot(b*x+a))/(f*x+e),x)
[Out] int(arccoth(cot(b*x+a))/(f*x+e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arccoth(cot(b*x + a))/(f*x + e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(\cot\left(a + bx\right)\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(cot(b*x+a))/(f*x+e),x)
[Out] Integral(acoth(cot(a + b*x))/(e + f*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(cot(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] integrate(arccoth(cot(b*x + a))/(f*x + e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{acoth}(\cot(a + b\,x))}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x))/(e + f*x),x)
[Out] int(acoth(cot(a + b*x))/(e + f*x), x)
```

3.253 $\int x^2 \coth^{-1}(c + d\cot(a + bx))\, dx$

Optimal. Leaf size=391

$$\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{6}x^3\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{ix^2]}{}$$

```
[Out] 1/3*x^3*arccoth(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1+c+
I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*a+2*
I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d
))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/8*I*po
lylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

Rubi [A]
time = 0.35, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6405, 2221, 2611, 6744, 2320, 6724}

$$\frac{i\text{Li}_4\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix^2\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{6}x^3\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{6}x^3\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6405

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 - c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 + c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{3}(b(i(1+c)-d))\int\frac{e^{2ia+}}{1+c-id+(-1}$$
$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$
$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$
$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$
$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$
$$=\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

Mathematica [A]
time = 0.23, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3\coth^{-1}(c+d\cot(a+bx))+\frac{4b^3x^3\log\left(1-\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-4b^3x^3\log\left(1-\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)-6ib^2x^2\text{PolyLog}\left(2,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)+6ib^2x^2\text{PolyLog}\left(2,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)+6bx\text{PolyLog}\left(3,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-6bx\text{PolyLog}\left(3,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)+3i\text{PolyLog}\left(4,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-3i\text{PolyLog}\left(4,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[c + d*Cot[a + b*x]])/3 + (4*b^3*x^3*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 4*b^3*x^3*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] + (6*I)*b^2*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*(a + b*x)))/(1 + c - I*d)] + 6*b*x*PolyLog[3, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] - 6*b*x*PolyLog[3, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + (3*I)*PolyLog[4, ((-1 + c + I*d)*E^((2*I)*
(a + b*x)))/(-1 + c - I*d)] - (3*I)*PolyLog[4, ((1 + c + I*d)*E^((2*I)*(a +
 b*x)))/(1 + c - I*d)])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.16, size = 6698, normalized size = 17.13

method	result	size
risch	Expression too large to display	6698

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*a
)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^2
 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) + (c^
2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
+ 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4
 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (
c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
+ 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*
(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4
 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c
^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin
(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1798 vs. 2(275) = 550.
time = 0.56, size = 1798, normalized size = 4.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*
cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c^2
 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 +
 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2
```

```
+ 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin
(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c
^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2
- 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^
2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*s
in(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(1/2*c^2 +
 I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(
I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^3*log(1/2*c^2
+ I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4*a^3*log(-1/2*c^
2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/
2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 4*a^3*log(-1/2*
c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 6*b*x*polylog
(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c
 + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - 6*b
*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I
*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c
+ 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 +
d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*c
os(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*
a))/(c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I*
(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2
 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 4*(b^3*x
^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c +
 1)/(c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I*
(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2
 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 4*(b^3*x
^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c +
 1)/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c -
I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c
 - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 -
2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2
- 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d +
 I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(c + d \cot(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(c+d*cot(b*x+a)),x)
[Out] Integral(x**2*acoth(c + d*cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \cot(a + b\,x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(c + d*cot(a + b*x)),x)
[Out] int(x^2*acoth(c + d*cot(a + b*x)), x)
```

3.254 $\int x \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=293

$$\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{4}x^2\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{ixP}{}$$

```
[Out] 1/2*x^2*arccoth(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*poly
log(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I*d)
*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*b*x)
/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^2
```

Rubi [A]
time = 0.29, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6405, 2221, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{4}x^2\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6405

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + (-Dist[I*b*((1 - c - I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*
(E^(2*I*a + 2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x]
, x] + Dist[I*b*((1 + c + I*d)/(f*(m + 1))), Int[(e + f*x)^(m + 1)*(E^(2*I*
a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x))), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(c+d\cot(a+bx))\,dx=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{2}(b(i(1+c)-d))\int\frac{e^{2ia+}}{1+c-id+(-1}$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

$$=\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{1}{4}x^2\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-$$

Mathematica [A]
time = 0.09, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2\coth^{-1}(c+d\cot(a+bx))+\frac{2b^2x^2\log\left(1-\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-2b^2x^2\log\left(1-\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)-2ibx\text{PolyLog}\left(2,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)+2ibx\text{PolyLog}\left(2,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)+\text{PolyLog}\left(3,\frac{(-1+c+id)e^{2i(a+bx)}}{-1+c-id}\right)-\text{PolyLog}\left(3,\frac{(1+c+id)e^{2i(a+bx)}}{1+c-id}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[c + d*Cot[a + b*x]])/2 + (2*b^2*x^2*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 2*b^2*x^2*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (2*I)*b*x*PolyLog[2, ((-1 + c + I*d)*E^((2
*I)*(a + b*x)))/(-1 + c - I*d)] + (2*I)*b*x*PolyLog[2, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + PolyLog[3, ((-1 + c + I*d)*E^((2*I)*(a + b
*x)))/(-1 + c - I*d)] - PolyLog[3, ((1 + c + I*d)*E^((2*I)*(a + b*x)))/(1 +
 c - I*d)])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.86, size = 6348, normalized size = 21.67

method	result	size
risch	Expression too large to display	6348

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin(
2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x
^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*cos
(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 - d^2
- 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 +
 (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 +
 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d
```

```
)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x
 + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c
*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*
c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c
 + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*
(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2
 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(207) = 414$.
time = 0.54, size = 1462, normalized size = 4.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*
cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 + d
^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(
c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
+ 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c +
1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x +
 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (
c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)
*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) +
 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos
(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a)
- 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(c + 1)*d - 1
/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 +
2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 + I*(c - 1)*d -
1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 -
 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^2 + I*(c + 1)*d
+ 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2
 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/2*c^2 + I*(c - 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*log((c^2 +
d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
 + 1)) - 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b
*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 +
d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
```

```
(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c
 + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*
c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b
*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1
)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*
c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c
 + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 -
 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2
*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2
 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
+ I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}\left(c + d \cot\left(a + bx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*acoth(c + d*cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{acoth}\left(c+d\,\mathrm{cot}\left(a+b\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(c + d*cot(a + b*x)),x)
```

```
[Out] int(x*acoth(c + d*cot(a + b*x)), x)
```

3.255 $\int \coth^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=194

$$x\coth^{-1}(c+d\cot(a+bx))+\frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right)-\frac{1}{2}x\log\left(1-\frac{(1+c+id)e^{2ia+2ibx}}{1+c-id}\right)-\frac{i\text{PolyL}}{}$$

```
[Out] x*arccoth(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d)
)-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c-I
*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+2*I
*b*x)/(1+c-I*d))/b
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6397, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)+x\coth^{-1}(d\cot(a+bx)+c)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6397

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[x*(E^(2*I*a +
2*I*b*x)/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x))), x], x] + Dist[
I*b*(1 + c + I*d), Int[x*(E^(2*I*a + 2*I*b*x)/(1 + c - I*d - (1 + c + I*d)*
E^(2*I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2
, 1]
```

Rubi steps

$$\int \coth^{-1}(c+d\cot(a+bx))\,dx = x\coth^{-1}(c+d\cot(a+bx)) + (b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x}{1+c-id+(-1-c-$$

$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log$$

$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log$$

$$= x\coth^{-1}(c+d\cot(a+bx)) + \frac{1}{2}x\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{2}x\log$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4463 vs. $2(194) = 388$.
time = 12.62, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
 + Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*
c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))/
(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c +
c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))/(-1 +
 c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 +
 d^2])/d) + Tan[(a + b*x)/2]] + (a + b*x)*Log[-((1 + c + Sqrt[1 + 2*c + c^2
 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c
- I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2
])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d +
```

```
 Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) +
 Tan[(a + b*x)/2]] - (a + b*x)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d] - I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sq
rt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(
a + b*x)/2])/d] + I*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1
- 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] + (a + b*x)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a +
 b*x)/2])/d] + I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 +
 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] - I*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*
c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)
/2])/d] - I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x
)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 + c +
Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c + I*d + Sqrt[1 - 2*
c + c^2 + d^2])] - I*PolyLog[2, (1 + c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[
(a + b*x)/2])/(1 + c + I*d - Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1
+ c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1
 + 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d
*Tan[(a + b*x)/2])/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2
, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c - I*d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2
] + d*Tan[(a + b*x)/2])/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*Poly
Log[2, (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(-1 - c +
I*d + Sqrt[1 + 2*c + c^2 + d^2])])*((2*a)/(b*(1 - c^2 - d^2 - Cos[2*(a + b*
x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])
) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x
)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/(-Log[-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + Log[-((1 + c + Sqrt[1 + 2
*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - Log[(1 - c + Sqrt[1 - 2*c + c^2 +
 d^2] + d*Tan[(a + b*x)/2])/d] + Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] +
d*Tan[(a + b*x)/2])/d] - ((I/2)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/
d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)
*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a +
b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(1 - c + Sqrt[1 - 2*c + c^2
 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]
) + ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]
*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((-1 + c + Sqrt[
1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(
a + b*x)/2]) - ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[(1 - c
+ Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I
 + Tan[(a + b*x)/2]) - ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((a + b*x)
*Sec[(a + b*x)/2]^2)/(2*(-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2])) - ((I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[
```

```
1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sqrt[1 - 2*c + c^2
+ d^2])/d) + Tan[(a + b*x)/2]) + ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(-1
+ c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/
2]^2)/(2*(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2])) + (
(I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(
a + b*x)/2]) - ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1
+ 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d
^2])/d) + Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2
 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + S...
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(164) = 328$.
time = 1.23, size = 570, normalized size = 2.94

method	result
derivativedivides	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\ldots\right.}{\ldots}$
default	$\frac{-d\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))\right)\operatorname{arccoth}(c+d\cot(bx+a))-d^2\left(\frac{\arctan\left(-\frac{c+d\cot(bx+a)}{d}+\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2d}-\ldots\right.}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arccoth(c+d*cot(b*x+a))-d^2*(1/2*arct
an(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)-1/2*arctan
(-(c+d*cot(b*x+a))/d+c/d)/d*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+1/4*I*ln(d*(
(c+d*cot(b*x+a))/d-c/d)+c-1)*(ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I*d+c-1)
)-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d)))/d+1/4*I/d*dilog((I*d-d*((
c+d*cot(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/d*dilog((I*d+d*((c+d*cot(b*x+a))/d
-c/d))/(1-c+I*d))-1/4*I*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*(ln((I*d-d*((c+d
*cot(b*x+a))/d-c/d))/(1+c+I*d))-ln((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-
1)))/d-1/4*I/d*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/d*di
log((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(136) = 272$.

time = 0.54, size = 392, normalized size = 2.02

$$\frac{4\,(bx+a)\,\mathrm{arcoth}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+(c^2+2\,c+1)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d+(c^2-2\,c+1)\tan(bx+a)}{c^2+d^2-2\,c+1},\frac{(c-1)d\tan(bx+a)+d^2}{c^2+d^2-2\,c+1}\right)\right)\log\left(\tan(bx+a)^2+1\right)-(bx+a)\log\left(\frac{2\,(c+1)d\tan(bx+a)+(c^2+2\,c+1)\tan(bx+a)^2+d^2}{c^2+d^2+2\,c+1}\right)+(bx+a)\log\left(\frac{2\,(c-1)d\tan(bx+a)+(c^2-2\,c+1)\tan(bx+a)^2+d^2}{c^2+d^2-2\,c+1}\right)+i\,\mathrm{Li}_2\left(-\frac{(c+1)\tan(bx+a)-i\,c-i}{i\,c+d+i}\right)-i\,\mathrm{Li}_2\left(-\frac{(c-1)\tan(bx+a)-i\,c+i}{i\,c+d-i}\right)+i\,\mathrm{Li}_2\left(-\frac{(c-1)\tan(bx+a)+i\,c-i}{-i\,c+d+i}\right)-i\,\mathrm{Li}_2\left(-\frac{(c+1)\tan(bx+a)+i\,c+i}{-i\,c+d-i}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2 +
 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + d^
2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x +
a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x + a) +
(c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 +
d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c + d + I))
- I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dilog(-((c -
 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*tan(b*x + a
) + I*c + I)/(-I*c + d - I)))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(136) = 272$.
time = 0.55, size = 1098, normalized size = 5.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log((d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(2
*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1)*
d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c - 1)*d
 - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c + 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(c - 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2 + d^2 - (
c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)
*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1))
- 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*
b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2
*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 + 2*I*(c
 - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 +
 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x + a
```

```
)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2
 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c
 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b
*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*dilog(-(c^2 + d^2 - (c^
2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d
- I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1)
 - I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c +
1)/(c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d
- d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(c + d\cot\left(a + bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(acoth(c + d*cot(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}(c + d\cot(a + b\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*cot(a + b*x)),x)
```

```
[Out] int(acoth(c + d*cot(a + b*x)), x)
```

3.256 $\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{\coth^{-1}(c+d\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arccoth(c+d*cot(b*x+a))/x,x)

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcCoth[c + d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCoth[c + d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcCoth[c + d*Cot[a + b*x]]/x, x]

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(c+d*cot(b*x+a))/x,x)
```

```
[Out] int(arccoth(c+d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(d*cot(b*x + a) + c)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(c+d\cot\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(c+d*cot(b*x+a))/x,x)
```

```
[Out] Integral(acoth(c + d*cot(a + b*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + c)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c + d\cot(a + b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(c + d*cot(a + b*x))/x,x)
[Out] int(acoth(c + d*cot(a + b*x))/x, x)
```

3.257 $\int x^2 \coth^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=168

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccoth(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*poly
log(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+2*I
*b*x))/b^3
```

Rubi [A]

time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6401, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left((id+1)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\coth^{-1}(d\cot(a+bx)+id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6401

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1-id)e^{2ia+2}}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(b(i-d))\int \frac{}{1+}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+i\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+i\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+i\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+i\right.$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+i\right.$$

Mathematica [A]
time = 0.13, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\coth^{-1}(1+id+d\cot(a+bx)) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6ib^2x^2\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right) + 6bx\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right) - 3i\mathrm{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.23, size = 2449, normalized size = 14.58

method	result	size
risch	Expression too large to display	2449

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a))*x-1/2*I/b^3*a^3/(I-d)
*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^3*a^3*d/(I-d)*ln(1-I*exp(I*(b
*x+a))*(I*(I-d))^(1/2))+1/2/b^3*a^3*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))
```

^(1/2))-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/3
/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*d/(I-d)*polylog(3,-
I*(I-d)*exp(2*I*(b*x+a)))*x-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a))
)^2+1/12*I*b*x^4-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))
+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I/(I-d
)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*cs
gn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I
*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a
))-1)*exp(2*I*(b*x+a)))^3+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I
*(b*x+a))*d-I)+1/8*I/b^3*d/(I-d)*polylog(4,-I*(I-d)*exp(2*I*(b*x+a)))+1/3*I
/b^3/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*
(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*
x+a))-1))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))+1/2*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^
2-1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^
2/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^3*a^2*d/(I-d)*dilo
g(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp(I
*(b*x+a))*(I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+
a)))*x^2+1/4*I/b^3*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^2
*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/2/b^2*a^2*d/(I-d)*ln(1-I*ex
p(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(
I*(I-d))^(1/2))*x-1/6*x^3*ln(d)-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*
exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a)
)-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12
*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a)
)-1))^2-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2
I(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)
))^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(
exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))
/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/6*d/(I-
d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*
x+a))*(I*(I-d))^(1/2))-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))
^(1/2))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2*csg
n(I*d)+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*(exp(2
I(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(
b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(e
xp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*
d)+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*
(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))-1))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/8/b^3/(I-d)*polylog(4,-I*(I-d)*exp(2*

```
I*(b*x+a))-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I
-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*
(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*e
xp(2*I*(b*x+a))+I)
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(119) = 238$.
time = 0.28, size = 344, normalized size = 2.05

$$\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+id+1)}{b^2}-\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)-\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\mathrm{Li}_2\left((id+1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3(4bx+a)\mathrm{Li}_3\left((id+1)e^{(2ibx+2ia)}\right)+6i\,\mathrm{Li}_4\left((id+1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b*
x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*ar
ctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x
 + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((I*d +
 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a
^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d*s
in(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d +
 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b
^2)/b
```

Fricas [**A**]
time = 0.36, size = 179, normalized size = 1.07

$$\frac{2ib^4x^4+4b^3x^3\log\left(\frac{\left((d-i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)+6ib^2x^2\mathrm{Li}_2\left(-(-id-1)e^{(2ibx+2ia)}\right)-2ia^4+4a^3\log\left(\frac{(d-i)e^{(2ibx+2ia)}+i}{d-i}\right)-6bx\,\mathrm{polylog}\left(3,(id+1)e^{(2ibx+2ia)}\right)-4\left(b^3x^3+a^3\right)\log\left((-id-1)e^{(2ibx+2ia)}+1\right)-3i\,\mathrm{polylog}\left(4,(id+1)e^{(2ibx+2ia)}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylog
(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2*
I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d+d*cot(b*x+a)),x)
[Out] Integral(x**2*acoth(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d*1i + d*cot(a + b*x) + 1),x)
[Out] int(x^2*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.258 $\int x \coth^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal. Leaf size=132

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{\text{Po}}{}$$

[Out] 1/6*I*b*x^3+1/2*x^2*arccoth(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2

Rubi [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6401, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d\cot(a+bx)+id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6401

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1-id)e^{2ia+2\cdots}}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(b(i-d))\int\frac{\cdots}{1+(\cdots}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+i\cdots\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+i\cdots\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+i\cdots\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+i\cdots\right.$$

Mathematica [A]
time = 0.08, size = 119, normalized size = 0.90

$$\frac{1}{2}x^2\coth^{-1}(1+id+d\cot(a+bx))-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{-i+d}\right)+2ibx\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)+\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.80, size = 2351, normalized size = 17.81

method	result	size
risch	Expression too large to display	2351

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*cs
gn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/2*I/b^2*a^2/(I-d)*ln(1+I*ex
p(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*
(I-d))^(1/2))+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(
2*I*(b*x+a))-1))^2+1/4/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/4/b
^2*a^2*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csg
n(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(
b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+
a))-1))^2-1/4*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^2*a^2*d/
(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp
(I*(b*x+a))*(I*(I-d))^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*
(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi
*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2*csgn(I*d)-1/8*I*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2
+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2
*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a))-
exp(2*I*(b*x+a))*d-I)-1/2*I/b/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/2*
I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2*I/b^2*a*d/(I-d)*di
log(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a*d/(I-d)*dilog(1+I*exp(I
*(b*x+a))*(I*(I-d))^(1/2))-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x
+a))+I)/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*
exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a
)))^2+1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*
(b*x+a))-1))^3+1/2/b*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a-1/2/b*a*d/(
I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1+I*exp(I*
(b*x+a))*(I*(I-d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*ex
```

```
p(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))
)^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(ex
p(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)+1/8*I*x^2*Pi*csgn(I*exp(2*I*(
b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x
+a))-1))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))
*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))-1))+1/2*I/b*a/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2)
)*x-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4*I/b^2*d/(I-d
)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a-1/4*x^2*ln(d)-1/2*x^2*ln(exp(I*(b*
x+a)))-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1)
)^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/8*I*x^
2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/4*I/(I-d)*ln(1+I*(I-
d)*exp(2*I*(b*x+a)))*x^2-1/8*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)
))-1/4/b^2/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a-1/8*I*x^2*Pi*csgn((
exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/2/b^2*a/
(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^2*a/(I-d)*dilog(1+I*e
xp(I*(b*x+a))*(I*(I-d))^(1/2))+1/4*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x
^2-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x+1/8/b^2*d/(I-d)*polyl
og(3,-I*(I-d)*exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*ex
p(2*I*(b*x+a)))^3+1/4*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)*x^2
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(94) = 188$.
time = 0.28, size = 249, normalized size = 1.89

$$\frac{\frac{12\left((bx+a)^2-2(bx+a)a\right)\operatorname{arcoth}(d\cot(bx+a)+id+1)}{b}-\frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\mathrm{Li}_2\left((id+1)e^{(2ibx+2ia)}\right)-6\left(i(bx+a)^2-2i(bx+a)a\right)\arctan(d\cos(2bx+2a)+\sin(2bx+2a),d\sin(2bx+2a)-\cos(2bx+2a)+1)+3\left((bx+a)^2-2(bx+a)a\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2+2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3\mathrm{Li}_3\left((id+1)e^{(2ibx+2ia)}\right)}{b}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I*b
*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(d*cos(2*b*x + 2*
a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*((b*x
 + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2
*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*polylog(
3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [A]
time = 0.39, size = 156, normalized size = 1.18

$$\frac{4ib^3x^3+6b^2x^2\log\left(\frac{((d-i)e^{(2ibx+2ia)}+i)e^{(-2ibx-2ia)}}{d}\right)+4ia^3+6ibx\mathrm{Li}_2\left(-(-id-1)e^{(2ibx+2ia)}\right)-6a^2\log\left(\frac{(d-i)e^{(2ibx+2ia)}+i}{d-i}\right)-6(b^2x^2-a^2)\log\left((-id-1)e^{(2ibx+2ia)}+1\right)-3\,\mathrm{polylog}\left(3,(id+1)e^{(2ibx+2ia)}\right)}{24b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a))
 - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a^2)
*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*I*b*
x + 2*I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}\left(d \cot\left(a + bx\right) + id + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*acoth(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(d \cot(a + b\,x) + 1 + d\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(x*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.259 $\int \coth^{-1}(1+id+d\cot(a+bx))\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}ibx^2+x\coth^{-1}(1+id+d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccoth(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6393, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right)+x\coth^{-1}(d\cot(a+bx)+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6393

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1+id+d\cot(a+bx))\,dx = x\coth^{-1}(1+id+d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) + (b(i-d))\int \frac{e^{2ia+2\cdots}}{1+(-1-i\cdots}$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+\cdots}\right.$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+\cdots}\right.$$

$$= \frac{1}{2}ibx^2 + x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+\cdots}\right.$$

Mathematica [A]
time = 0.33, size = 83, normalized size = 0.89

$$x\coth^{-1}(1+id+d\cot(a+bx)) - \frac{2bx\log\left(1+\frac{e^{-2i(a+bx)}}{-1-id}\right) + i\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{-i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 + I*d + d*Cot[a + b*x]] - (2*b*x*Log[1 + 1/((-1 - I*d)*E^((2*I)
*(a + b*x)))] + I*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(76) = 152.
time = 1.25, size = 321, normalized size = 3.45

method	result

derivativedivides	$\frac{-\frac{i\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\ln(id+d\cot(bx+a}{4d}\right.}{}}{}$
default	$\frac{-\frac{i\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(id+d\cot(bx+a))}{2}+\frac{i\mathrm{arccoth}(1+id+d\cot(bx+a))d\ln(-id+d\cot(bx+a))}{2}+\frac{d^2\left(-\frac{i\ln(id+d\cot(bx+a}{4d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/d*(-1/2*I*arccoth(1+I*d+d*cot(b*x+a))*d*ln(I*d+d*cot(b*x+a))+1/2*I*arcc
oth(1+I*d+d*cot(b*x+a))*d*ln(-I*d+d*cot(b*x+a))+1/2*d^2*(-1/4*I/d*ln(I*d+d*
cot(b*x+a))^2+1/2*I/d*dilog(1+1/2*I*d+1/2*d*cot(b*x+a))+1/2*I/d*ln(I*d+d*co
t(b*x+a))*ln(1+1/2*I*d+1/2*d*cot(b*x+a))-1/2*I/d*dilog(I*(-I*d+d*cot(b*x+a)
-I*(2*I-2*d))/(2*I-2*d))-1/2*I/d*ln(-I*d+d*cot(b*x+a))*ln(I*(-I*d+d*cot(b*x
+a)-I*(2*I-2*d))/(2*I-2*d))+1/2*I/d*dilog(-1/2*I*(I*d+d*cot(b*x+a))/d)+1/2*
I/d*ln(-I*d+d*cot(b*x+a))*ln(-1/2*I*(I*d+d*cot(b*x+a))/d)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(66) = 132.
time = 0.47, size = 286, normalized size = 3.08

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d+2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((i\,d+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}+1\right)+\mathrm{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-i\,d}{2i\,d+2}\right)\right)}{d}-\frac{2i\left(\log\left(\frac{1}{2}\,(d-2i)\tan(bx+a)-\frac{1}{2}i\,d\right)\log(i\,\tan(bx+a)+1)+\mathrm{Li}_2\left(-\frac{1}{2}\,(d-2i)\tan(bx+a)+\frac{1}{2}i\,d+1\right)\right)}{d}+\frac{2i\,\log((i\,d+2)\tan(bx+a)+d)\log(i\,\tan(bx+a)+1)-i\,\log(i\,\tan(bx+a)+1)^2}{d}+\frac{2i\left(\log(i\,\tan(bx+a)+1)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)-8\,(bx+a)\,\mathrm{arcoth}\left(i\,d+\frac{d}{\tan(bx+a)}+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) + d*(-2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x
+ a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
 + 2)))/d - 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x +
a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*log((I
*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arccoth(I*d + d/tan(b*x + a)
 + 1))/b
```

Fricas [A]
time = 0.36, size = 121, normalized size = 1.30

$$\frac{2i\,b^2x^2+2\,bx\log\left(\frac{((d-i)e^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{d}\right)-2i\,a^2-2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+i}{d-i}\right)+i\,\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 + 2*b*x*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
- 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
 + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d - 1)
*e^(2*I*b*x + 2*I*a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(d\cot(a+bx)+id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(acoth(d*cot(a + b*x) + I*d + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(d*cot(b*x + a) + I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(acoth(d*1i + d*cot(a + b*x) + 1), x)
```

3.260 $\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\coth^{-1}(1+id+d\cot(a+bx))}{x},x\right)$$

```
[Out] CannotIntegrate(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]]/x, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1+id+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(arccoth(1+I*d+d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) + 1/2*I*integrate(arctan2(d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)
 - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(1/2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)/
x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}\left(d\cot\left(a+bx\right)+id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Integral(acoth(d*cot(a + b*x) + I*d + 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arccoth(d*cot(b*x + a) + I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(d\cot(a+b\,x)+1+d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*1i + d*cot(a + b*x) + 1)/x,x)
[Out] int(acoth(d*1i + d*cot(a + b*x) + 1)/x, x)
```

3.261 $\int x^2 \coth^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=169

$$\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arccoth(1-I*d-d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*poly
log(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a+2*I
*b*x))/b^3
```

Rubi [A]

time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6401, 2215, 2221, 2611, 6744, 2320, 6724}

$$-\frac{i\text{Li}_4\left((1-id)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3\coth^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{12}ibx^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6401

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{3}(ib)\int\frac{x^3}{1+(-1+id)e^{2ia+2\cdots}}$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{3}(b(i+d))\int\frac{\cdots}{1+\cdots}$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-\cdots\right.$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-\cdots\right.$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-\cdots\right.$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-\cdots\right.$$
$$=\frac{1}{12}ibx^4+\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{6}x^3\log\left(1-(1-\cdots\right.$$

Mathematica [A]
time = 0.16, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\coth^{-1}(1-id-d\cot(a+bx))-\frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+6ib^2x^2\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+6bx\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)-3i\text{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcCoth[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.28, size = 2339, normalized size = 13.84

method	result	size
risch	Expression too large to display	2339

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*
I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1)
)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(d/(e
```

xp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a
))-1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*b*x^4+1/2/b^2*d/(I+d)*ln(1+I*(
I+d)*exp(2*I*(b*x+a)))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(
I+d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x
+1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/b^3*d/(I+d)
polylog(2,-I(I+d)*exp(2*I*(b*x+a)))*a^2-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*
(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I
+d))^(1/2))*x+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2
))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(
2*I*(b*x+a))-1))+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(
1/2))+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I
*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csg
n(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+e
xp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/6*d/(I+d)*ln(1+I*(I+d)*exp(2
I(b*x+a)))*x^3-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2)
)-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b/(I+d)*p
olylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,-I*(I+d)*ex
p(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*
x+a)))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+
a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/6*x^3*ln(I*exp(2*I*(b*x
+a))+exp(2*I*(b*x+a))*d-I)-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+
d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+
1/3/b^3*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2/b^3*a^3*d/(I+d)*ln(1
-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*I/b^2/(I+d)*polylog(3,-I*(I+d)*exp(2
I(b*x+a)))*x-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1
/8*I/b^3*d/(I+d)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))+1/6*I/b^3*a^3/(I+d)*l
n(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/3*I/b^3/(I+d)*ln(1+I*(I+d)*exp
(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)
/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp
(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/2*I/b^2/(
I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/6*x^3*ln(d)-1/6*I/(I+d)*ln(1+I*
(I+d)*exp(2*I*(b*x+a)))*x^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp
(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*P
i*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-
1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b^2*d/(I+d)*
polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x
+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/1
2*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*cs
gn(I*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*
I*(b*x+a)))^2*csgn(I*d)-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b
*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d-I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*

```
(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)-1/12
*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-
1))^3+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I
*(b*x+a))-1))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/8/b^3/(I+d)*polylog(4,-I*(I+d)
*exp(2*I*(b*x+a)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(120) = 240.
time = 0.28, size = 345, normalized size = 2.04

$$-\frac{\frac{12\left((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2\right)\operatorname{arcoth}(d\cot(bx+a)+id-1)}{b^2}+\frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2-2\left(4i(bx+a)^3-9i(bx+a)^2a+9i(bx+a)a^2\right)\arctan(-d\cos(2bx+2a)+\sin(2bx+2a),-d\sin(2bx+2a)-\cos(2bx+2a)+1)-3\left(4i(bx+a)^2-6i(bx+a)a+3ia^2\right)\operatorname{Li}_2\left((-id+1)e^{(2ibx+2ia)}\right)+\left(4(bx+a)^3-9(bx+a)^2a+9(bx+a)a^2\right)\log\left((d^2+1)\cos(2bx+2a)^2+(d^2+1)\sin(2bx+2a)^2-2d\sin(2bx+2a)-2\cos(2bx+2a)+1\right)+3(4bx+a)\operatorname{Li}_3\left((-id+1)e^{(2ibx+2ia)}\right)+6i\operatorname{Li}_4\left((-id+1)e^{(2ibx+2ia)}\right)}{b^2}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*cot(b
*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)*a
rctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*
b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog((-I
*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

Fricas [A]
time = 0.36, size = 179, normalized size = 1.06

$$\frac{2ib^4x^4-4b^3x^3\log\left(\frac{de^{(2ibx+2ia)}}{(d+i)e^{(2ibx+2ia)}-i}\right)+6ib^2x^2\operatorname{Li}_2\left(-(id-1)e^{(2ibx+2ia)}\right)-2ia^4+4a^3\log\left(\frac{(d+i)e^{(2ibx+2ia)}-i}{d+i}\right)-6bx\operatorname{polylog}\left(3,(-id+1)e^{(2ibx+2ia)}\right)-4\left(b^3x^3+a^3\right)\log\left((id-1)e^{(2ibx+2ia)}+1\right)-3i\operatorname{polylog}\left(4,(-id+1)e^{(2ibx+2ia)}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x
 + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I*a
^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylog(3
, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(2*I*
b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2\operatorname{acoth}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -Integral(x**2*acoth(d*cot(a + b*x) + I*d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2\,\mathrm{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.262 $\int x \coth^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal. Leaf size=133

$$\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\text{Po}$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arccoth(1-I*d-d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3
,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A]
time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6401, 2215, 2221, 2611, 2320, 6724}

$$-\frac{\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\coth^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{6}ibx^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6401

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcCoth[c + d*Cot[a + b*x]]/(f*(m
+ 1))), x] + Dist[I*(b/(f*(m + 1))), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\coth^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1+id)e^{2ia+2}\cdots}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(b(i+d))\int\frac{\cdots}{1+(\cdots}$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-i\cdots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-i\cdots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-i\cdots\right.$$

$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-i\cdots\right.$$

Mathematica [A]

time = 0.11, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\coth^{-1}(1-id-d\cot(a+bx))-\frac{2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+2ibx\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcCoth[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.64, size = 2249, normalized size = 16.91

method	result	size
risch	Expression too large to display	2249

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*b*x^3+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/
b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2/b^2*a^2*d/(I+d)*ln(1+I*e
xp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))^2*csgn(I*d)-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp
(2*I*(b*x+a)))^2-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))
*d-I)-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*
I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))
*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*(I*e
xp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I
*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))-1/4/b^2*a^2*d/(I+d)*l
n(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a)
)+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*
I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b
*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))
*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1
/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*ex
p(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-
1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*
x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*d/(ex
p(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*
x+a)))-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/8*I*x^2*Pi*csgn
(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/
8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*
```

```
(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(I*d)+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b
*x+a))*(I*(I+d))^(1/2))+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))
^(1/2))-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I
*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*
x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*e
xp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))
^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*P
i*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(d/(ex
p(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/4*x^2*ln(d)-1/8*I/b^2/(I+d)*polylog
(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^
2-1/8/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/2*x^2*ln(exp(I*(b*
x+a)))+1/4*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)*x^2+1/2*I/b*a/(I+d)*
ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+
a))*(I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2
/b*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b^2*a*d/(I+d)*d
ilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+
a))*(I*(I+d))^(1/2))*x-1/4*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4/b
/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,-I*(I
+d)*exp(2*I*(b*x+a)))*a+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^
(1/2))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*
Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3
-1/2*I/b/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/4*I/b*d/(I+d)*polylog(2
,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*
(b*x+a)))*a-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(ex
p(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*c
sgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(95) = 190$.
time = 0.28, size = 250, normalized size = 1.88

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{arcoth}(d\cot(bx+a)+i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\arctan(-d\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a),-d\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)+1)+3\left((bx+a)^2-2\,(bx+a)a\right)\log\left((d^2+1)\cos(2\,bx+2\,a)^2+(d^2+1)\sin(2\,bx+2\,a)^2-2\,d\sin(2\,bx+2\,a)-2\,\cos(2\,bx+2\,a)+1\right)+3\operatorname{Li}_3\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)}{b}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arccoth(d*cot(b*x + a) + I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2*I
*b*x + 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

Fricas [**A**]

time = 0.34, size = 156, normalized size = 1.17

$$\frac{4i\,b^3x^3-6\,b^2x^2\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+4i\,a^3+6i\,bx\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)-6\,(b^2x^2-a^2)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)-3\,\mathrm{polylog}\left(3,(-i\,d+1)e^{(2i\,bx+2i\,a)}\right)}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x
 + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) -
 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2)*l
og((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*b*x
+ 2*I*a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int x\,\mathrm{acoth}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -Integral(x*acoth(d*cot(a + b*x) + I*d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -x\,\mathrm{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*acoth(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.263 $\int \coth^{-1}(1-id-d\cot(a+bx))\,dx$

Optimal. Leaf size=94

$$\frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}$$

```
[Out] 1/2*I*b*x^2+x*arccoth(1-I*d-d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A]

time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6393, 2215, 2221, 2317, 2438}

$$\frac{i\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\coth^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcCoth[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6393

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Coth[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \coth^{-1}(1-id-d\cot(a+bx))\,dx = x\coth^{-1}(1-id-d\cot(a+bx))+(ib)\int \frac{x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx))+(b(i+d))\int \frac{e^{2ia+2}}{1+(-1+i}$$
$$= \frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{2}ibx^2+x\coth^{-1}(1-id-d\cot(a+bx))-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+}\right.$$

Mathematica [A]
time = 0.54, size = 84, normalized size = 0.89

$$x\coth^{-1}(1-id-d\cot(a+bx))-\frac{2bx\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+i\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{i+d}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] x*ArcCoth[1 - I*d - d*Cot[a + b*x]] - (2*b*x*Log[1 + 1/((-1 + I*d)*E^((2*I)
*(a + b*x)))] + I*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x)))])/(4*b)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(77) = 154.
time = 1.04, size = 334, normalized size = 3.55

method	result

derivativedivides	$-\frac{-\frac{i\operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}+\frac{i\operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\ln(-id-d\cot(}{4d}\right.}{}}{}$
default	$-\frac{-\frac{i\operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(id-d\cot(bx+a))}{2}+\frac{i\operatorname{arccoth}(1-id-d\cot(bx+a))d\ln(-id-d\cot(bx+a))}{2}-\frac{d^2\left(-\frac{i\ln(-id-d\cot(}{4d}\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d-d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/d*(-1/2*I*arccoth(1-I*d-d*cot(b*x+a))*d*ln(I*d-d*cot(b*x+a))+1/2*I*arc
coth(1-I*d-d*cot(b*x+a))*d*ln(-I*d-d*cot(b*x+a))-1/2*d^2*(-1/4*I/d*ln(-I*d-
d*cot(b*x+a))^2+1/2*I/d*dilog(1-1/2*I*d-1/2*d*cot(b*x+a))+1/2*I/d*ln(-I*d-d
*cot(b*x+a))*ln(1-1/2*I*d-1/2*d*cot(b*x+a))+1/2*I/d*dilog(1/2*I*(-I*d-d*cot
(b*x+a))/d)+1/2*I/d*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/
2*I/d*dilog(I*(I*d-d*cot(b*x+a)-I*(2*d+2*I))/(2*d+2*I))-1/2*I/d*ln(I*d-d*co
t(b*x+a))*ln(I*(I*d-d*cot(b*x+a)-I*(2*d+2*I))/(2*d+2*I))))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(67) = 134$.
time = 0.48, size = 288, normalized size = 3.06

$$-\frac{4\,(bx+a)d\left(\frac{\log((i\,d-2)\tan(bx+a)+d)}{d}-\frac{\log(i\,\tan(bx+a)+1)}{d}\right)-d\left(\frac{2i\left(\log((i\,d-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}+1\right)+\mathrm{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-i\,d}{2i\,d-2}\right)\right)}{d}+\frac{2i\left(\log\left(-\frac{1}{2}\,(d+2i)\tan(bx+a)+\frac{1}{2}i\,d\right)\log(i\,\tan(bx+a)+1)+\mathrm{Li}_2\left(\frac{1}{2}\,(d+2i)\tan(bx+a)-\frac{1}{2}i\,d+1\right)\right)}{d}-\frac{2i\,\log((i\,d-2)\tan(bx+a)+d)\log(i\,\tan(bx+a)+1)-i\,\log(i\,\tan(bx+a)+1)^2}{d}-\frac{2i\left(\log(i\,\tan(bx+a)+1)\log\left(-\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}i\,\tan(bx+a)+\frac{1}{2}\right)\right)}{d}\right)+8\,(bx+a)\operatorname{arcoth}\left(i\,d+\frac{d}{\tan(bx+a)}-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) - d*(2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x +
 a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I*d
- 2)))/d + 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*x +
a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d - (2*I*log((I*
d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) + 1
)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dilo
g(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arccoth(I*d + d/tan(b*x + a)
- 1))/b
```

Fricas [A]
time = 0.34, size = 121, normalized size = 1.29

$$\frac{2i\,b^2x^2-2\,bx\log\left(\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)-2i\,a^2-2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right)+2\,a\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)+i\,\mathrm{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 - 2*b*x*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*
I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) +
 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)*e^
(2*I*b*x + 2*I*a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{acoth}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d-d*cot(b*x+a)),x)
```

```
[Out] -Integral(acoth(d*cot(a + b*x) + I*d - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*cot(b*x + a) - I*d + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-acoth(d*1i + d*cot(a + b*x) - 1), x)
```

