Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/191-7.2.5-Inverse-hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [296]. This is test number [191].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.66 (295)	0.34 (1)
Mathematica	96.96 (287)	3.04 (9)
Maple	65.54 (194)	34.46 (102)
Fricas	43.92 (130)	56.08 (166)
Giac	28.38 (84)	71.62 (212)
Maxima	27.70 (82)	72.30 (214)
Sympy	27.36 (81)	72.64 (215)
Mupad	21.62 (64)	78.38 (232)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

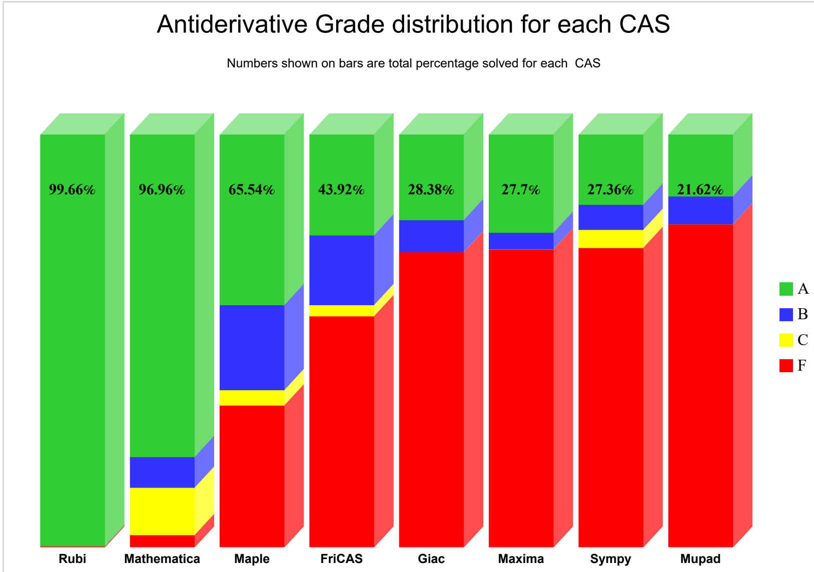
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

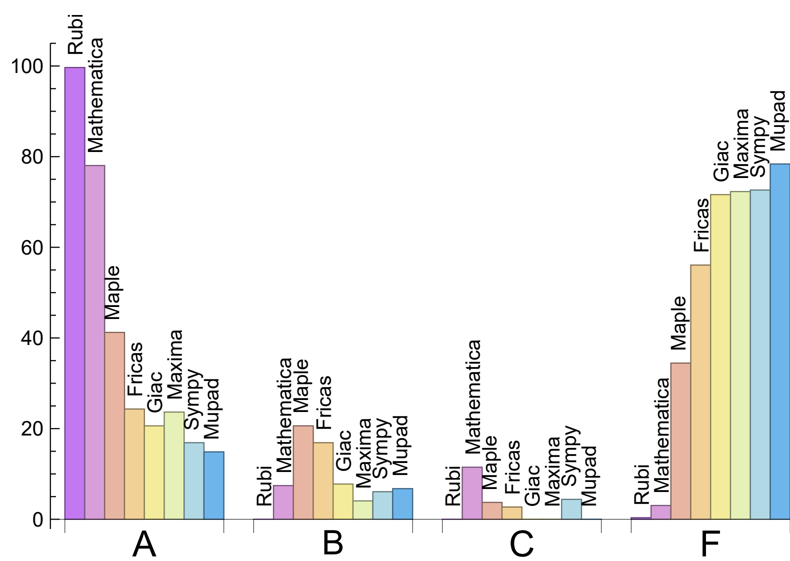
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.66	0.00	0.00	0.34
Mathematica	78.04	7.43	11.49	3.04
Maple	41.22	20.61	3.72	34.46
Fricas	24.32	16.89	2.70	56.08
Maxima	23.65	4.05	0.00	72.30
Giac	20.61	7.77	0.00	71.62
Sympy	16.89	6.08	4.39	72.64
Mupad	N/A	6.76	0.00	78.38

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	9	77.78 %	22.22 %	0.00 %
Maple	102	99.02 %	0.98 %	0.00 %
Fricas	166	62.65 %	0.00 %	37.35 %
Giac	212	72.64 %	0.94 %	26.42 %
Maxima	214	85.51 %	2.80 %	11.68 %
Sympy	215	91.16 %	5.12 %	3.72 %
Mupad	232	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

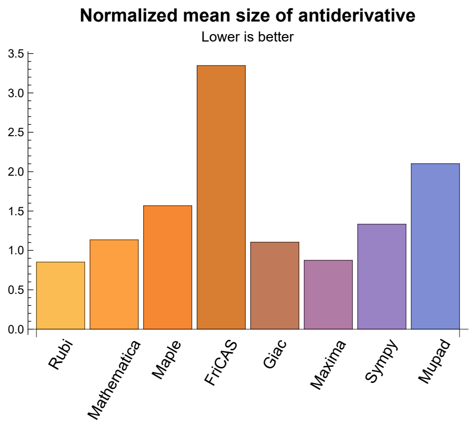
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

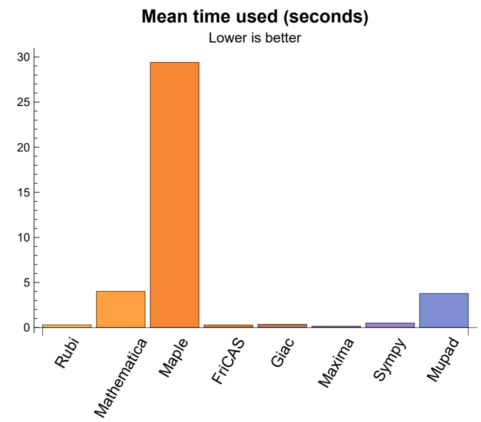
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.30	199.25	0.85	153.00	1.00
Mathematica	4.02	280.72	1.13	162.00	0.98
Maple	29.40	389.05	1.57	194.00	1.55
Maxima	0.15	92.98	0.87	0.00	0.00
Fricas	0.27	567.22	3.35	146.50	1.71
Sympy	0.49	277.85	1.33	46.00	0.82
Giac	0.36	127.73	1.10	57.50	0.81
Mupad	3.77	231.63	2.10	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
30
25
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 78, 82, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 268, 272, 273, 288, 289\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {3, 12, 13, 25, 26, 32, 33, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 86, 97, 112, 120, 121, 128, 129, 136, 137, 138, 139, 140, 161, 162, 163, 166, 167, 168, 170, 172, 180, 181, 182, 184, 186, 190, 192, 194, 195, 196, 232, 233, 252, 253, 294}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

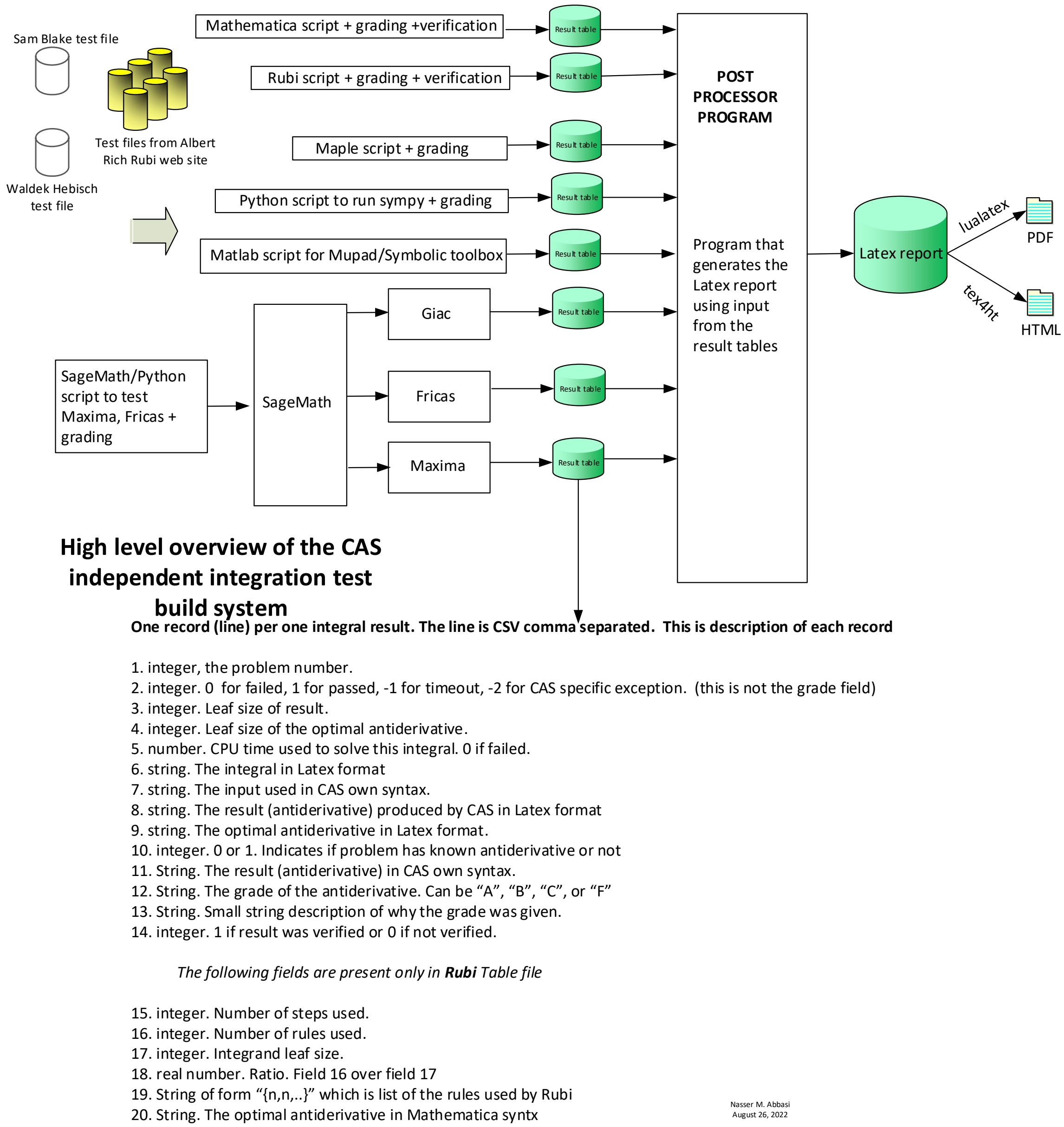
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296 }

B grade: { }

C grade: { }

F grade: { 61 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 53, 54, 55, 58, 59, 60, 62, 63, 64, 66, 67, 68, 71, 72, 73, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 163, 164, 169, 171, 173, 174, 175, 176, 179, 180, 181, 182, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 275, 276, 277, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296 }

B grade: { 121, 125, 127, 128, 129, 157, 162, 165, 166, 167, 168, 170, 172, 177, 183, 189, 195, 234, 238, 239, 278, 291 }

C grade: { 7, 12, 13, 20, 25, 26, 46, 49, 50, 51, 52, 56, 57, 61, 65, 69, 70, 74, 88, 89, 90, 198, 199, 200, 201, 202, 203, 204, 205, 279, 280, 281, 282, 283 }

F grade: { 79, 80, 178, 214, 220, 269, 270, 271, 274 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 10, 12, 13, 16, 17, 18, 23, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 56, 61, 66, 69, 78, 82, 85, 86, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 112, 116, 117, 120, 130, 131, 132, 133, 134, 135, 139, 140, 141, 145, 146, 147, 151, 152, 153, 159, 164, 169, 173, 179, 185, 191, 197, 199, 201, 203, 205, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 237, 238, 239, 243, 250, 268, 270, 271, 272, 273, 274, 288, 289, 290, 292, 294 }

B grade: { 6, 7, 14, 15, 19, 20, 21, 22, 26, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 70, 71, 72, 73, 74, 83, 84, 87, 89, 90, 104, 105, 106, 111, 113, 114, 115, 118, 122, 123, 124, 125, 126, 128, 136, 137, 138, 142, 143, 144, 148, 149, 150, 269, 278, 281, 282, 283, 291 }

C grade: { 45, 46, 198, 200, 202, 204, 275, 276, 277, 279, 280 }

F grade: { 8, 9, 11, 24, 38, 49, 50, 51, 52, 75, 76, 77, 79, 80, 81, 91, 92, 119, 121, 127, 129, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 230, 231, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 284, 285, 286, 287, 293, 295, 296 }

2.1.4 Maxima

A grade: { 1, 2, 3, 14, 15, 16, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 78, 82, 86, 97, 99, 100, 135, 141, 147, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 236, 237, 242, 243, 249, 250, 268, 272, 273, 276, 277, 288, 289, 292, 293 }

B grade: { 83, 84, 85, 93, 94, 95, 96, 102, 111, 238, 275, 278 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 88, 89, 90, 91, 92, 98, 101, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 235, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 294, 295, 296 }

2.1.5 FriCAS

A grade: { 2, 3, 9, 10, 16, 23, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 78, 82, 83, 84, 85, 86, 97, 135, 141, 147, 153, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 236, 237, 241, 242, 243, 249, 268, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 292, 296 }

B grade: { 1, 5, 6, 7, 8, 14, 15, 18, 19, 20, 21, 22, 50, 51, 52, 88, 89, 90, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 111, 113, 114, 115, 116, 117, 122, 123, 124, 125, 238, 240, 247, 248, 250, 293, 294, 295 }

C grade: { 198, 199, 200, 201, 202, 203, 204, 205 }

F grade: { 4, 11, 12, 13, 17, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 91, 92, 98, 109, 110, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 235, 239, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 284, 285, 286, 287, 290, 291 }

2.1.6 Sympy

A grade: { 8, 9, 10, 30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 82, 83, 84, 85, 86, 97, 135, 141, 147, 153, 159, 164, 169, 179, 185, 191, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 234, 272, 288, 289, 292, 293 }

B grade: { 93, 94, 95, 96, 104, 105, 106, 107, 108, 113, 114, 115, 116, 117, 122, 123, 124, 125 }

C grade: { 1, 2, 3, 14, 15, 16, 21, 22, 23, 41, 42, 43, 44 }

F grade: { 4, 5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 294, 295, 296 }

2.1.7 Giac

A grade: { 1, 2, 3, 16, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 82, 83, 84, 85, 88, 89, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 219, 220, 221, 222, 223, 225, 226, 227, 230, 232, 233, 234, 236, 237, 243, 279, 288, 289 }

B grade: { 5, 18, 52, 86, 90, 93, 94, 95, 96, 97, 238, 250, 275, 276, 277, 278, 280, 281, 282, 283, 292, 293, 294 }

C grade: { }

F grade: { 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 91, 92, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 224, 228, 229, 231, 235, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 284, 285, 286, 287, 290, 291, 295, 296 }

2.1.8 Mupad

A grade: { 30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 78, 82, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 268, 272, 273, 288, 289 }

B grade: { 3, 16, 86, 97, 234, 238, 243, 250, 275, 276, 277, 278, 279, 280, 281, 282, 283, 292, 293, 294 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 232, 233, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 284, 285, 286, 287, 290, 291, 295, 296 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	153	299	225	355	258	174	-1
N.S.	1	1.00	0.84	1.63	1.23	1.94	1.41	0.95	-0.01
time (sec)	N/A	0.105	0.149	1.358	0.275	0.365	0.264	0.443	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	213	140	188	155	133	-1
N.S.	1	1.00	0.92	1.73	1.14	1.53	1.26	1.08	-0.01
time (sec)	N/A	0.071	0.118	1.303	0.261	0.397	0.157	0.429	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	104	85	85	80	84	68
N.S.	1	1.00	0.75	1.07	0.88	0.88	0.82	0.87	0.70
time (sec)	N/A	0.030	0.053	1.927	0.267	0.372	0.099	0.430	0.706

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	176	304	0	0	0	0	-1
N.S.	1	1.00	0.99	1.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.009	3.458	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	134	0	1023	0	240	-1
N.S.	1	1.00	1.11	1.61	0.00	12.33	0.00	2.89	-0.01
time (sec)	N/A	0.064	0.069	5.672	0.000	0.388	0.000	0.579	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	190	266	0	3784	0	0	-1
N.S.	1	1.00	1.44	2.02	0.00	28.67	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.130	5.733	0.000	0.454	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	244	624	0	9111	0	0	-1
N.S.	1	1.00	1.25	3.20	0.00	46.72	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.386	6.083	0.000	0.768	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	191	0	0	560	371	0	-1
N.S.	1	1.00	0.57	0.00	0.00	1.68	1.11	0.00	-0.00
time (sec)	N/A	0.955	0.125	180.000	0.000	0.420	0.385	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	131	0	0	291	223	0	-1
N.S.	1	1.00	0.61	0.00	0.00	1.35	1.04	0.00	-0.00
time (sec)	N/A	0.674	0.101	180.000	0.000	0.351	0.248	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	105	100	0	128	110	0	-1
N.S.	1	1.00	0.86	0.82	0.00	1.05	0.90	0.00	-0.01
time (sec)	N/A	0.428	0.073	3.342	0.000	0.405	0.144	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	252	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.108	1.456	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	848	388	0	0	0	0	-1
N.S.	1	1.00	3.27	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	2.373	6.940	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	936	605	0	0	0	0	-1
N.S.	1	1.00	2.66	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	3.133	7.627	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	193	382	263	526	323	0	-1
N.S.	1	1.00	1.01	2.00	1.38	2.75	1.69	0.00	-0.01
time (sec)	N/A	0.105	0.181	1.645	0.265	0.370	0.301	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	262	167	282	197	0	-1
N.S.	1	1.00	1.08	1.98	1.27	2.14	1.49	0.00	-0.01
time (sec)	N/A	0.068	0.133	2.450	0.277	0.373	0.185	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	117	133	101	125	105	124	83
N.S.	1	1.00	1.10	1.25	0.95	1.18	0.99	1.17	0.78
time (sec)	N/A	0.030	0.072	2.095	0.269	0.415	0.108	0.461	1.029

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	183	324	0	0	0	0	-1
N.S.	1	1.00	0.94	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.095	2.679	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	121	155	0	1131	0	257	-1
N.S.	1	1.00	1.38	1.76	0.00	12.85	0.00	2.92	-0.01
time (sec)	N/A	0.048	0.139	4.943	0.000	0.413	0.000	0.574	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	184	365	0	4094	0	0	-1
N.S.	1	1.00	1.33	2.64	0.00	29.67	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.274	5.639	0.000	0.492	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	259	1141	0	9905	0	0	-1
N.S.	1	1.00	1.28	5.65	0.00	49.03	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.700	5.681	0.000	0.790	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	386	814	0	1187	750	0	-1
N.S.	1	1.00	0.97	2.05	0.00	2.98	1.88	0.00	-0.00
time (sec)	N/A	1.149	0.460	2.490	0.000	0.364	0.513	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	360	527	0	618	461	0	-1
N.S.	1	1.00	1.39	2.03	0.00	2.39	1.78	0.00	-0.00
time (sec)	N/A	0.761	0.409	1.639	0.000	0.401	0.331	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	174	240	0	263	240	0	-1
N.S.	1	1.00	1.16	1.60	0.00	1.75	1.60	0.00	-0.01
time (sec)	N/A	0.503	0.190	4.452	0.000	0.427	0.199	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	285	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.181	0.238	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	950	556	0	0	0	0	-1
N.S.	1	1.00	3.41	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	3.758	9.337	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1099	1174	0	0	0	0	-1
N.S.	1	1.00	2.89	3.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	5.640	9.194	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	287	394	0	0	0	0	-1
N.S.	1	1.00	0.73	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.880	0.416	10.834	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	187	254	0	0	0	0	-1
N.S.	1	1.00	0.76	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.235	9.183	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0	-1
N.S.	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.101	8.141	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.160	180.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.292	180.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	530	649	0	0	0	0	-1
N.S.	1	1.00	1.42	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	1.050	9.997	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	268	285	0	0	0	0	-1
N.S.	1	1.00	1.41	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.316	0.544	8.828	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	6.591	180.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.019	142.153	180.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	9.372	180.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.105	180.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	177	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.160	180.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.269	180.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.591	180.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	216	255	385	250	503	316	-1
N.S.	1	1.00	0.58	0.69	1.04	0.68	1.36	0.85	-0.00
time (sec)	N/A	0.317	0.208	1.572	0.265	0.399	1.476	0.414	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	154	176	257	179	328	214	-1
N.S.	1	1.00	0.58	0.66	0.96	0.67	1.23	0.80	-0.00
time (sec)	N/A	0.249	0.142	1.644	0.267	0.353	0.710	0.415	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	103	113	154	121	199	134	-1
N.S.	1	1.00	0.57	0.62	0.85	0.67	1.10	0.74	-0.01
time (sec)	N/A	0.135	0.111	1.665	0.271	0.405	0.332	0.422	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	62	74	71	90	70	-1
N.S.	1	1.00	0.71	0.74	0.88	0.85	1.07	0.83	-0.01
time (sec)	N/A	0.049	0.044	1.693	0.272	0.373	0.141	0.410	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	375	222	0	0	0	0	-1
N.S.	1	1.00	0.78	0.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	0.269	14.843	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	687	790	0	0	0	0	-1
N.S.	1	1.00	0.89	1.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.812	0.977	100.216	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	1.907	180.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.013	1.357	180.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	551	0	0	296	0	82	-1
N.S.	1	1.00	5.74	0.00	0.00	3.08	0.00	0.85	-0.01
time (sec)	N/A	0.119	12.298	180.000	0.000	0.368	0.000	0.427	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	609	0	0	613	0	190	-1
N.S.	1	1.00	3.38	0.00	0.00	3.41	0.00	1.06	-0.01
time (sec)	N/A	0.124	1.788	180.000	0.000	0.404	0.000	0.522	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	655	0	0	1098	0	411	-1
N.S.	1	1.00	2.43	0.00	0.00	4.08	0.00	1.53	-0.00
time (sec)	N/A	0.552	2.517	180.000	0.000	0.452	0.000	0.585	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	723	0	0	1752	0	876	-1
N.S.	1	1.00	1.96	0.00	0.00	4.75	0.00	2.37	-0.00
time (sec)	N/A	0.704	4.976	180.000	0.000	0.607	0.000	0.717	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	491	1430	0	0	0	0	-1
N.S.	1	1.00	0.69	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.957	1.191	7.532	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	356	1001	0	0	0	0	-1
N.S.	1	1.00	0.74	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.747	0.710	9.103	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	251	639	0	0	0	0	-1
N.S.	1	1.00	0.98	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.655	7.597	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	785	785	1121	1072	0	0	0	0	-1
N.S.	1	1.00	1.43	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.302	2.630	6.767	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	1139	1956	0	0	0	0	-1
N.S.	1	1.00	1.24	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.441	4.681	10.363	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1029	1029	901	2274	0	0	0	0	-1
N.S.	1	1.00	0.88	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.271	2.853	8.714	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	725	725	623	1726	0	0	0	0	-1
N.S.	1	1.00	0.86	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.053	1.811	8.483	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	432	1176	0	0	0	0	-1
N.S.	1	1.00	1.09	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.484	1.034	6.847	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1270	0	3068	1965	0	0	0	0	-1
N.S.	1	0.00	2.42	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.647	9.722	5.962	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1385	1385	1802	3170	0	0	0	0	-1
N.S.	1	1.00	1.30	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.533	7.222	8.918	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1015	1015	1282	2549	0	0	0	0	-1
N.S.	1	1.00	1.26	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.306	6.829	11.572	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	644	1856	0	0	0	0	-1
N.S.	1	1.00	1.13	3.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.570	4.193	7.115	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1744	1744	6244	4234	0	0	0	0	-1
N.S.	1	1.00	3.58	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.305	15.123	6.301	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	371	836	0	0	0	0	-1
N.S.	1	1.00	0.78	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.854	1.154	9.064	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	284	518	0	0	0	0	-1
N.S.	1	1.00	0.99	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	0.649	9.253	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	185	248	0	0	0	0	-1
N.S.	1	1.00	1.36	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.470	7.950	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	932	754	0	0	0	0	-1
N.S.	1	1.00	2.55	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.474	1.308	4.184	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	1115	1978	0	0	0	0	-1
N.S.	1	1.00	2.13	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	4.197	8.476	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	353	1238	0	0	0	0	-1
N.S.	1	1.00	0.64	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.001	1.125	9.006	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	281	879	0	0	0	0	-1
N.S.	1	1.00	0.61	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.829	0.617	8.738	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	178	133	499	0	0	0	0	-1
N.S.	1	1.25	0.94	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.362	7.118	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	1203	2484	0	0	0	0	-1
N.S.	1	1.00	1.56	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.242	8.644	5.722	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	204	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.407	180.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	204	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.173	180.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	219	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	1.454	180.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.433	0.122	180.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.847	3.070	180.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	1.257	180.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	0.018	180.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.486	0.214	180.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	285	321	110	255	163	-1
N.S.	1	1.00	0.80	1.88	2.11	0.72	1.68	1.07	-0.01
time (sec)	N/A	0.127	0.107	7.776	0.256	0.367	0.303	0.428	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	203	212	91	170	132	-1
N.S.	1	1.00	0.97	1.95	2.04	0.88	1.63	1.27	-0.01
time (sec)	N/A	0.082	0.074	3.301	0.260	0.380	0.171	0.446	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	113	151	75	104	112	-1
N.S.	1	1.00	0.97	1.26	1.68	0.83	1.16	1.24	-0.01
time (sec)	N/A	0.045	0.048	3.205	0.260	0.368	0.117	0.441	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	36	30	57	46	93	266
N.S.	1	1.00	1.44	0.88	0.73	1.39	1.12	2.27	6.49
time (sec)	N/A	0.011	0.089	0.416	0.252	0.418	0.074	0.429	3.998

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	153	431	0	0	0	0	-1
N.S.	1	1.00	1.17	3.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.010	10.849	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	83	101	0	322	0	73	-1
N.S.	1	1.00	1.30	1.58	0.00	5.03	0.00	1.14	-0.02
time (sec)	N/A	0.058	0.081	4.596	0.000	0.389	0.000	0.435	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	136	201	0	460	0	170	-1
N.S.	1	1.00	1.28	1.90	0.00	4.34	0.00	1.60	-0.01
time (sec)	N/A	0.073	0.192	5.017	0.000	0.400	0.000	0.461	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	162	467	0	566	0	340	-1
N.S.	1	1.00	1.05	3.03	0.00	3.68	0.00	2.21	-0.01
time (sec)	N/A	0.119	0.231	4.970	0.000	0.496	0.000	0.437	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.131	180.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	111	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.078	180.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	103	78	1231	1041	527	846	-1
N.S.	1	1.00	0.76	0.58	9.12	7.71	3.90	6.27	-0.01
time (sec)	N/A	0.057	0.089	3.897	0.283	0.361	0.503	1.060	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	144	789	655	394	617	-1
N.S.	1	1.00	0.97	1.21	6.63	5.50	3.31	5.18	-0.01
time (sec)	N/A	0.051	0.088	4.029	0.283	0.377	0.396	0.930	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	67	443	352	258	419	-1
N.S.	1	1.00	0.73	0.69	4.57	3.63	2.66	4.32	-0.01
time (sec)	N/A	0.042	0.059	4.082	0.280	0.372	0.259	0.768	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	107	207	171	148	245	-1
N.S.	1	1.00	1.08	1.43	2.76	2.28	1.97	3.27	-0.01
time (sec)	N/A	0.026	0.123	3.962	0.266	0.393	0.170	0.630	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	64	41	35	65	51	100	272
N.S.	1	1.00	1.39	0.89	0.76	1.41	1.11	2.17	5.91
time (sec)	N/A	0.016	0.110	1.160	0.257	0.401	0.216	0.423	3.962

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	103	0	0	0	0	-1
N.S.	1	1.00	0.85	1.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.042	8.787	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	78	83	72	168	0	0	-1
N.S.	1	1.00	1.39	1.48	1.29	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.092	4.029	0.510	0.391	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	65	110	210	0	0	-1
N.S.	1	1.00	0.83	0.98	1.67	3.18	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.039	8.156	0.491	0.469	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	112	0	445	0	0	-1
N.S.	1	1.00	1.02	1.13	0.00	4.49	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.161	4.026	0.000	0.409	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	76	246	475	0	0	-1
N.S.	1	1.00	0.83	0.73	2.37	4.57	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.058	8.348	0.305	0.381	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	136	141	0	803	0	0	-1
N.S.	1	1.00	0.99	1.03	0.00	5.86	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.164	4.005	0.000	0.447	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	220	540	0	2430	1268	0	-1
N.S.	1	1.00	1.01	2.48	0.00	11.15	5.82	0.00	-0.00
time (sec)	N/A	0.346	0.203	10.983	0.000	0.450	0.823	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	212	437	0	1457	916	0	-1
N.S.	1	1.00	1.14	2.35	0.00	7.83	4.92	0.00	-0.01
time (sec)	N/A	0.278	0.170	6.086	0.000	0.486	0.579	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	168	290	0	784	610	0	-1
N.S.	1	1.00	1.12	1.93	0.00	5.23	4.07	0.00	-0.01
time (sec)	N/A	0.225	0.142	8.611	0.000	0.377	0.381	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	167	182	0	367	335	0	-1
N.S.	1	1.00	1.52	1.65	0.00	3.34	3.05	0.00	-0.01
time (sec)	N/A	0.164	0.142	13.897	0.000	0.380	0.219	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	105	100	0	141	143	0	-1
N.S.	1	1.00	1.64	1.56	0.00	2.20	2.23	0.00	-0.02
time (sec)	N/A	0.082	0.058	16.316	0.000	0.398	0.116	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	140	243	0	0	0	0	-1
N.S.	1	1.00	1.19	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.275	12.264	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	161	270	0	0	0	0	-1
N.S.	1	1.00	1.46	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.538	19.842	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	176	215	413	0	0	-1
N.S.	1	1.00	0.88	1.91	2.34	4.49	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.138	16.990	0.519	0.409	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	251	352	0	0	0	0	-1
N.S.	1	1.00	1.35	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	0.693	16.743	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	404	1293	0	4401	2518	0	-1
N.S.	1	1.00	1.06	3.38	0.00	11.52	6.59	0.00	-0.00
time (sec)	N/A	0.477	0.370	31.689	0.000	0.427	1.382	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	359	647	0	2620	1828	0	-1
N.S.	1	1.00	1.17	2.11	0.00	8.53	5.95	0.00	-0.00
time (sec)	N/A	0.405	0.327	41.013	0.000	0.427	0.988	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	296	638	0	1389	1173	0	-1
N.S.	1	1.00	1.13	2.44	0.00	5.30	4.48	0.00	-0.00
time (sec)	N/A	0.310	0.243	37.656	0.000	0.394	0.628	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	244	301	0	642	685	0	-1
N.S.	1	1.00	1.39	1.72	0.00	3.67	3.91	0.00	-0.01
time (sec)	N/A	0.240	0.187	37.677	0.000	0.378	0.394	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	168	180	0	239	282	0	-1
N.S.	1	1.00	1.47	1.58	0.00	2.10	2.47	0.00	-0.01
time (sec)	N/A	0.120	0.117	49.309	0.000	0.371	0.192	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	217	436	0	0	0	0	-1
N.S.	1	1.00	1.36	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.385	43.264	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	327	0	0	0	0	0	-1
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.850	0.178	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	266	339	0	0	0	0	-1
N.S.	1	1.00	1.62	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.784	42.894	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	601	0	0	0	0	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	1.346	0.207	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	562	875	0	4034	2876	0	-1
N.S.	1	1.00	1.49	2.32	0.00	10.70	7.63	0.00	-0.00
time (sec)	N/A	0.745	0.730	40.880	0.000	0.451	1.518	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	475	1058	0	2094	1889	0	-1
N.S.	1	1.00	1.54	3.42	0.00	6.78	6.11	0.00	-0.00
time (sec)	N/A	0.522	0.324	40.374	0.000	0.443	0.952	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	360	434	0	959	1027	0	-1
N.S.	1	1.00	1.72	2.08	0.00	4.59	4.91	0.00	-0.00
time (sec)	N/A	0.355	0.261	39.493	0.000	0.403	0.578	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	261	275	0	344	444	0	-1
N.S.	1	1.00	2.02	2.13	0.00	2.67	3.44	0.00	-0.01
time (sec)	N/A	0.180	0.163	43.267	0.000	0.367	0.312	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	308	674	0	0	0	0	-1
N.S.	1	1.00	1.60	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.518	47.607	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	872	0	0	0	0	0	-1
N.S.	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	1.636	0.216	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	398	551	0	0	0	0	-1
N.S.	1	1.00	2.04	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	1.542	56.551	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	1309	0	0	0	0	0	-1
N.S.	1	1.00	3.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	7.418	0.205	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	151	194	0	0	0	0	-1
N.S.	1	1.00	0.71	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	0.214	58.105	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0	-1
N.S.	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.216	49.329	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	130	0	0	0	0	-1
N.S.	1	1.00	0.72	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.119	50.621	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0	-1
N.S.	1	1.00	0.88	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.096	51.918	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0	-1
N.S.	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.088	40.375	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.659	180.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	293	665	0	0	0	0	-1
N.S.	1	1.00	1.11	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	1.394	50.064	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	230	418	0	0	0	0	-1
N.S.	1	1.00	1.18	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	2.026	59.759	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	150	374	0	0	0	0	-1
N.S.	1	1.00	0.79	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	1.340	71.378	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	170	0	0	0	0	-1
N.S.	1	1.00	0.98	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.357	57.510	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	143	139	0	0	0	0	-1
N.S.	1	1.00	1.46	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.728	41.239	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	6.156	180.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	323	993	0	0	0	0	-1
N.S.	1	1.00	0.99	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.740	1.122	0.529	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	186	624	0	0	0	0	-1
N.S.	1	1.00	0.73	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.585	0.502	0.184	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	223	557	0	0	0	0	-1
N.S.	1	1.00	0.88	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.534	0.512	0.106	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	127	254	0	0	0	0	-1
N.S.	1	1.00	0.78	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.376	0.055	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	109	207	0	0	0	0	-1
N.S.	1	1.00	0.83	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.304	0.035	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.465	0.062	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	424	1375	0	0	0	0	-1
N.S.	1	1.00	0.98	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	1.832	0.214	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	330	860	0	0	0	0	-1
N.S.	1	1.00	0.92	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	1.279	0.137	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	272	777	0	0	0	0	-1
N.S.	1	1.00	0.77	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	1.011	0.119	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	195	353	0	0	0	0	-1
N.S.	1	1.00	0.89	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	0.764	0.052	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	144	295	0	0	0	0	-1
N.S.	1	1.00	0.83	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.528	0.043	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	11.508	0.063	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	0.519	0.241	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	0.403	0.174	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	237	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.360	0.096	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	437	0	0	0	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.354	1.994	0.023	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.247	0.016	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	1.750	0.050	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	558	0	0	0	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.941	3.011	0.123	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	592	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	2.020	0.100	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	515	0	0	0	0	0	-1
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	6.878	0.023	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	290	0	0	0	0	0	-1
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.558	0.017	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	1.269	0.062	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	968	0	0	0	0	0	-1
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.420	9.936	0.121	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	1008	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.175	8.438	0.105	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	1846	0	0	0	0	0	-1
N.S.	1	1.00	6.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	8.142	0.025	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	494	0	0	0	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.390	2.909	0.016	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.066	0.721	0.066	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1523	0	0	0	0	0	-1
N.S.	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.365	11.922	0.102	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	288	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.800	4.521	0.034	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	748	0	0	0	0	0	-1
N.S.	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	5.629	0.016	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.750	0.057	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	319	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.444	0.393	0.175	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.288	0.130	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	216	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.244	0.093	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	306	0	0	0	0	0	-1
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	1.008	0.025	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.030	0.016	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.063	0.054	0.050	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	396	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	1.323	0.159	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	265	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	0.746	0.111	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	265	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	1.094	0.083	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	314	0	0	0	0	0	-1
N.S.	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	5.298	0.023	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	145	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.233	0.355	0.018	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.073	0.064	0.047	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	615	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.187	2.516	0.163	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	391	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.937	1.763	0.122	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	391	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.813	2.391	0.086	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	687	0	0	0	0	0	-1
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	4.739	0.025	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	219	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	1.128	0.017	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.069	0.069	0.045	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	654	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	3.581	0.176	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	445	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	2.450	0.118	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	452	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.982	2.272	0.089	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	916	0	0	0	0	0	-1
N.S.	1	1.00	3.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	3.345	0.023	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	243	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	0.569	0.017	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.068	0.043	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	150	276	0	815	0	0	-1
N.S.	1	1.00	0.79	1.46	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.234	0.143	0.000	0.111	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	180	218	0	517	0	0	-1
N.S.	1	1.00	1.07	1.29	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.182	0.056	0.000	0.114	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	253	0	279	0	0	-1
N.S.	1	1.00	0.75	1.74	0.00	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.352	0.048	0.000	0.104	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	194	0	159	0	0	-1
N.S.	1	1.00	1.03	1.53	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.328	0.047	0.000	0.121	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	138	0	128	0	0	-1
N.S.	1	1.00	0.90	1.33	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.136	0.059	0.000	0.096	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	119	0	169	0	0	-1
N.S.	1	1.00	1.10	1.42	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.108	0.054	0.000	0.121	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	94	268	0	283	0	0	-1
N.S.	1	1.00	0.63	1.79	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.098	0.056	0.000	0.096	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	94	201	0	383	0	0	-1
N.S.	1	1.00	0.72	1.55	0.00	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.106	0.060	0.000	0.098	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.363	0.045	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.347	0.043	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.330	0.042	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.321	0.046	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	140	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.212	180.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	140	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.222	180.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.343	180.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	140	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.259	180.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	180.009	0.048	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	161.115	0.046	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	29.477	180.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	25.196	180.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	29.811	180.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	95.177	180.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	180.012	0.048	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	136.843	0.048	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	11.111	180.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	30.670	180.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	30.505	180.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	121.464	180.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	2.998	0.839	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	1.386	0.888	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	178	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.288	0.940	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	124	106	0	0	0	0	0	-1
N.S.	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.135	0.907	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.315	180.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.036	0.016	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	75	56	40	0	60	-1
N.S.	1	1.00	0.68	0.64	0.48	0.34	0.00	0.51	-0.01
time (sec)	N/A	0.046	0.043	0.037	0.275	0.357	0.000	1.594	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	65	46	35	0	55	-1
N.S.	1	1.00	0.86	0.76	0.53	0.41	0.00	0.64	-0.01
time (sec)	N/A	0.033	0.031	0.013	0.280	0.350	0.000	1.579	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	273	49	33	28	29	47	40
N.S.	1	1.00	5.46	0.98	0.66	0.56	0.58	0.94	0.80
time (sec)	N/A	0.019	3.211	0.023	0.297	0.348	0.092	1.635	1.429

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	65	0	0	0	0	-1
N.S.	1	1.00	1.00	1.41	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.028	0.041	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	29	19	26	0	45	-1
N.S.	1	1.00	1.00	0.72	0.48	0.65	0.00	1.12	-0.02
time (sec)	N/A	0.017	0.011	0.007	0.505	0.344	0.000	1.554	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	35	30	32	0	62	-1
N.S.	1	1.00	0.64	0.46	0.39	0.42	0.00	0.82	-0.01
time (sec)	N/A	0.027	0.019	0.012	0.497	0.353	0.000	1.551	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	59	38	17	72	0	22	23
N.S.	1	1.00	2.46	1.58	0.71	3.00	0.00	0.92	0.96
time (sec)	N/A	0.006	0.130	0.039	0.510	0.379	0.000	0.399	0.369

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	179	86	0	0	0	0	-1
N.S.	1	1.00	2.98	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.381	0.046	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	264	0	0	298	0	0	-1
N.S.	1	1.00	1.82	0.00	0.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.145	0.014	0.000	0.351	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	171	0	0	210	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.077	0.015	0.000	0.367	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	104	0	128	131	0	0	-1
N.S.	1	1.00	1.44	0.00	1.78	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.037	0.013	0.328	0.350	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	37	44	63	0	62	32
N.S.	1	1.00	0.76	0.76	0.90	1.29	0.00	1.27	0.65
time (sec)	N/A	0.023	0.042	0.008	0.311	0.328	0.000	0.409	0.591

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.095	0.013	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	130	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.626	0.014	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	152	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.298	0.015	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	264	0	0	298	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.144	0.014	0.000	0.427	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	171	0	0	210	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	1.91	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.080	0.016	0.000	0.336	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	104	0	128	131	0	0	-1
N.S.	1	1.00	1.42	0.00	1.75	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.038	0.019	0.348	0.331	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	37	44	63	0	67	32
N.S.	1	1.00	1.00	1.12	1.33	1.91	0.00	2.03	0.97
time (sec)	N/A	0.013	0.017	0.004	0.294	0.361	0.000	0.394	1.525

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.092	0.016	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	141	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.502	0.014	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	168	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.477	0.014	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	311	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	2.315	0.016	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	254	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	0.476	0.014	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	210	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.025	0.213	0.016	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.219	0.019	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	242	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.032	0.740	0.018	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	273	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.701	0.016	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	291	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	0.867	0.019	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	277	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.046	1.102	0.016	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.041	0.324	0.014	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	178	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.021	0.174	0.017	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.183	0.019	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	209	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.023	0.714	0.021	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	238	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.042	0.605	0.016	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	260	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.046	0.719	0.016	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.093	0.108	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	870	0	0	0	0	-1
N.S.	1	1.00	0.00	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.256	0.233	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	0	491	0	0	0	0	-1
N.S.	1	1.00	0.00	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.507	0.056	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	207	0	0	0	0	-1
N.S.	1	1.00	0.00	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	2.072	0.065	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.090	0.091	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	3.224	0.088	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	110	0	0	0	0	-1
N.S.	1	1.00	0.00	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.608	0.052	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	138	376	495	133	0	564	1408
N.S.	1	1.00	0.84	2.28	3.00	0.81	0.00	3.42	8.53
time (sec)	N/A	0.117	0.064	0.075	0.296	0.334	0.000	0.426	69.812

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	119	288	275	112	0	417	1067
N.S.	1	1.00	1.03	2.50	2.39	0.97	0.00	3.63	9.28
time (sec)	N/A	0.085	0.088	0.073	0.275	0.333	0.000	0.421	33.479

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	93	194	177	88	0	280	852
N.S.	1	1.00	1.39	2.90	2.64	1.31	0.00	4.18	12.72
time (sec)	N/A	0.050	0.105	0.073	0.291	0.343	0.000	0.414	16.716

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	69	147	143	66	0	151	79
N.S.	1	1.00	2.23	4.74	4.61	2.13	0.00	4.87	2.55
time (sec)	N/A	0.012	0.020	0.088	0.290	0.354	0.000	0.395	0.175

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	141	156	0	246	0	103	2500
N.S.	1	1.00	1.41	1.56	0.00	2.46	0.00	1.03	25.00
time (sec)	N/A	0.074	0.067	0.070	0.000	0.381	0.000	0.402	23.408

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	140	237	0	334	0	200	2500
N.S.	1	1.00	1.28	2.17	0.00	3.06	0.00	1.83	22.94
time (sec)	N/A	0.051	0.098	0.059	0.000	0.354	0.000	0.419	15.820

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	142	236	0	338	0	326	958
N.S.	1	1.00	1.03	1.71	0.00	2.45	0.00	2.36	6.94
time (sec)	N/A	0.074	0.196	0.066	0.000	0.375	0.000	0.465	14.149

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	179	374	0	431	0	487	1537
N.S.	1	1.00	0.95	1.98	0.00	2.28	0.00	2.58	8.13
time (sec)	N/A	0.116	0.183	0.066	0.000	0.398	0.000	0.543	18.889

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	198	603	0	569	0	817	2500
N.S.	1	1.00	0.83	2.53	0.00	2.39	0.00	3.43	10.50
time (sec)	N/A	0.152	0.320	0.058	0.000	0.444	0.000	0.571	28.031

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.538	0.226	0.005	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	136	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.163	0.003	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.088	0.002	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.031	0.002	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.029	0.097	0.005	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.026	0.335	0.004	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	92	0	0	0	0	-1
N.S.	1	1.00	0.88	1.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.034	0.065	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	19	78	0	0	0	0	-1
N.S.	1	1.00	6.33	26.00	0.00	0.00	0.00	0.00	-0.33
time (sec)	N/A	0.078	0.044	0.221	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	37	66	61	106	295
N.S.	1	1.00	0.93	0.83	0.69	1.22	1.13	1.96	5.46
time (sec)	N/A	0.038	0.051	0.006	0.278	0.334	0.211	0.405	4.683

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	0	39	152	76	124	303
N.S.	1	1.00	0.91	0.00	0.71	2.76	1.38	2.25	5.51
time (sec)	N/A	0.038	0.032	0.007	0.285	0.393	22.801	0.417	1.068

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	116	87	0	276	0	119	53
N.S.	1	1.00	2.00	1.50	0.00	4.76	0.00	2.05	0.91
time (sec)	N/A	0.068	0.239	0.158	0.000	0.433	0.000	2.291	1.512

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	108	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.74	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.109	0.052	0.000	0.371	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	33	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.61	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.062	0.049	0.000	0.384	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [269] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	12	0.417
2	A	4	4	1.00	12	0.333
3	A	4	4	1.00	10	0.400
4	A	8	5	1.00	12	0.417
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	6	6	1.00	12	0.500
8	A	18	7	1.00	14	0.500
9	A	13	7	1.00	14	0.500
10	A	9	7	1.00	12	0.583
11	A	10	6	1.00	14	0.429
12	A	10	7	1.00	14	0.500
13	A	13	10	1.00	14	0.714
14	A	5	5	1.00	16	0.312
15	A	4	4	1.00	16	0.250
16	A	4	4	1.00	14	0.286
17	A	8	5	1.00	16	0.312
18	A	3	3	1.00	16	0.188
19	A	4	4	1.00	16	0.250
20	A	6	6	1.00	16	0.375
21	A	18	7	1.00	18	0.389
22	A	13	7	1.00	18	0.389
23	A	9	7	1.00	16	0.438
24	A	10	6	1.00	18	0.333
25	A	10	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	13	10	1.00	18	0.556
27	A	27	7	1.00	18	0.389
28	A	17	6	1.00	18	0.333
29	A	11	7	1.00	16	0.438
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	19	7	1.00	18	0.389
33	A	11	7	1.00	16	0.438
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	3	3	1.00	16	0.188
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	6	6	1.00	14	0.429
42	A	6	6	1.00	14	0.429
43	A	6	6	1.00	14	0.429
44	A	3	3	1.00	12	0.250
45	A	18	6	1.00	14	0.429
46	A	26	9	1.00	14	0.643
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	7	8	1.00	16	0.500
50	A	8	10	1.00	16	0.625
51	A	9	11	1.00	16	0.688
52	A	10	12	1.00	16	0.750
53	A	17	13	1.00	31	0.419
54	A	14	9	1.00	31	0.290
55	A	9	7	1.00	29	0.241
56	A	23	22	1.00	31	0.710
57	A	38	22	1.00	31	0.710
58	A	27	19	1.00	31	0.613
59	A	23	14	1.00	31	0.452
60	A	14	11	1.00	29	0.379

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	F	0	0	N/A	0.	N/A
62	A	35	22	1.00	31	0.710
63	A	31	17	1.00	31	0.548
64	A	17	12	1.00	29	0.414
65	A	39	32	1.00	31	1.032
66	A	13	7	1.00	31	0.226
67	A	9	7	1.00	31	0.226
68	A	6	5	1.00	29	0.172
69	A	10	7	1.00	31	0.226
70	A	13	10	1.00	31	0.323
71	A	21	15	1.00	31	0.484
72	A	19	13	1.00	31	0.419
73	A	5	6	1.25	29	0.207
74	A	27	18	1.00	31	0.581
75	A	7	5	1.00	30	0.167
76	A	7	5	1.00	35	0.143
77	A	7	5	1.00	37	0.135
78	A	0	0	0.00	0	0.000
79	A	14	11	1.00	35	0.314
80	A	12	10	1.00	33	0.303
81	A	9	7	1.00	25	0.280
82	A	0	0	0.00	0	0.000
83	A	6	6	1.00	10	0.600
84	A	5	5	1.00	10	0.500
85	A	5	5	1.00	8	0.625
86	A	3	3	1.00	6	0.500
87	A	9	6	1.00	10	0.600
88	A	4	4	1.00	10	0.400
89	A	5	5	1.00	10	0.500
90	A	7	7	1.00	10	0.700
91	A	7	6	1.00	14	0.429
92	A	7	6	1.00	15	0.400
93	A	8	5	1.00	21	0.238
94	A	7	6	1.00	21	0.286
95	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	5	1.00	19	0.263
97	A	4	3	1.00	10	0.300
98	A	7	7	1.00	21	0.333
99	A	5	5	1.00	21	0.238
100	A	4	4	1.00	21	0.190
101	A	6	6	1.00	21	0.286
102	A	6	5	1.00	21	0.238
103	A	8	6	1.00	21	0.286
104	A	9	7	1.00	23	0.304
105	A	8	6	1.00	23	0.261
106	A	7	7	1.00	23	0.304
107	A	6	6	1.00	21	0.286
108	A	4	4	1.00	12	0.333
109	A	8	8	1.00	23	0.348
110	A	9	7	1.00	23	0.304
111	A	5	5	1.00	23	0.217
112	A	11	9	1.00	23	0.391
113	A	19	8	1.00	23	0.348
114	A	14	8	1.00	23	0.348
115	A	12	8	1.00	23	0.348
116	A	8	7	1.00	21	0.333
117	A	6	4	1.00	12	0.333
118	A	9	9	1.00	23	0.391
119	A	11	8	1.00	23	0.348
120	A	9	9	1.00	23	0.391
121	A	15	11	1.00	23	0.478
122	A	16	6	1.00	23	0.261
123	A	13	8	1.00	23	0.348
124	A	9	6	1.00	21	0.286
125	A	6	4	1.00	12	0.333
126	A	10	9	1.00	23	0.391
127	A	13	9	1.00	23	0.391
128	A	10	10	1.00	23	0.435
129	A	21	12	1.00	23	0.522
130	A	14	7	1.00	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	11	7	1.00	23	0.304
132	A	11	7	1.00	23	0.304
133	A	8	7	1.00	21	0.333
134	A	5	5	1.00	12	0.417
135	A	0	0	0.00	0	0.000
136	A	13	6	1.00	23	0.261
137	A	10	6	1.00	23	0.261
138	A	10	6	1.00	23	0.261
139	A	6	6	1.00	21	0.286
140	A	6	6	1.00	12	0.500
141	A	0	0	0.00	0	0.000
142	A	26	9	1.00	23	0.391
143	A	20	9	1.00	23	0.391
144	A	18	10	1.00	23	0.435
145	A	11	10	1.00	21	0.476
146	A	7	7	1.00	12	0.583
147	A	0	0	0.00	0	0.000
148	A	24	8	1.00	23	0.348
149	A	17	8	1.00	23	0.348
150	A	18	10	1.00	23	0.435
151	A	9	9	1.00	21	0.429
152	A	8	7	1.00	12	0.583
153	A	0	0	0.00	0	0.000
154	A	21	9	1.00	25	0.360
155	A	16	9	1.00	25	0.360
156	A	16	9	1.00	25	0.360
157	A	11	9	1.00	23	0.391
158	A	8	7	1.00	14	0.500
159	A	0	0	0.00	0	0.000
160	A	27	11	1.00	25	0.440
161	A	24	12	1.00	25	0.480
162	A	13	11	1.00	23	0.478
163	A	9	8	1.00	14	0.571
164	A	0	0	0.00	0	0.000
165	A	29	11	1.00	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	26	12	1.00	25	0.480
167	A	14	11	1.00	23	0.478
168	A	10	8	1.00	14	0.571
169	A	0	0	0.00	0	0.000
170	A	35	13	1.00	25	0.520
171	A	16	11	1.00	23	0.478
172	A	11	8	1.00	14	0.571
173	A	0	0	0.00	0	0.000
174	A	20	8	1.00	25	0.320
175	A	15	8	1.00	25	0.320
176	A	15	8	1.00	25	0.320
177	A	10	8	1.00	23	0.348
178	A	7	6	1.00	14	0.429
179	A	0	0	0.00	0	0.000
180	A	19	7	1.00	25	0.280
181	A	14	7	1.00	25	0.280
182	A	14	7	1.00	25	0.280
183	A	8	7	1.00	23	0.304
184	A	8	7	1.00	14	0.500
185	A	0	0	0.00	0	0.000
186	A	36	10	1.00	25	0.400
187	A	26	10	1.00	25	0.400
188	A	24	11	1.00	25	0.440
189	A	13	11	1.00	23	0.478
190	A	9	8	1.00	14	0.571
191	A	0	0	0.00	0	0.000
192	A	34	9	1.00	25	0.360
193	A	23	9	1.00	25	0.360
194	A	24	11	1.00	25	0.440
195	A	11	10	1.00	23	0.435
196	A	10	8	1.00	14	0.571
197	A	0	0	0.00	0	0.000
198	A	8	6	1.00	23	0.261
199	A	8	6	1.00	23	0.261
200	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	6	1.00	23	0.261
202	A	4	4	1.00	23	0.174
203	A	4	4	1.00	23	0.174
204	A	7	7	1.00	23	0.304
205	A	7	7	1.00	23	0.304
206	A	3	3	1.00	25	0.120
207	A	3	3	1.00	25	0.120
208	A	3	3	1.00	25	0.120
209	A	3	3	1.00	25	0.120
210	A	3	3	1.00	25	0.120
211	A	3	3	1.00	25	0.120
212	A	3	3	1.00	25	0.120
213	A	3	3	1.00	25	0.120
214	A	0	0	0.00	0	0.000
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	0	0	0.00	0	0.000
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	3	3	1.00	23	0.130
229	A	5	5	1.05	21	0.238
230	A	0	0	0.00	0	0.000
231	A	5	5	1.00	10	0.500
232	A	7	5	1.00	10	0.500
233	A	6	5	1.00	8	0.625
234	A	5	5	1.00	6	0.833
235	A	5	5	1.00	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	10	0.300
237	A	4	4	1.00	10	0.400
238	A	3	3	1.00	4	0.750
239	A	5	5	1.00	10	0.500
240	A	3	2	1.00	14	0.143
241	A	7	6	1.00	14	0.429
242	A	2	2	1.00	14	0.143
243	A	6	5	1.00	12	0.417
244	A	1	1	1.00	14	0.071
245	A	1	1	1.00	14	0.071
246	A	2	2	1.00	14	0.143
247	A	3	2	1.00	14	0.143
248	A	5	4	1.00	14	0.286
249	A	2	2	1.00	14	0.143
250	A	4	3	1.00	12	0.250
251	A	1	1	1.00	14	0.071
252	A	1	1	1.00	14	0.071
253	A	2	2	1.00	14	0.143
254	A	2	2	1.00	16	0.125
255	A	2	2	1.00	16	0.125
256	A	1	1	1.00	16	0.062
257	A	1	1	1.00	16	0.062
258	A	1	1	1.00	16	0.062
259	A	2	2	1.00	16	0.125
260	A	2	2	1.00	16	0.125
261	A	2	2	1.00	16	0.125
262	A	2	2	1.00	16	0.125
263	A	1	1	1.00	16	0.062
264	A	1	1	1.00	16	0.062
265	A	1	1	1.00	16	0.062
266	A	2	2	1.00	16	0.125
267	A	2	2	1.00	16	0.125
268	A	0	0	0.00	0	0.000
269	A	8	8	1.00	40	0.200
270	A	7	7	1.00	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	7	1.00	38	0.184
272	A	0	0	0.00	0	0.000
273	A	0	0	0.00	0	0.000
274	A	6	6	1.00	10	0.600
275	A	5	4	1.00	12	0.333
276	A	5	4	1.00	12	0.333
277	A	5	4	1.00	10	0.400
278	A	5	4	1.00	8	0.500
279	A	9	8	1.00	12	0.667
280	A	9	8	1.00	12	0.667
281	A	7	5	1.00	12	0.417
282	A	8	6	1.00	12	0.500
283	A	10	7	1.00	12	0.583
284	A	37	8	1.00	14	0.571
285	A	27	8	1.00	14	0.571
286	A	17	8	1.00	12	0.667
287	A	7	4	1.00	10	0.400
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	7	7	1.00	19	0.368
291	A	2	2	1.00	20	0.100
292	A	4	4	1.00	12	0.333
293	A	4	4	1.00	14	0.286
294	A	5	5	1.00	10	0.500
295	A	2	2	1.00	26	0.077
296	A	2	2	1.00	26	0.077

Chapter 3

Listing of integrals

Local contents

3.1 $\int (d+ex)^3 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=183

$$-\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(4d(19c^2d^2+16e}{96c^3}$$

```
[Out] -1/32*(8*c^4*d^4+24*c^2*d^2*e^2+3*e^4)*arccosh(c*x)/c^4/e+1/4*(e*x+d)^4*arc
cosh(c*x)/e-7/48*d*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*(e*x+d)^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/96*(4*d*(19*c^2*d^2+16*e^2)+e*(26*c^2*d^2+9*
e^2)*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5963, 102, 158, 152, 54}

$$-\frac{(8c^4d^4+24c^2d^2e^2+3e^4)\cosh^{-1}(cx)}{32c^4e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(ex(26c^2d^2+9e^2)+4d(19c^2d^2+16e^2))}{96c^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^3}{16c}-\frac{7d\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{48c}+\frac{\cosh^{-1}(cx)(d+ex)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] (-7*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(d + e*x)^3)/(16*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*(19*c^
2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - ((8*c^4*d^4 + 24*c^
2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*ArcCosh[c*x])/(4
*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \cosh^{-1}(cx)\,dx = \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{c\int \frac{(d+ex)^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{4e}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} + \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{\int \frac{(d+ex)^2(4c^2d^2+3e^2+7c}{\sqrt{-1+cx}\,\sqrt{1+}}}{16ce}$$

$$= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} + \frac{(d+ex)}{}$$

$$= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} - \frac{\sqrt{-1+}}{}$$

$$= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} - \frac{\sqrt{-1+}}{}$$

Mathematica [A]
time = 0.15, size = 153, normalized size = 0.84

$$-\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\,(e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3))-24c^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3)\cosh^{-1}(cx)+9(8c^2d^2e+e^3)\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] -1/96*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 7
2*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)) - 24*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*
e^2*x^2 + e^3*x^3)*ArcCosh[c*x] + 9*(8*c^2*d^2*e + e^3)*Log[c*x + Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]])/c^4
```

Maple [A]
time = 1.36, size = 299, normalized size = 1.63

method	result
derivativedivides	$\frac{\frac{c\,\mathrm{arccosh}(cx)d^4}{4e}+\mathrm{arccosh}(cx)d^3cx+\frac{3ce\,\mathrm{arccosh}(cx)d^2x^2}{2}+c\,e^2\mathrm{arccosh}(cx)d\,x^3+\frac{c\,e^3\mathrm{arccosh}(cx)x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+}}{}}{}$
default	$\frac{\frac{c\,\mathrm{arccosh}(cx)d^4}{4e}+\mathrm{arccosh}(cx)d^3cx+\frac{3ce\,\mathrm{arccosh}(cx)d^2x^2}{2}+c\,e^2\mathrm{arccosh}(cx)d\,x^3+\frac{c\,e^3\mathrm{arccosh}(cx)x^4}{4}-\frac{\sqrt{cx-1}\,\sqrt{cx+}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*c/e*arccosh(c*x)*d^4+arccosh(c*x)*d^3*c*x+3/2*c*e*arccosh(c*x)*d^2
*x^2+c*e^2*arccosh(c*x)*d*x^3+1/4*c*e^3*arccosh(c*x)*x^4-1/96/c^3/e*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(24*c^4*d^4*ln(c*x+(c^2*x^2-1)^(1/2))+96*c^3*d^3*e*(c^
2*x^2-1)^(1/2)+72*c^3*d^2*e^2*x*(c^2*x^2-1)^(1/2)+32*c^3*d*e^3*(c^2*x^2-1)^
(1/2)*x^2+6*e^4*(c^2*x^2-1)^(1/2)*c^3*x^3+72*c^2*d^2*e^2*ln(c*x+(c^2*x^2-1)
^(1/2))+64*c*d*e^3*(c^2*x^2-1)^(1/2)+9*e^4*c*x*(c^2*x^2-1)^(1/2)+9*e^4*ln(c
*x+(c^2*x^2-1)^(1/2)))/(c^2*x^2-1)^(1/2))
```

Maxima [A]
time = 0.28, size = 225, normalized size = 1.23

$$-\frac{1}{96}\left(\frac{6\sqrt{c^2x^2-1}\,x^3e^3}{c^2}+\frac{32\sqrt{c^2x^2-1}\,dx^2e^2}{c^2}+\frac{72\sqrt{c^2x^2-1}\,d^2xe}{c^2}+\frac{96\sqrt{c^2x^2-1}\,d^3}{c^2}+\frac{72\,d^2e\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}+\frac{9\sqrt{c^2x^2-1}\,xe^3}{c^4}+\frac{64\sqrt{c^2x^2-1}\,de^2}{c^4}+\frac{9\,e^3\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c+\frac{1}{4}\left(x^4e^3+4\,dx^3e^2+6\,d^2x^2e+4\,d^3x\right)\mathrm{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/96*(6*sqrt(c^2*x^2 - 1)*x^3*e^3/c^2 + 32*sqrt(c^2*x^2 - 1)*d*x^2*e^2/c^2
 + 72*sqrt(c^2*x^2 - 1)*d^2*x*e/c^2 + 96*sqrt(c^2*x^2 - 1)*d^3/c^2 + 72*d^2
```

```
*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 9*sqrt(c^2*x^2 - 1)*x*e^3/c^4
 + 64*sqrt(c^2*x^2 - 1)*d*e^2/c^4 + 9*e^3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)
*c)/c^5)*c + 1/4*(x^4*e^3 + 4*d*x^3*e^2 + 6*d^2*x^2*e + 4*d^3*x)*arccosh(c*
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(159) = 318$.
time = 0.37, size = 355, normalized size = 1.94

$$\frac{3\left(32\,c^4dx^3\cosh\left(1\right)^2+32\,c^4d^3x+\left(8\,c^4x^4-3\right)\cosh\left(1\right)^3+\left(8\,c^4x^4-3\right)\sinh\left(1\right)^3+\left(32\,c^4dx^3+3\left(8\,c^4x^4-3\right)\cosh\left(1\right)\right)\sinh\left(1\right)^2+24\left(2\,c^4d^2x^2-c^2d^2\right)\cosh\left(1\right)+\left(64\,c^4dx^3\cosh\left(1\right)+48\,c^4d^2x^2-24\,c^2d^2+3\left(8\,c^4x^4-3\right)\cosh\left(1\right)^2\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(72\,c^3d^2x\cosh\left(1\right)+96\,c^3d^3+3\left(2\,c^3x^3+3\,cx\right)\cosh\left(1\right)^3+3\left(2\,c^3x^3+3\,cx\right)\sinh\left(1\right)^3+32\left(c^3dx^2+2\,cd\right)\cosh\left(1\right)^2+\left(32\,c^3dx^2+64\,cd+9\left(2\,c^3x^3+3\,cx\right)\cosh\left(1\right)\right)\sinh\left(1\right)^2+\left(72\,c^3d^2x+9\left(2\,c^3x^3+3\,cx\right)\cosh\left(1\right)^2+64\left(c^3dx^2+2\,cd\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\sqrt{c^2x^2-1}}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(32*c^4*d*x^3*cosh(1)^2 + 32*c^4*d^3*x + (8*c^4*x^4 - 3)*cosh(1)^3
+ (8*c^4*x^4 - 3)*sinh(1)^3 + (32*c^4*d*x^3 + 3*(8*c^4*x^4 - 3)*cosh(1))*si
nh(1)^2 + 24*(2*c^4*d^2*x^2 - c^2*d^2)*cosh(1) + (64*c^4*d*x^3*cosh(1) + 48
*c^4*d^2*x^2 - 24*c^2*d^2 + 3*(8*c^4*x^4 - 3)*cosh(1)^2)*sinh(1))*log(c*x +
 sqrt(c^2*x^2 - 1)) - (72*c^3*d^2*x*cosh(1) + 96*c^3*d^3 + 3*(2*c^3*x^3 + 3
*c*x)*cosh(1)^3 + 3*(2*c^3*x^3 + 3*c*x)*sinh(1)^3 + 32*(c^3*d*x^2 + 2*c*d)*
cosh(1)^2 + (32*c^3*d*x^2 + 64*c*d + 9*(2*c^3*x^3 + 3*c*x)*cosh(1))*sinh(1)
^2 + (72*c^3*d^2*x + 9*(2*c^3*x^3 + 3*c*x)*cosh(1)^2 + 64*(c^3*d*x^2 + 2*c*
d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 0.26, size = 258, normalized size = 1.41

$$\begin{cases} d^3x\operatorname{acosh}(cx)+\frac{3d^2ex^2\operatorname{acosh}(cx)}{2}+de^2x^3\operatorname{acosh}(cx)+\frac{e^3x^4\operatorname{acosh}(cx)}{4}-\frac{d^3\sqrt{c^2x^2-1}}{c}-\frac{3d^2ex\sqrt{c^2x^2-1}}{4c}-\frac{de^2x^2\sqrt{c^2x^2-1}}{3c}-\frac{e^3x^3\sqrt{c^2x^2-1}}{16c}-\frac{3d^2e\operatorname{acosh}(cx)}{4c^2}-\frac{2de^2\sqrt{c^2x^2-1}}{3c^3}-\frac{3e^3x\sqrt{c^2x^2-1}}{32c^3}-\frac{3e^3\operatorname{acosh}(cx)}{32c^4} & \text{for}\: c\neq 0 \\ \frac{i\pi\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x),x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x) + 3*d**2*e*x**2*acosh(c*x)/2 + d*e**2*x**3*aco
sh(c*x) + e**3*x**4*acosh(c*x)/4 - d**3*sqrt(c**2*x**2 - 1)/c - 3*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - e**3*x*
*3*sqrt(c**2*x**2 - 1)/(16*c) - 3*d**2*e*acosh(c*x)/(4*c**2) - 2*d*e**2*sqr
t(c**2*x**2 - 1)/(3*c**3) - 3*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*e**3
*acosh(c*x)/(32*c**4), Ne(c, 0)), (I*pi*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*
x**3 + e**3*x**4/4)/2, True))
```

Giac [A]
time = 0.44, size = 174, normalized size = 0.95

$$\frac{(ex+d)^4\log\left(cx+\sqrt{c^2x^2-1}\right)}{4\,e}-\frac{\sqrt{c^2x^2-1}\left(\left(2\left(\frac{3\,e^4x}{c}+\frac{16\,de^3}{c}\right)x+\frac{9\left(8\,c^5d^2e^2+c^3e^4\right)}{c^6}\right)x+\frac{32\left(3\,c^5d^3e+2\,c^3de^3\right)}{c^6}\right)-\frac{3\left(8\,c^4d^4+24\,c^2d^2e^2+3\,e^4\right)\log\left(\left|-x|c|+\sqrt{c^2x^2-1}\right|\right)}{c^3|c|}}{96\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/4*(e*x + d)^4*log(c*x + sqrt(c^2*x^2 - 1))/e - 1/96*(sqrt(c^2*x^2 - 1)*((
2*(3*e^4*x/c + 16*d*e^3/c)*x + 9*(8*c^5*d^2*e^2 + c^3*e^4)/c^6)*x + 32*(3*c
^5*d^3*e + 2*c^3*d*e^3)/c^6) - 3*(8*c^4*d^4 + 24*c^2*d^2*e^2 + 3*e^4)*log(a
bs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^3*abs(c)))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}(c\,x)\,\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)*(d + e*x)^3, x)
```

3.2 $\int (d+ex)^2 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2)+5c^2dex)}{18c^3}-\frac{1}{6}d\left(\frac{2d^2}{e}+\frac{3e}{c^2}\right)\cosh^{-1}(cx$$

```
[Out] -1/6*d*(2*d^2/e+3*e/c^2)*arccosh(c*x)+1/3*(e*x+d)^3*arccosh(c*x)/e-1/9*(e*x
+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/18*(5*c^2*d*e*x+16*c^2*d^2+4*e^2)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5963, 102, 152, 54}

$$-\frac{1}{6}d\left(\frac{3e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(4(4c^2d^2+e^2)+5c^2dex)}{18c^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{9c}+\frac{\cosh^{-1}(cx)(d+ex)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -1/9*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/c - (Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (d*((2*d^2)/e + (3*e
)/c^2)*ArcCosh[c*x])/6 + ((d + e*x)^3*ArcCosh[c*x])/(3*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
```

```
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\cosh^{-1}(cx)\,dx &= \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{c\int \frac{(d+ex)^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3e}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c} + \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{\int \frac{(d+ex)(3c^2d^2+2e^2+5}{\sqrt{-1+cx}\,\sqrt{1-}}}{9ce}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2)+5c^2d}{18c^3}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2)+5c^2d}{18c^3}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 113, normalized size = 0.92

$$-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(4e^2+c^2(18d^2+9dex+2e^2x^2))-6c^3x(3d^2+3dex+e^2x^2)\cosh^{-1}(cx)+9cde\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -1/18*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*
x^2)) - 6*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCosh[c*x] + 9*c*d*e*Log[c*x
+ Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/c^3
```

Maple [A]
time = 1.30, size = 213, normalized size = 1.73

method	result
derivativedivides	$\frac{\frac{c\,\mathrm{arccosh}(cx)d^3}{3e}+\mathrm{arccosh}(cx)d^2cx+ce\,\mathrm{arccosh}(cx)d\,x^2+\frac{c\,e^2\mathrm{arccosh}(cx)x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6c^3d^3\ln\left(cx+\sqrt{c^2x\right.\right.}{}}{}$
default	$\frac{\frac{c\,\mathrm{arccosh}(cx)d^3}{3e}+\mathrm{arccosh}(cx)d^2cx+ce\,\mathrm{arccosh}(cx)d\,x^2+\frac{c\,e^2\mathrm{arccosh}(cx)x^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6c^3d^3\ln\left(cx+\sqrt{c^2x\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*c/e*arccosh(c*x)*d^3+arccosh(c*x)*d^2*c*x+c*e*arccosh(c*x)*d*x^2+1
/3*c*e^2*arccosh(c*x)*x^3-1/18/c^2/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(6*c^3*d^3
*ln(c*x+(c^2*x^2-1)^(1/2))+18*c^2*d^2*e*(c^2*x^2-1)^(1/2)+9*c^2*d*e^2*x*(c^
2*x^2-1)^(1/2)+2*e^3*(c^2*x^2-1)^(1/2)*c^2*x^2+9*c*d*e^2*ln(c*x+(c^2*x^2-1)
^(1/2))+4*e^3*(c^2*x^2-1)^(1/2))/(c^2*x^2-1)^(1/2))
```

Maxima [A]
time = 0.26, size = 140, normalized size = 1.14

$$-\frac{1}{18}\left(\frac{2\sqrt{c^2x^2-1}\,x^2e^2}{c^2}+\frac{9\sqrt{c^2x^2-1}\,dxe}{c^2}+\frac{18\sqrt{c^2x^2-1}\,d^2}{c^2}+\frac{9\,de\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}+\frac{4\sqrt{c^2x^2-1}\,e^2}{c^4}\right)c+\frac{1}{3}\left(x^3e^2+3\,dx^2e+3\,d^2x\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(c^2*x^2 - 1)*x^2*e^2/c^2 + 9*sqrt(c^2*x^2 - 1)*d*x*e/c^2 + 18
*sqrt(c^2*x^2 - 1)*d^2/c^2 + 9*d*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
 + 4*sqrt(c^2*x^2 - 1)*e^2/c^4)*c + 1/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*arc
cosh(c*x)
```

Fricas [A]
time = 0.40, size = 188, normalized size = 1.53

$$\frac{3\left(2\,c^3x^3\cosh\left(1\right)^2+2\,c^3x^3\sinh\left(1\right)^2+6\,c^3d^2x+3\left(2\,c^3dx^2-cd\right)\cosh\left(1\right)+\left(4\,c^3x^3\cosh\left(1\right)+6\,c^3dx^2-3\,cd\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,c^2dx\cosh\left(1\right)+18\,c^2d^2+2\left(c^2x^2+2\right)\cosh\left(1\right)^2+2\left(c^2x^2+2\right)\sinh\left(1\right)^2+\left(9\,c^2dx+4\left(c^2x^2+2\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\sqrt{c^2x^2-1}}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*c^3*x^3*cosh(1)^2 + 2*c^3*x^3*sinh(1)^2 + 6*c^3*d^2*x + 3*(2*c^3
*d*x^2 - c*d)*cosh(1) + (4*c^3*x^3*cosh(1) + 6*c^3*d*x^2 - 3*c*d)*sinh(1))*
log(c*x + sqrt(c^2*x^2 - 1)) - (9*c^2*d*x*cosh(1) + 18*c^2*d^2 + 2*(c^2*x^2
 + 2)*cosh(1)^2 + 2*(c^2*x^2 + 2)*sinh(1)^2 + (9*c^2*d*x + 4*(c^2*x^2 + 2)*
cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.16, size = 155, normalized size = 1.26

$$\begin{cases} d^2x\operatorname{acosh}(cx) + dex^2\operatorname{acosh}(cx) + \frac{e^2x^3\operatorname{acosh}(cx)}{3} - \frac{d^2\sqrt{c^2x^2-1}}{c} - \frac{dex\sqrt{c^2x^2-1}}{2c} - \frac{e^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{de\operatorname{acosh}(cx)}{2c^2} - \frac{2e^2\sqrt{c^2x^2-1}}{9c^3} & \text{for } c \neq 0 \\ \frac{i\pi\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x),x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x) + d*e*x**2*acosh(c*x) + e**2*x**3*acosh(c*x)/3
 - d**2*sqrt(c**2*x**2 - 1)/c - d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - e**2*x**2
*sqrt(c**2*x**2 - 1)/(9*c) - d*e*acosh(c*x)/(2*c**2) - 2*e**2*sqrt(c**2*x**
2 - 1)/(9*c**3), Ne(c, 0)), (I*pi*(d**2*x + d*e*x**2 + e**2*x**3/3)/2, True
))
```

Giac [A]
time = 0.43, size = 133, normalized size = 1.08

$$\frac{(ex+d)^3\log\left(cx+\sqrt{c^2x^2-1}\right)}{3\,e} - \frac{\sqrt{c^2x^2-1}\left(\left(\frac{2\,e^3x}{c}+\frac{9\,de^2}{c}\right)x+\frac{2\,(9\,c^3d^2e+2\,ce^3)}{c^4}\right) - \frac{3\,(2\,c^2d^3+3\,de^2)\log\left(\left|-x|c|+\sqrt{c^2x^2-1}\right|\right)}{c|c|}}{18\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/3*(e*x + d)^3*log(c*x + sqrt(c^2*x^2 - 1))/e - 1/18*(sqrt(c^2*x^2 - 1)*((
2*e^3*x/c + 9*d*e^2/c)*x + 2*(9*c^3*d^2*e + 2*c*e^3)/c^4) - 3*(2*c^2*d^3 +
3*d*e^2)*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c)))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(c\,x)\,\left(d+e\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)*(d + e*x)^2, x)
```

3.3 $\int (d+ex)\cosh^{-1}(cx)\,dx$

Optimal. Leaf size=97

$$-\frac{3d\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c}-\frac{1}{4}\left(\frac{2d^2}{e}+\frac{e}{c^2}\right)\cosh^{-1}(cx)+\frac{(d+ex)^2\cosh^{-1}(cx)}{2e}$$

```
[Out] -1/4*(2*d^2/e+e/c^2)*arccosh(c*x)+1/2*(e*x+d)^2*arccosh(c*x)/e-3/4*d*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c-1/4*(e*x+d)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5963, 92, 81, 54}

$$-\frac{1}{4}\left(\frac{e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)}{4c}+\frac{\cosh^{-1}(cx)(d+ex)^2}{2e}-\frac{3d\sqrt{cx-1}\,\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] (-3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
d + e*x))/(4*c) - (((2*d^2)/e + e/c^2)*ArcCosh[c*x])/4 + ((d + e*x)^2*ArcCo
sh[c*x])/(2*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\cosh^{-1}(cx)\,dx = \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{c\int \frac{(d+ex)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2e}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{\int \frac{2c^2d^2+e^2+3c^2dex}{\sqrt{-1+cx}\,\sqrt{1+c}}}{4ce}$$

$$= -\frac{3d\sqrt{-1+cx}\,\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(}{2e}$$

$$= -\frac{3d\sqrt{-1+cx}\,\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} - \frac{1}{4}\left(\frac{2d^2}{e}+\frac{e}{c^2}\right)\cos$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 0.75

$$-\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\,(4d+ex) - 2c^2x(2d+ex)\cosh^{-1}(cx) + 2e\tanh^{-1}\left(\sqrt{\frac{-1+cx}{1+cx}}\right)}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] -1/4*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x) - 2*c^2*x*(2*d + e*x)*ArcC
osh[c*x] + 2*e*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/c^2
```

Maple [A]
time = 1.93, size = 104, normalized size = 1.07

method	result

derivativedivides	$\frac{\operatorname{arccosh}(cx)dcx+\frac{c\operatorname{arccosh}(cx)e\,x^2}{2}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4dc\sqrt{c^2x^2-1}+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2}\right.\right.}{4c\sqrt{c^2x^2-1}}}{c}$
default	$\frac{\operatorname{arccosh}(cx)dcx+\frac{c\operatorname{arccosh}(cx)e\,x^2}{2}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4dc\sqrt{c^2x^2-1}+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2}\right.\right.}{4c\sqrt{c^2x^2-1}}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(arccosh(c*x)*d*c*x+1/2*c*arccosh(c*x)*e*x^2-1/4/c*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*(4*d*c*(c^2*x^2-1)^(1/2)+e*c*x*(c^2*x^2-1)^(1/2)+e*ln(c*x+(c^2*x^2-
1)^(1/2)))/(c^2*x^2-1)^(1/2))
```

Maxima [A]
time = 0.27, size = 85, normalized size = 0.88

$$-\frac{1}{4}\,c\left(\frac{\sqrt{c^2x^2-1}\,xe}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,d}{c^2}+\frac{e\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)+\frac{1}{2}\left(x^2e+2\,dx\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/4*c*(sqrt(c^2*x^2 - 1)*x*e/c^2 + 4*sqrt(c^2*x^2 - 1)*d/c^2 + e*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3) + 1/2*(x^2*e + 2*d*x)*arccosh(c*x)
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.88

$$\frac{\left(4\,c^2dx+\left(2\,c^2x^2-1\right)\cosh\left(1\right)+\left(2\,c^2x^2-1\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\left(cx\cosh\left(1\right)+cx\sinh\left(1\right)+4\,cd\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/4*((4*c^2*d*x + (2*c^2*x^2 - 1)*cosh(1) + (2*c^2*x^2 - 1)*sinh(1))*log(c*
x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*(c*x*cosh(1) + c*x*sinh(1) + 4*c
*d))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.10, size = 80, normalized size = 0.82

$$\begin{cases} dx\operatorname{acosh}\left(cx\right)+\frac{ex^2\operatorname{acosh}\left(cx\right)}{2}-\frac{d\sqrt{c^2x^2-1}}{c}-\frac{ex\sqrt{c^2x^2-1}}{4c}-\frac{e\operatorname{acosh}\left(cx\right)}{4c^2} & \text{for}\: c\neq 0\\ \frac{i\pi\left(dx+\frac{ex^2}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x),x)
```

```
[Out] Piecewise((d*x*acosh(c*x) + e*x**2*acosh(c*x)/2 - d*sqrt(c**2*x**2 - 1)/c -
 e*x*sqrt(c**2*x**2 - 1)/(4*c) - e*acosh(c*x)/(4*c**2), Ne(c, 0)), (I*pi*(d
*x + e*x**2/2)/2, True))
```

Giac [A]
time = 0.43, size = 84, normalized size = 0.87

$$\frac{1}{2}\left(ex^2+2\,dx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{1}{4}\sqrt{c^2x^2-1}\left(\frac{ex}{c}+\frac{4\,d}{c}\right)+\frac{e\log\left(\left|-x|c|+\sqrt{c^2x^2-1}\right|\right)}{4\,c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/4*sqrt(c^2*x^2 - 1)*(e
*x/c + 4*d/c) + 1/4*e*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c))
```

Mupad [B]
time = 0.71, size = 68, normalized size = 0.70

$$d\,x\,\mathrm{acosh}(c\,x)+e\,x\,\mathrm{acosh}(c\,x)\,\left(\frac{x}{2}-\frac{1}{4\,c^2\,x}\right)-\frac{d\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c}-\frac{e\,x\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x),x)
```

```
[Out] d*x*acosh(c*x) + e*x*acosh(c*x)*(x/2 - 1/(4*c^2*x)) - (d*(c*x - 1)^(1/2)*(c
*x + 1)^(1/2))/c - (e*x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/(4*c)
```

3.4 $\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=178

$$-\frac{\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\mathrm{PolyLog}\Big(2,}{}$$

```
[Out] -1/2*arccosh(c*x)^2/e+arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*d-(c^2*d^2-e^2)^(1/2)))/e+arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A]
time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5962, 5681, 2221, 2317, 2438}

$$\frac{\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-\frac{\cosh^{-1}(cx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcCosh[c*x]^2/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2
 - e^2]))]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx &= \text{Subst}\left(\int \frac{x\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)\\
&= -\frac{\cosh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd}\right.\\
&= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh}}{cd+\sqrt{c^2}}\right.}{e}\\
&= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh}}{cd+\sqrt{c^2}}\right.}{e}\\
&= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh}}{cd+\sqrt{c^2}}\right.}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 176, normalized size = 0.99

$$-\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\text{PolyLog}\left(2, \frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\text{PolyLog}\left(2, -\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcCosh[c*x]^2/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 - e^2])]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Maple [A]
time = 3.46, size = 304, normalized size = 1.71

method	result
derivativedivides	$-\frac{c\,\mathrm{arccosh}(cx)^2}{2e}+\frac{c\,\mathrm{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{c\,\mathrm{arccosh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{}\right.}{}\right.}{}$
default	$-\frac{c\,\mathrm{arccosh}(cx)^2}{2e}+\frac{c\,\mathrm{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{c\,\mathrm{arccosh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*c*arccosh(c*x)^2/e+1/e*c*arccosh(c*x)*ln((-c*d-e*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+1/e*c*ar
ccosh(c*x)*ln((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))
/(c*d+(c^2*d^2-e^2)^(1/2)))+1/e*c*dilog((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+1/e*c*dilog((-c*d-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(
1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(c*x)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d),x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*x)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x),x)
```

```
[Out] int(acosh(c*x)/(d + e*x), x)
```

3.5 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}$$

```
[Out] -arccosh(c*x)/e/(e*x+d)+2*c*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*d-e)^(1/
2)/(c*x-1)^(1/2))/e/(c*d-e)^(1/2)/(c*d+e)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5963, 95, 214}

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\,\sqrt{cd+e}}-\frac{\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^2,x]
```

```
[Out] -(ArcCosh[c*x]/(e*(d + e*x))) + (2*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/
(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{\cosh^{-1}(cx)}{e(d+ex)} + \frac{c\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{e}$$
$$= -\frac{\cosh^{-1}(cx)}{e(d+ex)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx, x, \frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}$$
$$= -\frac{\cosh^{-1}(cx)}{e(d+ex)} + \frac{2c\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}$$

Mathematica [A]
time = 0.07, size = 92, normalized size = 1.11

$$\frac{-\frac{\cosh^{-1}(cx)}{d+ex} + \frac{c\left(\log(d+ex)-\log\left(e+c^2dx-\sqrt{c^2d^2-e^2}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)\right)}{\sqrt{c^2d^2-e^2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^2,x]
```

```
[Out] (-(ArcCosh[c*x]/(d + e*x)) + (c*(Log[d + e*x] - Log[e + c^2*d*x - Sqrt[c^2*
d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/Sqrt[c^2*d^2 - e^2])/e
```

Maple [A]
time = 5.67, size = 134, normalized size = 1.61

method	result	size
derivativedivides	$\frac{-\frac{c^2\operatorname{arccosh}(cx)}{(ecx+cd)e} - \frac{c^2\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)}{e^2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}}}{c}$	134

default	$\frac{-\frac{c^{2}\operatorname{arccosh}(cx)}{(ecx+cd)e}-\frac{c^{2}\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^{2}dx-\sqrt{c^{2}x^{2}-1}\,\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,e+e\right)}{ecx+cd}\right)}{e^{2}\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,\sqrt{c^{2}x^{2}-1}}}{c}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-c^2/(c*e*x+c*d)/e*arccosh(c*x)-c^2/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/
((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(74) = 148.
time = 0.39, size = 1023, normalized size = 12.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((c*d*x*cosh(1) + c*d*x*sinh(1) + c*d^2)*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^
2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*log((c^3*d^2*x + c*d*cosh(1) + c*d
*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(
1))/(cosh(1) - sinh(1))) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)*sqrt(
c^2*x^2 - 1) + (c^2*d*x + cosh(1) + sinh(1))*sqrt(((c^2*d^2 - 1)*cosh(1) -
(c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*sinh(1) + d)) +
 (c^2*d^2*x*cosh(1) - x*cosh(1)^3 - 3*x*cosh(1)*sinh(1)^2 - x*sinh(1)^3 + (
c^2*d^2*x - 3*x*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + (c^2*d^2
*x*cosh(1) + c^2*d^3 - x*cosh(1)^3 - x*sinh(1)^3 - d*cosh(1)^2 - (3*x*cosh(
```

```
1) + d)*sinh(1)^2 + (c^2*d^2*x - 3*x*cosh(1)^2 - 2*d*cosh(1))*sinh(1))*log(
-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*x*cosh(1)^2 + c^2*d^4*cosh(1) - d*x*cos
h(1)^4 - d*x*sinh(1)^4 - d^2*cosh(1)^3 - (4*d*x*cosh(1) + d^2)*sinh(1)^3 +
(c^2*d^3*x - 6*d*x*cosh(1)^2 - 3*d^2*cosh(1))*sinh(1)^2 + (2*c^2*d^3*x*cosh
(1) + c^2*d^4 - 4*d*x*cosh(1)^3 - 3*d^2*cosh(1)^2)*sinh(1)), -(2*(c*d*x*cos
h(1) + c*d*x*sinh(1) + c*d^2)*sqrt(-((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*
sinh(1))/(cosh(1) - sinh(1)))*arctan(-(sqrt(c^2*x^2 - 1)*sqrt(-((c^2*d^2 -
1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*(cosh(1) + sinh(1)
) - (c*x*cosh(1) + c*x*sinh(1) + c*d)*sqrt(-((c^2*d^2 - 1)*cosh(1) - (c^2*d
^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(c^2*d^2 - cosh(1)^2 - 2*cosh(1)*sin
h(1) - sinh(1)^2)) - (c^2*d^2*x*cosh(1) - x*cosh(1)^3 - 3*x*cosh(1)*sinh(1)
^2 - x*sinh(1)^3 + (c^2*d^2*x - 3*x*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*
x^2 - 1)) - (c^2*d^2*x*cosh(1) + c^2*d^3 - x*cosh(1)^3 - x*sinh(1)^3 - d*co
sh(1)^2 - (3*x*cosh(1) + d)*sinh(1)^2 + (c^2*d^2*x - 3*x*cosh(1)^2 - 2*d*co
sh(1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*x*cosh(1)^2 + c^2*d
^4*cosh(1) - d*x*cosh(1)^4 - d*x*sinh(1)^4 - d^2*cosh(1)^3 - (4*d*x*cosh(1)
 + d^2)*sinh(1)^3 + (c^2*d^3*x - 6*d*x*cosh(1)^2 - 3*d^2*cosh(1))*sinh(1)^2
 + (2*c^2*d^3*x*cosh(1) + c^2*d^4 - 4*d*x*cosh(1)^3 - 3*d^2*cosh(1)^2)*sinh
(1))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(71) = 142.
time = 0.58, size = 240, normalized size = 2.89

$$-\frac{\log\left(cx+\sqrt{c^2x^2-1}\right)}{(ex+d)e}+\frac{\frac{ce^4\log\left(\left|c^2de-\sqrt{c^2d^2-e^2}\,|c||e|\right|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{c^2d^2-e^2}\,|e|}-\frac{ce^4\log\left(\left|c^2de-\sqrt{c^2d^2-e^2}\left(\sqrt{c^2-\frac{2c^2d}{ex+d}+\frac{c^2d^2}{(ex+d)^2}-\frac{e^2}{(ex+d)^2}}+\frac{\sqrt{c^2d^2e^2-e^4}}{(ex+d)e}\right)|e|\right|\right)}{\sqrt{c^2d^2-e^2}\,|e|\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -log(c*x + sqrt(c^2*x^2 - 1))/((e*x + d)*e) + (c*e^4*log(abs(c^2*d*e - sqrt
(c^2*d^2 - e^2)*abs(c)*abs(e)))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c^2*d^2 - e^2
)*abs(e)) - c*e^4*log(abs(c^2*d*e - sqrt(c^2*d^2 - e^2)*(sqrt(c^2 - 2*c^2*d
/(e*x + d) + c^2*d^2/(e*x + d)^2 - e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 - e^
```

```
4)/((e*x + d)*e))*abs(e)))/(sqrt(c^2*d^2 - e^2)*abs(e)*sgn(1/(e*x + d))*sgn
(e)))/e^4
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^2,x)
[Out] int(acosh(c*x)/(d + e*x)^2, x)
```

3.6 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=132

$$-\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}$$

```
[Out] -1/2*arccosh(c*x)/e/(e*x+d)^2+c^3*d*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*
d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(3/2)/e/(c*d+e)^(3/2)-1/2*c*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5963, 98, 95, 214}

$$\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{c\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] -1/2*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)*(d + e*x)) - ArcCosh
[c*x]/(2*e*(d + e*x)^2) + (c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/(Sqr
t[c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(cx)}{(d+ex)^3}\,dx &= -\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{c\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{2e}\\
&= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(c^3d\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{2e\left(c^2d^2-e^2\right)}\\
&= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(c^3d\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e\left(c^2d^2-e^2\right)}\\
&= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 190, normalized size = 1.44

$$\frac{-\left(c^2d^2-e^2\right)^{3/2}\cosh^{-1}(cx)+c(d+ex)\left(-e\sqrt{c^2d^2-e^2}\,\sqrt{-1+cx}\,\sqrt{1+cx}+c^2d(d+ex)\log(d+ex)-c^2d(d+ex)\log\left(e+c^2dx-\sqrt{c^2d^2-e^2}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)\right)}{2(cd-e)e(cd+e)\sqrt{c^2d^2-e^2}\,(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] (-((c^2*d^2 - e^2)^(3/2)*ArcCosh[c*x]) + c*(d + e*x)*(-(e*Sqrt[c^2*d^2 - e^
2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + c^2*d*(d + e*x)*Log[d + e*x] - c^2*d*(d
+ e*x)*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
)/(2*(c*d - e)*e*(c*d + e)*Sqrt[c^2*d^2 - e^2]*(d + e*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(112) = 224$.
time = 5.73, size = 266, normalized size = 2.02

method	result
derivativedivides	$\frac{-\frac{c^3\operatorname{arccosh}(cx)}{2(ecx+cd)^2e}+\frac{c^3\left(-\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)c^2d^2-\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-}{e^2}}\right.}{ecx+cd}\right.\right.}{2e^2\sqrt{c^2x^2-1}\,(cd-e)(cd+e)(ecx+cd}}{c}$
default	$\frac{-\frac{c^3\operatorname{arccosh}(cx)}{2(ecx+cd)^2e}+\frac{c^3\left(-\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)c^2d^2-\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-}{e^2}}\right.}{ecx+cd}\right.\right.}{2e^2\sqrt{c^2x^2-1}\,(cd-e)(cd+e)(ecx+cd}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arccosh(c*x)+1/2*c^3/e^2*(-ln(-2*(c^2*d*x-(c^
2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*c^2*d^2-ln(-2*(c
^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*c^2*d*
e*x-e^2*(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/(c^2*x^2-1)^(1/2)/(c*d-e)/(c*d+e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/
2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1882 vs. $2(115) = 230$.
time = 0.45, size = 3784, normalized size = 28.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c^4*d^5*x*cosh(1) + c^4*d^6 - c^2*d^2*x^2*cosh(1)^4 - c^2*d^2*x^2*
sinh(1)^4 - 2*c^2*d^3*x*cosh(1)^3 - 2*(2*c^2*d^2*x^2*cosh(1) + c^2*d^3*x)*s
inh(1)^3 + (c^4*d^4*x^2 - c^2*d^4)*cosh(1)^2 + (c^4*d^4*x^2 - 6*c^2*d^2*x^2
*cosh(1)^2 - 6*c^2*d^3*x*cosh(1) - c^2*d^4)*sinh(1)^2 + (c^3*d^3*x^2*cosh(1
)^2 + c^3*d^3*x^2*sinh(1)^2 + 2*c^3*d^4*x*cosh(1) + c^3*d^5 + 2*(c^3*d^3*x^
2*cosh(1) + c^3*d^4*x)*sinh(1))*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)
*sinh(1))/(cosh(1) - sinh(1)))*log((c^3*d^2*x + c*d*cosh(1) + c*d*sinh(1) +
 (c^2*d^2 - c*d*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(
1) - sinh(1))) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)*sqrt(c^2*x^2 -
1) - (c^2*d*x + cosh(1) + sinh(1))*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 +
 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*sinh(1) + d)) - (c^4*d^4*
x^2*cosh(1)^2 + 2*c^4*d^5*x*cosh(1) - 2*c^2*d^2*x^2*cosh(1)^4 - 4*c^2*d^3*x
*cosh(1)^3 + x^2*cosh(1)^6 + x^2*sinh(1)^6 + 2*d*x*cosh(1)^5 + 2*(3*x^2*cos
h(1) + d*x)*sinh(1)^5 - (2*c^2*d^2*x^2 - 15*x^2*cosh(1)^2 - 10*d*x*cosh(1))
*sinh(1)^4 - 4*(2*c^2*d^2*x^2*cosh(1) + c^2*d^3*x - 5*x^2*cosh(1)^3 - 5*d*x
*cosh(1)^2)*sinh(1)^3 + (c^4*d^4*x^2 - 12*c^2*d^2*x^2*cosh(1)^2 - 12*c^2*d^
3*x*cosh(1) + 15*x^2*cosh(1)^4 + 20*d*x*cosh(1)^3)*sinh(1)^2 + 2*(c^4*d^4*x
^2*cosh(1) + c^4*d^5*x - 4*c^2*d^2*x^2*cosh(1)^3 - 6*c^2*d^3*x*cosh(1)^2 +
3*x^2*cosh(1)^5 + 5*d*x*cosh(1)^4)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) -
(2*c^4*d^5*x*cosh(1) + c^4*d^6 - 4*c^2*d^3*x*cosh(1)^3 + x^2*cosh(1)^6 + x^
2*sinh(1)^6 + 2*d*x*cosh(1)^5 + 2*(3*x^2*cosh(1) + d*x)*sinh(1)^5 - (2*c^2*
d^2*x^2 - d^2)*cosh(1)^4 - (2*c^2*d^2*x^2 - 15*x^2*cosh(1)^2 - 10*d*x*cosh(
1) - d^2)*sinh(1)^4 - 4*(c^2*d^3*x - 5*x^2*cosh(1)^3 - 5*d*x*cosh(1)^2 + (2
*c^2*d^2*x^2 - d^2)*cosh(1))*sinh(1)^3 + (c^4*d^4*x^2 - 2*c^2*d^4)*cosh(1)^
2 + (c^4*d^4*x^2 - 12*c^2*d^3*x*cosh(1) - 2*c^2*d^4 + 15*x^2*cosh(1)^4 + 20
*d*x*cosh(1)^3 - 6*(2*c^2*d^2*x^2 - d^2)*cosh(1)^2)*sinh(1)^2 + 2*(c^4*d^5*
x - 6*c^2*d^3*x*cosh(1)^2 + 3*x^2*cosh(1)^5 + 5*d*x*cosh(1)^4 - 2*(2*c^2*d^
2*x^2 - d^2)*cosh(1)^3 + (c^4*d^4*x^2 - 2*c^2*d^4)*cosh(1))*sinh(1))*log(-c
*x + sqrt(c^2*x^2 - 1)) + 2*(c^4*d^5*x - 2*c^2*d^2*x^2*cosh(1)^3 - 3*c^2*d^
3*x*cosh(1)^2 + (c^4*d^4*x^2 - c^2*d^4)*cosh(1))*sinh(1) + (c^3*d^4*x*cosh(
1)^2 + c^3*d^5*cosh(1) - c*d^2*x*cosh(1)^4 - c*d^2*x*sinh(1)^4 - c*d^3*cosh
(1)^3 - (4*c*d^2*x*cosh(1) + c*d^3)*sinh(1)^3 + (c^3*d^4*x - 6*c*d^2*x*cosh
(1)^2 - 3*c*d^3*cosh(1))*sinh(1)^2 + (2*c^3*d^4*x*cosh(1) + c^3*d^5 - 4*c*d
^2*x*cosh(1)^3 - 3*c*d^3*cosh(1)^2)*sinh(1))*sqrt(c^2*x^2 - 1))/(2*c^4*d^7*
x*cosh(1)^2 + c^4*d^8*cosh(1) - 4*c^2*d^5*x*cosh(1)^4 + d^2*x^2*cosh(1)^7 +
 d^2*x^2*sinh(1)^7 + 2*d^3*x*cosh(1)^6 + (7*d^2*x^2*cosh(1) + 2*d^3*x)*sinh
(1)^6 - (2*c^2*d^4*x^2 - d^4)*cosh(1)^5 - (2*c^2*d^4*x^2 - 21*d^2*x^2*cosh(
1)^2 - 12*d^3*x*cosh(1) - d^4)*sinh(1)^5 - (4*c^2*d^5*x - 35*d^2*x^2*cosh(1
)^3 - 30*d^3*x*cosh(1)^2 + 5*(2*c^2*d^4*x^2 - d^4)*cosh(1))*sinh(1)^4 + (c^
4*d^6*x^2 - 2*c^2*d^6)*cosh(1)^3 + (c^4*d^6*x^2 - 16*c^2*d^5*x*cosh(1) - 2*
c^2*d^6 + 35*d^2*x^2*cosh(1)^4 + 40*d^3*x*cosh(1)^3 - 10*(2*c^2*d^4*x^2 - d
^4)*cosh(1)^2)*sinh(1)^3 + (2*c^4*d^7*x - 24*c^2*d^5*x*cosh(1)^2 + 21*d^2*x
^2*cosh(1)^5 + 30*d^3*x*cosh(1)^4 - 10*(2*c^2*d^4*x^2 - d^4)*cosh(1)^3 + 3*
```

```
(c^4*d^6*x^2 - 2*c^2*d^6)*cosh(1))*sinh(1)^2 + (4*c^4*d^7*x*cosh(1) + c^4*d
^8 - 16*c^2*d^5*x*cosh(1)^3 + 7*d^2*x^2*cosh(1)^6 + 12*d^3*x*cosh(1)^5 - 5*
(2*c^2*d^4*x^2 - d^4)*cosh(1)^4 + 3*(c^4*d^6*x^2 - 2*c^2*d^6)*cosh(1)^2)*si
nh(1)), -1/2*(2*c^4*d^5*x*cosh(1) + c^4*d^6 - c^2*d^2*x^2*cosh(1)^4 - c^2*d
^2*x^2*sinh(1)^4 - 2*c^2*d^3*x*cosh(1)^3 - 2*(2*c^2*d^2*x^2*cosh(1) + c^2*d
^3*x)*sinh(1)^3 + (c^4*d^4*x^2 - c^2*d^4)*cosh(1)^2 + (c^4*d^4*x^2 - 6*c^2*
d^2*x^2*cosh(1)^2 - 6*c^2*d^3*x*cosh(1) - c^2*d^4)*sinh(1)^2 + 2*(c^3*d^3*x
^2*cosh(1)^2 + c^3*d^3*x^2*sinh(1)^2 + 2*c^3*d^4*x*cosh(1) + c^3*d^5 + 2*(c
^3*d^3*x^2*cosh(1) + c^3*d^4*x)*sinh(1))*sqrt(-((c^2*d^2 - 1)*cosh(1) - (c^
2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*arctan(-(sqrt(c^2*x^2 - 1)*sqrt(-(
(c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*(cosh(1
) + sinh(1)) - (c*x*cosh(1) + c*x*sinh(1) + c*d)*sqrt(-((c^2*d^2 - 1)*cosh(
1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(c^2*d^2 - cosh(1)^2 - 2*
cosh(1)*sinh(1) - sinh(1)^2)) - (c^4*d^4*x^2*cosh(1)^2 + 2*c^4*d^5*x*cosh(1
) - 2*c^2*d^2*x^2*cosh(1)^4 - 4*c^2*d^3*x*cosh(1)^3 + x^2*cosh(1)^6 + x^2*s
inh(1)^6 + 2*d*x*cosh(1)^5 + 2*(3*x^2*cosh(1) + d*x)*sinh(1)^5 - (2*c^2*d^2
*x^2 - 15*x^2*cosh(1)^2 - 10*d*x*cosh(1))*sinh(1)^4 - 4*(2*c^2*d^2*x^2*cosh
(1) + c^2*d^3*x - 5*x^2*cosh(1)^3 - 5*d*x*cosh(1)^2)*sinh(1)^3 + (c^4*d^4*x
^2 - 12*c^2*d^2*x^2*cosh(1)^2 - 12*c^2*d^3*x*cosh(1) + 15*x^2*cosh(1)^4 + 2
0*d*x*cosh(1)^3)*sinh(1)^2 + 2*(c^4*d^4*x^2*cosh(1) + c^4*d^5*x - 4*c^2*d^2
*x^2*cosh(1)^3 - 6*c^2*d^3*x*cosh(1)^2 + 3*x^2*cosh(1)^5 + 5*d*x*cosh(1)^4)
*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - (2*c^4*d^5*x*cosh(1) + c^4*d^6 - 4
*c^2*d^3*x*cosh(1)^3 + x^2*cosh(1)^6 + x^2*sinh(1)^6 + 2*d*x*cosh(1)^5 + 2*
(3*x^2*cosh(1) + d*x)*sinh(1)^5 - (2*c^2*d^2*x^...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**3, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
```

```
itution variable should perhaps be purged.Warning, integration of abs or si
gn ass
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^3,x)
[Out] int(acosh(c*x)/(d + e*x)^3, x)
```

3.7 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=195

$$-\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{c^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{\ldots}}{\sqrt{cd-e}\,\sqrt{-\ldots}}\right)}{3(cd-e)^{5/2}e(cd+e)^{5/2}}$$

```
[Out] -1/3*arccosh(c*x)/e/(e*x+d)^3+1/3*c^3*(2*c^2*d^2+e^2)*arctanh((c*d+e)^(1/2)
*(c*x+1)^(1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(5/2)/e/(c*d+e)^(5/2)-1
/6*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)^2-1/2*c^3*d*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)
```

Rubi [A]
time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5963, 105, 156, 12, 95, 214}

$$-\frac{c^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{c\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] -1/6*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)*(d + e*x)^2) - (c^3*
d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - Arc
Cosh[c*x]/(3*e*(d + e*x)^3) + (c^3*(2*c^2*d^2 + e^2)*ArcTanh[(Sqrt[c*d + e]
*Sqrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(5/2)*e*(c*d
+ e)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}dx}{3e}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} - \frac{c\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}dx}{6e(c^2d^2-e^2)}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2(c^2d^2-e^2)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c\int \frac{c^2(2c^2 \cdots}{\sqrt{-1+cx}\sqrt{\cdots}}}{6e(c^2d^2 \cdots}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2(c^2d^2-e^2)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(c^3(2c^2d^2+e^2))\cdots}{\cdots}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2(c^2d^2-e^2)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(c^3(2c^2d^2+e^2))\cdots}{\cdots}$$

$$= -\frac{c\sqrt{-1+cx}\sqrt{1+cx}}{6(c^2d^2-e^2)(d+ex)^2} - \frac{c^3d\sqrt{-1+cx}\sqrt{1+cx}}{2(c^2d^2-e^2)^2(d+ex)} - \frac{\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{c^3(2c^2d^2+e^2)\tan\cdots}{3(cd \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 244, normalized size = 1.25

$$\frac{1}{6}\left(\frac{c\sqrt{-1+cx}\sqrt{1+cx}(e^2-c^2d(4d+3ex))}{(-c^2d^2+e^2)^2(d+ex)^2} - \frac{2\cosh^{-1}(cx)}{e(d+ex)^3} - \frac{ic^3(2c^2d^2+e^2)\log\left(\frac{12e^2(-cd+e)^2(cd+e)^2\left(-ie-ic^2dx+\sqrt{-c^2d^2+e^2}\sqrt{-1+cx}\sqrt{1+cx}\right)}{c^3\sqrt{-c^2d^2+e^2}(2c^2d^2+e^2)(d+ex)}\right)}{e(-cd+e)^2(cd+e)^2\sqrt{-c^2d^2+e^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] ((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2 - c^2*d*(4*d + 3*e*x)))/((-(c^2*d^2)
+ e^2)^2*(d + e*x)^2) - (2*ArcCosh[c*x])/(e*(d + e*x)^3) - (I*c^3*(2*c^2*d^
2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*((-I)*e - I*c^2*d*x + Sqrt[
-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*Sqrt[-(c^2*d^2) + e^2
]*(2*c^2*d^2 + e^2)*(d + e*x))])/(e*(-(c*d) + e)^2*(c*d + e)^2*Sqrt[-(c^2*d
^2) + e^2]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(166) = 332$.
time = 6.08, size = 624, normalized size = 3.20

method	result

derivativedivides	$\frac{-\frac{c^4 \operatorname{arccosh}(cx)}{3(ecx+cd)^3 e}-\frac{c^4\left(2\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)c^4d^4+4\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e}{e^2}}\right.}{ecx+cd}\right.\right.}{}}{}$
default	$\frac{-\frac{c^4 \operatorname{arccosh}(cx)}{3(ecx+cd)^3 e}-\frac{c^4\left(2\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)c^4d^4+4\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e}{e^2}}\right.}{ecx+cd}\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*c^4/(c*e*x+c*d)^3/e*arccosh(c*x)-1/6*c^4/e^2*(2*ln(-2*(c^2*d*x-(c
^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*c^4*d^4+4*ln(-2
*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*c^4
*d^3*e*x+2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/
(c*e*x+c*d))*c^4*d^2*e^2*x^2+4*c^2*d^2*e^2*(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2)+3*c^2*d*e^3*(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*x+ln(-2
*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*c^2
*d^2*e^2+2*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/
(c*e*x+c*d))*c^2*d*e^3*x+ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^
2)^(1/2)*e+e)/(c*e*x+c*d))*e^4*c^2*x^2-e^4*(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/
((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)^2)
```

Maxima **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas **[B]** Leaf count of result is larger than twice the leaf count of optimal. 4546 vs. $2(168) = 336$.

time = 0.77, size = 9111, normalized size = 46.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(9*c^6*d^8*x*cosh(1) - 3*c^4*d^4*x^3*cosh(1)^5 - 3*c^4*d^4*x^3*sinh(1
)^5 + 3*c^6*d^9 - 9*c^4*d^5*x^2*cosh(1)^4 - 3*(5*c^4*d^4*x^3*cosh(1) + 3*c^
4*d^5*x^2)*sinh(1)^4 + 3*(c^6*d^6*x^3 - 3*c^4*d^6*x)*cosh(1)^3 + 3*(c^6*d^6
*x^3 - 10*c^4*d^4*x^3*cosh(1)^2 - 12*c^4*d^5*x^2*cosh(1) - 3*c^4*d^6*x)*sin
h(1)^3 + 3*(3*c^6*d^7*x^2 - c^4*d^7)*cosh(1)^2 + 3*(3*c^6*d^7*x^2 - 10*c^4*
d^4*x^3*cosh(1)^3 - 18*c^4*d^5*x^2*cosh(1)^2 - c^4*d^7 + 3*(c^6*d^6*x^3 - 3
*c^4*d^6*x)*cosh(1))*sinh(1)^2 - (6*c^5*d^7*x*cosh(1) + c^3*d^3*x^3*cosh(1)
^5 + c^3*d^3*x^3*sinh(1)^5 + 2*c^5*d^8 + 3*c^3*d^4*x^2*cosh(1)^4 + (5*c^3*d
^3*x^3*cosh(1) + 3*c^3*d^4*x^2)*sinh(1)^4 + (2*c^5*d^5*x^3 + 3*c^3*d^5*x)*c
osh(1)^3 + (2*c^5*d^5*x^3 + 10*c^3*d^3*x^3*cosh(1)^2 + 12*c^3*d^4*x^2*cosh(
1) + 3*c^3*d^5*x)*sinh(1)^3 + (6*c^5*d^6*x^2 + c^3*d^6)*cosh(1)^2 + (6*c^5*
d^6*x^2 + 10*c^3*d^3*x^3*cosh(1)^3 + 18*c^3*d^4*x^2*cosh(1)^2 + c^3*d^6 + 3
*(2*c^5*d^5*x^3 + 3*c^3*d^5*x)*cosh(1))*sinh(1)^2 + (6*c^5*d^7*x + 5*c^3*d^
3*x^3*cosh(1)^4 + 12*c^3*d^4*x^2*cosh(1)^3 + 3*(2*c^5*d^5*x^3 + 3*c^3*d^5*x
)*cosh(1)^2 + 2*(6*c^5*d^6*x^2 + c^3*d^6)*cosh(1))*sinh(1))*sqrt(((c^2*d^2
- 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*log((c^3*d^2*x +
 c*d*cosh(1) + c*d*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 - 1)*cosh(1) - (
c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))) - cosh(1)^2 - 2*cosh(1)*sinh(1)
- sinh(1)^2)*sqrt(c^2*x^2 - 1) + (c^2*d*x + cosh(1) + sinh(1))*sqrt(((c^2*d
^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) +
 x*sinh(1) + d)) - 2*(3*c^6*d^7*x^2*cosh(1)^2 + 3*c^6*d^8*x*cosh(1) - 9*c^4
*d^5*x^2*cosh(1)^4 + 9*c^2*d^3*x^2*cosh(1)^6 - x^3*cosh(1)^9 - x^3*sinh(1)^
9 - 3*d*x^2*cosh(1)^8 - 3*(3*x^3*cosh(1) + d*x^2)*sinh(1)^8 + 3*(c^2*d^2*x^
3 - d^2*x)*cosh(1)^7 + 3*(c^2*d^2*x^3 - 12*x^3*cosh(1)^2 - 8*d*x^2*cosh(1)
- d^2*x)*sinh(1)^7 + 3*(3*c^2*d^3*x^2 - 28*x^3*cosh(1)^3 - 28*d*x^2*cosh(1)
^2 + 7*(c^2*d^2*x^3 - d^2*x)*cosh(1))*sinh(1)^6 - 3*(c^4*d^4*x^3 - 3*c^2*d^
4*x)*cosh(1)^5 - 3*(c^4*d^4*x^3 - 18*c^2*d^3*x^2*cosh(1) - 3*c^2*d^4*x + 42
*x^3*cosh(1)^4 + 56*d*x^2*cosh(1)^3 - 21*(c^2*d^2*x^3 - d^2*x)*cosh(1)^2)*s
inh(1)^5 - 3*(3*c^4*d^5*x^2 - 45*c^2*d^3*x^2*cosh(1)^2 + 42*x^3*cosh(1)^5 +
 70*d*x^2*cosh(1)^4 - 35*(c^2*d^2*x^3 - d^2*x)*cosh(1)^3 + 5*(c^4*d^4*x^3 -
 3*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (c^6*d^6*x^3 - 9*c^4*d^6*x)*cosh(1)^3 +
(c^6*d^6*x^3 - 36*c^4*d^5*x^2*cosh(1) - 9*c^4*d^6*x + 180*c^2*d^3*x^2*cosh(
1)^3 - 84*x^3*cosh(1)^6 - 168*d*x^2*cosh(1)^5 + 105*(c^2*d^2*x^3 - d^2*x)*c
osh(1)^4 - 30*(c^4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1)^2)*sinh(1)^3 + 3*(c^6*d^7
*x^2 - 18*c^4*d^5*x^2*cosh(1)^2 + 45*c^2*d^3*x^2*cosh(1)^4 - 12*x^3*cosh(1)
^7 - 28*d*x^2*cosh(1)^6 + 21*(c^2*d^2*x^3 - d^2*x)*cosh(1)^5 - 10*(c^4*d^4*
x^3 - 3*c^2*d^4*x)*cosh(1)^3 + (c^6*d^6*x^3 - 9*c^4*d^6*x)*cosh(1))*sinh(1)
^2 + 3*(2*c^6*d^7*x^2*cosh(1) + c^6*d^8*x - 12*c^4*d^5*x^2*cosh(1)^3 + 18*c
```

```
^2*d^3*x^2*cosh(1)^5 - 3*x^3*cosh(1)^8 - 8*d*x^2*cosh(1)^7 + 7*(c^2*d^2*x^3
 - d^2*x)*cosh(1)^6 - 5*(c^4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1)^4 + (c^6*d^6*x^
3 - 9*c^4*d^6*x)*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(3*c^
6*d^8*x*cosh(1) + c^6*d^9 - x^3*cosh(1)^9 - x^3*sinh(1)^9 - 3*d*x^2*cosh(1)
^8 - 3*(3*x^3*cosh(1) + d*x^2)*sinh(1)^8 + 3*(c^2*d^2*x^3 - d^2*x)*cosh(1)^
7 + 3*(c^2*d^2*x^3 - 12*x^3*cosh(1)^2 - 8*d*x^2*cosh(1) - d^2*x)*sinh(1)^7
+ (9*c^2*d^3*x^2 - d^3)*cosh(1)^6 + (9*c^2*d^3*x^2 - 84*x^3*cosh(1)^3 - 84*
d*x^2*cosh(1)^2 - d^3 + 21*(c^2*d^2*x^3 - d^2*x)*cosh(1))*sinh(1)^6 - 3*(c^
4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1)^5 - 3*(c^4*d^4*x^3 - 3*c^2*d^4*x + 42*x^3*
cosh(1)^4 + 56*d*x^2*cosh(1)^3 - 21*(c^2*d^2*x^3 - d^2*x)*cosh(1)^2 - 2*(9*
c^2*d^3*x^2 - d^3)*cosh(1))*sinh(1)^5 - 3*(3*c^4*d^5*x^2 - c^2*d^5)*cosh(1)
^4 - 3*(3*c^4*d^5*x^2 + 42*x^3*cosh(1)^5 - c^2*d^5 + 70*d*x^2*cosh(1)^4 - 3
5*(c^2*d^2*x^3 - d^2*x)*cosh(1)^3 - 5*(9*c^2*d^3*x^2 - d^3)*cosh(1)^2 + 5*(
c^4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (c^6*d^6*x^3 - 9*c^4*d^6*x)
*cosh(1)^3 + (c^6*d^6*x^3 - 9*c^4*d^6*x - 84*x^3*cosh(1)^6 - 168*d*x^2*cosh
(1)^5 + 105*(c^2*d^2*x^3 - d^2*x)*cosh(1)^4 + 20*(9*c^2*d^3*x^2 - d^3)*cosh
(1)^3 - 30*(c^4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1)^2 - 12*(3*c^4*d^5*x^2 - c^2*
d^5)*cosh(1))*sinh(1)^3 + 3*(c^6*d^7*x^2 - c^4*d^7)*cosh(1)^2 + 3*(c^6*d^7*
x^2 - c^4*d^7 - 12*x^3*cosh(1)^7 - 28*d*x^2*cosh(1)^6 + 21*(c^2*d^2*x^3 - d
^2*x)*cosh(1)^5 + 5*(9*c^2*d^3*x^2 - d^3)*cosh(1)^4 - 10*(c^4*d^4*x^3 - 3*c
^2*d^4*x)*cosh(1)^3 - 6*(3*c^4*d^5*x^2 - c^2*d^5)*cosh(1)^2 + (c^6*d^6*x^3
- 9*c^4*d^6*x)*cosh(1))*sinh(1)^2 + 3*(c^6*d^8*x - 3*x^3*cosh(1)^8 - 8*d*x^
2*cosh(1)^7 + 7*(c^2*d^2*x^3 - d^2*x)*cosh(1)^6 + 2*(9*c^2*d^3*x^2 - d^3)*c
osh(1)^5 - 5*(c^4*d^4*x^3 - 3*c^2*d^4*x)*cosh(1)^4 - 4*(3*c^4*d^5*x^2 - c^2
*d^5)*cosh(1)^3 + (c^6*d^6*x^3 - 9*c^4*d^6*x)*cosh(1)^2 + 2*(c^6*d^7*x^2 -
c^4*d^7)*cosh(1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)) + 3*(3*c^6*d^8*x -
 5*c^4*d^4*x^3*cosh(1)^4 - 12*c^4*d^5*x^2*cosh(1)^3 + 3*(c^6*d^6*x^3 - 3*c^
4*d^6*x)*cosh(1)^2 + 2*(3*c^6*d^7*x^2 - c^4*d^7)*cosh(1))*sinh(1) + (7*c^5*
d^7*x*cosh(1)^2 + 4*c^5*d^8*cosh(1) - 8*c^3*d^5...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**4,x)
[Out] Integral(acosh(c*x)/(d + e*x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, integration of abs or si
gn ass
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}(c\,x)}{{(d+e\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^4,x)
```

```
[Out] int(acosh(c*x)/(d + e*x)^4, x)
```

3.8 $\int (d+ex)^3 \cosh^{-1}(cx)^2 \, dx$

Optimal. Leaf size=334

$$2d^3x+\frac{4de^2x}{3c^2}+\frac{3}{4}d^2ex^2+\frac{3e^3x^2}{32c^2}+\frac{2}{9}de^2x^3+\frac{e^3x^4}{32}-\frac{2d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c}-\frac{4de^2\sqrt{-1+cx}\,\sqrt{1+c}}{3c^3}$$

```
[Out] 2*d^3*x+4/3*d*e^2*x/c^2+3/4*d^2*e*x^2+3/32*e^3*x^2/c^2+2/9*d*e^2*x^3+1/32*e
^3*x^4-1/4*d^4*arccosh(c*x)^2/e-3/4*d^2*e*arccosh(c*x)^2/c^2-3/32*e^3*arcco
sh(c*x)^2/c^4+1/4*(e*x+d)^4*arccosh(c*x)^2/e-2*d^3*arccosh(c*x)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c-4/3*d*e^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3
/2*d^2*e*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/16*e^3*x*arccosh(c*
x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/3*d*e^2*x^2*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c-1/8*e^3*x^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.95, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{3e^3\cosh^{-1}(cx)^2}{32c^4}-\frac{4de^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^3}-\frac{3e^3x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{16c^3}-\frac{3d^2e\cosh^{-1}(cx)^2}{4c^2}+\frac{4de^2x}{3c^2}+\frac{3e^3x^2}{32c^2}-\frac{d^4\cosh^{-1}(cx)^2}{4e}-\frac{2d^3\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}-\frac{3d^2ex\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{2c}-\frac{2de^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c}+\frac{\cosh^{-1}(cx)^2(d+ex)^4}{4e}-\frac{e^3x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{8c}+2d^3x+\frac{3}{4}d^2ex^2+\frac{2}{9}de^2x^3+\frac{e^3x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d^3*x + (4*d*e^2*x)/(3*c^2) + (3*d^2*e*x^2)/4 + (3*e^3*x^2)/(32*c^2) + (2
*d*e^2*x^3)/9 + (e^3*x^4)/32 - (2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[
c*x])/c - (4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3) - (3*
d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (3*e^3*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(16*c^3) - (2*d*e^2*x^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c) - (e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*A
rcCosh[c*x])/(8*c) - (d^4*ArcCosh[c*x]^2)/(4*e) - (3*d^2*e*ArcCosh[c*x]^2)/
(4*c^2) - (3*e^3*ArcCosh[c*x]^2)/(32*c^4) + ((d + e*x)^4*ArcCosh[c*x]^2)/(4
*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
```

```
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \frac{(d+ex)^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \left(\frac{d^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4d^3ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \ldots\right.}{\ldots}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \left(2cd^3\right)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx - \frac{\left(cd^4\right)\int \frac{\cosh\ldots}{\sqrt{-1+\ldots}}}{2e\ldots}$$

$$= -\frac{2d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{3d^2ex\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)\ldots}{2c}$$

$$= 2d^3x + \frac{3}{4}d^2ex^2 + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{4de^2\ldots}{\ldots}$$

$$= 2d^3x + \frac{4de^2x}{3c^2} + \frac{3}{4}d^2ex^2 + \frac{3e^3x^2}{32c^2} + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\ldots}{c}$$

Mathematica [**A**]
time = 0.13, size = 191, normalized size = 0.57

$$\frac{c^2x\left(3e^2(128d+9ex)+c^2\left(576d^3+216d^2ex+64de^2x^2+9e^3x^3\right)\right)-6c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(e^2(64d+9ex)+c^2\left(96d^3+72d^2ex+32de^2x^2+6e^3x^3\right)\right)\cosh^{-1}(cx)+9\left(-24c^2d^2e-3e^3+8c^4x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)\right)\cosh^{-1}(cx)^2}{288c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] (c^2*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*e*x + 64*d*e^2*x^2 +
 9*e^3*x^3)) - 6*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(
96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))*ArcCosh[c*x] + 9*(-24*c^2*
d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*ArcCos
h[c*x]^2)/(288*c^4)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3\operatorname{arccosh}(cx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x)^2,x)
```

```
[Out] int((e*x+d)^3*arccosh(c*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^4*e^3 + 4*d*x^3*e^2 + 6*d^2*x^2*e + 4*d^3*x)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2 - integrate(1/2*(c^3*x^6*e^3 + 4*c^3*d*x^5*e^2 - 6*c*d^2*
x^2*e - 4*c*d^3*x + (6*c^3*d^2*e - c*e^3)*x^4 + 4*(c^3*d^3 - c*d*e^2)*x^3 +
 (c^2*x^5*e^3 + 4*c^2*d*x^4*e^2 + 6*c^2*d^2*x^3*e + 4*c^2*d^3*x^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2
*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(275) = 550.
time = 0.42, size = 560, normalized size = 1.68

$$\frac{216\,c^4 d^2 x^2 \cosh\left(1\right) + 576\,c^4 d^3 x + 9\,{\left(c^4 x^4 + 3\,c^2 x^2\right)} \cosh\left(1\right)^3 + 9\,{\left(c^4 x^4 + 3\,c^2 x^2\right)} \sinh\left(1\right)^3 + 64\,{\left(c^4 d x^3 + 6\,c^2 d x\right)} \cosh\left(1\right)^2 + 9\,{\left(32\,c^4 d x^3 \cosh\left(1\right)^2 + 32\,c^4 d^3 x + {\left(8\,c^4 x^4 - 3\right)} \cosh\left(1\right)^3 + {\left(8\,c^4 x^4 - 3\right)} \sinh\left(1\right)^3 + {\left(32\,c^4 d x^3 + 3\,{\left(8\,c^4 x^4 - 3\right)} \cosh\left(1\right)\right)} \sinh\left(1\right)^2 + 24\,{\left(2\,c^4 d^2 x^2 - c^2 d^2\right)} \cosh\left(1\right) + {\left(64\,c^4 d x^3 \cosh\left(1\right) + 48\,c^4 d^2 x^2 - 24\,c^2 d^2 + 3\,{\left(8\,c^4 x^4 - 3\right)} \cosh\left(1\right)^2\right)} \sinh\left(1\right)\right)} \log\left(c x + \sqrt{c^2 x^2 - 1}\right)^2 + {\left(64\,c^4 d x^3 + 384\,c^2 d x + 27\,{\left(c^4 x^4 + 3\,c^2 x^2\right)} \cosh\left(1\right)\right)} \sinh\left(1\right)^2 - 6\,{\left(72\,c^3 d^2 x \cosh\left(1\right) + 96\,c^3 d^3 + 3\,{\left(2\,c^3 x^3 + 3\,c x\right)} \cosh\left(1\right)^3 + 3\,{\left(2\,c^3 x^3 + 3\,c x\right)} \sinh\left(1\right)^3 + 32\,{\left(c^3 d x^2 + 2\,c d\right)} \cosh\left(1\right)^2 + {\left(32\,c^3 d x^2 + 64\,c d + 9\,{\left(2\,c^3 x^3 + 3\,c x\right)} \cosh\left(1\right)\right)} \sinh\left(1\right)^2 + {\left(72\,c^3 d^2 x + 9\,{\left(2\,c^3 x^3 + 3\,c x\right)} \cosh\left(1\right)^2 + 64\,{\left(c^3 d x^2 + 2\,c d\right)} \cosh\left(1\right)\right)} \sinh\left(1\right)\right)} \sqrt{c^2 x^2 - 1} \log\left(c x + \sqrt{c^2 x^2 - 1}\right) + {\left(216\,c^4 d^2 x^2 + 27\,{\left(c^4 x^4 + 3\,c^2 x^2\right)} \cosh\left(1\right)^2 + 128\,{\left(c^4 d x^3 + 6\,c^2 d x\right)} \cosh\left(1\right)\right)} \sinh\left(1\right)}{288\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(216*c^4*d^2*x^2*cosh(1) + 576*c^4*d^3*x + 9*(c^4*x^4 + 3*c^2*x^2)*co
sh(1)^3 + 9*(c^4*x^4 + 3*c^2*x^2)*sinh(1)^3 + 64*(c^4*d*x^3 + 6*c^2*d*x)*co
sh(1)^2 + 9*(32*c^4*d*x^3*cosh(1)^2 + 32*c^4*d^3*x + (8*c^4*x^4 - 3)*cosh(1
)^3 + (8*c^4*x^4 - 3)*sinh(1)^3 + (32*c^4*d*x^3 + 3*(8*c^4*x^4 - 3)*cosh(1)
)*sinh(1)^2 + 24*(2*c^4*d^2*x^2 - c^2*d^2)*cosh(1) + (64*c^4*d*x^3*cosh(1)
+ 48*c^4*d^2*x^2 - 24*c^2*d^2 + 3*(8*c^4*x^4 - 3)*cosh(1)^2)*sinh(1))*log(c
*x + sqrt(c^2*x^2 - 1))^2 + (64*c^4*d*x^3 + 384*c^2*d*x + 27*(c^4*x^4 + 3*c
^2*x^2)*cosh(1))*sinh(1)^2 - 6*(72*c^3*d^2*x*cosh(1) + 96*c^3*d^3 + 3*(2*c^
3*x^3 + 3*c*x)*cosh(1)^3 + 3*(2*c^3*x^3 + 3*c*x)*sinh(1)^3 + 32*(c^3*d*x^2
+ 2*c*d)*cosh(1)^2 + (32*c^3*d*x^2 + 64*c*d + 9*(2*c^3*x^3 + 3*c*x)*cosh(1)
)*sinh(1)^2 + (72*c^3*d^2*x + 9*(2*c^3*x^3 + 3*c*x)*cosh(1)^2 + 64*(c^3*d*x
^2 + 2*c*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2 - 1)
) + (216*c^4*d^2*x^2 + 27*(c^4*x^4 + 3*c^2*x^2)*cosh(1)^2 + 128*(c^4*d*x^3
+ 6*c^2*d*x)*cosh(1))*sinh(1))/c^4
```

Sympy [A]
time = 0.39, size = 371, normalized size = 1.11

$$\begin{cases} d^{3} x \operatorname{acosh}^{2}{\left(c x \right)} + 2 d^{3} x + \frac{3 d^{2} e x^{2} \operatorname{acosh}^{2}{\left(c x \right)}}{2} + \frac{3 d^{2} e x^{2}}{4} + d e^{2} x^{3} \operatorname{acosh}^{2}{\left(c x \right)} + \frac{2 d e^{2} x^{3}}{9} + \frac{e^{3} x^{4} \operatorname{acosh}^{2}{\left(c x \right)}}{4} + \frac{e^{3} x^{4}}{32} - \frac{2 d^{3} \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{c} - \frac{3 d^{2} e x \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{2 c} - \frac{2 d e^{2} x^{2} \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{3 c} - \frac{e^{3} x^{3} \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{8 c} - \frac{3 d^{2} e \operatorname{acosh}^{2}{\left(c x \right)}}{4 c^{2}} + \frac{4 d e^{2} x}{3 c^{2}} + \frac{3 e^{3} x^{2}}{32 c^{2}} - \frac{4 d e^{2} \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{3 c^{3}} - \frac{3 e^{3} x \sqrt{c^{2} x^{2} - 1} \operatorname{acosh}{\left(c x \right)}}{16 c^{3}} - \frac{3 e^{3} \operatorname{acosh}^{2}{\left(c x \right)}}{32 c^{4}} & \text{for}\: c \neq 0 \\- \frac{\pi^{2} \left(d^{3} x + \frac{3 d^{2} e x^{2}}{2} + d e^{2} x^{3} + \frac{e^{3} x^{4}}{4}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x)**2 + 2*d**3*x + 3*d**2*e*x**2*acosh(c*x)**2/2
+ 3*d**2*e*x**2/4 + d*e**2*x**3*acosh(c*x)**2 + 2*d*e**2*x**3/9 + e**3*x**4
*acosh(c*x)**2/4 + e**3*x**4/32 - 2*d**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c -
 3*d**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - 2*d*e**2*x**2*sqrt(c**2*
x**2 - 1)*acosh(c*x)/(3*c) - e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c)
 - 3*d**2*e*acosh(c*x)**2/(4*c**2) + 4*d*e**2*x/(3*c**2) + 3*e**3*x**2/(32*
c**2) - 4*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*e**3*x*sqrt(c*
*2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*e**3*acosh(c*x)**2/(32*c**4), Ne(c, 0
)), (-pi**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)/4, True)
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}(c\,x)^2\,{(d+e\,x)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x)^3, x)
```

3.9 $\int (d+ex)^2 \cosh^{-1}(cx)^2 \, dx$

Optimal. Leaf size=215

$$2d^2x+\frac{4e^2x}{9c^2}+\frac{1}{2}dex^2+\frac{2e^2x^3}{27}-\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c}-\frac{4e^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{9c^3}-\frac{de}{}$$

```
[Out] 2*d^2*x+4/9*e^2*x/c^2+1/2*d*e*x^2+2/27*e^2*x^3-1/3*d^3*arccosh(c*x)^2/e-1/2
*d*e*arccosh(c*x)^2/c^2+1/3*(e*x+d)^3*arccosh(c*x)^2/e-2*d^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-4/9*e^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c^3-d*e*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-2/9*e^2*x^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.67, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{4e^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{9c^3}-\frac{de\cosh^{-1}(cx)^2}{2c^2}+\frac{4e^2x}{9c^2}-\frac{d^3\cosh^{-1}(cx)^2}{3e}-\frac{2d^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}-\frac{dex\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}+\frac{\cosh^{-1}(cx)^2(d+ex)^3}{3e}-\frac{2e^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{9c}+2d^2x+\frac{1}{2}dex^2+\frac{2e^2x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d^2*x + (4*e^2*x)/(9*c^2) + (d*e*x^2)/2 + (2*e^2*x^3)/27 - (2*d^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (4*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*ArcCosh[c*x])/(9*c^3) - (d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/
c - (2*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(9*c) - (d^3*ArcC
osh[c*x]^2)/(3*e) - (d*e*ArcCosh[c*x]^2)/(2*c^2) + ((d + e*x)^3*ArcCosh[c*x
]^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2 \cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \frac{(d+ex)^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \left(\frac{d^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3d^2ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right.}{}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - (2cd^2)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx - \frac{(2cd^3)\int \frac{}{\sqrt{-1}}}{}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{dex\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c}$$

$$= 2d^2x + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{4e^2\sqrt{-1+c}}{}$$

$$= 2d^2x + \frac{4e^2x}{9c^2} + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{4e^2}{}$$

Mathematica [A]
time = 0.10, size = 131, normalized size = 0.61

$$\frac{cx(24e^2+c^2(108d^2+27dex+4e^2x^2)) - 6\sqrt{-1+cx}\,\sqrt{1+cx}\,(4e^2+c^2(18d^2+9dex+2e^2x^2))\cosh^{-1}(cx) + 9(-3cde+2c^3x(3d^2+3dex+e^2x^2))\cosh^{-1}(cx)^2}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x]^2,x]
```

```
[Out] (c*x*(24*e^2 + c^2*(108*d^2 + 27*d*e*x + 4*e^2*x^2)) - 6*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))*ArcCosh[c*x] + 9*(-
3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*x]^2)/(54*c^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^2 \operatorname{arccosh}(cx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x)^2,x)
```

```
[Out] int((e*x+d)^2*arccosh(c*x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(x^3*e^2 + 3*d*x^2*e + 3*d^2*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - integrate(2/3*(c^3*x^5*e^2 + 3*c^3*d*x^4*e - 3*c*d*x^2*e - 3*c*d^2*x +
(3*c^3*d^2 - c*e^2)*x^3 + (c^2*x^4*e^2 + 3*c^2*d*x^3*e + 3*c^2*d^2*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
 (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A]
time = 0.35, size = 291, normalized size = 1.35

$$\frac{27\,c^3dx^2\cosh\left(1\right)+108\,c^3d^2x+4\,(c^3x^3+6\,cx)\cosh\left(1\right)^2+9\,\left(2\,c^3x^3\cosh\left(1\right)^2+2\,c^3x^3\sinh\left(1\right)^2+6\,c^3d^2x+3\,(2\,c^3dx^2-cd)\cosh\left(1\right)+(4\,c^3x^3\cosh\left(1\right)+6\,c^3dx^2-3\,cd)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+4\,(c^3x^3+6\,cx)\sinh\left(1\right)^2-6\,\left(9\,c^2dx\cosh\left(1\right)+18\,c^2d^2+2\,(c^2x^2+2)\cosh\left(1\right)^2+2\,(c^2x^2+2)\sinh\left(1\right)^2+(9\,c^2dx+4\,(c^2x^2+2)\cosh\left(1\right))\sinh\left(1\right)\right)\sqrt{c^2x^2-1}\log\left(cx+\sqrt{c^2x^2-1}\right)+(27\,c^3dx^2+8\,(c^3x^3+6\,cx)\cosh\left(1\right))\sinh\left(1\right)}{54\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(27*c^3*d*x^2*cosh(1) + 108*c^3*d^2*x + 4*(c^3*x^3 + 6*c*x)*cosh(1)^2
+ 9*(2*c^3*x^3*cosh(1)^2 + 2*c^3*x^3*sinh(1)^2 + 6*c^3*d^2*x + 3*(2*c^3*d*x
^2 - c*d)*cosh(1) + (4*c^3*x^3*cosh(1) + 6*c^3*d*x^2 - 3*c*d)*sinh(1))*log(
c*x + sqrt(c^2*x^2 - 1))^2 + 4*(c^3*x^3 + 6*c*x)*sinh(1)^2 - 6*(9*c^2*d*x*c
osh(1) + 18*c^2*d^2 + 2*(c^2*x^2 + 2)*cosh(1)^2 + 2*(c^2*x^2 + 2)*sinh(1)^2
 + (9*c^2*d*x + 4*(c^2*x^2 + 2)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1)*log(c*x
 + sqrt(c^2*x^2 - 1)) + (27*c^3*d*x^2 + 8*(c^3*x^3 + 6*c*x)*cosh(1))*sinh(1
))/c^3
```

Sympy [A]
time = 0.25, size = 223, normalized size = 1.04

$$\begin{cases} d^2x\operatorname{acosh}^2\left(cx\right)+2d^2x+dex^2\operatorname{acosh}^2\left(cx\right)+\frac{dex^2}{2}+\frac{e^2x^3\operatorname{acosh}^2\left(cx\right)}{3}+\frac{2e^2x^3}{27}-\frac{2d^2\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{c}-\frac{dex\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{c}-\frac{2e^2x^2\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{9c}-\frac{de\operatorname{acosh}^2\left(cx\right)}{2c^2}+\frac{4e^2x}{9c^2}-\frac{4e^2\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{9c^3} & \text{for}\: c\neq 0\\ -\frac{\pi^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x)**2 + 2*d**2*x + d*e*x**2*acosh(c*x)**2 + d*e*x
**2/2 + e**2*x**3*acosh(c*x)**2/3 + 2*e**2*x**3/27 - 2*d**2*sqrt(c**2*x**2
- 1)*acosh(c*x)/c - d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*e**2*x**2*sq
rt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - d*e*acosh(c*x)**2/(2*c**2) + 4*e**2*x/
(9*c**2) - 4*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), (-pi*
*2*(d**2*x + d*e*x**2 + e**2*x**3/3)/4, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(c\,x)^2\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x)^2, x)
```

3.10 $\int (d+ex)\cosh^{-1}(cx)^2\,dx$

Optimal. Leaf size=122

$$2dx+\frac{ex^2}{4}-\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{2c}-\frac{d^2\cosh^{-1}(cx)^2}{2e}-\frac{e\cosh^{-}}{4}$$

```
[Out] 2*d*x+1/4*e*x^2-1/2*d^2*arccosh(c*x)^2/e-1/4*e*arccosh(c*x)^2/c^2+1/2*(e*x+
d)^2*arccosh(c*x)^2/e-2*d*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*e*
x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.43, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{e\cosh^{-1}(cx)^2}{4c^2}-\frac{d^2\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)^2(d+ex)^2}{2e}-\frac{2d\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{2c}+2dx+\frac{ex^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (d^2*ArcCosh[c*x]^2)/(
2*e) - (e*ArcCosh[c*x]^2)/(4*c^2) + ((d + e*x)^2*ArcCosh[c*x]^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
```

```
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \frac{(d+ex)^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e}$$
$$= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \left(\frac{d^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2dex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{}{\sqrt{}}\right.}{e}$$
$$= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - (2cd)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{(cd^2)\int \frac{\cosh^{-1}}{\sqrt{-1+cx}}}{e}$$
$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{d^2}{}$$
$$= 2dx + \frac{ex^2}{4} - \frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh}{2c}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 0.86

$$2dx + \frac{ex^2}{4} - \frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} + dx\cosh^{-1}(cx)^2 + \frac{e(-1+2c^2x^2)\cosh^{-1}(cx)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) + d*x*ArcCosh[c*x]^2 + (
e*(-1 + 2*c^2*x^2)*ArcCosh[c*x]^2)/(4*c^2)
```