3.264 $\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\coth^{-1}(1-id-d\cot(a+bx))}{x},x\right)$$

[Out] CannotIntegrate(arccoth(1-I*d-d*cot(b*x+a))/x,x)

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Int[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Mathematica [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcCoth[1 - I*d - d*Cot[a + b*x]]/x, x]

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(1-id-d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

```
[Out] int(arccoth(1-I*d-d*cot(b*x+a))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a
) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(-1/2*log(d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))/
x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{acoth}\left(d\cot\left(a+bx\right)+id-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d-d*cot(b*x+a))/x,x)
```

```
[Out] -Integral(acoth(d*cot(a + b*x) + I*d - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1-I*d-d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(-d*cot(b*x + a) - I*d + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{\operatorname{acoth}(d\cot(a+b\,x)-1+d\,1\mathrm{i})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-acoth(d*1i + d*cot(a + b*x) - 1)/x,x)
```

```
[Out] int(-acoth(d*1i + d*cot(a + b*x) - 1)/x, x)
```

3.265 $\int \frac{(a+b\coth^{-1}(cx^n))(d+e\log(fx^m))}{x}\,dx$

Optimal. Leaf size=160

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{bd\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}+\frac{be\log(fx^m)\text{PolyLog}\left(2,-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\text{PolyLog}\left(2,\frac{x^{-n}}{c}\right)}{2n}-\frac{be\cdots}{}$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*b*d*polylog(2,-1/c/(x^n))/n+1/2*b*e*ln(
f*x^m)*polylog(2,-1/c/(x^n))/n-1/2*b*d*polylog(2,1/c/(x^n))/n-1/2*b*e*ln(f*
x^m)*polylog(2,1/c/(x^n))/n+1/2*b*e*m*polylog(3,-1/c/(x^n))/n^2-1/2*b*e*m*p
olylog(3,1/c/(x^n))/n^2
```

Rubi [A]

time = 0.41, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 6874, 6036, 6032, 6219, 6217, 2421, 6724}

$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{bd\text{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}-\frac{bd\text{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n}+\frac{be\text{Li}_2\left(-\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}-\frac{be\text{Li}_2\left(\frac{x^{-n}}{c}\right)\log(fx^m)}{2n}+\frac{bem\text{Li}_3\left(-\frac{x^{-n}}{c}\right)}{2n^2}-\frac{bem\text{Li}_3\left(\frac{x^{-n}}{c}\right)}{2n^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + (b*d*PolyLog[2, -(1/(c*x^n))])/(2*n
) + (b*e*Log[f*x^m]*PolyLog[2, -(1/(c*x^n))])/(2*n) - (b*d*PolyLog[2, 1/(c*
x^n)])/(2*n) - (b*e*Log[f*x^m]*PolyLog[2, 1/(c*x^n)])/(2*n) + (b*e*m*PolyLo
g[3, -(1/(c*x^n))])/(2*n^2) - (b*e*m*PolyLog[3, 1/(c*x^n)])/(2*n^2)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6036

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6217

```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[Log[d*x^m]*(Log[1 + 1/(c*x^n)]/x), x], x] - Dist[1/2, Int[Log[d
*x^m]*(Log[1 - 1/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 6219

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcCoth[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_),
x_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*Arc
Coth[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}\left(cx^n\right)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx = \int \left(\frac{d\left(a+b\coth^{-1}\left(cx^n\right)\right)}{x}+\frac{e\left(a+b\coth^{-1}\left(cx^n\right)\right)\log\left(fx^m\right)}{x}\right.$$

$$= d\int \frac{a+b\coth^{-1}\left(cx^n\right)}{x}\,dx + e\int \frac{\left(a+b\coth^{-1}\left(cx^n\right)\right)\log\left(fx^{\dots}\right.}{x}$$

$$= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx + (be)\int \frac{\coth^{-1}\left(cx^n\right)\log\left(fx^m\right)}{x}\,dx + \frac{dS\dots}{\dots}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} + \frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} - \frac{bd\mathrm{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n} - \dots$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} + \frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be\log\left(fx^m\right)\mathrm{Li}_2\dots}{2n}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} + \frac{bd\mathrm{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be\log\left(fx^m\right)\mathrm{Li}_2\dots}{2n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.21, size = 131, normalized size = 0.82

$$-\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}+\frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)\left(d+e\log\left(fx^m\right)\right)}{n}-\frac{1}{2}\left(a+b\coth^{-1}\left(cx^n\right)-b\tanh^{-1}\left(cx^n\right)\right)\log(x)\left(em\log(x)-2\left(d+e\log\left(fx^m\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*x
^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x
^(2*n)]*(d + e*Log[f*x^m]))/n - ((a + b*ArcCoth[c*x^n] - b*ArcTanh[c*x^n])*
Log[x]*(e*m*Log[x] - 2*(d + e*Log[f*x^m])))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 920, normalized size = 5.75

method	result	size
risch	Expression too large to display	920

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*b*m/n*ln(x)*polylog(2,c*x^n)+1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*
csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*f)*csgn(I*x^m)*csgn
```

```
(I*f*x^m)-1/4*I/n*ln(c*x^n-1)*ln(c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+
1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I/n*ln(
c*x^n-1)*ln(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/n*ln(x^n)*a*d+1/2*e*b
*ln(1-c*x^n)*ln(x^m)*ln(x)-1/2/n*ln(c*x^n-1)*ln(c*x^n)*b*d+1/4*e*b*ln(c*x^n
-1)*m*ln(x)^2-1/2*e*b*ln(c*x^n-1)*ln(x^m)*ln(x)-1/2/n*dilog(c*x^n)*b*d-1/2/
n*dilog(c*x^n+1)*b*d-1/2*e*b*m/n^2*polylog(3,c*x^n)-1/2*e*b/n*dilog(c*x^n)*
ln(x^m)+1/n*ln(f)*ln(x^n)*a*e+1/2*e*a/m*ln(x^m)^2+1/2*e*b*m/n^2*polylog(3,-
c*x^n)-1/2*e*b/n*dilog(c*x^n+1)*ln(x^m)-1/2/n*ln(f)*dilog(c*x^n+1)*b*e-1/4*
e*b*m*ln(x)^2*ln(1-c*x^n)+1/2*I/n*Pi*ln(x^n)*a*e*csgn(I*x^m)*csgn(I*f*x^m)^
2-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*Pi*ln(x
^n)*a*e*csgn(I*f)*csgn(I*f*x^m)^2-1/2*e*b*m/n*ln(x)*polylog(2,-c*x^n)+1/2*e
*b/n*dilog(c*x^n+1)*m*ln(x)-1/4*I/n*Pi*dilog(c*x^n+1)*b*e*csgn(I*f)*csgn(I*
f*x^m)^2-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*e*b/n*
ln(1-c*x^n)*ln(c*x^n)*m*ln(x)+1/4*I/n*ln(c*x^n-1)*ln(c*x^n)*Pi*b*e*csgn(I*f
*x^m)^3-1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^2-1/2*I/n*Pi*ln
(x^n)*a*e*csgn(I*f*x^m)^3-1/2/n*ln(c*x^n-1)*ln(c*x^n)*ln(f)*b*e+1/4*I/n*Pi*
dilog(c*x^n+1)*b*e*csgn(I*f*x^m)^3-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)+
1/2*e*b/n*dilog(c*x^n)*m*ln(x)+1/4*I/n*dilog(c*x^n)*Pi*b*e*csgn(I*f*x^m)^3-
1/2/n*dilog(c*x^n)*ln(f)*b*e+1/4*I/n*ln(c*x^n-1)*ln(c*x^n)*Pi*b*e*csgn(I*f)
*csgn(I*x^m)*csgn(I*f*x^m)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*m*e*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(b*e*log(f) + b*d)*log(x))*log(c*x^n + 1) + 1/4*(b*m*e*log(x)^2
- 2*b*e*log(x)*log(x^m) - 2*(b*e*log(f) + b*d)*log(x))*log(c*x^n - 1) + int
egrate(1/2*(2*b*c*n*e^(n*log(x) + 1)*log(x)*log(x^m) - (b*c*m*n*e*log(x)^2
- 2*(b*c*n*e*log(f) + b*c*d*n)*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(147) = 294$.
time = 0.41, size = 460, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*m*n^2*cosh(1) + a*m*n^2*sinh(1))*log(x)^2 + 2*(b*d*n + (b*n*cosh(
1) + b*n*sinh(1))*log(f) + (b*m*n*cosh(1) + b*m*n*sinh(1))*log(x))*dilog(c*
```

```
cosh(n*log(x)) + c*sinh(n*log(x))) - 2*(b*d*n + (b*n*cosh(1) + b*n*sinh(1))
*log(f) + (b*m*n*cosh(1) + b*m*n*sinh(1))*log(x))*dilog(-c*cosh(n*log(x)) -
 c*sinh(n*log(x))) - ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 + 2*(b*d
*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log(c*cosh(n*log(x))
 + c*sinh(n*log(x)) + 1) + ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 +
2*(b*d*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log(-c*cosh(n*
log(x)) - c*sinh(n*log(x)) + 1) + 4*(a*d*n^2 + (a*n^2*cosh(1) + a*n^2*sinh(
1))*log(f))*log(x) + ((b*m*n^2*cosh(1) + b*m*n^2*sinh(1))*log(x)^2 + 2*(b*d
*n^2 + (b*n^2*cosh(1) + b*n^2*sinh(1))*log(f))*log(x))*log((c*cosh(n*log(x)
) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)) - 2*(b
*m*cosh(1) + b*m*sinh(1))*polylog(3, c*cosh(n*log(x)) + c*sinh(n*log(x))) +
 2*(b*m*cosh(1) + b*m*sinh(1))*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(
x))))/n^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acoth}(c\,x^n))\,\left(d + e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

```
[Out] int(((a + b*acoth(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

3.266 $\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=297

$$\frac{b(3d-e)x}{18c^5}-\frac{137bex}{180c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{47bex^3}{540c^3}+\frac{b(3d-e)x^5}{90c}-\frac{bex^5}{75c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}\right.}{12c^2}$$

```
[Out] 1/36*b*(6*d-11*e)*x/c^5-23/45*b*e*x/c^5+1/108*b*(6*d-5*e)*x^3/c^3-8/135*b*e
*x^3/c^3+1/90*b*(3*d-e)*x^5/c-1/75*b*e*x^5/c-1/6*e*x^2*(a+b*arccoth(c*x))/c
^4-1/12*e*x^4*(a+b*arccoth(c*x))/c^2-1/18*e*x^6*(a+b*arccoth(c*x))-1/36*b*(
6*d-11*e)*arctanh(c*x)/c^6+23/45*b*e*arctanh(c*x)/c^6+1/6*b*e*x*ln(-c^2*x^2
+1)/c^5+1/18*b*e*x^3*ln(-c^2*x^2+1)/c^3+1/30*b*e*x^5*ln(-c^2*x^2+1)/c-1/6*e
*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^6+1/6*x^6*(a+b*arccoth(c*x))*(d+e*ln(-
c^2*x^2+1))
```

Rubi [A]
time = 0.27, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2504, 2442, 45, 6231, 327, 213, 308, 2608, 2498, 212, 2505}

$$-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}+\frac{1}{6}x^6\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{6c^6}-\frac{1}{18}ex^6\left(a+b\coth^{-1}(cx)\right)-\frac{b(3d-e)\tanh^{-1}(cx)}{18c^6}+\frac{137be\tanh^{-1}(cx)}{180c^6}+\frac{bx(3d-e)}{18c^5}-\frac{137bex}{180c^5}+\frac{bx^3(3d-e)}{54c^3}-\frac{47bex^3}{540c^3}+\frac{bex^5\log\left(1-c^2x^2\right)}{30c}+\frac{bex\log\left(1-c^2x^2\right)}{6c^5}+\frac{bex^3\log\left(1-c^2x^2\right)}{18c^3}+\frac{bx^5(3d-e)}{90c}-\frac{bex^5}{75c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(3*d - e)*x)/(18*c^5) - (137*b*e*x)/(180*c^5) + (b*(3*d - e)*x^3)/(54*c^
3) - (47*b*e*x^3)/(540*c^3) + (b*(3*d - e)*x^5)/(90*c) - (b*e*x^5)/(75*c) -
 (e*x^2*(a + b*ArcCoth[c*x]))/(6*c^4) - (e*x^4*(a + b*ArcCoth[c*x]))/(12*c^
2) - (e*x^6*(a + b*ArcCoth[c*x]))/18 - (b*(3*d - e)*ArcTanh[c*x])/(18*c^6)
+ (137*b*e*ArcTanh[c*x])/(180*c^6) + (b*e*x*Log[1 - c^2*x^2])/(6*c^5) + (b*
e*x^3*Log[1 - c^2*x^2])/(18*c^3) + (b*e*x^5*Log[1 - c^2*x^2])/(30*c) - (e*(
a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(6*c^6) + (x^6*(a + b*ArcCoth[c*x])*(
d + e*Log[1 - c^2*x^2]))/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6231

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x^5\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{1}{18}$$

$$=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}-\frac{1}{18}$$

$$=-\frac{bex}{6c^5}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{6c^4}-\frac{ex^4\left(a+b\coth^{-1}(cx)\right)}{12c^2}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{bex}{4c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)x^5}{90c}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{bex}{4c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)x^5}{90c}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{7bex}{12c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{bex^3}{36c^3}+\frac{b(3d-e)}{90c}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{137bex}{180c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{47bex^3}{540c^3}+\frac{b(3d}{9}$$

$$=\frac{b(3d-e)x}{18c^5}-\frac{137bex}{180c^5}+\frac{b(3d-e)x^3}{54c^3}-\frac{47bex^3}{540c^3}+\frac{b(3d}{9}$$

Mathematica [A]
time = 0.10, size = 236, normalized size = 0.79

$$\frac{30bc(10d-49e)x-300ac^2ex^2+10bc^3(10d-19e)x^3-150ac^4ex^4+4bc^5(15d-11e)x^5+100ac^6(3d-e)x^6-50bc^2x^2\left(-6c^4dx^4+e\left(6+3c^2x^2+2c^4x^4\right)\right)\coth^{-1}(cx)+15(10bd-20ae-49be)\log(1-cx)-15(10bd+20ae-49be)\log(1+cx)+20e\left(15ac^6x^6+bcx\left(15+5c^2x^2+3c^4x^4\right)+15b\left(-1+c^6x^6\right)\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{1800c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (30*b*c*(10*d - 49*e)*x - 300*a*c^2*e*x^2 + 10*b*c^3*(10*d - 19*e)*x^3 - 15
0*a*c^4*e*x^4 + 4*b*c^5*(15*d - 11*e)*x^5 + 100*a*c^6*(3*d - e)*x^6 - 50*b*
c^2*x^2*(-6*c^4*d*x^4 + e*(6 + 3*c^2*x^2 + 2*c^4*x^4))*ArcCoth[c*x] + 15*(1
0*b*d - 20*a*e - 49*b*e)*Log[1 - c*x] - 15*(10*b*d + 20*a*e - 49*b*e)*Log[1
 + c*x] + 20*e*(15*a*c^6*x^6 + b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + 15*b*(-
1 + c^6*x^6)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(1800*c^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.36, size = 4034, normalized size = 13.58

method	result	size
default	Expression too large to display	4034
risch	Expression too large to display	7318

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -49/60*b*e*x/c^5-19/180*b*e*x^3/c^3-11/450*b*e*x^5/c-23/45/c^6*b*e*ln(2)+1/
12*I*b*arccoth(c*x)*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+
1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^6*e+1/12*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)
*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(c*x-1)*(c*x+1))*x^6*e-1/6*I/c^6
*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1)
)^2+1/60*I/c*b*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1
/(c*x-1)*(c*x+1)-1)^2)^2*x^5*e+1/36*I/c^3*b*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)
^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^3*e+1/12*I/c^5*b*Pi
*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-
1)^2)^2*x*e+1/36*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)
^2*csgn(I/(c*x-1)*(c*x+1))*x^3*e+23/180*I/c^6*b*e*Pi*csgn(I/(1/(c*x-1)*(c*x
+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)
-1)^2)-1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(
I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/12*I/c^6*b*arccoth(c*x)*e*Pi
*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-
1/12*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(
1/2))^2*x^6*e+1/12*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn
(I*(1/(c*x-1)*(c*x+1)-1)^2)*x^6*e-1/6*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)
*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x^6*e+1/6*I*b*arccoth(c*x)*P
i*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x^6*e+1/12*I/c^
6*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x
+1))-1/12*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*
(1/(c*x-1)*(c*x+1)-1)^2)+1/6*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*(1/(c*x-1)*(c
*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2-1/60*I/c*b*Pi*csgn(I/(c*x-1)*(c
```

```
*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*x^5*e-1/12*I/c^5*b*Pi*csgn(I/(c*x-
1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*x*e-1/30*I/c*b*Pi*csgn(I*(1/(
c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x^5*e-1/18*I/c^3*b*Pi*
csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x^3*e-1/6*I
/c^5*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x
*e-1/36*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^
2*x^3*e+1/60*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csg
n(I/(c*x-1)*(c*x+1))*x^5*e+23/180*I/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/
2))^2*csgn(I/(c*x-1)*(c*x+1))+23/90*I/c^6*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-
1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2-1/60*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))
^3*x^5*e-1/12*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1))^3*x^6*e-1/36*I/c^
3*b*Pi*csgn(I/(c*x-1)*(c*x+1))^3*x^3*e-23/180*I/c^6*b*e*Pi*csgn(I/(1/(c*x-1
)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-23/180*I/
c^6*b*e*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1
)-1)^2)^2-23/90/c^6*b*d-23/90*I/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*
csgn(I/(c*x-1)*(c*x+1))^2-23/180*I/c^6*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))
^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)-1/12*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I*(1
/(c*x-1)*(c*x+1)-1)^2)^3-1/12*I/c^5*b*Pi*csgn(I/(c*x-1)*(c*x+1))^3*x*e-1/30
*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^5*e-1/18*I/c^
3*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^3*e-1/6*I/c^5*b*
Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x*e+1/12*I*b*arccoth(c
*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*x^6*e+1/12*I*b*arccoth(c*x)*Pi*csg
n(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*x^6*e-1/18*a*e*x^6-1/12*I*b*
arccoth(c*x)*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(
c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*x^6*e-1/60*I/c*b*Pi*csgn(I/(1/
(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(
I/(c*x-1)*(c*x+1))*x^5*e-1/36*I/c^3*b*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*cs
gn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*x^3*e
-1/12*I/c^5*b*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/
(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*x*e+1/12*I/c^6*b*arccoth(c*x)
*Pi*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1
)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)+1/6*I/c^6*b*arccoth(c*x)*Pi*e*csgn(I/(c*
x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/12*I/c^5*b*Pi*csgn(I/(c*x-1)*(c*x
+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(c*x-1)*(c*x+1))*x*e+1/30*I/c*b*Pi*cs
gn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x^5*e+1/18*I/c^3*b*
Pi*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x^3*e+1/6*I/c^
5*b*Pi*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x*e+1/60*I
/c*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*x^5
*e+1/36*I/c^3*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1
)-1)^2)*x^3*e+1/12*I/c^5*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*
x-1)*(c*x+1)-1)^2)*x*e+1/12*I/c^5*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*x*
e+1/60*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*x^5*e+1/3
6*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*...
```

Maxima [C] Result contains complex when optimal does not.

time = 0.26, size = 334, normalized size = 1.12

$$\frac{1}{6}\,adx^{6}+\frac{1}{36}\left(6\,x^{6}\log\left(-c^{2}x^{2}+1\right)-c^{2}\left(\frac{2\,c^{4}x^{6}+3\,c^{2}x^{4}+6\,x^{2}}{c^{6}}+\frac{6\,\log\left(c^{2}x^{2}-1\right)}{c^{8}}\right)\right)b\,\mathrm{arcoth}\left(cx\right)e+\frac{1}{180}\left(30\,x^{6}\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{2\left(3\,c^{4}x^{5}+5\,c^{2}x^{3}+15\,x\right)}{c^{6}}-\frac{15\,\log\left(cx+1\right)}{c^{7}}+\frac{15\,\log\left(cx-1\right)}{c^{7}}\right)\right)bd+\frac{1}{36}\left(6\,x^{6}\log\left(-c^{2}x^{2}+1\right)-c^{2}\left(\frac{2\,c^{4}x^{6}+3\,c^{2}x^{4}+6\,x^{2}}{c^{6}}+\frac{6\,\log\left(c^{2}x^{2}-1\right)}{c^{8}}\right)\right)ae-\frac{\left(4\left(-15i\,\pi c^{5}+11\,c^{5}\right)x^{5}+10\left(-10i\,\pi c^{3}+19\,c^{3}\right)x^{3}+30\left(-10i\,\pi c+49\,c\right)x+5\left(30i\,\pi-12\,c^{5}x^{5}-20\,c^{3}x^{3}-60\,cx-147\right)\log\left(cx+1\right)+5\left(-30i\,\pi-12\,c^{5}x^{5}-20\,c^{3}x^{3}-60\,cx+147\right)\log\left(cx-1\right)\right)be}{1800\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/6*a*d*x^6 + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^6 + 3*c^2*x^4 +
 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*b*arccoth(c*x)*e + 1/180*(30*x^6*arc
coth(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 +
 15*log(c*x - 1)/c^7))*b*d + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^
6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*e - 1/1800*(4*(-15*
I*pi*c^5 + 11*c^5)*x^5 + 10*(-10*I*pi*c^3 + 19*c^3)*x^3 + 30*(-10*I*pi*c +
49*c)*x + 5*(30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x - 147)*log(c*x + 1)
 + 5*(-30*I*pi - 12*c^5*x^5 - 20*c^3*x^3 - 60*c*x + 147)*log(c*x - 1))*b*e/
c^6
```

Fricas [**A**]
time = 0.35, size = 389, normalized size = 1.31

$$\frac{300\,ac^{6}dx^{6}+60\,bc^{5}dx^{5}+100\,bc^{3}dx^{3}+300\,bcdx-2\left(50\,ac^{6}x^{6}+22\,bc^{5}x^{5}+75\,ac^{4}x^{4}+95\,bc^{3}x^{3}+150\,ac^{2}x^{2}+735\,bcx\right)\cosh\left(1\right)+20\left(\left(15\,ac^{6}x^{6}+3\,bc^{5}x^{5}+5\,bc^{3}x^{3}+15\,bcx-15\,a\right)\cosh\left(1\right)+\left(15\,ac^{6}x^{6}+3\,bc^{5}x^{5}+5\,bc^{3}x^{3}+15\,bcx-15\,a\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right)+5\left(30\,bc^{6}dx^{6}-30\,bd-\left(10\,bc^{6}x^{6}+15\,bc^{4}x^{4}+30\,bc^{2}x^{2}-147\,b\right)\cosh\left(1\right)+30\left(\left(bc^{6}x^{6}-b\right)\cosh\left(1\right)+\left(bc^{6}x^{6}-b\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right)-\left(10\,bc^{6}x^{6}+15\,bc^{4}x^{4}+30\,bc^{2}x^{2}-147\,b\right)\sinh\left(1\right)\right)\log\left(\frac{cx+1}{cx-1}\right)-2\left(50\,ac^{6}x^{6}+22\,bc^{5}x^{5}+75\,ac^{4}x^{4}+95\,bc^{3}x^{3}+150\,ac^{2}x^{2}+735\,bcx\right)\sinh\left(1\right)}{1800\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/1800*(300*a*c^6*d*x^6 + 60*b*c^5*d*x^5 + 100*b*c^3*d*x^3 + 300*b*c*d*x -
2*(50*a*c^6*x^6 + 22*b*c^5*x^5 + 75*a*c^4*x^4 + 95*b*c^3*x^3 + 150*a*c^2*x^
2 + 735*b*c*x)*cosh(1) + 20*((15*a*c^6*x^6 + 3*b*c^5*x^5 + 5*b*c^3*x^3 + 15
*b*c*x - 15*a)*cosh(1) + (15*a*c^6*x^6 + 3*b*c^5*x^5 + 5*b*c^3*x^3 + 15*b*c
*x - 15*a)*sinh(1))*log(-c^2*x^2 + 1) + 5*(30*b*c^6*d*x^6 - 30*b*d - (10*b*
c^6*x^6 + 15*b*c^4*x^4 + 30*b*c^2*x^2 - 147*b)*cosh(1) + 30*((b*c^6*x^6 - b
)*cosh(1) + (b*c^6*x^6 - b)*sinh(1))*log(-c^2*x^2 + 1) - (10*b*c^6*x^6 + 15
*b*c^4*x^4 + 30*b*c^2*x^2 - 147*b)*sinh(1))*log((c*x + 1)/(c*x - 1)) - 2*(5
0*a*c^6*x^6 + 22*b*c^5*x^5 + 75*a*c^4*x^4 + 95*b*c^3*x^3 + 150*a*c^2*x^2 +
735*b*c*x)*sinh(1))/c^6
```

Sympy [**C**] Result contains complex when optimal does not.
time = 3.59, size = 362, normalized size = 1.22

$$\begin{cases} \frac{adx^{6}}{6}+\frac{aex^{6}\log\left(-c^{2}x^{2}+1\right)}{6}-\frac{aex^{6}}{18}-\frac{aex^{4}}{12c^{2}}-\frac{aex^{2}}{6c^{4}}-\frac{ae\log\left(-c^{2}x^{2}+1\right)}{6c^{6}}+\frac{bdx^{6}\operatorname{acoth}\left(cx\right)}{6}+\frac{bex^{6}\log\left(-c^{2}x^{2}+1\right)\operatorname{acoth}\left(cx\right)}{6}-\frac{bex^{6}\operatorname{acoth}\left(cx\right)}{18}+\frac{bdx^{5}}{30c}+\frac{bex^{5}\log\left(-c^{2}x^{2}+1\right)}{30c}-\frac{11bex^{5}}{450c}-\frac{bex^{4}\operatorname{acoth}\left(cx\right)}{12c^{2}}+\frac{bdx^{3}}{18c^{3}}+\frac{bex^{3}\log\left(-c^{2}x^{2}+1\right)}{18c^{3}}-\frac{19bex^{3}}{180c^{3}}-\frac{bex^{2}\operatorname{acoth}\left(cx\right)}{6c^{4}}+\frac{bdx}{6c^{5}}+\frac{bex\log\left(-c^{2}x^{2}+1\right)}{6c^{5}}-\frac{49bex}{60c^{5}}-\frac{bd\operatorname{acoth}\left(cx\right)}{6c^{6}}-\frac{be\log\left(-c^{2}x^{2}+1\right)\operatorname{acoth}\left(cx\right)}{6c^{6}}+\frac{49be\operatorname{acoth}\left(cx\right)}{60c^{6}} & \text{for}\: c \neq 0 \\ \frac{dx^{6}\left(a+\frac{i\pi b}{2}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**6/6 + a*e*x**6*log(-c**2*x**2 + 1)/6 - a*e*x**6/18 - a*e*
x**4/(12*c**2) - a*e*x**2/(6*c**4) - a*e*log(-c**2*x**2 + 1)/(6*c**6) + b*d
*x**6*acoth(c*x)/6 + b*e*x**6*log(-c**2*x**2 + 1)*acoth(c*x)/6 - b*e*x**6*a
coth(c*x)/18 + b*d*x**5/(30*c) + b*e*x**5*log(-c**2*x**2 + 1)/(30*c) - 11*b
*e*x**5/(450*c) - b*e*x**4*acoth(c*x)/(12*c**2) + b*d*x**3/(18*c**3) + b*e*
x**3*log(-c**2*x**2 + 1)/(18*c**3) - 19*b*e*x**3/(180*c**3) - b*e*x**2*acot
h(c*x)/(6*c**4) + b*d*x/(6*c**5) + b*e*x*log(-c**2*x**2 + 1)/(6*c**5) - 49*
b*e*x/(60*c**5) - b*d*acoth(c*x)/(6*c**6) - b*e*log(-c**2*x**2 + 1)*acoth(c
*x)/(6*c**6) + 49*b*e*acoth(c*x)/(60*c**6), Ne(c, 0)), (d*x**6*(a + I*pi*b/
2)/6, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 2.29, size = 510, normalized size = 1.72

$$\ln\left(1-c^2x^2\right)\left(\frac{a\,e\,x^6}{6}+\frac{b\,e\,x}{6\,c^5}+\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}\right)-\ln\left(\frac{1}{c\,x}+1\right)\left(\ln\left(1-c^2x^2\right)\left(\frac{b\,e}{12\,c^6}-\frac{b\,e\,x^6}{12}\right)-\frac{b\,d\,x^6}{12}+\frac{b\,e\,x^6}{36}+\frac{b\,e\,x^4}{24\,c^2}+\frac{b\,e\,x^2}{12\,c^4}\right)+\ln\left(1-\frac{1}{c\,x}\right)\left(\frac{\frac{b\,d\,x^7}{6}-\frac{b\,c^2\,d\,x^9}{6}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,e\,x^7}{36}+\frac{b\,e\,x^5}{12\,c^2}-\frac{b\,e\,x^3}{6\,c^4}+\frac{b\,c^2\,e\,x^9}{18}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2x^2\right)\left(\frac{b\,e\,x^7}{6}-\frac{b\,c^2\,e\,x^9}{6}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2x^2\right)\,\left(x-c^2x^3\right)}{12\,c^6\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)+x^4\left(\frac{a\,\left(3\,d-e\right)}{12\,c^2}-\frac{a\,d}{4\,c^2}\right)+x^3\left(\frac{b\,\left(15\,d-11\,e\right)}{270\,c^3}-\frac{7\,b\,e}{108\,c^3}\right)+x\left(\frac{\frac{b\,\left(15\,d-11\,e\right)}{90\,c^3}-\frac{7\,b\,e}{36\,c^3}}{c^2}-\frac{b\,e}{2\,c^5}\right)+\frac{a\,x^6\,\left(3\,d-e\right)}{18}+\frac{x^2\left(\frac{a\,\left(3\,d-e\right)}{3\,c^2}-\frac{a\,d}{c^2}\right)}{2\,c^2}-\frac{\ln\left(c\,x-1\right)\,\left(20\,a\,e-10\,b\,d+49\,b\,e\right)}{120\,c^6}-\frac{\ln\left(c\,x+1\right)\,\left(20\,a\,e+10\,b\,d-49\,b\,e\right)}{120\,c^6}+\frac{b\,x^5\,\left(15\,d-11\,e\right)}{450\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - c^2*x^2)*((a*e*x^6)/6 + (b*e*x)/(6*c^5) + (b*e*x^5)/(30*c) + (b*e*x
^3)/(18*c^3)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)*((b*e)/(12*c^6) - (b*e*x
^6)/12) - (b*d*x^6)/12 + (b*e*x^6)/36 + (b*e*x^4)/(24*c^2) + (b*e*x^2)/(12*
c^4)) + log(1 - 1/(c*x))*(((b*d*x^7)/6 - (b*c^2*d*x^9)/6)/(2*(x + c*x^2)*(c
*x - 1)) + ((b*e*x^7)/36 + (b*e*x^5)/(12*c^2) - (b*e*x^3)/(6*c^4) + (b*c^2*
e*x^9)/18)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^7)/6 - (b*
c^2*e*x^9)/6))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x - c^2*x
^3))/(12*c^6*(x + c*x^2)*(c*x - 1))) + x^4*((a*(3*d - e))/(12*c^2) - (a*d)/
(4*c^2)) + x^3*((b*(15*d - 11*e))/(270*c^3) - (7*b*e)/(108*c^3)) + x*(((b*(
15*d - 11*e))/(90*c^3) - (7*b*e)/(36*c^3))/c^2 - (b*e)/(2*c^5)) + (a*x^6*(3
*d - e))/18 + (x^2*((a*(3*d - e))/(3*c^2) - (a*d)/c^2))/(2*c^2) - (log(c*x
- 1)*(20*a*e - 10*b*d + 49*b*e))/(120*c^6) - (log(c*x + 1)*(20*a*e + 10*b*d
 - 49*b*e))/(120*c^6) + (b*x^5*(15*d - 11*e))/(450*c)
```

3.267 $\int x^3 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx$

Optimal. Leaf size=225

$$\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)-\frac{b(2d-3e)\tanh^{-1}}{8c^4}$$

```
[Out] 1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3+1/24*b*(2*d-e)*x^3/c-1/18*b*e*x^3/c-1/4
*e*x^2*(a+b*arccoth(c*x))/c^2-1/8*e*x^4*(a+b*arccoth(c*x))-1/8*b*(2*d-3*e)*
arctanh(c*x)/c^4+2/3*b*e*arctanh(c*x)/c^4+1/4*b*e*x*ln(-c^2*x^2+1)/c^3+1/12
*b*e*x^3*ln(-c^2*x^2+1)/c-1/4*e*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^4+1/4*x
^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))
```

Rubi [A]
time = 0.18, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2504, 2442, 45, 6231, 470, 327, 212, 2521, 2498, 2505, 308}

$$\frac{1}{4}x^4\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)-\frac{b(2d-3e)\tanh^{-1}(cx)}{8c^4}+\frac{2be\tanh^{-1}(cx)}{3c^4}+\frac{bx(2d-3e)}{8c^3}-\frac{2bex}{3c^3}+\frac{bex^3\log\left(1-c^2x^2\right)}{12c}+\frac{bex\log\left(1-c^2x^2\right)}{4c^3}+\frac{bx^3(2d-e)}{24c}-\frac{bex^3}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (e*x^2*(a + b*ArcCoth[c*x]))/(4*c^2) - (e*x^4*(a + b*Arc
Coth[c*x]))/8 - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*x])
/(3*c^4) + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcCoth[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcC
oth[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 6231

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x^3\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)-\frac{e(\cdots}{\cdots}$$

$$=-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\coth^{-1}(cx)\right)-\frac{e(\cdots}{\cdots}$$

$$=\frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\frac{1}{8}ex^4\left(a+b\,\mathrm{coth}\cdots\right.$$

$$=\frac{b(2d-3e)x}{8c^3}+\frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\cdots$$

$$=\frac{b(2d-3e)x}{8c^3}+\frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\coth^{-1}(cx)\right)}{4c^2}-\cdots$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{bex}{2c^3}+\frac{b(2d-e)x^3}{24c}-\frac{ex^2\left(a+b\coth^{-1}(c\cdots\right.}{4c^2}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{ex^2\left(a+b\cdots\right.}{4\cdots}$$

$$=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}+\frac{b(2d-e)x^3}{24c}-\frac{bex^3}{18c}-\frac{ex^2\left(a+b\cdots\right.}{4\cdots}$$

Mathematica [A]
time = 0.08, size = 192, normalized size = 0.85

$$\frac{6bc(6d-25e)x-36ac^2ex^2+2bc^3(6d-7e)x^3+18ac^4(2d-e)x^4-18bc^2x^2\left(-2c^2dx^2+e(2+c^2x^2)\right)\coth^{-1}(cx)+3(6bd-12ae-25be)\log(1-cx)-3(6bd+12ae-25be)\log(1+cx)+12e\left(3ac^4x^4+bcx(3+c^2x^2)+3b(-1+c^4x^4)\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{144c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^4
*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcCoth[c*x]
 + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e)*L
og[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x^4)*
ArcCoth[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.30, size = 3320, normalized size = 14.76

method	result	size
default	Expression too large to display	3320
risch	Expression too large to display	7050

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/c*b*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2
*Pi*x^3*e+1/24*I/c*b*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)
/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^3*e+1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1))^2*c
sgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x^3*e+1/24*I/c*b*csgn(I/(c*x-1)*(c*x+1))*
csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^3*e-1/24*I/c*b*csgn(
I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*Pi*x^3*e+1/8*I/c^3*b*c
sgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x*e-1/4*I
/c^3*b*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x
*e+1/8*I/c^3*b*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c
*x-1)*(c*x+1)-1)^2)^2*Pi*x*e+1/4*I/c^3*b*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/(
(c*x-1)/(c*x+1))^(1/2))*Pi*x*e+1/8*I/c^3*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(
c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x*e-25/24*b*e*x/c^3-7/72*b*e*x
^3/c-1/6*I/c^4*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*
x+1)-1)^2)+1/3*I/c^4*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)
*(c*x+1)-1)^2)^2-1/6*I/c^4*b*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c
*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/3*I/c^4*b*e*Pi*csgn(I/((c*x-1)/(
c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2+1/24*I/c*b*csgn(I*(1/(c*x-1)*(c*x+
1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x^3*e-1/6*I/c^4*b*e*Pi*csgn(I/(
c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/6*I/c^4
*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))+1/8*I*b*a
rccoth(c*x)*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*Pi*x^4*e-1/8*I*b*arccoth(c*x)
*csgn(I/(c*x-1)*(c*x+1))^3*Pi*x^4*e+1/8*I*b*arccoth(c*x)*csgn(I/(c*x-1)*(c*
x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*Pi*x^4*e-1/4*I*b*arccoth(c*x)*csgn(I/(c*x-1
)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^4*e+1/24*I/c*b*csgn(I*(1/(c*x-1)*
(c*x+1)-1)^2)^3*Pi*x^3*e-1/24*I/c*b*csgn(I/(c*x-1)*(c*x+1))^3*Pi*x^3*e-1/8*
```

```
I*b*arccoth(c*x)*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*cs
gn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x^4*e-1/24*I/c*b*csgn(I/(1
/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c
*x-1)*(c*x+1)-1)^2)*Pi*x^3*e-1/8*I/c^3*b*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*cs
gn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x*
e+1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x
-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)-1/8*I/c^3*b*cs
gn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*Pi*x*e-1/8*I/c^4*b*
arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)
-1)^2)+1/4*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(
1/(c*x-1)*(c*x+1)-1)^2)^2-1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I/(1/(c*x-1)*(
c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/4*I/c^4*b*
arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2
-1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+
1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)
/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))+1/6*I/c^4*b*e*Pi*csgn(I/(1/(c*x-
1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*
(c*x+1)-1)^2)+1/8*I*b*arccoth(c*x)*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(
1/(c*x-1)*(c*x+1)-1)^2)*Pi*x^4*e-1/4*I*b*arccoth(c*x)*csgn(I*(1/(c*x-1)*(c*
x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^4*e+1/8*I*b*arccoth(c*x)*cs
gn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^
2)^2*Pi*x^4*e+1/4*I*b*arccoth(c*x)*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1
)/(c*x+1))^(1/2))*Pi*x^4*e+1/8*I*b*arccoth(c*x)*csgn(I/(c*x-1)*(c*x+1))*csg
n(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^4*e-1/8*I*b*arccoth(c*x
)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*Pi*x^4*e-1/3*I/
c^4*b*e*Pi+1/12/c*b*x^3*d+1/4/c^3*b*x*d+1/4*b*arccoth(c*x)*x^4*d-1/8*b*arcc
oth(c*x)*x^4*e-1/4/c^4*b*arccoth(c*x)*d+41/24/c^4*b*arccoth(c*x)*e-1/3/c^4*
b*d+1/24*I/c*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*Pi*x^3*e-1
/12*I/c*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^3*e+1/8*I/
c^3*b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*Pi*x*e-1/8*I/c^3*b*csgn(I/(c*x-1)*(
c*x+1))^3*Pi*x*e+1/8*I/c^3*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2
)^3*Pi*x*e-1/8*I/c^4*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3+
1/8*I/c^4*b*arccoth(c*x)*Pi*e*csgn(I/(c*x-1)*(c*x+1))^3-1/8*I/c^4*b*arccoth
(c*x)*Pi*e*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3-1/4*I/c^3*b*cs
gn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x*e+1/4*I/c^4*b*arccoth(
c*x)*Pi*e*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+41/36*e/c^4*b+1
/4*I*b*arccoth(c*x)*Pi*x^4*e+1/12*I/c*b*Pi*x^3*e-1/6*I/c^4*b*e*Pi*csgn(I*(1
/(c*x-1)*(c*x+1)-1)^2)^3+1/6*I/c^4*b*e*Pi*csgn(I/(c*x-1)*(c*x+1))^3-1/6*I/c
^4*b*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3+1/3*I/c^4*b*e*P
i*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/4*I/c^3*b*Pi*x*e-1/4*
I/c^4*b*Pi*e*arccoth(c*x)-1/12/c^4*b*e*(3*arccoth(c*x)*x^3*c^3+3*arccoth(c*
x)*x^2*c^2+c^2*x^2+3*arccoth(c*x)*x*c+c*x+3*arc...
```

Maxima [C] Result contains complex when optimal does not.

time = 0.26, size = 274, normalized size = 1.22

$$\frac{1}{4}\,adx^{4}+\frac{1}{8}\left(2\,x^{4}\log\left(-c^{2}x^{2}+1\right)-c^{2}\left(\frac{c^{2}x^{4}+2\,x^{2}}{c^{4}}+\frac{2\,\log\left(c^{2}x^{2}-1\right)}{c^{6}}\right)\right)b\,\mathrm{arcoth}\left(cx\right)e+\frac{1}{24}\left(6\,x^{4}\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{2\left(c^{2}x^{3}+3\,x\right)}{c^{4}}-\frac{3\,\log\left(cx+1\right)}{c^{5}}+\frac{3\,\log\left(cx-1\right)}{c^{5}}\right)\right)bd+\frac{1}{8}\left(2\,x^{4}\log\left(-c^{2}x^{2}+1\right)-c^{2}\left(\frac{c^{2}x^{4}+2\,x^{2}}{c^{4}}+\frac{2\,\log\left(c^{2}x^{2}-1\right)}{c^{6}}\right)\right)ae-\frac{\left(2\left(-6i\,\pi c^{3}+7\,c^{3}\right)x^{3}+6\left(-6i\,\pi c+25\,c\right)x+3\left(6i\,\pi-4\,c^{3}x^{3}-12\,cx-25\right)\log\left(cx+1\right)+3\left(-6i\,\pi-4\,c^{3}x^{3}-12\,cx+25\right)\log\left(cx-1\right)\right)be}{144\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2
*log(c^2*x^2 - 1)/c^6))*b*arccoth(c*x)*e + 1/24*(6*x^4*arccoth(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/8*(
2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c
^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x + 3*
(6*I*pi - 4*c^3*x^3 - 12*c*x - 25)*log(c*x + 1) + 3*(-6*I*pi - 4*c^3*x^3 -
12*c*x + 25)*log(c*x - 1))*b*e/c^4
```

Fricas [A]
time = 0.35, size = 305, normalized size = 1.36

$$\frac{36\,ac^{4}dx^{4}+12\,bc^{3}dx^{3}+36\,bcdx-2\left(9\,ac^{4}x^{4}+7\,bc^{3}x^{3}+18\,ac^{2}x^{2}+75\,bcx\right)\cosh\left(1\right)+12\left(\left(3\,ac^{4}x^{4}+bc^{3}x^{3}+3\,bcx-3\,a\right)\cosh\left(1\right)+\left(3\,ac^{4}x^{4}+bc^{3}x^{3}+3\,bcx-3\,a\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right)+3\left(6\,bc^{4}dx^{4}-6\,bd-\left(3\,bc^{4}x^{4}+6\,bc^{2}x^{2}-25\,b\right)\cosh\left(1\right)+6\left(\left(bc^{4}x^{4}-b\right)\cosh\left(1\right)+\left(bc^{4}x^{4}-b\right)\sinh\left(1\right)\right)\log\left(-c^{2}x^{2}+1\right)-\left(3\,bc^{4}x^{4}+6\,bc^{2}x^{2}-25\,b\right)\sinh\left(1\right)\right)\log\left(\frac{cx+1}{cx-1}\right)-2\left(9\,ac^{4}x^{4}+7\,bc^{3}x^{3}+18\,ac^{2}x^{2}+75\,bcx\right)\sinh\left(1\right)}{144\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/144*(36*a*c^4*d*x^4 + 12*b*c^3*d*x^3 + 36*b*c*d*x - 2*(9*a*c^4*x^4 + 7*b*
c^3*x^3 + 18*a*c^2*x^2 + 75*b*c*x)*cosh(1) + 12*((3*a*c^4*x^4 + b*c^3*x^3 +
 3*b*c*x - 3*a)*cosh(1) + (3*a*c^4*x^4 + b*c^3*x^3 + 3*b*c*x - 3*a)*sinh(1)
)*log(-c^2*x^2 + 1) + 3*(6*b*c^4*d*x^4 - 6*b*d - (3*b*c^4*x^4 + 6*b*c^2*x^2
 - 25*b)*cosh(1) + 6*((b*c^4*x^4 - b)*cosh(1) + (b*c^4*x^4 - b)*sinh(1))*lo
g(-c^2*x^2 + 1) - (3*b*c^4*x^4 + 6*b*c^2*x^2 - 25*b)*sinh(1))*log((c*x + 1)
/(c*x - 1)) - 2*(9*a*c^4*x^4 + 7*b*c^3*x^3 + 18*a*c^2*x^2 + 75*b*c*x)*sinh(
1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.55, size = 286, normalized size = 1.27

$$\begin{cases} \frac{adx^{4}}{4}+\frac{aex^{4}\log\left(-c^{2}x^{2}+1\right)}{4}-\frac{aex^{4}}{8}-\frac{aex^{2}}{4c^{2}}-\frac{ae\log\left(-c^{2}x^{2}+1\right)}{4c^{4}}+\frac{bdx^{4}\,\mathrm{acoth}\left(cx\right)}{4}+\frac{bex^{4}\log\left(-c^{2}x^{2}+1\right)\mathrm{acoth}\left(cx\right)}{4}-\frac{bex^{4}\,\mathrm{acoth}\left(cx\right)}{8}+\frac{bdx^{3}}{12c}+\frac{bex^{3}\log\left(-c^{2}x^{2}+1\right)}{12c}-\frac{7bex^{3}}{72c}-\frac{bex^{2}\,\mathrm{acoth}\left(cx\right)}{4c^{2}}+\frac{bdx}{4c^{3}}+\frac{bex\log\left(-c^{2}x^{2}+1\right)}{4c^{3}}-\frac{25bex}{24c^{3}}-\frac{bd\,\mathrm{acoth}\left(cx\right)}{4c^{4}}-\frac{be\log\left(-c^{2}x^{2}+1\right)\mathrm{acoth}\left(cx\right)}{4c^{4}}+\frac{25be\,\mathrm{acoth}\left(cx\right)}{24c^{4}} & \text{for}\: c \neq 0 \\ \frac{dx^{4}\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x
**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*acoth(c*x)/4 + b
*e*x**4*log(-c**2*x**2 + 1)*acoth(c*x)/4 - b*e*x**4*acoth(c*x)/8 + b*d*x**3
```

```
/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**
2*acoth(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3)
 - 25*b*e*x/(24*c**3) - b*d*acoth(c*x)/(4*c**4) - b*e*log(-c**2*x**2 + 1)*a
coth(c*x)/(4*c**4) + 25*b*e*acoth(c*x)/(24*c**4), Ne(c, 0)), (d*x**4*(a + I
*pi*b/2)/4, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**B**]
time = 2.28, size = 414, normalized size = 1.84

$$\ln\left(1-\frac{1}{c\,x}\right)\,\left(\frac{\frac{b\,e\,x^5}{8}-\frac{b\,e\,x^3}{4\,c^2}+\frac{b\,c^2\,e\,x^7}{8}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,d\,x^5}{4}-\frac{b\,c^2\,d\,x^7}{4}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^5}{4}-\frac{b\,c^2\,e\,x^7}{4}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(1-c^2\,x^2\right)\,\left(x-c^2\,x^3\right)}{8\,c^4\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}\right)+x\,\left(\frac{b\,\left(6\,d-7\,e\right)}{24\,c^3}-\frac{3\,b\,e}{4\,c^3}\right)+\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^4}{4}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,e\,x^3}{12\,c}\right)-\ln\left(\frac{1}{c\,x}+1\right)\,\left(\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e}{8\,c^4}-\frac{b\,e\,x^4}{8}\right)-\frac{b\,d\,x^4}{8}+\frac{b\,e\,x^4}{16}+\frac{b\,e\,x^2}{8\,c^2}\right)+x^2\,\left(\frac{a\,\left(2\,d-e\right)}{4\,c^2}-\frac{a\,d}{2\,c^2}\right)+\frac{a\,x^4\,\left(2\,d-e\right)}{8}-\frac{\ln\left(c\,x-1\right)\,\left(12\,a\,e-6\,b\,d+25\,b\,e\right)}{48\,c^4}-\frac{\ln\left(c\,x+1\right)\,\left(12\,a\,e+6\,b\,d-25\,b\,e\right)}{48\,c^4}+\frac{b\,x^3\,\left(6\,d-7\,e\right)}{72\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - 1/(c*x))*(((b*e*x^5)/8 - (b*e*x^3)/(4*c^2) + (b*c^2*e*x^7)/8)/(2*(x
 + c*x^2)*(c*x - 1)) + ((b*d*x^5)/4 - (b*c^2*d*x^7)/4)/(2*(x + c*x^2)*(c*x
- 1)) + (log(1 - c^2*x^2)*((b*e*x^5)/4 - (b*c^2*e*x^7)/4))/(2*(x + c*x^2)*(
c*x - 1)) - (b*e*log(1 - c^2*x^2)*(x - c^2*x^3))/(8*c^4*(x + c*x^2)*(c*x -
1))) + x*((b*(6*d - 7*e))/(24*c^3) - (3*b*e)/(4*c^3)) + log(1 - c^2*x^2)*((
a*e*x^4)/4 + (b*e*x)/(4*c^3) + (b*e*x^3)/(12*c)) - log(1/(c*x) + 1)*(log(1
- c^2*x^2)*((b*e)/(8*c^4) - (b*e*x^4)/8) - (b*d*x^4)/8 + (b*e*x^4)/16 + (b*
e*x^2)/(8*c^2)) + x^2*((a*(2*d - e))/(4*c^2) - (a*d)/(2*c^2)) + (a*x^4*(2*d
 - e))/8 - (log(c*x - 1)*(12*a*e - 6*b*d + 25*b*e))/(48*c^4) - (log(c*x + 1
)*(12*a*e + 6*b*d - 25*b*e))/(48*c^4) + (b*x^3*(6*d - 7*e))/(72*c)
```

3.268 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=140

$$\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\tanh^{-1}(cx)}{c^2}+$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccoth
(c*x))-1/2*b*(d-e)*arctanh(c*x)/c^2+b*e*arctanh(c*x)/c^2+1/2*b*e*x*ln(-c^2*
x^2+1)/c-1/2*e*(-c^2*x^2+1)*(a+b*arccoth(c*x))*ln(-c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2504, 2436, 2332, 6231, 327, 213, 2498, 212}

$$-\frac{e(1-c^2x^2)\log\left(1-c^2x^2\right)\left(a+b\coth^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{bex\log\left(1-c^2x^2\right)}{2c}+\frac{be\tanh^{-1}(cx)}{c^2}+\frac{bx(d-e)}{2c}-\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*ArcTanh[c*
x])/c^2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcCoth[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6231

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{e(}{}$$

$$=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)-\frac{e(}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\operatorname{coth}\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+\right.$$

Mathematica [A]
time = 0.06, size = 129, normalized size = 0.92

$$\frac{2bc(d-3e)x+2ac^2(d-e)x^2+2bc^2(d-e)x^2\coth^{-1}(cx)+(b(d-3e)-2ae)\log(1-cx)-(b(d-3e)+2ae)\log(1+cx)+2e\left(cx(b+acx)+b(-1+c^2x^2)\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcCoth[c*x]
 + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x]
+ 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcCoth[c*x])*Log[1 - c^2*x^2])/(
4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.54, size = 2616, normalized size = 18.69

method	result	size
default	Expression too large to display	2616
risch	Expression too large to display	6696

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*b*e*(arccoth(c*x)*x*c+arccoth(c*x)+1)*(c*x-1)*ln((c*x-1)/(c*x+1))+
1/2*I*b*arccoth(c*x)*Pi*x^2*e+1/4*I/c^2*b*e*Pi*csgn(I/(c*x-1)*(c*x+1))^3-1/
4*I/c^2*b*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3-1/4*I/c^2*
b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3+1/2*I/c^2*b*e*Pi*csgn(I/(c*x-1)*(c
*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/2*I/c*b*Pi*x*e-1/2*I/c^2*b*arccoth(c*x)*
e*Pi+1/2*a*e/c^2-1/4*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))^3*x*e+1/4*I/c*b*Pi*cs
```

gn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*x*e+1/4*I/c*b*Pi*csgn(I*(1/
(c*x-1)*(c*x+1)-1)^2)^3*x*e+1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c
*x+1))^3-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c
*x+1)-1)^2)^3-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)
^3-1/2*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x*e+1/2*I
/c^2*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-
1/4*I/c^2*b*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/
(c*x-1)*(c*x+1)-1)^2)^2-1/2*I/c^2*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*cs
gn(I/(c*x-1)*(c*x+1))^2+1/2/c*b*x*d-1/2/c^2*b*arccoth(c*x)*d+5/2/c^2*b*arcc
oth(c*x)*e+1/2*b*arccoth(c*x)*x^2*d-1/2*b*arccoth(c*x)*x^2*e-1/4*I*b*arccot
h(c*x)*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c
x-1)(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*x^2*e-1/4*I/c*b*Pi*csgn(I/(1/(c*x-1)
*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c
*x+1)-1)^2)*x*e+1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^
2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)-
1/4*I/c^2*b*e*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*
(c*x+1)-1)^2)^2+1/4*I/c^2*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(
c*x-1)*(c*x+1))+1/2*I/c^2*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c
x-1)(c*x+1)-1)^2)^2-1/4*I/c^2*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn
(I*(1/(c*x-1)*(c*x+1)-1)^2)-1/4*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1))
^3*x^2*e+1/4*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-
1)^2)^3*x^2*e+1/4*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*x^2
*e-1/2*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^
2*x^2*e+3/2*e/c^2*b-3/2*b*e*x/c-1/c^2*b*e*ln(2)-1/4*I/c^2*b*arccoth(c*x)*e*
Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1
)-1)^2)^2-1/2*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csg
n(I/(c*x-1)*(c*x+1))^2-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)
)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/4*I/c^2*b*arccoth(c*x
)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))+1/2*I/c^2*
b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)
-1)^2)^2-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn
(I*(1/(c*x-1)*(c*x+1)-1)^2)+1/4*I/c*b*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*cs
gn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x*e+1/2*I/c*b*Pi*csgn(I/(c*
x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x*e+1/4*I/c^2*b*e*Pi*csgn(I
/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1
/(c*x-1)*(c*x+1)-1)^2)+1/4*I*b*arccoth(c*x)*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)
^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^2*e+1/2*I*b*arccoth
(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*x^2*e-1/
2*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I*(1/(c*x-1)*(
c*x+1)-1))*x^2*e+1/4*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*cs
gn(I*(1/(c*x-1)*(c*x+1)-1))^2*x^2*e-1/2/c^2*b*d+1/2*a*d*x^2+1/2*a*e*ln(-c^2
*x^2+1)*x^2-1/2*a*e/c^2*ln(-c^2*x^2+1)+1/4*I*b*arccoth(c*x)*Pi*csgn(I/(c*x-
1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^2*e-1/4*I*b
*arccoth(c*x)*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*
x^2*e+1/4*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1

```
)*(c*x+1)-1)^2)^2*x*e-1/4*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/
(c*x+1))^(1/2))^2*x*e-1/2*I/c*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I
*(1/(c*x-1)*(c*x+1)-1))*x*e+1/4*I/c*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*cs
gn(I*(1/(c*x-1)*(c*x+1)-1))^2*x*e-1/2*I/c^2*b*e*Pi+1/c*b*ln(2)*x*e-1/c*b*ln
(1/(c*x-1)*(c*x+1)-1)*x*e-1/c^2*b*arccoth(c*x)*e*ln(2)+1/c^2*b*arccoth(c*x)
*e*ln(1/(c*x-1)*(c*x+1)-1)+b*arccoth(c*x)*ln(2)*x^2*e-b*arccoth(c*x)*ln(1/(
c*x-1)*(c*x+1)-1)*x^2*e-1/2*a*e*x^2
```

Maxima [A]
time = 0.27, size = 174, normalized size = 1.24

$$\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{2\,x}{c^2}-\frac{\log\left(cx+1\right)}{c^3}+\frac{\log\left(cx-1\right)}{c^3}\right)\right)bd-\frac{\left(c^2x^2-\left(c^2x^2-1\right)\log\left(-c^2x^2+1\right)-1\right)b\,\mathrm{arcoth}\left(cx\right)e}{2\,c^2}-\frac{\left(c^2x^2-\left(c^2x^2-1\right)\log\left(-c^2x^2+1\right)-1\right)ae}{2\,c^2}-\frac{\left(3\,cx-\left(cx+1\right)\log\left(cx+1\right)-\left(cx-1\right)\log\left(-cx+1\right)\right)be}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*arccoth(c*x)*e/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2
```

Fricas [A]
time = 0.36, size = 214, normalized size = 1.53

$$\frac{2\,ac^2dx^2+2\,bcdx-2\left(ac^2x^2+3\,bcx\right)\cosh\left(1\right)+2\left(\left(ac^2x^2+bcx-a\right)\cosh\left(1\right)+\left(ac^2x^2+bcx-a\right)\sinh\left(1\right)\right)\log\left(-c^2x^2+1\right)+\left(bc^2dx^2-bd-\left(bc^2x^2-3\,b\right)\cosh\left(1\right)+\left(\left(bc^2x^2-b\right)\cosh\left(1\right)+\left(bc^2x^2-b\right)\sinh\left(1\right)\right)\log\left(-c^2x^2+1\right)-\left(bc^2x^2-3\,b\right)\sinh\left(1\right)\right)\log\left(\frac{cx+1}{cx-1}\right)-2\left(ac^2x^2+3\,bcx\right)\sinh\left(1\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*d*x^2 + 2*b*c*d*x - 2*(a*c^2*x^2 + 3*b*c*x)*cosh(1) + 2*((a*c^
2*x^2 + b*c*x - a)*cosh(1) + (a*c^2*x^2 + b*c*x - a)*sinh(1))*log(-c^2*x^2
+ 1) + (b*c^2*d*x^2 - b*d - (b*c^2*x^2 - 3*b)*cosh(1) + ((b*c^2*x^2 - b)*co
sh(1) + (b*c^2*x^2 - b)*sinh(1))*log(-c^2*x^2 + 1) - (b*c^2*x^2 - 3*b)*sinh
(1))*log((c*x + 1)/(c*x - 1)) - 2*(a*c^2*x^2 + 3*b*c*x)*sinh(1))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 209, normalized size = 1.49

$$\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log\left(-c^2x^2+1\right)}{2}-\frac{aex^2}{2}-\frac{ae\log\left(-c^2x^2+1\right)}{2c^2}+\frac{bdx^2\,\mathrm{acoth}\left(cx\right)}{2}+\frac{bex^2\log\left(-c^2x^2+1\right)\mathrm{acoth}\left(cx\right)}{2}-\frac{bex^2\,\mathrm{acoth}\left(cx\right)}{2}+\frac{bdx}{2c}+\frac{bex\log\left(-c^2x^2+1\right)}{2c}-\frac{3bex}{2c}-\frac{bd\,\mathrm{acoth}\left(cx\right)}{2c^2}-\frac{be\log\left(-c^2x^2+1\right)\mathrm{acoth}\left(cx\right)}{2c^2}+\frac{3be\,\mathrm{acoth}\left(cx\right)}{2c^2} & \text{for}\ c\neq 0\\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*l
og(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*acoth(c*x)/2 + b*e*x**2*log(-c**2*x*
```

```
*2 + 1)*acoth(c*x)/2 - b*e*x**2*acoth(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**
2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*acoth(c*x)/(2*c**2) - b*e*log(-c**2
*x**2 + 1)*acoth(c*x)/(2*c**2) + 3*b*e*acoth(c*x)/(2*c**2), Ne(c, 0)), (d*x
**2*(a + I*pi*b/2)/2, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.51, size = 241, normalized size = 1.72

$$-\frac{1}{4}bex^2\log(-cx+1)^2-\frac{1}{4}(-i\pi bd+i\pi be-2ad+2ae)x^2+\frac{1}{4}\left(bex^2-\frac{be}{c^2}\right)\log(cx+1)^2-\frac{1}{4}\left((-i\pi be-bd-2ae+be)x^2-\frac{2bex}{c}\right)\log(cx+1)-\frac{be\log(cx-1)^2}{4c^2}-\frac{1}{4}\left((-i\pi be+bd-2ae-be)x^2-\frac{2bex}{c}-\frac{2be\log(cx-1)}{c^2}\right)\log(-cx+1)+\frac{(bd-3be)x}{2c}+\frac{(-i\pi be-bd-2ae+3be)\log(cx+1)}{4c^2}+\frac{(-i\pi be+bd-2ae-3be)\log(cx-1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/4*b*e*x^2*log(-c*x + 1)^2 - 1/4*(-I*pi*b*d + I*pi*b*e - 2*a*d + 2*a*e)*x
^2 + 1/4*(b*e*x^2 - b*e/c^2)*log(c*x + 1)^2 - 1/4*((-I*pi*b*e - b*d - 2*a*e
 + b*e)*x^2 - 2*b*e*x/c)*log(c*x + 1) - 1/4*b*e*log(c*x - 1)^2/c^2 - 1/4*((
-I*pi*b*e + b*d - 2*a*e - b*e)*x^2 - 2*b*e*x/c - 2*b*e*log(c*x - 1)/c^2)*lo
g(-c*x + 1) + 1/2*(b*d - 3*b*e)*x/c + 1/4*(-I*pi*b*e - b*d - 2*a*e + 3*b*e)
*log(c*x + 1)/c^2 + 1/4*(-I*pi*b*e + b*d - 2*a*e - 3*b*e)*log(c*x - 1)/c^2
```

Mupad [B]
time = 2.05, size = 329, normalized size = 2.35

$$\ln\left(1-\frac{1}{cx}\right)\left(\frac{\frac{bdx^3}{2}-\frac{bc^2dx^5}{2}}{2(cx^2+x)(cx-1)}-\frac{\frac{bex^3}{2}-\frac{bc^2ex^5}{2}}{2(cx^2+x)(cx-1)}+\frac{\ln(1-c^2x^2)\left(\frac{bex^3}{2}-\frac{bc^2ex^5}{2}\right)}{2(cx^2+x)(cx-1)}-\frac{be\ln(1-c^2x^2)(x-c^2x^3)}{4c^2(cx^2+x)(cx-1)}\right)+\ln(1-c^2x^2)\left(\frac{aex^2}{2}+\frac{bex}{2c}\right)-\ln\left(\frac{1}{cx}+1\right)\left(\ln(1-c^2x^2)\left(\frac{be}{4c^2}-\frac{bex^2}{4}\right)-\frac{bdx^2}{4}+\frac{bex^2}{4}\right)+\frac{ax^2(d-e)}{2}-\frac{\ln(cx+1)(2ae+bd-3be)}{4c^2}-\frac{\ln(cx-1)(2ae-bd+3be)}{4c^2}+\frac{bx(d-3e)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1 - 1/(c*x))*(((b*d*x^3)/2 - (b*c^2*d*x^5)/2)/(2*(x + c*x^2)*(c*x - 1))
 - ((b*e*x^3)/2 - (b*c^2*e*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2
*x^2)*((b*e*x^3)/2 - (b*c^2*e*x^5)/2))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log
(1 - c^2*x^2)*(x - c^2*x^3))/(4*c^2*(x + c*x^2)*(c*x - 1))) + log(1 - c^2*x
^2)*((a*e*x^2)/2 + (b*e*x)/(2*c)) - log(1/(c*x) + 1)*(log(1 - c^2*x^2)*((b*
e)/(4*c^2) - (b*e*x^2)/4) - (b*d*x^2)/4 + (b*e*x^2)/4) + (a*x^2*(d - e))/2
- (log(c*x + 1)*(2*a*e + b*d - 3*b*e))/(4*c^2) - (log(c*x - 1)*(2*a*e - b*d
 + 3*b*e))/(4*c^2) + (b*x*(d - 3*e))/(2*c)
```

3.269 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x}\,dx$

Optimal. Leaf size=381

$$-\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log\left(\frac{1}{cx}\right)+ad\log(x)-be\log\left(\frac{c+\frac{1}{x}}{c}\right)\text{PolyLog}\left(2,\frac{c+\cdots}{\cdots}\right.$$

```
[Out] -1/2*b*e*ln(1+1/c/x)^2*ln(-1/c/x)+1/2*b*e*ln(1-1/c/x)^2*ln(1/c/x)+a*d*ln(x)
-b*e*ln((c+1/x)/c)*polylog(2,(c+1/x)/c)+b*e*ln(1-1/c/x)*polylog(2,1-1/c/x)+
1/2*b*d*polylog(2,-1/c/x)+1/2*b*e*ln(-c^2*x^2)*polylog(2,-1/c/x)-1/2*b*e*(l
n(1-1/c/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,-1/c/x)-1/2*b
*d*polylog(2,1/c/x)-1/2*b*e*ln(-c^2*x^2)*polylog(2,1/c/x)+1/2*b*e*(ln(1-1/c
/x)+ln(1+1/c/x)+ln(-c^2*x^2)-ln(-c^2*x^2+1))*polylog(2,1/c/x)-1/2*a*e*polyl
og(2,c^2*x^2)+b*e*polylog(3,(c+1/x)/c)-b*e*polylog(3,1-1/c/x)+b*e*polylog(3
,-1/c/x)-b*e*polylog(3,1/c/x)
```

Rubi [A]
time = 0.31, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6227, 6032, 6225, 2438, 6223, 2504, 2443, 2481, 2421, 6724, 6217}

$$-\frac{1}{2}ae\text{Li}_2(c^2x^2)+ad\log(x)+\frac{1}{2}be\text{Li}_2\left(-\frac{1}{cx}\right)\log\left(-c^2x^2\right)-\frac{1}{2}be\text{Li}_2\left(-\frac{1}{cx}\right)\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)-\frac{1}{2}be\text{Li}_2\left(\frac{1}{cx}\right)\log\left(-c^2x^2\right)+\frac{1}{2}be\text{Li}_2\left(\frac{1}{cx}\right)\left(\log\left(-c^2x^2\right)-\log\left(1-c^2x^2\right)+\log\left(1-\frac{1}{cx}\right)+\log\left(\frac{1}{cx}+1\right)\right)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)+be\text{Li}_3\left(\frac{c+\frac{1}{x}}{c}\right)-be\text{Li}_3\left(1-\frac{1}{cx}\right)+be\text{Li}_3\left(-\frac{1}{cx}\right)-be\text{Li}_3\left(\frac{1}{cx}\right)+be\text{Li}_2\left(1-\frac{1}{cx}\right)\log\left(1-\frac{1}{cx}\right)-be\text{Li}_2\left(\frac{c+\frac{1}{x}}{c}\right)\log\left(\frac{c+\frac{1}{x}}{c}\right)+\frac{1}{2}be\log\left(\frac{1}{cx}\right)\log^2\left(1-\frac{1}{cx}\right)-\frac{1}{2}be\log^2\left(\frac{1}{cx}+1\right)\log\left(-\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] -1/2*(b*e*Log[1 + 1/(c*x)]^2*Log[-(1/(c*x))]) + (b*e*Log[1 - 1/(c*x)]^2*Log
[1/(c*x)])/2 + a*d*Log[x] - b*e*Log[(c + x^(-1))/c]*PolyLog[2, (c + x^(-1))
/c] + b*e*Log[1 - 1/(c*x)]*PolyLog[2, 1 - 1/(c*x)] + (b*d*PolyLog[2, -(1/(c
*x))])/2 + (b*e*Log[-(c^2*x^2)]*PolyLog[2, -(1/(c*x))])/2 - (b*e*(Log[1 - 1
/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^2)] - Log[1 - c^2*x^2])*PolyLog[2,
 -(1/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 - (b*e*Log[-(c^2*x^2)]*PolyLo
g[2, 1/(c*x)])/2 + (b*e*(Log[1 - 1/(c*x)] + Log[1 + 1/(c*x)] + Log[-(c^2*x^
2)] - Log[1 - c^2*x^2])*PolyLog[2, 1/(c*x)])/2 - (a*e*PolyLog[2, c^2*x^2])/
2 + b*e*PolyLog[3, (c + x^(-1))/c] - b*e*PolyLog[3, 1 - 1/(c*x)] + b*e*Poly
Log[3, -(1/(c*x))] - b*e*PolyLog[3, 1/(c*x)]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6217

```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[Log[d*x^m]*(Log[1 + 1/(c*x^n)]/x), x], x] - Dist[1/2, Int[Log[d
*x^m]*(Log[1 - 1/(c*x^n)]/x), x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 6223

```
Int[(ArcCoth[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist
[Log[f + g*x^2] - Log[(-c^2)*x^2] - Log[1 - 1/(c*x)] - Log[1 + 1/(c*x)], In
t[ArcCoth[c*x]/x, x], x] + (Dist[1/2, Int[Log[1 + 1/(c*x)]^2/x, x], x] - Di
st[1/2, Int[Log[1 - 1/(c*x)]^2/x, x], x] + Int[Log[(-c^2)*x^2]*(ArcCoth[c*x
```

```
]/x), x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 6225

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[Log[f + g*x^2
]*(ArcCoth[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 6227

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcCoth[c*x])/x, x], x] + Dis
t[e, Int[Log[f + g*x^2]*((a + b*ArcCoth[c*x])/x), x], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\coth^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\coth^{-1}(cx)\right)\log\left(1-\right.}{x}$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\mathrm{Li}_2\left(\frac{1}{cx}\right)+(ae)\int\frac{\log}{}$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\mathrm{Li}_2\left(\frac{1}{cx}\right)-\frac{1}{2}ae\mathrm{Li}_2(c^2x$$

$$= ad\log(x)+\frac{1}{2}bd\mathrm{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}be\left(\log\left(1-\frac{1}{cx}\right)+\log\left(\right.\right.$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log$$

$$= -\frac{1}{2}be\log^2\left(1+\frac{1}{cx}\right)\log\left(-\frac{1}{cx}\right)+\frac{1}{2}be\log^2\left(1-\frac{1}{cx}\right)\log$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.80, size = 864, normalized size = 2.27

method	result	size
risch	Expression too large to display	864

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

```
[Out] dilog(c*x)*a*e-1/2*dilog(c*x)*b*d+polylog(3,-c*x+1)*b*e+ln(c*x)*a*d-1/2*I*d
ilog(c*x)*Pi*b*e-1/4*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*
x+1))^2-1/4*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1
/2*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/2*I*Pi*ln(c*x)*ln
(c*x-1)*b*e-1/4*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/2*I*Pi*ln(c
*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^3+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1))*csgn
(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(
c*x+1))^2-(-1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*
x+1))+1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*c
sgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1
))^3-1/2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*b*e+a*e+1/2*b*d)*dilog
(c*x+1)+1/2*ln(c*x+1)^2*ln(-c*x)*b*e+ln(c*x+1)*polylog(2,c*x+1)*b*e-I*Pi*ln
(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*dilog(c*x)*Pi*b*e*csgn(I*(c*x-1)*
(c*x+1))^2+1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*c
sgn(I*(c*x-1)*(c*x+1))-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1)*(c*x+1
))^3+I*Pi*ln(c*x)*a*e-polylog(3,c*x+1)*b*e-1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x
-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e
*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*c
sgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*dilog(c*x)*Pi*b*e*csgn(I*(c*
x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+ln(c*x)*ln(c*x-1)*a*e-1/2*ln(
c*x)*ln(c*x-1)*b*d-1/2*ln(c*x)*ln(c*x-1)^2*b*e-polylog(2,-c*x+1)*ln(c*x-1)*
b*e
```

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 173, normalized size = 0.45

$$i\,\pi a e\log\left(x\right)-\frac{1}{2}\left(\log\left(cx-1\right)^2\log\left(cx\right)+2\,\mathrm{Li}_2(-cx+1)\log\left(cx-1\right)-2\,\mathrm{Li}_3(-cx+1)\right)be+\frac{1}{2}\left(\log\left(cx+1\right)^2\log\left(-cx\right)+2\,\mathrm{Li}_2(cx+1)\log\left(cx+1\right)-2\,\mathrm{Li}_3(cx+1)\right)be+ad\log\left(x\right)-\frac{1}{2}\left(bd+(i\,\pi b-2\,a)e\right)\left(\log\left(cx-1\right)\log\left(cx\right)+\mathrm{Li}_2(-cx+1)\right)+\frac{1}{2}\left(bd+(i\,\pi b+2\,a)e\right)\left(\log\left(cx+1\right)\log\left(-cx\right)+\mathrm{Li}_2(cx+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] I*pi*a*e*log(x) - 1/2*(log(c*x - 1)^2*log(c*x) + 2*dilog(-c*x + 1)*log(c*x
- 1) - 2*polylog(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilo
g(c*x + 1)*log(c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(b*
d + (I*pi*b - 2*a)*e)*(log(c*x - 1)*log(c*x) + dilog(-c*x + 1)) + 1/2*(b*d
+ (I*pi*b + 2*a)*e)*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2x^2 + 1))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(c\,x))\, (d + e\, \ln(1 - c^2\,x^2))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x, x)
```

3.270 $\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^3}\,dx$

Optimal. Leaf size=247

$$-\frac{1}{2}bc^2e\coth^{-1}(cx)^2-\frac{1}{2}bc^2e\tanh^{-1}(cx)^2-ac^2e\log(x)+bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right)+\frac{1}{2}(a+b)c^2e\log(1-cx)+$$