Maple [A]
time = 3.34, size = 100, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{e\left(-2\sqrt{cx-1}\,\mathrm{arccosh}(cx)\sqrt{cx+1}\,xc+2\mathrm{arccosh}(cx)^2x^2c^2-\mathrm{arccosh}(cx)^2+c^2x^2\right)}{4c}+d\left(\mathrm{arccosh}(cx)^2xc-2\,\mathrm{arccosh}(cx)\sqrt{}\right.}{c}$
default	$\frac{\frac{e\left(-2\sqrt{cx-1}\,\mathrm{arccosh}(cx)\sqrt{cx+1}\,xc+2\mathrm{arccosh}(cx)^2x^2c^2-\mathrm{arccosh}(cx)^2+c^2x^2\right)}{4c}+d\left(\mathrm{arccosh}(cx)^2xc-2\,\mathrm{arccosh}(cx)\sqrt{}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*e*(-2*(c*x-1)^(1/2)*arccosh(c*x)*(c*x+1)^(1/2)*x*c+2*arccosh(c*x)^
2*x^2*c^2-arccosh(c*x)^2+c^2*x^2)/c+d*(arccosh(c*x)^2*x*c-2*arccosh(c*x)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - integrate((c
^3*x^4*e + 2*c^3*d*x^3 - c*x^2*e - 2*c*d*x + (c^2*x^3*e + 2*c^2*d*x^2)*sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
(c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [A]
time = 0.41, size = 128, normalized size = 1.05

$$\frac{c^2x^2\cosh(1) + c^2x^2\sinh(1) + 8\,c^2dx + (4\,c^2dx + (2\,c^2x^2 - 1)\cosh(1) + (2\,c^2x^2 - 1)\sinh(1))\log\left(cx + \sqrt{c^2x^2 - 1}\right)^2 - 2\,\sqrt{c^2x^2 - 1}\,(cx\cosh(1) + cx\sinh(1) + 4\,cd)\log\left(cx + \sqrt{c^2x^2 - 1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^2*cosh(1) + c^2*x^2*sinh(1) + 8*c^2*d*x + (4*c^2*d*x + (2*c^2*x^
2 - 1)*cosh(1) + (2*c^2*x^2 - 1)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1))^2 -
2*sqrt(c^2*x^2 - 1)*(c*x*cosh(1) + c*x*sinh(1) + 4*c*d)*log(c*x + sqrt(c^2*
x^2 - 1)))/c^2
```

Sympy [A]
time = 0.14, size = 110, normalized size = 0.90

$$\begin{cases} dx\operatorname{acosh}^2(cx) + 2dx + \frac{ex^2\operatorname{acosh}^2(cx)}{2} + \frac{ex^2}{4} - \frac{2d\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{ex\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{2c} - \frac{e\operatorname{acosh}^2(cx)}{4c^2} & \text{for } c \neq 0 \\ -\frac{\pi^2\left(dx+\frac{ex^2}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d*x*acosh(c*x)**2 + 2*d*x + e*x**2*acosh(c*x)**2/2 + e*x**2/4 -
2*d*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(
2*c) - e*acosh(c*x)**2/(4*c**2), Ne(c, 0)), (-pi**2*(d*x + e*x**2/2)/4, Tru
e))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(c\,x)^2\,(d+e\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x),x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x), x)
```

3.11 $\int \frac{\cosh^{-1}(cx)^2}{d+ex} dx$

Optimal. Leaf size=272

$$-\frac{\cosh^{-1}(cx)^3}{3e}+\frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(}{}$$

```
[Out] -1/3*arccosh(c*x)^3/e+arccosh(c*x)^2*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+arccosh(c*x)^2*ln(1+e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+2*arccosh(c*x)*polylog(2,-e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+2*arccosh(c*x)*po
lylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e-2
*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/
e-2*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)
))/e
```

Rubi [A]
time = 0.30, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5962, 5681, 2221, 2611, 2320, 6724}

$$\frac{2\cosh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\cosh^{-1}(cx)^2\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)^2\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-\frac{\cosh^{-1}(cx)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*ArcCosh[c*x]^3/e + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d +
 Sqrt[c^2*d^2 - e^2])])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])
/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCo
sh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd}\right.$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee}{cd+\sqrt{}}\right.}{e}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee}{cd+\sqrt{}}\right.}{e}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee}{cd+\sqrt{}}\right.}{e}$$

$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee}{cd+\sqrt{}}\right.}{e}$$

Mathematica [A]
time = 0.11, size = 252, normalized size = 0.93

$$-\frac{\cosh^{-1}(cx)^3 - 3\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right) - 3\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right) - 6\cosh^{-1}(cx)\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right) - 6\cosh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right) + 6\text{PolyLog}\left(3,\frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right) + 6\text{PolyLog}\left(3,-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*(ArcCosh[c*x]^3 - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])] - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))] + 6*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sq
rt[c^2*d^2 - e^2])] + 6*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
 - e^2]))])/e
```

Maple [F]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d),x)
[Out] int(arccosh(c*x)^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(arccosh(c*x)^2/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="fricas")
[Out] integral(arccosh(c*x)^2/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{2}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d),x)
[Out] Integral(acosh(c*x)**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="giac")
[Out] integrate(arccosh(c*x)^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{acosh}\left(c\,x\right)}^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x),x)
[Out] int(acosh(c*x)^2/(d + e*x), x)
```

3.12 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=259

$$-\frac{\cosh^{-1}(cx)^2}{e(d+ex)}+\frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\text{Poly}}{}$$

```
[Out] -arccosh(c*x)^2/e/(e*x+d)+2*c*arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*c*arccosh(c*x)*
ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*
d^2-e^2)^(1/2)+2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2
*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5963, 5980, 3401, 2296, 2221, 2317, 2438}

$$\frac{2c\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{\cosh^{-1}(cx)^2}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -(ArcCosh[c*x]^2/(e*(d + e*x))) + (2*c*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*ArcCosh[c*
x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2
 - e^2]) + (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])
)])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
```

```
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2}\,dx = -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\text{Subst}\left(\int \frac{x}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\text{Subst}\left(\int \frac{e^x x}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\text{Subst}\left(\int \frac{e^x x}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2d^2-e^2}} - \frac{(4c)\text{Subst}\Big(}{}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\Big(1+\frac{}{cd+}}{e\sqrt{c^2d^2-e^2}}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\Big(1+\frac{}{cd+}}{e\sqrt{c^2d^2-e^2}}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\Big(1+\frac{}{cd+}}{e\sqrt{c^2d^2-e^2}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 2.37, size = 848, normalized size = 3.27

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -((c*(ArcCosh[c*x]^2/(c*d + c*e*x) + (2*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*C
oth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*Arc
Tan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-
((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e
^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))
*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d +
c*e*x])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])
/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[
-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[
2]*Sqrt[e]*Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) +
```

```
e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e +
 I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqr
t[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTa
n[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d +
e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(
c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2,
((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[
ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/
2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^
2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]
*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*d^2) + e^2]))/e)
```

Maple [A]
time = 6.94, size = 388, normalized size = 1.50

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(cx)^2 c^2}{e(ecx+cd)}+\frac{2c^2\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c^2\operatorname{arccosh}(cx)\ln\left(\frac{cd+e}{}\right.}{}$
default	$-\frac{\operatorname{arccosh}(cx)^2 c^2}{e(ecx+cd)}+\frac{2c^2\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c^2\operatorname{arccosh}(cx)\ln\left(\frac{cd+e}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-arccosh(c*x)^2*c^2/e/(c*e*x+c*d)+2/e*c^2*arccosh(c*x)/(c^2*d^2-e^2)^(
1/2)*ln((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*
d+(c^2*d^2-e^2)^(1/2)))-2/e*c^2*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln((c*d+e*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(
1/2)))+2/e*c^2/(c^2*d^2-e^2)^(1/2)*dilog((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2/e*c^2/(c^2*d^2-e
^2)^(1/2)*dilog((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2
))/(c*d+(c^2*d^2-e^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(c\,x)^2}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)^2/(d + e*x)^2, x)
```

3.13 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3} \, dx$

Optimal. Leaf size=352

$$-\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)\,(d+ex)}-\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2}+\frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\,(c^2d^2-e^2)^{3/2}}-\frac{c^3d\cosh^{-1}(c$$

```
[Out] -1/2*arccosh(c*x)^2/e/(e*x+d)^2+c^2*ln(e*x+d)/e/(c^2*d^2-e^2)+c^3*d*arccosh
(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e
/(c^2*d^2-e^2)^(3/2)-c^3*d*arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)+c^3*d*polylog(2,-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)
^(3/2)-c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e
^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-c*(c*x+1)*arccosh(c*x)*((c*x-1)/(c*x+1))^
(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A]
time = 0.48, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5963, 5980, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{c^2\log(d+ex)}{e\,(c^2d^2-e^2)}-\frac{c\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\cosh^{-1}(cx)}{(c^2d^2-e^2)\,(d+ex)}+\frac{c^3d\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\,(c^2d^2-e^2)^{3/2}}-\frac{c^3d\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\,(c^2d^2-e^2)^{3/2}}+\frac{c^3d\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\,(c^2d^2-e^2)^{3/2}}-\frac{c^3d\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e\,(c^2d^2-e^2)^{3/2}}-\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] -((c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*ArcCosh[c*x])/((c^2*d^2 - e^2)*
(d + e*x))) - ArcCosh[c*x]^2/(2*e*(d + e*x)^2) + (c^3*d*ArcCosh[c*x]*Log[1
+ (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)
) - (c^3*d*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^
2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) +
(c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c
^2*d^2 - e^2)^(3/2)) - (c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c
^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3}\,dx = -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\text{Subst}\left(\int \frac{x}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\text{Subst}\left(\int \frac{\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-}\right.}{c^2d^2-e^2}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\text{Subst}\left(\int \frac{1}{cd+x}\,dx, x, cex\right)}{e(c^2d^2-e^2)} + \frac{(2c^3}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\log(d+ex)}{e(c^2d^2-e^2)} + \frac{(2c^3d)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}}}{cd-\sqrt{c^2d^2}}\right.}{e(c^2d^2-e^2)^{3/2}}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}}}{cd-\sqrt{c^2d^2}}\right.}{e(c^2d^2-e^2)^{3/2}}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\cosh^{-1}(cx)}{(c^2d^2-e^2)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}}}{cd-\sqrt{c^2d^2}}\right.}{e(c^2d^2-e^2)^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.13, size = 936, normalized size = 2.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*d - e)*(c*d
+ e)*(c*d + c*e*x))) - ArcCosh[c*x]^2/(2*e*(c*d + c*e*x)^2) + Log[1 + (e*x)
/d]/(c^2*d^2*e - e^3) + (c*d*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d
) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)]
```

```
+ 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcT
an[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-
(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d + c*e*x])] +
(ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2
*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2)
+ e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*
Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c
^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2
) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d)
+ e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d)
+ e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I
*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*S
qrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x
]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - Pol
yLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^
2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCo
sh[c*x]/2]))])))/(e*(-(c^2*d^2) + e^2)^(3/2)))
```

Maple [A]
time = 7.63, size = 605, normalized size = 1.72

method	result
derivativedivides	$-\frac{c^{3}\operatorname{arccosh}(cx)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,cde+2\sqrt{cx-1}\,\sqrt{cx+1}\,e^{2}cx+c^{2}d^{2}\operatorname{arccosh}(cx)-2c^{2}d^{2}-4c^{2}dex-2e^{2}c^{2}x^{2}\right.}{2e\left(c^{2}d^{2}-e^{2}\right)(ecx+cd)^{2}}$
default	$-\frac{c^{3}\operatorname{arccosh}(cx)\left(2\sqrt{cx-1}\,\sqrt{cx+1}\,cde+2\sqrt{cx-1}\,\sqrt{cx+1}\,e^{2}cx+c^{2}d^{2}\operatorname{arccosh}(cx)-2c^{2}d^{2}-4c^{2}dex-2e^{2}c^{2}x^{2}\right.}{2e\left(c^{2}d^{2}-e^{2}\right)(ecx+cd)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*c^3*arccosh(c*x)*(2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*d*e+2*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*e^2*c*x+c^2*d^2*arccosh(c*x)-2*c^2*d^2-4*c^2*d*e*x-2*e^2
*c^2*x^2-e^2*arccosh(c*x))/e/(c^2*d^2-e^2)/(c*e*x+c*d)^2+1/(c^2*d^2-e^2)^(3
/2)/e*c^4*arccosh(c*x)*ln((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^
2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-1/(c^2*d^2-e^2)^(3/2)/e*c^4*arc
cosh(c*x)*ln((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/
(c*d+(c^2*d^2-e^2)^(1/2)))*d+1/(c^2*d^2-e^2)^(3/2)/e*c^4*dilog((-c*d-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2
)))*d-1/(c^2*d^2-e^2)^(3/2)/e*c^4*dilog((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
```

```
1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+1/(c^2*d^2-e^2)/e*c
^3*ln(2*d*c*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2+e)-2/(c^2*d^2-e^2)/e*c^3*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**3, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, replacing 0 by ‘ u‘, a subst
itution variable should perhaps be purged.Warning, integration of abs or si
gn ass
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}(c\,x)^2}{{(d+e\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)^2/(d + e*x)^3, x)
```

3.14 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=191

$$-\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4d(19c^2d^2+16}{96c^3}$$

```
[Out] -1/32*b*(8*c^4*d^4+24*c^2*d^2*e^2+3*e^4)*arccosh(c*x)/c^4/e+1/4*(e*x+d)^4*(
a+b*arccosh(c*x))/e-7/48*b*d*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*b
*(e*x+d)^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2+16*e^2)+e*
(26*c^2*d^2+9*e^2)*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5963, 102, 158, 152, 54}

$$\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e}-\frac{b\left(8c^4d^4+24c^2d^2e^2+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^3}{16c}-\frac{7bd\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{48c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(d + e*x)^3)/(16*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*
(19*c^2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - (b*(8*c^4*d^4
 + 24*c^2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*A
rcCosh[c*x]))/(4*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{4e}$$

$$= -\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} + \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{b}{}$$

$$= -\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c}$$

$$= -\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c}$$

$$= -\frac{7bd\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c}$$

Mathematica [A]
time = 0.18, size = 193, normalized size = 1.01

$$\frac{24ac^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3))+24bc^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3)\cosh^{-1}(cx)-9be(8c^2d^2+e^2)\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x
^2 + 6*e^3*x^3)) + 24*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*A
rcCosh[c*x] - 9*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(96*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(169) = 338.
time = 1.64, size = 382, normalized size = 2.00

method	result
derivativedivides	$\frac{(ecx+cd)^4a}{4c^3e}+\frac{bc\,\mathrm{arccosh}(cx)d^4}{4e}+b\,\mathrm{arccosh}(cx)d^3cx+\frac{3bce\,\mathrm{arccosh}(cx)d^2x^2}{2}+bc\,e^2\mathrm{arccosh}(cx)d\,x^3+\frac{bc\,e^3\mathrm{arccosh}(cx)x^4}{4}-bc\sqrt{cx}$
default	$\frac{(ecx+cd)^4a}{4c^3e}+\frac{bc\,\mathrm{arccosh}(cx)d^4}{4e}+b\,\mathrm{arccosh}(cx)d^3cx+\frac{3bce\,\mathrm{arccosh}(cx)d^2x^2}{2}+bc\,e^2\mathrm{arccosh}(cx)d\,x^3+\frac{bc\,e^3\mathrm{arccosh}(cx)x^4}{4}-bc\sqrt{cx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+1/4*b*c/e*arccosh(c*x)*d^4+b*arccosh(c*x)*d^
3*c*x+3/2*b*c*e*arccosh(c*x)*d^2*x^2+b*c*e^2*arccosh(c*x)*d*x^3+1/4*b*c*e^3
*arccosh(c*x)*x^4-1/4*b*c/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d
^4*ln(c*x+(c^2*x^2-1)^(1/2))-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-3/4*b*e*(c*x
-1)^(1/2)*(c*x+1)^(1/2)*d^2*x-1/3*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x^2-1
/16*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-3/4*b/c*e*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-2/3*b/c^2*e^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*d-3/32*b/c^2*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-3/32*b/c
^3*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/
2)))
```

Maxima [A]
time = 0.27, size = 263, normalized size = 1.38

$$\frac{1}{4}ax^4e^3+adx^3e^2+\frac{3}{2}ad^2x^2e+ad^3x+\frac{3}{4}\left(2x^2\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^2e+\frac{\left(cx\,\mathrm{arcosh}\left(cx\right)-\sqrt{c^2x^2-1}\right)bd^3}{c}+\frac{1}{3}\left(3x^3\,\mathrm{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bde^2+\frac{1}{32}\left(8x^4\,\mathrm{arcosh}\left(cx\right)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x + 3/4*(2*x^2*arccos
h(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/
c^3))*b*d^2*e + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c + 1/3*(3*x^3
*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*
d*e^2 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^
2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(167) = 334$.
time = 0.37, size = 526, normalized size = 2.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*x^4*cosh(1)^3 + 24*a*c^4*x^4*sinh(1)^3 + 96*a*c^4*d*x^3*cosh
(1)^2 + 144*a*c^4*d^2*x^2*cosh(1) + 96*a*c^4*d^3*x + 24*(3*a*c^4*x^4*cosh(1
) + 4*a*c^4*d*x^3)*sinh(1)^2 + 3*(32*b*c^4*d*x^3*cosh(1)^2 + 32*b*c^4*d^3*x
 + (8*b*c^4*x^4 - 3*b)*cosh(1)^3 + (8*b*c^4*x^4 - 3*b)*sinh(1)^3 + (32*b*c^
4*d*x^3 + 3*(8*b*c^4*x^4 - 3*b)*cosh(1))*sinh(1)^2 + 24*(2*b*c^4*d^2*x^2 -
b*c^2*d^2)*cosh(1) + (64*b*c^4*d*x^3*cosh(1) + 48*b*c^4*d^2*x^2 - 24*b*c^2*
d^2 + 3*(8*b*c^4*x^4 - 3*b)*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)
) + 24*(3*a*c^4*x^4*cosh(1)^2 + 8*a*c^4*d*x^3*cosh(1) + 6*a*c^4*d^2*x^2)*si
nh(1) - (72*b*c^3*d^2*x*cosh(1) + 96*b*c^3*d^3 + 3*(2*b*c^3*x^3 + 3*b*c*x)*
cosh(1)^3 + 3*(2*b*c^3*x^3 + 3*b*c*x)*sinh(1)^3 + 32*(b*c^3*d*x^2 + 2*b*c*d
)*cosh(1)^2 + (32*b*c^3*d*x^2 + 64*b*c*d + 9*(2*b*c^3*x^3 + 3*b*c*x)*cosh(1
))*sinh(1)^2 + (72*b*c^3*d^2*x + 9*(2*b*c^3*x^3 + 3*b*c*x)*cosh(1)^2 + 64*(
b*c^3*d*x^2 + 2*b*c*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [C] Result contains complex when optimal does not.
time = 0.30, size = 323, normalized size = 1.69

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{acosh}(cx) + \frac{3bd^2ex^2\operatorname{acosh}(cx)}{2} + bde^2x^3\operatorname{acosh}(cx) + \frac{be^3x^4\operatorname{acosh}(cx)}{4} - \frac{bd^3\sqrt{c^2x^2-1}}{c} - \frac{3bd^2ex\sqrt{c^2x^2-1}}{4c} - \frac{bde^2x^2\sqrt{c^2x^2-1}}{3c} - \frac{be^3x^3\sqrt{c^2x^2-1}}{16c} - \frac{3bd^2e\operatorname{acosh}(cx)}{4c^2} - \frac{2bde^2\sqrt{c^2x^2-1}}{3c^3} - \frac{3be^3x\sqrt{c^2x^2-1}}{32c^3} - \frac{3be^3\operatorname{acosh}(cx)}{32c^4} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*acosh(c*x) + 3*b*d**2*e*x**2*acosh(c*x)/2 + b*d*e**2*x**3*acosh(c*x
) + b*e**3*x**4*acosh(c*x)/4 - b*d**3*sqrt(c**2*x**2 - 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 - 1)/(16*c) - 3*b*d**2*e*acosh(c*x)/(4*c**2) - 2*b*d*
e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*b*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3
```

```
) - 3*b*e**3*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + 3*d
**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}(c\,x))\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^3, x)
```

3.15 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=132

$$-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2)+5c^2dex)}{18c^3}-\frac{bd\left(2d^2+\frac{3e^2}{c^2}\right)\cosh^{-1}(c}{6e}$$

```
[Out] -1/6*b*d*(2*d^2+3*e^2/c^2)*arccosh(c*x)/e+1/3*(e*x+d)^3*(a+b*arccosh(c*x))/
e-1/9*b*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*
d^2+4*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5963, 102, 152, 54}

$$\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{bd\left(\frac{3e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{6e}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(4(4c^2d^2+e^2)+5c^2dex)}{18c^3}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/9*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/c - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (b*d*(2*d^2 + (3
*e^2)/c^2)*ArcCosh[c*x])/(6*e) + ((d + e*x)^3*(a + b*ArcCosh[c*x]))/(3*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
```

```
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{(bc)\int\frac{(d+ex)^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3e}$$
$$=-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c}+\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{b\int$$
$$=-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2}{18c^3}$$
$$=-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{9c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4(4c^2d^2+e^2}{18c^3}$$

Mathematica [A]
time = 0.13, size = 142, normalized size = 1.08

$$ad^2x+adex^2+\frac{1}{3}ae^2x^3-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4e^2+c^2(18d^2+9dex+2e^2x^2))}{18c^3}+\frac{1}{3}bx(3d^2+3dex+e^2x^2)\cosh^{-1}(cx)-\frac{bde\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d^2*x + a*d*e*x^2 + (a*e^2*x^3)/3 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^
2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))/(18*c^3) + (b*x*(3*d^2 + 3*d*e*x +
 e^2*x^2)*ArcCosh[c*x])/3 - (b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
/(2*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(115) = 230$.
time = 2.45, size = 262, normalized size = 1.98

method	result
derivativedivides	$\frac{\frac{(ecx+cd)^3 a}{3c^2 e}+\frac{bc \operatorname{arccosh}(cx) d^3}{3e}+b \operatorname{arccosh}(cx) d^2 cx+bce \operatorname{arccosh}(cx) d x^2+\frac{bc e^2 \operatorname{arccosh}(cx) x^3}{3}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{3e\sqrt{c^2x}}}{}$
default	$\frac{\frac{(ecx+cd)^3 a}{3c^2 e}+\frac{bc \operatorname{arccosh}(cx) d^3}{3e}+b \operatorname{arccosh}(cx) d^2 cx+bce \operatorname{arccosh}(cx) d x^2+\frac{bc e^2 \operatorname{arccosh}(cx) x^3}{3}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{3e\sqrt{c^2x}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+1/3*b*c/e*arccosh(c*x)*d^3+b*arccosh(c*x)*d^
2*c*x+b*c*e*arccosh(c*x)*d*x^2+1/3*b*c*e^2*arccosh(c*x)*x^3-1/3*b*c/e*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-b*(c
*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-1/2*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x-1/9*b
*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/2*b/c*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-2/9*b/c^2*e^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2))
```

Maxima [A]
time = 0.28, size = 167, normalized size = 1.27

$$\frac{1}{3}ax^3e^2 + adx^2e + ad^2x + \frac{1}{2}\left(2x^2 \operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2} + \frac{\log\left(2c^2x + 2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bde + \frac{\left(cx \operatorname{arcosh}(cx) - \sqrt{c^2x^2-1}\right)bd^2}{c} + \frac{1}{9}\left(3x^3 \operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2} + \frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 + a*d*x^2*e + a*d^2*x + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2
*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d*e + (c*x*a
rccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2/c + 1/9*(3*x^3*arccosh(c*x) - c*(sqr
t(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(114) = 228$.
time = 0.37, size = 282, normalized size = 2.14

$$\frac{6ac^3x^3\cosh(1)^2 + 6ac^3x^3\sinh(1)^2 + 18ac^3dx^2\cosh(1) + 18ac^3d^2x + 3\left(2bc^3x^3\cosh(1)^2 + 2bc^3x^3\sinh(1)^2 + 6bc^3d^2x + 3\left(2bc^3dx^2 - bcd\right)\cosh(1) + \left(4bc^3x^3\cosh(1) + 6bc^3dx^2 - 3bcd\right)\sinh(1)\right)\log\left(cx + \sqrt{c^2x^2-1}\right) + 6\left(2ac^3x^3\cosh(1) + 3ac^3dx^2\right)\sinh(1) - \left(9bc^2dx\cosh(1) + 18bc^2d^2 + 2\left(bc^2x^2 + 2b\right)\cosh(1)^2 + 2\left(bc^2x^2 + 2b\right)\sinh(1)^2 + \left(9bc^2dx + 4\left(bc^2x^2 + 2b\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2-1}}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*x^3*cosh(1)^2 + 6*a*c^3*x^3*sinh(1)^2 + 18*a*c^3*d*x^2*cosh(1
) + 18*a*c^3*d^2*x + 3*(2*b*c^3*x^3*cosh(1)^2 + 2*b*c^3*x^3*sinh(1)^2 + 6*b
*c^3*d^2*x + 3*(2*b*c^3*d*x^2 - b*c*d)*cosh(1) + (4*b*c^3*x^3*cosh(1) + 6*b
*c^3*d*x^2 - 3*b*c*d)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) + 6*(2*a*c^3*x^
3*cosh(1) + 3*a*c^3*d*x^2)*sinh(1) - (9*b*c^2*d*x*cosh(1) + 18*b*c^2*d^2 +
2*(b*c^2*x^2 + 2*b)*cosh(1)^2 + 2*(b*c^2*x^2 + 2*b)*sinh(1)^2 + (9*b*c^2*d*
x + 4*(b*c^2*x^2 + 2*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.19, size = 197, normalized size = 1.49

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{acosh}(cx) + bdex^2\operatorname{acosh}(cx) + \frac{be^2x^3\operatorname{acosh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2-1}}{c} - \frac{bdex\sqrt{c^2x^2-1}}{2c} - \frac{be^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{bde\operatorname{acosh}(cx)}{2c^2} - \frac{2be^2\sqrt{c^2x^2-1}}{9c^3} & \text{for}\ c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acosh(c*x) + b*
d*e*x**2*acosh(c*x) + b*e**2*x**3*acosh(c*x)/3 - b*d**2*sqrt(c**2*x**2 - 1)
/c - b*d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 - 1)/(9
*c) - b*d*e*acosh(c*x)/(2*c**2) - 2*b*e**2*sqrt(c**2*x**2 - 1)/(9*c**3), Ne
(c, 0)), ((a + I*pi*b/2)*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,{(d+e\,x)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^2, x)
```

3.16 $\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=106

$$-\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c}-\frac{b\left(2d^2+\frac{e^2}{c^2}\right)\cosh^{-1}(cx)}{4e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}$$

```
[Out] -1/4*b*(2*d^2+e^2/c^2)*arccosh(c*x)/e+1/2*(e*x+d)^2*(a+b*arccosh(c*x))/e-3/
4*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*(e*x+d)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5963, 92, 81, 54}

$$\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\left(\frac{e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{4e}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{cx-1}\,\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(d + e*x))/(4*c) - (b*(2*d^2 + e^2/c^2)*ArcCosh[c*x])/(4*e) + ((d + e*x)
^2*(a + b*ArcCosh[c*x]))/(2*e)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2e}$$

$$= -\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} + \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{b\int -}{\sqrt{}}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}}{4c} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} + \frac{(d+ex)^2}{}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}}{4c} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}{4c} - \frac{b\left(2d^2+\right.}{}$$

Mathematica [A]
time = 0.07, size = 117, normalized size = 1.10

$$adx+\frac{1}{2}aex^2-\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{c}-\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}+bdx\cosh^{-1}(cx)+\frac{1}{2}bex^2\cosh^{-1}(cx)-\frac{be\tanh^{-1}\left(\frac{\sqrt{-1+cx}}{\sqrt{1+cx}}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*e*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])/(4*c) + b*d*x*ArcCosh[c*x] + (b*e*x^2*ArcCosh[c*x])/
2 - (b*e*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2)
```

Maple [A]
time = 2.10, size = 133, normalized size = 1.25

method	result

derivativedivides	$\frac{\frac{a\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+b\,\mathrm{arccosh}(cx)dcx+\frac{bc\,\mathrm{arccosh}(cx)e\,x^2}{2}-b\sqrt{cx-1}\,\sqrt{cx+1}\,d-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,ex}{4}-}{c}$
default	$\frac{\frac{a\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+b\,\mathrm{arccosh}(cx)dcx+\frac{bc\,\mathrm{arccosh}(cx)e\,x^2}{2}-b\sqrt{cx-1}\,\sqrt{cx+1}\,d-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,ex}{4}-}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c*(c^2*d*x+1/2*c^2*e*x^2)+b*arccosh(c*x)*d*c*x+1/2*b*c*arccosh(c*x)*
e*x^2-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d-1/4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x
-1/4*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)
^(1/2)))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 0.95

$$\frac{1}{2}\,ax^2e+adx+\frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)be+\frac{\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*x + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2
+ log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*e + (c*x*arccosh(c*x) - sqrt
(c^2*x^2 - 1))*b*d/c
```

Fricas [A]
time = 0.41, size = 125, normalized size = 1.18

$$\frac{2\,ac^2x^2\cosh\left(1\right)+2\,ac^2x^2\sinh\left(1\right)+4\,ac^2dx+\left(4\,bc^2dx+\left(2\,bc^2x^2-b\right)\cosh\left(1\right)+\left(2\,bc^2x^2-b\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,\left(bcx\cosh\left(1\right)+bcx\sinh\left(1\right)+4\,bcd\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2*cosh(1) + 2*a*c^2*x^2*sinh(1) + 4*a*c^2*d*x + (4*b*c^2*d*x
 + (2*b*c^2*x^2 - b)*cosh(1) + (2*b*c^2*x^2 - b)*sinh(1))*log(c*x + sqrt(c^
2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*(b*c*x*cosh(1) + b*c*x*sinh(1) + 4*b*c*d))/
c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.11, size = 105, normalized size = 0.99

$$\begin{cases} adx+\frac{aex^2}{2}+bdx\,\mathrm{acosh}\,(cx)+\frac{bex^2\,\mathrm{acosh}\,(cx)}{2}-\frac{bd\sqrt{c^2x^2-1}}{c}-\frac{bex\sqrt{c^2x^2-1}}{4c}-\frac{be\,\mathrm{acosh}\,(cx)}{4c^2} & \text{for}\ c\neq 0\\ \left(a+\frac{i\pi b}{2}\right)\left(dx+\frac{ex^2}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*acosh(c*x) + b*e*x**2*acosh(c*x)/2 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*acosh(c*x
)/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**2/2), True))
```

Giac [A]
time = 0.46, size = 124, normalized size = 1.17

$$\frac{1}{2}aex^2 + \left(x\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{\sqrt{c^2x^2-1}}{c}\right)bd + \frac{1}{4}\left(2x^2\log\left(cx + \sqrt{c^2x^2-1}\right) - c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2} - \frac{\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{c^2|c|}\right)\right)be + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*e*x^2 + (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d +
1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 - log(
abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b*e + a*d*x
```

Mupad [B]
time = 1.03, size = 83, normalized size = 0.78

$$\frac{a\,x\,(2\,d+e\,x)}{2} + b\,d\,x\,\mathrm{acosh}(c\,x) + b\,e\,x\,\mathrm{acosh}(c\,x)\,\left(\frac{x}{2} - \frac{1}{4\,c^2\,x}\right) - \frac{b\,d\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c} - \frac{b\,e\,x\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 + b*d*x*acosh(c*x) + b*e*x*acosh(c*x)*(x/2 - 1/(4*c^2*x
)) - (b*d*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c - (b*e*x*(c*x - 1)^(1/2)*(c*x
+ 1)^(1/2))/(4*c)
```

3.17 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=195

$$-\frac{(a+b\cosh^{-1}(cx))^2}{2be}+\frac{(a+b\cosh^{-1}(cx))\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{(a+b\cosh^{-1}(cx))\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2/b/e+(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+(a+b*arccosh(c*x))*ln(1+e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+b*polylog(2,-e*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+b*polylog(2,-e
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A]
time = 0.19, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5962, 5681, 2221, 2317, 2438}

$$\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{(a+b\cosh^{-1}(cx))\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-\frac{(a+b\cosh^{-1}(cx))^2}{2be}+\frac{b\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{b\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] -1/2*(a + b*ArcCosh[c*x])^2/(b*e) + ((a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])*Log[1 + (
e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (b*PolyLog[2, -((e*E^Ar
cCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (b*PolyLog[2, -((e*E^ArcCosh
[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Su}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+b\,\text{co}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+b\,\text{co}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+b\,\text{co}}{}$$

Mathematica [A]
time = 0.09, size = 183, normalized size = 0.94

$$\frac{-\left(\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)-2b\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)\right)\right)+2b^2\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right)+2b^2\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 + (e*E^ArcCosh[c*x]
)/(c*d - Sqrt[c^2*d^2 - e^2])] - 2*b*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt
[c^2*d^2 - e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 - e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/(2*b*e)
```

Maple [A]
time = 2.68, size = 324, normalized size = 1.66

method	result
derivativedivides	$\frac{\frac{ac\ln(ecx+cd)}{e}-\frac{bc\operatorname{arccosh}(cx)^2}{2e}+\frac{bc\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\cdots}{\cdots}$
default	$\frac{\frac{ac\ln(ecx+cd)}{e}-\frac{bc\operatorname{arccosh}(cx)^2}{2e}+\frac{bc\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c*ln(c*e*x+c*d)/e-1/2*b*c*arccosh(c*x)^2/e+b*c/e*arccosh(c*x)*ln((-c
*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-
e^2)^(1/2)))+b*c/e*arccosh(c*x)*ln((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+b*c/e*dilog((c*d+e*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+
b*c/e*dilog((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/
(-c*d+(c^2*d^2-e^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/
(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}\left(cx\right)}{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{d + e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x), x)
```

3.18 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}$$

```
[Out] (-a-b*arccosh(c*x))/e/(e*x+d)+2*b*c*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*
d-e)^(1/2)/(c*x-1)^(1/2))/e/(c*d-e)^(1/2)/(c*d+e)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5963, 95, 214}

$$\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\,\sqrt{cd+e}}-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(e*(d + e*x))) + (2*b*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[
1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)}\,dx}{e}\\ &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(2bc)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}\\ &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 121, normalized size = 1.38

$$-\frac{\frac{a}{d+ex}+\frac{b\cosh^{-1}(cx)}{d+ex}-\frac{bc\log(d+ex)}{\sqrt{c^2d^2-e^2}}+\frac{bc\log\left(e+c^2dx-\sqrt{c^2d^2-e^2}\sqrt{-1+cx}\sqrt{1+cx}\right)}{\sqrt{c^2d^2-e^2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
[Out] -((a/(d + e*x) + (b*ArcCosh[c*x])/(d + e*x) - (b*c*Log[d + e*x])/Sqrt[c^2*d
^2 - e^2] + (b*c*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]])/Sqrt[c^2*d^2 - e^2])/e)
```

Maple [A]
time = 4.94, size = 155, normalized size = 1.76

method	result	size
derivativedivides	$\frac{-\frac{a\,c^2}{(ecx+cd)e}-\frac{b\,c^2\operatorname{arccosh}(cx)}{(ecx+cd)e}-\frac{b\,c^2\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)}{e^2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}}}{c}$	155

default	$\frac{-\frac{a c^2}{(ecx+cd)e}-\frac{b c^2 \operatorname{arccosh}(cx)}{(ecx+cd)e}-\frac{b c^2 \sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)}{e^2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}}}{c}$	155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a*c^2/(c*e*x+c*d)/e-b*c^2/(c*e*x+c*d)/e*arccosh(c*x)-b*c^2/e^2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^
(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

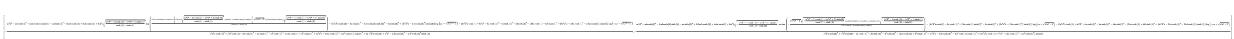
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(79) = 158.
time = 0.41, size = 1131, normalized size = 12.85



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*cosh(1)^2 - 2*a*d*cosh(1)*sinh(1) - a*d*sinh(1)^2 - (b*c
*d*x*cosh(1) + b*c*d*x*sinh(1) + b*c*d^2)*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^
2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*log((c^3*d^2*x + c*d*cosh(1) + c*d
*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(
1))/(cosh(1) - sinh(1))) - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)*sqrt(
c^2*x^2 - 1) + (c^2*d*x + cosh(1) + sinh(1))*sqrt(((c^2*d^2 - 1)*cosh(1) -
(c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*sinh(1) + d)) -
 (b*c^2*d^2*x*cosh(1) - b*x*cosh(1)^3 - 3*b*x*cosh(1)*sinh(1)^2 - b*x*sinh(
1)^3 + (b*c^2*d^2*x - 3*b*x*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)
```

```
) - (b*c^2*d^2*x*cosh(1) + b*c^2*d^3 - b*x*cosh(1)^3 - b*x*sinh(1)^3 - b*d*
cosh(1)^2 - (3*b*x*cosh(1) + b*d)*sinh(1)^2 + (b*c^2*d^2*x - 3*b*x*cosh(1)^
2 - 2*b*d*cosh(1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*x*cosh(
1)^2 + c^2*d^4*cosh(1) - d*x*cosh(1)^4 - d*x*sinh(1)^4 - d^2*cosh(1)^3 - (4
*d*x*cosh(1) + d^2)*sinh(1)^3 + (c^2*d^3*x - 6*d*x*cosh(1)^2 - 3*d^2*cosh(1
))*sinh(1)^2 + (2*c^2*d^3*x*cosh(1) + c^2*d^4 - 4*d*x*cosh(1)^3 - 3*d^2*cos
h(1)^2)*sinh(1)), -(a*c^2*d^3 - a*d*cosh(1)^2 - 2*a*d*cosh(1)*sinh(1) - a*d
*sinh(1)^2 + 2*(b*c*d*x*cosh(1) + b*c*d*x*sinh(1) + b*c*d^2)*sqrt(-((c^2*d^
2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*arctan(-(sqrt(
c^2*x^2 - 1)*sqrt(-((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1)
 - sinh(1)))*(cosh(1) + sinh(1)) - (c*x*cosh(1) + c*x*sinh(1) + c*d)*sqrt(-
((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(c^2*
d^2 - cosh(1)^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)) - (b*c^2*d^2*x*cosh(1) -
b*x*cosh(1)^3 - 3*b*x*cosh(1)*sinh(1)^2 - b*x*sinh(1)^3 + (b*c^2*d^2*x - 3*
b*x*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^2*x*cosh(1)
 + b*c^2*d^3 - b*x*cosh(1)^3 - b*x*sinh(1)^3 - b*d*cosh(1)^2 - (3*b*x*cosh(
1) + b*d)*sinh(1)^2 + (b*c^2*d^2*x - 3*b*x*cosh(1)^2 - 2*b*d*cosh(1))*sinh(
1))*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*x*cosh(1)^2 + c^2*d^4*cosh(1) -
 d*x*cosh(1)^4 - d*x*sinh(1)^4 - d^2*cosh(1)^3 - (4*d*x*cosh(1) + d^2)*sinh
(1)^3 + (c^2*d^3*x - 6*d*x*cosh(1)^2 - 3*d^2*cosh(1))*sinh(1)^2 + (2*c^2*d^
3*x*cosh(1) + c^2*d^4 - 4*d*x*cosh(1)^3 - 3*d^2*cosh(1)^2)*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\text{acosh}\,(cx)}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(76) = 152.
time = 0.57, size = 257, normalized size = 2.92

$$-b\left(\frac{\log\left(cx+\sqrt{c^2x^2-1}\right)}{(ex+d)e}-\frac{\frac{ce^4\log\left(\left|c^2de-\sqrt{c^2d^2-e^2}\,|c||e|\right|\right)\text{sgn}\left(\frac{1}{ex+d}\right)\text{sgn}(e)}{\sqrt{c^2d^2-e^2}\,|e|}-\frac{ce^4\log\left(\left|c^2de-\sqrt{c^2d^2-e^2}\left(\sqrt{c^2-\frac{2c^2d}{ex+d}+\frac{c^2d^2}{(ex+d)^2}-\frac{e^2}{(ex+d)^2}}+\frac{\sqrt{c^2d^2e^2-e^4}}{(ex+d)e}\right)|e|\right|\right)}{\sqrt{c^2d^2-e^2}\,|e|\text{sgn}\left(\frac{1}{ex+d}\right)\text{sgn}(e)}}{e^4}\right)-\frac{a}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -b*(log(c*x + sqrt(c^2*x^2 - 1))/((e*x + d)*e) - (c*e^4*log(abs(c^2*d*e - s
qrt(c^2*d^2 - e^2)*abs(c)*abs(e)))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c^2*d^2 -
```

```
e^2)*abs(e)) - c*e^4*log(abs(c^2*d*e - sqrt(c^2*d^2 - e^2)*(sqrt(c^2 - 2*c^
2*d/(e*x + d) + c^2*d^2/(e*x + d)^2 - e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 -
 e^4)/((e*x + d)*e))*abs(e)))/(sqrt(c^2*d^2 - e^2)*abs(e)*sgn(1/(e*x + d))*
sgn(e)))/e^4) - a/((e*x + d)*e)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x)^2, x)
```

3.19 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/e/(e*x+d)^2+b*c^3*d*arctanh((c*d+e)^(1/2)*(c*x+1)^(
1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(3/2)/e/(c*d+e)^(3/2)-1/2*b*c*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5963, 98, 95, 214}

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)*(d + e*x)) - (a +
b*ArcCosh[c*x])/(2*e*(d + e*x)^2) + (b*c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1
+ c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2)
)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{2e}\\ &= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc^3d)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+e}}{2e\left(c^2d^2-e^2\right)}\\ &= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc^3d)\,\mathrm{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx, x\right.}{e\left(c^2d^2-e^2\right)}\\ &= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right.}{(cd-e)^{3/2}e(cd+e)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 184, normalized size = 1.33

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{\left(c^2d^2-e^2\right)(d+ex)}-\frac{b\cosh^{-1}(cx)}{e(d+ex)^2}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc^3d\log\left(e+c^2dx-\sqrt{c^2d^2-e^2}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcCosh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*
(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]])/(e*(c^2*d^2 - e^2)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(121) = 242$.

time = 5.64, size = 365, normalized size = 2.64

method	result
derivativedivides	$\frac{-\frac{a c^3}{2(ecx+cd)^2 e}-\frac{b c^3 \operatorname{arccosh}(cx)}{2(ecx+cd)^2 e}-\frac{b c^5 \sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(-\frac{2\left(c^2 dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)d^2}{2e^2\sqrt{c^2x^2-1}\,(cd-e)(cd+e)(ecx+cd)\sqrt{\frac{c^2d^2-e^2}{e^2}}}-\frac{b c^5}{}}{c}$
default	$\frac{-\frac{a c^3}{2(ecx+cd)^2 e}-\frac{b c^3 \operatorname{arccosh}(cx)}{2(ecx+cd)^2 e}-\frac{b c^5 \sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(-\frac{2\left(c^2 dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{ecx+cd}\right)d^2}{2e^2\sqrt{c^2x^2-1}\,(cd-e)(cd+e)(ecx+cd)\sqrt{\frac{c^2d^2-e^2}{e^2}}}-\frac{b c^5}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a*c^3/(c*e*x+c*d)^2/e-1/2*b*c^3/(c*e*x+c*d)^2/e*arccosh(c*x)-1/2*
b*c^5/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d-e)/(c*d+e)/(c*
e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d
^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2-1/2*b*c^5/e*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/(c^2*x^2-1)^(1/2)/(c*d-e)/(c*d+e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/
2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c
*d))*d*x-1/2*b*c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c*d-e)/(c*d+e)/(c*e*x+c*d))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(121) = 242$.

time = 0.49, size = 4094, normalized size = 29.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*b*c^4*d^5*x*cosh(1) + (a + b)*c^4*d^6 - 2*b*c^2*d^3*x*cosh(1)^3 -
(b*c^2*d^2*x^2 - a*d^2)*cosh(1)^4 - (b*c^2*d^2*x^2 - a*d^2)*sinh(1)^4 - 2*(
b*c^2*d^3*x + 2*(b*c^2*d^2*x^2 - a*d^2)*cosh(1))*sinh(1)^3 + (b*c^4*d^4*x^2
 - (2*a + b)*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 - 6*b*c^2*d^3*x*cosh(1) -
(2*a + b)*c^2*d^4 - 6*(b*c^2*d^2*x^2 - a*d^2)*cosh(1)^2)*sinh(1)^2 + (b*c^3
*d^3*x^2*cosh(1)^2 + b*c^3*d^3*x^2*sinh(1)^2 + 2*b*c^3*d^4*x*cosh(1) + b*c^
3*d^5 + 2*(b*c^3*d^3*x^2*cosh(1) + b*c^3*d^4*x)*sinh(1))*sqrt(((c^2*d^2 - 1
)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*log((c^3*d^2*x + c*
d*cosh(1) + c*d*sinh(1) + (c^2*d^2 - c*d*sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2
*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))) - cosh(1)^2 - 2*cosh(1)*sinh(1) - s
inh(1)^2)*sqrt(c^2*x^2 - 1) - (c^2*d*x + cosh(1) + sinh(1))*sqrt(((c^2*d^2
- 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*
sinh(1) + d)) - (b*c^4*d^4*x^2*cosh(1)^2 + 2*b*c^4*d^5*x*cosh(1) - 2*b*c^2*
d^2*x^2*cosh(1)^4 - 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*cosh(1)^6 + b*x^2*sinh(
1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*x)*sinh(1)^5 - (2*b*c^2
*d^2*x^2 - 15*b*x^2*cosh(1)^2 - 10*b*d*x*cosh(1))*sinh(1)^4 - 4*(2*b*c^2*d^
2*x^2*cosh(1) + b*c^2*d^3*x - 5*b*x^2*cosh(1)^3 - 5*b*d*x*cosh(1)^2)*sinh(1
)^3 + (b*c^4*d^4*x^2 - 12*b*c^2*d^2*x^2*cosh(1)^2 - 12*b*c^2*d^3*x*cosh(1)
+ 15*b*x^2*cosh(1)^4 + 20*b*d*x*cosh(1)^3)*sinh(1)^2 + 2*(b*c^4*d^4*x^2*cos
h(1) + b*c^4*d^5*x - 4*b*c^2*d^2*x^2*cosh(1)^3 - 6*b*c^2*d^3*x*cosh(1)^2 +
3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(1)^4)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)
) - (2*b*c^4*d^5*x*cosh(1) + b*c^4*d^6 - 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*co
sh(1)^6 + b*x^2*sinh(1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*x)
*sinh(1)^5 - (2*b*c^2*d^2*x^2 - b*d^2)*cosh(1)^4 - (2*b*c^2*d^2*x^2 - 15*b*
x^2*cosh(1)^2 - 10*b*d*x*cosh(1) - b*d^2)*sinh(1)^4 - 4*(b*c^2*d^3*x - 5*b*
x^2*cosh(1)^3 - 5*b*d*x*cosh(1)^2 + (2*b*c^2*d^2*x^2 - b*d^2)*cosh(1))*sinh
(1)^3 + (b*c^4*d^4*x^2 - 2*b*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 - 12*b*c^2
*d^3*x*cosh(1) - 2*b*c^2*d^4 + 15*b*x^2*cosh(1)^4 + 20*b*d*x*cosh(1)^3 - 6*
(2*b*c^2*d^2*x^2 - b*d^2)*cosh(1)^2)*sinh(1)^2 + 2*(b*c^4*d^5*x - 6*b*c^2*d
^3*x*cosh(1)^2 + 3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(1)^4 - 2*(2*b*c^2*d^2*x^2
 - b*d^2)*cosh(1)^3 + (b*c^4*d^4*x^2 - 2*b*c^2*d^4)*cosh(1))*sinh(1))*log(-
c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^4*d^5*x - 3*b*c^2*d^3*x*cosh(1)^2 - 2*(b*
c^2*d^2*x^2 - a*d^2)*cosh(1)^3 + (b*c^4*d^4*x^2 - (2*a + b)*c^2*d^4)*cosh(1
))*sinh(1) + (b*c^3*d^4*x*cosh(1)^2 + b*c^3*d^5*cosh(1) - b*c*d^2*x*cosh(1)
^4 - b*c*d^2*x*sinh(1)^4 - b*c*d^3*cosh(1)^3 - (4*b*c*d^2*x*cosh(1) + b*c*d
^3)*sinh(1)^3 + (b*c^3*d^4*x - 6*b*c*d^2*x*cosh(1)^2 - 3*b*c*d^3*cosh(1))*s
inh(1)^2 + (2*b*c^3*d^4*x*cosh(1) + b*c^3*d^5 - 4*b*c*d^2*x*cosh(1)^3 - 3*b
*c*d^3*cosh(1)^2)*sinh(1))*sqrt(c^2*x^2 - 1))/(2*c^4*d^7*x*cosh(1)^2 + c^4*
d^8*cosh(1) - 4*c^2*d^5*x*cosh(1)^4 + d^2*x^2*cosh(1)^7 + d^2*x^2*sinh(1)^7
 + 2*d^3*x*cosh(1)^6 + (7*d^2*x^2*cosh(1) + 2*d^3*x)*sinh(1)^6 - (2*c^2*d^4
*x^2 - d^4)*cosh(1)^5 - (2*c^2*d^4*x^2 - 21*d^2*x^2*cosh(1)^2 - 12*d^3*x*co
```

```
sh(1) - d^4)*sinh(1)^5 - (4*c^2*d^5*x - 35*d^2*x^2*cosh(1)^3 - 30*d^3*x*cos
h(1)^2 + 5*(2*c^2*d^4*x^2 - d^4)*cosh(1))*sinh(1)^4 + (c^4*d^6*x^2 - 2*c^2*
d^6)*cosh(1)^3 + (c^4*d^6*x^2 - 16*c^2*d^5*x*cosh(1) - 2*c^2*d^6 + 35*d^2*x
^2*cosh(1)^4 + 40*d^3*x*cosh(1)^3 - 10*(2*c^2*d^4*x^2 - d^4)*cosh(1)^2)*sin
h(1)^3 + (2*c^4*d^7*x - 24*c^2*d^5*x*cosh(1)^2 + 21*d^2*x^2*cosh(1)^5 + 30*
d^3*x*cosh(1)^4 - 10*(2*c^2*d^4*x^2 - d^4)*cosh(1)^3 + 3*(c^4*d^6*x^2 - 2*c
^2*d^6)*cosh(1))*sinh(1)^2 + (4*c^4*d^7*x*cosh(1) + c^4*d^8 - 16*c^2*d^5*x*
cosh(1)^3 + 7*d^2*x^2*cosh(1)^6 + 12*d^3*x*cosh(1)^5 - 5*(2*c^2*d^4*x^2 - d
^4)*cosh(1)^4 + 3*(c^4*d^6*x^2 - 2*c^2*d^6)*cosh(1)^2)*sinh(1)), -1/2*(2*b*
c^4*d^5*x*cosh(1) + (a + b)*c^4*d^6 - 2*b*c^2*d^3*x*cosh(1)^3 - (b*c^2*d^2*
x^2 - a*d^2)*cosh(1)^4 - (b*c^2*d^2*x^2 - a*d^2)*sinh(1)^4 - 2*(b*c^2*d^3*x
 + 2*(b*c^2*d^2*x^2 - a*d^2)*cosh(1))*sinh(1)^3 + (b*c^4*d^4*x^2 - (2*a + b
)*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 - 6*b*c^2*d^3*x*cosh(1) - (2*a + b)*c
^2*d^4 - 6*(b*c^2*d^2*x^2 - a*d^2)*cosh(1)^2)*sinh(1)^2 + 2*(b*c^3*d^3*x^2*
cosh(1)^2 + b*c^3*d^3*x^2*sinh(1)^2 + 2*b*c^3*d^4*x*cosh(1) + b*c^3*d^5 + 2
*(b*c^3*d^3*x^2*cosh(1) + b*c^3*d^4*x)*sinh(1))*sqrt(-((c^2*d^2 - 1)*cosh(1
) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*arctan(-(sqrt(c^2*x^2 - 1)*
sqrt(-((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*
(cosh(1) + sinh(1)) - (c*x*cosh(1) + c*x*sinh(1) + c*d)*sqrt(-((c^2*d^2 - 1
)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/(c^2*d^2 - cosh(1)
^2 - 2*cosh(1)*sinh(1) - sinh(1)^2)) - (b*c^4*d^4*x^2*cosh(1)^2 + 2*b*c^4*d
^5*x*cosh(1) - 2*b*c^2*d^2*x^2*cosh(1)^4 - 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*
cosh(1)^6 + b*x^2*sinh(1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*
x)*sinh(1)^5 - (2*b*c^2*d^2*x^2 - 15*b*x^2*cosh(1)^2 - 10*b*d*x*cosh(1))*si
nh(1)^4 - 4*(2*b*c^2*d^2*x^2*cosh(1) + b*c^2*d^3*x - 5*b*x^2*cosh(1)^3 - 5*
b*d*x*cosh(1)^2)*sinh(1)^3 + (b*c^4*d^4*x^2 - 1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x)^3,x)
[Out] int((a + b*acosh(c*x))/(d + e*x)^3, x)
```

3.20 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{3(cd-e)^{5/2}e(cd+e)^{5/2}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/e/(e*x+d)^3+1/3*b*c^3*(2*c^2*d^2+e^2)*arctanh((c*d+
e)^(1/2)*(c*x+1)^(1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(5/2)/e/(c*d+e)
^(5/2)-1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)^2-1/2*b*c^
3*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)
```

Rubi [A]
time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5963, 105, 156, 12, 95, 214}

$$-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)*(d + e*x)^2) - (b*
c^3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) -
 (a + b*ArcCosh[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(2*c^2*d^2 + e^2)*ArcTanh[
(Sqrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)
^(5/2)*e*(c*d + e)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}dx}{3e}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} - \frac{(bc)\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}d}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc)\int}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc^3(2c}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{(bc^3(2c}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2} - \frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)} - \frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3} + \frac{bc^3(2c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 259, normalized size = 1.28

$$-\frac{\frac{2a+\frac{bce\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)\left(-e^2+c^2d(4d+3ex)\right)}{\left(-c^2d^2+e^2\right)^2}}{(d+ex)^3}+\frac{2b\cosh^{-1}(cx)}{(d+ex)^3}+\frac{ibc^3\left(2c^2d^2+e^2\right)\log\left(\frac{12e^2(-cd+e)^2(cd+e)^2\left(-ie-ic^2dx+\sqrt{-c^2d^2+e^2}\,\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{bc^3\sqrt{-c^2d^2+e^2}\left(2c^2d^2+e^2\right)(d+ex)}\right)}{(-cd+e)^2(cd+e)^2\sqrt{-c^2d^2+e^2}}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*((2*a + (b*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)*(-e^2 + c^2*d*(4
*d + 3*e*x)))/(-(c^2*d^2) + e^2)^2)/(d + e*x)^3 + (2*b*ArcCosh[c*x])/(d + e
*x)^3 + (I*b*c^3*(2*c^2*d^2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*(
(-I)*e - I*c^2*d*x + Sqrt[-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/
(b*c^3*Sqrt[-(c^2*d^2) + e^2]*(2*c^2*d^2 + e^2)*(d + e*x))])/((-(c*d) + e)^
2*(c*d + e)^2*Sqrt[-(c^2*d^2) + e^2]))/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. 2(176) = 352.
time = 5.68, size = 1141, normalized size = 5.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*a*c^4/(c*e*x+c*d)^3/e-1/3*b*c^4/(c*e*x+c*d)^3/e*arccosh(c*x)-1/3*
b*c^8/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^
2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-
1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^4-2/3*b*c^8/e*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*
x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d
^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^3*x-1/3*b*c^8*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d
^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/
2)*e+e)/(c*e*x+c*d))*d^2*x^2-2/3*b*c^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c*d+e)/
(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2-1/2*b*c^6*e*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d*x-1/6*b*c^6*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)
^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2
)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2-1/3*b*c^6*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2
)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e
)/(c*e*x+c*d))*d*x-1/6*b*c^6*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1
/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*l
n(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))
*x^2+1/6*b*c^4*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2
)/(c*e*x+c*d)^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*integrate(1/3/(c^3*x^6*e^4 + 3*c^3*d*x^5*e^3 - 3*c*d^2*x^2*e^2 -
c*d^3*x*e + (3*c^3*d^2*e^2 - c*e^4)*x^4 + (c^3*d^3*e - 3*c*d*e^3)*x^3 + (c^
2*x^5*e^4 + 3*c^2*d*x^4*e^3 + (3*c^2*d^2*e^2 - e^4)*x^3 - 3*d^2*x*e^2 - d^3
*e + (c^2*d^3*e - 3*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x
) + 2*(c^6*d^3 + 3*c^4*d*e^2)*log(x*e + d)/(c^6*d^6*e - 3*c^4*d^4*e^3 + 3*c
^2*d^2*e^5 - e^7) - (3*c^6*d^6 - 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e
^2 - c^2*e^6)*x^2 + (5*c^6*d^5*e - 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x - 2*(c^6*
d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (c^6*d^6 + 3*c^5*d^5*e + 3*c^4*d^4*e^2 + c^3*d^3*e^3 + (c^6*d^3*e^
3 + 3*c^5*d^2*e^4 + 3*c^4*d*e^5 + c^3*e^6)*x^3 + 3*(c^6*d^4*e^2 + 3*c^5*d^3
*e^3 + 3*c^4*d^2*e^4 + c^3*d*e^5)*x^2 + 3*(c^6*d^5*e + 3*c^5*d^4*e^2 + 3*c^
4*d^3*e^3 + c^3*d^2*e^4)*x)*log(c*x + 1) + (c^6*d^6 - 3*c^5*d^5*e + 3*c^4*d
^4*e^2 - c^3*d^3*e^3 + (c^6*d^3*e^3 - 3*c^5*d^2*e^4 + 3*c^4*d*e^5 - c^3*e^6
)*x^3 + 3*(c^6*d^4*e^2 - 3*c^5*d^3*e^3 + 3*c^4*d^2*e^4 - c^3*d*e^5)*x^2 + 3
```

```
*(c^6*d^5*e - 3*c^5*d^4*e^2 + 3*c^4*d^3*e^3 - c^3*d^2*e^4)*x)*log(c*x - 1))
/(c^6*d^9*e - 3*c^4*d^7*e^3 + 3*c^2*d^5*e^5 + (c^6*d^6*e^4 - 3*c^4*d^4*e^6
+ 3*c^2*d^2*e^8 - e^10)*x^3 - d^3*e^7 + 3*(c^6*d^7*e^3 - 3*c^4*d^5*e^5 + 3*
c^2*d^3*e^7 - d*e^9)*x^2 + 3*(c^6*d^8*e^2 - 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 -
 d^2*e^8)*x))*b - 1/3*a/(x^3*e^4 + 3*d*x^2*e^3 + 3*d^2*x*e^2 + d^3*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4943 vs. $2(175) = 350$.
time = 0.79, size = 9905, normalized size = 49.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(9*b*c^6*d^8*x*cosh(1) - 3*b*c^4*d^4*x^3*cosh(1)^5 + (2*a + 3*b)*c^6*
d^9 - 2*a*d^3*cosh(1)^6 - 2*a*d^3*sinh(1)^6 - 3*(b*c^4*d^4*x^3 + 4*a*d^3*co
sh(1))*sinh(1)^5 - 3*(3*b*c^4*d^5*x^2 - 2*a*c^2*d^5)*cosh(1)^4 - 3*(5*b*c^4
*d^4*x^3*cosh(1) + 3*b*c^4*d^5*x^2 - 2*a*c^2*d^5 + 10*a*d^3*cosh(1)^2)*sinh
(1)^4 + 3*(b*c^6*d^6*x^3 - 3*b*c^4*d^6*x)*cosh(1)^3 + (3*b*c^6*d^6*x^3 - 30
*b*c^4*d^4*x^3*cosh(1)^2 - 9*b*c^4*d^6*x - 40*a*d^3*cosh(1)^3 - 12*(3*b*c^4
*d^5*x^2 - 2*a*c^2*d^5)*cosh(1))*sinh(1)^3 + 3*(3*b*c^6*d^7*x^2 - (2*a + b)
*c^4*d^7)*cosh(1)^2 + 3*(3*b*c^6*d^7*x^2 - 10*b*c^4*d^4*x^3*cosh(1)^3 - (2*
a + b)*c^4*d^7 - 10*a*d^3*cosh(1)^4 - 6*(3*b*c^4*d^5*x^2 - 2*a*c^2*d^5)*cos
h(1)^2 + 3*(b*c^6*d^6*x^3 - 3*b*c^4*d^6*x)*cosh(1))*sinh(1)^2 - (6*b*c^5*d^
7*x*cosh(1) + b*c^3*d^3*x^3*cosh(1)^5 + b*c^3*d^3*x^3*sinh(1)^5 + 2*b*c^5*d
^8 + 3*b*c^3*d^4*x^2*cosh(1)^4 + (5*b*c^3*d^3*x^3*cosh(1) + 3*b*c^3*d^4*x^2
)*sinh(1)^4 + (2*b*c^5*d^5*x^3 + 3*b*c^3*d^5*x)*cosh(1)^3 + (2*b*c^5*d^5*x^
3 + 10*b*c^3*d^3*x^3*cosh(1)^2 + 12*b*c^3*d^4*x^2*cosh(1) + 3*b*c^3*d^5*x)*
sinh(1)^3 + (6*b*c^5*d^6*x^2 + b*c^3*d^6)*cosh(1)^2 + (6*b*c^5*d^6*x^2 + 10
*b*c^3*d^3*x^3*cosh(1)^3 + 18*b*c^3*d^4*x^2*cosh(1)^2 + b*c^3*d^6 + 3*(2*b*
c^5*d^5*x^3 + 3*b*c^3*d^5*x)*cosh(1))*sinh(1)^2 + (6*b*c^5*d^7*x + 5*b*c^3*
d^3*x^3*cosh(1)^4 + 12*b*c^3*d^4*x^2*cosh(1)^3 + 3*(2*b*c^5*d^5*x^3 + 3*b*c
^3*d^5*x)*cosh(1)^2 + 2*(6*b*c^5*d^6*x^2 + b*c^3*d^6)*cosh(1))*sinh(1))*sqr
t(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1)))*log(
(c^3*d^2*x + c*d*cosh(1) + c*d*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 - 1)
*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))) - cosh(1)^2 - 2*cosh
(1)*sinh(1) - sinh(1)^2)*sqrt(c^2*x^2 - 1) + (c^2*d*x + cosh(1) + sinh(1))*
sqrt(((c^2*d^2 - 1)*cosh(1) - (c^2*d^2 + 1)*sinh(1))/(cosh(1) - sinh(1))))/
(x*cosh(1) + x*sinh(1) + d)) - 2*(3*b*c^6*d^7*x^2*cosh(1)^2 + 3*b*c^6*d^8*x
*cosh(1) - 9*b*c^4*d^5*x^2*cosh(1)^4 + 9*b*c^2*d^3*x^2*cosh(1)^6 - b*x^3*co
sh(1)^9 - b*x^3*sinh(1)^9 - 3*b*d*x^2*cosh(1)^8 - 3*(3*b*x^3*cosh(1) + b*d*
x^2)*sinh(1)^8 + 3*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^7 + 3*(b*c^2*d^2*x^3 -
 12*b*x^3*cosh(1)^2 - 8*b*d*x^2*cosh(1) - b*d^2*x)*sinh(1)^7 + 3*(3*b*c^2*d
^3*x^2 - 28*b*x^3*cosh(1)^3 - 28*b*d*x^2*cosh(1)^2 + 7*(b*c^2*d^2*x^3 - b*d
```

```
^2*x)*cosh(1))*sinh(1)^6 - 3*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x)*cosh(1)^5 - 3*
(b*c^4*d^4*x^3 - 18*b*c^2*d^3*x^2*cosh(1) - 3*b*c^2*d^4*x + 42*b*x^3*cosh(1
)^4 + 56*b*d*x^2*cosh(1)^3 - 21*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^2)*sinh(1
)^5 - 3*(3*b*c^4*d^5*x^2 - 45*b*c^2*d^3*x^2*cosh(1)^2 + 42*b*x^3*cosh(1)^5
+ 70*b*d*x^2*cosh(1)^4 - 35*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^3 + 5*(b*c^4*
d^4*x^3 - 3*b*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (b*c^6*d^6*x^3 - 9*b*c^4*d^6*
x)*cosh(1)^3 + (b*c^6*d^6*x^3 - 36*b*c^4*d^5*x^2*cosh(1) - 9*b*c^4*d^6*x +
180*b*c^2*d^3*x^2*cosh(1)^3 - 84*b*x^3*cosh(1)^6 - 168*b*d*x^2*cosh(1)^5 +
105*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^4 - 30*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x
)*cosh(1)^2)*sinh(1)^3 + 3*(b*c^6*d^7*x^2 - 18*b*c^4*d^5*x^2*cosh(1)^2 + 45
*b*c^2*d^3*x^2*cosh(1)^4 - 12*b*x^3*cosh(1)^7 - 28*b*d*x^2*cosh(1)^6 + 21*(
b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^5 - 10*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x)*cos
h(1)^3 + (b*c^6*d^6*x^3 - 9*b*c^4*d^6*x)*cosh(1))*sinh(1)^2 + 3*(2*b*c^6*d^
7*x^2*cosh(1) + b*c^6*d^8*x - 12*b*c^4*d^5*x^2*cosh(1)^3 + 18*b*c^2*d^3*x^2
*cosh(1)^5 - 3*b*x^3*cosh(1)^8 - 8*b*d*x^2*cosh(1)^7 + 7*(b*c^2*d^2*x^3 - b
*d^2*x)*cosh(1)^6 - 5*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x)*cosh(1)^4 + (b*c^6*d^
6*x^3 - 9*b*c^4*d^6*x)*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1)) - 2
*(3*b*c^6*d^8*x*cosh(1) + b*c^6*d^9 - b*x^3*cosh(1)^9 - b*x^3*sinh(1)^9 - 3
*b*d*x^2*cosh(1)^8 - 3*(3*b*x^3*cosh(1) + b*d*x^2)*sinh(1)^8 + 3*(b*c^2*d^2
*x^3 - b*d^2*x)*cosh(1)^7 + 3*(b*c^2*d^2*x^3 - 12*b*x^3*cosh(1)^2 - 8*b*d*x
^2*cosh(1) - b*d^2*x)*sinh(1)^7 + (9*b*c^2*d^3*x^2 - b*d^3)*cosh(1)^6 + (9*
b*c^2*d^3*x^2 - 84*b*x^3*cosh(1)^3 - 84*b*d*x^2*cosh(1)^2 - b*d^3 + 21*(b*c
^2*d^2*x^3 - b*d^2*x)*cosh(1))*sinh(1)^6 - 3*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x
)*cosh(1)^5 - 3*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x + 42*b*x^3*cosh(1)^4 + 56*b*
d*x^2*cosh(1)^3 - 21*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^2 - 2*(9*b*c^2*d^3*x
^2 - b*d^3)*cosh(1))*sinh(1)^5 - 3*(3*b*c^4*d^5*x^2 - b*c^2*d^5)*cosh(1)^4
- 3*(3*b*c^4*d^5*x^2 + 42*b*x^3*cosh(1)^5 - b*c^2*d^5 + 70*b*d*x^2*cosh(1)^
4 - 35*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^3 - 5*(9*b*c^2*d^3*x^2 - b*d^3)*co
sh(1)^2 + 5*(b*c^4*d^4*x^3 - 3*b*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (b*c^6*d^6
*x^3 - 9*b*c^4*d^6*x)*cosh(1)^3 + (b*c^6*d^6*x^3 - 9*b*c^4*d^6*x - 84*b*x^3
*cosh(1)^6 - 168*b*d*x^2*cosh(1)^5 + 105*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^
4 + 20*(9*b*c^2*d^3*x^2 - b*d^3)*cosh(1)^3 - 30*(b*c^4*d^4*x^3 - 3*b*c^2*d^
4*x)*cosh(1)^2 - 12*(3*b*c^4*d^5*x^2 - b*c^2*d^5)*cosh(1))*sinh(1)^3 + 3*(b
*c^6*d^7*x^2 - b*c^4*d^7)*cosh(1)^2 + 3*(b*c^6*d^7*x^2 - b*c^4*d^7 - 12*b*x
^3*cosh(1)^7 - 28*b*d*x^2*cosh(1)^6 + 21*(b*c^2*d^2*x^3 - b*d^2*x)*cosh(1)^
5 + 5*(9*b*c^2*d^3*x^2 - b*d^3)*cosh(1)^4 - 10*(b*c^4*d^4*x^3 - 3*b*c^2*d^4
*x)*cosh(1)^3 - 6*(3*b*c^4*d^5*x^2 - b*c^2*d^5)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{(d + ex)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{{(d+e\,x)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x)^4,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x)^4, x)
```

3.21 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=398

$$2b^2d^3x+\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2+\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-$$

```
[Out] 2*b^2*d^3*x+4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2+3/32*b^2*e^3*x^2/c^2+2/9*
b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arccosh(c*x))^2/e-3/4*d^2*e*(a+
b*arccosh(c*x))^2/c^2-3/32*e^3*(a+b*arccosh(c*x))^2/c^4+1/4*(e*x+d)^4*(a+b*
arccosh(c*x))^2/e-2*b*d^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-
4/3*b*d*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/2*b*d^2*e*
x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/16*b*e^3*x*(a+b*arccos
h(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/3*b*d*e^2*x^2*(a+b*arccosh(c*x))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/8*b*e^3*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c
```

Rubi [A]

time = 1.15, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{3e^3\left(a+b\cosh^{-1}(cx)\right)^2}{32c^4}-\frac{4bde^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{3be^3x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{16c^3}-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e}-\frac{2bd^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{3bd^2ex\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{2c}-\frac{2bde^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c}+\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e}-\frac{be^3x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{8c}+\frac{4b^2de^2x}{3c^2}+\frac{3b^2e^3x^2}{32c^2}+2b^2d^3x+\frac{3}{4}b^2d^2ex^2+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x + (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 + (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(2*c) - (3*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x]))/(16*c^3) - (2*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(8*c) - (d^4*(a + b*ArcCosh[c*x])^2)/(4*e) - (3*d^2*e
*(a + b*ArcCosh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcCosh[c*x])^2)/(32*c^4)
+ ((d + e*x)^4*(a + b*ArcCosh[c*x])^2)/(4*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
```

```
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{d^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4d^3}{\sqrt{-}}\right.}{}$$

$$= \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \left(2bcd^3\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$

$$= -\frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{3bd^2ex\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{-1+cx}\sqrt{1+c}}{c}$$

$$= 2b^2d^3x + \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 + \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2}{}$$

Mathematica [A]
time = 0.46, size = 386, normalized size = 0.97

$$\frac{c\left(72a^2c^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-6ab\sqrt{-1+cx}\sqrt{1+cx}\,(e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3))+b^2cx(3e^2(128d+9ex)+c^2(576d^3+216d^2ex+64de^2x^2+9e^3x^3))\right)-6bc\left(-24ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+b\sqrt{-1+cx}\sqrt{1+cx}\,(e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3))\right)\cosh^{-1}(cx)+9b^2(-24c^2d^2e-3e^3+8c^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3))\cosh^{-1}(cx)^2-54abe(8c^2d^2+e^2)\log\left(cx+\sqrt{-1+cx}\sqrt{1+cx}\right)}{288c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*
d*e^2*x^2 + 6*e^3*x^3)) + b^2*c*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 2
16*d^2*e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*c*(-24*a*c^3*x*(4*d^3 + 6*d^
2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d
+ 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)))*ArcCosh[c
*x] + 9*b^2*(-24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x
^2 + e^3*x^3))*ArcCosh[c*x]^2 - 54*a*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-
1 + c*x]*Sqrt[1 + c*x]])/(288*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(346) = 692$.

time = 2.49, size = 814, normalized size = 2.05

$$2b^2d^3x+\frac{ab\,\mathrm{arccosh}(cx)\,d^4}{2e}+2ab\,\mathrm{arccosh}(cx)\,x\,d^3+b^2\mathrm{arccosh}(cx)^2\,x^3d\,e^2+\frac{3b^2\mathrm{arccosh}(cx)^2\,x^2d^2e}{2}-\frac{3b^2\mathrm{arccosh}(}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 2*b^2*d^3*x+1/2*a*b/e*arccosh(c*x)*d^4+2*a*b*arccosh(c*x)*x*d^3+b^2*arccosh
(c*x)^2*x^3*d*e^2+3/2*b^2*arccosh(c*x)^2*x^2*d^2*e-3/4/c^2*b^2*arccosh(c*x)
^2*d^2*e+1/2*a*b*e^3*arccosh(c*x)*x^4-2/3/c*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*x^2*d-3/2/c*a*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*x-3/2/c*b^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*x*d^2*e-3/16/c^4*a*b*e^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))-2/3/c*b^2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^2*d*e^2-1/2*a*b/e*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-3/2/c^2*a*b*e*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-15/256/c
^4*b^2*e^3+1/4*a^2*e^3*x^4+a^2*x*d^3-3/16/c^3*a*b*e^3*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x-1/8/c*a*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-1/8/c*b^2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*arccosh(c*x)*x^3*e^3-3/16/c^3*b^2*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*arccosh(c*x)*x*e^3-4/3/c^3*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x
)*d*e^2-4/3/c^3*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d+1/4*a^2/e*d^4+1/4*b^2
*arccosh(c*x)^2*x^4*e^3+b^2*arccosh(c*x)^2*x*d^3-3/32/c^4*b^2*arccosh(c*x)^
2*e^3+a^2*e^2*x^3*d+3/2*a^2*e*x^2*d^2+3/4*b^2*d^2*e*x^2+3/32*b^2*e^3*x^2/c^
2+2/9*b^2*d*e^2*x^3-2/c*b^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)*d^3-2/
c*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3+3*a*b*e*arccosh(c*x)*x^2*d^2+2*a*b*e^
2*arccosh(c*x)*x^3*d+4/3*b^2*d*e^2*x/c^2+1/32*b^2*e^3*x^4-3/8/c^2*b^2*d^2*e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^3*x*arccosh(c*x)^2 + 1/4*a^2*x^4*e^3 + a^2*d*x^3*e^2 + 3/2*a^2*d^2*x^
2*e + 2*b^2*d^3*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^3*x + 3/2*(2
*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x
^2 - 1)*c)/c^3))*a*b*d^2*e + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d
^3/c + 2/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*
x^2 - 1)/c^4))*a*b*d*e^2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*
x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c
)/c^5)*c)*a*b*e^3 + 1/4*(b^2*x^4*e^3 + 4*b^2*d*x^3*e^2 + 6*b^2*d^2*x^2*e)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - integrate(1/2*(b^2*c^3*x^6*e^3 +
```

```
4*b^2*c^3*d*x^5*e^2 - 4*b^2*c*d*x^3*e^2 - 6*b^2*c*d^2*x^2*e + (6*b^2*c^3*d^
2*e - b^2*c*e^3)*x^4 + (b^2*c^2*x^5*e^3 + 4*b^2*c^2*d*x^4*e^2 + 6*b^2*c^2*d
^2*x^3*e)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. $2(339) = 678$.
time = 0.36, size = 1187, normalized size = 2.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(216*(2*a^2 + b^2)*c^4*d^2*x^2*cosh(1) + 288*(a^2 + 2*b^2)*c^4*d^3*x
+ 9*((8*a^2 + b^2)*c^4*x^4 + 3*b^2*c^2*x^2)*cosh(1)^3 + 9*((8*a^2 + b^2)*c^
4*x^4 + 3*b^2*c^2*x^2)*sinh(1)^3 + 32*((9*a^2 + 2*b^2)*c^4*d*x^3 + 12*b^2*c
^2*d*x)*cosh(1)^2 + 9*(32*b^2*c^4*d*x^3*cosh(1)^2 + 32*b^2*c^4*d^3*x + (8*b
^2*c^4*x^4 - 3*b^2)*cosh(1)^3 + (8*b^2*c^4*x^4 - 3*b^2)*sinh(1)^3 + (32*b^2
*c^4*d*x^3 + 3*(8*b^2*c^4*x^4 - 3*b^2)*cosh(1))*sinh(1)^2 + 24*(2*b^2*c^4*d
^2*x^2 - b^2*c^2*d^2)*cosh(1) + (64*b^2*c^4*d*x^3*cosh(1) + 48*b^2*c^4*d^2*
x^2 - 24*b^2*c^2*d^2 + 3*(8*b^2*c^4*x^4 - 3*b^2)*cosh(1)^2)*sinh(1))*log(c*
x + sqrt(c^2*x^2 - 1))^2 + (32*(9*a^2 + 2*b^2)*c^4*d*x^3 + 384*b^2*c^2*d*x
+ 27*((8*a^2 + b^2)*c^4*x^4 + 3*b^2*c^2*x^2)*cosh(1))*sinh(1)^2 + 6*(96*a*b
*c^4*d*x^3*cosh(1)^2 + 96*a*b*c^4*d^3*x + 3*(8*a*b*c^4*x^4 - 3*a*b)*cosh(1)
^3 + 3*(8*a*b*c^4*x^4 - 3*a*b)*sinh(1)^3 + 3*(32*a*b*c^4*d*x^3 + 3*(8*a*b*c
^4*x^4 - 3*a*b)*cosh(1))*sinh(1)^2 + 72*(2*a*b*c^4*d^2*x^2 - a*b*c^2*d^2)*c
osh(1) + 3*(64*a*b*c^4*d*x^3*cosh(1) + 48*a*b*c^4*d^2*x^2 - 24*a*b*c^2*d^2
+ 3*(8*a*b*c^4*x^4 - 3*a*b)*cosh(1)^2)*sinh(1) - (72*b^2*c^3*d^2*x*cosh(1)
+ 96*b^2*c^3*d^3 + 3*(2*b^2*c^3*x^3 + 3*b^2*c*x)*cosh(1)^3 + 3*(2*b^2*c^3*x
^3 + 3*b^2*c*x)*sinh(1)^3 + 32*(b^2*c^3*d*x^2 + 2*b^2*c*d)*cosh(1)^2 + (32*
b^2*c^3*d*x^2 + 64*b^2*c*d + 9*(2*b^2*c^3*x^3 + 3*b^2*c*x)*cosh(1))*sinh(1)
^2 + (72*b^2*c^3*d^2*x + 9*(2*b^2*c^3*x^3 + 3*b^2*c*x)*cosh(1)^2 + 64*(b^2*
c^3*d*x^2 + 2*b^2*c*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(
c^2*x^2 - 1)) + (216*(2*a^2 + b^2)*c^4*d^2*x^2 + 27*((8*a^2 + b^2)*c^4*x^4
+ 3*b^2*c^2*x^2)*cosh(1)^2 + 64*((9*a^2 + 2*b^2)*c^4*d*x^3 + 12*b^2*c^2*d*x
)*cosh(1))*sinh(1) - 6*(72*a*b*c^3*d^2*x*cosh(1) + 96*a*b*c^3*d^3 + 3*(2*a*
b*c^3*x^3 + 3*a*b*c*x)*cosh(1)^3 + 3*(2*a*b*c^3*x^3 + 3*a*b*c*x)*sinh(1)^3
+ 32*(a*b*c^3*d*x^2 + 2*a*b*c*d)*cosh(1)^2 + (32*a*b*c^3*d*x^2 + 64*a*b*c*d
 + 9*(2*a*b*c^3*x^3 + 3*a*b*c*x)*cosh(1))*sinh(1)^2 + (72*a*b*c^3*d^2*x + 9
*(2*a*b*c^3*x^3 + 3*a*b*c*x)*cosh(1)^2 + 64*(a*b*c^3*d*x^2 + 2*a*b*c*d)*cos
h(1))*sinh(1))*sqrt(c^2*x^2 - 1))/c^4
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.51, size = 750, normalized size = 1.88

$$\begin{cases} a^{2}d^{3}x + \frac{3a^{2}d^{2}ex^{2}}{2} + a^{2}de^{2}x^{3} + \frac{a^{2}e^{3}x^{4}}{4} + 2abd^{3}x\operatorname{acosh}(cx) + 3abd^{2}ex^{2}\operatorname{acosh}(cx) + 2abde^{2}x^{3}\operatorname{acosh}(cx) + \frac{abe^{3}x^{4}\operatorname{acosh}(cx)}{2} - \frac{2abd^{3}\sqrt{c^{2}x^{2}-1}}{c} - \frac{3abd^{2}ex\sqrt{c^{2}x^{2}-1}}{2c} - \frac{2abde^{2}x^{2}\sqrt{c^{2}x^{2}-1}}{3c} - \frac{abe^{3}x^{3}\sqrt{c^{2}x^{2}-1}}{8c} - \frac{3abd^{2}e\operatorname{acosh}(cx)}{2c^{2}} - \frac{4abde^{2}\sqrt{c^{2}x^{2}-1}}{3c^{3}} - \frac{3abe^{3}x\sqrt{c^{2}x^{2}-1}}{16c^{3}} - \frac{3abe^{3}\operatorname{acosh}(cx)}{16c^{4}} + b^{2}d^{3}x\operatorname{acosh}^{2}(cx) + 2b^{2}d^{3}x + \frac{3b^{2}d^{2}ex^{2}\operatorname{acosh}^{2}(cx)}{2} + \frac{3b^{2}d^{2}ex^{2}}{4} + b^{2}de^{2}x^{3}\operatorname{acosh}^{2}(cx) + \frac{2b^{2}de^{2}x^{3}}{9} + \frac{b^{2}e^{3}x^{4}\operatorname{acosh}^{2}(cx)}{4} + \frac{b^{2}e^{3}x^{4}}{32} - \frac{2b^{2}d^{3}\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{c} - \frac{3b^{2}d^{2}ex\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{2c} - \frac{2b^{2}de^{2}x^{2}\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{3c} - \frac{b^{2}e^{3}x^{3}\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{8c} - \frac{3b^{2}d^{2}e\operatorname{acosh}^{2}(cx)}{4c^{2}} + \frac{4b^{2}de^{2}x}{3c^{2}} + \frac{3b^{2}e^{3}x^{2}}{32c^{2}} - \frac{4b^{2}de^{2}\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{3c^{3}} - \frac{3b^{2}e^{3}x\sqrt{c^{2}x^{2}-1}\operatorname{acosh}(cx)}{16c^{3}} - \frac{3b^{2}e^{3}\operatorname{acosh}^{2}(cx)}{32c^{4}} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)^{2}\left(d^{3}x + \frac{3d^{2}ex^{2}}{2} + de^{2}x^{3} + \frac{e^{3}x^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*acosh(c*x) + 3*a*b*d**2*e*x**2*acosh(c*x) + 2*a*b*d
*e**2*x**3*acosh(c*x) + a*b*e**3*x**4*acosh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 - 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 - 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(8*c) - 3*a*b*
d**2*e*acosh(c*x)/(2*c**2) - 4*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*
a*b*e**3*x*sqrt(c**2*x**2 - 1)/(16*c**3) - 3*a*b*e**3*acosh(c*x)/(16*c**4)
+ b**2*d**3*x*acosh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*acosh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*acosh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*acosh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 - 1)*ac
osh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c) - 3*b**2*d**2*e*acosh(c
*x)**2/(4*c**2) + 4*b**2*d*e**2*x/(3*c**2) + 3*b**2*e**3*x**2/(32*c**2) - 4
*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*b**2*e**3*x*sqrt(c
**2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*b**2*e**3*acosh(c*x)**2/(32*c**4), N
e(c, 0)), ((a + I*pi*b/2)**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3
*x**4/4), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c\,x))^2\,(d + e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^3, x)
```

3.22 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=259

$$2b^2d^2x+\frac{4b^2e^2x}{9c^2}+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3-\frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4be^2\sqrt{-1+cx}\,\sqrt{1+c}}{9c}$$

```
[Out] 2*b^2*d^2*x+4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+b
*arccosh(c*x))^2/e-1/2*d*e*(a+b*arccosh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcc
osh(c*x))^2/e-2*b*d^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-4/9*
b*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-b*d*e*x*(a+b*arcco
sh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-2/9*b*e^2*x^2*(a+b*arccosh(c*x))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A]
time = 0.76, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{4be^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{de\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2bd^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2be^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c}+\frac{4b^2e^2x}{9c^2}+2b^2d^2x+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (2*b*e^2*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (d^3*(a + b*ArcCosh[c*x])^2)
/(3*e) - (d*e*(a + b*ArcCosh[c*x])^2)/(2*c^2) + ((d + e*x)^3*(a + b*ArcCosh
[c*x])^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
```

```
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \frac{(d+ex)^3 (a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \left(\frac{d^3 (a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\ldots}{\sqrt{\ldots}}\right.}{\ldots}$$

$$= \frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \left(2bcd^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx \ldots$$

$$= -\frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bdex\sqrt{-1+cx}\sqrt{1\ldots}}{\ldots}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh\ldots\right.}{c}$$

$$= 2b^2d^2x + \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(\ldots\right.}{c}$$

Mathematica [A]
time = 0.41, size = 360, normalized size = 1.39

$$a^2d^2x + 2b^2d^2x + \frac{4b^2e^2x}{9c^2} + a^2dex^2 + \frac{1}{2}b^2dex^2 + \frac{1}{3}a^2e^2x^3 + \frac{2}{27}b^2e^2x^3 - \frac{2abd^2\sqrt{-1+cx}\sqrt{1+cx}}{c} - \frac{4abe^2\sqrt{-1+cx}\sqrt{1+cx}}{9c^3} - \frac{abdex\sqrt{-1+cx}\sqrt{1+cx}}{c} - \frac{2abe^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{9c} - \frac{b\left(-6ac^3x(3d^2+3dex+e^2x^2) + b\sqrt{-1+cx}\sqrt{1+cx}\,(4e^2+c^2(18d^2+9dex+2e^2x^2))\right)\cosh^{-1}(cx)}{9c^3} + \frac{1}{6}b^2\left(-\frac{3de}{c^2} + 2x(3d^2+3dex+e^2x^2)\right)\cosh^{-1}(cx)^2 - \frac{abde\log\left(cx+\sqrt{-1+cx}\sqrt{1+cx}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] a^2*d^2*x + 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + a^2*d*e*x^2 + (b^2*d*e*x^
2)/2 + (a^2*e^2*x^3)/3 + (2*b^2*e^2*x^3)/27 - (2*a*b*d^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/c - (4*a*b*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (a*b*d*e
*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (2*a*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/(9*c) - (b*(-6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))*ArcCosh[c*x])
/(9*c^3) + (b^2*((-3*d*e)/c^2 + 2*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*
x]^2)/6 - (a*b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(227) = 454$.
time = 1.64, size = 527, normalized size = 2.03

$$-\frac{b^2de}{4c^2} - \frac{2ab\sqrt{cx-1}\sqrt{cx+1}\,d^3\ln\left(cx+\sqrt{c^2x^2-1}\right)}{3e\sqrt{c^2x^2-1}} - \frac{2b^2\mathrm{arccosh}(cx)\sqrt{cx-1}\sqrt{cx+1}\,x^2e^2}{9c} - \frac{2ab\,e^2\sqrt{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/4/c^2*b^2*d*e-2/3*a*b/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^
3*ln(c*x+(c^2*x^2-1)^(1/2))-2/9/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*x^2*e^2-2/9/c*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/c*a*b*e*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*d*x+2*a*b*e*arccosh(c*x)*x^2*d-4/9/c^3*a*b*e^2*(c*x-1)
^(1/2)*(c*x+1)^(1/2)-2/c*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-2/c*b^2*arccos
h(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-4/9/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*e^2-1/c^2*a*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1
/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))+2/3*a*b/e*arccosh(c*x)*d^3+2*b^2*d^2*x+2*a*
b*arccosh(c*x)*x*d^2+2/3*a*b*e^2*arccosh(c*x)*x^3+b^2*arccosh(c*x)^2*x^2*d*
e-1/2/c^2*b^2*arccosh(c*x)^2*d*e-1/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x*d*e+4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+1/3*a^2/e*d^3+1/3*b^2*arccos
h(c*x)^2*x^3*e^2+b^2*arccosh(c*x)^2*x*d^2+a^2*e*x^2*d+2/27*b^2*e^2*x^3+a^2*
x*d^2+1/3*a^2*e^2*x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^2*x*arccosh(c*x)^2 + 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + 2*b^2*d^2*(x - s
qrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^2*x + (2*x^2*arccosh(c*x) - c*(sqr
t(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*a*b*d*e +
 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^2/c + 2/9*(3*x^3*arccosh(c*
x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*e^2 + 1/3
*(b^2*x^3*e^2 + 3*b^2*d*x^2*e)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - i
ntegrate(2/3*(b^2*c^3*x^5*e^2 + 3*b^2*c^3*d*x^4*e - b^2*c*x^3*e^2 - 3*b^2*c
*d*x^2*e + (b^2*c^2*x^4*e^2 + 3*b^2*c^2*d*x^3*e)*sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x
 + 1)*sqrt(c*x - 1) - c*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(225) = 450$.
time = 0.40, size = 618, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(27*(2*a^2 + b^2)*c^3*d*x^2*cosh(1) + 54*(a^2 + 2*b^2)*c^3*d^2*x + 2*(
(9*a^2 + 2*b^2)*c^3*x^3 + 12*b^2*c*x)*cosh(1)^2 + 9*(2*b^2*c^3*x^3*cosh(1)^
```

```
2 + 2*b^2*c^3*x^3*sinh(1)^2 + 6*b^2*c^3*d^2*x + 3*(2*b^2*c^3*d*x^2 - b^2*c*
d)*cosh(1) + (4*b^2*c^3*x^3*cosh(1) + 6*b^2*c^3*d*x^2 - 3*b^2*c*d)*sinh(1))
*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*((9*a^2 + 2*b^2)*c^3*x^3 + 12*b^2*c*x)*
sinh(1)^2 + 6*(6*a*b*c^3*x^3*cosh(1)^2 + 6*a*b*c^3*x^3*sinh(1)^2 + 18*a*b*c
^3*d^2*x + 9*(2*a*b*c^3*d*x^2 - a*b*c*d)*cosh(1) + 3*(4*a*b*c^3*x^3*cosh(1)
 + 6*a*b*c^3*d*x^2 - 3*a*b*c*d)*sinh(1) - (9*b^2*c^2*d*x*cosh(1) + 18*b^2*c
^2*d^2 + 2*(b^2*c^2*x^2 + 2*b^2)*cosh(1)^2 + 2*(b^2*c^2*x^2 + 2*b^2)*sinh(1
)^2 + (9*b^2*c^2*d*x + 4*(b^2*c^2*x^2 + 2*b^2)*cosh(1))*sinh(1))*sqrt(c^2*x
^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(2*a^2 + b^2)*c^3*d*x^2 + 4*((9
*a^2 + 2*b^2)*c^3*x^3 + 12*b^2*c*x)*cosh(1))*sinh(1) - 6*(9*a*b*c^2*d*x*cos
h(1) + 18*a*b*c^2*d^2 + 2*(a*b*c^2*x^2 + 2*a*b)*cosh(1)^2 + 2*(a*b*c^2*x^2
+ 2*a*b)*sinh(1)^2 + (9*a*b*c^2*d*x + 4*(a*b*c^2*x^2 + 2*a*b)*cosh(1))*sinh
(1))*sqrt(c^2*x^2 - 1))/c^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 461, normalized size = 1.78

$$\begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{acosh}(cx) + 2abdex^2\operatorname{acosh}(cx) + \frac{2abe^2x^3\operatorname{acosh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2-1}}{c} - \frac{abdex\sqrt{c^2x^2-1}}{c} - \frac{2abe^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{abde\operatorname{acosh}(cx)}{c^2} - \frac{4abe^2\sqrt{c^2x^2-1}}{9c^3} + b^2d^2x\operatorname{acosh}^2(cx) + 2b^2d^2x + b^2dex^2\operatorname{acosh}^2(cx) + \frac{b^2dex^2}{2} + \frac{b^2e^2x^3\operatorname{acosh}^2(cx)}{3} + \frac{2b^2e^2x^3}{27} - \frac{2b^2d^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{b^2dex\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{2b^2e^2x^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c} - \frac{b^2de\operatorname{acosh}^2(cx)}{2c^2} + \frac{4b^2e^2x}{9c^2} - \frac{4b^2e^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{9c^3} & \text{for}\: c \neq 0 \\ \left(a + \frac{i\pi b}{2}\right)^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*ac
osh(c*x) + 2*a*b*d*e*x**2*acosh(c*x) + 2*a*b*e**2*x**3*acosh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 - 1)/c - a*b*d*e*x*sqrt(c**2*x**2 - 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 - 1)/(9*c) - a*b*d*e*acosh(c*x)/c**2 - 4*a*b*e**2*sqrt(
c**2*x**2 - 1)/(9*c**3) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*acosh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*acosh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 -
1)*acosh(c*x)/(9*c) - b**2*d*e*acosh(c*x)**2/(2*c**2) + 4*b**2*e**2*x/(9*c*
*2) - 4*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), ((a +
 I*pi*b/2)**2*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\text{acosh}(c\,x))^2\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^2, x)
```

3.23 $\int(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=150

$$2b^2dx+\frac{1}{4}b^2ex^2-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{2c}-$$

```
[Out] 2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arccosh(c*x))^2/e-1/4*e*(a+b*arccosh(c
*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arccosh(c*x))^2/e-2*b*d*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*b*e*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c
```

Rubi [A]
time = 0.50, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5963, 5975, 5893, 5915, 8, 5939, 30}

$$-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}-\frac{2bd\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{2c}+2b^2dx+\frac{1}{4}b^2ex^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2
*c) - (d^2*(a + b*ArcCosh[c*x])^2)/(2*e) - (e*(a + b*ArcCosh[c*x])^2)/(4*c^
2) + ((d + e*x)^2*(a + b*ArcCosh[c*x])^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2de}{\sqrt{-1}}\right.}{}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx - $$

$$= -\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{-1+cx}\,\sqrt{1+c}}{2}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{}}{}$$

Mathematica [A]

time = 0.19, size = 174, normalized size = 1.16

$$\frac{c\left(2a^2cx(2d+ex)-2ab\sqrt{-1+cx}\,\sqrt{1+cx}\,(4d+ex)+b^2cx(8d+ex)\right)-2bc\left(-2acx(2d+ex)+b\sqrt{-1+cx}\,\sqrt{1+cx}\,(4d+ex)\right)\cosh^{-1}(cx)+b^2(4c^2dx+e(-1+2c^2x^2))\cosh^{-1}(cx)^2-2abe\log\left(cx+\sqrt{-1+cx}\,\sqrt{1+cx}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) - 2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x)
+ b^2*c*x*(8*d + e*x)) - 2*b*c*(-2*a*c*x*(2*d + e*x) + b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*d + e*x))*ArcCosh[c*x] + b^2*(4*c^2*d*x + e*(-1 + 2*c^2*x^2))
*ArcCosh[c*x]^2 - 2*a*b*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(4*c^2)
```

Maple [A]

time = 4.45, size = 240, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{a^2\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\operatorname{arccosh}(cx)^2xc-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}+2cx\right)+\frac{e\left(-2\sqrt{cx-1}\,\operatorname{arccosh}(cx)\sqrt{}\right.}{}\right.}{c}}{}$
default	$\frac{\frac{a^2\left(c^2dx+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\operatorname{arccosh}(cx)^2xc-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}+2cx\right)+\frac{e\left(-2\sqrt{cx-1}\,\operatorname{arccosh}(cx)\sqrt{}\right.}{}\right.}{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2/c*(c^2*d*x+1/2*c^2*e*x^2)+b^2/c*(d*c*(arccosh(c*x)^2*x*c-2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x)+1/4*e*(-2*(c*x-1)^(1/2)*arccosh(c*
x)*(c*x+1)^(1/2)*x*c+2*arccosh(c*x)^2*x^2*c^2-arccosh(c*x)^2+c^2*x^2))+2*a*
b*arccosh(c*x)*d*c*x+a*b*c*arccosh(c*x)*e*x^2-2*a*b*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*d-1/2*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x-1/2*a*b/c*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d*x*arccosh(c*x)^2 + 1/2*a^2*x^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 - 1)*arc
cosh(c*x)/c) + a^2*d*x + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c
^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*a*b*e + 1/2*(x^2*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*integrate((c^3*x^4 + sqrt(c*x + 1)*sqrt(
c*x - 1)*c^2*x^3 - c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
 (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x))*b^2*e + 2*(c*x*arcco
sh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d/c
```

Fricas [A]
time = 0.43, size = 263, normalized size = 1.75

$$\frac{(2\,a^2+b^2)c^2x^2\cosh\left(1\right)+(2\,a^2+b^2)c^2x^2\sinh\left(1\right)+4\,(a^2+2\,b^2)c^2dx+(4\,b^2c^2dx+(2\,b^2c^2x^2-b^2)\cosh\left(1\right)+(2\,b^2c^2x^2-b^2)\sinh\left(1\right))\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\left(4\,abc^2dx+(2\,abc^2x^2-ab)\cosh\left(1\right)+(2\,abc^2x^2-ab)\sinh\left(1\right)-(b^2cx\cosh\left(1\right)+b^2cx\sinh\left(1\right)+4\,b^2cd)\sqrt{c^2x^2-1}\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-2\,(abcx\cosh\left(1\right)+abcx\sinh\left(1\right)+4\,abcd)\sqrt{c^2x^2-1}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*x^2*cosh(1) + (2*a^2 + b^2)*c^2*x^2*sinh(1) + 4*(a^2
 + 2*b^2)*c^2*d*x + (4*b^2*c^2*d*x + (2*b^2*c^2*x^2 - b^2)*cosh(1) + (2*b^2
*c^2*x^2 - b^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(4*a*b*c^2*d*x
+ (2*a*b*c^2*x^2 - a*b)*cosh(1) + (2*a*b*c^2*x^2 - a*b)*sinh(1) - (b^2*c*x*
cosh(1) + b^2*c*x*sinh(1) + 4*b^2*c*d)*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^
2*x^2 - 1)) - 2*(a*b*c*x*cosh(1) + a*b*c*x*sinh(1) + 4*a*b*c*d)*sqrt(c^2*x^
2 - 1))/c^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.20, size = 240, normalized size = 1.60

$$\begin{cases} a^2dx+\frac{a^2ex^2}{2}+2abdx\operatorname{acosh}\left(cx\right)+abex^2\operatorname{acosh}\left(cx\right)-\frac{2abd\sqrt{c^2x^2-1}}{c}-\frac{abex\sqrt{c^2x^2-1}}{2c}-\frac{abe\operatorname{acosh}\left(cx\right)}{2c^2}+b^2dx\operatorname{acosh}^2\left(cx\right)+2b^2dx+\frac{b^2ex^2\operatorname{acosh}^2\left(cx\right)}{2}+\frac{b^2ex^2}{4}-\frac{2b^2d\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{c}-\frac{b^2ex\sqrt{c^2x^2-1}\operatorname{acosh}\left(cx\right)}{2c}-\frac{b^2e\operatorname{acosh}^2\left(cx\right)}{4c^2} & \text{for}\: c\neq 0 \\ \left(a+\frac{i\pi b}{2}\right)^2\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*acosh(c*x) + a*b*e*x**2*aco
sh(c*x) - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - a*b*e*x*sqrt(c**2*x**2 - 1)/(2*c)
 - a*b*e*acosh(c*x)/(2*c**2) + b**2*d*x*acosh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*acosh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 - 1)*acosh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - b**2*e*acosh(c*x)*
*2/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**2/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\left(d+e\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x), x)
```

3.24 $\int \frac{(a+b\cosh^{-1}(cx))^2}{d+ex}\,dx$

Optimal. Leaf size=303

$$-\frac{(a+b\cosh^{-1}(cx))^3}{3be}+\frac{(a+b\cosh^{-1}(cx))^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{(a+b\cosh^{-1}(cx))^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

```
[Out] -1/3*(a+b*arccosh(c*x))^3/b/e+(a+b*arccosh(c*x))^2*ln(1+e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+(a+b*arccosh(c*x))^2*ln(1+e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+2*b*(a+b*arcc
osh(c*x))*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)
^(1/2)))/e+2*b*(a+b*arccosh(c*x))*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e-2*b^2*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e-2*b^2*polylog(3,-e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A]

time = 0.32, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5962, 5681, 2221, 2611, 2320, 6724}

$$\frac{2b(a+b\cosh^{-1}(cx))\,\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2b(a+b\cosh^{-1}(cx))\,\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{(a+b\cosh^{-1}(cx))^2\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{(a+b\cosh^{-1}(cx))^2\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-\frac{(a+b\cosh^{-1}(cx))^3}{3be}-\frac{2b^2\mathrm{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2b^2\mathrm{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] -1/3*(a + b*ArcCosh[c*x])^3/(b*e) + ((a + b*ArcCosh[c*x])^2*Log[1 + (e*E^Ar
cCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])^2*Log[1
 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (2*b*(a + b*ArcCosh
[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (
2*b*(a + b*ArcCosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^
2 - e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2
 - e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) -$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{(a+}{}$$

Mathematica [A]
time = 0.18, size = 285, normalized size = 0.94

$$\frac{-\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{b} + 3\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right) + 3\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right) + 6b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right) + 6b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right) - 6b^2\text{PolyLog}\left(3,\frac{ee^{\cosh^{-1}(cx)}}{-cd+\sqrt{c^2d^2-e^2}}\right) - 6b^2\text{PolyLog}\left(3,-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^3/b) + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])] + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*ArcCosh[c*x])*Pol
yLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*Arc
Cosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(3*e)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d),x)
[Out] int((a+b*arccosh(c*x))^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="maxima")
[Out] a^2*e^(-1)*log(x*e + d) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))^2/(x*e + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(x*e + d), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d),x)
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x), x)
```

3.25 $\int \frac{(a+b\cosh^{-1}(cx))^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=279

$$-\frac{(a+b\cosh^{-1}(cx))^2}{e(d+ex)}+\frac{2bc(a+b\cosh^{-1}(cx))\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2bc(a+b\cosh^{-1}(cx))\log\left(1+\right.}{e\sqrt{c^2d^2-e^2}}$$

```
[Out] -(a+b*arccosh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*b
*c*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^
2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*b^2*c*p
olylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/
(c^2*d^2-e^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5963, 5980, 3401, 2296, 2221, 2317, 2438}

$$\frac{2bc(a+b\cosh^{-1}(cx))\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2bc(a+b\cosh^{-1}(cx))\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{(a+b\cosh^{-1}(cx))^2}{e(d+ex)}+\frac{2b^2c\,\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c\,\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcCosh[c*x])*Log[1
 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2])
 - (2*b*c*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d
^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2
- e^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\text{Subst}\left(\int \frac{a+bx}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\text{Subst}\left(\int \frac{e^x(a+bx)}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx\right.}{\sqrt{c^2d^2-e^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \ldots$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \ldots$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.76, size = 950, normalized size = 3.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a^2/(d + e*x) + 2*a*b*c*(ArcCosh[c*x]/(c*d + c*e*x) + (2*ArcTan[(Sqrt[c*
d - e]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[-(c*d) - e]])/(Sqrt[-(c*d) - e]*Sqr
t[c*d - e])) + b^2*c*(ArcCosh[c*x]^2/(c*d + c*e*x) + (2*(2*ArcCosh[c*x]*Arc
Tan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos
[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e
^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqr
t[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x
]/2)*Sqrt[c*(d + e*x)])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[
```

ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]])*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*(d + e*x)])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*d^2) + e^2]))/e)

Maple [**A**]
time = 9.34, size = 556, normalized size = 1.99

method	result
derivativedivides	$-\frac{a^2c^2}{(ecx+cd)e}-\frac{b^2c^2\operatorname{arccosh}(cx)^2}{e(ecx+cd)}+\frac{2b^2c^2\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c^2\operatorname{arc}}{}$
default	$-\frac{a^2c^2}{(ecx+cd)e}-\frac{b^2c^2\operatorname{arccosh}(cx)^2}{e(ecx+cd)}+\frac{2b^2c^2\operatorname{arccosh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c^2\operatorname{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a^2*c^2/(c*e*x+c*d)/e-b^2*c^2*arccosh(c*x)^2/e/(c*e*x+c*d)+2*b^2*c^2/
e*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2*b^2*c^2/e*arccosh(c
*x)/(c^2*d^2-e^2)^(1/2)*ln((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^
2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+2*b^2*c^2/e/(c^2*d^2-e^2)^(1/2)*di
log((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c
^2*d^2-e^2)^(1/2)))-2*b^2*c^2/e/(c^2*d^2-e^2)^(1/2)*dilog((c*d+e*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))-2*a
*b*c^2/(c*e*x+c*d)/e*arccosh(c*x)-2*a*b*c^2/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)
```

```
*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d
))/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(x^2*e^2 + 2*d*x*e
 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x)^2,x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x)^2, x)
```

3.26 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=380

$$-\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cos}}{cd-\sqrt{c^2}}\right.}{e\left(c^2d^2-e^2\right)^{3/2}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2-e^2)+b*c
^3*d*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*
d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b*c^3*d*(a+b*arccosh(c*x))*ln(1+e*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^
(3/2)+b^2*c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^
2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b*c*(c*
x+1)*(a+b*arccosh(c*x))*((c*x-1)/(c*x+1))^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A]
time = 0.54, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5963, 5980, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{bc\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{b^2c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}+\frac{b^2c^3d\,\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{b^2c^3d\,\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*(a + b*ArcCosh[c*x]))/((c^2*d
^2 - e^2)*(d + e*x))) - (a + b*ArcCosh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d
*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]
)])/(e*(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*(a + b*ArcCosh[c*x])*Log[1 + (e*E^
ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (b^
2*c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2)) - (b^2*c
^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*(c^2
*d^2 - e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc^2)\,\text{Subst}\left(\int \frac{a+bx}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(b^2c^2)\,\text{Sub}}{}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(b^2c^2)\,\text{Sub}}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{b^2c^2\log(d}{e\,(c^2d^2-}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\big(a+}{}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\big(a+}{}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{(c^2d^2-e^2)\,(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\big(a+}{}$$

<u>**Mathematica**</u> [**C**] Result contains complex when optimal does not.
time = 5.64, size = 1099, normalized size = 2.89

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -1/2*a^2/(e*(d + e*x)^2) + a*b*c^2*(-((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c*d
- e)*(c*d + e)*(c*d + c*e*x))) - ArcCosh[c*x]/(e*(c*d + c*e*x)^2) + (2*c*d*
ArcTan[(Sqrt[c*d - e]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[-(c*d) - e]])/((-(c*
d) - e)^(3/2)*(c*d - e)^(3/2)*e)) + b^2*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)*ArcCosh[c*x])/((c*d - e)*(c*d + e)*(c*d + c*e*x))) - ArcCosh[c*x]
```

```
^2/(2*e*(c*d + c*e*x)^2) + Log[1 + (e*x)/d]/(c^2*d^2*e - e^3) + (c*d*(2*Arc
Cosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] -
 (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-
(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E
^(ArcCosh[c*x]/2)*Sqrt[c*(d + e*x)])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c
*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e
)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^
2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*(d + e*x)])] - (ArcCos[-((c*
d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2
]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[
c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] -
(ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c
^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1
+ Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCos
h[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I
*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d
^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) +
 e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d +
 e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))])))/(e*(-(c^2*d^2) + e
^2)^(3/2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(410) = 820$.
time = 9.19, size = 1174, normalized size = 3.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e-1/2*b^2*c^5*arccosh(c*x)^2/e/(c*e*x+c*d)^
2/(c^2*d^2-e^2)*d^2-b^2*c^4*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2-e^2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*d-b^2*c^4*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*x+b^2*c^5*arccosh(c*x)/e/(c*e*x+c*d)^2/(c^2*d^2
-e^2)*d^2+2*b^2*c^5*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d*x+b^2*c^5*ar
ccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*x^2+1/2*b^2*c^3*arccosh(c*x)^2*e/(
c*e*x+c*d)^2/(c^2*d^2-e^2)+b^2*c^4/e/(c^2*d^2-e^2)^(3/2)*arccosh(c*x)*ln((-
c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2
-e^2)^(1/2)))*d-b^2*c^4/e/(c^2*d^2-e^2)^(3/2)*arccosh(c*x)*ln((c*d+e*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))
*d+b^2*c^4/e/(c^2*d^2-e^2)^(3/2)*dilog((-c*d-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-b^2*c^4/e/(c^2*d^2
-e^2)^(3/2)*dilog((c*d+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+(c^2*d^2-e^2)^(1
/2))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+b^2*c^3/e/(c^2*d^2-e^2)*ln(2*d*c*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2+e)-2*b^2*c^
3/e/(c^2*d^2-e^2)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-a*b*c^3/(c*e*x+c*d)^2
```

```
/e*arccosh(c*x)-a*b*c^5/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(
c*d-e)/(c*d+e)/((c^2*d^2-e^2)/e^2)^(1/2)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^
2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2-a*b*c^5/e*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d-e)/(c*d+e)/((c^2*d^2-e^2)/e^2)
^(1/2)/(c*e*x+c*d)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/
2)*e+e)/(c*e*x+c*d))*d*x-a*b*c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c*d-e)/(c*d+e
)/(c*e*x+c*d))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e>0)', see `assume?` for more
details
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(x^3*e^3 + 3*d*x^2
*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\left(d + e x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))^2}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x)^3, x)
```

3.27 $\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=394

$$-\frac{d^3\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}-\frac{3de^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3}-\frac{3d^2e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right.}{2bc^2}$$

```
[Out] d^3*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+3/4*d*e^2*cosh(a/b)*Shi(a/b+arccosh
(c*x))/b/c^3+3/2*d^2*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^2+1/4*e^3*
cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^4+3/4*d*e^2*cosh(3*a/b)*Shi(3*a/b
+3*arccosh(c*x))/b/c^3+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arccosh(c*x))/b/c^4-
d^3*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arccosh(c*x))*sin
h(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arccosh(c*x))*sinh(2*a/b)/b/c^2-1/4*e^3*
Chi(2*a/b+2*arccosh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Chi(3*a/b+3*arccosh(c
*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arccosh(c*x))*sinh(4*a/b)/b/c^4
```

Rubi [A]
time = 0.88, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5965, 6874, 3384, 3379, 3382, 5556, 12}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{4bc^4}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^4}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{4bc^4}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^4}-\frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{3d^2e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^3*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (3*d*e^2*CoshIn
tegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*
a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (e^3*CoshIntegral[(2*a)/b
 + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (3*d*e^2*CoshIntegral[(3*a)/b
 + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) - (e^3*CoshIntegral[(4*a)/b + 4
*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) + (d^3*Cosh[a/b]*SinhIntegral[a/b +
 ArcCosh[c*x]])/(b*c) + (3*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(2*b*c^2) + (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(4
*b*c^4) + (3*d*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4
*b*c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c
^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5965

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]*(c*d + e*Cosh[x
])^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))^3\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\sinh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh^2(x)\sinh(x)}{a+bx} + \frac{e^3\cosh^3(x)\sinh(x)}{a+bx}\right)dx,\right.}{c^4}$$

$$= \frac{d^3\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{c^2}$$

$$= \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{(3de^2)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right)dx\right.}{c^3}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)}{bc} + \frac{(3d^2e)\,\text{Sub}}{}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)}{bc} + \frac{(3de^2\cosh}{}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{3de^2\text{Chi}\left(\frac{a}{b} + \cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{3d^2e\text{Chi}}{}$$

Mathematica [A]
time = 0.42, size = 287, normalized size = 0.73

$$\frac{-2cd(4c^2d^2+3e^2)\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) - 2e(6c^2d^2+e^2)\,\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right) - 6cde^2\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - e^3\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right) + 8c^3d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 6cde^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 12c^2d^2e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 2e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 6cde^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{8bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*(4*c^2*d^2 + 3*e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 2*
e*(6*c^2*d^2 + e^2)*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - 6*
c*d*e^2*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - e^3*CoshIntegr
al[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + 8*c^3*d^3*Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c*x]] + 6*c*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
 + 12*c^2*d^2*e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*e^3*
Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*c*d*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^3*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])])/(8*b*c^4)
```

Maple [A]
time = 10.83, size = 394, normalized size = 1.00

method	result

derivativedivides	$-\frac{e^{3} \mathrm{e}^{-\frac{4a}{b}} \operatorname{expIntegral}\left(1,-4 \operatorname{arccosh}(cx)-\frac{4a}{b}\right)}{16 c^{3} b}+\frac{e^{3} \mathrm{e}^{\frac{4a}{b}} \operatorname{expIntegral}\left(1,4 \operatorname{arccosh}(cx)+\frac{4a}{b}\right)}{16 c^{3} b}+\frac{3 e\, \mathrm{e}^{\frac{2a}{b}} \operatorname{expIntegral}\left(1,2 \operatorname{arccosh}(cx)+\frac{2a}{b}\right.}{4 c b}$
default	$-\frac{e^{3} \mathrm{e}^{-\frac{4a}{b}} \operatorname{expIntegral}\left(1,-4 \operatorname{arccosh}(cx)-\frac{4a}{b}\right)}{16 c^{3} b}+\frac{e^{3} \mathrm{e}^{\frac{4a}{b}} \operatorname{expIntegral}\left(1,4 \operatorname{arccosh}(cx)+\frac{4a}{b}\right)}{16 c^{3} b}+\frac{3 e\, \mathrm{e}^{\frac{2a}{b}} \operatorname{expIntegral}\left(1,2 \operatorname{arccosh}(cx)+\frac{2a}{b}\right.}{4 c b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*arccosh(c*x)+4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arccosh(
c*x)+2*a/b)*d^2+1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)*d^2-1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arccosh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-
3*a/b)+1/2*d^3/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+3/8/c^2*d/b*exp(a/b)*Ei(1,
arccosh(c*x)+a/b)*e^2-1/2*d^3/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-3/8/c^2*d
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2+3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arccosh(c*x)+3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{3}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x)^3/(a + b*acosh(c*x)), x)
```

3.28 $\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}-\frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3}-\frac{de\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2}-$$

```
[Out] d^2*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+1/4*e^2*cosh(a/b)*Shi(a/b+arccosh(c
*x))/b/c^3+d*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^2+1/4*e^2*cosh(3*a
/b)*Shi(3*a/b+3*arccosh(c*x))/b/c^3-d^2*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c
-1/4*e^2*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arccosh(c*x)
)*sinh(2*a/b)/b/c^2-1/4*e^2*Chi(3*a/b+3*arccosh(c*x))*sinh(3*a/b)/b/c^3
```

Rubi [A]

time = 0.52, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5965, 6874, 3384, 3379, 3382, 5556}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{bc^2}+\frac{de\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{bc^2}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^2*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e^2*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2
*ArcCosh[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (e^2*CoshIntegral[(3*a)/b + 3*ArcCo
sh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (d^2*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(b*c) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^3
) + (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b*c^2) + (e
^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5965

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]*(c*d + e*Cosh[x
])^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))^2\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\sinh(x)}{a+bx}+\frac{e^2\cosh^2(x)\sinh(x)}{a+bx}+\frac{cde\sinh(2x)}{a+bx}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{(de)\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{e}{}$$

$$= \frac{e^2\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+\right.}{a+bx}\right.}{c}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh}{}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh}{}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{de\text{Chi}\left(\frac{2a}{b}\right.}{}$$

Mathematica [A]
time = 0.24, size = 187, normalized size = 0.76

$$\frac{-\left(\left(4c^2d^2+e^2\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)\right)-4cde\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-e^2\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right)+4c^2d^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4cde\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d^2 + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - 4*c*d*e*
CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - e^2*CoshIntegral[3*(a/
b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d^2*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 4*c*d*e*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + e^2*Cosh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcCosh[c*x])])/(4*b*c^3)
```

Maple [A]
time = 9.18, size = 254, normalized size = 1.04

method	result
derivativedivides	$-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \operatorname{expIntegral}\left(1,-3\operatorname{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2b}+\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)d^2}{2b}+$
default	$-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \operatorname{expIntegral}\left(1,-3\operatorname{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2b}+\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \operatorname{expIntegral}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{\mathrm{e}^{\frac{a}{b}} \operatorname{expIntegral}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)d^2}{2b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
cosh(c*x)-a/b)*d^2-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
rccosh(c*x)+2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x)),x)
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{a+b\operatorname{acosh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*acosh(c*x)),x)
[Out] int((d + e*x)^2/(a + b*acosh(c*x)), x)
```

3.29 $\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$-\frac{d\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}-\frac{e\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{e\ c}{}$$

```
[Out] d*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh
(c*x))/b/c^2-d*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arccos
h(c*x))*sinh(2*a/b)/b/c^2
```

Rubi [A]

time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5965, 6874, 3384, 3379, 3382, 5556, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e*CoshIntegral[(
2*a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (d*Cosh[a/b]*SinhIntegr
al[a/b + ArcCosh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*A
rcCosh[c*x]])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5965

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Sinh[x]*(c*d + e*Cosh[x
])^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{cd\sinh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)\,dx,x,\cosh^{-1}(cx)\right)}{c^2}\\
&= \frac{d\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c}+\frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^2}\\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\cosh^{-1}(cx)\right)}{c^2}+\frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}\right.}{c}\\
&= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{e\text{Subst}\left(\int \right.}{}\\
&= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}+\frac{\left(e\cosh\left(\frac{2a}{b}\right)\right.}{}\\
&= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc}-\frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 98, normalized size = 0.84

$$\frac{-2cd\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-e\,\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)+2cd\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - e*CoshIntegral[2*(a/b
+ ArcCosh[c*x])]*Sinh[(2*a)/b] + 2*c*d*Cosh[a/b]*SinhIntegral[a/b + ArcCosh
[c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])])/(2*b*c^2)
```

Maple [A]
time = 8.14, size = 120, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{d\,e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b}-\frac{d\,e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b}-\frac{e\,e^{-\frac{2a}{b}}\,\text{expIntegral}\left(1,-2\,\text{arccosh}(cx)-\frac{2a}{b}\right)}{4cb}+\frac{e\,e^{\frac{2a}{b}}\cdots}{}}{c}$
default	$\frac{\frac{d\,e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b}-\frac{d\,e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b}-\frac{e\,e^{-\frac{2a}{b}}\,\text{expIntegral}\left(1,-2\,\text{arccosh}(cx)-\frac{2a}{b}\right)}{4cb}+\frac{e\,e^{\frac{2a}{b}}\cdots}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*d/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2*d/b*exp(-a/b)*Ei(1,-arccos
h(c*x)-a/b)-1/4*e/c/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/4*e/c/b*exp
(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(b*arccosh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)/(b*arccosh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{a + b\operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x)),x)
[Out] Integral((d + e*x)/(a + b*acosh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + e\,x}{a + b\operatorname{acosh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*acosh(c*x)),x)
[Out] int((d + e*x)/(a + b*acosh(c*x)), x)
```

3.30 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccosh(c*x) + a)*(x*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x*e + a*d + (b*x*e + b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx))(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\operatorname{acosh}(c\,x))\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x)), x)
```

3.31 $\int \frac{1}{(d+ex)^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2\left(a+b\,\mathrm{arccosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccosh(c*x) + a)*(x*e + d)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x^2*e^2 + 2*a*d*x*e + a*d^2 + (b*x^2*e^2 + 2*b*d*x*e + b*d^2)
*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(d+ex\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d + e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x)^2),x)
```

```
[Out] int(1/((a + b*acosh(c*x))*(d + e*x)^2), x)
```

3.32 $\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=374

$$-\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-}}{b}\right.}{b^2c}$$

```
[Out] d^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arccosh(c*x)
)/b)*cosh(a/b)/b^2/c^3+2*d*e*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)/b^2/c^
2+3/4*e^2*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3-d^2*Shi((a+b*arcc
osh(c*x))/b)*sinh(a/b)/b^2/c-1/4*e^2*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^
2/c^3-2*d*e*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Shi(3*(
a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/
c/(a+b*arccosh(c*x))-2*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c
*x))-e^2*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.52, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5964, 5880, 5953, 3384, 3379, 3382, 5885}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\operatorname{Shi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3}+\frac{2de\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^2}-\frac{2de\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^2}+\frac{d^2\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshInt
egral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e^2*Cosh[a/b]*CoshIntegral[(a + b
*ArcCosh[c*x])/b])/(4*b^2*c^3) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a +
b*ArcCosh[c*x]))/b])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a +
b*ArcCosh[c*x]))/b])/(4*b^2*c^3) - (d^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c*x])/b])/(b^2*c) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])
/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x]))/b
])/(b^2*c^2) - (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c*x]))/b
])/(4*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5964

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\cosh^{-1}(cx)\right)^2} + \frac{2dex}{\left(a+b\cosh^{-1}(cx)\right)^2} + \frac{e^2x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + (2de)\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + e^2\int \frac{}{\left(a+b\right.}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+}}{bc\left(a+b\cosh^{-1}(cx\right.}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+}}{bc\left(a+b\cosh^{-1}(cx\right.}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+}}{bc\left(a+b\cosh^{-1}(cx\right.}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+}}{bc\left(a+b\cosh^{-1}(cx\right.}$$

Mathematica [**A**]
time = 1.05, size = 530, normalized size = 1.42

$$-\frac{4bc^2d^2\sqrt{\frac{-1+cx}{1+cx}} + 4bc^3d^2x\sqrt{\frac{-1+cx}{1+cx}} + 8bc^2dex\sqrt{\frac{-1+cx}{1+cx}} + 8bc^3dex^2\sqrt{\frac{-1+cx}{1+cx}} + 4bc^2e^2x^2\sqrt{\frac{-1+cx}{1+cx}} + 4bc^3e^2x^3\sqrt{\frac{-1+cx}{1+cx}} - (4c^2d^2+e^2)\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - 8cde\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - 3ae^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) - 3be^2\cosh^{-1}(cx)\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 4ac^2d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + ae^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 4bc^2d^2\cosh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + be^2\cosh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 8acde\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 8bcde\cosh^{-1}(cx)\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 3ae^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 3be^2\cosh^{-1}(cx)\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{4b^2c^3\left(a+b\cosh^{-1}(cx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/4*(4*b*c^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*d^2*x*Sqrt[(-1 + c*x
)/(1 + c*x)] + 8*b*c^2*d*e*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b*c^3*d*e*x^2*S
qrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^2*e^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*
b*c^3*e^2*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] - (4*c^2*d^2 + e^2)*(a + b*ArcCosh
[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] - 8*c*d*e*(a + b*ArcCosh[
c*x])*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] - 3*a*e^2*Cosh[(3*
a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*b*e^2*ArcCosh[c*x]*Cosh[(3*a
)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + 4*a*c^2*d^2*Sinh[a/b]*SinhInteg
ral[a/b + ArcCosh[c*x]] + a*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
+ 4*b*c^2*d^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*e
^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*a*c*d*e*Sinh
[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 8*b*c*d*e*ArcCosh[c*x]*Sin
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 3*a*e^2*Sinh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*e^2*ArcCosh[c*x]*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcCosh[c*x])])/(b^2*c^3*(a + b*ArcCosh[c*x]))
```

Maple [**A**]
time = 10.00, size = 649, normalized size = 1.74

method	result
derivativedivides	$\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e^2}{8c^2b(a+b\,\mathrm{arccosh}(cx))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-$
default	$\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e^2}{8c^2b(a+b\,\mathrm{arccosh}(cx))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e^2/c^2/b/(a+b*arccosh(c*x))-3/8*e^2/c^2/b^2*exp(3*a/b)*
Ei(1,3*arccosh(c*x)+3*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8/b^
2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+
1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*
x)+a/b)+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^2/b/(a+b*arccosh(c*x))
-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b)-1/8/c^2/b*e^2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-
1/8/c^2/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)+1/2*(-2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+2*c^2*x^2-1)*d*e/c/b/(a+b*arccosh(c*x))-e*d/c/b^2*exp(2*a/b)
*Ei(1,2*arccosh(c*x)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2-1+2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c)/(a+b*arccosh(c*x))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arccosh(c*x
)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^5*e^2 + 2*c^3*d*x^4*e - 2*c*d*x^2*e - c*d^2*x + (c^3*d^2 - c*e^2)*x
^3 + (c^2*x^4*e^2 + 2*c^2*d*x^3*e + (c^2*d^2 - e^2)*x^2 - 2*d*x*e - d^2)*sq
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c
^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*x^6*e^2 + 4*c^
5*d*x^5*e - 8*c^3*d*x^3*e + (c^5*d^2 - 6*c^3*e^2)*x^4 + 4*c*d*x*e + (3*c^3*
x^4*e^2 + 4*c^3*d*x^3*e + c*d^2 + (c^3*d^2 - c*e^2)*x^2)*(c*x + 1)*(c*x - 1
) + c*d^2 - (2*c^3*d^2 - 3*c*e^2)*x^2 + (6*c^4*x^5*e^2 + 8*c^4*d*x^4*e - 8*
c^2*d*x^2*e + (2*c^4*d^2 - 7*c^2*e^2)*x^3 - (c^2*d^2 - 2*e^2)*x + 2*d*e)*sq
```

```
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 -
 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^
2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x)
 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x))**2,x)
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{(a+b\operatorname{acosh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*acosh(c*x))^2,x)
[Out] int((d + e*x)^2/(a + b*acosh(c*x))^2, x)
```

3.33 $\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=190

$$-\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-}}{b}\right.}{b^2c^2}$$

```
[Out] d*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+e*Chi(2*(a+b*arccosh(c*x))/b)*c
osh(2*a/b)/b^2/c^2-d*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a+b
*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+
b*arccosh(c*x))-e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A]
time = 0.32, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5964, 5880, 5953, 3384, 3379, 3382, 5885}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshInte
gral[(a + b*ArcCosh[c*x])/b])/(b^2*c) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a
 + b*ArcCosh[c*x]))/b])/(b^2*c^2) - (d*Sinh[a/b]*SinhIntegral[(a + b*ArcCos
h[c*x])/b])/(b^2*c) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c*x])
)/b])/(b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5964

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx \\
&= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + e\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{(cd)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{b} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cos\right.}{bc} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cos\right.}{b^2c^2} \\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-}\right.}{b^2c}
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 268, normalized size = 1.41

$$
-\frac{bcd\sqrt{\frac{-1+cx}{1+cx}}+bc^2dx\sqrt{\frac{-1+cx}{1+cx}}+bcex\sqrt{\frac{-1+cx}{1+cx}}+bc^2ex^2\sqrt{\frac{-1+cx}{1+cx}}-cd\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\left(a+b\cosh^{-1}(cx)\right)\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+acd\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+bcd\cosh^{-1}(cx)\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+ae\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+be\cosh^{-1}(cx)\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{b^2c^2\left(a+b\cosh^{-1}(cx)\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((b*c*d*Sqrt[(-1 + c*x)/(1 + c*x)] + b*c^2*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]
+ b*c*e*x*Sqrt[(-1 + c*x)/(1 + c*x)] + b*c^2*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)] - c*d*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] -
e*(a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] +
 a*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*c*d*ArcCosh[c*x]*Sinh
[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + a*e*Sinh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcCosh[c*x])] + b*e*ArcCosh[c*x]*Sinh[(2*a)/b]*SinhIntegral[2*(a/b +
 ArcCosh[c*x])])/(b^2*c^2*(a + b*ArcCosh[c*x])))
```

Maple [A]
time = 8.83, size = 285, normalized size = 1.50

method	result
derivativedivides	$\frac{\frac{\left(-\sqrt{cx-1}\,\sqrt{cx+1}+cx\right)d}{2b(a+b\,\text{arccosh}(cx))}-\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)d}{2b(a+b\,\text{arccosh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{expInteg}}{}}{}$

default	$\frac{\left(-\sqrt{cx-1}\,\sqrt{cx+1}+cx\right)d}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)d}{2b(a+b\,\mathrm{arccosh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expInt}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d/b/(a+b*arccosh(c*x))-1/2*d/b^
2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
a+b*arccosh(c*x))*d-1/2/b^2*d*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)+1/4*(-2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c+2*c^2*x^2-1)*e/c/(a+b*arccosh(c*x))/b-1/2*e/c/
b^2*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-1/4*e/c/b*(2*c^2*x^2-1+2*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c)/(a+b*arccosh(c*x))-1/2*e/c/b^2*exp(-2*a/b)*Ei(1,-2
*arccosh(c*x)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^4*e + c^3*d*x^3 - c*x^2*e - c*d*x + (c^2*x^3*e + c^2*d*x^2 - x*e -
d)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*
a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x -
b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((2*c^5*x^5*e + c
^5*d*x^4 - 4*c^3*x^3*e - 2*c^3*d*x^2 + (2*c^3*x^3*e + c^3*d*x^2 + c*d)*(c*x
 + 1)*(c*x - 1) + 2*c*x*e + (4*c^4*x^4*e + 2*c^4*d*x^3 - 4*c^2*x^2*e - c^2*
d*x + e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*(c*x -
 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2
*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{(a + b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*acosh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + e\,x}{(a + b\operatorname{acosh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x)/(a + b*acosh(c*x))^2, x)
```

3.34 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 6.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^3*e
 + a*b*c^3*d*x^2 - a*b*c*x*e - a*b*c*d + (a*b*c^2*x^2*e + a*b*c^2*d*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) + (b^2*c^3*x^3*e + b^2*c^3*d*x^2 - b^2*c*x*e - b^2*
c*d + (b^2*c^2*x^2*e + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (c^3*d
*x^2 + 2*c*x*e + c*d)*(c*x + 1)*(c*x - 1) + (2*c^4*d*x^3 + 2*c^2*x^2*e - c^
2*d*x - e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^6*e^2 + 2*a*b*c^5*
d*x^5*e - 4*a*b*c^3*d*x^3*e + 2*a*b*c*d*x*e + a*b*c*d^2 + (a*b*c^5*d^2 - 2*
a*b*c^3*e^2)*x^4 + (a*b*c^3*x^4*e^2 + 2*a*b*c^3*d*x^3*e + a*b*c^3*d^2*x^2)*
(c*x + 1)*(c*x - 1) - (2*a*b*c^3*d^2 - a*b*c*e^2)*x^2 + 2*(a*b*c^4*x^5*e^2
+ 2*a*b*c^4*d*x^4*e - 2*a*b*c^2*d*x^2*e - a*b*c^2*d^2*x + (a*b*c^4*d^2 - a*
b*c^2*e^2)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6*e^2 + 2*b^2*c^5*
d*x^5*e - 4*b^2*c^3*d*x^3*e + 2*b^2*c*d*x*e + b^2*c*d^2 + (b^2*c^5*d^2 - 2*
b^2*c^3*e^2)*x^4 + (b^2*c^3*x^4*e^2 + 2*b^2*c^3*d*x^3*e + b^2*c^3*d^2*x^2)*
(c*x + 1)*(c*x - 1) - (2*b^2*c^3*d^2 - b^2*c*e^2)*x^2 + 2*(b^2*c^4*x^5*e^2
+ 2*b^2*c^4*d*x^4*e - 2*b^2*c^2*d*x^2*e - b^2*c^2*d^2*x + (b^2*c^4*d^2 - b^
2*c^2*e^2)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x*e + a^2*d + (b^2*x*e + b^2*d)*arccosh(c*x)^2 + 2*(a*b*x*e
 + a*b*d)*arccosh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x)), x)
```

3.35 $\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 142.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2\,(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^4*e
^2 + 2*a*b*c^3*d*x^3*e - 2*a*b*c*d*x*e - a*b*c*d^2 + (a*b*c^3*d^2 - a*b*c*e
^2)*x^2 + (a*b*c^2*x^3*e^2 + 2*a*b*c^2*d*x^2*e + a*b*c^2*d^2*x)*sqrt(c*x +
1)*sqrt(c*x - 1) + (b^2*c^3*x^4*e^2 + 2*b^2*c^3*d*x^3*e - 2*b^2*c*d*x*e - b
^2*c*d^2 + (b^2*c^3*d^2 - b^2*c*e^2)*x^2 + (b^2*c^2*x^3*e^2 + 2*b^2*c^2*d*x
^2*e + b^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))) - integrate((c^5*x^5*e - c^5*d*x^4 - 2*c^3*x^3*e + 2*c^3*d*
x^2 + (c^3*x^3*e - c^3*d*x^2 - 3*c*x*e - c*d)*(c*x + 1)*(c*x - 1) + c*x*e +
 (2*c^4*x^4*e - 2*c^4*d*x^3 - 5*c^2*x^2*e + c^2*d*x + 2*e)*sqrt(c*x + 1)*sq
rt(c*x - 1) - c*d)/(a*b*c^5*x^7*e^3 + 3*a*b*c^5*d*x^6*e^2 + 3*a*b*c*d^2*x*e
 + a*b*c*d^3 + (3*a*b*c^5*d^2*e - 2*a*b*c^3*e^3)*x^5 + (a*b*c^5*d^3 - 6*a*b
*c^3*d*e^2)*x^4 - (6*a*b*c^3*d^2*e - a*b*c*e^3)*x^3 + (a*b*c^3*x^5*e^3 + 3*
a*b*c^3*d*x^4*e^2 + 3*a*b*c^3*d^2*x^3*e + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x -
 1) - (2*a*b*c^3*d^3 - 3*a*b*c*d*e^2)*x^2 + 2*(a*b*c^4*x^6*e^3 + 3*a*b*c^4*
d*x^5*e^2 - 3*a*b*c^2*d^2*x^2*e - a*b*c^2*d^3*x + (3*a*b*c^4*d^2*e - a*b*c^
2*e^3)*x^4 + (a*b*c^4*d^3 - 3*a*b*c^2*d*e^2)*x^3)*sqrt(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^7*e^3 + 3*b^2*c^5*d*x^6*e^2 + 3*b^2*c*d^2*x*e + b^2*c*d^3 +
 (3*b^2*c^5*d^2*e - 2*b^2*c^3*e^3)*x^5 + (b^2*c^5*d^3 - 6*b^2*c^3*d*e^2)*x^
4 - (6*b^2*c^3*d^2*e - b^2*c*e^3)*x^3 + (b^2*c^3*x^5*e^3 + 3*b^2*c^3*d*x^4*
e^2 + 3*b^2*c^3*d^2*x^3*e + b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) - (2*b^2*c
^3*d^3 - 3*b^2*c*d*e^2)*x^2 + 2*(b^2*c^4*x^6*e^3 + 3*b^2*c^4*d*x^5*e^2 - 3*
b^2*c^2*d^2*x^2*e - b^2*c^2*d^3*x + (3*b^2*c^4*d^2*e - b^2*c^2*e^3)*x^4 + (
b^2*c^4*d^3 - 3*b^2*c^2*d*e^2)*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^2*e^2 + 2*a^2*d*x*e + a^2*d^2 + (b^2*x^2*e^2 + 2*b^2*d*x*
e + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*x^2*e^2 + 2*a*b*d*x*e + a*b*d^2)*arcco
sh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(cx\right)\right)^{2} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x)^2),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x)^2), x)
```

3.36 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^3}{e(1+m)}-\frac{3bc\mathrm{Int}\left(\frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}},x\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))^3/e/(1+m)-3*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arccosh(c*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/e/(1+m)
```

Rubi [A]
time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^3)/(e*(1 + m)) - (3*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^3}{e(1+m)}-\frac{(3bc)\int \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} \, dx}{e(1+m)}$$

Mathematica [A]
time = 9.37, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m (a+b\,\mathrm{arccosh}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a^3*e^(-1)/(m + 1) + (b^3*x*e + b^3*d)*e^(m*log(x*e + d)
- 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^3/(m + 1) + integrate(-3*(((b^3
*c^2*d*x + a*b^2*(m + 1)*e - (a*b^2*c^2*(m + 1) - b^3*c^2)*x^2*e)*sqrt(c*x
+ 1)*sqrt(c*x - 1)*(x*e + d)^m + (b^3*c^3*d*x^2 - b^3*c*d - (a*b^2*c^3*(m +
 1) - b^3*c^3)*x^3*e + (a*b^2*c*(m + 1) - b^3*c)*x*e)*(x*e + d)^m)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))^2 - ((a^2*b*c^2*(m + 1)*x^2*e - a^2*b*(m + 1
)*e)*sqrt(c*x + 1)*sqrt(c*x - 1)*(x*e + d)^m + (a^2*b*c^3*(m + 1)*x^3*e - a
^2*b*c*(m + 1)*x*e)*(x*e + d)^m)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^3*(m + 1)*x^3*e - c*(m + 1)*x*e + (c^2*(m + 1)*x^2*e - (m + 1)*e)*sqrt(c*x
 + 1)*sqrt(c*x - 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*x
) + a^3)*(x*e + d)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{acosh}\,(cx))^3 \, (d+ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**3*(d + e*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(e*x + d)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))^3\,(d + e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^3*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^3*(d + e*x)^m, x)
```

3.37 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=80

$$\frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{e(1+m)}-\frac{2bc\text{Int}\left(\frac{(d+ex)^{1+m}(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\sqrt{1+cx}},x\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/e/(1+m)
```

Rubi [A]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
)/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}(a+b\cosh^{-1}(cx))}{\sqrt{-1+cx}\sqrt{1+cx}} \, dx}{e(1+m)}$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m (a+b\,\mathrm{arccosh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a^2*e^(-1)/(m + 1) + (b^2*x*e + b^2*d)*e^(m*log(x*e + d)
- 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(m + 1) + integrate(-2*((b^2*
c^2*d*x + a*b*(m + 1)*e - (a*b*c^2*(m + 1) - b^2*c^2)*x^2*e)*sqrt(c*x + 1)*
sqrt(c*x - 1)*(x*e + d)^m + (b^2*c^3*d*x^2 - (a*b*c^3*(m + 1) - b^2*c^3)*x^
3*e - b^2*c*d + (a*b*c*(m + 1) - b^2*c)*x*e)*(x*e + d)^m)*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/(c^3*(m + 1)*x^3*e - c*(m + 1)*x*e + (c^2*(m + 1)*x^2
*e - (m + 1)*e)*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*(x*e + d)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{acosh}\,(cx))^2\,(d+ex)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*(d + e*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2*(e*x + d)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))^2\,(d + e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^m,x)
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^m, x)
```

3.38 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{2}\,b(cd+e)\sqrt{-1+cx}\,(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(1+m)}+\frac{(d+ex)^{1+m}\,(a+}{e(1+}$$

[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))/e/(1+m)-b*(c*d+e)*(e*x+d)^m*AppellF1(1/2,-1-m,1/2,3/2,e*(-c*x+1)/(c*d+e),-1/2*c*x+1/2)*2^(1/2)*(c*x-1)^(1/2)/c/e/(1+m)/((c*(e*x+d)/(c*d+e))^m)

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5963, 144, 143}

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{e(m+1)}-\frac{\sqrt{2}\,b\sqrt{cx-1}\,(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((Sqrt[2]*b*(c*d + e)*Sqrt[-1 + c*x]*(d + e*x)^m*AppellF1[1/2, 1/2, -1 - m
, 3/2, (1 - c*x)/2, (e*(1 - c*x))/(c*d + e)])/(c*e*(1 + m)*((c*(d + e*x))/(
c*d + e))^m)) + ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(e*(1 + m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(b(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\right)}{e(1+m)}$$

$$= -\frac{\sqrt{2}\,b(cd+e)\sqrt{-1+cx}\,(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m} F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}\right.}{ce(1+m)}$$