```
[Out] -1/2*b*c^2*e*arccoth(c*x)^2-1/2*b*c^2*e*arctanh(c*x)^2-a*c^2*e*ln(x)+b*c^2*
e*arctanh(c*x)*ln(2/(-c*x+1))+1/2*(a+b)*c^2*e*ln(-c*x+1)+1/2*(a-b)*c^2*e*ln
(c*x+1)-1/2*b*c*(d+e*ln(-c^2*x^2+1))/x-1/2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*
x^2+1))/x^2+1/2*b*c^2*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-b*c^2*e*arccoth(c*x
)*ln(2-2/(c*x+1))+1/2*b*c^2*e*polylog(2,1-2/(-c*x+1))+1/2*b*c^2*e*polylog(2
,-1+2/(c*x+1))
```

Rubi [A]
time = 0.34, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$, Rules used = {6038, 331, 212, 6233, 6857, 815, 6136, 6080, 2497, 6131, 6055, 2449, 2352}

$$-\frac{(a+b\coth^{-1}(cx))\,(e\log(1-c^2x^2)+d)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2}c^2e(a-b)\log(cx+1)-ac^2e\log(x)-\frac{bc(e\log(1-c^2x^2)+d)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\,(e\log(1-c^2x^2)+d)+\frac{1}{2}bc^2e\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{1}{2}bc^2e\text{Li}_2\left(\frac{2}{cx+1}-1\right)-\frac{1}{2}bc^2e\tanh^{-1}(cx)^2-\frac{1}{2}bc^2e\coth^{-1}(cx)^2+bc^2e\log\left(\frac{2}{1-cx}\right)\tanh^{-1}(cx)-bc^2e\log\left(2-\frac{2}{cx+1}\right)\coth^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] -1/2*(b*c^2*e*ArcCoth[c*x]^2) - (b*c^2*e*ArcTanh[c*x]^2)/2 - a*c^2*e*Log[x]
 + b*c^2*e*ArcTanh[c*x]*Log[2/(1 - c*x)] + ((a + b)*c^2*e*Log[1 - c*x])/2 +
 ((a - b)*c^2*e*Log[1 + c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) - ((
a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(2*x^2) + (b*c^2*ArcTanh[c*x]
*(d + e*Log[1 - c^2*x^2]))/2 - b*c^2*e*ArcCoth[c*x]*Log[2 - 2/(1 + c*x)] +
(b*c^2*e*PolyLog[2, 1 - 2/(1 - c*x)])/2 + (b*c^2*e*PolyLog[2, -1 + 2/(1 + c
*x)])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6233

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = -\frac{bc(d+e\log\left(1-c^2x^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\right.}{2x^2}$$

$$= -\frac{bc(d+e\log\left(1-c^2x^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\right.}{2x^2}$$

$$= -\frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - \frac{bc(d+e\log\left(1-c^2x^2\right))}{2x} - \frac{\left(a+b\,\mathrm{c}\right.}{}$$

$$= -\frac{1}{2}bc^2e\tanh^{-1}(cx)^2 + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \frac{bc}{}$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 + bc^2e\tanh^{-1}(c$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - ac^2e\log(x) +$$

$$= -\frac{1}{2}bc^2e\coth^{-1}(cx)^2 - \frac{1}{2}bc^2e\tanh^{-1}(cx)^2 - ac^2e\log(x) +$$

Mathematica [A]
time = 0.10, size = 161, normalized size = 0.65

$$\frac{1}{2}\left(-\frac{ad}{x^2} - 2ac^2e\log(x) + (a+b)c^2e\log(1-cx) + (a-b)c^2e\log(1+cx) - \frac{bd\left(2\coth^{-1}(cx) + cx(2+cx\log(1-cx) - cx\log(1+cx))\right)}{2x^2} - \frac{e\left(a+bcx+\left(b-bc^2x^2\right)\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x^2} - bc^2e\left(\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right) - \mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] (-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^2
*e*Log[1 + c*x] - (b*d*(2*ArcCoth[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Lo
g[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcCoth[c*x])*Log[1
 - c^2*x^2])/x^2 - b*c^2*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x)]))/
2
```

Maple [F]
time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccoth}(cx)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d +
1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e - 1/4*b
*(log(c*x + 1)^2/x^2 - 2*integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*
pi*c + c)*x)*log(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^4 + x^3),
 x))*e - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(cx))(d + e\log(-c^2x^2 + 1))}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3,x)
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)
```

3.271 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^5}\,dx$

Optimal. Leaf size=339

$$\frac{ac^2e}{4x^2}+\frac{5bc^3e}{12x}+\frac{bc^2e\coth^{-1}(cx)}{4x^2}-\frac{1}{4}bc^4e\coth^{-1}(cx)^2-\frac{1}{4}bc^4e\tanh^{-1}(cx)-\frac{1}{4}bc^4e\tanh^{-1}(cx)^2-\frac{1}{2}ac^4e\log(x)+\frac{1}{2}bc$$

```
[Out] 1/4*a*c^2*e/x^2+5/12*b*c^3*e/x+1/4*b*c^2*e*arccoth(c*x)/x^2-1/4*b*c^4*e*arc
coth(c*x)^2-1/4*b*c^4*e*arctanh(c*x)-1/4*b*c^4*e*arctanh(c*x)^2-1/2*a*c^4*e
*ln(x)+1/2*b*c^4*e*arctanh(c*x)*ln(2/(-c*x+1))+1/12*(3*a+4*b)*c^4*e*ln(-c*x
+1)+1/12*(3*a-4*b)*c^4*e*ln(c*x+1)-1/12*b*c*(d+e*ln(-c^2*x^2+1))/x^3-1/4*b*
c^3*(d+e*ln(-c^2*x^2+1))/x-1/4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4+
1/4*b*c^4*arctanh(c*x)*(d+e*ln(-c^2*x^2+1))-1/2*b*c^4*e*arccoth(c*x)*ln(2-2
/(c*x+1))+1/4*b*c^4*e*polylog(2,1-2/(-c*x+1))+1/4*b*c^4*e*polylog(2,-1+2/(c
*x+1))
```

Rubi [A]
time = 0.49, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {6038, 331, 212, 6233, 6857, 1816, 6130, 6136, 6080, 2497, 6131, 6055, 2449, 2352}

$$\frac{1}{12}c^4e(3a+4b)\log(1-cx)+\frac{1}{12}c^4e(3a-4b)\log(cx+1)-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}-\frac{1}{2}ac^4e\log(x)+\frac{ac^2e}{4x^2}+\frac{1}{4}bc^4e\text{Li}_2\left(1-\frac{2}{1-cx}\right)+\frac{1}{4}bc^4e\text{Li}_2\left(\frac{2}{cx+1}-1\right)-\frac{1}{4}bc^4e\tanh^{-1}(cx)^2-\frac{1}{4}bc^4e\tanh^{-1}(cx)-\frac{1}{4}bc^4e\coth^{-1}(cx)^2+\frac{1}{2}bc^4e\log\left(\frac{2}{1-cx}\right)\tanh^{-1}(cx)-\frac{1}{2}bc^4e\log\left(2-\frac{2}{cx+1}\right)\coth^{-1}(cx)+\frac{5bc^3e}{12x}-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)}{12x^3}+\frac{bc^2e\coth^{-1}(cx)}{4x^2}+\frac{1}{4}bc^4\tanh^{-1}(cx)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{bc^3\left(e\log\left(1-c^2x^2\right)+d\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) + (b*c^2*e*ArcCoth[c*x])/(4*x^2) - (
b*c^4*e*ArcCoth[c*x]^2)/4 - (b*c^4*e*ArcTanh[c*x])/4 - (b*c^4*e*ArcTanh[c*x
]^2)/4 - (a*c^4*e*Log[x])/2 + (b*c^4*e*ArcTanh[c*x]*Log[2/(1 - c*x)])/2 + (
(3*a + 4*b)*c^4*e*Log[1 - c*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 -
(b*c*(d + e*Log[1 - c^2*x^2]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/
(4*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(4*x^4) + (b*c^4*Ar
cTanh[c*x]*(d + e*Log[1 - c^2*x^2]))/4 - (b*c^4*e*ArcCoth[c*x]*Log[2 - 2/(1
 + c*x)])/2 + (b*c^4*e*PolyLog[2, 1 - 2/(1 - c*x)])/4 + (b*c^4*e*PolyLog[2,
 -1 + 2/(1 + c*x)])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6080

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcCoth[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Dist[b*c*(p/d), Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/
(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6136

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6233

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+e\log\left(1-c^2x^2\right))}{4x} - \ldots$$

$$= -\frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+e\log\left(1-c^2x^2\right))}{4x} - \ldots$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 - \frac{bc(d+e\log\left(1-c^2x^2\right))}{12x^3} - \frac{bc^3(d+\ldots}{\ldots}$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \ldots$$

$$= -\frac{1}{4}bc^4e\tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e\tanh^{-1}(cx)\log\left(\frac{2}{1-cx}\right) - \ldots$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}b\ldots$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}\ldots$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e\coth^{-1}(cx)^2 - \frac{1}{4}\ldots$$

Mathematica [A]
time = 0.08, size = 307, normalized size = 0.91

$$-\frac{ad}{4x^4}+\frac{ac^2e}{4x^2}+\frac{bc^3e}{6x}-\frac{1}{2}ac^4e\log(x)+\frac{1}{12}(3ac^4e+4bc^4e)\log(1-cx)-\frac{1}{2}bc^4e\left(-\frac{\coth^{-1}(cx)}{2c^2x^2}+\frac{1}{2}\left(-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)+bc^4d\left(-\frac{\coth^{-1}(cx)}{4c^4x^4}+\frac{1}{4}\left(-\frac{1}{3c^3x^3}-\frac{1}{cx}-\frac{1}{2}\log(1-cx)+\frac{1}{2}\log(1+cx)\right)\right)+\frac{1}{12}(3ac^4e-4bc^4e)\log(1+cx)+\frac{e\left(-3a-bcx-3bc^3x^3-3b\coth^{-1}(cx)+3bc^4x^4\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{12x^4}-\frac{1}{4}bc^4e\left(\mathrm{PolyLog}\left(2,-\frac{1}{cx}\right)-\mathrm{PolyLog}\left(2,\frac{1}{cx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] -1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
 ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcCoth[c*x]/(c
^2*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d*(-
1/4*ArcCoth[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 +
 Log[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(-3*a
- b*c*x - 3*b*c^3*x^3 - 3*b*ArcCoth[c*x] + 3*b*c^4*x^4*ArcCoth[c*x])*Log[1
- c^2*x^2])/(12*x^4) - (b*c^4*e*(PolyLog[2, -(1/(c*x))] - PolyLog[2, 1/(c*x
)]))/4
```

Maple [F]
time = 5.94, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccoth}(cx)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxima
")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arccoth(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^
2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e - 1/8*b*(log(c*x + 1)^2/x^4 - 4*integra
te(-1/2*(2*(c*x + 1)*log(c*x - 1)^2 - (2*I*pi + (2*I*pi*c + c)*x)*log(c*x +
 1) + 2*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*x^6 + x^5), x))*e - 1/4*a*d/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(cx))\,(d + e\log(-c^2x^2 + 1))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^5, x)
```

3.272 $\int x^4 \left(a+b \coth^{-1}(cx)\right) \left(d+e \log \left(1-c^2 x^2\right)\right) \, dx$

Optimal. Leaf size=315

$$-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\coth^{-1}(cx)}{5c^4}-\frac{2bex^3\coth^{-1}(cx)}{15c^2}-\frac{2}{25}bex^5\coth^{-1}(cx)+\frac{be\coth}{5c}$$

```
[Out] -2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c-2/25*a*e
*x^5-2/5*b*e*x*arccoth(c*x)/c^4-2/15*b*e*x^3*arccoth(c*x)/c^2-2/25*b*e*x^5*
arccoth(c*x)+1/5*b*e*arccoth(c*x)^2/c^5-1/20*(4*a+3*b)*e*ln(-c*x+1)/c^5+1/2
0*(4*a-3*b)*e*ln(c*x+1)/c^5-23/75*b*e*ln(-c^2*x^2+1)/c^5-1/20*b*e*ln(-c^2*x
^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(-c^2*x^2+1))/c^3+1/20*b*x^4*(d+e*ln(-c^2*x^2
+1))/c+1/5*x^5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/10*b*ln(-c^2*x^2+1
)*(d+e*ln(-c^2*x^2+1))/c^5
```

Rubi [A]
time = 0.51, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6038, 272, 45, 6233, 6857, 1816, 647, 31, 6128, 6022, 266, 6096, 2525, 2437, 2338}

$$-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}+\frac{1}{5}x^5\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{5c^4}-\frac{2aex^3}{15c^2}-\frac{2}{25}aex^5+\frac{be\coth^{-1}(cx)^2}{5c^5}-\frac{2bex\coth^{-1}(cx)}{5c^4}-\frac{77bex^2}{300c^3}+\frac{bx^4\left(e\log\left(1-c^2x^2\right)+d\right)}{20c}-\frac{2bex^3\coth^{-1}(cx)}{15c^2}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^5}-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}-\frac{23be\log\left(1-c^2x^2\right)}{75c^5}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{10c^3}-\frac{2}{25}bex^5\coth^{-1}(cx)-\frac{9bex^4}{200c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcCoth[c*x])/(5*c^4) - (2*b*e*x^
3*ArcCoth[c*x])/(15*c^2) - (2*b*e*x^5*ArcCoth[c*x])/25 + (b*e*ArcCoth[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6233

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx &= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{10c^3}+\frac{bx^4(d+e\log\left(1-c^2x^2\right))}{20c}+ \\
&= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{10c^3}+\frac{bx^4(d+e\log\left(1-c^2x^2\right))}{20c}+ \\
&= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{10c^3}+\frac{bx^4(d+e\log\left(1-c^2x^2\right))}{20c}+ \\
&= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{10c^3}+\frac{bx^4(d+e\log\left(1-c^2x^2\right))}{20c}+ \\
&= -\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}+\frac{bx^2(d+e\log\left(1-c^2x^2\right))}{10c^3}+\frac{bx^4(\cdots}{} \\
&= -\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\,\mathrm{c}\cdots \\
&= -\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex^3\,\mathrm{co}\cdots}{15\cdots} \\
&= -\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex\,\mathrm{cot}\cdots}{5\cdots} \\
&= -\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\,\mathrm{c}\cdots}{} \\
&= -\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\,\mathrm{c}\cdots}{}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 236, normalized size = 0.75

$$
\frac{-240acex+2bc^2(30d-77e)x^2-80ac^3ex^3+3bc^4(10d-9e)x^4+24ac^5(5d-2e)x^5-8bcx\left(-15c^4dx^4+2e(15+5c^2x^2+3c^4x^4)\right)\coth^{-1}(cx)+120be\coth^{-1}(cx)^2+2(30bd-60ae-137be)\log(1-cx)+2(30bd+60ae-137be)\log(1+cx)+30c^2ex^2\left(4ac^3x^3+b(2+c^2x^2)+4bc^3x^3\coth^{-1}(cx)\right)\log\left(1-c^2x^2\right)+30be\log^2\left(1-c^2x^2\right)}{600c^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
- 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15 +
5*c^2*x^2 + 3*c^4*x^4))*ArcCoth[c*x] + 120*b*e*ArcCoth[c*x]^2 + 2*(30*b*d -
 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 + c*x
] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcCoth[c*x])
*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.56, size = 4194, normalized size = 13.31

method	result	size
risch	Expression too large to display	1727
default	Expression too large to display	4194

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I/c^5*b*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*l
n(1/(c*x-1)*(c*x+1)-1)*e+1/40*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c
*x+1)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I/
(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*
x^2*e+1/40*I/c*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+
1)-1)^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c
*x-1)*(c*x+1)-1)^2)*x^2*e+1/20*I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((
c*x-1)/(c*x+1))^(1/2))*x^4*e+1/10*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1))^2*csgn
(I/((c*x-1)/(c*x+1))^(1/2))*x^2*e+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/(1/
(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/1
0*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/
(1/(c*x-1)*(c*x+1)-1)^2)^2+1/5*I/c^5*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I*
(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2+1/10*I/c^5*b*Pi*ln
(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c
*x+1))-1/10*I/c^5*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I*(1/(c*x-1)*(c*x+1)-
1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)-1/10*I/c^5*b*csgn(I/(c*x-1)*(c*x+1)/(
1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(c*x-1)*(c*x+1))*Pi*ln(1/(c*x-1)*(c*x+1)-1
)*e+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(
1/(c*x-1)*(c*x+1)-1)^2)-1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)/(c*x
+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))+3/40*I/c^5*b*Pi*e*csgn(I/(1/(c*x-1)*(
c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x
+1)-1)^2)-1/5*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(
I*(1/(c*x-1)*(c*x+1)-1)^2)^2+1/5*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/
(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2-1/20*I/c*b*Pi*csgn(I*(1/(c*x-1)*(
c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x^4*e-1/10*I/c^3*b*Pi*csgn(I*(
1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*x^2*e-1/40*I/c*b*Pi
*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*x^4*e-1/20*I/c^3
*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*x^2*e+1/40*
I/c*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-
1)^2)^2*x^4*e+1/20*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1
)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x^2*e-3/10/c^5*b*e*ln(2)+1/20/c*b*x^4*d+1/10/c
^3*b*x^2*d+1/5/c^5*b*arccoth(c*x)*d+1/5/c^5*b*ln(1/(c*x-1)*(c*x+1)-1)^2*e+1
37/150/c^5*b*e*ln(1/(c*x-1)*(c*x+1)-1)-1/5/c^5*b*d*ln(1/(c*x-1)*(c*x+1)-1)+
1/5*b*arccoth(c*x)*x^5*d-46/75/c^5*b*arccoth(c*x)*e+181/600*e/c^5*b-2/5*b*e
*x*arccoth(c*x)/c^4-2/15*b*e*x^3*arccoth(c*x)/c^2-2/25*a*e*x^5-1/10*I/c^5*b
*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3+1/10*I/c^5*
```

```
b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3+1/5*I/c^5*b*Pi*ln(1/(
c*x-1)*(c*x+1)-1)*e*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-3/40*
I/c^5*b*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x
-1)*(c*x+1)-1)^2)^2-3/40*I/c^5*b*Pi*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(
I*(1/(c*x-1)*(c*x+1)-1)^2)-3/20*I/c^5*b*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2)
)*csgn(I/(c*x-1)*(c*x+1))^2+3/20*I/c^5*b*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))
*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2+3/40*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1
))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I/(
c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3-1/10*I/c^5*b*arccoth(c*x)*Pi*e*cs
gn(I/(c*x-1)*(c*x+1))^3-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9
/200*b*e*x^4/c-2/25*b*e*x^5*arccoth(c*x)-1/10*I*b*arccoth(c*x)*Pi*csgn(I/(c
*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(1/(c
*x-1)*(c*x+1)-1)^2)*x^5*e+1/5*a*e*x^5*ln(-c^2*x^2+1)-1/5*a*e/c^5*ln(c*x-1)+
1/5*a*e/c^5*ln(c*x+1)-1/10/c*b*ln(1/(c*x-1)*(c*x+1)-1)*x^4*e-1/5/c^3*b*ln(1
/(c*x-1)*(c*x+1)-1)*x^2*e+1/10/c*b*ln(2)*x^4*e+1/5/c^3*b*ln(2)*x^2*e+2/5/c^
5*b*arccoth(c*x)*e*ln(2)-2/5/c^5*b*ln(2)*ln(1/(c*x-1)*(c*x+1)-1)*e-2/5*b*ar
ccoth(c*x)*ln(1/(c*x-1)*(c*x+1)-1)*x^5*e+2/5*b*arccoth(c*x)*ln(2)*x^5*e-1/2
0/c^5*b*e*(4*arccoth(c*x)*x^5*c^5+x^4*c^4+2*c^2*x^2+4*arccoth(c*x)-4*ln(1/(
c*x-1)*(c*x+1)-1)-3)*ln((c*x-1)/(c*x+1))-3/20*I/c^5*b*Pi*e+1/10*I/c^5*b*Pi*
ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c
*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)-1/40*I/c*b*Pi*csgn(I
/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(1
/(c*x-1)*(c*x+1)-1)^2)*x^4*e-1/20*I/c^3*b*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I
/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*x
^2*e-1/10*I/c^5*b*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/
(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)-1/10*I/c^5
*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*ln(1/(c*x-1)*(c*x+1
)-1)*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)+1/10*I*b*arccoth(c*x)*Pi*csgn(I/(c*x
-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*x^5*
e+1/10*I*b*arccoth(c*x)*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1
)*(c*x+1)-1)^2)*x^5*e-1/10*I*b*arccoth(c*x)*Pi*...
```

<u>**Maxima**</u> [C] Result contains complex when optimal does not.
time = 0.27, size = 320, normalized size = 1.02

$$\frac{1}{5}adx^5 + \frac{1}{75}\left(15x^5\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6} - \frac{15\log\left(cx+1\right)}{c^7} + \frac{15\log\left(cx-1\right)}{c^7}\right)\right)b\,\mathrm{arccoth}\left(cx\right)e + \frac{1}{20}\left(4x^5\,\mathrm{arccoth}\left(cx\right) + c\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bd + \frac{1}{75}\left(15x^5\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6} - \frac{15\log\left(cx+1\right)}{c^7} + \frac{15\log\left(cx-1\right)}{c^7}\right)\right)ae - \frac{\left(3\left(-10i\pi c^4+9c^4\right)x^4 + 2\left(-30i\pi c^2+77c^2\right)x^2 + 2\left(-30i\pi - 15c^4x^4 - 30c^2x^2 - 60\log\left(cx-1\right) + 137\right)\log\left(cx+1\right) + 2\left(-30i\pi - 15c^4x^4 - 30c^2x^2 + 137\right)\log\left(cx-1\right)\right)be}{600c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*arccoth(c*x)*
e + 1/20*(4*x^5*arccoth(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1
)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
```

```
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/600*(3*
(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 + 2*(-30*I*pi -
15*c^4*x^4 - 30*c^2*x^2 - 60*log(c*x - 1) + 137)*log(c*x + 1) + 2*(-30*I*pi
 - 15*c^4*x^4 - 30*c^2*x^2 + 137)*log(c*x - 1))*b*e/c^5
```

Fricas [A]
time = 0.35, size = 388, normalized size = 1.23

$$\frac{120\,ac^5dx^5 + 30\,bc^4dx^4 + 60\,bc^2dx^2 + 30\,(b\cosh\left(1\right) + b\sinh\left(1\right))\log\left(-c^2x^2+1\right)^2 + 30\,(b\cosh\left(1\right) + b\sinh\left(1\right))\log\left(\frac{cx+1}{cx-1}\right)^2 - (48\,ac^5x^5 + 27\,bc^4x^4 + 80\,ac^3x^3 + 154\,bc^2x^2 + 240\,acx)\cosh\left(1\right) + 2\,(30\,bd + (60\,ac^5x^5 + 15\,bc^4x^4 + 30\,bc^2x^2 - 137\,b)\cosh\left(1\right) + (60\,ac^5x^5 + 15\,bc^4x^4 + 30\,bc^2x^2 - 137\,b)\sinh\left(1\right))\log\left(-c^2x^2+1\right) + 4\,(15\,bc^5dx^5 - 2\,(3\,bc^5x^5 + 5\,bc^3x^3 + 15\,bcx - 15\,a)\cosh\left(1\right) + 15\,(bc^5x^5\cosh\left(1\right) + bc^5x^5\sinh\left(1\right))\log\left(-c^2x^2+1\right) - 2\,(3\,bc^5x^5 + 5\,bc^3x^3 + 15\,bcx - 15\,a)\sinh\left(1\right))\log\left(\frac{cx+1}{cx-1}\right) - (48\,ac^5x^5 + 27\,bc^4x^4 + 80\,ac^3x^3 + 154\,bc^2x^2 + 240\,acx)\sinh\left(1\right)}{600\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/600*(120*a*c^5*d*x^5 + 30*b*c^4*d*x^4 + 60*b*c^2*d*x^2 + 30*(b*cosh(1) +
b*sinh(1))*log(-c^2*x^2 + 1)^2 + 30*(b*cosh(1) + b*sinh(1))*log((c*x + 1)/(
c*x - 1))^2 - (48*a*c^5*x^5 + 27*b*c^4*x^4 + 80*a*c^3*x^3 + 154*b*c^2*x^2 +
 240*a*c*x)*cosh(1) + 2*(30*b*d + (60*a*c^5*x^5 + 15*b*c^4*x^4 + 30*b*c^2*x
^2 - 137*b)*cosh(1) + (60*a*c^5*x^5 + 15*b*c^4*x^4 + 30*b*c^2*x^2 - 137*b)*
sinh(1))*log(-c^2*x^2 + 1) + 4*(15*b*c^5*d*x^5 - 2*(3*b*c^5*x^5 + 5*b*c^3*x
^3 + 15*b*c*x - 15*a)*cosh(1) + 15*(b*c^5*x^5*cosh(1) + b*c^5*x^5*sinh(1))*
log(-c^2*x^2 + 1) - 2*(3*b*c^5*x^5 + 5*b*c^3*x^3 + 15*b*c*x - 15*a)*sinh(1)
)*log((c*x + 1)/(c*x - 1)) - (48*a*c^5*x^5 + 27*b*c^4*x^4 + 80*a*c^3*x^3 +
154*b*c^2*x^2 + 240*a*c*x)*sinh(1))/c^5
```

Sympy [C] Result contains complex when optimal does not.
time = 2.32, size = 345, normalized size = 1.10

$$\begin{cases} \frac{adx^5}{5} + \frac{aex^5\log\left(-c^2x^2+1\right)}{5} - \frac{2aex^5}{25} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae\operatorname{acoth}\left(cx\right)}{5c^5} + \frac{bdx^5\operatorname{acoth}\left(cx\right)}{5} + \frac{bex^5\log\left(-c^2x^2+1\right)\operatorname{acoth}\left(cx\right)}{5} - \frac{2bex^5\operatorname{acoth}\left(cx\right)}{25} + \frac{bdx^4}{20c} + \frac{bex^4\log\left(-c^2x^2+1\right)}{20c} - \frac{9bex^4}{200c} - \frac{2bex^3\operatorname{acoth}\left(cx\right)}{15c^2} + \frac{bdx^2}{10c^3} + \frac{bex^2\log\left(-c^2x^2+1\right)}{10c^3} - \frac{77bex^2}{300c^3} - \frac{2bex\operatorname{acoth}\left(cx\right)}{5c^4} + \frac{bd\log\left(-c^2x^2+1\right)}{10c^5} + \frac{be\log\left(-c^2x^2+1\right)^2}{20c^5} - \frac{137be\log\left(-c^2x^2+1\right)}{300c^5} + \frac{be\operatorname{acoth}^2\left(cx\right)}{5c^5} & \text{for}\: c \neq 0 \\ \frac{dx^5\left(a + \frac{i\pi b}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*acoth(c*x)/(5*c**5) + b*d*x**
5*acoth(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*acoth(c*x)/5 - 2*b*e*x**5*aco
th(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*acoth(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*acoth(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*acoth(c*x)**2/
(5*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.63, size = 359, normalized size = 1.14

$$-\frac{1}{10}\,bex^5\log\left(-cx+1\right)^2 - \frac{1}{50}\,(-5i\,\pi bd + 2i\,\pi be - 10\,ad + 4\,ae)x^5 + \frac{(10\,bd - 9\,be)x^4}{200\,c} + \frac{1}{10}\left(bex^5 + \frac{be}{c^5}\right)\log\left(cx+1\right)^2 - \frac{(i\,\pi be + 2\,ae)x^3}{15\,c^2} - \frac{1}{300}\left(6\,(-5i\,\pi be - 5\,bd - 10\,ae + 2\,be)x^5 - \frac{15\,bex^4}{c} + \frac{20\,bex^3}{c^2} - \frac{30\,bex^2}{c^3} + \frac{60\,bex}{c^4}\right)\log\left(cx+1\right) - \frac{1}{300}\left(6\,(-5i\,\pi be + 5\,bd - 10\,ae - 2\,be)x^5 - \frac{15\,bex^4}{c} - \frac{20\,bex^3}{c^2} - \frac{30\,bex^2}{c^3} - \frac{60\,bex}{c^4} - \frac{60\,be\log\left(cx-1\right)}{c^5}\right)\log\left(-cx+1\right) + \frac{(30\,bd - 77\,be)x^2}{300\,c^3} - \frac{be\log\left(cx-1\right)^2}{10\,c^5} - \frac{(i\,\pi be + 2\,ae)x}{5\,c^4} + \frac{(30i\,\pi be + 30\,bd + 60\,ae - 137\,be)\log\left(cx+1\right)}{300\,c^5} + \frac{(-30i\,\pi be + 30\,bd - 60\,ae - 137\,be)\log\left(cx-1\right)}{300\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")

[Out] -1/10*b*e*x^5*log(-c*x + 1)^2 - 1/50*(-5*I*pi*b*d + 2*I*pi*b*e - 10*a*d + 4*a*e)*x^5 + 1/200*(10*b*d - 9*b*e)*x^4/c + 1/10*(b*e*x^5 + b*e/c^5)*log(c*x + 1)^2 - 1/15*(I*pi*b*e + 2*a*e)*x^3/c^2 - 1/300*(6*(-5*I*pi*b*e - 5*b*d - 10*a*e + 2*b*e)*x^5 - 15*b*e*x^4/c + 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 + 60*b*e*x/c^4)*log(c*x + 1) - 1/300*(6*(-5*I*pi*b*e + 5*b*d - 10*a*e - 2*b*e)*x^5 - 15*b*e*x^4/c - 20*b*e*x^3/c^2 - 30*b*e*x^2/c^3 - 60*b*e*x/c^4 - 60*b*e*log(c*x - 1)/c^5)*log(-c*x + 1) + 1/300*(30*b*d - 77*b*e)*x^2/c^3 - 1/10*b*e*log(c*x - 1)^2/c^5 - 1/5*(I*pi*b*e + 2*a*e)*x/c^4 + 1/300*(30*I*pi*b*e + 30*b*d + 60*a*e - 137*b*e)*log(c*x + 1)/c^5 + 1/300*(-30*I*pi*b*e + 30*b*d - 60*a*e - 137*b*e)*log(c*x - 1)/c^5

Mupad [B]
time = 2.34, size = 497, normalized size = 1.58

$$\ln\left(\frac{1}{c\,x}+1\right)\,\left(\frac{b\,d\,x^5}{10}-\frac{\frac{2\,b\,e\,c^5\,x^5}{5}+\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{10\,c^5}+\frac{b\,e\,x^5\,\ln\left(1-c^2\,x^2\right)}{10}\right)+\ln\left(1-\frac{1}{c\,x}\right)\,\left(\frac{\frac{b\,d\,x^6}{5}-\frac{b\,c^2\,d\,x^8}{5}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{4\,b\,e\,x^6}{75}+\frac{4\,b\,e\,x^4}{15\,c^2}-\frac{2\,b\,e\,x^2}{5\,c^4}+\frac{2\,b\,c^2\,e\,x^8}{25}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^6}{5}-\frac{b\,c^2\,e\,x^8}{5}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{10\,c^5}\right)+x^3\,\left(\frac{a\,\left(5\,d-2\,e\right)}{15\,c^2}-\frac{a\,d}{3\,c^2}\right)+x^2\,\left(\frac{b\,\left(10\,d-9\,e\right)}{100\,c^3}-\frac{b\,e}{6\,c^3}\right)+\frac{x\,\left(\frac{a\,\left(5\,d-2\,e\right)}{5\,c^2}-\frac{a\,d}{c^2}\right)}{c^2}+\frac{a\,x^5\,\left(5\,d-2\,e\right)}{25}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^5}{5\,c^2}+\frac{b\,e\,x^4}{20\,c^3}+\frac{b\,e\,x^2}{10\,c^5}\right)-\frac{\ln\left(c\,x-1\right)\,\left(60\,a\,e-30\,b\,d+137\,b\,e\right)}{300\,c^5}+\frac{\ln\left(c\,x+1\right)\,\left(60\,a\,e+30\,b\,d-137\,b\,e\right)}{300\,c^5}+\frac{b\,e\,{\ln\left(\frac{1}{c\,x}+1\right)}^2}{20\,c^5}+\frac{b\,e\,{\ln\left(1-\frac{1}{c\,x}\right)}^2}{20\,c^5}+\frac{b\,e\,{\ln\left(1-c^2\,x^2\right)}^2}{20\,c^5}+\frac{b\,x^4\,\left(10\,d-9\,e\right)}{200\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)

[Out] log(1/(c*x) + 1)*((b*d*x^5)/10 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3 + (2*b*c^5*e*x^5)/5)/(10*c^5) + (b*e*x^5*log(1 - c^2*x^2))/10) + log(1 - 1/(c*x))*(((b*d*x^6)/5 - (b*c^2*d*x^8)/5)/(2*(x + c*x^2)*(c*x - 1)) + ((4*b*e*x^6)/75 + (4*b*e*x^4)/(15*c^2) - (2*b*e*x^2)/(5*c^4) + (2*b*c^2*e*x^8)/25)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^6)/5 - (b*c^2*e*x^8)/5))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(10*c^5)) + x^3*((a*(5*d - 2*e))/(15*c^2) - (a*d)/(3*c^2)) + x^2*((b*(10*d - 9*e))/(100*c^3) - (b*e)/(6*c^3)) + (x*((a*(5*d - 2*e))/(5*c^2) - (a*d)/c^2))/c^2 + (a*x^5*(5*d - 2*e))/25 + c^2*log(1 - c^2*x^2)*((a*e*x^5)/(5*c^2) + (b*e*x^4)/(20*c^3) + (b*e*x^2)/(10*c^5)) - (log(c*x - 1)*(60*a*e - 30*b*d + 137*b*e))/(300*c^5) + (log(c*x + 1)*(60*a*e + 30*b*d - 137*b*e))/(300*c^5) + (b*e*log(1/(c*x) + 1)^2)/(20*c^5) + (b*e*log(1 - 1/(c*x))^2)/(20*c^5) + (b*e*log(1 - c^2*x^2)^2)/(20*c^5) + (b*x^4*(10*d - 9*e))/(200*c)

3.273 $\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=247

$$-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2bex\coth^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\coth^{-1}(cx)+\frac{be\coth^{-1}(cx)^2}{3c^3}-\frac{(2a+b)e\log(1-cx)}{6c^3}+\frac{(2a-$$

```
[Out] -2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*b*e*x*arccoth(c*x)/c^2-2/9*b*
e*x^3*arccoth(c*x)+1/3*b*e*arccoth(c*x)^2/c^3-1/6*(2*a+b)*e*ln(-c*x+1)/c^3+
1/6*(2*a-b)*e*ln(c*x+1)/c^3-4/9*b*e*ln(-c^2*x^2+1)/c^3-1/12*b*e*ln(-c^2*x^2
+1)^2/c^3+1/6*b*x^2*(d+e*ln(-c^2*x^2+1))/c+1/3*x^3*(a+b*arccoth(c*x))*(d+e*
ln(-c^2*x^2+1))+1/6*b*ln(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/c^3
```

Rubi [A]
time = 0.43, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6038, 272, 45, 6233, 6857, 815, 647, 31, 6128, 6022, 266, 6096, 2525, 2437, 2338}

$$-\frac{e(2a+b)\log(1-cx)}{6c^3}+\frac{e(2a-b)\log(cx+1)}{6c^3}+\frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{2aex}{3c^2}-\frac{2}{9}aex^3+\frac{be\coth^{-1}(cx)^2}{3c^3}+\frac{bx^2\left(e\log\left(1-c^2x^2\right)+d\right)}{6c}-\frac{2bex\coth^{-1}(cx)}{3c^2}+\frac{b\log\left(1-c^2x^2\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{6c^3}-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}-\frac{4be\log\left(1-c^2x^2\right)}{9c^3}-\frac{2}{9}bex^3\coth^{-1}(cx)-\frac{5bex^2}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcCoth[
c*x])/(3*c^2) - (2*b*e*x^3*ArcCoth[c*x])/9 + (b*e*ArcCoth[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcCoth[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6233

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int x^2\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+\right.$$

$$= \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3\left(a+b\coth^{-1}(cx)\right)\left(d+\right.$$

$$= -\frac{be\log^2\left(1-c^2x^2\right)}{12c^3} + \frac{bx^2(d+e\log\left(1-c^2x^2\right))}{6c} + \frac{1}{3}x^3($$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2}{9}bex^3\coth^{-1}(cx) - \frac{be\log^2}{1}$$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex\coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\,\mathrm{co}$$

$$= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex\coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\,\mathrm{co}$$

$$= -\frac{2aex}{3c^2} - \frac{5bex^2}{18c} - \frac{2}{9}aex^3 - \frac{2bex\coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3\,\mathrm{c}$$

Mathematica [A]
time = 0.07, size = 183, normalized size = 0.74

$$\frac{-24acex+2bc^2(3d-5e)x^2+4ac^3(3d-2e)x^3+4bcx(3c^2dx^2-2e(3+c^2x^2))\coth^{-1}(cx)+12be\coth^{-1}(cx)^2+2(3bd-6ae-11be)\log(1-cx)+2(3bd+6ae-11be)\log(1+cx)+6c^2ex^2(b+2acx+2bcx\coth^{-1}(cx))\log\left(1-c^2x^2\right)+3be\log^2\left(1-c^2x^2\right)}{36c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x*
(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcCoth[c*x] + 12*b*e*ArcCoth[c*x]^2 + 2*
(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1 +
c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcCoth[c*x])*Log[1 - c^2*x^2] +
3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.94, size = 3514, normalized size = 14.23

method	result	size

risch	Expression too large to display	1459
default	Expression too large to display	3514

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)

[Out] 1/3*a*e*x^3*ln(-c^2*x^2+1)-1/3*a*e/c^3*ln(c*x-1)+1/3*a*e/c^3*ln(c*x+1)-5/18
*b*e*x^2/c-2/9*b*e*x^3*arccoth(c*x)-2/3*b*e*x*arccoth(c*x)/c^2-2/9*a*e*x^3+
1/6*I/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*
csgn(I/(c*x-1)*(c*x+1))+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I/(1/(c*x-1)*(c*
x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/6*I/c^3*b*Pi
*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(
c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/12*I/c^3*b*e*Pi*csgn(I/(1/(c*x-1)*(c*x+
1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-
1)^2)+1/6*I*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*
x+1)-1)^2)^2*arccoth(c*x)*Pi*x^3*e-1/6*I*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(
(c*x-1)/(c*x+1))^(1/2))^2*arccoth(c*x)*Pi*x^3*e+1/6*I*b*csgn(I/(c*x-1)*(c*x
+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*arccoth(c*x)
*Pi*x^3*e-1/3*I*b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I*(1/(c*x-1)*(c*x+
1)-1))*arccoth(c*x)*Pi*x^3*e+1/6*I*b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I
*(1/(c*x-1)*(c*x+1)-1))^2*arccoth(c*x)*Pi*x^3*e+1/3*I*b*csgn(I/(c*x-1)*(c*x
+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*arccoth(c*x)*Pi*x^3*e-2/3*a*e*x/c^2-
1/6/c^3*b*e*(2*arccoth(c*x)*x^3*c^3+c^2*x^2+2*arccoth(c*x)-2*ln(1/(c*x-1)*(
c*x+1)-1)-1)*ln((c*x-1)/(c*x+1))+1/3*a*d*x^3+5/18*e/c^3*b-1/6*I/c*b*csgn(I/
(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*Pi*x^2*e+1/6*I/c^3*b*arccoth(c*x
)*Pi*e*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3-1/6*I/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1
)-1)*e*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3-1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn
(I/(c*x-1)*(c*x+1))^3+1/6*I/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(c*x-
1)*(c*x+1))^3+1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-
1)*(c*x+1)-1)^2)^3-1/6*I/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(c*x-1)*
(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3+1/6*I/c^3*b*Pi*e*csgn(I*(1/(c*x-1)*(c*x+
1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2-1/12*I/c^3*b*e*Pi*csgn(I*(1/(c*x-1
)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)+1/6/c*b*x^2*d+1/3*b*arccoth
(c*x)*x^3*d+1/3/c^3*b*ln(1/(c*x-1)*(c*x+1)-1)^2*e+1/3/c^3*b*arccoth(c*x)*d-
8/9/c^3*b*e*arccoth(c*x)-1/3/c^3*b*d*ln(1/(c*x-1)*(c*x+1)-1)+11/9/c^3*b*e*l
n(1/(c*x-1)*(c*x+1)-1)-1/6*I/c^3*b*e*Pi-1/6*I*b*csgn(I/(c*x-1)*(c*x+1))*csg
n(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2
)*arccoth(c*x)*Pi*x^3*e-1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(
c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x^2*e-1/
6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*
(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)+1/6*I/c^3*b*Pi*ln(
1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+
1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)+1/3/c*b*ln(2)*x^2*e-1/3

```
/c*b*ln(1/(c*x-1)*(c*x+1)-1)*x^2*e+2/3/c^3*b*arccoth(c*x)*ln(2)*e-2/3/c^3*b
*ln(2)*ln(1/(c*x-1)*(c*x+1)-1)*e+2/3*b*arccoth(c*x)*ln(2)*x^3*e-2/3*b*arcco
th(c*x)*ln(1/(c*x-1)*(c*x+1)-1)*x^3*e+1/6*I*b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^
2)^3*arccoth(c*x)*Pi*x^3*e-1/6*I*b*csgn(I/(c*x-1)*(c*x+1))^3*arccoth(c*x)*P
i*x^3*e+1/12*I/c*b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*Pi*x^2*e-1/12*I/c*b*cs
gn(I/(c*x-1)*(c*x+1))^3*Pi*x^2*e+1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-
1)*(c*x+1)-1)^2)^3*Pi*x^2*e-1/3/c^3*b*e*ln(2)-1/6/c^3*b*d-1/6*I/c*b*csgn(I*
(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I*(1/(c*x-1)*(c*x+1)-1))*Pi*x^2*e+1/12*I/c*
b*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*Pi*x^2*e+
1/6*I/c*b*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x^2*
e+1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x
+1)-1)^2)^2*Pi*x^2*e-1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1))*csgn(I/((c*x-1)/(c*
x+1))^(1/2))^2*Pi*x^2*e+1/12*I/c*b*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1
)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*Pi*x^2*e-1/3*I/c^3*b*arccoth(c*x)
*Pi*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2+1/3*I
/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1
/(c*x-1)*(c*x+1)-1)^2)^2+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I*(1/(c*x-1)*(c
*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)-1/6*I/c^3*b*csgn(I*(1/(c*x-1)*(
c*x+1)-1)^2)*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e+1
/3*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I/(c*x-1)
*(c*x+1))^2-1/3*I/c^3*b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/((c*x-1)/(c*x+1
))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I/(c
*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/6*I/c^3*
b*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+
1)/(1/(c*x-1)*(c*x+1)-1)^2)^2-1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I/((c*x-1)
/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))-1/6*I/c^3*b*Pi*e*csgn(I/((c*x-1)
/(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2-1/12*I/c^3*b*e*Pi*csgn(I/(c*x-1)
*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/12*I/c^3*b*Pi
*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))-1/12*I/c^3*b*e
*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x...
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 255, normalized size = 1.03

$$\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)b\,\mathrm{arcoth}\left(cx\right)e+\frac{1}{6}\left(2\,x^3\,\mathrm{arcoth}\left(cx\right)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bd+\frac{1}{9}\left(3\,x^3\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)ae+\frac{((3i\,\pi c^2-5\,c^2)x^2+(3i\,\pi+3\,c^2x^2+6\,\log\left(cx-1\right)-11)\log\left(cx+1\right)+(3i\,\pi+3\,c^2x^2-11)\log\left(cx-1\right))be}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*arccoth(c*x)*e + 1/6*(2*x^3*arcc
oth(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
 - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3
```

Fricas [A]
time = 0.37, size = 304, normalized size = 1.23

$$\frac{12\,ac^3dx^3 + 6\,bc^2dx^2 + 3\,(b\cosh(1) + b\sinh(1))\log\left(-c^2x^2 + 1\right)^2 + 3\,(b\cosh(1) + b\sinh(1))\log\left(\frac{cx+1}{cx-1}\right)^2 - 2\,(4\,ac^3x^3 + 5\,bc^2x^2 + 12\,acx)\cosh(1) + 2\,(3\,bd + (6\,ac^3x^3 + 3\,bc^2x^2 - 11\,b)\cosh(1) + (6\,ac^3x^3 + 3\,bc^2x^2 - 11\,b)\sinh(1))\log\left(-c^2x^2 + 1\right) + 2\,(3\,bc^3dx^3 - 2\,(bc^3x^3 + 3\,bcx - 3\,a)\cosh(1) + 3\,(bc^3x^3\cosh(1) + bc^3x^3\sinh(1))\log\left(-c^2x^2 + 1\right) - 2\,(bc^3x^3 + 3\,bcx - 3\,a)\sinh(1))\log\left(\frac{cx+1}{cx-1}\right) - 2\,(4\,ac^3x^3 + 5\,bc^2x^2 + 12\,acx)\sinh(1)}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] 1/36*(12*a*c^3*d*x^3 + 6*b*c^2*d*x^2 + 3*(b*cosh(1) + b*sinh(1))*log(-c^2*x
^2 + 1)^2 + 3*(b*cosh(1) + b*sinh(1))*log((c*x + 1)/(c*x - 1))^2 - 2*(4*a*c
^3*x^3 + 5*b*c^2*x^2 + 12*a*c*x)*cosh(1) + 2*(3*b*d + (6*a*c^3*x^3 + 3*b*c^
2*x^2 - 11*b)*cosh(1) + (6*a*c^3*x^3 + 3*b*c^2*x^2 - 11*b)*sinh(1))*log(-c^
2*x^2 + 1) + 2*(3*b*c^3*d*x^3 - 2*(b*c^3*x^3 + 3*b*c*x - 3*a)*cosh(1) + 3*(
b*c^3*x^3*cosh(1) + b*c^3*x^3*sinh(1))*log(-c^2*x^2 + 1) - 2*(b*c^3*x^3 + 3
*b*c*x - 3*a)*sinh(1))*log((c*x + 1)/(c*x - 1)) - 2*(4*a*c^3*x^3 + 5*b*c^2*
x^2 + 12*a*c*x)*sinh(1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 265, normalized size = 1.07

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3\log\left(-c^2x^2+1\right)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae\operatorname{acoth}(cx)}{3c^3} + \frac{bdx^3\operatorname{acoth}(cx)}{3} + \frac{bex^3\log\left(-c^2x^2+1\right)\operatorname{acoth}(cx)}{3} - \frac{2bex^3\operatorname{acoth}(cx)}{9} + \frac{bdx^2}{6c} + \frac{bex^2\log\left(-c^2x^2+1\right)}{6c} - \frac{5bex^2}{18c} - \frac{2bex\operatorname{acoth}(cx)}{3c^2} + \frac{bd\log\left(-c^2x^2+1\right)}{6c^3} + \frac{be\log\left(-c^2x^2+1\right)^2}{12c^3} - \frac{11be\log\left(-c^2x^2+1\right)}{18c^3} + \frac{be\operatorname{acoth}^2(cx)}{3c^3} & \text{for}\: c \neq 0 \\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*acoth
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)**2/(3
*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.51, size = 282, normalized size = 1.14

$$-\frac{1}{6}\,bex^3\log\left(-cx + 1\right)^2 - \frac{1}{18}\,(-3i\,\pi bd + 2i\,\pi be - 6\,ad + 4\,ae)x^3 + \frac{1}{6}\left(bex^3 + \frac{be}{c^3}\right)\log\left(cx + 1\right)^2 + \frac{(3\,bd - 5\,be)x^2}{18\,c} - \frac{1}{18}\left((-3i\,\pi be - 3\,bd - 6\,ae + 2\,be)x^3 - \frac{3\,bex^2}{c} + \frac{6\,bex}{c^2}\right)\log\left(cx + 1\right) - \frac{1}{18}\left((-3i\,\pi be + 3\,bd - 6\,ae - 2\,be)x^3 - \frac{3\,bex^2}{c} - \frac{6\,bex}{c^2} - \frac{6\,be\log\left(cx - 1\right)}{c^3}\right)\log\left(-cx + 1\right) - \frac{be\log\left(cx - 1\right)^2}{6\,c^3} - \frac{(i\,\pi be + 2\,ae)x}{3\,c^2} + \frac{(3i\,\pi be + 3\,bd + 6\,ae - 11\,be)\log\left(cx + 1\right)}{18\,c^3} + \frac{(-3i\,\pi be + 3\,bd - 6\,ae - 11\,be)\log\left(cx - 1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/6*b*e*x^3*log(-c*x + 1)^2 - 1/18*(-3*I*pi*b*d + 2*I*pi*b*e - 6*a*d + 4*a
*e)*x^3 + 1/6*(b*e*x^3 + b*e/c^3)*log(c*x + 1)^2 + 1/18*(3*b*d - 5*b*e)*x^2
/c - 1/18*((-3*I*pi*b*e - 3*b*d - 6*a*e + 2*b*e)*x^3 - 3*b*e*x^2/c + 6*b*e*
```

```
x/c^2)*log(c*x + 1) - 1/18*((-3*I*pi*b*e + 3*b*d - 6*a*e - 2*b*e)*x^3 - 3*b
*e*x^2/c - 6*b*e*x/c^2 - 6*b*e*log(c*x - 1)/c^3)*log(-c*x + 1) - 1/6*b*e*lo
g(c*x - 1)^2/c^3 - 1/3*(I*pi*b*e + 2*a*e)*x/c^2 + 1/18*(3*I*pi*b*e + 3*b*d
+ 6*a*e - 11*b*e)*log(c*x + 1)/c^3 + 1/18*(-3*I*pi*b*e + 3*b*d - 6*a*e - 11
*b*e)*log(c*x - 1)/c^3
```

Mupad [B]
time = 2.33, size = 414, normalized size = 1.68

$$\ln\left(\frac{1}{c\,x}+1\right)\,\left(\frac{b\,d\,x^3}{6}-\frac{\frac{2\,b\,e\,c^3\,x^3}{3}+2\,b\,e\,c\,x}{6\,c^3}+\frac{b\,e\,x^3\,\ln\left(1-c^2\,x^2\right)}{6}\right)+x\,\left(\frac{a\,\left(3\,d-2\,e\right)}{3\,c^2}-\frac{a\,d}{c^2}\right)+\ln\left(1-\frac{1}{c\,x}\right)\,\left(\frac{\frac{4\,b\,e\,x^4}{9}-\frac{2\,b\,e\,x^2}{3\,c^2}+\frac{2\,b\,c^2\,e\,x^6}{9}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\frac{b\,d\,x^4}{3}-\frac{b\,c^2\,d\,x^6}{3}}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(\frac{b\,e\,x^4}{3}-\frac{b\,c^2\,e\,x^6}{3}\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{6\,c^3}\right)+\frac{a\,x^3\,\left(3\,d-2\,e\right)}{9}+c^2\,\ln\left(1-c^2\,x^2\right)\,\left(\frac{a\,e\,x^3}{3\,c^2}+\frac{b\,e\,x^2}{6\,c^3}\right)-\frac{\ln\left(c\,x-1\right)\,\left(6\,a\,e-3\,b\,d+11\,b\,e\right)}{18\,c^3}+\frac{\ln\left(c\,x+1\right)\,\left(6\,a\,e+3\,b\,d-11\,b\,e\right)}{18\,c^3}+\frac{b\,e\,{\ln\left(\frac{1}{c\,x}+1\right)}^2}{12\,c^3}+\frac{b\,e\,{\ln\left(1-\frac{1}{c\,x}\right)}^2}{12\,c^3}+\frac{b\,e\,{\ln\left(1-c^2\,x^2\right)}^2}{12\,c^3}+\frac{b\,x^2\,\left(3\,d-5\,e\right)}{18\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1/(c*x) + 1)*((b*d*x^3)/6 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3)/(6*c^3) + (
b*e*x^3*log(1 - c^2*x^2))/6) + x*((a*(3*d - 2*e))/(3*c^2) - (a*d)/c^2) + lo
g(1 - 1/(c*x))*(((4*b*e*x^4)/9 - (2*b*e*x^2)/(3*c^2) + (2*b*c^2*e*x^6)/9)/(
2*(x + c*x^2)*(c*x - 1)) + ((b*d*x^4)/3 - (b*c^2*d*x^6)/3)/(2*(x + c*x^2)*(
c*x - 1)) + (log(1 - c^2*x^2)*((b*e*x^4)/3 - (b*c^2*e*x^6)/3))/(2*(x + c*x^
2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(6*c^3)) + (a*x^3*(3*d - 2*e))/9 + c
^2*log(1 - c^2*x^2)*((a*e*x^3)/(3*c^2) + (b*e*x^2)/(6*c^3)) - (log(c*x - 1)
*(6*a*e - 3*b*d + 11*b*e))/(18*c^3) + (log(c*x + 1)*(6*a*e + 3*b*d - 11*b*e
))/(18*c^3) + (b*e*log(1/(c*x) + 1)^2)/(12*c^3) + (b*e*log(1 - 1/(c*x))^2)/
(12*c^3) + (b*e*log(1 - c^2*x^2)^2)/(12*c^3) + (b*x^2*(3*d - 5*e))/(18*c)
```

3.274 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=104

$$-2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c}+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)$$

```
[Out] -2*a*e*x-2*b*e*x*arccoth(c*x)+e*(a+b*arccoth(c*x))^2/b/c-b*e*ln(-c^2*x^2+1)
/c+x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))+1/4*b*(d+e*ln(-c^2*x^2+1))^2/c
/e
```

Rubi [A]

time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6221, 2525, 2437, 2338, 6128, 6022, 266, 6096}

$$x\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-2aex+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be\log\left(1-c^2x^2\right)}{c}-2bex\coth^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (e*(a + b*ArcCoth[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6221

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[
2*e*g, Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx = x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-(bc)\int \frac{x(d+$$

$$= x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{1}{2}(bc)\text{Subst}\Big($$

$$= -2aex+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}+x\left(a+b\coth^{-1}(cx)\right)(d+$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}+x(a+$$

$$= -2aex-2bex\coth^{-1}(cx)+\frac{e\left(a+b\coth^{-1}(cx)\right)^2}{bc}-\frac{be\log$$

Mathematica [A]
time = 0.02, size = 144, normalized size = 1.38

$$adx-2aex+bdx\coth^{-1}(cx)-2bex\coth^{-1}(cx)+\frac{be\coth^{-1}(cx)^2}{c}+\frac{2ae\tanh^{-1}(cx)}{c}+\frac{bd\log\left(1-c^2x^2\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}+aex\log\left(1-c^2x^2\right)+bex\coth^{-1}(cx)\log\left(1-c^2x^2\right)+\frac{be\log^2\left(1-c^2x^2\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] - 2*b*e*x*ArcCoth[c*x] + (b*e*ArcCoth[
c*x]^2)/c + (2*a*e*ArcTanh[c*x])/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*Lo
g[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcCoth[c*x]*Log[1 - c^2
*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.01, size = 2210, normalized size = 21.25

method	result	size
risch	Expression too large to display	1104
default	Expression too large to display	2210

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] b*arccoth(c*x)*d*x+b/c*ln(1/(c*x-1)*(c*x+1)-1)^2*e+b/c*arccoth(c*x)*d-2*b/c
*e*arccoth(c*x)-b/c*d*ln(1/(c*x-1)*(c*x+1)-1)+2*b/c*e*ln(1/(c*x-1)*(c*x+1)-
1)-2*b*ln(1/(c*x-1)*(c*x+1)-1)*arccoth(c*x)*e*x+2*b*arccoth(c*x)*e*ln(2)*x+
a*d*x-2*a*e*x-2*b*e*x*arccoth(c*x)+I*b/c*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*
(c*x+1)-1)^2)^2*e*ln(1/(c*x-1)*(c*x+1)-1)*Pi-1/2*I*b/c*arccoth(c*x)*e*Pi*cs
```

```
gn(I/(c*x-1)*(c*x+1))^3+1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/
(1/(c*x-1)*(c*x+1)-1)^2)^3+1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c
*x+1)-1)^2)^3+1/2*I*b/c*csgn(I/(c*x-1)*(c*x+1))^3*e*ln(1/(c*x-1)*(c*x+1)-1)
*Pi-1/2*I*b/c*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^3*e*ln(1/(c*x
-1)*(c*x+1)-1)*Pi-1/2*I*b/c*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*e*ln(1/(c*x-1
)*(c*x+1)-1)*Pi-I*b/c*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(
c*x+1)-1)^2)^2-ln((c*x-1)/(c*x+1))*(arccoth(c*x)*x*c+arccoth(c*x)-ln(1/(c*x
-1)*(c*x+1)-1))*b*e/c+1/2*I*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1))*csg
n(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x-1/2*I*b*arccoth(c*x)*e*Pi*
csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I/(c*x-1)*(c*x+1))*x+1/2*I*b*arccoth
(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)*
x-I*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*
x+1)-1)^2)^2*x-1/2*I*b/c*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I/(c*x-1)*(c*x+1
))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/2*I*b/c*csgn(I/(c*x-
1)*(c*x+1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*e*ln(1/(c*x-1)*(c*x+1)-1)*Pi-
1/2*I*b/c*Pi*ln(1/(c*x-1)*(c*x+1)-1)*e*csgn(I*(1/(c*x-1)*(c*x+1)-1))^2*csgn
(I*(1/(c*x-1)*(c*x+1)-1)^2)+1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(
c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2+1/2*I*b/c*ar
ccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(
c*x+1)-1)^2)^2-1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^
2*csgn(I/(c*x-1)*(c*x+1))+1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*
x+1)-1))^2*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)-I*b/c*arccoth(c*x)*e*Pi*csgn(I*(
1/(c*x-1)*(c*x+1)-1))*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^2-1/2*I*b/c*csgn(I/(c
*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*e*
ln(1/(c*x-1)*(c*x+1)-1)*Pi-I*b/c*csgn(I/(c*x-1)*(c*x+1))^2*csgn(I/((c*x-1)/
(c*x+1))^(1/2))*e*ln(1/(c*x-1)*(c*x+1)-1)*Pi+I*b*arccoth(c*x)*e*Pi*csgn(I/(
(c*x-1)/(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*x+1))^2*x+I*b/c*csgn(I*(1/(c*x-1)
*(c*x+1)-1)^2)^2*csgn(I*(1/(c*x-1)*(c*x+1)-1))*e*ln(1/(c*x-1)*(c*x+1)-1)*Pi
+I*b/c*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I/(c*x-1)*(c*
x+1))^2+1/2*I*b*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c
*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x-a*e/c*ln(c*x-1)+a*e/c*ln(c*x+1)-
1/2*I*b/c*arccoth(c*x)*e*Pi*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*
(c*x+1))*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)+1/2*I*b/c*csgn(I/(
c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(1/(
c*x-1)*(c*x+1)-1)^2)*e*ln(1/(c*x-1)*(c*x+1)-1)*Pi-1/2*I*b*arccoth(c*x)*e*Pi
*csgn(I/(1/(c*x-1)*(c*x+1)-1)^2)*csgn(I/(c*x-1)*(c*x+1))*csgn(I/(c*x-1)*(c*
x+1)/(1/(c*x-1)*(c*x+1)-1)^2)*x-1/2*I*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c
*x+1))^3*x+1/2*I*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x
+1)-1)^2)^3*x+1/2*I*b*arccoth(c*x)*e*Pi*csgn(I*(1/(c*x-1)*(c*x+1)-1)^2)^3*x
-I*b*arccoth(c*x)*e*Pi*csgn(I/(c*x-1)*(c*x+1)/(1/(c*x-1)*(c*x+1)-1)^2)^2*x+
a*e*x*ln(-c^2*x^2+1)+2*b/c*arccoth(c*x)*e*ln(2)-2*b/c*ln(2)*ln(1/(c*x-1)*(c
*x+1)-1)*e+I*b*Pi*e*arccoth(c*x)*x+I*b/c*Pi*e*arccoth(c*x)-I*b/c*e*ln(1/(c*
x-1)*(c*x+1)-1)*Pi
```

Maxima [C] Result contains complex when optimal does not.

time = 0.26, size = 181, normalized size = 1.74

$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)b\,\mathrm{arcoth}\,(cx)\,e+adx-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)ae+\frac{(2\,cx\,\mathrm{arcoth}\,(cx)+\log(-c^2x^2+1))bd}{2\,c}+\frac{((i\,\pi+2\,\log(cx-1)-2)\log(cx+1)+(i\,\pi-2)\log(cx-1))be}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1)
)*b*arccoth(c*x)*e + a*d*x - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1
)/c^3) - x*log(-c^2*x^2 + 1))*a*e + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x^2
+ 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2)*lo
g(c*x - 1))*b*e/c
```

Fricas [A]
time = 0.34, size = 187, normalized size = 1.80

$$\frac{4\,acdx-8\,acx\cosh(1)-8\,acx\sinh(1)+(b\cosh(1)+b\sinh(1))\log\left(-c^2x^2+1\right)^2+(b\cosh(1)+b\sinh(1))\log\left(\frac{cx+1}{cx-1}\right)^2+2\,(bd+2\,(acx-b)\cosh(1)+2\,(acx-b)\sinh(1))\log\left(-c^2x^2+1\right)+2\,(bcdx-2\,(bcx-a)\cosh(1)+(bcx\cosh(1)+bcx\sinh(1))\log\left(-c^2x^2+1\right)-2\,(bcx-a)\sinh(1))\log\left(\frac{cx+1}{cx-1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*c*d*x - 8*a*c*x*cosh(1) - 8*a*c*x*sinh(1) + (b*cosh(1) + b*sinh(1)
)*log(-c^2*x^2 + 1)^2 + (b*cosh(1) + b*sinh(1))*log((c*x + 1)/(c*x - 1))^2
+ 2*(b*d + 2*(a*c*x - b)*cosh(1) + 2*(a*c*x - b)*sinh(1))*log(-c^2*x^2 + 1)
 + 2*(b*c*d*x - 2*(b*c*x - a)*cosh(1) + (b*c*x*cosh(1) + b*c*x*sinh(1))*log
(-c^2*x^2 + 1) - 2*(b*c*x - a)*sinh(1))*log((c*x + 1)/(c*x - 1)))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 155, normalized size = 1.49

$$\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\,\mathrm{acoth}\,(cx)}{c}+bdx\,\mathrm{acoth}\,(cx)+bex\log\left(-c^2x^2+1\right)\mathrm{acoth}\,(cx)-2bex\,\mathrm{acoth}\,(cx)+\frac{bd\log(-c^2x^2+1)}{2c}+\frac{be\log(-c^2x^2+1)^2}{4c}-\frac{be\log(-c^2x^2+1)}{c}+\frac{be\,\mathrm{acoth}^2\,(cx)}{c} & \text{for}\ c\neq 0\\ dx\left(a+\frac{i\pi b}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/c
 + b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acoth(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*pi*b
/2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.47, size = 198, normalized size = 1.90

$$-\frac{1}{2}\,bex\log\left(-cx+1\right)^2-\frac{1}{2}\,(-i\,\pi be-bd-2\,ae+2\,be)x\log(cx+1)+\frac{1}{2}\left(bex+\frac{be}{c}\right)\log(cx+1)^2-\frac{be\log(cx-1)^2}{2\,c}-\frac{1}{2}\,(-i\,\pi bd+2i\,\pi be-2\,ad+4\,ae)x-\frac{1}{2}\left((-i\,\pi be+bd-2\,ae-2\,be)x-\frac{2\,be\log(cx-1)}{c}\right)\log(-cx+1)+\frac{(i\,\pi be+bd+2\,ae-2\,be)\log(cx+1)}{2\,c}+\frac{(-i\,\pi be+bd-2\,ae-2\,be)\log(cx-1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] -1/2*b*e*x*log(-c*x + 1)^2 - 1/2*(-I*pi*b*e - b*d - 2*a*e + 2*b*e)*x*log(c*
x + 1) + 1/2*(b*e*x + b*e/c)*log(c*x + 1)^2 - 1/2*b*e*log(c*x - 1)^2/c - 1/
2*(-I*pi*b*d + 2*I*pi*b*e - 2*a*d + 4*a*e)*x - 1/2*((-I*pi*b*e + b*d - 2*a*
e - 2*b*e)*x - 2*b*e*log(c*x - 1)/c)*log(-c*x + 1) + 1/2*(I*pi*b*e + b*d +
2*a*e - 2*b*e)*log(c*x + 1)/c + 1/2*(-I*pi*b*e + b*d - 2*a*e - 2*b*e)*log(c
*x - 1)/c
```

Mupad [B]
time = 2.09, size = 315, normalized size = 3.03

$$\ln\left(\frac{1}{c\,x}+1\right)\,\left(\frac{b\,d\,x}{2}-b\,e\,x+\frac{b\,e\,x\,\ln\left(1-c^2\,x^2\right)}{2}\right)+\ln\left(1-\frac{1}{c\,x}\right)\,\left(\frac{b\,d\,x^2-b\,c^2\,d\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{2\,b\,e\,x^2-2\,b\,c^2\,e\,x^4}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}+\frac{\ln\left(1-c^2\,x^2\right)\,\left(b\,e\,x^2-b\,c^2\,e\,x^4\right)}{2\,\left(c\,x^2+x\right)\,\left(c\,x-1\right)}-\frac{b\,e\,\ln\left(\frac{1}{c\,x}+1\right)}{2\,c}\right)+a\,x\,\left(d-2\,e\right)+\frac{\ln\left(c\,x+1\right)\,\left(2\,a\,e+b\,d-2\,b\,e\right)}{2\,c}-\frac{\ln\left(c\,x-1\right)\,\left(2\,a\,e-b\,d+2\,b\,e\right)}{2\,c}+\frac{b\,e\ln\left(\frac{1}{c\,x}+1\right)^2}{4\,c}+\frac{b\,e\ln\left(1-\frac{1}{c\,x}\right)^2}{4\,c}+\frac{b\,e\ln\left(1-c^2\,x^2\right)^2}{4\,c}+a\,e\,x\,\ln\left(1-c^2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)
```

```
[Out] log(1/(c*x) + 1)*((b*d*x)/2 - b*e*x + (b*e*x*log(1 - c^2*x^2))/2) + log(1 -
 1/(c*x))*((b*d*x^2 - b*c^2*d*x^4)/(2*(x + c*x^2)*(c*x - 1)) - (2*b*e*x^2 -
 2*b*c^2*e*x^4)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*(b*e*x^2 - b*
c^2*e*x^4))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(2*c)) + a*x
*(d - 2*e) + (log(c*x + 1)*(2*a*e + b*d - 2*b*e))/(2*c) - (log(c*x - 1)*(2*
a*e - b*d + 2*b*e))/(2*c) + (b*e*log(1/(c*x) + 1)^2)/(4*c) + (b*e*log(1 - 1
/(c*x))^2)/(4*c) + (b*e*log(1 - c^2*x^2)^2)/(4*c) + a*e*x*log(1 - c^2*x^2)
```

3.275 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}+\frac{1}{2}bc\left(d+e\log\left(1-c^2x^2\right)\right)\log\left(1-\frac{1}{1-c\right.$$

```
[Out] -c*e*(a+b*arccoth(c*x))^2/b-(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x+1/2*b
*c*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(-c^2*x^
2+1))
```

Rubi [A]

time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6229, 2525, 2458, 2379, 2438, 6096}

$$-\frac{\left(a+b\coth^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{1-c^2x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{1}{2}bce\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -((c*e*(a + b*ArcCoth[c*x])^2)/b) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^
2*x^2]))/x + (b*c*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/2 -
 (b*c*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/2
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6229

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x^2}\,dx = -\frac{(a+b\coth^{-1}(cx))\,(d+e\log(1-c^2x^2))}{x} + (bc)\int \frac{d+e}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} - \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} - \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} - \frac{(a+b\coth^{-1}(cx))\,(d+e\log(1}{x}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{(a+b\coth^{-1}(cx)}{}$$

$$= -\frac{ce\left(a+b\coth^{-1}(cx)\right)^2}{b} + bcd\log(x) - \frac{(a+b\coth^{-1}(cx)}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

time = 0.13, size = 332, normalized size = 3.16

$$-\frac{4ad+4bd\coth^{-1}(cx)+4bcex\coth^{-1}(cx)^2+8acex\tanh^{-1}(cx)-4bcdx\log(x)-bcex\log^2\left(-\frac{1}{c}+x\right)-bcex\log^2\left(\frac{1}{c}+x\right)-2bcex\log\left(\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1-cx)\right)+4bcex\log(x)\log(1-cx)-2bcex\log\left(-\frac{1}{c}+x\right)\log\left(\frac{1}{2}(1+cx)\right)+4bcex\log(x)\log(1+cx)+4ae\log\left(1-c^2x^2\right)+2bcdx\log\left(1-c^2x^2\right)+4be\coth^{-1}(cx)\log\left(1-c^2x^2\right)-4bcex\log(x)\log\left(1-c^2x^2\right)+2bcex\log\left(-\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)+2bcex\log\left(\frac{1}{c}+x\right)\log\left(1-c^2x^2\right)+4bcex\mathrm{PolyLog}(2,-cx)+4bcex\mathrm{PolyLog}(2,cx)-2bcex\mathrm{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)-2bcex\mathrm{PolyLog}\left(2,\frac{1}{2}(1+cx)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -1/4*(4*a*d + 4*b*d*ArcCoth[c*x] + 4*b*c*e*x*ArcCoth[c*x]^2 + 8*a*c*e*x*Arc
Tanh[c*x] - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c^(
-1) + x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*
Log[1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[
x]*Log[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b
*e*ArcCoth[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*
c*e*x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*Log[1 -
 c^2*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*
c*e*x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/x
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccoth}(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d - (c^2*(log(c
*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e - 1/2*b*(log(c*x + 1
)^2/x - integrate(-((c*x + 1)*log(c*x - 1)^2 - (I*pi + (I*pi*c + 2*c)*x)*lo
g(c*x + 1) - (-I*pi - I*pi*c*x)*log(c*x - 1))/(c*x^3 + x^2), x))*e - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e\log(-c^2x^2 + 1))}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acoth}(c\,x))\,(d + e\,\ln(1 - c^2\,x^2))}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)
```

3.276 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

Optimal. Leaf size=197

$$\frac{2c^2e(a+b\coth^{-1}(cx))}{3x}-\frac{c^3e(a+b\coth^{-1}(cx))^2}{3b}-bc^3e\log(x)+\frac{1}{3}bc^3e\log\left(1-c^2x^2\right)-\frac{bc(1-c^2x^2)\left(d+e\log\left(\right.\right.}{6x^2}$$

```
[Out] 2/3*c^2*e*(a+b*arccoth(c*x))/x-1/3*c^3*e*(a+b*arccoth(c*x))^2/b-b*c^3*e*ln(
x)+1/3*b*c^3*e*ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*ln(-c^2*x^2+1))/x^2
-1/3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*ln(-c^2*x^2
+1))*ln(1-1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))
```

Rubi [A]
time = 0.29, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$, Rules used = {6229, 2525, 2458, 2389, 2379, 2438, 2351, 31, 6130, 6038, 272, 36, 29, 6096}

$$-\frac{c^3e(a+b\coth^{-1}(cx))^2}{3b}-\frac{(a+b\coth^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}+\frac{2c^2e(a+b\coth^{-1}(cx))}{3x}-bc^3e\log(x)-\frac{bc(1-c^2x^2)\left(e\log\left(1-c^2x^2\right)+d\right)}{6x^2}+\frac{1}{6}bc^3\log\left(1-\frac{1}{1-c^2x^2}\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{1}{6}bc^3e\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)+\frac{1}{3}bc^3e\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] (2*c^2*e*(a + b*ArcCoth[c*x]))/(3*x) - (c^3*e*(a + b*ArcCoth[c*x])^2)/(3*b)
 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*c*(1 - c^2*x^2)*(d +
e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2
]))/(3*x^3) + (b*c^3*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/
6 - (b*c^3*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6229

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int \frac{d+}{}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\Big($$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{(a+}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{(a+}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}-\frac{bc(1}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\coth^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 457 vs. 2(197) = 394.
time = 0.23, size = 457, normalized size = 2.32

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - (2*b*d*ArcCoth[c*x])/x^3 + (4
*b*c^2*e*ArcCoth[c*x])/x - 2*b*c^3*e*ArcCoth[c*x]^2 - 4*a*c^3*e*ArcTanh[c*x
] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*e*L
og[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2 + b*
c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x] + b*
c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*x] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Maple [F]
time = 2.86, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccoth}(cx))\,(d+e\ln(-c^2x^2+1))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arccoth(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e - 1/6*b*(log(c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x + 1)*log(c
*x - 1)^2 - (3*I*pi + (3*I*pi*c + 2*c)*x)*log(c*x + 1) + 3*(I*pi + I*pi*c*x
)*log(c*x - 1))/(c*x^5 + x^4), x))*e - 1/3*a*d/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acoth}(cx))\,(d+e\log(-c^2x^2+1))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac")
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acoth}(c\,x))\,\left(d + e\,\ln\left(1 - c^2\,x^2\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4,x)
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^4, x)
```

3.277 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal. Leaf size=256

$$\frac{7bc^3e}{60x^2}+\frac{2c^2e(a+b\coth^{-1}(cx))}{15x^3}+\frac{2c^4e(a+b\coth^{-1}(cx))}{5x}-\frac{c^5e(a+b\coth^{-1}(cx))^2}{5b}-\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log($$

```
[Out] 7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*arccoth(c*x))/x^3+2/5*c^4*e*(a+b*arccoth(c
*x))/x-1/5*c^5*e*(a+b*arccoth(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(
-c^2*x^2+1)-1/20*b*c*(d+e*ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d+e*
ln(-c^2*x^2+1))/x^2-1/5*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^5+1/10*b*
c^5*(d+e*ln(-c^2*x^2+1))*ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(-c^
2*x^2+1))
```

Rubi [A]
time = 0.43, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {6229, 2525, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46, 6130, 6038, 272, 36, 29, 6096}

$$-\frac{c^5e(a+b\coth^{-1}(cx))^2}{5b}+\frac{2c^4e(a+b\coth^{-1}(cx))}{5x}-\frac{(a+b\coth^{-1}(cx))(e\log(1-c^2x^2)+d)}{5x^5}+\frac{2c^2e(a+b\coth^{-1}(cx))}{15x^3}-\frac{5}{6}bc^5e\log(x)+\frac{7bc^3e}{60x^2}-\frac{bc(e\log(1-c^2x^2)+d)}{20x^4}+\frac{1}{10}bc^5\log\left(1-\frac{1}{1-c^2x^2}\right)(e\log(1-c^2x^2)+d)-\frac{1}{10}bc^5e\text{Li}_2\left(\frac{1}{1-c^2x^2}\right)+\frac{19}{60}bc^5e\log(1-c^2x^2)-\frac{bc^3(1-c^2x^2)(e\log(1-c^2x^2)+d)}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcCoth[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcCoth[c*x]))/(5*x) - (c^5*e*(a + b*ArcCoth[c*x])^2)/(5*b) - (5*b*c^
5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*c*(d + e*Log[1 - c^2*
x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(10*x^2) -
 ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Lo
g[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 -
c^2*x^2)^(-1)])/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6130

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcCoth[c*x])^p/(d + e*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6229

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
```

```
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{5}(bc)\int \frac{d+}{}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}+\frac{1}{10}(bc)\text{Subst}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}-\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log(1\right.}{5x^5}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e(a}{}$$

$$= \frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}-\frac{c^5e(a}{}$$

$$= \frac{bc^3e}{15x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}$$

$$= \frac{7bc^3e}{60x^2}+\frac{2c^2e\left(a+b\coth^{-1}(cx)\right)}{15x^3}+\frac{2c^4e\left(a+b\coth^{-1}(cx)\right)}{5x}$$

Mathematica [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Maple [F]
time = 5.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccoth}(cx)\right)\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxima
")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arccoth(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) -
2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e - 1/10*b*(log(c*x
 + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x + 1)*log(c*x - 1)^2 - (5*I*pi + (5*I
*pi*c + 2*c)*x)*log(c*x + 1) + 5*(I*pi + I*pi*c*x)*log(c*x - 1))/(c*x^7 + x
^6), x))*e - 1/5*a*d/x^5
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(-c^2*x^2 +
1))/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acoth}(cx))\,(d + e\log{(-c^2x^2 + 1)})}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(1-c^2\,x^2\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^6, x)
```

3.278 $\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=512

$$\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh}{2c^2}$$