Mathematica [A]
time = 0.16, size = 177, normalized size = 1.42

$$\frac{(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\left(-2be\sqrt{-2+2cx}\,F_1\left(\frac{1}{2};-\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}-\frac{cx}{2},\frac{e-cex}{cd+e}\right)+b(-cd+e)\sqrt{-2+2cx}\,F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}-\frac{cx}{2},\frac{e-cex}{cd+e}\right)+c(d+ex)\left(\frac{c(d+ex)}{cd+e}\right)^m\left(a+b\cosh^{-1}(cx)\right)\right)}{ce(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] ((d + e*x)^m*(-2*b*e*Sqrt[-2 + 2*c*x]*AppellF1[1/2, -1/2, -m, 3/2, 1/2 - (c
*x)/2, (e - c*e*x)/(c*d + e)] + b*(-(c*d) + e)*Sqrt[-2 + 2*c*x]*AppellF1[1/
2, 1/2, -m, 3/2, 1/2 - (c*x)/2, (e - c*e*x)/(c*d + e)] + c*(d + e*x)*((c*(d
 + e*x))/(c*d + e))^m*(a + b*ArcCosh[c*x])))/(c*e*(1 + m)*((c*(d + e*x))/(c
*d + e))^m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(a+b\,\mathrm{arccosh}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] ((x*e + d)*e^(m*log(x*e + d) - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m
 + 1) - integrate((c^2*x^2*e + c^2*d*x)*(x*e + d)^m/(c^2*(m + 1)*x^2*e - (m
 + 1)*e), x) + integrate((c*x*e + c*d)*(x*e + d)^m/(c^3*(m + 1)*x^3*e - c*(
m + 1)*x*e + (c^2*(m + 1)*x^2*e - (m + 1)*e)*sqrt(c*x + 1)*sqrt(c*x - 1)),
x))*b + (x*e + d)^(m + 1)*a*e^(-1)/(m + 1)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(x*e + d)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acosh}(cx)) (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(e*x + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c\,x))\,\,(d + e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^m, x)
```

3.39 $\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(d+ex)^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^m/(b*arccosh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m/(b*arccosh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*acosh(c*x)),x)
[Out] int((d + e*x)^m/(a + b*acosh(c*x)), x)
```

3.40 $\int \frac{(d+ex)^m}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1)*(x*e + d)^m + (c^3*x^3 - c*x)*(
x*e + d)^m)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
(b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))) + integrate(((c^3*(m + 1)*x^3*e + c^3*d*x^2 - c*
(m - 1)*x*e + c*d)*(c*x + 1)*(c*x - 1)*(x*e + d)^m + (2*c^4*(m + 1)*x^4*e +
 2*c^4*d*x^3 - c^2*(3*m + 1)*x^2*e - c^2*d*x + m*e)*sqrt(c*x + 1)*sqrt(c*x
- 1)*(x*e + d)^m + (c^5*(m + 1)*x^5*e + c^5*d*x^4 - 2*c^3*(m + 1)*x^3*e - 2
*c^3*d*x^2 + c*(m + 1)*x*e + c*d)*(x*e + d)^m)/(a*b*c^5*x^5*e + a*b*c^5*d*x
^4 - 2*a*b*c^3*x^3*e - 2*a*b*c^3*d*x^2 + a*b*c*x*e + a*b*c*d + (a*b*c^3*x^3
*e + a*b*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*x^4*e + a*b*c^4*d*x^3
- a*b*c^2*x^2*e - a*b*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5*e
 + b^2*c^5*d*x^4 - 2*b^2*c^3*x^3*e - 2*b^2*c^3*d*x^2 + b^2*c*x*e + b^2*c*d
+ (b^2*c^3*x^3*e + b^2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*x^4*e +
b^2*c^4*d*x^3 - b^2*c^2*x^2*e - b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x)^m/(a + b*acosh(c*x))^2, x)
```

3.41 $\int (c+dx^2)^4 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=370

$$\frac{(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)\,(1-a^2x^2)}{315a^9\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{4d(105a^6c^3+189a^4c^2d+135a^2cd^2+35d}{945a^9\sqrt{-1+ax}\,\sqrt{1+ax}}$$

```
[Out] c^4*x*arccosh(a*x)+4/3*c^3*d*x^3*arccosh(a*x)+6/5*c^2*d^2*x^5*arccosh(a*x)+
4/7*c*d^3*x^7*arccosh(a*x)+1/9*d^4*x^9*arccosh(a*x)+1/315*(315*a^8*c^4+420*
a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*(-a^2*x^2+1)/a^9/(a*x-1)^(1
/2)/(a*x+1)^(1/2)-4/945*d*(105*a^6*c^3+189*a^4*c^2*d+135*a^2*c*d^2+35*d^3)*
(-a^2*x^2+1)^2/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/525*d^2*(63*a^4*c^2+90*a^2
*c*d+35*d^2)*(-a^2*x^2+1)^3/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)-4/441*d^3*(9*a^
2*c+7*d)*(-a^2*x^2+1)^4/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/81*d^4*(-a^2*x^2+
1)^5/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {200, 5908, 12, 1624, 1813, 1864}

$$-\frac{4d^3(1-a^2x^2)^4(9a^2c+7d)}{441a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{d^4(1-a^2x^2)^5}{81a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{2d^2(1-a^2x^2)^3(63a^4c^2+90a^2cd+35d^2)}{525a^9\sqrt{ax-1}\sqrt{ax+1}}-\frac{4d(1-a^2x^2)^2(105a^6c^3+189a^4c^2d+135a^2cd^2+35d^3)}{945a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{(1-a^2x^2)(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4)}{315a^9\sqrt{ax-1}\sqrt{ax+1}}+c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(ax)+\frac{1}{9}d^4x^9\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*(
1 - a^2*x^2))/(315*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d*(105*a^6*c^3 +
189*a^4*c^2*d + 135*a^2*c*d^2 + 35*d^3)*(1 - a^2*x^2)^2)/(945*a^9*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]) + (2*d^2*(63*a^4*c^2 + 90*a^2*c*d + 35*d^2)*(1 - a^2*x
^2)^3)/(525*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d^3*(9*a^2*c + 7*d)*(1 -
 a^2*x^2)^4)/(441*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (d^4*(1 - a^2*x^2)^5)
/(81*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^4*x*ArcCosh[a*x] + (4*c^3*d*x^3*
ArcCosh[a*x])/3 + (6*c^2*d^2*x^5*ArcCosh[a*x])/5 + (4*c*d^3*x^7*ArcCosh[a*x
])/7 + (d^4*x^9*ArcCosh[a*x])/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \cosh^{-1}(ax)\,dx = c^4 x \cosh^{-1}(ax) + \frac{4}{3}c^3dx^3\cosh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cosh^{-1}(ax) + \frac{4}{7}cd^3x^7\cosh^{-1}$$

$$= c^4 x \cosh^{-1}(ax) + \frac{4}{3}c^3dx^3\cosh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cosh^{-1}(ax) + \frac{4}{7}cd^3x^7\cosh^{-1}$$

$$= c^4 x \cosh^{-1}(ax) + \frac{4}{3}c^3dx^3\cosh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cosh^{-1}(ax) + \frac{4}{7}cd^3x^7\cosh^{-1}$$

$$= c^4 x \cosh^{-1}(ax) + \frac{4}{3}c^3dx^3\cosh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cosh^{-1}(ax) + \frac{4}{7}cd^3x^7\cosh^{-1}$$

$$= c^4 x \cosh^{-1}(ax) + \frac{4}{3}c^3dx^3\cosh^{-1}(ax) + \frac{6}{5}c^2d^2x^5\cosh^{-1}(ax) + \frac{4}{7}cd^3x^7\cosh^{-1}$$

$$= \frac{\left(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4\right)\left(1-a^2x^2\right)}{315a^9\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{4d(105a}{}$$

Mathematica [A]

time = 0.21, size = 216, normalized size = 0.58

$$-\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,(4480d^4+320a^2d^3(81c+7dx^2)+48a^4d^2(1323c^2+270cdx^2+35d^2x^4)+8a^6d(11025c^3+3969c^2dx^2+1215cd^2x^4+175d^3x^6)+a^8(99225c^4+44100c^3dx^2+23814c^2d^2x^4+8100cd^3x^6+1225d^4x^8))}{99225a^9}+\frac{1}{315}x\left(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8\right)\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] -1/99225*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(4480*d^4 + 320*a^2*d^3*(81*c + 7*d*
x^2) + 48*a^4*d^2*(1323*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 8*a^6*d*(11025*c^
3 + 3969*c^2*d*x^2 + 1215*c*d^2*x^4 + 175*d^3*x^6) + a^8*(99225*c^4 + 44100
*c^3*d*x^2 + 23814*c^2*d^2*x^4 + 8100*c*d^3*x^6 + 1225*d^4*x^8)))/a^9 + (x*
(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*Ar
cCosh[a*x])/315
```

Maple [A]

time = 1.57, size = 255, normalized size = 0.69

method	result
derivativedivides	$\frac{\operatorname{arccosh}(ax)c^4ax+\frac{4a\operatorname{arccosh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arccosh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccosh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arccosh}(ax)d^4x^9}{9}-\frac{\sqrt{ax-1}\,\sqrt{}}{}}{}$
default	$\frac{\operatorname{arccosh}(ax)c^4ax+\frac{4a\operatorname{arccosh}(ax)c^3d\,x^3}{3}+\frac{6a\operatorname{arccosh}(ax)c^2d^2x^5}{5}+\frac{4a\operatorname{arccosh}(ax)c\,d^3x^7}{7}+\frac{a\operatorname{arccosh}(ax)d^4x^9}{9}-\frac{\sqrt{ax-1}\,\sqrt{}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccosh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccosh(a*x)*c^4*a*x+4/3*a*arccosh(a*x)*c^3*d*x^3+6/5*a*arccosh(a*x)*c
^2*d^2*x^5+4/7*a*arccosh(a*x)*c*d^3*x^7+1/9*a*arccosh(a*x)*d^4*x^9-1/99225/
a^8*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(1225*a^8*d^4*x^8+8100*a^8*c*d^3*x^6+23814*
a^8*c^2*d^2*x^4+1400*a^6*d^4*x^6+44100*a^8*c^3*d*x^2+9720*a^6*c*d^3*x^4+992
25*a^8*c^4+31752*a^6*c^2*d^2*x^2+1680*a^4*d^4*x^4+88200*a^6*c^3*d+12960*a^4
*c*d^3*x^2+63504*a^4*c^2*d^2+2240*a^2*d^4*x^2+25920*a^2*c*d^3+4480*d^4))
```

Maxima [A]
time = 0.26, size = 385, normalized size = 1.04

$$-\frac{1}{99225}\left(\frac{1225\sqrt{a^2x^2-1}\,d^4x^8}{a^2}+\frac{8100\sqrt{a^2x^2-1}\,cd^3x^6}{a^2}+\frac{23814\sqrt{a^2x^2-1}\,c^2d^2x^4}{a^2}+\frac{1400\sqrt{a^2x^2-1}\,d^4x^6}{a^4}+\frac{44100\sqrt{a^2x^2-1}\,c^3dx^2}{a^2}+\frac{9720\sqrt{a^2x^2-1}\,cd^3x^4}{a^4}+\frac{99225\sqrt{a^2x^2-1}\,c^4}{a^2}+\frac{31752\sqrt{a^2x^2-1}\,c^2d^2x^2}{a^4}+\frac{1680\sqrt{a^2x^2-1}\,d^4x^4}{a^6}+\frac{88200\sqrt{a^2x^2-1}\,c^3d}{a^4}+\frac{12960\sqrt{a^2x^2-1}\,cd^3x^2}{a^6}+\frac{63504\sqrt{a^2x^2-1}\,c^2d^2}{a^6}+\frac{2240\sqrt{a^2x^2-1}\,d^4x^2}{a^8}+\frac{25920\sqrt{a^2x^2-1}\,cd^3}{a^8}+\frac{4480\sqrt{a^2x^2-1}\,d^4}{a^{10}}\right)a+\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\operatorname{arcosh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/99225*(1225*sqrt(a^2*x^2 - 1)*d^4*x^8/a^2 + 8100*sqrt(a^2*x^2 - 1)*c*d^3
*x^6/a^2 + 23814*sqrt(a^2*x^2 - 1)*c^2*d^2*x^4/a^2 + 1400*sqrt(a^2*x^2 - 1)
*d^4*x^6/a^4 + 44100*sqrt(a^2*x^2 - 1)*c^3*d*x^2/a^2 + 9720*sqrt(a^2*x^2 -
1)*c*d^3*x^4/a^4 + 99225*sqrt(a^2*x^2 - 1)*c^4/a^2 + 31752*sqrt(a^2*x^2 - 1
)*c^2*d^2*x^2/a^4 + 1680*sqrt(a^2*x^2 - 1)*d^4*x^4/a^6 + 88200*sqrt(a^2*x^2
 - 1)*c^3*d/a^4 + 12960*sqrt(a^2*x^2 - 1)*c*d^3*x^2/a^6 + 63504*sqrt(a^2*x^
2 - 1)*c^2*d^2/a^6 + 2240*sqrt(a^2*x^2 - 1)*d^4*x^2/a^8 + 25920*sqrt(a^2*x^
2 - 1)*c*d^3/a^8 + 4480*sqrt(a^2*x^2 - 1)*d^4/a^10)*a + 1/315*(35*d^4*x^9 +
 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*arccosh(a*x)
```

Fricas [A]
time = 0.40, size = 250, normalized size = 0.68

$$\frac{315\left(35\,a^9d^4x^9+180\,a^9cd^3x^7+378\,a^9c^2d^2x^5+420\,a^9c^3dx^3+315\,a^9c^4x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(1225\,a^8d^4x^8+99225\,a^8c^4+88200\,a^6c^3d+63504\,a^4c^2d^2+100\left(81\,a^8cd^3+14\,a^6d^4\right)x^6+25920\,a^2cd^3+6\left(3969\,a^8c^2d^2+1620\,a^6cd^3+280\,a^4d^4\right)x^4+4480\,d^4+4\left(11025\,a^8c^3d+7938\,a^6c^2d^2+3240\,a^4cd^3+560\,a^2d^4\right)x^2\right)\sqrt{a^2x^2-1}}{99225\,a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/99225*(315*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 42
0*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (1225*a^8*d
^4*x^8 + 99225*a^8*c^4 + 88200*a^6*c^3*d + 63504*a^4*c^2*d^2 + 100*(81*a^8*
c*d^3 + 14*a^6*d^4)*x^6 + 25920*a^2*c*d^3 + 6*(3969*a^8*c^2*d^2 + 1620*a^6*
c*d^3 + 280*a^4*d^4)*x^4 + 4480*d^4 + 4*(11025*a^8*c^3*d + 7938*a^6*c^2*d^2
 + 3240*a^4*c*d^3 + 560*a^2*d^4)*x^2)*sqrt(a^2*x^2 - 1))/a^9
```

Sympy [C] Result contains complex when optimal does not.
time = 1.48, size = 503, normalized size = 1.36

$$\begin{cases} c^{4} x \operatorname{acosh}{\left(a x \right)} + \frac{4 c^{3} d x^{3} \operatorname{acosh}{\left(a x \right)}}{3} + \frac{6 c^{2} d^{2} x^{5} \operatorname{acosh}{\left(a x \right)}}{5} + \frac{4 c d^{3} x^{7} \operatorname{acosh}{\left(a x \right)}}{7} + \frac{d^{4} x^{9} \operatorname{acosh}{\left(a x \right)}}{9} - \frac{c^{4} \sqrt{a^{2} x^{2} - 1}}{a} - \frac{4 c^{3} d x^{2} \sqrt{a^{2} x^{2} - 1}}{9 a} - \frac{6 c^{2} d^{2} x^{4} \sqrt{a^{2} x^{2} - 1}}{25 a} - \frac{4 c d^{3} x^{6} \sqrt{a^{2} x^{2} - 1}}{49 a} - \frac{d^{4} x^{8} \sqrt{a^{2} x^{2} - 1}}{81 a} - \frac{8 c^{3} d \sqrt{a^{2} x^{2} - 1}}{9 a^{3}} - \frac{8 c^{2} d^{2} x^{2} \sqrt{a^{2} x^{2} - 1}}{25 a^{3}} - \frac{24 c d^{3} x^{4} \sqrt{a^{2} x^{2} - 1}}{245 a^{3}} - \frac{8 d^{4} x^{6} \sqrt{a^{2} x^{2} - 1}}{567 a^{3}} - \frac{16 c^{2} d^{2} \sqrt{a^{2} x^{2} - 1}}{25 a^{5}} - \frac{32 c d^{3} x^{2} \sqrt{a^{2} x^{2} - 1}}{245 a^{5}} - \frac{16 d^{4} x^{4} \sqrt{a^{2} x^{2} - 1}}{945 a^{5}} - \frac{64 c d^{3} \sqrt{a^{2} x^{2} - 1}}{245 a^{7}} - \frac{64 d^{4} x^{2} \sqrt{a^{2} x^{2} - 1}}{2835 a^{7}} - \frac{128 d^{4} \sqrt{a^{2} x^{2} - 1}}{2835 a^{9}} & \text{for}\: a \neq 0 \\ \frac{i \pi \left(c^{4} x + \frac{4 c^{3} d x^{3}}{3} + \frac{6 c^{2} d^{2} x^{5}}{5} + \frac{4 c d^{3} x^{7}}{7} + \frac{d^{4} x^{9}}{9}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acosh(a*x),x)
```

```
[Out] Piecewise((c**4*x*acosh(a*x) + 4*c**3*d*x**3*acosh(a*x)/3 + 6*c**2*d**2*x**
5*acosh(a*x)/5 + 4*c*d**3*x**7*acosh(a*x)/7 + d**4*x**9*acosh(a*x)/9 - c**4
*sqrt(a**2*x**2 - 1)/a - 4*c**3*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 6*c**2*d
**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d**3*x**6*sqrt(a**2*x**2 - 1)/(49
*a) - d**4*x**8*sqrt(a**2*x**2 - 1)/(81*a) - 8*c**3*d*sqrt(a**2*x**2 - 1)/(
9*a**3) - 8*c**2*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 24*c*d**3*x**4*s
qrt(a**2*x**2 - 1)/(245*a**3) - 8*d**4*x**6*sqrt(a**2*x**2 - 1)/(567*a**3)
- 16*c**2*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 32*c*d**3*x**2*sqrt(a**2*x**
2 - 1)/(245*a**5) - 16*d**4*x**4*sqrt(a**2*x**2 - 1)/(945*a**5) - 64*c*d**3
*sqrt(a**2*x**2 - 1)/(245*a**7) - 64*d**4*x**2*sqrt(a**2*x**2 - 1)/(2835*a*
*7) - 128*d**4*sqrt(a**2*x**2 - 1)/(2835*a**9), Ne(a, 0)), (I*pi*(c**4*x +
4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, Tr
ue))
```

Giac [A]
time = 0.41, size = 316, normalized size = 0.85

$$\frac{1}{315} \, {\left(35 \, d^{4} x^{9} + 180 \, c d^{3} x^{7} + 378 \, c^{2} d^{2} x^{5} + 420 \, c^{3} d x^{3} + 315 \, c^{4} x\right)} \log \left(a x + \sqrt{a^{2} x^{2} - 1}\right) - \frac{{\left(315 \, a^{8} c^{4} + 420 \, a^{6} c^{3} d + 378 \, a^{4} c^{2} d^{2} + 180 \, a^{2} c d^{3} + 35 \, d^{4}\right)} \sqrt{a^{2} x^{2} - 1}}{315 \, a^{9}} - \frac{44100 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{3}{2}} a^{6} c^{3} d + 23814 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{5}{2}} a^{4} c^{2} d^{2} + 79380 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{3}{2}} a^{4} c^{2} d^{2} + 8100 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{7}{2}} a^{2} c d^{3} + 34020 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{5}{2}} a^{2} c d^{3} + 1225 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{9}{2}} d^{4} + 56700 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{3}{2}} a^{2} c d^{3} + 6300 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{7}{2}} d^{4} + 13230 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{5}{2}} d^{4} + 14700 \, {\left(a^{2} x^{2} - 1\right)}^{\frac{3}{2}} d^{4}}{99225 \, a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/315*(315*a^8*c^4 + 420*a^6*c^3*d + 3
78*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*sqrt(a^2*x^2 - 1)/a^9 - 1/99225*(4
4100*(a^2*x^2 - 1)^(3/2)*a^6*c^3*d + 23814*(a^2*x^2 - 1)^(5/2)*a^4*c^2*d^2
+ 79380*(a^2*x^2 - 1)^(3/2)*a^4*c^2*d^2 + 8100*(a^2*x^2 - 1)^(7/2)*a^2*c*d^
3 + 34020*(a^2*x^2 - 1)^(5/2)*a^2*c*d^3 + 1225*(a^2*x^2 - 1)^(9/2)*d^4 + 56
700*(a^2*x^2 - 1)^(3/2)*a^2*c*d^3 + 6300*(a^2*x^2 - 1)^(7/2)*d^4 + 13230*(a
^2*x^2 - 1)^(5/2)*d^4 + 14700*(a^2*x^2 - 1)^(3/2)*d^4)/a^9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)\,{\left(d\,x^2+c\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^4,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^4, x)
```

3.42 $\int (c+dx^2)^3 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=267

$$\frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)(1-a^2x^2)}{35a^7\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{d(35a^4c^2+42a^2cd+15d^2)(1-a^2x^2)^2}{105a^7\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{3d^2(7a^2c+5d)(1-}{175a^7\sqrt{-1+ax}\,\sqrt{}}$$

```
[Out] c^3*x*arccosh(a*x)+c^2*d*x^3*arccosh(a*x)+3/5*c*d^2*x^5*arccosh(a*x)+1/7*d^
3*x^7*arccosh(a*x)+1/35*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c*d^2+5*d^3)*(-a^2*
x^2+1)/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/105*d*(35*a^4*c^2+42*a^2*c*d+15*d^
2)*(-a^2*x^2+1)^2/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/175*d^2*(7*a^2*c+5*d)*(
-a^2*x^2+1)^3/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/49*d^3*(-a^2*x^2+1)^4/a^7/(
a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {200, 5908, 12, 1624, 1813, 1864}

$$\frac{3d^2(1-a^2x^2)^3(7a^2c+5d)}{175a^7\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{d^3(1-a^2x^2)^4}{49a^7\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{d(1-a^2x^2)^2(35a^4c^2+42a^2cd+15d^2)}{105a^7\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{(1-a^2x^2)(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)}{35a^7\sqrt{ax-1}\,\sqrt{ax+1}}+c^3x\cosh^{-1}(ax)+c^2dx^3\cosh^{-1}(ax)+\frac{3}{5}cd^2x^5\cosh^{-1}(ax)+\frac{1}{7}d^3x^7\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCosh[a*x],x]
```

```
[Out] ((35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*(1 - a^2*x^2))/(35*a^7*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d*(35*a^4*c^2 + 42*a^2*c*d + 15*d^2)*(1 -
a^2*x^2)^2)/(105*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*d^2*(7*a^2*c + 5*d)
*(1 - a^2*x^2)^3)/(175*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d^3*(1 - a^2*x^
2)^4)/(49*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^3*x*ArcCosh[a*x] + c^2*d*x^
3*ArcCosh[a*x] + (3*c*d^2*x^5*ArcCosh[a*x])/5 + (d^3*x^7*ArcCosh[a*x])/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int (c+dx^2)^3 \cosh^{-1}(ax)\,dx = c^3x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax$$

$$= c^3x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax$$

$$= c^3x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax$$

$$= c^3x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax$$

$$= c^3x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax$$

$$= \frac{(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3)\,(1-a^2x^2)}{35a^7\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{d(35a^4c^2+42a^2cd+15d^2}{105a^7\sqrt{-1+ax}\,\sqrt{1}$$

Mathematica [A]
time = 0.14, size = 154, normalized size = 0.58

$$-\frac{\sqrt{-1+ax}\sqrt{1+ax}\left(240d^3+24a^2d^2(49c+5dx^2)+2a^4d(1225c^2+294cdx^2+45d^2x^4)+a^6(3675c^3+1225c^2dx^2+441cd^2x^4+75d^3x^6)\right)}{3675a^7}+\frac{1}{35}x\left(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6\right)\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCosh[a*x],x]
```

```
[Out] -1/3675*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(240*d^3 + 24*a^2*d^2*(49*c + 5*d*x^2
) + 2*a^4*d*(1225*c^2 + 294*c*d*x^2 + 45*d^2*x^4) + a^6*(3675*c^3 + 1225*c^
2*d*x^2 + 441*c*d^2*x^4 + 75*d^3*x^6)))/a^7 + (x*(35*c^3 + 35*c^2*d*x^2 + 2
1*c*d^2*x^4 + 5*d^3*x^6)*ArcCosh[a*x])/35
```

Maple [A]
time = 1.64, size = 176, normalized size = 0.66

method	result
derivativedivides	$\frac{\operatorname{arccosh}(ax)c^3ax+a\operatorname{arccosh}(ax)c^2dx^3+\frac{3a\operatorname{arccosh}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccosh}(ax)d^3x^7}{7}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,(75d^3a^6x^6+44}{}}{a}$
default	$\frac{\operatorname{arccosh}(ax)c^3ax+a\operatorname{arccosh}(ax)c^2dx^3+\frac{3a\operatorname{arccosh}(ax)c\,d^2x^5}{5}+\frac{a\operatorname{arccosh}(ax)d^3x^7}{7}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,(75d^3a^6x^6+44}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccosh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccosh(a*x)*c^3*a*x+a*arccosh(a*x)*c^2*d*x^3+3/5*a*arccosh(a*x)*c*d^2
*x^5+1/7*a*arccosh(a*x)*d^3*x^7-1/3675/a^6*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(75*
a^6*d^3*x^6+441*a^6*c*d^2*x^4+1225*a^6*c^2*d*x^2+90*a^4*d^3*x^4+3675*a^6*c^
3+588*a^4*c*d^2*x^2+2450*a^4*c^2*d+120*a^2*d^3*x^2+1176*a^2*c*d^2+240*d^3))
```

Maxima [A]
time = 0.27, size = 257, normalized size = 0.96

$$-\frac{1}{3675}\left(\frac{75\sqrt{a^2x^2-1}\,d^3x^6}{a^2}+\frac{441\sqrt{a^2x^2-1}\,cd^2x^4}{a^2}+\frac{1225\sqrt{a^2x^2-1}\,c^2dx^2}{a^2}+\frac{90\sqrt{a^2x^2-1}\,d^3x^4}{a^4}+\frac{3675\sqrt{a^2x^2-1}\,c^3}{a^2}+\frac{588\sqrt{a^2x^2-1}\,cd^2x^2}{a^4}+\frac{2450\sqrt{a^2x^2-1}\,c^2d}{a^4}+\frac{120\sqrt{a^2x^2-1}\,d^3x^2}{a^6}+\frac{1176\sqrt{a^2x^2-1}\,cd^2}{a^6}+\frac{240\sqrt{a^2x^2-1}\,d^3}{a^8}\right)a+\frac{1}{35}\left(5d^3x^7+21cd^2x^5+35c^2dx^3+35c^3x\right)\operatorname{arcosh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/3675*(75*sqrt(a^2*x^2 - 1)*d^3*x^6/a^2 + 441*sqrt(a^2*x^2 - 1)*c*d^2*x^4
/a^2 + 1225*sqrt(a^2*x^2 - 1)*c^2*d*x^2/a^2 + 90*sqrt(a^2*x^2 - 1)*d^3*x^4/
a^4 + 3675*sqrt(a^2*x^2 - 1)*c^3/a^2 + 588*sqrt(a^2*x^2 - 1)*c*d^2*x^2/a^4
+ 2450*sqrt(a^2*x^2 - 1)*c^2*d/a^4 + 120*sqrt(a^2*x^2 - 1)*d^3*x^2/a^6 + 11
76*sqrt(a^2*x^2 - 1)*c*d^2/a^6 + 240*sqrt(a^2*x^2 - 1)*d^3/a^8)*a + 1/35*(5
*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*arccosh(a*x)
```

Fricas [A]
time = 0.35, size = 179, normalized size = 0.67

$$\frac{105\,(5\,a^7d^3x^7+21\,a^7cd^2x^5+35\,a^7c^2dx^3+35\,a^7c^3x)\log\left(ax+\sqrt{a^2x^2-1}\right)-(75\,a^6d^3x^6+3675\,a^6c^3+2450\,a^4c^2d+1176\,a^2cd^2+9\,(49\,a^6cd^2+10\,a^4d^3)x^4+240\,d^3+(1225\,a^6c^2d+588\,a^4cd^2+120\,a^2d^3)x^2)\sqrt{a^2x^2-1}}{3675\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7*c^2*d*x^3 + 35*a^7*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (75*a^6*d^3*x^6 + 3675*a^6*c^3 + 2450*
a^4*c^2*d + 1176*a^2*c*d^2 + 9*(49*a^6*c*d^2 + 10*a^4*d^3)*x^4 + 240*d^3 +
(1225*a^6*c^2*d + 588*a^4*c*d^2 + 120*a^2*d^3)*x^2)*sqrt(a^2*x^2 - 1))/a^7
```

Sympy [C] Result contains complex when optimal does not.
time = 0.71, size = 328, normalized size = 1.23

$$\begin{cases} c^3x\operatorname{acosh}(ax)+c^2dx^3\operatorname{acosh}(ax)+\frac{3cd^2x^5\operatorname{acosh}(ax)}{5}+\frac{d^3x^7\operatorname{acosh}(ax)}{7}-\frac{c^3\sqrt{a^2x^2-1}}{a}-\frac{c^2dx^2\sqrt{a^2x^2-1}}{3a}-\frac{3cd^2x^4\sqrt{a^2x^2-1}}{25a}-\frac{d^3x^6\sqrt{a^2x^2-1}}{49a}-\frac{2c^2d\sqrt{a^2x^2-1}}{3a^3}-\frac{4cd^2x^2\sqrt{a^2x^2-1}}{25a^3}-\frac{6d^3x^4\sqrt{a^2x^2-1}}{245a^3}-\frac{8cd^2\sqrt{a^2x^2-1}}{25a^5}-\frac{8d^3x^2\sqrt{a^2x^2-1}}{245a^5}-\frac{16d^3\sqrt{a^2x^2-1}}{245a^7} & \text{for}\ a\neq 0 \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acosh(a*x),x)
```

```
[Out] Piecewise((c**3*x*acosh(a*x) + c**2*d*x**3*acosh(a*x) + 3*c*d**2*x**5*acosh
(a*x)/5 + d**3*x**7*acosh(a*x)/7 - c**3*sqrt(a**2*x**2 - 1)/a - c**2*d*x**2
*sqrt(a**2*x**2 - 1)/(3*a) - 3*c*d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - d**
3*x**6*sqrt(a**2*x**2 - 1)/(49*a) - 2*c**2*d*sqrt(a**2*x**2 - 1)/(3*a**3) -
 4*c*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 6*d**3*x**4*sqrt(a**2*x**2 -
 1)/(245*a**3) - 8*c*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 8*d**3*x**2*sqrt(
a**2*x**2 - 1)/(245*a**5) - 16*d**3*sqrt(a**2*x**2 - 1)/(245*a**7), Ne(a, 0
)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7)/2, True))
```

Giac [A]
time = 0.42, size = 214, normalized size = 0.80

$$\frac{1}{35}\,(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3)\sqrt{a^2x^2-1}}{35\,a^7}-\frac{1225\,(a^2x^2-1)^{\frac{3}{2}}a^4c^2d+441\,(a^2x^2-1)^{\frac{5}{2}}a^2cd^2+1470\,(a^2x^2-1)^{\frac{3}{2}}a^2cd^2+75\,(a^2x^2-1)^{\frac{7}{2}}d^3+315\,(a^2x^2-1)^{\frac{5}{2}}d^3+525\,(a^2x^2-1)^{\frac{3}{2}}d^3}{3675\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - 1/35*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*sqrt(
a^2*x^2 - 1)/a^7 - 1/3675*(1225*(a^2*x^2 - 1)^(3/2)*a^4*c^2*d + 441*(a^2*x^
2 - 1)^(5/2)*a^2*c*d^2 + 1470*(a^2*x^2 - 1)^(3/2)*a^2*c*d^2 + 75*(a^2*x^2 -
 1)^(7/2)*d^3 + 315*(a^2*x^2 - 1)^(5/2)*d^3 + 525*(a^2*x^2 - 1)^(3/2)*d^3)/
a^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^3,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^3, x)
```

3.43 $\int (c+dx^2)^2 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=181

$$\frac{(15a^4c^2+10a^2cd+3d^2)(1-a^2x^2)}{15a^5\sqrt{-1+ax}\sqrt{1+ax}}-\frac{2d(5a^2c+3d)(1-a^2x^2)^2}{45a^5\sqrt{-1+ax}\sqrt{1+ax}}+\frac{d^2(1-a^2x^2)^3}{25a^5\sqrt{-1+ax}\sqrt{1+ax}}+c^2x\cosh^{-1}(ax$$

```
[Out] c^2*x*arccosh(a*x)+2/3*c*d*x^3*arccosh(a*x)+1/5*d^2*x^5*arccosh(a*x)+1/15*(
15*a^4*c^2+10*a^2*c*d+3*d^2)*(-a^2*x^2+1)/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2
/45*d*(5*a^2*c+3*d)*(-a^2*x^2+1)^2/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/25*d^2
*(-a^2*x^2+1)^3/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {200, 5908, 12, 534, 1261, 712}

$$-\frac{2d(1-a^2x^2)^2(5a^2c+3d)}{45a^5\sqrt{ax-1}\sqrt{ax+1}}+\frac{d^2(1-a^2x^2)^3}{25a^5\sqrt{ax-1}\sqrt{ax+1}}+\frac{(1-a^2x^2)(15a^4c^2+10a^2cd+3d^2)}{15a^5\sqrt{ax-1}\sqrt{ax+1}}+c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCosh[a*x],x]
```

```
[Out] ((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*(1 - a^2*x^2))/(15*a^5*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) - (2*d*(5*a^2*c + 3*d)*(1 - a^2*x^2)^2)/(45*a^5*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]) + (d^2*(1 - a^2*x^2)^3)/(25*a^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]) + c^2*x*ArcCosh[a*x] + (2*c*d*x^3*ArcCosh[a*x])/3 + (d^2*x^5*ArcCosh[a*x
])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 534

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[(a1 + b1*x^(n/2))^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 +
b1*b2*x^n)^FracPart[p]), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \cosh^{-1}(ax)\,dx = c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-a\int \frac{x(15c^2+10cd}{15\sqrt{-1+ax}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{1}{15}a\int \frac{x(15c^2+1}{\sqrt{-1+}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)}{15\sqrt{-1+}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)}{30}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)}{}$$

$$= \frac{(15a^4c^2+10a^2cd+3d^2)(1-a^2x^2)}{15a^5\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{2d(5a^2c+3d)(1-a^2x^2)^2}{45a^5\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{d^2(}{25a^5\sqrt{-1}}$$

Mathematica [A]

time = 0.11, size = 103, normalized size = 0.57

$$-\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,(24d^2+4a^2d(25c+3dx^2)+a^4(225c^2+50cdx^2+9d^2x^4))}{225a^5}+\left(c^2x+\frac{2}{3}cdx^3+\frac{d^2x^5}{5}\right)\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCosh[a*x],x]
```

```
[Out] -1/225*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(24*d^2 + 4*a^2*d*(25*c + 3*d*x^2) + a
^4*(225*c^2 + 50*c*d*x^2 + 9*d^2*x^4)))/a^5 + (c^2*x + (2*c*d*x^3)/3 + (d^2
*x^5)/5)*ArcCosh[a*x]
```

Maple [A]
time = 1.66, size = 113, normalized size = 0.62

method	result
derivativedivides	$\frac{\operatorname{arccosh}(ax)c^2ax+\frac{2a\operatorname{arccosh}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccosh}(ax)d^2x^5}{5}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(9d^2a^4x^4+50c\,a^4d\,x^2+225a^4c^2+12a^2\right.}{225a^4}}{a}$
default	$\frac{\operatorname{arccosh}(ax)c^2ax+\frac{2a\operatorname{arccosh}(ax)cd\,x^3}{3}+\frac{a\operatorname{arccosh}(ax)d^2x^5}{5}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(9d^2a^4x^4+50c\,a^4d\,x^2+225a^4c^2+12a^2\right.}{225a^4}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccosh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccosh(a*x)*c^2*a*x+2/3*a*arccosh(a*x)*c*d*x^3+1/5*a*arccosh(a*x)*d^2
*x^5-1/225/a^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(9*a^4*d^2*x^4+50*a^4*c*d*x^2+22
5*a^4*c^2+12*a^2*d^2*x^2+100*a^2*c*d+24*d^2))
```

Maxima [A]
time = 0.27, size = 154, normalized size = 0.85

$$-\frac{1}{225}\left(\frac{9\sqrt{a^2x^2-1}\,d^2x^4}{a^2}+\frac{50\sqrt{a^2x^2-1}\,cdx^2}{a^2}+\frac{225\sqrt{a^2x^2-1}\,c^2}{a^2}+\frac{12\sqrt{a^2x^2-1}\,d^2x^2}{a^4}+\frac{100\sqrt{a^2x^2-1}\,cd}{a^4}+\frac{24\sqrt{a^2x^2-1}\,d^2}{a^6}\right)a+\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\operatorname{arcosh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/225*(9*sqrt(a^2*x^2 - 1)*d^2*x^4/a^2 + 50*sqrt(a^2*x^2 - 1)*c*d*x^2/a^2
+ 225*sqrt(a^2*x^2 - 1)*c^2/a^2 + 12*sqrt(a^2*x^2 - 1)*d^2*x^2/a^4 + 100*sq
rt(a^2*x^2 - 1)*c*d/a^4 + 24*sqrt(a^2*x^2 - 1)*d^2/a^6)*a + 1/15*(3*d^2*x^5
 + 10*c*d*x^3 + 15*c^2*x)*arccosh(a*x)
```

Fricas [A]
time = 0.40, size = 121, normalized size = 0.67

$$\frac{15\left(3\,a^5d^2x^5+10\,a^5cdx^3+15\,a^5c^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(9\,a^4d^2x^4+225\,a^4c^2+100\,a^2cd+2\left(25\,a^4cd+6\,a^2d^2\right)x^2+24\,d^2\right)\sqrt{a^2x^2-1}}{225\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a^5*c^2*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - (9*a^4*d^2*x^4 + 225*a^4*c^2 + 100*a^2*c*d + 2*(25*a^4*c*d +
6*a^2*d^2)*x^2 + 24*d^2)*sqrt(a^2*x^2 - 1))/a^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.33, size = 199, normalized size = 1.10

$$\begin{cases} c^2x\operatorname{acosh}(ax) + \frac{2cdx^3\operatorname{acosh}(ax)}{3} + \frac{d^2x^5\operatorname{acosh}(ax)}{5} - \frac{c^2\sqrt{a^2x^2-1}}{a} - \frac{2cdx^2\sqrt{a^2x^2-1}}{9a} - \frac{d^2x^4\sqrt{a^2x^2-1}}{25a} - \frac{4cd\sqrt{a^2x^2-1}}{9a^3} - \frac{4d^2x^2\sqrt{a^2x^2-1}}{75a^3} - \frac{8d^2\sqrt{a^2x^2-1}}{75a^5} & \text{for } a \neq 0 \\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acosh(a*x),x)
```

```
[Out] Piecewise((c**2*x*acosh(a*x) + 2*c*d*x**3*acosh(a*x)/3 + d**2*x**5*acosh(a*
x)/5 - c**2*sqrt(a**2*x**2 - 1)/a - 2*c*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) -
d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d*sqrt(a**2*x**2 - 1)/(9*a**3) -
 4*d**2*x**2*sqrt(a**2*x**2 - 1)/(75*a**3) - 8*d**2*sqrt(a**2*x**2 - 1)/(75
*a**5), Ne(a, 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

Giac [A]
time = 0.42, size = 134, normalized size = 0.74

$$\frac{1}{15}\left(3\,d^2x^5 + 10\,cdx^3 + 15\,c^2x\right)\log\left(ax + \sqrt{a^2x^2-1}\right) - \frac{\left(15\,a^4c^2 + 10\,a^2cd + 3\,d^2\right)\sqrt{a^2x^2-1}}{15\,a^5} - \frac{50\left(a^2x^2-1\right)^{\frac{3}{2}}a^2cd + 9\left(a^2x^2-1\right)^{\frac{5}{2}}d^2 + 30\left(a^2x^2-1\right)^{\frac{3}{2}}d^2}{225\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/1
5*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*sqrt(a^2*x^2 - 1)/a^5 - 1/225*(50*(a^2*
x^2 - 1)^(3/2)*a^2*c*d + 9*(a^2*x^2 - 1)^(5/2)*d^2 + 30*(a^2*x^2 - 1)^(3/2)
*d^2)/a^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)\,\left(d\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^2,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^2, x)
```

3.44 $\int (c+dx^2)\cosh^{-1}(ax)\,dx$

Optimal. Leaf size=84

$$-\frac{(9a^2c+2d)\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3}-\frac{dx^2\sqrt{-1+ax}\,\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)$$

```
[Out] c*x*arccosh(a*x)+1/3*d*x^3*arccosh(a*x)-1/9*(9*a^2*c+2*d)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a^3-1/9*d*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5908, 471, 75}

$$-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,(9a^2c+2d)}{9a^3}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)-\frac{dx^2\sqrt{ax-1}\,\sqrt{ax+1}}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCosh[a*x],x]
```

```
[Out] -1/9*((9*a^2*c + 2*d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/a^3 - (d*x^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x])/(9*a) + c*x*ArcCosh[a*x] + (d*x^3*ArcCosh[a*x])/3
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*((a2 + b2*x^(n/2))^(p + 1)/(b1*b2*e*(m + n
*(p + 1) + 1))), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
```

```
LtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^2\right)\cosh^{-1}(ax)\,dx &= cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)-a\int \frac{x\left(c+\frac{dx^2}{3}\right)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx\\ &= -\frac{dx^2\sqrt{-1+ax}\,\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)+\frac{1}{9}\Bigg(a\Bigg(-9c-\\ &= -\frac{\left(9a^2c+2d\right)\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3}-\frac{dx^2\sqrt{-1+ax}\,\sqrt{1+ax}}{9a}+cx\cosh^{-1}(a\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.71

$$-\frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(2d+a^2\left(9c+dx^2\right)\right)}{9a^3}+\left(cx+\frac{dx^3}{3}\right)\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCosh[a*x],x]
[Out] -1/9*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2*d + a^2*(9*c + d*x^2)))/a^3 + (c*x +
(d*x^3)/3)*ArcCosh[a*x]
```

Maple [A]
time = 1.69, size = 62, normalized size = 0.74

method	result	size
derivativedivides	$\frac{\operatorname{arccosh}(ax)cax+\frac{a\operatorname{arccosh}(ax)d\,x^3}{3}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(d\,a^2x^2+9a^2c+2d\right)}{9a^2}}{a}$	62
default	$\frac{\operatorname{arccosh}(ax)cax+\frac{a\operatorname{arccosh}(ax)d\,x^3}{3}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(d\,a^2x^2+9a^2c+2d\right)}{9a^2}}{a}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccosh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a*(arccosh(a*x)*c*a*x+1/3*a*arccosh(a*x)*d*x^3-1/9/a^2*(a*x-1)^(1/2)*(a*x
+1)^(1/2)*(a^2*d*x^2+9*a^2*c+2*d))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.88

$$-\frac{1}{9}\left(\frac{\sqrt{a^2x^2-1}\,dx^2}{a^2}+\frac{9\sqrt{a^2x^2-1}\,c}{a^2}+\frac{2\sqrt{a^2x^2-1}\,d}{a^4}\right)a+\frac{1}{3}\left(dx^3+3\,cx\right)\operatorname{arcosh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/9*(sqrt(a^2*x^2 - 1)*d*x^2/a^2 + 9*sqrt(a^2*x^2 - 1)*c/a^2 + 2*sqrt(a^2*
x^2 - 1)*d/a^4)*a + 1/3*(d*x^3 + 3*c*x)*arccosh(a*x)
```

Fricas [A]
time = 0.37, size = 71, normalized size = 0.85

$$\frac{3\left(a^{3}dx^{3}+3\,a^{3}cx\right)\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)-\left(a^{2}dx^{2}+9\,a^{2}c+2\,d\right)\sqrt{a^{2}x^{2}-1}}{9\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(a^3*d*x^3 + 3*a^3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (a^2*d*x^2 +
9*a^2*c + 2*d)*sqrt(a^2*x^2 - 1))/a^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.14, size = 90, normalized size = 1.07

$$\begin{cases} cx\operatorname{acosh}\left(ax\right)+\frac{dx^{3}\operatorname{acosh}\left(ax\right)}{3}-\frac{c\sqrt{a^{2}x^{2}-1}}{a}-\frac{dx^{2}\sqrt{a^{2}x^{2}-1}}{9a}-\frac{2d\sqrt{a^{2}x^{2}-1}}{9a^{3}} & \text{for}\ a\neq 0\\ \frac{i\pi\left(cx+\frac{dx^{3}}{3}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acosh(a*x),x)
```

```
[Out] Piecewise((c*x*acosh(a*x) + d*x**3*acosh(a*x)/3 - c*sqrt(a**2*x**2 - 1)/a -
 d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 2*d*sqrt(a**2*x**2 - 1)/(9*a**3), Ne(a,
 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

Giac [A]
time = 0.41, size = 70, normalized size = 0.83

$$\frac{1}{3}\left(dx^{3}+3\,cx\right)\log\left(ax+\sqrt{a^{2}x^{2}-1}\right)-\frac{\left(a^{2}x^{2}-1\right)^{\frac{3}{2}}d}{9\,a^{3}}-\frac{\sqrt{a^{2}x^{2}-1}\left(3\,a^{2}c+d\right)}{3\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*(d*x^3 + 3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/9*(a^2*x^2 - 1)^(3/2)*
d/a^3 - 1/3*sqrt(a^2*x^2 - 1)*(3*a^2*c + d)/a^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2),x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2), x)
```

3.45 $\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=481

$$\frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(\right.}{}$$

```
[Out] 1/2*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/
2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)-1/2*arccosh(a*x)*ln(1+(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d
^(1/2)+1/2*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(
-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)-1/2*arccosh(a*x)*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^
(1/2)/d^(1/2)-1/2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(
-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)+1/2*polylog(2,(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^
(1/2)-1/2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2
)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)+1/2*polylog(2,(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5909, 5962, 5681, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{-c}\,a+\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{-c}\,a+\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}+1\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}+1\right)}{2\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c)
- d])])/(2*Sqrt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x
])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) + (ArcCosh[a*x]
*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sq
rt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c
] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^Ar
cCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + Poly
Log[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(2*Sqrt[
-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a
^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a
*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(2*Sqrt[-c]*Sqrt[d])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\,\cosh^{-1}(ax)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cosh^{-1}(ax)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}-\sqrt{d}\,\cosh(x)}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}+\sqrt{d}\,\cosh(x)}\,dx, x,\right.}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}-\sqrt{-a^2c-d}-\sqrt{d}\,e^x}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}+\sqrt{-a^2c}\right.}{}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}\right.}{2\sqrt{-c}\,\sqrt{d}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}\right.}{2\sqrt{-c}\,\sqrt{d}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}\right.}{2\sqrt{-c}\,\sqrt{d}}$$

Mathematica [A]
time = 0.27, size = 375, normalized size = 0.78

$$\frac{-\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)+\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-a\sqrt{-c}+\sqrt{-a^2c-d}}\right)+\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}+\sqrt{-a^2c-d}}\right)-\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}+\sqrt{-a^2c-d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-a\sqrt{-c}+\sqrt{-a^2c-d}}\right)-\text{PolyLog}\left(2,-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}+\sqrt{-a^2c-d}}\right)+\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}+\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c
) - d])]) + ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(-(a*Sqrt[-c]) +
Sqrt[-(a^2*c) - d])] + ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqr
t[-c] + Sqrt[-(a^2*c) - d])] - ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x]
)/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])] - PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(-
(a*Sqrt[-c]) + Sqrt[-(a^2*c) - d])] - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])
```

```
/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d]))] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 14.84, size = 222, normalized size = 0.46

method	result
derivativedivides	$\frac{a^2\left(\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^4+\left(4a^2c+2d\right)\_Z^2+d\right)}\frac{\_R1\left(\mathrm{arccosh}(ax)\ln\left(\frac{\_R1-ax-\sqrt{ax-1}\,\sqrt{ax+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R\cdots}{}\right)\right)}{d\_R1^2+2a^2c+d}\right.}{2}$
default	$\frac{a^2\left(\sum_{\_R1=\mathrm{RootOf}\left(d\_Z^4+\left(4a^2c+2d\right)\_Z^2+d\right)}\frac{\_R1\left(\mathrm{arccosh}(ax)\ln\left(\frac{\_R1-ax-\sqrt{ax-1}\,\sqrt{ax+1}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R\cdots}{}\right)\right)}{d\_R1^2+2a^2c+d}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*a^2*sum(_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^
(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1))
,_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))-1/2*a^2*sum(1/_R1/(_R1^2*d+2*a^2*
c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1
-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^
2+d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(d*x^2 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c),x)
[Out] Integral(acosh(a*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(arccosh(a*x)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{d\,x^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2),x)
[Out] int(acosh(a*x)/(c + d*x^2), x)
```

3.46 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=774

$$-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\,\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\,\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\,\sqrt{a\sqrt{-c}+\sqrt{d}}\,\sqrt{d}}-\frac{a\tanh^{-1}}{2c\sqrt{a\sqrt{}}}$$

```
[Out] -1/4*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1
/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)+1/4*arccosh(a*x)*ln(1+(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/
d^(1/2)-1/4*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*
(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)+1/4*arccosh(a*x)*ln(1+(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)
^(3/2)/d^(1/2)+1/4*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*
(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1/4*polylog(2,(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d
^(1/2)+1/4*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/
2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1/4*polylog(2,(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1
/4*arccosh(a*x)/c/d^(1/2)/((-c)^(1/2)-x*d^(1/2))+1/4*arccosh(a*x)/c/d^(1/2)
/((-c)^(1/2)+x*d^(1/2))+1/2*a*arctanh((a*x+1)^(1/2)*(a*(-c)^(1/2)-d^(1/2))^
(1/2)/(a*x-1)^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2))/c/d^(1/2)/(a*(-c)^(1/2)-d
^(1/2))^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2)-1/2*a*arctanh((a*x+1)^(1/2)*(a*(
-c)^(1/2)+d^(1/2))^(1/2)/(a*x-1)^(1/2)/(a*(-c)^(1/2)-d^(1/2))^(1/2))/c/d^(1
/2)/(a*(-c)^(1/2)-d^(1/2))^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2)
```

Rubi [A]

time = 0.81, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5909, 5963, 95, 214, 5962, 5681, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\text{Li}_2\left(-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}+1\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}+1\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{ax+1}\sqrt{a\sqrt{-c}-\sqrt{d}}}{\sqrt{ax-1}\sqrt{a\sqrt{-c}+\sqrt{d}}}\right)}{2c\sqrt{d}\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{ax+1}\sqrt{a\sqrt{-c}+\sqrt{d}}}{\sqrt{ax-1}\sqrt{a\sqrt{-c}-\sqrt{d}}}\right)}{2c\sqrt{d}\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -1/4*ArcCosh[a*x]/(c*Sqrt[d]*(Sqrt[-c] - Sqrt[d]*x)) + ArcCosh[a*x]/(4*c*Sq
rt[d]*(Sqrt[-c] + Sqrt[d]*x)) + (a*ArcTanh[(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt
[1 + a*x])/(Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[-1 + a*x])])/(2*c*Sqrt[a*Sqrt[-
c] - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (a*ArcTanh[(Sqrt[a*Sqrt
[-c] + Sqrt[d]]*Sqrt[1 + a*x])/(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[-1 + a*x])]
```

```
)/(2*c*Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (Ar
cCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d
])])/(4*(-c)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x]
)/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) - (ArcCosh[a*x
]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(4*(
-c)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt
[-c] + Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]
*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d])
 - PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(
4*(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] +
 Sqrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) - PolyLog[2, (Sqrt[d]*E^ArcCo
sh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(4*(-c)^(3/2)*Sqrt[d])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5909

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^2}\,dx = \int \left(-\frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\,\sqrt{d}\,-dx\right)^2} - \frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\,\sqrt{d}\,+dx\right)^2} - \frac{d\cosh^{-1}(ax)}{2c\,(-cd-d^2x^2)} \right) dx$$

$$= -\frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\,\sqrt{d}\,-dx\right)^2}\,dx}{4c} - \frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\,\sqrt{d}\,+dx\right)^2}\,dx}{4c} - \frac{d\int \frac{\cosh^{-1}(ax)}{-cd-d^2x^2}\,dx}{2c}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\int \frac{1}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(\sqrt{-\right.}}}{4c}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}\,-\sqrt{d}\,x}\,dx}{4(-c)^{3/2}} + \frac{\int \frac{\cosh^{-}}{\sqrt{-c}\,-}}{4(-}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}\,-\sqrt{d}}}{\sqrt{a\sqrt{-c}\,+\sqrt{d}}}\right.}{2c\sqrt{a\sqrt{-c}\,-\sqrt{d}}\,\sqrt{a\sqrt{-c}}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}\,-\sqrt{d}}}{\sqrt{a\sqrt{-c}\,+\sqrt{d}}}\right.}{2c\sqrt{a\sqrt{-c}\,-\sqrt{d}}\,\sqrt{a\sqrt{-c}}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}\,-\sqrt{d}}}{\sqrt{a\sqrt{-c}\,+\sqrt{d}}}\right.}{2c\sqrt{a\sqrt{-c}\,-\sqrt{d}}\,\sqrt{a\sqrt{-c}}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}\,-\sqrt{d}}}{\sqrt{a\sqrt{-c}\,+\sqrt{d}}}\right.}{2c\sqrt{a\sqrt{-c}\,-\sqrt{d}}\,\sqrt{a\sqrt{-c}}}$$

$$= -\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,-\sqrt{d}\,x\right)} + \frac{\cosh^{-1}(ax)}{4c\sqrt{d}\,\left(\sqrt{-c}\,+\sqrt{d}\,x\right)} + \frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}\,-\sqrt{d}}}{\sqrt{a\sqrt{-c}\,+\sqrt{d}}}\right.}{2c\sqrt{a\sqrt{-c}\,-\sqrt{d}}\,\sqrt{a\sqrt{-c}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.98, size = 687, normalized size = 0.89

$$\frac{2\sqrt{c}\left(\frac{\cosh^{-1}(ax)}{-i\sqrt{c}+\sqrt{d}\,x}+\frac{a\log\left(\frac{2d\left(i\sqrt{d}+a^2\sqrt{c}\,x-i\sqrt{-a^2c-d}\,\sqrt{-1+ax}\,\sqrt{1+ax}\right)}{a\sqrt{-a^2c-d}\left(\sqrt{c}+i\sqrt{d}\,x\right)}\right)}{\sqrt{-a^2c-d}}\right)-2\sqrt{c}\left(-\frac{\cosh^{-1}(ax)}{i\sqrt{c}+\sqrt{d}\,x}-\frac{a\log\left(\frac{2d\left(-\sqrt{d}-ia^2\sqrt{c}\,x+\sqrt{-a^2c-d}\,\sqrt{-1+ax}\,\sqrt{1+ax}\right)}{a\sqrt{-a^2c-d}\left(i\sqrt{c}+\sqrt{d}\,x\right)}\right)}{\sqrt{-a^2c-d}}\right)+i\left(\cosh^{-1}(ax)\left(-\cosh^{-1}(ax)+2\left(\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{ia\sqrt{c}-\sqrt{-a^2c-d}}\right)+\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{ia\sqrt{c}+\sqrt{-a^2c-d}}\right)\right)\right)+2\,\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-ia\sqrt{c}+\sqrt{-a^2c-d}}\right)+2\,\text{PolyLog}\left(2,-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{ia\sqrt{c}+\sqrt{-a^2c-d}}\right)\right)-i\left(\cosh^{-1}(ax)\left(-\cosh^{-1}(ax)+2\left(\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-ia\sqrt{c}+\sqrt{-a^2c-d}}\right)+\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{ia\sqrt{c}+\sqrt{-a^2c-d}}\right)\right)\right)+2\,\text{PolyLog}\left(2,-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-ia\sqrt{c}+\sqrt{-a^2c-d}}\right)+2\,\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{ia\sqrt{c}+\sqrt{-a^2c-d}}\right)\right)}{8c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*(ArcCosh[a*x]/((-I)*Sqrt[c] + Sqrt[d]*x) + (a*Log[(2*d*(I*Sqrt[d
] + a^2*Sqrt[c]*x - I*Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*
Sqrt[-(a^2*c) - d]*(Sqrt[c] + I*Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) - 2*Sqrt[
c]*(-(ArcCosh[a*x]/(I*Sqrt[c] + Sqrt[d]*x)) - (a*Log[(2*d*(-Sqrt[d] - I*a^2
*Sqrt[c]*x + Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*Sqrt[-(a^
2*c) - d]*(I*Sqrt[c] + Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) + I*(ArcCosh[a*x]*
(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] - Sqrt[-(
a^2*c) - d])] + Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*
c) - d])])) + 2*PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[
-(a^2*c) - d])] + 2*PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sq
rt[-(a^2*c) - d]))]) - I*(ArcCosh[a*x]*(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]
*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d])] + Log[1 - (Sqrt[d]*
E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])])) + 2*PolyLog[2, -((Sqr
t[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d]))] + 2*PolyLog[2,
 (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])]))/(8*c^(3/2)*
Sqrt[d])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 100.22, size = 790, normalized size = 1.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*arccosh(a*x)*a^3*x/c/(a^2*d*x^2+a^2*c)+1/4/c*a^2*sum(_R1/(_R1^2*d+
2*a^2*c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilo
g((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*
d)*_Z^2+d))+1/2*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*(2*(a^2*c*
(a^2*c+d))^(1/2)*a^2*c+2*a^4*c^2+2*a^2*c*d+(a^2*c*(a^2*c+d))^(1/2)*d)*a^2*a
rctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(
1/2)-d)*d)^(1/2))/c/(a^2*c+d)/d^3-1/2*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+
d)*d)^(1/2)*(2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*arctanh(d*(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))*a^2/c/
d^3+1/2*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*(-2*(a^2*c*(a^2*c+d
))^(1/2)*a^2*c+2*a^4*c^2+2*a^2*c*d-(a^2*c*(a^2*c+d))^(1/2)*d)*a^2*arctan(d*
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)
^(1/2))/c/(a^2*c+d)/d^3-1/2*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)
*(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^
```

```
(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))*a^2/c/d^3-1/4/c*a^2
*sum(1/_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x
+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootO
f(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(arccosh(a*x)/(d*x^2 + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] integral(arccosh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{(c + dx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**2,x)
[Out] Integral(acosh(a*x)/(c + d*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(arccosh(a*x)/(d*x^2 + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}(a\,x)}{{\left(d\,x^2+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^2,x)
[Out] int(acosh(a*x)/(c + d*x^2)^2, x)
```

3.47 $\int \sqrt{c+dx^2} \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\sqrt{c+dx^2} \cosh^{-1}(ax), x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax)\,dx = \int \sqrt{c+dx^2} \cosh^{-1}(ax)\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c}\,\operatorname{arccosh}(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
[Out] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \operatorname{acosh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acosh(a*x),x)
[Out] Integral(sqrt(c + d*x**2)*acosh(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{acosh}(a\,x)\,\sqrt{d\,x^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^(1/2), x)
```

3.48 $\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{arccosh}(ax)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}{\left(ax \right)}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/sqrt(c + d*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acosh}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(1/2),x)
[Out] int(acosh(a*x)/(c + d*x^2)^(1/2), x)
```

3.49 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{-1+a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{-1+a^2x^2}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{-1+ax}\,\sqrt{1+ax}}$$

```
[Out] -arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c/d
^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+x*arccosh(a*x)/c/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {197, 5908, 12, 533, 455, 65, 223, 212}

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x])/(c*Sqrt[c + d*x^2]) - (Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]
*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(c*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1
 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{3/2}}\,dx &= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}\,dx \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x}{\sqrt{-1+a^2x^2}\,\sqrt{c+dx^2}}\,dx}{c\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-1+a^2x}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{2c\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+\frac{d}{a^2}+\frac{dx^2}{a^2}}}\,dx, x, \sqrt{-1+a^2x^2}\right)}{ac\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a^2}}\,dx, x, \frac{\sqrt{-1+a^2x^2}}{\sqrt{c+dx^2}}\right)}{ac\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{-1+a^2x^2}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{-1+ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 12.30, size = 551, normalized size = 5.74

$$
\frac{x\cosh^{-1}(ax) + \frac{2(-1+ax)^{3/2}\sqrt{\frac{\left(a\sqrt{c}-i\sqrt{d}\right)(1+ax)}{\left(a\sqrt{c}+i\sqrt{d}\right)(-1+ax)}}\left(\frac{a\left(-ia\sqrt{c}+\sqrt{d}\right)\left(i\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{1+\frac{ia\sqrt{c}}{\sqrt{d}}-ax+\frac{i\sqrt{d}x}{\sqrt{c}}}{1-ax}}\,F\left(\mathrm{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{-1+ax}+a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{(a^2c+d)(c+dx^2)}{cd(-1+ax)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{a\sqrt{c}+i\sqrt{d}};\mathrm{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)\right)}{a(a^2c+d)\sqrt{1+ax}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}}}{c\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x] + (2*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1 + a*
x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*((a*((-I)*a*Sqrt[c] + Sqrt[d])*(I
*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*x + (I*Sqrt[d]*x)
/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] +
a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*S
```

```
qrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*S
qrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((-1 + (I*Sqrt[d]*x
)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*EllipticPi[(2*a*Sqrt[c
])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*(
(I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt
[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-((-1 + (I*Sqrt[d]*
x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(c*Sqrt[c + d*x^2])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{(d\,x^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.37, size = 296, normalized size = 3.08

$$\left[\frac{4\,\sqrt{dx^2+c}\,dx\log\left(ax+\sqrt{a^2x^2-1}\right)+(dx^2+c)\sqrt{d}\,\log\left(8\,a^4d^2x^4+a^4c^2-6\,a^2cd+8\,(a^4cd-a^2d^2)x^2-4\,(2\,a^3dx^2+a^3c-ad)\sqrt{a^2x^2-1}\,\sqrt{dx^2+c}\,\sqrt{d}+d^2\right)}{4\,(cd^2x^2+c^2d)},\ \frac{2\,\sqrt{dx^2+c}\,dx\log\left(ax+\sqrt{a^2x^2-1}\right)+(dx^2+c)\sqrt{-d}\,\arctan\left(\frac{(2\,a^2dx^2+a^2c-d)\sqrt{a^2x^2-1}\,\sqrt{dx^2+c}\,\sqrt{-d}}{2\,(a^3d^2x^4-acd+(a^3cd-ad^2)x^2)}\right)}{2\,(cd^2x^2+c^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2 - 1)) + (d*x^2 + c)*sqrt
(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^2 - 4
*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d) + d^
2))/(c*d^2*x^2 + c^2*d), 1/2*(2*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2
- 1)) + (d*x^2 + c)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d)*sqrt(a^2*
```

```
x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*d - a*d^2)*
x^2)))/(c*d^2*x^2 + c^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [**A**]
time = 0.43, size = 82, normalized size = 0.85

$$\frac{x\log\left(ax+\sqrt{a^2x^2-1}\right)}{\sqrt{dx^2+c}\,c}+\frac{a\log\left(\left|-\sqrt{a^2x^2-1}\,\sqrt{d}+\sqrt{a^2c+(a^2x^2-1)d+d}\right|\right)}{c\sqrt{d}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))/(sqrt(d*x^2 + c)*c) + a*log(abs(-sqrt(a^2*x^
2 - 1)*sqrt(d) + sqrt(a^2*c + (a^2*x^2 - 1)*d + d)))/(c*sqrt(d)*abs(a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(3/2), x)
```

3.50 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=180

$$\frac{a(1-a^2x^2)}{3c\,(a^2c+d)\,\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}-\frac{2\sqrt{-1+a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{-1+a^2x^2}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\,\sqrt{-1+ax}\,\sqrt{1+ax}}$$

```
[Out] 1/3*x*arccosh(a*x)/c/(d*x^2+c)^(3/2)-2/3*arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/
a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c^2/d^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2
)+2/3*x*arccosh(a*x)/c^2/(d*x^2+c)^(1/2)+1/3*a*(-a^2*x^2+1)/c/(a^2*c+d)/(a*
x-1)^(1/2)/(a*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {198, 197, 5908, 12, 533, 585, 79, 65, 223, 212}

$$-\frac{2\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{a(1-a^2x^2)}{3c\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)\sqrt{c+dx^2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(3*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*
x^2]) + (x*ArcCosh[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCosh[a*x])/(3*c^
2*Sqrt[c + d*x^2]) - (2*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x
^2])/(a*Sqrt[c + d*x^2])])/(3*c^2*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx = \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x(3c+2dx^2)}{3c^2\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}}\,dx$$

$$= \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x(3c+2dx^2)}{\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x(3c+2dx^2)}{\sqrt{-1+a^2x^2}\,(c+dx^2)^{3/2}}\,dx}{3c^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\Big(\int \frac{3c+2dx}{\sqrt{-1+a^2x}\,(c+dx)^{3/2}}\,dx,}{6c^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{3c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\big(a\sqrt{}}{}$$

$$= \frac{a(1-a^2x^2)}{3c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\big(2\sqrt{}}{}$$

$$= \frac{a(1-a^2x^2)}{3c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\big(2\sqrt{}}{}$$

$$= \frac{a(1-a^2x^2)}{3c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\,(c+dx^2)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{2\sqrt{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 1.79, size = 609, normalized size = 3.38

$$\frac{-\frac{ac\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)}{a^2c+d}+x(3c+2dx^2)\cosh^{-1}(ax)+\frac{4(-1+ax)^{3/2}\sqrt{\frac{(a\sqrt{c}-i\sqrt{d})(1+ax)}{(a\sqrt{c}+i\sqrt{d})(-1+ax)}}(c+dx^2)\left(\frac{a(-ia\sqrt{c}+\sqrt{d})(i\sqrt{c}+\sqrt{d}x)\sqrt{\frac{1+\frac{ia\sqrt{c}}{\sqrt{d}}-ax+\frac{i\sqrt{d}x}{\sqrt{c}}}{1-ax}}\,F\left(\operatorname{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{(a\sqrt{c}+i\sqrt{d})^2}\right)}{-1+ax}+a\sqrt{c}(-a\sqrt{c}+i\sqrt{d})\sqrt{\frac{(a^2c+d)(c+dx^2)}{cd(-1+ax)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{a\sqrt{c}+i\sqrt{d}};\operatorname{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{(a\sqrt{c}+i\sqrt{d})^2}\right)\right)}{a(a^2c+d)\sqrt{1+ax}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}}}{3c^2(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (-((a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2))/(a^2*c + d)) + x*(3*c + 2
*d*x^2)*ArcCosh[a*x] + (4*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)*((a*((-I)*a*Sqrt
[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*
x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt
[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt
[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c
]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((
-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ellip
ticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]
*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]
*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-(
(-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(3
*c^2*(c + d*x^2)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{(d\,x^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(147) = 294$.
time = 0.40, size = 613, normalized size = 3.41

$$\left[\frac{(a^2c^3 + (a^2cd^2 + d^3)x^4 + c^2d + 2\,(a^2c^2d + cd^2)x^2)\sqrt{d}\,\log\left(8\,a^4d^2x^4 + a^4c^2 - 6\,a^2cd + 8\,(a^4cd - a^2d^2)x^2 - 4\,(2\,a^3dx^2 + a^3c - ad)\sqrt{a^2x^2 - 1}\,\sqrt{dx^2 + c}\,\sqrt{d} + d^2\right) + 2\,(2\,(a^2cd^2 + d^3)x^3 + 3\,(a^2c^2d + cd^2)x)\sqrt{dx^2 + c}\,\log\left(ax + \sqrt{a^2x^2 - 1}\right) - 2\,(acd^2x^2 + ac^2d)\sqrt{a^2x^2 - 1}\,\sqrt{dx^2 + c}}{6\,(a^2c^5d + c^4d^2 + (a^2c^3d^3 + c^2d^4)x^4 + 2\,(a^2c^4d^2 + c^3d^3)x^2)}, \frac{(a^2c^3 + (a^2cd^2 + d^3)x^4 + c^2d + 2\,(a^2c^2d + cd^2)x^2)\sqrt{-d}\,\arctan\left(\frac{(2\,a^2dx^2 + a^2c - d)\sqrt{a^2x^2 - 1}\,\sqrt{dx^2 + c}\,\sqrt{-d}}{2\,(a^3d^2x^4 - acd + (a^3cd - ad^2)x^2)}\right) + (2\,(a^2cd^2 + d^3)x^3 + 3\,(a^2c^2d + cd^2)x)\sqrt{dx^2 + c}\,\log\left(ax + \sqrt{a^2x^2 - 1}\right) - (acd^2x^2 + ac^2d)\sqrt{a^2x^2 - 1}\,\sqrt{dx^2 + c}}{3\,(a^2c^5d + c^4d^2 + (a^2c^3d^3 + c^2d^4)x^4 + 2\,(a^2c^4d^2 + c^3d^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2*d + 2*(a^2*c^2*d + c*d^2)*x^2)
*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^
2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d)
 + d^2) + 2*(2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqrt(d*x^2
+ c)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a*c*d^2*x^2 + a*c^2*d)*sqrt(a^2*x^2
- 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*x^4 +
2*(a^2*c^4*d^2 + c^3*d^3)*x^2), 1/3*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2
*d + 2*(a^2*c^2*d + c*d^2)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c -
d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c
*d - a*d^2)*x^2)) + (2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqr
t(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (a*c*d^2*x^2 + a*c^2*d)*sqrt(a^
2*x^2 - 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*
x^4 + 2*(a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(ax\right)}{\left(c + dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [A]
time = 0.52, size = 190, normalized size = 1.06

$$-\frac{1}{3}\,a\left(\frac{2\,a^2|d|}{\left(a^2cd + \left(\sqrt{a^2d}\,\sqrt{dx^2 + c} - \sqrt{(dx^2 + c)a^2d - a^2cd - d^2}\right)^2 + d^2\right)c\sqrt{d}\,|a|} - \frac{|d|\log\left(\left(\sqrt{a^2d}\,\sqrt{dx^2 + c} - \sqrt{(dx^2 + c)a^2d - a^2cd - d^2}\right)^2\right)}{c^2d^{\frac{3}{2}}|a|}\right) + \frac{x\left(\frac{2\,dx^2}{c^2} + \frac{3}{c}\right)\log\left(ax + \sqrt{a^2x^2 - 1}\right)}{3\,(dx^2 + c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*a*(2*a^2*abs(d)/((a^2*c*d + (sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2
 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)*c*sqrt(d)*abs(a)) - abs(d)*log((sqrt
(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2)/(c^2*
d^(3/2)*abs(a))) + 1/3*x*(2*d*x^2/c^2 + 3/c)*log(a*x + sqrt(a^2*x^2 - 1))/(
d*x^2 + c)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(5/2), x)
```

3.51 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=269

$$\frac{a(1-a^2x^2)}{15c\,(a^2c+d)\,\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{3/2}}+\frac{2a(3a^2c+2d)\,(1-a^2x^2)}{15c^2\,(a^2c+d)^2\,\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(\ldots}{5c\,(c+dx\ldots}$$

```
[Out] 1/5*x*arccosh(a*x)/c/(d*x^2+c)^(5/2)+4/15*x*arccosh(a*x)/c^2/(d*x^2+c)^(3/2
)+1/15*a*(-a^2*x^2+1)/c/(a^2*c+d)/(d*x^2+c)^(3/2)/(a*x-1)^(1/2)/(a*x+1)^(1/
2)-8/15*arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1
/2)/c^3/d^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+8/15*x*arccosh(a*x)/c^3/(d*x^2+
c)^(1/2)+2/15*a*(3*a^2*c+2*d)*(-a^2*x^2+1)/c^2/(a^2*c+d)^2/(a*x-1)^(1/2)/(a
*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {198, 197, 5908, 12, 533, 6847, 963, 79, 65, 223, 212}

$$-\frac{8\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{15c^3\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2a(1-a^2x^2)\,(3a^2c+2d)}{15c^2\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)^2\,\sqrt{c+dx^2}}+\frac{a(1-a^2x^2)}{15c\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)\,(c+dx^2)^{3/2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}+\frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(15*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(3/2)) + (2*a*(3*a^2*c + 2*d)*(1 - a^2*x^2))/(15*c^2*(a^2*c + d)^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(5*c*(c + d*x^2)
^(5/2)) + (4*x*ArcCosh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCosh[a*x]
)/(15*c^3*Sqrt[c + d*x^2]) - (8*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1
 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(15*c^3*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x(15c^2+20cdx^2+}{15c^3\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x(15c^2+20cdx^2+8d^2x^4)}{\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx}}{15c^3}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x(15c^2+20}{\sqrt{-1+a}}}{15c^3\sqrt{-1+ax}\,\sqrt{1}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{30c^3\sqrt{-1+a}}$$

$$= \frac{a(1-a^2x^2)}{15c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}} + \frac{x\cosh^{-1}(ax)}{5c\,(c+dx^2)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\,(c+dx^2)^{3/2}}$$

$$= \frac{a(1-a^2x^2)}{15c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}} + \frac{2a(3a^2c+2d)\,(1-a^2x^2)}{15c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{15c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}} + \frac{2a(3a^2c+2d)\,(1-a^2x^2)}{15c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{15c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}} + \frac{2a(3a^2c+2d)\,(1-a^2x^2)}{15c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{15c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}} + \frac{2a(3a^2c+2d)\,(1-a^2x^2)}{15c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.52, size = 655, normalized size = 2.43

$$\frac{-\frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)\left(d(5c+4dx^2)+a^2c(7c+6dx^2)\right)}{c^2(a^2c+d)^2} + \frac{x(15c^2+20cdx^2+8d^2x^4)\cosh^{-1}(ax)}{c^3} + \frac{16(-1+ax)^{3/2}\sqrt{\frac{\left(a\sqrt{c}-i\sqrt{d}\right)(1+ax)}{\left(a\sqrt{c}+i\sqrt{d}\right)(-1+ax)}}\,(c+dx^2)^2\left(\frac{a\left(-ia\sqrt{c}+\sqrt{d}\right)\left(i\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{1+\frac{ia\sqrt{c}}{\sqrt{d}}-ax+\frac{i\sqrt{d}x}{\sqrt{c}}}{1-ax}}\,F\left(\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)}{-1+ax} + a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{(a^2c+d)(c+dx^2)}{cd(-1+ax)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{a\sqrt{c}+i\sqrt{d}};\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{\left(a\sqrt{c}+i\sqrt{d}\right)^2}\right)\right)}{ac^3(a^2c+d)\sqrt{1+ax}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}}}{15\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (-((a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(
7*c + 6*d*x^2)))/(c^2*(a^2*c + d)^2)) + (x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4
)*ArcCosh[a*x])/c^3 + (16*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)^2*((a*((-I)*a*Sq
rt[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] -
a*x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sq
rt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sq
rt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt
[c]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-
((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ell
ipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[
d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[
c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*c^3*(a^2*c + d)*Sqrt[1 + a*x]*S
qrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))
]))/(15*(c + d*x^2)^(5/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{(d\,x^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(223) = 446.
time = 0.45, size = 1098, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^6 + c
^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d + 2*a^2
*c^3*d^2 + c^2*d^3)*x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d +
8*(a^4*c*d - a^2*d^2)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)
*sqrt(d*x^2 + c)*sqrt(d) + d^2) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5
+ 20*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*
d^2 + c^2*d^3)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4
*d + 5*a*c^3*d^2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*
a*c^2*d^3)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d
^2 + c^6*d^3 + (a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3
 + 2*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)
*x^2), 1/15*(4*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x
^6 + c^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d +
 2*a^2*c^3*d^2 + c^2*d^3)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d
)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*
d - a*d^2)*x^2)) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5 + 20*(a^4*c^3*d
^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*d^2 + c^2*d^3)*
x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4*d + 5*a*c^3*d^
2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*a*c^2*d^3)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d^2 + c^6*d^3 +
(a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3 + 2*a^2*c^5*d^
4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(7/2), x)
```

Giac [A]
time = 0.58, size = 411, normalized size = 1.53

$$\frac{4}{15}\,a\left(\frac{|d|\log\left(\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{(dx^2+c)a^2d-a^2cd-d^2}\right)^2\right)}{c^3d^{\frac{3}{2}}|a|}-\frac{3\,a^6c^2d^{\frac{5}{2}}|d|+7\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{(dx^2+c)a^2d-a^2cd-d^2}\right)^2a^4cd^{\frac{3}{2}}|d|+5\,a^4cd^{\frac{7}{2}}|d|+2\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{(dx^2+c)a^2d-a^2cd-d^2}\right)^4a^2\sqrt{d}\,|d|+4\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{(dx^2+c)a^2d-a^2cd-d^2}\right)^2a^2d^{\frac{5}{2}}|d|+2\,a^2d^{\frac{9}{2}}|d|}{\left(a^2cd+\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{(dx^2+c)a^2d-a^2cd-d^2}\right)^2+d^2\right)^3c^2d|a|}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2-1}\right)}{15\,(dx^2+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 4/15*a*(abs(d)*log((sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d -
a^2*c*d - d^2))^2)/(c^3*d^(3/2)*abs(a)) - (3*a^6*c^2*d^(5/2)*abs(d) + 7*(sq
rt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^4*
c*d^(3/2)*abs(d) + 5*a^4*c*d^(7/2)*abs(d) + 2*(sqrt(a^2*d)*sqrt(d*x^2 + c)
- sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^2*sqrt(d)*abs(d) + 4*(sqrt(a
^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^2*d^(5
/2)*abs(d) + 2*a^2*d^(9/2)*abs(d))/((a^2*c*d + (sqrt(a^2*d)*sqrt(d*x^2 + c)
 - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)^3*c^2*d*abs(a))) + 1/1
5*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(
d*x^2 + c)^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{{\left(d\,x^2+c\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(7/2), x)
```

3.52 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=369

$$\frac{a(1-a^2x^2)}{35c\,(a^2c+d)\,\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}}+\frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\,\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{3/2}}+\frac{4}{105c^3}$$

```
[Out] 1/7*x*arccosh(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccosh(a*x)/c^2/(d*x^2+c)^(5/2
)+8/35*x*arccosh(a*x)/c^3/(d*x^2+c)^(3/2)+1/35*a*(-a^2*x^2+1)/c/(a^2*c+d)/(
d*x^2+c)^(5/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/105*a*(5*a^2*c+3*d)*(-a^2*x^2+
1)/c^2/(a^2*c+d)^2/(d*x^2+c)^(3/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-16/35*arctan
h(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c^4/d^(1/2
)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+16/35*x*arccosh(a*x)/c^4/(d*x^2+c)^(1/2)+4/10
5*a*(11*a^4*c^2+15*a^2*c*d+6*d^2)*(-a^2*x^2+1)/c^3/(a^2*c+d)^3/(a*x-1)^(1/2
)/(a*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {198, 197, 5908, 12, 533, 6847, 1636, 963, 79, 65, 223, 212}

$$-\frac{16\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{35c^4\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2a(1-a^2x^2)\,(5a^2c+3d)}{105c^2\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)^2\,(c+dx^2)^{3/2}}+\frac{a(1-a^2x^2)}{35c\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)\,(c+dx^2)^{5/2}}+\frac{4a(1-a^2x^2)\,(11a^4c^2+15a^2cd+6d^2)}{105c^3\sqrt{ax-1}\,\sqrt{ax+1}\,(a^2c+d)^3\,\sqrt{c+dx^2}}+\frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\cosh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}}+\frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}}+\frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(35*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(5/2)) + (2*a*(5*a^2*c + 3*d)*(1 - a^2*x^2))/(105*c^2*(a^2*c + d)^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)^(3/2)) + (4*a*(11*a^4*c^2 + 15*a^2*c*d
+ 6*d^2)*(1 - a^2*x^2))/(105*c^3*(a^2*c + d)^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCosh
[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCosh[a*x])/(35*c^3*(c + d*x^2)^
(3/2)) + (16*x*ArcCosh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (16*Sqrt[-1 + a^2*x
^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(35*c^4*Sqrt
[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 533

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
), Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 963

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1636

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && GtQ[Expon[Px, x], 2
]
```

Rule 5908

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x(35c^3}{35c^4\sqrt{}}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x(35c^3}{\sqrt{-1+}}}{}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+}\right.}{}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\,(c+dx^2)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+}\right.}{}$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{x\cosh^{-1}(ax)}{7c\,(c+dx^2)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\,(c+dx^2)^{5/2}} +$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a(1-a^2x^2)}{35c\,(a^2c+d)\sqrt{-1+ax}\,\sqrt{1+ax}\,(c+dx^2)^{5/2}} + \frac{2a(5a^2c+3d)\,(1-a^2x^2)}{105c^2\,(a^2c+d)^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 4.98, size = 723, normalized size = 1.96

$$\frac{-\frac{a\sqrt{-1+ax}\sqrt{1+ax}\,(c+dx^2)\left(3d^2(11c^2+18cdx^2+8d^2x^4)+2a^2cd(41c^2+68cdx^2+30d^2x^4)+a^4c^2(57c^2+98cdx^2+44d^2x^4)\right)}{3c^3(a^2c+d)^3}+\frac{x(35c^3+70c^2dx^2+56cd^2x^4+16d^3x^6)\cosh^{-1}(ax)}{c^4}+\frac{32(-1+ax)^{3/2}\sqrt{\frac{(a\sqrt{c}-i\sqrt{d})(1+ax)}{(a\sqrt{c}+i\sqrt{d})(-1+ax)}}\,(c+dx^2)^3\left(\frac{a(-ia\sqrt{c}+\sqrt{d})(i\sqrt{c}+\sqrt{d}x)\sqrt{\frac{1+\frac{ia\sqrt{c}}{\sqrt{d}}-ax+\frac{i\sqrt{d}x}{\sqrt{c}}}{1-ax}}\,F\left(\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{(a\sqrt{c}+i\sqrt{d})^2}\right)}{-1+ax}+a\sqrt{c}(-a\sqrt{c}+i\sqrt{d})\sqrt{\frac{(a^2c+d)(c+dx^2)}{cd(-1+ax)^2}}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{a\sqrt{c}+i\sqrt{d}};\text{ArcSin}\left(\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{2-2ax}}\right)\middle|\frac{4ia\sqrt{c}\sqrt{d}}{(a\sqrt{c}+i\sqrt{d})^2}\right)\right)}{ac^4(a^2c+d)\sqrt{1+ax}\sqrt{-\frac{-1+\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)}{1-ax}}}}{35(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (-1/3*(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)*(3*d^2*(11*c^2 + 18*c*d*x
^2 + 8*d^2*x^4) + 2*a^2*c*d*(41*c^2 + 68*c*d*x^2 + 30*d^2*x^4) + a^4*c^2*(5
7*c^2 + 98*c*d*x^2 + 44*d^2*x^4)))/(c^3*(a^2*c + d)^3) + (x*(35*c^3 + 70*c^
2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*ArcCosh[a*x])/c^4 + (32*(-1 + a*x)^(3/
2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 +
a*x))]*(c + d*x^2)^3*((a*((-I)*a*Sqrt[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)
*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*
EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d
] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2
])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c +
 d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqr
t[c])/Sqrt[d] + x))/(1 - a*x))]*EllipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqr
t[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] +
 x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))
/(a*c^4*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((
I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(35*(c + d*x^2)^(7/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\left(d\,x^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(310) = 620$.

time = 0.61, size = 1752, normalized size = 4.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
[Out] [1/105*(12*(a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^
2*d^5 + 3*a^2*c*d^6 + d^7)*x^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 +
 3*a^2*c^2*d^5 + c*d^6)*x^6 + 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^
4 + c^2*d^5)*x^4 + 4*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*
x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2
)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqr
t(d) + d^2) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x^7 +
 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^6*c^5
*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^6*c^6*d + 3*a^4
*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x
^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*c^4*d^3 + 4*(11*a^5*c^3*d^
4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c^4*d^3 + 98*a^3*c^3*d^4 +
39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d + 3*a^4*c^10*d^2 + 3*a^2*c^
9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 + 3*a^2*c^5*d^7 + c^4*d^8)*x
^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^6
*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^6*c^10*d^2 +
 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/105*(24*(a^6*c^7 + 3*a^4*
c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x
^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^6
+ 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^4 + 4*(a^6*c^
6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x^2)*sqrt(-d)*arctan(1/2*(2*
a^2*d*x^2 + a^2*c - d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*
x^4 - a*c*d + (a^3*c*d - a*d^2)*x^2)) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5
+ 3*a^2*c*d^6 + d^7)*x^7 + 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5
+ c*d^6)*x^5 + 70*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x
^3 + 35*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2
 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*
c^4*d^3 + 4*(11*a^5*c^3*d^4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c
^4*d^3 + 98*a^3*c^3*d^4 + 39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^
4*d^3 + 87*a*c^3*d^4)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d +
 3*a^4*c^10*d^2 + 3*a^2*c^9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 +
3*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d
^6 + c^5*d^7)*x^6 + 6*(a^6*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^
6)*x^4 + 4*(a^6*c^10*d^2 + 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 876 vs. 2(310) = 620.
time = 0.72, size = 876, normalized size = 2.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105*a*(3*abs(d)*log((sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d
 - a^2*c*d - d^2))^2)/(c^4*d^(3/2)*abs(a)) - (11*a^10*c^4*d^(9/2)*abs(d) +
49*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^
2*a^8*c^3*d^(7/2)*abs(d) + 37*a^8*c^3*d^(11/2)*abs(d) + 77*(sqrt(a^2*d)*sqr
t(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^6*c^2*d^(5/2)*a
bs(d) + 112*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d
 - d^2))^2*a^6*c^2*d^(9/2)*abs(d) + 47*a^6*c^2*d^(13/2)*abs(d) + 33*(sqrt(a
^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^6*a^4*c*d^
(3/2)*abs(d) + 93*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a
^2*c*d - d^2))^4*a^4*c*d^(7/2)*abs(d) + 87*(sqrt(a^2*d)*sqrt(d*x^2 + c) - s
qrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^4*c*d^(11/2)*abs(d) + 27*a^4*c*
d^(15/2)*abs(d) + 6*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d -
 a^2*c*d - d^2))^8*a^2*sqrt(d)*abs(d) + 24*(sqrt(a^2*d)*sqrt(d*x^2 + c) - s
qrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^6*a^2*d^(5/2)*abs(d) + 36*(sqrt(a^2
*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^2*d^(9/2
)*abs(d) + 24*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c
*d - d^2))^2*a^2*d^(13/2)*abs(d) + 6*a^2*d^(17/2)*abs(d))/((a^2*c*d + (sqrt
(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)
^5*c^3*d*abs(a))) + 1/35*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*
x^2 + 35/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(d*x^2 + c)^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(9/2), x)
```

3.53 $\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=713

$$\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bfg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcf^2g}{3\sqrt{-1}}$$

```
[Out] 1/2*f^3*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/8*f*g^2*x*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2
+d)^(1/2)-f^2*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^
2-2/15*g^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-1/5
*g^3*x^2*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+b*f^2
*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/15*b*g^3*x*(-c^2*
d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f^3*x^2*(-c^2*d*x^2+
d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/16*b*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/
c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+1/45*b*g^3*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-3/16*b*c*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/25*b*c*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4
*f^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-3/16*f*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.96, antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.419$, Rules used = {5972, 5975, 5896, 5893, 30, 5915, 41, 5927, 5939, 102, 12, 75, 5923}

$$\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{f^2g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2}-\frac{3fg^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{g^3x^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}-\frac{2g^3(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{3fg^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bfg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bcfg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcg^3x^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bg^3x^3\sqrt{d-c^2dx^2}}{45c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*g^3
*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^3*x^
2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*f*g^2*x^2*Sq
rt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^2*g*x^3*Sqr
t[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^3*x^3*Sqrt[d - c^
2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*f*g^2*x^4*Sqrt[d - c
^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^3*x^5*Sqrt[d - c^2*d*
x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8
*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (f^2*g*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/c^2 - (2*g^3*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (g^
```

```
3*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
 - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5923

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(d1 + e1*x
)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c*Simp[Sqrt
[d1 + e1*x]*(Sqrt[d2 + e2*x]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], Int[Simplify
Integrand[u/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x], x]] /; FreeQ[{a, b,
c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IntegerQ[p -
 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]
)
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
```

```
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^3\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int\left(f^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(f^3\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2gx^3}{3\sqrt{-1+}}$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3}{4\sqrt{-}}$$

Mathematica [**A**]
time = 1.19, size = 491, normalized size = 0.69

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (240*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-16*g^3 -
c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*
f*g^2*x^2 + 4*g^3*x^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
c^2*x^2))] + 2400*b*c^2*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c
^3*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[
c*x] - Sinh[2*ArcCosh[c*x]])) - 675*b*c*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCos
h[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + 8*
b*g^3*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*Arc
Cosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28
800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. 2(613) = 1226.
time = 7.53, size = 1430, normalized size = 2.01

method	result	size
default	Expression too large to display	1430

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/5*a*g^3*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15*a*g^3/d/c^4*(-c^2*d*x^2+d)^(
3/2)-3/4*a*f*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^2*x*(-c^2*d*x^2
+d)^(1/2)+3/8*a*f*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^
2+d)^(1/2))-a*f^2*g*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f^3*x*(-c^2*d*x^2+d)^(
1/2)+1/2*a*f^3*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
+b*(-1/16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*f*arccosh(
c*x)^2*(4*c^2*f^2+3*g^2)+1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^
4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*g^3*(-1+5*arccosh(c*x)
)/(c*x+1)/c^4/(c*x-1)+3/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*g^2*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*
x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(36*arccosh(c*x)*c
^2*f^2-12*c^2*f^2+3*arccosh(c*x)*g^2-g^2)/(c*x+1)/c^4/(c*x-1)+1/16*(-d*(c^2
*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-
1)^(1/2)*(c*x+1)^(1/2))*f^3*(2*arccosh(c*x)-1)/(c*x+1)/c/(c*x-1)-1/16*(-d*(
c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(6*arccosh(
c*x)*c^2*f^2-6*c^2*f^2+arccosh(c*x)*g^2-g^2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(6*arccosh
(c*x)*c^2*f^2+6*c^2*f^2+arccosh(c*x)*g^2+g^2)/(c*x+1)/c^4/(c*x-1)+1/16*(-d*
(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1
)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f^3*(2*arccosh(c*x)+1)/(c*x+1)/c/(c*x-1)+1/288
*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*g*(36*arccosh(c*x)*c^2*f^2+12
*c^2*f^2+3*arccosh(c*x)*g^2+g^2)/(c*x+1)/c^4/(c*x-1)+3/256*(-d*(c^2*x^2-1))
^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*f*g^2*(1+
4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*g^3*(1+5*a
rccosh(c*x))/(c*x+1)/c^4/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^3 - 1/15*a*g^3*(3*
(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 3/
8*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d)
+ sqrt(d)*arcsin(c*x)/c^3) - (-c^2*d*x^2 + d)^(3/2)*a*f^2*g/(c^2*d) + integ
rate(sqrt(-c^2*d*x^2 + d)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
+ 3*sqrt(-c^2*d*x^2 + d)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
 + 3*sqrt(-c^2*d*x^2 + d)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
+ sqrt(-c^2*d*x^2 + d)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))\,(f+gx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.54 $\int (f+gx)^2 \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=479

$$\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcg^2x^4}{16\sqrt{-1}}$$

```
[Out] 1/2*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/8*g^2*x*(a+b*arccosh(c*
x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^
(1/2)-2/3*f*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+
2/3*b*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f^2*
x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*g^2*x^2*(-c^2*d
*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/9*b*c*f*g*x^3*(-c^2*d*x^2+d)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*c*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-1/4*f^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/
b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d
)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {5972, 5975, 5896, 5893, 30, 5915, 41, 5927, 5939}

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2fg(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}-\frac{g^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{g^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f
^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^2*x^2*S
qrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*f*g*x^3*Sq
rt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^2*x^4*Sqrt[d -
 c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (2*f*g*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2) - (f^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2\left(a+b\cosh\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(f^2\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcf}{9\sqrt{-}}$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bg}{16c\sqrt{}}$$

Mathematica [A]
time = 0.71, size = 356, normalized size = 0.74

$$\frac{48ac\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}\,(12c^2f^2x+16fg(-1+c^2x^2)+3g^2x(-1+2c^2x^2))-144a\sqrt{d}\,(4c^2f^2+g^2)\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+64bcfg\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh\left(3\cosh^{-1}(cx)\right)\right)-144bc^2f^2\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)-9bg^2\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)}{1152c^3\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(12*c^2*f^
2*x + 16*f*g*(-1 + c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*
c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x
 + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCos
h[c*x]]) - 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcC
osh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) - 9*b*g^2*Sqrt[d - c^2*d*x^
2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh
[c*x]]))/(1152*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(407) = 814$.
time = 9.10, size = 1001, normalized size = 2.09

method	result

default	$-\frac{a\,g^{2}x\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4c^{2}d}+\frac{a\,g^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{8c^{2}}+\frac{a\,g^{2}d\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{8c^{2}\sqrt{c^{2}d}}-\frac{2afg\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{3c^{2}d}+\frac{a\,f^{2}x\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/4*a*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2
)+1/8*a*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)-2/3*a*f*g*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f^2*x*(-c^2*d*x^2+d)^(1/2)+1/2
*a*f^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/1
6*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*(4*
c^2*f^2+g^2)+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*g^2*(-1+4*arccosh(c*x))/(c*x+1)/c^3/(c*x-1)+1/36*(
-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*f*g*(-1+3*arccosh(c*x))/(c*x+1)/
c^2/(c*x-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2*(2*arccosh(c*x)-1)/(c*
x+1)/c/(c*x-1)-1/4*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*f*g*(-1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/4*(-d*(c^2*x^2-1))^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(1+arccosh(c*x))/(c*x
+1)/c^2/(c*x-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f^2*(2*arccosh(c*x)+1
)/(c*x+1)/c/(c*x-1)+1/36*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*f*g*(
1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*g^2*(1+4*arccosh(c*x))/(
c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^2 + 1/8*a*g^2*(sqr
t(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcs
```

```
in(c*x)/c^3) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*f*g/(c^2*d) + integrate(sqrt(-c
^2*d*x^2 + d)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*sqrt(-c^
2*d*x^2 + d)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(-c^2*d*x
^2 + d)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.55 $\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=255

$$\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] 1/2*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/3*g*(-c*x+1)*(c*x+1)*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/3*b*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-1/9*b*c*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-1/4*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5972, 5975, 5896, 5893, 30, 5915, 41}

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}-\frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f*x^2
*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g*x^3*Sqrt[d
- c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x]))/2 - (g*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)+\right.}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{\left(f\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{2}fx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right) - \frac{g(1-cx)(1+cx)\sqrt{d}}{}$$

$$= \frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcgx^3}{9\sqrt{-1+}}$$

Mathematica [**A**]
time = 0.65, size = 251, normalized size = 0.98

$$\frac{12a\sqrt{d-c^2dx^2}\,(3c^2fx+2g(-1+c^2x^2)) - 36ac\sqrt{d}\,f\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right) + \frac{2bg\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh(3\cosh^{-1}(cx))\right)}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)} - \frac{9bcf\sqrt{d-c^2dx^2}\,(2\cosh^{-1}(cx)^2+\cosh(2\cosh^{-1}(cx))-2\cosh^{-1}(cx)\sinh(2\cosh^{-1}(cx)))}{\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)}}{72c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d]
*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sqrt
[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCos
h[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (9
*b*c*f*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*Arc
Cosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/(7
2*c^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(215) = 430$.
time = 7.60, size = 639, normalized size = 2.51

method	result
default	$-\frac{ag(-c^2d\,x^2+d)^{\frac{3}{2}}}{3c^2d} + \frac{afx\sqrt{-c^2d\,x^2+d}}{2} + \frac{afd\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}} + b\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,f\mathrm{arccosh}(}{4\sqrt{cx-1}\,\sqrt{cx+1}\,c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/3*a*g*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*
d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c
^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2+1/72*(-d*(c
^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*
x-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(2*arccosh(c*x)-1)/(c*x+1)/(c*x
-1)/c-1/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1
)*g*(-1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-
1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^
3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f*(2*arccosh(c*x)+1)/(c*x+1)/(c*x-
1)/c+1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*
c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*g*(1+3*arccosh(c*x))
/(c*x+1)/c^2/(c*x-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f - 1/3*(-c^2*d*x^2
+ d)^(3/2)*a*g/(c^2*d) + integrate(sqrt(-c^2*d*x^2 + d)*b*g*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + sqrt(-c^2*d*x^2 + d)*b*f*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x)), x
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))\,(f+gx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.56 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=785

$$-\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)}+\frac{b\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g}-\frac{cx\sqrt{d-c^2dx^2}\,(a+b\cosh}{2bg\sqrt{-1+cx}\,\sqrt{1+c}$$

```
[Out] a*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/g/(-c*x+1)/(c*x+1)+b*arccosh(c*x)*(-c^2
*d*x^2+d)^(1/2)/g-b*c*x*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-
1/2*c*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+1/2*(1-c^2*f^2/g^2)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(g
*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/b/c/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*arccosh(c*x)*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-
g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^
2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2
*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*poly
log(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*
f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-a*arcta
nh((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*f^2-g^2)^(1/2)*(
c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c*x+1)/(c*x+1)
```

Rubi [A]
time = 2.30, antiderivative size = 785, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.710$, Rules used = {5972, 5976, 697, 5970, 6874, 95, 214, 1624, 1668, 12, 739, 212, 5993, 5992, 5915, 8, 5980, 3401, 2296, 2221, 2317, 2438}

$$\frac{\sqrt{d-c^2dx^2}\left(1-\frac{c^2f^2}{g^2}\right)(a+b\cosh^{-1}(cx))^2}{2bc\sqrt{cx-1}\,\sqrt{cx+1}\,(f+gx)}-\frac{(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2bc\sqrt{cx-1}\,\sqrt{cx+1}\,(f+gx)}-\frac{cx\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2bg\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{a\sqrt{c^2x^2-1}\,\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\tanh^{-1}\left(\frac{c^2fx+g}{\sqrt{c^2x^2-1}\,\sqrt{c^2f^2-g^2}}\right)}{g^2(1-cx)(cx+1)}+\frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(cx+1)}+\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\mathrm{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\mathrm{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\cosh^{-1}(cx)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{g^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}\,\cosh^{-1}(cx)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}+1\right)}{g^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2])/(g*(1 - c*x)*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*
g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - ((
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2]*Sq
rt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*
```

```
x^2])])/(g^2*(1 - c*x)*(1 + c*x)) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*Pol
yLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5970

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5976

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[(f + g*x
```

```
)^m*(d1*d2 + e1*e2*x^2)*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*Sqrt[(-d1)*d2]*(
n + 1))), x] - Dist[1/(b*c*Sqrt[(-d1)*d2]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5992

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5993

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d-c^2dx^2}\,\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)} - \frac{\sqrt{d-c^2dx^2}\,\int \frac{(g+2c}{}}{2bc\sqrt{-1}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\,(a+}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(}{}$$

$$= \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2d}}{2bg\sqrt{-}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a(1-c^2x^2)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}}{g}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.63, size = 1121, normalized size = 1.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d - c^2*d*x^2] - 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[f + g
*x] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt
[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((2*c*g*x*S
qrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + 2*g*ArcCosh[c*x] + (c*f*Sqrt[(-1 + c
*x)/(1 + c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + (2*(-(c*f) + g)*(c*f + g)*(2*Arc
Cosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] -
 (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-
(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCos
h[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c
*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g
)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*S
qrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*
f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[
c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] -
(ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1
+ Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I
*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f
^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) +
 g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f +
 g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) +
 g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2)
```

Maple [A]
time = 6.77, size = 1072, normalized size = 1.37

method	result

default	$\frac{a\sqrt{-\left(x+\frac{f}{g}\right)^{2} c^{2} d+\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}-\frac{d \left(c^{2} f^{2}-g^{2}\right)}{g^{2}}}}{g}+\frac{a\,c^{2} d f \arctan\left(\frac{\sqrt{c^{2} d}\,x}{\sqrt{-\left(x+\frac{f}{g}\right)^{2} c^{2} d+\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}-\frac{d(c^{2}}{}}}\right.}{g^{2}\sqrt{c^{2} d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE
)
```

```
[Out] a/g*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*
c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln
((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2
)/(c*x+1)/(c*x-1)/g*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(
1/2)/(c*x-1)^(1/2)/g*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)/g*arccosh
(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)/g^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-
g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g
^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(
-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*d
ilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(
c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*
f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x), x)
```

3.57 $\int \frac{\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{(f+gx)^2}\,dx$

Optimal. Leaf size=918

$$-\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)}+\frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\,(c^2f^2-g^2)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{b\sqrt{-\frac{1-cx}{1+cx}}\,\sqrt{1+cx}\,\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g\sqrt{-1+cx}\,(f+gx)}+\frac{b\cdots}{2g^2\,(\cdots}$$

```
[Out] -a*(-c^2*d*x^2+d)^(1/2)/g/(g*x+f)+a*c^3*f^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/
2)/g^2/(c^2*f^2-g^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c^3*f^2*arccosh(c*x)
^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(
c^2*f*x+g)^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c^2*f^2-g^2)/(g
*x+f)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(
-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*ln(g*x+f)
*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*a*c^2*f*arctanh((c*
f+g)^(1/2)*(c*x+1)^(1/2)/(c*f-g)^(1/2)/(c*x-1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/
g^2/(c*f-g)^(1/2)/(c*f+g)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^2*f*arccosh
(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(
-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c
^2*f*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^
2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f
^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(
c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-b*arccosh(c*x)*((c*x-1)/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c
^2*d*x^2+d)^(1/2)/g/(g*x+f)/(c*x-1)^(1/2)
```

Rubi [A]

time = 2.44, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.710$, Rules used = {5972, 5976, 37, 5969, 12, 186, 54, 98, 95, 214, 5993, 5992, 5893, 5980, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{bf^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)^2c^3}{2g^2(c^2f^2-g^2)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{af^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)c^3}{g^2(c^2f^2-g^2)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2af\sqrt{d-c^2dx^2}\,\tanh^{-1}\left(\frac{\sqrt{cf+g}\,\sqrt{cx+1}}{\sqrt{cf-g}\,\sqrt{cx-1}}\right)c^2}{\sqrt{cf-g}\,g^2\sqrt{cf+g}\,\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bf\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)\log\left(\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)c^2}{g^2\sqrt{c^2f^2-g^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bf\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)\log\left(\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}+1\right)c^2}{g^2\sqrt{c^2f^2-g^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bf\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bf\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)c^2}{g^2\sqrt{c^2f^2-g^2}\,\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{b\sqrt{d-c^2dx^2}\,\log(f+gx)c}{g^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{-\frac{1-cx}{cx+1}}\,\sqrt{cx+1}\,\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g\sqrt{cx-1}\,(f+gx)}-\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)}-\frac{(1-c^2x^2)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2b\sqrt{cx-1}\,\sqrt{cx+1}\,(f+gx)^2c}-\frac{(fxc^2+g)^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))^2}{2b(c^2f^2-g^2)\sqrt{cx-1}\,\sqrt{cx+1}\,(f+gx)^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) + (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-
((1 - c*x)/(1 + c*x))]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(g*S
qrt[-1 + c*x]*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(
```

```
2*g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((g + c^2*f*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(f + g*x)^2) - ((1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)^2) - (2*a*c^2
*f*Sqrt[d - c^2*d*x^2]*ArcTanh[(Sqrt[c*f + g]*Sqrt[1 + c*x])/(Sqrt[c*f - g]
*Sqrt[-1 + c*x])])/(Sqrt[c*f - g]*g^2*Sqrt[c*f + g]*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[
c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5969

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[u*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x
], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && IL
tQ[m, 0] && LtQ[m + p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5976

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[(f + g*x
)^m*(d1*d2 + e1*e2*x^2)*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*Sqrt[(-d1)*d2]*(
n + 1))), x] - Dist[1/(b*c*Sqrt[(-d1)*d2]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5992

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5993

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{(f+gx)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-c^2x^2)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{\sqrt{d-c^2dx^2}\int \frac{(2g\cdots}{\cdots}}{2bc\sqrt{-1\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}\,(f+gx)^2} - \frac{(1-c^2x^2)\sqrt{d\cdots}}{2bc\sqrt{-1+\cdots}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3f^2\cdots}{2g^2\left(c^2f^2\cdots\right.}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-\cdots}{1+\cdots}}}{\cdots}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-\cdots}{1+\cdots}}}{\cdots}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-\cdots}{1+\cdots}}}{\cdots}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-\cdots}{1+\cdots}}}{\cdots}$$

$$\sqrt{\frac{1\cdots}{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.68, size = 1139, normalized size = 1.24

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/(f + g*x) + 2*a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d
- c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*a*c^2*Sqrt[d]*f*Log[f + g*x])/
Sqrt[-(c^2*f^2) + g^2] - (2*a*c^2*Sqrt[d]*f*Log[d*(g + c^2*f*x) + Sqrt[d]*S
qrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[-(c^2*f^2) + g^2] + b*c*Sq
rt[d - c^2*d*x^2]*((-2*g*ArcCosh[c*x])/(c*f + c*g*x) + ArcCosh[c*x]^2/(Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*Log[1 + (g*x)/f])/(Sqrt[(-1 + c*x)/(
1 + c*x)]*(1 + c*x)) + (2*c*f*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*
f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)]
 + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Arc
Tan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[
-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] +
 (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]
*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[Ar
cCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(
c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^
2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f)
 + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g +
I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Po
lyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g
^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))))/(2*g^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1955 vs. $2(864) = 1728$.
time = 10.36, size = 1956, normalized size = 2.13

method	result	size
default	Expression too large to display	1956

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2-g^2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g^2*c^4*f^2/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))-a/g^3*c^4*f^3/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln(
(-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+
a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/(c^2*f^2-g^
2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+a*c^2
/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1
)/(c*x-1)/g^2/(g*x+f)*x^3*c^4*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+
1)^(1/2)/(c*x-1)^(1/2)/g/(g*x+f)*x*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/
(c*x+1)/(c*x-1)/g/(g*x+f)*x^2*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*
x+1)^(1/2)/(c*x-1)^(1/2)/g^2/(g*x+f)*c*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x+1)/(c*x-1)/g^2/(g*x+f)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*
x)/(c*x+1)/(c*x-1)/g/(g*x+f)-b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*f*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*
(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*
f*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1
/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-2*b*(-d*(c^2*x^2-1))^(1/2)*c^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2+b
*(-d*(c^2*x^2-1))^(1/2)*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*l
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+g)*f^2-b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c
^2*f^2-g^2)^(1/2)*f*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^
2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*f*dilog(((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+2*b*(
-d*(c^2*x^2-1))^(1/2)*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(c^2*f^2-g^2)*ln(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))-b*(-d*(c^2*x^2-1))^(1/2)*c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/(c^2*f^2-g^2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+g)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\operatorname{acosh}(cx))}{(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{(f+g\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

3.58 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=1029

$$\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bdfg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcc}{5\sqrt{}}$$

[Out] 3/8*d*f^3*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/16*d*f*g^2*x*(a+b*arc
cosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arccosh(c*x))*(-c^
2*d*x^2+d)^(1/2)+1/4*d*f^3*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)+1/2*d*f*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+
d)^(1/2)-3/5*d*f^2*g*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/c^2-2/35*d*g^3*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c^4-1/7*d*g^3*x^2*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x
^2+d)^(1/2)/c^2+3/5*b*d*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+2/35*b*d*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-5/16*b*c*d*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/32*b
*d*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/5*b*c*d*f
^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/105*b*d*g^3*x^3
*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*c^3*d*f^3*x^4*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/32*b*c*d*f*g^2*x^4*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/25*b*c^3*d*f^2*g*x^5*(-c^2*d*x^
2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/175*b*c*d*g^3*x^5*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/12*b*c^3*d*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/49*b*c^3*d*g^3*x^7*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-3/16*d*f^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/32*d*f*g^2*(a+b*arccosh(c*x))^2*(-c^2*
d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)

Rubi [A]
time = 1.27, antiderivative size = 1029, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 19, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 5915, 41, 200, 5931, 5927, 5939, 102, 12, 75, 5923, 380}

Antiderivative was successfully verified.

[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]

[Out] (3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
*b*d*g^3*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*
b*c*d*f^3*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*

```
b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^
3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c
*d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*
c^3*d*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8
*b*c*d*g^3*x^5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3
*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d*f*g^2*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/4 + (d*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (3*d*f^2*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (2*d*g^3*(1 - c*x)^2*(1 + c
*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*g^3*x^2*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (3*d
*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 14</u>

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 41</u>

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

<u>Rule 74</u>

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

```
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
```

```
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5923

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(d1 + e1*x
)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c*Simp[Sqrt
[d1 + e1*x]*(Sqrt[d2 + e2*x]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], Int[Simplify
Integrand[u/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x], x]] /; FreeQ[{a, b,
c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IntegerQ[p -
 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]
)
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
```

```
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^3}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\right.\right.}{}$$

$$= -\frac{\left(df^3\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\,c\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}df^3x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{3}{8}df^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d-}$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2}{5}$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}}-$$

<u>Mathematica</u> [A]
time = 2.85, size = 901, normalized size = 0.88

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5040*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(32*g^3
 + c^2*g*(336*f^2 + 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*
x + 70*f*g^2*x^2 + 20*g^3*x^3) - 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2
*x^2 + 64*g^3*x^3)) - 529200*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] + 235200*b*c^2*d*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 352800*b
*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcC
osh[c*x] - Sinh[2*ArcCosh[c*x]])) + 22050*b*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 66150*b*c*d*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcC
osh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 2352*b*c^2*d*f^2*g*Sqrt[
d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
```

```
c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*S
inh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) + 784*b*d*g^3*S
qrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*
x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) - 3675*b*c*
d*f*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] -
 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh
[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) - 4*b*d*
g^3*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c*x]] -
 225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 6615*A
rcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]]))
/(2822400*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2273 vs. $2(885) = 1770$.
time = 8.71, size = 2274, normalized size = 2.21

method	result	size
default	Expression too large to display	2274

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/7*a*g^3*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35*a*g^3/d/c^4*(-c^2*d*x^2+d)^(
5/2)-1/2*a*f*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/8*a*f*g^2/c^2*x*(-c^2*d*x^2
+d)^(3/2)+3/16*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/16*a*f*g^2/c^2*d^2/(c
^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/5*a*f^2*g*(-c^2*
d*x^2+d)^(5/2)/c^2/d+1/4*a*f^3*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f^3*d*x*(-c^2*d
*x^2+d)^(1/2)+3/8*a*f^3*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^
2+d)^(1/2))+b*(-3/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3
*f*arccosh(c*x)^2*(2*c^2*f^2+g^2)*d-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x
^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g^3*(-1+7*arccosh(c*x))*d/(c*x+1
)/c^4/(c*x-1)-1/768*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4
*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/
2))*f*g^2*(-1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/3200*(-d*(c^2*x^2-1))
^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2
*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-1)*g*(60*arccosh(c*x)*c^2*f^2-12*c^2*f^2-5*arccosh(c*x)*g^2+g^2)*d/(c*x+
1)/c^4/(c*x-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c
```

```
*x-1)^(1/2)*(c*x+1)^(1/2))*f*(8*arccosh(c*x)*c^2*f^2-2*c^2*f^2-12*arccosh(c
*x)*g^2+3*g^2)*d/(c*x+1)/c^3/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^
4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*g*(36*arccosh(c*x)*c^2*f^2-12*c^2*f^2+3*arccosh(c*x)*g^2-g^2)*d
/(c*x+1)/c^4/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(32*arccosh(c*
x)*c^2*f^2-16*c^2*f^2+6*arccosh(c*x)*g^2-3*g^2)*d/(c*x+1)/c^3/(c*x-1)-3/128
*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(8*ar
ccosh(c*x)*c^2*f^2-8*c^2*f^2+arccosh(c*x)*g^2-g^2)*d/(c*x+1)/c^4/(c*x-1)-3/
128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(
8*arccosh(c*x)*c^2*f^2+8*c^2*f^2+arccosh(c*x)*g^2+g^2)*d/(c*x+1)/c^4/(c*x-1
)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^
3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f*(32*arccosh(c*x)*c^2*f^2+16*c^2*
f^2+6*arccosh(c*x)*g^2+3*g^2)*d/(c*x+1)/c^3/(c*x-1)+1/384*(-d*(c^2*x^2-1))^
(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c-5*c^2*x^2+1)*g*(36*arccosh(c*x)*c^2*f^2+12*c^2*f^2+3*arccosh
(c*x)*g^2+g^2)*d/(c*x+1)/c^4/(c*x-1)-1/512*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*
c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*f*(8*arccosh(c*x)*c^2*f^2
+2*c^2*f^2-12*arccosh(c*x)*g^2-3*g^2)*d/(c*x+1)/c^3/(c*x-1)-1/3200*(-d*(c^2
*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
+13*c^2*x^2-1)*g*(60*arccosh(c*x)*c^2*f^2+12*c^2*f^2-5*arccosh(c*x)*g^2-g^2
)*d/(c*x+1)/c^4/(c*x-1)-1/768*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5
*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1
)^(1/2)-6*c*x)*f*g^2*(1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/6272*(-d*(c
^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*g^3*(
1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^3 - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*g^3 + 1/16*a*f*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c
^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 +
```

```
3*d^(3/2)*arcsin(c*x)/c^3) - 3/5*(-c^2*d*x^2 + d)^(5/2)*a*f^2*g/(c^2*d) + i
ntegrate((-c^2*d*x^2 + d)^(3/2)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1)) + 3*(-c^2*d*x^2 + d)^(3/2)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)) + 3*(-c^2*d*x^2 + d)^(3/2)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)) + (-c^2*d*x^2 + d)^(3/2)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
+ (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*c^
2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d*f^
2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c
^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x)**3, x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f + g\,x)^3\,(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.59 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=725

$$\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3df}{16\sqrt{-}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/16*d*g^2*x*(a+b*arcco
sh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)+1/4*d*f^2*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)+1/6*d*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)-2/5*d*f*g*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c
^2+2/5*b*d*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/16*b*
c*d*f^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/32*b*d*g^2*x
^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/15*b*c*d*f*g*x^3*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*c^3*d*f^2*x^4*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/96*b*c*d*g^2*x^4*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/25*b*c^3*d*f*g*x^5*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c^3*d*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-3/16*d*f^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.05, antiderivative size = 725, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$,
Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 5915, 41, 200, 5931, 5927, 5939}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}df^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{2dfg(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}-\frac{dg^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^2}+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}dg^2x^3(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dg^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{cx-1}\sqrt{cx+1}}-\frac{7bcdg^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3df^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dfgx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dg^2x^6\sqrt{d-c^2dx^2}}{36\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b
*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d
*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*d*g
^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*
f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g
^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f^2*x*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/8 + (d*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
```

```
osh[c*x]))/4 + (d*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (2*d*f*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
```

```
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
```

```
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^2 \,(\ldots}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(f^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\,\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(df^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cos\ldots\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{4}df^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\ldots$$

$$= \frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d-c^2\ldots}$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4b\ldots}{15\ldots}$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b\ldots}{32c\ldots}$$

Mathematica [A]
time = 1.81, size = 623, normalized size = 0.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-240*a*c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(96*f*
g*(-1 + c^2*x^2)^2 + 30*c^2*f^2*x*(-5 + 2*c^2*x^2) + 5*g^2*x*(3 - 14*c^2*x^
2 + 8*c^4*x^4)) - 3600*a*d^(3/2)*(6*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3
200*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*
(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 7200*b*c^2*d*f^2*Sqrt[d
 - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])) + 450*b*c^2*d*f^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 450*b*d*g^2*Sq
rt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]
*Sinh[4*ArcCosh[c*x]]) - 32*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] -
 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh
```

```
[c*x]*Sinh[5*ArcCosh[c*x]]) - 25*b*d*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c
*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh
[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]]
+ Sinh[6*ArcCosh[c*x]])))/(57600*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(621) = 1242$.

time = 8.48, size = 1726, normalized size = 2.38

method	result	size
default	Expression too large to display	1726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/6*a*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^(3/
2)+1/16*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a*g^2/c^2*d^2/(c^2*d)^(1/2)
*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/5*a*f*g*(-c^2*d*x^2+d)^(5/2
)/c^2/d+1/4*a*f^2*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f^2*d*x*(-c^2*d*x^2+d)^(1/2)
+3/8*a*f^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b
*(-1/32*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)
^2*(6*c^2*f^2+g^2)*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+3
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))*g^2*(-1+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1)-1/400*(-d*(c^2*x^
2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+1
3*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c-1)*f*g*(-1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/512*(-d*(c^2*x^2
-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*
x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(8*arc
cosh(c*x)*c^2*f^2-2*c^2*f^2-4*arccosh(c*x)*g^2+g^2)*d/(c*x+1)/c^3/(c*x-1)+1
/48*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*f*g*(-1+3*arccosh(c*x))*d/(
c*x+1)/c^2/(c*x-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(32*arccosh(c*x)*c
^2*f^2-16*c^2*f^2+2*arccosh(c*x)*g^2-g^2)*d/(c*x+1)/c^3/(c*x-1)-1/8*(-d*(c^
2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(-1+arccosh
(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/256*
(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c
*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*(32*arccosh(c*x)*c^2*f^2+16*c^2*f^2+2*arcc
osh(c*x)*g^2+g^2)*d/(c*x+1)/c^3/(c*x-1)+1/48*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)*f*g*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/512*(-d*(c^2*
```

```
x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(8
*arccosh(c*x)*c^2*f^2+2*c^2*f^2-4*arccosh(c*x)*g^2-g^2)*d/(c*x+1)/c^3/(c*x-
1)-1/400*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16
*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*f*g*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1
)-1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32
*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*g^2*(1
+6*arccosh(c*x))*d/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^2 + 1/48*a*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d
*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcs
in(c*x)/c^3) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*f*g/(c^2*d) + integrate((-c^2*d
*x^2 + d)^(3/2)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*(-c^2*
d*x^2 + d)^(3/2)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x
^2 + d)^(3/2)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (a
*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*
g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
+ d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x)**2, x
)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(a+b\,\mathrm{acosh}(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.60 $\int (f+gx) \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=398

$$\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dfx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dg}{25\sqrt{-1}}$$

```
[Out] 3/8*d*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(-c*x+1)*(c*x+1
)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/5*d*g*(-c*x+1)^2*(c*x+1)^2*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/5*b*d*g*x*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/16*b*c*d*f*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2/15*b*c*d*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+1/16*b*c^3*d*f*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+1/25*b*c^3*d*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/16
*d*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A]
time = 0.48, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 5915, 41, 200}

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{dg(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}-\frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dfx^4\sqrt{d-c^2dx^2}}{16\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3dgx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d
*f*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*g*
x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*f*x^4
*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f*x*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
```

```
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)\,(a}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\,\mathrm{c}\right.\right.}{\sqrt{-1}}$$

$$= -\frac{\left(df\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{4}dfx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) -$$

$$= \frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(1$$

$$= \frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2}{15}$$

Mathematica [A]
time = 1.03, size = 432, normalized size = 1.09

$$\frac{-720ad\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\sqrt{d-c^2dx^2}\left(8g(-1+c^2x^2)^2+5c^2fx(-5+2c^2x^2)\right)-10800acd^{3/2}f\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+800bdg\sqrt{d-c^2dx^2}\left(9cx+12\left(\frac{-1+cx}{1+cx}\right)^{3/2}(1+cx)^3\cosh^{-1}(cx)-\cosh\left(3\cosh^{-1}(cx)\right)\right)-3600bcdf\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)+225bcdf\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx)\sinh\left(4\cosh^{-1}(cx)\right)\right)-8bdg\sqrt{d-c^2dx^2}\left(450cx-450\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)-25\cosh\left(3\cosh^{-1}(cx)\right)-9\cosh\left(5\cosh^{-1}(cx)\right)+75\cosh^{-1}(cx)\sinh\left(3\cosh^{-1}(cx)\right)+45\cosh^{-1}(cx)\sinh\left(5\cosh^{-1}(cx)\right)\right)}{28800c^2\sqrt{\frac{-1+cx}{1+cx}}(1+cx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-720*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(8*g*(-1
 + c^2*x^2)^2 + 5*c^2*f*x*(-5 + 2*c^2*x^2)) - 10800*a*c*d^(3/2)*f*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
 c^2*x^2))] + 800*b*d*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*
x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c*d*f*S
qrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - S
inh[2*ArcCosh[c*x]])) + 225*b*c*d*f*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 +
 Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 8*b*d*g*Sqrt
[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh
[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*
Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28800*c^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(338) = 676$.
time = 6.85, size = 1176, normalized size = 2.95

method	result
default	$-\frac{ag(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+\frac{afx(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{-c^2dx^2+d}}{8}+\frac{3afd^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}+b\left(-\frac{3\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/5*a*g*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*
d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))+b*(-3/16*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/c*f*arccosh(c*x)^2*d-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*
x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*g*(-1+5*arccosh(c*x)
)*d/(c*x+1)/c^2/(c*x-1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(-1+4*arccosh(c*x))*d/(c*x+1)/(c*x-1)
/c+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(-1+3*arccosh(c*x))*d
/(c*x+1)/c^2/(c*x-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(2*arccosh(c*x)
-1)*d/(c*x+1)/(c*x-1)/c-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*g*(-1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^
2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(1+arccosh(c
*x))*d/(c*x+1)/c^2/(c*x-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f*(2*arcco
sh(c*x)+1)*d/(c*x+1)/(c*x-1)/c+1/96*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x
^2+1)*g*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/256*(-d*(c^2*x^2-1))^(1/
2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*f*(1+4*arccos
h(c*x))*d/(c*x+1)/(c*x-1)/c-1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28
*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*g*(1+5*arccosh(c*x
))*d/(c*x+1)/c^2/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f - 1/5*(-c^2*d*x^2 + d)^(5/2)*a*g/(c^2*d) + integrate((-c^2
*d*x^2 + d)^(3/2)*b*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x^
2 + d)^(3/2)*b*f*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-d(cx-1)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.61 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1270

$$-\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3}+\frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{ad(2$$

```
[Out] -a*d*(c*f-g)*(c*f+g)*(-c^2*d*x^2+d)^(1/2)/g^3+1/6*a*d*(-2*c^2*x^2+3*c*x+2)*
(-c^2*d*x^2+d)^(1/2)/g-b*d*(c*f-g)*(c*f+g)*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2
)/g^3+1/6*b*d*(-2*c^2*x^2+3*c*x+2)*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/g+1/2*
c*d*(c*f-g)*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2+b*c*d*(c*f-g)*(c*
f+g)*x*(-c^2*d*x^2+d)^(1/2)/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c^2*d*(c*
f-g)*x^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c*d*x*
(4*c^2*x^2-9*c*x-12)*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2
*a*d*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*
d*arccosh(c*x)^2*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*d*(
c*f-g)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/2*c*d*(c*f-g)*(c*f+g)*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)
/b/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*d*(c*f-g)^2*(c*f+g)^2*(a+b*arccosh(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2
*d*(c*f-g)*(c*f+g)*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b
/c/g^2/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*a*d*(c*f-g)^(3/2)*(c*f+g)^(3/2
)*arctanh((c*f+g)^(1/2)*(c*x+1)^(1/2)/(c*f-g)^(1/2)/(c*x-1)^(1/2))*(-c^2*d*
x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d*(c*f-g)*(c*f+g)*arccosh(c*
x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2
*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*(c
*f-g)*(c*f+g)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c
^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-b*d*(c*f-g)*(c*f+g)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/
2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*(c*f-g)*(c*f+g)*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(
-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [F]

time = 2.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (b*c^2*d*(c*f - g)*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (a*d*(c*f - g)*(c*f + g)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x
^2])/(g^3*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g^3 + (c*d*(c*f - g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(2*g^2) - (d*(c*f - g)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
4*b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (d*(c*f - g)^2*(c*f + g)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2
*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (d*(c*f - g)*(c*f + g)*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (a*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 -
g^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2
*f^2 - g^2]*Sqrt[-1 + c^2*x^2])]])/(g^4*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g
)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E
^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])]])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])]])/(g^4*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]
*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 -
g^2]))]])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt
[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
Sqrt[c^2*f^2 - g^2]))]])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d*Sqrt[d -
c^2*d*x^2]*Defer[Int][(-1 + c*x)^(3/2)*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]),
x])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}(a+b\cosh^{-1}(cx))}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(-\frac{c(cf-g)\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}{g^2}\right.}{}$$

$$= \frac{\left(cd(cf-g)\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(c\right.}{g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^2} + \frac{d(cf-g)(cf+g)}{2bcg}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\left(a+b\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)}{g^3(1-cx)(1+cx)}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)}{g^3(1-cx)(1+cx)}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{} - \frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.72, size = 3068, normalized size = 2.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f*
x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*ArcT
an[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) + (a*
d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*f^2)
 + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d
*(-1 + c^2*x^2))]])/g^4 + (b*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*((-2*c*g*x)/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + 2*g*ArcCosh[c*x] - (c*f*ArcCosh[c*
x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*(-(c*f) + g)*(c*f + g)*(2
*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c
*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(Ar
cCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan
[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/
2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-
((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
 g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcC
osh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))
] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])
*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[Ar
cCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g
 - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c
^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^
2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c
*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^
2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2) - (b*d*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*((-9*(-2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (ArcCos[-((c*f)/g)] +
 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTa
n[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(
c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] - (
ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*
f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
```

```
 g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*S
qrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcC
osh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^
2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] + (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) +
 g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) +
 g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - I*(PolyLog[2, ((c*f - I*Sq
rt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Poly
Log[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2
]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))])))/Sqrt[-(c^2*f^2) + g^2] - (-18*c*g*(-4*c^2*f^2 + g^2)*x + 18
*g*(-4*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 1
8*c*f*(2*c^2*f^2 - g^2)*ArcCosh[c*x]^2 - 9*c*f*g^2*Cosh[2*ArcCosh[c*x]] + 2
*g^3*Cosh[3*ArcCosh[c*x]] + (9*(8*c^4*f^4 - 8*c^2*f^2*g^2 + g^4)*(2*ArcCosh
[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*
I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2
*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*
x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f +
 g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Ta
nh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[
-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-((c*f)/g
)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*
Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Arc
Cos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f
^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqr...
```

Maple [A]
time = 5.96, size = 1965, normalized size = 1.55

method	result	size
default	Expression too large to display	1965

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*c^4*d/(c*x+1)/(c*x-1)/g^2*arccosh(c*x)*x^3+2
*a/g^3*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*
f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x
+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2+1/2*a/g^2*c^2*d*f*(-(x+f
```

```
/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+3/2*a/g^2*c^2*
d^2*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x
+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-a/g^4*d^2*c^4*f^3/(c^2*d)^(1/2)*arctan((c
^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1
/2))-a/g^5*d^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4+1/3*a/g*(-(x+f/g)^2*
c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+1/2*b*(-d*(c^2*x^2-1))
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*arccosh(c*x)^2*c^3*d/g^4-a/g^3*d*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2-a/g*d
^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f
/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*
(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*c^2*d/(c*
x+1)/(c*x-1)/g^2*arccosh(c*x)*x-b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)/
g^3*arccosh(c*x)*x^2*c^4*f^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(
c*x-1)^(1/2)/g*x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)
^(1/2)/g*x*c+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)
/g^2-b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)/g^4*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2
))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-3/4*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c*d/g^2-1/3*b*(-d*(c^
2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)/g*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2
-1))^(1/2)*d/(c*x+1)/(c*x-1)/g*arccosh(c*x)*x^2*c^2+b*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)/(c*x-1)/g^3*arccosh(c*x)*c^2*f^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f
*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g^2*x^2+b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+
1)^(1/2)/(c*x-1)^(1/2)/g^3*x*c^3*f^2-b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*
(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^
4*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1
/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x
-1)/g*arccosh(c*x)+a/g*d*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g
^2)/g^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x), x)
```

3.62 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=1385

$$\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{25bcd^2f^3x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{15bd^2fg^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3b}{7}$$

```
[Out] 3/7*b*d^2*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/256
*b*d^2*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/7*b*c
*d^2*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/256*b*c*
d^2*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+9/35*b*c^3*d
^2*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/96*b*c^3*d
^2*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/49*b*c^5*d^
2*f^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/64*b*c^5*d^2
*f*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/256*d^2*f*g^
2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+5/16*d^2*f^3*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-15/128*d^2*f*g^2*
x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/24*d^2*f^3*x*(-c*x+1)*(c*x+
1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f^3*x*(-c*x+1)^2*(c*x+1)
^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-2/63*d^2*g^3*(-c*x+1)^3*(c*x+1)^
3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+15/64*d^2*f*g^2*x^3*(a+b*arcc
osh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/16*d^2*f*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arc
cosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(-c*x+1)^2*(c*x+1)^2*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/7*d^2*f^2*g*(-c*x+1)^3*(c*x+1)^3*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/9*d^2*g^3*x^2*(-c*x+1)^3*(c*x+1)^
3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+2/63*b*d^2*g^3*x*(-c^2*d*x^2+
d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-25/96*b*c*d^2*f^3*x^2*(-c^2*d*x^2+
d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/189*b*d^2*g^3*x^3*(-c^2*d*x^2+d)^(1/
2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d^2*f^3*x^4*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/21*b*c*d^2*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+19/441*b*c^3*d^2*g^3*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)-1/81*b*c^5*d^2*g^3*x^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*f^3*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*f^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)
^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.53, antiderivative size = 1385, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.710$, Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 267, 5915, 41, 200, 5931, 5927, 5939, 272, 45, 102, 12, 75, 5923, 380}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])
/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d
*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*
d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f^3*(1 - c^2*x^2)^3*Sqrt
[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^3*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/64 + (5*d^2*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (d^2*f^3*x*(1 - c*x)^2*(1 + c*x)^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d^2*f^2*g*(1 -
c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (2*d
^2*g^3*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(6
3*c^4) - (d^2*g^3*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/(9*c^2) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
```

```
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5923

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(d1 + e1*x
)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c*Simp[Sqrt
[d1 + e1*x]*(Sqrt[d2 + e2*x]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], Int[Simplify
Integrand[u/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x], x]] /; FreeQ[{a, b,
c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IntegerQ[p -
 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]
)
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^3 (a}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^3(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\,\mathrm{co}\right.\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2f^3x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right.$$

$$= \frac{bd^2f^3\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{24}d^2f^3x(1-cx)(1+c$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9b}{3}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2}{9}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2}{9}$$

Mathematica [A]
time = 7.22, size = 1802, normalized size = 1.30

Too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-1/63*(a*d^2*g*(27*c^2*f^2 + 2*g^2))/c^4 + (a*d^
2*f*(88*c^2*f^2 - 15*g^2)*x)/(128*c^2) - (a*d^2*g*(-81*c^2*f^2 + g^2)*x^2)/
(63*c^2) - (a*d^2*f*(104*c^2*f^2 - 177*g^2)*x^3)/192 + (a*d^2*g*(-27*c^2*f^
2 + 5*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 - 51*g^2)*x^5)/48 - (a*c^2*d^2
*g*(-27*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4*d^2*
g^3*x^8)/9) - (5*a*d^(5/2)*f*(8*c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 +
 c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))]])/(128*c^3) - (b*d^2*f^2*g*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3
*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(12*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)) - (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]
```

```
] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])))/(8*c*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*
ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]
))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (3*b*d^2*f*g^2*Sqrt[-(d*(-
1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c
*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
 (b*d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*A
rcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[
5*ArcCosh[c*x]]))/(600*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g
^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]
] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c
*x]]))/(3600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(
d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]]
- 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcC
osh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*Ar
cCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g^2
*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sin
h[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*S
inh[6*ArcCosh[c*x]])))/(384*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh
[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*C
osh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Ar
cCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sin
h[7*ArcCosh[c*x]])))/(235200*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b
*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh[
c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*Co
sh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Arc
Cosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sinh
[7*ArcCosh[c*x]])))/(352800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(1440*ArcCosh[c*x]^2 - 576*Cosh[2
*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x]] + 9*Cos
h[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 576*ArcCosh[c*
x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] - 72*ArcCos
h[c*x]*Sinh[8*ArcCosh[c*x]]))/(24576*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)) - (b*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-1389150*c*x + 31752*Cosh
[5*ArcCosh[c*x]] + 5*(2025*Cosh[7*ArcCosh[c*x]] + 245*Cosh[9*ArcCosh[c*x]]
- 63*ArcCosh[c*x]*(-4410*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 504*Sinh[5*
ArcCosh[c*x]] + 225*Sinh[7*ArcCosh[c*x]] + 35*Sinh[9*ArcCosh[c*x]]))))/(254
01600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3169 vs. $2(1201) = 2402$.

time = 8.92, size = 3170, normalized size = 2.29

method	result	size
default	Expression too large to display	3170

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/9*a*g^3*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63*a*g^3/d/c^4*(-c^2*d*x^2+d)^(
7/2)-3/8*a*f*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/16*a*f*g^2/c^2*x*(-c^2*d*x^
2+d)^(5/2)+5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+15/128*a*f*g^2/c^2*d^2
*x*(-c^2*d*x^2+d)^(1/2)+15/128*a*f*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/7*a*f^2*g*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a
*f^3*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f^3*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f^3*
d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2))+b*(-5/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*(8*c^2*f^2+3*g^2)*d^2+1/41472*(-d*(c^2*x^2
-1))^(1/2)*(256*x^10*c^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c
^9+688*x^6*c^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*g^3*(-1+9*arccosh(c*x))*d^2/(c
*x+1)/c^4/(c*x-1)+3/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x^9-320*c^7*x^7+1
28*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^6*c^6-88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c*x-32
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*g^2*(-1
+8*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+3/25088*(-d*(c^2*x^2-1))^(1/2)*(64
*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(28*arccosh(c*x)*c^2*f^2
-4*c^2*f^2-7*arccosh(c*x)*g^2+g^2)*d^2/(c*x+1)/c^4/(c*x-1)+1/2304*(-d*(c^2*
x^2-1))^(1/2)*(32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6
+38*c^3*x^3-48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(6*arccosh(c*x)*c^2*f^2
-c^2*f^2-18*arccosh(c*x)*g^2+3*g^2)*d^2/(c*x+1)/c^3/(c*x-1)-3/640*(-d*(c^2*
x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-1)*f^2*g*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-3/1024*(-d*(
c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c
^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*f*(8*arccosh(c*x)*c^2*f^2-2*c^2*f^2-4*arccosh(c*x)*g^2+g^2)*d^2/(c*x+1)/c^
3/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(81*arccosh(
c*x)*c^2*f^2-27*c^2*f^2+6*arccosh(c*x)*g^2-2*g^2)*d^2/(c*x+1)/c^4/(c*x-1)+3
```

```
/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(10*arccosh(c*x)*c^2*f^2-5*c^2*f^2+2
*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^3/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(10*arccosh(c*x)*c^2*f^2-10*
c^2*f^2+arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1)
)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(10*arccosh(c*x)*c^2
*f^2+10*c^2*f^2+arccosh(c*x)*g^2+g^2)*d^2/(c*x+1)/c^4/(c*x-1)+3/256*(-d*(c^
2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(
1/2)*(c*x+1)^(1/2)-2*c*x)*f*(10*arccosh(c*x)*c^2*f^2+5*c^2*f^2+2*arccosh(c*
x)*g^2+g^2)*d^2/(c*x+1)/c^3/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-
5*c^2*x^2+1)*g*(81*arccosh(c*x)*c^2*f^2+27*c^2*f^2+6*arccosh(c*x)*g^2+2*g^2
)*d^2/(c*x+1)/c^4/(c*x-1)-3/1024*(-d*(c^2*x^2-1))^(1/2)*(-8*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3
*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*f*(8*arccosh(c*x)*c^2*f^2+2*c^2*f^2
-4*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^3/(c*x-1)-3/640*(-d*(c^2*x^2-1))^(1/
2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-
1)*f^2*g*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/2304*(-d*(c^2*x^2-1))
^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38
*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*f*(6*arccosh(c*x)*c^2*f^2+c^2*f
^2-18*arccosh(c*x)*g^2-3*g^2)*d^2/(c*x+1)/c^3/(c*x-1)+3/25088*(-d*(c^2*x^2-
1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*g*(28*arccos
h(c*x)*c^2*f^2+4*c^2*f^2-7*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^4/(c*x-1)+3/
16384*(-d*(c^2*x^2-1))^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*
c^9*x^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2
-88*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)*f*g^2*(1+8*arccosh(c*x))*d^2
/(c*x+1)/c^3/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^3 + 1/128*(8*(-c^2*d*
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
```

```
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*f*g^2 - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*g^3 - 3/7*(-c^2*d*x^2 + d)^(7/2)*a*f^2*g/(c^2*d)
 + integrate((-c^2*d*x^2 + d)^(5/2)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)) + 3*(-c^2*d*x^2 + d)^(5/2)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sq
rt(c*x - 1)) + 3*(-c^2*d*x^2 + d)^(5/2)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)) + (-c^2*d*x^2 + d)^(5/2)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a*c
^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f^3
- 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*
b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.63 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=1015

$$\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bd^2g^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bc^3}{96\sqrt{}}$$

```
[Out] 5/16*d^2*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-5/128*d^2*g^2*x*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f^2*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)+5/48*d^2*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*x*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)-2/7*d^2*f*g*(-c*x+1)^3*(c*x+1)^3*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/c^2+2/7*b*d^2*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+5/256*b*d^2*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2/7*b*c*d^2*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
5/96*b*c^3*d^2*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/
768*b*c*d^2*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+6/35*b
*c^3*d^2*f*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/288*b*
c^3*d^2*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/49*b*c^5
*d^2*f*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/64*b*c^5*d^
2*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*f^2*(
-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*f
^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-5/256*d^2*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A]
time = 1.31, antiderivative size = 1015, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 267, 5915, 41, 200, 5931, 5927, 5939, 272, 45}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2
5*b*c*d^2*f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*b*d^2*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (2*b*c*d^2*f*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

```
+ (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*d^2*f^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128
*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^
2*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 +
(5*d^2*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/48 + (d^2*f^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/6 + (d^2*g^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/8 - (2*d^2*f*g*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(7*c^2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*g^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
```

```
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5931

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^p*(d2 + e2*x)^p*((a + b*ArcCosh[c*x])^n/(f*(m + 2*p + 1))), x]
 + (Dist[2*d1*d2*(p/(m + 2*p + 1)), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m + 2*p + 1))
)*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*
x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
```

```
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^2 (}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(f^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\right.\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\,\mathrm{co}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2f^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.$$

$$= \frac{bd^2f^2(1-c^2x^2)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{24}d^2f^2x(1-cx)(1+$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{6b}{3}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2}{7}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{}{2}$$