```
[Out] 1/2*b*(d-e)*x/c-b*e*x/c+1/2*d*x^2*(a+b*arccoth(c*x))-1/2*e*x^2*(a+b*arccoth
(c*x))-1/2*b*(d-e)*arctanh(c*x)/c^2-b*e*(c^2*f+g)*arctanh(c*x)*ln(2/(c*x+1)
)/c^2/g+1/2*b*e*x*ln(g*x^2+f)/c+1/2*e*(g*x^2+f)*(a+b*arccoth(c*x))*ln(g*x^2
+f)/g-1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(g*x^2+f)/c^2/g+1/2*b*e*(c^2*f+g)*ar
ctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/c^
2/g+1/2*b*e*(c^2*f+g)*arctanh(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c
*(-f)^(1/2)+g^(1/2)))/c^2/g+1/2*b*e*(c^2*f+g)*polylog(2,1-2/(c*x+1))/c^2/g-
1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1
/2)-g^(1/2)))/c^2/g-1/4*b*e*(c^2*f+g)*polylog(2,1-2*c*((-f)^(1/2)+x*g^(1/2)
)/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/c^2/g+b*e*arctan(x*g^(1/2)/f^(1/2))*f^(1/
2)/c/g^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {2504, 2436, 2332, 6231, 327, 213, 531, 2608, 2498, 211, 2520, 12, 6139, 6057, 2449, 2352, 2497}

$$\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)\left(a+b\coth^{-1}(cx)\right)}{2g}-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\left(c^2f+g\right)\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{Li}_2\left(1-\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{\left(c\sqrt{-f}-\sqrt{g}\right)(cx+1)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{Li}_2\left(1-\frac{2c\left(\sqrt{g}\,x+\sqrt{-f}\right)}{\left(\sqrt{-f}\,c+\sqrt{g}\right)(cx+1)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(f+gx^2\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\log\left(\frac{2}{cx+1}\right)\tanh^{-1}(cx)}{c^2g}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(\frac{2c\left(\sqrt{-f}-\sqrt{g}\,x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{2c^2g}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(\frac{2c\left(\sqrt{-f}+\sqrt{g}\,x\right)}{(cx+1)\left(c\sqrt{-f}+\sqrt{g}\right)}\right)}{2c^2g}+\frac{bx(d-e)}{2c}+\frac{bex\log\left(f+gx^2\right)}{2c}-\frac{bex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*ArcCoth[c*x]))/2 - (e*x^2*(
a + b*ArcCoth[c*x]))/2 + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(c*Sqrt[
g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))]])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) + (e*(f + g*x^2)*(a + b
*ArcCoth[c*x])*Log[f + g*x^2])/(2*g) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[f
+ g*x^2])/(2*c^2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))]])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]])/(4*c^2*g)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
```

```
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6231

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps

$$\int x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{e(f}{}$$

$$=\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}(cx)\right)+\frac{e(f}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}\right.$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}\right.$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}\right.$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\coth^{-1}\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{1}{2}dx^2\left(a+b\coth^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\right.$$

Mathematica [A]

time = 3.67, size = 677, normalized size = 1.32

$$\frac{2bcdgx - 6bcegx + 2ac^2dgx^2 - 2ac^2egx^2 - 2bdg\coth^{-1}(cx) + 2beg\coth^{-1}(cx) + 2bc^2dgx^2\coth^{-1}(cx) - 2bc^2egx^2\coth^{-1}(cx) - 4bc^2ef\coth^{-1}(cx)^2 - 4beg\coth^{-1}(cx)^2 + 4bce\sqrt{f}\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right) - 4bc^2ef\coth^{-1}(cx)\log\left(1-e^{-2\coth^{-1}(cx)}\right) - 4beg\coth^{-1}(cx)\log\left(1-e^{-2\coth^{-1}(cx)}\right) + 2bc^2ef\coth^{-1}(cx)\log\left(1+\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{-c^2f-2c\sqrt{-f}\sqrt{g}+g}\right) + 2beg\coth^{-1}(cx)\log\left(1+\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{-c^2f-2c\sqrt{-f}\sqrt{g}+g}\right) + 2bc^2ef\coth^{-1}(cx)\log\left(1+\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{-c^2f+2c\sqrt{-f}\sqrt{g}+g}\right) + 2beg\coth^{-1}(cx)\log\left(1+\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{-c^2f+2c\sqrt{-f}\sqrt{g}+g}\right) + 2ac^2ef\log(f+gx^2) + 2bcegx\log(f+gx^2) + 2ac^2egx^2\log(f+gx^2) - 2beg\coth^{-1}(cx)\log(f+gx^2) + 2bc^2egx^2\coth^{-1}(cx)\log(f+gx^2) + 2be(c^2f+g)\,\text{PolyLog}\left(2,e^{-2\coth^{-1}(cx)}\right) + be(c^2f+g)\,\text{PolyLog}\left(2,\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{c^2f-2c\sqrt{-f}\sqrt{g}-g}\right) + bc^2ef\,\text{PolyLog}\left(2,\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{c^2f+2c\sqrt{-f}\sqrt{g}-g}\right) + beg\,\text{PolyLog}\left(2,\frac{e^{2\coth^{-1}(cx)}(c^2f+g)}{c^2f+2c\sqrt{-f}\sqrt{g}-g}\right)}{4c^2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (2*b*c*d*g*x - 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 - 2*b*d*g*Ar
cCoth[c*x] + 2*b*e*g*ArcCoth[c*x] + 2*b*c^2*d*g*x^2*ArcCoth[c*x] - 2*b*c^2*
e*g*x^2*ArcCoth[c*x] - 4*b*c^2*e*f*ArcCoth[c*x]^2 - 4*b*e*g*ArcCoth[c*x]^2
+ 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - 4*b*c^2*e*f*ArcCoth
[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] - 4*b*e*g*ArcCoth[c*x]*Log[1 - E^(-2*Arc
Coth[c*x])] + 2*b*c^2*e*f*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f +
 g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*b*e*g*ArcCoth[c*x]*Log[1 +
(E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2
*b*c^2*e*f*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f)
+ 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*b*e*g*ArcCoth[c*x]*Log[1 + (E^(2*ArcCoth[c
*x])*(c^2*f + g))/(-(c^2*f) + 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*a*c^2*e*f*Log[
f + g*x^2] + 2*b*c*e*g*x*Log[f + g*x^2] + 2*a*c^2*e*g*x^2*Log[f + g*x^2] -
2*b*e*g*ArcCoth[c*x]*Log[f + g*x^2] + 2*b*c^2*e*g*x^2*ArcCoth[c*x]*Log[f +
g*x^2] + 2*b*e*(c^2*f + g)*PolyLog[2, E^(-2*ArcCoth[c*x])] + b*e*(c^2*f + g
)*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g]
 - g)] + b*c^2*e*f*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))/(c^2*f + 2*c
*Sqrt[-f]*Sqrt[g] - g)] + b*e*g*PolyLog[2, (E^(2*ArcCoth[c*x])*(c^2*f + g))
/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)])/(4*c^2*g)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.58, size = 8491, normalized size = 16.58

method	result
risch	$-\frac{eb\,\text{dilog}\left(\frac{c\sqrt{-fg}+g(cx-1)+g}{c\sqrt{-fg}+g}\right)}{4c^2}+\frac{eb\,\text{dilog}\left(\frac{c\sqrt{-fg}-(cx+1)g+g}{c\sqrt{-fg}+g}\right)f}{4g}+\frac{bxd}{2c}-\frac{3bex}{2c}-\frac{bd\,x^2\ln(cx-1)}{4}+\frac{be\,x^2\ln(cx-}{4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/4*(2*c^2*g*integrate(x^3*log(c*x + 1)/(c^2*g*x^2 +
c^2*f), x) - 2*c^2*g*integrate(x^3*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x) - 2
*c*g*(-I*f*(log(I*g*x/sqrt(f*g) + 1) - log(-I*g*x/sqrt(f*g) + 1))/(sqrt(f*g
)*c^2*g) - 2*x/(c^2*g)) - 2*g*integrate(x*log(c*x + 1)/(c^2*g*x^2 + c^2*f),
 x) + 2*g*integrate(x*log(c*x - 1)/(c^2*g*x^2 + c^2*f), x) - (2*c*x + (c^2*
x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/c^2)*b*e
 - 1/2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arccoth(c*x) + a*d*x + (b*x*arccoth(c*x)*e + a*x*e)*log(g*x^
2 + f), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acoth}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int(x*(a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

3.279 $\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=546

$$-2aex-2bex\coth^{-1}(cx)+\frac{2ae\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{g}}+\frac{be\sqrt{f}\,\text{ArcTan}\cdots}{\cdots}$$

```
[Out] -2*a*e*x-2*b*e*x*arccoth(c*x)-b*e*ln(-c^2*x^2+1)/c+x*(a+b*arccoth(c*x))*(d+
e*ln(g*x^2+f))+1/2*b*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))/c+1/2*b
*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))/c+2*a*e*arctan(x*g^(1/2)/f^(1/2))*f^(
1/2)/g^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1-1/c/x)*f^(1/2)/g^(1/2)+b*e*
arctan(x*g^(1/2)/f^(1/2))*ln(1+1/c/x)*f^(1/2)/g^(1/2)+b*e*arctan(x*g^(1/2)/
f^(1/2))*ln(-2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*x*
g^(1/2)))*f^(1/2)/g^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(2*(c*x+1)*f^(1/2
)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(1/2)-1/2*
I*b*e*polylog(2,1+2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)
-I*x*g^(1/2)))*f^(1/2)/g^(1/2)+1/2*I*b*e*polylog(2,1-2*(c*x+1)*f^(1/2)*g^(1
/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 20, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$, Rules used = {6221, 2525, 2441, 2440, 2438, 6128, 6022, 266, 6122, 211, 6120, 2520, 12, 6820, 4996, 4940, 4966, 2449, 2352, 2497}

$$\frac{2ae\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-2aex-\frac{be\sqrt{f}\,\log\left(1-\frac{1}{cx}\right)\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\sqrt{f}\,\log\left(\frac{1}{cx}+1\right)\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}}+\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\,(1-cx)}{\left(-\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{g}}-\frac{be\sqrt{f}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\,(cx+1)}{\left(\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{g}}+\frac{b\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}+\frac{be\text{Li}_2\left(\frac{c^2(gx^2+f)}{fc^2+g}\right)}{2c}-\frac{be\log\left(1-c^2x^2\right)}{c}-\frac{ibe\sqrt{f}\,\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}\,(1-cx)}{\left(ic\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}+1\right)}{2\sqrt{g}}+\frac{ibe\sqrt{f}\,\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\,(cx+1)}{\left(i\sqrt{f}\,c+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2\sqrt{g}}-2bex\coth^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]
])/Sqrt[g] - (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqr
t[g] + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[g] +
 (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x)
)/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[
f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g] - (b*e*Log[1 - c^2*x^2])/c
 + x*(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))
/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/(2*c) + (b*e*PolyLog[2, (c^2*(f + g*x
^2))/(c^2*f + g)])/(2*c) - ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 + (2*Sqrt[f]*Sqr
t[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
 + ((I/2)*b*e*Sqrt[f]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sq
rt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[g]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
```

```
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6120

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6122

```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 6128

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcCoth[c*x])^p/(d
 + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 6221

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x]
 + (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[
2*e*g, Int[x^2*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx = x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x(d+\ldots$$

$$= x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\text{Subst}\Big(\ldots$$

$$= -2aex + x\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\,\text{lo}\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} + x\ldots$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$= -2aex - 2bex\coth^{-1}(cx) + \frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{g}} - \frac{be\ldots}{\ldots}$$

$$\frac{2ae\sqrt{f}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\ldots} \quad \frac{be\ldots}{\ldots}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1287 vs. $2(546) = 1092$.
time = 2.44, size = 1287, normalized size = 2.36

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + b*d*x*ArcCoth[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]])/Sqrt[g] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e
*(x*ArcCoth[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*(-4*c*x*Ar
cCoth[c*x] + 4*Log[1/(c*Sqrt[1 - 1/(c^2*x^2)]*x)] + (Sqrt[c^2*f*g]*((-2*I)*
ArcCos[(c^2*f - g)/(c^2*f + g)]*ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + 4*ArcCoth[c
*x]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*Ar
cTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[((2*I)*g*(I*c^2*f + Sqrt[c^2*f*g])*(-1 + 1
/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] - (ArcCos[(c^2*f - g)
/(c^2*f + g)] - 2*ArcTan[Sqrt[c^2*f*g]/(c*g*x)])*Log[(2*g*(c^2*f + I*Sqrt[c
^2*f*g])*(1 + 1/(c*x)))/((c^2*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + (Arc
Cos[(c^2*f - g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c
*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c*x]*Sqrt[c^2
*f + g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])] + (ArcCos[(
c^2*f - g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x)
/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcCoth[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f +
g]*Sqrt[-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]]])] + I*(-PolyLog[2,
 ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(g - (I*Sqrt[c^2*f*g])/(c*x)))/((c^2
*f + g)*(g + (I*Sqrt[c^2*f*g])/(c*x)))] + PolyLog[2, ((c^2*f - g + (2*I)*Sq
rt[c^2*f*g])*(I*g + Sqrt[c^2*f*g]/(c*x)))/((c^2*f + g)*((-I)*g + Sqrt[c^2*f
*g]/(c*x)))])))/g))/(2*c) - (b*e*g*(((-Log[-c^(-1) + x] - Log[c^(-1) + x] +
 Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g) + (Log[-c^(-1) + x]*Log[1 - (Sqrt[
g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1
) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (Log[-c^(-1) + x]*Log[1 - (Sqr
t[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(-c^(-1)
 + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*
(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x
))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(2*g) + (Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(
c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]*(c^(-1) + x))/(
I*Sqrt[f] + Sqrt[g]/c)])/(2*g)))/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.22, size = 3508, normalized size = 6.42

method	result	size
risch	Expression too large to display	3508

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*e*b/c*Pi*ln(c*x-1)*csgn(I*c)^2*csgn(I*c^2)-1/2*I*e*b/c*Pi*ln(c*x-1)*c
sgn(I*c)*csgn(I*c^2)^2+1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))
*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2*ln(c*x-1)+1/4*I*e*b/c*Pi*csgn(
I/c^2)*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2
*c*x-1)*g))+1/4*I*e*b/c*Pi*ln(c*x+1)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*
csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*csgn(I/c^2)*csgn
(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))
+1/4*I*e*b/c*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*csgn(
I/c^2)+1/2*I*e*b*Pi*ln(c*x-1)*csgn(I*c)*csgn(I*c^2)^2*x-1/4*I*e*b*Pi*csgn(I
*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2
*ln(c*x-1)*x-1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*
(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2)*x-1/4*I*e*b*Pi*csgn(I/c^2*(c^2*f
+((c*x-1)^2+2*c*x-1)*g))^2*ln(c*x-1)*csgn(I/c^2)*x-1/4*I*e*b*Pi*ln(c*x-1)*c
sgn(I*c)^2*csgn(I*c^2)*x+1/4*I*e*b/c*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-
1)*g))^2*ln(c*x-1)*csgn(I/c^2)+1/4*I*e*b*Pi*ln(c*x+1)*csgn(I/c^2*(c^2*f+((c
*x+1)^2-2*c*x-1)*g))^2*csgn(I/c^2)*x+e*b/(f*g)^(1/2)*arctan(1/2*(2*g*(c*x-1
)+2*g)/c/(f*g)^(1/2))*f-1/2*e*b/c*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)
/(c*(-f*g)^(1/2)-g))-1/2*e*b/c*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c
*(-f*g)^(1/2)+g))+a*e*x*ln(g*x^2+f)-1/2*e*b/c*ln(c^2*f+((c*x-1)^2+2*c*x-1)*
g)-1/2*e*b/c*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))-1/2*e*b
/c*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))+1/2*e*b/c*ln(c^2*
f+2*g*(c*x-1)+g*(c*x-1)^2+g)+1/2*e*b/c*ln(c^2*f-2*(c*x+1)*g+g*(c*x+1)^2+g)-
1/2*e*b/c*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))+4*b*e/c+a*
d*x-2*a*e*x-1/2*e*b/c*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(
1/2)+g))-1/2*e*b/c*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2
)-g))-e*b/(f*g)^(1/2)*arctan(1/2*(2*(c*x+1)*g-2*g)/c/(f*g)^(1/2))*f+1/2*e*b
/(-f*g)^(1/2)*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*f-1/2*
e*b/(-f*g)^(1/2)*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*f+1
/2*e*b/c*ln(c^2*f+((c*x+1)^2-2*c*x-1)*g)*ln(c*x+1)+1/2*e*b/c*ln(c^2*f+((c*x
-1)^2+2*c*x-1)*g)*ln(c*x-1)-1/2*e*b*ln(c^2*f+((c*x-1)^2+2*c*x-1)*g)*ln(c*x-
1)*x-e*b/c*ln(c)*ln(c*x-1)-1/2*I*e*b/c*Pi*csgn(I*c^2)^3-b*e/c*ln(c*x-1)+1/2
*e*b*ln(c*x+1)/(-f*g)^(1/2)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)
+g))*f-1/2*e*b*ln(c*x+1)/(-f*g)^(1/2)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-
f*g)^(1/2)-g))*f-1/2*e*b*ln(c*x-1)/(-f*g)^(1/2)*ln((c*(-f*g)^(1/2)-g*(c*x-1
)-g)/(c*(-f*g)^(1/2)-g))*f+1/2*e*b*ln(c*x-1)/(-f*g)^(1/2)*ln((c*(-f*g)^(1/2
)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))*f-1/4*I*e*b*Pi*csgn(I/c^2*(c^2*f+((c*x-1
)^2+2*c*x-1)*g))^3*x+1/4*I*e*b*Pi*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))
^3*x+1/4*I*e*b/c*Pi*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^3+1/4*I*e*b/c
*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3-d*b/c+I*e*b/c*Pi*csgn(I*c)*
csgn(I*c^2)^2+1/2/c*ln(c*x-1)*b*d+b*e*x*ln(c*x-1)-1/4*I*e*b/c*Pi*csgn(I*(c^
```

```
2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*ln(c*
x-1)*csgn(I/c^2)+1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/
c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*ln(c*x-1)*csgn(I/c^2)*x-1/4*I*e*b/c*Pi*l
n(c*x+1)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2
-2*c*x-1)*g))*csgn(I/c^2)-1/4*I*e*b*Pi*ln(c*x+1)*csgn(I*(c^2*f+((c*x+1)^2-2
*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2)*x-1/4*I*e
*b/c*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3*ln(c*x-1)-1/4*I*e*b*Pi*
csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*csgn(I/c^2)*x+1/4*I*e*b*Pi*ln(c
*x+1)*csgn(I*c)^2*csgn(I*c^2)*x-1/2*I*e*b*Pi*ln(c*x+1)*csgn(I*c)*csgn(I*c^2
)^2*x+1/4*I*e*b*Pi*ln(c*x+1)*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c
^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))^2*x+1/4*I*e*b*Pi*csgn(I*(c^2*f+((c*x+1)^2
-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2)*x+1/4*I
*e*b/c*Pi*ln(c*x+1)*csgn(I*c)^2*csgn(I*c^2)-1/2*I*e*b/c*Pi*ln(c*x+1)*csgn(I
*c)*csgn(I*c^2)^2-1/2*e*b/c*ln(c^2*f+((c*x+1)^2-2*c*x-1)*g)-e*b*ln(c*x+1)*x
-1/4*I*e*b/c*Pi*csgn(I*(c^2*f+((c*x+1)^2-2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c
*x+1)^2-2*c*x-1)*g))^2+1/4*I*e*b*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g
))^2*csgn(I/c^2)*x-1/4*I*e*b*Pi*ln(c*x-1)*csgn(I*c^2)^3*x+1/4*I*e*b*Pi*csgn
(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))
^2*x+1/4*I*e*b*Pi*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^3*ln(c*x-1)*x-1
/4*I*e*b/c*Pi*csgn(I/c^2)*csgn(I/c^2*(c^2*f+((c*x-1)^2+2*c*x-1)*g))^2-1/4*I
*e*b/c*Pi*csgn(I*(c^2*f+((c*x-1)^2+2*c*x-1)*g))*csgn(I/c^2*(c^2*f+((c*x-1)^
2+2*c*x-1)*g))^2+1/4*I*e*b/c*Pi*ln(c*x-1)*csgn(I*c^2)^3-1/2*I*e*b/c*Pi*csgn
(I*c)^2*csgn(I*c^2)+2*e*a*f/(f*g)^(1/2)*arctan(x*g/(f*g)^(1/2))-e*b/c*ln(c)
*ln(c*x+1)-e*b*ln(c)*ln(c*x+1)*x+1/2*e*b*ln(c^2*f+((c*x+1)^2-2*c*x-1)*g)*ln
(c*x+1)*x+1/2*e*b/(-f*g)^(1/2)*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)
^(1/2)+g))*f-1/2*e*b/(-f*g)^(1/2)*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f
*g)^(1/2)-g))*f+e*b*ln(c)*ln(c*x-1)*x-e*b/c*ln(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] a*d*x + (2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 +
f))*a*e + 1/2*b*(((c*x + 1)*log(c*x + 1) - (c*x - 1)*log(c*x - 1))*log(g*x^
2 + f)/c - integrate(2*((c*g*x^2 + g*x)*log(c*x + 1) - (c*g*x^2 - g*x)*log(
c*x - 1))/(c*g*x^2 + c*f), x))*e + 1/2*(2*c*x*arccoth(c*x) + log(-c^2*x^2 +
 1))*b*d/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(g*x^2 + f),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

3.280 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=101

$$ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}bd\text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}bd\text{PolyLog}\left(2,\frac{1}{cx}\right)+\frac{1}{2}ae\text{PolyLog}\left(2,1\right.$$

```
[Out] b*e*CannotIntegrate(arccoth(c*x)*ln(g*x^2+f)/x,x)+a*d*ln(x)+1/2*a*e*ln(-g*x
^2/f)*ln(g*x^2+f)+1/2*b*d*polylog(2,-1/c/x)-1/2*b*d*polylog(2,1/c/x)+1/2*a*
e*polylog(2,1+g*x^2/f)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 + (b*d*PolyLog[2, -(1
/(c*x))])/2 - (b*d*PolyLog[2, 1/(c*x)])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])
/2 + b*e*Defer[Int][(ArcCoth[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx = d\int \frac{a+b\coth^{-1}(cx)}{x}\,dx + e\int \frac{(a+b\coth^{-1}(cx))\log(f+\ldots}{x}$$

$$= ad\log(x)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)+(ae)\int \frac{\log(f\ldots}{}$$

$$= ad\log(x)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right)-\frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right)+\frac{1}{2}(ae)\text{Subst}\left(\ldots\right.$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\ldots\right.$$

$$= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)+\frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\ldots\right.$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccoth}(cx))\left(d+e\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))*e*log(g
*x^2 + f)/x + 1/2*b*d*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))/x + a*e*log(g*
x^2 + f)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(g*x^2 + f))
/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acoth}(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Integral((a + b*acoth(c*x))*(d + e*log(f + g*x**2))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

3.281 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^2}\,dx$

Optimal. Leaf size=560

$$\frac{2ae\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right)}{\sqrt{f}}+\frac{be\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1+\frac{1}{cx}\right)}{\sqrt{f}}+\frac{be\sqrt{g}}{}$$

[Out] -(a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f))-1/2*b*c*ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*ln(g*x^2+f))-1/2*b*c*e*polylog(2,c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*e*polylog(2,1+g*x^2/f)+2*a*e*arctan(x*g^(1/2)/f^(1/2))*g^(1/2)/f^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1-1/c/x)*g^(1/2)/f^(1/2)+b*e*arctan(x*g^(1/2)/f^(1/2))*ln(1+1/c/x)*g^(1/2)/f^(1/2)+b*e*arctan(x*g^(1/2)/f^(1/2))*ln(-2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)-b*e*arctan(x*g^(1/2)/f^(1/2))*ln(2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)-1/2*I*b*e*polylog(2,1+2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)+1/2*I*b*e*polylog(2,1-2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2))/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(1/2)

Rubi [A]

time = 0.79, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6229, 2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438, 6122, 211, 6120, 2520, 12, 266, 6820, 4996, 4940, 4966, 2449, 2497}

$$\frac{2ae\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}-\frac{(a+b\coth^{-1}(cx))\,(d+e\log(f+gx^2))}{x}-\frac{be\sqrt{g}\log\left(1-\frac{1}{cx}\right)\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{be\sqrt{g}\log\left(\frac{1}{cx}+1\right)\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{be\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(-\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}}-\frac{be\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(cx+1)}{\left(\sqrt{g}+ic\sqrt{f}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{\sqrt{f}}-\frac{1}{2}bc\log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e\log(f+gx^2))-\frac{1}{2}bce\text{Li}_2\left(\frac{c^2(gx^2+f)}{fc^2+g}\right)+\frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+gx^2))+\frac{1}{2}bce\text{Li}_2\left(\frac{gx^2}{f}+1\right)-\frac{ibe\sqrt{g}\,\text{Li}_2\left(\frac{2\sqrt{f}\sqrt{g}(1-cx)}{\left(ic\sqrt{f}-\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}+1\right)}{2\sqrt{f}}+\frac{ibe\sqrt{g}\,\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(cx+1)}{\left(i\sqrt{f}c+\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}\,x\right)}\right)}{2\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]

[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 - 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[1 + 1/(c*x)])/Sqrt[f] + (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - (b*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*x^2]))/2 - (b*c*e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f]

```
 - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[
f]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6120

```
Int[ArcCoth[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 6122

```
Int[(ArcCoth[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 6229

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
 b*ArcCoth[c*x])/(m + 1)), x] + (-Dist[b*(c/(m + 1)), Int[x^(m + 1)*((d + e
*Log[f + g*x^2])/(1 - c^2*x^2)), x], x] - Dist[2*e*(g/(m + 1)), Int[x^(m +
2)*((a + b*ArcCoth[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + (bc)\int \frac{d+e\log}{x\,(1}$$

$$= -\frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\text{Subst}\Big(\int$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f-\right.\right.}{x}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

$$= \frac{2ae\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)\log\left(1-\frac{1}{cx}\right.}{\sqrt{f}}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1236 vs. 2(560) = 1120.
time = 2.92, size = 1236, normalized size = 2.21

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] -((a*d)/x) - (b*d*ArcCoth[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])
/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2]/
x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f +
 g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*Sqrt
[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (I*Sqr
t[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g]*x))/(c
*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2*
x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x))/(I*c*Sqr
t[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt[f] + Sqrt[
g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g])] + PolyLog
[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyLog[2, (I*Sqrt
[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) - (c*g*((2*I)*ArcCos[(c^2*f - g)/(
c^2*f + g)]*ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] - 4*ArcCoth[c*x]*ArcTan[(c*g*x)
/Sqrt[c^2*f*g]] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcTan[(c*f)/(Sqrt[c
^2*f*g]*x)])*Log[(2*g*(c^2*f - I*Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f + g)*(I
*Sqrt[c^2*f*g] + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*ArcTan[(c*
f)/(Sqrt[c^2*f*g]*x)])*Log[(2*g*(c^2*f + I*Sqrt[c^2*f*g])*(1 + c*x))/((c^2*
f + g)*(I*Sqrt[c^2*f*g] + c*g*x))] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(
ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt
[2]*Sqrt[c^2*f*g])/(E^ArcCoth[c*x]*Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2
*f + g)*Cosh[2*ArcCoth[c*x]]])] - (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(Arc
Tan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]
*E^ArcCoth[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[-(c^2*f) + g + (c^2*f
+ g)*Cosh[2*ArcCoth[c*x]]])] + I*(PolyLog[2, ((c^2*f - g - (2*I)*Sqrt[c^2*f
*g])*(Sqrt[c^2*f*g] + I*c*g*x))/((c^2*f + g)*(Sqrt[c^2*f*g] - I*c*g*x))] -
PolyLog[2, ((c^2*f - g + (2*I)*Sqrt[c^2*f*g])*(Sqrt[c^2*f*g] + I*c*g*x))/((
c^2*f + g)*(Sqrt[c^2*f*g] - I*c*g*x))])))/Sqrt[c^2*f*g]))/2
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccoth}(cx)\right)\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arccoth(c*x)/x)*b*d + (2*g*arctan
(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + 1/2*b*e*integrate((log(
1/(c*x) + 1) - log(-1/(c*x) + 1))*log(g*x^2 + f)/x^2, x) - a*d/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(g*x^2 + f))
/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
[Out] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

3.282 $\int \frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{x^3}\,dx$

Optimal. Leaf size=712

$$\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{beg\coth^{-1}(cx)\log\left(\frac{2}{1+cx}\right)}{f}+bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right)-\frac{beg\coth^{-1}}{}$$

```
[Out] a*e*g*ln(x)/f+b*e*g*arccoth(c*x)*ln(2/(c*x+1))/f+b*c^2*e*arctanh(c*x)*ln(2/
(c*x+1))-1/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x-1/2*(a+b*arcco
th(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*b*c^2*arctanh(c*x)*(d+e*ln(g*x^2+f))-1/2
*b*e*g*arccoth(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(
1/2)))/f-1/2*b*c^2*e*arctanh(c*x)*ln(2*c*((-f)^(1/2)-x*g^(1/2))/(c*x+1)/(c*
(-f)^(1/2)-g^(1/2)))-1/2*b*e*g*arccoth(c*x)*ln(2*c*((-f)^(1/2)+x*g^(1/2))/(
c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f-1/2*b*c^2*e*arctanh(c*x)*ln(2*c*((-f)^(1/2
)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))+1/2*b*e*g*polylog(2,-1/c/x)/f-
1/2*b*e*g*polylog(2,1/c/x)/f-1/2*b*c^2*e*polylog(2,1-2/(c*x+1))-1/2*b*e*g*p
olylog(2,1-2/(c*x+1))/f+1/4*b*c^2*e*polylog(2,1-2*c*((-f)^(1/2)-x*g^(1/2))/
(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))+1/4*b*e*g*polylog(2,1-2*c*((-f)^(1/2)-x*g^(
1/2))/(c*x+1)/(c*(-f)^(1/2)-g^(1/2)))/f+1/4*b*c^2*e*polylog(2,1-2*c*((-f)^(
1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))+1/4*b*e*g*polylog(2,1-2*c*(
(-f)^(1/2)+x*g^(1/2))/(c*x+1)/(c*(-f)^(1/2)+g^(1/2)))/f+b*c*e*arctan(x*g^(1
/2)/f^(1/2))*g^(1/2)/f^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {6038, 331, 212, 6233, 6857, 815, 649, 211, 266, 6140, 6032, 6058, 2449, 2352, 2497, 6139, 6057}

$$-\frac{(a+b\coth^{-1}(cx))(d+e\log(f+gx^2))}{2x^2}-\frac{aeg\log(f+gx^2)}{2f}+\frac{aeg\log(x)}{f}+\frac{bce\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{1}{2}bc^2\tanh^{-1}(cx)(d+e\log(f+gx^2))+\frac{1}{4}bc^2e\text{Li}_2\left(1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(cx+1)}\right)+\frac{1}{4}bc^2e\text{Li}_2\left(1-\frac{2c(\sqrt{g}x+\sqrt{-f})}{(\sqrt{-f}c+\sqrt{g})(cx+1)}\right)-\frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)-\frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(cx+1)(c\sqrt{-f}+\sqrt{g})}\right)-\frac{1}{2}bc^2e\text{Li}_2\left(1-\frac{2}{cx+1}\right)+bc^2e\log\left(\frac{2}{cx+1}\right)\tanh^{-1}(cx)-\frac{bc(d+e\log(f+gx^2))}{2x}+\frac{beg\text{Li}_2\left(-\frac{1}{cx}\right)}{2f}-\frac{beg\text{Li}_2\left(\frac{1}{cx}\right)}{2f}-\frac{beg\text{Li}_2\left(1-\frac{2}{cx+1}\right)}{2f}+\frac{beg\text{Li}_2\left(1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-\sqrt{g})(cx+1)}\right)}{4f}+\frac{beg\text{Li}_2\left(1-\frac{2c(\sqrt{g}x+\sqrt{-f})}{(\sqrt{-f}c+\sqrt{g})(cx+1)}\right)}{4f}+\frac{beg\log\left(\frac{2}{cx+1}\right)\coth^{-1}(cx)}{f}-\frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(cx+1)(c\sqrt{-f}-\sqrt{g})}\right)}{2f}-\frac{beg\coth^{-1}(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(cx+1)(c\sqrt{-f}+\sqrt{g})}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*g*ArcCoth[c*x]*Log[2/(1 + c*x)])/f + b*c^2*e*ArcTanh[c*x]*Log[2/(1 + c*x
)] - (b*e*g*ArcCoth[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sq
rt[g])*(1 + c*x))])/(2*f) - (b*c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt
[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/2 - (b*e*g*ArcCoth[c*x]*Log[(2
*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (b*
c^2*e*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])
*(1 + c*x))])/2 - (a*e*g*Log[f + g*x^2])/(2*f) - (b*c*(d + e*Log[f + g*x^2]
```

```
))/(2*x) - ((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) + (b*c^2*A
rcTanh[c*x]*(d + e*Log[f + g*x^2]))/2 + (b*e*g*PolyLog[2, -(1/(c*x))])/(2*f
) - (b*e*g*PolyLog[2, 1/(c*x)])/(2*f) - (b*c^2*e*PolyLog[2, 1 - 2/(1 + c*x)
])/2 - (b*e*g*PolyLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*c^2*e*PolyLog[2, 1 -
(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/4 + (b*e*
g*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 +
c*x))])/(4*f) + (b*c^2*e*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sq
rt[-f] + Sqrt[g])*(1 + c*x))])/4 + (b*e*g*PolyLog[2, 1 - (2*c*(Sqrt[-f] + S
qrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(4*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rule 6038

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcCoth[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m
+ 1)), Int[x^(m + n)*((a + b*ArcCoth[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x]
, x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1]
 && IntegerQ[m])) && NeQ[m, -1]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6140

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcCoth[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6233

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[x*
(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc(d+e\log\left(f+gx^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f\right.\right.}{2x^2}$$

$$= -\frac{bc(d+e\log\left(f+gx^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f\right.\right.}{2x^2}$$

$$= -\frac{bc(d+e\log\left(f+gx^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f\right.\right.}{2x^2}$$

$$= -\frac{bc(d+e\log\left(f+gx^2\right))}{2x} - \frac{\left(a+b\coth^{-1}(cx)\right)\left(d+e\log\left(f\right.\right.}{2x^2}$$

$$= bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}(cx)\log\left(\frac{}{\left(c\right.}\right.$$

$$= \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}$$

$$= \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log\left(\frac{2}{1+cx}\right) - \frac{1}{2}bc^2e\tanh^{-1}$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + bc^2e\tanh^{-1}(cx)\log$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\right.}{f}$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\right.}{f}$$

$$= \frac{bce\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g}\,x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg\log(x)}{f} + \frac{beg\coth^{-1}(cx)\log\left(\right.}{f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.48, size = 1318, normalized size = 1.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcCoth[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -1/4*(2*a*d*f - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - 4
*a*e*g*x^2*Log[x] + 2*a*e*g*x^2*Log[f + g*x^2] + 2*e*f*(a + b*c*x + (b - b*
c^2*x^2)*ArcCoth[c*x])*Log[f + g*x^2] + b*c^2*e*f*x^2*(-4*ArcCoth[c*x]^2 -
4*ArcCoth[c*x]*Log[1 - E^(-2*ArcCoth[c*x])] + 2*ArcCoth[c*x]*Log[1 + (E^(2*
ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + 2*ArcCo
th[c*x]*Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-(c^2*f) + 2*c*Sqrt[-f]*S
qrt[g] + g)] + 2*PolyLog[2, E^(-2*ArcCoth[c*x])] + PolyLog[2, (E^(2*ArcCoth
[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + PolyLog[2, (E^(2*
ArcCoth[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)]) - b*d*(-2*(
c*f*x + g*x^2*ArcCoth[c*x]^2 + ArcCoth[c*x]*(f - c^2*f*x^2 + 2*g*x^2*Log[1
+ E^(-2*ArcCoth[c*x])]) - g*x^2*PolyLog[2, -E^(-2*ArcCoth[c*x])]) + g*x^2*(
2*ArcCoth[c*x]*(-ArcCoth[c*x] + Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f + g))/(-
(c^2*f) - 2*c*Sqrt[-f]*Sqrt[g] + g)] + Log[1 + (E^(2*ArcCoth[c*x])*(c^2*f +
 g))/(-(c^2*f) + 2*c*Sqrt[-f]*Sqrt[g] + g)]) + PolyLog[2, (E^(2*ArcCoth[c*x
])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] + PolyLog[2, (E^(2*ArcC
oth[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)])) + b*d*g*x^2*(2
*ArcCoth[c*x]^2 - (4*I)*ArcSin[Sqrt[g/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c
^2*f*g)]*x)] - 2*ArcCoth[c*x]*(ArcCoth[c*x] + 2*Log[1 + E^(-2*ArcCoth[c*x])
]) + 2*(ArcCoth[c*x] - I*ArcSin[Sqrt[g/(c^2*f + g)]])*Log[(c^2*(-1 + E^(2*A
rcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcC
oth[c*x])*(c^2*f + g))] + 2*(ArcCoth[c*x] + I*ArcSin[Sqrt[g/(c^2*f + g)]])*
Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(
c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*PolyLog[2, -E^(-2*ArcCoth[
c*x])] - PolyLog[2, (c^2*f - g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c
^2*f + g))] - PolyLog[2, -((-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCot
h[c*x])*(c^2*f + g)))]) + b*e*g*x^2*(2*ArcCoth[c*x]^2 - (4*I)*ArcSin[Sqrt[g
/(c^2*f + g)]]*ArcTanh[(c*f)/(Sqrt[-(c^2*f*g)]*x)] - 2*ArcCoth[c*x]*(ArcCot
h[c*x] + 2*Log[1 + E^(-2*ArcCoth[c*x])]) + 2*(ArcCoth[c*x] - I*ArcSin[Sqrt[
g/(c^2*f + g)]])*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f + g + E^(2*ArcCoth[c*
x])*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] + 2*(ArcCoth[
c*x] + I*ArcSin[Sqrt[g/(c^2*f + g)]])*Log[(c^2*(-1 + E^(2*ArcCoth[c*x]))*f
+ g + E^(2*ArcCoth[c*x])*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f
 + g))] + 2*PolyLog[2, -E^(-2*ArcCoth[c*x])] - PolyLog[2, (c^2*f - g + 2*Sq
rt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g))] - PolyLog[2, -((-(c^2*f)
+ g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcCoth[c*x])*(c^2*f + g)))]))/(f*x^2)
```

Maple [A]
time = 4.64, size = 936, normalized size = 1.31

method	result
risch	$-\frac{ad}{2x^2}+\frac{be\ln(cx-1)\ln\left(\frac{c\sqrt{-fg}+g(cx-1)+g}{c\sqrt{-fg}+g}\right)c^2}{4}+\frac{aeg\ln(x)}{f}-\frac{aeg\ln(g\,x^2+f)}{2f}+\frac{gbe\,\mathrm{dilog}\left(\frac{c\sqrt{-fg}-g(cx-1)-g}{c\sqrt{-fg}-g}\right)}{4f}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*d/x^2+1/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1
/2)+g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-
g))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))+a*e*
g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f-1/2*g*b*e*dilog(c*x+1)/f-1/4*g*b*e/f*ln(c
*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*ln(c*
x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+g*e*b*c/(f*g)^(1/2
)*arctan(x*g/(f*g)^(1/2))-1/4*b*e*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f
*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2
)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*c^
2+1/4*b*e*dilog((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*
e*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e
*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1/4*g*b*
e/f*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*dilo
g((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))-1/2*g*b*e/f*dilog(c*x)+1
/4*b*e*ln(c*x-1)*ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))*c^2-1/
2*d*b*c/x+1/4*d*b*c^2*ln(c*x+1)-1/4*d*b*ln(c*x+1)/x^2-1/4*d*b*c^2*ln(c*x-1)
+1/4*d*b*ln(c*x-1)/x^2-1/2*g*b*e/f*ln(c*x-1)*ln(c*x)+1/4*g*b*e/f*ln(c*x-1)*
ln((c*(-f*g)^(1/2)-g*(c*x-1)-g)/(c*(-f*g)^(1/2)-g))+1/4*g*b*e/f*ln(c*x-1)*l
n((c*(-f*g)^(1/2)+g*(c*x-1)+g)/(c*(-f*g)^(1/2)+g))+(-1/4*b*e/x^2*ln(c*x+1)+
1/4*e*(b*c^2*ln(c*x+1)*x^2-b*x^2*ln(c*x-1)*c^2-2*x*b*c+b*ln(c*x-1)-2*a)/x^2
)*ln(g*x^2+f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arccoth(c*x)/x^2)*b*d -
1/2*(g*(log(g*x^2 + f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e - 1/4*(2*c
^2*g*integrate(x^2*log(c*x + 1)/(g*x^3 + f*x), x) - 2*c^2*g*integrate(x^2*l
og(c*x - 1)/(g*x^3 + f*x), x) + 2*I*c*g*(log(I*g*x/sqrt(f*g) + 1) - log(-I*
g*x/sqrt(f*g) + 1))/sqrt(f*g) - 2*g*integrate(log(c*x + 1)/(g*x^3 + f*x), x
) + 2*g*integrate(log(c*x - 1)/(g*x^3 + f*x), x) + (2*c*x - (c^2*x^2 - 1)*l
```

```
og(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))*log(g*x^2 + f)/x^2)*b*e - 1/2*a*d
/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arccoth(c*x) + a*d + (b*arccoth(c*x)*e + a*e)*log(g*x^2 + f))
/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acoth}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

3.283 $\int \coth^{-1}(e^x)\, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{-x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{-x}\right)$$

```
[Out] 1/2*polylog(2,-1/exp(x))-1/2*polylog(2,exp(-x))
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 6032}

$$\frac{\text{Li}_2(-e^{-x})}{2}-\frac{\text{Li}_2(e^{-x})}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[E^x],x]
```

```
[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}(e^x)\, dx &= \text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\, dx, x, e^x\right)\\ &= \frac{\text{Li}_2(-e^{-x})}{2}-\frac{\text{Li}_2(e^{-x})}{2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.84

$$x\coth^{-1}(e^x)+\frac{1}{2}(-x(-\log(1-e^x)+\log(1+e^x))-\text{PolyLog}(2,-e^x)+\text{PolyLog}(2,e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[E^x],x]
```

```
[Out] x*ArcCoth[E^x] + (-(x*(-Log[1 - E^x] + Log[1 + E^x])) - PolyLog[2, -E^x] +
PolyLog[2, E^x])/2
```

Maple [A]
time = 0.07, size = 31, normalized size = 1.24

method	result	size
risch	$-\frac{\operatorname{dilog}(e^x+1)}{2}-\frac{\operatorname{dilog}(e^x)}{2}-\frac{\ln(e^x-1)\ln(e^x)}{2}$	24
derivativedivides	$\ln(e^x)\operatorname{arccoth}(e^x)-\frac{\operatorname{dilog}(e^x)}{2}-\frac{\operatorname{dilog}(e^x+1)}{2}-\frac{\ln(e^x)\ln(e^x+1)}{2}$	31
default	$\ln(e^x)\operatorname{arccoth}(e^x)-\frac{\operatorname{dilog}(e^x)}{2}-\frac{\operatorname{dilog}(e^x+1)}{2}-\frac{\ln(e^x)\ln(e^x+1)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x))*arccoth(exp(x))-1/2*dilog(exp(x))-1/2*dilog(exp(x)+1)-1/2*ln(exp
(x))*ln(exp(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.
time = 0.26, size = 58, normalized size = 2.32

$$-\frac{1}{2}x(\log(e^x+1)-\log(e^x-1))+x\operatorname{arcoth}(e^x)+\frac{1}{2}\log(-e^x)\log(e^x+1)-\frac{1}{2}x\log(e^x-1)+\frac{1}{2}\operatorname{Li}_2(e^x+1)-\frac{1}{2}\operatorname{Li}_2(-e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arccoth(e^x) + 1/2*log(-e^x)*log(e
^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.
time = 0.34, size = 64, normalized size = 2.56

$$\frac{1}{2}x\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}\operatorname{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}\operatorname{Li}_2(-\cosh(x)-\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cosh
(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x)
+ sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(e^{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(exp(x)),x)
```

```
[Out] Integral(acoth(exp(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e^x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{acoth}(\mathrm{e}^{x})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(exp(x)),x)
```

```
[Out] int(acoth(exp(x)), x)
```

3.284 $\int x \coth^{-1}(e^x)\, dx$

Optimal. Leaf size=51

$$\frac{1}{2}x\text{PolyLog}(2,-e^{-x})-\frac{1}{2}x\text{PolyLog}(2,e^{-x})+\frac{1}{2}\text{PolyLog}(3,-e^{-x})-\frac{1}{2}\text{PolyLog}(3,e^{-x})$$

```
[Out] 1/2*x*polylog(2,-1/exp(x))-1/2*x*polylog(2,exp(-x))+1/2*polylog(3,-1/exp(x)
)-1/2*polylog(3,exp(-x))
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6349, 2611, 2320, 6724}

$$\frac{1}{2}x\text{Li}_2(-e^{-x})-\frac{1}{2}x\text{Li}_2(e^{-x})+\frac{\text{Li}_3(-e^{-x})}{2}-\frac{\text{Li}_3(e^{-x})}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[E^x],x]
```

```
[Out] (x*PolyLog[2, -E^(-x)])/2 - (x*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/
2 - PolyLog[3, E^(-x)]/2
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\coth^{-1}\left(e^x\right)\,dx &= -\left(\frac{1}{2}\int x\log\left(1-e^{-x}\right)\,dx\right)+\frac{1}{2}\int x\log\left(1+e^{-x}\right)\,dx \\
&= \frac{1}{2}x\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x\text{Li}_2\left(e^{-x}\right)-\frac{1}{2}\int \text{Li}_2\left(-e^{-x}\right)\,dx+\frac{1}{2}\int \text{Li}_2\left(e^{-x}\right)\,dx \\
&= \frac{1}{2}x\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x\text{Li}_2\left(e^{-x}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}\,dx,x,e^{-x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}\,\ldots\right. \\
&= \frac{1}{2}x\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x\text{Li}_2\left(e^{-x}\right)+\frac{\text{Li}_3\left(-e^{-x}\right)}{2}-\frac{\text{Li}_3\left(e^{-x}\right)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.39

$$\frac{1}{4}\left(2x^2\coth^{-1}\left(e^x\right)+x^2\log\left(1-e^x\right)-x^2\log\left(1+e^x\right)-2x\text{PolyLog}(2,-e^x)+2x\text{PolyLog}(2,e^x)+2\text{PolyLog}(3,-e^x)-2\text{PolyLog}(3,e^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[E^x],x]
[Out] (2*x^2*ArcCoth[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
-E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

Maple [A]
time = 0.05, size = 62, normalized size = 1.22

method	result	si
risch	$-\frac{x^2\ln(e^x-1)}{4}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\,\mathrm{polylog}(2,e^x)}{2}-\frac{\mathrm{polylog}(3,e^x)}{2}-\frac{x\,\mathrm{polylog}(2,-e^x)}{2}+\frac{\mathrm{polylog}(3,-e^x)}{2}$	54
default	$\frac{x^2\mathrm{arccoth}(e^x)}{2}-\frac{x^2\ln(e^x+1)}{4}-\frac{x\,\mathrm{polylog}(2,-e^x)}{2}+\frac{\mathrm{polylog}(3,-e^x)}{2}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\,\mathrm{polylog}(2,e^x)}{2}-\frac{\mathrm{polylog}(3,e^x)}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(exp(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*arccoth(exp(x))-1/4*x^2*ln(exp(x)+1)-1/2*x*polylog(2,-exp(x))+1/2*p
olylog(3,-exp(x))+1/4*x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(
3,exp(x))
```

Maxima [A]
time = 0.26, size = 59, normalized size = 1.16

$$\frac{1}{2}\,x^2\operatorname{arcoth}\left(e^x\right)-\frac{1}{4}\,x^2\log\left(e^x+1\right)+\frac{1}{4}\,x^2\log\left(-e^x+1\right)-\frac{1}{2}\,x\mathrm{Li}_2(-e^x)+\frac{1}{2}\,x\mathrm{Li}_2(e^x)+\frac{1}{2}\,\mathrm{Li}_3(-e^x)-\frac{1}{2}\,\mathrm{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*x
*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e^x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(37) = 74.
time = 0.34, size = 94, normalized size = 1.84

$$\frac{1}{4}\,x^2\log\left(\frac{\cosh\left(x\right)+\sinh\left(x\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)-1}\right)-\frac{1}{4}\,x^2\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\frac{1}{4}\,x^2\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+\frac{1}{2}\,x\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\frac{1}{2}\,x\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))-\frac{1}{2}\,\mathrm{polylog}(3,\cosh\left(x\right)+\sinh\left(x\right))+\frac{1}{2}\,\mathrm{polylog}(3,-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log(
cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilog(
cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, cosh(
x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acoth}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(x)),x)
```

```
[Out] Integral(x*acoth(exp(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(e^x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acoth}(e^x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(exp(x)),x)
```

```
[Out] int(x*acoth(exp(x)), x)
```

3.285 $\int x^2 \coth^{-1}(e^x)\, dx$

Optimal. Leaf size=70

$$\frac{1}{2}x^2\text{PolyLog}(2,-e^{-x})-\frac{1}{2}x^2\text{PolyLog}(2,e^{-x})+x\text{PolyLog}(3,-e^{-x})-x\text{PolyLog}(3,e^{-x})+\text{PolyLog}(4,-e^{-x})-$$

```
[Out] 1/2*x^2*polylog(2,-1/exp(x))-1/2*x^2*polylog(2,exp(-x))+x*polylog(3,-1/exp(
x))-x*polylog(3,exp(-x))+polylog(4,-1/exp(x))-polylog(4,exp(-x))
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6349, 2611, 6744, 2320, 6724}

$$\frac{1}{2}x^2\text{Li}_2(-e^{-x})-\frac{1}{2}x^2\text{Li}_2(e^{-x})+x\text{Li}_3(-e^{-x})-x\text{Li}_3(e^{-x})+\text{Li}_4(-e^{-x})-\text{Li}_4(e^{-x})$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[E^x],x]
```

```
[Out] (x^2*PolyLog[2, -E^(-x)])/2 - (x^2*PolyLog[2, E^(-x)])/2 + x*PolyLog[3, -E^
(-x)] - x*PolyLog[3, E^(-x)] + PolyLog[4, -E^(-x)] - PolyLog[4, E^(-x)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}\left(e^x\right)\,dx &= -\left(\frac{1}{2}\int x^2\log\left(1-e^{-x}\right)\,dx\right)+\frac{1}{2}\int x^2\log\left(1+e^{-x}\right)\,dx\\ &= \frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)-\int x\text{Li}_2\left(-e^{-x}\right)\,dx+\int x\text{Li}_2\left(e^{-x}\right)\,dx\\ &= \frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)-\int \text{Li}_3\left(-e^{-x}\right)\,dx+\int \text{Li}\\ &= \frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)+\text{Subst}\left(\int\frac{\text{Li}_3(-x)}{x}\,dx,x\right.\\ &= \frac{1}{2}x^2\text{Li}_2\left(-e^{-x}\right)-\frac{1}{2}x^2\text{Li}_2\left(e^{-x}\right)+x\text{Li}_3\left(-e^{-x}\right)-x\text{Li}_3\left(e^{-x}\right)+\text{Li}_4\left(-e^{-x}\right)-\text{Li}_4\left(e^{-x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 93, normalized size = 1.33

$$\frac{1}{6}\left(2x^3\coth^{-1}\left(e^x\right)+x^3\log\left(1-e^x\right)-x^3\log\left(1+e^x\right)-3x^2\text{PolyLog}\left(2,-e^x\right)+3x^2\text{PolyLog}\left(2,e^x\right)+6x\text{PolyLog}\left(3,-e^x\right)-6x\text{PolyLog}\left(3,e^x\right)-6\text{PolyLog}\left(4,-e^x\right)+6\text{PolyLog}\left(4,e^x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^x],x]
```

```
[Out] (2*x^3*ArcCoth[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

Maple [A]
time = 0.05, size = 79, normalized size = 1.13

method	result
risch	$-\frac{x^3\ln(\mathrm{e}^x-1)}{6}+\frac{x^3\ln(1-\mathrm{e}^x)}{6}+\frac{x^2\,\mathrm{polylog}(2,\mathrm{e}^x)}{2}-x\,\mathrm{polylog}(3,\mathrm{e}^x)+\mathrm{polylog}(4,\mathrm{e}^x)-\frac{x^2\,\mathrm{polylog}(2,-\mathrm{e}^x)}{2}+x$
default	$\frac{x^3\mathrm{arccoth}(\mathrm{e}^x)}{3}-\frac{x^3\ln(\mathrm{e}^x+1)}{6}-\frac{x^2\,\mathrm{polylog}(2,-\mathrm{e}^x)}{2}+x\,\mathrm{polylog}(3,-\mathrm{e}^x)-\mathrm{polylog}(4,-\mathrm{e}^x)+\frac{x^3\ln(1-\mathrm{e}^x)}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccoth(exp(x))-1/6*x^3*ln(exp(x)+1)-1/2*x^2*polylog(2,-exp(x))+x*p
olylog(3,-exp(x))-polylog(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2
,exp(x))-x*polylog(3,exp(x))+polylog(4,exp(x))
```

Maxima [A]
time = 0.26, size = 76, normalized size = 1.09

$$\frac{1}{3}x^3\operatorname{arcoth}(e^x)-\frac{1}{6}x^3\log(e^x+1)+\frac{1}{6}x^3\log(-e^x+1)-\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Li}_4(-e^x)+\mathrm{Li}_4(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*x
^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e^x
) - polylog(4, -e^x) + polylog(4, e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(58) = 116.
time = 0.36, size = 119, normalized size = 1.70

$$\frac{1}{6}x^3\log\left(\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{6}x^3\log(\cosh(x)+\sinh(x)+1)+\frac{1}{6}x^3\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x^2\mathrm{Li}_2(\cosh(x)+\sinh(x))-\frac{1}{2}x^2\mathrm{Li}_2(-\cosh(x)-\sinh(x))-x\mathrm{polylog}(3,\cosh(x)+\sinh(x))+x\mathrm{polylog}(3,-\cosh(x)-\sinh(x))+\mathrm{polylog}(4,\cosh(x)+\sinh(x))-\mathrm{polylog}(4,-\cosh(x)-\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log(
cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dilo
g(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, cos
h(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) + s
inh(x)) - polylog(4, -cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{acoth}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(x)),x)
```

```
[Out] Integral(x**2*acoth(exp(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(e^x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acoth}(\mathrm{e}^x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(exp(x)),x)
```

```
[Out] int(x^2*acoth(exp(x)), x)
```

3.286 $\int \coth^{-1}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=41

$$\frac{\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}$$

```
[Out] 1/2*polylog(2,-exp(-b*x-a))/b-1/2*polylog(2,exp(-b*x-a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2320, 6032}

$$\frac{\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{\text{Li}_2\left(e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[E^(a + b*x)],x]
```

```
[Out] PolyLog[2, -E^(-a - b*x)]/(2*b) - PolyLog[2, E^(-a - b*x)]/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6032

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (Simp[(b/2)*PolyLog[2, -(c*x)^(-1)], x] - Simp[(b/2)*PolyLog[2, 1/(c*x)
], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \coth^{-1}\left(e^{a+bx}\right)\,dx&=\frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x}\,dx,x,e^{a+bx}\right)}{b}\\&=\frac{\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{\text{Li}_2\left(e^{-a-bx}\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 69, normalized size = 1.68

$$x\coth^{-1}\left(e^{a+bx}\right)+\frac{bx\left(\log\left(1-e^{a+bx}\right)-\log\left(1+e^{a+bx}\right)\right)-\text{PolyLog}\left(2,-e^{a+bx}\right)+\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[E^(a + b*x)],x]
```

```
[Out] x*ArcCoth[E^(a + b*x)] + (b*x*(Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)])
 - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

Maple [A]
time = 0.08, size = 59, normalized size = 1.44

method	result	size
risch	$-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2b}-\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)\ln\left(\mathrm{e}^{bx+a}\right)}{2b}-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}\right)}{2b}$	49
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\mathrm{arccoth}\left(\mathrm{e}^{bx+a}\right)-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$	59
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)\mathrm{arccoth}\left(\mathrm{e}^{bx+a}\right)-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}\right)}{2}-\frac{\mathrm{dilog}\left(\mathrm{e}^{bx+a}+1\right)}{2}-\frac{\ln\left(\mathrm{e}^{bx+a}\right)\ln\left(\mathrm{e}^{bx+a}+1\right)}{2}}{b}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(ln(exp(b*x+a))*arccoth(exp(b*x+a))-1/2*dilog(exp(b*x+a))-1/2*dilog(exp
(b*x+a)+1)-1/2*ln(exp(b*x+a))*ln(exp(b*x+a)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(33) = 66$.
time = 0.25, size = 107, normalized size = 2.61

$$\frac{(bx+a)\,\mathrm{arcoth}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx+a)\log\left(e^{(bx+a)}-1\right)-\mathrm{Li}_2\left(e^{(bx+a)}+1\right)+\mathrm{Li}_2\left(-e^{(bx+a)}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arccoth(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) - l
og(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)*l
og(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(33) = 66$.
time = 0.35, size = 137, normalized size = 3.34

$$\frac{bx\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+(bx+a)\log\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)+1\right)+\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-\mathrm{Li}_2(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
+ a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x +
a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
 + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b*x +
 a)))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}\left(e^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(exp(b*x+a)),x)
```

```
[Out] Integral(acoth(exp(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e^(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(exp(a + b*x)),x)
```

```
[Out] int(acoth(exp(a + b*x)), x)
```

3.287 $\int x \coth^{-1}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=83

$$\frac{x\text{PolyLog}\left(2,-e^{-a-bx}\right)}{2b}-\frac{x\text{PolyLog}\left(2,e^{-a-bx}\right)}{2b}+\frac{\text{PolyLog}\left(3,-e^{-a-bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{-a-bx}\right)}{2b^2}$$

```
[Out] 1/2*x*polylog(2,-exp(-b*x-a))/b-1/2*x*polylog(2,exp(-b*x-a))/b+1/2*polylog(
3,-exp(-b*x-a))/b^2-1/2*polylog(3,exp(-b*x-a))/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6349, 2611, 2320, 6724}

$$\frac{\text{Li}_3\left(-e^{-a-bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{-a-bx}\right)}{2b^2}+\frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (x*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x*PolyLog[2, E^(-a - b*x)])/(2*b) +
PolyLog[3, -E^(-a - b*x)]/(2*b^2) - PolyLog[3, E^(-a - b*x)]/(2*b^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\coth^{-1}\left(e^{a+bx}\right)\,dx&=-\left(\frac{1}{2}\int x\log\left(1-e^{-a-bx}\right)\,dx\right)+\frac{1}{2}\int x\log\left(1+e^{-a-bx}\right)\,dx\\&=\frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}-\frac{\int\text{Li}_2\left(-e^{-a-bx}\right)\,dx}{2b}+\frac{\int\text{Li}_2\left(e^{-a-bx}\right)\,dx}{2b}\\&=\frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}\,dx,x,e^{-a-bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int\frac{\text{Li}_2(}{x}\right.}{}\\&=\frac{x\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{\text{Li}_3\left(-e^{-a-bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{-a-bx}\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 113, normalized size = 1.36

$$\frac{2b^2x^2\coth^{-1}\left(e^{a+bx}\right)+b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(1+e^{a+bx}\right)-2bx\text{PolyLog}\left(2,-e^{a+bx}\right)+2bx\text{PolyLog}\left(2,e^{a+bx}\right)+2\text{PolyLog}\left(3,-e^{a+bx}\right)-2\text{PolyLog}\left(3,e^{a+bx}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (2*b^2*x^2*ArcCoth[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*Lo
g[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a
 + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(71) = 142.
time = 0.16, size = 179, normalized size = 2.16

method	result
default	$\frac{x^2\operatorname{arccoth}\left(e^{bx+a}\right)}{2}+\frac{-a^2\operatorname{arctanh}\left(e^{bx+a}\right)-\frac{(bx+a)^2\ln\left(e^{bx+a}+1\right)}{2}-(bx+a)\operatorname{polylog}\left(2,-e^{bx+a}\right)+\operatorname{polylog}\left(3,-e^{bx+a}\right)+\frac{(bx+a)^2\ln\left(1-e\right.}{2}}{}$
risch	$-\frac{x^2\ln\left(e^{bx+a}-1\right)}{4}+\frac{\ln\left(1-e^{bx+a}\right)x^2}{4}+\frac{\ln\left(1-e^{bx+a}\right)xa}{2b}+\frac{a^2\ln\left(e^{bx+a}-1\right)}{4b^2}+\frac{a^2\ln\left(1-e^{bx+a}\right)}{4b^2}+\frac{x\operatorname{polylog}\left(2,e^{bx+a}\right)}{2b}+\frac{a\,\mathrm{di}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccoth(exp(b*x+a))+1/2/b^2*(-a^2*arctanh(exp(b*x+a))-1/2*(b*x+a)^2
*ln(exp(b*x+a)+1)-(b*x+a)*polylog(2,-exp(b*x+a))+polylog(3,-exp(b*x+a))+1/2
*(b*x+a)^2*ln(1-exp(b*x+a))+(b*x+a)*polylog(2,exp(b*x+a))-polylog(3,exp(b*x
+a))+a*(b*x+a)*ln(exp(b*x+a)+1)-a*(b*x+a)*ln(1-exp(b*x+a))-a*polylog(2,exp(
b*x+a))+a*polylog(2,-exp(b*x+a)))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 1.30

$$\frac{1}{2}\,x^2\,\operatorname{arcoth}\left(e^{(bx+a)}\right)-\frac{1}{4}\,b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(-e^{(bx+a)}\right)}{b^3}-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2\,bx\mathrm{Li}_2\left(e^{(bx+a)}\right)-2\,\mathrm{Li}_3\left(e^{(bx+a)}\right)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccoth(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b
*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(69) = 138.
time = 0.34, size = 198, normalized size = 2.39

$$\frac{b^2x^2\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\mathrm{Li}_2(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-2\,bx\mathrm{Li}_2(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))+a^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-1\right)+\left(b^2x^2-a^2\right)\log\left(-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)+1\right)-2\,\mathrm{polylog}(3,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+2\,\mathrm{polylog}(3,-\cosh\left(bx+a\right)-\sinh\left(bx+a\right))}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(
b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*dil
og(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*x +
 a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log(-co
sh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x +
a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acoth}\left(e^{a}e^{bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(b*x+a)),x)
```

```
[Out] Integral(x*acoth(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x*arccoth(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(exp(a + b*x)),x)
[Out] int(x*acoth(exp(a + b*x)), x)
```

3.288 $\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=119

$$\frac{x^2\text{PolyLog}(2,-e^{-a-bx})}{2b}-\frac{x^2\text{PolyLog}(2,e^{-a-bx})}{2b}+\frac{x\text{PolyLog}(3,-e^{-a-bx})}{b^2}-\frac{x\text{PolyLog}(3,e^{-a-bx})}{b^2}+\frac{\text{PolyLog}(4}{b^3}$$

```
[Out] 1/2*x^2*polylog(2,-exp(-b*x-a))/b-1/2*x^2*polylog(2,exp(-b*x-a))/b+x*polylo
g(3,-exp(-b*x-a))/b^2-x*polylog(3,exp(-b*x-a))/b^2+polylog(4,-exp(-b*x-a))/
b^3-polylog(4,exp(-b*x-a))/b^3
```

Rubi [A]

time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6349, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4(-e^{-a-bx})}{b^3}-\frac{\text{Li}_4(e^{-a-bx})}{b^3}+\frac{x\text{Li}_3(-e^{-a-bx})}{b^2}-\frac{x\text{Li}_3(e^{-a-bx})}{b^2}+\frac{x^2\text{Li}_2(-e^{-a-bx})}{2b}-\frac{x^2\text{Li}_2(e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (x^2*PolyLog[2, -E^(-a - b*x)])/(2*b) - (x^2*PolyLog[2, E^(-a - b*x)])/(2*b
) + (x*PolyLog[3, -E^(-a - b*x)])/b^2 - (x*PolyLog[3, E^(-a - b*x)])/b^2 +
PolyLog[4, -E^(-a - b*x)]/b^3 - PolyLog[4, E^(-a - b*x)]/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

```
IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \coth^{-1}\left(e^{a+bx}\right)\,dx &= -\left(\frac{1}{2}\int x^2 \log\left(1-e^{-a-bx}\right)\,dx\right)+\frac{1}{2}\int x^2 \log\left(1+e^{-a-bx}\right)\,dx\\ &= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b}-\frac{\int x\text{Li}_2\left(-e^{-a-bx}\right)\,dx}{b}+\frac{\int x\text{Li}_2\left(e^{-a-bx}\right)\,dx}{b}\\ &= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2}-\frac{\int \text{Li}_3(-e}{b}\\ &= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2}+\frac{\text{Subst}\Big(\int}{}\\ &= \frac{x^2\text{Li}_2\left(-e^{-a-bx}\right)}{2b}-\frac{x^2\text{Li}_2\left(e^{-a-bx}\right)}{2b}+\frac{x\text{Li}_3\left(-e^{-a-bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{-a-bx}\right)}{b^2}+\frac{\text{Li}_4(-e}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 149, normalized size = 1.25

$$\frac{2b^3x^3\coth^{-1}\left(e^{a+bx}\right)+b^3x^3\log\left(1-e^{a+bx}\right)-b^3x^3\log\left(1+e^{a+bx}\right)-3b^2x^2\text{PolyLog}\left(2,-e^{a+bx}\right)+3b^2x^2\text{PolyLog}\left(2,e^{a+bx}\right)+6bx\text{PolyLog}\left(3,-e^{a+bx}\right)-6bx\text{PolyLog}\left(3,e^{a+bx}\right)-6\text{PolyLog}\left(4,-e^{a+bx}\right)+6\text{PolyLog}\left(4,e^{a+bx}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^(a + b*x)],x]
```

```
[Out] (2*b^3*x^3*ArcCoth[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Lo
g[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog
[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a +
 b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(109) = 218$.

time = 0.16, size = 325, normalized size = 2.73

method	result
risch	$-\frac{x^3 \ln(e^{bx+a}-1)}{6}+\frac{\ln(1-e^{bx+a})x^3}{6}-\frac{\ln(1-e^{bx+a})x\,a^2}{2b^2}+\frac{x^2\,\mathrm{polylog}(2,e^{bx+a})}{2b}-\frac{a^3\ln(1-e^{bx+a})}{3b^3}-\frac{a^3\ln(e^{bx+a}-1)}{6b^3}-$
default	$\frac{x^3\mathrm{arccoth}(e^{bx+a})}{3}+\frac{-\frac{(bx+a)^3\ln\left(e^{bx+a}+1\right)}{2}-\frac{3(bx+a)^2\,\mathrm{polylog}\left(2,-e^{bx+a}\right)}{2}+3(bx+a)\,\mathrm{polylog}(3,-e^{bx+a})-3\,\mathrm{polylog}(4,-e^{bx+a})+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arccoth(exp(b*x+a))+1/3/b^3*(-1/2*(b*x+a)^3*ln(exp(b*x+a)+1)-3/2*(b
*x+a)^2*polylog(2,-exp(b*x+a))+3*(b*x+a)*polylog(3,-exp(b*x+a))-3*polylog(4
,-exp(b*x+a))+1/2*(b*x+a)^3*ln(1-exp(b*x+a))+3/2*(b*x+a)^2*polylog(2,exp(b*
x+a))-3*(b*x+a)*polylog(3,exp(b*x+a))+3*polylog(4,exp(b*x+a))+a^3*arctanh(e
xp(b*x+a))+3/2*a*(b*x+a)^2*ln(exp(b*x+a)+1)+3*a*(b*x+a)*polylog(2,-exp(b*x+
a))-3*a*polylog(3,-exp(b*x+a))-3/2*a*(b*x+a)^2*ln(1-exp(b*x+a))-3*a*(b*x+a)
*polylog(2,exp(b*x+a))+3*a*polylog(3,exp(b*x+a))-3/2*a^2*(b*x+a)*ln(exp(b*x
+a)+1)-3/2*a^2*polylog(2,-exp(b*x+a))+3/2*a^2*(b*x+a)*ln(1-exp(b*x+a))+3/2*
a^2*polylog(2,exp(b*x+a)))
```

Maxima [A]

time = 0.27, size = 142, normalized size = 1.19

$$\frac{1}{3}\,x^3\,\mathrm{arcoth}\left(e^{(bx+a)}\right)-\frac{1}{6}\,b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2(-e^{(bx+a)})-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}-\frac{b^3x^3\log\left(-e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2(e^{(bx+a)})-6\,bx\mathrm{Li}_3(e^{(bx+a)})+6\,\mathrm{Li}_4(e^{(bx+a)})}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2
*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e
^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x
+ a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(107) = 214$.

time = 0.35, size = 247, normalized size = 2.08

$$\frac{b^3x^3\log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^3x^3\log(\cosh(bx+a)+\sinh(bx+a)+1)+3\,b^2x^2\mathrm{Li}_2(\cosh(bx+a)+\sinh(bx+a))-3\,b^2x^2\mathrm{Li}_2(-\cosh(bx+a)-\sinh(bx+a))-a^3\log(\cosh(bx+a)+\sinh(bx+a)-1)-6\,bx\mathrm{polylog}(3,\cosh(bx+a)+\sinh(bx+a))+6\,bx\mathrm{polylog}(3,-\cosh(bx+a)-\sinh(bx+a))+(b^3x^3+a^3)\log(-\cosh(bx+a)-\sinh(bx+a)+1)+6\,\mathrm{polylog}(4,\cosh(bx+a)+\sinh(bx+a))-6\,\mathrm{polylog}(4,-\cosh(bx+a)-\sinh(bx+a))}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log((cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(
b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^2
*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) - si
nh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polylog(3
, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - sinh(b
*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*poly
log(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a) - sinh(
b*x + a)))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(e^a e^{bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(b*x+a)),x)
[Out] Integral(x**2*acoth(exp(a)*exp(b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="giac")
[Out] integrate(x^2*arccoth(e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(exp(a + b*x)),x)
[Out] int(x^2*acoth(exp(a + b*x)), x)
```

3.289 $\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=168

$$-\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)}+\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right)}\right)}{d\log(f)}+\frac{\text{PolyLog}\left(2,1-\frac{2}{1+a+bf}\right.}{2d\log(f)}$$

```
[Out] -arccoth(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arccoth(a+b*f^(d*x+
c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(1+
a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c
)))/d/ln(f)
```

Rubi [A]
time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 6247, 6058, 2449, 2352, 2497}

$$\frac{\text{Li}_2\left(1-\frac{2}{bf^{c+dx}+a+1}\right)}{2d\log(f)}-\frac{\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\coth^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)\coth^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcCoth[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcCoth[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6058

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcCoth[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcCoth[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6247

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \coth^{-1}\left(a+bf^{c+dx}\right)\,dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}$$

$$= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+b}\right.}{d\log(f)}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+b}\right.}{d\log(f)}$$

$$= -\frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\coth^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+b}\right.}{d\log(f)}$$

Mathematica [A]
time = 0.05, size = 108, normalized size = 0.64

$$\frac{dx\log(f)\left(2\coth^{-1}\left(a+bf^{c+dx}\right)+\log\left(\frac{-1+a+bf^{c+dx}}{-1+a}\right)-\log\left(\frac{1+a+bf^{c+dx}}{1+a}\right)\right)+\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (d*x*Log[f]*(2*ArcCoth[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(-
1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c + d*
x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

Maple [A]
time = 0.52, size = 160, normalized size = 0.95

method	result
derivativedivides	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccoth}\left(a+b\,f^{dx+c}\right)-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}-\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a+1}{1-a}\right)}{2}+}{d\ln(f)}$
default	$\frac{\ln\left(-b\,f^{dx+c}\right)\operatorname{arccoth}\left(a+b\,f^{dx+c}\right)-\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}-\frac{\ln\left(-b\,f^{dx+c}\right)\ln\left(\frac{-b\,f^{dx+c}-a-1}{-a-1}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{-b\,f^{dx+c}-a+1}{1-a}\right)}{2}+}{d\ln(f)}$
risch	$-\frac{x\ln\left(a+b\,f^{dx+c}-1\right)}{2}+\frac{\operatorname{dilog}\left(\frac{b\,f^{dx}\,f^{c}+a-1}{-1+a}\right)}{2\ln(f)d}+\frac{\ln\left(\frac{b\,f^{dx}\,f^{c}+a-1}{-1+a}\right)x}{2}+\frac{\ln\left(\frac{b\,f^{dx}\,f^{c}+a-1}{-1+a}\right)c}{2d}-\frac{c\ln\left(b\,f^{dx}\,f^{c}+a-1\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/ln(f)*(ln(-b*f^(d*x+c))*arccoth(a+b*f^(d*x+c))-1/2*dilog((-b*f^(d*x+c)-
a-1)/(-a-1))-1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a-1)/(-a-1))+1/2*dilog((
-b*f^(d*x+c)-a+1)/(1-a))+1/2*ln(-b*f^(d*x+c))*ln((-b*f^(d*x+c)-a+1)/(1-a)))
```

Maxima [A]
time = 0.26, size = 202, normalized size = 1.20

$$\frac{(dx+c)\operatorname{arcoth}\left(bf^{dx+c}+a\right)}{d}-\frac{(dx+c)b\left(\frac{\log(bf^{dx+c}+a+1)}{b}-\frac{\log(bf^{dx+c}+a-1)}{b}\right)\log(f)-b\left(\frac{\log(bf^{dx+c}+a+1)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right)+\text{Li}_2\left(\frac{bf^{dx+c}+a+1}{a+1}\right)}{b}-\frac{\log(bf^{dx+c}+a-1)\log\left(-\frac{bf^{dx+c}+a-1}{a-1}+1\right)+\text{Li}_2\left(\frac{bf^{dx+c}+a-1}{a-1}\right)}{b}\right)}{2\,d\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arccoth(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x + c
) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x + c)
 + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x + c)
```

```
+ a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c) + a
- 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log(f))
```

Fricas [A]
time = 0.36, size = 283, normalized size = 1.68

$$\frac{dx\log\left(f\right)\log\left(\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}\right)+c\log\left(b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1\right)\log\left(f\right)-c\log\left(b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1\right)\log\left(f\right)-\left(dx+c\right)\log\left(f\right)\log\left(\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{a+1}\right)+\left(dx+c\right)\log\left(f\right)\log\left(\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}{a-1}\right)-\mathrm{Li}_2\left(-\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a+1}{a+1}+1\right)+\mathrm{Li}_2\left(-\frac{b\cosh\left(\left(dx+c\right)\log\left(f\right)\right)+b\sinh\left(\left(dx+c\right)\log\left(f\right)\right)+a-1}{a-1}+1\right)}{2\,d\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c*l
og(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) - c*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) - (
d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*s
inh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log(
f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}{\left(a + b f^{c + d x} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(acoth(a + b*f**(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.55Unable to divide, perhaps due to rou
nding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,
0,0]%%%}+
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acoth}\left(a+b\,f^{c+d\,x}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*f^(c + d*x)),x)
[Out] int(acoth(a + b*f^(c + d*x)), x)
```

3.290 $\int x \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=216

$$\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{x\mathrm{L}}{}$$