Mathematica [A]
time = 6.83, size = 1282, normalized size = 1.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-2*a*d^2*f*g)/(7*c^2) + (a*d^2*(88*c^2*f^2 - 5*
g^2)*x)/(128*c^2) + (6*a*d^2*f*g*x^2)/7 + (a*d^2*(-104*c^2*f^2 + 59*g^2)*x^
3)/192 - (6*a*c^2*d^2*f*g*x^4)/7 + (a*c^2*d^2*(8*c^2*f^2 - 17*g^2)*x^5)/48
+ (2*a*c^4*d^2*f*g*x^6)/7 + (a*c^4*d^2*g^2*x^7)/8) - (5*a*d^(5/2)*(8*c^2*f^
2 + g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/
(128*c^3) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(18
*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)
*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^2*Sq
```

```
rt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[
4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-
450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3
*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*
x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(900*c^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)) + (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*Ar
cCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCos
h[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*
ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh
[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(1152*c^3*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
*(-55125*c*x + 1225*Cosh[3*ArcCosh[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x] + 441*Cosh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c
*x]] - 1225*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*Ar
cCosh[c*x]] - 525*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]])))/(352800*c^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*
(1440*ArcCosh[c*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]]
+ 64*Cosh[6*ArcCosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh
[2*ArcCosh[c*x]] - 576*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]
*Sinh[6*ArcCosh[c*x]] - 72*ArcCosh[c*x]*Sinh[8*ArcCosh[c*x]]))/(73728*c^3*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(879) = 1758$.
time = 11.57, size = 2549, normalized size = 2.51

method	result	size
default	Expression too large to display	2549

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/8*a*g^2*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^(5/
2)+5/192*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a*g^2/c^2*d^2*x*(-c^2*d*x
^2+d)^(1/2)+5/128*a*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*
d*x^2+d)^(1/2))-2/7*a*f*g*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f^2*x*(-c^2*d*x^
2+d)^(5/2)+5/24*a*f^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(-c^2*d*x^2
+d)^(1/2)+5/16*a*f^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
```

)^(1/2))+b*(-5/256*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*a
rccosh(c*x)^2*(8*c^2*f^2+g^2)*d^2+1/16384*(-d*(c^2*x^2-1))^(1/2)*(128*c^9*x
^9-320*c^7*x^7+128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+272*c^5*x^5-256*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-88*c^3*x^3+160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+8*c*x-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*g^2*(-1+8*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1)+1/3136*(-d*(c^2*x^2-
1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+10
4*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*f*g*(-1+7*arc
cosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*c^7*x^
7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*
x-1)^(1/2)*(c*x+1)^(1/2))*(6*arccosh(c*x)*c^2*f^2-c^2*f^2-6*arccosh(c*x)*g^
2+g^2)*d^2/(c*x+1)/c^3/(c*x-1)-1/320*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*
c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*f*g*(-1+5*arccos
h(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/1024*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12
*c^3*x^3+8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(24*arccosh(c*x)*c^2*f^2-6*c^2
*f^2-4*arccosh(c*x)*g^2+g^2)*d^2/(c*x+1)/c^3/(c*x-1)+1/64*(-d*(c^2*x^2-1))^
(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c+1)*f*g*(-1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+
1/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(30*arccosh(c*x)*c^2*f^2-15*c^2*f^2+2
*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^3/(c*x-1)-5/64*(-d*(c^2*x^2-1))^(1/2)*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(-1+arccosh(c*x))*d^2/(c*x+
1)/c^2/(c*x-1)-5/64*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c+c^2*x^2-1)*f*g*(1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/256*(-d*(c^2*x^
2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)
*(c*x+1)^(1/2)-2*c*x)*(30*arccosh(c*x)*c^2*f^2+15*c^2*f^2+2*arccosh(c*x)*g^
2+g^2)*d^2/(c*x+1)/c^3/(c*x-1)+1/64*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x
^2+1)*f*g*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/1024*(-d*(c^2*x^2-1)
)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*(24*arcc
osh(c*x)*c^2*f^2+6*c^2*f^2-4*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^3/(c*x-1)-
1/320*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^
6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+13*c^2*x^2-1)*f*g*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)
+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*
c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)*(6*arcc
osh(c*x)*c^2*f^2+c^2*f^2-6*arccosh(c*x)*g^2-g^2)*d^2/(c*x+1)/c^3/(c*x-1)+1/
3136*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8
x^8+112(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(

```
c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x
^2+1)*f*g*(1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/16384*(-d*(c^2*x^2-1
))^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*c^9*x^9+256*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4
*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*c^3*x^3-(c*x-1)^
(1/2)*(c*x+1)^(1/2)+8*c*x)*g^2*(1+8*arccosh(c*x))*d^2/(c*x+1)/c^3/(c*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^2 + 1/384*(8*(-c^2*d*
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*g^2 - 2/7*(-c^2*d*x^2 + d)^(7/2)*a*f*g/(c^2*d) + integrate((-c
^2*d*x^2 + d)^(5/2)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*(-
c^2*d*x^2 + d)^(5/2)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2
*d*x^2 + d)^(5/2)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2
*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*
g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
 + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.64 $\int (f+gx) \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=568

$$\frac{bd^2 gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{5bc^3d^2fx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3bc^3d^2g\ldots}{35\sqrt{-1\ldots}}$$

```
[Out] 5/16*d^2*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(-c*x+1)*
(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(-c*x+1)^2*(c*x
+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/7*d^2*g*(-c*x+1)^3*(c*x+1)^
3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/7*b*d^2*g*x*(-c^2*d*x^2+d)^
(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-25/96*b*c*d^2*f*x^2*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c*d^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d^2*f*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+3/35*b*c^3*d^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/49*b*c^5*d^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+1/36*b*d^2*f*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-5/32*d^2*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {5972, 5975, 5898, 5896, 5893, 30, 74, 14, 267, 5915, 41, 200}

$$\frac{1}{6}d^2fx(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2fx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5d^2f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d^2g(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2f\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc^5d^2gx^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d^2fx^4\sqrt{d-c^2dx^2}}{96\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bc^3d^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*b*
c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d
^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c^3*d
^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*
d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f*
(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*f*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*f*x*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*g*(1
- c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (5
*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
```

```
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5898

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[x*(d1 + e1*x)^p*(d2 + e2*x)^p
*((a + b*ArcCosh[c*x])^n/(2*p + 1)), x] + (Dist[2*d1*d2*(p/(2*p + 1)), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[b*c*(n/(2*p + 1))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1
 + c*x)^p], Int[x*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)\left(a\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\,\mathrm{co}\right.\right.}{\sqrt{-1+}}$$

$$= \frac{\left(d^2f\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2fx(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

$$= \frac{bd^2f\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{5}{24}d^2fx(1-cx)(1+cx$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc}{35\sqrt{}}$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc}{7\sqrt{}}$$

Mathematica [A]
time = 4.19, size = 644, normalized size = 1.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(8400*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(48*g
*(-1 + c^2*x^2)^3 + 7*c^2*f*x*(33 - 26*c^2*x^2 + 8*c^4*x^4)) - 882000*a*c*S
qrt[d]*f*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + 78400*b*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]
) - 352800*b*c*f*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]
*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 44100*b*c*f*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 1568*b*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]]
 + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*
x]]) + 1225*b*c*f*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCo
sh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x
```

```
]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])
) + 4*b*g*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c
*x]] - 225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] +
6615*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c
*x]])))/(2822400*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1855 vs. $2(488) = 976$.
time = 7.12, size = 1856, normalized size = 3.27

method	result	size
default	Expression too large to display	1856

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/7*a*g*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f
*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-5/32*(-d*(c^
2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2*d^2+1/6272*(
-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*x^6*c^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g
*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*
(32*c^7*x^7-64*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+38*c^3*x^3-48
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-6*c*x+18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(-1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-
1)/c-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*x^6*c^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*g*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x
-1)-3/512*(-d*(c^2*x^2-1))^(1/2)*(8*c^5*x^5-12*c^3*x^3+8*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^4*c^4+4*c*x-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)
*(c*x+1)^(1/2))*f*(-1+4*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+1/128*(-d*(c^2*
x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*g*(-1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c
*x-1)+15/256*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*(2*arccosh(c*x)-1)*d^2/(c*x+
1)/(c*x-1)/c-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*g*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1
))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(1+arccosh(c*x))*d^
2/(c*x+1)/c^2/(c*x-1)+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*f*(2*arccosh(
c*x)+1)*d^2/(c*x+1)/(c*x-1)/c+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2
```

```
)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x
^2+1)*g*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-3/512*(-d*(c^2*x^2-1))^(
1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)*f*(1+4*arcc
osh(c*x))*d^2/(c*x+1)/(c*x-1)/c-1/640*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5+16*x^6*c^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*g*(1+5*arccosh
(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(-32*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^
4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2
)*(c*x+1)^(1/2)-6*c*x)*f*(1+6*arccosh(c*x))*d^2/(c*x+1)/(c*x-1)/c+1/6272*(-
d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+11
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*x^6*c^6-56*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*g
*(1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f - 1/7*(-c^2*d*x^2 + d
)^(7/2)*a*g/(c^2*d) + integrate((-c^2*d*x^2 + d)^(5/2)*b*g*x*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x^2 + d)^(5/2)*b*f*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c
^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccosh(c*x))*sqrt(
-c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-d(cx-1)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.65 $\int \frac{(d-c^2dx^2)^{5/2}(a+b\cosh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=1744

$$\frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bcd^2(c^2f^2-2g^2)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2(c^2f^2-g^2)^2x\sqrt{d-c^2dx^2}}{g^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^3d^2fx^2\sqrt{d}}{16g^2\sqrt{-1+c}}$$

```
[Out] 1/4*c*d^2*f*(c^2*f^2-2*g^2)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^4
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*c*d^2*(c^2*f^2-g^2)^2*x*(a+b*arccosh(c*x))
^2*(-c^2*d*x^2+d)^(1/2)/b/g^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*d^2*(c^2*f^2-
g^2)^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+1/4*b*c^3*d^2*f*(c^2*f^2-2*g^2)*x^2*(-c^2*d*x^2+d)^(1/2)/
g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*d^2*(c^2*f^2-g^2)^2*(-c^2*x^2+1)*(a+b*a
rccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/8*c^2*d^2*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^
2*f*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2-1/3*d^2*(c^2*f^2-2*g^2)
*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^3+1/16*b*c^5*d^
2*f*x^4*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*c*d^2*f*(
a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
b*d^2*(c^2*f^2-g^2)^(5/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-b*d^2*(c^2*f^2-g^2)^(5/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-a*d^2*(c^2*f^2-g^2)^(5/2)*arctanh((c^2*f*x+g)/(c^2*f^2-
g^2)^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^6/(-
c*x+1)/(c*x+1)+1/3*b*c*d^2*(c^2*f^2-2*g^2)*x*(-c^2*d*x^2+d)^(1/2)/g^3/(c*x-
1)^(1/2)/(c*x+1)^(1/2)-b*c*d^2*(c^2*f^2-g^2)^2*x*(-c^2*d*x^2+d)^(1/2)/g^5/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*c^3*d^2*f*x^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c^3*d^2*(c^2*f^2-2*g^2)*x^3*(-c^2*d*x^2+d)^
(1/2)/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/15*d^2*(-c*x+1)*(c*x+1)*(a+b*arccos
h(c*x))*(-c^2*d*x^2+d)^(1/2)/g+b*d^2*(c^2*f^2-g^2)^2*arccosh(c*x)*(-c^2*d*x
^2+d)^(1/2)/g^5+a*d^2*(c^2*f^2-g^2)^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/g^5
/(-c*x+1)/(c*x+1)-1/2*c^2*d^2*f*(c^2*f^2-2*g^2)*x*(a+b*arccosh(c*x))*(-c^2*
d*x^2+d)^(1/2)/g^4-1/5*c^2*d^2*x^2*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^
2*d*x^2+d)^(1/2)/g+2/15*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+1/45*b*c^3*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
b*d^2*(c^2*f^2-g^2)^(5/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c
*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^(1/
2)-b*d^2*(c^2*f^2-g^2)^(5/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^
(1/2)
```

Rubi [A]

time = 3.30, antiderivative size = 1744, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 32, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 1.032$, Rules used = {5972, 5977, 5896, 5893, 30, 5915, 41, 5927, 5939, 102, 12, 75, 5923, 5976, 697, 5970, 6874, 95, 214, 1624, 1668, 739, 212, 5993, 5992, 8, 5980, 3401, 2296, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]

[Out] (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2])/(3*g^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2])/(g^5*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d - c^2*d
*x^2])/(4*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^3*Sqrt[d - c^2*d
*x^2])/(45*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x
^3*Sqrt[d - c^2*d*x^2])/(9*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^5*d^2*f
*x^4*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a*d^2*(c^
2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(g^5*(1 - c*x)*(1 + c*x))
 + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/g^5 + (c^2*d^
2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*g^2) - (c^2*d^2*f*(c^2*f
^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*g^4) - (c^4*d^2*
f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*g^2) - (2*d^2*(1 - c*x)*
(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*g) - (d^2*(c^2*f^2
- 2*g^2)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*g
^3) - (c^2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(5*g) + (c*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*g^
2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d
^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(c^2*f^2 - g^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x))
 - (d^2*(c^2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(2*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*d^2*(c^2*f
^2 - g^2)^(5/2)*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2])])/(g^6*(1 - c*x)*(1 + c*x)) + (b
*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcC
osh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^6*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
```

```
 c*x]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^A
rcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5896

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]
```

```
*((a + b*ArcCosh[c*x])^n/2), x] + (-Dist[(1/2)*Simp[Sqrt[d1 + e1*x]/Sqrt[1
+ c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(a + b*ArcCosh[c*x])^n/(S
qrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d1 + e1*x]/
Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[x*(a + b*ArcCosh[c
*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d
1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5923

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(d1 + e1*x
)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c*Simp[Sqrt
[d1 + e1*x]*(Sqrt[d2 + e2*x]/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]))], Int[Simplify
Integrand[u/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x], x]] /; FreeQ[{a, b,
c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IntegerQ[p -
 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]
)
```

Rule 5927

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(f*x)^(m + 1)*S
qrt[d1 + e1*x]*Sqrt[d2 + e2*x]*((a + b*ArcCosh[c*x])^n/(f*(m + 2))), x] + (
-Dist[(1/(m + 2))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 + c*x]]*Simp[Sqrt[d2 + e2*x]/
Sqrt[-1 + c*x]], Int[(f*x)^m*((a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]*Sqrt[-1
 + c*x])), x], x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d1 + e1*x]/Sqrt[1 +
c*x]]*Simp[Sqrt[d2 + e2*x]/Sqrt[-1 + c*x]], Int[(f*x)^(m + 1)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && Eq
Q[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
```

```
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5970

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5976

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[(f + g*x
)^m*(d1*d2 + e1*e2*x^2)*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*Sqrt[(-d1)*d2]*(
n + 1))), x] - Dist[1/(b*c*Sqrt[(-d1)*d2]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5977

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n, (f + g*x)
^m*(d1 + e1*x)^(p - 1/2)*(d2 + e2*x)^(p - 1/2), x], x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Integ
erQ[m] && IGtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5992

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5993

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(-\frac{c^2f(c^2f^2-2g^2)\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-}}{g^4}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d-c^2dx^2}\right)\int x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx\right.}{g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c^2d^2f(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2g^4} - \frac{c^4d^2fx^3}{}$$

$$= \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2f(c^2f^2-2g^2)\,x^2\sqrt{d-c^2}}{4g^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{16}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{16}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{16}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{16}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b}{16}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - bc$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - bc$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}} - bc$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{} + \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{} - bc$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.12, size = 6244, normalized size = 3.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4233 vs. $2(1608) = 3216$.
time = 6.30, size = 4234, normalized size = 2.43

method	result	size
default	Expression too large to display	4234

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4*a/g^2*c^2*d*f*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2
)^(3/2)*x+7/8*a/g^2*c^2*d^2*f*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*
f^2-g^2)/g^2)^(1/2)*x+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-23/15*b*(-d*(c^
2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)/g*arccosh(c*x)+3*a/g^3*d^3/(-d*(c^2*f^2
-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*
f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g
^2)^(1/2))/(x+f/g))*c^2*f^2-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*d^2*(-d*(c^2*x^2-1)
)^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*dilog((-(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(
1/2)))+33/128*b*(-d*(c^2*x^2-1))^(1/2)*f*c*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/
g^2-1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g^
4-1/25*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g*x^5*c^5+1
1/45*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g*x^3*c^3-23/
15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g*x*c+b*d^2*(-d
*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*arc
cosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))
/(-c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-
g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*arccosh(c*x)*ln(((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^
4*f^4-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2
```

```
*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g
*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-2*b*d^2*(-d*(c^2*x^2-1))^(1/2
)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog((-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))
*c^2*f^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*c^6*d^2/(c*x+1)/(c*x-1)/g^2*arccosh
(c*x)*x^5+11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*c^4*d^2/(c*x+1)/(c*x-1)/g^2*arcco
sh(c*x)*x^3-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*c^2*d^2/(c*x+1)/(c*x-1)/g^2*arcc
osh(c*x)*x+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-
g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+b*d^2*(-d*(c^2*x^2-1))^(1/2)
*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*dilog((-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*
c^4*f^4+7/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g^3*x*
c^3*f^2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^3*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2
)/g^4*x^2+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/
2)/g^2*x^4-9/16*b*(-d*(c^2*x^2-1))^(1/2)*f*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)/g^2*x^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)/g^3*arccosh(c*
x)*x^4*c^6*f^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)/g^3*arccosh
(c*x)*x^2*c^4*f^2+15/8*a/g^2*c^2*d^3*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-1/2*a/g^
4*d^2*c^4*f^3*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1
/2)*x-5/2*a/g^4*d^3*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^
2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^6*d^3*c^6*f^5/(
c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d
*(c^2*f^2-g^2)/g^2)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3*c^6*d^2/(c*x+1)
/(c*x-1)/g^4*arccosh(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3*c^4*d^2/(c*x
+1)/(c*x-1)/g^4*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)
/g^5*arccosh(c*x)*x^2*c^6*f^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-1/9*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g^3*x^3*c^5*f^2-b*(-d*(c^
2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/g^5*x*c^5*f^4-1/2*b*(-d*(c^
2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^5*arccosh(c*x)^2*c^5*d^2/g^6+
5/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*arccosh(c*x)^2
*c^3*d^2/g^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f
+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x+1)/(c*x-1)/g^5*arccosh(c*x)*c^4*f^4-15/16*b*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c*d^2/g^2+1/5*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x+1)/(c*x-1)/g*arccosh(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(c*x+1)/(c*x-1)/g*arccosh(c*x)*x^4*c^4+2*b*d^2*(-d*(c^2*x^2-1))
^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln(
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f...
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(d - c^2\,d\,x^2\right)^{5/2}}{f + g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x), x)
```

3.66 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=478

$$-\frac{3bf^2gx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{2bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}}-\frac{3bfg^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}}-\frac{bg^3x^3\sqrt{-1+cx}}{9c\sqrt{d-c^2}}$$

```
[Out] -3*f^2*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2/3*g
^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^4/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*x
*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/3*g^3*x^2*(
-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*b*g^3*x*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-3/4*b*f*g^2*x^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(
-c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/b/c/(-c^2*d*x^2+d)^(1/2)+3/4*f*g^2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5972, 5975, 5893, 5915, 8, 5939, 30}

$$\frac{f^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{3f^2g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{3fg^2x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{g^3x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}}-\frac{2g^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}}+\frac{3fg^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{3bf^2gx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{3bfg^2x^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}-\frac{bg^3x^3\sqrt{cx-1}\,\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bg^3x\sqrt{cx-1}\,\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*
g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^
2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (3*f^2*g*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*g^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (3*f*g^2*
x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) - (
g^3*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2
]) + (f^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[
d - c^2*d*x^2]) + (3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
```

```
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\left(\frac{f^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3f^2gx\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\,\sqrt{1+c}}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3f^2g\sqrt{-1+\right.}{}$$

$$= -\frac{3f^2g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-cx)(1+cx)\left(\right.}{2c^2\sqrt{d-c^2}}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3}{}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{3bfg^2x}{}$$

Mathematica [A]

time = 1.15, size = 371, normalized size = 0.78

$$\frac{18bc\sqrt{d}\,f(2c^2f^2+3g^2)(-1+cx)\cosh^{-1}(cx)^2 - 4\left(\sqrt{d}\,g(-1+c^2x^2)\left(2b(7g^2-cg^2x+c^2(27f^2+g^2x^2)) - 3a\sqrt{\frac{-1+cx}{1+cx}}\,(4g^2+c^2(18f^2+9fgx+2g^2x^2))\right) + 9acf(2c^2f^2+3g^2)\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)\right) - 27bc\sqrt{d}\,fg^2(-1+cx)\cosh\left(2\cosh^{-1}(cx)\right) + 6b\sqrt{d}\,g(-1+cx)\cosh^{-1}(cx)\left(4\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)(2g^2+c^2(9f^2+g^2x^2)) + 9cfg\sinh\left(2\cosh^{-1}(cx)\right)\right)}{72c^4\sqrt{d}\,\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (18*b*c*Sqrt[d]*f*(2*c^2*f^2 + 3*g^2)*(-1 + c*x)*ArcCosh[c*x]^2 - 4*(Sqrt[d
]*g*(-1 + c^2*x^2)*(2*b*(7*g^2 - c*g^2*x + c^2*(27*f^2 + g^2*x^2)) - 3*a*Sq
rt[(-1 + c*x)/(1 + c*x)]*(4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 9*
a*c*f*(2*c^2*f^2 + 3*g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*Ar
cTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]) - 27*b*c*Sqrt[d]*
f*g^2*(-1 + c*x)*Cosh[2*ArcCosh[c*x]] + 6*b*Sqrt[d]*g*(-1 + c*x)*ArcCosh[c*
x]*(4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(2*g^2 + c^2*(9*f^2 + g^2*x^2))
```

```
+ 9*c*f*g*Sinh[2*ArcCosh[c*x]]))/(72*c^4*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2])
```

Maple [A]
time = 9.06, size = 836, normalized size = 1.75

method	result
default	$-\frac{a\,g^{3}x^{2}\sqrt{-c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2a\,g^{3}\sqrt{-c^{2}d\,x^{2}+d}}{3d\,c^{4}}-\frac{3af\,g^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{2c^{2}d}+\frac{3af\,g^{2}\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2c^{2}\sqrt{c^{2}d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/3*a*g^3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(-c^2*d*x^2+d)^(1
/2)-3/2*a*f*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+3/2*a*f*g^2/c^2/(c^2*d)^(1/2)*
arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3*a*f^2*g/c^2/d*(-c^2*d*x^2+d)
^(1/2)+a*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(
-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*f
*arccosh(c*x)^2*(2*c^2*f^2+3*g^2)-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*
c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+1)*g^3*(-1+3*arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/16*(-d*(c^2*x^2-1))^(1/
2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))*f*g^2*(2*arccosh(c*x)-1)/d/c^3/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))
^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(4*arccosh(c*x)*c^2*f^
2-4*c^2*f^2+arccosh(c*x)*g^2-g^2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(4*arccosh(c*x)*c^2*f^2+
4*c^2*f^2+arccosh(c*x)*g^2+g^2)/c^4/d/(c^2*x^2-1)-3/16*(-d*(c^2*x^2-1))^(1/
2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^
(1/2)-2*c*x)*f*g^2*(2*arccosh(c*x)+1)/d/c^3/(c^2*x^2-1)-1/72*(-d*(c^2*x^2-1
))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*g^3*(1+3*arccosh(c*x))/c^4/d/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/3*a*g^3*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*
d)) - 3/2*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d
```

```
))) + a*f^3*arcsin(c*x)/(c*sqrt(d)) - 3*sqrt(-c^2*d*x^2 + d)*a*f^2*g/(c^2*d
) + integrate(b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*
x^2 + d) + 3*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d
*x^2 + d) + 3*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*
x^2 + d) + b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^2
*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))(f + gx)^3}{\sqrt{-d(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^3\,(a + b\,\mathrm{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
[Out] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.67 $\int \frac{(f+gx)^2(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=288

$$-\frac{2bfgx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{bg^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2x(1-$$

```
[Out] -2*f*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/2*g^2
*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/2*f^2*(a+b*arccosh(c*x))^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)+1/4*g^2*(a+b*arccosh(c*x))^2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5972, 5975, 5893, 5915, 8, 5939, 30}

$$\frac{f^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}+\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{2bfgx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{bg^2x^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 -
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (f^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^
2]) + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sq
rt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(\frac{f^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2fgx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+c}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(2fg\sqrt{-1+c\right.}{}$$

$$= -\frac{2fg(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x(1-cx)(1+cx)\left(a+\right.}{2c^2\sqrt{d-c^2dx}}$$

$$= -\frac{2bfgx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg(1-}{}$$

Mathematica [A]
time = 0.65, size = 284, normalized size = 0.99

$$\frac{4c\sqrt{d}\,g(-1+c^2x^2)\left(-4bf+a\sqrt{\frac{-1+cx}{1+cx}}\,(4f+gx)\right)+2b\sqrt{d}\,(2c^2f^2+g^2)\,(-1+cx)\cosh^{-1}(cx)^2-4a(2c^2f^2+g^2)\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)-b\sqrt{d}\,g^2(-1+cx)\cosh\left(2\cosh^{-1}(cx)\right)+2b\sqrt{d}\,g(-1+cx)\cosh^{-1}(cx)\left(8cf\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)+g\sinh\left(2\cosh^{-1}(cx)\right)\right)}{8c^3\sqrt{d}\,\sqrt{\frac{-1+cx}{1+cx}}\,\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (4*c*Sqrt[d]*g*(-1 + c^2*x^2)*(-4*b*f + a*Sqrt[(-1 + c*x)/(1 + c*x)]*(4*f +
 g*x)) + 2*b*Sqrt[d]*(2*c^2*f^2 + g^2)*(-1 + c*x)*ArcCosh[c*x]^2 - 4*a*(2*c
^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sq
rt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - b*Sqrt[d]*g^2*(-1 + c*x)*Cos
h[2*ArcCosh[c*x]] + 2*b*Sqrt[d]*g*(-1 + c*x)*ArcCosh[c*x]*(8*c*f*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x) + g*Sinh[2*ArcCosh[c*x]]))/(8*c^3*Sqrt[d]*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(252) = 504$.
time = 9.25, size = 518, normalized size = 1.80

method	result
default	$-\frac{a\,g^2x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\,g^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}-\frac{2afg\sqrt{-c^2d\,x^2+d}}{c^2d}+\frac{a\,f^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+}}\right.}{\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2*(2*c^2*f^2+g^2)-1/16*(-d*(c^2*x^2-1))^(1/2)*(2*c^3*x^3-2*c*x+2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))*g^2*(2*arccosh(c*x)-1)/d/c^3/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(-1+arccosh(c*x))/c^2/(c^2*x^2-1)/d-(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*f*g*(1+arccosh(c*x))/c^2/(c^2*x^2-1)/d-1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)*g^2*(2*arccosh(c*x)+1)/d/c^3/(c^2*x^2-1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")

[Out] -1/2*a*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + a*f^2*arcsin(c*x)/(c*sqrt(d)) - 2*sqrt(-c^2*d*x^2 + d)*a*f*g/(c^2*d) + integrate(b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d) + 2*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d) + b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^2*d*x^2 - d), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))(f + gx)^2}{\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^2\,(a + b\operatorname{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.68 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=136

$$-\frac{bgx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{d-c^2dx^2}}$$

```
[Out] -g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-b*g*x*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arccosh(c*x))^2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5972, 5975, 5893, 5915, 8}

$$\frac{f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) + (f*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
```

```
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5975

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(\frac{f\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{gx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+ca}}\right.}{\sqrt{d-c^2dx^2}}\\
&= \frac{\left(f\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}+\frac{\left(g\sqrt{-1+cx}\,\sqrt{\right.}{}\\
&= -\frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\right.}{2bc\sqrt{d-c^2dx^2}}\\
&= -\frac{bgx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f}{}
\end{aligned}$$

Mathematica [A]

time = 0.47, size = 185, normalized size = 1.36

$$\frac{-\frac{2ag\sqrt{d-c^2dx^2}}{d}+\frac{bcf\sqrt{\frac{-1+cx}{1+cx}}(1+cx)\cosh^{-1}(cx)^2}{\sqrt{d-c^2dx^2}}-\frac{2bg\sqrt{d-c^2dx^2}\left(-1+\sqrt{\frac{-1+cx}{1+cx}}\cosh^{-1}(cx)\right)}{d\sqrt{\frac{-1+cx}{1+cx}}}-\frac{2acf\operatorname{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)}{\sqrt{d}}}{2c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/d + (b*c*f*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x]^2)/Sqrt[d - c^2*d*x^2] - (2*b*g*Sqrt[d - c^2*d*x^2]*(-1 + S
qrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]))/(d*Sqrt[(-1 + c*x)/(1 + c*x)]) - (
2*a*c*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d]
)/(2*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(120) = 240$.
time = 7.95, size = 248, normalized size = 1.82

method	result
default	$-\frac{ag\sqrt{-c^2d\,x^2+d}}{c^2d}+\frac{af\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+b\left(-\frac{\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,f\operatorname{arcc}}{2dc(c^2x^2-1)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -a*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d/c/(c^2*x^2-1)*f*arccosh(c*x)^2-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(-1+arccosh(c*x))/c^2/d/(c^2*x^2-1)-1/2*
(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*g*(1+ar
ccosh(c*x))/c^2/d/(c^2*x^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] a*f*arcsin(c*x)/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*a*g/(c^2*d) + integrate(
b*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d) + b*f*log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(
c^2*d*x^2 - d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))(f + gx)}{\sqrt{-d(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f + g\,x)\,(a + b\,\mathrm{acosh}(c\,x))}{\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
[Out] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.69 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}}-\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx}{\sqrt{c^2f^2-g^2}\,\sqrt{d}}$$

```
[Out] (a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g
^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^
(1/2)-(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2
*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x
^2+d)^(1/2)+b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-
g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)
^(1/2)-b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^
(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2
)
```

Rubi [A]
time = 0.47, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5972, 5980, 3401, 2296, 2221, 2317, 2438}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(a+b\cosh^{-1}(cx))\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}+1\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
- (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x
]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f -
Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf+g+e^{2x}g}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2-g^2}}\,dx, x, \cosh^{-1}(cx)\right.}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1}}{}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1}}{}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.31, size = 932, normalized size = 2.55

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2
) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^
2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*
Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[A
rcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2
```

```
*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[
((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCo
sh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (
ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2)
 + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]
*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*
(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[
-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]
))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[d
 - c^2*d*x^2])/Sqrt[-(c^2*f^2) + g^2]
```

Maple [A]
time = 4.18, size = 754, normalized size = 2.07

method	result
default	$-\frac{a\ln\left(\frac{-\frac{2d\left(c^2f^2-g^2\right)}{g^2}+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{-\frac{d(c^2f^2-g^2)}{g^2}}\sqrt{-\left(x+\frac{f}{g}\right)^2c^2d+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}-\frac{d(c^2f^2-g^2)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{-\frac{d(c^2f^2-g^2)}{g^2}}}-\frac{b\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-
c^2*f^2+g^2)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f
^2+g^2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c
*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog((-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2
)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog(((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*
x^2 - d*g*x - d*f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{\sqrt{-d \left(cx - 1\right) \left(cx + 1\right)} \left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{(f+g\,x)\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)), x)
```

3.70 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=523

$$-\frac{g\sqrt{-1+cx}\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}}+\frac{c^2f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\Big(}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}$$

```
[Out] -g*(c*x+1)^(3/2)*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*((c*x-1)/(c*x+1))^(1/2)/(
c^2*f^2-g^2)/(g*x+f)/(-c^2*d*x^2+d)^(1/2)+b*c*ln(g*x+f)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*f^2-g^2)/(-c^2*d*x^2+d)^(1/2)+c^2*f*(a+b*arccosh(c*x))*ln(1+(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*arccosh(
c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+b*c^2*f
*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-b*c^2*
f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {5972, 5980, 3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{g\sqrt{cx-1}\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)\,(f+gx)}+\frac{c^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}-\frac{c^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}+1\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}+\frac{bc^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}-\frac{bc^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)^{3/2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log(f+gx)}{\sqrt{d-c^2dx^2}\,(c^2f^2-g^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((g*Sqrt[-1 + c*x]*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)^(3/2)*(a + b*Arc
Cosh[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2])) + (c^2*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (c
^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*
x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[
d - c^2*d*x^2]) - (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^Arc
Cosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d -
 c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)^2}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(c\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(c^2f\sqrt{-1+cx}\right.}{}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{\right.}{(c^2}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1}}{(c^2f^2-g^2)}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.20, size = 1115, normalized size = 2.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Lo
g[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*
x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g
```

```
)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*(-((g*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*f - g)*(c
*f + g)*(c*f + c*g*x))) + Log[1 + (g*x)/f]/(c^2*f^2 - g^2) + (c*f*(2*ArcCos
h[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2
*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*
x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])
/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c
*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt
[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/
g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])
*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x
]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Ar
cCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*
f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sq
rt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^
2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
+ I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(-(c^2*f^2) + g^2)^(3
/2)))/Sqrt[d - c^2*d*x^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1977 vs. $2(519) = 1038$.
time = 8.48, size = 1978, normalized size = 3.78

method	result	size
default	Expression too large to display	1978

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x
-1)*(c*x+1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^
2*f^2-g^2)/(g*x+f)*x^3*c^4*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x
^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*c*g+b*(-d*(c^2*x^
2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-b*(-
```

```
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2
-1)/(c^2*f^2-g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(
c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*g-b*c^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c
^2*x^2-1))^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f
^2*g^2-g^4)/d*f*arccosh(c*x)*(c^2*f^2-g^2)^(1/2)*ln((-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*c^2*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^
2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*f*arccosh(c*x)*(c^2*f^2-g^2)
^(1/2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*
f+(c^2*f^2-g^2)^(1/2)))+2*b*c^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1)
)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^
4)/d*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2-b*c^3*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*(-d*(c^2*x^2-1))^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x
^2+2*c^2*f^2*g^2-g^4)/d*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))+g)*f^2-b*c^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*
(c^2*x^2-1))^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2
*f^2*g^2-g^4)/d*f*(c^2*f^2-g^2)^(1/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*c^2*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^
4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*f*(c^2*f^2-g^2)^(1/2)*dilog(((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1
/2)))-2*b*c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/(c^6*f^4*x^2
-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*ln(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*g^2+b*c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(-d*(c^2*x^2-1))^(
1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/
d*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))+g)*g^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*
d*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{\sqrt{-d(cx-1)(cx+1)}\,(f+gx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{(f+g\,x)^2\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.71 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=549

$$\frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}}-\frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^3(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{g^3(}{}$$

```
[Out] -1/2*(c*f-g)^3*(-c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+1/2*(
c*f+g)^3*(c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+g^3*(-c*x+1)
*(c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+b*g^3*x*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*(a+b*arccosh(c*x))^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f-g)^3*ln(
2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-
1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f+g)^3*ln(2/(c*x+1))*((-c*x
+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)
/(-c^2*d*x^2+d)^(1/2)+1/2*b*(c*f+g)^3*ln((c*x-1)/(c*x+1))*((-c*x+1)*(c*x+1)
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A]
time = 1.00, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {5972, 5981, 37, 5987, 12, 1986, 15, 266, 272, 36, 31, 29, 5893, 5915, 8}

$$-\frac{(1-cx)(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{g^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}}-\frac{3fg^2\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf-g)^3\log\left(\frac{2}{cx+1}\right)}{2c^4d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf+g)^3\log\left(\sqrt{-\frac{1-cx}{cx+1}}\right)}{c^4d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf+g)^3\log\left(\frac{2}{cx+1}\right)}{2c^4d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}+\frac{bg^3x\sqrt{cx-1}\,\sqrt{cx+1}}{c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - ((c*f
- g)^3*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + ((c*
f + g)^3*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + (g
^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) -
(3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sq
rt[d - c^2*d*x^2]) + (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*
x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]
*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(
1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sq
rt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
+ c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5981

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5987

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[b*c*(Sqrt[1 - c^2*x^2]/(Sqr
```

```
t[-1 + c*x]*Sqrt[1 + c*x])), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^3(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(-\frac{(cf-g)^3(a+b\cosh^{-1}(cx))}{2c^3\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{(cf+g)^3(a+b\cosh}{2c^3(-1+cx)^{3/2}\sqrt{1}}\right.}{d\sqrt{d-c^2}}$$

$$= \frac{\left((cf-g)^3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2c^3d\sqrt{d-c^2dx^2}} - \frac{\left(3fg^2\sqrt{-}\right.}{}$$

$$= -\frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)\left(a+b\cosh\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(c}{}$$

Mathematica [**A**]
time = 1.12, size = 353, normalized size = 0.64

$$\frac{-3bc\sqrt{d}\,fg^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)^2+6acfg^2\sqrt{d-c^2dx^2}\,\text{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+b\sqrt{d}\,\cosh^{-1}(cx)\left(3g^3+2c^4f^3x+6c^2fg(f+gx)-g^3\cosh\left(2\cosh^{-1}(cx)\right)\right)-2\sqrt{d}\left(-a(2g^3+c^4f^3x+c^2g(3f^2+3fgx-g^2x^2))+bcf(c^2f^2+3g^2)\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)+bg(3c^2f^2+g^2)\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)\right)+b\sqrt{d}\,g^3\sinh\left(2\cosh^{-1}(cx)\right)}{2c^4d^{3/2}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*b*c*Sqrt[d]*f*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 +
 6*a*c*f*g^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*
(-1 + c^2*x^2))] + b*Sqrt[d]*ArcCosh[c*x]*(3*g^3 + 2*c^4*f^3*x + 6*c^2*f*g*
(f + g*x) - g^3*Cosh[2*ArcCosh[c*x]]) - 2*Sqrt[d]*(-(a*(2*g^3 + c^4*f^3*x +
 c^2*g*(3*f^2 + 3*f*g*x - g^2*x^2))) + b*c*f*(c^2*f^2 + 3*g^2)*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + b*g*(3
*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/
2]]) + b*Sqrt[d]*g^3*Sinh[2*ArcCosh[c*x]])/(2*c^4*d^(3/2)*Sqrt[d - c^2*d*x^
2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(486) = 972$.
time = 9.01, size = 1238, normalized size = 2.26

method	result
default	$-\frac{a\,g^3x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2a\,g^3}{d\,c^4\sqrt{-c^2d\,x^2+d}}+\frac{3af\,g^2x}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{3af\,g^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -a*g^3*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a*g^3/d/c^4/(-c^2*d*x^2+d)^(1/2)+3*
a*f*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-3*a*f*g^2/c^2/d/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3*a*f^2*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a
*f^3*x/d/(-c^2*d*x^2+d)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g^3+b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/c/d^2/(c^2*x^2-1)*f^3-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d^2/(c^2*x^2-1)*g^3+3/
2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*
arccosh(c*x)^2*f*g^2-3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^2/c^3/(c^2*x^2-1)*f*arccosh(c*x)*g^2+3*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(
c*x)/c^2/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/
```

```
d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f*g^2-b*(-d*(c^2*x^2-
1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f^3-b*(-d*(c^2*x^2-1))^(1/2)*g^3/c
^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*f^3*arccosh(c*x)-3*b*(-d*(c^2*
x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*f^2*g+b*(-d*(c^2*x^2-1))^(1/
2)*arccosh(c*x)/c^4/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*g^3-3*b*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c
^2/d^2/(c^2*x^2-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2/(c^2*x^2-1)*f*g^2+b*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^3-3*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2
/d^2/(c^2*x^2-1)*x*f*g^2-b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/d^2/(c^2*x^2-1)*a
rccosh(c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*b*c*f^3*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d - a*g^3*(x^2/(sqrt(-c^2*d*
x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) + 3*a*f*g^2*(x/(sqrt(-c^2
*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*f^3*x*arccosh(c*x)/(sqr
t(-c^2*d*x^2 + d)*d) + a*f^3*x/(sqrt(-c^2*d*x^2 + d)*d) + 3*a*f^2*g/(sqrt(-
c^2*d*x^2 + d)*c^2*d) + integrate(b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))/(-c^2*d*x^2 + d)^(3/2) + 3*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1))/(-c^2*d*x^2 + d)^(3/2) + 3*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^4*
d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\,(f + gx)^3}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^3\,(a + b\,\mathrm{acosh}(c\,x))}{(d - c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.72 $\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$-\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b}{2bc^3d\sqrt{d-c^2dx}}$$

```
[Out] -1/2*(c*f-g)^2*(-c*x+1)*(a+b*arccosh(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+1/2*(
c*f+g)^2*(c*x+1)*(a+b*arccosh(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*g^2*(a+b
*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1
/2*b*(c*f-g)^2*ln(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^
3/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f+g)^2*ln
(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c*x+1)/((c*x
-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*b*(c*f+g)^2*ln((c*x-1)/(c*x+1))
*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c*x+1)/((c*x-1)/(c*x+1)
)^(1/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.83, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {5972, 5981, 37, 5987, 12, 1986, 15, 266, 272, 36, 31, 29, 5893}

$$-\frac{(1-cx)(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf-g)^2\log\left(\frac{2}{cx+1}\right)}{2c^3d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf+g)^2\log\left(\sqrt{-\frac{1-cx}{cx+1}}\right)}{c^3d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,(cf+g)^2\log\left(\frac{2}{cx+1}\right)}{2c^3d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -1/2*((c*f - g)^2*(1 - c*x)*(a + b*ArcCosh[c*x]))/(c^3*d*Sqrt[d - c^2*d*x^2
]) + ((c*f + g)^2*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x
^2]) - (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d
*Sqrt[d - c^2*d*x^2]) + (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c
^2*x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^3*d*Sqrt[-((1 - c*x)/(1 + c*x
))]*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^2*Sqrt[(1 - c*x)*(1 + c*x
)]*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))
]*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]
*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*
(1 + c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5981

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5987

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[b*c*(Sqrt[1 - c^2*x^2]/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x])), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(f+gx)^2 \left(a+b\cosh^{-1}(cx)\right)}{(d-c^2dx^2)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^2(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(-\frac{(cf-g)^2(a+b\cosh^{-1}(cx))}{2c^2\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{(cf+g)^2(a+b\,\mathrm{co}}{2c^2(-1+cx)^{3/2}\sqrt{}}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left((cf-g)^2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2c^2d\sqrt{d-c^2dx^2}} - \frac{\left(g^2\sqrt{-}\right.}{}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

Mathematica [A]
time = 0.62, size = 281, normalized size = 0.61

$$\frac{2bc\sqrt{d}\,(2fg+c^2f^2x+g^2x)\cosh^{-1}(cx)-b\sqrt{d}\,g^2\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)^2+2ag^2\sqrt{d-c^2dx^2}\,\mathrm{ArcTan}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(-1+c^2x^2)}\right)+2\sqrt{d}\left(ac(2fg+c^2f^2x+g^2x)-b(c^2f^2+g^2)\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\right)-2bcfg\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)\right)}{2c^3d^{3/2}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*b*c*Sqrt[d]*(2*f*g + c^2*f^2*x + g^2*x)*ArcCosh[c*x] - b*Sqrt[d]*g^2*Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 + 2*a*g^2*Sqrt[d - c^2*d*x
^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*Sqrt[d]*
(a*c*(2*f*g + c^2*f^2*x + g^2*x) - b*(c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] - 2*b*c*f*g*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]]))/(2*c^3*d^(3/2)*S
qrt[d - c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(404) = 808$.
time = 8.74, size = 879, normalized size = 1.92

method	result
default	$\frac{a\,g^2x}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{a\,g^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+\frac{2afg}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{a\,f^2x}{d\sqrt{-c^2d\,x^2+d}}+\frac{b\sqrt{-d(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBO
SE)
```

```
[Out] a*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a*g^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+2*a*f*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a*f^2*x/d
/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d^2/c^3/(c^2*x^2-1)*g^2*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*f^2-b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*g^2+2*b*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*(c*x-1)*(c*x+1)*f*g
-2*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*f*g-b*(-d*(c^2
*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f^2-b*(-d*(c^2*x^2-1))^(1/2)*
arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^
2-2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1
)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^
```

```
2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^2+2*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)-1)*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/
(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*b*c*f^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + a*g^2*(x/(sqrt(-c^2*d*x^
2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*f^2*x*arccosh(c*x)/(sqrt(-c^
2*d*x^2 + d)*d) + a*f^2*x/(sqrt(-c^2*d*x^2 + d)*d) + 2*a*f*g/(sqrt(-c^2*d*x
^2 + d)*c^2*d) + integrate(b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/(-c^2*d*x^2 + d)^(3/2) + 2*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/
(-c^2*d*x^2 + d)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))(f + gx)^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,(a+b\,\mathrm{acosh}(c\,x))}{(d-c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.73 $\int \frac{(f+gx)(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{(g+c^2fx)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b(cf-g)\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(1-}{cd\sqrt{d-c^2dx^2}}$$

```
[Out] (c^2*f*x+g)*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(c*f-g)*arctanh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*f*ln(-c*x+1)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5972, 79, 37, 5974, 35, 212}

$$-\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,(cf-g)\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{cx-1}\,\sqrt{cx+1}\,\log(1-cx)}{cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((c*f - g)*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2])) + (f*(1 + c
*x)*(a + b*ArcCosh[c*x]))/(c*d*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (b*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c*x])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5974

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d1_) + (e1_.)*(x_))^(p_)*((d2_) +
 (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHid
e[(f + g*x)^m*(d1 + e1*x)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u,
 x] - Dist[b*c, Int[Dist[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), u, x], x], x]] /
; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2
+ c*d2, 0] && IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] &&
 (GtQ[m, 3] || LtQ[m, -2*p - 1])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-\right.}{}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{bf\sqrt{-}}{}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{bf\sqrt{-}}{}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}} - \frac{b(cf-}{}$$

Mathematica [A]
time = 0.36, size = 133, normalized size = 0.94

$$-\frac{(g+c^2fx)\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}(cx))}{c^2d^2(-1+c^2x^2)}+\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d-c^2dx^2}\,((cf+g)\log(-1+cx)+(cf-g)\log(1+cx))}{2c^2d^2(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((g + c^2*f*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c
^2*x^2))) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*((c*f + g)*
Log[-1 + c*x] + (c*f - g)*Log[1 + c*x]))/(2*c^2*d^2*(-1 + c^2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(128) = 256$.
time = 7.12, size = 499, normalized size = 3.51

method	result
default	$a\left(\frac{g}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{fx}{d\sqrt{-c^2d\,x^2+d}}\right)-\frac{b\sqrt{-d\,(c^2x^2-1)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,f\,\mathrm{arccosh}(cx)}{d^2c(c^2x^2-1)}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] a*(g/c^2/d/(-c^2*d*x^2+d)^(1/2)+f*x/d/(-c^2*d*x^2+d)^(1/2))-b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*f*arccosh(c*x)+b*(-
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*(c*x-1)*(c*x+1)*g-b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*g-b*(-d*(c^2*x^2-1))
^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2/(c^2*x^2-1)*f
-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/c^2/d^2/(c^2*x^2-1)*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f+
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/2*b*c*f*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*g*(((c*sqrt(d)*x + sqrt(
c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt
(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sqrt(-c*x + 1))/(sqrt(c*x
+ 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) - integrate((c^2*x^3 + c*
x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(sqrt(-c*x + 1)*((c^2*d^(3
/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^3*d^(3/2)*x^3
 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1)) + (c^4*d^(3/2)*x^4 - c^
2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + b*f*x*arccosh(c*x)/(sqrt(-c^2*d*x^2 +
 d)*d) + a*f*x/(sqrt(-c^2*d*x^2 + d)*d) + a*g/(sqrt(-c^2*d*x^2 + d)*c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(c
^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))(f + gx)}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f + g\,x)\,(a + b\,\mathrm{acosh}(c\,x))}{(d - c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
[Out] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

3.74 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=773

$$-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\Big(}{d\left(c^2f^2-g^2\right)^{3/2}\sqrt{d-c^2dx}}$$

```
[Out] -1/2*(-c*x+1)*(a+b*arccosh(c*x))/d/(c*f-g)/(-c^2*d*x^2+d)^(1/2)+1/2*(c*x+1)
*(a+b*arccosh(c*x))/d/(c*f+g)/(-c^2*d*x^2+d)^(1/2)-g^2*(a+b*arccosh(c*x))*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+g^2*(a+b*arc
cosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2
)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-
b*g^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/
2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
+b*g^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2
)-1/2*b*ln(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f-g)
/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(2/(c*x+1))*(
(-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f+g)/(c*x+1)/((c*x-1)/(c*x+
1))^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*b*ln((c*x-1)/(c*x+1))*((-c*x+1)*(c*x+1))
^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f+g)/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d
*x^2+d)^(1/2)
```

Rubi [A]
time = 1.24, antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 18, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {5972, 5981, 37, 5987, 12, 1986, 15, 266, 272, 36, 31, 29, 5980, 3401, 2296, 2221, 2317, 2438}

$$-\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}+\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}+1\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}\,(cf-g)}+\frac{(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}\,(cf+g)}-\frac{bg^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}+\frac{bg^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}+\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,\log\left(\sqrt{-\frac{1-cx}{cx+1}}\right)}{d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}\,(cf+g)}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,\log\left(\frac{2}{cx+1}\right)}{2d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}\,(cf-g)}-\frac{b\sqrt{(1-cx)(cx+1)}\,\sqrt{1-c^2x^2}\,\log\left(\frac{2}{cx+1}\right)}{2d\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\sqrt{d-c^2dx^2}\,(cf+g)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -1/2*((1 - c*x)*(a + b*ArcCosh[c*x]))/(d*(c*f - g)*Sqrt[d - c^2*d*x^2]) + (
(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f + g)*Sqrt[d - c^2*d*x^2]) - (g^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g
)/(c*f - Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2
]) + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcC
osh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d -
```

```
c^2*d*x^2]) + (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[Sqrt[-((1
 - c*x)/(1 + c*x))]])/(d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*S
qrt[d - c^2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/
(1 + c*x)])/(2*d*(c*f - g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d -
c^2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x
)])/(2*d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^2*d*x^
2]) - (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(
c*f - Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
 + (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f
 + Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5981

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5987

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[b*c*(Sqrt[1 - c^2*x^2]/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x])), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(-\frac{c\left(a+b\cosh^{-1}(cx)\right)}{2(cf-g)\sqrt{-1+cx}\,(1+cx)^{3/2}}+\frac{c\left(a+b\cosh^{-1}(\cdots\right.}{2(cf+g)(-1+cx)^{3/2}\cdots}\cdots\right.}{d\sqrt{d-c^2dx^2}}$$

$$=\frac{\left(c\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2d(cf-g)\sqrt{d-c^2dx^2}}-\frac{\left(c\sqrt{-1+cx}\,\sqrt{1+cx}\cdots\right.}{2d(cf+g)\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{\left(g^2\sqrt{-1+c\cdots\right.}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{\left(2g^2\sqrt{-1+\cdots\right.}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{\left(2g^3\sqrt{-1+\cdots\right.}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\cdots}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\cdots}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\cdots}{\cdots}$$

$$=-\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}}+\frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{-1+cx}\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.64, size = 1203, normalized size = 1.56

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((-(a*g) + a*c^2*f*x)*Sqrt[-(d*(-1 + c^2*x^2))])/(d^2*(-(c^2*f^2) + g^2)*(-
1 + c^2*x^2)) + (a*g^2*Log[f + g*x])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-
(c^2*f^2) + g^2]) - (a*g^2*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) +
g^2]*Sqrt[-(d*(-1 + c^2*x^2))]]])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2
*f^2) + g^2]) - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((ArcCosh[c*x]*Co
th[ArcCosh[c*x]/2])/(c*f + g)) + (2*c*f*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)])/(c^2*f^2 - g^2) + (2*g*Log[Tanh[ArcCosh[c*x]/2]])/(-(c^2*f^2) + g^2
) + (2*g^2*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(
Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)
] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Ar
cTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])
] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-
1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2
*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) +
 g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g
^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
 + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(
(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (ArcCosh[c*x]*Tanh[ArcCosh
[c*x]/2])/(c*f - g)))/(2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2483 vs. $2(738) = 1476$.
time = 5.72, size = 2484, normalized size = 3.21

method	result	size
default	Expression too large to display	2484

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -a*g/d/(c^2*f^2-g^2)/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/
g^2)^(1/2)+a*f/(c^2*f^2-g^2)/d/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2
*f^2-g^2)/g^2)^(1/2)*x*c^2+a*g/d/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1
/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/
g))-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*(c*
x+1)*(c*x-1)*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f
^2-g^2)*x^2*c^2*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^
2*f^2-g^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh
(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*x*c^2*f-2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*
x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3*f^3+b*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2
*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)-1)*c^3*f^3+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^
6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3*f^3-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^
2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2*f^2*g+b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2
-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*c^2*f^2*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*
f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*arccosh(c*
x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+
(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*
x^2+2*c^2*f^2*g^2-g^4)*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^
2+2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2
*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*c*f*g^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4
)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c*f*g^2-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^
4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c*f*g^2+b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2
*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*dilog((-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*
f^2-g^2)^(1/2)*g^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2
```

```
/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*dilo
g(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f
^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*
c^2*f^2*g^2-g^4)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g^3-b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4
*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*g*x^5 + c^4*d^2
*f*x^4 - 2*c^2*d^2*g*x^3 - 2*c^2*d^2*f*x^2 + d^2*g*x + d^2*f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{(-d(cx-1)(cx+1))^{\frac{3}{2}}(f+gx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/((-d*(c*x - 1)*(c*x + 1))**(3/2)*(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}(c\,x)}{(f+g\,x)\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(3/2)), x)
```

3.75 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=239

$$\frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}+\frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right.}{2c^2\sqrt{1-c^2x^2}}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(-c^2*x^2+
1)^(1/2)+1/2*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*x^2+1)
^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(-c^2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.33, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5972, 5980, 3398, 3388, 2212}

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{ge^{a/b}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}+\frac{f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[1 - c^2*x^2]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*
(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 -
c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5972

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d + e*x^2)^Fra
cPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(
-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2] && IntegerQ[m]
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\operatorname{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh\right.}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\right.}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\,\sqrt{1+c\right.}{}$$

$$= \frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\,\sqrt{1+c\right.}{}$$

$$= \frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\,\sqrt{1+}{}$$

Mathematica [A]
time = 0.41, size = 204, normalized size = 0.85

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{-n}\left(2ce^{a/b}f\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n - be^{\frac{2a}{b}}g(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right) + bg(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2bc^2(1+n)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\,\mathrm{arccosh}(cx)\right)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(cx))^n (f+gx)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.76 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=200

$$\frac{f\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}}+\frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c/(1+n)/(-c*x+1)^(1/2)+1/2*g*(a+
b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/exp(a/
b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c
*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/(((a+b*arccosh(c*x
))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5982, 5980, 3398, 3388, 2212}

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{ge^{a/b}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}+\frac{f\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (f*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*x])
 + (g*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c
*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^
(a/b)*g*Sqrt[-1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c
*x])/b])/(2*c^2*Sqrt[1 - c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5982

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-d1
)*d2)^IntPart[p]*(d1 + e1*x)^FracPart[p]*((d2 + e2*x)^FracPart[p]/((-1 + c*
x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(-1 + c*x)^p*(1 + c
*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
g, n}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && Integ
erQ[p - 1/2] &&  !(GtQ[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\,\sqrt{1+cx}}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\,dx, x, \cosh^{-1}\right.}{c^2\sqrt{1-cx}\,\sqrt{1+cx}}\\
&= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\,\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int (a+b\right.}{c^2\sqrt{1-cx}}\\
&= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\,\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int e^{-x}(a\right.}{2c^2\sqrt{1-cx}}\\
&= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\,\sqrt{1+cx}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(\right.}{2}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 204, normalized size = 1.02

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\,(1+cx)\,(a+b\cosh^{-1}(cx))^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{-n}\left(2ce^{a/b}f(a+b\cosh^{-1}(cx))\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n-be^{\frac{2a}{b}}g(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)+bg(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2bc^2(1+n)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\,(a+b\,\mathrm{arccosh}(cx))^n}{\sqrt{-cx+1}\,\sqrt{cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*x + 1)*sqrt(-c*x + 1)),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(-sqrt(c*x + 1)*sqrt(-c*x + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^
2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n (f + gx)}{\sqrt{-cx + 1}\sqrt{cx + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c*x+1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/(sqrt(-c*x + 1)*sqrt(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*x + 1)*sqrt(-c*x + 1)),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)\,(a + b\operatorname{acosh}(c\,x))^n}{\sqrt{1 - c\,x}\,\sqrt{c\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/((1 - c*x)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/((1 - c*x)^(1/2)*(c*x + 1)^(1/2)), x)
```

3.77 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\,\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=260

$$\frac{f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}+\frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(c*d1*x+d1
)^(1/2)/(-c*d2*x+d2)^(1/2)+1/2*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arcco
sh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/
b)^n)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c*x)
)^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(((a+b*
arccosh(c*x))/b)^n)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {5982, 5980, 3398, 3388, 2212}

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{\mathrm{cd1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-\mathrm{cd2}x}}-\frac{ge^{a/b}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{\mathrm{cd1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-\mathrm{cd2}x}}+\frac{f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{\mathrm{cd1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-\mathrm{cd2}x}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]
),x]
```

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*
Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/
b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a +
b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5980

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[(-d1)*d2]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5982

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-d1
)*d2)^IntPart[p]*(d1 + e1*x)^FracPart[p]*((d2 + e2*x)^FracPart[p]/((-1 + c*
x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[(f + g*x)^m*(-1 + c*x)^p*(1 + c
*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f,
g, n}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && Integ
erQ[p - 1/2] &&  !(GtQ[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)(a+b\cosh^{-1}(cx))^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\,dx, x, \cosh^{-1}(\ldots}{c^2\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int (a+b\ldots\right.}{c^2\sqrt{\text{d1}+c\text{d1}x}\ldots}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int e^{-x}(a\ldots\right.}{2c^2\sqrt{\text{d1}+c\text{d1}x}\ldots}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(\ldots\right.}{2c^2\sqrt{\ldots}}$$

Mathematica [A]
time = 1.45, size = 219, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{-1+cx}{1+cx}}\sqrt{\text{d1}+c\text{d1}x}\,\sqrt{\text{d2}-c\text{d2}x}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^{-n}\left(-2ce^{a/b}f\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{(a+b\cosh^{-1}(cx))^2}{b^2}\right)^n+be^{\frac{2a}{b}}g(1+n)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\cosh^{-1}(cx)\right)-bg(1+n)\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2bc^2\text{d1}\text{d2}(1+n)(-1+cx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcC
osh[c*x])^n*(-2*c*E^(a/b)*f*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^n + b*E^((2*a)/b)*g*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, a/b + ArcCosh[c*x]] - b*g*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((
a + b*ArcCosh[c*x])/b)]))/(2*b*c^2*d1*d2*E^(a/b)*(1 + n)*(-1 + c*x)*(-((a +
 b*ArcCosh[c*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\left(a+b\,\text{arccosh}(cx)\right)^n}{\sqrt{cd1x+d1}\,\sqrt{-cd2x+d2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x
+ d2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(g*x + f)*(b*arccosh(c*x) +
a)^n/(c^2*d1*d2*x^2 - d1*d2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}{(cx)})^n (f + gx)}{\sqrt{d_1 (cx + 1)}\sqrt{-d_2 (cx - 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2
),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x
- 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="giac")
```

[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,\left(a+b\,\mathrm{acosh}(c\,x)\right)^n}{\sqrt{d_1+c\,d_1\,x}\,\sqrt{d_2-c\,d_2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)),x)

[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)), x)

3.78 $\int \frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}} dx$

Optimal. Leaf size=75

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{-1+cx}\,\sqrt{1+cx}},x\right)}{\sqrt{1-c^2x^2}}$$

[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*Unintegrable((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/(-c^2*x^2+1)^(1/2)

Rubi [A]
time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

[In] Int[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]

[Out] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^n \ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/(c^2
*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARc near
 0Simplification assuming sageVARc near 0Simplification assuming sageVARc n
ear 0S
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.79 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=774

$$\frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}}-\frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-}}\right.}{3bc\sqrt{1-c^2x^2}}$$

```
[Out] 1/12*m*(a+b*arccosh(c*x))^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b^2/c/(-c^2*x^2+1)^
(1/2)+1/3*(a+b*arccosh(c*x))^3*ln(h*(g*x+f)^m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
b/c/(-c^2*x^2+1)^(1/2)-1/3*m*(a+b*arccosh(c*x))^3*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/
(-c^2*x^2+1)^(1/2)-1/3*m*(a+b*arccosh(c*x))^3*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^
2*x^2+1)^(1/2)-m*(a+b*arccosh(c*x))^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^
2+1)^(1/2)-m*(a+b*arccosh(c*x))^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)
^(1/2)+2*b*m*(a+b*arccosh(c*x))*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(
1/2)+2*b*m*(a+b*arccosh(c*x))*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/
2)-2*b^2*m*polylog(4,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2
)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2*b^2*m*polylog(
4,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {5892, 5973, 5893, 5983, 5962, 5681, 2221, 2611, 6744, 2320, 6724}

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^4)/(12*b^2*c*Sqrt[1 -
c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 +
(E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2])
- (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c*Sqr
```

```
t[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Po
lyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^
2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*PolyLog[2,
 -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2])
+ (2*b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (2*b
*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
 - c^2*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5973

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d
+ e*x^2)^FracPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[L
og[h*(f + g*x)^m]*(-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 5983

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[Log[h*(f + g*x)^m]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*Sqrt[(-d1)*d
2]*(n + 1))), x] - Dist[g*(m/(b*c*Sqrt[(-d1)*d2]*(n + 1))), Int[(a + b*ArcC
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} -$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} -$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1}}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{}}{}$$

Mathematica [F]

time = 3.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(cx)\right)^2\ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2
)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\,(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1
)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.54sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^2)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^2)/(1 - c^2*x^2)^(1/2), x)
```

3.80 $\int \frac{(a+b\cosh^{-1}(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=600

$$\frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}}-\frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-}}\right.}{2bc\sqrt{1-c^2x^2}}$$

```
[Out] 1/6*m*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b^2/c/(-c^2*x^2+1)^(
1/2)+1/2*(a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b
/c/(-c^2*x^2+1)^(1/2)-1/2*m*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(
-c^2*x^2+1)^(1/2)-1/2*m*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2
*x^2+1)^(1/2)-m*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1
)^(1/2)-m*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2
)+b*m*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/
2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)+b*m*polylog(3,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {5892, 5973, 5893, 5983, 5962, 5681, 2221, 2611, 2320, 6724}

$$\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{2bc\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}+1\right)}{2bc\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc\sqrt{1-c^2x^2}}+\frac{bm\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_3\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}+\frac{bm\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_3\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b^2*c*Sqrt[1 - c
^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) -
 (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (E^ArcCosh[
c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c*Sqrt
[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyL
og[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x
^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, -((E
^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*
m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[
```

```
c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
- c^2*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5892

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]*(Sqrt[-1 + c*x]/Sqrt[d
 + e*x^2])]*(a + b*ArcCosh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x
] && EqQ[c^2*d + e, 0] && NeQ[n, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5973

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(-d)^IntPart[p]*((d
+ e*x^2)^FracPart[p]/((-1 + c*x)^FracPart[p]*(1 + c*x)^FracPart[p])), Int[L
og[h*(f + g*x)^m]*(-1 + c*x)^p*(1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 5983

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[Log[h*(f + g*x)^m]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*Sqrt[(-d1)*d
2]*(n + 1))), x] - Dist[g*(m/(b*c*Sqrt[(-d1)*d2]*(n + 1))), Int[(a + b*ArcC
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(a+b\cosh^{-1}(cx))\log(h(f+gx)^m)}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\log(h(f+gx)^m)}{2bc\sqrt{1-c^2x^2}} -$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\log(h(f+gx)^m)}{2bc\sqrt{1-c^2x^2}} -$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1-}}{}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{}}{}$$

$$= \frac{m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\sqrt{}}{}$$

Mathematica [F]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))\ln(h(gx+f)^m)}{\sqrt{-c^2x^2+1}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*log((g*x + f)^m*h)/(c^2*x
^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}{(cx)}) \log{(h(f + gx)^m)}}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x)))/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x)))/(1 - c^2*x^2)^(1/2), x)
```

3.81 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=237

$$\frac{im\text{ArcSin}(cx)^2}{2c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{\text{ArcS}}{}$$

```
[Out] 1/2*I*m*arcsin(c*x)^2/c+arcsin(c*x)*ln(h*(g*x+f)^m)/c-m*arcsin(c*x)*ln(1-I*
(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-m*arcsin(c*x)*ln(
1-I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2
,I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2,
I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

Rubi [A]
time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {222, 2451, 4825, 4615, 2221, 2317, 2438}

$$\frac{im\text{Li}_2\left(\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{Li}_2\left(\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ige^{i\text{ArcSin}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\text{ArcSin}(cx)\log\left(1-\frac{ige^{i\text{ArcSin}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}+\frac{\text{ArcSin}(cx)\log\left(h(f+gx)^m\right)}{c}+\frac{im\text{ArcSin}(cx)^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sin^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{x\cos(x)}{c^2f+cg\sin(x)}\,dx, x, \sin^{-1}(cx)\right)$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{e^{ix}x}{c^2f-ice^{ix}g-c\sqrt{c}\ldots}\right.$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\ldots\right.}{c}$$

Mathematica [A]
time = 0.02, size = 246, normalized size = 1.04

$$\frac{im\text{ArcSin}(cx)^2}{2c} - \frac{m\text{ArcSin}(cx)\log\left(1-\frac{ice^{i\text{ArcSin}(cx)}g}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\text{ArcSin}(cx)\log\left(1-\frac{ice^{i\text{ArcSin}(cx)}g}{c^2f+c\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\text{ArcSin}(cx)\log\left(h(f+gx)^m\right)}{c} + \frac{im\text{PolyLog}\left(2,\frac{ie^{i\text{ArcSin}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\text{PolyLog}\left(2,\frac{ie^{i\text{ArcSin}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*g
)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])]])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*Ar
cSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])]])/c + (ArcSin[c*x]*Log[h*(f +
 g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 -
 g^2])]])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2])]])/c
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.82 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Int}\left(\frac{\log(h(f+gx)^m)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))},x\right)}{\sqrt{1-c^2x^2}}$$

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*Unintegrable(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))
/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/(-c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\log(h(f+gx)^m)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^{m}\right)}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b
)*arccosh(c*x) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^{m}\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))
), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.83 $\int x^3 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(4a(16+}{96b^4}$$

```
[Out] -1/32*(8*a^4+24*a^2+3)*arccosh(b*x+a)/b^4+1/4*x^4*arccosh(b*x+a)+7/48*a*x^2
*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/16*x^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/
2)/b+1/96*(4*a*(19*a^2+16)-(26*a^2+9)*(b*x+a))*(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2)/b^4
```

Rubi [A]
time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5996, 5963, 102, 158, 152, 54}

$$\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}\,(4a(19a^2+16)-(26a^2+9)\,(a+bx))}{96b^4}-\frac{(8a^4+24a^2+3)\cosh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{48b^2}+\frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{x^3\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(48*b^2) - (x^3*Sqrt[-1 + a
+ b*x]*Sqrt[1 + a + b*x])/(16*b) + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4
*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x)))/(96*b^4) - ((3 + 24*a^2 + 8*a^4
)*ArcCosh[a + b*x])/(32*b^4) + (x^4*ArcCosh[a + b*x])/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \cosh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \cosh^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b} + \frac{1}{4}x^4\cosh^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(\frac{3+4a^2}{b^2} -\right.}{\sqrt{-1}}\right.$$

$$= \frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2} - \frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b} + \frac{1}{4}x^4\cosh^{-}$$

$$= \frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2} - \frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b} + \frac{\sqrt{-1+a}}{}$$

$$= \frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2} - \frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b} + \frac{\sqrt{-1+a}}{}$$

Mathematica [A]
time = 0.11, size = 121, normalized size = 0.80

$$\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(55a+50a^3-9bx-26a^2bx+14ab^2x^2-6b^3x^3)+24b^4x^4\cosh^{-1}(a+bx)-3(3+24a^2+8a^4)\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x],x]
[Out] (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x +
 14*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCosh[a + b*x] - 3*(3 + 24*a^2 +
8*a^4)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(96*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(130) = 260$.
time = 7.78, size = 285, normalized size = 1.88

method	result
derivativedivides	$\frac{\frac{\operatorname{arccosh}(bx+a)a^4}{4} - \operatorname{arccosh}(bx+a)a^3(bx+a) + \frac{3\operatorname{arccosh}(bx+a)a^2(bx+a)^2}{2} - \operatorname{arccosh}(bx+a)a(bx+a)^3 + \frac{\operatorname{arccosh}(bx+a)(bx+a)^4}{4} - }{}$

default	$\frac{\operatorname{arccosh}(bx+a)a^4}{4}-\operatorname{arccosh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arccosh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arccosh}(bx+a)a(bx+a)^3+\frac{\operatorname{arccosh}(bx+a)(bx+a)^4}{4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/4*arccosh(b*x+a)*a^4-arccosh(b*x+a)*a^3*(b*x+a)+3/2*arccosh(b*x+a)
*a^2*(b*x+a)^2-arccosh(b*x+a)*a*(b*x+a)^3+1/4*arccosh(b*x+a)*(b*x+a)^4-1/96
*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(24*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))-96*a
^3*((b*x+a)^2-1)^(1/2)+72*a^2*(b*x+a)*((b*x+a)^2-1)^(1/2)-32*a*((b*x+a)^2-1
)^(1/2)*(b*x+a)^2+6*((b*x+a)^2-1)^(1/2)*(b*x+a)^3+72*a^2*ln(b*x+a+((b*x+a)^
2-1)^(1/2))-64*a*((b*x+a)^2-1)^(1/2)+9*(b*x+a)*((b*x+a)^2-1)^(1/2)+9*ln(b*x
+a+((b*x+a)^2-1)^(1/2)))/((b*x+a)^2-1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(129) = 258$.
time = 0.26, size = 321, normalized size = 2.11

$$\frac{1}{4}x^4\operatorname{arcosh}(bx+a)-\frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2abx+a^2-1}\,x^3}{b^2}-\frac{14\sqrt{b^2x^2+2abx+a^2-1}\,ax^2}{b^3}+\frac{105a^4\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^5}+\frac{35\sqrt{b^2x^2+2abx+a^2-1}\,a^2x}{b^4}-\frac{90(a^2-1)a^2\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^5}-\frac{105\sqrt{b^2x^2+2abx+a^2-1}\,a^3}{b^5}-\frac{9\sqrt{b^2x^2+2abx+a^2-1}\,(a^2-1)x}{b^4}+\frac{9(a^2-1)^2\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^5}+\frac{55\sqrt{b^2x^2+2abx+a^2-1}\,(a^2-1)a}{b^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccosh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^3/b^
2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x^2/b^3 + 105*a^4*log(2*b^2*x +
2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 35*sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)*a^2*x/b^4 - 90*(a^2 - 1)*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^
2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a
^3/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*x/b^4 + 9*(a^2 - 1)^
2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 55*sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*a/b^5)*b
```

Fricas [A]
time = 0.37, size = 110, normalized size = 0.72

$$\frac{3\left(8b^4x^4-8a^4-24a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2-1}\right)-\left(6b^3x^3-14ab^2x^2-50a^3+(26a^2+9)bx-55a\right)\sqrt{b^2x^2+2abx+a^2-1}}{96b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^4
```

Sympy [A]
time = 0.30, size = 255, normalized size = 1.68

$$\begin{cases} -\frac{a^4 \operatorname{acosh}(a+bx)}{4b^4} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2-1}}{48b^4} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2-1}}{48b^3} - \frac{3a^2 \operatorname{acosh}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2-1}}{48b^2} + \frac{55a\sqrt{a^2+2abx+b^2x^2-1}}{96b^4} + \frac{x^4 \operatorname{acosh}(a+bx)}{4} - \frac{x^3\sqrt{a^2+2abx+b^2x^2-1}}{16b} - \frac{3x\sqrt{a^2+2abx+b^2x^2-1}}{32b^3} - \frac{3\operatorname{acosh}(a+bx)}{32b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acosh}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acosh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(48
*b**3) - 3*a**2*acosh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**2) + 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(96*b**4
) + x**4*acosh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(16*b
) - 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(32*b**3) - 3*acosh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

Giac [A]
time = 0.43, size = 163, normalized size = 1.07

$$\frac{1}{4}x^4\log\left(bx+a+\sqrt{(bx+a)^2-1}\right) - \frac{1}{96}\left(\sqrt{b^2x^2+2abx+a^2-1}\left(\left(2x\left(\frac{3x}{b^2}-\frac{7a}{b^3}\right)+\frac{26a^2b^3+9b^3}{b^7}\right)x - \frac{5(10a^3b^2+11ab^2)}{b^7}\right) - \frac{3(8a^4+24a^2+3)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)|b|\right|\right)}{b^4|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 + 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 + 11*a*b^2)/b^7) - 3*(8*a^4 + 24*a^2 + 3)*log(abs(-a*b - (x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^4*abs(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a + b*x),x)
```

```
[Out] int(x^3*acosh(a + b*x), x)
```

3.84 $\int x^2 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=104

$$-\frac{x^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(4+11a^2-5abx)}{18b^3}+\frac{a(3+2a^2)\cosh^{-1}(a+$$

```
[Out] 1/6*a*(2*a^2+3)*arccosh(b*x+a)/b^3+1/3*x^3*arccosh(b*x+a)-1/9*x^2*(b*x+a-1)
^(1/2)*(b*x+a+1)^(1/2)/b-1/18*(-5*a*b*x+11*a^2+4)*(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2)/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5996, 5963, 102, 152, 54}

$$-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}\,(11a^2-5abx+4)}{18b^3}+\frac{a(2a^2+3)\cosh^{-1}(a+bx)}{6b^3}+\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{x^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a + b*x],x]
```

```
[Out] -1/9*(x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/b - (Sqrt[-1 + a + b*x]*Sqr
t[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x))/(18*b^3) + (a*(3 + 2*a^2)*ArcCosh[a
+ b*x])/(6*b^3) + (x^3*ArcCosh[a + b*x])/3
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
```

```
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)$$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}+\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(\frac{2+3a^2}{b^2}-5\right.}{\sqrt{-1+}}\right.$$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\,(4+11a^2-5ab}{18b^3}$$

$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\,(4+11a^2-5ab}{18b^3}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 0.97

$$\frac{-\sqrt{-1+a+bx}\sqrt{1+a+bx}\,(4+11a^2-5abx+2b^2x^2)+6b^3x^3\cosh^{-1}(a+bx)+(9a+6a^3)\log\left(a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a + b*x],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2))
 + 6*b^3*x^3*ArcCosh[a + b*x] + (9*a + 6*a^3)*Log[a + b*x + Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x]])/(18*b^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(88) = 176$.
time = 3.30, size = 203, normalized size = 1.95

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(bx+a)a^3}{3}+\operatorname{arccosh}(bx+a)a^2(bx+a)-\operatorname{arccosh}(bx+a)a(bx+a)^2+\frac{\operatorname{arccosh}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{bx+a-1}\,\sqrt{b}}{}$
default	$-\frac{\operatorname{arccosh}(bx+a)a^3}{3}+\operatorname{arccosh}(bx+a)a^2(bx+a)-\operatorname{arccosh}(bx+a)a(bx+a)^2+\frac{\operatorname{arccosh}(bx+a)(bx+a)^3}{3}+\frac{\sqrt{bx+a-1}\,\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(-1/3*arccosh(b*x+a)*a^3+arccosh(b*x+a)*a^2*(b*x+a)-arccosh(b*x+a)*a*
(b*x+a)^2+1/3*arccosh(b*x+a)*(b*x+a)^3+1/18*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)
*(6*a^3*ln(b*x+a+((b*x+a)^2-1)^(1/2))-18*a^2*((b*x+a)^2-1)^(1/2)+9*a*(b*x+a
)*((b*x+a)^2-1)^(1/2)-2*((b*x+a)^2-1)^(1/2)*(b*x+a)^2+9*a*ln(b*x+a+((b*x+a)
^2-1)^(1/2))-4*((b*x+a)^2-1)^(1/2))/((b*x+a)^2-1)^(1/2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.
time = 0.26, size = 212, normalized size = 2.04

$$\frac{1}{3}\,x^3\operatorname{arcosh}\left(bx+a\right)-\frac{1}{18}\,b\left(\frac{2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,x^2}{b^2}-\frac{15\,a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{b^4}-\frac{5\,\sqrt{b^2x^2+2\,abx+a^2-1}\,ax}{b^3}+\frac{9\,(a^2-1)a\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{b^4}+\frac{15\,\sqrt{b^2x^2+2\,abx+a^2-1}\,a^2}{b^4}-\frac{4\,\sqrt{b^2x^2+2\,abx+a^2-1}\,(a^2-1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^2/
b^2 - 15*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b
^4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x/b^3 + 9*(a^2 - 1)*a*log(2*b^2*
x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^4 + 15*sqrt(b^2*x^2 +
2*a*b*x + a^2 - 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)/
b^4)
```

Fricas [A]
time = 0.38, size = 91, normalized size = 0.88

$$\frac{3\,(2\,b^3x^3+2\,a^3+3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-(2\,b^2x^2-5\,abx+11\,a^2+4)\sqrt{b^2x^2+2\,abx+a^2-1}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 - 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 + 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/b^3
```

Sympy [A]
time = 0.17, size = 170, normalized size = 1.63

$$\begin{cases} \frac{a^3\operatorname{acosh}(a+bx)}{3b^3}-\frac{11a^2\sqrt{a^2+2abx+b^2x^2-1}}{18b^3}+\frac{5ax\sqrt{a^2+2abx+b^2x^2-1}}{18b^2}+\frac{a\operatorname{acosh}(a+bx)}{2b^3}+\frac{x^3\operatorname{acosh}(a+bx)}{3}-\frac{x^2\sqrt{a^2+2abx+b^2x^2-1}}{9b}-\frac{2\sqrt{a^2+2abx+b^2x^2-1}}{9b^3} & \text{for } b\neq 0 \\ \frac{x^3\operatorname{acosh}(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(b*x+a),x)
```

```
[Out] Piecewise((a**3*acosh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 - 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(18*b**2
) + a*acosh(a + b*x)/(2*b**3) + x**3*acosh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 - 1)/(9*b) - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(9*b*
*3), Ne(b, 0)), (x**3*acosh(a)/3, True))
```

Giac [A]
time = 0.45, size = 132, normalized size = 1.27

$$\frac{1}{3}\,x^3\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b+4\,b}{b^5}\right)+\frac{3\,(2\,a^3+3\,a)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)|b|\right|\right)}{b^3|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b + 4*b)/b^5) + 3*(2*a^3 + 3*a
)*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b
^3*abs(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acosh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a + b*x),x)
```

```
[Out] int(x^2*acosh(a + b*x), x)
```

3.85 $\int x \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{3a\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b}-\frac{(1+2a^2)\cosh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a$$

```
[Out] -1/4*(2*a^2+1)*arccosh(b*x+a)/b^2+1/2*x^2*arccosh(b*x+a)+3/4*a*(b*x+a-1)^(1
/2)*(b*x+a+1)^(1/2)/b^2-1/4*x*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5996, 5963, 92, 81, 54}

$$-\frac{(2a^2+1)\cosh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{x\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*b^2) - (x*Sqrt[-1 + a + b*x]*
Sqrt[1 + a + b*x])/(4*b) - ((1 + 2*a^2)*ArcCosh[a + b*x])/(4*b^2) + (x^2*Ar
cCosh[a + b*x])/2
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\cosh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\cosh^{-1}(a+bx) - \frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, a+bx\right)$$

$$= -\frac{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b} + \frac{1}{2}x^2\cosh^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int\frac{\frac{1+2a^2}{b^2}-\text{3}}{\sqrt{-1+x}\,\sqrt{}}\right.$$

$$= \frac{3a\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b^2} - \frac{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b} + \frac{1}{2}x^2\cosh^{-1}(a$$

$$= \frac{3a\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b^2} - \frac{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4b} - \frac{(1+2a^2)\cosh}{4b^2}$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 0.97

$$\frac{(3a-bx)\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+2b^2x^2\cosh^{-1}(a+bx)-(1+2a^2)\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*b^2*x^2*ArcCosh[a + b
*x] - (1 + 2*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(4*b
^2)
```

Maple [A]
time = 3.20, size = 113, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{\operatorname{arccosh}(bx+a)(bx+a)^2}{2}-\operatorname{arccosh}(bx+a)a(bx+a)-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-4a\sqrt{(bx+a)^2-1}+(bx\right.}{4\sqrt{(bx+a)}}}{b^2}$
default	$\frac{\frac{\operatorname{arccosh}(bx+a)(bx+a)^2}{2}-\operatorname{arccosh}(bx+a)a(bx+a)-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-4a\sqrt{(bx+a)^2-1}+(bx\right.}{4\sqrt{(bx+a)}}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arccosh(b*x+a)*(b*x+a)^2-arccosh(b*x+a)*a*(b*x+a)-1/4*(b*x+a-1)^
(1/2)*(b*x+a+1)^(1/2)*(-4*a*((b*x+a)^2-1)^(1/2)+(b*x+a)*((b*x+a)^2-1)^(1/2)
+ln(b*x+a+((b*x+a)^2-1)^(1/2)))/((b*x+a)^2-1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.
time = 0.26, size = 151, normalized size = 1.68

$$\frac{1}{2}x^2\operatorname{arcosh}(bx+a)-\frac{1}{4}b\left(\frac{3a^2\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^3}+\frac{\sqrt{b^2x^2+2abx+a^2-1}\,x}{b^2}-\frac{(a^2-1)\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^3}-\frac{3\sqrt{b^2x^2+2abx+a^2-1}\,a}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(b*x + a) - 1/4*b*(3*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*b)/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x/b^2 - (
a^2 - 1)*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^3 -
 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a/b^3)
```

Fricas [A]
time = 0.37, size = 75, normalized size = 0.83

$$\frac{(2b^2x^2-2a^2-1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2-1}\right)-\sqrt{b^2x^2+2abx+a^2-1}\,(bx-3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 3*a))/b^2
```

Sympy [A]
time = 0.12, size = 104, normalized size = 1.16

$$\begin{cases} -\frac{a^2\operatorname{acosh}(a+bx)}{2b^2} + \frac{3a\sqrt{a^2+2abx+b^2x^2-1}}{4b^2} + \frac{x^2\operatorname{acosh}(a+bx)}{2} - \frac{x\sqrt{a^2+2abx+b^2x^2-1}}{4b} - \frac{\operatorname{acosh}(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2\operatorname{acosh}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acosh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 - 1)/(4*b**2) + x**2*acosh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 - 1)/(4*b) - acosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acosh(a)/2, True)
)
```

Giac [A]
time = 0.44, size = 112, normalized size = 1.24

$$\frac{1}{2}x^2\log\left(bx+a+\sqrt{(bx+a)^2-1}\right) - \frac{1}{4}\left(\sqrt{b^2x^2+2abx+a^2-1}\left(\frac{x}{b^2}-\frac{3a}{b^3}\right) - \frac{(2a^2+1)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)|b|\right|\right)}{b^2|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*(x/b^2 - 3*a/b^3) - (2*a^2 + 1)*log(abs(-a*b - (x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^2*abs(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\operatorname{acosh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(a + b*x),x)
```

```
[Out] int(x*acosh(a + b*x), x)
```

3.86 $\int \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{b}+\frac{(a+bx)\cosh^{-1}(a+bx)}{b}$$

```
[Out] (b*x+a)*arccosh(b*x+a)/b-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5995, 5879, 75}

$$\frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x],x]
```

```
[Out] -((Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/b) + ((a + b*x)*ArcCosh[a + b*x])/
b
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \cosh^{-1}(x)\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\cosh^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{b} + \frac{(a+bx)\cosh^{-1}(a+bx)}{b}$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 1.44

$$x\cosh^{-1}(a+bx) - \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx} - 2a\tanh^{-1}\left(\sqrt{\frac{-1+a+bx}{1+a+bx}}\right)}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a + b*x],x]
[Out] x*ArcCosh[a + b*x] - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] - 2*a*ArcTanh[Sq
rt[(-1 + a + b*x)/(1 + a + b*x)]])/b
```

Maple [A]
time = 0.42, size = 36, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(bx+a)\operatorname{arccosh}(bx+a)-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}$	36
default	$\frac{(bx+a)\operatorname{arccosh}(bx+a)-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*((b*x+a)*arccosh(b*x+a)-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))
```

Maxima [A]
time = 0.25, size = 30, normalized size = 0.73

$$\frac{(bx+a)\operatorname{arcosh}(bx+a) - \sqrt{(bx+a)^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arccosh(b*x + a) - sqrt((b*x + a)^2 - 1))/b
```

Fricas [**A**]
time = 0.42, size = 57, normalized size = 1.39

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1))/b
```

Sympy [**A**]
time = 0.07, size = 46, normalized size = 1.12

$$\begin{cases} \frac{a\operatorname{acosh}(a+bx)}{b}+x\operatorname{acosh}(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2-1}}{b} & \text{for}\ b\neq 0\\ x\operatorname{acosh}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x)/b + x*acosh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/b, Ne(b, 0)), (x*acosh(a), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.
time = 0.43, size = 93, normalized size = 2.27

$$-b\left(\frac{a\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b))
)/(b*abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)/b^2) + x*log(b*x + a + sqr
t((b*x + a)^2 - 1))
```

Mupad [**B**]
time = 4.00, size = 266, normalized size = 6.49

$$x\operatorname{acosh}(a+b\,x)-\frac{\frac{4\,a\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{4\,a\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^3}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^3}-\frac{8\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^2\sqrt{a-1}\,\sqrt{a+1}}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^2}}{\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^4}-\frac{2\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^2}+1}+\frac{4\,a\,\operatorname{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x-1}}{\sqrt{a+1}-\sqrt{a+b\,x+1}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x),x)
```

```
[Out] x*acosh(a + b*x) - ((4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/(b*((a + 1)
^(1/2) - (a + b*x + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^3)/(b*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) - (8*((a - 1)^(1/2) - (a +
b*x - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^2))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4/((a + 1)^(1/2) -
(a + b*x + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^2 + 1) + (4*a*atanh(((a - 1)^(1/2) - (a + b
*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))))/b
```

3.87 $\int \frac{\cosh^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{2}\cosh^{-1}(a+bx)^2+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a+\sqrt{-1+a^2}}\right)+$$

```
[Out] -1/2*arccosh(b*x+a)^2+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^
(1/2))/(a-(a^2-1)^(1/2)))+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a
+1)^(1/2))/(a+(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2))/(a-(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(
a+(a^2-1)^(1/2)))
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5996, 5962, 5681, 2221, 2317, 2438}

$$\text{Li}_2\left(\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{Li}_2\left(\frac{e^{\cosh^{-1}(a+bx)}}{a+\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}+a}\right)-\frac{1}{2}\cosh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcCosh[a + b*x]^2 + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a -
Sqrt[-1 + a^2])] + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a + Sqrt[-1
 + a^2])] + PolyLog[2, E^ArcCosh[a + b*x]/(a - Sqrt[-1 + a^2])] + PolyLog[2
, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5962

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Sinh[x]/(c*d + e*Cosh[x])), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{-\frac{a}{b}+\frac{\cosh(x)}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{-1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b} + \text{Sub}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx$$

Mathematica [A]
time = 0.01, size = 153, normalized size = 1.17

$$-\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1+\frac{e^{\cosh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{-1+a^2}}{b}\right)b}\right) + \cosh^{-1}(a+bx)\log\left(1+\frac{e^{\cosh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{-1+a^2}}{b}\right)b}\right) + \text{PolyLog}\left(2, -\frac{e^{\cosh^{-1}(a+bx)}}{-a+\sqrt{-1+a^2}}\right) + \text{PolyLog}\left(2, \frac{e^{\cosh^{-1}(a+bx)}}{a+\sqrt{-1+a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcCosh[a + b*x]^2 + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/((-(a
/b) - Sqrt[-1 + a^2]/b)*b)] + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/(
(-(a/b) + Sqrt[-1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcCosh[a + b*x]/(-a + Sqr
t[-1 + a^2]))] + PolyLog[2, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(177) = 354$.
time = 10.85, size = 431, normalized size = 3.29

method	result
derivativedivides	$-\frac{\operatorname{arccosh}(bx+a)^2}{2}+\frac{a\operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{a+\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{a\operatorname{arccosh}(bx+$
default	$-\frac{\operatorname{arccosh}(bx+a)^2}{2}+\frac{a\operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{a+\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{a\operatorname{arccosh}(bx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccosh(b*x+a)^2+a*arccosh(b*x+a)/(a^2-1)^(1/2)*ln(((a^2-1)^(1/2)-b*x-
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-a*arccosh(b*x+a)/(a^2-1
)^(1/2)*ln(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^
(1/2)))+(a^2-1+a*(a^2-1)^(1/2))/(a^2-1)*arccosh(b*x+a)*(ln(((a^2-1)^(1/2)+b
*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1/2)))-2*ln(((a^2-1)^(1/2)
-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))*a^2+ln(((a^2-1)^(1
/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))+2*a*(a^2-1)^(1/
2)*ln(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2))
))+dilog(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1
/2)))+dilog(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^
(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(b*x + a)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(b*x + a)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x,x)
[Out] Integral(acosh(a + b*x)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(arccosh(b*x + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x,x)
[Out] int(acosh(a + b*x)/x, x)
```

3.88 $\int \frac{\cosh^{-1}(a+bx)}{x^2} dx$

Optimal. Leaf size=64

$$-\frac{\cosh^{-1}(a+bx)}{x}-\frac{2b\text{ArcTan}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}$$

```
[Out] -arccosh(b*x+a)/x-2*b*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a
-1)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5996, 5963, 95, 211}

$$-\frac{2b\text{ArcTan}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\cosh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(ArcCosh[a + b*x]/x) - (2*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
 + a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(a+bx)}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\cosh^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx\right)\\
&= -\frac{\cosh^{-1}(a+bx)}{x} + 2\text{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{-1+a+bx}}\right)\\
&= -\frac{\cosh^{-1}(a+bx)}{x} - \frac{2b\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 83, normalized size = 1.30

$$-\frac{\cosh^{-1}(a+bx)}{x} - \frac{ib\log\left(\frac{2\left(\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+\frac{i\left(-1+a^2+abx\right)}{\sqrt{1-a^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^2,x]
[Out] -(ArcCosh[a + b*x]/x) - (I*b*Log[(2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] +
 (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(b*x)])/Sqrt[1 - a^2]
```

Maple [A]
time = 4.60, size = 101, normalized size = 1.58

method	result

derivativedivides	$b\left(-\frac{\operatorname{arccosh}(bx+a)}{bx}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+}{bx}\right.}{\sqrt{(bx+a)^2-1}\,(-1+a)(1+a)}\right.$
default	$b\left(-\frac{\operatorname{arccosh}(bx+a)}{bx}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+}{bx}\right.}{\sqrt{(bx+a)^2-1}\,(-1+a)(1+a)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arccosh(b*x+a)-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(a^2-1)^(1/2)*ln(2
*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)/((b*x+a)^2-1)^(1/2)/(
-1+a)/(1+a))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(54) = 108.
time = 0.39, size = 322, normalized size = 5.03

$$\left[\frac{\sqrt{a^2-1}\,bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\,\left(a^2-\sqrt{a^2-1}\,a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)+\left(a^2-1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(a^2-\left(a^2-1\right)x-1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)}{\left(a^2-1\right)x},\ \frac{2\sqrt{-a^2+1}\,bx\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{b^2x^2+2\,abx+a^2-1}\,\sqrt{-a^2+1}}{a^2-1}\right)+\left(a^2-1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(a^2-\left(a^2-1\right)x-1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)}{\left(a^2-1\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*
(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^
```

```
2 - 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 -
1)*x - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x),
(2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a^2 - 1)*x*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 - 1)*x - 1)*log(b*x + a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a + bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**2,x)
```

```
[Out] Integral(acosh(a + b*x)/x**2, x)
```

Giac [**A**]
time = 0.43, size = 73, normalized size = 1.14

$$\frac{2\,b\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}}-\frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1))/
sqrt(-a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 - 1))/x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a + b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^2,x)
```

```
[Out] int(acosh(a + b*x)/x^2, x)
```

3.89 $\int \frac{\cosh^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=106

$$\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}-\frac{ab^2\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{-1+a+bx}}\right)}{(1-a^2)^{3/2}}$$

```
[Out] -1/2*arccosh(b*x+a)/x^2-a*b^2*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2
)/(b*x+a-1)^(1/2))/(-a^2+1)^(3/2)+1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a
^2+1)/x
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5996, 5963, 98, 95, 211}

$$-\frac{ab^2\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{(1-a^2)^{3/2}}+\frac{b\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2(1-a^2)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x) - ArcCosh[a + b*x]
/(2*x^2) - (a*b^2*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
-1 + a + b*x])])/(1 - a^2)^(3/2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right) \\
&= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{(ab)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\ldots}\right.}{2\left(1-a^2\ldots\right.} \\
&= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{(ab)\text{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx\ldots\right.}{1-a^2} \\
&= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}-\frac{ab^2\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+\ldots}}{\sqrt{1+a}\,\sqrt{-1+a-\ldots}}\right.}{\left(1-a^2\right)^{3/2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.19, size = 136, normalized size = 1.28

$$-\cosh^{-1}(a+bx)+\frac{\frac{bx\left(-\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+\frac{iabx\log\left(\frac{4i\sqrt{1-a^2}\left(-1+a^2+abx-i\sqrt{1-a^2}\,\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{ab^2x}\right)}{\sqrt{1-a^2}}\right)}{-1+a^2}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-ArcCosh[a + b*x] + (b*x*(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) + (I*a*b
*x*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a
 + b*x]*Sqrt[1 + a + b*x]))/(a*b^2*x)])/Sqrt[1 - a^2]))/(-1 + a^2))/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(88) = 176$.
time = 5.02, size = 201, normalized size = 1.90

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arccosh}(bx+a)}{2b^2x^2}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2(bx+a)a-}{bx}\right.\right.}{}\right.$
default	$b^2\left(-\frac{\operatorname{arccosh}(bx+a)}{2b^2x^2}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left(\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2(bx+a)a-}{bx}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arccosh(b*x+a)-1/2*(b*x+a+1)^(1/2)*(b*x+a-1)^(1/2)*(ln(2*
((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)^(1/2)*a^2-ln(2
*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)^(1/2)*a*(b*x+
a)+a^2*((b*x+a)^2-1)^(1/2)-((b*x+a)^2-1)^(1/2))/b/x/(a^2-1)/(1+a)/(-1+a)/((
b*x+a)^2-1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(86) = 172$.
time = 0.40, size = 460, normalized size = 4.34

$$\left[\frac{\sqrt{a^2-1}\,ab^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\,\left(a^2+\sqrt{a^2-1}\,a-1\right)+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-(a^2-1)b^2x^2+(a^4-2\,a^2+1)x^2\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}\,(a^2-1)bx-(a^4-(a^4-2\,a^2+1)x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)}{2\,(a^4-2\,a^2+1)x^2}, -\frac{2\sqrt{-a^2+1}\,ab^2x^2\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{b^2x^2+2\,abx+a^2-1}\,\sqrt{-a^2+1}}{a^2-1}\right)+(a^2-1)b^2x^2-(a^4-2\,a^2+1)x^2\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{b^2x^2+2\,abx+a^2-1}\,(a^2-1)bx+(a^4-(a^4-2\,a^2+1)x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)}{2\,(a^4-2\,a^2+1)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - 1)*a*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
)/x) - (a^2 - 1)*b^2*x^2 + (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*b*x -
 (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*a*b^2*x^2
*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2
+ 1))/(a^2 - 1)) + (a^2 - 1)*b^2*x^2 - (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^
2 - 1)*b*x + (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**3,x)
```

```
[Out] Integral(acosh(a + b*x)/x**3, x)
```

Giac [A]
time = 0.46, size = 170, normalized size = 1.60

$$-\left(\frac{ab\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{(a^2-1)\sqrt{-a^2+1}}-\frac{\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)ab+a^2|b|-|b|}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^2-a^2+1\right)(a^2-1)}\right)b-\frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -(a*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1)
)/((a^2 - 1)*sqrt(-a^2 + 1)) - ((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
1))*a*b + a^2*abs(b) - abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))^2 - a^2 + 1)*(a^2 - 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 - 1))/
x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^3,x)
```

```
[Out] int(acosh(a + b*x)/x^3, x)
```

3.90 $\int \frac{\cosh^{-1}(a+bx)}{x^4} \, dx$

Optimal. Leaf size=154

$$\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2}+\frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)^2x}-\frac{\cosh^{-1}(a+bx)}{3x^3}-\frac{(1+2a^2)\,b^3\text{ArcTan}\Big(\frac{-}{\sqrt{}}}{3(1-}$$

```
[Out] -1/3*arccosh(b*x+a)/x^3-1/3*(2*a^2+1)*b^3*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2
)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(5/2)+1/6*b*(b*x+a-1)^(1/2)*(b*x+a+
1)^(1/2)/(-a^2+1)/x^2+1/2*a*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/
x
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5996, 5963, 105, 156, 12, 95, 211}

$$-\frac{(2a^2+1)\,b^3\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{3(1-a^2)^{5/2}}+\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2(1-a^2)^2x}+\frac{b\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{6(1-a^2)x^2}-\frac{\cosh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[-1
 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)^2*x) - ArcCosh[a + b*x]/(3*x^3)
 - ((1 + 2*a^2)*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqr
t[-1 + a + b*x])])/(3*(1 - a^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5963

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n
 - 1)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^4}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{b^2\operatorname{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\sqrt{-1+x}\,\sqrt{1+x}\,(}\right.}{6(1-a^2)}$$

$$= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx}{3x^3}$$

$$= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx}{3x^3}$$

$$= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx}{3x^3}$$

$$= \frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2(1-a^2)^2x} - \frac{\cosh^{-1}(a+bx}{3x^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 162, normalized size = 1.05

$$\frac{1}{6}\left(\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(1-a^2+3abx)}{(-1+a^2)^2x^2} - \frac{2\cosh^{-1}(a+bx)}{x^3} - \frac{i(1+2a^2)\,b^3\log\left(\frac{12(1-a^2)^{3/2}\left(-i+ia^2+iabx+\sqrt{1-a^2}\,\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{b^3(x+2a^2x)}\right)}{(1-a^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(1 - a^2 + 3*a*b*x))/((-1 + a^2)^2
*x^2) - (2*ArcCosh[a + b*x])/x^3 - (I*(1 + 2*a^2)*b^3*Log[(12*(1 - a^2)^(3/
2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]))/(b^3*(x + 2*a^2*x))])/(1 - a^2)^(5/2))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(128) = 256$.
time = 4.97, size = 467, normalized size = 3.03

method	result
derivativedivides	$b^3\left(-\frac{\operatorname{arccosh}(bx+a)}{3b^3x^3}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,\left(2\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2(bx+a)a}{bx}\right.\right.}{}\right.$
default	$b^3\left(-\frac{\operatorname{arccosh}(bx+a)}{3b^3x^3}-\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,\left(2\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2(bx+a)a}{bx}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arccosh(b*x+a)-1/6*(b*x+a+1)^(1/2)*(b*x+a-1)^(1/2)*(2*ln(
2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)^(1/2)*a^4-4*
ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)^(1/2)*a^3
*(b*x+a)+2*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1
)^(1/2)*a^2*(b*x+a)^2+4*((b*x+a)^2-1)^(1/2)*a^4-3*((b*x+a)^2-1)^(1/2)*a^3*(
b*x+a)+ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)^(1
/2)*a^2-2*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*(a^2-1)
^(1/2)*a*(b*x+a)+ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+(b*x+a)*a-1)/b/x)*
(a^2-1)^(1/2)*(b*x+a)^2-5*a^2*((b*x+a)^2-1)^(1/2)+3*a*(b*x+a)*((b*x+a)^2-1)
^(1/2)+((b*x+a)^2-1)^(1/2))/b^2/x^2/(a^2-1)^2/(1+a)/(-1+a)/((b*x+a)^2-1)^(1
/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(124) = 248$.

time = 0.50, size = 566, normalized size = 3.68

$$\left[\frac{(2a^2+1)\sqrt{a^2-1}\,b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a-1\right)-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)+3(a^3-a)b^3x^3+2(a^6-3a^4+3a^2-1)x^3\log\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-2(a^6-3a^4-(a^6-3a^4+3a^2-1)x^3+3a^2-1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2-1}\right)+(3(a^3-a)b^2x^2-(a^4-2a^2+1)bx)\sqrt{b^2x^2+2abx+a^2-1}}{6(a^6-3a^4+3a^2-1)x^3},\ \frac{2(2a^2+1)\sqrt{-a^2+1}\,b^3x^3\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{b^2x^2+2abx+a^2-1}\sqrt{-a^2+1}}{a^2-1}\right)+3(a^3-a)b^3x^3+2(a^6-3a^4+3a^2-1)x^3\log\left(-bx-a+\sqrt{b^2x^2+2abx+a^2-1}\right)-2(a^6-3a^4-(a^6-3a^4+3a^2-1)x^3+3a^2-1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2-1}\right)+(3(a^3-a)b^2x^2-(a^4-2a^2+1)bx)\sqrt{b^2x^2+2abx+a^2-1}}{6(a^6-3a^4+3a^2-1)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*a^2 + 1)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*a^4 - (a^6 - 3*a
^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), 1/6*(2*(2*a^2 + 1)*sqrt(-a^
2 + 1)*b^3*x^3*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a
^2 - 1)*x^3*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*
a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**4,x)
```

```
[Out] Integral(acosh(a + b*x)/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(124) = 248.
time = 0.44, size = 340, normalized size = 2.21

$$\frac{1}{3}b\left(\frac{(2a^2b^2+b^2)\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{(a^4-2a^2+1)\sqrt{-a^2+1}}-\frac{2\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)^3a^2b^2-6\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)a^4b^2-4a^5b|b|+\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)^3b^2+7\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)a^2b^2+8a^3b|b|-\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)b^2-4ab|b|}{(a^4-2a^2+1)\left(\left(x|b|-\sqrt{b^2x^2+2abx+a^2-1}\right)^2-a^2+1\right)^2}\right)-\frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b*((2*a^2*b^2 + b^2)*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - (2*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 - 1))*a^4*b^2 - 4*a^5*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*
```

```
a*b*x + a^2 - 1))^3*b^2 + 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*
a^2*b^2 + 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*b
^2 - 4*a*b*abs(b))/((a^4 - 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1))^2 - a^2 + 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 - 1))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a + b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^4,x)
```

```
[Out] int(acosh(a + b*x)/x^4, x)
```

3.91 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5995, 5881, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/2*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(Sqrt[b]*
d) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(
a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x\right.}{}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}\right.}{2\sqrt{b}\,d}$$

Mathematica [A]

time = 0.13, size = 110, normalized size = 1.20

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x
])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}\left(c + d\, x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

3.92 $\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=94

$$-\frac{e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5995, 5881, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/2*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(Sqrt[b]*
d) + (Sqrt[Pi]*Erfi[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(
a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c}}{\sqrt{b}}\right.}{2\sqrt{b}\,d}$$

Mathematica [**A**]

time = 0.08, size = 111, normalized size = 1.18

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}-\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}-\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},-\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)}{2d\sqrt{a-b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b - ArcCosh[c + d*x]]*Gamma[1/2, a/b - ArcCosh[c + d*x]
] + Sqrt[-(a/b) + ArcCosh[c + d*x]]*Gamma[1/2, -(a/b) + ArcCosh[c + d*x]])/
(2*d*E^(a/b)*Sqrt[a - b*ArcCosh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-b\,\mathrm{arccosh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - b\operatorname{acosh}\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a - b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a - b\operatorname{acosh}\left(c + d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a - b*acosh(c + d*x))^(1/2), x)
```

3.93 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=135

$$-\frac{8be^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{75d}-\frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{75d}-\frac{be^4\sqrt{-1+c+dx}\,(c+d}{25d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))/d-8/75*b*e^4*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)/d-4/75*b*e^4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/25*b*e^
4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5996, 12, 5883, 102, 75}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}-\frac{be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4}{25d}-\frac{4be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{75d}-\frac{8be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{75d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(75*d) - (4*b*e^4*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(75*d) - (b*e^4*Sqrt[-1 + c + d*x]
*(c + d*x)^4*Sqrt[1 + c + d*x])/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcCosh[c
+ d*x]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right) dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right) dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{-1+x}\,\sqrt{}}\right.}{5d}$$

$$= -\frac{be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\right.}{5d}$$

$$= -\frac{be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\right.}{5d}$$

$$= -\frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}}{}$$

$$= -\frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}}{}$$

$$= -\frac{8be^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{75d} - \frac{4be^4\sqrt{-1+c+dx}\,(c+dx}{75d}$$

Mathematica [A]

time = 0.09, size = 103, normalized size = 0.76

$$\frac{e^4\left(-\frac{1}{5}b\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}-\frac{4}{15}b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(2+c^2+2cdx+d^2x^2)+(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-1/5*(b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x]) - (4*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2*x^2))/15 + (c + d
*x)^5*(a + b*ArcCosh[c + d*x])))/(5*d)
```

Maple [A]
time = 3.90, size = 78, normalized size = 0.58

method	result	size
derivativedivides	$\frac{\frac{e^4(dx+c)^5a}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$	78
default	$\frac{\frac{e^4(dx+c)^5a}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*e^4*(d*x+c)^5*a+e^4*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(111) = 222$.
time = 0.28, size = 1231, normalized size = 9.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d^4*x^5*e^4 + a*c*d^3*x^4*e^4 + 2*a*c^2*d^2*x^3*e^4 + 2*a*c^3*d*x^2*e
^4 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (
c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 -
 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*arcco
sh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2
*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt
```

```
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*c^2*d^2*e^
4 + 1/24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^
3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*
x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 -
 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*c*d^3*e^4 + 1/600*
(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^4/d^2 -
 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3*x/d^5
- 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 + 1050*(c^2 - 1)*c
^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 + 945*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(
c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)^2/d^6)*d)
*b*d^4*e^4 + a*c^4*x*e^4 + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 -
 1))*b*c^4*e^4/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(111) = 222$.

time = 0.36, size = 1041, normalized size = 7.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^2 +
 5*a*c^4*d*x)*cosh(1)^4 + 60*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3
+ 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*cosh(1)^3*sinh(1) + 90*(a*d^5*x^5 + 5*a*c
*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*cosh(1)^2*sin
h(1)^2 + 60*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^
2 + 5*a*c^4*d*x)*cosh(1)*sinh(1)^3 + 15*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c
^2*d^3*x^3 + 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*sinh(1)^4 + 15*((b*d^5*x^5 + 5
*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*c
osh(1)^4 + 4*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x
^2 + 5*b*c^4*d*x + b*c^5)*cosh(1)^3*sinh(1) + 6*(b*d^5*x^5 + 5*b*c*d^4*x^4
+ 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*cosh(1)^2*sinh
(1)^2 + 4*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2
+ 5*b*c^4*d*x + b*c^5)*cosh(1)*sinh(1)^3 + (b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*
```

```
b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*sinh(1)^4)*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - ((3*b*d^4*x^4 + 12*b*c*d^3*x^3 +
 3*b*c^4 + 2*(9*b*c^2 + 2*b)*d^2*x^2 + 4*b*c^2 + 4*(3*b*c^3 + 2*b*c)*d*x +
8*b)*cosh(1)^4 + 4*(3*b*d^4*x^4 + 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 + 2
*b)*d^2*x^2 + 4*b*c^2 + 4*(3*b*c^3 + 2*b*c)*d*x + 8*b)*cosh(1)^3*sinh(1) +
6*(3*b*d^4*x^4 + 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 + 2*b)*d^2*x^2 + 4*b
*c^2 + 4*(3*b*c^3 + 2*b*c)*d*x + 8*b)*cosh(1)^2*sinh(1)^2 + 4*(3*b*d^4*x^4
+ 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 + 2*b)*d^2*x^2 + 4*b*c^2 + 4*(3*b*c
^3 + 2*b*c)*d*x + 8*b)*cosh(1)*sinh(1)^3 + (3*b*d^4*x^4 + 12*b*c*d^3*x^3 +
3*b*c^4 + 2*(9*b*c^2 + 2*b)*d^2*x^2 + 4*b*c^2 + 4*(3*b*c^3 + 2*b*c)*d*x + 8
*b)*sinh(1)^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(126) = 252$.
time = 0.50, size = 527, normalized size = 3.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*acosh(c + d*x)/(5*d)
 + b*c**4*e**4*x*acosh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)/(25*d) + 2*b*c**3*d*e**4*x**2*acosh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 2*b*c**2*d**2*e**4*x**3*acosh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(75*d) + b*c*d**3*e**4*x**4*acos
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 + b*d**4*e**4*x**5*ac
osh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(
c)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(115) = 230$.
time = 1.06, size = 846, normalized size = 6.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*ab
```

```
s(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^4*e^4 + (2*x^2*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2
- 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c^3*d*e^4 + 1/3*(6*x^3*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(
x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*
d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d
)*b*c^2*d^2*e^4 + 1/24*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c
^2*d^3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2
 + 3)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d))
)/(d^4*abs(d)))*d)*b*c*d^3*e^4 + 1/600*(120*x^5*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*(3*x*(4*x/d^
2 - 9*c/d^3) + (47*c^2*d^5 + 16*d^5)/d^9)*x - 7*(22*c^3*d^4 + 23*c*d^4)/d^9
)*x + (274*c^4*d^3 + 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 + 40*c^3 + 15*c
)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d
^5*abs(d)))*d)*b*d^4*e^4 + a*c^4*e^4*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{acosh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)), x)
```

3.94 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=119

$$-\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{32d}-\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{16d}-\frac{3be^3\cosh^{-1}(c+d$$

```
[Out] -3/32*b*e^3*arccosh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))/d-3/32*
b*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5996, 12, 5883, 102, 92, 54}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3}{16d}-\frac{3be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{32d}-\frac{3be^3\cosh^{-1}(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(32*d) - (b*e^3*S
qrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(16*d) - (3*b*e^3*ArcCosh[
c + d*x])/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right) dx &= \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right) dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right) dx,x,c+dx\right)}{d}\\
&= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x}\,\sqrt{}}\right.}{4d}\\
&= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4\left(a+b\right.}{4d}\\
&= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4\left(a+b\right.}{4d}\\
&= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}}{}\\
&= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}}{}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 115, normalized size = 0.97

$$\frac{e^3\left(-\frac{1}{4}b\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}+(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)-\frac{3}{8}b\left(\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}+2\tanh^{-1}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}\right)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-1/4*(b*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]) + (c + d*x)
^4*(a + b*ArcCosh[c + d*x]) - (3*b*(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c
 + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/8))/(4*d)
```

Maple [A]
time = 4.03, size = 144, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{b\,e^3(dx+c)^4\operatorname{arccosh}(dx+c)}{4}-\frac{b\,e^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3}{16}-\frac{3b\,e^3\sqrt{dx+c-1}\,\sqrt{d}}{32}}{d}$
default	$\frac{\frac{e^3(dx+c)^4 a}{4}+\frac{b\,e^3(dx+c)^4\operatorname{arccosh}(dx+c)}{4}-\frac{b\,e^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)^3}{16}-\frac{3b\,e^3\sqrt{dx+c-1}\,\sqrt{d}}{32}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a+1/4*b*e^3*(d*x+c)^4*arccosh(d*x+c)-1/16*b*e^3*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c)^3-3/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*(d*x+c)-3/32*b*e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2
)*ln(d*x+c+((d*x+c)^2-1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(99) = 198$.
time = 0.28, size = 789, normalized size = 6.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4*e^3 + a*c*d^2*x^3*e^3 + 3/2*a*c^2*d*x^2*e^3 + 3/4*(2*x^2*arcc
osh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^
```

```
2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d
^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arccosh(d*x + c) - d*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d +
 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*c*d^2*e^3 + 1/96*(24*x^4*a
rccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 - 14*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d^5 - 9*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*log(2*d^2*x
+ 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*x*e^3 + ((d*x + c)*a
rccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c^3*e^3/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(99) = 198$.
time = 0.38, size = 655, normalized size = 5.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)*cosh(1)
^3 + 24*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)*cosh(1)
^2*sinh(1) + 24*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)
*cosh(1)*sinh(1)^2 + 8*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c
^3*d*x)*sinh(1)^3 + ((8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*
b*c^3*d*x + 8*b*c^4 - 3*b)*cosh(1)^3 + 3*(8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48
*b*c^2*d^2*x^2 + 32*b*c^3*d*x + 8*b*c^4 - 3*b)*cosh(1)^2*sinh(1) + 3*(8*b*d
^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*b*c^3*d*x + 8*b*c^4 - 3*b)*
cosh(1)*sinh(1)^2 + (8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*b
*c^3*d*x + 8*b*c^4 - 3*b)*sinh(1)^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*b*d^3*x^3 + 6*b*c*d^2*x^
2 + 2*b*c^3 + 3*(2*b*c^2 + b)*d*x + 3*b*c)*cosh(1)^3 + 3*(2*b*d^3*x^3 + 6*b
*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 + b)*d*x + 3*b*c)*cosh(1)^2*sinh(1) + 3*(
2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 + b)*d*x + 3*b*c)*cosh(1
)*sinh(1)^2 + (2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 + b)*d*x
+ 3*b*c)*sinh(1)^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(110) = 220$.

time = 0.40, size = 394, normalized size = 3.31

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{acosh}(c+dx)}{4d} + bc^3e^3x\operatorname{acosh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2-1}}{16d} + \frac{3bc^2de^3x^2\operatorname{acosh}(c+dx)}{2} - \frac{3bc^2e^3x\sqrt{c^2+2cdx+d^2x^2-1}}{16} + bcd^2e^3x^3\operatorname{acosh}(c+dx) - \frac{3bcde^3x^2\sqrt{c^2+2cdx+d^2x^2-1}}{16} - \frac{3bce^3\sqrt{c^2+2cdx+d^2x^2-1}}{32d} + \frac{bd^3e^3x^4\operatorname{acosh}(c+dx)}{4} - \frac{bd^2e^3x^3\sqrt{c^2+2cdx+d^2x^2-1}}{16} - \frac{3be^3x\sqrt{c^2+2cdx+d^2x^2-1}}{32} - \frac{3be^3\operatorname{acosh}(c+dx)}{32d} & \text{for } d \neq 0 \\ c^3e^3x(a+b\operatorname{acosh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*acosh(c + d*x)/(4*d) + b*c**3*e**3*x*acosh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*acosh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/16 + b*c*d**2*e**3*x**3*acosh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(32*d) + b*d**3*e**3*x**4*acosh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/32 - 3*b*e**3*acosh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*acosh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(103) = 206.
time = 0.93, size = 617, normalized size = 5.18

$$\frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2 - \left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2cdx+c^2-1}}{d^2}\right) - x\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right)\right)bc^3e^3 + \frac{3}{4}\left(2x^2\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(\frac{x}{d^2}-\frac{3c}{d^3}\right) - \frac{(2c^2+1)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^2|d|}\right)d\right)bc^2de^3 + \frac{1}{6}\left(6x^3\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(x\left(\frac{2x}{d^2}-\frac{5c}{d^3}\right)+\frac{11c^2d+4d}{d^5}\right) + \frac{3(2c^3+3c)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^3|d|}\right)d\right)bcd^2e^3 + \frac{1}{96}\left(24x^4\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(\left(2x\left(\frac{3x}{d^2}-\frac{7c}{d^3}\right)+\frac{26c^2d^3+9d^3}{d^7}\right)x - \frac{5(10c^3d^2+11cd^2)}{d^7}\right) - \frac{3(8c^4+24c^2+3)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^4|d|}\right)d\right)bd^3e^3 + ac^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 - (d*(c*log(abs(-
c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)))*b*c^3*e^3 + 3/4*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*
c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(
d)))/(d^2*abs(d)))*d)*b*c^2*d*e^3 + 1/6*(6*x^3*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c
/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*abs(d) -
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*c*d^2*e^3 +
1/96*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 + 9*d^3)/d^
7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)*log(abs(-c*d
 - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^4*abs(d)))*d)
*b*d^3*e^3 + a*c^3*e^3*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)), x)
```

3.95 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=97

$$-\frac{2be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{9d}-\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{9d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(\right.}{3d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))/d-2/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)/d-1/9*b*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5996, 12, 5883, 102, 75}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{9d}-\frac{2be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(9*d) - (b*e^2*Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcCo
sh[c + d*x]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\,\sqrt{}}\right.}{3d}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{9d} + \frac{e^2(c+dx)^3\left(a+b\right.}{3d}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{9d} + \frac{e^2(c+dx)^3\left(a+b\right.}{3d}$$

$$= -\frac{2be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{9d} - \frac{be^2\sqrt{-1+c+dx}\,(c+dx)}{9d}$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 0.73

$$\frac{e^2\left(-\frac{1}{9}b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(2+c^2+2cdx+d^2x^2)+\frac{1}{3}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-1/9*(b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2
*x^2)) + ((c + d*x)^3*(a + b*ArcCosh[c + d*x]))/3))/d
```

Maple [A]
time = 4.08, size = 67, normalized size = 0.69

method	result	size
derivativedivides	$\frac{\frac{e^{2}(dx+c)^{3}a}{3}+b\,e^{2}\left(\frac{(dx+c)^{3}\operatorname{arccosh}(dx+c)}{3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^{2}+2\right)}{9}\right)}{d}$	67
default	$\frac{\frac{e^{2}(dx+c)^{3}a}{3}+b\,e^{2}\left(\frac{(dx+c)^{3}\operatorname{arccosh}(dx+c)}{3}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^{2}+2\right)}{9}\right)}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a+b*e^2*(1/3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(80) = 160$.
time = 0.28, size = 443, normalized size = 4.57

$$\frac{1}{3}ad^{2}x^{3}e^{2}+acdx^{2}e^{2}+\frac{1}{2}\left(2x^{2}\operatorname{arcosh}(dx+c)-d\left(\frac{3c^{2}\log\left(2d^{2}x+2cd+2\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,d\right)}{d^{3}}+\frac{\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,x}{d^{2}}-\frac{(c^{2}-1)\log\left(2d^{2}x+2cd+2\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,d\right)}{d^{3}}-\frac{3\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,c}{d^{3}}\right)\right)bcde^{2}+\frac{1}{18}\left(6x^{3}\operatorname{arcosh}(dx+c)-d\left(\frac{2\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,x^{2}}{d^{2}}-\frac{15c^{3}\log\left(2d^{2}x+2cd+2\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,d\right)}{d^{4}}-\frac{5\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,cx}{d^{3}}+\frac{9(c^{2}-1)c\log\left(2d^{2}x+2cd+2\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,d\right)}{d^{4}}+\frac{15\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,c^{2}}{d^{4}}-\frac{4\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\,(c^{2}-1)}{d^{4}}\right)\right)bd^{2}e^{2}+ac^{2}xe^{2}+\frac{\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^{2}-1}\right)bc^{2}e^{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3*e^2 + a*c*d*x^2*e^2 + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d
^3))*b*c*d*e^2 + 1/18*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 -
 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*(c^2 - 1)/d^4))*b*d^2*e^2 + a*c^2*x*e^2 + ((d*x + c)*arccosh(d*x + c)
 - sqrt((d*x + c)^2 - 1))*b*c^2*e^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(80) = 160$.
time = 0.37, size = 352, normalized size = 3.63

$$\frac{3\left(ad^{3}x^{3}+3acd^{2}x^{2}+3ac^{2}dx\right)\cosh(1)^{2}+6\left(ad^{3}x^{3}+3acd^{2}x^{2}+3ac^{2}dx\right)\cosh(1)\sinh(1)+3\left(ad^{3}x^{3}+3acd^{2}x^{2}+3ac^{2}dx\right)\sinh(1)^{2}+3\left(\left(bd^{3}x^{3}+3bcd^{2}x^{2}+3bc^{2}dx+bc^{3}\right)\cosh(1)^{2}+2\left(bd^{3}x^{3}+3bcd^{2}x^{2}+3bc^{2}dx+bc^{3}\right)\cosh(1)\sinh(1)+\left(bd^{3}x^{3}+3bcd^{2}x^{2}+3bc^{2}dx+bc^{3}\right)\sinh(1)^{2}\right)\log\left(dx+c+\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\right)-\sqrt{d^{2}x^{2}+2cdx+c^{2}-1}\left(\left(bd^{2}x^{2}+2bcdx+bc^{2}+2b\right)\cosh(1)^{2}+2\left(bd^{2}x^{2}+2bcdx+bc^{2}+2b\right)\cosh(1)\sinh(1)+\left(bd^{2}x^{2}+2bcdx+bc^{2}+2b\right)\sinh(1)^{2}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x)*cosh(1)^2 + 6*(a*d^3*x^3 +
 3*a*c*d^2*x^2 + 3*a*c^2*d*x)*cosh(1)*sinh(1) + 3*(a*d^3*x^3 + 3*a*c*d^2*x^
2 + 3*a*c^2*d*x)*sinh(1)^2 + 3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3)*cosh(1)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cosh
(1)*sinh(1) + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sinh(1)^2)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + 2*b)*cosh(1)^2 + 2*(b*d^2*x^2 +
 2*b*c*d*x + b*c^2 + 2*b)*cosh(1)*sinh(1) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
+ 2*b)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(88) = 176$.
time = 0.26, size = 258, normalized size = 2.66

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{acosh}(c+dx)}{3d} + bc^2e^2x\operatorname{acosh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} + bcde^2x^2\operatorname{acosh}(c+dx) - \frac{2bce^2x\sqrt{c^2+2cdx+d^2x^2-1}}{9} + \frac{bd^2e^2x^3\operatorname{acosh}(c+dx)}{3} - \frac{bde^2x^2\sqrt{c^2+2cdx+d^2x^2-1}}{9} - \frac{2be^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} & \text{for } d \neq 0 \\ c^2e^2x(a+b\operatorname{acosh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*acosh(c + d*x)/(3*d) + b*c**2*e**2*x*acosh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + b*c*d*e**2*x**2*acosh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 + b*d**2*e**2*x**3*acosh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*acosh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(83) = 166$.
time = 0.77, size = 419, normalized size = 4.32

$$\frac{1}{3}ad^2e^2x^3 + acde^2x^2 - \left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2cdx+c^2-1}}{d^2}\right) - x\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right)\right)bc^2e^2 + \frac{1}{2}\left(2x^2\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(\frac{x}{d^2} - \frac{3c}{d^3}\right) - \frac{(2c^2+1)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^2|d|}\right)d\right)bcde^2 + \frac{1}{18}\left(6x^3\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(x\left(\frac{2x}{d^2} - \frac{5c}{d^3}\right) + \frac{11c^2d+4d}{d^5}\right) + \frac{3(2c^3+3c)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^3|d|}\right)d\right)bd^2e^2 + ac^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^2*
e^2 + 1/2*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d -
(x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*
```

```
c*d*e^2 + 1/18*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/
d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*d^2*e^2 + a*c^2*e^2*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)), x)
```

3.96 $\int (ce + dex) \left(a + b\cosh^{-1}(c + dx)\right) dx$

Optimal. Leaf size=75

$$-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{4d}-\frac{be\cosh^{-1}(c+dx)}{4d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d}$$

```
[Out] -1/4*b*e*arccosh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d-1/4*b*e*(d
*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5996, 12, 5883, 92, 54}

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{4d}-\frac{be\cosh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/4*(b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/d - (b*e*ArcCosh[
c + d*x])/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1}}\right.}{2d}$$
$$= -\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{4d} + \frac{e(c+dx)^2\left(a+b\,\text{co}\right.}{2d}$$
$$= -\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{4d} - \frac{be\cosh^{-1}(c+dx)}{4d} +$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 1.08

$$\frac{e\left(\frac{1}{2}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)-\frac{1}{4}b\left(\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}+2\tanh^{-1}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2 - (b*(Sqrt[-1 + c + d*x]*(c +
d*x)*Sqrt[1 + c + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/4)
)/d
```

Maple [A]
time = 3.96, size = 107, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2 a}{2}+\frac{be(dx+c)^2\operatorname{arccosh}(dx+c)}{2}-\frac{be\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)}{4}-\frac{be\sqrt{dx+c-1}\,\sqrt{dx+c+}}{4\sqrt{(dx}}}{d}$
default	$\frac{\frac{e(dx+c)^2 a}{2}+\frac{be(dx+c)^2\operatorname{arccosh}(dx+c)}{2}-\frac{be\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,(dx+c)}{4}-\frac{be\sqrt{dx+c-1}\,\sqrt{dx+c+}}{4\sqrt{(dx}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a+1/2*b*e*(d*x+c)^2*arccosh(d*x+c)-1/4*b*e*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*(d*x+c)-1/4*b*e*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+
c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(68) = 136$.
time = 0.27, size = 207, normalized size = 2.76

$$\frac{1}{2}adx^2e+\frac{1}{4}\left(2x^2\operatorname{arcosh}(dx+c)-d\left(\frac{3c^2\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+2cdx+c^2-1}\,x}{d^2}-\frac{(c^2-1)\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cdx+c^2-1}\,d\right)}{d^3}-\frac{3\sqrt{d^2x^2+2cdx+c^2-1}\,c}{d^3}\right)\right)bde+acxe+\frac{\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^2-1}\right)bce}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2*e + 1/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d
+ 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*d*e + a*c*x*e
 + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c*e/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(68) = 136$.
time = 0.39, size = 171, normalized size = 2.28

$$\frac{2(ad^2x^2+2acdx)\cosh(1)+((2bd^2x^2+4bcdx+2bc^2-b)\cosh(1)+(2bd^2x^2+4bcdx+2bc^2-b)\sinh(1))\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right)+2(ad^2x^2+2acdx)\sinh(1)-\sqrt{d^2x^2+2cdx+c^2-1}\,((bdx+bc)\cosh(1)+(bdx+bc)\sinh(1))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*d^2*x^2 + 2*a*c*d*x)*cosh(1) + ((2*b*d^2*x^2 + 4*b*c*d*x + 2*b*c^
2 - b)*cosh(1) + (2*b*d^2*x^2 + 4*b*c*d*x + 2*b*c^2 - b)*sinh(1))*log(d*x +
```

```
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(a*d^2*x^2 + 2*a*c*d*x)*sinh(1)
 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((b*d*x + b*c)*cosh(1) + (b*d*x + b*c)
*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(66) = 132$.
time = 0.17, size = 148, normalized size = 1.97

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\operatorname{acosh}(c+dx)}{2d} + bcex\operatorname{acosh}(c+dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2-1}}{4d} + \frac{bdex^2\operatorname{acosh}(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+d^2x^2-1}}{4} - \frac{be\operatorname{acosh}(c+dx)}{4d} & \text{for } d \neq 0 \\ cex(a+b\operatorname{acosh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*acosh(c + d*x)/(2*d) + b*c*e*x
*acosh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(4*d) + b*d*e*
x**2*acosh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - b*e*
acosh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(65) = 130$.
time = 0.63, size = 245, normalized size = 3.27

$$\frac{1}{2}adex^2 - \left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2cdx+c^2-1}}{d^2}\right) - x\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right)\right)bce + \frac{1}{4}\left(2x^2\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2-1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2-1}\left(\frac{x}{d^2} - \frac{3c}{d^3}\right) - \frac{(2c^2+1)\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2-1}\right)|d|\right|\right)}{d^2|d|}\right)d\right)bde + acex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*e*x^2 - (d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c*e + 1/4*(2*x^2*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*d*e + a*c*e*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x)), x)
```

3.97 $\int (a+b\cosh^{-1}(c+dx))\ dx$

Optimal. Leaf size=46

$$ax-\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arccosh(d*x+c)/d-b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {5995, 5879, 75}

$$ax-\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (b*(c + d*x)*ArcCosh[c +
 d*x])/d
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)\,dx = ax+b\int \cosh^{-1}(c+dx)\,dx$$
$$= ax+\frac{b\text{Subst}\left(\int \cosh^{-1}(x)\,dx,x,c+dx\right)}{d}$$
$$= ax+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+d\right.}{d}$$
$$= ax-\frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 1.39

$$ax+bx\cosh^{-1}(c+dx)-\frac{b\left(\sqrt{-1+c+dx}\sqrt{1+c+dx}-2c\tanh^{-1}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcCosh[c + d*x] - (b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*c
*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/d
```

Maple [A]
time = 1.16, size = 41, normalized size = 0.89

method	result	size
default	$ax+\frac{b\left((dx+c)\operatorname{arccosh}(dx+c)-\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)}{d}$	41
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\operatorname{arccosh}(dx+c)-\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)}{d}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.76

$$ax+\frac{\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^2-1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b/d
```

Fricas [A]
time = 0.40, size = 65, normalized size = 1.41

$$\frac{adx + (bdx + bc)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b)/d
```

Sympy [A]
time = 0.22, size = 51, normalized size = 1.11

$$ax + b\left(\begin{cases} \frac{c\operatorname{acosh}(c+dx)}{d} + x\operatorname{acosh}(c+dx) - \frac{\sqrt{c^2 + 2cdx + d^2x^2 - 1}}{d} & \text{for}\ d \neq 0 \\ x\operatorname{acosh}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acosh(c + d*x)/d + x*acosh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*acosh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(42) = 84.
time = 0.42, size = 100, normalized size = 2.17

$$-\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{(dx + c)^2 - 1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)
))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sq
rt((d*x + c)^2 - 1)))*b + a*x
```

Mupad [B]
time = 3.96, size = 272, normalized size = 5.91

$$a\,x+b\,x\,\mathrm{acosh}(c+d\,x)-\frac{b\left(\frac{4\,c\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}+\frac{4\,c\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^3}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^3}-\frac{8\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^2\sqrt{c-1}\,\sqrt{c+1}}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^2}\right)}{\frac{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^4}{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^4}-\frac{2\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^2}{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^2}+1}+\frac{4\,b\,c\,\mathrm{atanh}\left(\frac{\sqrt{c-1}-\sqrt{c+d\,x-1}}{\sqrt{c+1}-\sqrt{c+d\,x+1}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(c + d*x),x)
```

```
[Out] a*x + b*x*acosh(c + d*x) - (b*((4*c*((c - 1)^(1/2) - (c + d*x - 1)^(1/2)))/
(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))) + (4*c*((c - 1)^(1/2) - (c + d*x
- 1)^(1/2))^3)/(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^3) - (8*((c - 1)^(1
/2) - (c + d*x - 1)^(1/2))^2*(c - 1)^(1/2)*(c + 1)^(1/2))/(d*((c + 1)^(1/2)
 - (c + d*x + 1)^(1/2))^2)))/((((c - 1)^(1/2) - (c + d*x - 1)^(1/2))^4/((c +
 1)^(1/2) - (c + d*x + 1)^(1/2))^4 - (2*((c - 1)^(1/2) - (c + d*x - 1)^(1/2
))^2)/((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^2 + 1) + (4*b*c*atanh(((c - 1)^
(1/2) - (c + d*x - 1)^(1/2))/((c + 1)^(1/2) - (c + d*x + 1)^(1/2))))/d
```

3.98 $\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)}{de}-\frac{b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

```
[Out] 1/2*(a+b*arccosh(d*x+c))^2/b/d/e+(a+b*arccosh(d*x+c))*ln(1+1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-1/2*b*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5996, 12, 5882, 3799, 2221, 2317, 2438}

$$\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcCosh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcCosh[c + d*x])*Log[1 + E^(-2*ArcCosh[c + d*x])])/(d*e) - (b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Dist[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{ex}\,dx,x,c+dx\right)}{d}\\
&= \frac{\mathrm{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx,x,c+dx\right)}{de}\\
&= \frac{\mathrm{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{2\mathrm{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 0.85

$$\frac{b\cosh^{-1}(c+dx)^2+2b\cosh^{-1}(c+dx)\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+2a\log(c+dx)-b\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (b*ArcCosh[c + d*x]^2 + 2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x]
)] + 2*a*Log[c + d*x] - b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2*d*e)
```

Maple [A]
time = 8.79, size = 103, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}-\frac{b\operatorname{arccosh}(dx+c)^2}{2e}+\frac{b\operatorname{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{b\operatorname{polylog}\left(2,-\left(dx+c\right.\right.}{}}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}-\frac{b\operatorname{arccosh}(dx+c)^2}{2e}+\frac{b\operatorname{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{b\operatorname{polylog}\left(2,-\left(dx+c\right.\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/e*ln(d*x+c)-1/2*b/e*arccosh(d*x+c)^2+b/e*arccosh(d*x+c)*ln(1+(d*x+c+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2*b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] b*integrate(log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d*x*e + c*e
), x) + a*e^(-1)*log(d*x*e + c*e)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx}\, dx + \int \frac{b \operatorname{acosh}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acosh}(c + d\,x)}{c\,e + d\,e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x), x)
```

3.99 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\mathrm{ArcTan}\left(\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{de^2}$$

```
[Out] (-a-b*arccosh(d*x+c))/d/e^2/(d*x+c)+b*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))/d/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5996, 12, 5883, 94, 209}

$$\frac{b\mathrm{ArcTan}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{de^2}-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])/(d*e^2*(c + d*x))) + (b*ArcTan[Sqrt[-1 + c + d*x
]*Sqrt[1 + c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{de^2}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\tan^{-1}\left(\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{de^2}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 1.39

$$\frac{\frac{-a-b\cosh^{-1}(c+dx)}{c+dx}+\frac{b\sqrt{-1+(c+dx)^2}\,\text{ArcTan}\left(\sqrt{-1+(c+dx)^2}\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] ((-a - b*ArcCosh[c + d*x])/(c + d*x) + (b*Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqr
t[-1 + (c + d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(d*e^2)
```

Maple [A]
time = 4.03, size = 83, normalized size = 1.48

method	result	size
derivativedivides	$\frac{-\frac{a}{e^2(dx+c)}-\frac{b\,\mathrm{arccosh}(dx+c)}{e^2(dx+c)}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{e^2\sqrt{(dx+c)^2-1}}}{d}$	83
default	$\frac{-\frac{a}{e^2(dx+c)}-\frac{b\,\mathrm{arccosh}(dx+c)}{e^2(dx+c)}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{e^2\sqrt{(dx+c)^2-1}}}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)-b/e^2/(d*x+c)*arccosh(d*x+c)-b/e^2*(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2)))
```

Maxima [A]
time = 0.51, size = 72, normalized size = 1.29

$$-b\left(\frac{\arcsin\left(\frac{de^2}{|d^2xe^2+cde^2|}\right)e^{(-2)}}{d}+\frac{\mathrm{arcosh}\,(dx+c)}{d^2xe^2+cde^2}\right)-\frac{a}{d^2xe^2+cde^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b*(arcsin(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arccosh(d*x + c)/(d^2
*x*e^2 + c*d*e^2)) - a/(d^2*x*e^2 + c*d*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(50) = 100.
time = 0.39, size = 168, normalized size = 3.00

$$\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-ac+2\,(bcdx+bc^2)\arctan\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+(bdx+bc)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)}{(cd^2x+c^2d)\cosh(1)^2+2\,(cd^2x+c^2d)\cosh(1)\sinh(1)+(cd^2x+c^2d)\sinh(1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - a*c + 2*(b*c*d*x
+ b*c^2)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (b*d*x + b*
c)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))/((c*d^2*x + c^2*d)*co
```

```
sh(1)^2 + 2*(c*d^2*x + c^2*d)*cosh(1)*sinh(1) + (c*d^2*x + c^2*d)*sinh(1)^2
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*acosh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\operatorname{acosh}(c+d\,x)}{{(c\,e+d\,e\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2, x)
```

3.100 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=66

$$\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] 1/2*(-a-b*arccosh(d*x+c))/d/e^3/(d*x+c)^2+1/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5996, 12, 5883, 97}

$$\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}\\ &= -\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{2de^3}\\ &= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.83

$$-\frac{a-b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
[Out] -1/2*(a - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] + b*ArcCosh[c +
d*x])/(d*e^3*(c + d*x)^2)
```

Maple [A]
time = 8.16, size = 65, normalized size = 0.98

method	result	size
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3}}{d}$	65
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3}}{d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Maxima [A]
time = 0.49, size = 110, normalized size = 1.67

$$\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d}{d^3xe^3+cd^2e^3}-\frac{\operatorname{arcosh}\left(dx+c\right)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)-\frac{a}{2\,(d^3x^2e^3+2\,cd^2xe^3+c^2de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*x*e^3 + c*d^2*e^3) - arccos
h(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 1/2*a/(d^3*x^2*e^3
+ 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(56) = 112.
time = 0.47, size = 210, normalized size = 3.18

$$\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+(bc^2dx+bc^3)\sqrt{d^2x^2+2\,cdx+c^2-1}}{2\left((c^2d^3x^2+2\,c^3d^2x+c^4d)\cosh\left(1\right)^3+3\,(c^2d^3x^2+2\,c^3d^2x+c^4d)\cosh\left(1\right)^2\sinh\left(1\right)+3\,(c^2d^3x^2+2\,c^3d^2x+c^4d)\cosh\left(1\right)\sinh\left(1\right)^2+(c^2d^3x^2+2\,c^3d^2x+c^4d)\sinh\left(1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) + (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/((c^2*d^3
*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^3 + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*
d)*cosh(1)^2*sinh(1) + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)*sinh(1
)^2 + (c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*sinh(1)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b\operatorname{acosh}\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3, x)
```

3.101 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx$

Optimal. Leaf size=99

$$\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2}-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{ArcTan}\left(\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{6de^4}$$