```
[Out] 1/4*x^2*ln(1-b*f^(d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))-1/4*x^2*ln(
1-1/(a+b*f^(d*x+c)))+1/4*x^2*ln(1+1/(a+b*f^(d*x+c)))+1/2*x*polylog(2,b*f^(d
*x+c)/(1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*polylo
g(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln
(f)^2
```

Rubi [A]
time = 1.89, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6349, 2631, 12, 6874, 2221, 2611, 2320, 6724}

$$-\frac{\mathrm{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\mathrm{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\mathrm{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\mathrm{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(\frac{bf^{c+dx}}{a+1}+1\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(\frac{1}{a+bf^{c+dx}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 - (x^2*Log[1 - (a + b*f^(c + d*x))^(-1)])/4 + (x^2*Log[1 + (a + b*
f^(c + d*x))^(-1)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]
) - (x*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b
*f^(c + d*x))/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 +
 a))]/(2*d^2*Log[f]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x\coth^{-1}\left(a+bf^{c+dx}\right)\,dx &= -\left(\frac{1}{2}\int x\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\,dx\right)+\frac{1}{2}\int x\log\left(1+\frac{1}{a+bf^{c+dx}}\right)\,dx \\
&= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}\int\frac{bdf}{(-1+a+} \\
&= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int \\
&= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int \\
&= -\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{4}(bd\log(f))\int \\
&= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right. \\
&= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right. \\
&= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right. \\
&= \frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{4}x^2\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{4}x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 177, normalized size = 0.82

$$
\frac{2d^2x^2\coth^{-1}\left(a+bf^{c+dx}\right)\log^2(f)+d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^2x^2\log^2(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-2dx\log(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)-2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{-1+a}\right)+2\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)}{4d^2\log^2(f)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^2*x^2*ArcCoth[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*Pol
yLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-1 +
a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(200) = 400$.
time = 0.07, size = 590, normalized size = 2.73

method	result
risch	$-\frac{x^2 \ln(a+b f^{dx+c}-1)}{4}+\frac{x^2 \ln(1+a+b f^{dx+c})}{4}-\frac{\ln\left(1-\frac{b f^{dx} f^c}{-a-1}\right) x^2}{4}-\frac{\ln\left(1-\frac{b f^{dx} f^c}{-a-1}\right) x c}{2d}-\frac{\ln\left(1-\frac{b f^{dx} f^c}{-a-1}\right) c^2}{4d^2}-\frac{\operatorname{polylog}\left(2,\right.}{2 \ln}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2*ln(a+b*f^(d*x+c)-1)+1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*ln(1-b*f^(d*x)
*f^c/(-a-1))*x^2-1/2/d*ln(1-b*f^(d*x)*f^c/(-a-1))*x*c-1/4/d^2*ln(1-b*f^(d*x
)*f^c/(-a-1))*c^2-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-a-1))*x-1/2/ln(f)/d
^2*polylog(2,b*f^(d*x)*f^c/(-a-1))*c+1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^
c/(-a-1))-1/4/d^2*c^2*ln(1+a+b*f^(d*x)*f^c)+1/2/ln(f)/d^2*c*dilog((1+a+b*f^
(d*x)*f^c)/(1+a))+1/2/d*c*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x+1/2/d^2*c^2*ln((1
+a+b*f^(d*x)*f^c)/(1+a))+1/4*ln(1-b*f^(d*x)*f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(
d*x)*f^c/(1-a))*x*c+1/4/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*c^2+1/2/ln(f)/d*polyl
og(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/(1-a))*c-
1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))+1/4/d^2*c^2*ln(b*f^(d*x)*f^c
+a-1)-1/2/ln(f)/d^2*c*dilog((b*f^(d*x)*f^c+a-1)/(-1+a))-1/2/d*c*ln((b*f^(d*
x)*f^c+a-1)/(-1+a))*x-1/2/d^2*c^2*ln((b*f^(d*x)*f^c+a-1)/(-1+a))
```

Maxima [A]
time = 0.30, size = 194, normalized size = 0.90

$$-\frac{1}{4} bd \left(\frac{d^2x^2 \log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)-2\,\mathrm{Li}_3\left(-\frac{bf^{dx}f^c}{a+1}\right)}{bd^3\log(f)^3}-\frac{d^2x^2 \log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a-1}\right)\log(f)-2\,\mathrm{Li}_3\left(-\frac{bf^{dx}f^c}{a-1}\right)}{bd^3\log(f)^3}\right)\log(f)+\frac{1}{2}x^2\operatorname{arcoth}\left(bf^{dx+c}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-b
*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d^3
*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*dilog
(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))/(b*
d^3*log(f)^3))*log(f) + 1/2*x^2*arccoth(b*f^(d*x + c) + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(193) = 386.
time = 0.35, size = 395, normalized size = 1.83

$$\frac{d^2x^2\log(f)^2\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)-c^2\log\left(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1\right)\log(f)^2+c^2\log\left(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1\right)\log(f)^2-2\,dx\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}+1\right)\log(f)+2\,dx\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}+1\right)\log(f)-(d^2x^2-c^2)\log(f)^2\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}\right)+(d^2x^2-c^2)\log(f)^2\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}\right)+2\operatorname{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a+1}\right)-2\operatorname{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a-1}\right)}{4\,d^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^2*x^2*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1))
 - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log
(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1
)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) +
 b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 - c^2)*lo
g(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)/(a
 + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d
*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x + c)*log(f))
 + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}\left(a + bf^{c}f^{dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x*acoth(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(b*f^(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}\left(a + b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a + b*f^(c + d*x)),x)
```

```
[Out] int(x*acoth(a + b*f^(c + d*x)), x)
```

3.291 $\int x^2 \coth^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=269

$$\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{x^2\mathrm{P}}{}$$

```
[Out] 1/6*x^3*ln(1-b*f^(d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))-1/6*x^3*ln(
1-1/(a+b*f^(d*x+c)))+1/6*x^3*ln(1+1/(a+b*f^(d*x+c)))+1/2*x^2*polylog(2,b*f^
(d*x+c)/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*poly
log(3,b*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln
(f)^2+polylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/(1+a)
)/d^3/ln(f)^3
```

Rubi [A]
time = 1.74, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6349, 2631, 12, 6874, 2221, 2611, 6744, 2320, 6724}

$$\frac{\text{Li}_4\left(\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{Li}_4\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}-\frac{x\text{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{x^2\text{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x^2\text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(\frac{bf^{c+dx}}{a+1}+1\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(\frac{1}{a+bf^{c+dx}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 - (x^3*Log[1 - (a + b*f^(c + d*x))^(-1)])/6 + (x^3*Log[1 + (a + b*
f^(c + d*x))^(-1)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[
f]) - (x^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLo
g[3, (b*f^(c + d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d
*x))/(1 + a))])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*L
og[f]^3) - PolyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 6349

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
 IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^2\coth^{-1}\left(a+bf^{c+dx}\right)\,dx &= -\left(\frac{1}{2}\int x^2\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\,dx\right)+\frac{1}{2}\int x^2\log\left(1+\frac{1}{a+bf^{c+dx}}\right)\,dx\\
&= -\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}\int\frac{bdf^{c}}{(-1+a+}\\
&= -\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\\
&= -\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\\
&= -\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}x^3\log\left(1+\frac{1}{a+bf^{c+dx}}\right)+\frac{1}{6}(bd\log(f))\int\\
&= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\\
&= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\\
&= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\\
&= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)\\
&= \frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-\frac{1}{6}x^3\log\left(1+\frac{bf^{c+dx}}{1+a}\right)-\frac{1}{6}x^3\log\left(1-\frac{1}{a+bf^{c+dx}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 235, normalized size = 0.87

$$\frac{2d^3x^3\coth^{-1}\left(a+bf^{c+dx}\right)\log^3(f)+d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{-1+a}\right)-d^3x^3\log^3(f)\log\left(1+\frac{bf^{c+dx}}{1+a}\right)+3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{-1+a}\right)-3d^2x^2\log^2(f)\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{1+a}\right)-6dx\log(f)\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{-1+a}\right)+6dx\log(f)\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{1+a}\right)+6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{-1+a}\right)-6\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{1+a}\right)}{6d^3\log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^3*x^3*ArcCoth[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 + a)
```

```
] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x^2*
Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[3, -
((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(1
+ a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((b*f^(c
 + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(257) = 514$.
time = 0.07, size = 666, normalized size = 2.48

method	result
risch	$-\frac{x^3\ln\left(a+b\,f^{dx+c}-1\right)}{6}+\frac{x^3\ln\left(1+a+b\,f^{dx+c}\right)}{6}-\frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{-a-1}\right)x^3}{6}+\frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{-a-1}\right)x\,c^2}{2d^2}+\frac{\ln\left(1-\frac{b\,f^{dx}\,f^{c}}{-a-1}\right)c^3}{3d^3}-\frac{\text{polylog}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^3*ln(a+b*f^(d*x+c)-1)+1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*ln(1-b*f^(d*x)
*f^c/(-a-1))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-a-1))*x*c^2+1/3/d^3*ln(1-b*f^
(d*x)*f^c/(-a-1))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(-a-1))*x^2+1/2/l
n(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-a-1))*c^2+1/ln(f)^2/d^2*polylog(3,b*f^(d
*x)*f^c/(-a-1))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(-a-1))+1/6/d^3*c^3
*ln(1+a+b*f^(d*x)*f^c)-1/2/ln(f)/d^3*c^2*dilog((1+a+b*f^(d*x)*f^c)/(1+a))-1
/2/d^2*c^2*ln((1+a+b*f^(d*x)*f^c)/(1+a))*x-1/2/d^3*c^3*ln((1+a+b*f^(d*x)*f^
c)/(1+a))+1/6*ln(1-b*f^(d*x)*f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a
))*x*c^2-1/3/d^3*ln(1-b*f^(d*x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d
*x)*f^c/(1-a))*x^2-1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)
^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f
^c/(1-a))-1/6/d^3*c^3*ln(b*f^(d*x)*f^c+a-1)+1/2/ln(f)/d^3*c^2*dilog((b*f^(d
*x)*f^c+a-1)/(-1+a))+1/2/d^2*c^2*ln((b*f^(d*x)*f^c+a-1)/(-1+a))*x+1/2/d^3*c
^3*ln((b*f^(d*x)*f^c+a-1)/(-1+a))
```

Maxima [A]
time = 0.29, size = 254, normalized size = 0.94

$$\frac{1}{3}\,x^3\,\mathrm{arcoth}\left(bf^{dx+c}+a\right)-\frac{1}{6}\,bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log\left(f\right)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})+6\,\mathrm{Li}_4(-\frac{bf^{dx}f^c}{a+1})}{bd^4\log\left(f\right)^4}-\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log\left(f\right)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a-1}\right)\log\left(f\right)^2-6\,dx\log\left(f\right)\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a-1})+6\,\mathrm{Li}_4(-\frac{bf^{dx}f^c}{a-1})}{bd^4\log\left(f\right)^4}\right)\log\left(f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(a
 + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 - 6*
d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*f^c
/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f
)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*polyl
```

```
og(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^
4*log(f)^4))*log(f)
```

Fricas [A]
time = 0.39, size = 479, normalized size = 1.78

$$\frac{d^3x^3\log(f)^3\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}\right)-3\,d^2x^2\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}+1\right)\log(f)^2+3\,d^2x^2\mathrm{Li}_2\left(-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}+1\right)\log(f)^2+c^3\log(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1)\log(f)^3-c^3\log(b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1)\log(f)^3-(d^3x^3+c^3)\log(f)^3\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a+1}{a+1}\right)+(d^3x^3+c^3)\log(f)^3\log\left(\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))+a-1}{a-1}\right)+6\,dx\log(f)\,\mathrm{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a+1}\right)-6\,dx\log(f)\,\mathrm{polylog}\left(3,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a-1}\right)-6\,\mathrm{polylog}\left(4,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a+1}\right)+6\,\mathrm{polylog}\left(4,-\frac{b\cosh((dx+c)\log(f))+b\sinh((dx+c)\log(f))}{a-1}\right)}{6\,d^3\log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(d^3*x^3*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(
f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1))
 - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) +
 b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*cosh((
d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*log(b*c
osh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 - (d^3*x
^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f))
 + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f))
 + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*polylog(3, -(b
*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6*d*x*log(f)
*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1))
 - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a +
 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/
(a - 1)))/(d^3*log(f)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}\left(a + bf^c f^{dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Integral(x**2*acoth(a + b*f**c*f**(d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*f^(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}\left(a + b\, f^{c+d\,x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a + b*f^(c + d*x)),x)
[Out] int(x^2*acoth(a + b*f^(c + d*x)), x)
```

3.292 $\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)} \, dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

```
[Out] -1/2*ln(1-2*arccoth(x))/a/b
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6094}

$$-\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

```
[Out] -1/2*Log[1 - 2*ArcCoth[x]]/(a*b)
```

Rule 6094

```
Int[1/(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(a-ax^2)\left(b-2b\coth^{-1}(x)\right)} \, dx = -\frac{\log\left(1-2\coth^{-1}(x)\right)}{2ab}$$

Mathematica [A]
time = 0.04, size = 17, normalized size = 1.00

$$-\frac{\log\left(-1+2\coth^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])),x]
```

```
[Out] -1/2*Log[-1 + 2*ArcCoth[x]]/(a*b)
```

Maple [A]
time = 0.10, size = 19, normalized size = 1.12

method	result	size
default	$-\frac{\ln(2b\,\mathrm{arccoth}(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-1+\ln(1+x)-\ln(-1+x))}{2ba}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*ln(2*b*arccoth(x)-b)/b
```

Maxima [A]
time = 0.27, size = 21, normalized size = 1.24

$$-\frac{\log\left(\log\left(x+1\right)-\log\left(x-1\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(log(x + 1) - log(x - 1) - 1)/(a*b)
```

Fricas [A]
time = 0.33, size = 21, normalized size = 1.24

$$-\frac{\log\left(\log\left(\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(log((x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A]
time = 0.23, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{acoth}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)
```

```
[Out] -log(acoth(x) - 1/2)/(2*a*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.
time = 0.41, size = 44, normalized size = 2.59

$$-\frac{\log\left(\frac{1}{4}\pi^2(\operatorname{sgn}(x+1)\operatorname{sgn}(x-1)-1)^2+\left(\log\left(\frac{|x+1|}{|x-1|}\right)-1\right)^2\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(1/4*pi^2*(sgn(x + 1)*sgn(x - 1) - 1)^2 + (log(abs(x + 1)/abs(x - 1
)) - 1)^2)/(a*b)
```

Mupad [B]
time = 1.40, size = 15, normalized size = 0.88

$$-\frac{\ln\left(2\,\mathrm{acoth}(x)-1\right)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x^2)*(b - 2*b*acoth(x))),x)
```

```
[Out] -log(2*acoth(x) - 1)/(2*a*b)
```

3.293 $\int x^3 \coth^{-1}(a+bx^4)\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^4)\coth^{-1}(a+bx^4)}{4b}+\frac{\log\left(1-(a+bx^4)^2\right)}{8b}$$

```
[Out] 1/4*(b*x^4+a)*arccoth(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6847, 6239, 6022, 266}

$$\frac{\log\left(1-(a+bx^4)^2\right)}{8b}+\frac{(a+bx^4)\coth^{-1}(a+bx^4)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCoth[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcCoth[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \coth^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \coth^{-1}(a+bx)\,dx,x,x^4\right)\\
&= \frac{\text{Subst}\left(\int \coth^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\coth^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{(a+bx^4)\coth^{-1}\left(a+bx^4\right)}{4b}+\frac{\log\left(1-(a+bx^4)^2\right)}{8b}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 39, normalized size = 0.89

$$\frac{2(a+bx^4)\coth^{-1}\left(a+bx^4\right)+\log\left(1-(a+bx^4)^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x^4],x]
[Out] (2*(a + b*x^4)*ArcCoth[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

Maple [**A**]
time = 0.05, size = 37, normalized size = 0.84

method	result
derivativedivides	$\frac{(b\,x^4+a)\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\operatorname{arccoth}(b\,x^4+a)+\frac{\ln\left((b\,x^4+a)^2-1\right)}{2}}{4b}$
risch	$\frac{x^4\ln(b\,x^4+a+1)}{8}-\frac{x^4\ln(b\,x^4+a-1)}{8}+\frac{\ln(b\,x^4+a+1)a}{8b}-\frac{\ln(-b\,x^4-a+1)a}{8b}+\frac{\ln(b\,x^4+a+1)}{8b}+\frac{\ln(-b\,x^4-a+1)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arccoth(b*x^4+a)+1/2*ln((b*x^4+a)^2-1))
```

Maxima [**A**]
time = 0.27, size = 37, normalized size = 0.84

$$\frac{2\left(bx^4+a\right)\operatorname{arcoth}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arccoth(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

Fricas [A]
time = 0.35, size = 58, normalized size = 1.32

$$\frac{bx^4\log\left(\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(b*x^4*log((b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a + 1
) - (a - 1)*log(b*x^4 + a - 1))/b
```

Sympy [A]
time = 0.95, size = 60, normalized size = 1.36

$$\begin{cases}\frac{a\operatorname{acoth}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acoth}\left(a+bx^4\right)}{4}+\frac{\log\left(a+bx^4+1\right)}{4b}-\frac{\operatorname{acoth}\left(a+bx^4\right)}{4b} & \text{for } b\neq 0\\ \frac{x^4\operatorname{acoth}(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x**4+a),x)
```

```
[Out] Piecewise((a*acoth(a + b*x**4)/(4*b) + x**4*acoth(a + b*x**4)/4 + log(a + b
*x**4 + 1)/(4*b) - acoth(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*acoth(a)/4, Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(40) = 80.
time = 0.42, size = 225, normalized size = 5.11

$$\frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx^4+a+1|}{|bx^4+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^4+a+1}{bx^4+a-1}-1\right|\right)}{b^2}+\frac{\log\left(-\frac{\frac{1}{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}}+1}{\frac{1}{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")

[Out] 1/8*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^4 + a + 1)/abs(b*x^4 + a - 1))/b^2 - log(abs((b*x^4 + a + 1)/(b*x^4 + a - 1) - 1))/b^2 + log(-(1/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a - 1) - b)) + 1)/(1/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a - 1) - b)) - 1))/(b^2*((b*x^4 + a + 1)/(b*x^4 + a - 1) - 1)))

Mupad [B]
time = 1.56, size = 107, normalized size = 2.43

$$\frac{x^4\,\ln\left(\frac{b\,x^4+a+1}{b\,x^4+a}\right)}{8}-\frac{x^4\,\ln\left(\frac{b\,x^4+a-1}{b\,x^4+a}\right)}{8}+\frac{\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{\ln\left(b\,x^4+a+1\right)}{8\,b}-\frac{a\,\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{a\,\ln\left(b\,x^4+a+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*acoth(a + b*x^4),x)

[Out] (x^4*log((a + b*x^4 + 1)/(a + b*x^4)))/8 - (x^4*log((a + b*x^4 - 1)/(a + b*x^4)))/8 + log(a + b*x^4 - 1)/(8*b) + log(a + b*x^4 + 1)/(8*b) - (a*log(a + b*x^4 - 1))/(8*b) + (a*log(a + b*x^4 + 1))/(8*b)

3.294 $\int x^{-1+n} \coth^{-1}(a+bx^n)\, dx$

Optimal. Leaf size=47

$$\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

```
[Out] (a+b*x^n)*arccoth(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6847, 6239, 6022, 266}

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcCoth[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6022

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcCoth[c*x^n])^
(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
&& (EqQ[n, 1] || EqQ[p, 1])
```

Rule 6239

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int x^{-1+n}\coth^{-1}(a+bx^n)\,dx=\frac{\text{Subst}\left(\int\coth^{-1}(a+bx)\,dx,x,x^n\right)}{n}$$
$$=\frac{\text{Subst}\left(\int\coth^{-1}(x)\,dx,x,a+bx^n\right)}{bn}$$
$$=\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int\frac{x}{1-x^2}\,dx,x,a+bx^n\right)}{bn}$$
$$=\frac{(a+bx^n)\coth^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.89

$$\frac{2(a+bx^n)\coth^{-1}(a+bx^n)+\log\left(1-(a+bx^n)^2\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCoth[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcCoth[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(45) = 90.
time = 0.05, size = 118, normalized size = 2.51

method	result	size
risch	$\frac{x^n\ln(a+b\,x^n+1)}{2n}-\frac{x^n\ln(a+b\,x^n-1)}{2n}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)a}{2nb}-\frac{\ln\left(x^n+\frac{-1+a}{b}\right)a}{2nb}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)}{2nb}+\frac{\ln\left(x^n+\frac{-1+a}{b}\right)}{2nb}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arccoth(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*x^n*ln(a+b*x^n+1)-1/2/n*x^n*ln(a+b*x^n-1)+1/2/n/b*ln(x^n+(1+a)/b)*a-1
/2/n/b*ln(x^n+(-1+a)/b)*a+1/2/n/b*ln(x^n+(1+a)/b)+1/2/n/b*ln(x^n+(-1+a)/b)
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.85

$$\frac{2\,(bx^n+a)\,\text{arcoth}\,(bx^n+a)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arccoth(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(45) = 90$.
time = 0.36, size = 108, normalized size = 2.30

$$\frac{(a+1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a+1\right)-(a-1)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a-1\right)+\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)\right)\log\left(\frac{b\cosh(n\log(x))+b\sinh(n\log(x))+a+1}{b\cosh(n\log(x))+b\sinh(n\log(x))+a-1}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*log
(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sinh(
n*log(x)))*log((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*log(
x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acoth(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.
time = 0.42, size = 119, normalized size = 2.53

$$\frac{\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|bx^n+a+1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|\frac{bx^n+a+1}{bx^n+a-1}-1\right|\right)}{b^2}+\frac{\log\left(\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - (a - 1)*b)*(log(abs(b*x^n + a + 1)/abs(b*x^n + a - 1))/b^2
 - log(abs((b*x^n + a + 1)/(b*x^n + a - 1) - 1))/b^2 + log((b*x^n + a + 1)/
(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

Mupad [B]
time = 2.51, size = 58, normalized size = 1.23

$$\frac{\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n-1\right)}{2}+a\,\mathrm{acoth}\left(a+b\,x^n\right)}{b\,n}+\frac{x^n\,\mathrm{acoth}\left(a+b\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acoth(a + b*x^n),x)
```

```
[Out] (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n - 1)/2 + a*acoth(a + b*x^n))/(b*n) + (x^
n*acoth(a + b*x^n))/n
```

3.295 $\int e^{c(a+bx)} \coth^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccoth(sinh(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 6411, 2320, 12, 1261, 646, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\coth^{-1}(\sinh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\sinh(x))\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{e^x\cosh(x)}{1-\sinh^2(x)}\,dx, x, ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{2\text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx\right.}{2bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\sinh(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2}\right.}{2bc}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 153, normalized size = 1.43

$$\frac{-2e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+\log\left(1-2e^{c(a+bx)}-e^{2c(a+bx)}\right)+\log\left(1+2e^{c(a+bx)}-e^{2c(a+bx)}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Sinh[a*c + b*c*x]],x]
```

```
[Out] (-2*E^(c*(a + b*x))*ArcCoth[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2*
Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 + E^
(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))] +
Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 794, normalized size = 7.42

method	result
risch	$\frac{e^{c(bx+a)}\ln\left(e^{2c(bx+a)}+2e^{c(bx+a)}-1\right)}{2cb}+\frac{i\pi\,\mathrm{csgn}\left(ie^{-c(bx+a)}\right)\mathrm{csgn}\left(ie^{-c(bx+a)}\left(e^{2c(bx+a)}+2e^{c(bx+a)}-1\right)\right)^{2}e^{c(bx+a)}}{4bc}-\frac{i\pi\,\mathrm{csgn}\left(i\left(e\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi
*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(
b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*
(b*x+a))-1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a
))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a
))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(
-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))
^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*e
xp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*
exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a)
)-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+
a))+1))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*ex
p(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c
*(b*x+a))+1))^3*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2
*exp(c*(b*x+a))-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b
/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2/b*a+1/2/b/c*ln(exp(2*c*(b*x
+a))-(1+2^(1/2))^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

time = 0.47, size = 184, normalized size = 1.72

$$\frac{\operatorname{arcoth}\left(\sinh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(90) = 180$.
time = 0.39, size = 233, normalized size = 2.18

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\,\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^{2}-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}+3\right)\sinh(bcx+ac)^{2}-2\sqrt{2}-3}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3\right)}{\cosh(bcx+ac)^{2}-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(s
inh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2
 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3
)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(
b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^
2)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acoth}\left(\sinh\left(ac+bcx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(sinh(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acoth(sinh(a*c + b*c*x)), x)
```

Giac [A]
time = 0.56, size = 167, normalized size = 1.56

$$\frac{e^{\left((bx+a)c\right)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\sqrt{2}\,\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sq
rt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) -
6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B]
time = 1.67, size = 187, normalized size = 1.75

$$\frac{\ln\left(6\sqrt{2}\,\mathrm{e}^{2c(a+bx)}-2\sqrt{2}-8\,\mathrm{e}^{2c(a+bx)}\right)\left(\sqrt{2}+1\right)}{2bc}-\frac{\mathrm{e}^{ac+bcx}\ln\left(1-\frac{1}{\frac{\mathrm{e}^{bcx}\,\mathrm{e}^{ac}}{2}-\frac{\mathrm{e}^{-bcx}\,\mathrm{e}^{-ac}}{2}}\right)}{2bc}-\frac{\ln\left(2\sqrt{2}-8\,\mathrm{e}^{2c(a+bx)}-6\sqrt{2}\,\mathrm{e}^{2c(a+bx)}\right)\left(\sqrt{2}-1\right)}{2bc}+\frac{\ln\left(\frac{1}{\frac{\mathrm{e}^{bcx}\,\mathrm{e}^{ac}}{2}-\frac{\mathrm{e}^{-bcx}\,\mathrm{e}^{-ac}}{2}}+1\right)\mathrm{e}^{ac+bcx}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acoth(sinh(a*c + b*c*x)),x)
```

```
[Out] (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(1
/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2 -
(exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)
) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp(b*c
*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c)
```

3.296 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx))\, dx$

Optimal. Leaf size=49

$$\frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arccoth(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2225, 6411, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\cosh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\cosh(x))\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{csch}(x)\,dx, x, ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 1.22

$$\frac{e^{c(a+bx)}\coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcCoth[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 824, normalized size = 16.82

method	result	size
risch	Expression too large to display	824

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*ex
```

```
p(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a)
)+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*
exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(e
xp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)
)*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*
(b*x+a))*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*
(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4
*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/4
*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-
c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*
(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))+1/4*I/b/c*Pi*csg
n(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+
a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(
exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(
b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(e
xp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))
-1)-2/b*a+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

Maxima [**A**]
time = 0.26, size = 64, normalized size = 1.31

$$\frac{\operatorname{arcoth}\left(\cosh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [**A**]
time = 0.37, size = 92, normalized size = 1.88

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(\frac{\cosh\left(bcx+ac\right)+1}{\cosh\left(bcx+ac\right)-1}\right)+2\,\log\left(\frac{2\,\sinh\left(bcx+ac\right)}{\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(c
osh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))))/(b*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(47) = 94.
time = 0.48, size = 98, normalized size = 2.00

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a*c
) - 1))/(b*c)
```

Mupad [B]
time = 1.83, size = 119, normalized size = 2.43

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c}-\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{2\,b\,c}+\frac{\ln\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}+1\right)\,\mathrm{e}^{a\,c+b\,c\,x}}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acoth(cosh(a*c + b*c*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(b*
c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c
)
```

3.297 $\int e^{c(a+bx)} \coth^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=45

$$-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(tanh(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2225, 6411}

$$\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Tanh[c*(a + b*x)]])/(b*
c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\tanh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\ &= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\tanh(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(-1+\coth^{-1}\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcCoth[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 349, normalized size = 7.76

method	result
risch	$\frac{e^{c(bx+a)}\ln(e^{c(bx+a)})}{cb}-\frac{i\left(\pi\,\mathrm{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\mathrm{csgn}\left(ie^{2c(bx+a)}\right)\mathrm{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right)-\pi\,\mathrm{csgn}\left(\frac{i}{e^{2c(bx+a)}+1}\right)\mathrm{csgn}\left(\frac{ie^{2c(bx+a)}}{e^{2c(bx+a)}+1}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*exp(c*(b*x+a))*ln(exp(c*(b*x+a)))-1/4*I*(Pi*csgn(I/(exp(2*c*(b*x+a))+
1))*csgn(I*exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))-
Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1
))^2+2*Pi*csgn(I/(exp(2*c*(b*x+a))+1))^3-2*Pi*csgn(I/(exp(2*c*(b*x+a))+1))^
2+Pi*csgn(I*exp(c*(b*x+a)))^2*csgn(I*exp(2*c*(b*x+a)))-2*Pi*csgn(I*exp(c*(b
*x+a)))*csgn(I*exp(2*c*(b*x+a)))^2+Pi*csgn(I*exp(2*c*(b*x+a)))^3-Pi*csgn(I*
exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))^2+Pi*csgn(I
*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))+1))^3-4*I+2*Pi)/c/b*exp(c*(b*x+a))
```

Maxima [A]
time = 0.26, size = 43, normalized size = 0.96

$$\frac{\operatorname{arcoth}\left(\tanh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}-\frac{e^{\left(bcx+ac\right)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.56

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.76, size = 63, normalized size = 1.40

$$\begin{cases} \frac{i\pi x}{2} & \text{for}\ c = 0 \wedge (b = 0 \vee c = 0) \\ xe^{ac} \operatorname{acoth}(\tanh(ac)) & \text{for}\ b = 0 \\ \frac{e^{ac}e^{bcx} \operatorname{acoth}(\tanh(ac+bcx))}{bc} - \frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(tanh(b*c*x+a*c)),x)
```

```
[Out] Piecewise((I*pi*x/2, Eq(c, 0) & (Eq(b, 0) | Eq(c, 0))), (x*exp(a*c)*acoth(t
anh(a*c)), Eq(b, 0)), (exp(a*c)*exp(b*c*x)*acoth(tanh(a*c + b*c*x))/(b*c) -
 exp(a*c)*exp(b*c*x)/(b*c), True))
```

Giac [A]
time = 0.40, size = 40, normalized size = 0.89

$$\frac{\left(e^{(bcx)} \log\left(-e^{(2\,bcx+2\,ac)}\right) - 2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

Mupad [B]
time = 0.10, size = 28, normalized size = 0.62

$$\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\left(\mathrm{acoth}(\tanh(a\,c+b\,c\,x))-1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acoth(tanh(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*(acoth(tanh(a*c + b*c*x)) - 1))/(b*c)
```

3.298 $\int e^{c(a+bx)} \coth^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=45

$$-\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arccoth(coth(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2225, 6411}

$$\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcCoth[Coth[c*(a + b*x)]])/(b*
c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\coth^{-1}(\coth(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\coth^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\
&= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\coth(c(a+bx)))}{bc}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(-1+\coth^{-1}\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcCoth[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

Maple [A]
time = 0.10, size = 68, normalized size = 1.51

method	result
default	$\frac{e^{xbc+ac}(xbc+ac)-e^{xbc+ac}+e^{xbc+ac}(\operatorname{arccoth}(\coth(xbc+ac))-xbc-ac)}{bc}$
risch	$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{cb}-\frac{i\left(\pi\operatorname{csgn}\left(ie^{c(bx+a)}\right)^{2}\operatorname{csgn}\left(ie^{2c(bx+a)}\right)-2\pi\operatorname{csgn}\left(ie^{c(bx+a)}\right)\operatorname{csgn}\left(ie^{2c(bx+a)}\right)^{2}+\pi\operatorname{csgn}\left(ie^{2c(bx+a)}\right)^{3}+\pi\operatorname{csgn}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/c*(exp(b*c*x+a*c)*(b*c*x+a*c)-exp(b*c*x+a*c)+exp(b*c*x+a*c)*(arccoth(co
th(b*c*x+a*c))-x*b*c-a*c))
```

Maxima [A]
time = 0.25, size = 42, normalized size = 0.93

$$\frac{ae^{(bcx+ac)}}{b}+\frac{\left(bcxe^{(ac)}-e^{(ac)}\right)e^{(bcx)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] a*e^(b*c*x + a*c)/b + (b*c*x*e^(a*c) - e^(a*c))*e^(b*c*x)/(b*c)
```

Fricas [A]
time = 0.32, size = 25, normalized size = 0.56

$$\frac{(bcx+ac-1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acoth}\left(\coth\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acoth(coth(a*c + b*c*x)), x)
```

Giac [A]
time = 0.42, size = 35, normalized size = 0.78

$$\frac{(b^2c^2x+abc^2-bc)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(coth(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] (b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

Mupad [B]
time = 1.21, size = 28, normalized size = 0.62

$$\frac{\mathrm{e}^{a\,c+b\,c\,x}\,\left(\mathrm{acoth}(\mathrm{coth}(a\,c+b\,c\,x))-1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*acoth(coth(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*(acoth(coth(a*c + b*c*x)) - 1))/(b*c)
```

3.299 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

```
[Out] exp(b*c*x+a*c)*arccoth(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2225, 6411, 2320, 12, 266}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\coth^{-1}(\operatorname{sech}(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^x\coth^{-1}(\operatorname{sech}(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int e^x\operatorname{csch}(x)\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{2\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}\\ &= \frac{e^{ac+bcx}\coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 59, normalized size = 1.20

$$\frac{e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Sech[a*c + b*c*x]],x]
[Out] (E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 939, normalized size = 19.16

method	result	size
risch	Expression too large to display	939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
[Out] -1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp
(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+
```

```
1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(ex
p(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b
*x+a))*Pi+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+
1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x
+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c
*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(
exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a))-1/4*I/b/c*
Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c
*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(
exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I/b/c*
Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a
))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I/(e
xp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))*exp(c
*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)^3*exp(c*(b*x+a))-1/4*I/
b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b
*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(
I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))*e
xp(c*(b*x+a))-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-1)+1/c/b*exp(c*(b*x+a)
)*ln(exp(c*(b*x+a))+1)-2/b*a+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 1.31

$$\frac{\operatorname{arcoth}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.90

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))))/(b*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(47) = 94.
time = 0.47, size = 147, normalized size = 3.00

$$\frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2\,bcx+2\,ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{2\,e^{(bcx+ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{1}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}\right)+2\,e^{(-ac)}\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x +
 a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1
) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(abs(e
^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

Mupad [**B**]
time = 1.44, size = 111, normalized size = 2.27

$$\frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1\right)}{b\,c}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(1-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}-\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}\right)}{2\,b\,c}+\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\ln\left(\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp(
-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*exp(
a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*c)
```

3.300 $\int e^{c(a+bx)} \coth^{-1}(\mathbf{csch}(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2c(a+bx)}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(3+2\sqrt{2}-e^{2c(a+\cdots}\right.}{2bc}$$

```
[Out] exp(b*c*x+a*c)*arccoth(csch(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2225, 6411, 2320, 12, 1261, 646, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcCoth[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6411

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(
1 - u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\coth^{-1}(\operatorname{csch}(ac+bcx))\,dx=\frac{\text{Subst}\left(\int e^x\coth^{-1}(\operatorname{csch}(x))\,dx,x,ac+bcx\right)}{bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{e^x\coth(x)\operatorname{csch}(x)}{1-\operatorname{csch}^2(x)}\,dx,x,ac\right.}{bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{2x(1+x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{2\text{Subst}\left(\int\frac{x(1+x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{1+x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int\frac{1}{-3+2\sqrt{2}+x}\right.}{2bc}$$

$$=\frac{e^{ac+bcx}\coth^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac}\right.}{2bc}$$

Mathematica [A]
time = 0.15, size = 150, normalized size = 1.40

$$\frac{2e^{c(a+bx)}\coth^{-1}\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right)-2\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{c(a+bx)}}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{1+e^{c(a+bx)}}{\sqrt{2}}\right)+\log\left(1-2e^{c(a+bx)}-e^{2c(a+bx)}\right)+\log\left(1+2e^{c(a+bx)}-e^{2c(a+bx)}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(c*(a + b*x))*ArcCoth[Csch[a*c + b*c*x]],x]
```

```
[Out] (2*E^(c*(a + b*x))*ArcCoth[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] -
2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 +
E^(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
+ Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 920, normalized size = 8.60

method	result	size
risch	Expression too large to display	920

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/b*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)-1/4*I/b/c*Pi
*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))
*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(
b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-
1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csg
n(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1))*cs
gn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c*(b*
x+a))+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(ex
p(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1
/2*I/b/c*exp(c*(b*x+a))*Pi+1/2*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(
b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(
2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*
x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(
b*x+a))+1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))
-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*
x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(
b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/2/b/c*
exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)-1/2/b/c*ln(exp(2*c*(
b*x+a))-(2^(1/2)-1)^2)*2^(1/2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2
^(1/2)-2/b*a+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)+1/2/b/c*ln(exp(2*c*
(b*x+a))-(1+2^(1/2))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.
time = 0.47, size = 184, normalized size = 1.72

$$\frac{\operatorname{arcoth}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc}+\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+ac)}-1\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}-1\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(90) = 180$.
time = 0.37, size = 234, normalized size = 2.19

$$\frac{\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^{2}-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh(bcx+ac)+3\left(2\sqrt{2}+3\right)\sinh(bcx+ac)^{2}-2\sqrt{2}-3}{\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3}\right)+\log\left(\frac{2\left(\cosh(bcx+ac)^{2}+\sinh(bcx+ac)^{2}-3\right)}{\cosh(bcx+ac)^{2}-2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^{2}}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^
2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) +
3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
 a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh
(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)
^2)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{acoth}\left(\operatorname{csch}\left(ac+bcx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*acoth(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*acoth(csch(a*c + b*c*x)), x)
```

Giac [A]
time = 0.54, size = 157, normalized size = 1.47

$$\frac{e^{\left((bx+a)c\right)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc}+\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2) +
2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) + lo
g(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

Mupad [B]
time = 1.60, size = 179, normalized size = 1.67

$$\frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c}-\frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+1\right)}{2\,b\,c}+\frac{\ln\left(6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,e^{2\,c\,(a+b\,x)}-6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2 +
 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x
)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2)
 - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*
c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```