```
[Out] 1/3*(-a-b*arccosh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctan((d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))/d/e^4+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5996, 12, 5883, 105, 94, 209}

$$-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{ArcTan}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{6de^4}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^4*(c + d*x)^2) - (a + b*Arc
Cosh[c + d*x])/(3*d*e^4*(c + d*x)^3) + (b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(6*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^4x^4}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^4}\,dx,x,c+dx\right)}{de^4}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx,x,c+dx\right)}{3de^4}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2}-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\cdots}\right.}{}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2}-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x\cdots\right.}{}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2}-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\tan^{-1}\left(\sqrt{-1+c+\cdots}\right.}{6d\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 101, normalized size = 1.02

$$\frac{-\frac{2(a+b\cosh^{-1}(c+dx))}{(c+dx)^3}+\frac{b\left(\frac{(-1+c+dx)(1+c+dx)}{(c+dx)^2}+\sqrt{-1+(c+dx)^2}\,\mathrm{ArcTan}\left(\sqrt{-1+(c+dx)^2}\right)\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*(a + b*ArcCosh[c + d*x]))/(c + d*x)^3 + (b*(((-1 + c + d*x)*(1 + c + d
*x))/(c + d*x)^2 + Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(6*d*e^4)
```

Maple [A]
time = 4.03, size = 112, normalized size = 1.13

method	result
derivativedivides	$\frac{-\frac{a}{3e^4(dx+c)^3}-\frac{b\,\mathrm{arccosh}(dx+c)}{3e^4(dx+c)^3}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{6e^4\sqrt{(dx+c)^2-1}}+\frac{b\sqrt{dx+c-}}{6}}{d}$
default	$\frac{-\frac{a}{3e^4(dx+c)^3}-\frac{b\,\mathrm{arccosh}(dx+c)}{3e^4(dx+c)^3}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{6e^4\sqrt{(dx+c)^2-1}}+\frac{b\sqrt{dx+c-}}{6}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a/e^4/(d*x+c)^3-1/3*b/e^4/(d*x+c)^3*arccosh(d*x+c)-1/6*b/e^4*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2
))+1/6*b/e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*((2*d^2*x^2 + 4*c*d*x + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d*x + c + 1) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*log(d*x +
 c - 1) - 2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*x^3*e^
4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 6*integrate(1/3/(d^6*x
^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^
3)*x^3*e^4 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^
4 + (c^6 - c^4)*e^4 + (d^5*x^5*e^4 + 5*c*d^4*x^4*e^4 + (10*c^2*d^3 - d^3)*x
^3*e^4 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^4 + (5*c^4*d - 3*c^2*d)*x*e^4 + (c^5
- c^3)*e^4)*e^(1/2*log(d*x + c + 1) + 1/2*log(d*x + c - 1))), x)) - 1/3*a/(
d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(82) = 164$.
time = 0.41, size = 445, normalized size = 4.49

$$-\frac{2\,ac^3 - 2\,(bc^3d^3x^3 + 3\,bc^4d^2x^2 + 3\,bc^5dx + bc^6)\arctan\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - 2\,(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - 2\,(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3)\log\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - (bc^3dx + bc^4)\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{6\left((c^3d^4x^3 + 3\,c^4d^3x^2 + 3\,c^5d^2x + c^6d)\cosh(1)^4 + 4\,(c^3d^4x^3 + 3\,c^4d^3x^2 + 3\,c^5d^2x + c^6d)\cosh(1)^3\sinh(1) + 6\,(c^3d^4x^3 + 3\,c^4d^3x^2 + 3\,c^5d^2x + c^6d)\cosh(1)^2\sinh(1)^2 + 4\,(c^3d^4x^3 + 3\,c^4d^3x^2 + 3\,c^5d^2x + c^6d)\cosh(1)\sinh(1)^3 + (c^3d^4x^3 + 3\,c^4d^3x^2 + 3\,c^5d^2x + c^6d)\sinh(1)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x + b*c^6)*a
rctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2
*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)) - (b*c^3*d*x + b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))/((c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x + c^6*d)*cosh(1)^4 + 4*
(c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x + c^6*d)*cosh(1)^3*sinh(1) + 6*(
c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x + c^6*d)*cosh(1)^2*sinh(1)^2 + 4*
(c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x + c^6*d)*cosh(1)*sinh(1)^3 + (c^
3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x + c^6*d)*sinh(1)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{(c\,e+d\,e\,x)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4, x)
```

3.102 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx$

Optimal. Leaf size=104

$$\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3}+\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^5(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}$$

```
[Out] 1/4*(-a-b*arccosh(d*x+c))/d/e^5/(d*x+c)^4+1/12*b*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)/d/e^5/(d*x+c)^3+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^5/(d*x+c)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5996, 12, 5883, 105, 97}

$$-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^5(c+dx)}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^5*(c + d*x)) - (a + b*ArcCosh[c + d*
x])/(4*d*e^5*(c + d*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^5x^5}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^5}\,dx,x,c+dx\right)}{de^5}$$

$$= -\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^4\sqrt{1+x}}\,dx,x,c+dx\right)}{4de^5}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{2}{\sqrt{-1+x}}\right.}{1}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}}\right.}{6}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^5(c+dx)} - \frac{a+b\cosh^{-1}}{4de^5(c+}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.83

$$\frac{-3a+b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(c+2c^3+dx+6c^2dx+6cd^2x^2+2d^3x^3)-3b\cosh^{-1}(c+dx)}{12de^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] (-3*a + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(c + 2*c^3 + d*x + 6*c^2*d*x
 + 6*c*d^2*x^2 + 2*d^3*x^3) - 3*b*ArcCosh[c + d*x])/(12*d*e^5*(c + d*x)^4)
```

Maple [A]
time = 8.35, size = 76, normalized size = 0.73

method	result	size
derivativedivides	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5}}{d}$	76
default	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5}}{d}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arccosh(d*x+c)+1/12*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^2+1)/(d*x+c)^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(87) = 174.
time = 0.30, size = 246, normalized size = 2.37

$$\frac{1}{12}\,b\left(\frac{(2\,d^4x^4+8\,cd^3x^3+2\,c^4+(12\,c^2d^2-d^2)x^2-c^2+2\,(4\,c^3d-cd)x-1)d}{(d^5x^3e^5+3\,cd^4x^2e^5+3\,c^2d^3xe^5+c^3d^2e^5)\sqrt{dx+c+1}\,\sqrt{dx+c-1}}-\frac{3\,\operatorname{arcosh}(dx+c)}{d^5x^4e^5+4\,cd^4x^3e^5+6\,c^2d^3x^2e^5+4\,c^3d^2xe^5+c^4de^5}\right)-\frac{a}{4\,(d^5x^4e^5+4\,cd^4x^3e^5+6\,c^2d^3x^2e^5+4\,c^3d^2xe^5+c^4de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 - d^2)*x^2 - c^2 + 2
*(4*c^3*d - c*d)*x - 1)*d/((d^5*x^3*e^5 + 3*c*d^4*x^2*e^5 + 3*c^2*d^3*x*e^5
 + c^3*d^2*e^5)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)) - 3*arccosh(d*x + c)/(
d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c^4*d
*e^5)) - 1/4*a/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d
^2*x*e^5 + c^4*d*e^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(87) = 174.
time = 0.38, size = 475, normalized size = 4.57

$$\frac{3\,ad^4x^4+12\,acd^3x^3+18\,ac^2d^2x^2+12\,ac^3dx-3\,bc^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+(2\,bc^4d^3x^3+6\,bc^5d^2x^2+2\,bc^7+bc^5+(6\,bc^6+bc^4)dx)\sqrt{d^2x^2+2\,cdx+c^2-1}}{12\left((c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\cosh(1)^5+5\,(c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\cosh(1)^4\sinh(1)+10\,(c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\cosh(1)^3\sinh(1)^2+10\,(c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\cosh(1)^2\sinh(1)^3+5\,(c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\cosh(1)\sinh(1)^4+(c^4d^5x^4+4\,c^5d^4x^3+6\,c^6d^3x^2+4\,c^7d^2x+c^8d)\sinh(1)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 + b*c^5 + (6*b*c^6 + b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))/((c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2
*x + c^8*d)*cosh(1)^5 + 5*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*
c^7*d^2*x + c^8*d)*cosh(1)^4*sinh(1) + 10*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*
c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(1)^3*sinh(1)^2 + 10*(c^4*d^5*x^4 +
4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(1)^2*sinh(1)^3 +
5*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(
1)*sinh(1)^4 + (c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x +
 c^8*d)*sinh(1)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx+\int \frac{b\operatorname{acosh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*acosh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5, x)
```

3.103 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=137

$$\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}+\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2}-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\text{ArcTan}\left(\sqrt{-1+\right.}$$

```
[Out] 1/5*(-a-b*arccosh(d*x+c))/d/e^6/(d*x+c)^5+3/40*b*arctan((d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))/d/e^6+1/20*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^4+
3/40*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^2
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5996, 12, 5883, 105, 94, 209}

$$-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\text{ArcTan}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{40de^6}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{40de^6(c+dx)^2}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{20de^6(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcCosh[c
 + d*x])/(5*d*e^6*(c + d*x)^5) + (3*b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]])/(40*d*e^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx=\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^6x^6}\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^6}\,dx,x,c+dx\right)}{de^6}$$

$$=-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^5\sqrt{1+x}}\,dx,x,c+dx\right)}{5de^6}$$

$$=\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{b\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\cdots}\right.}{\cdots}$$

$$=\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{(3b)\text{Subst}\left(\int \frac{}{\sqrt{-1}\cdots}\right.}{\cdots}$$

$$=\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}+\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2}-\frac{a+b\cosh\cdots}{5de^6(\cdots}$$

$$=\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}+\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2}-\frac{a+b\cosh\cdots}{5de^6(\cdots}$$

$$=\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4}+\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2}-\frac{a+b\cosh\cdots}{5de^6(\cdots}$$

Mathematica [A]
time = 0.16, size = 136, normalized size = 0.99

$$\frac{\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{4(c+dx)^4}-\frac{a+b\cosh^{-1}(c+dx)}{(c+dx)^5}+\frac{3b\left(\frac{(-1+c+dx)(1+c+dx)}{(c+dx)^2}+\sqrt{-1+(c+dx)^2}\,\text{ArcTan}\left(\sqrt{-1+(c+dx)^2}\right)\right)}{8\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}}{5de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] ((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(4*(c + d*x)^4) - (a + b*ArcCosh[
c + d*x])/(c + d*x)^5 + (3*b*(((-1 + c + d*x)*(1 + c + d*x))/(c + d*x)^2 +
Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/(8*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]))/(5*d*e^6)
```

Maple [A]
time = 4.00, size = 141, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{a}{5e^6(dx+c)^5}-\frac{b\,\mathrm{arccosh}(dx+c)}{5e^6(dx+c)^5}-\frac{3b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{40e^6\sqrt{(dx+c)^2-1}}+\frac{3b\sqrt{dx+c}}{}}{d}$
default	$\frac{-\frac{a}{5e^6(dx+c)^5}-\frac{b\,\mathrm{arccosh}(dx+c)}{5e^6(dx+c)^5}-\frac{3b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{40e^6\sqrt{(dx+c)^2-1}}+\frac{3b\sqrt{dx+c}}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a/e^6/(d*x+c)^5-1/5*b/e^6/(d*x+c)^5*arccosh(d*x+c)-3/40*b/e^6*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/
2))+3/40*b/e^6*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)^2+1/20*b/e^6*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*((6*d^4*x^4 + 24*c*d^3*x^3 + 6*c^4 + 2*(18*c^2*d^2 + d^2)*x^2 + 2*c^
2 + 4*(6*c^3*d + c*d)*x - 3*(d^5*x^5 + 5*c*d^4*x^4 + 10*c^2*d^3*x^3 + 10*c^
3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c + 1) + 3*(d^5*x^5 + 5*c*d^4*x^4 +
10*c^2*d^3*x^3 + 10*c^3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c - 1) - 6*log
(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^6*x^5*e^6 + 5*c*d^5*x^4
*e^6 + 10*c^2*d^4*x^3*e^6 + 10*c^3*d^3*x^2*e^6 + 5*c^4*d^2*x*e^6 + c^5*d*e^
6) - 30*integrate(1/5/(d^8*x^8*e^6 + 8*c*d^7*x^7*e^6 + (28*c^2*d^6 - d^6)*x
^6*e^6 + 2*(28*c^3*d^5 - 3*c*d^5)*x^5*e^6 + 5*(14*c^4*d^4 - 3*c^2*d^4)*x^4*
e^6 + 4*(14*c^5*d^3 - 5*c^3*d^3)*x^3*e^6 + (28*c^6*d^2 - 15*c^4*d^2)*x^2*e^
6 + 2*(4*c^7*d - 3*c^5*d)*x*e^6 + (c^8 - c^6)*e^6 + (d^7*x^7*e^6 + 7*c*d^6*
x^6*e^6 + (21*c^2*d^5 - d^5)*x^5*e^6 + 5*(7*c^3*d^4 - c*d^4)*x^4*e^6 + 5*(7
*c^4*d^3 - 2*c^2*d^3)*x^3*e^6 + (21*c^5*d^2 - 10*c^3*d^2)*x^2*e^6 + (7*c^6*
d - 5*c^4*d)*x*e^6 + (c^7 - c^5)*e^6)*e^(1/2*log(d*x + c + 1) + 1/2*log(d*x
 + c - 1))), x)) - 1/5*a/(d^6*x^5*e^6 + 5*c*d^5*x^4*e^6 + 10*c^2*d^4*x^3*e^
6 + 10*c^3*d^3*x^2*e^6 + 5*c^4*d^2*x*e^6 + c^5*d*e^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(113) = 226$.
time = 0.45, size = 803, normalized size = 5.86

$$-\frac{8\,ac^{5} - 6\,(bc^{5}d^{5}x^{5} + 5\,bc^{6}d^{4}x^{4} + 10\,bc^{7}d^{3}x^{3} + 10\,bc^{8}d^{2}x^{2} + 5\,bc^{9}dx + bc^{10})\arctan\left(-dx - c + \sqrt{d^{2}x^{2} + 2\,cdx + c^{2} - 1}\right) - 8\,(bd^{5}x^{5} + 5\,bcd^{4}x^{4} + 10\,bc^{2}d^{3}x^{3} + 10\,bc^{3}d^{2}x^{2} + 5\,bc^{4}dx)\log\left(dx + c + \sqrt{d^{2}x^{2} + 2\,cdx + c^{2} - 1}\right) - 8\,(bd^{5}x^{5} + 5\,bcd^{4}x^{4} + 10\,bc^{2}d^{3}x^{3} + 10\,bc^{3}d^{2}x^{2} + 5\,bc^{4}dx + bc^{5})\log\left(-dx - c + \sqrt{d^{2}x^{2} + 2\,cdx + c^{2} - 1}\right) - (3\,bc^{5}d^{3}x^{3} + 9\,bc^{6}d^{2}x^{2} + 3\,bc^{8} + 2\,bc^{6} + (9\,bc^{7} + 2\,bc^{5})dx)\sqrt{d^{2}x^{2} + 2\,cdx + c^{2} - 1}}{40\,\left((c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)^{6} + 6\,(c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)^{5}\sinh(1) + 15\,(c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)^{4}\sinh(1)^{2} + 20\,(c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)^{3}\sinh(1)^{3} + 15\,(c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)^{2}\sinh(1)^{4} + 6\,(c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\cosh(1)\sinh(1)^{5} + (c^{5}d^{6}x^{5} + 5\,c^{6}d^{5}x^{4} + 10\,c^{7}d^{4}x^{3} + 10\,c^{8}d^{3}x^{2} + 5\,c^{9}d^{2}x + c^{10}d)\sinh(1)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 6*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10
*b*c^8*d^2*x^2 + 5*b*c^9*d*x + b*c^10)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c
^3*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*
c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*c
^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 + 2*b*c^6 + (9*b*c^7 + 2*b*c^5)*d*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/((c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*
d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^6 + 6*(c^5*d^6*x^5
 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*
cosh(1)^5*sinh(1) + 15*(c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c
^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^4*sinh(1)^2 + 20*(c^5*d^6*x^5 +
5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cos
h(1)^3*sinh(1)^3 + 15*(c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^
8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^2*sinh(1)^4 + 6*(c^5*d^6*x^5 + 5*
c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(
1)*sinh(1)^5 + (c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x
^2 + 5*c^9*d^2*x + c^10*d)*sinh(1)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^{6}+6c^{5}dx+15c^{4}d^{2}x^{2}+20c^{3}d^{3}x^{3}+15c^{2}d^{4}x^{4}+6cd^{5}x^{5}+d^{6}x^{6}}\,dx + \int \frac{b\operatorname{acosh}{(c+dx)}}{c^{6}+6c^{5}dx+15c^{4}d^{2}x^{2}+20c^{3}d^{3}x^{3}+15c^{2}d^{4}x^{4}+6cd^{5}x^{5}+d^{6}x^{6}}\,dx}{e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*acosh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6, x)
```

3.104 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=218

$$\frac{16}{75}b^2e^4x+\frac{8b^2e^4(c+dx)^3}{225d}+\frac{2b^2e^4(c+dx)^5}{125d}-\frac{16be^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{75d}-\frac{8be^4}{}$$

```
[Out] 16/75*b^2*e^4*x+8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^4
*(d*x+c)^5*(a+b*arccosh(d*x+c))^2/d-16/75*b*e^4*(a+b*arccosh(d*x+c))*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d-8/75*b*e^4*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arccosh(d*x+c))*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.35, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5883, 5939, 5915, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{25d}-\frac{8be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{75d}-\frac{16be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{2b^2e^4(c+dx)^5}{125d}+\frac{8b^2e^4(c+dx)^3}{225d}+\frac{16}{75}b^2e^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (16*b^2*e^4*x)/75 + (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x]))/(75*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*
x]*(a + b*ArcCosh[c + d*x]))/(75*d) - (2*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)
^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(25*d) + (e^4*(c + d*x)^5*(a
 + b*ArcCosh[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d} \\
&= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d} - \frac{(2be^4)\,\text{Subst}\left(\int \frac{x^5(a+b}{\sqrt{-1+}}\right.}{} \\
&= -\frac{2be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{25d} \\
&= \frac{2b^2e^4(c+dx)^5}{125d} - \frac{8be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,(a}{75d} \\
&= \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c+dx}\,\sqrt{1+}}{} \\
&= \frac{16}{75}b^2e^4x + \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c-}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 220, normalized size = 1.01

$$
\frac{e^4\left(240b^2(c+dx)+40b^2(c+dx)^3+9(25a^2+2b^2)(c+dx)^5+30ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(-8-4(c+dx)^2-3(c+dx)^4)+30b\left(15a(c+dx)^5-8b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,-4b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,-3b\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}\,\right)\cosh^{-1}(c+dx)+225b^2(c+dx)^5\cosh^{-1}(c+dx)^2\right)}{1125d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) + 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8 - 4*(c + d*x)^2 - 3*(c
 + d*x)^4) + 30*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x] - 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 3*b*Sqrt[-1 +
c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 225*b^2*(c + d*x
)^5*ArcCosh[c + d*x]^2))/(1125*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(192) = 384$.
time = 10.98, size = 540, normalized size = 2.48

method	result

default	$\frac{e^4(dx+c)^5a^2}{5d}+\frac{e^4b^2\left(225\mathrm{arccosh}(dx+c)^2c^5+18x^5d^5+240c-90\,\mathrm{arccosh}(dx+c)\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,c^4-120\,\mathrm{arccosh}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/5*e^4*(d*x+c)^5*a^2/d+1/1125*e^4*b^2*(225*arccosh(d*x+c)^2*c^5+18*x^5*d^5 +240*c-90*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^4-120*arccosh(d* x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+1125*arccosh(d*x+c)^2*x^4*c*d^4+22 50*arccosh(d*x+c)^2*x^3*c^2*d^3+2250*arccosh(d*x+c)^2*x^2*c^3*d^2+1125*arcc osh(d*x+c)^2*x*c^4*d+40*c^3+40*d^3*x^3+18*c^5+240*d*x-90*arccosh(d*x+c)*(d* x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^4*d^4-120*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d* x+c-1)^(1/2)*x^2*d^2-540*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2 *c^2*d^2-360*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c^3*d-240*arc cosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d-360*arccosh(d*x+c)*(d*x+c +1)^(1/2)*(d*x+c-1)^(1/2)*x^3*c*d^3+120*c^2*d*x+90*x*c^4*d+180*x^3*c^2*d^3+ 90*x^4*c*d^4+180*x^2*c^3*d^2+120*x^2*c*d^2+225*arccosh(d*x+c)^2*x^5*d^5-240 *arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d+2*e^4*a*b/d*(1/5*(d*x+c) ^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+ c)^2+8))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")

[Out] 1/5*a^2*d^4*x^5*e^4 + a^2*c*d^3*x^4*e^4 + 2*a^2*c^2*d^2*x^3*e^4 + 2*a^2*c^3 *d*x^2*e^4 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*s qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1) *x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^3*d*e^4 + 2/3* (6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5 *sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2* c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d* x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))* a*b*c^2*d^2*e^4 + 1/12*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105* c^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*s qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x +

```
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/
d^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a*b*c*
d^3*e^4 + 1/300*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*c^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 +
1050*(c^2 - 1)*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^6 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x + 2
*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*(c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^
2 - 1)^2/d^6)*d)*a*b*d^4*e^4 + a^2*c^4*x*e^4 + 2*((d*x + c)*arccosh(d*x + c
) - sqrt((d*x + c)^2 - 1))*a*b*c^4*e^4/d + 1/5*(b^2*d^4*x^5*e^4 + 5*b^2*c*d
^3*x^4*e^4 + 10*b^2*c^2*d^2*x^3*e^4 + 10*b^2*c^3*d*x^2*e^4 + 5*b^2*c^4*x*e^
4)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(2/5*(b^
2*d^7*x^7*e^4 + 7*b^2*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*b^2*x^5*e^4 + 5*(7
*c^3*d^4 - c*d^4)*b^2*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*b^2*x^3*e^4 + 10*
(2*c^5*d^2 - c^3*d^2)*b^2*x^2*e^4 + 5*(c^6*d - c^4*d)*b^2*x*e^4 + (b^2*d^6*
x^6*e^4 + 6*b^2*c*d^5*x^5*e^4 + 15*b^2*c^2*d^4*x^4*e^4 + 20*b^2*c^3*d^3*x^3
*e^4 + 15*b^2*c^4*d^2*x^2*e^4 + 5*b^2*c^5*d*x*e^4)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 +
3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2430 vs. $2(185) = 370$.
time = 0.45, size = 2430, normalized size = 11.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*((9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 + 10*(9
*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 + 4*b^2
*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^2)*d*x)*cosh(1)
^4 + 4*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 + 10*(9*
(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 + 4*b^2*
c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^2)*d*x)*cosh(1)^
3*sinh(1) + 6*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 +
 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 +
 4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^2)*d*x)*c
```

osh(1)^2*sinh(1)^2 + 4*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*
d^4*x^4 + 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b
^2)*c^3 + 4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^
2)*d*x)*cosh(1)*sinh(1)^3 + (9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^
2)*c*d^4*x^4 + 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*x^3 + 30*(3*(25*a^2
+ 2*b^2)*c^3 + 4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 +
16*b^2)*d*x)*sinh(1)^4 + 225*((b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d
^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)^4 + 4*(b^2*d
^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*
c^4*d*x + b^2*c^5)*cosh(1)^3*sinh(1) + 6*(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 1
0*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)^2
sinh(1)^2 + 4(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2
*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)*sinh(1)^3 + (b^2*d^5*x^5 +
5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x +
 b^2*c^5)*sinh(1)^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3
0*(15*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*
x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh(1)^4 + 60*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^
4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh
(1)^3*sinh(1) + 90*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10
*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh(1)^2*sinh(1)^2 + 60*(a*b*d
^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*
c^4*d*x + a*b*c^5)*cosh(1)*sinh(1)^3 + 15*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 +
10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*sinh(1)^
4 - ((3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 + 2*b^2)*
d^2*x^2 + 4*b^2*c^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*x + 8*b^2)*cosh(1)^4 + 4*(3
*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 + 2*b^2)*d^2*x^2
 + 4*b^2*c^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*x + 8*b^2)*cosh(1)^3*sinh(1) + 6*(
3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 + 2*b^2)*d^2*x^
2 + 4*b^2*c^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*x + 8*b^2)*cosh(1)^2*sinh(1)^2 +
4*(3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 + 2*b^2)*d^2
*x^2 + 4*b^2*c^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*x + 8*b^2)*cosh(1)*sinh(1)^3 +
 (3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 + 2*b^2)*d^2*
x^2 + 4*b^2*c^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*x + 8*b^2)*sinh(1)^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- 30*((3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 + 2*a*b)
*d^2*x^2 + 4*a*b*c^2 + 4*(3*a*b*c^3 + 2*a*b*c)*d*x + 8*a*b)*cosh(1)^4 + 4*(
3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 + 2*a*b)*d^2*x^
2 + 4*a*b*c^2 + 4*(3*a*b*c^3 + 2*a*b*c)*d*x + 8*a*b)*cosh(1)^3*sinh(1) + 6*
(3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 + 2*a*b)*d^2*x
^2 + 4*a*b*c^2 + 4*(3*a*b*c^3 + 2*a*b*c)*d*x + 8*a*b)*cosh(1)^2*sinh(1)^2 +
 4*(3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 + 2*a*b)*d^
2*x^2 + 4*a*b*c^2 + 4*(3*a*b*c^3 + 2*a*b*c)*d*x + 8*a*b)*cosh(1)*sinh(1)^3
+ (3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 + 2*a*b)*d^2
*x^2 + 4*a*b*c^2 + 4*(3*a*b*c^3 + 2*a*b*c)*d*x + 8*a*b)*sinh(1)^4)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(209) = 418$.

time = 0.82, size = 1268, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
cosh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*acosh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*acosh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*acosh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/25 - 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*acosh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/75 + 2*a*b*d**4*e**4*x**5*acosh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)/(75*d) + b**2*c**5*e**4*acosh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*acosh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*acosh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*acosh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*c**2
*e**4*x/75 - 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)/25 + 8*b**2*c*d*e**4*x**2/75 - 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + b**2*d**4*e**4*x**5*acosh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*d**2*e**4*x**3/225 - 8*b**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**2, True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{acosh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2, x)
```

3.105 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=186

$$\frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}-\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{16d}-\frac{be^3\sqrt{-1}}{}$$

```
[Out] 3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arccosh(d*x
+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d-3/16*b*e^3*(d*x+c)*(a+b
*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b
*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 12, 5883, 5939, 5893, 30}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{8d}-\frac{3be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{16d}-\frac{3e^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}+\frac{3b^2e^3(c+dx)^2}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) - (3*b*e^3*Sq
rt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(16*
d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh
[c + d*x]))/(8*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^2)/(32*d) + (e^3*(c + d
*x)^4*(a + b*ArcCosh[c + d*x])^2)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\text{c}}{\sqrt{-1+\text{:}}}\right.}{2}$$

$$= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{8d}$$

$$= \frac{b^2e^3(c+dx)^4}{32d} - \frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,(a+}{16d}$$

$$= \frac{3b^2e^3(c+dx)^2}{32d} + \frac{b^2e^3(c+dx)^4}{32d} - \frac{3be^3\sqrt{-1+c+dx}\,(c+dx)}{}$$

Mathematica [**A**]
time = 0.17, size = 212, normalized size = 1.14

$$\frac{e^3\left(3b^2(c+dx)^2+(8a^2+b^2)(c+dx)^4+2ab\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,(-3-2(c+dx)^2)+2b(c+dx)\left(8a(c+dx)^3-3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}-2b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+b^2(-3+8(c+dx)^4)\cosh^{-1}(c+dx)^2-6ab\log\left(c+dx+\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*(3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*Sqrt[-1 + c + d
*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3 - 2*(c + d*x)^2) + 2*b*(c + d*x)*(8*a*(
c + d*x)^3 - 3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-3 + 8*(c + d*x)
^4)*ArcCosh[c + d*x]^2 - 6*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]]))/(32*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(166) = 332$.
time = 6.09, size = 437, normalized size = 2.35

method	result
default	$\frac{e^3(dx+c)^4a^2}{4d} + \frac{e^3b^2\left(\frac{\left(8\text{arccosh}(dx+c)^2+1\right)\left(\cosh^2(2\,\text{arccosh}(dx+c))\right)}{128}+\frac{\left(4\text{arccosh}(dx+c)^2-\text{arccosh}(dx+c)\sinh(2\,\text{arccosh}(dx+c))+2\right)\cosh(2\,\text{ar}}{32}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^2/d+e^3*b^2/d*(1/128*(8*arccosh(d*x+c)^2+1)*cosh(2*arcc
osh(d*x+c))^2+1/32*(4*arccosh(d*x+c)^2-arccosh(d*x+c)*sinh(2*arccosh(d*x+c)
)+2)*cosh(2*arccosh(d*x+c))-1/32*arccosh(d*x+c)^2-1/8*arccosh(d*x+c)*sinh(2
*arccosh(d*x+c))-1/256)+1/2*e^3*a*b*d^3*arccosh(d*x+c)*x^4+2*e^3*a*b*d^2*ar
ccosh(d*x+c)*x^3*c+3*e^3*a*b*d*arccosh(d*x+c)*x^2*c^2+2*e^3*a*b*arccosh(d*x
+c)*x*c^3+1/2*e^3*a*b/d*arccosh(d*x+c)*c^4-1/8*e^3*a*b*d^2*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*x^3-3/8*e^3*a*b*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/8
*e^3*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-1/8*e^3*a*b/d*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*c^3-3/16*e^3*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/16*e
^3*a*b/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/16*e^3*a*b/d*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4*e^3 + a^2*c*d^2*x^3*e^3 + 3/2*a^2*c^2*d*x^2*e^3 + 3/2*(2*x^
2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*l
og(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arccosh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4
 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a*b*c*d^2*e^3 + 1/48
*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 -
14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d
^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*lo
g(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2*c^3*x*e^3
+ 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c^3*e^3/d + 1/
4*(b^2*d^3*x^4*e^3 + 4*b^2*c*d^2*x^3*e^3 + 6*b^2*c^2*d*x^2*e^3 + 4*b^2*c^3*
```

```
x*e^3)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(1/2
*(b^2*d^6*x^6*e^3 + 6*b^2*c*d^5*x^5*e^3 + (15*c^2*d^4 - d^4)*b^2*x^4*e^3 +
4*(5*c^3*d^3 - c*d^3)*b^2*x^3*e^3 + 2*(7*c^4*d^2 - 3*c^2*d^2)*b^2*x^2*e^3 +
 4*(c^5*d - c^3*d)*b^2*x*e^3 + (b^2*d^5*x^5*e^3 + 5*b^2*c*d^4*x^4*e^3 + 10*
b^2*c^2*d^3*x^3*e^3 + 10*b^2*c^3*d^2*x^2*e^3 + 4*b^2*c^4*d*x*e^3)*sqrt(d*x
+ c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(160) = 320$.
time = 0.49, size = 1457, normalized size = 7.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^
2)*c^2 + b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*x)*cosh(1)^3 +
3*((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^2)*c
^2 + b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*x)*cosh(1)^2*sinh(1
) + 3*((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^
2)*c^2 + b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*x)*cosh(1)*sinh
(1)^2 + ((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 +
b^2)*c^2 + b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*x)*sinh(1)^3
+ ((8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x
+ 8*b^2*c^4 - 3*b^2)*cosh(1)^3 + 3*(8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b
^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8*b^2*c^4 - 3*b^2)*cosh(1)^2*sinh(1) + 3*
(8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8
*b^2*c^4 - 3*b^2)*cosh(1)*sinh(1)^2 + (8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 4
8*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8*b^2*c^4 - 3*b^2)*sinh(1)^3)*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 2*((8*a*b*d^4*x^4 + 32*a*b*c*d
^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*x + 8*a*b*c^4 - 3*a*b)*cosh(1)^3
 + 3*(8*a*b*d^4*x^4 + 32*a*b*c*d^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*
x + 8*a*b*c^4 - 3*a*b)*cosh(1)^2*sinh(1) + 3*(8*a*b*d^4*x^4 + 32*a*b*c*d^3*
x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*x + 8*a*b*c^4 - 3*a*b)*cosh(1)*sinh
(1)^2 + (8*a*b*d^4*x^4 + 32*a*b*c*d^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3
*d*x + 8*a*b*c^4 - 3*a*b)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2
*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 + 3*b^2*c + 3*(2*b^2*c^2 + b^2)*
d*x)*cosh(1)^3 + 3*(2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 + 3*b^2*c +
 3*(2*b^2*c^2 + b^2)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^2*d^3*x^3 + 6*b^2*c*d^
2*x^2 + 2*b^2*c^3 + 3*b^2*c + 3*(2*b^2*c^2 + b^2)*d*x)*cosh(1)*sinh(1)^2 +
(2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 + 3*b^2*c + 3*(2*b^2*c^2 + b^2
)*d*x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*sqr
```

```
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*c^
3 + 3*a*b*c + 3*(2*a*b*c^2 + a*b)*d*x)*cosh(1)^3 + 3*(2*a*b*d^3*x^3 + 6*a*b
*c*d^2*x^2 + 2*a*b*c^3 + 3*a*b*c + 3*(2*a*b*c^2 + a*b)*d*x)*cosh(1)^2*sinh(
1) + 3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*c^3 + 3*a*b*c + 3*(2*a*b*c^
2 + a*b)*d*x)*cosh(1)*sinh(1)^2 + (2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*
c^3 + 3*a*b*c + 3*(2*a*b*c^2 + a*b)*d*x)*sinh(1)^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(172) = 344$.
time = 0.58, size = 916, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*acosh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*acosh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/8 + 2*a*b*c*d**2*e**3*x**3*acosh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + a*b*d**3*e**3*x**4*acosh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*a*b*e**3*acosh(c + d*x)/
(16*d) + b**2*c**4*e**3*acosh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*acosh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*acosh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)*acosh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/8 + 3*b**2*c*e**3*x/16 - 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*acosh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 3*b**2*d*e**3*x**2/32 - 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/16 - 3*b**2*e**3*a
cosh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**2, True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2, x)
```

3.106 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=150

$$\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{2be^2\sqrt{-1+c+dx}\,(c+$$

```
[Out] 4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c
))^2/d-4/9*b*e^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/9
*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5883, 5939, 5915, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{4be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4}{9}b^2e^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) - (4*b*e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 +
c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) + (e
^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx &= \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d} \\
&= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d} \\
&= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\text{c}}{\sqrt{-1+x}}\right.}{3\text{.}} \\
&= -\frac{2be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{9d} \\
&= \frac{2b^2e^2(c+dx)^3}{27d} - \frac{4be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}\right.}{9d} \\
&= \frac{4}{9}b^2e^2x + \frac{2b^2e^2(c+dx)^3}{27d} - \frac{4be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a\right.}{9d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 168, normalized size = 1.12

$$\frac{e^2\left(12b^2(c+dx)+(9a^2+2b^2)(c+dx)^3+6ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(-2-(c+dx)^2)+6b\left(3a(c+dx)^3-2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,-b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+9b^2(c+dx)^3\cosh^{-1}(c+dx)^2\right)}{27d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*(12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(-2 - (c + d*x)^2) + 6*b*(3*a*(c + d*x)^3 - 2*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x] - b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
+ c + d*x])*ArcCosh[c + d*x] + 9*b^2*(c + d*x)^3*ArcCosh[c + d*x]^2))/(27*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(132) = 264$.
time = 8.61, size = 290, normalized size = 1.93

method	result
default	$\frac{e^2(dx+c)^3a^2}{3d} + \frac{e^2b^2\left(9\text{arccosh}(dx+c)^2x^3d^3+27\text{arccosh}(dx+c)^2x^2c\,d^2-6\,\text{arccosh}(dx+c)\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,x^2d^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^2/d+1/27*e^2*b^2*(9*arccosh(d*x+c)^2*x^3*d^3+27*arccosh
(d*x+c)^2*x^2*c*d^2-6*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^
2+27*arccosh(d*x+c)^2*x*c^2*d-12*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(
1/2)*x*c*d+2*d^3*x^3+9*arccosh(d*x+c)^2*c^3-6*arccosh(d*x+c)*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*c^2+6*x^2*c*d^2+6*c^2*d*x-12*arccosh(d*x+c)*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)+2*c^3+12*d*x+12*c)/d+2*a*b*e^2/d*(1/3*(d*x+c)^3*arccosh(
d*x+c)-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3*e^2 + a^2*c*d*x^2*e^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d
^3))*a*b*c*d*e^2 + 1/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2
- 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 1
5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*(c^2 - 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*x*e^2 + 2*((d*x + c)*arccosh(d
*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*x^3*e^2 + 3*b
^2*c*d*x^2*e^2 + 3*b^2*c^2*x*e^2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)^2 - integrate(2/3*(b^2*d^5*x^5*e^2 + 5*b^2*c*d^4*x^4*e^2 + (10*c^
2*d^3 - d^3)*b^2*x^3*e^2 + 3*(3*c^3*d^2 - c*d^2)*b^2*x^2*e^2 + 3*(c^4*d - c
^2*d)*b^2*x*e^2 + (b^2*d^4*x^4*e^2 + 4*b^2*c*d^3*x^3*e^2 + 6*b^2*c^2*d^2*x^
2*e^2 + 3*b^2*c^3*d*x*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + s
qrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2
*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d -
d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(127) = 254$.
time = 0.38, size = 784, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

[Out] 1/27*(((9*a^2 + 2*b^2)*d^3*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*a^2 + 2*b^2)*c^2 + 4*b^2)*d*x)*cosh(1)^2 + 9*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(1)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(1)*sinh(1) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(1)^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 2*((9*a^2 + 2*b^2)*d^3*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*a^2 + 2*b^2)*c^2 + 4*b^2)*d*x)*cosh(1)*sinh(1) + ((9*a^2 + 2*b^2)*d^3*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*a^2 + 2*b^2)*c^2 + 4*b^2)*d*x)*sinh(1)^2 + 6*(3*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*cosh(1)^2 + 6*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*cosh(1)*sinh(1) + 3*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*sinh(1)^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b^2)*cosh(1)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b^2)*cosh(1)*sinh(1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b^2)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 + 2*a*b)*cosh(1)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 + 2*a*b)*cosh(1)*sinh(1) + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 + 2*a*b)*sinh(1)^2))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(143) = 286$.
time = 0.38, size = 610, normalized size = 4.07

$$\begin{cases} a^{2} c^{2} e^{2} x + a^{2} c d e^{2} x^{2} + \frac{a^{2} d^{2} e^{2} x^{3}}{3} + \frac{2 a b c^{3} e^{2} \operatorname{acosh}{\left(c + d x \right)}}{3 d} + 2 a b c^{2} e^{2} x \operatorname{acosh}{\left(c + d x \right)} - \frac{2 a b c^{2} e^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1}}{9 d} + 2 a b c d e^{2} x^{2} \operatorname{acosh}{\left(c + d x \right)} - \frac{4 a b c e^{2} x \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1}}{9} + \frac{2 a b d^{2} e^{2} x^{3} \operatorname{acosh}{\left(c + d x \right)}}{3} - \frac{2 a b d e^{2} x^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1}}{9} - \frac{4 a b e^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1}}{9 d} + \frac{b^{2} c^{3} e^{2} \operatorname{acosh}^{2}{\left(c + d x \right)}}{3 d} + b^{2} c^{2} e^{2} x \operatorname{acosh}^{2}{\left(c + d x \right)} + \frac{2 b^{2} c^{2} e^{2} x}{9} - \frac{2 b^{2} c^{2} e^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1} \operatorname{acosh}{\left(c + d x \right)}}{9 d} + b^{2} c d e^{2} x^{2} \operatorname{acosh}^{2}{\left(c + d x \right)} + \frac{2 b^{2} c d e^{2} x^{2}}{9} - \frac{4 b^{2} c e^{2} x \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1} \operatorname{acosh}{\left(c + d x \right)}}{9} + \frac{b^{2} d^{2} e^{2} x^{3} \operatorname{acosh}^{2}{\left(c + d x \right)}}{3} + \frac{2 b^{2} d^{2} e^{2} x^{3}}{27} - \frac{2 b^{2} d e^{2} x^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1} \operatorname{acosh}{\left(c + d x \right)}}{9} + \frac{4 b^{2} e^{2} x}{9} - \frac{4 b^{2} e^{2} \sqrt{c^{2} + 2 c d x + d^{2} x^{2} - 1} \operatorname{acosh}{\left(c + d x \right)}}{9 d} & \text{for}\: d \neq 0 \\ c^{2} e^{2} x \left(a + b \operatorname{acosh}{\left(c \right)}\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**2,x)

[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 + 2*a*b*c**3*e**2*acosh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*acosh(c + d*x) - 2*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a*b*c*d*e**2*x**2*acosh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 + 2*a*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + b**2*c**3*e**2*acosh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*acosh(c + d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*acosh(c + d*x)**2 + 2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/9 + b**2*d**2*e**2*x**3*acosh(c + d*x)**2/3 + 2*b**2*d**2*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/9 + 4*b**2*e**2*x/9 - 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**2, True))

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2, x)
```

3.107 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=110

$$\frac{b^2e(c+dx)^2}{4d}-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(\ldots}{}$$

```
[Out] 1/4*b^2*e*(d*x+c)^2/d-1/4*e*(a+b*arccosh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*a
rccosh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5996, 12, 5883, 5939, 5893, 30}

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{b^2e(c+dx)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c +
d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (e*(a + b*ArcCosh[c + d*x])^2)/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2(a+b\cosh^{-1}}{\sqrt{-1+x}\,\sqrt{}}\right.}{d}$$

$$= -\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}$$

$$= \frac{b^2e(c+dx)^2}{4d} - \frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\,\text{co}\right.}{2d}$$

Mathematica [A]

time = 0.14, size = 167, normalized size = 1.52

$$\frac{e\left((c+dx)\left(2a^2(c+dx)+b^2(c+dx)-2ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)-2b(c+dx)\left(-2a(c+dx)+b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+b^2(-1+2c^2+4cdx+2d^2x^2)\cosh^{-1}(c+dx)^2-2ab\log\left(c+dx+\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*((c + d*x)*(2*a^2*(c + d*x) + b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]) - 2*b*(c + d*x)*(-2*a*(c + d*x) + b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-1 + 2*c^2 + 4*c*d*x + 2*d^2*x^2)*
ArcCosh[c + d*x]^2 - 2*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]]))/(4*d)
```

Maple [A]
time = 13.90, size = 182, normalized size = 1.65

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a^2}{2}+e\,b^2\left(\frac{\operatorname{arccosh}(dx+c)^2(dx+c)^2}{2}-\frac{\operatorname{arccosh}(dx+c)(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2}-\frac{\operatorname{arccosh}(dx+c)^2}{4}+\frac{(d\ldots}{}\right.}{}$
default	$\frac{\frac{e(dx+c)^2a^2}{2}+e\,b^2\left(\frac{\operatorname{arccosh}(dx+c)^2(dx+c)^2}{2}-\frac{\operatorname{arccosh}(dx+c)(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2}-\frac{\operatorname{arccosh}(dx+c)^2}{4}+\frac{(d\ldots}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^2+e*b^2*(1/2*arccosh(d*x+c)^2*(d*x+c)^2-1/2*arccosh(
d*x+c)*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*arccosh(d*x+c)^2+1/4*(d*
x+c)^2)+e*a*b*(d*x+c)^2*arccosh(d*x+c)-1/2*e*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*(d*x+c)-1/2*e*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)
*ln(d*x+c+((d*x+c)^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2*e + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*d*e + a^2
*c*x*e + 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c*e/d +
```

```
1/2*(b^2*d*x^2*e + 2*b^2*c*x*e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)^2 - integrate((b^2*d^4*x^4*e + 4*b^2*c*d^3*x^3*e + (5*c^2*d^2 - d^
2)*b^2*x^2*e + 2*(c^3*d - c*d)*b^2*x*e + (b^2*d^3*x^3*e + 3*b^2*c*d^2*x^2*e
 + 2*b^2*c^2*d*x*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x
 + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 +
2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x -
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(102) = 204$.
time = 0.38, size = 367, normalized size = 3.34

$$\frac{\left(\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-b^2\right)\cosh\left(1\right)+\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-b^2\right)\sinh\left(1\right)\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2+\left(\left(2\,a^2+b^2\right)d^2x^2+2\,\left(2\,a^2+b^2\right)cdx\right)\cosh\left(1\right)+2\,\left(\left(2\,abd^2x^2+4\,abcdx+2\,abc^2-ab\right)\cosh\left(1\right)+\left(2\,abd^2x^2+4\,abcdx+2\,abc^2-ab\right)\sinh\left(1\right)-\sqrt{d^2x^2+2\,cdx+c^2-1}\left(\left(b^2dx+b^2c\right)\cosh\left(1\right)+\left(b^2dx+b^2c\right)\sinh\left(1\right)\right)\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\left(\left(2\,a^2+b^2\right)d^2x^2+2\,\left(2\,a^2+b^2\right)cdx\right)\sinh\left(1\right)-2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\left(\left(abdx+abc\right)\cosh\left(1\right)+\left(abdx+abc\right)\sinh\left(1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - b^2)*cosh(1) + (2*b^2*d^2*
x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - b^2)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1))^2 + ((2*a^2 + b^2)*d^2*x^2 + 2*(2*a^2 + b^2)*c*d*x)*cos
h(1) + 2*((2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 - a*b)*cosh(1) + (2*a*b*
d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 - a*b)*sinh(1) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)*((b^2*d*x + b^2*c)*cosh(1) + (b^2*d*x + b^2*c)*sinh(1)))*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + ((2*a^2 + b^2)*d^2*x^2 + 2*(2*a^
2 + b^2)*c*d*x)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((a*b*d*x + a
*b*c)*cosh(1) + (a*b*d*x + a*b*c)*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(97) = 194$.
time = 0.22, size = 335, normalized size = 3.05

$$\begin{cases} a^2cex+\frac{a^2dex^2}{2}+\frac{abc^2e\operatorname{acosh}\left(c+dx\right)}{d}+2abcex\operatorname{acosh}\left(c+dx\right)-\frac{abce\sqrt{c^2+2cdx+d^2x^2-1}}{2d}+abdex^2\operatorname{acosh}\left(c+dx\right)-\frac{abex\sqrt{c^2+2cdx+d^2x^2-1}}{2}-\frac{abe\operatorname{acosh}\left(c+dx\right)}{2d}+\frac{b^2c^2e\operatorname{acosh}^2\left(c+dx\right)}{2d}+b^2cex\operatorname{acosh}^2\left(c+dx\right)+\frac{b^2cex}{2}-\frac{b^2ce\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}\left(c+dx\right)}{2d}+\frac{b^2dex^2\operatorname{acosh}^2\left(c+dx\right)}{2}+\frac{b^2dex^2}{4}-\frac{b^2ex\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}\left(c+dx\right)}{2}-\frac{b^2e\operatorname{acosh}^2\left(c+dx\right)}{4d} & \text{for}\: d \neq 0 \\ cex\left(a+b\operatorname{acosh}\left(c\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*acosh(c + d*x)/d + 2*a
*b*c*e*x*acosh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d
) + a*b*d*e*x**2*acosh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/2 - a*b*e*acosh(c + d*x)/(2*d) + b**2*c**2*e*acosh(c + d*x)**2/(2*d) +
b**2*c*e*x*acosh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(2*d) + b**2*d*e*x**2*acosh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/2 - b**2*e*acosh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))
**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2, x)
```

3.108 $\int \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$2b^2x-\frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}$$

```
[Out] 2*b^2*x+(d*x+c)*(a+b*arccosh(d*x+c))^2/d-2*b*(a+b*arccosh(d*x+c))*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5995, 5879, 5915, 8}

$$-\frac{2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]
))/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, c\right.}{d}$$

$$= -\frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\,c\right.}{}$$

$$= 2b^2x - \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(c\right.}{}$$

Mathematica [A]
time = 0.06, size = 105, normalized size = 1.64

$$\frac{a^2(c+dx)+2b^2(c+dx)-2ab\sqrt{-1+c+dx}\sqrt{1+c+dx}-2b\left(-a(c+dx)+b\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+b^2(c+dx)\cosh^{-1}(c+dx)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x] - 2*b*(-(a*(c + d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[
c + d*x] + b^2*(c + d*x)*ArcCosh[c + d*x]^2)/d
```

Maple [A]
time = 16.32, size = 100, normalized size = 1.56

method	result
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\text{arccosh}(dx+c)^2(dx+c)-2\,\text{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2dx+2c\right)+2ab\left((dx+c)a\right.}{d}$
default	$\frac{(dx+c)a^2+b^2\left(\text{arccosh}(dx+c)^2(dx+c)-2\,\text{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2dx+2c\right)+2ab\left((dx+c)a\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2+b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(2*(d^3*
x^3 + 2*c*d^2*x^2 + (d^2*x^2 + c*d*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 (c^2*d - d)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3
 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + (3*c^2*d - d)*x - c), x))*b^2 + a^2*x + 2*((d*x + c)*arccos
h(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.
time = 0.40, size = 141, normalized size = 2.20

$$\frac{(a^2+2\,b^2)dx+(b^2dx+b^2c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2-2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\,ab+2\left(abdx+abc-\sqrt{d^2x^2+2\,cdx+c^2-1}\,b^2\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(60) = 120$.
time = 0.12, size = 143, normalized size = 2.23

$$\begin{cases} a^2x+\frac{2abc\operatorname{acosh}(c+dx)}{d}+2abx\operatorname{acosh}(c+dx)-\frac{2ab\sqrt{c^2+2cdx+d^2x^2-1}}{d}+\frac{b^2c\operatorname{acosh}^2(c+dx)}{d}+b^2x\operatorname{acosh}^2(c+dx)+2b^2x-\frac{2b^2\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}(c+dx)}{d} & \text{for}\ d\neq 0\\ x(a+b\operatorname{acosh}(c))^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*acosh(c + d*x)/d + 2*a*b*x*acosh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + b**2*c*acosh(c + d*x)**2/d + b*
*2*x*acosh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b\,\mathrm{acosh}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2, x)
```

3.109 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=118

$$\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)}{de}-\frac{b\left(a+b\cosh^{-1}(c+dx)\right)\text{PolyL}}{de}$$

```
[Out] 1/3*(a+b*arccosh(d*x+c))^3/b/d/e+(a+b*arccosh(d*x+c))^2*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-b*(a+b*arccosh(d*x+c))*polylog(2,-1/(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-1/2*b^2*polylog(3,-1/(d*x+c+(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5882, 3799, 2221, 2611, 2320, 6724}

$$-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}-\frac{b^2\text{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcCosh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcCosh[c + d*x])^2*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (b*(a + b*ArcCosh[c + d*x])*PolyLog[2, -E
^(-2*ArcCosh[c + d*x])])/(d*e) - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])])
/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{ex}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x}\,dx,x,c+dx\right)}{de}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de}$$

Mathematica [A]
time = 0.28, size = 140, normalized size = 1.19

$$\frac{ab\cosh^{-1}(c+dx)^2+\frac{1}{3}b^2\cosh^{-1}(c+dx)^3+2ab\cosh^{-1}(c+dx)\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+b^2\cosh^{-1}(c+dx)^2\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+a^2\log(c+dx)-b\left(a+b\cosh^{-1}(c+dx)\right)\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)-\frac{1}{2}b^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a*b*ArcCosh[c + d*x]^2 + (b^2*ArcCosh[c + d*x]^3)/3 + 2*a*b*ArcCosh[c + d*
x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*
ArcCosh[c + d*x])] + a^2*Log[c + d*x] - b*(a + b*ArcCosh[c + d*x])*PolyLog[
2, -E^(-2*ArcCosh[c + d*x])] - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])])/2
)/(d*e)
```

Maple [A]
time = 12.26, size = 243, normalized size = 2.06

method	result

derivativedivides	$\frac{\frac{a^2 \ln(dx+c)}{e} - \frac{b^2 \operatorname{arccosh}(dx+c)^3}{3e} + \frac{b^2 \operatorname{arccosh}(dx+c)^2 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e} + \frac{b^2 \operatorname{arccosh}(dx+c}{}$
default	$\frac{a^2 \ln(dx+c)}{e} - \frac{b^2 \operatorname{arccosh}(dx+c)^3}{3e} + \frac{b^2 \operatorname{arccosh}(dx+c)^2 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e} + \frac{b^2 \operatorname{arccosh}(dx+c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)-1/3*b^2/e*arccosh(d*x+c)^3+b^2/e*arccosh(d*x+c)^2*ln(1
+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+b^2/e*arccosh(d*x+c)*polylog(2,
-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2*b^2/e*polylog(3,-(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-a*b/e*arccosh(d*x+c)^2+2*a*b/e*arccosh(d*x
+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+a*b/e*polylog(2,-(d*x+c
+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(d*x*e + c*e)/d + integrate(b^2*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)^2/(d*x*e + c*e) + 2*a*b*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)/(d*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*e^(-1)/(d*
x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*acosh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}(c + d\,x))^2}{c\,e + d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x), x)
```

3.110 $\int \frac{(a+b\cosh^{-1}(c+dx))^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(a+b\cosh^{-1}(c+dx))^2}{de^2(c+dx)}+\frac{4b(a+b\cosh^{-1}(c+dx))\,\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{2ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}$$

```
[Out] -(a+b*arccosh(d*x+c))^2/d/e^2/(d*x+c)+4*b*(a+b*arccosh(d*x+c))*arctan(d*x+c
+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-2*I*b^2*polylog(2,-I*(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+2*I*b^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)))/d/e^2
```

Rubi [A]
time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5883, 5947, 4265, 2317, 2438}

$$\frac{4b\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^2}-\frac{(a+b\cosh^{-1}(c+dx))^2}{de^2(c+dx)}-\frac{2ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{2ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^2/(d*e^2*(c + d*x))) + (4*b*(a + b*ArcCosh[c + d
*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((2*I)*b^2*PolyLog[2, (-I)*E^Arc
Cosh[c + d*x]])/(d*e^2) + ((2*I)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e
^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^2x^2}\,dx,x,c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^2}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx,x,c+dx\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx,x,\cosh^{-1}(c+\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2}
\end{aligned}
$$

Mathematica [A]
time = 0.54, size = 161, normalized size = 1.46

$$
\frac{-\frac{a^2}{c+dx} + 2ab\left(-\frac{\cosh^{-1}(c+dx)}{c+dx} + 2\text{ArcTan}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right)\right) - ib^2\left(\cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx} + 2\log\left(1-ie^{-\cosh^{-1}(c+dx)}\right) - 2\log\left(1+ie^{-\cosh^{-1}(c+dx)}\right)\right) + 2\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right) - 2\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + 2*a*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[Ar
cCosh[c + d*x]/2]]) - I*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c + d*x])/(c +
 d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c + d*x]])
+ 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCosh[c + d*x]
]))/(d*e^2)
```

Maple [A]
time = 19.84, size = 270, normalized size = 2.45

method	result

derivativedivides	$$\frac{-\frac{a^2}{e^2(dx+c)}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{e^2(dx+c)}-\frac{2ib^2\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)}{e^2}+\frac{2ib^2\operatorname{arccosh}(dx}{}}{}$$
default	$$\frac{-\frac{a^2}{e^2(dx+c)}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{e^2(dx+c)}-\frac{2ib^2\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)}{e^2}+\frac{2ib^2\operatorname{arccosh}(dx}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)-b^2/e^2*arccosh(d*x+c)^2/(d*x+c)-2*I*b^2/e^2*arccosh(
d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I*b^2/e^2*arccosh(
d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2*I*b^2/e^2*dilog(1+
I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I*b^2/e^2*dilog(1-I*(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2*a*b/e^2/(d*x+c)*arccosh(d*x+c)-2*a*b/e^2*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arcsin(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arccosh(d*x + c)/
(d^2*x*e^2 + c*d*e^2)) - b^2*(log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
 + c)^2/(d^2*x*e^2 + c*d*e^2) - integrate(2*(d^2*x^2 + 2*c*d*x + sqrt(d*x +
 c + 1)*(d*x + c)*sqrt(d*x + c - 1) + c^2 - 1)*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)/(d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*x
^2*e^2 + 2*(2*c^3*d - c*d)*x*e^2 + (d^3*x^3*e^2 + 3*c*d^2*x^2*e^2 + (3*c^2*
d - d)*x*e^2 + (c^3 - c)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^4 -
c^2)*e^2), x)) - a^2/(d^2*x*e^2 + c*d*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*e^(-2)/(d^
2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*acosh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*acosh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.111 $\int \frac{(a+b\cosh^{-1}(c+dx))^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(a+b\cosh^{-1}(c+dx))}{de^3(c+dx)}-\frac{(a+b\cosh^{-1}(c+dx))^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -1/2*(a+b*arccosh(d*x+c))^2/d/e^3/(d*x+c)^2-b^2*ln(d*x+c)/d/e^3+b*(a+b*arcc
osh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5996, 12, 5883, 5918, 29}

$$\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(a+b\cosh^{-1}(c+dx))}{de^3(c+dx)}-\frac{(a+b\cosh^{-1}(c+dx))^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(d*e^3*(c
 + d*x)) - (a + b*ArcCosh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) - (b^2*Log[c +
d*x])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
```

```
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{de^3}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+a\right.}{2de^3(c+dx)^2}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+a\right.}{2de^3(c+dx)^2}$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 0.88

$$\frac{-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2(c+dx)^2}+b\left(\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{c+dx}-b\log(c+dx)\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (-1/2*(a + b*ArcCosh[c + d*x])^2/(c + d*x)^2 + b*((Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(c + d*x) - b*Log[c + d*x]))/(d*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(86) = 172$.
time = 16.99, size = 176, normalized size = 1.91

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\operatorname{arccosh}(dx+c)}{e^3}+\frac{b^2\operatorname{arccosh}(dx+c)\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{e^3(dx+c)}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{2e^3(dx+c)^2}-\frac{b^2\ln\left(1+\left(dx\right.\right.}{}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}+\frac{b^2\operatorname{arccosh}(dx+c)}{e^3}+\frac{b^2\operatorname{arccosh}(dx+c)\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{e^3(dx+c)}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{2e^3(dx+c)^2}-\frac{b^2\ln\left(1+\left(dx\right.\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2+b^2/e^3*arccosh(d*x+c)+b^2/e^3*arccosh(d*x+c)/(
d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)-1/2*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)
^2-b^2/e^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+2*a*b/e^3*(-1/2/
(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(83) = 166$.
time = 0.52, size = 215, normalized size = 2.34

$$\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d\,\operatorname{arcosh}\left(dx+c\right)}{d^3xe^3+cd^2e^3}-\frac{e^{(-3)}\log\left(dx+c\right)}{d}\right)b^2+ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d}{d^3xe^3+cd^2e^3}-\frac{\operatorname{arcosh}\left(dx+c\right)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)-\frac{b^2\operatorname{arcosh}\left(dx+c\right)^2}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}-\frac{a^2}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*x*e^3 + c*d^2*e^
3) - e^(-3)*log(d*x + c)/d)*b^2 + a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/
(d^3*x*e^3 + c*d^2*e^3) - arccosh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c
^2*d*e^3)) - 1/2*b^2*arccosh(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*
d*e^3) - 1/2*a^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(83) = 166$.
time = 0.41, size = 413, normalized size = 4.49

$$\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4-b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2-a^2c^2+2\left(abd^2x^2+2\,abcdx+\left(b^2c^2dx+b^2c^3\right)\sqrt{d^2x^2+2\,cdx+c^2-1}\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-2\left(b^2c^2d^2x^2+2\,b^2c^3dx+b^2c^4\right)\log\left(dx+c\right)+2\left(abd^2x^2+2\,abcdx+abc^2\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+2\left(abc^2dx+abc^3\right)\sqrt{d^2x^2+2\,cdx+c^2-1}}{2\left(\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^3+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^2\sinh\left(1\right)+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)\sinh\left(1\right)^2+\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\sinh\left(1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 - b^2*c^2*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - a^2*c^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d
*x + (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
+ b^2*c^4)*log(d*x + c) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
- c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1))/((c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^3
 + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^2*sinh(1) + 3*(c^2*d^3*x^2
 + 2*c^3*d^2*x + c^4*d)*cosh(1)*sinh(1)^2 + (c^2*d^3*x^2 + 2*c^3*d^2*x + c^
4*d)*sinh(1)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ab\operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

3.112 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=186

$$\frac{b^2}{3de^4(c+dx)}+\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{2b\left(a+b\cosh\right.}{}$$

```
[Out] 1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+b
*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-1/3*I*
b^2*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/3*I*b^2*p
olylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/3*b*(a+b*arccos
h(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.25, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5883, 5933, 5947, 4265, 2317, 2438, 30}

$$\frac{2b\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}-\frac{ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{b^2}{3de^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] b^2/(3*d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcCosh[c + d*x])^2/(3*d*e^
4*(c + d*x)^3) + (2*b*(a + b*ArcCosh[c + d*x])*ArcTan[E^ArcCosh[c + d*x]])/
(3*d*e^4) - ((I/3)*b^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^4) + ((I/3
)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
 Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx, x, c+ \right.}{3de^4}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+\right.}{3de^4(c+dx)}$$

$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{(}{}$$

$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{(}{}$$

$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{(}{}$$

$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{(}{}$$

Mathematica [A]
time = 0.69, size = 251, normalized size = 1.35

$$\frac{-\frac{a^2}{(c+dx)^3}+ab\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx)}{(c+dx)^2}-\frac{2\cosh^{-1}(c+dx)}{(c+dx)^3}+2\text{ArcTan}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right)\right)+b^2\left(\frac{1}{c+dx}+\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx)\cosh^{-1}(c+dx)}{(c+dx)^2}-\frac{\cosh^{-1}(c+dx)^2}{(c+dx)^3}-i\cosh^{-1}(c+dx)\log\left(1-ie^{-\cosh^{-1}(c+dx)}\right)+i\cosh^{-1}(c+dx)\log\left(1+ie^{-\cosh^{-1}(c+dx)}\right)-i\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)+i\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)\right)}{3de^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] (-(a^2/(c + d*x)^3) + a*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x
))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCosh[c
 + d*x]/2]]) + b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c + d*x)^3
- I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c + d*x]*Log
```

[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]))/(3*d*e^4)

Maple [A]
time = 16.74, size = 352, normalized size = 1.89

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\operatorname{arccosh}(dx+c)}{3e^4(dx+c)^2}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{3e^4(dx+c)^3}+\frac{b^2}{3e^4(dx+c)}-\frac{ib^2\operatorname{arccosh}(dx+c)\ln(1}{}$
default	$-\frac{a^2}{3e^4(dx+c)^3}+\frac{b^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\operatorname{arccosh}(dx+c)}{3e^4(dx+c)^2}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{3e^4(dx+c)^3}+\frac{b^2}{3e^4(dx+c)}-\frac{ib^2\operatorname{arccosh}(dx+c)\ln(1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3+1/3*b^2/e^4/(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*arccosh(d*x+c)-1/3*b^2/e^4/(d*x+c)^3*arccosh(d*x+c)^2+1/3*b^2/e^4/(d
*x+c)-1/3*I*b^2/e^4*arccosh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)))+1/3*I*b^2/e^4*arccosh(d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)))-1/3*I*b^2/e^4*dilog(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+
1/3*I*b^2/e^4*dilog(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2/3*a*b/e^
4/(d*x+c)^3*arccosh(d*x+c)-1/3*a*b/e^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*
x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))+1/3*a*b/e^4*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/(d*x+c)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^4*x^3*e^4
+ 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^2/(d^4*x^3*e^4 + 3
*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 +
 b^2*d^3)*x^3 + 3*(c^3 - c)*a*b + (c^3 - c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^
2)*x^2 + (b^2*c^2 + 3*(c^2 - 1)*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*
```

```
c*d + b^2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*d - d)*a*
b + (3*c^2*d - d)*b^2)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*x^5*e^4 + 5*(7*c^3*d^
4 - c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 - 10*c
^3*d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^
4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^3)*x^3*e^4 + 3*(5*c^4*d
^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^4 + (c^6 - c^4)*e^4)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)*e^4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*e^(-4)/(d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{2ab\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*acosh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4,x)
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.113 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=382

$$\frac{16}{25}ab^2e^4x-\frac{4144b^3e^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5625d}-\frac{272b^3e^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5625d}-\frac{6b^3e^4\sqrt{}}{}$$

```
[Out] 16/25*a*b^2*e^4*x+16/25*b^3*e^4*(d*x+c)*arccosh(d*x+c)/d+8/75*b^2*e^4*(d*x+
c)^3*(a+b*arccosh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))/d+
1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^3/d-4144/5625*b^3*e^4*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/d-272/5625*b^3*e^4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/d-6/625*b^3*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-8/25*b*e^4*
(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-4/25*b*e^4*(d*x+c)
^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/25*b*e^4*(d*x
+c)^4*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.48, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5883, 5939, 5915, 5879, 75, 102}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d}+\frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^3}{5d}-\frac{3be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{25d}-\frac{4be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{25d}-\frac{8be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{25d}-\frac{6b^3e^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4}{625d}-\frac{272b^3e^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{5625d}-\frac{4144b^3e^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{5625d}+\frac{16b^3e^4(c+dx)\cosh^{-1}(c+dx)}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (16*a*b^2*e^4*x)/25 - (4144*b^3*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(
5625*d) - (272*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5
625*d) - (6*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(625*
d) + (16*b^3*e^4*(c + d*x)*ArcCosh[c + d*x])/(25*d) + (8*b^2*e^4*(c + d*x)^
3*(a + b*ArcCosh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*ArcCosh[
c + d*x]))/(125*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/(25*d) - (4*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) - (3*b*e^4*Sqrt[-1 + c + d*x
]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) + (e^4*(
c + d*x)^5*(a + b*ArcCosh[c + d*x])^3)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
```

```
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^3}{5d} - \frac{(3be^4)\,\text{Subst}\left(\int \frac{x^5(a+b}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{3be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{25d}$$

$$= \frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d} - \frac{4be^4\sqrt{-1+c+dx}\,(c}{}$$

$$= -\frac{6b^3e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{625d} + \frac{8b^2e^4(c+dx)^3}{}$$

$$= \frac{16}{25}ab^2e^4x - \frac{8b^3e^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{225d} - \frac{6b^3}{}$$

$$= \frac{16}{25}ab^2e^4x - \frac{272b^3e^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5625d} - \frac{6}{}$$

$$= \frac{16}{25}ab^2e^4x - \frac{32b^3e^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{45d} - \frac{272b^3e^4\sqrt{-}}{}$$

$$= \frac{16}{25}ab^2e^4x - \frac{4144b^3e^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5625d} - \frac{272b^3e^4\sqrt{}}{}$$

Mathematica [A]
time = 0.37, size = 404, normalized size = 1.06

$$\frac{e^4\left(240ab^2(c+dx)+40ab^2(c+dx)^3+3a(25a^2+6b^2)(c+dx)^5+\frac{1}{15}b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(-8(225a^2+518b^2)-4(225a^2+68b^2)(c+dx)^2-27(25a^2+2b^2)(c+dx)^4\right)-b\left(-240b^2(c+dx)-40b^2(c+dx)^3-225a^2(c+dx)^5-18b^2(c+dx)^5+240ab\sqrt{-1+c+dx}\sqrt{1+c+dx}+120ab\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}+90ab\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)-15b^2\left(-15a(c+dx)^5+8b\sqrt{-1+c+dx}\sqrt{1+c+dx}+4b\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+dx}+3b\sqrt{-1+c+dx}(c+dx)^4\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+75b^3(c+dx)^5\cosh^{-1}(c+dx)^3\right)}{375d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) + 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8*(225*a^2 + 518*b^2)
- 4*(225*a^2 + 68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 -
 b*(-240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*
(c + d*x)^5 + 240*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 120*a*b*Sqrt[-
1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] + 90*a*b*Sqrt[-1 + c + d*x]*(c +
 d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 15*b^2*(-15*a*(c + d*x)^5 + 8
*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^
2*Sqrt[1 + c + d*x] + 3*b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])
*ArcCosh[c + d*x]^2 + 75*b^3*(c + d*x)^5*ArcCosh[c + d*x]^3))/(375*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(336) = 672$.
time = 31.69, size = 1293, normalized size = 3.38

method	result	size
default	Expression too large to display	1293

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^4*(d*x+c)^5*a^3/d+1/5625*e^4*b^3*(-4144*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)-2700*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^3*c*d^3-272*(d*x
+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2+5625*arccosh(d*x+c)^3*x^4*c*d^4+11250*a
rccosh(d*x+c)^3*x^3*c^2*d^3+11250*arccosh(d*x+c)^3*x^2*c^3*d^2+1350*arccosh
(d*x+c)*x^4*c*d^4+5625*arccosh(d*x+c)^3*x*c^4*d+2700*arccosh(d*x+c)*x^3*c^2
*d^3+2700*arccosh(d*x+c)*x^2*c^3*d^2+1350*arccosh(d*x+c)*x*c^4*d+1800*arcco
sh(d*x+c)*x^2*c*d^2+1800*arccosh(d*x+c)*x*c^2*d-675*arccosh(d*x+c)^2*(d*x+c
+1)^(1/2)*(d*x+c-1)^(1/2)*c^4-900*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1
)^(1/2)*c^2-216*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^3*c*d^3-324*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)*x^2*c^2*d^2-216*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c^3*d-
544*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d-675*arccosh(d*x+c)^2*(d*x+c+1)^(1
/2)*(d*x+c-1)^(1/2)*x^4*d^4-900*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^
(1/2)*x^2*d^2-4050*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*c^2
*d^2-2700*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c^3*d-1800*arc
cosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d-54*(d*x+c+1)^(1/2)*(d*x
+c-1)^(1/2)*c^4-54*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^4*d^4+1125*arccosh(d*x
```

```
+c)^3*c^5+270*arccosh(d*x+c)*c^5+600*arccosh(d*x+c)*c^3+3600*arccosh(d*x+c)
*c-272*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+1125*arccosh(d*x+c)^3*x^5*d^5+27
0*arccosh(d*x+c)*x^5*d^5+600*arccosh(d*x+c)*x^3*d^3+3600*arccosh(d*x+c)*x*d
-1800*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2))/d+1/375*e^4*a*b^2*(
225*arccosh(d*x+c)^2*c^5+18*x^5*d^5+240*c-90*arccosh(d*x+c)*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*c^4-120*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2
+1125*arccosh(d*x+c)^2*x^4*c*d^4+2250*arccosh(d*x+c)^2*x^3*c^2*d^3+2250*arc
cosh(d*x+c)^2*x^2*c^3*d^2+1125*arccosh(d*x+c)^2*x*c^4*d+40*c^3+40*d^3*x^3+1
8*c^5+240*d*x-90*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^4*d^4-120
*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2-540*arccosh(d*x+c)*
(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*c^2*d^2-360*arccosh(d*x+c)*(d*x+c+1)^(1
/2)*(d*x+c-1)^(1/2)*x*c^3*d-240*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1
/2)*x*c*d-360*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^3*c*d^3+120*
c^2*d*x+90*x*c^4*d+180*x^3*c^2*d^3+90*x^4*c*d^4+180*x^2*c^3*d^2+120*x^2*c*d
^2+225*arccosh(d*x+c)^2*x^5*d^5-240*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2))/d+3*e^4*a^2*b/d*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*d^4*x^5*e^4 + a^3*c*d^3*x^4*e^4 + 2*a^3*c^2*d^2*x^3*e^4 + 2*a^3*c^3
*d*x^2*e^4 + 3*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^3*d*e^4 + (6
*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15
*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^
2*b*c^2*d^2*e^4 + 1/8*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c
^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d
^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^2*b*c
*d^3*e^4 + 1/200*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c
```

^2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*c^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 +
 1050*(c^2 - 1)*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*d)/d^6 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*(c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c
^2 - 1)^2/d^6)*d)*a^2*b*d^4*e^4 + a^3*c^4*x*e^4 + 3*((d*x + c)*arccosh(d*x
+ c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^4*e^4/d + 1/5*(b^3*d^4*x^5*e^4 + 5*b^
3*c*d^3*x^4*e^4 + 10*b^3*c^2*d^2*x^3*e^4 + 10*b^3*c^3*d*x^2*e^4 + 5*b^3*c^4
*x*e^4)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + integrate(3/
5*((5*a*b^2*d^7 - b^3*d^7)*x^7*e^4 + 7*(5*a*b^2*c*d^6 - b^3*c*d^6)*x^6*e^4
+ (5*(21*c^2*d^5 - d^5)*a*b^2 - (21*c^2*d^5 - d^5)*b^3)*x^5*e^4 + 5*(5*(7*c
^3*d^4 - c*d^4)*a*b^2 - (7*c^3*d^4 - c*d^4)*b^3)*x^4*e^4 + 5*(c^7 - c^5)*a*
b^2*e^4 + 5*(5*(7*c^4*d^3 - 2*c^2*d^3)*a*b^2 - (7*c^4*d^3 - 2*c^2*d^3)*b^3)
*x^3*e^4 + 5*((21*c^5*d^2 - 10*c^3*d^2)*a*b^2 - 2*(2*c^5*d^2 - c^3*d^2)*b^3
)*x^2*e^4 + 5*((7*c^6*d - 5*c^4*d)*a*b^2 - (c^6*d - c^4*d)*b^3)*x*e^4 + ((5
*a*b^2*d^6 - b^3*d^6)*x^6*e^4 + 6*(5*a*b^2*c*d^5 - b^3*c*d^5)*x^5*e^4 - 5*(
3*b^3*c^2*d^4 - (15*c^2*d^4 - d^4)*a*b^2)*x^4*e^4 + 5*(c^6 - c^4)*a*b^2*e^4
 - 20*(b^3*c^3*d^3 - (5*c^3*d^3 - c*d^3)*a*b^2)*x^3*e^4 - 15*(b^3*c^4*d^2 -
 (5*c^4*d^2 - 2*c^2*d^2)*a*b^2)*x^2*e^4 - 5*(b^3*c^5*d - 2*(3*c^5*d - 2*c^3
*d)*a*b^2)*x*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 +
2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x -
c), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4401 vs. $2(325) = 650$.
time = 0.43, size = 4401, normalized size = 11.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")

[Out] 1/5625*(15*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4
+ 10*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 + 30*(4*a*b^2*c + (25*a^3
 + 6*a*b^2)*c^3)*d^2*x^2 + 15*(8*a*b^2*c^2 + (25*a^3 + 6*a*b^2)*c^4 + 16*a*
b^2)*d*x)*cosh(1)^4 + 60*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b
^2)*c*d^4*x^4 + 10*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 + 30*(4*a*b
^2*c + (25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 + 15*(8*a*b^2*c^2 + (25*a^3 + 6*a*b^
2)*c^4 + 16*a*b^2)*d*x)*cosh(1)^3*sinh(1) + 90*(3*(25*a^3 + 6*a*b^2)*d^5*x^
5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4 + 10*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^

```
2)*d^3*x^3 + 30*(4*a*b^2*c + (25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 + 15*(8*a*b^2*
c^2 + (25*a^3 + 6*a*b^2)*c^4 + 16*a*b^2)*d*x)*cosh(1)^2*sinh(1)^2 + 60*(3*(
25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4 + 10*(4*a*b^2 +
 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 + 30*(4*a*b^2*c + (25*a^3 + 6*a*b^2)*c^3
)*d^2*x^2 + 15*(8*a*b^2*c^2 + (25*a^3 + 6*a*b^2)*c^4 + 16*a*b^2)*d*x)*cosh(
1)*sinh(1)^3 + 15*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d
^4*x^4 + 10*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 + 30*(4*a*b^2*c +
(25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 + 15*(8*a*b^2*c^2 + (25*a^3 + 6*a*b^2)*c^4
+ 16*a*b^2)*d*x)*sinh(1)^4 + 1125*((b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*
c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh(1)^4 + 4*(
b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5
*b^3*c^4*d*x + b^3*c^5)*cosh(1)^3*sinh(1) + 6*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^
4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh
(1)^2*sinh(1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 1
0*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh(1)*sinh(1)^3 + (b^3*d^5*x
^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*
d*x + b^3*c^5)*sinh(1)^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^
3 + 225*(15*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*
a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*cosh(1)^4 + 60*(a*b^2*d^5*
x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a
*b^2*c^4*d*x + a*b^2*c^5)*cosh(1)^3*sinh(1) + 90*(a*b^2*d^5*x^5 + 5*a*b^2*c
*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x +
a*b^2*c^5)*cosh(1)^2*sinh(1)^2 + 60*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10
*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*co
sh(1)*sinh(1)^3 + 15*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*
x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*sinh(1)^4 - ((3*b
^3*d^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 + 4*b^3*c^2 + 2*(9*b^3*c^2 + 2*b^
3)*d^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 + 2*b^3*c)*d*x)*cosh(1)^4 + 4*(3*b^3*d^4*
x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 + 4*b^3*c^2 + 2*(9*b^3*c^2 + 2*b^3)*d^2*
x^2 + 8*b^3 + 4*(3*b^3*c^3 + 2*b^3*c)*d*x)*cosh(1)^3*sinh(1) + 6*(3*b^3*d^4
*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 + 4*b^3*c^2 + 2*(9*b^3*c^2 + 2*b^3)*d^2
*x^2 + 8*b^3 + 4*(3*b^3*c^3 + 2*b^3*c)*d*x)*cosh(1)^2*sinh(1)^2 + 4*(3*b^3*
d^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 + 4*b^3*c^2 + 2*(9*b^3*c^2 + 2*b^3)*
d^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 + 2*b^3*c)*d*x)*cosh(1)*sinh(1)^3 + (3*b^3*d
^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 + 4*b^3*c^2 + 2*(9*b^3*c^2 + 2*b^3)*d
^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 + 2*b^3*c)*d*x)*sinh(1)^4)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 15*((
9*(25*a^2*b + 2*b^3)*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 + 10*(4*b^3
+ 9*(25*a^2*b + 2*b^3)*c^2)*d^3*x^3 + 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5
 + 30*(4*b^3*c + 3*(25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c + 15*(8*b^3*
c^2 + 3*(25*a^2*b + 2*b^3)*c^4 + 16*b^3)*d*x)*cosh(1)^4 + 4*(9*(25*a^2*b +
2*b^3)*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 + 10*(4*b^3 + 9*(25*a^2*b
+ 2*b^3)*c^2)*d^3*x^3 + 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 30*(4*b^3*c
 + 3*(25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c + 15*(8*b^3*c^2 + 3*(25*a^
2*b + 2*b^3)*c^4 + 16*b^3)*d*x)*cosh(1)^3*sinh(1) + 6*(9*(25*a^2*b + 2*b^3)
```

```
*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^
3)*c^2)*d^3*x^3 + 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 30*(4*b^3*c + 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c + 15*(8*b^3*c^2 + 3*(25*a^2*b +
2*b^3)*c^4 + 16*b^3)*d*x)*cosh(1)^2*sinh(1)^2 + 4*(9*(25*a^2*b + 2*b^3)*d^5
*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^3)*c
^2)*d^3*x^3 + 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 30*(4*b^3*c + 3*(25*a
^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c + 15*(8*b^3*c^2 + 3*(25*a^2*b + 2*b^
3)*c^4 + 16*b^3)*d*x)*cosh(1)*sinh(1)^3 + (9*(25*a^2*b + 2*b^3)*d^5*x^5 + 4
5*(25*a^2*b + 2*b^3)*c*d^4*x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^3)*c^2)*d^3*
x^3 + 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 30*(4*b^3*c + 3*(25*a^2*b + 2
*b^3)*c^3)*d^2*x^2 + 240*b^3*c + 15*(8*b^3*c^2 + 3*(25*a^2*b + 2*b^3)*c^4 +
 16*b^3)*d*x)*sinh(1)^4 - 30*((3*a*b^2*d^4*x^4 + 12*a*b^2*c*d^3*x^3 + 3*a*b
^2*c^4 + 4*a*b^2*c^2 + 2*(9*a*b^2*c^2 + 2*a*b^2)*d^2*x^2 + 8*a*b^2 + 4*(3*a
*b^2*c^3 + 2*a*b^2*c)*d*x)*cosh(1)^4 + 4*(3*a*b...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(371) = 742$.
time = 1.38, size = 2518, normalized size = 6.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*acosh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*acosh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*acosh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*acosh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*acosh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25
- 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 3*a**2*b*d**4
*e**4*x**5*acosh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/25 - 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a*b*
*2*c**5*e**4*acosh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*acosh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*acosh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*acosh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*aco
sh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c**2*
e**4*x/25 - 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
```

```
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c*d*e**4*x**2/25 - 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*acosh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*d*
*2*e**4*x**3/75 - 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + b**3*c**5*e**4*acosh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*acosh(c + d*x)/(125*d) + b**3*c**4*e**4*x*acosh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*acosh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*aco
sh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*acosh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**3*e**4*aco
sh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*acosh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*acosh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**2*e**4*x*acosh(c + d*
x)/25 - 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x
)**2/(25*d) - 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*acosh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*aco
sh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)*acosh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/625 + 8*b**3*c*d*e**4*x**2*acosh(c + d*x)/25 - 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*c*e**4*acosh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*acosh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*acosh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)*acosh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/625 + 8*b**3*d**2*e**4*x**3*acosh(c + d*x)/75 - 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*e**4*x*acos
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{acosh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3, x)
```

3.114 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=307

$$-\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{256d}-\frac{3b^3e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{128d}-\frac{45b^3e^3\cosh^{-1}}{256d}$$

```
[Out] -45/256*b^3*e^3*arccosh(d*x+c)/d+9/32*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c)
)/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))/d-3/32*e^3*(a+b*arccosh(d*x
+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^3/d-45/256*b^3*e^3*(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-9/32*b*e^3*(d*x+c)*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)/d-3/16*b*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.41, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5883, 5939, 5893, 92, 54, 102}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d}-\frac{3be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d}-\frac{9be^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{32d}-\frac{3e^3\left(a+b\cosh^{-1}(c+dx)\right)^3}{32d}-\frac{3b^3e^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3}{128d}-\frac{45b^3e^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{256d}-\frac{45b^3e^3\cosh^{-1}(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (-45*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(256*d) - (3*b
^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(128*d) - (45*b^3*
e^3*ArcCosh[c + d*x])/(256*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d
*x]))/(32*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(32*d) - (9
*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*
x])^2)/(32*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^3)/(32*
d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^3)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4(a+b}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{16d}$$

$$= \frac{3b^2e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)}{32d} - \frac{9be^3\sqrt{-1+c+dx}\,(c}{}$$

$$= -\frac{3b^3e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{128d} + \frac{9b^2e^3(c+dx)^{}}{}$$

$$= -\frac{9b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{64d} - \frac{3b^3e^3\sqrt{-1+c}}{}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+}}{}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+}}{}$$

Mathematica [A]
time = 0.33, size = 359, normalized size = 1.17

$$\frac{e^3\left(72ab^2(c+dx)^2+8a(8a^2+3b^2)(c+dx)^4+3b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,(-3(8a^2+5b^2)-2(8a^2+b^2)(c+dx)^2)-24b(c+dx)\left(-3b^2(c+dx)-8a^2(c+dx)^3-b^2(c+dx)^3+6ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}+4ab\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+24b^2\left(-3a+8a(c+dx)^4-3b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}-2b\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+8b^3(-3+8(c+dx)^4)\cosh^{-1}(c+dx)^3-9b(8a^2+5b^2)\log\left(c+dx+\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\right)}{256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^3*(72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*Sqrt[-1
+ c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2
)*(c + d*x)^2) - 24*b*(c + d*x)*(-3*b^2*(c + d*x) - 8*a^2*(c + d*x)^3 - b^2
*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*a*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 24*b^2*(-3*a +
8*a*(c + d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*
Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 8*b^
3*(-3 + 8*(c + d*x)^4)*ArcCosh[c + d*x]^3 - 9*b*(8*a^2 + 5*b^2)*Log[c + d*x
 + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(256*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(273) = 546$.
time = 41.01, size = 647, normalized size = 2.11

method	result
default	$\frac{e^{3}(dx+c)^{4}a^{3}}{4d} - \frac{b^{3}e^{3}\left(-32(\cosh^{2}(2\,\mathrm{arccosh}(dx+c)))\,\mathrm{arccosh}(dx+c)^{3}+24\sinh(2\,\mathrm{arccosh}(dx+c))\cosh(2\,\mathrm{arccosh}(dx+c))\,\mathrm{arccosh}(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^3/d-1/512*b^3*e^3*(-32*cosh(2*arccosh(d*x+c))^2*arccosh
(d*x+c)^3+24*sinh(2*arccosh(d*x+c))*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^2
-64*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^3+96*arccosh(d*x+c)^2*sinh(2*arcc
osh(d*x+c))-12*arccosh(d*x+c)*cosh(2*arccosh(d*x+c))^2+16*arccosh(d*x+c)^3+
3*sinh(2*arccosh(d*x+c))*cosh(2*arccosh(d*x+c))-96*cosh(2*arccosh(d*x+c))*a
rccosh(d*x+c)+48*sinh(2*arccosh(d*x+c))+6*arccosh(d*x+c))/d+3*a*b^2*e^3/d*(
1/128*(8*arccosh(d*x+c)^2+1)*cosh(2*arccosh(d*x+c))^2+1/32*(4*arccosh(d*x+c
)^2-arccosh(d*x+c)*sinh(2*arccosh(d*x+c))+2)*cosh(2*arccosh(d*x+c))-1/32*ar
ccosh(d*x+c)^2-1/8*arccosh(d*x+c)*sinh(2*arccosh(d*x+c))-1/256)+3/4*a^2*b*e
^3*d^3*arccosh(d*x+c)*x^4+3*a^2*b*e^3*d^2*arccosh(d*x+c)*x^3*c+9/2*a^2*b*e^
3*d*arccosh(d*x+c)*x^2*c^2+3*a^2*b*e^3*arccosh(d*x+c)*x*c^3+3/4*a^2*b*e^3/d
*arccosh(d*x+c)*c^4-3/16*a^2*b*e^3*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-
9/16*a^2*b*e^3*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-9/16*a^2*b*e^3*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/16*a^2*b*e^3/d*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*c^3-9/32*a^2*b*e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-9/32*a^2*b*e^3/
d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-9/32*a^2*b*e^3/d*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*x^4*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + 9/4*(2*x^
2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*l
og(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arccosh(d*x
 + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x
 + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d
^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*c*d^2*e^3 +
1/32*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^
2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c
^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^
2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^2*b*d^3*e^3 + a^3*c^3*
x*e^3 + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^3*e^
3/d + 1/4*(b^3*d^3*x^4*e^3 + 4*b^3*c*d^2*x^3*e^3 + 6*b^3*c^2*d*x^2*e^3 + 4*
b^3*c^3*x*e^3)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + integ
rate(3/4*((4*a*b^2*d^6 - b^3*d^6)*x^6*e^3 + 6*(4*a*b^2*c*d^5 - b^3*c*d^5)*x
^5*e^3 + (4*(15*c^2*d^4 - d^4)*a*b^2 - (15*c^2*d^4 - d^4)*b^3)*x^4*e^3 + 4*
(c^6 - c^4)*a*b^2*e^3 + 4*(4*(5*c^3*d^3 - c*d^3)*a*b^2 - (5*c^3*d^3 - c*d^3
)*b^3)*x^3*e^3 + 2*(6*(5*c^4*d^2 - 2*c^2*d^2)*a*b^2 - (7*c^4*d^2 - 3*c^2*d^
2)*b^3)*x^2*e^3 + 4*(2*(3*c^5*d - 2*c^3*d)*a*b^2 - (c^5*d - c^3*d)*b^3)*x*e
^3 + ((4*a*b^2*d^5 - b^3*d^5)*x^5*e^3 + 5*(4*a*b^2*c*d^4 - b^3*c*d^4)*x^4*e
^3 + 4*(c^5 - c^3)*a*b^2*e^3 - 2*(5*b^3*c^2*d^3 - 2*(10*c^2*d^3 - d^3)*a*b^
2)*x^3*e^3 - 2*(5*b^3*c^3*d^2 - 2*(10*c^3*d^2 - 3*c*d^2)*a*b^2)*x^2*e^3 - 4
*(b^3*c^4*d - (5*c^4*d - 3*c^2*d)*a*b^2)*x*e^3)*sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3
*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2620 vs. $2(264) = 528$.
time = 0.43, size = 2620, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*((8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 + 3*(3*
a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 + 2*(9*a*b^2*c + 2*(8*a^3 + 3*a*b^
2)*c^3)*d*x)*cosh(1)^3 + 8*((8*b^3*d^4*x^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*
d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^3)*cosh(1)^3 + 3*(8*b^3*d^4*x^4
+ 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^
3)*cosh(1)^2*sinh(1) + 3*(8*b^3*d^4*x^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2
*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^3)*cosh(1)*sinh(1)^2 + (8*b^3*d^4*x
^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3
*b^3)*sinh(1)^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 24*((
8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 + 3*(3*a*b^2 + 2*(
8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 + 2*(9*a*b^2*c + 2*(8*a^3 + 3*a*b^2)*c^3)*d*x
)*cosh(1)^2*sinh(1) + 24*((8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c
*d^3*x^3 + 3*(3*a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 + 2*(9*a*b^2*c + 2
*(8*a^3 + 3*a*b^2)*c^3)*d*x)*cosh(1)*sinh(1)^2 + 8*((8*a^3 + 3*a*b^2)*d^4*x
^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 + 3*(3*a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*
d^2*x^2 + 2*(9*a*b^2*c + 2*(8*a^3 + 3*a*b^2)*c^3)*d*x)*sinh(1)^3 + 24*((8*a
*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x
 + 8*a*b^2*c^4 - 3*a*b^2)*cosh(1)^3 + 3*(8*a*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x
^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x + 8*a*b^2*c^4 - 3*a*b^2)*cosh(
1)^2*sinh(1) + 3*(8*a*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x
^2 + 32*a*b^2*c^3*d*x + 8*a*b^2*c^4 - 3*a*b^2)*cosh(1)*sinh(1)^2 + (8*a*b^2
*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x + 8
*a*b^2*c^4 - 3*a*b^2)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b^3*c + 3*(2*b^3*c^2 + b^3)*d*x)
*cosh(1)^3 + 3*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b^3*c + 3*(
2*b^3*c^2 + b^3)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^
2 + 2*b^3*c^3 + 3*b^3*c + 3*(2*b^3*c^2 + b^3)*d*x)*cosh(1)*sinh(1)^2 + (2*b
^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b^3*c + 3*(2*b^3*c^2 + b^3)*d*
x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*((8*(
8*a^2*b + b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 + 24*b^3*c^2 + 8*(8*a
^2*b + b^3)*c^4 + 24*(b^3 + 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*
b^3 + 16*(3*b^3*c + 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^2*b +
 b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 + 24*b^3*c^2 + 8*(8*a^2*b + b^
3)*c^4 + 24*(b^3 + 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 + 16*
(3*b^3*c + 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*(8*(8*a^2*b +
b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 + 24*b^3*c^2 + 8*(8*a^2*b + b^3
)*c^4 + 24*(b^3 + 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 + 16*(
3*b^3*c + 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)*sinh(1)^2 + (8*(8*a^2*b + b^3
)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 + 24*b^3*c^2 + 8*(8*a^2*b + b^3)*c
^4 + 24*(b^3 + 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 + 16*(3*b
^3*c + 2*(8*a^2*b + b^3)*c^3)*d*x)*sinh(1)^3 - 16*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*((2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2*x^2 + 2*a*b^2*c^3 + 3*a*b^2*c +
3*(2*a*b^2*c^2 + a*b^2)*d*x)*cosh(1)^3 + 3*(2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2
*x^2 + 2*a*b^2*c^3 + 3*a*b^2*c + 3*(2*a*b^2*c^2 + a*b^2)*d*x)*cosh(1)^2*sin
h(1) + 3*(2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2*x^2 + 2*a*b^2*c^3 + 3*a*b^2*c + 3
```

```
*(2*a*b^2*c^2 + a*b^2)*d*x)*cosh(1)*sinh(1)^2 + (2*a*b^2*d^3*x^3 + 6*a*b^2*
c*d^2*x^2 + 2*a*b^2*c^3 + 3*a*b^2*c + 3*(2*a*b^2*c^2 + a*b^2)*d*x)*sinh(1)^
3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*((2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 +
 2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b + 5*b^3 + 2*(8*a^2*b + b^3)*c^2)*d*x +
3*(8*a^2*b + 5*b^3)*c)*cosh(1)^3 + 3*(2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*
b + b^3)*c*d^2*x^2 + 2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b + 5*b^3 + 2*(8*a^2*
b + b^3)*c^2)*d*x + 3*(8*a^2*b + 5*b^3)*c)*cosh(1)^2*sinh(1) + 3*(2*(8*a^2*
b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 + 2*(8*a^2*b + b^3)*c^3 + 3*
(8*a^2*b + 5*b^3 + 2*(8*a^2*b + b^3)*c^2)*d*x + 3*(8*a^2*b + 5*b^3)*c)*cosh
(1)*sinh(1)^2 + (2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 +
2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b + 5*b^3 + 2*(8*a^2*b + b^3)*c^2)*d*x + 3
*(8*a^2*b + 5*b^3)*c)*sinh(1)^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1828 vs. $2(294) = 588$.
time = 0.99, size = 1828, normalized size = 5.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*acosh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*acosh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*acosh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*acosh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/16 - 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*acosh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/32 - 9*a**2*b*e**3*acosh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*acosh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*c*e**3*x/16 - 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*acosh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*d
*e**3*x**2/32 - 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
c + d*x)/16 - 9*a*b**2*e**3*acosh(c + d*x)**2/(32*d) + b**3*c**4*e**3*acosh
```

```
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*acosh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*acosh(c + d*x)**3 + 3*b**3*c**3*e**3*x*acosh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*acosh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*acosh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c**2*
e**3*acosh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*acosh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c*e**3*x*acosh(c + d*x)/16 - 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(32*d) - 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(256*d) + b**3*d**3*e**3*
x**4*acosh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*acosh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*d*
e**3*x**2*acosh(c + d*x)/32 - 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**2/32 - 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/256 - 3*b**3*e**3*acosh(c + d*x)**3/(32*d) - 45*b**3*e**3*acosh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3, x)
```

3.115 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=262

$$\frac{4}{3}ab^2e^2x-\frac{40b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{27d}-\frac{2b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{27d}+\frac{4b^3e^2(c+dx)}{}$$

```
[Out] 4/3*a*b^2*e^2*x+4/3*b^3*e^2*(d*x+c)*arccosh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*
(a+b*arccosh(d*x+c))/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^3/d-40/27*b^3
*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/27*b^3*e^2*(d*x+c)^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/d-2/3*b*e^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.31, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5883, 5939, 5915, 5879, 75, 102}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{4}{3}ab^2e^2x-\frac{be^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^3}{3d}-\frac{2b^3e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}}{27d}-\frac{40b^3e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{27d}+\frac{4b^3e^2(c+dx)\cosh^{-1}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (4*a*b^2*e^2*x)/3 - (40*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(27*d
) - (2*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(27*d) + (
4*b^3*e^2*(c + d*x)*ArcCosh[c + d*x])/(3*d) + (2*b^2*e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x])^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^3}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\,c}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3d}$$

$$= \frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d} - \frac{2be^2\sqrt{-1+c+dx}\,\sqrt{}}{}$$

$$= \frac{4}{3}ab^2e^2x - \frac{2b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{27d} + \frac{2b^2e}{}$$

$$= \frac{4}{3}ab^2e^2x - \frac{2b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{27d} + \frac{4b^3e}{}$$

$$= \frac{4}{3}ab^2e^2x - \frac{40b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{27d} - \frac{2b^3e^2\sqrt{-1+}}{}$$

Mathematica [A]
time = 0.24, size = 296, normalized size = 1.13

$$\frac{e^2\left(12ab^2(c+dx)+a(3a^2+2b^2)(c+dx)^3+\frac{1}{3}b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(-2(9a^2+20b^2)-(9a^2+2b^2)(c+dx)^2)-b\left(-12b^2(c+dx)-9a^2(c+dx)^3-2b^2(c+dx)^3+12ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,+6ab\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)-3b^2\left(-3a(c+dx)^3+2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,+b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+3b^3(c+dx)^3\cosh^{-1}(c+dx)^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^2*(12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]*(-2*(9*a^2 + 20*b^2) - (9*a^2 + 2*b^2)*(c + d*x)^2)
)/3 - b*(-12*b^2*(c + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 + 12*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*a*b*Sqrt[-1 + c + d*x]*(c + d*x)^
```

```
2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-3*a*(c + d*x)^3 + 2*b*Sqrt[
-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 +
 c + d*x])*ArcCosh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcCosh[c + d*x]^3))/(9*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(230) = 460$.
time = 37.66, size = 638, normalized size = 2.44

method	result
default	$\frac{e^2(dx+c)^3a^3}{3d} + \frac{b^3e^2\left(9\mathrm{arccosh}(dx+c)^3x^3d^3+27\mathrm{arccosh}(dx+c)^3x^2c\,d^2-9\mathrm{arccosh}(dx+c)^2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,x^2d^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^3/d+1/27*b^3*e^2*(9*arccosh(d*x+c)^3*x^3*d^3+27*arccosh
(d*x+c)^3*x^2*c*d^2-9*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*
d^2+27*arccosh(d*x+c)^3*x*c^2*d-18*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-
1)^(1/2)*x*c*d+6*arccosh(d*x+c)*x^3*d^3+9*arccosh(d*x+c)^3*c^3-9*arccosh(d*
x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+18*arccosh(d*x+c)*x^2*c*d^2-2*(d
*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2+18*arccosh(d*x+c)*x*c^2*d-4*(d*x+c+1)
^(1/2)*(d*x+c-1)^(1/2)*x*c*d-18*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^
(1/2)+6*arccosh(d*x+c)*c^3-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+36*arccosh
(d*x+c)*x*d+36*arccosh(d*x+c)*c-40*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d+1/9*a
*b^2*e^2*(9*arccosh(d*x+c)^2*x^3*d^3+27*arccosh(d*x+c)^2*x^2*c*d^2-6*arccos
h(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2+27*arccosh(d*x+c)^2*x*c^2*
d-12*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d+2*d^3*x^3+9*arcco
sh(d*x+c)^2*c^3-6*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+6*x^2*
c*d^2+6*c^2*d*x-12*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*c^3+12*
d*x+12*c)/d+3*a^2*b*e^2/d*(1/3*(d*x+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*((d*x+c)^2+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*x^3*e^2 + a^3*c*d*x^2*e^2 + 3/2*(2*x^2*arccosh(d*x + c) - d*(3*
c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqr
```

```
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9
*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d
^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*x*e^2 + 3*((d*x + c)*a
rccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*d^2*x^3
*e^2 + 3*b^3*c*d*x^2*e^2 + 3*b^3*c^2*x*e^2)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^3 + integrate(((3*a*b^2*d^5 - b^3*d^5)*x^5*e^2 + 5*(3*a
*b^2*c*d^4 - b^3*c*d^4)*x^4*e^2 + 3*(c^5 - c^3)*a*b^2*e^2 + (3*(10*c^2*d^3
- d^3)*a*b^2 - (10*c^2*d^3 - d^3)*b^3)*x^3*e^2 + 3*((10*c^3*d^2 - 3*c*d^2)*
a*b^2 - (3*c^3*d^2 - c*d^2)*b^3)*x^2*e^2 + 3*((5*c^4*d - 3*c^2*d)*a*b^2 - (
c^4*d - c^2*d)*b^3)*x*e^2 + ((3*a*b^2*d^4 - b^3*d^4)*x^4*e^2 + 3*(c^4 - c^2
)*a*b^2*e^2 + 4*(3*a*b^2*c*d^3 - b^3*c*d^3)*x^3*e^2 - 3*(2*b^3*c^2*d^2 - (6
*c^2*d^2 - d^2)*a*b^2)*x^2*e^2 - 3*(b^3*c^3*d - 2*(2*c^3*d - c*d)*a*b^2)*x*
e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2
 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(222) = 444$.
time = 0.39, size = 1389, normalized size = 5.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cosh(1)^
2 + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cosh(1)*sin
h(1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sinh(1)^2)
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*((3*a^3 + 2*a*b^2)*
d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2 + 3*(4*a*b^2 + (3*a^3 + 2*a*b^2)*c^
2)*d*x)*cosh(1)^2 + 9*(3*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d
*x + a*b^2*c^3)*cosh(1)^2 + 6*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*
c^2*d*x + a*b^2*c^3)*cosh(1)*sinh(1) + 3*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2
 + 3*a*b^2*c^2*d*x + a*b^2*c^3)*sinh(1)^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*((b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 + 2*b^3)*cosh(1)^2 + 2*(b^3*d^2*x^
2 + 2*b^3*c*d*x + b^3*c^2 + 2*b^3)*cosh(1)*sinh(1) + (b^3*d^2*x^2 + 2*b^3*c
*d*x + b^3*c^2 + 2*b^3)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1))^2 + 6*((3*a^3 + 2*a*b^2)*d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2
+ 3*(4*a*b^2 + (3*a^3 + 2*a*b^2)*c^2)*d*x)*cosh(1)*sinh(1) + 3*((3*a^3 + 2*
a*b^2)*d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2 + 3*(4*a*b^2 + (3*a^3 + 2*a*
b^2)*c^2)*d*x)*sinh(1)^2 + 3*(((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2*b
```

```
^3)*c*d^2*x^2 + 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 + 3*(4*b^3 + (9*a^2*b + 2*
b^3)*c^2)*d*x)*cosh(1)^2 + 2*((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2*b^
3)*c*d^2*x^2 + 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 + 3*(4*b^3 + (9*a^2*b + 2*b
^3)*c^2)*d*x)*cosh(1)*sinh(1) + ((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2
*b^3)*c*d^2*x^2 + 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 + 3*(4*b^3 + (9*a^2*b +
2*b^3)*c^2)*d*x)*sinh(1)^2 - 6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((a*b^2*d^
2*x^2 + 2*a*b^2*c*d*x + a*b^2*c^2 + 2*a*b^2)*cosh(1)^2 + 2*(a*b^2*d^2*x^2 +
 2*a*b^2*c*d*x + a*b^2*c^2 + 2*a*b^2)*cosh(1)*sinh(1) + (a*b^2*d^2*x^2 + 2*
a*b^2*c*d*x + a*b^2*c^2 + 2*a*b^2)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(((9*a^2*b + 2*b^3
)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*x + 18*a^2*b + 40*b^3 + (9*a^2*b + 2*b^
3)*c^2)*cosh(1)^2 + 2*((9*a^2*b + 2*b^3)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*
x + 18*a^2*b + 40*b^3 + (9*a^2*b + 2*b^3)*c^2)*cosh(1)*sinh(1) + ((9*a^2*b
+ 2*b^3)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*x + 18*a^2*b + 40*b^3 + (9*a^2*b
 + 2*b^3)*c^2)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(252) = 504$.

time = 0.63, size = 1173, normalized size = 4.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*acosh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*acosh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(3*d) + 3*a**2*b*c*d*e
**2*x**2*acosh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/3 + a**2*b*d**2*e**2*x**3*acosh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/3 - 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(3*d) + a*b**2*c**3*e**2*acosh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*acosh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
acosh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*acosh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + 4*a*b**2*e**2*x/3 - 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + b**3*c**3*e**
2*acosh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*acosh(c + d*x)**3 + 2*b**3*c**2*e**2*x*acosh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**3*c*d*e**2*x*
*2*acosh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 4*b**3*c*e**
```

```
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 4*b**3*c*e**2*acosh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*aco
sh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27
 + 4*b**3*e**2*x*acosh(c + d*x)/3 - 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*acosh(c + d*x)**2/(3*d) - 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**3, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3, x)
```

3.116 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=175

$$-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{8d}-\frac{3b^3e\cosh^{-1}(c+dx)}{8d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-3b$$

```
[Out] -3/8*b^3*e*arccosh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d-1/4*
e*(a+b*arccosh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^3/d-3/8*b^3
*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arccosh(d
*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5996, 12, 5883, 5939, 5893, 92, 54}

$$\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{3be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^3}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d}-\frac{3b^3e\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}}{8d}-\frac{3b^3e\cosh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(8*d) - (3*b^3*e*
ArcCosh[c + d*x])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(4
*d) - (3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x])^2)/(4*d) - (e*(a + b*ArcCosh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*
(a + b*ArcCosh[c + d*x])^3)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\cosh^{-}}{\sqrt{-1+x}\,\sqrt{}}\right.}{2d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$

$$= \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{3be\sqrt{-1+c+dx}\,(c+d}{}$$

$$= -\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{8d} + \frac{3b^2e(c+dx)^2\,(a+}{4}$$

$$= -\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{8d} - \frac{3b^3e\cosh^{-1}(c+dx}{8d}$$

Mathematica [A]
time = 0.19, size = 244, normalized size = 1.39

$$\frac{e\left(2a(2a^2+3b^2)(c+dx)^2-3b(2a^2+b^2)\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}-6b(c+dx)\left(-2a^2(c+dx)-b^2(c+dx)+2ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+6b^2\left(-a+2a(c+dx)^2-b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+2b^3(-1+2(c+dx)^2)\cosh^{-1}(c+dx)^3-3b(2a^2+b^2)\log\left(c+dx+\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*Sqrt[-1 + c + d*x]*
(c + d*x)*Sqrt[1 + c + d*x] - 6*b*(c + d*x)*(-2*a^2*(c + d*x) - b^2*(c + d*
x) + 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 6*b^2*(
-a + 2*a*(c + d*x)^2 - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*Ar
cCosh[c + d*x]^2 + 2*b^3*(-1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^3 - 3*b*(2*a
^2 + b^2)*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(8*d)
```

Maple [A]
time = 37.68, size = 301, normalized size = 1.72

method	result

derivativedivides	$\frac{\frac{e(dx+c)^2a^3}{2}+e\,b^3\left(\frac{\operatorname{arccosh}(dx+c)^3(dx+c)^2}{2}-\frac{3\operatorname{arccosh}(dx+c)^2(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4}-\frac{\operatorname{arccosh}(dx+c)^3}{4}\right.}{}$
default	$\frac{\frac{e(dx+c)^2a^3}{2}+e\,b^3\left(\frac{\operatorname{arccosh}(dx+c)^3(dx+c)^2}{2}-\frac{3\operatorname{arccosh}(dx+c)^2(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4}-\frac{\operatorname{arccosh}(dx+c)^3}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^3+e*b^3*(1/2*arccosh(d*x+c)^3*(d*x+c)^2-3/4*arccosh(
d*x+c)^2*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*arccosh(d*x+c)^3+3/4*(
d*x+c)^2*arccosh(d*x+c)-3/8*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/8*arc
cosh(d*x+c))+3*e*a*b^2*(1/2*arccosh(d*x+c)^2*(d*x+c)^2-1/2*arccosh(d*x+c)*(
d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*arccosh(d*x+c)^2+1/4*(d*x+c)^2)+
3/2*e*a^2*b*(d*x+c)^2*arccosh(d*x+c)-3/4*e*a^2*b*(d*x+c)*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)-3/4*e*a^2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/
2)*ln(d*x+c+((d*x+c)^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2*e + 3/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*d*e + a
^3*c*x*e + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c*e
/d + 1/2*(b^3*d*x^2*e + 2*b^3*c*x*e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)^3 + integrate(3/2*((2*a*b^2*d^4 - b^3*d^4)*x^4*e + 2*(c^4 - c^
2)*a*b^2*e + 4*(2*a*b^2*c*d^3 - b^3*c*d^3)*x^3*e + (2*(6*c^2*d^2 - d^2)*a*b
^2 - (5*c^2*d^2 - d^2)*b^3)*x^2*e + 2*(2*(2*c^3*d - c*d)*a*b^2 - (c^3*d - c
*d)*b^3)*x*e + (2*(c^3 - c)*a*b^2*e + (2*a*b^2*d^3 - b^3*d^3)*x^3*e + 3*(2*
a*b^2*c*d^2 - b^3*c*d^2)*x^2*e - 2*(b^3*c^2*d - (3*c^2*d - d)*a*b^2)*x*e)*s
qrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(161) = 322$.
time = 0.38, size = 642, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*((2*b^3*d^2*x^2 + 4*b^3*c*d*x + 2*b^3*c^2 - b^3)*cosh(1) + (2*b^3*d^
2*x^2 + 4*b^3*c*d*x + 2*b^3*c^2 - b^3)*sinh(1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))^3 + 6*((2*a*b^2*d^2*x^2 + 4*a*b^2*c*d*x + 2*a*b^2*c^2
 - a*b^2)*cosh(1) + (2*a*b^2*d^2*x^2 + 4*a*b^2*c*d*x + 2*a*b^2*c^2 - a*b^2)
*sinh(1) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((b^3*d*x + b^3*c)*cosh(1) + (
b^3*d*x + b^3*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
^2 + 2*((2*a^3 + 3*a*b^2)*d^2*x^2 + 2*(2*a^3 + 3*a*b^2)*c*d*x)*cosh(1) + 3*
((2*(2*a^2*b + b^3)*d^2*x^2 + 4*(2*a^2*b + b^3)*c*d*x - 2*a^2*b - b^3 + 2*(
2*a^2*b + b^3)*c^2)*cosh(1) + (2*(2*a^2*b + b^3)*d^2*x^2 + 4*(2*a^2*b + b^3
)*c*d*x - 2*a^2*b - b^3 + 2*(2*a^2*b + b^3)*c^2)*sinh(1) - 4*sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*((a*b^2*d*x + a*b^2*c)*cosh(1) + (a*b^2*d*x + a*b^2*c)*
sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*((2*a^3 + 3*
a*b^2)*d^2*x^2 + 2*(2*a^3 + 3*a*b^2)*c*d*x)*sinh(1) - 3*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*(((2*a^2*b + b^3)*d*x + (2*a^2*b + b^3)*c)*cosh(1) + ((2*a^2
*b + b^3)*d*x + (2*a^2*b + b^3)*c)*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(165) = 330$.
time = 0.39, size = 685, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*acosh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*acosh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/(4*d) + 3*a**2*b*d*e*x**2*acosh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - 3*a**2*b*e*acosh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*acosh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*acosh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*acosh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/2 - 3*a*
b**2*e*acosh(c + d*x)**2/(4*d) + b**3*c**2*e*acosh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*acosh(c + d*x)/(4*d) + b**3*c*e*x*acosh(c + d*x)**3 + 3*b**3*c*e*
x*acosh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
```

```
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) +
 b**3*d*e*x**2*acosh(c + d*x)**3/2 + 3*b**3*d*e*x**2*acosh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - b**3*e*acosh(c + d*x)**3/(4*d)
- 3*b**3*e*acosh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**3, Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3, x)
```

3.117 $\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=114

$$6ab^2x-\frac{6b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{d}+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(a+b}{d}$$

```
[Out] 6*a*b^2*x+6*b^3*(d*x+c)*arccosh(d*x+c)/d+(d*x+c)*(a+b*arccosh(d*x+c))^3/d-6
*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3*b*(a+b*arccosh(d*x+c))^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]

time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5995, 5879, 5915, 75}

$$6ab^2x-\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{d}+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (6*b^3*(c + d*
x)*ArcCosh[c + d*x])/d - (3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^3)/d
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
```

```
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{(3b)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x\right.}{d}$$

$$=-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+\right.}{}$$

$$=6ab^2x-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx}{}$$

$$=6ab^2x+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+\right.}{d}$$

$$=6ab^2x-\frac{6b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d}+\frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d}-$$

Mathematica [A]
time = 0.12, size = 168, normalized size = 1.47

$$\frac{a(a^2+6b^2)(c+dx)-3b(a^2+2b^2)\sqrt{-1+c+dx}\sqrt{1+c+dx}-3b\left(-a^2(c+dx)-2b^2(c+dx)+2ab\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)-3b^2\left(-a(c+dx)+b\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+b^3(c+dx)\cosh^{-1}(c+dx)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x] - 3*b*(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + b^3*(c + d*x)*ArcCosh[c
+ d*x]^3)/d
```

Maple [A]
time = 49.31, size = 180, normalized size = 1.58

method	result
derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccosh}(dx+c)^3(dx+c)-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c+1}\,\sqrt{dx+c-1}+6(dx+c)\operatorname{arccosh}(dx+c)-\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arccosh}(dx+c)^3(dx+c)-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c+1}\,\sqrt{dx+c-1}+6(dx+c)\operatorname{arccosh}(dx+c)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*(arccosh(d*x+c)^3*(d*x+c)-3*arccosh(d*x+c)^2*(d*x+c+1)
^(1/2)*(d*x+c-1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))+3*a*b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)+2*d*x+2*c)+3*a^2*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + a^3*x + 3*((d*
x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b/d + integrate(3*((c^
3 - c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^
2 + ((c^2 - 1)*a*b^2 + (a*b^2*d^2 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*
x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^2 - (c^2*d - d)
*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3*
c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(106) = 212$.
time = 0.37, size = 239, normalized size = 2.10

$$\frac{(b^3dx+b^3c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^3+(a^3+6\,ab^2)dx+3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,cdx+c^2-1}\,b^3\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2-3\left(2\sqrt{d^2x^2+2\,cdx+c^2-1}\,ab^2-(a^2b+2\,b^3)dx-(a^2b+2\,b^3)c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-3\sqrt{d^2x^2+2\,cdx+c^2-1}\,(a^2b+2\,b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - 3*(2*sqrt(d^2*
```

```
x^2 + 2*c*d*x + c^2 - 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*(a^2*b + 2*b^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(109) = 218$.
time = 0.19, size = 282, normalized size = 2.47

$$\begin{cases} a^3x + \frac{3a^2bc\,\mathrm{acosh}\,(c+dx)}{d} + 3a^2bx\,\mathrm{acosh}\,(c+dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2-1}}{d} + \frac{3ab^2c\,\mathrm{acosh}^2\,(c+dx)}{d} + 3ab^2x\,\mathrm{acosh}^2\,(c+dx) + 6ab^2x - \frac{6ab^2\sqrt{c^2+2cdx+d^2x^2-1}\,\mathrm{acosh}\,(c+dx)}{d} + \frac{b^3c\,\mathrm{acosh}^3\,(c+dx)}{d} + \frac{6b^3c\,\mathrm{acosh}\,(c+dx)}{d} + b^3x\,\mathrm{acosh}^3\,(c+dx) + 6b^3x\,\mathrm{acosh}\,(c+dx) - \frac{3b^3\sqrt{c^2+2cdx+d^2x^2-1}\,\mathrm{acosh}^2\,(c+dx)}{d} - \frac{6b^3\sqrt{c^2+2cdx+d^2x^2-1}}{d} & \text{for } d \neq 0 \\ x(a+b\,\mathrm{acosh}\,(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*acosh(c + d*x)/d + 3*a**2*b*x*acosh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 3*a*b**2*c*acosh(c + d
*x)**2/d + 3*a*b**2*x*acosh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + b**3*c*acosh(c + d*x)**3/d + 6
*b**3*c*acosh(c + d*x)/d + b**3*x*acosh(c + d*x)**3 + 6*b**3*x*acosh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*(a + b*acosh(c))**
3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3, x)
```

3.118 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=159

$$\frac{(a+b\cosh^{-1}(c+dx))^4}{4bde}+\frac{(a+b\cosh^{-1}(c+dx))^3\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)}{de}-\frac{3b(a+b\cosh^{-1}(c+dx))^2\,\mathrm{PolyI}}{2de}$$

```
[Out] 1/4*(a+b*arccosh(d*x+c))^4/b/d/e+(a+b*arccosh(d*x+c))^3*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*b*(a+b*arccosh(d*x+c))^2*polylog(2,-
1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*b^2*(a+b*arccosh(d*x+c
))*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/4*b^3*poly
log(4,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5882, 3799, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3b^2\text{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{2de}-\frac{3b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^2}{2de}+\frac{(a+b\cosh^{-1}(c+dx))^4}{4bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)(a+b\cosh^{-1}(c+dx))^3}{de}-\frac{3b^3\text{Li}_4\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcCosh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcCosh[c + d*x])^3*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b*(a + b*ArcCosh[c + d*x])^2*PolyLog[2
, -E^(-2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])*Poly
Log[3, -E^(-2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^3*PolyLog[4, -E^(-2*ArcCos
h[c + d*x])])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{ce+dex}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{ex}\,dx,x,c+dx\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx,x,c+dx\right)}{de} \\
&= \frac{\operatorname{Subst}\left(\int (a+bx)^3\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{2\operatorname{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right.}{de}
\end{aligned}
$$

Mathematica [**A**]
time = 0.38, size = 217, normalized size = 1.36

$$
\frac{6a^2b\cosh^{-1}(c+dx)^2+4ab^2\cosh^{-1}(c+dx)^3+b^3\cosh^{-1}(c+dx)^4+12a^2b\cosh^{-1}(c+dx)\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+12ab^2\cosh^{-1}(c+dx)^2\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+4b^3\cosh^{-1}(c+dx)^3\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)+4a^3\log(c+dx)-6b\left(a+b\cosh^{-1}(c+dx)\right)^2\operatorname{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)-6b^2\left(a+b\cosh^{-1}(c+dx)\right)\operatorname{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)-3b^3\operatorname{PolyLog}\left(4,-e^{-2\cosh^{-1}(c+dx)}\right)}{4de}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (6*a^2*b*ArcCosh[c + d*x]^2 + 4*a*b^2*ArcCosh[c + d*x]^3 + b^3*ArcCosh[c +
d*x]^4 + 12*a^2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + 12*a*
b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*b^3*ArcCosh[c +
 d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*a^3*Log[c + d*x] - 6*b*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] - 6*b^2*(a + b*Arc
Cosh[c + d*x])*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*b^3*PolyLog[4, -E^(
-2*ArcCosh[c + d*x])])/(4*d*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(207) = 414$.
time = 43.26, size = 436, normalized size = 2.74

method	result
derivativedivides	$\frac{\frac{a^3\ln(dx+c)}{e}-\frac{b^3\operatorname{arccosh}(dx+c)^4}{4e}+\frac{b^3\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{3b^3\operatorname{arccosh}(dx}{}}{}$
default	$\frac{\frac{a^3\ln(dx+c)}{e}-\frac{b^3\operatorname{arccosh}(dx+c)^4}{4e}+\frac{b^3\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{3b^3\operatorname{arccosh}(dx}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/e*ln(d*x+c)-1/4*b^3/e*arccosh(d*x+c)^4+b^3/e*arccosh(d*x+c)^3*ln(1
+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/2*b^3/e*arccosh(d*x+c)^2*poly
log(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*b^3/e*arccosh(d*x+c)*
polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/4*b^3/e*polylog(4,-
(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-a*b^2/e*arccosh(d*x+c)^3+3*a*b^2
/e*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3*a*b^2
/e*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2
*a*b^2/e*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*a^2*b/e*
arccosh(d*x+c)^2+3*a^2*b/e*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2))^2)+3/2*a^2*b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*e^(-1)*log(d*x*e + c*e)/d + integrate(b^3*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)^3/(d*x*e + c*e) + 3*a*b^2*log(d*x + sqrt(d*x + c + 1)
*sqrt(d*x + c - 1) + c)^2/(d*x*e + c*e) + 3*a^2*b*log(d*x + sqrt(d*x + c +
1)*sqrt(d*x + c - 1) + c)/(d*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx + \int \frac{b^3 \operatorname{acosh}^3(c+dx)}{c+dx}\,dx + \int \frac{3ab^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{3a^2 b \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*acosh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*acosh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
cosh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x), x)
```

3.119 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{(a+b\cosh^{-1}(c+dx))^3}{de^2(c+dx)}+\frac{6b(a+b\cosh^{-1}(c+dx))^2\operatorname{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{6ib^2(a+b\cosh^{-1}(c+dx))\ \mathrm{F}}{de}$$

```
[Out] -(a+b*arccosh(d*x+c))^3/d/e^2/(d*x+c)+6*b*(a+b*arccosh(d*x+c))^2*arctan(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-6*I*b^2*(a+b*arccosh(d*x+c))*poly
log(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*I*b^2*(a+b*arccos
h(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*I*b^
3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2-6*I*b^3*polyl
og(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2
```

Rubi [A]

time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5883, 5947, 4265, 2611, 2320, 6724}

$$\frac{6b\operatorname{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^2}{de^2}-\frac{6ib^2\operatorname{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^2}+\frac{6ib^2\operatorname{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^2}-\frac{(a+b\cosh^{-1}(c+dx))^3}{de^2(c+dx)}+\frac{6ib^3\operatorname{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{6ib^3\operatorname{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^3/(d*e^2*(c + d*x))) + (6*b*(a + b*ArcCosh[c + d
*x])^2*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^2*(a + b*ArcCosh[c +
d*x])*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^2*(a + b*ArcC
osh[c + d*x])*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^3*PolyLo
g[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^3*PolyLog[3, I*E^ArcCosh[
c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx, x, c+d\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \cosh^{-1}(c\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx}\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx}\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx}\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx}\right.}{de^2}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.85, size = 327, normalized size = 1.76

$$
-\frac{\frac{a^3}{c+dx}+\frac{3a^2b\cosh^{-1}(c+dx)}{c+dx}+3a^2b\text{ArcTan}\left(\frac{1}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}\right)+3iab^2\left(\cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx}+2\log\left(1-ie^{-\cosh^{-1}(c+dx)}\right)-2\log\left(1+ie^{-\cosh^{-1}(c+dx)}\right)\right)+2\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)\right)+b^3\left(\frac{\cosh^{-1}(c+dx)^3}{c+dx}-3i\left(-\cosh^{-1}(c+dx)^2\left(\log\left(1-ie^{-\cosh^{-1}(c+dx)}\right)-\log\left(1+ie^{-\cosh^{-1}(c+dx)}\right)\right)-2\cosh^{-1}(c+dx)\left(\text{PolyLog}\left(2,-ie^{-\cosh^{-1}(c+dx)}\right)-\text{PolyLog}\left(2,ie^{-\cosh^{-1}(c+dx)}\right)\right)-2\text{PolyLog}\left(3,-ie^{-\cosh^{-1}(c+dx)}\right)+2\text{PolyLog}\left(3,ie^{-\cosh^{-1}(c+dx)}\right)\right)\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a^3/(c + d*x) + (3*a^2*b*ArcCosh[c + d*x])/(c + d*x) + 3*a^2*b*ArcTan[1/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])] + (3*I)*a*b^2*(ArcCosh[c + d*x]*(((
-I)*ArcCosh[c + d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1
 + I/E^ArcCosh[c + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyL
og[2, I/E^ArcCosh[c + d*x]]) + b^3*(ArcCosh[c + d*x]^3/(c + d*x) - (3*I)*(-
(ArcCosh[c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c
+ d*x]])) - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyL
og[2, I/E^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*Po
lyLog[3, I/E^ArcCosh[c + d*x]])))/(d*e^2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^3}{(dex + ce)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
[Out] -b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^2*x*e^2 + c*d*
e^2) - 3*a^2*b*(arcsin(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arccosh(d
*x + c)/(d^2*x*e^2 + c*d*e^2)) - a^3/(d^2*x*e^2 + c*d*e^2) + integrate(3*((
c^3 - c)*a*b^2 + (c^3 - c)*b^3 + (a*b^2*d^3 + b^3*d^3)*x^3 + 3*(a*b^2*c*d^2
 + b^3*c*d^2)*x^2 + (b^3*c^2 + (c^2 - 1)*a*b^2 + (a*b^2*d^2 + b^3*d^2)*x^2
+ 2*(a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*
d - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)^2/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 - d^3)*x^3*e^2
+ (10*c^3*d^2 - 3*c*d^2)*x^2*e^2 + (5*c^4*d - 3*c^2*d)*x*e^2 + (d^4*x^4*e^2
 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*x^2*e^2 + 2*(2*c^3*d - c*d)*x*e^2 +
(c^4 - c^2)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^5 - c^3)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*e^(-2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\, dx + \int \frac{b^3 \operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\, dx + \int \frac{3ab^2 \operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\, dx + \int \frac{3a^2b \operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\, dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*acosh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*acosh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.120 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3}+\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx\right.}{2de^3(c+dx)^2}$$

```
[Out] -3/2*b*(a+b*arccosh(d*x+c))^2/d/e^3-1/2*(a+b*arccosh(d*x+c))^3/d/e^3/(d*x+c
)^2-3*b^2*(a+b*arccosh(d*x+c))*ln(1+1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2)/d/e^3+3/2*b^3*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
/d/e^3+3/2*b*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^3/(
d*x+c)
```

Rubi [A]

time = 0.26, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5883, 5918, 5882, 3799, 2221, 2317, 2438}

$$-\frac{3b^2\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}+\frac{3b^3\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-3*b*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3) + (3*b*Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])^3/(2*d*e^3*(c + d*x)^2) - (3*b^2*(a + b*ArcCosh[c + d*x])*Log[
1 + E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^3*PolyLog[2, -E^(-2*ArcCosh[c
+ d*x])])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx, x, c+d\right.}{2de^3}$$

$$= \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx}$$

$$= \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx}$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(\right.}{2de^3(c+dx)}$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(\right.}{2de^3(c+dx)}$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(\right.}{2de^3(c+dx)}$$

$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(\right.}{2de^3(c+dx)}$$

Mathematica [A]
time = 0.78, size = 266, normalized size = 1.62

$$\frac{-\frac{a^3}{(c+dx)^2} + \frac{3a^2b\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}(c+c^2+2cdx+dx(1+dx)) - \cosh^{-1}(c+dx)\right)}{(c+dx)^2} - \frac{b^3\cosh^{-1}(c+dx)^3}{(c+dx)^2} + 6ab^2\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\cosh^{-1}(c+dx)}{c+dx} - \frac{\cosh^{-1}(c+dx)^2}{2(c+dx)^2} - \log(c+dx)\right) + 3b^3\left(\cosh^{-1}(c+dx)\left(-\cosh^{-1}(c+dx) + \frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\cosh^{-1}(c+dx)}{c+dx} - 2\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)\right) + \text{PolyLog}\left(2, -e^{-2\cosh^{-1}(c+dx)}\right)\right)}{2de^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^3/(c + d*x)^2) + (3*a^2*b*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2
 + 2*c*d*x + d*x*(1 + d*x)) - ArcCosh[c + d*x]))/(c + d*x)^2 - (b^3*ArcCosh
[c + d*x]^3)/(c + d*x)^2 + 6*a*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*ArcCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2)
 - Log[c + d*x]) + 3*b^3*(ArcCosh[c + d*x]*(-ArcCosh[c + d*x] + (Sqrt[(-1 +
 c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) - 2*Log[
```

```
1 + E^(-2*ArcCosh[c + d*x])]) + PolyLog[2, -E^(-2*ArcCosh[c + d*x])]))/(2*d
*e^3)
```

Maple [A]
time = 42.89, size = 339, normalized size = 2.07

method	result
derivativedivides	$\frac{-\frac{a^3}{2e^3(dx+c)^2}+\frac{3b^3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{2e^3(dx+c)}+\frac{3b^3\operatorname{arccosh}(dx+c)^2}{2e^3}-\frac{b^3\operatorname{arccosh}(dx+c)^3}{2e^3(dx+c)^2}-\frac{3b^3\operatorname{arccos}}{}}{}$
default	$\frac{-\frac{a^3}{2e^3(dx+c)^2}+\frac{3b^3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{2e^3(dx+c)}+\frac{3b^3\operatorname{arccosh}(dx+c)^2}{2e^3}-\frac{b^3\operatorname{arccosh}(dx+c)^3}{2e^3(dx+c)^2}-\frac{3b^3\operatorname{arccos}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/e^3/(d*x+c)^2+3/2*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^
(1/2)*(d*x+c-1)^(1/2)+3/2*b^3/e^3*arccosh(d*x+c)^2-1/2*b^3/e^3*arccosh(d*x+
c)^3/(d*x+c)^2-3*b^3/e^3*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))^2)-3/2*b^3/e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
^2)+3*a*b^2/e^3*arccosh(d*x+c)+3*a*b^2/e^3*arccosh(d*x+c)/(d*x+c)*(d*x+c+1)
^(1/2)*(d*x+c-1)^(1/2)-3/2*a*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^2-3*a*b^2/e^3
*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3*a^2*b/e^3*(-1/2/(d*x+c)^
2*arccosh(d*x+c)+1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*x*e^3 + c*d^2*
e^3) - e^(-3)*log(d*x + c)/d)*a*b^2 - 1/2*(log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2*integrat
e(3/2*(d^2*x^2 + 2*c*d*x + sqrt(d*x + c + 1)*(d*x + c)*sqrt(d*x + c - 1) +
c^2 - 1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^5*x^5*e^3
+ 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*x^3*e^3 + (10*c^3*d^2 - 3*c*d^2)*x^2
*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 + (d^4*x^4*e^3 + 4*c*d^3*x^3*e^3 + (6*c^2*
d^2 - d^2)*x^2*e^3 + 2*(2*c^3*d - c*d)*x*e^3 + (c^4 - c^2)*e^3)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + (c^5 - c^3)*e^3), x))*b^3 + 3/2*a^2*b*(sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*x*e^3 + c*d^2*e^3) - arccosh(d*x + c)/(d^3*
x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 3/2*a*b^2*arccosh(d*x + c)^2/(d^3*x
```

```
^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 1/2*a^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3
+ c^2*d*e^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*e^(-3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.121 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=297

$$\frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)}+\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}$$

```
[Out] b^2*(a+b*arccosh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^3/d/e^4/(d*
x+c)^3+b*(a+b*arccosh(d*x+c))^2*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))/d/e^4-b^3*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-I*b^2*(a+b*arcco
sh(d*x+c))*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+I*b^
2*(a+b*arccosh(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
/d/e^4+I*b^3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-I*
b^3*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/2*b*(a+b*a
rccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.40, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5996, 12, 5883, 5933, 5947, 4265, 2611, 2320, 6724, 94, 209}

$$\frac{b\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4}-\frac{ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}-\frac{b^3\text{ArcTan}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{de^4}+\frac{ib^3\text{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^4}-\frac{ib^3\text{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] (b^2*(a + b*ArcCosh[c + d*x]))/(d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*
ArcCosh[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcCosh[c + d*x])^2*A
rcTan[E^ArcCosh[c + d*x]])/(d*e^4) - (b^3*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(d*e^4) - (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, (-I)*E^Ar
cCosh[c + d*x]])/(d*e^4) + (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, I*E^A
rcCosh[c + d*x]])/(d*e^4) + (I*b^3*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*
e^4) - (I*b^3*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
```

```
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
 Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{de^4}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+\right.}{3de^4(c+dx)^3}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^4(c+dx)^2}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 601 vs. $2(297) = 594$.
time = 1.35, size = 601, normalized size = 2.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*a^3)/(c + d*x)^3 + (3*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c +
 d*x)^2 - (6*a^2*b*ArcCosh[c + d*x])/(c + d*x)^3 - 3*a^2*b*ArcTan[1/(Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])] + 6*a*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c
+ d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh
[c + d*x]^2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]]
```

```
+ I*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^Ar
cCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + 6*b^3*(ArcCosh[c +
d*x]/(c + d*x) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[
c + d*x]^2)/(2*(c + d*x)^2) - ArcCosh[c + d*x]^3/(3*(c + d*x)^3) - 2*ArcTan
[c + d*x + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)] + ArcCosh[c +
d*x]^2*ArcTan[c + d*x + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)] -
 I*ArcCosh[c + d*x]*PolyLog[2, (-I)*(c + d*x + Sqrt[(-1 + c + d*x)/(1 + c +
 d*x)]*(1 + c + d*x))] + I*ArcCosh[c + d*x]*PolyLog[2, I*(c + d*x + Sqrt[(-
1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))] + I*PolyLog[3, (-I)*(c + d*x +
Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))] - I*PolyLog[3, I*(c + d*
x + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))]))/(6*d*e^4)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^4*x^3*e^4
+ 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^3/(d^4*x^3*e^4 + 3
*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(((3*(c^3 - c)*a*b
^2 + (c^3 - c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c
*d^2)*x^2 + (b^3*c^2 + 3*(c^2 - 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*
(3*a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*
d - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d - d)*a^2*b
*x + (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*a^2*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + c))/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*x^5*e^4 + 5
*(7*c^3*d^4 - c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*x^3*e^4 + (21*c^5*
d^2 - 10*c^3*d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (d^6*x^6*e^4 + 6*c*
d^5*x^5*e^4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^3)*x^3*e^4 +
```

```
3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^4 + (c^6 - c^
4)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)*e^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*e^(-4)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*
d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{3a^2b\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*acosh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.122 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=377

$$\frac{45b^4e^3(c+dx)^2}{128d}+\frac{3b^4e^3(c+dx)^4}{128d}-\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{64d}-\frac{3b^3e}{}$$

[Out] 45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*arccosh(d*x+c))^2/d+9/16*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c))^2/d+3/16*b^2*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d-3/32*e^3*(a+b*arccosh(d*x+c))^4/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^4/d-45/64*b^3*e^3*(d*x+c)*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/8*b*e^3*(d*x+c)*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d

Rubi [A]
time = 0.75, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 12, 5883, 5939, 5893, 30}

$$-\frac{3b^3e^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{32d}-\frac{45b^3e^3\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{64d}+\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d}-\frac{45b^2e^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{128d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d}-\frac{be^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d}-\frac{3be^3\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{8d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^4}{4d}-\frac{3e^3\left(a+b\cosh^{-1}(c+dx)\right)^4}{32d}+\frac{3b^4e^3(c+dx)^4}{128d}+\frac{45b^4e^3(c+dx)^2}{128d}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]

[Out] (45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) - (45*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(64*d) - (3*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(32*d) - (45*b^2*e^3*(a + b*ArcCosh[c + d*x])^2)/(128*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(8*d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^4)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^4)/(4*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^4}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\,\mathrm{c}}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{4d}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d} - \frac{3be^3\sqrt{-1+c+dx}\,(}{}$$

$$= -\frac{3b^3e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{32d}$$

$$= \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,(}{64d}$$

$$= \frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}\,(c+}{}$$

Mathematica [A]
time = 0.73, size = 562, normalized size = 1.49

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^3*(9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
+ d*x)^4 + 2*a*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2
+ 15*b^2) - 2*(8*a^2 + 3*b^2)*(c + d*x)^2) + 2*b*(c + d*x)*(72*a*b^2*(c + d
*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 - 72*a^2*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x] - 45*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 48*a^2
*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 6*b^3*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-24*a^2 - 15*b
^2 + 24*b^2*(c + d*x)^2 + 64*a^2*(c + d*x)^4 + 8*b^2*(c + d*x)^4 - 48*a*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 32*a*b*Sqrt[-1 + c + d*x]*(
```

```
c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 16*b^3*(-3*a + 8*a*(c +
d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 4*b^4*(-3 + 8
*(c + d*x)^4)*ArcCosh[c + d*x]^4 - 6*a*b*(8*a^2 + 15*b^2)*Log[c + d*x + Sqr
t[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(128*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(339) = 678$.
time = 40.88, size = 875, normalized size = 2.32

method	result	size
default	Expression too large to display	875

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^4/d-1/1024*e^3*b^4*(-64*cosh(2*arccosh(d*x+c))^2*arccos
h(d*x+c)^4+64*sinh(2*arccosh(d*x+c))*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^
3-128*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^4+256*sinh(2*arccosh(d*x+c))*ar
ccosh(d*x+c)^3-48*cosh(2*arccosh(d*x+c))^2*arccosh(d*x+c)^2+32*arccosh(d*x+
c)^4+24*sinh(2*arccosh(d*x+c))*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)-384*co
sh(2*arccosh(d*x+c))*arccosh(d*x+c)^2+384*arccosh(d*x+c)*sinh(2*arccosh(d*x
+c))-6*cosh(2*arccosh(d*x+c))^2+24*arccosh(d*x+c)^2-192*cosh(2*arccosh(d*x+
c))+3)/d-1/128*e^3*a*b^3*(-32*cosh(2*arccosh(d*x+c))^2*arccosh(d*x+c)^3+24*
sinh(2*arccosh(d*x+c))*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)^2-64*cosh(2*ar
ccosh(d*x+c))*arccosh(d*x+c)^3+96*arccosh(d*x+c)^2*sinh(2*arccosh(d*x+c))-1
2*arccosh(d*x+c)*cosh(2*arccosh(d*x+c))^2+16*arccosh(d*x+c)^3+3*sinh(2*arcc
osh(d*x+c))*cosh(2*arccosh(d*x+c))-96*cosh(2*arccosh(d*x+c))*arccosh(d*x+c)
+48*sinh(2*arccosh(d*x+c))+6*arccosh(d*x+c))/d+6*e^3*a^2*b^2/d*(1/128*(8*ar
ccosh(d*x+c)^2+1)*cosh(2*arccosh(d*x+c))^2+1/32*(4*arccosh(d*x+c)^2-arccosh
(d*x+c)*sinh(2*arccosh(d*x+c))+2)*cosh(2*arccosh(d*x+c))-1/32*arccosh(d*x+c
)^2-1/8*arccosh(d*x+c)*sinh(2*arccosh(d*x+c))-1/256)+e^3*a^3*b*d^3*arccosh(
d*x+c)*x^4+4*e^3*a^3*b*d^2*arccosh(d*x+c)*x^3*c+6*e^3*a^3*b*d*arccosh(d*x+c
)*x^2*c^2+4*e^3*a^3*b*arccosh(d*x+c)*x*c^3+e^3*a^3*b/d*arccosh(d*x+c)*c^4-1
/4*e^3*a^3*b*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-3/4*e^3*a^3*b*d*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-3/4*e^3*a^3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*c^2-1/4*e^3*a^3*b/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-3/8*e^3*a^3*b*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/8*e^3*a^3*b/d*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*c-3/8*e^3*a^3*b/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2
)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*d^3*x^4*e^3 + a^4*c*d^2*x^3*e^3 + 3/2*a^4*c^2*d*x^2*e^3 + 3*(2*x^2*
arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log
(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arccosh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4
 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*c*d^2*e^3 + 1/
24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2
- 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*
c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3
/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^3*b*d^3*e^3 + a^4*c^3*x*
e^3 + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c^3*e^3/
d + 1/4*(b^4*d^3*x^4*e^3 + 4*b^4*c*d^2*x^3*e^3 + 6*b^4*c^2*d*x^2*e^3 + 4*b^
4*c^3*x*e^3)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4 + integra
te((((4*a*b^3*d^6 - b^4*d^6)*x^6*e^3 + 6*(4*a*b^3*c*d^5 - b^4*c*d^5)*x^5*e^
3 + 4*(c^6 - c^4)*a*b^3*e^3 + (4*(15*c^2*d^4 - d^4)*a*b^3 - (15*c^2*d^4 - d
^4)*b^4)*x^4*e^3 + 4*(4*(5*c^3*d^3 - c*d^3)*a*b^3 - (5*c^3*d^3 - c*d^3)*b^4
)*x^3*e^3 + 2*(6*(5*c^4*d^2 - 2*c^2*d^2)*a*b^3 - (7*c^4*d^2 - 3*c^2*d^2)*b^
4)*x^2*e^3 + 4*(2*(3*c^5*d - 2*c^3*d)*a*b^3 - (c^5*d - c^3*d)*b^4)*x*e^3 +
((4*a*b^3*d^5 - b^4*d^5)*x^5*e^3 + 4*(c^5 - c^3)*a*b^3*e^3 + 5*(4*a*b^3*c*d
^4 - b^4*c*d^4)*x^4*e^3 - 2*(5*b^4*c^2*d^3 - 2*(10*c^2*d^3 - d^3)*a*b^3)*x^
3*e^3 - 2*(5*b^4*c^3*d^2 - 2*(10*c^3*d^2 - 3*c*d^2)*a*b^3)*x^2*e^3 - 4*(b^4
*c^4*d - (5*c^4*d - 3*c^2*d)*a*b^3)*x*e^3)*sqrt(d*x + c + 1)*sqrt(d*x + c -
 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 6*(a^2*b^2*d^6*
x^6*e^3 + 6*a^2*b^2*c*d^5*x^5*e^3 + (15*c^2*d^4 - d^4)*a^2*b^2*x^4*e^3 + 4*
(5*c^3*d^3 - c*d^3)*a^2*b^2*x^3*e^3 + 3*(5*c^4*d^2 - 2*c^2*d^2)*a^2*b^2*x^2
*e^3 + 2*(3*c^5*d - 2*c^3*d)*a^2*b^2*x*e^3 + (c^6 - c^4)*a^2*b^2*e^3 + (a^2
*b^2*d^5*x^5*e^3 + 5*a^2*b^2*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*a^2*b^2*x^3
*e^3 + (10*c^3*d^2 - 3*c*d^2)*a^2*b^2*x^2*e^3 + (5*c^4*d - 3*c^2*d)*a^2*b^2
*x*e^3 + (c^5 - c^3)*a^2*b^2*e^3)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(
d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (
3*c^2*d - d)*x - c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4034 vs.

2(328) = 656.
time = 0.45, size = 4034, normalized size = 10.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
[Out] 1/128*(4*((8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c
^3*d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)^3 + 3*(8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3
 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)^2*sinh(
1) + 3*(8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*
d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)*sinh(1)^2 + (8*b^4*d^4*x^4 + 32*b^4*c*d^3*
x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*sinh(1)^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + ((32*a^4 + 24*a^2*b^2 +
 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 + 3*(24*a^2*b^2
 + 15*b^4 + 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 2
4*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*cosh(1)^3 + 16*((8*a
*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x
 + 8*a*b^3*c^4 - 3*a*b^3)*cosh(1)^3 + 3*(8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x
^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*cosh(
1)^2*sinh(1) + 3*(8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x
^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*cosh(1)*sinh(1)^2 + (8*a*b^3
*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8
*a*b^3*c^4 - 3*a*b^3)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*b^4
*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 + 3*b^4*c + 3*(2*b^4*c^2 + b^4)*d*x)
*cosh(1)^3 + 3*(2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 + 3*b^4*c + 3*(
2*b^4*c^2 + b^4)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^
2 + 2*b^4*c^3 + 3*b^4*c + 3*(2*b^4*c^2 + b^4)*d*x)*cosh(1)*sinh(1)^2 + (2*b
^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 + 3*b^4*c + 3*(2*b^4*c^2 + b^4)*d*
x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*((32*
a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x
^3 + 3*(24*a^2*b^2 + 15*b^4 + 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2
+ 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*co
sh(1)^2*sinh(1) + 3*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24
*a^2*b^2 + 3*b^4)*c*d^3*x^3 + 3*(24*a^2*b^2 + 15*b^4 + 2*(32*a^4 + 24*a^2*b
^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a^
2*b^2 + 5*b^4)*c)*d*x)*cosh(1)*sinh(1)^2 + ((32*a^4 + 24*a^2*b^2 + 3*b^4)*d
^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 + 3*(24*a^2*b^2 + 15*b^4
 + 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2
 + 3*b^4)*c^3 + 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*sinh(1)^3 + 3*((8*(8*a^2*b^2
+ b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 + 24*b^4*c^2 + 8*(8*a^2*b^2
 + b^4)*c^4 + 24*(b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 - 15*
b^4 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^2*b
^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 + 24*b^4*c^2 + 8*(8*a^2*
```

```
b^2 + b^4)*c^4 + 24*(b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 -
15*b^4 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*
(8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 + 24*b^4*c^2
+ 8*(8*a^2*b^2 + b^4)*c^4 + 24*(b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24
*a^2*b^2 - 15*b^4 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)*sin
h(1)^2 + (8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 + 24
*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 + 24*(b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d^2
*x^2 - 24*a^2*b^2 - 15*b^4 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*si
nh(1)^3 - 16*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*a*b^3*d^3*x^3 + 6*a*b^3*
c*d^2*x^2 + 2*a*b^3*c^3 + 3*a*b^3*c + 3*(2*a*b^3*c^2 + a*b^3)*d*x)*cosh(1)^
3 + 3*(2*a*b^3*d^3*x^3 + 6*a*b^3*c*d^2*x^2 + 2*a*b^3*c^3 + 3*a*b^3*c + 3*(2
*a*b^3*c^2 + a*b^3)*d*x)*cosh(1)^2*sinh(1) + 3*(2*a*b^3*d^3*x^3 + 6*a*b^3*c
*d^2*x^2 + 2*a*b^3*c^3 + 3*a*b^3*c + 3*(2*a*b^3*c^2 + a*b^3)*d*x)*cosh(1)*s
inh(1)^2 + (2*a*b^3*d^3*x^3 + 6*a*b^3*c*d^2*x^2 + 2*a*b^3*c^3 + 3*a*b^3*c +
 3*(2*a*b^3*c^2 + a*b^3)*d*x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1))^2 + 2*((8*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a*b
^3)*c*d^3*x^3 + 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*(3*a*b^3 + 2*
(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3 + 16*(9*a*b^3*c + 2*
(8*a^3*b + 3*a*b^3)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^3*b + 3*a*b^3)*d^4*x^4
+ 32*(8*a^3*b + 3*a*b^3)*c*d^3*x^3 + 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c
^4 + 24*(3*a*b^3 + 2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3
 + 16*(9*a*b^3*c + 2*(8*a^3*b + 3*a*b^3)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*(8
*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*x^3 + 72*a*b^3*
c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*(3*a*b^3 + 2*(8*a^3*b + 3*a*b^3)*c^2)*
d^2*x^2 - 24*a^3*b - 45*a*b^3 + 16*(9*a*b^3*c + 2*(8*a^3*b + 3*a*b^3)*c^3)*
d*x)*cosh(1)*sinh(1)^2 + (8*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a
*b^3)*c*d^3*x^3 + 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 + 24*(3*a*b^3 +
2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3 + 16*(9*a*b^3*c +
2*(8*a^3*b + 3*a*b^3)*c^3)*d*x)*sinh(1)^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*((2*(8*a^2*b^2 + b^4)*d^3*x^3 + 6*(8*a^2*b...
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2876 vs. $2(359) = 718$.
time = 1.52, size = 2876, normalized size = 7.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*acosh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*acosh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*ac
```

osh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/
4 - 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) + a**3*b*d**
3*e**3*x**4*acosh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/4 - 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3
*a**3*b*e**3*acosh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*acosh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2 + 9*a**2*b**2*c
2*d*e3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/4 + 9*a**2*b**2*c*e**3*x/8 - 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*acosh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*
x)/4 + 9*a**2*b**2*d*e**3*x**2/16 - 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)/8 - 9*a**2*b**2*e**3*acosh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*acosh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*acosh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*acosh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*ac
osh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*acosh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*acosh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)/32 + 9*a*b**3*c**2*e**3*acosh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*acosh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32
 + 9*a*b**3*c*e**3*x*acosh(c + d*x)/4 - 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)**2/(8*d) - 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/(64*d) + a*b**3*d**3*e**3*x**4*acosh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*acosh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32 + 9*a*b**3*d*e**3*x**2*acosh(c + d*x
)/8 - 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**
2/8 - 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/64 - 3*a*b**3*e
3*acosh(c + d*x)3/(8*d) - 45*a*b**3*e**3*acosh(c + d*x)/(64*d) + b**4*c
4*e3*acosh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*acosh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*acosh(c + d*x)**4 + 3*b**4*c**3*e**3*x*acosh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*acosh(c + d*x)
4/2 + 9*b4*c**2*d*e**3*x**2*acosh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c

```
+ d*x)/32 + 9*b**4*c**2*e**3*acosh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*acosh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*acosh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*c*e**3*x*acosh(c + d*x)**2/8 + 45*b**
4*c*e**3*x/64 - 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)**3/(8*d) - 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*acosh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*acosh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*d
*e**3*x**2*acosh(c + d*x)**2/16 + 45*b**4*d*e**3*x**2/128 - 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/8 - 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/64 - 3*b**4*e**3*acosh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*acosh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*acosh(c))**4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4, x)
```

3.123 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=309

$$\frac{160}{27}b^4e^2x+\frac{8b^4e^2(c+dx)^3}{81d}-\frac{160b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2\sqrt{-1+c+dx}}{}$$

```
[Out] 160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d+8/3*b^2*e^2*(d*x+c)*(a+b*arccosh(
d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^2/d+1/3*e^2*(d*x+c)^
3*(a+b*arccosh(d*x+c))^4/d-160/27*b^3*e^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d-8/9*b*e^2*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-4/9*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.52, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5883, 5939, 5915, 5879, 8, 30}

$$-\frac{8b^3e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{160b^3e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{9d}+\frac{8b^2e^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{4be^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{9d}-\frac{8be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{9d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^4}{3d}+\frac{8b^4e^2(c+dx)^3}{81d}+\frac{160}{27}b^4e^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) - (160*b^3*e^2*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(27*d) - (8*b^3*e^2*
Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/
(27*d) + (8*b^2*e^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (4*b^2*e^
2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(9*d) - (8*b*e^2*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) - (4*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) +
 (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^4)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^4}{3d} - \frac{(4be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\,\text{co}}{\sqrt{-1+x}}\right.}{3}$$

$$= -\frac{4be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{9d}$$

$$= \frac{4b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{9d} - \frac{8be^2\sqrt{-1+c+dx}\,\sqrt{}}{}$$

$$= -\frac{8b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{27d}$$

$$= \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{27d}$$

$$= \frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+}}{27d}$$

Mathematica [A]
time = 0.32, size = 475, normalized size = 1.54

$$\frac{e^2\left(24b^2(9a^2+20b^2)(c+dx)+(27a^4+36a^2b^2+8b^4)(c+dx)^3+12ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(-6a^2-40b^2-(3a^2+2b^2)(c+dx)^2)+12b\left(36ab^2(c+dx)+9a^3(c+dx)^3+6ab^2(c+dx)^3-18a^2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}-40b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}-9a^2b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}-2b^3\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+18b^2\left(12b^2(c+dx)+9a^2(c+dx)^3+2b^2(c+dx)^3-12ab\sqrt{-1+c+dx}\,\sqrt{1+c+dx}-6ab\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2-36b^3\left(-3a(c+dx)^3+2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}+b\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^3+27b^4(c+dx)^3\cosh^{-1}(c+dx)^4\right)}{81d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*(24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
+ d*x)^3 + 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-6*a^2 - 40*b^2 - (
3*a^2 + 2*b^2)*(c + d*x)^2) + 12*b*(36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3
+ 6*a*b^2*(c + d*x)^3 - 18*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 40*
b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 9*a^2*b*Sqrt[-1 + c + d*x]*(c +
d*x)^2*Sqrt[1 + c + d*x] - 2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c
+ d*x])*ArcCosh[c + d*x] + 18*b^2*(12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 + 2
*b^2*(c + d*x)^3 - 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 6*a*b*Sqrt
[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 36*b^3*(
-3*a*(c + d*x)^3 + 2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c
```

```
 + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 27*b^4*(c + d*x
)^3*ArcCosh[c + d*x]^4))/(81*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(275) = 550$.
time = 40.37, size = 1058, normalized size = 3.42

method	result
default	$\frac{e^2(dx+c)^3 a^4}{3d} + \frac{e^2 b^4 \left(27\operatorname{arccosh}(dx+c)^4 x^3 d^3 - 36\sqrt{dx+c-1}\,\operatorname{arccosh}(dx+c)^3\sqrt{dx+c+1}\,x^2 d^2 + 81\operatorname{arccosh}(dx+c)^4 x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^4/d+1/81*e^2*b^4*(27*arccosh(d*x+c)^4*x^3*d^3-36*(d*x+c
-1)^(1/2)*arccosh(d*x+c)^3*(d*x+c+1)^(1/2)*x^2*d^2+81*arccosh(d*x+c)^4*x^2*
c*d^2-72*(d*x+c-1)^(1/2)*arccosh(d*x+c)^3*(d*x+c+1)^(1/2)*x*c*d+81*arccosh(
d*x+c)^4*x*c^2*d+36*arccosh(d*x+c)^2*x^3*d^3-36*(d*x+c-1)^(1/2)*arccosh(d*x
+c)^3*(d*x+c+1)^(1/2)*c^2-24*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)
*x^2*d^2+27*arccosh(d*x+c)^4*c^3+108*arccosh(d*x+c)^2*x^2*c*d^2-48*arccosh(
d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d+108*arccosh(d*x+c)^2*x*c^2*d+8
*d^3*x^3-72*(d*x+c-1)^(1/2)*arccosh(d*x+c)^3*(d*x+c+1)^(1/2)-24*arccosh(d*x
+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+36*arccosh(d*x+c)^2*c^3+24*x^2*c*d^
2+216*arccosh(d*x+c)^2*x*d+24*c^2*d*x-480*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)+216*arccosh(d*x+c)^2*c+8*c^3+480*d*x+480*c)/d+4/27*e^2*a*b^3*
(9*arccosh(d*x+c)^3*x^3*d^3+27*arccosh(d*x+c)^3*x^2*c*d^2-9*arccosh(d*x+c)^
2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2+27*arccosh(d*x+c)^3*x*c^2*d-18*ar
ccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d+6*arccosh(d*x+c)*x^3*d
^3+9*arccosh(d*x+c)^3*c^3-9*arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2
)*c^2+18*arccosh(d*x+c)*x^2*c*d^2-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x^2*d^2
+18*arccosh(d*x+c)*x*c^2*d-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*x*c*d-18*arcco
sh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)+6*arccosh(d*x+c)*c^3-2*(d*x+c+1
)^(1/2)*(d*x+c-1)^(1/2)*c^2+36*arccosh(d*x+c)*x*d+36*arccosh(d*x+c)*c-40*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d+2/9*e^2*a^2*b^2*(9*arccosh(d*x+c)^2*x^3*d^
3+27*arccosh(d*x+c)^2*x^2*c*d^2-6*arccosh(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^
(1/2)*x^2*d^2+27*arccosh(d*x+c)^2*x*c^2*d-12*arccosh(d*x+c)*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*x*c*d+2*d^3*x^3+9*arccosh(d*x+c)^2*c^3-6*arccosh(d*x+c)*(d
*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*c^2+6*x^2*c*d^2+6*c^2*d*x-12*arccosh(d*x+c)*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*c^3+12*d*x+12*c)/d+4*e^2*a^3*b/d*(1/3*(d*x
+c)^3*arccosh(d*x+c)-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*d^2*x^3*e^2 + a^4*c*d*x^2*e^2 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c^
2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c
/d^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(
c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4
 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*x*e^2 + 4*((d*x + c)*arc
cosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^2*x^3*e
^2 + 3*b^4*c*d*x^2*e^2 + 3*b^4*c^2*x*e^2)*log(d*x + sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + c)^4 + integrate(2/3*(2*((3*a*b^3*d^5 - b^4*d^5)*x^5*e^2 + 3
*(c^5 - c^3)*a*b^3*e^2 + 5*(3*a*b^3*c*d^4 - b^4*c*d^4)*x^4*e^2 + (3*(10*c^2
*d^3 - d^3)*a*b^3 - (10*c^2*d^3 - d^3)*b^4)*x^3*e^2 + 3*((10*c^3*d^2 - 3*c*
d^2)*a*b^3 - (3*c^3*d^2 - c*d^2)*b^4)*x^2*e^2 + 3*((5*c^4*d - 3*c^2*d)*a*b^
3 - (c^4*d - c^2*d)*b^4)*x*e^2 + (3*(c^4 - c^2)*a*b^3*e^2 + (3*a*b^3*d^4 -
b^4*d^4)*x^4*e^2 + 4*(3*a*b^3*c*d^3 - b^4*c*d^3)*x^3*e^2 - 3*(2*b^4*c^2*d^2
 - (6*c^2*d^2 - d^2)*a*b^3)*x^2*e^2 - 3*(b^4*c^3*d - 2*(2*c^3*d - c*d)*a*b^
3)*x*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^5*x^5*e^2 + 5*a^2*b^2*c*d^4*x^4*e^2
 + (10*c^2*d^3 - d^3)*a^2*b^2*x^3*e^2 + (10*c^3*d^2 - 3*c*d^2)*a^2*b^2*x^2*
e^2 + (5*c^4*d - 3*c^2*d)*a^2*b^2*x*e^2 + (c^5 - c^3)*a^2*b^2*e^2 + (a^2*b^
2*d^4*x^4*e^2 + 4*a^2*b^2*c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*a^2*b^2*x^2*e^2
 + 2*(2*c^3*d - c*d)*a^2*b^2*x*e^2 + (c^4 - c^2)*a^2*b^2*e^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^
2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(266) = 532$.
time = 0.44, size = 2094, normalized size = 6.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*((b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(1)
^2 + 2*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(1)*si
nh(1) + (b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*sinh(1)^2
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 36*(3*(a*b^3*d^3*x^3
```

```
 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*cosh(1)^2 + 6*(a*b^3*d^
3*x^3 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*cosh(1)*sinh(1) +
3*(a*b^3*d^3*x^3 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*sinh(1)
^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^
2 + 2*b^4)*cosh(1)^2 + 2*(b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2 + 2*b^4)*cosh
(1)*sinh(1) + (b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2 + 2*b^4)*sinh(1)^2))*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + ((27*a^4 + 36*a^2*b^2 + 8
*b^4)*d^3*x^3 + 3*(27*a^4 + 36*a^2*b^2 + 8*b^4)*c*d^2*x^2 + 3*(72*a^2*b^2 +
 160*b^4 + (27*a^4 + 36*a^2*b^2 + 8*b^4)*c^2)*d*x)*cosh(1)^2 + 18*(((9*a^2*
b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 + 12*b^4*c + (9*a^2*
b^2 + 2*b^4)*c^3 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*x)*cosh(1)^2 + 2*(
(9*a^2*b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 + 12*b^4*c +
(9*a^2*b^2 + 2*b^4)*c^3 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*x)*cosh(1)*
sinh(1) + ((9*a^2*b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 +
12*b^4*c + (9*a^2*b^2 + 2*b^4)*c^3 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*
x)*sinh(1)^2 - 6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((a*b^3*d^2*x^2 + 2*a*b^
3*c*d*x + a*b^3*c^2 + 2*a*b^3)*cosh(1)^2 + 2*(a*b^3*d^2*x^2 + 2*a*b^3*c*d*x
 + a*b^3*c^2 + 2*a*b^3)*cosh(1)*sinh(1) + (a*b^3*d^2*x^2 + 2*a*b^3*c*d*x +
a*b^3*c^2 + 2*a*b^3)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1))^2 + 2*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*x^3 + 3*(27*a^4 + 36*a^2*b^
2 + 8*b^4)*c*d^2*x^2 + 3*(72*a^2*b^2 + 160*b^4 + (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*x)*cosh(1)*sinh(1) + ((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*x^3 + 3*
(27*a^4 + 36*a^2*b^2 + 8*b^4)*c*d^2*x^2 + 3*(72*a^2*b^2 + 160*b^4 + (27*a^4
 + 36*a^2*b^2 + 8*b^4)*c^2)*d*x)*sinh(1)^2 + 12*(3*((3*a^3*b + 2*a*b^3)*d^3
*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 + 12*a*b^3*c + (3*a^3*b + 2*a*b^3)*c
^3 + 3*(4*a*b^3 + (3*a^3*b + 2*a*b^3)*c^2)*d*x)*cosh(1)^2 + 6*((3*a^3*b + 2
*a*b^3)*d^3*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 + 12*a*b^3*c + (3*a^3*b +
 2*a*b^3)*c^3 + 3*(4*a*b^3 + (3*a^3*b + 2*a*b^3)*c^2)*d*x)*cosh(1)*sinh(1)
+ 3*((3*a^3*b + 2*a*b^3)*d^3*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 + 12*a*b
^3*c + (3*a^3*b + 2*a*b^3)*c^3 + 3*(4*a*b^3 + (3*a^3*b + 2*a*b^3)*c^2)*d*x)
*sinh(1)^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(((9*a^2*b^2 + 2*b^4)*d^2*x^
2 + 18*a^2*b^2 + 40*b^4 + 2*(9*a^2*b^2 + 2*b^4)*c*d*x + (9*a^2*b^2 + 2*b^4)
*c^2)*cosh(1)^2 + 2*((9*a^2*b^2 + 2*b^4)*d^2*x^2 + 18*a^2*b^2 + 40*b^4 + 2*
(9*a^2*b^2 + 2*b^4)*c*d*x + (9*a^2*b^2 + 2*b^4)*c^2)*cosh(1)*sinh(1) + ((9*
a^2*b^2 + 2*b^4)*d^2*x^2 + 18*a^2*b^2 + 40*b^4 + 2*(9*a^2*b^2 + 2*b^4)*c*d*
x + (9*a^2*b^2 + 2*b^4)*c^2)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)) - 12*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(((3*a^3*b + 2*a*b^3)
*d^2*x^2 + 6*a^3*b + 40*a*b^3 + 2*(3*a^3*b + 2*a*b^3)*c*d*x + (3*a^3*b + 2*
a*b^3)*c^2)*cosh(1)^2 + 2*((3*a^3*b + 2*a*b^3)*d^2*x^2 + 6*a^3*b + 40*a*b^3
 + 2*(3*a^3*b + 2*a*b^3)*c*d*x + (3*a^3*b + 2*a*b^3)*c^2)*cosh(1)*sinh(1) +
 ((3*a^3*b + 2*a*b^3)*d^2*x^2 + 6*a^3*b + 40*a*b^3 + 2*(3*a^3*b + 2*a*b^3)*
c*d*x + (3*a^3*b + 2*a*b^3)*c^2)*sinh(1)^2))/d
```

Sympy **[B]** Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(298) = 596$.

time = 0.95, size = 1889, normalized size = 6.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*acosh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*acosh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*acosh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/9 + 4*a**3*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a**2*b**2*c**3*e**2*acosh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*acosh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*acosh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*acosh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*aco
sh(c + d*x)/3 + 8*a**2*b**2*e**2*x/3 - 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*acosh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
cosh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*acosh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*acosh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 8*a*b**3*
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 16*a*b**3*c*e**2*acos
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*acosh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/27 + 16*a*b**3*e**2*x*acosh(c + d*x)/3 - 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**4*c**3*e**2*acosh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*acosh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*acosh(c + d*x)**4 + 4*b**4*c**2*e**2*x*acosh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acos
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*acosh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*acosh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*c*e**2*acosh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*acosh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*acosh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
```

```
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*e**2*x*ac
osh(c + d*x)**2/3 + 160*b**4*e**2*x/27 - 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)*acosh(c + d*x)**3/(9*d) - 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acos
h(c))**4, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4, x)
```

3.124 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=209

$$\frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{3b^2e\left(a+b\cosh^{-1}(c+dx)\right.}{4d}$$

```
[Out] 3/4*b^4*e*(d*x+c)^2/d-3/4*b^2*e*(a+b*arccosh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arccosh(d*x+c))^2/d-1/4*e*(a+b*arccosh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(
a+b*arccosh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/d-b*e*(d*x+c)*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5996, 12, 5883, 5939, 5893, 30}

$$-\frac{3b^3e\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{3b^2e\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^4}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^4}{4d}+\frac{3b^4e(c+dx)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (3*b^2*e*(a + b*ArcCosh[c + d*
x])^2)/(4*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d) - (b*
e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3
)/d - (e*(a + b*ArcCosh[c + d*x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[
c + d*x])^4)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
 NeQ[m, -1]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^4}{2d}-\frac{(2be)\text{Subst}\left(\int\frac{x^2(a+b\cosh^{-}}{\sqrt{-1+x}\,\sqrt{}}\right.}{d}$$

$$=-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}$$

$$=\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{-1+c+dx}\,(c+d}{}$$

$$=-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right.}{2d}$$

$$=\frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,(a+b}{2d}$$

Mathematica [**A**]
time = 0.26, size = 360, normalized size = 1.72

$$\frac{e\left((2a^4+6a^2b^2+3b^4)(c+dx)^2-2ab(2a^2+3b^2)\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}-2b(c+dx)\left(-4a^3(c+dx)-6ab^2(c+dx)+6a^2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}+3b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)+3b^2\left(-2a^2-b^2+4a^2(c+dx)^2+2b^2(c+dx)^2-4ab\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2+4b^3\left(-a+2a(c+dx)^2-b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^3+b^4(-1+2(c+dx)^2)\cosh^{-1}(c+dx)^4-2ab(2a^2+3b^2)\log\left(c+dx+\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*Sqrt[-1
 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*(c + d*x)*(-4*a^3*(c + d*x) -
 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 3*b^3*S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-2*a^2 - b^2
 + 4*a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*Sqrt[-1 + c + d*x]*(c + d*
x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 4*b^3*(-a + 2*a*(c + d*x)^2 - b*
Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(-
1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^4 - 2*a*b*(2*a^2 + 3*b^2)*Log[c + d*x +
 Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(189) = 378$.
time = 39.49, size = 434, normalized size = 2.08

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a^4}{2}+e\,b^4\left(\frac{\operatorname{arccosh}(dx+c)^4(dx+c)^2}{2}-\operatorname{arccosh}(dx+c)^3(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}-\frac{\operatorname{arccosh}(dx+c)^4}{4}\right.}{}$
default	$\frac{\frac{e(dx+c)^2a^4}{2}+e\,b^4\left(\frac{\operatorname{arccosh}(dx+c)^4(dx+c)^2}{2}-\operatorname{arccosh}(dx+c)^3(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}-\frac{\operatorname{arccosh}(dx+c)^4}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^4+e*b^4*(1/2*arccosh(d*x+c)^4*(d*x+c)^2-arccosh(d*x+
c)^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*arccosh(d*x+c)^4+3/2*arcco
sh(d*x+c)^2*(d*x+c)^2-3/2*arccosh(d*x+c)*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)-3/4*arccosh(d*x+c)^2+3/4*(d*x+c)^2)+4*e*a*b^3*(1/2*arccosh(d*x+c)^3*(
d*x+c)^2-3/4*arccosh(d*x+c)^2*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*a
rccosh(d*x+c)^3+3/4*(d*x+c)^2*arccosh(d*x+c)-3/8*(d*x+c)*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)-3/8*arccosh(d*x+c))+6*e*a^2*b^2*(1/2*arccosh(d*x+c)^2*(d*x+c)
^2-1/2*arccosh(d*x+c)*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/4*arccosh(d
*x+c)^2+1/4*(d*x+c)^2)+2*e*a^3*b*(d*x+c)^2*arccosh(d*x+c)-e*a^3*b*(d*x+c)*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-e*a^3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*
x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*a^4*d*x^2*e + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d +
2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*d*e + a^4*c
*x*e + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c*e/d +
 1/2*(b^4*d*x^2*e + 2*b^4*c*x*e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)^4 + integrate(2*((2*(c^4 - c^2)*a*b^3*e + (2*a*b^3*d^4 - b^4*d^4)*
x^4*e + 4*(2*a*b^3*c*d^3 - b^4*c*d^3)*x^3*e + (2*(6*c^2*d^2 - d^2)*a*b^3 -
(5*c^2*d^2 - d^2)*b^4)*x^2*e + 2*(2*(2*c^3*d - c*d)*a*b^3 - (c^3*d - c*d)*b
^4)*x*e + (2*(c^3 - c)*a*b^3*e + (2*a*b^3*d^3 - b^4*d^3)*x^3*e + 3*(2*a*b^3
```

```
*c*d^2 - b^4*c*d^2)*x^2*e - 2*(b^4*c^2*d - (3*c^2*d - d)*a*b^3)*x*e)*sqrt(d
*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^3 + 3*(a^2*b^2*d^4*x^4*e + 4*a^2*b^2*c*d^3*x^3*e + (6*c^2*d^2 - d^2)
*a^2*b^2*x^2*e + 2*(2*c^3*d - c*d)*a^2*b^2*x*e + (c^4 - c^2)*a^2*b^2*e + (a
^2*b^2*d^3*x^3*e + 3*a^2*b^2*c*d^2*x^2*e + (3*c^2*d - d)*a^2*b^2*x*e + (c^3
 - c)*a^2*b^2*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 +
2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x -
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(196) = 392$.
time = 0.40, size = 959, normalized size = 4.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(((2*b^4*d^2*x^2 + 4*b^4*c*d*x + 2*b^4*c^2 - b^4)*cosh(1) + (2*b^4*d^2*
x^2 + 4*b^4*c*d*x + 2*b^4*c^2 - b^4)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1))^4 + 4*((2*a*b^3*d^2*x^2 + 4*a*b^3*c*d*x + 2*a*b^3*c^2 -
 a*b^3)*cosh(1) + (2*a*b^3*d^2*x^2 + 4*a*b^3*c*d*x + 2*a*b^3*c^2 - a*b^3)*s
inh(1) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((b^4*d*x + b^4*c)*cosh(1) + (b^
4*d*x + b^4*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3
 + 3*((2*(2*a^2*b^2 + b^4)*d^2*x^2 - 2*a^2*b^2 - b^4 + 4*(2*a^2*b^2 + b^4)*
c*d*x + 2*(2*a^2*b^2 + b^4)*c^2)*cosh(1) + (2*(2*a^2*b^2 + b^4)*d^2*x^2 - 2
*a^2*b^2 - b^4 + 4*(2*a^2*b^2 + b^4)*c*d*x + 2*(2*a^2*b^2 + b^4)*c^2)*sinh(
1) - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((a*b^3*d*x + a*b^3*c)*cosh(1) + (
a*b^3*d*x + a*b^3*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1))^2 + ((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^
4)*c*d*x)*cosh(1) + 2*((2*(2*a^3*b + 3*a*b^3)*d^2*x^2 - 2*a^3*b - 3*a*b^3 +
 4*(2*a^3*b + 3*a*b^3)*c*d*x + 2*(2*a^3*b + 3*a*b^3)*c^2)*cosh(1) + (2*(2*a
^3*b + 3*a*b^3)*d^2*x^2 - 2*a^3*b - 3*a*b^3 + 4*(2*a^3*b + 3*a*b^3)*c*d*x +
 2*(2*a^3*b + 3*a*b^3)*c^2)*sinh(1) - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(
((2*a^2*b^2 + b^4)*d*x + (2*a^2*b^2 + b^4)*c)*cosh(1) + ((2*a^2*b^2 + b^4)*
d*x + (2*a^2*b^2 + b^4)*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 - 1)) + ((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*x^2 + 2*(2*a^4 + 6*a^2*b^2 +
3*b^4)*c*d*x)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(((2*a^3*b + 3*
a*b^3)*d*x + (2*a^3*b + 3*a*b^3)*c)*cosh(1) + ((2*a^3*b + 3*a*b^3)*d*x + (2
*a^3*b + 3*a*b^3)*c)*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(196) = 392$.
time = 0.58, size = 1027, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*acosh(c + d*x)/d
+ 4*a**3*b*c*e*x*acosh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)/d + 2*a**3*b*d*e*x**2*acosh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1) - a**3*b*e*acosh(c + d*x)/d + 3*a**2*b**2*c**2*e*acosh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*acosh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*acosh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x) - 3*a**2*b**2*e*acosh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*acosh(c + d*x)**3/d + 3*a*b**3*c**2*e*
acosh(c + d*x)/d + 4*a*b**3*c*e*x*acosh(c + d*x)**3 + 6*a*b**3*c*e*x*acosh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d) + 2*a*b**3*
d*e*x**2*acosh(c + d*x)**3 + 3*a*b**3*d*e*x**2*acosh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/2 - a*b**3*e*acosh(c + d*x)**3/d - 3*a*b*
*3*e*acosh(c + d*x)/(2*d) + b**4*c**2*e*acosh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*acosh(c + d*x)**2/(2*d) + b**4*c*e*x*acosh(c + d*x)**4 + 3*b**4*c*e*x*
acosh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
1)*acosh(c + d*x)/(2*d) + b**4*d*e*x**2*acosh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*acosh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/2 - b**4*e*acosh(c + d*x)**4/(4*d) - 3*b**4*e*acosh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,{\left(a+b\,\mathrm{acosh}(c+d\,x)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4, x)
```

3.125 $\int \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=129

$$24b^4x-\frac{24b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{}}{}$$

```
[Out] 24*b^4*x+12*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^2/d+(d*x+c)*(a+b*arccosh(d*x+c
))^4/d-24*b^3*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-4*b*(a
+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{5995, 5879, 5915, 8\}$

$$-\frac{24b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4}{d}+24b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x]))/d + (12*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d - (4*b*Sqrt[-1 +
 c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/d + ((c + d*x)*(a +
 b*ArcCosh[c + d*x])^4)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^4\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4}{d} - \frac{(4b)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x\right.}{d}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+\right.}{}$$

$$= \frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a\right.}{d}$$

$$= -\frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)}{}$$

$$= 24b^4x - \frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(129) = 258.
time = 0.16, size = 261, normalized size = 2.02

$$\frac{(a^4+12a^2b^2+24b^4)(c+dx) - 4ab(a^2+6b^2)\sqrt{-1+c+dx}\sqrt{1+c+dx} - 4b\left(-a^3(c+dx) - 6ab^2(c+dx) + 3a^2b\sqrt{-1+c+dx}\sqrt{1+c+dx} + 6b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx) + 6b^2\left(a^2(c+dx)+2b^2(c+dx)-2ab\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^2 - 4b^3\left(-a(c+dx)+b\sqrt{-1+c+dx}\sqrt{1+c+dx}\right)\cosh^{-1}(c+dx)^3 + b^4(c+dx)\cosh^{-1}(c+dx)^4}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*
b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x])*ArcCosh[c + d*x] + 6*b^2*(a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 4*b^3*(-(a*(c +
 d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(
c + d*x)*ArcCosh[c + d*x]^4)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(121) = 242$.
time = 43.27, size = 275, normalized size = 2.13

method	result
derivativedivides	$\frac{(dx+c)a^4+b^4\left(\operatorname{arccosh}(dx+c)^4(dx+c)-4\sqrt{dx+c-1}\,\operatorname{arccosh}(dx+c)^3\sqrt{dx+c+1}+12\operatorname{arccosh}(dx+c)^2(dx+c\right.}{}$
default	$\frac{(dx+c)a^4+b^4\left(\operatorname{arccosh}(dx+c)^4(dx+c)-4\sqrt{dx+c-1}\,\operatorname{arccosh}(dx+c)^3\sqrt{dx+c+1}+12\operatorname{arccosh}(dx+c)^2(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*(arccosh(d*x+c)^4*(d*x+c)-4*(d*x+c-1)^(1/2)*arccosh(d*
x+c)^3*(d*x+c+1)^(1/2)+12*arccosh(d*x+c)^2*(d*x+c)-24*arccosh(d*x+c)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+24*d*x+24*c)+4*a*b^3*(arccosh(d*x+c)^3*(d*x+c)-3*
arccosh(d*x+c)^2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))+6*a^2*b^2*(arccosh(d*x+c)^2*(d*x+c)-2*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+4*a^3*b*((d*x+c)*arc
cosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4 + a^4*x + 4*((d*
x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b/d + integrate(2*(2*(
(c^3 - c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2)
*x^2 + ((c^2 - 1)*a*b^3 + (a*b^3*d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*
d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^3 - (c^2*d -
 d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*(a^2*b
^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d)*a^2*b^2*x + (c^3 - c)*a^2*
b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 - 1)*a^2*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^
2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(121) = 242$.
time = 0.37, size = 344, normalized size = 2.67

$$\frac{(b^4dx+b^4c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^4+4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2-1}\,b^4\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^3+(a^4+12\,a^2b^2+24\,b^4)dx-6\left(2\sqrt{d^2x^2+2\,cdx+c^2-1}\,ab^3-(a^2b^2+2\,b^4)dx-(a^2b^2+2\,b^4)c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2+4\left((a^3b+6\,ab^3)dx+(a^3b+6\,ab^3)c-3\,(a^2b^2+2\,b^4)\sqrt{d^2x^2+2\,cdx+c^2-1}\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-4\,(a^3b+6\,ab^3)\sqrt{d^2x^2+2\,cdx+c^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(122) = 244$.
time = 0.31, size = 444, normalized size = 3.44

$$\begin{cases} a^4x + \frac{4a^3bc\operatorname{acosh}(c+dx)}{d} + 4a^3bx\operatorname{acosh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2-1}}{d} + \frac{6a^2b^2c\operatorname{acosh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{acosh}^2(c+dx) + 12a^2b^2x - \frac{12a^2b^2\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}(c+dx)}{d} + \frac{4ab^3c\operatorname{acosh}^3(c+dx)}{d} + \frac{24ab^3c\operatorname{acosh}(c+dx)}{d} + 4ab^3x\operatorname{acosh}^3(c+dx) + 24ab^3x\operatorname{acosh}(c+dx) - \frac{12ab^3\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}^2(c+dx)}{d} - \frac{24ab^3\sqrt{c^2+2cdx+d^2x^2-1}}{d} + \frac{b^4c\operatorname{acosh}^4(c+dx)}{d} + \frac{12b^4c\operatorname{acosh}^2(c+dx)}{d} + b^4x\operatorname{acosh}^4(c+dx) + 12b^4x\operatorname{acosh}^2(c+dx) + 24b^4x - \frac{4b^4\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}^3(c+dx)}{d} - \frac{24b^4\sqrt{c^2+2cdx+d^2x^2-1}\operatorname{acosh}(c+dx)}{d} & \text{for}\: d \neq 0 \\ x(a+b\operatorname{acosh}(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*acosh(c + d*x)/d + 4*a**3*b*x*acosh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 6*a**2*b**2*c*acosh(c
+ d*x)**2/d + 6*a**2*b**2*x*acosh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 4*a*b**3*c*acosh
(c + d*x)**3/d + 24*a*b**3*c*acosh(c + d*x)/d + 4*a*b**3*x*acosh(c + d*x)**
3 + 24*a*b**3*x*acosh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d
 + b**4*c*acosh(c + d*x)**4/d + 12*b**4*c*acosh(c + d*x)**2/d + b**4*x*acos
h(c + d*x)**4 + 12*b**4*x*acosh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**4, Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4,x)
[Out] int((a + b*acosh(c + d*x))^4, x)
```

3.126 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{ce+dex}\,dx$

Optimal. Leaf size=192

$$\frac{(a+b\cosh^{-1}(c+dx))^5}{5bde}+\frac{(a+b\cosh^{-1}(c+dx))^4\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)}{de}-\frac{2b(a+b\cosh^{-1}(c+dx))^3\,\mathrm{Pol}}{de}$$

```
[Out] 1/5*(a+b*arccosh(d*x+c))^5/b/d/e+(a+b*arccosh(d*x+c))^4*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-2*b*(a+b*arccosh(d*x+c))^3*polylog(2,-1/
(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3*b^2*(a+b*arccosh(d*x+c))^2
*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3*b^3*(a+b*arc
cosh(d*x+c))*polylog(4,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/
2*b^4*polylog(5,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.21, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5882, 3799, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3b^3\mathrm{Li}_4\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de}-\frac{3b^2\mathrm{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^2}{de}-\frac{2b\mathrm{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^3}{de}+\frac{(a+b\cosh^{-1}(c+dx))^5}{5bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)(a+b\cosh^{-1}(c+dx))^4}{de}-\frac{3b^4\mathrm{Li}_5\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcCosh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcCosh[c + d*x])^4*Log[1 +
E^(-2*ArcCosh[c + d*x])])/(d*e) - (2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2
, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])^2*Poly
Log[3, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^3*(a + b*ArcCosh[c + d*x])*P
olyLog[4, -E^(-2*ArcCosh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, -E^(-2*ArcCo
sh[c + d*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x}\,dx, x, c+dx\right)}{de} \\
&= \frac{\text{Subst}\left(\int (a+bx)^4 \tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4 \log\left(1+e^{2\cosh^{-1}(c+dx}\right.}{de}
\end{aligned}
$$

Mathematica [**A**]
time = 0.52, size = 308, normalized size = 1.60

$$
\frac{2a^3b\cosh^{-1}(c+dx)^2 + 2a^2b^2\cosh^{-1}(c+dx)^3 + ab^3\cosh^{-1}(c+dx)^4 + \frac{1}{5}b^4\cosh^{-1}(c+dx)^5 + 4a^3b\cosh^{-1}(c+dx)\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right) + 6a^2b^2\cosh^{-1}(c+dx)^2\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right) + 4ab^3\cosh^{-1}(c+dx)^3\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right) + b^4\cosh^{-1}(c+dx)^4\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right) + a^4\log(c+dx) - 2b\left(a+b\cosh^{-1}(c+dx)\right)^3\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right) - 3b^2\left(a+b\cosh^{-1}(c+dx)\right)^2\text{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right) - 3ab^3\text{PolyLog}\left(4,-e^{-2\cosh^{-1}(c+dx)}\right) - 3b^4\cosh^{-1}(c+dx)\text{PolyLog}\left(4,-e^{-2\cosh^{-1}(c+dx)}\right) - \frac{3}{2}b^4\text{PolyLog}\left(5,-e^{-2\cosh^{-1}(c+dx)}\right)}{de}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (2*a^3*b*ArcCosh[c + d*x]^2 + 2*a^2*b^2*ArcCosh[c + d*x]^3 + a*b^3*ArcCosh[
c + d*x]^4 + (b^4*ArcCosh[c + d*x]^5)/5 + 4*a^3*b*ArcCosh[c + d*x]*Log[1 +
E^(-2*ArcCosh[c + d*x])] + 6*a^2*b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCo
sh[c + d*x])] + 4*a*b^3*ArcCosh[c + d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])]
 + b^4*ArcCosh[c + d*x]^4*Log[1 + E^(-2*ArcCosh[c + d*x])] + a^4*Log[c + d*
```

x] - 2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] - 3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*a*b^3*PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - 3*b^4*ArcCosh[c + d*x]*PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - (3*b^4*PolyLog[5, -E^(-2*ArcCosh[c + d*x])])/2)/(d*e)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(258) = 516$.
time = 47.61, size = 674, normalized size = 3.51

method	result
derivativedivides	$\frac{\frac{a^4\ln(dx+c)}{e}-\frac{b^4\operatorname{arccosh}(dx+c)^5}{5e}+\frac{b^4\operatorname{arccosh}(dx+c)^4\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{2b^4\operatorname{arccosh}(dx+}{}}{}$
default	$\frac{\frac{a^4\ln(dx+c)}{e}-\frac{b^4\operatorname{arccosh}(dx+c)^5}{5e}+\frac{b^4\operatorname{arccosh}(dx+c)^4\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{e}+\frac{2b^4\operatorname{arccosh}(dx+}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4/e*ln(d*x+c)-1/5*b^4/e*arccosh(d*x+c)^5+b^4/e*arccosh(d*x+c)^4*ln(1
+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+2*b^4/e*arccosh(d*x+c)^3*polylo
g(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3*b^4/e*arccosh(d*x+c)^2*po
lylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3*b^4/e*arccosh(d*x+c)*
polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2*b^4/e*polylog(5,-
(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-a*b^3/e*arccosh(d*x+c)^4+4*a*b^3
/e*arccosh(d*x+c)^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+6*a*b^3
/e*arccosh(d*x+c)^2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-6
*a*b^3/e*arccosh(d*x+c)*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^
2)+3*a*b^3/e*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-2*a^2*b^
2/e*arccosh(d*x+c)^3+6*a^2*b^2/e*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2))^2)+6*a^2*b^2/e*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))^2)-3*a^2*b^2/e*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)-2*a^3*b/e*arccosh(d*x+c)^2+4*a^3*b/e*arccosh(d*x+c)*ln(
1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+2*a^3*b/e*polylog(2,-(d*x+c+(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^4*e^(-1)*log(d*x*e + c*e)/d + integrate(b^4*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)^4/(d*x*e + c*e) + 4*a*b^3*log(d*x + sqrt(d*x + c + 1)
*sqrt(d*x + c - 1) + c)^3/(d*x*e + c*e) + 6*a^2*b^2*log(d*x + sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + c)^2/(d*x*e + c*e) + 4*a^3*b*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c)/(d*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*acosh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*acosh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*acosh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*acosh(c + d*x)/(c + d
*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x), x)
```

3.127 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^2}\,dx$

Optimal. Leaf size=264

$$-\frac{(a+b\cosh^{-1}(c+dx))^4}{de^2(c+dx)}+\frac{8b(a+b\cosh^{-1}(c+dx))^3\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{12ib^2(a+b\cosh^{-1}(c+dx))^2}{de^2}$$

```
[Out] -(a+b*arccosh(d*x+c))^4/d/e^2/(d*x+c)+8*b*(a+b*arccosh(d*x+c))^3*arctan(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-12*I*b^2*(a+b*arccosh(d*x+c))^2*p
olylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+12*I*b^2*(a+b*ar
ccosh(d*x+c))^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+
24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)))/d/e^2-24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,I*(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)))/d/e^2-24*I*b^4*polylog(4,-I*(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)))/d/e^2+24*I*b^4*polylog(4,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)))/d/e^2
```

Rubi [A]
time = 0.28, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5883, 5947, 4265, 2611, 6744, 2320, 6724}

$$\frac{8b\,\mathrm{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^3}{de^2}+\frac{24ib^3\mathrm{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^2}-\frac{24ib^3\mathrm{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^2}-\frac{12ib^2\mathrm{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^2}{de^2}+\frac{12ib^2\mathrm{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))^2}{de^2}-\frac{(a+b\cosh^{-1}(c+dx))^4}{de^2(c+dx)}-\frac{24ib^4\mathrm{Li}_4\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{24ib^4\mathrm{Li}_4\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^4/(d*e^2*(c + d*x))) + (8*b*(a + b*ArcCosh[c + d
*x])^3*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((12*I)*b^2*(a + b*ArcCosh[c +
 d*x])^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((12*I)*b^2*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^3
*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((
24*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^2
) - ((24*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^4*
PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^2x^2}\,dx,x,c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^2}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{(4b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx,x,c+dx\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{(4b)\text{Subst}\left(\int (a+bx)^3\text{sech}(x)\,dx,x,\cosh^{-1}(c+\right.}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} \\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)}+\frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. 2(264) = 528.

time = 1.64, size = 872, normalized size = 3.30

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^4/(c + d*x)) + 4*a^3*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[
ArcCosh[c + d*x]/2]]) - (6*I)*a^2*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c +
d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c
 + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCos
h[c + d*x]]) + 4*a*b^3*(-(ArcCosh[c + d*x]^3/(c + d*x)) + (3*I)*(-(ArcCosh[
c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c + d*x]]))
 - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyLog[2, I/E
^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*PolyLog[3,
I/E^ArcCosh[c + d*x]])) + b^4*(((-7*I)/16)*Pi^4 + (Pi^3*ArcCosh[c + d*x])/2
 - ((3*I)/2)*Pi^2*ArcCosh[c + d*x]^2 - 2*Pi*ArcCosh[c + d*x]^3 + I*ArcCosh[
c + d*x]^4 - ArcCosh[c + d*x]^4/(c + d*x) + (Pi^3*Log[1 + I/E^ArcCosh[c + d
*x]])/2 - (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - 6*Pi*
ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + (4*I)*ArcCosh[c + d*x]^3
*Log[1 + I/E^ArcCosh[c + d*x]] + (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 - I*E^Ar
cCosh[c + d*x]] + 6*Pi*ArcCosh[c + d*x]^2*Log[1 - I*E^ArcCosh[c + d*x]] - (
Pi^3*Log[1 + I*E^ArcCosh[c + d*x]])/2 - (4*I)*ArcCosh[c + d*x]^3*Log[1 + I*
E^ArcCosh[c + d*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[c + d*x])/4]])/2 +
(3*I)*(Pi - (2*I)*ArcCosh[c + d*x])^2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] -
 (12*I)*ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + (3*I)*Pi^2
*PolyLog[2, I*E^ArcCosh[c + d*x]] + 12*Pi*ArcCosh[c + d*x]*PolyLog[2, I*E^A
rcCosh[c + d*x]] + 12*Pi*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - (24*I)*ArcCo
sh[c + d*x]*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + (24*I)*ArcCosh[c + d*x]*P
olyLog[3, (-I)*E^ArcCosh[c + d*x]] - 12*Pi*PolyLog[3, I*E^ArcCosh[c + d*x]]
 - (24*I)*PolyLog[4, (-I)/E^ArcCosh[c + d*x]] - (24*I)*PolyLog[4, (-I)*E^Ar
cCosh[c + d*x]]))/(d*e^2)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^4}{(dex + ce)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^2*x*e^2 + c*d*
e^2) - 4*a^3*b*(arcsin(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arccosh(d
*x + c)/(d^2*x*e^2 + c*d*e^2)) - a^4/(d^2*x*e^2 + c*d*e^2) + integrate(2*(2
*((c^3 - c)*a*b^3 + (c^3 - c)*b^4 + (a*b^3*d^3 + b^4*d^3)*x^3 + 3*(a*b^3*c*
d^2 + b^4*c*d^2)*x^2 + (b^4*c^2 + (c^2 - 1)*a*b^3 + (a*b^3*d^2 + b^4*d^2)*x
^2 + 2*(a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c
^2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) + c)^3 + 3*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d
)*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2
 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + c)^2)/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3
 - d^3)*x^3*e^2 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^2 + (5*c^4*d - 3*c^2*d)*x*e^
2 + (d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*x^2*e^2 + 2*(2*c^3*d
 - c*d)*x*e^2 + (c^4 - c^2)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^5
 - c^3)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*e^(-2)/(d^2*x^2 + 2*c*d
*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4\operatorname{acosh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3\operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*acosh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*acosh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*acosh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^4}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

3.128 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=195

$$-\frac{2b(a+b\cosh^{-1}(c+dx))^3}{de^3}+\frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(a+b\cosh^{-1}(c+dx))^3}{de^3(c+dx)}-\frac{(a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] -2*b*(a+b*arccosh(d*x+c))^3/d/e^3-1/2*(a+b*arccosh(d*x+c))^4/d/e^3/(d*x+c)^
2-6*b^2*(a+b*arccosh(d*x+c))^2*ln(1+1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2)/d/e^3+6*b^3*(a+b*arccosh(d*x+c))*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)/d/e^3+3*b^4*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))^2)/d/e^3+2*b*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.32, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5996, 12, 5883, 5918, 5882, 3799, 2221, 2611, 2320, 6724}

$$\frac{6b^3\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)(a+b\cosh^{-1}(c+dx))}{de^3}-\frac{6b^2\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)(a+b\cosh^{-1}(c+dx))^2}{de^3}+\frac{2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(a+b\cosh^{-1}(c+dx))^3}{de^3(c+dx)}-\frac{2b(a+b\cosh^{-1}(c+dx))^3}{de^3}-\frac{(a+b\cosh^{-1}(c+dx))^4}{2de^3(c+dx)^2}+\frac{3b^4\text{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-2*b*(a + b*ArcCosh[c + d*x])^3)/(d*e^3) + (2*b*Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcCosh[c
 + d*x])^4/(2*d*e^3*(c + d*x)^2) - (6*b^2*(a + b*ArcCosh[c + d*x])^2*Log[1
+ E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (6*b^3*(a + b*ArcCosh[c + d*x])*PolyL
og[2, -E^(-2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^4*PolyLog[3, -E^(-2*ArcCosh
[c + d*x])])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Sim
p[(d2 + e2*x)^p/(-1 + c*x)^p], Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 +
c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2, f, m, p}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0]
&& EqQ[m + 2*p + 3, 0] && NeQ[p, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^3x^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^3}\,dx,x,c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,x,c+dx\right.}{de^3}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{de^3(c+dx)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(195) = 390.
time = 1.54, size = 398, normalized size = 2.04

$$\frac{-\frac{a^4}{(c+dx)^2}+\frac{4a^3b\sqrt{-1+c+dx}\sqrt{1+c+dx}}{c+dx}-\frac{4a^3b\cosh^{-1}(c+dx)}{(c+dx)^2}-\frac{b^4\cosh^{-1}(c+dx)^4}{(c+dx)^2}+12a^2b^2\left(\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\cosh^{-1}(c+dx)}{c+dx}-\frac{\cosh^{-1}(c+dx)^2}{2(c+dx)^2}-\log(c+dx)\right)+4ab^3\left(-\cosh^{-1}(c+dx)\left(3\cosh^{-1}(c+dx)-\frac{3\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\cosh^{-1}(c+dx)}{c+dx}+\frac{\cosh^{-1}(c+dx)^2}{(c+dx)^2}+6\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)\right)+3\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)\right)+2b^4\left(2\cosh^{-1}(c+dx)^2\left(-\cosh^{-1}(c+dx)+\frac{\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\cosh^{-1}(c+dx)}{c+dx}-3\log\left(1+e^{-2\cosh^{-1}(c+dx)}\right)\right)+6\cosh^{-1}(c+dx)\mathrm{PolyLog}\left(2,-e^{-2\cosh^{-1}(c+dx)}\right)+3\mathrm{PolyLog}\left(3,-e^{-2\cosh^{-1}(c+dx)}\right)\right)}{2de^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^4/(c + d*x)^2) + (4*a^3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d
*x) - (4*a^3*b*ArcCosh[c + d*x])/(c + d*x)^2 - (b^4*ArcCosh[c + d*x]^4)/(c
+ d*x)^2 + 12*a^2*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Ar
cCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2) - Log[c + d*
x]) + 4*a*b^3*(-(ArcCosh[c + d*x]*(3*ArcCosh[c + d*x] - (3*Sqrt[(-1 + c + d
*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) + ArcCosh[c +
d*x]^2/(c + d*x)^2 + 6*Log[1 + E^(-2*ArcCosh[c + d*x])])) + 3*PolyLog[2, -E
^(-2*ArcCosh[c + d*x])]) + 2*b^4*(2*ArcCosh[c + d*x]^2*(-ArcCosh[c + d*x] +
 (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d
*x) - 3*Log[1 + E^(-2*ArcCosh[c + d*x])]) + 6*ArcCosh[c + d*x]*PolyLog[2, -
E^(-2*ArcCosh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcCosh[c + d*x])]))/(2*d*e^
3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(231) = 462.
time = 56.55, size = 551, normalized size = 2.83

method	result
derivativedivides	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{2b^4\,\mathrm{arccosh}(dx+c)^3\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{e^3(dx+c)}+\frac{2b^4\,\mathrm{arccosh}(dx+c)^3}{e^3}-\frac{b^4\,\mathrm{arccosh}(dx+c)^4}{2e^3(dx+c)^2}-\frac{6b^4\,\mathrm{arccos}}{}$
default	$-\frac{a^4}{2e^3(dx+c)^2}+\frac{2b^4\,\mathrm{arccosh}(dx+c)^3\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{e^3(dx+c)}+\frac{2b^4\,\mathrm{arccosh}(dx+c)^3}{e^3}-\frac{b^4\,\mathrm{arccosh}(dx+c)^4}{2e^3(dx+c)^2}-\frac{6b^4\,\mathrm{arccos}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^4/e^3/(d*x+c)^2+2*b^4/e^3*arccosh(d*x+c)^3/(d*x+c)*(d*x+c+1)^(1
/2)*(d*x+c-1)^(1/2)+2*b^4/e^3*arccosh(d*x+c)^3-1/2*b^4/e^3*arccosh(d*x+c)^4
/(d*x+c)^2-6*b^4/e^3*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))^2)-6*b^4/e^3*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2))^2)+3*b^4/e^3*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
^2)+6*a*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)+6*
a*b^3/e^3*arccosh(d*x+c)^2-2*a*b^3/e^3*arccosh(d*x+c)^3/(d*x+c)^2-12*a*b^3/
```

```
e^3*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-6*a*b^3/
e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+6*a^2*b^2/e^3*arc
cosh(d*x+c)+6*a^2*b^2/e^3*arccosh(d*x+c)/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^
(1/2)-3*a^2*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^2-6*a^2*b^2/e^3*ln(1+(d*x+c+(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+4*a^3*b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c
)+1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^3*x^2*e^3
+ 2*c*d^2*x*e^3 + c^2*d*e^3) + 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arcco
sh(d*x + c)/(d^3*x*e^3 + c*d^2*e^3) - e^(-3)*log(d*x + c)/d)*a^2*b^2 + 2*a^
3*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*x*e^3 + c*d^2*e^3) - arccosh(
d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 3*a^2*b^2*arccosh(d*x
 + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 1/2*a^4/(d^3*x^2*e^3 +
2*c*d^2*x*e^3 + c^2*d*e^3) + integrate(2*(2*(c^3 - c)*a*b^3 + (c^3 - c)*b^4
 + (2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (b^4*c
^2 + 2*(c^2 - 1)*a*b^3 + (2*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4
*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (2*(3*c^2*d - d)*a*b^3 + (3*
c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^
6*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 - d^4)*x^4*e^3 + 4*(5*c^3*d^3 - c
*d^3)*x^3*e^3 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d - 2*c^3*d)*x
*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*x^3*e^3 + (10*c^
3*d^2 - 3*c*d^2)*x^2*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 + (c^5 - c^3)*e^3)*sqr
t(d*x + c + 1)*sqrt(d*x + c - 1) + (c^6 - c^4)*e^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*e^(-3)/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{4}}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int \frac{b^{4}\operatorname{acosh}^{4}(c+dx)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int \frac{4ab^{3}\operatorname{acosh}^{3}(c+dx)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int \frac{6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx+\int \frac{4a^{3}b\operatorname{acosh}(c+dx)}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\,dx}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*acosh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^4}{(c\,e+d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

3.129 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^4}\,dx$

Optimal. Leaf size=432

$$\frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)}+\frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3}$$

```
[Out] 2*b^2*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^4/d/e^4
/(d*x+c)^3-8*b^3*(a+b*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2))/d/e^4+4/3*b*(a+b*arccosh(d*x+c))^3*arctan(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))/d/e^4+4*I*b^4*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)))/d/e^4-2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,-I*(d*x+c+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^4*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)))/d/e^4+2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,I*(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*b^3*(a+b*arccosh(d*x+c))*polylog(3
,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^3*(a+b*arccosh(d*x
+c))*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^4*pol
ylog(4,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*b^4*polylog(4,
I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+2/3*b*(a+b*arccosh(d*x+c))
^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.55, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$,
Rules used = {5996, 12, 5883, 5933, 5947, 4265, 2611, 6744, 2320, 6724, 2317, 2438}

$$-\frac{8b^3\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{4b\text{ArcTan}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4}+\frac{4ib^3\text{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{4ib^3\text{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{2ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4}+\frac{2ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4}+\frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)}+\frac{2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3}+\frac{4ib^4\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^4}-\frac{4ib^4\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^4}-\frac{4ib^4\text{Li}_4\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^4}+\frac{4ib^4\text{Li}_4\left(ie^{\cosh^{-1}(c+dx)}\right)}{de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (2*b^2*(a + b*ArcCosh[c + d*x])^2)/(d*e^4*(c + d*x)) + (2*b*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (
a + b*ArcCosh[c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcCosh[c +
 d*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcCosh[c + d*x])^
3*ArcTan[E^ArcCosh[c + d*x]])/(3*d*e^4) + ((4*I)*b^4*PolyLog[2, (-I)*E^ArcC
osh[c + d*x]])/(d*e^4) - ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, (
-I)*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[2, I*E^ArcCosh[c + d*
x]])/(d*e^4) + ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh
[c + d*x]])/(d*e^4) + ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E
^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3
, I*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c +
 d*x]])/(d*e^4) + ((4*I)*b^4*PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5933

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)
*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(d1*d2*f*(
m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[b*c*(n/(f*
(m + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p],
 Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[
c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}, x] && EqQ[
e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5947

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(1/c^(m + 1))*Simp[
Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Subst[
Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx, x, c+\right.}{3de^4}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c\right.}{3de^4(c+d}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-}\right.}{3de^4(c+dx)^2}$$

<u>Mathematica</u> [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1309 vs. 2(432) = 864.
time = 7.42, size = 1309, normalized size = 3.03

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (-(a^4/(c + d*x)^3) + 2*a^3*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c +
 d*x))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCo
```

```
sh[c + d*x]/2]]) + 6*a^2*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c
+ d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c
 + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c
+ d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]
] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + 12*a*b^3*(ArcCosh[c + d*x]/(c + d
*x) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^2)
/(2*(c + d*x)^2) - ArcCosh[c + d*x]^3/(3*(c + d*x)^3) - 2*ArcTan[c + d*x +
Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)] + ArcCosh[c + d*x]^2*ArcT
an[c + d*x + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)] - I*ArcCosh[
c + d*x]*PolyLog[2, (-I)*(c + d*x + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x))] + I*ArcCosh[c + d*x]*PolyLog[2, I*(c + d*x + Sqrt[(-1 + c + d*x
)/(1 + c + d*x)]*(1 + c + d*x))] + I*PolyLog[3, (-I)*(c + d*x + Sqrt[(-1 +
c + d*x)/(1 + c + d*x)]*(1 + c + d*x))] - I*PolyLog[3, I*(c + d*x + Sqrt[(-
1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))]) + 3*b^4*(((-7*I)/96)*Pi^4 + (P
i^3*ArcCosh[c + d*x])/12 - (I/4)*Pi^2*ArcCosh[c + d*x]^2 + (2*ArcCosh[c + d
*x]^2)/(c + d*x) - (Pi*ArcCosh[c + d*x]^3)/3 + (2*Sqrt[(-1 + c + d*x)/(1 +
c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^3)/(3*(c + d*x)^2) + (I/6)*ArcCosh
[c + d*x]^4 - ArcCosh[c + d*x]^4/(3*(c + d*x)^3) + (4*I)*ArcCosh[c + d*x]*L
og[1 - I/E^ArcCosh[c + d*x]] + (Pi^3*Log[1 + I/E^ArcCosh[c + d*x]])/12 - (4
*I)*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - (I/2)*Pi^2*ArcCosh[c +
 d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - Pi*ArcCosh[c + d*x]^2*Log[1 + I/E^Arc
Cosh[c + d*x]] + ((2*I)/3)*ArcCosh[c + d*x]^3*Log[1 + I/E^ArcCosh[c + d*x]]
 + (I/2)*Pi^2*ArcCosh[c + d*x]*Log[1 - I*E^ArcCosh[c + d*x]] + Pi*ArcCosh[c
 + d*x]^2*Log[1 - I*E^ArcCosh[c + d*x]] - (Pi^3*Log[1 + I*E^ArcCosh[c + d*x
]])/12 - ((2*I)/3)*ArcCosh[c + d*x]^3*Log[1 + I*E^ArcCosh[c + d*x]] + (Pi^3
*Log[Tan[(Pi + (2*I)*ArcCosh[c + d*x])/4]])/12 + (I/2)*(8 + Pi^2 - (4*I)*Pi
*ArcCosh[c + d*x] - 4*ArcCosh[c + d*x]^2)*PolyLog[2, (-I)/E^ArcCosh[c + d*x
]] - (4*I)*PolyLog[2, I/E^ArcCosh[c + d*x]] - (2*I)*ArcCosh[c + d*x]^2*Poly
Log[2, (-I)*E^ArcCosh[c + d*x]] + (I/2)*Pi^2*PolyLog[2, I*E^ArcCosh[c + d*x
]] + 2*Pi*ArcCosh[c + d*x]*PolyLog[2, I*E^ArcCosh[c + d*x]] + 2*Pi*PolyLog[
3, (-I)/E^ArcCosh[c + d*x]] - (4*I)*ArcCosh[c + d*x]*PolyLog[3, (-I)/E^ArcC
osh[c + d*x]] + (4*I)*ArcCosh[c + d*x]*PolyLog[3, (-I)*E^ArcCosh[c + d*x]]
- 2*Pi*PolyLog[3, I*E^ArcCosh[c + d*x]] - (4*I)*PolyLog[4, (-I)/E^ArcCosh[c
 + d*x]] - (4*I)*PolyLog[4, (-I)*E^ArcCosh[c + d*x]]))/(3*d*e^4)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccosh}(dx + c))^4}{(dex + ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^4*x^3*e^4
+ 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^4/(d^4*x^3*e^4 + 3
*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 -
c)*a*b^3 + (c^3 - c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 +
 b^4*c*d^2)*x^2 + (b^4*c^2 + 3*(c^2 - 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^
2 + 2*(3*a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(
3*c^2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d
- d)*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (
c^2 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (
3*c^2*d - d)*a^3*b*x + (c^3 - c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (
c^2 - 1)*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c))/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5
- d^5)*x^5*e^4 + 5*(7*c^3*d^4 - c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*
x^3*e^4 + (21*c^5*d^2 - 10*c^3*d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (
d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 -
 c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)
*x*e^4 + (c^6 - c^4)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)
*e^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*e^(-4)/(d^4*x^4 + 4*c*d
^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\operatorname{acosh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{6a^2b^2\operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4a^3b\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*acosh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*acosh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c + d\,x))^4}{(c\,e + d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

3.130 $\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$-\frac{e^4\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8bd}-\frac{3e^4\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16bd}-\frac{e^4\mathrm{Chi}\left(\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{5a}{b}\right.}{16bd}$$

```
[Out] 1/8*e^4*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d+3/16*e^4*cosh(3*a/b)*Shi(
3*(a+b*arccosh(d*x+c))/b)/b/d+1/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arccosh(d*x+c
))/b)/b/d-1/8*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b/d-3/16*e^4*Chi(3*
(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Chi(5*(a+b*arccosh(d*x+c))
/b)*sinh(5*a/b)/b/d
```

Rubi [A]
time = 0.30, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5887, 5556, 3384, 3379, 3382}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8bd}+\frac{3e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/8*(e^4*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d) - (3*e^
4*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(16*b*d) - (e
^4*CoshIntegral[(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/(16*b*d) + (
e^4*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(8*b*d) + (3*e^4*Co
sh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(16*b*d) + (e^4*C
osh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c + d*x]))/b])/(16*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{x^4}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}+\frac{\sinh(5x)}{16(a+bx)}\right)\,dx,x,\cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{16d}+\frac{e^4\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{8d} \\
&= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d}+\frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Sub}}{} \\
&= -\frac{e^4\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8bd}-\frac{3e^4\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh}{16bd}
\end{aligned}
$$

Mathematica [A]

time = 0.21, size = 151, normalized size = 0.71

$$
\frac{e^4\left(-2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{5a}{b}\right)+2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{16bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])]))/(16*b*d)
```

Maple [A]

time = 58.10, size = 194, normalized size = 0.91

method	result
derivativedivides	$\frac{e^4 e^{\frac{5a}{b}}\,\text{expIntegral}\left(1,5\,\text{arccosh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(dx+c)+\frac{3a}{b}\right)}{32b}+\frac{e^4 e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right.}{16b}$
default	$\frac{e^4 e^{\frac{5a}{b}}\,\text{expIntegral}\left(1,5\,\text{arccosh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arccosh}(dx+c)+\frac{3a}{b}\right)}{32b}+\frac{e^4 e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right.}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/32*e^4/b*exp(5*a/b)*Ei(1,5*arccosh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*e^4/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-3/32*e^4/b*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="maxima")

[Out] integrate((d*x*e + c*e)^4/(b*arccosh(d*x + c) + a), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="fricas")

[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b*arccosh(d*x + c) + a), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int\frac{c^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{d^4x^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{4cd^3x^3}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{6c^2d^2x^2}{a+b\operatorname{acosh}(c+dx)}\,dx+\int\frac{4c^3dx}{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c)),x)

[Out] e**4*(Integral(c**4/(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*acosh(c + d*x)), x) + Integral(6*c**2*d**2*x**2/(a + b*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(a + b*acosh(c + d*x)), x))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)), x)
```

3.131 $\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{e^3\text{Chi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)}{4bd}$$

```
[Out] 1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*S
hi(4*(a+b*arccosh(d*x+c))/b)/b/d-1/4*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*sinh
(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b/d
```

Rubi [A]
time = 0.24, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5887, 5556, 3384, 3379, 3382}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)}{4bd}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/4*(e^3*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(b*d)
 - (e^3*CoshIntegral[(4*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(4*a)/b])/(8*b*d)
 + (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(4*b*d)
 + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/(8*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d}+\frac{e^3\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{4d}\\
&= \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Su}}{}\\
&= -\frac{e^3\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)\text{sin}}{8bd}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 109, normalized size = 0.75

$$\frac{e^3\left(-2\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)+2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-2*CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - CoshInteg
ral[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegra
l[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh
[c + d*x])]))/(8*b*d)
```

Maple [A]
time = 49.33, size = 134, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{e^3 e^{\frac{4a}{b}} \text{expIntegral}\left(1,4\,\text{arccosh}(dx+c)+\frac{4a}{b}\right)}{16b}+\frac{e^3 e^{\frac{2a}{b}} \text{expIntegral}\left(1,2\,\text{arccosh}(dx+c)+\frac{2a}{b}\right)}{8b}-\frac{e^3 e^{-\frac{2a}{b}} \text{expIntegral}\left(1,-2\,\text{arccosh}(dx+c)-\right.}{8b}}{d}$
default	$\frac{\frac{e^3 e^{\frac{4a}{b}} \text{expIntegral}\left(1,4\,\text{arccosh}(dx+c)+\frac{4a}{b}\right)}{16b}+\frac{e^3 e^{\frac{2a}{b}} \text{expIntegral}\left(1,2\,\text{arccosh}(dx+c)+\frac{2a}{b}\right)}{8b}-\frac{e^3 e^{-\frac{2a}{b}} \text{expIntegral}\left(1,-2\,\text{arccosh}(dx+c)-\right.}{8b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="fricas")

[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b*arccosh(d*x + c) + a), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^{3}x^{3}}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{3cd^{2}x^{2}}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{3c^{2}dx}{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c)),x)

[Out] e**3*(Integral(c**3/(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*acosh(c + d*x)), x) + Integral(3*c**2*d*x/(a + b*acosh(c + d*x)), x))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="giac")

[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)),x)

[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)), x)

3.132 $\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4bd}-\frac{e^2\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2}{}$$

```
[Out] 1/4*e^2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d+1/4*e^2*cosh(3*a/b)*Shi(3
*(a+b*arccosh(d*x+c))/b)/b/d-1/4*e^2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/
b/d-1/4*e^2*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b/d
```

Rubi [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5996, 12, 5887, 5556, 3384, 3379, 3382}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/4*(e^2*CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d) - (e^2*
CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(4*b*d) + (e^2*
Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(4*b*d) + (e^2*Cosh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{e^2\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{4d}\\
&= \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subs}}{}\\
&= -\frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4bd}-\frac{e^2\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh\left(\right.}{4bd}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 102, normalized size = 0.72

$$\frac{e^2\left(-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) - CoshIntegral[3*(a
/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]))/(4*b
*d)
```

Maple [A]
time = 50.62, size = 130, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{e^2 e^{\frac{3a}{b}} \text{expIntegral}\left(1,3\,\text{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{a}{b}} \text{expIntegral}\left(1,-\text{arccosh}(dx+c)-\frac{a}{b}\right)}{8b}-}{d}$
default	$\frac{\frac{e^2 e^{\frac{3a}{b}} \text{expIntegral}\left(1,3\,\text{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}} \text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right)}{8b}-\frac{e^2 e^{-\frac{a}{b}} \text{expIntegral}\left(1,-\text{arccosh}(dx+c)-\frac{a}{b}\right)}{8b}-}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*E
i(1,arccosh(d*x+c)+a/b)-1/8*e^2/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8*e
^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b*arccosh(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^2x^2}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{2cdx}{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**2*(Integral(c**2/(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
acosh(c + d*x)), x) + Integral(2*c*d*x/(a + b*acosh(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)), x)
```

3.133 $\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{e\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

```
[Out] 1/2*e*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arccos
h(d*x+c))/b)*sinh(2*a/b)/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5996, 12, 5887, 5556, 3384, 3379, 3382}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/2*(e*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(b*d) +
 (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} \\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subs}}{} \\
&= -\frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+\right.}{2bd}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 61, normalized size = 0.88

$$-\frac{e\left(\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]))/(b*d)
```

Maple [A]
time = 51.92, size = 66, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{e\,e^{\frac{2a}{b}}\,\text{expIntegral}\left(1,2\,\text{arccosh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,e^{-\frac{2a}{b}}\,\text{expIntegral}\left(1,-2\,\text{arccosh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66
default	$\frac{\frac{e\,e^{\frac{2a}{b}}\,\text{expIntegral}\left(1,2\,\text{arccosh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,e^{-\frac{2a}{b}}\,\text{expIntegral}\left(1,-2\,\text{arccosh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arccosh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b*arccosh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{dx}{a+b\operatorname{acosh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

```
[Out] e*(Integral(c/(a + b*acosh(c + d*x)), x) + Integral(d*x/(a + b*acosh(c + d*
x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{a+b\operatorname{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x)), x)
```

3.134 $\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bd}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d-Chi((a+b*arccosh(d*x+c))/b)*sinh(
a/b)/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5995, 5881, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d)) + (Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh}{}\right.}{}$$
$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bd} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.84

$$\frac{-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-1),x]
[Out] (-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) + Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c + d*x]])/(b*d)
```

Maple [A]
time = 40.38, size = 60, normalized size = 1.03

method	result	size
derivativedivides	$\frac{\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arccosh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

default	$\frac{\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arccosh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(
d*x+c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acosh}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x)),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x)), x)
```

3.135 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\operatorname{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c)),x)/e
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arccosh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*d*x + b*c)*arccosh(d*x + c)*e + (a*d*x + a*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac+adx+bc\operatorname{acosh}(c+dx)+bdx\operatorname{acosh}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral(1/(a*c + a*d*x + b*c*acosh(c + d*x) + b*d*x*acosh(c + d*x)), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))), x)
```

3.136 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}}{b}\right.}{16b^2d}$$

```
[Out] 1/8*e^4*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d+9/16*e^4*Chi(3*(a+b*arc
cosh(d*x+c))/b)*cosh(3*a/b)/b^2/d+5/16*e^4*Chi(5*(a+b*arccosh(d*x+c))/b)*co
sh(5*a/b)/b^2/d-1/8*e^4*Shi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^2/d-9/16*e^
4*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Shi(5*(a+b*arcco
sh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.27, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 12, 5885, 3384, 3379, 3382}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)}{16b^2d}-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^2d}-\frac{9e^4\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)}{16b^2d}-\frac{e^4\sqrt{c+dx-1}\,(c+dx)^4\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(8
*b^2*d) + (9*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]
)/(16*b^2*d) + (5*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcCosh[c + d*x]
))/b])/(16*b^2*d) - (e^4*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b]
)/(8*b^2*d) - (9*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x])
)/b])/(16*b^2*d) - (5*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c +
d*x]))/b])/(16*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^4\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)} - \frac{9\cosh}{16(a}\right.\right.}{}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh\right.}{8bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}\right.}{a+b}\right.}{8bc}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(\right.}{8b^2d}$$

Mathematica [A]
time = 1.39, size = 293, normalized size = 1.11

$$\frac{e^4\left(-\frac{16b(c+dx)^4\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\cosh^{-1}(c+dx)} - 16\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - 3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - \sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right) + 5\left(10\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + 5\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + \cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - 10\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - 5\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{16b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/
(a + b*ArcCosh[c + d*x]) - 16*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d
*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c + d*x])]) + 5*(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
 + 5*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]
*CoshIntegral[5*(a/b + ArcCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b +
 ArcCosh[c + d*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x]
)] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(16*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(247) = 494$.
time = 50.06, size = 665, normalized size = 2.53

method	result

derivativedivides	$\frac{\left(-16\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^4+12\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2-\sqrt{dx+c-1}\right.}{32b(a+b\,\mathrm{arccosh}(dx+c))}$
default	$\frac{\left(-16\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^4+12\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2-\sqrt{dx+c-1}\right.}{32b(a+b\,\mathrm{arccosh}(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32*(-16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+12*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20*
(d*x+c)^3+5*d*x+5*c)*e^4/b/(a+b*arccosh(d*x+c))-5/32*e^4/b^2*exp(5*a/b)*Ei(
1,5*arccosh(d*x+c)+5*a/b)+3/32*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^
2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4/b/(a+b*arccosh
(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*(-(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^4/b/(a+b*arccosh(d*x+c))-1/16*e^4/b^2*ex
p(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/
b)-3/32/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x
+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/32/b^2*e^4*ex
p(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32/b*e^4*(16*(d*x+c)^5-20*(d*x+c)
^3+16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d
*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*x^5*e^4 + 5*(7*c^3*d^4
 - c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 - 10*c^
3*d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^4
 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^3)*x^3*e^4 + 3*(5*c^4*d^
2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^4 + (c^6 - c^4)*e^4)*sqr
t(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)*e^4)/(a*b*d^3*x^2 + 2*a*b*c*
d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^
2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)) + integrate((5*d^8*x^8*e^4 + 40*c*d^7*x^7*e^4 + 10*(14
```

```
*c^2*d^6 - d^6)*x^6*e^4 + 20*(14*c^3*d^5 - 3*c*d^5)*x^5*e^4 + 5*(70*c^4*d^4
 - 30*c^2*d^4 + d^4)*x^4*e^4 + 20*(14*c^5*d^3 - 10*c^3*d^3 + c*d^3)*x^3*e^4
 + 10*(14*c^6*d^2 - 15*c^4*d^2 + 3*c^2*d^2)*x^2*e^4 + (5*d^6*x^6*e^4 + 30*c
*d^5*x^5*e^4 + 3*(25*c^2*d^4 - d^4)*x^4*e^4 + 4*(25*c^3*d^3 - 3*c*d^3)*x^3*
e^4 + 3*(25*c^4*d^2 - 6*c^2*d^2)*x^2*e^4 + 6*(5*c^5*d - 2*c^3*d)*x*e^4 + (5
*c^6 - 3*c^4)*e^4)*(d*x + c + 1)*(d*x + c - 1) + 20*(2*c^7*d - 3*c^5*d + c^
3*d)*x*e^4 + (10*d^7*x^7*e^4 + 70*c*d^6*x^6*e^4 + (210*c^2*d^5 - 13*d^5)*x^
5*e^4 + 5*(70*c^3*d^4 - 13*c*d^4)*x^4*e^4 + 2*(175*c^4*d^3 - 65*c^2*d^3 + 2
*d^3)*x^3*e^4 + 2*(105*c^5*d^2 - 65*c^3*d^2 + 6*c*d^2)*x^2*e^4 + (70*c^6*d
- 65*c^4*d + 12*c^2*d)*x*e^4 + (10*c^7 - 13*c^5 + 4*c^3)*e^4)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + 5*(c^8 - 2*c^6 + c^4)*e^4)/(a*b*d^4*x^4 + 4*a*b*c*
d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^
2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*
a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*
b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2
*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x
 + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^2
*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{d^{4}x^{4}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{4cd^{3}x^{3}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{6c^{2}d^{2}x^{2}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{4c^{3}dx}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x)
+ b**2*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*acosh(c +
 d*x) + b**2*acosh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2, x)
```

3.137 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$-\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh}{}\right.}{2b^2d}$$

```
[Out] 1/2*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+b*
arccosh(d*x+c))/b)*cosh(4*a/b)/b^2/d-1/2*e^3*Shi(2*(a+b*arccosh(d*x+c))/b)*
sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b^2/d-e
^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 12, 5885, 3384, 3379, 3382}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x])
)/b])/(2*b^2*d) + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c + d*x
]))/b])/(2*b^2*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d
*x]))/b])/(2*b^2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCosh[c +
 d*x]))/b])/(2*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{(a+b\cosh^{-1}(x))^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{(a+b\cosh^{-1}(x))^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^3\text{Subst}\Big(\int \Big(-\frac{\cosh(2x)}{2(a+bx)} - \frac{\cosh(4x}{2(a+bx}}{bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\text{Subst}\Big(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-}}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\Big(\int \frac{\cosh(\frac{2a}{b}}{a+b}}{2bc}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\Big(\frac{2a}{b}+2\cosh^{-}}{2b^2d}$$

Mathematica [**A**]
time = 2.03, size = 230, normalized size = 1.18

$$\frac{e^3\left(-\frac{2b(c+dx)^3\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\cosh^{-1}(c+dx)}+4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+3\log\left(a+b\cosh^{-1}(c+dx)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-3\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\log\left(a+b\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{2b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*((-2*b*(c + d*x)^3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c +
 d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a
+ b*ArcCosh[c + d*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d
*x])] - 3*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a +
 b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x]
)]) - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c + d*x])]))/(2*b^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(183) = 366$.
time = 59.76, size = 418, normalized size = 2.14

method	result
derivativedivides	$\frac{\left(-8\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^3+4(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+8(dx+c)^4-8(dx+c)^2+1\right.}{16b(a+b\,\text{arccosh}(dx+c))}$
default	$\frac{\left(-8\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^3+4(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+8(dx+c)^4-8(dx+c)^2+1\right.}{16b(a+b\,\text{arccosh}(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*(-8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3+4*(d*x+c)*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3/b/(a+b*arccosh(d*x+c)
)-1/4*e^3/b^2*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*(-2*(d*x+c)*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)+2*(d*x+c)^2-1)*e^3/b/(a+b*arccosh(d*x+c))-1/4*e^
3/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e^3*(2*(d*x+c)^2-1+2*(d
*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/4/b^2*e^3*exp
(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/16/b*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+
8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)+1)/(a+b*arccosh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(
d*x+c)-4*a/b))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 - d^4)*x^4*e^3 + 4*(5*c^3*d^3
 - c*d^3)*x^3*e^3 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d - 2*c^3*
d)*x*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*x^3*e^3 + (1
0*c^3*d^2 - 3*c*d^2)*x^2*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 + (c^5 - c^3)*e^3)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^6 - c^4)*e^3)/(a*b*d^3*x^2 + 2*a*
b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x
+ b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)
*sqrt(d*x + c - 1) + c)) + integrate((4*d^7*x^7*e^3 + 28*c*d^6*x^6*e^3 + 4*
(21*c^2*d^5 - 2*d^5)*x^5*e^3 + 20*(7*c^3*d^4 - 2*c*d^4)*x^4*e^3 + 4*(35*c^4
*d^3 - 20*c^2*d^3 + d^3)*x^3*e^3 + 4*(21*c^5*d^2 - 20*c^3*d^2 + 3*c*d^2)*x^
2*e^3 + 2*(2*d^5*x^5*e^3 + 10*c*d^4*x^4*e^3 + (20*c^2*d^3 - d^3)*x^3*e^3 +
(20*c^3*d^2 - 3*c*d^2)*x^2*e^3 + (10*c^4*d - 3*c^2*d)*x*e^3 + (2*c^5 - c^3)
*e^3)*(d*x + c + 1)*(d*x + c - 1) + 4*(7*c^6*d - 10*c^4*d + 3*c^2*d)*x*e^3
+ (8*d^6*x^6*e^3 + 48*c*d^5*x^5*e^3 + 10*(12*c^2*d^4 - d^4)*x^4*e^3 + 40*(4
*c^3*d^3 - c*d^3)*x^3*e^3 + 3*(40*c^4*d^2 - 20*c^2*d^2 + d^2)*x^2*e^3 + 2*(
24*c^5*d - 20*c^3*d + 3*c*d)*x*e^3 + (8*c^6 - 10*c^4 + 3*c^2)*e^3)*sqrt(d*x
 + c + 1)*sqrt(d*x + c - 1) + 4*(c^7 - 2*c^5 + c^3)*e^3)/(a*b*d^4*x^4 + 4*a
*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d
^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2
+ 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 -
c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 +
2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqr
t(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^2*arccosh(d*x + c
)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{d^{3}x^{3}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{3cd^{2}x^{2}}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{3c^{2}dx}{a^{2} + 2ab\operatorname{acosh}{\left(c + dx \right)} + b^{2}\operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2
*acosh(c + d*x)**2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2, x)
```

3.138 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=191

$$-\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{4b^2d}$$

```
[Out] 1/4*e^2*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d+3/4*e^2*Chi(3*(a+b*arcc
osh(d*x+c))/b)*cosh(3*a/b)/b^2/d-1/4*e^2*Shi((a+b*arccosh(d*x+c))/b)*sinh(a
/b)/b^2/d-3/4*e^2*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 12, 5885, 3384, 3379, 3382}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{4b^2d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)}{4b^2d}-\frac{e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(4
*b^2*d) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b]
)/(4*b^2*d) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(4*b
^2*d) - (3*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/
(4*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^2\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} - \frac{3\cosh}{4(a+}\right.\right.}{b}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh\right.}{4bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}\right.}{a+b}\right.}{4bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(\right.}{4b^2d}$$

Mathematica [A]
time = 1.34, size = 150, normalized size = 0.79

$$\frac{e^2\left(-\frac{4b(c+dx)^2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)}{a+b\cosh^{-1}(c+dx)}+\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{4b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]] - 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcC
osh[c + d*x])]))/(4*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(179) = 358$.
time = 71.38, size = 374, normalized size = 1.96

method	result
derivativedivides	$\frac{\left(-4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2}{8b(a+b\,\text{arccosh}(dx+c))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}}{}$
default	$\frac{\left(-4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2}{8b(a+b\,\text{arccosh}(dx+c))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2/b/(a+b*arccosh(d*x+c))-3/8*e^2/b^2*e
xp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+d*x+c)*e^2/b/(a+b*arccosh(d*x+c))-1/8*e^2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c
)+a/b)-1/8/b*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))-1/8/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8/b*e^2*(4*(d*x+c)^3-3
*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))/(a+b*arccosh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x
+c)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 - d^3)*x^3*e^2 + (10*c^3*d^2
- 3*c*d^2)*x^2*e^2 + (5*c^4*d - 3*c^2*d)*x*e^2 + (d^4*x^4*e^2 + 4*c*d^3*x^3
*e^2 + (6*c^2*d^2 - d^2)*x^2*e^2 + 2*(2*c^3*d - c*d)*x*e^2 + (c^4 - c^2)*e^
2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^5 - c^3)*e^2)/(a*b*d^3*x^2 + 2*
a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*
x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c +
1)*sqrt(d*x + c - 1) + c)) + integrate((3*d^6*x^6*e^2 + 18*c*d^5*x^5*e^2 +
3*(15*c^2*d^4 - 2*d^4)*x^4*e^2 + 12*(5*c^3*d^3 - 2*c*d^3)*x^3*e^2 + 3*(15*c
^4*d^2 - 12*c^2*d^2 + d^2)*x^2*e^2 + (3*d^4*x^4*e^2 + 12*c*d^3*x^3*e^2 + (1
8*c^2*d^2 - d^2)*x^2*e^2 + 2*(6*c^3*d - c*d)*x*e^2 + (3*c^4 - c^2)*e^2)*(d*
x + c + 1)*(d*x + c - 1) + 6*(3*c^5*d - 4*c^3*d + c*d)*x*e^2 + (6*d^5*x^5*e
^2 + 30*c*d^4*x^4*e^2 + (60*c^2*d^3 - 7*d^3)*x^3*e^2 + 3*(20*c^3*d^2 - 7*c*
d^2)*x^2*e^2 + (30*c^4*d - 21*c^2*d + 2*d)*x*e^2 + (6*c^5 - 7*c^3 + 2*c)*e^
2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 3*(c^6 - 2*c^4 + c^2)*e^2)/(a*b*d^
4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b
*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c
^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*
x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b
^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2
+ 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 -
c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^2*arccosh(d*x + c)^2 + 2*a*b*arcc
osh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{2} + 2ab\operatorname{acosh}\left(c + dx\right) + b^{2}\operatorname{acosh}^{2}\left(c + dx\right)}\, dx + \int \frac{d^{2}x^{2}}{a^{2} + 2ab\operatorname{acosh}\left(c + dx\right) + b^{2}\operatorname{acosh}^{2}\left(c + dx\right)}\, dx + \int \frac{2cdx}{a^{2} + 2ab\operatorname{acosh}\left(c + dx\right) + b^{2}\operatorname{acosh}^{2}\left(c + dx\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d
*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2, x)
```

3.139 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}$$

```
[Out] e*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arccosh(d*x+
c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b
*arccosh(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5996, 12, 5885, 3384, 3379, 3382}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c
+ d*x]))) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/
(b^2*d) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(b
^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\cosh^{-1}(c\right.}{bd}\\ &= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right.}{a+bx}\right.}{bd}\\ &= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c\right.}{b^2d}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 108, normalized size = 0.98

$$\frac{e\left(-\frac{b\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(c+c^2+2cdx+dx(1+dx))}{a+b\cosh^{-1}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2 + 2*c*d*x + d*x*(1 + d
*x)))/(a + b*ArcCosh[c + d*x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCo
sh[c + d*x])] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]))/(b
^2*d)
```

Maple [A]
time = 57.51, size = 170, normalized size = 1.55

method	result
derivativedivides	$\frac{\frac{\left(-2(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2(dx+c)^{2}-1\right)e}{4b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^{2}}-\frac{e\left(2(dx+c)^{2}-\cdots\right.}{\cdots}}{d}$
default	$\frac{\frac{\left(-2(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}+2(dx+c)^{2}-1\right)e}{4b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^{2}}-\frac{e\left(2(dx+c)^{2}-\cdots\right.}{\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(-2*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*(d*x+c)^2-1)*e/b/(a+
b*arccosh(d*x+c))-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4/b*e
*(2*(d*x+c)^2-1+2*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-1/2/b^2*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*x^4*e + 4*c*d^3*x^3*e + (6*c^2*d^2 - d^2)*x^2*e + 2*(2*c^3*d - c*d)*x
*e + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d - d)*x*e + (c^3 - c)*e)*sqrt(d*x
 + c + 1)*sqrt(d*x + c - 1) + (c^4 - c^2)*e)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x +
 (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)) + integrate((2*d^5*x^5*e + 10*c*d^4*x^4*e + 4*(5*c^2*d^3 - d^
3)*x^3*e + 4*(5*c^3*d^2 - 3*c*d^2)*x^2*e + 2*(d^3*x^3*e + 3*c*d^2*x^2*e + 3
*c^2*d*x*e + c^3*e)*(d*x + c + 1)*(d*x + c - 1) + 2*(5*c^4*d - 6*c^2*d + d)
*x*e + (4*d^4*x^4*e + 16*c*d^3*x^3*e + 4*(6*c^2*d^2 - d^2)*x^2*e + 8*(2*c^3
*d - c*d)*x*e + (4*c^4 - 4*c^2 + 1)*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
```

```
+ 2*(c^5 - 2*c^3 + c)*e)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^
2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*
(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*
b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2
+ 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c +
1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
[Out] integral((d*x + c)*e/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2+2ab\operatorname{acosh}(c+dx)+b^2\operatorname{acosh}^2(c+dx)}\,dx+\int \frac{dx}{a^2+2ab\operatorname{acosh}(c+dx)+b^2\operatorname{acosh}^2(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
[Out] e*(Integral(c/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\operatorname{acosh}\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2, x)
```

3.140 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}$$

```
[Out] Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d-Shi((a+b*arccosh(d*x+c))/b)*sin
h(a/b)/b^2/d-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5995, 5880, 5953, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x]))) +
(Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(b^2*d) - (Sinh[a/b]*S
inhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b^2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\cosh^{-1}(x))^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh^{-1}(x)}\right.}{bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}\right.}{bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d} - \text{si}$$

Mathematica [A]
time = 0.73, size = 143, normalized size = 1.46

$$\frac{-\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{a+b\cosh^{-1}(c+dx)} + \log\left(1+\frac{b\cosh^{-1}(c+dx)}{a}\right) + \sqrt{\frac{-1+c+dx}{1+c+dx}}\,\coth\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - \log\left(a+b\cosh^{-1}(c+dx)\right) - \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])) + Log
[1 + (b*ArcCosh[c + d*x])/a] + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Coth[ArcC
osh[c + d*x]/2]*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] - Log[a + b
*ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]))/(b^2*
d)
```

Maple [A]
time = 41.24, size = 139, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c}{2b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c}}{2b(a+b\,\mathrm{arccosh}(dx+c))}}{d}$
default	$\frac{\frac{-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c}{2b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c}}{2b(a+b\,\mathrm{arccosh}(dx+c))}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)/b/(a+b*arccosh(d*x+c))-1/
2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))/(a+b*arccosh(d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x
+ (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)) + integrate((d^4*x^4 + 4*c*d^3*x^3 + c^4 + (d^2*x^2 + 2*c*d*
x + c^2 + 1)*(d*x + c + 1)*(d*x + c - 1) + 2*(3*c^2*d^2 - d^2)*x^2 + (2*d^3
*x^3 + 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d - d)*x - c)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) - 2*c^2 + 4*(c^3*d - c*d)*x + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3
+ 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*
```

```
b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*
(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*
d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1
)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**2,x)
[Out] Integral((a + b*acosh(c + d*x))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^2,x)
[Out] int(1/(a + b*acosh(c + d*x))^2, x)
```

3.141 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^2,x)/e
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 6.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\mathrm{arccosh}\,(dx+c)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^4*x^3*e + 3*a*b*c*d^3*
x^2*e + (3*c^2*d^2 - d^2)*a*b*x*e + (c^3*d - c*d)*a*b*e + (a*b*d^3*x^2*e +
2*a*b*c*d^2*x*e + a*b*c^2*d*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d
^4*x^3*e + 3*b^2*c*d^3*x^2*e + (3*c^2*d^2 - d^2)*b^2*x*e + (c^3*d - c*d)*b^
2*e + (b^2*d^3*x^2*e + 2*b^2*c*d^2*x*e + b^2*c^2*d*e)*sqrt(d*x + c + 1)*sqr
t(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integ
rate((2*(d*x + c + 1)*(d*x + c)*(d*x + c - 1) + (2*d^2*x^2 + 4*c*d*x + 2*c^
2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))/(a*b*d^6*x^6*e + 6*a*b*c*d^5*x^
5*e + (15*c^2*d^4 - 2*d^4)*a*b*x^4*e + 4*(5*c^3*d^3 - 2*c*d^3)*a*b*x^3*e +
(15*c^4*d^2 - 12*c^2*d^2 + d^2)*a*b*x^2*e + 2*(3*c^5*d - 4*c^3*d + c*d)*a*b
*x*e + (c^6 - 2*c^4 + c^2)*a*b*e + (a*b*d^4*x^4*e + 4*a*b*c*d^3*x^3*e + 6*a
*b*c^2*d^2*x^2*e + 4*a*b*c^3*d*x*e + a*b*c^4*e)*(d*x + c + 1)*(d*x + c - 1)
 + 2*(a*b*d^5*x^5*e + 5*a*b*c*d^4*x^4*e + (10*c^2*d^3 - d^3)*a*b*x^3*e + (1
0*c^3*d^2 - 3*c*d^2)*a*b*x^2*e + (5*c^4*d - 3*c^2*d)*a*b*x*e + (c^5 - c^3)*
a*b*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^6*x^6*e + 6*b^2*c*d^5*x
^5*e + (15*c^2*d^4 - 2*d^4)*b^2*x^4*e + 4*(5*c^3*d^3 - 2*c*d^3)*b^2*x^3*e +
 (15*c^4*d^2 - 12*c^2*d^2 + d^2)*b^2*x^2*e + 2*(3*c^5*d - 4*c^3*d + c*d)*b^
2*x*e + (c^6 - 2*c^4 + c^2)*b^2*e + (b^2*d^4*x^4*e + 4*b^2*c*d^3*x^3*e + 6*
b^2*c^2*d^2*x^2*e + 4*b^2*c^3*d*x*e + b^2*c^4*e)*(d*x + c + 1)*(d*x + c - 1
) + 2*(b^2*d^5*x^5*e + 5*b^2*c*d^4*x^4*e + (10*c^2*d^3 - d^3)*b^2*x^3*e + (
10*c^3*d^2 - 3*c*d^2)*b^2*x^2*e + (5*c^4*d - 3*c^2*d)*b^2*x*e + (c^5 - c^3)
*b^2*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*d*x + b^2*c)*arccosh(d*x + c)^2*e + 2*(a*b*d*x + a*b*c)*ar
ccosh(d*x + c)*e + (a^2*d*x + a^2*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{acosh}(c+dx)+2abdx\operatorname{acosh}(c+dx)+b^2c\operatorname{acosh}^2(c+dx)+b^2dx\operatorname{acosh}^2(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*acosh(c + d*x) + 2*a*b*d*x*acosh(c
+ d*x) + b**2*c*acosh(c + d*x)**2 + b**2*d*x*acosh(c + d*x)**2), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\operatorname{acosh}\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2), x)
```

3.142 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=327

$$-\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{5e^4(c+dx)^5}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^4\mathrm{C}}{}$$

```
[Out] 2*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*arc
cosh(d*x+c))+1/16*e^4*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^3/d+27/32*e^4
*cosh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^3/d+25/32*e^4*cosh(5*a/b)*Shi(
5*(a+b*arccosh(d*x+c))/b)/b^3/d-1/16*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a
/b)/b^3/d-27/32*e^4*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*e
^4*Chi(5*(a+b*arccosh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2
```

Rubi [A]
time = 0.74, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3384, 3379, 3382}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{16b^3d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}-\frac{5e^4(c+dx)^5}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^2) + (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcCosh[c + d*x])) -
(5*e^4*(c + d*x)^5)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^4*CoshIntegral[
(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(16*b^3*d) - (27*e^4*CoshIntegral[(3
*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(32*b^3*d) - (25*e^4*CoshInteg
ral[(5*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(5*a)/b])/(32*b^3*d) + (e^4*Cosh[a
/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(16*b^3*d) + (27*e^4*Cosh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(32*b^3*d) + (25*e^4*Co
sh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcCosh[c + d*x]))/b])/(32*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{(2e^4)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{1}}\right.}{}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

Mathematica [A]
time = 1.12, size = 323, normalized size = 0.99

$$\frac{e^4\left(-\frac{16b^2\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{16b\left(4(c+dx)^3-5(c+dx)^5\right)}{a+b\cosh^{-1}(c+dx)}+48\left(\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)+\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)+25\left(-2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{5a}{b}\right)+2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^2 + (16*b*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[c
+ d*x]) + 48*(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] + CoshIntegral
[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c + d*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])
+ 25*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(32*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(307) = 614$.

time = 0.53, size = 993, normalized size = 3.04 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/64*(-16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+12*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(5*b*arccosh(d*x+c)+5*a-b)/b^2/(b^2*arccosh(d*x+c
)^2+2*a*b*arccosh(d*x+c)+a^2)+25/64*e^4/b^3*exp(5*a/b)*Ei(1,5*arccosh(d*x+c
)+5*a/b)-3/64*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(3*b*arccosh(d*x+c)+3*a-b)/b^2/
(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+27/64*e^4/b^3*exp(3*a/b)*Ei
(1,3*arccosh(d*x+c)+3*a/b)-1/32*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^
4*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2
)+1/32*e^4/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/32/b*e^4*(d*x+c+(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,
-arccosh(d*x+c)-a/b)-3/64/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d
*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-9/64/b^2*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*
(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-27/64/b^3*e
^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/64/b*e^4*(16*(d*x+c)^5-20*(d
*x+c)^3+16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(d*x+c+1)
^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcc
osh(d*x+c))^2-5/64/b^2*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c+1)^(1/2)*(d
*x+c-1)^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c
)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-25/64/b^3*e^4*exp
(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((5*a*d^11 + b*d^11)*x^11*e^4 + 11*(5*a*c*d^10 + b*c*d^10)*x^10*e^4 + (5*(55*c^2*d^9 - 3*d^9)*a + (55*c^2*d^9 - 3*d^9)*b)*x^9*e^4 + 3*(5*(55*c^3*d^8 - 9*c*d^8)*a + (55*c^3*d^8 - 9*c*d^8)*b)*x^8*e^4 + 3*(5*(110*c^4*d^7 - 36*c^2*d^7 + d^7)*a + (110*c^4*d^7 - 36*c^2*d^7 + d^7)*b)*x^7*e^4 + 21*(5*(22*c^5*d^6 - 12*c^3*d^6 + c*d^6)*a + (22*c^5*d^6 - 12*c^3*d^6 + c*d^6)*b)*x^6*e^4 + (5*(462*c^6*d^5 - 378*c^4*d^5 + 63*c^2*d^5 - d^5)*a + (462*c^6*d^5 - 378*c^4*d^5 + 63*c^2*d^5 - d^5)*b)*x^5*e^4 + (5*(330*c^7*d^4 - 378*c^5*d^4 + 105*c^3*d^4 - 5*c*d^4)*a + (330*c^7*d^4 - 378*c^5*d^4 + 105*c^3*d^4 - 5*c*d^4)*b)*x^4*e^4 + (5*(165*c^8*d^3 - 252*c^6*d^3 + 105*c^4*d^3 - 10*c^2*d^3)*a + (165*c^8*d^3 - 252*c^6*d^3 + 105*c^4*d^3 - 10*c^2*d^3)*b)*x^3*e^4 + ((5*a*d^8 + b*d^8)*x^8*e^4 + 8*(5*a*c*d^7 + b*c*d^7)*x^7*e^4 + (4*(35*c^2*d^6 - 2*d^6)*a + (28*c^2*d^6 - d^6)*b)*x^6*e^4 + 2*(4*(35*c^3*d^5 - 6*c*d^5)*a + (28*c^3*d^5 - 3*c*d^5)*b)*x^5*e^4 + ((350*c^4*d^4 - 120*c^2*d^4 + 3*d^4)*a + 5*(14*c^4*d^4 - 3*c^2*d^4)*b)*x^4*e^4 + 4*((70*c^5*d^3 - 40*c^3*d^3 + 3*c*d^3)*a + (14*c^5*d^3 - 5*c^3*d^3)*b)*x^3*e^4 + (2*(70*c^6*d^2 - 60*c^4*d^2 + 9*c^2*d^2)*a + (28*c^6*d^2 - 15*c^4*d^2)*b)*x^2*e^4 + 2*(2*(10*c^7*d - 12*c^5*d + 3*c^3*d)*a + (4*c^7*d - 3*c^5*d)*b)*x*e^4 + ((5*c^8 - 8*c^6 + 3*c^4)*a + (c^8 - c^6)*b)*e^4)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (5*(55*c^9*d^2 - 108*c^7*d^2 + 63*c^5*d^2 - 10*c^3*d^2)*a + (55*c^9*d^2 - 108*c^7*d^2 + 63*c^5*d^2 - 10*c^3*d^2)*b)*x^2*e^4 + (3*(5*a*d^9 + b*d^9)*x^9*e^4 + 27*(5*a*c*d^8 + b*c*d^8)*x^8*e^4 + ((540*c^2*d^7 - 31*d^7)*a + (108*c^2*d^7 - 5*d^7)*b)*x^7*e^4 + 7*((180*c^3*d^6 - 31*c*d^6)*a + (36*c^3*d^6 - 5*c*d^6)*b)*x^6*e^4 + ((1890*c^4*d^5 - 651*c^2*d^5 + 20*d^5)*a + (378*c^4*d^5 - 105*c^2*d^5 + 2*d^5)*b)*x^5*e^4 + (5*(378*c^5*d^4 - 217*c^3*d^4 + 20*c*d^4)*a + (378*c^5*d^4 - 175*c^3*d^4 + 10*c*d^4)*b)*x^4*e^4 + ((1260*c^6*d^3 - 1085*c^4*d^3 + 200*c^2*d^3 - 4*d^3)*a + (252*c^6*d^3 - 175*c^4*d^3 + 20*c^2*d^3)*b)*x^3*e^4 + ((540*c^7*d^2 - 651*c^5*d^2 + 200*c^3*d^2 - 12*c*d^2)*a + (108*c^7*d^2 - 105*c^5*d^2 + 20*c^3*d^2)*b)*x^2*e^4 + ((135*c^8*d - 217*c^6*d + 100*c^4*d - 12*c^2*d)*a + (27*c^8*d - 35*c^6*d + 10*c^4*d)*b)*x*e^4 + ((15*c^9 - 31*c^7 + 20*c^5 - 4*c^3)*a + (3*c^9 - 5*c^7 + 2*c^5)*b)*e^4)*(d*x + c + 1)*(d*x + c - 1) + (5*(11*c^10*d - 27*c^8*d + 21*c^6*d - 5*c^4*d)*a + (11*c^10*d - 27*c^8*d + 21*c^6*d - 5*c^4*d)*b)*x*e^4 + (3*(5*a*d^10 + b*d^10)*x^10*e^4 + 30*(5*a*c*d^9 + b*c*d^9)*x^9*e^4 + ((675*c^2*d^8 - 38*d^8)*a + (135*c^2*d^8 - 7*d^8)*b)*x^8*e^4 + 8*((225*c^3*d^7 - 38*c*d^7)*a + (45*c^3*d^7 - 7*c*d^7)*b)*x^7*e^4 + (2*(1575*c^4*d^6 - 532*c^2*d^6 + 16*d^6)*a + (630*c^4*d^6 - 196*c^2*d^6 + 5*d^6)*b)*x^6*e^4 + 2*(2*(945*c^5*d^5 - 532*c^3*d^5 + 48*c*d^5)*a + (378*c^5*d^5 - 196*c^3*d^5 + 15*c*d^5)*b)*x^5*e^4 + ((3150*c^6*d^4 - 2660*c^4*d^4 + 480*c^2*d^4 - 9*d^4)*a + (630*c^6*d^4 - 490*c^4*d^4 + 75*c^2*d^4 - d^4)*b)*x^4*e^4 + 4*((450*c^7*d^3 - 532*c^5*d^3 + 160*c^3*d^3 - 9*c*d^3)*a + (90*c^7*d^3 - 98*c^5*d^3 + 25*c^3*d^3 - c*d^3)*b)*x^3*e^4 + ((675*c^8*d^2 - 1064*c^6*d^2 + 480*c^4*d^2 - 54*c^2*d

```
^2)*a + (135*c^8*d^2 - 196*c^6*d^2 + 75*c^4*d^2 - 6*c^2*d^2)*b)*x^2*e^4 + 2
*((75*c^9*d - 152*c^7*d + 96*c^5*d - 18*c^3*d)*a + (15*c^9*d - 28*c^7*d + 1
5*c^5*d - 2*c^3*d)*b)*x*e^4 + ((15*c^10 - 38*c^8 + 32*c^6 - 9*c^4)*a + (3*c
^10 - 7*c^8 + 5*c^6 - c^4)*b)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5
*(c^11 - 3*c^9 + 3*c^7 - c^5)*a + (c^11 - 3*c^9 + 3*c^7 - c^5)*b)*e^4 + (5*
b*d^11*x^11*e^4 + 55*b*c*d^10*x^10*e^4 + 5*(55*c^2*d^9 - 3*d^9)*b*x^9*e^4 +
 15*(55*c^3*d^8 - 9*c*d^8)*b*x^8*e^4 + 15*(110*c^4*d^7 - 36*c^2*d^7 + d^7)*
b*x^7*e^4 + 105*(22*c^5*d^6 - 12*c^3*d^6 + c*d^6)*b*x^6*e^4 + 5*(462*c^6*d^
5 - 378*c^4*d^5 + 63*c^2*d^5 - d^5)*b*x^5*e^4 + 5*(330*c^7*d^4 - 378*c^5*d^
4 + 105*c^3*d^4 - 5*c*d^4)*b*x^4*e^4 + 5*(165*c^8*d^3 - 252*c^6*d^3 + 105*c
^4*d^3 - 10*c^2*d^3)*b*x^3*e^4 + 5*(55*c^9*d^2 - 108*c^7*d^2 + 63*c^5*d^2 -
 10*c^3*d^2)*b*x^2*e^4 + (5*b*d^8*x^8*e^4 + 40*b*c*d^7*x^7*e^4 + 4*(35*c^2*
d^6 - 2*d^6)*b*x^6*e^4 + 8*(35*c^3*d^5 - 6*c*d^5)*b*x^5*e^4 + (350*c^4*d^4
- 120*c^2*d^4 + 3*d^4)*b*x^4*e^4 + 4*(70*c^5*d^3 - 40*c^3*d^3 + 3*c*d^3)*b*
x^3*e^4 + 2*(70*c^6*d^2 - 60*c^4*d^2 + 9*c^2*d^2)*b*x^2*e^4 + 4*(10*c^7*d -
 12*c^5*d + 3*c^3*d)*b*x*e^4 + (5*c^8 - 8*c^6 + 3*c^4)*b*e^4)*(d*x + c + 1)
^(3/2)*(d*x + c - 1)^(3/2) + 5*(11*c^10*d - 27*c^8*d + 21*c^6*d - 5*c^4*d)*
b*x*e^4 + (15*b*d^9*x^9*e^4 + 135*b*c*d^8*x^8*e^4 + (540*c^2*d^7 - 31*d^7)*
b*x^7*e^4 + 7*(180*c^3*d^6 - 31*c*d^6)*b*x^6*e^4 + (1890*c^4*d^5 - 651*c^2*
d^5 + 20*d^5)*b*x^5*e^4 + 5*(378*c^5*d^4 - 217*c^3*d^4 + 20*c*d^4)*b*x^4*e^
4 + (1260*c^6*d^3 - 1085*c^4*d^3 + 200*c^2*d^3 - 4*d^3)*b*x^3*e^4 + (540*c^
7*d^2 - 651*c^5*d^2 + 200*c^3*d^2 - 12*c*d^2)*b*x^2*e^4 + (135*c^8*d - 217*
c^6*d + 100*c^4*d - 12*c^2*d)*b*x*e^4 + (15*c^9 - 31*c^7 + 20*c^5 - 4*c^3)*
b*e^4)*(d*x + c + 1)*(d*x + c - 1) + 5*(c^11 - 3*c^9 + 3*c^7 - c^5)*b*e^4 +
 (15*b*d^10*x^10*e^4 + 150*b*c*d^9*x^9*e^4 + (675*c^2*d^8 - 38*d^8)*b*x^8*e
^4 + 8*(225*c^3*d^7 - 38*c*d^7)*b*x^7*e^4 + 2*(...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^3
*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c)
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{d^{4}x^{4}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{4cd^{3}x^{3}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{6c^{2}d^{2}x^{2}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{4c^{3}dx}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**4*(Integral(c**4/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c +
 d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a**3 +
3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)*
*3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*ac
osh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3, x)
```

3.143 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{2e^3(c+dx)^4}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^3\mathrm{Chi}}{}$$

```
[Out] 3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*arc
cosh(d*x+c))+1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^3/d+e^3*co
sh(4*a/b)*Shi(4*(a+b*arccosh(d*x+c))/b)/b^3/d-1/2*e^3*Chi(2*(a+b*arccosh(d*
x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b^
3/d-1/2*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+
c))^2
```

Rubi [A]
time = 0.58, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3384, 3379, 3382}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)}{b^3d}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)}{2b^3d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)}{b^3d}-\frac{2e^3(c+dx)^4}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^2) + (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x]))
- (2*e^3*(c + d*x)^4)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (e^3*CoshIntegral[
(2*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegra
l[(4*(a + b*ArcCosh[c + d*x]))/b]*Sinh[(4*a)/b])/(b^3*d) + (e^3*Cosh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(2*b^3*d) + (e^3*Cosh[(4*a
)/b]*SinhIntegral[(4*(a + b*ArcCosh[c + d*x]))/b])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{(3e^3)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{1+\cdots}}\right.}{2\cdots}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 186, normalized size = 0.73

$$\frac{e^3\left(-\frac{b^2\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{b\left(3(c+dx)^2-4(c+dx)^4\right)}{a+b\cosh^{-1}(c+dx)}-\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
[Out] (e^3*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcCo
sh[c + d*x])^2) + (b*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c + d*
```

```
x]) - CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegr
al[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcCosh[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c + d*x])]))/(2*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(242) = 484$.

time = 0.18, size = 624, normalized size = 2.46

method	result
derivativedivides	$-\frac{\left(-8\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^3+4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2\cdots\right.}{32b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}$
default	$-\frac{\left(-8\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^3+4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2\cdots\right.}{32b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32*(-8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3+4*(d*x+c+1)^(1/2)*
(d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(4*b*arccosh(d*x+c)+
4*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/2*e^3/b^3*exp(
4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)-1/16*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2
)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*
x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4*e^3/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+
c)+2*a/b)-1/16/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))^2-1/8/b^2*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*
x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4/b^3*e^3*exp(-2*a/b)*Ei(1,-2*
arccosh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+
b*arccosh(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c+1)^(1/2)*
(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b
*arccosh(d*x+c))-1/2/b^3*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*d^10 + b*d^10)*x^10*e^3 + 10*(4*a*c*d^9 + b*c*d^9)*x^9*e^3 + 3*(
4*(15*c^2*d^8 - d^8)*a + (15*c^2*d^8 - d^8)*b)*x^8*e^3 + 24*(4*(5*c^3*d^7 -
```

c*d^7)*a + (5*c^3*d^7 - c*d^7)*b)*x^7*e^3 + 3*(4*(70*c^4*d^6 - 28*c^2*d^6
+ d^6)*a + (70*c^4*d^6 - 28*c^2*d^6 + d^6)*b)*x^6*e^3 + 6*(4*(42*c^5*d^5 -
28*c^3*d^5 + 3*c*d^5)*a + (42*c^5*d^5 - 28*c^3*d^5 + 3*c*d^5)*b)*x^5*e^3 +
(4*(210*c^6*d^4 - 210*c^4*d^4 + 45*c^2*d^4 - d^4)*a + (210*c^6*d^4 - 210*c^
4*d^4 + 45*c^2*d^4 - d^4)*b)*x^4*e^3 + 4*(4*(30*c^7*d^3 - 42*c^5*d^3 + 15*c
^3*d^3 - c*d^3)*a + (30*c^7*d^3 - 42*c^5*d^3 + 15*c^3*d^3 - c*d^3)*b)*x^3*e
^3 + ((4*a*d^7 + b*d^7)*x^7*e^3 + 7*(4*a*c*d^6 + b*c*d^6)*x^6*e^3 + (6*(14*
c^2*d^5 - d^5)*a + (21*c^2*d^5 - d^5)*b)*x^5*e^3 + 5*(2*(14*c^3*d^4 - 3*c*d
^4)*a + (7*c^3*d^4 - c*d^4)*b)*x^4*e^3 + (2*(70*c^4*d^3 - 30*c^2*d^3 + d^3)
a + 5(7*c^4*d^3 - 2*c^2*d^3)*b)*x^3*e^3 + (6*(14*c^5*d^2 - 10*c^3*d^2 + c
*d^2)*a + (21*c^5*d^2 - 10*c^3*d^2)*b)*x^2*e^3 + (2*(14*c^6*d - 15*c^4*d +
3*c^2*d)*a + (7*c^6*d - 5*c^4*d)*b)*x*e^3 + (2*(2*c^7 - 3*c^5 + c^3)*a + (c
^7 - c^5)*b)*e^3)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(4*(15*c^8*d^
2 - 28*c^6*d^2 + 15*c^4*d^2 - 2*c^2*d^2)*a + (15*c^8*d^2 - 28*c^6*d^2 + 15*
c^4*d^2 - 2*c^2*d^2)*b)*x^2*e^3 + (3*(4*a*d^8 + b*d^8)*x^8*e^3 + 24*(4*a*c*
d^7 + b*c*d^7)*x^7*e^3 + (24*(14*c^2*d^6 - d^6)*a + (84*c^2*d^6 - 5*d^6)*b)
*x^6*e^3 + 6*(8*(14*c^3*d^5 - 3*c*d^5)*a + (28*c^3*d^5 - 5*c*d^5)*b)*x^5*e^
3 + (15*(56*c^4*d^4 - 24*c^2*d^4 + d^4)*a + (210*c^4*d^4 - 75*c^2*d^4 + 2*d
^4)*b)*x^4*e^3 + 4*(3*(56*c^5*d^3 - 40*c^3*d^3 + 5*c*d^3)*a + (42*c^5*d^3 -
25*c^3*d^3 + 2*c*d^3)*b)*x^3*e^3 + 3*((112*c^6*d^2 - 120*c^4*d^2 + 30*c^2*
d^2 - d^2)*a + (28*c^6*d^2 - 25*c^4*d^2 + 4*c^2*d^2)*b)*x^2*e^3 + 2*(3*(16*
c^7*d - 24*c^5*d + 10*c^3*d - c*d)*a + (12*c^7*d - 15*c^5*d + 4*c^3*d)*b)*x
e^3 + (3(4*c^8 - 8*c^6 + 5*c^4 - c^2)*a + (3*c^8 - 5*c^6 + 2*c^4)*b)*e^3)
*(d*x + c + 1)*(d*x + c - 1) + 2*(4*(5*c^9*d - 12*c^7*d + 9*c^5*d - 2*c^3*d
)*a + (5*c^9*d - 12*c^7*d + 9*c^5*d - 2*c^3*d)*b)*x*e^3 + (3*(4*a*d^9 + b*d
^9)*x^9*e^3 + 27*(4*a*c*d^8 + b*c*d^8)*x^8*e^3 + (6*(72*c^2*d^7 - 5*d^7)*a
+ (108*c^2*d^7 - 7*d^7)*b)*x^7*e^3 + 7*(6*(24*c^3*d^6 - 5*c*d^6)*a + (36*c^
3*d^6 - 7*c*d^6)*b)*x^6*e^3 + ((1512*c^4*d^5 - 630*c^2*d^5 + 25*d^5)*a + (3
78*c^4*d^5 - 147*c^2*d^5 + 5*d^5)*b)*x^5*e^3 + ((1512*c^5*d^4 - 1050*c^3*d^
4 + 125*c*d^4)*a + (378*c^5*d^4 - 245*c^3*d^4 + 25*c*d^4)*b)*x^4*e^3 + ((10
08*c^6*d^3 - 1050*c^4*d^3 + 250*c^2*d^3 - 7*d^3)*a + (252*c^6*d^3 - 245*c^4
*d^3 + 50*c^2*d^3 - d^3)*b)*x^3*e^3 + ((432*c^7*d^2 - 630*c^5*d^2 + 250*c^3
*d^2 - 21*c*d^2)*a + (108*c^7*d^2 - 147*c^5*d^2 + 50*c^3*d^2 - 3*c*d^2)*b)*
x^2*e^3 + ((108*c^8*d - 210*c^6*d + 125*c^4*d - 21*c^2*d)*a + (27*c^8*d - 4
9*c^6*d + 25*c^4*d - 3*c^2*d)*b)*x*e^3 + ((12*c^9 - 30*c^7 + 25*c^5 - 7*c^3
)*a + (3*c^9 - 7*c^7 + 5*c^5 - c^3)*b)*e^3)*sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + (4*(c^10 - 3*c^8 + 3*c^6 - c^4)*a + (c^10 - 3*c^8 + 3*c^6 - c^4)*b)*
e^3 + (4*b*d^10*x^10*e^3 + 40*b*c*d^9*x^9*e^3 + 12*(15*c^2*d^8 - d^8)*b*x^8
e^3 + 96(5*c^3*d^7 - c*d^7)*b*x^7*e^3 + 12*(70*c^4*d^6 - 28*c^2*d^6 + d^6
)*b*x^6*e^3 + 24*(42*c^5*d^5 - 28*c^3*d^5 + 3*c*d^5)*b*x^5*e^3 + 4*(210*c^6
*d^4 - 210*c^4*d^4 + 45*c^2*d^4 - d^4)*b*x^4*e^3 + 16*(30*c^7*d^3 - 42*c^5*
d^3 + 15*c^3*d^3 - c*d^3)*b*x^3*e^3 + 12*(15*c^8*d^2 - 28*c^6*d^2 + 15*c^4*
d^2 - 2*c^2*d^2)*b*x^2*e^3 + 2*(2*b*d^7*x^7*e^3 + 14*b*c*d^6*x^6*e^3 + 3*(1
4*c^2*d^5 - d^5)*b*x^5*e^3 + 5*(14*c^3*d^4 - 3*c*d^4)*b*x^4*e^3 + (70*c^4*d
^3 - 30*c^2*d^3 + d^3)*b*x^3*e^3 + 3*(14*c^5*d^2 - 10*c^3*d^2 + c*d^2)*b*x^

```
2*e^3 + (14*c^6*d - 15*c^4*d + 3*c^2*d)*b*x*e^3 + (2*c^7 - 3*c^5 + c^3)*b*e
^3)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 8*(5*c^9*d - 12*c^7*d + 9*c^5
*d - 2*c^3*d)*b*x*e^3 + 3*(4*b*d^8*x^8*e^3 + 32*b*c*d^7*x^7*e^3 + 8*(14*c^2
*d^6 - d^6)*b*x^6*e^3 + 16*(14*c^3*d^5 - 3*c*d^5)*b*x^5*e^3 + 5*(56*c^4*d^4
 - 24*c^2*d^4 + d^4)*b*x^4*e^3 + 4*(56*c^5*d^3 - 40*c^3*d^3 + 5*c*d^3)*b*x^
3*e^3 + (112*c^6*d^2 - 120*c^4*d^2 + 30*c^2*d^2 - d^2)*b*x^2*e^3 + 2*(16*c^
7*d - 24*c^5*d + 10*c^3*d - c*d)*b*x*e^3 + (4*c^8 - 8*c^6 + 5*c^4 - c^2)*b*
e^3)*(d*x + c + 1)*(d*x + c - 1) + 4*(c^10 - 3*c^8 + 3*c^6 - c^4)*b*e^3 + (
12*b*d^9*x^9*e^3 + 108*b*c*d^8*x^8*e^3 + 6*(72*c^2*d^7 - 5*d^7)*b*x^7*e^3 +
 42*(24*c^3*d^6 - 5*c*d^6)*b*x^6*e^3 + (1512*c^4*d^5 - 630*c^2*d^5 + 25*d^5
)*b*x^5*e^3 + (1512*c^5*d^4 - 1050*c^3*d^4 + 125*c*d^4)*b*x^4*e^3 + (1008*c
^6*d^3 - 1050*c^4*d^3 + 250*c^2*d^3 - 7*d^3)*b*x^3*e^3 + (432*c^7*d^2 - 630
*c^5*d^2 + 250*c^3*d^2 - 21*c*d^2)*b*x^2*e^3 + (108*c^8*d - 210*c^6*d + 125
*c^4*d - 21*c^2*d)*b*x*e^3 + (12*c^9 - 30*c^7 + 25*c^5 - 7*c^3)*b*e^3)*sqrt
(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c))/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*a^2*
b^2*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 +
d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d - 3*c
^4*d + 3*c^2*d - d)*a^2*b^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^
2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d*x + c + 1)^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^3*arccosh(d*x + c
)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{d^{3}x^{3}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{3cd^{2}x^{2}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{3c^{2}dx}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c +
 d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
```

```
*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3, x)
```

3.144 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=252

$$-\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{3e^2(c+dx)^3}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^2\mathrm{C}}{}$$

```
[Out] e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arccosh
(d*x+c))+1/8*e^2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^3/d+9/8*e^2*cosh(3
*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^3/d-1/8*e^2*Chi((a+b*arccosh(d*x+c))/
b)*sinh(a/b)/b^3/d-9/8*e^2*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d-
1/2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^
2
```

Rubi [A]
time = 0.53, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3384, 3379, 3382, 5881}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}-\frac{3e^2(c+dx)^3}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^2) + (e^2*(c + d*x))/(b^2*d*(a + b*ArcCosh[c + d*x])) - (3*e
^2*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^2*CoshIntegral[(a +
 b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(8*b^3*d) - (9*e^2*CoshIntegral[(3*(a +
b*ArcCosh[c + d*x]))/b]*Sinh[(3*a)/b])/(8*b^3*d) + (e^2*Cosh[a/b]*SinhInteg
ral[(a + b*ArcCosh[c + d*x])/b])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*SinhInteg
ral[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
```

```
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2\text{Subst}\Big(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+}}}{}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} -$$

Mathematica [A]
time = 0.51, size = 223, normalized size = 0.88

$$\frac{e^2\left(-\frac{4b^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{4b\left(2(c+dx)-3(c+dx)^3\right)}{a+b\cosh^{-1}(c+dx)}+8\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-8\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+9\left(-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{8b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]

[Out] (e^2*((-4*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])^2 + (4*b*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c + d*x]) + 8*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 8*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + 9*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])))/(8*b^3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(236) = 472$.
time = 0.11, size = 557, normalized size = 2.21

method	result
derivativedivides	$-\frac{\left(-4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2(3b\,\mathrm{arccos}}{16b^2\left(b^2\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}$
default	$-\frac{\left(-4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)^2+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2(3b\,\mathrm{arccos}}{16b^2\left(b^2\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/16*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(3*b*arccosh(d*x+c)+3*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)-1/16*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^2*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/16/b^2*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^3*e^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-3/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((3*a*d^9 + b*d^9)*x^9*e^2 + 9*(3*a*c*d^8 + b*c*d^8)*x^8*e^2 + 3*(3*(1
2*c^2*d^7 - d^7)*a + (12*c^2*d^7 - d^7)*b)*x^7*e^2 + 21*(3*(4*c^3*d^6 - c*d
^6)*a + (4*c^3*d^6 - c*d^6)*b)*x^6*e^2 + 3*(3*(42*c^4*d^5 - 21*c^2*d^5 + d^
5)*a + (42*c^4*d^5 - 21*c^2*d^5 + d^5)*b)*x^5*e^2 + 3*(3*(42*c^5*d^4 - 35*c
^3*d^4 + 5*c*d^4)*a + (42*c^5*d^4 - 35*c^3*d^4 + 5*c*d^4)*b)*x^4*e^2 + (3*(
84*c^6*d^3 - 105*c^4*d^3 + 30*c^2*d^3 - d^3)*a + (84*c^6*d^3 - 105*c^4*d^3
+ 30*c^2*d^3 - d^3)*b)*x^3*e^2 + ((3*a*d^6 + b*d^6)*x^6*e^2 + 6*(3*a*c*d^5
+ b*c*d^5)*x^5*e^2 + ((45*c^2*d^4 - 4*d^4)*a + (15*c^2*d^4 - d^4)*b)*x^4*e^
2 + 4*((15*c^3*d^3 - 4*c*d^3)*a + (5*c^3*d^3 - c*d^3)*b)*x^3*e^2 + ((45*c^4
*d^2 - 24*c^2*d^2 + d^2)*a + 3*(5*c^4*d^2 - 2*c^2*d^2)*b)*x^2*e^2 + 2*((9*c
^5*d - 8*c^3*d + c*d)*a + (3*c^5*d - 2*c^3*d)*b)*x*e^2 + ((3*c^6 - 4*c^4 +
c^2)*a + (c^6 - c^4)*b)*e^2)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(3
*(12*c^7*d^2 - 21*c^5*d^2 + 10*c^3*d^2 - c*d^2)*a + (12*c^7*d^2 - 21*c^5*d^
2 + 10*c^3*d^2 - c*d^2)*b)*x^2*e^2 + (3*(3*a*d^7 + b*d^7)*x^7*e^2 + 21*(3*a
*c*d^6 + b*c*d^6)*x^6*e^2 + ((189*c^2*d^5 - 17*d^5)*a + (63*c^2*d^5 - 5*d^5
)*b)*x^5*e^2 + 5*((63*c^3*d^4 - 17*c*d^4)*a + (21*c^3*d^4 - 5*c*d^4)*b)*x^4
*e^2 + (5*(63*c^4*d^3 - 34*c^2*d^3 + 2*d^3)*a + (105*c^4*d^3 - 50*c^2*d^3 +
 2*d^3)*b)*x^3*e^2 + ((189*c^5*d^2 - 170*c^3*d^2 + 30*c*d^2)*a + (63*c^5*d^
2 - 50*c^3*d^2 + 6*c*d^2)*b)*x^2*e^2 + ((63*c^6*d - 85*c^4*d + 30*c^2*d - 2
*d)*a + (21*c^6*d - 25*c^4*d + 6*c^2*d)*b)*x*e^2 + ((9*c^7 - 17*c^5 + 10*c^
3 - 2*c)*a + (3*c^7 - 5*c^5 + 2*c^3)*b)*e^2)*(d*x + c + 1)*(d*x + c - 1) +
3*(3*(3*c^8*d - 7*c^6*d + 5*c^4*d - c^2*d)*a + (3*c^8*d - 7*c^6*d + 5*c^4*d
 - c^2*d)*b)*x*e^2 + (3*(3*a*d^8 + b*d^8)*x^8*e^2 + 24*(3*a*c*d^7 + b*c*d^7
)*x^7*e^2 + (2*(126*c^2*d^6 - 11*d^6)*a + 7*(12*c^2*d^6 - d^6)*b)*x^6*e^2 +
 6*(2*(42*c^3*d^5 - 11*c*d^5)*a + 7*(4*c^3*d^5 - c*d^5)*b)*x^5*e^2 + (6*(10
5*c^4*d^4 - 55*c^2*d^4 + 3*d^4)*a + 5*(42*c^4*d^4 - 21*c^2*d^4 + d^4)*b)*x^
4*e^2 + 4*(2*(63*c^5*d^3 - 55*c^3*d^3 + 9*c*d^3)*a + (42*c^5*d^3 - 35*c^3*d
^3 + 5*c*d^3)*b)*x^3*e^2 + ((252*c^6*d^2 - 330*c^4*d^2 + 108*c^2*d^2 - 5*d^
2)*a + (84*c^6*d^2 - 105*c^4*d^2 + 30*c^2*d^2 - d^2)*b)*x^2*e^2 + 2*((36*c^
7*d - 66*c^5*d + 36*c^3*d - 5*c*d)*a + (12*c^7*d - 21*c^5*d + 10*c^3*d - c*
d)*b)*x*e^2 + ((9*c^8 - 22*c^6 + 18*c^4 - 5*c^2)*a + (3*c^8 - 7*c^6 + 5*c^4
 - c^2)*b)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(c^9 - 3*c^7 + 3*c
^5 - c^3)*a + (c^9 - 3*c^7 + 3*c^5 - c^3)*b)*e^2 + (3*b*d^9*x^9*e^2 + 27*b*
c*d^8*x^8*e^2 + 9*(12*c^2*d^7 - d^7)*b*x^7*e^2 + 63*(4*c^3*d^6 - c*d^6)*b*x
^6*e^2 + 9*(42*c^4*d^5 - 21*c^2*d^5 + d^5)*b*x^5*e^2 + 9*(42*c^5*d^4 - 35*c
^3*d^4 + 5*c*d^4)*b*x^4*e^2 + 3*(84*c^6*d^3 - 105*c^4*d^3 + 30*c^2*d^3 - d^
3)*b*x^3*e^2 + 9*(12*c^7*d^2 - 21*c^5*d^2 + 10*c^3*d^2 - c*d^2)*b*x^2*e^2 +
 (3*b*d^6*x^6*e^2 + 18*b*c*d^5*x^5*e^2 + (45*c^2*d^4 - 4*d^4)*b*x^4*e^2 + 4
*(15*c^3*d^3 - 4*c*d^3)*b*x^3*e^2 + (45*c^4*d^2 - 24*c^2*d^2 + d^2)*b*x^2*e
^2 + 2*(9*c^5*d - 8*c^3*d + c*d)*b*x*e^2 + (3*c^6 - 4*c^4 + c^2)*b*e^2)*(d*
x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 9*(3*c^8*d - 7*c^6*d + 5*c^4*d - c^2
*d)*b*x*e^2 + (9*b*d^7*x^7*e^2 + 63*b*c*d^6*x^6*e^2 + (189*c^2*d^5 - 17*d^5
)*b*x^5*e^2 + 5*(63*c^3*d^4 - 17*c*d^4)*b*x^4*e^2 + 5*(63*c^4*d^3 - 34*c^2*
```

```
d^3 + 2*d^3)*b*x^3*e^2 + (189*c^5*d^2 - 170*c^3*d^2 + 30*c*d^2)*b*x^2*e^2 +
 (63*c^6*d - 85*c^4*d + 30*c^2*d - 2*d)*b*x*e^2 + (9*c^7 - 17*c^5 + 10*c^3
- 2*c)*b*e^2)*(d*x + c + 1)*(d*x + c - 1) + 3*(c^9 - 3*c^7 + 3*c^5 - c^3)*b
*e^2 + (9*b*d^8*x^8*e^2 + 72*b*c*d^7*x^7*e^2 + 2*(126*c^2*d^6 - 11*d^6)*b*x
^6*e^2 + 12*(42*c^3*d^5 - 11*c*d^5)*b*x^5*e^2 + 6*(105*c^4*d^4 - 55*c^2*d^4
 + 3*d^4)*b*x^4*e^2 + 8*(63*c^5*d^3 - 55*c^3*d^3 + 9*c*d^3)*b*x^3*e^2 + (25
2*c^6*d^2 - 330*c^4*d^2 + 108*c^2*d^2 - 5*d^2)*b*x^2*e^2 + 2*(36*c^7*d - 66
*c^5*d + 36*c^3*d - 5*c*d)*b*x*e^2 + (9*c^8 - 22*c^6 + 18*c^4 - 5*c^2)*b*e^
2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) + c))/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 - d^
5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 - 6*c^2
*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*
d - 3*c^4*d + 3*c^2*d - d)*a^2*b^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2
 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(
3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a^2*b^2
*x^2 + 2*(2*c^3*d^2 - c*d^2)*a^2*b^2*x + (c^4*d - c^2*d)*a^2*b^2)*(d*x + c
+ 1)*(d*x + c - 1) + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*b
^4*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*
b^4*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d - 3*c^4*d + 3*c^2*
d - d)*b^4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*
(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3
+ (6*c^2*d^3 - d^3)*b^4*x^2 + 2*(2*c^3*d^2 - c*d^2)*b^4*x + (c^4*d - c^2*d)
*b^4)*(d*x + c + 1)*(d*x + c - 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5
*c^2*d^4 - d^4)*b^4*x^3 + 2*(5*c^3*d^3 - 3*c*d^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*ar
ccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{d^{2}x^{2}}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{2cdx}{a^{3} + 3a^{2}b\operatorname{acosh}{\left(c + dx \right)} + 3ab^{2}\operatorname{acosh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{acosh}^{3}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
```

```
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)*
*2 + b**3*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3, x)
```

3.145 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e\text{Chi}\Big(2}{}$$

```
[Out] 1/2*e/b^2/d/(a+b*arccosh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))+e*c
osh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arccosh(d*x+c))
/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+
b*arccosh(d*x+c))^2
```

Rubi [A]
time = 0.34, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3384, 3379, 3382, 5893}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -1/2*(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh
[c + d*x])^2) + e/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e*(c + d*x)^2)/(b^2
*d*(a + b*ArcCosh[c + d*x])) - (e*CoshIntegral[(2*(a + b*ArcCosh[c + d*x]))
/b]*Sinh[(2*a)/b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCos
h[c + d*x]))/b])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{(a+b\cosh^{-1}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\cosh^{-1}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,(a\cdots}\right.}{2bd}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \cdots$$

Mathematica [A]
time = 0.38, size = 127, normalized size = 0.78

$$\frac{e\left(-\frac{b^2\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{b\left(1-2(c+dx)^2\right)}{a+b\cosh^{-1}(c+dx)} - 2\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c
 + d*x])^2) + (b*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x]) - 2*CoshInte
gral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c + d*x])]))/(2*b^3*d)
```

Maple [A]
time = 0.06, size = 254, normalized size = 1.56

method	result
derivativedivides	$-\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e(2b\operatorname{arccosh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{expIntegral}\left(1,2\operatorname{arccosh}(d\right.}{2b^3}$
default	$-\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e(2b\operatorname{arccosh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{expIntegral}\left(1,2\operatorname{arccosh}(d\right.}{2b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+
1/2*e/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2-1+2*
(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/4/b^2*e*(
2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c
))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*d^8 + b*d^8)*x^8*e + 8*(2*a*c*d^7 + b*c*d^7)*x^7*e + (2*(28*c^2*
d^6 - 3*d^6)*a + (28*c^2*d^6 - 3*d^6)*b)*x^6*e + 2*(2*(28*c^3*d^5 - 9*c*d^5
)*a + (28*c^3*d^5 - 9*c*d^5)*b)*x^5*e + (2*(70*c^4*d^4 - 45*c^2*d^4 + 3*d^4
)*a + (70*c^4*d^4 - 45*c^2*d^4 + 3*d^4)*b)*x^4*e + 4*(2*(14*c^5*d^3 - 15*c^
3*d^3 + 3*c*d^3)*a + (14*c^5*d^3 - 15*c^3*d^3 + 3*c*d^3)*b)*x^3*e + ((2*a*d
^5 + b*d^5)*x^5*e + 5*(2*a*c*d^4 + b*c*d^4)*x^4*e + (2*(10*c^2*d^3 - d^3)*a
 + (10*c^2*d^3 - d^3)*b)*x^3*e + (2*(10*c^3*d^2 - 3*c*d^2)*a + (10*c^3*d^2
- 3*c*d^2)*b)*x^2*e + (2*(5*c^4*d - 3*c^2*d)*a + (5*c^4*d - 3*c^2*d)*b)*x*e
 + (2*(c^5 - c^3)*a + (c^5 - c^3)*b)*e)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(
3/2) + (2*(28*c^6*d^2 - 45*c^4*d^2 + 18*c^2*d^2 - d^2)*a + (28*c^6*d^2 - 45
```

```
*c^4*d^2 + 18*c^2*d^2 - d^2)*b)*x^2*e + (3*(2*a*d^6 + b*d^6)*x^6*e + 18*(2*
a*c*d^5 + b*c*d^5)*x^5*e + 5*(2*(9*c^2*d^4 - d^4)*a + (9*c^2*d^4 - d^4)*b)*
x^4*e + 20*(2*(3*c^3*d^3 - c*d^3)*a + (3*c^3*d^3 - c*d^3)*b)*x^3*e + (5*(18
*c^4*d^2 - 12*c^2*d^2 + d^2)*a + (45*c^4*d^2 - 30*c^2*d^2 + 2*d^2)*b)*x^2*e
 + 2*((18*c^5*d - 20*c^3*d + 5*c*d)*a + (9*c^5*d - 10*c^3*d + 2*c*d)*b)*x*e
 + ((6*c^6 - 10*c^4 + 5*c^2 - 1)*a + (3*c^6 - 5*c^4 + 2*c^2)*b)*e)*(d*x + c
 + 1)*(d*x + c - 1) + 2*(2*(4*c^7*d - 9*c^5*d + 6*c^3*d - c*d)*a + (4*c^7*d
 - 9*c^5*d + 6*c^3*d - c*d)*b)*x*e + (3*(2*a*d^7 + b*d^7)*x^7*e + 21*(2*a*c
*d^6 + b*c*d^6)*x^6*e + 7*(2*(9*c^2*d^5 - d^5)*a + (9*c^2*d^5 - d^5)*b)*x^5
*e + 35*(2*(3*c^3*d^4 - c*d^4)*a + (3*c^3*d^4 - c*d^4)*b)*x^4*e + ((210*c^4
*d^3 - 140*c^2*d^3 + 11*d^3)*a + 5*(21*c^4*d^3 - 14*c^2*d^3 + d^3)*b)*x^3*e
 + ((126*c^5*d^2 - 140*c^3*d^2 + 33*c*d^2)*a + (63*c^5*d^2 - 70*c^3*d^2 + 1
5*c*d^2)*b)*x^2*e + ((42*c^6*d - 70*c^4*d + 33*c^2*d - 3*d)*a + (21*c^6*d -
 35*c^4*d + 15*c^2*d - d)*b)*x*e + ((6*c^7 - 14*c^5 + 11*c^3 - 3*c)*a + (3*
c^7 - 7*c^5 + 5*c^3 - c)*b)*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (2*(c^
8 - 3*c^6 + 3*c^4 - c^2)*a + (c^8 - 3*c^6 + 3*c^4 - c^2)*b)*e + (2*b*d^8*x^
8*e + 16*b*c*d^7*x^7*e + 2*(28*c^2*d^6 - 3*d^6)*b*x^6*e + 4*(28*c^3*d^5 - 9
*c*d^5)*b*x^5*e + 2*(70*c^4*d^4 - 45*c^2*d^4 + 3*d^4)*b*x^4*e + 8*(14*c^5*d
^3 - 15*c^3*d^3 + 3*c*d^3)*b*x^3*e + 2*(28*c^6*d^2 - 45*c^4*d^2 + 18*c^2*d^
2 - d^2)*b*x^2*e + 2*(b*d^5*x^5*e + 5*b*c*d^4*x^4*e + (10*c^2*d^3 - d^3)*b*
x^3*e + (10*c^3*d^2 - 3*c*d^2)*b*x^2*e + (5*c^4*d - 3*c^2*d)*b*x*e + (c^5 -
 c^3)*b*e)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 4*(4*c^7*d - 9*c^5*d +
 6*c^3*d - c*d)*b*x*e + (6*b*d^6*x^6*e + 36*b*c*d^5*x^5*e + 10*(9*c^2*d^4 -
 d^4)*b*x^4*e + 40*(3*c^3*d^3 - c*d^3)*b*x^3*e + 5*(18*c^4*d^2 - 12*c^2*d^2
 + d^2)*b*x^2*e + 2*(18*c^5*d - 20*c^3*d + 5*c*d)*b*x*e + (6*c^6 - 10*c^4 +
 5*c^2 - 1)*b*e)*(d*x + c + 1)*(d*x + c - 1) + 2*(c^8 - 3*c^6 + 3*c^4 - c^2
)*b*e + (6*b*d^7*x^7*e + 42*b*c*d^6*x^6*e + 14*(9*c^2*d^5 - d^5)*b*x^5*e +
70*(3*c^3*d^4 - c*d^4)*b*x^4*e + (210*c^4*d^3 - 140*c^2*d^3 + 11*d^3)*b*x^3
*e + (126*c^5*d^2 - 140*c^3*d^2 + 33*c*d^2)*b*x^2*e + (42*c^6*d - 70*c^4*d
+ 33*c^2*d - 3*d)*b*x*e + (6*c^7 - 14*c^5 + 11*c^3 - 3*c)*b*e)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)
)/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*a^2*b^2*x^4
+ 4*(5*c^3*d^4 - 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*a^2
*b^2*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d - 3*c^4*d + 3
*c^2*d - d)*a^2*b^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^
2*d^2*x + a^2*b^2*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(a^2*b
^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a^2*b^2*x^2 + 2*(2*c^3
*d^2 - c*d^2)*a^2*b^2*x + (c^4*d - c^2*d)*a^2*b^2)*(d*x + c + 1)*(d*x + c -
 1) + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*b^4*x^4 + 4*(5*c
^3*d^4 - 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^
5*d^2 - 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*b^4 + (b
^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d*x + c + 1)^(
3/2)*(d*x + c - 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 -
d^3)*b^4*x^2 + 2*(2*c^3*d^2 - c*d^2)*b^4*x + (c^4*d - c^2*d)*b^4)*(d*x + c
+ 1)*(d*x + c - 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)
```

```
*b^4*x^3 + 2*(5*c^3*d^3 - 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*
b^4*x + (c^5*d - 2*c^3*d + c*d)*b^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*l
og(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2*b^2*d^6*x^5 +
5*a^2*b^2*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 - 3*c*
d^3)*a^2*b^2*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d - 2*c^3
*d + c*d)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(a*b^3*d^7*x^6 +
 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 - 3*c*d^4
)*a*b^3*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 - 2*c^
3*d^2 + c*d^2)*a*b^3*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*a*b^3 + (a*b^3*d^4
*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d*x + c + 1)^(
3/2)*(d*x + c - 1)^(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^
3 - d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 - c*d^2)*a*b^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 +
 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acosh}^2(c+dx) + b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{dx}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acosh}^2(c+dx) + b^3\operatorname{acosh}^3(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2
+ b**3*acosh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*acosh(c + d*x
) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3, x)
```

3.146 $\int \frac{1}{(a+b\cosh^{-1}(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}$$

```
[Out] 1/2*(-d*x-c)/b^2/d/(a+b*arccosh(d*x+c))+1/2*cosh(a/b)*Shi((a+b*arccosh(d*x+
c))/b)/b^3/d-1/2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2
```

Rubi [A]
time = 0.19, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5995, 5880, 5951, 5881, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -1/2*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x])^2
) - (c + d*x)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (CoshIntegral[(a + b*Arc
Cosh[c + d*x])/b]*Sinh[a/b])/(2*b^3*d) + (Cosh[a/b]*SinhIntegral[(a + b*Arc
Cosh[c + d*x])/b])/(2*b^3*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\cosh^{-1}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh^{-1}(x)}\right.}{2bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)}{}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\text{Chi}\left(\frac{a+b}{}\right.}{}$$

Mathematica [A]
time = 0.30, size = 109, normalized size = 0.83

$$-\frac{\frac{b\left(ac+adx+b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}+b(c+dx)\cosh^{-1}(c+dx)\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + \text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -1/2*((b*(a*c + a*d*x + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*(c + d*x
)*ArcCosh[c + d*x]))/(a + b*ArcCosh[c + d*x])^2 + CoshIntegral[a/b + ArcCos
h[c + d*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(b^
3*d)
```

Maple [A]
time = 0.04, size = 207, normalized size = 1.57

method	result
derivativedivides	$-\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)(b\,\text{arccosh}(dx+c)+a-b)}{4b^2\left(b^2\text{arccosh}(dx+c)^2+2ab\,\text{arccosh}(dx+c)+a^2\right)} + \frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right)}{4b^3} - \frac{dx+c+\sqrt{}}{}$

default	$\frac{-\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)(b\,\mathrm{arccosh}(dx+c)+a-b)}{4b^2\left(b^2\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arccosh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{d}}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b*arccosh(d*x+c)+a-b)/b
^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4/b^3*exp(a/b)*Ei(1,ar
ccosh(d*x+c)+a/b)-1/4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccos
h(d*x+c))^2-1/4/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*
x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 - d^5
)*a + (7*c^2*d^5 - d^5)*b)*x^5 + 5*((7*c^3*d^4 - 3*c*d^4)*a + (7*c^3*d^4 -
3*c*d^4)*b)*x^4 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c*d^3)*x^3 + (6*a*c
^2*d^2 + (6*c^2*d^2 - d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 - c^2)*b + 2*(2*a*c^
3*d + (2*c^3*d - c*d)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + ((35*
c^4*d^3 - 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b)*x^3
+ (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d^3 - d^
3)*a + 5*(6*c^2*d^3 - d^3)*b)*x^3 + 3*((10*c^3*d^2 - 3*c*d^2)*a + 5*(2*c^3*
d^2 - c*d^2)*b)*x^2 + 3*(c^5 - c^3)*a + (3*c^5 - 5*c^3 + 2*c)*b + (3*(5*c^4
*d - 3*c^2*d)*a + (15*c^4*d - 15*c^2*d + 2*d)*b)*x)*(d*x + c + 1)*(d*x + c
- 1) + 3*((7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d^2 - 10*c^3*d^2 +
3*c*d^2)*b)*x^2 + (3*(a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*
(15*c^2*d^4 - 2*d^4)*a + (45*c^2*d^4 - 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 - 2*
c*d^3)*a + (15*c^3*d^3 - 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 - 36*c^2*d^2 + 4*d^
2)*a + (45*c^4*d^2 - 42*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 - 6*c^4 + 4*c^2 -
1)*a + (3*c^6 - 7*c^4 + 5*c^2 - 1)*b + 2*((9*c^5*d - 12*c^3*d + 4*c*d)*a +
(9*c^5*d - 14*c^3*d + 5*c*d)*b)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c
^7 - 3*c^5 + 3*c^3 - c)*a + (c^7 - 3*c^5 + 3*c^3 - c)*b + ((7*c^6*d - 15*c^
4*d + 9*c^2*d - d)*a + (7*c^6*d - 15*c^4*d + 9*c^2*d - d)*b)*x + (b*d^7*x^7
 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 - d^5)*b*x^5 + 5*(7*c^3*d^4 - 3*c*d^4)*b*x^
4 + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b*x^3 + (b*d^4*x^4 + 4*b*c*d^3*x^3 +
6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(d*x + c + 1)^(3/2)*(d*x + c -
 1)^(3/2) + 3*(7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*b*x^2 + 3*(b*d^5*x^5 + 5*b
*c*d^4*x^4 + (10*c^2*d^3 - d^3)*b*x^3 + (10*c^3*d^2 - 3*c*d^2)*b*x^2 + (5*c
```

```
^4*d - 3*c^2*d)*b*x + (c^5 - c^3)*b)*(d*x + c + 1)*(d*x + c - 1) + (7*c^6*d
 - 15*c^4*d + 9*c^2*d - d)*b*x + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*
d^4 - 2*d^4)*b*x^4 + 12*(5*c^3*d^3 - 2*c*d^3)*b*x^3 + (45*c^4*d^2 - 36*c^2*
d^2 + 4*d^2)*b*x^2 + 2*(9*c^5*d - 12*c^3*d + 4*c*d)*b*x + (3*c^6 - 6*c^4 +
4*c^2 - 1)*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - 3*c^5 + 3*c^3 -
c)*b)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(a^2*b^2*d^7*x^6
+ 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 - 3*
c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d
^2 - 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*a^2*b^2
 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c
^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*
b^2*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 - c*d^2)*a^2*b
^2*x + (c^4*d - c^2*d)*a^2*b^2)*(d*x + c + 1)*(d*x + c - 1) + (b^4*d^7*x^6
+ 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*b^4*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*b
^4*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 +
 c*d^2)*b^4*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*b^4 + (b^4*d^4*x^3 + 3*b^4*
c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^
(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 - d^3)*b^4*x^2 + 2*(2
*c^3*d^2 - c*d^2)*b^4*x + (c^4*d - c^2*d)*b^4)*(d*x + c + 1)*(d*x + c - 1)
+ 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)*b^4*x^3 + 2*(5*c^3
*d^3 - 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*b^4*x + (c^5*d - 2*
c^3*d + c*d)*b^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4
 + 2*(5*c^2*d^4 - d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 - 3*c*d^3)*a^2*b^2*x^2 +
(5*c^4*d^2 - 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d - 2*c^3*d + c*d)*a^2*b^2)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5
+ 3*(5*c^2*d^5 - d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*a*b^3*x^3 + 3*(5*
c^4*d^3 - 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*a*b^
3*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^
3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^
(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a*b^3*x^2
+ 2*(2*c^3*d^2 - c*d^2)*a*b^3*x + (c^4*d - c^2*d)*a*b^3)*(d*x + c + 1)*(d*x
 + c - 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)*a*b^
3*x^3 + 2*(5*c^3*d^3 - 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*a
*b^3*x + (c^5*d - 2*c^3*d + c*d)*a*b^3)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate(1/2*(d^8*
x^8 + 8*c*d^7*x^7 + c^8 + 4*(7*c^2*d^6 - d^6)*x^6 - 4*c^6 + 8*(7*c^3*d^5 -
3*c*d^5)*x^5 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3
)*(d*x + c + 1)^2*(d*x + c - 1)^2 + 2*(35*c^4*d^4 - 30*c^2*d^4 + 3*d^4)*x^4
 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 - d^3)*x^3 - 4*c^3 + 4
*(10*c^3*d^2 - 3*c*d^2)*x^2 + (20*c^4*d - 12*c^2*d + 3*d)*x + 3*c)*(d*x + c
 + 1)^(3/2)*(d*x + c - 1)^(3/2) + 6*c^4 + 8*(7*c^5*d^3 - 10*c^3*d^3 + 3*c*d
^3)*x^3 + 3*(2*d^6*x^6 + 12*c*d^5*x^5 + 2*c^6 +...
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*a
rccosh(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{acosh}(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{acosh}(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^3, x)
```

3.147 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^3,x)/e

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^3), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 1.47, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3), x]

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b\,\text{arccosh}\,(dx + c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 - 3*d^6)*b*x^6 + 2*(28*c^3*d^
5 - 9*c*d^5)*b*x^5 + (70*c^4*d^4 - 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*d^
3 - 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (b*d^5*x^5 + 5*b*c*d^4*x^4 + (2*a*d^3 + (
10*c^2*d^3 - d^3)*b)*x^3 + (6*a*c*d^2 + (10*c^3*d^2 - 3*c*d^2)*b)*x^2 + 2*(
c^3 - c)*a + (c^5 - c^3)*b + (2*(3*c^2*d - d)*a + (5*c^4*d - 3*c^2*d)*b)*x)
*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (28*c^6*d^2 - 45*c^4*d^2 + 18*c^
2*d^2 - d^2)*b*x^2 + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + (4*a*d^4 + 5*(9*c^2*d^
4 - d^4)*b)*x^4 + 4*(4*a*c*d^3 + 5*(3*c^3*d^3 - c*d^3)*b)*x^3 + ((24*c^2*d^
2 - 5*d^2)*a + (45*c^4*d^2 - 30*c^2*d^2 + 2*d^2)*b)*x^2 + (4*c^4 - 5*c^2 +
1)*a + (3*c^6 - 5*c^4 + 2*c^2)*b + 2*((8*c^3*d - 5*c*d)*a + (9*c^5*d - 10*c
^3*d + 2*c*d)*b)*x)*(d*x + c + 1)*(d*x + c - 1) + 2*(4*c^7*d - 9*c^5*d + 6*
c^3*d - c*d)*b*x + (3*b*d^7*x^7 + 21*b*c*d^6*x^6 + (2*a*d^5 + 7*(9*c^2*d^5
- d^5)*b)*x^5 + 5*(2*a*c*d^4 + 7*(3*c^3*d^4 - c*d^4)*b)*x^4 + ((20*c^2*d^3
- 3*d^3)*a + 5*(21*c^4*d^3 - 14*c^2*d^3 + d^3)*b)*x^3 + ((20*c^3*d^2 - 9*c*
d^2)*a + (63*c^5*d^2 - 70*c^3*d^2 + 15*c*d^2)*b)*x^2 + (2*c^5 - 3*c^3 + c)*
a + (3*c^7 - 7*c^5 + 5*c^3 - c)*b + ((10*c^4*d - 9*c^2*d + d)*a + (21*c^6*d
 - 35*c^4*d + 15*c^2*d - d)*b)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^
8 - 3*c^6 + 3*c^4 - c^2)*b + (2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + (3*c^2*d - d)*
b*x + (c^3 - c)*b)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + (4*b*d^4*x^4 +
 16*b*c*d^3*x^3 + (24*c^2*d^2 - 5*d^2)*b*x^2 + 2*(8*c^3*d - 5*c*d)*b*x + (4
*c^4 - 5*c^2 + 1)*b)*(d*x + c + 1)*(d*x + c - 1) + (2*b*d^5*x^5 + 10*b*c*d^
4*x^4 + (20*c^2*d^3 - 3*d^3)*b*x^3 + (20*c^3*d^2 - 9*c*d^2)*b*x^2 + (10*c^4
*d - 9*c^2*d + d)*b*x + (2*c^5 - 3*c^3 + c)*b)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(a^2*b^2*d^9*x
^8*e + 8*a^2*b^2*c*d^8*x^7*e + (28*c^2*d^7 - 3*d^7)*a^2*b^2*x^6*e + 2*(28*c
^3*d^6 - 9*c*d^6)*a^2*b^2*x^5*e + (70*c^4*d^5 - 45*c^2*d^5 + 3*d^5)*a^2*b^2
```

```
*x^4*e + 4*(14*c^5*d^4 - 15*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3*e + (28*c^6*d^3
- 45*c^4*d^3 + 18*c^2*d^3 - d^3)*a^2*b^2*x^2*e + 2*(4*c^7*d^2 - 9*c^5*d^2 +
 6*c^3*d^2 - c*d^2)*a^2*b^2*x*e + (c^8*d - 3*c^6*d + 3*c^4*d - c^2*d)*a^2*b
^2*e + (a^2*b^2*d^6*x^5*e + 5*a^2*b^2*c*d^5*x^4*e + 10*a^2*b^2*c^2*d^4*x^3*
e + 10*a^2*b^2*c^3*d^3*x^2*e + 5*a^2*b^2*c^4*d^2*x*e + a^2*b^2*c^5*d*e)*(d*
x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(a^2*b^2*d^7*x^6*e + 6*a^2*b^2*c*d
^6*x^5*e + (15*c^2*d^5 - d^5)*a^2*b^2*x^4*e + 4*(5*c^3*d^4 - c*d^4)*a^2*b^2
*x^3*e + 3*(5*c^4*d^3 - 2*c^2*d^3)*a^2*b^2*x^2*e + 2*(3*c^5*d^2 - 2*c^3*d^2
)*a^2*b^2*x*e + (c^6*d - c^4*d)*a^2*b^2*e)*(d*x + c + 1)*(d*x + c - 1) + (b
^4*d^9*x^8*e + 8*b^4*c*d^8*x^7*e + (28*c^2*d^7 - 3*d^7)*b^4*x^6*e + 2*(28*c
^3*d^6 - 9*c*d^6)*b^4*x^5*e + (70*c^4*d^5 - 45*c^2*d^5 + 3*d^5)*b^4*x^4*e +
 4*(14*c^5*d^4 - 15*c^3*d^4 + 3*c*d^4)*b^4*x^3*e + (28*c^6*d^3 - 45*c^4*d^3
 + 18*c^2*d^3 - d^3)*b^4*x^2*e + 2*(4*c^7*d^2 - 9*c^5*d^2 + 6*c^3*d^2 - c*d
^2)*b^4*x*e + (c^8*d - 3*c^6*d + 3*c^4*d - c^2*d)*b^4*e + (b^4*d^6*x^5*e +
5*b^4*c*d^5*x^4*e + 10*b^4*c^2*d^4*x^3*e + 10*b^4*c^3*d^3*x^2*e + 5*b^4*c^4
*d^2*x*e + b^4*c^5*d*e)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(b^4*d^
7*x^6*e + 6*b^4*c*d^6*x^5*e + (15*c^2*d^5 - d^5)*b^4*x^4*e + 4*(5*c^3*d^4 -
 c*d^4)*b^4*x^3*e + 3*(5*c^4*d^3 - 2*c^2*d^3)*b^4*x^2*e + 2*(3*c^5*d^2 - 2*
c^3*d^2)*b^4*x*e + (c^6*d - c^4*d)*b^4*e)*(d*x + c + 1)*(d*x + c - 1) + 3*(
b^4*d^8*x^7*e + 7*b^4*c*d^7*x^6*e + (21*c^2*d^6 - 2*d^6)*b^4*x^5*e + 5*(7*c
^3*d^5 - 2*c*d^5)*b^4*x^4*e + (35*c^4*d^4 - 20*c^2*d^4 + d^4)*b^4*x^3*e + (
21*c^5*d^3 - 20*c^3*d^3 + 3*c*d^3)*b^4*x^2*e + (7*c^6*d^2 - 10*c^4*d^2 + 3*
c^2*d^2)*b^4*x*e + (c^7*d - 2*c^5*d + c^3*d)*b^4*e)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2
*b^2*d^8*x^7*e + 7*a^2*b^2*c*d^7*x^6*e + (21*c^2*d^6 - 2*d^6)*a^2*b^2*x^5*e
 + 5*(7*c^3*d^5 - 2*c*d^5)*a^2*b^2*x^4*e + (35*c^4*d^4 - 20*c^2*d^4 + d^4)*
a^2*b^2*x^3*e + (21*c^5*d^3 - 20*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2*e + (7*c^6*
d^2 - 10*c^4*d^2 + 3*c^2*d^2)*a^2*b^2*x*e + (c^7*d - 2*c^5*d + c^3*d)*a^2*b
^2*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(a*b^3*d^9*x^8*e + 8*a*b^3*c*
d^8*x^7*e + (28*c^2*d^7 - 3*d^7)*a*b^3*x^6*e + 2*(28*c^3*d^6 - 9*c*d^6)*a*b
^3*x^5*e + (70*c^4*d^5 - 45*c^2*d^5 + 3*d^5)*a*b^3*x^4*e + 4*(14*c^5*d^4 -
15*c^3*d^4 + 3*c*d^4)*a*b^3*x^3*e + (28*c^6*d^3 - 45*c^4*d^3 + 18*c^2*d^3 -
 d^3)*a*b^3*x^2*e + 2*(4*c^7*d^2 - 9*c^5*d^2 + 6*c^3*d^2 - c*d^2)*a*b^3*x*e
 + (c^8*d - 3*c^6*d + 3*c^4*d - c^2*d)*a*b^3*e + (a*b^3*d^6*x^5*e + 5*a*b^3
*c*d^5*x^4*e + 10*a*b^3*c^2*d^4*x^3*e + 10*a*b^3*c^3*d^3*x^2*e + 5*a*b^3*c^
4*d^2*x*e + a*b^3*c^5*d*e)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(a*b
^3*d^7*x^6*e + 6*a*b^3*c*d^6*x^5*e + (15*c^2*d^5 - d^5)*a*b^3*x^4*e + 4*(5*
c^3*d^4 - c*d^4)*a*b^3*x^3*e + 3*(5*c^4*d^3 - 2*c^2*d^3)*a*b^3*x^2*e + 2*(3
*c^5*d^2 - 2*c^3*d^2)*a*b^3*x*e + (c^6*d - c^4*d)*a*b^3*e)*(d*x + c + 1)*(d
*x + c - 1) + 3*(a*b^3*d^8*x^7*e + 7*a*b^3*c*d^7*x^6*e + (21*c^2*d^6 - 2*d^
6)*a*b^3*x^5*e + 5*(7*c^3*d^5 - 2*c*d^5)*a*b^3*...
```

Fricas [A]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/((b^3*d*x + b^3*c)*arccosh(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c
)*arccosh(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arccosh(d*x + c)*e + (a^3*
d*x + a^3*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^3c+a^3dx+3a^2bc\operatorname{acosh}(c+dx)+3a^2bdx\operatorname{acosh}(c+dx)+3ab^2c\operatorname{acosh}^2(c+dx)+3ab^2dx\operatorname{acosh}^2(c+dx)+b^3c\operatorname{acosh}^3(c+dx)+b^3dx\operatorname{acosh}^3(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*acosh(c + d*x) + 3*a**2*b*d*x*ac
osh(c + d*x) + 3*a*b**2*c*acosh(c + d*x)**2 + 3*a*b**2*d*x*acosh(c + d*x)**
2 + b**3*c*acosh(c + d*x)**3 + b**3*d*x*acosh(c + d*x)**3), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3), x)
```

3.148 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=431

$$-\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{5e^4(c+dx)^5}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{2}{\cdots}$$

```
[Out] 2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a+b
*arccosh(d*x+c))^2+1/48*e^4*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^4/d+27/
32*e^4*Chi(3*(a+b*arccosh(d*x+c))/b)*cosh(3*a/b)/b^4/d+125/96*e^4*Chi(5*(a+
b*arccosh(d*x+c))/b)*cosh(5*a/b)/b^4/d-1/48*e^4*Shi((a+b*arccosh(d*x+c))/b)
*sinh(a/b)/b^4/d-27/32*e^4*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^4/d-
125/96*e^4*Shi(5*(a+b*arccosh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(d*x+c)^
4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3+2*e^4*(d*x+c)^
2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))-25/6*e^4*(d*x+
c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.79, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5886, 5951, 5885, 3384, 3379, 3382}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{48b^4d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}-\frac{25e^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{6b^3d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{2e^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{5e^4(c+dx)^5}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^3) + (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2
) - (5*e^4*(c + d*x)^5)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (2*e^4*Sqrt[
-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x]
)) - (25*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(6*b^3*d*(a
+ b*ArcCosh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x
])/b])/(48*b^4*d) + (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c
+ d*x]))/b])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*Arc
Cosh[c + d*x]))/b])/(96*b^4*d) - (e^4*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh
[c + d*x])/b])/(48*b^4*d) - (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*Ar
cCosh[c + d*x]))/b])/(32*b^4*d) - (125*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*(a
 + b*ArcCosh[c + d*x]))/b])/(96*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{(4e^4)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{1}}\right.}{}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Mathematica [A]
time = 1.83, size = 424, normalized size = 0.98

$$\frac{e^4\left(-\frac{32b^3\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{16b^2\left(4(c+dx)^3-5(c+dx)^5\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{16b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(-12(c+dx)^2+25(c+dx)^4\right)}{a+b\cosh^{-1}(c+dx)}+384\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-544\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)+125\left(10\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+5\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-10\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-5\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^4*((-32*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^3 + (16*b^2*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[
```

```
c + d*x])^2 - (16*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-12*(c + d*x)^2 +
 25*(c + d*x)^4))/(a + b*ArcCosh[c + d*x]) + 384*(Cosh[a/b]*CoshIntegral[a/
b + ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]) - 5
44*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh
[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]) + 125*
(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + 5*Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - 5*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[(5*a)/b]*SinhIn
tegral[5*(a/b + ArcCosh[c + d*x])])))/(96*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(399) = 798$.
time = 0.21, size = 1375, normalized size = 3.19

method	result	size
derivativedivides	Expression too large to display	1375
default	Expression too large to display	1375

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/192*(-16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+12*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(25*b^2*arccosh(d*x+c)^2+50*a*b*arccosh(d*x+c)-5*
b^2*arccosh(d*x+c)+25*a^2-5*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*ar
ccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-125/192*e^4/b^4*exp(5*a/b)*Ei(1,
5*arccosh(d*x+c)+5*a/b)+1/64*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(9*b^2*arccosh(d
*x+c)^2+18*a*b*arccosh(d*x+c)-3*b^2*arccosh(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(
b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2
7/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/96*(-(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+d*x+c)*e^4*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-b^2*
arccosh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x
+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/96*e^4/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)
+a/b)-1/48/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^3-1/96/b^2*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+
c))^2-1/96/b^3*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/32/b*e^4*(4*(d*x+c)
^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-3/64/b^2*e^4*(4*(d*x+c)^3-3*d*x-3*c+4
*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
/(a+b*arccosh(d*x+c))^2-9/64/b^3*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(
```

```
d*x+c))-27/64/b^4*e^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/96/b*e^4*
(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+5*d
*x+5*c-12*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))/(a+b*arccosh(d*x+c))^3-5/192/b^2*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+1
6*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(d*x+c+1)^(1/2)*(d
*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-25/192/b^3*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c+1)^(1/2)*(d*x+c-1)
^(1/2)*(d*x+c)^4+5*d*x+5*c-12*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-125/192/b^4*e^4*exp(-5*a
/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^4
*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x +
c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{4}+4a^{3}b\operatorname{acosh}(c+dx)+6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)+4ab^{3}\operatorname{acosh}^{3}(c+dx)+b^{4}\operatorname{acosh}^{4}(c+dx)}\,dx+\int \frac{d^{4}x^{4}}{a^{4}+4a^{3}b\operatorname{acosh}(c+dx)+6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)+4ab^{3}\operatorname{acosh}^{3}(c+dx)+b^{4}\operatorname{acosh}^{4}(c+dx)}\,dx+\int \frac{4cd^{3}x^{3}}{a^{4}+4a^{3}b\operatorname{acosh}(c+dx)+6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)+4ab^{3}\operatorname{acosh}^{3}(c+dx)+b^{4}\operatorname{acosh}^{4}(c+dx)}\,dx+\int \frac{6c^{2}d^{2}x^{2}}{a^{4}+4a^{3}b\operatorname{acosh}(c+dx)+6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)+4ab^{3}\operatorname{acosh}^{3}(c+dx)+b^{4}\operatorname{acosh}^{4}(c+dx)}\,dx+\int \frac{4c^{3}dx}{a^{4}+4a^{3}b\operatorname{acosh}(c+dx)+6a^{2}b^{2}\operatorname{acosh}^{2}(c+dx)+4ab^{3}\operatorname{acosh}^{3}(c+dx)+b^{4}\operatorname{acosh}^{4}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**4*(Integral(c**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**4*x**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(4*
c*d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2
```

```
 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(6*c*
*2*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**
2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(4*c
**3*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4
*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4, x)
```

3.149 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=360

$$-\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{2e^3(c+dx)^4}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e}{}$$

```
[Out] 1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a+b
*arccosh(d*x+c))^2+1/3*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^4/d+
4/3*e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*cosh(4*a/b)/b^4/d-1/3*e^3*Shi(2*(a+b*
arccosh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arccosh(d*x+c))/b)*
sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+
b*arccosh(d*x+c))^3+e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*
arccosh(d*x+c))-8/3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+
b*arccosh(d*x+c))
```

Rubi [A]
time = 0.57, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5886, 5951, 5885, 3384, 3379, 3382}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{4e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{8e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{3b^3d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{2e^3(c+dx)^4}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^3) + (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2)
- (2*e^3*(c + d*x)^4)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^3*Sqrt[-1 +
 c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x])) - (
8*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*Arc
Cosh[c + d*x])) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcCosh[c + d*x
]))/b])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcCosh[c +
 d*x]))/b])/(3*b^4*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcCosh[c
 + d*x]))/b])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcCo
sh[c + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e^3\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1+}\right.}{}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}\\
&= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}
\end{aligned}
$$

Mathematica [A]
time = 1.28, size = 330, normalized size = 0.92

$$
\frac{e^3\left(-\frac{2b^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{b^2\left(3(c+dx)^2-4(c+dx)^4\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\left(-3(c+dx)+8(c+dx)^3\right)}{a+b\cosh^{-1}(c+dx)}+6\log\left(a+b\cosh^{-1}(c+dx)\right)-30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\log\left(a+b\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)+8\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+3\log\left(a+b\cosh^{-1}(c+dx)\right)-4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{6b^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]

[Out] (e^3*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (b^2*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c +
```

```
d*x])^2 - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-3*(c + d*x) + 8*(c +
d*x)^3))/(a + b*ArcCosh[c + d*x]) + 6*Log[a + b*ArcCosh[c + d*x]] - 30*(Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a + b*ArcCosh[c +
 d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]) + 8*(4*Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*CoshInt
egral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a + b*ArcCosh[c + d*x]] - 4*Sinh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])] - Sinh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(332) = 664$.
time = 0.14, size = 860, normalized size = 2.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3+4*(d*x+c+1)^(1/2)*(
d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(8*b^2*arccosh(d*x+c)
^2+16*a*b*arccosh(d*x+c)-2*b^2*arccosh(d*x+c)+8*a^2-2*a*b+b^2)/b^3/(b^3*arc
cosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2/3*e^3/
b^4*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/24*(-2*(d*x+c+1)^(1/2)*(d*x+c
-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b^2*arccosh(d*x+c)^2+4*a*b*arccosh(
d*x+c)-b^2*arccosh(d*x+c)+2*a^2-a*b+b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*
arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*
arccosh(d*x+c)+2*a/b)-1/24/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)
^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/24/b^2*e^3*(2*(d*x+c)^2-1+2*(d*x+c
+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/12/b^3*e^3*(2*(
d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-
1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/48/b*e^3*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^2-1/6/b^3*e^3*(8*(d*x+c)^
4-8*(d*x+c)^2+8*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^3-4*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-2/3/b^4*e^3*exp(-4*a/b)*Ei
(1,-4*arccosh(d*x+c)-4*a/b))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^4*arccosh(d*x + c
)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*a
rccosh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{4} + 4a^{3}b\operatorname{acosh}\left(c + dx\right) + 6a^{2}b^{2}\operatorname{acosh}^{2}\left(c + dx\right) + 4ab^{3}\operatorname{acosh}^{3}\left(c + dx\right) + b^{4}\operatorname{acosh}^{4}\left(c + dx\right)}\, dx + \int \frac{d^{3}x^{3}}{a^{4} + 4a^{3}b\operatorname{acosh}\left(c + dx\right) + 6a^{2}b^{2}\operatorname{acosh}^{2}\left(c + dx\right) + 4ab^{3}\operatorname{acosh}^{3}\left(c + dx\right) + b^{4}\operatorname{acosh}^{4}\left(c + dx\right)}\, dx + \int \frac{3cd^{2}x^{2}}{a^{4} + 4a^{3}b\operatorname{acosh}\left(c + dx\right) + 6a^{2}b^{2}\operatorname{acosh}^{2}\left(c + dx\right) + 4ab^{3}\operatorname{acosh}^{3}\left(c + dx\right) + b^{4}\operatorname{acosh}^{4}\left(c + dx\right)}\, dx + \int \frac{3c^{2}dx}{a^{4} + 4a^{3}b\operatorname{acosh}\left(c + dx\right) + 6a^{2}b^{2}\operatorname{acosh}^{2}\left(c + dx\right) + 4ab^{3}\operatorname{acosh}^{3}\left(c + dx\right) + b^{4}\operatorname{acosh}^{4}\left(c + dx\right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**3*(Integral(c**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(3*
c*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2
 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(3*c*
*2*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*
a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4, x)
```

3.150 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=352

$$-\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e^2(c+dx)^3}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+e^2$$

```
[Out] 1/3*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b*a
rccosh(d*x+c))^2+1/24*e^2*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^4/d+9/8*e
^2*Chi(3*(a+b*arccosh(d*x+c))/b)*cosh(3*a/b)/b^4/d-1/24*e^2*Shi((a+b*arccos
h(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a
/b)/b^4/d-1/3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccos
h(d*x+c))^3+1/3*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+
c))-3/2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*
x+c))
```

Rubi [A]
time = 0.61, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5996, 12, 5886, 5951, 5885, 3384, 3379, 3382, 5880, 5953}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{3e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{2b^3d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e^2(c+dx)^3}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*Arc
Cosh[c + d*x])^3) + (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) -
(e^2*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^2*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) - (3*e^2*Sqrt[-
1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b^3*d*(a + b*ArcCosh[c + d*x
])) + (e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcCosh[c + d*x])/b])/(24*b^4*d) +
 (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^4*
d) - (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(24*b^4*d) -
(9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcCosh[c + d*x]))/b])/(8*b^4*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
```

```
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{(2e^2)\,\text{Subst}\Big(\int \frac{}{\sqrt{-1+x}\,\sqrt{1}}}{}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}$$

Mathematica [A]
time = 1.01, size = 272, normalized size = 0.77

$$\frac{e^2\left(-\frac{8b^3\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{4b^2\left(2(c+dx)-3(c+dx)^3\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(-2+9(c+dx)^2\right)}{a+b\cosh^{-1}(c+dx)}-80\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+80\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+27\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (4*b^2*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c +
d*x])^2 - (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-2 + 9*(c + d*x)^2))/(
a + b*ArcCosh[c + d*x]) - 80*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
 + 80*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + 27*(3*Cosh[a/b]*Cosh
Integral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcC
osh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])))/(24*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(322) = 644$.
time = 0.12, size = 777, normalized size = 2.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(9*b^2*arccosh(d*x+c)^2+18*a*b*arcc
osh(d*x+c)-3*b^2*arccosh(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^
3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-9/16*e^2/b^4*exp(3*a
/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/48*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x
+c)*e^2*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)+a^2-a
*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccos
h(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/24/b*e^2*(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-1/48/b^2*e^2*(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/48/b^3*e^2*(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/48/b^4*e^2*ex
p(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+
c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*
arccosh(d*x+c))^3-1/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*
x+c-1)^(1/2)*(d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c)
)^2-3/16/b^3*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(
d*x+c)^2-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16/b^4*e^2
*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*ar
ccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c)
+ a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{4}+4a^{3}b\operatorname{acosh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{acosh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{acosh}^{3}\left(c+dx\right)+b^{4}\operatorname{acosh}^{4}\left(c+dx\right)}\,dx+\int \frac{d^{2}x^{2}}{a^{4}+4a^{3}b\operatorname{acosh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{acosh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{acosh}^{3}\left(c+dx\right)+b^{4}\operatorname{acosh}^{4}\left(c+dx\right)}\,dx+\int \frac{2cdx}{a^{4}+4a^{3}b\operatorname{acosh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{acosh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{acosh}^{3}\left(c+dx\right)+b^{4}\operatorname{acosh}^{4}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**2*(Integral(c**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(2*
c*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*a
*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4, x)
```

3.151 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=218

$$-\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e(c+dx)^2}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{2e\sqrt{}}{}$$

```
[Out] 1/6*e/b^2/d/(a+b*arccosh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c
))^2+2/3*e*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(a+b
*arccosh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3-2/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.34, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5996, 12, 5886, 5951, 5885, 3384, 3379, 3382, 5893}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{3b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{e(c+dx)^2}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}+\frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -1/3*(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh
[c + d*x])^3) + e/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e*(c + d*x)^2)/(3
*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt
[1 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) + (2*e*Cosh[(2*a)/b]*Cosh
Integral[(2*(a + b*ArcCosh[c + d*x]))/b])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*Si
nhIntegral[(2*(a + b*ArcCosh[c + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\,(a\text{-}}\right.}{3bd}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} -$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} -$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} -$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} -$$

Mathematica [A]
time = 0.76, size = 195, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^3}+\frac{b^2\left(1-2(c+dx)^2\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{4b\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{a+b\cosh^{-1}(c+dx)}-4\log\left(a+b\cosh^{-1}(c+dx)\right)+4\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\log\left(a+b\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[
c + d*x])^3 + (b^2*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x])^2 - (4*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - 4
*Log[a + b*ArcCosh[c + d*x]] + 4*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCo
sh[c + d*x])] + Log[a + b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*
(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

Maple [A]
time = 0.05, size = 353, normalized size = 1.62

method	result
derivativedivides	$\frac{\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e\left(2b^2\operatorname{arccosh}(dx+c)^2+4ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+2\right.}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}}{}$
default	$\frac{\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e\left(2b^2\operatorname{arccosh}(dx+c)^2+4ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+2\right.}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
^2*arccosh(d*x+c)^2+4*a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)+2*a^2-a*b+b^2)/
b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a
^3)-1/3*e/b^4*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2
-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/12/b
^2*e*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh
(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b
))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 +
 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{dx}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e*(Integral(c/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(d*
x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*a*b**
3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4, x)
```

3.152 $\int \frac{1}{(a+b\cosh^{-1}(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6b^3d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Ch}}{}$$

```
[Out] 1/6*(-d*x-c)/b^2/d/(a+b*arccosh(d*x+c))^2+1/6*Chi((a+b*arccosh(d*x+c))/b)*c
osh(a/b)/b^4/d-1/6*Shi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3-1/6*(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A]
time = 0.29, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5995, 5880, 5951, 5953, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -1/3*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x])^3
) - (c + d*x)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x])/(6*b^3*d*(a + b*ArcCosh[c + d*x])) + (Cosh[a/b]*CoshIntegr
al[(a + b*ArcCosh[c + d*x])/b])/(6*b^4*d) - (Sinh[a/b]*SinhIntegral[(a + b*
ArcCosh[c + d*x])/b])/(6*b^4*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\cosh^{-1}(x))^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh^{-1}(x)}\right.}{3bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+}}{6b^3d\,(a}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+}}{6b^3d\,(a}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+}}{6b^3d\,(a}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+}}{6b^3d\,(a}$$

Mathematica [A]
time = 0.53, size = 144, normalized size = 0.83

$$-\frac{\frac{2b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2(c+dx)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{a+b\cosh^{-1}(c+dx)} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -1/6*((2*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])
^3 + (b^2*(c + d*x))/(a + b*ArcCosh[c + d*x])^2 + (b*Sqrt[-1 + c + d*x]*Sqr
t[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + Arc
Cosh[c + d*x]] + Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(b^4*d)
```

Maple [A]
time = 0.04, size = 295, normalized size = 1.70

method	result

derivativedivides	$\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}-\frac{e^{\frac{a}{b}}\,\mathrm{ex}}{}$
default	$\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)-b^2\operatorname{arccosh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}-\frac{e^{\frac{a}{b}}\,\mathrm{ex}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b^2*arccosh(d*x+c)^2+2*
a*b*arccosh(d*x+c)-b^2*arccosh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c
)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/12/b^4*exp(a/b)*
Ei(1,arccosh(d*x+c)+a/b)-1/6/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^3-1/12/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*ar
ccosh(d*x+c))^2-1/12/b^3*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcco
sh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2
*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^4, x)
```

3.153 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^4,x)/e
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^4), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 11.51, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\left(a + b\,\mathrm{arccosh}\left(dx + c\right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/((b^4*d*x + b^4*c)*arccosh(d*x + c)^4*e + 4*(a*b^3*d*x + a*b^3*c
)*arccosh(d*x + c)^3*e + 6*(a^2*b^2*d*x + a^2*b^2*c)*arccosh(d*x + c)^2*e +
 4*(a^3*b*d*x + a^3*b*c)*arccosh(d*x + c)*e + (a^4*d*x + a^4*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} c + a^{4} d x + 4 a^{3} b c \operatorname{acosh}{\left(c + d x \right)} + 4 a^{3} b d x \operatorname{acosh}{\left(c + d x \right)} + 6 a^{2} b^{2} c \operatorname{acosh}^{2}{\left(c + d x \right)} + 6 a^{2} b^{2} d x \operatorname{acosh}^{2}{\left(c + d x \right)} + 4 a b^{3} c \operatorname{acosh}^{3}{\left(c + d x \right)} + 4 a b^{3} d x \operatorname{acosh}^{3}{\left(c + d x \right)} + b^{4} c \operatorname{acosh}^{4}{\left(c + d x \right)} + b^{4} d x \operatorname{acosh}^{4}{\left(c + d x \right)}}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*acosh(c + d*x) + 4*a**3*b*d*x*ac
osh(c + d*x) + 6*a**2*b**2*c*acosh(c + d*x)**2 + 6*a**2*b**2*d*x*acosh(c +
d*x)**2 + 4*a*b**3*c*acosh(c + d*x)**3 + 4*a*b**3*d*x*acosh(c + d*x)**3 + b
**4*c*acosh(c + d*x)**4 + b**4*d*x*acosh(c + d*x)**4), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4), x)
```

3.154 $\int (ce+dex)^4 \sqrt{a+b\cosh^{-1}(c+dx)}\, dx$

Optimal. Leaf size=361

$$\frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d}-\frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{b}\,e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\cdots\right.}{\cdots}$$

```
[Out] -1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b
^(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^(1
/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d-1/192*e^
4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)
/d/exp(3*a/b)-1/32*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(
1/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*P
i^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.67, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5996, 12, 5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\frac{\sqrt{\pi}\sqrt{b}\,e^4e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^4e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^4e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}+\frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcCosh[c + d*x]])/(5*d) - (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(64*d) - (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Cosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) - (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh^5(x)}{\sqrt{a+bx}}\right.}{10}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \left(\frac{5\cosh(x}{8\sqrt{a+b}}\right.\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\right.}{16}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\right.}{32}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{e^4\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x\right.}{1}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-}}{}\right.}{32d}$$

Mathematica [A]

time = 0.52, size = 342, normalized size = 0.95

$$\frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)+25\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+150e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+25\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+3\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)\right)}{2400d\sqrt{-\frac{(a+b\cosh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcCosh[c + d*x]]*(150*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c
+ d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcCo
sh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcCosh[c + d*x]))/b] + 25*Sqrt[3]*E^((2
*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x])
)/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*Arc
Cosh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])
/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c + d*x]))/b] + 3*Sqrt[5]*E^((10*a)/b)*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcCosh[c + d*x]))/b
]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \sqrt{a + b \operatorname{arccosh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int c^4\sqrt{a + b\operatorname{acosh}(c + dx)}\,dx + \int d^4x^4\sqrt{a + b\operatorname{acosh}(c + dx)}\,dx + \int 4cd^3x^3\sqrt{a + b\operatorname{acosh}(c + dx)}\,dx + \int 6c^2d^2x^2\sqrt{a + b\operatorname{acosh}(c + dx)}\,dx + \int 4c^3dx\sqrt{a + b\operatorname{acosh}(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^4\,\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2), x)
```

3.155 $\int (ce+dex)^3 \sqrt{a+b\cosh^{-1}(c+dx)}\, dx$

Optimal. Leaf size=272

$$-\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}+\frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{\sqrt{b}\,e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+}}{\sqrt{b}}\right.}{256d}$$

```
[Out] -1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*2^(1/2)*Pi^(1/2)/d-1/64*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b*a
rccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*arcc
osh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.53, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5996, 12, 5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcCosh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d) - (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) - (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}}\right.\right.}{}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

<u>Mathematica</u> [A]
time = 0.40, size = 223, normalized size = 0.82

$$\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)+4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\left(4\sqrt{2}\,\Gamma\left(\frac{3}{2},\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)}{128d\sqrt{-\frac{(a+b\cosh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh
```

```
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c + d
*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(128*d
*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 \sqrt{a + b\operatorname{arccosh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int c^3\sqrt{a + b\operatorname{acosh}(c + dx)}\, dx + \int d^3x^3\sqrt{a + b\operatorname{acosh}(c + dx)}\, dx + \int 3cd^2x^2\sqrt{a + b\operatorname{acosh}(c + dx)}\, dx + \int 3c^2dx\sqrt{a + b\operatorname{acosh}(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(1/2),x)
[Out] e**3*(Integral(c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2), x)
```

3.156 $\int (ce+dex)^2 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d}-\frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{b}\,e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\Big(}{}$$

```
[Out] -1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/16*e^2*erfi((a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b*arc
cosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.50, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5996, 12, 5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}-\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(48*d) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \frac{}{\sqrt{-1+x}}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \frac{\cosh^3(x)}{\sqrt{a+bx}}}{6}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \Big(\frac{3\cosh(x}{4\sqrt{a+}}}{}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \frac{\cosh(3x)}{\sqrt{a+bx}}}{24}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \frac{e^{-3x}}{\sqrt{a+bx}}}{48}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{e^2\text{Subst}\Big(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x}{2}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\text{erf}\Big(\frac{\sqrt{a-}}{}}{16d}$$

Mathematica [A]
time = 0.36, size = 237, normalized size = 0.97

$$\frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+9e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)\right)}{72d\sqrt{-\frac{(a+b\cosh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c
```

```
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqrt[a
/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcCos
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]
)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(dex + ce\right)^{2} \sqrt{a + b \,\mathrm{arccosh}\left(dx + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2} \left(\int c^{2} \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}\, dx + \int d^{2} x^{2} \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}\, dx + \int 2 c d x \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*acosh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2), x)
```

3.157 $\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}-\frac{\sqrt{b}\,ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+}{\sqrt{b}}\right.}{16d}$$

```
[Out] -1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/4*e*(a+b*arccosh(d*x+c))^(1/2)/d+1/
2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.35, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5996, 12, 5884, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}-\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/4*(e*Sqrt[a + b*ArcCosh[c + d*x]])/d + (e*(c + d*x)^2*Sqrt[a + b*ArcCosh
[c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int ex\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int x\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d} \\
&= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\sqrt{1}} \right.}{} \\
&= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx,\right.}{4d} \\
&= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}+\right.\right.}{} \\
&= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} \\
&= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} \\
&= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} \\
&= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 437 vs. $2(164) = 328$.

time = 1.99, size = 437, normalized size = 2.66

$$
\frac{e\left(-32c(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}+8\sqrt{a+b\cosh^{-1}(c+dx)}\cosh\left(2\cosh^{-1}(c+dx)\right)+8\sqrt{b}c\sqrt{\pi}\cosh\left(\frac{a}{b}\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-\sqrt{b}\sqrt{2\pi}\cosh\left(\frac{2a}{b}\right)\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)+16ce^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)-8\sqrt{b}c\sqrt{\pi}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{a}{b}\right)+8\sqrt{b}c\sqrt{\pi}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+\sqrt{b}\sqrt{2\pi}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{2a}{b}\right)-\sqrt{b}\sqrt{2\pi}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)\right)}{32d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCosh[c +
 d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[
```

```
a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(S
qrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] + (16*c*Sqrt[a + b*ArcCosh[c
+ d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCos
h[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 8*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - Sqrt[b]*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[
(2*a)/b])))/(32*d)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int c\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int dx\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x*sqrt(a + b*acos
h(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2), x)
```

3.158 $\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cdots}}{\cdots}\right)}{4d}$$

```
[Out] -1/4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/
4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x
+c)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5995, 5879, 5953, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/d - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\text{c}}}\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+\right.}{4d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d} - \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx}}{\sqrt{b}}\right.}{4d}$$

Mathematica [A]
time = 0.25, size = 110, normalized size = 0.96

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*
x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b
)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{arccosh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(1/2),x)
[Out] int((a + b*acosh(c + d*x))^(1/2), x)
```

3.159 $\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

[Out] Unintegrable((a+b*arccosh(d*x+c))^(1/2)/(d*x+c),x)/e

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcCosh[x]]/x, x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]
[Out] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
[Out] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccosh(d*x + c) + a)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2)/(d*e*x+c*e),x)
[Out] Integral(sqrt(a + b*acosh(c + d*x))/(c + d*x), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x),x)
[Out] int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

3.160 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=374

$$-\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}-\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+}}{32d}$$

```
[Out] -3/32*e^3*(a+b*arccosh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c
))^(3/2)/d-3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^3*ex
p(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b^(3/2
)*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)-9/64
*b*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d
-3/32*b*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.94, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5996, 12, 5884, 5939, 5893, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}b^{3/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}b^{3/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{9be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}-\frac{3e^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-9*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh
[c + d*x]])/(64*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d
*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^
(3/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (3
*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(2048*d) - (3*b^(3/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2048*d*E^((4*a)/b)) + (3*b^(3/2)*e
^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d*
E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
```

```
c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4\sqrt{c}}{\sqrt{-}}\right.}{}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c-}}{32d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{64d}$$

Mathematica [A]

time = 3.01, size = 558, normalized size = 1.49

$$e^3\left(\frac{ae^{-\frac{4a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)+4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\left(4\sqrt{2}\,\Gamma\left(\frac{3}{2},\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)}{128d\sqrt{-\frac{(a+b\cosh^{-1}(c+dx))^2}{b^2}}}+\frac{\sqrt{b}\left((8a+3b)\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{4a}{b}\right)-\sinh\left(\frac{4a}{b}\right)\right)+(8a-3b)\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{4a}{b}\right)+\sinh\left(\frac{4a}{b}\right)\right)+8\left((4a+3b)\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)+(4a-3b)\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+8\sqrt{b}\sqrt{a+b\cosh^{-1}(c+dx)}\left(4\cosh^{-1}(c+dx)\cosh\left(2\cosh^{-1}(c+dx)\right)-3\sinh\left(2\cosh^{-1}(c+dx)\right)\right)\right)+8\sqrt{b}\sqrt{a+b\cosh^{-1}(c+dx)}\left(8\cosh^{-1}(c+dx)\cosh\left(4\cosh^{-1}(c+dx)\right)-3\sinh\left(4\cosh^{-1}(c+dx)\right)\right)\right)}{2048d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^3*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/
2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqr
t[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c
+ d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(12
8*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (Sqrt[b]*((8*a +
 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b
] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2
*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c
+ d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c +
 d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*Cosh[
4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(2048*d))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\,(a+b\,\mathrm{arccosh}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int ac^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int ad^{3}x^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int bc^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int 3acd^{2}x^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int 3ac^{2}dx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int bd^{3}x^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int 3bcd^{2}x^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int 3bc^{2}dx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*acosh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^3\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2), x)
```

3.161 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=342

$$-\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d}-\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\sqrt{a+}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)/d-1/288*b^(3/2)*e^2*exp(3*a/b)
*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288*b
^(3/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2
)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d+3/32*b^(3/2)*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/d/exp(a/b)-1/3*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(
d*x+c))^(1/2)/d-1/6*b*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*ar
ccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.72, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5996, 12, 5884, 5939, 5915, 5881, 3389, 2211, 2236, 2235, 5887, 5556}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{3\sqrt{\pi}\,b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{3d}-\frac{be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{6d}-\frac{be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x
]])/d - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*
ArcCosh[c + d*x]])/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))
/(3*d) - (3*b^(3/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/S
qrt[b]])/(32*d) - (b^(3/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d) + (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} \, dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{3/2} \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{3/2} \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3\sqrt{a}}{\sqrt{-1}} \right.}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+}}{6d}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{b}{}$$

Mathematica [A]
time = 2.02, size = 592, normalized size = 1.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^2*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCos
h[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqr
t[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[
3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*Arc
Cosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^
2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c +
 d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*x]*S
qrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[
3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] -
 Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*Arc
Cosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*ArcCos
h[c + d*x]])))/(288*d))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b\,\mathrm{arccosh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int ac^2\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int ad^2x^2\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int bc^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int 2acdx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int bd^2x^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int 2bcdx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*acosh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2), x)
```

3.162 $\int (ce+dex) \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=212

$$-\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d}+\frac{e(c+dx)}{}$$

[Out] -1/4*e*(a+b*arccosh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(3/2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d

Rubi [A]
time = 0.45, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5996, 12, 5884, 5939, 5893, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}-\frac{3be\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{8d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]

[Out] (-3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(8*d) - (e*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2\sqrt{a+}}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+d}}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(212) = 424.
time = 6.88, size = 515, normalized size = 2.43

$$\frac{e\left(-128ac^2\left(a+b\cosh^{-1}(c+dx)\right)^2-128acdx\left(a+b\cosh^{-1}(c+dx)\right)^2+32a\left(a+b\cosh^{-1}(c+dx)\right)^2\cosh\left(2\cosh^{-1}(c+dx)\right)+32b\cosh^{-1}(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2\cosh\left(2\cosh^{-1}(c+dx)\right)+32a\sqrt{b}\,ce^{a/b}\sqrt{\pi}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-3b^{3/2}e^{\frac{2a}{b}}\sqrt{2\pi}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)+32a\sqrt{b}\,ce^{-\frac{a}{b}}\sqrt{\pi}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)+3b^{3/2}e^{-\frac{2a}{b}}\sqrt{2\pi}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-64ab^2ce^{a/b}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\sqrt{-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{b^2}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-64ab^2ce^{-\frac{a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{b^2}}\,\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)-24b\left(a+b\cosh^{-1}(c+dx)\right)^2\sinh\left(2\cosh^{-1}(c+dx)\right)\right)}{128d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e*(-128*a*c^2*(a + b*ArcCosh[c + d*x])^2 - 128*a*c*d*x*(a + b*ArcCosh[c +
d*x])^2 + 32*a*(a + b*ArcCosh[c + d*x])^2*Cosh[2*ArcCosh[c + d*x]] + 32*b*A
rcCosh[c + d*x]*(a + b*ArcCosh[c + d*x])^2*Cosh[2*ArcCosh[c + d*x]] + 32*a*
Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]] - 3*b^(3/2)*E^((2*a)/b)*Sqrt[2*Pi]*(a + b*ArcCosh[
c + d*x])^(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] + (32*a
*Sqrt[b]*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]])/E^(a/b) + (3*b^(3/2)*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x]
)^(3/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b) -
 64*a*b^2*c*E^(a/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Sqrt[-((a + b*ArcCo
sh[c + d*x])^2/b^2)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] - (64*a*b^2*c*Sqrt[
a/b + ArcCosh[c + d*x]]*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]*Gamma[3/2,
-((a + b*ArcCosh[c + d*x])/b)])/E^(a/b) - 24*b*(a + b*ArcCosh[c + d*x])^2*S
inh[2*ArcCosh[c + d*x]]))/(128*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(dex+ce\right)\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int ac\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int adx\sqrt{a+b\operatorname{acosh}(c+dx)}\,dx+\int bc\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx+\int bdx\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(a*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
acosh(c + d*x)), x) + Integral(b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2), x)
```

3.163 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=157

$$-\frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{3b^{3/2}e^{a/b}\sqrt{}}{}$$

```
[Out] (d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d-3/8*b^(3/2)*exp(a/b)*erf((a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*
arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5995, 5879, 5915, 5881, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(2
*d) + ((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/d - (3*b^(3/2)*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{d} - \frac{(3b)\text{Subst}\left(\int \frac{x\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\sqrt{1+x}}\right.}{2d}$$

$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} + \frac{(c+dx)(a}{}$$

$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} + \frac{(c+dx)(a}{}$$

$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} + \frac{(c+dx)(a}{}$$

$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} + \frac{(c+dx)(a}{}$$

$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} + \frac{(c+dx)(a}{}$$

Mathematica [A]
time = 0.56, size = 290, normalized size = 1.85

$$\frac{-12b\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}+8b(c+dx)\cosh^{-1}(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}+4ae^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)+\sqrt{b}(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(2a-3b)\sqrt{b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-12*b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[
c + d*x]] + 8*b*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (
4*a*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c +
 d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*(2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b])
+ (2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(C
osh[a/b] + Sinh[a/b]))/(8*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{arccosh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acosh}\,(c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(3/2), x)
```

3.164 $\int \frac{(a+b\cosh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(3/2)/(d*x+c),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cosh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(c+dx))^{3/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx+\int \frac{b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

3.165 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=469

$$-\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d}+\frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-}}}{256d}$$

```
[Out] -3/32*e^3*(a+b*arccosh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c
))^(5/2)/d-15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*arc
cosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5/2)*
e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/1638
4*b^(5/2)*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a
/b)-15/64*b*e^3*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)/d-5/32*b*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-225/2048*b^2*e^3*(a+b*arccosh(d*x+c))^(1/2)/d+45/256*b^2*e
^3*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arc
cosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.42, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5996, 12, 5884, 5939, 5893, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}+\frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d}-\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{4d}-\frac{5be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d}-\frac{15be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{64d}-\frac{3e^3\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(2048*d) + (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcCosh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcCosh[c + d*x]])/(256*d) - (15*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*Sqrt[-1 +
c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(32*
d) - (3*e^3*(a + b*ArcCosh[c + d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a +
b*ArcCosh[c + d*x])^(5/2))/(4*d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf
[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16384*d) - (15*b^(5/2)*e^3*E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5
12*d) - (15*b^(5/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(16384*d*E^((4*a)/b)) - (15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(512*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
```

```
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{(5be^3)\,\text{Subst}\left(\int \frac{x^4(a+}{\sqrt{-1}}\right.}{}$$

$$= -\frac{5be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{32d}$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} - \frac{15be^3\sqrt{-1+c+dx}}{}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a}}{25}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a}}{25}$$

$$= -\frac{45b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cos}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\text{co}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\text{co}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\text{co}}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\text{co}}}{256d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 968 vs. $2(469) =$

938.
time = 9.94, size = 968, normalized size = 2.06

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^3*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[
3/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[
c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (a*Sqrt[b]*((8
*a + 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*
a)/b] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi
]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sin
h[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCos
h[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh
[c + d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*C
osh[4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(1024*d) + (-(Sqrt[
b]*(64*a^2 + 48*a*b + 15*b^2)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]]*(Cosh[(4*a)/b] - Sinh[(4*a)/b])) - Sqrt[b]*(64*a^2 - 48*a*b + 15
*b^2)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b]
 + Sinh[(4*a)/b]) - 16*(Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)
/b]) + Sqrt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*
x]] + 4*(a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])) + 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 64*ArcCosh[c + d*x]^2)*Cosh[4*ArcCosh[c + d*
x]] + 8*(a - 5*b*ArcCosh[c + d*x])*Sinh[4*ArcCosh[c + d*x]]))/(16384*d))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^3\,\left(a + b\,\mathrm{acosh}(c + d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2), x)
```

3.166 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=408

$$\frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{36d}-\frac{5be^2\sqrt{-1+c+dx}\,\sqrt{1+c}}{}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(5/2)/d-5/1728*b^(5/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/1728
*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d-15/64*b^(5/2)*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/d/exp(a/b)-5/9*b*e^2*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)/d+5/6*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d
+5/36*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.18, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5996, 12, 5884, 5939, 5915, 5879, 5953, 3388, 2211, 2236, 2235, 3393}

$$-\frac{15\sqrt{\pi}b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}-\frac{15\sqrt{\pi}b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{36d}+\frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{3d}-\frac{5be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{18d}-\frac{5be^2\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(36*d) - (5*b*e^2*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(18
*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*
e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(64*d) - (5
*b^(5/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(576*d) - (15*b^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]])/(64*d*E^(a/b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
```

```
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{3d} - \frac{(5be^2)\,\text{Subst}\left(\int \frac{x^3(a+}{\sqrt{-1}}\right.}{}$$

$$= -\frac{5be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{18d}$$

$$= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{36d} - \frac{5be^2\sqrt{-1+c+dx}}{}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+}}{36d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1008 vs. $2(408) = 816$.

time = 8.44, size = 1008, normalized size = 2.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^2*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*
x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*S
qrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/
b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b
*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*Ar
cCosh[c + d*x]])))/(144*d) + (-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqr
t[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) +
b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*b^2)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Erf[(S
qrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b
]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*Cosh[
3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d*x]]
))/(1728*d))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int a^{2}c^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\,dx+\int a^{2}d^{2}x^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\,dx+\int b^{2}c^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}^{2}{(c+dx)}\,dx+\int 2abc^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}{(c+dx)}\,dx+\int 2a^{2}cdx\sqrt{a+b\operatorname{acosh}{(c+dx)}}\,dx+\int b^{2}d^{2}x^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}^{2}{(c+dx)}\,dx+\int 2abd^{2}x^{2}\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}{(c+dx)}\,dx+\int 2b^{2}cdx\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}^{2}{(c+dx)}\,dx+\int 4abcdx\sqrt{a+b\operatorname{acosh}{(c+dx)}}\operatorname{acosh}{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(a**2*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a**2*d**
2*x**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*acosh(c + d*x)
), x) + Integral(b**2*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**
2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x),
 x) + Integral(2*b**2*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2, x
) + Integral(4*a*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2), x)
```

3.167 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=269

$$-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c}}{8a}$$

```
[Out] -1/4*e*(a+b*arccosh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(5
/2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arccosh
(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-15/64*b^2*e*(a+b*arccosh(d
*x+c))^(1/2)/d+15/32*b^2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.71, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5996, 12, 5884, 5939, 5893, 5953, 3393, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}-\frac{5be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{8d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-15*b^2*e*Sqrt[a + b*ArcCosh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqr
t[a + b*ArcCosh[c + d*x]])/(32*d) - (5*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(8*d) - (e*(a + b*ArcCosh[c
+ d*x])^(5/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d)
 - (15*b^(5/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
```

```
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d}-\frac{(5be)\text{Subst}\left(\int \frac{x^2(a+b\cdots}{\sqrt{-1+\cdots}}\right.\cdots}{\cdots}$$

$$=-\frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\cdots\right.}{8d}\cdots$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}\,(\cdots}{\cdots}$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}\,(\cdots}{\cdots}$$

$$=\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}-\frac{5be\sqrt{-1+c+dx}\,(\cdots}{\cdots}$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh\cdots}}{32d}\cdots$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh\cdots}}{32d}\cdots$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh\cdots}}{32d}\cdots$$

$$=-\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh\cdots}}{32d}\cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1846 vs. $2(269) = 538$.
time = 8.14, size = 1846, normalized size = 6.86

Too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]

[Out] e*((a^2*c*Sqrt[-1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gam
ma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2,
-((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E
^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a*b*c*Sqrt[
-1 + c + d*x]*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a
+ b*ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c +
d*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(4*d*Sqrt[(-1 + c
+ d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (c*Sqrt[-1 + c + d*x]*(-4*b*Sqr
t[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*
x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) +
Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(1
6*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a^2*Sqrt[-1 +
c + d*x]*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCo
sh[c + d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - 8*Sqrt[b]*c*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]
*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b
]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a
)/b] - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(32*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x
)]*Sqrt[1 + c + d*x]) + (a*Sqrt[-1 + c + d*x]*(-16*c*(-12*b*Sqrt[(-1 + c +
d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*b*(c + d
*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + Sqrt[b]*(2*a + 3*b)*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
(2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cos
h[a/b] + Sinh[a/b])) + Sqrt[b]*(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*
b)*Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(
Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*b*Sqrt[a + b*ArcCosh[c + d*x]]*(4*ArcCos
h[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c + d*x]])))/(64*d*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (Sqrt[-1 + c + d*x]*
(-32*c*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d
x)](1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh
[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*
b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] +

```
 Sinh[a/b])) + Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sq
rt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a + b*ArcCo
sh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*x]] + 4*(
a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])))/(512*d*Sqrt[(-1 + c +
 d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(dex+ce\right) \left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e \left(\int a^{2} c \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}}\, dx + \int a^{2} d x \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}}\, dx + \int b^{2} c \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}} \operatorname{acosh}^{2}{\left(c + d x \right)}\, dx + \int 2 a b c \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}} \operatorname{acosh}{\left(c + d x \right)}\, dx + \int b^{2} d x \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}} \operatorname{acosh}^{2}{\left(c + d x \right)}\, dx + \int 2 a b d x \sqrt{a + b \operatorname{acosh}{\left(c + d x \right)}} \operatorname{acosh}{\left(c + d x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(a**2*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a**2*d*x*sqrt(
a + b*acosh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*acosh(c + d*x))*acos
h(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x), x) + Integral(b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2, x
) + Integral(2*a*b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2), x)
```

3.168 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=186

$$\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)}{}$$

```
[Out] (d*x+c)*(a+b*arccosh(d*x+c))^(5/2)/d-15/16*b^(5/2)*exp(a/b)*erf((a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.39, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5995, 5879, 5915, 5953, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) - (5*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(
a + b*ArcCosh[c + d*x])^(5/2))/d - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}} dx \right.}{2d}$$

$$= -\frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(\right.}{}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(\right.}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 494 vs. $2(186) = 372$.

time = 2.91, size = 494, normalized size = 2.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x
]^2)) + (8*a^2*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b +
ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCo
sh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) - Sqrt[b]*(4
```

```
*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 4*a*b*(-12*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]
] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*b
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(16*d)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{arccosh}(dx + c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acosh}\,(c + dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(5/2), x)
```

3.169 $\int \frac{(a+b\cosh^{-1}(c+dx))^{5/2}}{ce+dex} dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(5/2)/(d*x+c),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(c+dx))^{5/2}}{ce+dex} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cosh^{-1}(c+dx))^{5/2}}{ce+dex} dx &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^{5/2}}{ex} dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^{5/2}}{x} dx, x, c+dx\right)}{de}
\end{aligned}$$

Mathematica [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(c+dx))^{5/2}}{ce+dex} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

Maple [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^{\frac{5}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx+\int \frac{b^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)}{c+dx}\,dx+\int \frac{2ab\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b**2*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c + d\,x))^{5/2}}{c\,e + d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

3.170 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=509

$$-\frac{175b^3e^2\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}-\frac{35b^3e^2\sqrt{-1+c+dx}(c+dx)^2\sqrt{1+c+d}}{216d}$$

```
[Out] 35/18*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)^3
*(a+b*arccosh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(7/2)/
d-35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^2*exp(
a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+105/128*b^(7/2)*e^2
*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-7/9*b*e^2*(a+
b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-7/18*b*e^2*(d*x+c
)^2*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-175/54*b^3
*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d-35/216*b^
3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/
d
```

Rubi [A]
time = 1.37, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5996, 12, 5884, 5939, 5915, 5879, 5881, 3389, 2211, 2236, 2235, 5887, 5556}

$$-\frac{105\sqrt{\pi}b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{105\sqrt{\pi}b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}b^{7/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{35b^3e^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\sqrt{a+b\cosh^{-1}(c+dx)}}{216d}-\frac{175b^3e^2\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}+\frac{35b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{108d}+\frac{35b^2e^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{18d}-\frac{7be^2\sqrt{c+dx-1}(c+dx)^2\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{18d}-\frac{7be^2\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{9d}+\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-175*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d
*x]])/(54*d) - (35*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]
*Sqrt[a + b*ArcCosh[c + d*x]])/(216*d) + (35*b^2*e^2*(c + d*x)*(a + b*ArcCo
sh[c + d*x])^(3/2))/(18*d) + (35*b^2*e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x
])^(3/2))/(108*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(18*d) + (e^2*(c + d*x)^3*(a
+ b*ArcCosh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(128*d) - (35*b^(7/2)*e^2*E^((3*a)/b
)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3456*d)
+ (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12
8*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
```

```
, c, n}, x]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{(7be^2)\,\text{Subst}\left(\int \frac{x^3(a+}{\sqrt{-1}}\right.}{}$$

$$= -\frac{7be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{18d}$$

$$= \frac{35b^2e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{108d} - \frac{7be^2\sqrt{-1+c+d}}{}$$

$$= -\frac{35b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(}}{216d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1523 vs. $2(509) = 1018$.

time = 11.92, size = 1523, normalized size = 2.99

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] e^2*((a^3*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
b^2)]) + (a^2*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c +
d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a +
 b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*
ArcCosh[c + d*x]])))/(96*d) + (a*(-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2
*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]
) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15
*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh
[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*
b^2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[
(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*
a)/b]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*C
osh[3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d
*x]])))/(576*d) + (-81*(4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[(-1 + c + d*
x)/(1 + c + d*x)]*(1 + c + d*x)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15
 + 4*ArcCosh[c + d*x]^2)) - 2*b*(c + d*x)*(-10*a + b*ArcCosh[c + d*x]*(35 +
 4*ArcCosh[c + d*x]^2))) + Sqrt[b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + Sqrt[b]*(-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(72*a^3 + 1
08*a^2*b + 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(-72*a^3 + 108
*a^2*b - 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
```

```
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) - 12*b*Sqrt[a + b*ArcCosh[c
 + d*x]]*(-2*b*(-10*a + b*ArcCosh[c + d*x]*(35 + 36*ArcCosh[c + d*x]^2))*Co
sh[3*ArcCosh[c + d*x]] + (12*a^2 - 12*a*b*ArcCosh[c + d*x] + 7*b^2*(5 + 12*
ArcCosh[c + d*x]^2))*Sinh[3*ArcCosh[c + d*x]]))/(10368*d))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2), x)
```

3.171 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=319

$$-\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}-\frac{35b^2e\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{64d}+\frac{35b}{}$$

```
[Out] -35/64*b^2*e*(a+b*arccosh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arccos
h(d*x+c))^(3/2)/d-1/4*e*(a+b*arccosh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*a
rccosh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8*b*e
*(d*x+c)*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-105/1
28*b^3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)
/d
```

Rubi [A]
time = 0.80, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5996, 12, 5884, 5939, 5893, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}+\frac{35b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d}-\frac{35b^2e\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{64d}-\frac{7be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{8d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{2d}-\frac{e\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCo
sh[c + d*x]])/(128*d) - (35*b^2*e*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) +
(35*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/(32*d) - (7*b*e*Sqrt[
-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(
8*d) - (e*(a + b*ArcCosh[c + d*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*Arc
Cosh[c + d*x])^(7/2))/(2*d) - (105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2
*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5884

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcCosh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
```

```
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5939

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[f*(f*x)^(m -
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(e1*e2*(
m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p + 1))), Int[(f*x)^(m
- 2)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] - Dist[b*f*
(n/(c*(m + 2*p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-
1 + c*x)^p], Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a
+ b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, p}
, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && GtQ[n, 0] && IGtQ[m, 1] && NeQ
[m + 2*p + 1, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{2d} - \frac{(7be)\text{Subst}\left(\int \frac{x^2(a+b\cdots}{\sqrt{-1+\cdots}}\right.}{\cdots}$$

$$= -\frac{7be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\cdots\right.}{8d}$$

$$= \frac{35b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d} - \frac{7be\sqrt{-1+c+dx}\cdots}{\cdots}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(\cdots}}{128d}$$

Mathematica [A]

time = 4.52, size = 288, normalized size = 0.90

$$\frac{e\left(105b^{7/2}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)-105b^{7/2}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+8\sqrt{a+b\cosh^{-1}(c+dx)}\left(4a(16a^2+35b^2)\cosh\left(2\cosh^{-1}(c+dx)\right)+64b^3\cosh^{-1}(c+dx)^3\cosh\left(2\cosh^{-1}(c+dx)\right)-7b(16a^2+15b^2)\sinh\left(2\cosh^{-1}(c+dx)\right)+16b^2\cosh^{-1}(c+dx)^2\left(12a\cosh\left(2\cosh^{-1}(c+dx)\right)-7b\sinh\left(2\cosh^{-1}(c+dx)\right)\right)+4b\cosh^{-1}(c+dx)\left((48a^2+35b^2)\cosh\left(2\cosh^{-1}(c+dx)\right)-56ab\sinh\left(2\cosh^{-1}(c+dx)\right)\right)\right)\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e*(105*b^(7/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 105*b^(7/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*
Sqrt[a + b*ArcCosh[c + d*x]]*(4*a*(16*a^2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]
] + 64*b^3*ArcCosh[c + d*x]^3*Cosh[2*ArcCosh[c + d*x]] - 7*b*(16*a^2 + 15*b
^2)*Sinh[2*ArcCosh[c + d*x]] + 16*b^2*ArcCosh[c + d*x]^2*(12*a*Cosh[2*ArcCo
sh[c + d*x]] - 7*b*Sinh[2*ArcCosh[c + d*x]]) + 4*b*ArcCosh[c + d*x]*((48*a^
2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]] - 56*a*b*Sinh[2*ArcCosh[c + d*x]]))))/
(2048*d)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dex + ce)\,(a + b\,\mathrm{arccosh}(dx + c))^{\frac{7}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2), x)
```

3.172 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=230

$$-\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{-}\cdots}{}$$

```
[Out] 35/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arccosh(d*x+c))^
(7/2)/d-105/32*b^(7/2)*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/32*b^(7/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/
exp(a/b)-7/2*b*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
-105/8*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.43, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5995, 5879, 5915, 5881, 3389, 2211, 2236, 2235}

$$-\frac{105\sqrt{\pi}\,b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\,b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]
)/(8*d) + (35*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (7*b*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d) +
((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2))/d - (105*b^(7/2)*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5915

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(d1 + e1*x)^(p + 1)*(d2 +
e2*x)^(p + 1)*((a + b*ArcCosh[c*x])^n/(2*e1*e2*(p + 1))), x] - Dist[b*(n/(2
*c*(p + 1)))*Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^
p], Int[(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n -
1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x] && EqQ[e1, c*d1] && Eq
Q[e2, (-c)*d2] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{7/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{d} - \frac{(7b)\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,\right.}{2d}$$

$$= -\frac{7b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d} + \frac{(c+dx)\,(a}{}$$

$$= \frac{35b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{7b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,(}{2d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+d}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+d}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+d}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+d}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+d}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 748 vs. 2(230) = 460.

time = 5.63, size = 748, normalized size = 3.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(-2*b*(c + d*x)*(-10*a + 35*b*ArcCosh[c
+ d*x] + 4*b*ArcCosh[c + d*x]^3) + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
```

```
c + d*x)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15 + 4*ArcCosh[c + d*x]^2
))) + (16*a^3*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + A
rcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCos
h[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*(8*
a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(-8*a^3 + 36*a^2*b - 90*a*b
^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b]
 + Sinh[a/b]) + 12*a^2*b*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d
*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*
ArcCosh[c + d*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]) + 6*a*(
4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]
^2)) - Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^
2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(32*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2),x)
[Out] int((a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c + d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(7/2), x)
```

3.173 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(7/2)/(d*x+c),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}(c + d\,x))^{7/2}}{c\,e + d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

3.174 $\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$-\frac{e^4 e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{e^4 e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{e^4 e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{Er}$$

```
[Out] -1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/
2)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)-1/16*e^4*exp(a/b)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*erf(3^
(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)+1/32*e
^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(
3*a/b)/b^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5996, 12, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^4 e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{5}}\,e^4 e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^4 e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}+\frac{\sqrt{3\pi}\,e^4 e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{5}}\,e^4 e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/16*(e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(Sqr
t[b]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d
*x]])/Sqrt[b]])/(32*Sqrt[b]*d) - (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*S
qrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) + (e^4*Sqrt[3
*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^
((3*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)\,dx, x, \cosh^{-1}(c+dx\right.}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^4\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x\right.}{8d}$$

$$= -\frac{e^4\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{32d} + \frac{e^4\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx,\right.}{32d}$$

$$= -\frac{e^4\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{16bd} + \frac{e^4\text{Subst}\left(\int e^{-\frac{5a}{b}+\ldots}\right.}{}$$

$$= -\frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d} - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\ldots}}{}\right.}{32\sqrt{b}\,d}$$

<u>**Mathematica**</u> [A]
time = 0.39, size = 319, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)+5\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+10e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+5\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)\right)}{160d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*(10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[
c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a +
 b*ArcCosh[c + d*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c +
```

```
d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqrt
[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] +
 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*Ar
cCosh[c + d*x]))/b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gam
ma[1/2, (5*(a + b*ArcCosh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Arc
Cosh[c + d*x]])
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4 \left(\int \frac{c^4}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{d^4 x^4}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{4 c d^3 x^3}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{6 c^2 d^2 x^2}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{4 c^3 dx}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2), x)
```

3.175 $\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{E}}{}$$

```
[Out] -1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/16*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*ar
ccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5996, 12, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\,\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/32*(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/(Sqrt[b]*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) + (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} + \frac{e^3\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, \right.}{4d}$$

$$= -\frac{e^3\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^3\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx\right.}{16d}$$

$$= -\frac{e^3\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\text{Subst}\left(\int e^{-\frac{4a}{b}+}\right.}{}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+}}{}\right.}{8\sqrt{b}\,d}$$

Mathematica [A]

time = 0.29, size = 205, normalized size = 0.94

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)+2\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(2\sqrt{2}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{1}{2},\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)}{32d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcCosh[c
```

```
+ d*x]]*(2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)
*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a + b
*ArcCosh[c + d*x]])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{\sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}}\,dx + \int \frac{d^3x^3}{\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}}\,dx + \int \frac{3cd^2x^2}{\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}}\,dx + \int \frac{3c^2dx}{\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2), x)
```

3.176 $\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{e^2 e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{e^2 e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{e^2 e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erf}}{}$$

```
[Out] -1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/8*e^2*erfi((a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5996, 12, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^2 e^{a/b}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{3}}\,e^2 e^{\frac{3a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^2 e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2 e^{-\frac{3a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/8*(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(Sqrt
[b]*d) - (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*
x]]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e^2\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x \right.}{4d}$$

$$= -\frac{e^2\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} - \frac{e^2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, \right.}{8d}$$

$$= -\frac{e^2\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{4bd} - \frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}} \right.}{}$$

$$= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c \right.}{8\sqrt{b}\,d}$$

<u>Mathematica</u> [A]
time = 0.24, size = 216, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+3e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)\right)}{24d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]
```

```
*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{d^2x^2}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{2cdx}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*acosh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2), x)
```

3.177 $\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$-\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

```
[Out] -1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5996, 12, 5887, 5556, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/4*(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d
*x]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{e\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, \cdots\right.}{4d}$$

$$= -\frac{e\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{2bd} + \frac{e\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x}{b}\cdots}\right.}{\cdots}$$

$$= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right.}{4\sqrt{\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 306 vs. 2(113) = 226.
time = 1.01, size = 306, normalized size = 2.71

$$\frac{e\left(\frac{4ce^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{4ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} + \frac{e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} + \frac{4ce^{a/b}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{4ce^{-\frac{a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*((4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/Sqrt[b
] - (E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
```

```
[b]])/Sqrt[b] - (4*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(
Sqrt[b]*E^(a/b)) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/
Sqrt[b]])/(Sqrt[b]*E^((2*a)/b)) + (4*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]
*Gamma[1/2, a/b + ArcCosh[c + d*x]])/Sqrt[a + b*ArcCosh[c + d*x]] + (4*c*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]
)/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(8*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{a + b \operatorname{arccosh}\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e \left(\int \frac{c}{\sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}}\, dx + \int \frac{dx}{\sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x/sqrt(a + b*acos
h(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2), x)
```

3.178 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5995, 5881, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -1/2*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(Sqrt[b]*
d) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(
a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,\right.}{}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{b}}\right.}{2\sqrt{b}\,d}$$

Mathematica [F]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cosh^{-1}(c + dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]], x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

3.179 $\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(1/2),x)/e

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]

[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcCosh[x]]), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+dx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*acosh(c + d*x)) + d*x*sqrt(a + b*acosh(c + d*x))),
 x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)), x)
```

3.180 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$-\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{e^4e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\right.}{}$$

```
[Out] 1/8*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
+1/8*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/d+3/16*e^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)+1/16*e^4*exp(5*a/b)*erf(5^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16*e^4*er
fi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d/e
xp(5*a/b)-2*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(
d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5996, 12, 5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}\,e^4e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}\,e^4e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{2e^4\sqrt{c+dx-1}\,(c+dx)^4\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(8*b^(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (
e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/
b)) + (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
])/(16*b^(3/2)*d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{(a+b\cosh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{(a+b\cosh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{(2e^4)\,\text{Subst}\left(\int \left(-\frac{\cosh(\cdots}{8\sqrt{a+\cdots}}\right.\right.}{\cdots}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, \cdots\right.}{4bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, \cdots\right.}{8bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{\cdots}\right.}{4b^2\cdots}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh\cdots}}{\cdots}\right.}{8b^{3/2}d}$$

Mathematica [A]
time = 1.32, size = 396, normalized size = 1.06

$$\frac{e^4e^{-\frac{5a}{b}}\left(-4e^{\frac{5a}{b}}\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)+3\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)-3\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)-\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\cosh^{-1}(c+dx))}{b}\right)-6e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(c+dx)\right)-2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(c+dx)\right)\right)}{16bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-4*E^((5*a)/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - 2*E
^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]]
+ Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh
[c + d*x]))/b] + 3*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*A
```

```
rcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - 3*Sqrt[3]*
E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d
*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (5*
(a + b*ArcCosh[c + d*x]))/b] - 6*E^((5*a)/b)*Sinh[3*ArcCosh[c + d*x]] - 2*E
^((5*a)/b)*Sinh[5*ArcCosh[c + d*x]]))/(16*b*d*E^((5*a)/b)*Sqrt[a + b*ArcCos
h[c + d*x]])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^4x^4}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{4cd^3x^3}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{6c^2d^2x^2}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{4c^3dx}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt
(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2), x)
```

3.181 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{\quad}}{\quad}\right.}{\quad}$$

```
[Out] 1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arccosh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5996, 12, 5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}-\frac{2e^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[
(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) + (e^3
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2
)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{(2e^3)\,\text{Subst}\left(\int \left(-\frac{\cosh(2x}{2\sqrt{a+b}}\right.\right.}{}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x\right.}{2bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{\right.}{b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cos}}{\sqrt{}}\right.}{4b^{3/2}d}$$

Mathematica [A]

time = 0.75, size = 265, normalized size = 0.99

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-e^{\frac{4a}{b}}\left(8(c+dx)^3\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx)+\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)\right)\right)}{4bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] - E^((4*a)/b)*(8*(c + d*x)^3*Sqrt[(-1
 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((4*a)/b)*S
```

```
qrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
4*b*d*E^((4*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^{3}}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^{3}x^{3}}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{3cd^{2}x^{2}}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{3c^{2}dx}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(3*c*d**2*x*
```

```
*2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b
*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2), x)
```

3.182 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\cdots\right.}{\cdots}$$

```
[Out] 1/4*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
+1/4*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)+1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/
2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5996, 12, 5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}\,e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{2e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(4*b^(3/2)*d) + (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{(a+b\cosh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{(a+b\cosh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{(2e^2)\,\text{Subst}\left(\int \left(-\frac{\cosh(}{4\sqrt{a+}}\right.\right.}{}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,\right.}{2bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,\right.}{4bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \right.}{2b^2}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\,\text{co}}}{}\right.}{4b^{3/2}d}$$

Mathematica [A]
time = 1.09, size = 265, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(-e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)-\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)-2e^{\frac{3a}{b}}\left(\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)+\sinh\left(3\cosh^{-1}(c+dx)\right)\right)\right)}{4bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-(E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]]) + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
 b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b] - 2*E^((3*a)/b
```

```
)*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sinh[3*ArcCosh[c + d*
x]])))/(4*b*d*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\, dx + \int \frac{d^2x^2}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\, dx + \int \frac{2cdx}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(2*c*d*x/(a*
```

```
sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2), x)
```

3.183 $\int \frac{ce+dex}{(a+b\cosh^{-1}(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\right.}{}$$

```
[Out] 1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 12, 5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCo
sh[c + d*x]]) + (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x\right.}{bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \text{co}\right.}{bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{}\right.}{b^2d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{co}}}{\sqrt{}}\right.}{b^{3/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 314 vs. 2(155) = 310.

time = 5.30, size = 314, normalized size = 2.03

$$\frac{e\left(-2ce^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)+e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-2c\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)-\frac{2\sqrt{b}\,e^{-\frac{a}{b}}\left(ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-c\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+e^{a/b}\sinh\left(2\cosh^{-1}(c+dx)\right)\right)}{\sqrt{a+b\cosh^{-1}(c+dx)}}\right)}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e*((-2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b) + (S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)
 - 2*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Co
sh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*(c*E^((2*a)/b)*Sqrt[a/b + ArcCosh
[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] - c*Sqrt[-((a + b*ArcCosh[c +
 d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^(a/b)*Sinh[2*ArcCo
sh[c + d*x]]))/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(2*b^(3/2)*d)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\,dx + \int \frac{dx}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
[Out] e*(Integral(c/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*
acosh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a
 + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2), x)
```

3.184 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5995, 5880, 5953, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCosh[c + d*x]]
) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d
) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{a+\cdots}}\right.}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\cdots\right.}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+\cdots\right.}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{\cdots}}\right.}{b^2d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right.}{b^{3/2}d}$$

Mathematica [A]
time = 0.36, size = 145, normalized size = 1.13

$$\frac{e^{-\frac{a}{b}}\left(-2e^{a/b}\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - E^((2*a)/b)*
Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] + Sqrt[-((a
 + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/(b*d*
E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
[Out] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(3/2),x)
[Out] Integral((a + b*acosh(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(3/2), x)
```

3.185 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(3/2),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(3/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac\sqrt{a + b\,\mathrm{acosh}\,(c + dx)} + adx\sqrt{a + b\,\mathrm{acosh}\,(c + dx)} + bc\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,\mathrm{acosh}\,(c+dx) + bdx\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,\mathrm{acosh}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
[Out] Integral(1/(a*c*sqrt(a + b*acosh(c + d*x)) + a*d*x*sqrt(a + b*acosh(c + d*x
)) + b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b*d*x*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x)), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)), x)
```

3.186 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-$$

```
[Out] -1/12*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)
/d+1/12*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp
(a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/b^(5/2)/d+3/8*e^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-5/24*e^4*exp(5*a/b)*erf(5^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*e^4*e
rfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d/
exp(5*a/b)-2/3*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arcco
sh(d*x+c))^(3/2)+16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-20/3*e
^4*(d*x+c)^5/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 1.19, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}+\frac{3\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcCosh[c
+ d*x]]) - (20*e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e
^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*
d) - (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*b^(5/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) + (3*e^4*Sqrt[3
*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^(
(3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(24*b^(5/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
```

```
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(8e^4)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{}}\right.}{}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Mathematica [A]

time = 2.52, size = 615, normalized size = 1.39

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-10*Sqrt[5]*b*E^(5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^
(3/2)*Gamma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b] - 18*Sqrt[3]*b*E^((2*a)/b
 + 5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3
*(a + b*ArcCosh[c + d*x]))/b] + 2*E^(4*(a/b + ArcCosh[c + d*x]))*(2*E^((2*a
)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x
])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c + d*x]*Sq
rt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCosh[c + d*x]
))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*ArcCosh[c + d
*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])) + 3*E^((5*a)/b +
 2*ArcCosh[c + d*x])*(b - 6*a*(1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c
+ d*x] - b*E^(6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3
*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c +
d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]) + 2*E^((5*a)/b)*(-1/2*(b*
(-1 + E^(10*ArcCosh[c + d*x]))) - 5*(1 + E^(10*ArcCosh[c + d*x]))*(a + b*Ar
cCosh[c + d*x]) + 5*Sqrt[5]*E^(5*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCo
sh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c + d*x]
))/b])))/(48*b^2*d*E^(5*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^
(3/2))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)}\,dx+\int \frac{d^{4}x^{4}}{a^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)}\,dx+\int \frac{4cd^{3}x^{3}}{a^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)}\,dx+\int \frac{6c^{2}d^{2}x^{2}}{a^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)}\,dx+\int \frac{4c^{3}dx}{a^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*
x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b
**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2), x)
```

3.187 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^3}{}$$

```
[Out] -2/3*e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5
/2)/d+2/3*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
/exp(4*a/b)-1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d+1/3*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+4*e^3*(d*x+c)^2/b^
2/d/(a+b*arccosh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arccosh(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3389, 2211, 2236, 2235}

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (16*e^3*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e^
3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^
(5/2)*d) - (e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((4*a)/b)) + (e^3*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(2e^3)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx}}$$

Mathematica [A]

time = 1.76, size = 391, normalized size = 1.17

$$\frac{e^3 e^{-4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-16be^{4\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)-8\sqrt{2}\,be^{\frac{2a}{b}+4\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\left(-\left(1+e^{2\cosh^{-1}(c+dx)}\right)^2\left(b\left(-1+e^{4\cosh^{-1}(c+dx)}\right)+8a\left(1-e^{2\cosh^{-1}(c+dx)}+e^{4\cosh^{-1}(c+dx)}\right)+8b\left(1-e^{2\cosh^{-1}(c+dx)}+e^{4\cosh^{-1}(c+dx)}\right)\cosh^{-1}(c+dx)\right)+8\sqrt{2}\,e^{\frac{2a}{b}+4\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{2(a+b\cosh^{-1}(c+dx))}{b}\right)+16e^{4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{4(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)}{24b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] - 8*Sqrt[2]*b*E^((2*a)/b + 4*ArcC
osh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*A
rcCosh[c + d*x]))/b] + E^((4*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))^2*(b*(-1
 + E^(4*ArcCosh[c + d*x])) + 8*a*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh
[c + d*x])) + 8*b*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh[c + d*x]))*Arc
Cosh[c + d*x])) + 8*Sqrt[2]*E^((2*a)/b + 4*ArcCosh[c + d*x])*Sqrt[a/b + Arc
Cosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*ArcCosh[c + d*
x]))/b] + 16*E^(4*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(24*b^
2*d*E^(4*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\operatorname{arccosh}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int\frac{c^{3}}{a^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}+2ab\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}\left(c+dx\right)+b^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}^{2}\left(c+dx\right)}\,dx+\int\frac{d^{3}x^{3}}{a^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}+2ab\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}\left(c+dx\right)+b^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}^{2}\left(c+dx\right)}\,dx+\int\frac{3cd^{2}x^{2}}{a^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}+2ab\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}\left(c+dx\right)+b^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}^{2}\left(c+dx\right)}\,dx+\int\frac{3c^{2}dx}{a^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}+2ab\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}\left(c+dx\right)+b^{2}\sqrt{a+b\operatorname{acosh}\left(c+dx\right)}\operatorname{acosh}^{2}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*
x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2), x)
```

3.188 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{e^2e}{}$$

```
[Out] -1/6*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d+1/6*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a
/b)-1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+8/3*e^2*(d*x+c)/b^2/d/(a+b
*arccosh(d*x+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3389, 2211, 2236, 2235, 5881}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}-\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\,e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (4*e^2*(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e^2*E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) - (e
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/(2*b^(5/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
])/(6*b^(5/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
```

```
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(4e^2)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+d}}$$

Mathematica [A]
time = 2.39, size = 391, normalized size = 1.19

$$\frac{e^2 e^{-3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(2e^{\frac{4a}{b}+3\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-6\sqrt{3}\,be^{3\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)-2be^{\frac{2a}{b}+3\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+e^{\frac{3a}{b}}\left(-\left(\left(1+e^{2\cosh^{-1}(c+dx)}\right)\left(a\left(6-4e^{2\cosh^{-1}(c+dx)}+6e^{4\cosh^{-1}(c+dx)}\right)+b\left(-1+6\cosh^{-1}(c+dx)-4e^{2\cosh^{-1}(c+dx)}\cosh^{-1}(c+dx)+e^{4\cosh^{-1}(c+dx)}(1+6\cosh^{-1}(c+dx))\right)\right)\right)+6\sqrt{3}\,e^{3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)}{12b^2d\,(a+b\cosh^{-1}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a +
b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] - 2*b*E^((2*a)/b + 3*ArcCosh[c + d*x])*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^((3
*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))*(a*(6 - 4*E^(2*ArcCosh[c + d*x]) + 6
*E^(4*ArcCosh[c + d*x])) + b*(-1 + 6*ArcCosh[c + d*x] - 4*E^(2*ArcCosh[c +
d*x])*ArcCosh[c + d*x] + E^(4*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]))))
 + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b])))/(12*b^
2*d*E^(3*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)}\,dx+\int \frac{d^2x^2}{a^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)}\,dx+\int \frac{2cdx}{a^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*acosh(c + d*x)) +
2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2), x)
```

3.189 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2ee}{}$$

```
[Out] -2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+4/3*e/b^2/d/(a+b*arccosh(d*x+c))^(1/2
)-8/3*e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5996, 12, 5886, 5951, 5887, 5556, 3389, 2211, 2236, 2235, 5893}

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh
[c + d*x])^(3/2)) + (4*e)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (8*e*(c
+ d*x)^2)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2
*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5
/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5887

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b], x], x,
a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
```

```
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(2e)\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{1}}\right.}{}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 687 vs. 2(216) =

432.
time = 4.74, size = 687, normalized size = 3.18

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e*(4*a*Sqrt[b]*c*(c + d*x) + 4*b^(3/2)*c*(c + d*x)*ArcCosh[c + d*x] - (2*S
qrt[b]*c*(1 + E^(2*ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x]))/E^ArcCosh[c
 + d*x] - 4*a*Sqrt[b]*Cosh[2*ArcCosh[c + d*x]] - 4*b^(3/2)*ArcCosh[c + d*x]
*Cosh[2*ArcCosh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cos
h[a/b]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - 2*Sqrt[2*Pi]*(a + b*ArcC
osh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]] - 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cosh[a/b]*Erfi[Sqr
t[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] +
2*Sqrt[b]*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]] - (2*b^(3/2)*c*(-((a + b*ArcCosh[c + d*x]
)/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/E^(a/b) + 2*c*Sqrt[P
i]*(a + b*ArcCosh[c + d*x])^(3/2)*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*Sinh[a/b] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 2
*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - b^(3/2)*Sinh[2*ArcCosh[c + d*x]]))/(3*
b^(5/2)*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)}\,dx+\int \frac{dx}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acos
h(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2), x)
```

3.190 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+$$

```
[Out] -2/3*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/
3*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*(d*x+c)/
b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5995, 5880, 5951, 5881, 3389, 2211, 2236, 2235}

$$-\frac{2\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^(
3/2)) - (4*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*E^(a/b)*S
qrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5881

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Sinh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh^{-1}}\right.}{3bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{4\text{Subst}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4\text{Subst}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{2\text{Subst}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4\text{Subst}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{2e^{a/b}\sqrt{}}{}$$

Mathematica [A]
time = 1.13, size = 219, normalized size = 1.33

$$\frac{e^{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\left(2e^{\frac{2a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-2\left(e^{a/b}\left(be^{\cosh^{-1}(c+dx)}\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx)+\left(1+e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)\right)+be^{\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)\right)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

```
[Out] (2*E^((2*a)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCo
sh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c
 + d*x]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCos
```

```
h[c + d*x]))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*Arc
Cosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(3*b^2
*d*E^((a + b*ArcCosh[c + d*x])/b)*(a + b*ArcCosh[c + d*x])^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \,\mathrm{arccosh}\left(dx + c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(5/2), x)
```

3.191 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\operatorname{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(5/2),x)/e

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(5/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)), x]

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\left(a + b \operatorname{arccosh}\left(dx + c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{2} c \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} + a^{2} dx \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} + 2abc \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} \operatorname{acosh}{\left(c + dx \right)} + 2abdx \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} \operatorname{acosh}{\left(c + dx \right)} + b^{2} c \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} \operatorname{acosh}^{2}{\left(c + dx \right)} + b^{2} dx \sqrt{a + b \operatorname{acosh}{\left(c + dx \right)}} \operatorname{acosh}^{2}{\left(c + dx \right)}}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*acosh(c + d*x)) + a**2*d*x*sqrt(a + b*acosh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*c*sqrt(a + b*acosh(c + d*x))
*acosh(c + d*x)**2 + b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2)
, x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)), x)
```

3.192 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=552

$$-\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}+\frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^4(c+dx)^5}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/}}$$

```
[Out] 16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^2/
d/(a+b*arccosh(d*x+c))^(3/2)+1/30*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+9/20*e^4*erfi(3^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)+
5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)
*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)+32/5*e^4*(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5996, 12, 5886, 5951, 5885, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}\,e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}\,e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}-\frac{40e^4\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^4}{3b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{32e^4\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^2}{5b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4e^4(c+dx)^5}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^4\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^4}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcCosh[c + d
*x])^(3/2)) - (4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (32*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a
 + b*ArcCosh[c + d*x]]) - (40*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c
 + d*x])/(3*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d) + (9*e^4*E^((3*a)/b)
*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2
)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) + (9*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
```

```
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2)*d*E^((3*a)/b)) + (5*e^4*Sq
rt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*
d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
```

```
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(8e^4)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}} \right.}{}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

Mathematica [A]
time = 3.58, size = 654, normalized size = 1.18

[illegible]

Warning: Unable to verify antiderivative.

[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]

[Out] (e^4*(-4*(3*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + ((a + b*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] + ((a + b*ArcCosh[c + d*x])*(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b)) - 9*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b + 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 5*(a + b*ArcCosh[c + d*x])*((2*(b + 10*a*(-1 + E^(10*ArcCosh[c + d*x])) - 10*b*ArcCosh[c + d*x] + b*E^(10*ArcCosh[c + d*x])*(1 + 10*ArcCosh[c + d*x])))/E^(5*ArcCosh[c + d*x]) + (20*Sqrt[5]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b])/E^((5*a)/b) + 20*Sqrt[5]*E^((5*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c + d*x]))/b]) - 18*b^2*Sinh[3*ArcCosh[c + d*x]] - 6*b^2*Sinh[5*ArcCosh[c + d*x]]))/(240*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)

[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((d*x*e + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**4*x**4/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
4*c*d**3*x**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(6
*c**2*d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acos
h(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c +
d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral
(4*c**3*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c
+ d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2), x)
```

3.193 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=441

$$-\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}+\frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^3(c+dx)^4}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/}}$$

```
[Out] 4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^2/
d/(a+b*arccosh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)+4/15*e^3*exp(2*a/b)*erf(2^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+4/15*e^3*er
fi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/e
xp(2*a/b)-2/5*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccos
h(d*x+c))^(5/2)+16/5*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b
*arccosh(d*x+c))^(1/2)-128/15*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$,
Rules used = {5996, 12, 5886, 5951, 5885, 3388, 2211, 2236, 2235}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{128e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{16e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{5b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{16e^3(c+dx)^4}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcCosh[c + d*x
])^(3/2)) - (16*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (16*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a +
 b*ArcCosh[c + d*x]]) - (128*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c
+ d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (16*e^3*E^((4*a)/b)*Sqrt[
Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (4*e^3*
E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) + (4*e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(6e^3)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx\right.}$$

Mathematica [A]
time = 2.45, size = 445, normalized size = 1.01

$$\frac{e^3\left(-4e^{-4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(a+b\cosh^{-1}(c+dx)\right)\left(16be^{4\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{4a}{b}}\left(b+8a\left(-1+e^{8\cosh^{-1}(c+dx)}\right)-8b\cosh^{-1}(c+dx)+be^{8\cosh^{-1}(c+dx)}\left(1+8\cosh^{-1}(c+dx)\right)+16e^{4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)\right)\right)-2\left(\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{-2\cosh^{-1}(c+dx)}\left(b+4a\left(-1+e^{4\cosh^{-1}(c+dx)}\right)-4b\cosh^{-1}(c+dx)+be^{4\cosh^{-1}(c+dx)}\left(1+4\cosh^{-1}(c+dx)\right)\right)+8\sqrt{2}be^{-\frac{2a}{b}}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+8\sqrt{2}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)\right)+3b^2\sinh\left(2\cosh^{-1}(c+dx)\right)\right)-3b^2\sinh\left(4\cosh^{-1}(c+dx)\right)\right)}{60b^3d\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*((-4*(a + b*ArcCosh[c + d*x])*(16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*A
rcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + E^
((4*a)/b)*(b + 8*a*(-1 + E^(8*ArcCosh[c + d*x])) - 8*b*ArcCosh[c + d*x] + b
*E^(8*ArcCosh[c + d*x])*(1 + 8*ArcCosh[c + d*x]) + 16*E^(4*(a/b + ArcCosh[c
 + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2,
(4*(a + b*ArcCosh[c + d*x]))/b])))/E^(4*(a/b + ArcCosh[c + d*x])) - 2*((a +
 b*ArcCosh[c + d*x])*((2*(b + 4*a*(-1 + E^(4*ArcCosh[c + d*x])) - 4*b*ArcCo
sh[c + d*x] + b*E^(4*ArcCosh[c + d*x])*(1 + 4*ArcCosh[c + d*x])))/E^(2*ArcC
osh[c + d*x]) + (8*Sqrt[2]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcCosh[c + d*x]))/b])/E^((2*a)/b) + 8*Sqrt[2]*E^((2*a)/b)*Sq
rt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*Ar
cCosh[c + d*x]))/b]) + 3*b^2*Sinh[2*ArcCosh[c + d*x]]) - 3*b^2*Sinh[4*ArcCo
sh[c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{3}(c+dx)}\,dx+\int \frac{d^{3}x^{3}}{a^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{3}(c+dx)}\,dx+\int \frac{3cd^{2}x^{2}}{a^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{3}(c+dx)}\,dx+\int \frac{3c^{2}dx}{a^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^{3}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**3*x**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
3*c*d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(3
*c**2*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**
2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2), x)
```

3.194 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=431

$$-\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}+\frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^2(c+dx)^3}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

```
[Out] 8/15*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d/(
a+b*arccosh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/15*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2)*(a+b
*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)-2/5*e
^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)
+16/15*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
-24/5*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+
c))^(1/2)
```

Rubi [A]

time = 0.98, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5996, 12, 5886, 5951, 5885, 3388, 2211, 2236, 2235, 5880, 5953}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{24e^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{5b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{16e^2\sqrt{c+dx-1}\sqrt{c+dx+1}}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4e^2(c+dx)^3}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x]
)^(3/2)) - (4*e^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) + (
16*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c
 + d*x]]) - (24*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^
3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (3*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d) + (e^2*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b)) +
(3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*
b^(7/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(4e^2)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}}\right.}{}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+d\right.}$$

Mathematica [A]
time = 2.27, size = 452, normalized size = 1.05

$$\frac{e^2\left(-6b^2\sqrt{\frac{-1+c+dx}{1+c+dx}}(1+c+dx)-2e^{-\cosh^{-1}(c+dx)}(a+b\cosh^{-1}(c+dx))\left(-2a+b-2b\cosh^{-1}(c+dx)+2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-2e^{-\frac{a}{b}}(a+b\cosh^{-1}(c+dx))\left(e^{\frac{a}{b}+\cosh^{-1}(c+dx)}(2a+b+2b\cosh^{-1}(c+dx))+2b\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)-3(a+b\cosh^{-1}(c+dx))\left(12\sqrt{3}\,be^{-\frac{3a}{b}}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)+2e^{-3\cosh^{-1}(c+dx)}\left(b+6a\left(-1+e^{6\cosh^{-1}(c+dx)}\right)-6b\cosh^{-1}(c+dx)+be^{6\cosh^{-1}(c+dx)}(1+6\cosh^{-1}(c+dx))+6\sqrt{3}\,e^{3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\Gamma\left(\frac{1}{2},\frac{3(a+b\cosh^{-1}(c+dx))}{b}\right)\right)\right)-6b^2\sinh\left(3\cosh^{-1}(c+dx)\right)\right)}{60b^3d\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(-6*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*A
rcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c +
d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b
+ ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*(a + b*ArcCosh[c + d*x])*(E^(
a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/
b) - 3*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/
b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b
+ 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh
[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x
]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 6*b^2*Sinh[3*ArcCosh[
c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)+b^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^3\,(c+dx)}\,dx+\int \frac{d^2x^2}{a^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)+b^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^3\,(c+dx)}\,dx+\int \frac{2cdx}{a^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+3a^2b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+3ab^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+dx)+b^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^3\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
2*c*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2
+ b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2), x)
```

3.195 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}+\frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

```
[Out] 4/15*e/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arccosh
(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5996, 12, 5886, 5951, 5885, 3388, 2211, 2236, 2235, 5893}

$$\frac{8\sqrt{2\pi}\,ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{32e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}+\frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{2e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh
[c + d*x])^(5/2)) + (4*e)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*e*
(c + d*x)^2)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (32*e*Sqrt[-1 + c
+ d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]])
 + (8*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(15*b^(7/2)*d) + (8*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5885

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + Dist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n +
1), Cosh[-a/b + x/b]^(m - 1)*(m - (m + 1)*Cosh[-a/b + x/b]^2), x], x], x, a
 + b*ArcCosh[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] &
& LtQ[n, -1]
```

Rule 5886

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)
)), x] + (-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCosh[c*x]
)^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])), x], x] + Dist[m/(b*c*(n + 1)), I
nt[x^(m - 1)*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x])),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{(a+b\cosh^{-1}(x))^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{(a+b\cosh^{-1}(x))^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(2e)\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\,\sqrt{1}}\right.}{}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right.}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 916 vs. 2(266) = 532.
time = 3.34, size = 916, normalized size = 3.44

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
[Out] (e*(4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[(-1 + c + d*x)/(1 + c +
d*x)]*(1 + c + d*x) + 4*b^(5/2)*c*(c + d*x)*ArcCosh[c + d*x] + 16*a*b^(3/2)
```

```
*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x] + 8*b^
(5/2)*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^2
 - 4*a*b^(3/2)*Cosh[2*ArcCosh[c + d*x]] - 4*b^(5/2)*ArcCosh[c + d*x]*Cosh[2
*ArcCosh[c + d*x]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*
Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c +
 d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*Erfi[Sqrt[a + b
*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*C
osh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - (2*Sqrt
[b]*c*(a + b*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b
+ ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*Sqrt[b]*c*(a +
b*ArcCosh[c + d*x])*(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c +
d*x]) + 2*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Er
f[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*Sqrt[Pi]*(a + b*Arc
Cosh[c + d*x])^(5/2)*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] +
 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])
^(5/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] -
 16*a^2*Sqrt[b]*Sinh[2*ArcCosh[c + d*x]] - 3*b^(5/2)*Sinh[2*ArcCosh[c + d*x
]] - 32*a*b^(3/2)*ArcCosh[c + d*x]*Sinh[2*ArcCosh[c + d*x]] - 16*b^(5/2)*Ar
cCosh[c + d*x]^2*Sinh[2*ArcCosh[c + d*x]]))/(15*b^(7/2)*d*(a + b*ArcCosh[c
+ d*x])^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^3\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)+b^3\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^3(c+dx)}\,dx+\int \frac{dx}{a^3\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c+dx)+b^3\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^3(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e*(Integral(c/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(d
*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c + d*x))*a
cosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2 + b**
3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2), x)
```

3.196 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}-\frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{4e^{a/b}\sqrt{\pi}}{}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)+4/15*exp(a/b)*erf((a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)-8/15*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^
3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5995, 5880, 5951, 5953, 3388, 2211, 2236, 2235}

$$\frac{4\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh[c + d*x])^(
5/2)) - (4*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (
4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d
) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E
^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c*
x]*Sqrt[-1 + c*x]*((a + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c
/(b*(n + 1)), Int[x*((a + b*ArcCosh[c*x])^(n + 1)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x])), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5951

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*x)^m*((a
 + b*ArcCosh[c*x])^(n + 1)/(b*c*(n + 1)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 + e1*x
]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], x] - Dist[f*(m/(b*c*(n + 1)))*Simp
[Sqrt[1 + c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]], Int[(
f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1
, d2, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] && LtQ[n, -1]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\cosh^{-1}(x))^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh^{-}}\right.}{5bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4\text{Su}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{}}{15b}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{}}{15b}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{}}{15b}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{}}{15b}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{}}{15b}$$

Mathematica [A]
time = 0.57, size = 243, normalized size = 1.16

$$\frac{-6\sqrt{\frac{-1+c+dx}{1+c+dx}}\,(1+c+dx) - \frac{2e^{-\cosh^{-1}(c+dx)}(a+b\cosh^{-1}(c+dx))\left(-2a+b-2b\cosh^{-1}(c+dx)+2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,(a+b\cosh^{-1}(c+dx))\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)}{b^2} - \frac{2e^{-\frac{a}{b}}(a+b\cosh^{-1}(c+dx))\left(e^{\frac{a}{b}+\cosh^{-1}(c+dx)}(2a+b+2b\cosh^{-1}(c+dx))+2b\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{b^2}}{15bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*ArcCosh[c
+ d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c + d*x])*Sqr
t[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh
[c + d*x]]))/(b^2*E^ArcCosh[c + d*x]) - (2*(a + b*ArcCosh[c + d*x])*(E^(a/b
 + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*ArcCo
sh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(b^2*E^(
a/b)))/(15*b*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{arccosh}\left(dx + c\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
[Out] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(c + dx\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(7/2), x)
```

3.197 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}} \, dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(7/2),x)/e

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}} \, dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(7/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{7/2}} \, dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{7/2}} \, dx, x, c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}} \, dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)), x]

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)), x)
```

3.198 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=189

$$-\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d}-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d}+\frac{2(e(c+c}{}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))/d/e-28/135*b*e^3*EllipticE(1/2*(
d*x+c+1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/(-d*x-
c)^(1/2)/(d*x+c-1)^(1/2)-28/405*b*e^2*(e*(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 5883, 104, 12, 115, 114}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de}-\frac{28be^3\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{135d\sqrt{-c-dx}\,\sqrt{c+dx-1}}-\frac{28be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}{405d}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{7/2}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-28*b*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(405*d
) - (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(7/2)*Sqrt[1 + c + d*x])/(81*d) +
 (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x]))/(9*d*e) - (28*b*e^3*Sqrt[
1 - c - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]],
 2])/(135*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int (ex)^{7/2} \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2} \left(a+b\cosh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\text{Subst}\left(\int \frac{(e}{\sqrt{-1+}}\right.}{}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9}}{}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9}}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+}}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+}}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+}}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.23, size = 150, normalized size = 0.79

$$\frac{2(e(c+dx))^{7/2}\left((c+dx)^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)+\frac{2b(c+dx)^{3/2}\left(7(1-(c+dx)^2)+5(c+dx)^2(1-(c+dx)^2)-7\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\right)}{45\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{9d(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*((c + d*x)^(9/2)*(a + b*ArcCosh[c + d*x]) + (2*b*(c
+ d*x)^(3/2)*(7*(1 - (c + d*x)^2) + 5*(c + d*x)^2*(1 - (c + d*x)^2) - 7*Sqr
t[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2]))/(45*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(9*d*(c + d*x)^(7/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 276, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{9}+\frac{-\frac{2\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{11}{2}}}{81}-\frac{4\sqrt{-\frac{1}{e}}\,e^{2}(dex+ce)^{\frac{7}{2}}}{405}+\frac{14e^{5}\sqrt{\frac{dex+ce+e}{e}}}{}}{}\right.}{}$
default	$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{9}+\frac{-\frac{2\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{11}{2}}}{81}-\frac{4\sqrt{-\frac{1}{e}}\,e^{2}(dex+ce)^{\frac{7}{2}}}{405}+\frac{14e^{5}\sqrt{\frac{dex+ce+e}{e}}}{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arccosh((d*e*x+c*e)
/e)+2/405/e*(-5*(-1/e)^(1/2)*(d*e*x+c*e)^(11/2)-2*(-1/e)^(1/2)*e^2*(d*e*x+c
*e)^(7/2)+21*e^5*((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticE
((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-21*e^5*((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-
c*e+e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)+7*(-1/e)^(1/2)*
e^4*(d*e*x+c*e)^(3/2))/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e
)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/9*(d*x*e + c*e)^(9/2)*a*e^(-1)/d + 1/405*(90*(d^4*x^4*e^(7/2) + 4*c*d^3*x
^3*e^(7/2) + 6*c^2*d^2*x^2*e^(7/2) + 4*c^3*d*x*e^(7/2) + c^4*e^(7/2))*sqrt(
d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/d + (-45*I*(log
(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(7/2) + 45*e^(7/2)*log
(sqrt(d*x + c) + 1) - 45*e^(7/2)*log(sqrt(d*x + c) - 1) - 20*e^(9/2*log(d*x
 + c) + 7/2) - 36*e^(5/2*log(d*x + c) + 7/2) - 180*e^(1/2*log(d*x + c) + 7/
2))/d + 405*integrate(2/9*(d^4*x^4*e^(7/2) + 4*c*d^3*x^3*e^(7/2) + 6*c^2*d^
2*x^2*e^(7/2) + 4*c^3*d*x*e^(7/2) + c^4*e^(7/2))*sqrt(d*x + c)/(d^3*x^3 + 3
*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + (3*c^2*d - d)*x - c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 815, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/405*(45*((b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^3*d^2*x + b
*c^4*d)*cosh(1)^3 + 3*(b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^
3*d^2*x + b*c^4*d)*cosh(1)^2*sinh(1) + 3*(b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c
^2*d^3*x^2 + 4*b*c^3*d^2*x + b*c^4*d)*cosh(1)*sinh(1)^2 + (b*d^5*x^4 + 4*b*
c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^3*d^2*x + b*c^4*d)*sinh(1)^3)*sqrt((d*x
 + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) + 42*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*cosh(1)^3 + 3*b*cosh(1)^2*
sinh(1) + 3*b*cosh(1)*sinh(1)^2 + b*sinh(1)^3)*weierstrassZeta(4/d^2, 0, we
ierstrassPInverse(4/d^2, 0, (d*x + c)/d)) + (45*(a*d^5*x^4 + 4*a*c*d^4*x^3
+ 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c^4*d)*cosh(1)^3 + 135*(a*d^5*x^4 + 4
*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c^4*d)*cosh(1)^2*sinh(1)
 + 135*(a*d^5*x^4 + 4*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c^4
*d)*cosh(1)*sinh(1)^2 + 45*(a*d^5*x^4 + 4*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 + 4
*a*c^3*d^2*x + a*c^4*d)*sinh(1)^3 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((5
*b*d^4*x^3 + 15*b*c*d^3*x^2 + (15*b*c^2 + 7*b)*d^2*x + (5*b*c^3 + 7*b*c)*d)
*cosh(1)^3 + 3*(5*b*d^4*x^3 + 15*b*c*d^3*x^2 + (15*b*c^2 + 7*b)*d^2*x + (5*
b*c^3 + 7*b*c)*d)*cosh(1)^2*sinh(1) + 3*(5*b*d^4*x^3 + 15*b*c*d^3*x^2 + (15
*b*c^2 + 7*b)*d^2*x + (5*b*c^3 + 7*b*c)*d)*cosh(1)*sinh(1)^2 + (5*b*d^4*x^3
 + 15*b*c*d^3*x^2 + (15*b*c^2 + 7*b)*d^2*x + (5*b*c^3 + 7*b*c)*d)*sinh(1)^3
))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/d^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x)), x)
```

3.199 $\int (ce+dex)^{5/2} \left(a+b\cosh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=169

$$-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d}+\frac{2(e(c+d\cdots}{}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))/d/e-20/147*b*e^(5/2)*EllipticF((
e*(d*x+c))^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/(d*x+c-1)^(1/2)-4/49*b*(e*(d
*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-20/147*b*e^2*(d*x+c-1)^(1/2)
*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 5883, 104, 12, 118, 117}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{20be^{5/2}\sqrt{-c-dx+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{147d\sqrt{c+dx-1}}-\frac{20be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{e(c+dx)}}{147d}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{5/2}}{49d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-20*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(147*d)
- (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqrt[1 + c + d*x])/(49*d) + (
2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x]))/(7*d*e) - (20*b*e^(5/2)*Sqr
t[1 - c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(147*d*Sqr
t[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sq
rt[c]*Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c
, 0] && GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

Rule 118

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e
 + f*x])), Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)\,dx=\frac{\operatorname{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{(2b)\operatorname{Subst}\left(\int \frac{(e}{\sqrt{-1+}}\right.}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d}+\frac{2(e(c+dx))^{7}}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d}+\frac{2(e(c+dx))^{7}}{}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c}}{}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c}}{}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c}}{}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.18, size = 180, normalized size = 1.07

$$\frac{2(e(c+dx))^{5/2}\left(10b-4b(c+dx)^2-6b(c+dx)^4+21a\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}+21b\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\cosh^{-1}(c+dx)-10b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\right)}{147d\sqrt{\frac{-1+c+dx}{c+dx}}\,(c+dx)^{5/2}\sqrt{1+c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(10*b - 4*b*(c + d*x)^2 - 6*b*(c + d*x)^4 + 21*a*Sqr
t[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x] + 21*b*Sqrt[-1 + c + d*x]*(c
+ d*x)^3*Sqrt[1 + c + d*x]*ArcCosh[c + d*x] - 10*b*Sqrt[1 - (c + d*x)^2]*Hy
pergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(147*d*Sqrt[(-1 + c + d*x)/(c
 + d*x)]*(c + d*x)^(5/2)*Sqrt[1 + c + d*x])
```

Maple [A]
time = 0.06, size = 218, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(3\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}\,e^{2}(dex+ce)^{\frac{5}{2}}+5e^{4}\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\cdots}\right.}{147e\sqrt{-\frac{1}{e}}\,\sqrt{\cdots}}\right.}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(3\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}\,e^{2}(dex+ce)^{\frac{5}{2}}+5e^{4}\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\cdots}\right.}{147e\sqrt{-\frac{1}{e}}\,\sqrt{\cdots}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arccosh((d*e*x+c*e)
/e)-2/147/e*(3*(-1/e)^(1/2)*(d*e*x+c*e)^(9/2)+2*(-1/e)^(1/2)*e^2*(d*e*x+c*e
)^(5/2)+5*e^4*((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticF((d
*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-5*(-1/e)^(1/2)*e^4*(d*e*x+c*e)^(1/2))/(-1/e
)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a*e^(-1)/d + 1/147*(42*(d^3*x^3*e^(5/2) + 3*c*d^2*x
^2*e^(5/2) + 3*c^2*d*x*e^(5/2) + c^3*e^(5/2))*sqrt(d*x + c)*log(d*x + sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + c)/d + (21*I*(log(I*sqrt(d*x + c) + 1) - l
og(-I*sqrt(d*x + c) + 1))*e^(5/2) + 21*e^(5/2)*log(sqrt(d*x + c) + 1) - 21*
e^(5/2)*log(sqrt(d*x + c) - 1) - 12*e^(7/2*log(d*x + c) + 5/2) - 28*e^(3/2*
log(d*x + c) + 5/2))/d + 147*integrate(2/7*(d^3*x^3*e^(5/2) + 3*c*d^2*x^2*e
^(5/2) + 3*c^2*d*x*e^(5/2) + c^3*e^(5/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*
x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (3*c^2*d - d)*x - c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 517, normalized size = 3.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/147*(21*((b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cosh(1)^
2 + 2*(b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cosh(1)*sinh(
1) + (b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*sinh(1)^2)*sqr
t((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)) - 10*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*cosh(1)^2 + 2*b*cosh
(1)*sinh(1) + b*sinh(1)^2)*weierstrassPInverse(4/d^2, 0, (d*x + c)/d) + (21
*(a*d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*cosh(1)^2 + 42*(a*
d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*cosh(1)*sinh(1) + 21*(
a*d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*sinh(1)^2 - 2*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*((3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 + 5*b)*d
^2)*cosh(1)^2 + 2*(3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 + 5*b)*d^2)*cosh(1)
*sinh(1) + (3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 + 5*b)*d^2)*sinh(1)^2))*sq
rt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/d^3
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{acosh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)), x)
```

3.200 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=145

$$-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d}+\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{12be\sqrt{1-c-dx}}{2\cdots}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))/d/e-12/25*b*e*EllipticE(1/2*(d*x
+c+1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/(-d*x-c)^
(1/2)/(d*x+c-1)^(1/2)-4/25*b*(e*(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 5883, 104, 12, 115, 114}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{12be\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{25d\sqrt{-c-dx}\,\sqrt{c+dx-1}}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(25*d) + (2
*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x]))/(5*d*e) - (12*b*e*Sqrt[1 - c
 - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/
(25*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{(2b)\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d}+\frac{2(e(c+dx)}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d}+\frac{2(e(c+dx)}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d}+\frac{2(e(c+dx)}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d}+\frac{2(e(c+dx)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.35, size = 109, normalized size = 0.75

$$\frac{2(e(c+dx))^{3/2}\left(5(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b\left(-1+c^2+2cdx+d^2x^2+\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*(c + d*x)*(a + b*ArcCosh[c + d*x]) - (2*b*(-1 + c
^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4,
7/4, (c + d*x)^2]))/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(25*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 253, normalized size = 1.74

method	result

derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{\right.}\right.}{}\right.}{}$
default	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{\right.}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c*e)
/e)-2/25/e*((-1/e)^(1/2)*(d*e*x+c*e)^(7/2)+3*((d*e*x+c*e+e)/e)^(1/2)*((-d*e
*x-c*e+e)/e)^(1/2)*e^3*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-3*e^3*((
d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)*
(-1/e)^(1/2),I)-(-1/e)^(1/2)*e^2*(d*e*x+c*e)^(3/2))/(-1/e)^(1/2)/((d*e*x+c*
e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a*e^(-1)/d + 1/25*(10*(d^2*x^2*e^(3/2) + 2*c*d*x*e^
(3/2) + c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + c)/d + (-5*I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))
*e^(3/2) + 5*e^(3/2)*log(sqrt(d*x + c) + 1) - 5*e^(3/2)*log(sqrt(d*x + c) -
 1) - 4*e^(5/2*log(d*x + c) + 3/2) - 20*e^(1/2*log(d*x + c) + 3/2))/d + 25*
integrate(2/5*(d^2*x^2*e^(3/2) + 2*c*d*x*e^(3/2) + c^2*e^(3/2))*sqrt(d*x +
c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 279, normalized size = 1.92

$$\frac{2\left(5\left((bd^{3}x^{2}+2\,bcd^{2}x+bc^{2}d)\cosh(1)+(bd^{3}x^{2}+2\,bcd^{2}x+bc^{2}d)\sinh(1)\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\right)+6\sqrt{d^{3}\cosh(1)+d^{3}\sinh(1)}\,(b\cosh(1)+b\sinh(1))\operatorname{weierstrassZeta}\left(\frac{4}{d^{2}},0,\operatorname{weierstrassPInverse}\left(\frac{4}{d^{2}},0,\frac{dx+c}{d}\right)\right)+\left(5\,(ad^{3}x^{2}+2\,acd^{2}x+ac^{2}d)\cosh(1)+5\,(ad^{3}x^{2}+2\,acd^{2}x+ac^{2}d)\sinh(1)-2\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\,((bd^{2}x+bcd)\cosh(1)+(bd^{2}x+bcd)\sinh(1))\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{25\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/25*(5*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cosh(1) + (b*d^3*x^2 + 2*b*c*d
^2*x + b*c^2*d)*sinh(1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 6*sqrt(d^3*cosh(1) + d^3*sinh(
1))*(b*cosh(1) + b*sinh(1))*weierstrassZeta(4/d^2, 0, weierstrassPInverse(4
/d^2, 0, (d*x + c)/d)) + (5*(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)*cosh(1) + 5
*(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*((b*d^2*x + b*c*d)*cosh(1) + (b*d^2*x + b*c*d)*sinh(1)))*sqrt((d*x
+ c)*cosh(1) + (d*x + c)*sinh(1)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{acosh}(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)), x)
```

3.201 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=127

$$-\frac{4b\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{e}\,\sqrt{1-c-dx}}{9\ldots}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))/d/e-4/9*b*EllipticF((e*(d*x+c))^
(1/2)/e^(1/2),I)*e^(1/2)*(-d*x-c+1)^(1/2)/d/(d*x+c-1)^(1/2)-4/9*b*(d*x+c-1)
^(1/2)*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5996, 5883, 104, 12, 118, 117}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{e}\,\sqrt{-c-dx+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{9d\sqrt{c+dx-1}}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{e(c+dx)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(9*d) + (2*(e
*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x]))/(3*d*e) - (4*b*Sqrt[e]*Sqrt[1 -
 c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(9*d*Sqrt[-1 +
c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sq
```

```
rt[c]*Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c
, 0] && GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

Rule 118

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e
 + f*x])), Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\,\left(a+b\cosh^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int\sqrt{ex}\,\left(a+b\cosh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{(2b)\text{Subst}\left(\int\frac{(e}{\sqrt{-1+}}\right.}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/}}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/}}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/}}{}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{9d}+\frac{2(e(c+dx))^{3/}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.33, size = 131, normalized size = 1.03

$$\frac{\sqrt{e(c+dx)}\left(\frac{2}{3}(c+dx)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)-\frac{4b\left(-1+c^2+2cdx+d^2x^2+\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\right)}{9\sqrt{\frac{-1+c+dx}{c+dx}}\sqrt{1+c+dx}}\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (Sqrt[e*(c + d*x)]*((2*(c + d*x)^(3/2)*(a + b*ArcCosh[c + d*x]))/3 - (4*b*(
-1 + c^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/4,
 1/2, 5/4, (c + d*x)^2]))/(9*Sqrt[(-1 + c + d*x)/(c + d*x)]*Sqrt[1 + c + d*
x])))/(d*Sqrt[c + d*x])
```

Maple [A]
time = 0.05, size = 194, normalized size = 1.53

method	result
derivativedivides	$\frac{\frac{2a(dex+ce)^{\frac{3}{2}}}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,\operatorname{EllipticF}\left(\sqrt{de\right.\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-}}\right.}{de}$
default	$\frac{\frac{2a(dex+ce)^{\frac{3}{2}}}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{-dex-ce+e}{e}}\,\operatorname{EllipticF}\left(\sqrt{de\right.\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/3*a*(d*e*x+c*e)^(3/2)+b*(1/3*(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c*e)
/e)-2/9/e*((-1/e)^(1/2)*(d*e*x+c*e)^(5/2)+((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-
c*e+e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e^2-(-1/e)^(1/2
)*e^2*(d*e*x+c*e)^(1/2))/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e
+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a*e^(-1)/d + 1/9*(6*(d*x*e^(1/2) + c*e^(1/2))*sqrt(
d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/d + (3*I*(log(I
*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(1/2) + 3*e^(1/2)*log(sq
rt(d*x + c) + 1) - 3*e^(1/2)*log(sqrt(d*x + c) - 1) - 4*e^(3/2*log(d*x + c)
 + 1/2))/d + 9*integrate(2/3*(d*x*e^(1/2) + c*e^(1/2))*sqrt(d*x + c)/(d^3*x
^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + (3*c^2*d - d)*x - c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 159, normalized size = 1.25

$$\frac{2\left(3\left(bd^3x+bcd^2\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-2\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,b\mathrm{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)+\left(3\,ad^3x+3\,acd^2-2\sqrt{d^2x^2+2\,cdx+c^2-1}\,bd^2\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*(b*d^3*x + b*c*d^2)*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*sqrt(d^3*cosh(1) + d^3*sin
h(1))*b*weierstrassPInverse(4/d^2, 0, (d*x + c)/d) + (3*a*d^3*x + 3*a*c*d^2
 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b*d^2)*sqrt((d*x + c)*cosh(1) + (d*x
 + c)*sinh(1)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,(a+b\operatorname{acosh}(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)), x)
```

3.202 $\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))}{de}-\frac{4b\sqrt{1-c-dx}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1+c+dx}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\,\sqrt{-1+c+dx}}$$

```
[Out] 2*(a+b*arccosh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*EllipticE(1/2*(d*x+c+1)^(1
/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/e/(-d*x-c)^(1/2)/
(d*x+c-1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5996, 5883, 115, 114}

$$\frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))}{de}-\frac{4b\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\,\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x]))/(d*e) - (4*b*Sqrt[1 - c - d*
x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(d*e*
Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{(2b)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x\right.}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{\left(\sqrt{2}\,b\sqrt{1-c-dx}\,\sqrt{e(c+dx)}\right)\text{Sub}}{de\sqrt{-c-dx}}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{4b\sqrt{1-c-dx}\,\sqrt{e(c+dx)}\,E\left(\sin^{-1}\right.}{de\sqrt{-c-dx}\,\sqrt{-1+c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 94, normalized size = 0.90

$$\frac{2\sqrt{e(c+dx)}\left(3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

[Out] (2*Sqrt[e*(c + d*x)]*(3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(3*d*e)

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 138, normalized size = 1.33

method	result
derivativedivides	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{de\cdots}\right.\right.}{\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-\cdots}{e}}}\right.}{de}$
default	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-\operatorname{EllipticE}\left(\sqrt{de\cdots}\right.\right.}{\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-\cdots}{e}}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/e*((d*e*x+c*e)^(1/2)*a+b*((d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)-2*(EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I))*((-d*e*x-c*e+e)/e)^(1/2)/(-1/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")

[Out] (2*(d*x*e^(1/2) + c*e^(1/2))*e^(-1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(sqrt(d*x + c)*d) - (I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(-1/2) - e^(-1/2)*log(sqrt(d*x + c) + 1) + e^(-1/2)*log(sqrt(d*x + c) - 1) + 4*e^(1/2*log(d*x + c) - 1/2))/d + integrate(2*(d*x*e^(1/2) + c*e^(1/2))/((d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d - d)*x*e + (d^2*x^2*e + 2*c*d*x*e + (c^2 - 1)*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^3 - c)*e)*sqrt(d*x + c)), x))*b + 2*sqrt(d*x*e + c*e)*a*e^(-1)/d

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 128, normalized size = 1.23

$$\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,ad+2\,\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,b\mathrm{weierstrassZeta}\left(\frac{4}{d^2},0,\mathrm{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)\right)}{d^2\cosh(1)+d^2\sinh(1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)) + sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*a*d
+ 2*sqrt(d^3*cosh(1) + d^3*sinh(1))*b*weierstrassZeta(4/d^2, 0, weierstrass
PInverse(4/d^2, 0, (d*x + c)/d)))/(d^2*cosh(1) + d^2*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}\left(c + dx\right)}{\sqrt{e\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/sqrt(e*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acosh}(c + d\,x)}{\sqrt{c\,e + d\,e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

3.203 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}+\frac{4b\sqrt{1-c-dx}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{-1+c+dx}}$$

```
[Out] 4*b*EllipticF((e*(d*x+c))^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/e^(3/2)/(d*x+
c-1)^(1/2)-2*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5996, 5883, 118, 117}

$$\frac{4b\sqrt{-c-dx+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[1 - c - d
*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(d*e^(3/2)*Sqrt[-1 +
c + d*x])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sq
rt[c]*Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c
, 0] && GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

Rule 118

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e
 + f*x])), Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{ex}\,\sqrt{1+x}}\,dx, x, c+d\right.}{de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{\left(2b\sqrt{1-c-dx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{ex}\,\sqrt{1+}}\right.}{de\sqrt{-1+c+dx}}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{4b\sqrt{1-c-dx}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{-1+c+dx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 92, normalized size = 1.10

$$\frac{2\left(-a-b\cosh^{-1}(c+dx)+\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] + (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
])))/(d*e*Sqrt[e*(c + d*x)])
```

Maple [A]
time = 0.05, size = 119, normalized size = 1.42

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{\frac{-dex-ce+e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right)}{de}$	119
default	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{\frac{-dex-ce+e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{-\frac{-dex-ce+e}{e}}}\right)}{de}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)+
2/e*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*((-d*e*x-c*e+e)/e)^(1/2)/(-
1/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*(2*e^(-3/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(sqrt(d*x
 + c)*d) - (-I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(-3
/2) - e^(-3/2)*log(sqrt(d*x + c) + 1) + e^(-3/2)*log(sqrt(d*x + c) - 1))/d
+ 2*integrate(1/((d^2*x^2*e^(3/2) + 2*c*d*x*e^(3/2) + (c^2 - 1)*e^(3/2))*sq
rt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^3*x^3*e^(3/2) + 3*c*d^
2*x^2*e^(3/2) + (3*c^2*d - d)*x*e^(3/2) + (c^3 - c)*e^(3/2))*sqrt(d*x + c))
, x)) - 2*a*e^(-1)/(sqrt(d*x*e + c*e)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 169, normalized size = 2.01

$$-\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,ad^2-2\,\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,(bdx+bc)\mathrm{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)}{(d^4x+cd^3)\cosh(1)^2+2\,(d^4x+cd^3)\cosh(1)\sinh(1)+(d^4x+cd^3)\sinh(1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d^2*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)) + sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*a
```

```
*d^2 - 2*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*d*x + b*c)*weierstrassPInverse(
4/d^2, 0, (d*x + c)/d))/((d^4*x + c*d^3)*cosh(1)^2 + 2*(d^4*x + c*d^3)*cosh
(1)*sinh(1) + (d^4*x + c*d^3)*sinh(1)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(c + dx)}{(e\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c + d\,x)}{{(c\,e + d\,e\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

3.204 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=150

$$\frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{1-c-dx}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1+c+dx}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\,\sqrt{-1+c+dx}}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*EllipticE(1/2*(d*x+c+
1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/e^3/(-d*x-c)
^(1/2)/(d*x+c-1)^(1/2)+4/3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 5883, 106, 12, 16, 115, 114}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\,\sqrt{c+dx-1}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3de^2\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*d*e^2*Sqrt[e*(c + d*x)]) - (2
*(a + b*ArcCosh[c + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*Sqrt[1 - c -
d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(3*
d*e^3*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx, x, c+\right.}{3de}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\text{Subst}\left(\int -\right.}{}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{\left(\sqrt{2}\,b\sqrt{1-c-\right.}{}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{4b\sqrt{1-c-dx}}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.10, size = 94, normalized size = 0.63

$$\frac{2\left(-a-b\cosh^{-1}(c+dx)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 268, normalized size = 1.79

method	result
derivativedivides	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\right.}{3}}{e^{3}}\right.}{}$
default	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\right.}{3}}{e^{3}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c*
e)/e)+2/3/e^3*(-((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*(d*e*x+c*e
)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e+((d*e*x+c*e+e)/e)^(1/
2)*((-d*e*x-c*e+e)/e)^(1/2)*(d*e*x+c*e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)*(
-1/e)^(1/2),I)*e+(-1/e)^(1/2)*(d*e*x+c*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/
(d*e*x+c*e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*e^(1/2)*integrate(1/3/((d^4*x^4*e^3 + 4*c*d^3*x^3*e^3 + (6*c^2*d^2
- d^2)*x^2*e^3 + 2*(2*c^3*d - c*d)*x*e^3 + (d^3*x^3*e^3 + 3*c*d^2*x^2*e^3 +
 (3*c^2*d - d)*x*e^3 + (c^3 - c)*e^3)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 (c^4 - c^2)*e^3)*sqrt(d*x + c)), x) + 2*e^(1/2)*log(d*x + sqrt(d*x + c + 1
)*sqrt(d*x + c - 1) + c)/((d^2*x*e^3 + c*d*e^3)*sqrt(d*x + c)) + (-I*(log(I
*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(-5/2) - e^(-5/2)*log(-(
e^(1/2) - e^(1/2*log(d*x + c) + 1/2))/(e^(1/2) + e^(1/2*log(d*x + c) + 1/2)
)))/d)*b - 2/3*a*e^(-1)/((d*x*e + c*e)^(3/2)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 283, normalized size = 1.89

$$-\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,\mathrm{weierstrassZeta}\left(\frac{4}{d^2},0,\mathrm{weierstrassPInverse}\left(\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+\left(ad-2\left(bd^2x+bcd\right)\sqrt{d^2x^2+2\,cdx+c^2-1}\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{3\left(\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)^3+3\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)^2\sinh(1)+3\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)\sinh(1)^2+\left(d^4x^2+2\,cd^3x+c^2d^2\right)\sinh(1)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(d^3*co
sh(1) + d^3*sinh(1))*weierstrassZeta(4/d^2, 0, weierstrassPInverse(4/d^2, 0
, (d*x + c)/d)) + (a*d - 2*(b*d^2*x + b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/((d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*cosh(1)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(1)^2*sinh(1) + 3
*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(1)*sinh(1)^2 + (d^4*x^2 + 2*c*d^3*x +
 c^2*d^2)*sinh(1)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(c + dx)}{(e (c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acosh}(c + d\,x)}{{(c\,e + d\,e\,x)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

3.205 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=130

$$\frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{1-c-dx}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{15de^{7/2}\sqrt{-1+c+dx}}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(5/2)+4/15*b*EllipticF((e*(d*x+c)
)^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/e^(7/2)/(d*x+c-1)^(1/2)+4/15*b*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5996, 5883, 106, 12, 16, 118, 117}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{-c-dx+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{15de^{7/2}\sqrt{c+dx-1}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{15de^2(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*d*e^2*(e*(c + d*x))^(3/2)) -
 (2*(a + b*ArcCosh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) + (4*b*Sqrt[1 - c
 - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(15*d*e^(7/2)*Sqr
t[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sq
rt[c]*Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c
, 0] && GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

Rule 118

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e
 + f*x])), Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx, x, c+\cdots\right.}{5de}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(4b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{\left(2b\sqrt{1-c-dx}\cdots\right.}{\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{4b\sqrt{1-c-dx}\cdots}{15d\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 94, normalized size = 0.72

$$\frac{2\left(-3\left(a+b\cosh^{-1}(c+dx)\right) - \frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hype
rgeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x])))/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [A]
time = 0.06, size = 201, normalized size = 1.55

method	result
derivativedivides	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)(dex+}{15\,e^{3}\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-d}{}}}\right.}{de}$
default	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{2\sqrt{\frac{dex+ce+e}{e}}\sqrt{\frac{-dex-ce+e}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)(dex+}{15\,e^{3}\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\sqrt{-\frac{-d}{}}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c*
e)/e)+2/15/e^3*(((d*e*x+c*e+e)/e)^(1/2)*((-d*e*x-c*e+e)/e)^(1/2)*EllipticF(
(d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(d*e*x+c*e)^(3/2)+(-1/e)^(1/2)*(d*e*x+c*e
)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/(d*e*x+c*e)^(3/2)/((d*e*x+c*e+e)/e)^(1/2
)/(-(-d*e*x-c*e+e)/e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(10*e^(1/2)*integrate(1/5/((d^5*x^5*e^4 + 5*c*d^4*x^4*e^4 + (10*c^2*d^
3 - d^3)*x^3*e^4 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^4 + (5*c^4*d - 3*c^2*d)*x*e
^4 + (d^4*x^4*e^4 + 4*c*d^3*x^3*e^4 + (6*c^2*d^2 - d^2)*x^2*e^4 + 2*(2*c^3*
d - c*d)*x*e^4 + (c^4 - c^2)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^
5 - c^3)*e^4)*sqrt(d*x + c)), x) + 2*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)/((d^3*x^2*e^4 + 2*c*d^2*x*e^4 + c^2*d*e^4)*sqrt(d*x +
c)) + (I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))*e^(-7/2) -
e^(-7/2)*log(-(e^(1/2) - e^(1/2*log(d*x + c) + 1/2))/(e^(1/2) + e^(1/2*log(
d*x + c) + 1/2))) - 4*e^(-1/2*log(d*x + c) - 7/2))/d)*b - 2/5*a*e^(-1)/((d*
x*e + c*e)^(5/2)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 383, normalized size = 2.95

$$-\frac{2\left(3\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd^{2}\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\right)-2\,(bd^{3}x^{3}+3\,bcd^{2}x^{2}+3\,bc^{2}dx+bc^{3})\sqrt{d^{3}\cosh(1)+d^{3}\sinh(1)}\,\operatorname{weierstrassPInverse}\left(\frac{4}{d^{2}},0,\frac{dx+c}{d}\right)+\left(3\,ad^{2}-2\,(bd^{3}x+bcd^{2})\sqrt{d^{2}x^{2}+2\,cdx+c^{2}-1}\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{15\left((d^{6}x^{3}+3\,cd^{5}x^{2}+3\,c^{2}d^{4}x+c^{3}d^{3})\cosh(1)^{4}+4\,(d^{6}x^{3}+3\,cd^{5}x^{2}+3\,c^{2}d^{4}x+c^{3}d^{3})\cosh(1)^{3}\sinh(1)+6\,(d^{6}x^{3}+3\,cd^{5}x^{2}+3\,c^{2}d^{4}x+c^{3}d^{3})\cosh(1)^{2}\sinh(1)^{2}+4\,(d^{6}x^{3}+3\,cd^{5}x^{2}+3\,c^{2}d^{4}x+c^{3}d^{3})\cosh(1)\sinh(1)^{3}+(d^{6}x^{3}+3\,cd^{5}x^{2}+3\,c^{2}d^{4}x+c^{3}d^{3})\sinh(1)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d^2*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3)*sqrt(d^3*cosh(1) + d^3*sinh(1))*weierstrassPInverse(4/d^2, 0, (
d*x + c)/d) + (3*a*d^2 - 2*(b*d^3*x + b*c*d^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/((d^6*x^3 + 3*c*d^5*x^2
 + 3*c^2*d^4*x + c^3*d^3)*cosh(1)^4 + 4*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*
x + c^3*d^3)*cosh(1)^3*sinh(1) + 6*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c
^3*d^3)*cosh(1)^2*sinh(1)^2 + 4*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*
d^3)*cosh(1)*sinh(1)^3 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*si
nh(1)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

3.206 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=153

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de}-\frac{8b\sqrt{1-c-dx}\,(e(c+dx))^{11/2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\right.}{99de^2\sqrt{-1+c+dx}}$$

[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))^2/d/e-16/1287*b^2*(e*(d*x+c))^(13/2)*hypergeom([1, 13/4, 13/4],[15/4, 17/4],(d*x+c)^2)/d/e^3-8/99*b*(e*(d*x+c))^(11/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 11/4],[15/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right)}{1287de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{99de^2\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]

[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])^2)/(9*d*e) - (8*b*Sqrt[1 - c - d*x]*(e*(c + d*x))^(11/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, (c + d*x)^2])/(99*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, (c + d*x)^2])/(1287*d*e^3)

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^{7/2} \left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2} \left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{-1}}\right.}{}$$

$$= \frac{2(e(c+dx))^{9/2} \left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{8b(e(c+dx))^{11/2}\sqrt{}}{}$$

Mathematica [A]
time = 0.36, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{9/2} \left(143\left(a+b\cosh^{-1}(c+dx)\right)^2 - 4b(c+dx)\left(\frac{13\sqrt{1-(c+dx)^2}\,(a+b\cosh^{-1}(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}} + 2b(c+dx)\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right)\right)\right)}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((13
*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 11/4
, 15/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x
)*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, (c + d*x)^2])))/(1287*d*
e)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}} \left(a+b\,\mathrm{arccosh}(dx+c)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/9*(d*x*e + c*e)^(9/2)*a^2*e^(-1)/d + 2/9*(b^2*d^4*x^4*e^(7/2) + 4*b^2*c*d
^3*x^3*e^(7/2) + 6*b^2*c^2*d^2*x^2*e^(7/2) + 4*b^2*c^3*d*x*e^(7/2) + b^2*c^
4*e^(7/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)
^2/d + integrate(2/9*(((9*a*b*d^5 - 2*b^2*d^5)*x^5*e^(7/2) + 5*(9*a*b*c*d^4
 - 2*b^2*c*d^4)*x^4*e^(7/2) - (20*b^2*c^2*d^3 - 9*(10*c^2*d^3 - d^3)*a*b)*x
^3*e^(7/2) - (20*b^2*c^3*d^2 - 9*(10*c^3*d^2 - 3*c*d^2)*a*b)*x^2*e^(7/2) -
(10*b^2*c^4*d - 9*(5*c^4*d - 3*c^2*d)*a*b)*x*e^(7/2) - (2*b^2*c^5 - 9*(c^5
- c^3)*a*b)*e^(7/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((
9*a*b*d^6 - 2*b^2*d^6)*x^6*e^(7/2) + 6*(9*a*b*c*d^5 - 2*b^2*c*d^5)*x^5*e^(7
/2) + (9*(15*c^2*d^4 - d^4)*a*b - 2*(15*c^2*d^4 - d^4)*b^2)*x^4*e^(7/2) + 4
*(9*(5*c^3*d^3 - c*d^3)*a*b - 2*(5*c^3*d^3 - c*d^3)*b^2)*x^3*e^(7/2) + 3*(9
*(5*c^4*d^2 - 2*c^2*d^2)*a*b - 2*(5*c^4*d^2 - 2*c^2*d^2)*b^2)*x^2*e^(7/2) +
 2*(9*(3*c^5*d - 2*c^3*d)*a*b - 2*(3*c^5*d - 2*c^3*d)*b^2)*x*e^(7/2) + (9*(
c^6 - c^4)*a*b - 2*(c^6 - c^4)*b^2)*e^(7/2))*sqrt(d*x + c))*log(d*x + sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2
 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x
 - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*arccosh
(d*x + c)^2*e^3 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^
3)*arccosh(d*x + c)*e^3 + (a^2*d^3*x^3 + 3*a^2*c*d^2*x^2 + 3*a^2*c^2*d*x +
a^2*c^3)*e^3)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{acosh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x))^2, x)
```

3.207 $\int (ce+dex)^{5/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de}-\frac{8b\sqrt{1-c-dx}\,(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4}\right.}{63de^2\sqrt{-1+c+dx}}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))^2/d/e-16/693*b^2*(e*(d*x+c))^(11
/2)*hypergeom([1, 11/4, 11/4],[13/4, 15/4],(d*x+c)^2)/d/e^3-8/63*b*(e*(d*x+
c))^(9/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 9/4],[13/4],(d*x+c)^2)*(-d*x
-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{693de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{63de^2\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])^2)/(7*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 9/4, 13/4, (c + d*x)^2])/(63*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c +
d*x))^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])
/(693*d*e^3)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{-1}\right.}{}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{8b(e(c+dx))^{9/2}\sqrt{1}}{}$$

Mathematica [A]
time = 0.35, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{7/2}\left(99\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(\frac{11\sqrt{1-(c+dx)^2}\,(a+b\cosh^{-1}(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)\right)\right)}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((11*
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 9/4,
13/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*
HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])))/(693*d*e)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a^2*e^(-1)/d + 2/7*(b^2*d^3*x^3*e^(5/2) + 3*b^2*c*d
^2*x^2*e^(5/2) + 3*b^2*c^2*d*x*e^(5/2) + b^2*c^3*e^(5/2))*sqrt(d*x + c)*log
(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/d + integrate(2/7*(((7*a*
b*d^4 - 2*b^2*d^4)*x^4*e^(5/2) + 4*(7*a*b*c*d^3 - 2*b^2*c*d^3)*x^3*e^(5/2)
- (12*b^2*c^2*d^2 - 7*(6*c^2*d^2 - d^2)*a*b)*x^2*e^(5/2) - 2*(4*b^2*c^3*d -
 7*(2*c^3*d - c*d)*a*b)*x*e^(5/2) - (2*b^2*c^4 - 7*(c^4 - c^2)*a*b)*e^(5/2)
)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((7*a*b*d^5 - 2*b^2*d
^5)*x^5*e^(5/2) + 5*(7*a*b*c*d^4 - 2*b^2*c*d^4)*x^4*e^(5/2) + (7*(10*c^2*d^
3 - d^3)*a*b - 2*(10*c^2*d^3 - d^3)*b^2)*x^3*e^(5/2) + (7*(10*c^3*d^2 - 3*c
*d^2)*a*b - 2*(10*c^3*d^2 - 3*c*d^2)*b^2)*x^2*e^(5/2) + (7*(5*c^4*d - 3*c^2
*d)*a*b - 2*(5*c^4*d - 3*c^2*d)*b^2)*x*e^(5/2) + (7*(c^5 - c^3)*a*b - 2*(c^
5 - c^3)*b^2)*e^(5/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*arccosh(d*x + c)^2*e^2 + 2*
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*arccosh(d*x + c)*e^2 + (a^2*d^2*x^2 +
 2*a^2*c*d*x + a^2*c^2)*e^2)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{5}{2}} (a+b\operatorname{acosh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(5/2)*(a + b*acosh(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,{\left(a+b\,\mathrm{acosh}(c+d\,x)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2, x)
```

3.208 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=153

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}-\frac{8b\sqrt{1-c-dx}\,(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4}\right.}{35de^2\sqrt{-1+c+dx}}$$

[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^2/d/e-16/315*b^2*(e*(d*x+c))^(9/2)*hypergeom([1, 9/4, 9/4],[11/4, 13/4],(d*x+c)^2)/d/e^3-8/35*b*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 7/4],[11/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{315de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{35de^2\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(5*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 7/4, 11/4, (c + d*x)^2])/(35*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c +
d*x))^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])/(3
15*d*e^3)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}-\frac{(4b)\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{-1}}\right.}{}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}-\frac{8b(e(c+dx))^{7/2}\sqrt{1}}{}$$

Mathematica [A]
time = 0.33, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{5/2}\left(63\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(\frac{9\sqrt{1-(c+dx)^2}\,\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)\right)\right)}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((9*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 1
1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*H
ypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])))/(315*d*e)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\,\text{arccosh}(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^2*e^(-1)/d + 2/5*(b^2*d^2*x^2*e^(3/2) + 2*b^2*c*d
*x*e^(3/2) + b^2*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^2/d + integrate(2/5*(((5*a*b*d^3 - 2*b^2*d^3)*x^3*e^(3/
2) + 3*(5*a*b*c*d^2 - 2*b^2*c*d^2)*x^2*e^(3/2) - (6*b^2*c^2*d - 5*(3*c^2*d
- d)*a*b)*x*e^(3/2) - (2*b^2*c^3 - 5*(c^3 - c)*a*b)*e^(3/2))*sqrt(d*x + c +
 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((5*a*b*d^4 - 2*b^2*d^4)*x^4*e^(3/2)
+ 4*(5*a*b*c*d^3 - 2*b^2*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2 - d^2)*a*b - 2*
(6*c^2*d^2 - d^2)*b^2)*x^2*e^(3/2) + 2*(5*(2*c^3*d - c*d)*a*b - 2*(2*c^3*d
- c*d)*b^2)*x*e^(3/2) + (5*(c^4 - c^2)*a*b - 2*(c^4 - c^2)*b^2)*e^(3/2))*sq
rt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 +
3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d*x + b^2*c)*arccosh(d*x + c)^2*e + 2*(a*b*d*x + a*b*c)*arcc
osh(d*x + c)*e + (a^2*d*x + a^2*c)*e)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{acosh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{acosh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2, x)
```

3.209 $\int \sqrt{ce+dex}\,\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=153

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de}-\frac{8b\sqrt{1-c-dx}\,(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};\right.}{15de^2\sqrt{-1+c+dx}}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^2/d/e-16/105*b^2*(e*(d*x+c))^(7/
2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],(d*x+c)^2)/d/e^3-8/15*b*(e*(d*x+c))^
(5/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 5/4],[9/4],(d*x+c)^2)*(-d*x-c+1)
^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{105de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{15de^2\sqrt{c+dx-1}}+\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(3*d*e) - (8*b*Sqrt[1 -
c - d*x]*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, (c + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c + d
*x))^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])/(105
*d*e^3)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{(4b)\text{Subst}\left(\int \frac{(ex)^{3/2}(a}{\sqrt{-1+}}\right.}{}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{8b(e(c+dx))^{5/2}\sqrt{1-}}{}$$

Mathematica [A]
time = 0.32, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{3/2}\left(35\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(\frac{7\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)\right)\right)}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((7*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9
/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hy
pergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])))/(105*d*e)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{arccosh}(dx+c)\right)^2\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a^2*e^(-1)/d + 2/3*(b^2*d*x*e^(1/2) + b^2*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/d + in
tegrate(2/3*(((3*a*b*d^2 - 2*b^2*d^2)*x^2*e^(1/2) + 2*(3*a*b*c*d - 2*b^2*c*
d)*x*e^(1/2) - (2*b^2*c^2 - 3*(c^2 - 1)*a*b)*e^(1/2))*sqrt(d*x + c + 1)*sqr
t(d*x + c)*sqrt(d*x + c - 1) + ((3*a*b*d^3 - 2*b^2*d^3)*x^3*e^(1/2) + 3*(3*
a*b*c*d^2 - 2*b^2*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d - d)*a*b - 2*(3*c^2*d -
d)*b^2)*x*e^(1/2) + (3*(c^3 - c)*a*b - 2*(c^3 - c)*b^2)*e^(1/2))*sqrt(d*x +
 c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*
x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (3*c^2*d - d)*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2, x)
```

3.210 $\int \frac{(a+b\cosh^{-1}(c+dx))^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}-\frac{8b\sqrt{1-c-dx}\,(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c\right.}{3de^2\sqrt{-1+c+dx}}$$

```
[Out] -16/15*b^2*(e*(d*x+c))^(5/2)*hypergeom([1, 5/4, 5/4],[7/4, 9/4],(d*x+c)^2)/
d/e^3-8/3*b*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 3/4],[7/
4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)+2*(a+b*arccosh(d*x+c))
^2*(e*(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^2\sqrt{c+dx-1}}+\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/(d*e) - (8*b*Sqrt[1 - c -
d*x]*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/
4, 7/4, (c + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]) - (16*b^2*(e*(c + d*x))^
(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])/(15*d*e^3)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\text{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\cosh^{-1}(x))}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\sqrt{1-(c+dx)^2}\,(}{3de^2\sqrt{-1+}}$$

Mathematica [A]
time = 0.21, size = 140, normalized size = 0.93

$$\frac{2\sqrt{e(c+dx)}\left(15\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(\frac{5\sqrt{1-(c+dx)^2}\,(a+b\cosh^{-1}(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)\right)\right)}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((5*Sqr
t[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4
, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hype
rgeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])))/(15*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(dx+c)\right)^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^2*e^(-1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^2/d + 2*sqrt(d*x*e + c*e)*a^2*e^(-1)/d + integrate(2*(((a*b*d^2 - 2*b^
2*d^2)*x^2*e^(1/2) + 2*(a*b*c*d - 2*b^2*c*d)*x*e^(1/2) - (2*b^2*c^2 - (c^2
- 1)*a*b)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((a*
b*d^3 - 2*b^2*d^3)*x^3*e^(1/2) + 3*(a*b*c*d^2 - 2*b^2*c*d^2)*x^2*e^(1/2) +
((3*c^2*d - d)*a*b - 2*(3*c^2*d - d)*b^2)*x*e^(1/2) + ((c^3 - c)*a*b - 2*(c
^3 - c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)/(d^4*x^4*e + 4*c*d^3*x^3*e + (6*c^2*d^2 - d^2)*x^2*e + 2*(2*c^
3*d - c*d)*x*e + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d - d)*x*e + (c^3 - c)
*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^4 - c^2)*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*e^(-1/2)/s
qrt(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(c + dx))^2}{\sqrt{e(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

3.211 $\int \frac{(a+b\cosh^{-1}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}}+\frac{8b\sqrt{1-c-dx}\ \sqrt{e(c+dx)}\ \left(a+b\cosh^{-1}(c+dx)\right)\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)}{de^2\sqrt{-1+c+dx}}+\frac{1}{}$$

```
[Out] 16/3*b^2*(e*(d*x+c))^(3/2)*hypergeom([3/4, 3/4, 1],[5/4, 7/4],(d*x+c)^2)/d/
e^3-2*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arccosh(d*x+c))
*hypergeom([1/4, 1/2],[5/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d
/e^2/(d*x+c-1)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{-c-dx+1}\ \sqrt{e(c+dx)}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[1 - c -
 d*x]*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2
, 5/4, (c + d*x)^2])/(d*e^2*Sqrt[-1 + c + d*x]) + (16*b^2*(e*(c + d*x))^(3/
2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])/(3*d*e^3)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
```

```
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}}+\frac{(4b)\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c\right.}{de}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}}+\frac{8b\sqrt{e(c+dx)}\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}\right.}{de^2\sqrt{-1+c+dx}\sqrt{1}}$$

Mathematica [A]
time = 0.22, size = 140, normalized size = 0.94

$$\frac{2\left(-3\left(a+b\cosh^{-1}(c+dx)\right)^2+4b(c+dx)\left(\frac{3\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)\right)\right)}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*((3*Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*HypergeometricPFQ[{3/4
, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*x)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}(dx+c)\right)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^2/(d^2*x*e^2 + c*d*e^2) - 2*a^2*e^(-1)/(sqrt(d*x*e + c*e)*d) + integra
te(2*(((a*b*d^2 + 2*b^2*d^2)*x^2*e^(1/2) + 2*(a*b*c*d + 2*b^2*c*d)*x*e^(1/2
) + (2*b^2*c^2 + (c^2 - 1)*a*b)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sq
rt(d*x + c - 1) + ((a*b*d^3 + 2*b^2*d^3)*x^3*e^(1/2) + 3*(a*b*c*d^2 + 2*b^2
*c*d^2)*x^2*e^(1/2) + ((3*c^2*d - d)*a*b + 2*(3*c^2*d - d)*b^2)*x*e^(1/2) +
 ((c^3 - c)*a*b + 2*(c^3 - c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d
*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2
*d^3 - d^3)*x^3*e^2 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^2 + (5*c^4*d - 3*c^2*d)*
x*e^2 + (d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*x^2*e^2 + 2*(2*c
^3*d - c*d)*x*e^2 + (c^4 - c^2)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
(c^5 - c^3)*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-3/2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^2}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2}{\left(c\,e+d\,e\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}-\frac{8b\sqrt{1-c-dx}\,\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)}{3de^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}}-\frac{16b^2\sqrt{e(c+}{}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arccosh(d*x+c)
)*hypergeom([-1/4, 1/2],[3/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(
1/2)/(e*(d*x+c))^(1/2)-16/3*b^2*hypergeom([1/4, 1/4, 1],[3/4, 5/4],(d*x+c)^
2)*(e*(d*x+c))^(1/2)/d/e^3
```

Rubi [A]
time = 0.22, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$-\frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{3de^3}-\frac{8b\sqrt{-c-dx+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^2\sqrt{c+dx-1}\,\sqrt{e(c+dx)}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*Sqrt[1 -
 c - d*x]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d
*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]) - (16*b^2*Sqrt[e*(c
+ d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])/(3*d*e^3
)
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
```

```
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^2}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx, x, c\right.}{3de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} - \frac{8b\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F}{3de^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{}}$$

Mathematica [A]
time = 0.34, size = 140, normalized size = 0.92

$$\frac{2\left(-\left(a+b\cosh^{-1}(c+dx)\right)^2+4b(c+dx)\left(-\frac{\sqrt{1-(c+dx)^2}\,\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}-2b(c+dx)\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)\right)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]*(
a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x])) - 2*b*(c + d*x)*HypergeometricPFQ[{1/4
, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccosh}(dx+c)\right)^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2/3*a^2*e^(-1)/((d*x*e
 + c*e)^(3/2)*d) + integrate(2/3*(((3*a*b*d^2 + 2*b^2*d^2)*x^2*e^(1/2) + 2*
(3*a*b*c*d + 2*b^2*c*d)*x*e^(1/2) + (2*b^2*c^2 + 3*(c^2 - 1)*a*b)*e^(1/2))*
sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((3*a*b*d^3 + 2*b^2*d^3
)*x^3*e^(1/2) + 3*(3*a*b*c*d^2 + 2*b^2*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d - d
)*a*b + 2*(3*c^2*d - d)*b^2)*x*e^(1/2) + (3*(c^3 - c)*a*b + 2*(c^3 - c)*b^2
)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)/(d^6*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 - d^4)*x^4*e^3 + 4*(5*c^3*d^
3 - c*d^3)*x^3*e^3 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d - 2*c^3
*d)*x*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*x^3*e^3 + (
10*c^3*d^2 - 3*c*d^2)*x^2*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 + (c^5 - c^3)*e^3
)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^6 - c^4)*e^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-5/2)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(c + dx))^2}{(e(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

3.213 $\int \frac{(a+b\cosh^{-1}(c+dx))^2}{(ce+dex)^{7/2}} dx$

Optimal. Leaf size=153

$$-\frac{2(a+b\cosh^{-1}(c+dx))^2}{5de(e(c+dx))^{5/2}}-\frac{8b\sqrt{1-c-dx}\,(a+b\cosh^{-1}(c+dx))\ {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)}{15de^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}}+\frac{16b^2\ {}_3F_2\left(-\frac{1}{4}\right.}{15de}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arccosh(d*x+c
))*hypergeom([-3/4, 1/2],[1/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(e*(d*x+c)
)^(3/2)/(d*x+c-1)^(1/2)+16/15*b^2*hypergeom([-1/4, -1/4, 1],[1/4, 3/4],(d*x
+c)^2)/d/e^3/(e*(d*x+c))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5996, 5883, 5949}

$$\frac{16b^2\ {}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\sqrt{-c-dx+1}\ {}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)(a+b\cosh^{-1}(c+dx))}{15de^2\sqrt{c+dx-1}\,(e(c+dx))^{3/2}}-\frac{2(a+b\cosh^{-1}(c+dx))^2}{5de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*Sqrt[1 -
 c - d*x]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d
*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)) + (16*b^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])/(15*d*e^3*Sqrt[e*(c +
 d*x)])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
```

```
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^2}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx, x, c\right.}{5de}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}} - \frac{8b\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F}{15de^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{}}$$

Mathematica [A]
time = 0.26, size = 140, normalized size = 0.92

$$\frac{2\left(-3\left(a+b\cosh^{-1}(c+dx)\right)^2+4b(c+dx)\left(-\frac{\sqrt{1-(c+dx)^2}\,(a+b\cosh^{-1}(c+dx))\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}+2b(c+dx)\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)\right)\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])) + 2*b*(c + d*x)*HypergeometricPFQ[{-
1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])))/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccosh}(dx+c)\right)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 2/
5*a^2*e^(-1)/((d*x*e + c*e)^(5/2)*d) + integrate(2/5*(((5*a*b*d^2 + 2*b^2*d
^2)*x^2*e^(1/2) + 2*(5*a*b*c*d + 2*b^2*c*d)*x*e^(1/2) + (2*b^2*c^2 + 5*(c^2
 - 1)*a*b)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((5
*a*b*d^3 + 2*b^2*d^3)*x^3*e^(1/2) + 3*(5*a*b*c*d^2 + 2*b^2*c*d^2)*x^2*e^(1/
2) + (5*(3*c^2*d - d)*a*b + 2*(3*c^2*d - d)*b^2)*x*e^(1/2) + (5*(c^3 - c)*a
*b + 2*(c^3 - c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c)/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*x
^5*e^4 + 5*(7*c^3*d^4 - c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*x^3*e^4
+ (21*c^5*d^2 - 10*c^3*d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (d^6*x^6*
e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^3)*
x^3*e^4 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^4 +
 (c^6 - c^4)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)*e^4), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-7/2)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}\left(c + dx\right)\right)^{2}}{\left(e\left(c + dx\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)
```

3.214 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3 dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^3 dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de}-\frac{(6b)\text{Subst}\left(\int \frac{(ex)^{5/}}{\sqrt{-}}\right.}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

[Out] $Aborted

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (a + b \operatorname{arccosh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^3*e^(-1)/d + 2/5*(b^3*d^2*x^2*e^(3/2) + 2*b^3*c*d
*x*e^(3/2) + b^3*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^3/d + integrate(3/5*((((5*a*b^2*d^3 - 2*b^3*d^3)*x^3*e^
(3/2) + 3*(5*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2*e^(3/2) - (6*b^3*c^2*d - 5*(3*c
^2*d - d)*a*b^2)*x*e^(3/2) - (2*b^3*c^3 - 5*(c^3 - c)*a*b^2)*e^(3/2))*sqrt(
d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((5*a*b^2*d^4 - 2*b^3*d^4)*x
^4*e^(3/2) + 4*(5*a*b^2*c*d^3 - 2*b^3*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2 -
d^2)*a*b^2 - 2*(6*c^2*d^2 - d^2)*b^3)*x^2*e^(3/2) + 2*(5*(2*c^3*d - c*d)*a*
b^2 - 2*(2*c^3*d - c*d)*b^3)*x*e^(3/2) + (5*(c^4 - c^2)*a*b^2 - 2*(c^4 - c^
2)*b^3)*e^(3/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + c)^2 + 5*((a^2*b*d^3*x^3*e^(3/2) + 3*a^2*b*c*d^2*x^2*e^(3/2) + (3*c^2*
d - d)*a^2*b*x*e^(3/2) + (c^3 - c)*a^2*b*e^(3/2))*sqrt(d*x + c + 1)*sqrt(d*
x + c)*sqrt(d*x + c - 1) + (a^2*b*d^4*x^4*e^(3/2) + 4*a^2*b*c*d^3*x^3*e^(3/
2) + (6*c^2*d^2 - d^2)*a^2*b*x^2*e^(3/2) + 2*(2*c^3*d - c*d)*a^2*b*x*e^(3/2
) + (c^4 - c^2)*a^2*b*e^(3/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*s
qrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x +
c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(((b^3*d*x + b^3*c)*arccosh(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c)*
arccosh(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arccosh(d*x + c)*e + (a^3*d*
x + a^3*c)*e)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{acosh}(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{acosh}(c+d\,x))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3, x)
```

3.215 $\int \sqrt{ce+dex} \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=87

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+c
))^(3/2)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{ce+dex} \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(d*e)
```

Rubi steps

$$\int \sqrt{ce+dex} \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \sqrt{ex} \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{3/2}(a}{\sqrt{-1+}}\right.}{}$$

Mathematica [A]
time = 161.12, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex} \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3, x]

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(dx + c))^3 \sqrt{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)

[Out] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")

[Out] 2/3*(d*x*e + c*e)^(3/2)*a^3*e^(-1)/d + 2/3*(b^3*d*x*e^(1/2) + b^3*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/d + in
tegrate(((((3*a*b^2*d^2 - 2*b^3*d^2)*x^2*e^(1/2) + 2*(3*a*b^2*c*d - 2*b^3*c
*d)*x*e^(1/2) - (2*b^3*c^2 - 3*(c^2 - 1)*a*b^2)*e^(1/2))*sqrt(d*x + c + 1)*
sqrt(d*x + c)*sqrt(d*x + c - 1) + ((3*a*b^2*d^3 - 2*b^3*d^3)*x^3*e^(1/2) +
3*(3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d - d)*a*b^2 - 2*(3
*c^2*d - d)*b^3)*x*e^(1/2) + (3*(c^3 - c)*a*b^2 - 2*(c^3 - c)*b^3)*e^(1/2))
*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*((
a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b*e^(1/2))*sq
rt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b*d^3*x^3*e^(1/2) +
3*a^2*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b*x*e^(1/2) + (c^3 - c)*a^2*b
*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)
)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")

[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*x + c)*e^(1/2), x)

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,(a+b\operatorname{acosh}{(c+dx)})^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**3, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\;{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3, x)
```

3.216 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^3}{de}-\frac{6b\text{Int}\left(\frac{\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^2}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] 2*(a+b*arccosh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arccos
h(d*x+c))^2*(e*(d*x+c))^(1/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\ &= \frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^3}{de}-\frac{(6b)\text{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\cosh^{-1}(x))^2}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{de}\end{aligned}$$

Mathematica [A]
time = 29.48, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]

[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x], x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{arccosh}(dx + c))^3}{\sqrt{dex + ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)

[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(d*x*e + c*e)*a^3*e^(-1)/d + 2*(b^3*d*x*e^(1/2) + b^3*c*e^(1/2))*e^(-1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(sqrt(d*x + c)*d) + integrate(3*(((a*b^2*d^3 - 2*b^3*d^3)*x^3*e^(1/2) + 3*(a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2*e^(1/2) + ((3*c^2*d - d)*a*b^2 - 2*(3*c^2*d - d)*b^3)*x*e^(1/2) + ((a*b^2*d^2 - 2*b^3*d^2)*x^2*e^(1/2) + 2*(a*b^2*c*d - 2*b^3*c*d)*x*e^(1/2) - (2*b^3*c^2 - (c^2 - 1)*a*b^2)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((c^3 - c)*a*b^2 - 2*(c^3 - c)*b^3)*e^(1/2))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + (a^2*b*d^3*x^3*e^(1/2) + 3*a^2*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b*x*e^(1/2) + (c^3 - c)*a^2*b*e^(1/2) + (a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/((d^2*x^2*e + 2*c*d*x*e + (c^2 - 1)*e)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d - d)*x*e + (c^3 - c)*e)*sqrt(d*x + c)), x)

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*e^(-1/2)/sqrt(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(c+dx))^3}{\sqrt{e(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
[Out] Integral((a + b*acosh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^3}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

3.217 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{6b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] -2*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcco
sh(d*x+c))^2/(d*x+c-1)^(1/2)/(e*(d*x+c))^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,\sqrt{ex}\,\sqrt{1+x}}\,dx,x,c\right.}{de}\end{aligned}$$

Mathematica [A]
time = 25.20, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^{3}}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*e^(-3/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(sqrt(
d*x + c)*d) - 2*a^3*e^(-1)/(sqrt(d*x*e + c*e)*d) + integrate(3*(((c^3 - c)*
a*b^2 + 2*(c^3 - c)*b^3 + (a*b^2*d^3 + 2*b^3*d^3)*x^3 + 3*(a*b^2*c*d^2 + 2*
b^3*c*d^2)*x^2 + (2*b^3*c^2 + (c^2 - 1)*a*b^2 + (a*b^2*d^2 + 2*b^3*d^2)*x^2
 + 2*(a*b^2*c*d + 2*b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c
^2*d - d)*a*b^2 + 2*(3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + c)^2 + (a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d - d)*a^
2*b*x + (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*a^2*b)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c))/((d^3*x^3*e^(3/2) + 3*c*d^2*x^2*e^(3/2) + (3*c^2*d - d)*x*e^
(3/2) + (c^3 - c)*e^(3/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1
) + (d^4*x^4*e^(3/2) + 4*c*d^3*x^3*e^(3/2) + (6*c^2*d^2 - d^2)*x^2*e^(3/2)
+ 2*(2*c^3*d - c*d)*x*e^(3/2) + (c^4 - c^2)*e^(3/2))*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-3/2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^3}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{acosh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

3.218 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{2b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*arc
cosh(d*x+c))^2/(e*(d*x+c))^(3/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx,x,\right.}{de}$$

Mathematica [A]
time = 29.81, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*b^3*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/((d^2
*x*e^3 + c*d*e^3)*sqrt(d*x + c)) - 2/3*a^3*e^(-1)/((d*x*e + c*e)^(3/2)*d) +
 integrate((((3*a*b^2*d^3 + 2*b^3*d^3)*x^3*e^(1/2) + 3*(3*a*b^2*c*d^2 + 2*b
^3*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d - d)*a*b^2 + 2*(3*c^2*d - d)*b^3)*x*e^(
1/2) + ((3*a*b^2*d^2 + 2*b^3*d^2)*x^2*e^(1/2) + 2*(3*a*b^2*c*d + 2*b^3*c*d)
*x*e^(1/2) + (2*b^3*c^2 + 3*(c^2 - 1)*a*b^2)*e^(1/2))*sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + (3*(c^3 - c)*a*b^2 + 2*(c^3 - c)*b^3)*e^(1/2))*log(d*x + s
qrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2*b*d^3*x^3*e^(1/2) + 3*a^
2*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b*x*e^(1/2) + (c^3 - c)*a^2*b*e^(
1/2) + (a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b*e^(
1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c))/((d^4*x^4*e^3 + 4*c*d^3*x^3*e^3 + (6*c^2*d^2 - d^2)*x^2
*e^3 + 2*(2*c^3*d - c*d)*x*e^3 + (c^4 - c^2)*e^3)*sqrt(d*x + c + 1)*sqrt(d*
x + c)*sqrt(d*x + c - 1) + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d
^3)*x^3*e^3 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 +
(c^5 - c^3)*e^3)*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-5/2)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
d*x + c^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{acosh}\left(c + dx\right)\right)^{3}}{\left(e\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{6b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*a
rccosh(d*x+c))^2/(e*(d*x+c))^(5/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c\right.}{5de}\end{aligned}$$

Mathematica [A]
time = 95.18, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(dx+c))^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*b^3*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/((d^3
*x^2*e^4 + 2*c*d^2*x*e^4 + c^2*d*e^4)*sqrt(d*x + c)) - 2/5*a^3*e^(-1)/((d*x
*e + c*e)^(5/2)*d) + integrate(3/5*(((5*a*b^2*d^3 + 2*b^3*d^3)*x^3*e^(1/2)
+ 3*(5*a*b^2*c*d^2 + 2*b^3*c*d^2)*x^2*e^(1/2) + (5*(3*c^2*d - d)*a*b^2 + 2*
(3*c^2*d - d)*b^3)*x*e^(1/2) + ((5*a*b^2*d^2 + 2*b^3*d^2)*x^2*e^(1/2) + 2*(
5*a*b^2*c*d + 2*b^3*c*d)*x*e^(1/2) + (2*b^3*c^2 + 5*(c^2 - 1)*a*b^2)*e^(1/2
))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*(c^3 - c)*a*b^2 + 2*(c^3 - c)*b
^3)*e^(1/2))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 5*(a^2*
b*d^3*x^3*e^(1/2) + 3*a^2*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b*x*e^(1/
2) + (c^3 - c)*a^2*b*e^(1/2) + (a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/
2) + (c^2 - 1)*a^2*b*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x
+ sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/((d^5*x^5*e^4 + 5*c*d^4*x^4*e^4
 + (10*c^2*d^3 - d^3)*x^3*e^4 + (10*c^3*d^2 - 3*c*d^2)*x^2*e^4 + (5*c^4*d -
 3*c^2*d)*x*e^4 + (c^5 - c^3)*e^4)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x
 + c - 1) + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 - d^4)*x^4*e^4 + 4
*(5*c^3*d^3 - c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 - 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5
*d - 2*c^3*d)*x*e^4 + (c^6 - c^4)*e^4)*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-7/2)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*
d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\text{acosh}\,(c + dx))^3}{(e\,(c + dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(7/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\text{acosh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

3.220 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\text{Subst}\left(\int \frac{(ex)^{5/}}{\sqrt{-}}\right.}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

[Out] $Aborted

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(dex + ce\right)^{\frac{3}{2}} \left(a + b\,\mathrm{arccosh}\left(dx + c\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^4*e^(-1)/d + 2/5*(b^4*d^2*x^2*e^(3/2) + 2*b^4*c*d
*x*e^(3/2) + b^4*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^4/d + integrate(2/5*(2*(((5*a*b^3*d^3 - 2*b^4*d^3)*x^3*
e^(3/2) + 3*(5*a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2*e^(3/2) - (6*b^4*c^2*d - 5*(3
*c^2*d - d)*a*b^3)*x*e^(3/2) - (2*b^4*c^3 - 5*(c^3 - c)*a*b^3)*e^(3/2))*sqr
t(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((5*a*b^3*d^4 - 2*b^4*d^4)
*x^4*e^(3/2) + 4*(5*a*b^3*c*d^3 - 2*b^4*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2
- d^2)*a*b^3 - 2*(6*c^2*d^2 - d^2)*b^4)*x^2*e^(3/2) + 2*(5*(2*c^3*d - c*d)*
a*b^3 - 2*(2*c^3*d - c*d)*b^4)*x*e^(3/2) + (5*(c^4 - c^2)*a*b^3 - 2*(c^4 -
c^2)*b^4)*e^(3/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)^3 + 15*((a^2*b^2*d^3*x^3*e^(3/2) + 3*a^2*b^2*c*d^2*x^2*e^(3/2) +
(3*c^2*d - d)*a^2*b^2*x*e^(3/2) + (c^3 - c)*a^2*b^2*e^(3/2))*sqrt(d*x + c +
 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^4*x^4*e^(3/2) + 4*a^2*b^2*
c*d^3*x^3*e^(3/2) + (6*c^2*d^2 - d^2)*a^2*b^2*x^2*e^(3/2) + 2*(2*c^3*d - c*
d)*a^2*b^2*x*e^(3/2) + (c^4 - c^2)*a^2*b^2*e^(3/2))*sqrt(d*x + c))*log(d*x
+ sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 10*((a^3*b*d^3*x^3*e^(3/2) +
 3*a^3*b*c*d^2*x^2*e^(3/2) + (3*c^2*d - d)*a^3*b*x*e^(3/2) + (c^3 - c)*a^3*
b*e^(3/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^4*x
^4*e^(3/2) + 4*a^3*b*c*d^3*x^3*e^(3/2) + (6*c^2*d^2 - d^2)*a^3*b*x^2*e^(3/2
) + 2*(2*c^3*d - c*d)*a^3*b*x*e^(3/2) + (c^4 - c^2)*a^3*b*e^(3/2))*sqrt(d*x
 + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d
^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + (3*c^2*d - d)*x - c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(((b^4*d*x + b^4*c)*arccosh(d*x + c)^4*e + 4*(a*b^3*d*x + a*b^3*c)*
arccosh(d*x + c)^3*e + 6*(a^2*b^2*d*x + a^2*b^2*c)*arccosh(d*x + c)^2*e + 4
*(a^3*b*d*x + a^3*b*c)*arccosh(d*x + c)*e + (a^4*d*x + a^4*c)*e)*sqrt(d*x +
 c)*e^(1/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\,\mathrm{acosh}\,(c+dx))^4 \;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{acosh}(c+d\,x)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^4, x)
```

3.221 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{-1+c+dx}\sqrt{1+c+dx}},x\right)}{3e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*x
+c))^(3/2)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx &= \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^4 dx,x,c+dx\right)}{d}\\ &= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{(8b)\text{Subst}\left(\int \frac{(ex)^{3/2}(a}{\sqrt{-1+}}\right.}{}\end{aligned}$$

Mathematica [A]
time = 136.84, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{arccosh}(dx + c))^4 \sqrt{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*x*e + c*e)^(3/2)*a^4*e^(-1)/d + 2/3*(b^4*d*x*e^(1/2) + b^4*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/d + in
tegrate(2/3*(2*(((3*a*b^3*d^2 - 2*b^4*d^2)*x^2*e^(1/2) + 2*(3*a*b^3*c*d - 2
*b^4*c*d)*x*e^(1/2) - (2*b^4*c^2 - 3*(c^2 - 1)*a*b^3)*e^(1/2))*sqrt(d*x + c
 + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((3*a*b^3*d^3 - 2*b^4*d^3)*x^3*e^(1
/2) + 3*(3*a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d - d)*a*b^3
- 2*(3*c^2*d - d)*b^4)*x*e^(1/2) + (3*(c^3 - c)*a*b^3 - 2*(c^3 - c)*b^4)*e^
(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3
+ 9*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b^2
*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*
x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b^2*x*e^(1/2)
 + (c^3 - c)*a^2*b^2*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)^2 + 6*((a^3*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2)
+ (c^2 - 1)*a^3*b*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1
) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^3*
b*x*e^(1/2) + (c^3 - c)*a^3*b*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*
c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*x + c)*e^(1/2),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^4, x)
```

3.222 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^4}{de}-\frac{8b\text{Int}\left(\frac{\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^3}{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] 2*(a+b*arccosh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arccos
h(d*x+c))^3*(e*(d*x+c))^(1/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\cosh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{(a+b\cosh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx=\frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^4}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$
$$=\frac{2\sqrt{e(c+dx)}\,(a+b\cosh^{-1}(c+dx))^4}{de}-\frac{(8b)\text{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\cosh^{-1}(x))^3}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{de}$$

Mathematica [A]
time = 11.11, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^4}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^4*e^(-1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^4/d + 2*sqrt(d*x*e + c*e)*a^4*e^(-1)/d + integrate(2*(2*(((a*b^3*d^2 -
 2*b^4*d^2)*x^2*e^(1/2) + 2*(a*b^3*c*d - 2*b^4*c*d)*x*e^(1/2) - (2*b^4*c^2
- (c^2 - 1)*a*b^3)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c -
1) + ((a*b^3*d^3 - 2*b^4*d^3)*x^3*e^(1/2) + 3*(a*b^3*c*d^2 - 2*b^4*c*d^2)*x
^2*e^(1/2) + ((3*c^2*d - d)*a*b^3 - 2*(3*c^2*d - d)*b^4)*x*e^(1/2) + ((c^3
- c)*a*b^3 - 2*(c^3 - c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c
*d*x*e^(1/2) + (c^2 - 1)*a^2*b^2*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*s
qrt(d*x + c - 1) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) +
 (3*c^2*d - d)*a^2*b^2*x*e^(1/2) + (c^3 - c)*a^2*b^2*e^(1/2))*sqrt(d*x + c)
)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 2*((a^3*b*d^2*x^2*
e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 - 1)*a^3*b*e^(1/2))*sqrt(d*x + c + 1
)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*
x^2*e^(1/2) + (3*c^2*d - d)*a^3*b*x*e^(1/2) + (c^3 - c)*a^3*b*e^(1/2))*sqrt
(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*x^4*e +
 4*c*d^3*x^3*e + (6*c^2*d^2 - d^2)*x^2*e + 2*(2*c^3*d - c*d)*x*e + (d^3*x^3
*e + 3*c*d^2*x^2*e + (3*c^2*d - d)*x*e + (c^3 - c)*e)*sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + (c^4 - c^2)*e), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*e^(-1/2)/sqrt(d*x + c),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(c+dx))^4}{\sqrt{e(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^4}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

3.223 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{8b\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}},x\right)}{e}$$

```
[Out] -2*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcco
sh(d*x+c))^3/(d*x+c-1)^(1/2)/(e*(d*x+c))^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx=\frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^3}{\sqrt{-1+x}\,\sqrt{ex}\,\sqrt{1+x}}\,dx,x,c$$

Mathematica [A]
time = 30.67, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccosh}(dx + c))^4}{(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
[Out] -2*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^4/(d^2*x*e^2 + c*d*e^2) - 2*a^4*e^(-1)/(sqrt(d*x*e + c*e)*d) + integra
te(2*(2*(((a*b^3*d^2 + 2*b^4*d^2)*x^2*e^(1/2) + 2*(a*b^3*c*d + 2*b^4*c*d)*x
*e^(1/2) + (2*b^4*c^2 + (c^2 - 1)*a*b^3)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*
x + c)*sqrt(d*x + c - 1) + ((a*b^3*d^3 + 2*b^4*d^3)*x^3*e^(1/2) + 3*(a*b^3*
c*d^2 + 2*b^4*c*d^2)*x^2*e^(1/2) + ((3*c^2*d - d)*a*b^3 + 2*(3*c^2*d - d)*b
^4)*x*e^(1/2) + ((c^3 - c)*a*b^3 + 2*(c^3 - c)*b^4)*e^(1/2))*sqrt(d*x + c))
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*((a^2*b^2*d^2*x^2
*e^(1/2) + 2*a^2*b^2*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b^2*e^(1/2))*sqrt(d*x +
c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b
^2*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^2*b^2*x*e^(1/2) + (c^3 - c)*a^2*b^2*
e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^
2 + 2*((a^3*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 - 1)*a^3*b*e^(
1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*x^3*e^
(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^3*b*x*e^(1/2) + (c^3 -
c)*a^3*b*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c))/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 - d^3)*x^3*e^2 + (10
*c^3*d^2 - 3*c*d^2)*x^2*e^2 + (5*c^4*d - 3*c^2*d)*x*e^2 + (d^4*x^4*e^2 + 4*
c*d^3*x^3*e^2 + (6*c^2*d^2 - d^2)*x^2*e^2 + 2*(2*c^3*d - c*d)*x*e^2 + (c^4
- c^2)*e^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^5 - c^3)*e^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-3/2)/
(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^4}{(e(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{acosh}(c + d\,x))^4}{(c\,e + d\,e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

3.224 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{8b\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}},x\right)}{3e}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*a
rccosh(d*x+c))^3/(e*(d*x+c))^(3/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^3}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx,x,\right.}{3de}\end{aligned}$$

Mathematica [A]
time = 30.50, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(dx+c\right)\right)^{4}}{\left(dex+ce\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
[Out] -2/3*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^4/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2/3*a^4*e^(-1)/((d*x*e
 + c*e)^(3/2)*d) + integrate(2/3*(2*(((3*a*b^3*d^2 + 2*b^4*d^2)*x^2*e^(1/2)
 + 2*(3*a*b^3*c*d + 2*b^4*c*d)*x*e^(1/2) + (2*b^4*c^2 + 3*(c^2 - 1)*a*b^3)*
e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((3*a*b^3*d^3
+ 2*b^4*d^3)*x^3*e^(1/2) + 3*(3*a*b^3*c*d^2 + 2*b^4*c*d^2)*x^2*e^(1/2) + (3
*(3*c^2*d - d)*a*b^3 + 2*(3*c^2*d - d)*b^4)*x*e^(1/2) + (3*(c^3 - c)*a*b^3
+ 2*(c^3 - c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3 + 9*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c*d*x*e^(1/2
) + (c^2 - 1)*a^2*b^2*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c
 - 1) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) + (3*c^2*d -
 d)*a^2*b^2*x*e^(1/2) + (c^3 - c)*a^2*b^2*e^(1/2))*sqrt(d*x + c))*log(d*x +
 sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 6*((a^3*b*d^2*x^2*e^(1/2) + 2
*a^3*b*c*d*x*e^(1/2) + (c^2 - 1)*a^3*b*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x
+ c)*sqrt(d*x + c - 1) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2)
 + (3*c^2*d - d)*a^3*b*x*e^(1/2) + (c^3 - c)*a^3*b*e^(1/2))*sqrt(d*x + c))*
log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^6*x^6*e^3 + 6*c*d^5*
x^5*e^3 + (15*c^2*d^4 - d^4)*x^4*e^3 + 4*(5*c^3*d^3 - c*d^3)*x^3*e^3 + 3*(5
*c^4*d^2 - 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d - 2*c^3*d)*x*e^3 + (d^5*x^5*e^3
+ 5*c*d^4*x^4*e^3 + (10*c^2*d^3 - d^3)*x^3*e^3 + (10*c^3*d^2 - 3*c*d^2)*x^2
*e^3 + (5*c^4*d - 3*c^2*d)*x*e^3 + (c^5 - c^3)*e^3)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + (c^6 - c^4)*e^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-5/2)/
(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(c + dx))^4}{(e(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c + d\,x))^4}{(c\,e + d\,e\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

3.225 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{8b\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^3}{\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*a
rccosh(d*x+c))^3/(e*(d*x+c))^(5/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^3}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c\right.}{5de}\end{aligned}$$

Mathematica [A]
time = 121.46, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccosh}(dx + c))^4}{(dex + ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^4/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 2/
5*a^4*e^(-1)/((d*x*e + c*e)^(5/2)*d) + integrate(2/5*(2*(((5*a*b^3*d^2 + 2*
b^4*d^2)*x^2*e^(1/2) + 2*(5*a*b^3*c*d + 2*b^4*c*d)*x*e^(1/2) + (2*b^4*c^2 +
 5*(c^2 - 1)*a*b^3)*e^(1/2))*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c -
 1) + ((5*a*b^3*d^3 + 2*b^4*d^3)*x^3*e^(1/2) + 3*(5*a*b^3*c*d^2 + 2*b^4*c*d
^2)*x^2*e^(1/2) + (5*(3*c^2*d - d)*a*b^3 + 2*(3*c^2*d - d)*b^4)*x*e^(1/2) +
 (5*(c^3 - c)*a*b^3 + 2*(c^3 - c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 15*((a^2*b^2*d^2*x^2*e^(1/2) + 2
*a^2*b^2*c*d*x*e^(1/2) + (c^2 - 1)*a^2*b^2*e^(1/2))*sqrt(d*x + c + 1)*sqrt(
d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2
*e^(1/2) + (3*c^2*d - d)*a^2*b^2*x*e^(1/2) + (c^3 - c)*a^2*b^2*e^(1/2))*sqr
t(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 10*((a^3
*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 - 1)*a^3*b*e^(1/2))*sqrt(
d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*x^3*e^(1/2) + 3*a
^3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d - d)*a^3*b*x*e^(1/2) + (c^3 - c)*a^3*b*e^
(1/2))*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(
d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 - d^5)*x^5*e^4 + 5*(7*c^3*d^4 -
 c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 - 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 - 10*c^3*
d^2)*x^2*e^4 + (7*c^6*d - 5*c^4*d)*x*e^4 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 +
 (15*c^2*d^4 - d^4)*x^4*e^4 + 4*(5*c^3*d^3 - c*d^3)*x^3*e^4 + 3*(5*c^4*d^2
- 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d - 2*c^3*d)*x*e^4 + (c^6 - c^4)*e^4)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - c^5)*e^4), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-7/2)/
(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{acosh}(c+dx))^4}{(e(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(7/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^4}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

3.226 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=94

$$\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(1+m)}-\frac{4b\mathrm{Int}\left(\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{-1+c+dx}\sqrt{1+c+dx}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e/(1+m
)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\mathrm{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(1+m)} - \frac{(4b)\mathrm{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{-1}}\right.}{}$$

Mathematica [A]
time = 3.00, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4, x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int (dex + ce)^m (a + b \operatorname{arccosh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
[Out] (d*x*e + c*e)^(m + 1)*a^4*e^(-1)/(d*(m + 1)) + (b^4*d*x*e^m + b^4*c*e^m)*(d
*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d*(m + 1))
+ integrate(-2*(2*((b^4*c^2*e^m - ((m*e^m + e^m)*c^2 - m*e^m - e^m)*a*b^3 -
 ((m*e^m + e^m)*a*b^3*d^2 - b^4*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b^3*c*d -
 b^4*c*d*e^m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - (((m*e^m
 + e^m)*c^3 - (m*e^m + e^m)*c)*a*b^3 - (c^3*e^m - c*e^m)*b^4 + ((m*e^m + e^
m)*a*b^3*d^3 - b^4*d^3*e^m)*x^3 + 3*((m*e^m + e^m)*a*b^3*c*d^2 - b^4*c*d^2*
e^m)*x^2 + ((3*(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a*b^3 - (3*c^2*d*e^m
- d*e^m)*b^4)*x)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
 + c)^3 - 3*(((m*e^m + e^m)*a^2*b^2*d^2*x^2 + 2*(m*e^m + e^m)*a^2*b^2*c*d*x
 + ((m*e^m + e^m)*c^2 - m*e^m - e^m)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x +
c - 1)*(d*x + c)^m + ((m*e^m + e^m)*a^2*b^2*d^3*x^3 + 3*(m*e^m + e^m)*a^2*b
^2*c*d^2*x^2 + (3*(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a^2*b^2*x + ((m*e^
m + e^m)*c^3 - (m*e^m + e^m)*c)*a^2*b^2)*(d*x + c)^m)*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c)^2 - 2*(((m*e^m + e^m)*a^3*b*d^2*x^2 + 2*(m*e^
m + e^m)*a^3*b*c*d*x + ((m*e^m + e^m)*c^2 - m*e^m - e^m)*a^3*b)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + ((m*e^m + e^m)*a^3*b*d^3*x^3 + 3*(m*
e^m + e^m)*a^3*b*c*d^2*x^2 + (3*(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a^3*
b*x + ((m*e^m + e^m)*c^3 - (m*e^m + e^m)*c)*a^3*b)*(d*x + c)^m)*log(d*x + s
qrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)
*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m -
 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) - d*
(m + 1))*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*((d*x + c)*e)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4, x)
```

3.227 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=94

$$\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(1+m)}-\frac{3b\text{Int}\left(\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{-1+c+dx}\sqrt{1+c+dx}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e/(1+m
)
```

Rubi [A]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(1+m)} - \frac{(3b)\text{Subst}\left(\int \frac{(ex)^{1+m}(}{\sqrt{-1}}\right.}{de}$$

Mathematica [A]
time = 1.39, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3, x]
```

Maple [A]
time = 0.89, size = 0, normalized size = 0.00

$$\int (dex+ce)^{m} (a+b\,\mathrm{arccosh}(dx+c))^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (d*x*e + c*e)^(m + 1)*a^3*e^(-1)/(d*(m + 1)) + (b^3*d*x*e^m + b^3*c*e^m)*(d
*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d*(m + 1))
+ integrate(-3*(((b^3*c^2*e^m - ((m*e^m + e^m)*c^2 - m*e^m - e^m)*a*b^2 - (
(m*e^m + e^m)*a*b^2*d^2 - b^3*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b^2*c*d - b
^3*c*d*e^m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - (((m*e^m +
 e^m)*c^3 - (m*e^m + e^m)*c)*a*b^2 - (c^3*e^m - c*e^m)*b^3 + ((m*e^m + e^m)
*a*b^2*d^3 - b^3*d^3*e^m)*x^3 + 3*((m*e^m + e^m)*a*b^2*c*d^2 - b^3*c*d^2*e^
m)*x^2 + ((3*(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a*b^2 - (3*c^2*d*e^m -
d*e^m)*b^3)*x)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 c)^2 - ((((m*e^m + e^m)*a^2*b*d^2*x^2 + 2*(m*e^m + e^m)*a^2*b*c*d*x + ((m*e
^m + e^m)*c^2 - m*e^m - e^m)*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*
x + c)^m + ((m*e^m + e^m)*a^2*b*d^3*x^3 + 3*(m*e^m + e^m)*a^2*b*c*d^2*x^2 +
 (3*(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a^2*b*x + ((m*e^m + e^m)*c^3 - (
m*e^m + e^m)*c)*a^2*b)*(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + c))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m
 + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1)*sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) - d*(m + 1))*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*((d*x + c)*e)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3, x)
```

3.228 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=206

$$\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)}-\frac{2b\sqrt{1-c-dx}\,(e(c+dx))^{2+m}\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2}\right.}{de^2(1+m)(2+m)\sqrt{-1+c+dx}}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^2/d/e/(1+m)-2*b^2*(e*(d*x+c))^(3+m)*
hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],(d*x+c)^2)/d/e^3/(
3+m)/(m^2+3*m+2)-2*b*(e*(d*x+c))^(2+m)*(a+b*arccosh(d*x+c))*hypergeom([1/2,
 1+1/2*m],[2+1/2*m],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(1+m)/(2+m)/(d*x+c-1)
^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5996, 5883, 5949}

$$-\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b\sqrt{-c-dx+1}\,(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}}+\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*Sqr
t[1 - c - d*x]*(e*(c + d*x))^(2 + m)*(a + b*ArcCosh[c + d*x])*Hypergeometri
c2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-
1 + c + d*x]) - (2*b^2*(e*(c + d*x))^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/
2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, (c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(
3 + m))
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5949

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)/(f
*(m + 1)))*Simp[Sqrt[1 - c^2*x^2]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2], x] +
Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*Simp[Sqrt[1 + c*x]/Sqrt[d1 +
 e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*HypergeometricPFQ[{1, 1 + m/2,
```

```
 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d1, e1, d2
, e2, f, m}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2] &&  !IntegerQ[m]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)}-\frac{(2b)\text{Subst}\left(\int \frac{(ex)^{1+m}(}{\sqrt{-1+}}\right.}{de}$$

$$=\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)}-\frac{2b(e(c+dx))^{2+m}\sqrt{1}}{de}$$

Mathematica [**A**]
time = 0.29, size = 178, normalized size = 0.86

$$\frac{(c+dx)(e(c+dx))^m\left(\left(a+b\cosh^{-1}(c+dx)\right)^2-\frac{2b(c+dx)\left(\frac{\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};(c+dx)^2\right)}{\sqrt{-1+c+dx}\sqrt{1+c+dx}}+\frac{b(c+dx)\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};(c+dx)^2\right)}{3+m}\right)}{2+m}\right)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcCosh[c + d*x])^2 - (2*b*(c + d*x)*((S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, (2 + m
)/2, (4 + m)/2, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (b*(
c + d*x)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 (c + d*x)^2])/(3 + m)))/(2 + m)))/(d*(1 + m))
```

Maple [**F**]
time = 0.94, size = 0, normalized size = 0.00

$$\int (dex+ce)^m \left(a+b\,\text{arccosh}(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (d*x*e + c*e)^(m + 1)*a^2*e^(-1)/(d*(m + 1)) + (b^2*d*x*e^m + b^2*c*e^m)*(d
*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d*(m + 1))
+ integrate(-2*((b^2*c^2*e^m - ((m*e^m + e^m)*c^2 - m*e^m - e^m)*a*b - ((m*
e^m + e^m)*a*b*d^2 - b^2*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b*c*d - b^2*c*d*
e^m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - (((m*e^m + e^m)*a
*b*d^3 - b^2*d^3*e^m)*x^3 + ((m*e^m + e^m)*c^3 - (m*e^m + e^m)*c)*a*b - (c^
3*e^m - c*e^m)*b^2 + 3*((m*e^m + e^m)*a*b*c*d^2 - b^2*c*d^2*e^m)*x^2 + ((3*
(m*e^m + e^m)*c^2*d - (m*e^m + e^m)*d)*a*b - (3*c^2*d*e^m - d*e^m)*b^2)*x)*
(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*(m + 1
)*x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1
)*x + c^2*(m + 1) - m - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1)
+ (3*c^2*d*(m + 1) - d*(m + 1))*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*((d*x + c)
*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{acosh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^m\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2, x)
```

3.229 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=118

$$\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)}-\frac{b(e(c+dx))^{2+m}\left(1-(c+dx)^2\right)\,{}_2F_1\left(1,\frac{3+m}{2};\frac{4+m}{2};(c+dx)^2\right)}{de^2(1+m)(2+m)\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*(1-(d*
x+c)^2)*hypergeom([1, 3/2+1/2*m],[2+1/2*m],(d*x+c)^2)/d/e^2/(1+m)/(2+m)/(d*
x+c-1)^(1/2)/(d*x+c+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5996, 5883, 127, 372, 371}

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+dx)^2\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}\,\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
 d*x])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5883

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[1 +
c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{-1+x}\,\sqrt{\cdots}}\cdots\right.}{de(1\cdots}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{-1+(c+dx)^2}\right)\cdots}{de(1+m)\sqrt{\cdots}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{1-(c+dx)^2}\right)\text{S}\cdots}{de(1+m)\sqrt{-\cdots}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b(e(c+dx))^{2+m}\sqrt{1-\cdots}}{de^2(1+m)(2+\cdots}$$

Mathematica [A]
time = 0.13, size = 106, normalized size = 0.90

$$\frac{(c+dx)(e(c+dx))^m\left(a+b\cosh^{-1}(c+dx)-\frac{b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};(c+dx)^2\right)}{(2+m)\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}\right)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*(a + b*ArcCosh[c + d*x] - (b*(c + d*x)*Sqrt[1 -
(c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/((2
 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(d*(1 + m))
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int \left(dex+ce\right)^{m} \left(a+b\,\mathrm{arccosh}(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*((d*x*e^m + c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)/(d*(m + 1)) - integrate((d^2*x^2*e^m + 2*c*d*x*e^m + c^2*e^m)*(d*x
 + c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1), x) + int
egrate((d*x*e^m + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2
 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) - d*(m +
 1))*x), x)) + (d*x*e + c*e)^(m + 1)*a*e^(-1)/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)*((d*x + c)*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e(c+dx)\right)^{m} \left(a+b\,\mathrm{acosh}\left(c+dx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,\left(a+b\,\mathrm{acosh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)), x)
```

3.230 $\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e(c+dx))^m}{a+b\cosh^{-1}(c+dx)},x\right)$$

```
[Out] Unintegrable((e*(d*x+c))^m/(a+b*arccosh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcCosh[x]), x], x, c + d*x]/d
```

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]), x]
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
[Out] integral(((d*x + c)*e)^m/(b*arccosh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e(c+dx))^m}{a+b\operatorname{acosh}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*acosh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m/(a + b*acosh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,e+d\,e\,x)^m}{a+b\operatorname{acosh}(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)), x)
```

3.231 $\int \frac{\cosh^{-1}(ax^5)}{x}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}(ax^5)}\right)+\frac{1}{10}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^5)}\right)$$

```
[Out] -1/10*arccosh(a*x^5)^2+1/5*arccosh(a*x^5)*ln(1+(a*x^5+(a*x^5-1)^(1/2)*(a*x^
5+1)^(1/2))^2)+1/10*polylog(2,-(a*x^5+(a*x^5-1)^(1/2)*(a*x^5+1)^(1/2))^2)
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6011, 3799, 2221, 2317, 2438}

$$\frac{1}{10}\text{Li}_2\left(-e^{2\cosh^{-1}(ax^5)}\right)-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(e^{2\cosh^{-1}(ax^5)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^5]/x,x]
```

```
[Out] -1/10*ArcCosh[a*x^5]^2 + (ArcCosh[a*x^5]*Log[1 + E^(2*ArcCosh[a*x^5])])/5 +
 PolyLog[2, -E^(2*ArcCosh[a*x^5])]/10
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6011

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax^5)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax^5)\right)\\ &= -\frac{1}{10}\cosh^{-1}(ax^5)^2 + \frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax^5)\right)\\ &= -\frac{1}{10}\cosh^{-1}(ax^5)^2 + \frac{1}{5}\cosh^{-1}(ax^5)\log\left(1+e^{2\cosh^{-1}(ax^5)}\right) - \frac{1}{5}\text{Subst}\left(\int \log\left(1+e^{2x}\right)\right.\\ &= -\frac{1}{10}\cosh^{-1}(ax^5)^2 + \frac{1}{5}\cosh^{-1}(ax^5)\log\left(1+e^{2\cosh^{-1}(ax^5)}\right) - \frac{1}{10}\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= -\frac{1}{10}\cosh^{-1}(ax^5)^2 + \frac{1}{5}\cosh^{-1}(ax^5)\log\left(1+e^{2\cosh^{-1}(ax^5)}\right) + \frac{1}{10}\text{Li}_2\left(-e^{2\cosh^{-1}(ax^5)}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 50, normalized size = 0.93

$$\frac{1}{10}\left(\cosh^{-1}(ax^5)\left(\cosh^{-1}(ax^5)+2\log\left(1+e^{-2\cosh^{-1}(ax^5)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(ax^5)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x^5]/x,x]
```

```
[Out] (ArcCosh[a*x^5]*(ArcCosh[a*x^5] + 2*Log[1 + E^(-2*ArcCosh[a*x^5])]) - PolyL
og[2, -E^(-2*ArcCosh[a*x^5])])/10
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^5)/x,x)
```

```
[Out] int(arccosh(a*x^5)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arccosh(a*x^5)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arccosh(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**5)/x,x)
[Out] Integral(acosh(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arccosh(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x^5)/x,x)
[Out] int(acosh(a*x^5)/x, x)
```

3.232 $\int x^2 \cosh^{-1}\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=117

$$-\frac{5}{48}\sqrt{-1+\sqrt{x}}\ \sqrt{1+\sqrt{x}}\ \sqrt{x}-\frac{5}{72}\sqrt{-1+\sqrt{x}}\ \sqrt{1+\sqrt{x}}\ x^{3/2}-\frac{1}{18}\sqrt{-1+\sqrt{x}}\ \sqrt{1+\sqrt{x}}\ x^{5/2}-\frac{5}{48}\ c$$

```
[Out] -5/48*arccosh(x^(1/2))+1/3*x^3*arccosh(x^(1/2))-5/72*x^(3/2)*(-1+x^(1/2))^(
1/2)*(1+x^(1/2))^(1/2)-1/18*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-5/
48*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6017, 12, 329, 336, 54}

$$-\frac{1}{18}\sqrt{\sqrt{x}-1}\ \sqrt{\sqrt{x}+1}\ x^{5/2}-\frac{5}{72}\sqrt{\sqrt{x}-1}\ \sqrt{\sqrt{x}+1}\ x^{3/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{\sqrt{x}-1}\ \sqrt{\sqrt{x}+1}\ \sqrt{x}-\frac{5}{48}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/48 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/72 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/18 - (5*ArcCosh[Sqrt[x]])/48 + (x^3*ArcCosh[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 6017

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
+ u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int x^2\cosh^{-1}\left(\sqrt{x}\right)dx = \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x^{5/2}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{36}\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1}$$

$$= -\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{18}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2}+\frac{1}{3}x^3\cosh$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{18}\sqrt{-1}$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{18}\sqrt{-1}$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{72}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{1}{18}\sqrt{-1}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.68

$$\frac{1}{144}\left(-\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\left(15+10x+8x^2\right)+48x^3\cosh^{-1}\left(\sqrt{x}\right)-30\tanh^{-1}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*ArcCosh[Sqrt[x]],x]

[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2)) + 48*x^3*ArcCosh[Sqrt[x]] - 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/144

Maple [A]
time = 0.04, size = 75, normalized size = 0.64

method	result
derivativedivides	$\frac{x^3 \mathrm{arccosh}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(8\sqrt{-1+x}\,x^{\frac{5}{2}}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+\right.}{144\sqrt{-1+x}}$
default	$\frac{x^3 \mathrm{arccosh}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(8\sqrt{-1+x}\,x^{\frac{5}{2}}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+\right.}{144\sqrt{-1+x}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arccosh(x^(1/2)),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*arccosh(x^(1/2))-1/144*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*(-1+x)^(1/2)*x^(5/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)

Maxima [A]
time = 0.27, size = 56, normalized size = 0.48

$$\frac{1}{3}\,x^3\,\mathrm{arcosh}\left(\sqrt{x}\right) - \frac{1}{18}\,\sqrt{x-1}\,x^{\frac{5}{2}} - \frac{5}{72}\,\sqrt{x-1}\,x^{\frac{3}{2}} - \frac{5}{48}\,\sqrt{x-1}\,\sqrt{x} - \frac{5}{48}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="maxima")

[Out] 1/3*x^3*arccosh(sqrt(x)) - 1/18*sqrt(x - 1)*x^(5/2) - 5/72*sqrt(x - 1)*x^(3/2) - 5/48*sqrt(x - 1)*sqrt(x) - 5/48*log(2*sqrt(x - 1) + 2*sqrt(x))

Fricas [A]
time = 0.36, size = 40, normalized size = 0.34

$$-\frac{1}{144}\left(8\,x^2+10\,x+15\right)\sqrt{x-1}\,\sqrt{x} + \frac{1}{48}\left(16\,x^3-5\right)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="fricas")

[Out] -1/144*(8*x^2 + 10*x + 15)*sqrt(x - 1)*sqrt(x) + 1/48*(16*x^3 - 5)*log(sqrt(x - 1) + sqrt(x))

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acosh}\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(x**(1/2)),x)
```

```
[Out] Integral(x**2*acosh(sqrt(x)), x)
```

Giac [**A**]
time = 1.59, size = 60, normalized size = 0.51

$$\frac{1}{3}\,x^3 \log\left(\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{144}\,(2\,(4\,x+5)x+15)\sqrt{x-1}\,\sqrt{x}+\frac{5}{48}\,\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/144*(2*(4*x
+ 5)*x + 15)*sqrt(x - 1)*sqrt(x) + 5/48*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{acosh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(x^(1/2)),x)
```

```
[Out] int(x^2*acosh(x^(1/2)), x)
```

3.233 $\int x\cosh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=86

$$-\frac{3}{16}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -3/16*arccosh(x^(1/2))+1/2*x^2*arccosh(x^(1/2))-1/8*x^(3/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)-3/16*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6017, 12, 329, 336, 54}

$$-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/16 - (Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/8 - (3*ArcCosh[Sqrt[x]])/16 + (x^2*ArcCosh[Sqrt[
x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 6017

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
+ u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int x\cosh^{-1}\left(\sqrt{x}\right)\,dx=\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$=\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$=-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+}}$$

$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}$$

$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}$$

$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{3}{16}\cosh^{-1}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.86

$$\frac{1}{16}\left(-\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,(3+2x)+8x^2\cosh^{-1}\left(\sqrt{x}\right)-6\tanh^{-1}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x)) + 8*x^2*ArcCosh[
Sqrt[x]] - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/16
```

Maple [A]
time = 0.01, size = 65, normalized size = 0.76

method	result
derivativedivides	$\frac{x^2 \operatorname{arccosh}(\sqrt{x})}{2} - \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{\right.\right.}{16\sqrt{-1+x}}$
default	$\frac{x^2 \operatorname{arccosh}(\sqrt{x})}{2} - \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{\right.\right.}{16\sqrt{-1+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*arccosh(x^(1/2))-1/16*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3/
2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(
1/2)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.53

$$\frac{1}{2}\,x^2\operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{8}\,\sqrt{x-1}\,x^{\frac{3}{2}} - \frac{3}{16}\,\sqrt{x-1}\,\sqrt{x} - \frac{3}{16}\,\log\left(2\,\sqrt{x-1} + 2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(sqrt(x)) - 1/8*sqrt(x - 1)*x^(3/2) - 3/16*sqrt(x - 1)*sqrt(
x) - 3/16*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.41

$$-\frac{1}{16}\,(2\,x+3)\sqrt{x-1}\,\sqrt{x} + \frac{1}{16}\,\left(8\,x^2-3\right)\log\left(\sqrt{x-1} + \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x + 3)*sqrt(x - 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x - 1) + sq
rt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acosh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(x**(1/2)),x)
```

```
[Out] Integral(x*acosh(sqrt(x)), x)
```

Giac [A]
time = 1.58, size = 55, normalized size = 0.64

$$\frac{1}{2}x^2\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{16}(2x+3)\sqrt{x-1}\sqrt{x}+\frac{3}{16}\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/16*(2*x + 3)
*sqrt(x - 1)*sqrt(x) + 3/16*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{acosh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(x^(1/2)),x)
```

```
[Out] int(x*acosh(x^(1/2)), x)
```

3.234 $\int \cosh^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)+x\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/2*arccosh(x^(1/2))+x*arccosh(x^(1/2))-1/2*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6016, 12, 329, 336, 54}

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]],x]
```

```
[Out] -1/2*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) - ArcCosh[Sqrt[x]]/2 +
x*ArcCosh[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 6016

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \cosh^{-1}\left(\sqrt{x}\right)\,dx = x\cosh^{-1}\left(\sqrt{x}\right) - \int \frac{\sqrt{x}}{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}$$

$$= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} + x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\right.$$

$$= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right) + x\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 273 vs. 2(50) = 100.
time = 3.21, size = 273, normalized size = 5.46

$$-\frac{2\left(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\left(28+70\sqrt{x}+18x-14x^{3/2}-4x^2-4\sqrt{-1+\sqrt{x}}\left(-12-8\sqrt{x}+5x+3x^{3/2}\right)\right)\right)}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\sqrt{x}}\left(7+2\sqrt{x}\right)+80\sqrt{x}\right)+112\sqrt{x}+28x}+x\cosh^{-1}\left(\sqrt{x}\right)+2\tanh^{-1}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]],x]
```

```
[Out] (-2*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(28
 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8*S
qrt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sqrt
```

```
[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[x])
 + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + x*ArcCosh[Sqrt[x]] + 2*ArcTanh[(-1 +
 Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [A]
time = 0.02, size = 49, normalized size = 0.98

method	result	si
derivativedivides	$x \,\mathrm{arccosh}\left(\sqrt{x}\right)-\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(\sqrt{x}\, \sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2 \sqrt{-1+x}}$	49
default	$x \,\mathrm{arccosh}\left(\sqrt{x}\right)-\frac{\sqrt{-1+\sqrt{x}}\, \sqrt{1+\sqrt{x}}\, \left(\sqrt{x}\, \sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2 \sqrt{-1+x}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arccosh(x^(1/2))-1/2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(-1+x)
^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.66

$$x \operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{2} \sqrt{x-1} \sqrt{x} - \frac{1}{2} \log\left(2 \sqrt{x-1} + 2 \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="maxima")
[Out] x*arccosh(sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x) - 1/2*log(2*sqrt(x - 1) + 2*sq
rt(x))
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.56

$$\frac{1}{2} (2x-1) \log\left(\sqrt{x-1} + \sqrt{x}\right) - \frac{1}{2} \sqrt{x-1} \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x - 1)*log(sqrt(x - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x)
```

Sympy [A]
time = 0.09, size = 29, normalized size = 0.58

$$-\frac{\sqrt{x} \sqrt{x-1}}{2} + x \operatorname{acosh}\left(\sqrt{x}\right) - \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(x - 1)/2 + x*acosh(sqrt(x)) - acosh(sqrt(x))/2
```

Giac [A]
time = 1.63, size = 47, normalized size = 0.94

$$x\log\left(\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x-1}\,\sqrt{x}+\frac{1}{2}\,\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt
(x) + 1/2*log(-sqrt(x - 1) + sqrt(x))
```

Mupad [B]
time = 1.43, size = 40, normalized size = 0.80

$$-2\,\sqrt{x}\,\mathrm{acosh}\left(\sqrt{x}\right)\,\left(\frac{1}{4\,\sqrt{x}}-\frac{\sqrt{x}}{2}\right)-\frac{\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2)),x)
```

```
[Out] - 2*x^(1/2)*acosh(x^(1/2))*(1/(4*x^(1/2)) - x^(1/2)/2) - (x^(1/2)*(x^(1/2)
- 1)^(1/2)*(x^(1/2) + 1)^(1/2))/2
```

3.235 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x} \, dx$

Optimal. Leaf size=46

$$-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)+\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] -arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6011, 3799, 2221, 2317, 2438}

$$\text{Li}_2\left(-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(e^{2\cosh^{-1}\left(\sqrt{x}\right)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x,x]
```

```
[Out] -ArcCosh[Sqrt[x]]^2 + 2*ArcCosh[Sqrt[x]]*Log[1 + E^(2*ArcCosh[Sqrt[x]])] +
PolyLog[2, -E^(2*ArcCosh[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6011

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right)\\ &= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 4\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right)\\ &= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - 2\text{Subst}\left(\int \log\left(1+e^{2x}\right.\right.\\ &= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.\\ &= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.00

$$\cosh^{-1}\left(\sqrt{x}\right)\left(\cosh^{-1}\left(\sqrt{x}\right)+2\log\left(1+e^{-2\cosh^{-1}\left(\sqrt{x}\right)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x,x]
```

```
[Out] ArcCosh[Sqrt[x]]*(ArcCosh[Sqrt[x]] + 2*Log[1 + E^(-2*ArcCosh[Sqrt[x]])]) -
PolyLog[2, -E^(-2*ArcCosh[Sqrt[x]])]
```

Maple [A]
time = 0.04, size = 65, normalized size = 1.41

method	result
derivativedivides	$-\text{arccosh}\left(\sqrt{x}\right)^2+2\,\text{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\right)^2\right)+$
default	$-\text{arccosh}\left(\sqrt{x}\right)^2+2\,\text{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\right)^2\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(sqrt(x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x,x)
```

```
[Out] Integral(acosh(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2))/x,x)
```

```
[Out] int(acosh(x^(1/2))/x, x)
```

3.236 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arccosh(x^(1/2))/x+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6017, 12, 271}

$$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 6017

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
+ u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^2} dx = -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x} + \int \frac{1}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\, x^{3/2}} dx$$
$$= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x} + \frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\, x^{3/2}} dx$$
$$= \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}} - \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}} - \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^2,x]
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.72

method	result	size
derivativedivides	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x} + \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$	29
default	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x} + \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arccosh(x^(1/2))/x+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.51, size = 19, normalized size = 0.48

$$\frac{\sqrt{x-1}}{\sqrt{x}} - \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="maxima")
[Out] sqrt(x - 1)/sqrt(x) - arccosh(sqrt(x))/x
```

Fricas [A]
time = 0.34, size = 26, normalized size = 0.65

$$\frac{\sqrt{x-1}\,\sqrt{x}\,-\log\left(\sqrt{x-1}\,+\sqrt{x}\,\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="fricas")
[Out] (sqrt(x - 1)*sqrt(x) - log(sqrt(x - 1) + sqrt(x)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acosh}\left(\sqrt{x}\,\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**2,x)
[Out] Integral(acosh(sqrt(x))/x**2, x)
```

Giac [A]
time = 1.55, size = 45, normalized size = 1.12

$$-\frac{\log\left(\sqrt{\sqrt{x}\,+1}\,\sqrt{\sqrt{x}\,-1}\,+\sqrt{x}\,\right)}{x}+\frac{2}{\left(\sqrt{x-1}\,-\sqrt{x}\,\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="giac")
[Out] -log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x + 2/((sqrt(x - 1) - s
qrt(x))^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{acosh}\left(\sqrt{x}\,\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2))/x^2,x)
[Out] int(acosh(x^(1/2))/x^2, x)
```

3.237 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{6x^{3/2}}+\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}$$

```
[Out] -1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+
1/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6017, 12, 278, 271}

$$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{6x^{3/2}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(6*x^(3/2)) + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])/(3*Sqrt[x]) - ArcCosh[Sqrt[x]]/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
```

```
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 6017

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcCosh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1
+ u])), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx\\
&= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx\\
&= \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{6x^{3/2}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{6}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx\\
&= \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{6x^{3/2}}+\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.64

$$
\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,(1+2x)-3\cosh^{-1}\left(\sqrt{x}\right)}{6x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 + 2*x) - 3*ArcCosh[Sqrt[x]
])/(6*x^2)
```

Maple [A]
time = 0.01, size = 35, normalized size = 0.46

method	result	size
derivativedivides	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(2x+1)}{6x^{\frac{3}{2}}}$	35
default	$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^{2}}+\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(2x+1)}{6x^{\frac{3}{2}}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x+1)/
x^(3/2)
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.39

$$\frac{\sqrt{x-1}}{3\sqrt{x}}+\frac{\sqrt{x-1}}{6\,x^{\frac{3}{2}}}-\frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x - 1)/sqrt(x) + 1/6*sqrt(x - 1)/x^(3/2) - 1/2*arccosh(sqrt(x))/x^
2
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.42

$$\frac{(2\,x+1)\sqrt{x-1}\,\sqrt{x}-3\,\log\left(\sqrt{x-1}+\sqrt{x}\right)}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*((2*x + 1)*sqrt(x - 1)*sqrt(x) - 3*log(sqrt(x - 1) + sqrt(x)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**3,x)
```

[Out] Integral(acosh(sqrt(x))/x**3, x)

Giac [A]
time = 1.55, size = 62, normalized size = 0.82

$$-\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{2\,x^2}+\frac{2\left(3\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="giac")

[Out] -1/2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x - 1) - sqrt(x))^2 + 1)/((sqrt(x - 1) - sqrt(x))^2 + 1)^3

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(x^(1/2))/x^3,x)

[Out] int(acosh(x^(1/2))/x^3, x)

3.238 $\int \cosh^{-1}\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=24

$$x\text{sech}^{-1}(x)+\sqrt{\frac{1}{1+x}}\,\sqrt{1+x}\,\text{ArcSin}(x)$$

[Out] x*arcsech(x)+arcsin(x)*(1/(1+x))^(1/2)*(1+x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6012, 6412, 222}

$$\sqrt{\frac{1}{x+1}}\,\sqrt{x+1}\,\text{ArcSin}(x)+x\text{sech}^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[x^(-1)],x]

[Out] x*ArcSech[x] + Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcSin[x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6012

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6412

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{-1}\left(\frac{1}{x}\right)\,dx &= \int \text{sech}^{-1}(x)\,dx\\ &= x\text{sech}^{-1}(x)+\left(\sqrt{\frac{1}{1+x}}\,\sqrt{1+x}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x\text{sech}^{-1}(x)+\sqrt{\frac{1}{1+x}}\,\sqrt{1+x}\,\sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(24) = 48.
time = 0.13, size = 59, normalized size = 2.46

$$x\cosh^{-1}\left(\frac{1}{x}\right)-\frac{2\sqrt{1-x^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{-1+\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x^(-1)],x]
```

```
[Out] x*ArcCosh[x^(-1)] - (2*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/(Sqrt
[-1 + x^(-1)]*Sqrt[1 + x^(-1)]*x)
```

Maple [A]
time = 0.04, size = 38, normalized size = 1.58

method	result	size
derivativedivides	$x\,\mathrm{arccosh}\left(\frac{1}{x}\right)+\frac{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}\right)}{\sqrt{\frac{1}{x^2}-1}}$	38
default	$x\,\mathrm{arccosh}\left(\frac{1}{x}\right)+\frac{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}\right)}{\sqrt{\frac{1}{x^2}-1}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(1/x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccosh(1/x)+(1/x-1)^(1/2)*(1/x+1)^(1/2)/(1/x^2-1)^(1/2)*arctan(1/(1/x^2-
1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.
time = 0.51, size = 17, normalized size = 0.71

$$x\,\mathrm{arcosh}\left(\frac{1}{x}\right)-\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="maxima")
[Out] x*arccosh(1/x) - arctan(sqrt(1/x^2 - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(7) = 14$.
time = 0.38, size = 72, normalized size = 3.00

$$(x-2)\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}+1}{x}\right)-2\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)-2\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="fricas")
[Out] (x - 2)*log((x*sqrt(-(x^2 - 1)/x^2) + 1)/x) - 2*arctan((x*sqrt(-(x^2 - 1)/x
^2) - 1)/x) - 2*log((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(\frac{1}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(1/x),x)
[Out] Integral(acosh(1/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.
time = 0.40, size = 22, normalized size = 0.92

$$x\log\left(\sqrt{\frac{1}{x^2}-1}+\frac{1}{x}\right)+\frac{\arcsin(x)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="giac")
[Out] x*log(sqrt(1/x^2 - 1) + 1/x) + arcsin(x)/sgn(x)
```

Mupad [B]
time = 0.37, size = 23, normalized size = 0.96

$$\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}}\right)+x\operatorname{acosh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x),x)
[Out] atan(1/((1/x - 1)^(1/2)*(1/x + 1)^(1/2))) + x*acosh(1/x)
```

3.239 $\int \frac{\cosh^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=60

$$-\frac{\cosh^{-1}(ax^n)^2}{2n}+\frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n}+\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^n)}\right)}{2n}$$

```
[Out] -1/2*arccosh(a*x^n)^2/n+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1
)^(1/2))^2)/n+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2)/n
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6011, 3799, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}(ax^n)}\right)}{2n}-\frac{\cosh^{-1}(ax^n)^2}{2n}+\frac{\cosh^{-1}(ax^n)\log\left(e^{2\cosh^{-1}(ax^n)}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^n]/x,x]
```

```
[Out] -1/2*ArcCosh[a*x^n]^2/n + (ArcCosh[a*x^n]*Log[1 + E^(2*ArcCosh[a*x^n])])/n
+ PolyLog[2, -E^(2*ArcCosh[a*x^n])]/(2*n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6011

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax^n)}{x}\,dx &= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax^n)\right)}{n} \\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax^n)\right)}{n} \\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x,\right.}{n} \\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cos}\right.}{2n} \\
&= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(ax^n)}\right)}{2n}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(60) = 120.
time = 0.38, size = 179, normalized size = 2.98

$$\cosh^{-1}(ax^n)\log(x) + \frac{a\sqrt{1-a^2x^{2n}}\left(\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)^2 + 2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)\log\left(1-e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right) - 2n\log(x)\log\left(\sqrt{-a^2}\,x^n+\sqrt{1-a^2x^{2n}}\right) - \text{PolyLog}\left(2, e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right)\right)}{2\sqrt{-a^2}\,n\sqrt{-1+ax^n}\,\sqrt{1+ax^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x^n]/x,x]
```

```
[Out] ArcCosh[a*x^n]*Log[x] + (a*Sqrt[1 - a^2*x^(2*n)]*(ArcSinh[Sqrt[-a^2]*x^n]^2
 + 2*ArcSinh[Sqrt[-a^2]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] - 2*n*
Log[x]*Log[Sqrt[-a^2]*x^n + Sqrt[1 - a^2*x^(2*n)]] - PolyLog[2, E^(-2*ArcSi
nh[Sqrt[-a^2]*x^n])]))/(2*Sqrt[-a^2]*n*Sqrt[-1 + a*x^n]*Sqrt[1 + a*x^n])
```

Maple [A]
time = 0.05, size = 86, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccosh}(a\,x^{n})^{2}}{2}+\operatorname{arccosh}(a\,x^{n})\ln\left(1+\left(a\,x^{n}+\sqrt{a\,x^{n}-1}\,\sqrt{a\,x^{n}+1}\right)^{2}\right)+\frac{\operatorname{polylog}\left(2,-\left(a\,x^{n}+\sqrt{a\,x^{n}-1}\,\sqrt{\right.\right.}{2}}{n}$
default	$\frac{-\frac{\operatorname{arccosh}(a\,x^{n})^{2}}{2}+\operatorname{arccosh}(a\,x^{n})\ln\left(1+\left(a\,x^{n}+\sqrt{a\,x^{n}-1}\,\sqrt{a\,x^{n}+1}\right)^{2}\right)+\frac{\operatorname{polylog}\left(2,-\left(a\,x^{n}+\sqrt{a\,x^{n}-1}\,\sqrt{\right.\right.}{2}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/2*arccosh(a*x^n)^2+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^
n+1)^(1/2))^2)+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) - a*x*x^n + (a^2*x*x^(2*n) - x)*sqr
t(a*x^n + 1)*sqrt(a*x^n - 1)), x) - 1/2*n*log(x)^2 + n*integrate(1/2*log(x)
/(a*x*x^n + x), x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(a*x^n +
 sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax^{n})}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**n)/x,x)
```

```
[Out] Integral(acosh(a*x**n)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x^n)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x^n)/x,x)
```

```
[Out] int(acosh(a*x^n)/x, x)
```

3.240 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=145

$$384b^4x-\frac{192b^3(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+48b^2x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2-\frac{8b(2x^2+dx^4)\left(a+b\,\mathrm{c}\right.}{x\sqrt{dx^2}\,\sqrt{2}}$$

```
[Out] 384*b^4*x+48*b^2*x*(a+b*arccosh(d*x^2+1))^2+x*(a+b*arccosh(d*x^2+1))^4-192*
b^3*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-8*
b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^3/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {6001, 8}

$$-\frac{192b^3(dx^4+2x^2)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\,\sqrt{dx^2+2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^4-\frac{8b(dx^4+2x^2)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3}{x\sqrt{dx^2}\,\sqrt{dx^2+2}}+384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*
x^2]*Sqrt[2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[1 + d*x^2])^2 - (8*b*(2*x^2
 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x
*(a + b*ArcCosh[1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx = -\frac{8b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}{x\sqrt{dx^2}\,\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4$$

$$= -\frac{192b^3(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(1+\right.\right.$$

$$= 384b^4x - \frac{192b^3(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}} + 48b^2x\left(a+b\cosh\right.$$

Mathematica [A]
time = 0.15, size = 264, normalized size = 1.82

$$\frac{(a^4+48a^2b^2+384b^4)dx^2 - 8ab(a^2+24b^2)\sqrt{dx^2}\sqrt{2+dx^2} + 4b\left(a^3dx^2+24ab^2dx^2-6a^2b\sqrt{dx^2}\sqrt{2+dx^2}-48b^3\sqrt{dx^2}\sqrt{2+dx^2}\right)\cosh^{-1}(1+dx^2)+6b^2\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\sqrt{2+dx^2}\right)\cosh^{-1}(1+dx^2)^2+4b^3\left(adx^2-2b\sqrt{dx^2}\sqrt{2+dx^2}\right)\cosh^{-1}(1+dx^2)^3+b^4dx^2\cosh^{-1}(1+dx^2)^4}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[2
+ d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2] + 6*b^2*(
a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^
2]^3 + b^4*d*x^2*ArcCosh[1 + d*x^2]^4)/(d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^4 + 6*a^2*b^2*x*arccosh(d*
x^2 + 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log(d*x^2 + sq
rt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 4*(x*arccosh(d*x^2
+ 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a^3*b + a^4*x + int
egrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 + (3*a*b^3*d - 4*b^4*d)*x^2
 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 + 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt(d*x^2
 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3/(d^2*x^4 + 3*d*x^2 + (d
^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(137) = 274$.
time = 0.35, size = 298, normalized size = 2.06

$$\frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^4+(a^4+48\,a^2b^2+384\,b^4)dx^2+4\left(ab^3dx^2-2\,\sqrt{d^2x^4+2\,dx^2}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^3-6\left(4\,\sqrt{d^2x^4+2\,dx^2}\,ab^3-(a^2b^2+8\,b^4)dx^2\right)\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^2+4\left((a^3b+24\,ab^3)dx^2-6\,\sqrt{d^2x^4+2\,dx^2}\,(a^2b^2+8\,b^4)\right)\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)-8\,\sqrt{d^2x^4+2\,dx^2}\,(a^3b+24\,ab^3)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 - 6*(4*sqrt(d^2*x^4 + 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 + 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 8*sqrt(d^2*x^4 + 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
```

```
by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^4,x)
[Out] int((a + b*acosh(d*x^2 + 1))^4, x)
```

3.241 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3 \, dx$

Optimal. Leaf size=125

$$24ab^2x-\frac{48b^3\sqrt{\frac{dx^2}{2+dx^2}}\,(2+dx^2)}{dx}+24b^3x\cosh^{-1}\left(1+dx^2\right)-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+\right.$$

```
[Out] 24*a*b^2*x+24*b^3*x*arccosh(d*x^2+1)+x*(a+b*arccosh(d*x^2+1))^3-48*b^3*(d*x
^2+2)*(d*x^2/(d*x^2+2))^(1/2)/d/x-6*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^
2/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6001, 6016, 12, 1986, 15, 267}

$$24ab^2x+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3-\frac{6b(dx^4+2x^2)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{dx^2+2}}-\frac{48b^3\sqrt{\frac{dx^2}{dx^2+2}}\,(dx^2+2)}{dx}+24b^3x\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + 24*b^3*
x*ArcCosh[1 + d*x^2] - (6*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^2)/(
x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 6016

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3\,dx=-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3$$

$$=24ab^2x-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+\right.\right.$$

$$=24ab^2x+24b^3x\cosh^{-1}\left(1+dx^2\right)-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right.\right.}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}$$

$$=24ab^2x+24b^3x\cosh^{-1}\left(1+dx^2\right)-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right.\right.}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}$$

$$=24ab^2x+24b^3x\cosh^{-1}\left(1+dx^2\right)-\frac{6b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right.\right.}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}$$

$$=24ab^2x-\frac{48b^3\sqrt{\frac{dx^2}{2+dx^2}}\,(2+dx^2)}{dx}+24b^3x\cosh^{-1}\left(1+dx^2\right)-\frac{6b(2x^2}{}$$

Mathematica [A]

time = 0.08, size = 171, normalized size = 1.37

$$\frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\sqrt{dx^2}\,\sqrt{2+dx^2}+3b\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\,\sqrt{2+dx^2}\right)\cosh^{-1}(1+dx^2)+3b^2\left(adx^2-2b\sqrt{dx^2}\,\sqrt{2+dx^2}\right)\cosh^{-1}(1+dx^2)^2+b^3dx^2\cosh^{-1}(1+dx^2)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[2 + d*x^2] + 3
*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1
+ d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + b^3*d*x^2*ArcCosh[1 + d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b \,\mathrm{arccosh}\left(d \,x^{2}+1\right)\right)^{3}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x*arccosh(d*x^2 + 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 3
*(x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a
^2*b + (x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3 - integrate(6*(d^2*x
^4 + 2*d*x^2 + (d^(3/2)*x^3 + sqrt(d)*x)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(
d*x^2 + 2)*sqrt(d)*x + 1)^2/(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x
)*sqrt(d*x^2 + 2) + 2), x))*b^3 + a^3*x
```

Fricas [A]
time = 0.37, size = 210, normalized size = 1.68

$$\frac{b^{3}dx^{2}\log\left(dx^{2}+\sqrt{d^{2}x^{4}+2\,dx^{2}}+1\right)^{3}+(a^{3}+24\,ab^{2})dx^{2}+3\left(ab^{2}dx^{2}-2\sqrt{d^{2}x^{4}+2\,dx^{2}}\,b^{3}\right)\log\left(dx^{2}+\sqrt{d^{2}x^{4}+2\,dx^{2}}+1\right)^{2}+3\left((a^{2}b+8\,b^{3})dx^{2}-4\sqrt{d^{2}x^{4}+2\,dx^{2}}\,ab^{2}\right)\log\left(dx^{2}+\sqrt{d^{2}x^{4}+2\,dx^{2}}+1\right)-6\sqrt{d^{2}x^{4}+2\,dx^{2}}\,(a^{2}b+8\,b^{3})}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 + 2*d*x^2) + 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 + 2*d*x^2
```

```
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 6*sqrt(d^2*x^4 + 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(dx^2 + 1\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^3,x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^3, x)
```

3.242 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=72

$$8b^2x-\frac{4b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2$$

```
[Out] 8*b^2*x+x*(a+b*arccosh(d*x^2+1))^2-4*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))
/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {6001, 8}

$$x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2-\frac{4b(dx^4+2x^2)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\,\sqrt{dx^2+2}}+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*S
qrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2\,dx &= -\frac{4b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2+\\ &= 8b^2x-\frac{4b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 104, normalized size = 1.44

$$(a^2+8b^2)\,x-\frac{4ab\sqrt{dx^2}\,\sqrt{2+dx^2}}{dx}+\frac{2b\left(adx^2-2b\sqrt{dx^2}\,\sqrt{2+dx^2}\right)\cosh^{-1}\left(1+dx^2\right)}{dx}+b^2x\cosh^{-1}\left(1+dx^2\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2])/(d*x) + b^2*x*ArcCo
sh[1 + d*x^2]^2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^2,x)
```

Maxima [A]
time = 0.33, size = 128, normalized size = 1.78

$$b^2x\,\mathrm{arcosh}\left(dx^2+1\right)^2+4\,b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2+2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2+2}\,\sqrt{dx^2}+1\right)}{\sqrt{dx^2+2}\,d^2}\right)+2\left(x\,\mathrm{arcosh}\left(dx^2+1\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2\,\sqrt{d}\right)}{\sqrt{dx^2+2}\,d}\right)ab+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(d*x^2 + 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 2*(x*arc
cosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a*b + a^
2*x
```

Fricas [A]
time = 0.35, size = 131, normalized size = 1.82

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^2+(a^2+8\,b^2)dx^2-4\,\sqrt{d^2x^4+2\,dx^2}\,ab+2\left(abdx^2-2\,\sqrt{d^2x^4+2\,dx^2}\,b^2\right)\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 + 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1))/(d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{acosh}\left(dx^2 + 1\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^2,x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^2, x)
```

3.243 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)\,dx$

Optimal. Leaf size=49

$$ax-\frac{2b\sqrt{\frac{dx^2}{2+dx^2}}\,(2+dx^2)}{dx}+bx\cosh^{-1}\left(1+dx^2\right)$$

```
[Out] a*x+b*x*arccosh(d*x^2+1)-2*b*(d*x^2+2)*(d*x^2/(d*x^2+2))^(1/2)/d/x
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6016, 12, 1986, 15, 267}

$$ax-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\,(dx^2+2)}{dx}+bx\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 + d
*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 6016

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)\,dx &= ax+b\int \cosh^{-1}\left(1+dx^2\right)\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-b\int 2\sqrt{\frac{dx^2}{2+dx^2}}\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-(2b)\int \sqrt{\frac{dx^2}{2+dx^2}}\,dx\\
&= ax+bx\cosh^{-1}\left(1+dx^2\right)-\frac{\left(2b\sqrt{\frac{dx^2}{2+dx^2}}\,\sqrt{2+dx^2}\right)\int \frac{x}{\sqrt{2+dx^2}}\,dx}{x}\\
&= ax-\frac{2b\sqrt{\frac{dx^2}{2+dx^2}}\,(2+dx^2)}{dx}+bx\cosh^{-1}\left(1+dx^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.76

$$
ax-\frac{2bx}{\sqrt{\frac{dx^2}{2+dx^2}}}+bx\cosh^{-1}\left(1+dx^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[1 + d*x^2],x]
[Out] a*x - (2*b*x)/Sqrt[(d*x^2)/(2 + d*x^2)] + b*x*ArcCosh[1 + d*x^2]
```

Maple [A]
time = 0.01, size = 37, normalized size = 0.76

method	result	size

default	$ax+b\left(x\,\mathrm{arccosh}(d\,x^{2}+1)-\frac{2x\sqrt{d\,x^{2}+2}}{\sqrt{d\,x^{2}}}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*arccosh(d*x^2+1)-2/(d*x^2)^(1/2)*x*(d*x^2+2)^(1/2))
```

Maxima [A]
time = 0.31, size = 44, normalized size = 0.90

$$\left(x\,\mathrm{arcosh}\left(dx^{2}+1\right)-\frac{2\left(d^{\frac{3}{2}}x^{2}+2\sqrt{d}\right)}{\sqrt{dx^{2}+2}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*b
+ a*x
```

Fricas [A]
time = 0.33, size = 63, normalized size = 1.29

$$\frac{bdx^{2}\log\left(dx^{2}+\sqrt{d^{2}x^{4}+2\,dx^{2}}+1\right)+adx^{2}-2\sqrt{d^{2}x^{4}+2\,dx^{2}}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 + 2*d*x^2)*b)/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{acosh}\left(dx^{2}+1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2+1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 + 1), x)
```

Giac [A]
time = 0.41, size = 62, normalized size = 1.27

$$\left(x\log\left(dx^2+\sqrt{\left(dx^2+1\right)^2-1}+1\right)+\frac{2\sqrt{2}\,\mathrm{sgn}(x)}{\sqrt{d}}-\frac{2\sqrt{d^2x^2+2\,d}}{d\mathrm{sgn}\left(x\right)}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="giac")
```

```
[Out] (x*log(d*x^2 + sqrt((d*x^2 + 1)^2 - 1) + 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*
sqrt(d^2*x^2 + 2*d)/(d*sgn(x)))*b + a*x
```

Mupad [B]
time = 0.59, size = 32, normalized size = 0.65

$$a\,x+b\,x\,\mathrm{acosh}\left(d\,x^2+1\right)-\frac{2\,b\,\mathrm{sign}(x)\,\sqrt{d\,x^2+2}}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(d*x^2 + 1),x)
```

```
[Out] a*x + b*x*acosh(d*x^2 + 1) - (2*b*sign(x)*(d*x^2 + 2)^(1/2))/d^(1/2)
```

3.244 $\int \frac{1}{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

```
[Out] 1/2*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*cosh(1/2*a/b)/b*2^(1/2)/(d*x^2)^(1/
2)-1/2*x*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)*sinh(1/2*a/b)/b*2^(1/2)/(d*x^2)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {6002}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b
*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 6002

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos
h[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[
d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(1+dx^2)}\,dx=\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Mathematica [A]
time = 0.10, size = 118, normalized size = 1.20

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\right)}{b\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1
+ d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)]))/(b*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,\mathrm{arccosh}\left(d\,x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1)),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\,\mathrm{acosh}\left(dx^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1)),x)
```

```
[Out] Integral(1/(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{acosh}\left(d\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1)),x)
[Out] int(1/(a + b*acosh(d*x^2 + 1)), x)
```

3.245 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{dx^2}\,\sqrt{2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}-\frac{x\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}$$

```
[Out] 1/4*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)/b^2*2^(1/2)/(d*x^2)^(
1/2)-1/4*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*sinh(1/2*a/b)/b^2*2^(1/2)/(d*x
^2)^(1/2)-1/2*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))
```

Rubi [A]
time = 0.02, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6008}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\,\sqrt{dx^2+2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -1/2*(Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*(a + b*ArcCosh[1 + d*x^2])) - (x*
CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(2*Sqrt[2]*b^
2*Sqrt[d*x^2]) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(
2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 6008

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[(-Sqr
t[d*x^2])*(Sqrt[2 + d*x^2]/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2]))), x] + (-Si
mp[x*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(2*Sqrt[
2]*b^2*Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCos
h[1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b, d},
x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}\,dx=-\frac{\sqrt{dx^2}\,\sqrt{2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}-\frac{x\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\ldots$$

Mathematica [A]
time = 0.63, size = 130, normalized size = 0.87

$$-\frac{\frac{2b\sqrt{dx^2}\sqrt{2+dx^2}}{ad+bd\cosh^{-1}(1+dx^2)}+x^2\mathrm{csch}\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\left(\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\right)}{4b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -1/4*((2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(a*d + b*d*ArcCosh[1 + d*x^2]) + x^
2*Csch[ArcCosh[1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)
]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)]))/(b^2*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2)/
(a*b*d^2*x^3 + 2*a*b*d*x + (b^2*d^2*x^3 + 2*b^2*d*x + (b^2*d^(3/2)*x^2 + b^
2*sqrt(d))*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (a
*b*d^(3/2)*x^2 + a*b*sqrt(d))*sqrt(d*x^2 + 2)) + integrate(1/2*(d^3*x^6 + 3
*d^2*x^4 + (d^2*x^4 + d*x^2 + 2)*(d*x^2 + 2) + (2*d^(5/2)*x^5 + 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 + 2) - 4)/(a*b*d^3*x^6 + 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 + 2*a*b*d*x^2 + a*b)*(d*x^2 + 2) + (b^2*d^3*x^6 + 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 + 2*b^2*d*x^2 + b^2)*(d*x^2 + 2) + 2*
(b^2*d^(5/2)*x^5 + 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2))*lo
g(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 2*(a*b*d^(5/2)*x^5 + 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arccosh(d*x^2 + 1)^2 + 2*a*b*arccosh(d*x^2 + 1) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^2 + 1\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**2,x)
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(d\, x^2 + 1\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^2,x)
[Out] int(1/(a + b*acosh(d*x^2 + 1))^2, x)
```

3.246 $\int \frac{1}{\left(a+b\cosh^{-1}(1+dx^2)\right)^3} dx$

Optimal. Leaf size=180

$$-\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(1}{2b}\right.}{8\sqrt{2}\,b^3\sqrt{dx^2}}$$

```
[Out] -1/8*x/b^2/(a+b*arccosh(d*x^2+1))+1/16*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*
cosh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Shi(1/2*(a+b*arccosh(d*x^2+1
))/b)*sinh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d*x^4-2*x^2)/b/x/(a+b*a
rccosh(d*x^2+1))^2/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6010, 6002}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}-\frac{dx^4+2x^2}{4bx\sqrt{dx^2}\sqrt{dx^2+2}\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] -1/4*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*
x^2])^2) - x/(8*b^2*(a + b*ArcCosh[1 + d*x^2])) + (x*Cosh[a/(2*b)]*CoshInte
gral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) - (x*Si
nh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*
Sqrt[d*x^2])
```

Rule 6002

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos
h[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[
d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}\,dx = -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+\right.\right.}$$

$$= -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+\right.\right.}$$

Mathematica [**A**]
time = 0.30, size = 152, normalized size = 0.84

$$\frac{-\frac{2b^2\sqrt{dx^2}\,\sqrt{2+dx^2}}{d\left(a+b\cosh^{-1}(1+dx^2)\right)^2}-\frac{bx^2}{a+b\cosh^{-1}(1+dx^2)}+\frac{\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\right)}{d}}{8b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] ((-2*b^2*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*(a + b*ArcCosh[1 + d*x^2])^2) - (b
*x^2)/(a + b*ArcCosh[1 + d*x^2]) + (Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b
)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhInteg
ral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 + 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 + 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
```

*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 + (3*a*d^3 + 8*b*d^3)*x^
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 + 4*(a*d + b*d)*x)*(d*x^2 + 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 + 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 + 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 + 2) + (b*d^(11/2)*x^10 + 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 + 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 + 3*b*d^3*x^5 + 4*b*d^2*x^3 +
4*b*d*x)*(d*x^2 + 2)^(3/2) + (3*b*d^(9/2)*x^8 + 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 + 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2) + (3*b*d^5*x^9 + 15*
b*d^4*x^7 + 23*b*d^3*x^5 + 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2) - 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 + 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 + (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2) - 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 + 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 + 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 + 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 + b^4*d)*(d*x
^2 + 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
 + 2*b^4*d^(3/2)*x)*(d*x^2 + 2) + 3*(b^4*d^5*x^8 + 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 + 4*b^4*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*
x + 1)^2 + (a^2*b^2*d^4*x^6 + 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 + a^2*b
^2*d)*(d*x^2 + 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 + 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 + 2*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2) + 2*(a*b^3*d^(11/2
)*x^9 + 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 + 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 + 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 + a*b^3*d)*(d*x^2 + 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 + 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 + 2
*a*b^3*d^(3/2)*x)*(d*x^2 + 2) + 3*(a*b^3*d^5*x^8 + 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 + 4*a*b^3*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*s
qrt(d)*x + 1) + 3*(a^2*b^2*d^5*x^8 + 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
 + 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 + 2)) + integrate(1/8*(d^6*x^12 + 8*d^5*x^1
0 + 27*d^4*x^8 + 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*d^3*x^6 + 3*d^2*x^4
 - 8*d*x^2 + 4)*(d*x^2 + 2)^2 + 96*d*x^2 + 2*(2*d^(9/2)*x^9 + 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 - d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 3*(2*d
^5*x^10 + 12*d^4*x^8 + 26*d^3*x^6 + 24*d^2*x^4 + 3*d*x^2 - 10)*(d*x^2 + 2)
 + 2*(2*d^(11/2)*x^11 + 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 + 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 + 30*sqrt(d)*x)*sqrt(d*x^2 + 2) + 48)/(a*b^2*d^6*x^12 + 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 + 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 + 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 + 2)^2 + 4*(a*b^2*d^(9/2)*x^9 + 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 + 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 6*(a*b^2*
d^5*x^10 + 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 + 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 + 2) + (b^3*d^6*x^12 + 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 + 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 + 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
 + 4*b^3*d*x^2 + b^3)*(d*x^2 + 2)^2 + 4*(b^3*d^(9/2)*x^9 + 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 + 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 + 2)^(3/
2) + 6*(b^3*d^5*x^10 + 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 + 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 + 2) + 4*(b^3*d^(11/2)*x^11 + 7*b^3*d^(9/2)*x^9 + 18*b^3*

```
d^(7/2)*x^7 + 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 + 2))*log(
d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 4*(a*b^2*d^(11/2)*x^11 + 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 + 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 + 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x^2 + 1)^3 + 3*a*b^2*arccosh(d*x^2 + 1)^2 + 3*a^2
*b*arccosh(d*x^2 + 1) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^{2} + 1\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^3,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^3, x)
```

3.247 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=147

$$384b^4x+\frac{192b^3(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+48b^2x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2+\frac{8b(2x^2-dx^4)\,(a}{x\sqrt{dx^2}}$$

```
[Out] 384*b^4*x+48*b^2*x*(a+b*arccosh(d*x^2-1))^2+x*(a+b*arccosh(d*x^2-1))^4+192*
b^3*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)+8
*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^3/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6001, 8}

$$\frac{192b^3(2x^2-dx^4)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\,\sqrt{dx^2-2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^4+\frac{8b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3}{x\sqrt{dx^2}\,\sqrt{dx^2-2}}+384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d
*x^2]*Sqrt[-2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[-1 + d*x^2])^2 + (8*b*(2*
x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]
) + x*(a + b*ArcCosh[-1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\,dx = \frac{8b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)$$

$$= \frac{192b^3(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + 48b^2x\left(a+b\cosh^{-1}\left(-1\right.\right.$$

$$= 384b^4x + \frac{192b^3(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + 48b^2x\left(a+b\cos\right.$$

Mathematica [A]
time = 0.14, size = 264, normalized size = 1.80

$$\frac{(a^4+48a^2b^2+384b^4)\,dx^2-8ab(a^2+24b^2)\sqrt{dx^2}\sqrt{-2+dx^2}+4b\left(a^3dx^2+24ab^2dx^2-6a^2b\sqrt{dx^2}\sqrt{-2+dx^2}-48b^3\sqrt{dx^2}\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)+6b^2\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)^2+4b^3\left(adx^2-2b\sqrt{dx^2}\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)^3+b^4dx^2\cosh^{-1}\left(-1+dx^2\right)^4}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2] + 6*b
^2*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^3 + b^4*d*x^2*ArcCosh[-1 + d*x^2]^4)/(d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^4 + 6*a^2*b^2*x*arccosh(d*
x^2 - 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log(d*x^2 + sq
rt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 4*(x*arccosh(d*x^2
- 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a^3*b + a^4*x + int
egrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 - (3*a*b^3*d - 4*b^4*d)*x^2
 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 - 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt(d*x^2
 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3/(d^2*x^4 - 3*d*x^2 + (d
^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(137) = 274$.

time = 0.43, size = 298, normalized size = 2.03

$$\frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^4+(a^4+48\,a^2b^2+384\,b^4)dx^2+4\left(ab^3dx^2-2\sqrt{d^2x^4-2\,dx^2}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^3-6\left(4\sqrt{d^2x^4-2\,dx^2}\,ab^3-(a^2b^2+8\,b^4)dx^2\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+4\left((a^3b+24\,ab^3)dx^2-6\sqrt{d^2x^4-2\,dx^2}\,(a^2b^2+8\,b^4)\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)-8\sqrt{d^2x^4-2\,dx^2}\,(a^3b+24\,ab^3)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 - 6*(4*sqrt(d^2*x^4 - 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 - 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 8*sqrt(d^2*x^4 - 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**4, x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
```

```
 by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^4,x)
[Out] int((a + b*acosh(d*x^2 - 1))^4, x)
```

3.248 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=110

$$24ab^2x-48b^3\sqrt{1-\frac{2}{dx^2}}\,x+24b^3x\cosh^{-1}\left(-1+dx^2\right)+\frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\,\mathrm{c}\right.$$

```
[Out] 24*a*b^2*x+24*b^3*x*arccosh(d*x^2-1)+x*(a+b*arccosh(d*x^2-1))^3-48*b^3*x*(1
-2/d/x^2)^(1/2)+6*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^2/x/(d*x^2)^(1/2)
/(d*x^2-2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6001, 6016, 12, 197}

$$24ab^2x+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3+\frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}-48b^3x\sqrt{1-\frac{2}{dx^2}}+24b^3x\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - 48*b^3*Sqrt[1 - 2/(d*x^2)]*x + 24*b^3*x*ArcCosh[-1 + d*x^2] +
(6*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[-2
+ d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 6016

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] && !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3\,dx = \frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)$$

$$= 24ab^2x + \frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-\right.\right.$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+\right.\right.}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+\right.\right.}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}$$

$$= 24ab^2x - 48b^3\sqrt{1-\frac{2}{dx^2}}\,x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b(2x^2-dx^4)}{x\sqrt{}}$$

Mathematica [A]
time = 0.08, size = 171, normalized size = 1.55

$$\frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\sqrt{dx^2}\,\sqrt{-2+dx^2}+3b\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2}\,\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)+3b^2\left(adx^2-2b\sqrt{dx^2}\,\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)^2+b^3dx^2\cosh^{-1}\left(-1+dx^2\right)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[-2 + d*x^2] +
3*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[
-1 + d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1
 + d*x^2]^2 + b^3*d*x^2*ArcCosh[-1 + d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x*arccosh(d*x^2 - 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 3
*(x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a
^2*b + (x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3 - integrate(6*(d^2*x
^4 - 2*d*x^2 + (d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(
d*x^2 - 2)*sqrt(d)*x - 1)^2/(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x
)*sqrt(d*x^2 - 2) + 2), x))*b^3 + a^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(103) = 206$.
time = 0.34, size = 210, normalized size = 1.91

$$\frac{b^3dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^3+(a^3+24\,ab^2)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4-2\,dx^2}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+3\left((a^2b+8\,b^3)dx^2-4\sqrt{d^2x^4-2\,dx^2}\,ab^2\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)-6\sqrt{d^2x^4-2\,dx^2}\,(a^2b+8\,b^3)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 - 2*d*x^2) - 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 - 2*d*x^2
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 6*sqrt(d^2*x^4 - 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^3,x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^3, x)
```

3.249 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=73

$$8b^2x+\frac{4b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2$$

```
[Out] 8*b^2*x+x*(a+b*arccosh(d*x^2-1))^2+4*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1)
)/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6001, 8}

$$x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+\frac{4b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\,\sqrt{dx^2-2}}+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x + (4*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]*
Sqrt[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx=\frac{4b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right.$$

$$=8b^2x+\frac{4b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1\right.\right.$$

Mathematica [A]
time = 0.04, size = 104, normalized size = 1.42

$$(a^2+8b^2)\,x-\frac{4ab\sqrt{dx^2}\,\sqrt{-2+dx^2}}{dx}+\frac{2b\left(adx^2-2b\sqrt{dx^2}\,\sqrt{-2+dx^2}\right)\cosh^{-1}\left(-1+dx^2\right)}{dx}+b^2x\cosh^{-1}\left(-1+dx^2\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^2,x]
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*x) + (2*b*(a*d*x^
2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2])/(d*x) + b^2*x*Ar
cCosh[-1 + d*x^2]^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^2,x)
[Out] int((a+b*arccosh(d*x^2-1))^2,x)
```

Maxima [A]
time = 0.35, size = 128, normalized size = 1.75

$$b^2x\,\mathrm{arcosh}\left(dx^2-1\right)^2+4\,b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2-2}\,\sqrt{dx^2}-1\right)}{\sqrt{dx^2-2}\,d^2}\right)+2\left(x\,\mathrm{arcosh}\left(dx^2-1\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)}{\sqrt{dx^2-2}\,d}\right)ab+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
[Out] b^2*x*arccosh(d*x^2 - 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 2*(x*arc
cosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a*b + a^
2*x
```

Fricas [A]
time = 0.33, size = 131, normalized size = 1.79

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+(a^2+8\,b^2)dx^2-4\,\sqrt{d^2x^4-2\,dx^2}\,ab+2\left(abdx^2-2\,\sqrt{d^2x^4-2\,dx^2}\,b^2\right)\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 - 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1))/(d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]index
.cc index
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^2,x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^2, x)
```

3.250 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\,dx$

Optimal. Leaf size=33

$$ax-2b\sqrt{1-\frac{2}{dx^2}}\,x+bx\cosh^{-1}\left(-1+dx^2\right)$$

```
[Out] a*x+b*x*arccosh(d*x^2-1)-2*b*x*(1-2/d/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6016, 12, 197}

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[-1 + d*x^2],x]
```

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6016

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(Sqrt[-1 + u]*Sqrt[1 + u])), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\,dx &= ax+b\int \cosh^{-1}\left(-1+dx^2\right)\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-b\int \frac{2}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-(2b)\int \frac{1}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax-2b\sqrt{1-\frac{2}{dx^2}}\,x+bx\cosh^{-1}\left(-1+dx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$ax-2b\sqrt{1-\frac{2}{dx^2}}\,x+bx\cosh^{-1}\left(-1+dx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[-1 + d*x^2],x]
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Maple [A]
time = 0.00, size = 37, normalized size = 1.12

method	result	size
default	$ax+b\left(x\,\mathrm{arccosh}(d\,x^2-1)-\frac{2x\sqrt{d\,x^2-2}}{\sqrt{d\,x^2}}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2-1),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*arccosh(d*x^2-1)-2/(d*x^2)^(1/2)*x*(d*x^2-2)^(1/2))
```

Maxima [A]
time = 0.29, size = 44, normalized size = 1.33

$$\left(x\,\mathrm{arcosh}\left(dx^2-1\right)-\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{dx^2-2}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*b
+ a*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.
time = 0.36, size = 63, normalized size = 1.91

$$\frac{bdx^2 \log \left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right) + adx^2 - 2\,\sqrt{d^2x^4 - 2\,dx^2}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 - 2*d*x^2)*b)/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{acosh}\left(dx^2 - 1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2-1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(31) = 62$.
time = 0.39, size = 67, normalized size = 2.03

$$\left(x \log \left(dx^2 + \sqrt{\left(dx^2 - 1\right)^2 - 1} - 1\right) + \frac{2\,\sqrt{2}\,\sqrt{-d}\,\operatorname{sgn}(x)}{d} - \frac{2\,\sqrt{d^2x^2 - 2\,d}}{d\operatorname{sgn}\left(x\right)}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="giac")
```

```
[Out] (x*log(d*x^2 + sqrt((d*x^2 - 1)^2 - 1) - 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d -
 2*sqrt(d^2*x^2 - 2*d)/(d*sgn(x)))*b + a*x
```

Mupad [B]
time = 1.53, size = 32, normalized size = 0.97

$$a\,x + b\,x\operatorname{acosh}\left(d\,x^2 - 1\right) - \frac{2\,b\operatorname{sign}(x)\,\sqrt{d\,x^2 - 2}}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(d*x^2 - 1),x)
```

```
[Out] a*x + b*x*acosh(d*x^2 - 1) - (2*b*sign(x)*(d*x^2 - 2)^(1/2))/d^(1/2)
```

3.251 $\int \frac{1}{a+b\cosh^{-1}(-1+dx^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

```
[Out] 1/2*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)/b*2^(1/2)/(d*x^2)^(1/
2)-1/2*x*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*sinh(1/2*a/b)/b*2^(1/2)/(d*x^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6003}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((x*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[2
]*b*Sqrt[d*x^2])) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 6003

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x)
*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*
Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[-1 +
d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx=-\frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.88

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\right)}{bdx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((Cosh[ArcCosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2
*b)]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)]))/(b*d*x))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1)),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{acosh}\left(dx^{2}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1)),x)
```

```
[Out] Integral(1/(a + b*acosh(d*x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{acosh}\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1)),x)
[Out] int(1/(a + b*acosh(d*x^2 - 1)), x)
```

3.252 $\int \frac{1}{\left(a+b\cosh^{-1}(-1+dx^2)\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{dx^2}\,\sqrt{-2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}$$

```
[Out] 1/4*x*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*cosh(1/2*a/b)/b^2*2^(1/2)/(d*x^2)^(
1/2)-1/4*x*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)*sinh(1/2*a/b)/b^2*2^(1/2)/(d*x
^2)^(1/2)-1/2*(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))
```

Rubi [A]

time = 0.01, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6009}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\,\sqrt{dx^2-2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] -1/2*(Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*(a + b*ArcCosh[-1 + d*x^2])) + (
x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]
*b^2*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 6009

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[(-Sq
rt[d*x^2])*(Sqrt[-2 + d*x^2]/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2]))), x] + (
Simp[x*Cosh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(2*Sq
rt[2]*b^2*Sqrt[d*x^2])), x] - Simp[x*Sinh[a/(2*b)]*(SinhIntegral[(a + b*Arc
Cosh[-1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b,
d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}\,dx=-\frac{\sqrt{dx^2}\,\sqrt{-2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}$$

Mathematica [A]
time = 0.50, size = 141, normalized size = 0.94

$$\frac{-\frac{b\sqrt{dx^2}\sqrt{-2+dx^2}}{a+b\cosh^{-1}(-1+dx^2)}+\frac{\sinh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\left(\cosh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(-1+dx^2)}{2b}\right)\right)}{\sqrt{1-\frac{2}{dx^2}}}}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] (-((b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(a + b*ArcCosh[-1 + d*x^2])) + (Sinh[Ar
cCosh[-1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2]
)/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/
Sqrt[1 - 2/(d*x^2)])/(2*b^2*d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(d\,x^{2}-1\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2)/
(a*b*d^2*x^3 - 2*a*b*d*x + (b^2*d^2*x^3 - 2*b^2*d*x + (b^2*d^(3/2)*x^2 - b^
2*sqrt(d))*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (a
*b*d^(3/2)*x^2 - a*b*sqrt(d))*sqrt(d*x^2 - 2)) + integrate(1/2*(d^3*x^6 - 3
*d^2*x^4 + (d^2*x^4 - d*x^2 + 2)*(d*x^2 - 2) + (2*d^(5/2)*x^5 - 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 - 2) + 4)/(a*b*d^3*x^6 - 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 - 2*a*b*d*x^2 + a*b)*(d*x^2 - 2) + (b^2*d^3*x^6 - 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 - 2*b^2*d*x^2 + b^2)*(d*x^2 - 2) + 2*
(b^2*d^(5/2)*x^5 - 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2))*lo
g(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 2*(a*b*d^(5/2)*x^5 - 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x^2 - 1)^2 + 2*a*b*arccosh(d*x^2 - 1) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(dx^2 - 1))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{acosh}(d\,x^2 - 1))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^2,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^2, x)
```

3.253 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=181

$$\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\,\sqrt{-2+dx^2}\,\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}-\frac{x\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right.}{8\sqrt{2}\,b^3\sqrt{dx^2}}$$

```
[Out] -1/8*x/b^2/(a+b*arccosh(d*x^2-1))+1/16*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh
(d*x^2-1))/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Chi(1/2*(a+b*arccosh(d*x^2-1
))/b)*sinh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d*x^4+2*x^2)/b/x/(a+b*a
rccosh(d*x^2-1))^2/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6010, 6003}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}+\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\,\sqrt{dx^2-2}\,\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] (2*x^2 - d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^2) - x/(8*b^2*(a + b*ArcCosh[-1 + d*x^2])) - (x*CoshIntegral[(a + b*Ar
cCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*C
osh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^
3*Sqrt[d*x^2])
```

Rule 6003

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x)
*Sinh[a/(2*b)]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*
Sqrt[d*x^2])), x] + Simp[x*Cosh[a/(2*b)]*(SinhIntegral[(a + b*ArcCosh[-1 +
d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx = \frac{2x^2-dx^4}{4bx\sqrt{dx^2}\,\sqrt{-2+dx^2}\,\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(\right.\right.}$$

$$= \frac{2x^2-dx^4}{4bx\sqrt{dx^2}\,\sqrt{-2+dx^2}\,\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2} - \frac{x}{8b^2\left(a+b\cosh^{-1}\left(\right.\right.}$$

Mathematica [A]
time = 0.48, size = 168, normalized size = 0.93

$$-\frac{\frac{2b^2\sqrt{dx^2}\,\sqrt{-2+dx^2}}{d\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}+\frac{bx^2}{a+b\cosh^{-1}\left(-1+dx^2\right)}+\frac{1}{2}\sqrt{1-\frac{2}{dx^2}}\,x^2\operatorname{csch}\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\operatorname{Chi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)\right)}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] -1/8*((2*b^2*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*(a + b*ArcCosh[-1 + d*x^2])^2
) + (b*x^2)/(a + b*ArcCosh[-1 + d*x^2]) + (Sqrt[1 - 2/(d*x^2)]*x^2*Csch[Arc
Cosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a
/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/2
)/(b^3*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 - 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 - 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
```

```
*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*a*d^3 + 8*b*d^3)*x^
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(a*d + b*d)*x)*(d*x^2 - 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 - 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 - 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 - 2) + (b*d^(11/2)*x^10 - 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 - 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 - 3*b*d^3*x^5 + 4*b*d^2*x^3 -
4*b*d*x)*(d*x^2 - 2)^(3/2) + (3*b*d^(9/2)*x^8 - 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 - 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2) + (3*b*d^5*x^9 - 15*
b*d^4*x^7 + 23*b*d^3*x^5 - 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2) + 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 - (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2) + 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 - 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 - 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 - 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 - b^4*d)*(d*x
^2 - 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
- 2*b^4*d^(3/2)*x)*(d*x^2 - 2) + 3*(b^4*d^5*x^8 - 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 - 4*b^4*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*
x - 1)^2 + (a^2*b^2*d^4*x^6 - 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 - a^2*b
^2*d)*(d*x^2 - 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 - 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 - 2*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2) + 2*(a*b^3*d^(11/2
)*x^9 - 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 - 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 - 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 - a*b^3*d)*(d*x^2 - 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 - 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 - 2
*a*b^3*d^(3/2)*x)*(d*x^2 - 2) + 3*(a*b^3*d^5*x^8 - 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 - 4*a*b^3*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*s
qrt(d)*x - 1) + 3*(a^2*b^2*d^5*x^8 - 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
- 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 - 2)) + integrate(1/8*(d^6*x^12 - 8*d^5*x^1
0 + 27*d^4*x^8 - 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*d^3*x^6 + 3*d^2*x^4
 + 8*d*x^2 + 4)*(d*x^2 - 2)^2 - 96*d*x^2 + 2*(2*d^(9/2)*x^9 - 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 + d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 3*(2*d
^5*x^10 - 12*d^4*x^8 + 26*d^3*x^6 - 24*d^2*x^4 + 3*d*x^2 + 10)*(d*x^2 - 2)
+ 2*(2*d^(11/2)*x^11 - 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 - 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 - 30*sqrt(d)*x)*sqrt(d*x^2 - 2) + 48)/(a*b^2*d^6*x^12 - 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 - 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 - 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 - 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 - 2)^2 + 4*(a*b^2*d^(9/2)*x^9 - 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 - 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 6*(a*b^2*
d^5*x^10 - 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 - 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 - 2) + (b^3*d^6*x^12 - 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 - 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 - 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
- 4*b^3*d*x^2 + b^3)*(d*x^2 - 2)^2 + 4*(b^3*d^(9/2)*x^9 - 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 - 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 - 2)^(3/
2) + 6*(b^3*d^5*x^10 - 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 - 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 - 2) + 4*(b^3*d^(11/2)*x^11 - 7*b^3*d^(9/2)*x^9 + 18*b^3*
```

```
d^(7/2)*x^7 - 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 - 2))*log(
d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 4*(a*b^2*d^(11/2)*x^11 - 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 - 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 - 2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*arccosh(d*x^2 - 1)^3 + 3*a*b^2*arccosh(d*x^2 - 1)^2 + 3*a^2
*b*arccosh(d*x^2 - 1) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\,\text{acosh}\,(dx^2 - 1))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**3,x)
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,\text{acosh}\,(d\,x^2 - 1))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^3,x)
[Out] int(1/(a + b*acosh(d*x^2 - 1))^3, x)
```

3.254 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=280

$$-\frac{5b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}-\frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{2}}\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2+1))^(5/2)-15/2*b^(5/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))
^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^
2+1))*2^(1/2)*Pi^(1/2)/d/x+15/2*b^(5/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2
)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))
*2^(1/2)*Pi^(1/2)/d/x-5*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(3/2)/x/(d*x
^2)^(1/2)/(d*x^2+2)^(1/2)+30*b^2*sinh(1/2*arccosh(d*x^2+1))^2*(a+b*arccosh(
d*x^2+1))^(1/2)/d/x
```

Rubi [A]
time = 0.08, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6001, 5999}

$$\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{30b^2\sinh^2\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{dx}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{5/2}-\frac{5b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}}{x\sqrt{dx^2}\,\sqrt{dx^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

```
[Out] (-5*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(5/2) - (15*b^(5/2)*Sqrt[Pi/2]*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (15*b^(5/2)*Sqrt[Pi/2]*Erf[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(
2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (30*b^2*Sqrt[a + b*ArcCosh[1 + d
*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rule 5999

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCosh[1 + d*x^2]]*(Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2/(d*x)), x] + (
Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh[
1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x] -
 Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x]
) /; FreeQ[{a, b, d}, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx=-\frac{5b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)$$

$$=-\frac{5b(2x^2+dx^4)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)$$

Mathematica [A]
time = 2.31, size = 311, normalized size = 1.11

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\left(-15b^{5/2}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+15b^{5/2}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\left(-5ab\cosh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)+(a^2+15b^2)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)+b^2\cosh^{-1}\left(1+dx^2\right)^2\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)-b\cosh^{-1}\left(1+dx^2\right)\left(5b\cosh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)-2a\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\right)\right)\right)}{2\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCo
sh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[
a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-5*a*b*Cosh[A
rcCosh[1 + d*x^2]/2] + (a^2 + 15*b^2)*Sinh[ArcCosh[1 + d*x^2]/2] + b^2*ArcC
osh[1 + d*x^2]^2*Sinh[ArcCosh[1 + d*x^2]/2] - b*ArcCosh[1 + d*x^2]*(5*b*Cos
h[ArcCosh[1 + d*x^2]/2] - 2*a*Sinh[ArcCosh[1 + d*x^2]/2]))))/(2*Sqrt[d*x^2]
*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(dx^2 + 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(5/2),x)
[Out] Integral((a + b*acosh(d*x**2 + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(5/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^(5/2), x)
```

3.255 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx$

Optimal. Leaf size=238

$$-\frac{3b(2x^2+dx^4)\sqrt{a+b\cosh^{-1}(1+dx^2)}}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}+\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(}}{\sqrt{2}\sqrt{b}}\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2+1))^(3/2)+3/2*b^(3/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))^
(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2
+1))*2^(1/2)*Pi^(1/2)/d/x+3/2*b^(3/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*
2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2
^(1/2)*Pi^(1/2)/d/x-3*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(1/2)/x/(d*x^2
)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6001, 6004}

$$\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}-\frac{3b(dx^4+2x^2)\sqrt{a+b\cosh^{-1}(dx^2+1)}}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

```
[Out] (-3*b*(2*x^2 + d*x^4)*Sqrt[a + b*ArcCosh[1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[2
 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (3*b^(3/2)*Sqrt[Pi/2]*Erf[Sqrt
[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 6004

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
```

```
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx=-\frac{3b(2x^2+dx^4)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3$$

$$=-\frac{3b(2x^2+dx^4)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3$$

Mathematica [A]
time = 0.48, size = 254, normalized size = 1.07

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\left(3b^{3/2}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+3b^{3/2}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\left(-3b\cosh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)+a\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)+b\cosh^{-1}\left(1+dx^2\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\right)\right)}{2\sqrt{dx^2}\,\sqrt{\frac{dx^2}{2+dx^2}}\,\sqrt{2+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh
[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-3*b*Cosh[ArcCos
h[1 + d*x^2]/2] + a*Sinh[ArcCosh[1 + d*x^2]/2] + b*ArcCosh[1 + d*x^2]*Sinh[
ArcCosh[1 + d*x^2]/2])))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 +
d*x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(dx^{2} + 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(3/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^(3/2), x)
```

3.256 $\int \sqrt{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)}{dx}+\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{E}}{}$$

```
[Out] -1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-
sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+1/2*
erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1
/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+2*sinh(1/2
*arccosh(d*x^2+1))^2*(a+b*arccosh(d*x^2+1))^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{5999\}$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(dx^2+1)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(dx^2+1)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{2\sinh^2\left(\frac{1}{2}\cosh^{-1}(dx^2+1)\right)\sqrt{a+b\cosh^{-1}(dx^2+1)}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] -((Sqrt[b]*Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])
]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (Sqr
t[b]*Sqrt[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[a +
b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rule 5999

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCosh[1 + d*x^2]]*(Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2/(d*x)), x] + (
Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh[
1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x] -
 Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]]/(d*x)), x]
) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,dx = -\frac{\sqrt{b}\,\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\right.}{dx}$$

Mathematica [A]
time = 0.21, size = 210, normalized size = 1.02

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\left(\sqrt{b}\,\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{b}\,\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\right)}{2\sqrt{dx^2}\,\sqrt{\frac{dx^2}{2+dx^2}}\,\sqrt{2+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[1
 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[b]*Sq
rt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b
)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x
^2]/2]))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{arccosh}\left(d\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}\left(dx^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b \operatorname{acosh}\left(d\,x^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(1/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^(1/2), x)
```

3.257 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(1+dx^2)}}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)}{\sqrt{b}\,dx}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{a+}}{}\right.}{}$$

```
[Out] 1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)+1/2*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6004}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(dx^2+1)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(dx^2+1)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[P
i/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
 Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

Rule 6004

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(1+dx^2)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cos\right.}{\sqrt{b}\,dx}$$

Mathematica [A]
time = 0.22, size = 166, normalized size = 1.01

$$\frac{\sqrt{\frac{\pi}{2}}\,x\left(\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)}{\sqrt{b}\,\sqrt{dx^2}\,\sqrt{\frac{dx^2}{2+dx^2}}\,\sqrt{2+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*x*(Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] - Sinh[a/(2*b)]) + Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[
b]*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\operatorname{acosh}(dx^2 + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\operatorname{acosh}\left(d\,x^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(1/2), x)
```

3.258 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{dx^2}\,\sqrt{2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cos\right.}{b^{3/2}dx}$$

```
[Out] 1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-1/2*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-(d*x^2)^(1/
2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6006}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\,\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\,\sqrt{b}}\right)}{b^{3/2}dx}-\frac{\sqrt{dx^2}\,\sqrt{dx^2+2}}{bdx\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -((Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) + (
Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt[Pi
/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x)
```

Rule 6006

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-S
qrt[d*x^2])*(Sqrt[2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])), x] +
(-Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2
]*(Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + Simp[
Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; FreeQ[{a,
 b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{dx^2}\,\sqrt{2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)}{}$$

Mathematica [A]
time = 0.74, size = 242, normalized size = 1.14

$$-\frac{x\left(4\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)+\sqrt{2\pi}\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)}{2b^{3/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -1/2*(x*(4*Sqrt[b]*Cosh[ArcCosh[1 + d*x^2]/2] + Sqrt[2*Pi]*Sqrt[a + b*ArcCo
sh[1 + d*x^2]]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cos
h[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Erf
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/
(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 +
 d*x^2)]*Sqrt[2 + d*x^2]*Sqrt[a + b*ArcCosh[1 + d*x^2]])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^{2} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(3/2), x)
```

3.259 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-}}}{\sqrt{2}\,\sqrt{}}\right.}{}$$

```
[Out] 1/6*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/6*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/3*(-d*x^4
-2*x^2)/b/x/(a+b*arccosh(d*x^2+1))^(3/2)/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-1/3*
x/b^2/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6010, 6004}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}-\frac{dx^4+2x^2}{3bx\sqrt{dx^2}\sqrt{dx^2+2}\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

```
[Out] -1/3*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*
x^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erfi[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(
2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x) + (Sqrt[Pi/2]*Erf[Sqrt[a
 + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])
*Sinh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x)
```

Rule 6004

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqrt
[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] + Simp[Sqrt[Pi/2]
*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d}, x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
```

```
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx=-\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}}}$$

$$=-\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}}}$$

Mathematica [A]
time = 0.70, size = 273, normalized size = 1.08

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\left(\sqrt{2\pi}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{b}\left(-b\cosh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)-\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx^2\right)\right)\right)\right)}{6b^{5/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*Erf[Sqrt[a + b*Ar
cCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sq
rt[b]*(-(b*Cosh[ArcCosh[1 + d*x^2]/2]) - (a + b*ArcCosh[1 + d*x^2])*Sinh[Ar
cCosh[1 + d*x^2]/2])))/(6*b^(5/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqr
t[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^2])^(3/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(d x^{2} + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(5/2),x)
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(5/2),x)
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(5/2), x)
```

3.260 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}-\frac{\sqrt{dx^2}\,\sqrt{2+dx}}{15b^3dx\sqrt{a+b\cosh^{-1}}}$$

```
[Out] -1/15*x/b^2/(a+b*arccosh(d*x^2+1))^(3/2)+1/30*erfi(1/2*(a+b*arccosh(d*x^2+1
))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*
x^2+1))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x-1/30*erf(1/2*(a+b*arccosh(d*x^2+1))^(1
/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1
))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/5*(-d*x^4-2*x^2)/b/x/(a+b*arccosh(d*x^2+1
))^(5/2)/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-1/15*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b
^3/d/x/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6010, 6006}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}-\frac{\sqrt{dx^2}\,\sqrt{dx^2+2}}{15b^3dx\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}}-\frac{dx^4+2x^2}{5bx\sqrt{dx^2}\,\sqrt{dx^2+2}\,\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -1/5*(2*x^2 + d*x^4)/(b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*
x^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*S
qrt[2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*E
rfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh
[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x) - (Sqrt[Pi/2]*Erf[S
qrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2
*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x)
```

Rule 6006

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-S
qrt[d*x^2])*(Sqrt[2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])), x] +
(-Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2
]*(Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + Simp[
Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*(Erfi
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; FreeQ[{a,
 b, d}, x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx = -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+b\cosh^{-}\right.}$$

$$= -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+b\cosh^{-}\right.}$$

Mathematica [A]
time = 0.87, size = 291, normalized size = 0.97

$$-\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)\left(\sqrt{2\pi}\left(a+b\cosh^{-1}(1+dx^2)\right)^{5/2}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\left(a+b\cosh^{-1}(1+dx^2)\right)^{5/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{b}\left(\left(3b^2+\left(a+b\cosh^{-1}(1+dx^2)\right)^2\right)\cosh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)+b\left(a+b\cosh^{-1}(1+dx^2)\right)\sinh\left(\frac{1}{2}\cosh^{-1}(1+dx^2)\right)\right)\right)}{30b^{7/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}\left(a+b\cosh^{-1}(1+dx^2)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -1/30*(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^
(5/2)*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)
] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(5/2)*Erf[Sqrt[a
 + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])
 + 4*Sqrt[b]*((3*b^2 + (a + b*ArcCosh[1 + d*x^2])^2)*Cosh[ArcCosh[1 + d*x^2
]/2] + b*(a + b*ArcCosh[1 + d*x^2])*Sinh[ArcCosh[1 + d*x^2]/2])))/(b^(7/2)*
Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*
x^2])^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{acosh}(dx^2 + 1))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(7/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(d\,x^2+1))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(7/2),x)
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(7/2), x)
```

3.261 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$\frac{5b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}+\frac{30b^2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{d\ldots}$$

```
[Out] x*(a+b*arccosh(d*x^2-1))^(5/2)-15/2*b^(5/2)*cosh(1/2*arccosh(d*x^2-1))*erfi
(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*
a/b))*2^(1/2)*Pi^(1/2)/d/x-15/2*b^(5/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*
(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))
*2^(1/2)*Pi^(1/2)/d/x+5*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(3/2)/x/(d*
x^2)^(1/2)/(d*x^2-2)^(1/2)+30*b^2*cosh(1/2*arccosh(d*x^2-1))^2*(a+b*arccosh
(d*x^2-1))^(1/2)/d/x
```

Rubi [A]
time = 0.05, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6001, 6000}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{30b^2\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{dx}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{5/2}+\frac{5b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

```
[Out] (5*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(5/2) + (30*b^2*Sqrt[a + b*Ar
cCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (15*b^(5/2)*Sqrt[
Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (15*b^(5/2)*Sqrt[Pi
/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2
]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 6000

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqr
t[a + b*ArcCosh[-1 + d*x^2]]*(Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2/(d*x)), x]
+ (-Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*ArcC
osh[-1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d*x))
, x] - Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*A
rcCosh[-1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d
*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx=\frac{5b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\Big(a+b\cosh^{-1}\big(-1+$$

$$=\frac{5b(2x^2-dx^4)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\Big(a+b\cosh^{-1}\big(-1+$$

Mathematica [A]
time = 1.10, size = 277, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(-15b^{5/2}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)-15b^{5/2}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\left((a^2+15b^2)\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)+b^2\cosh^{-1}\left(-1+dx^2\right)^2\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)-5ab\sinh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)+b\cosh^{-1}\left(-1+dx^2\right)\left(2a\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)-5b\sinh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\right)\right)\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCos
h[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*((a^2 + 15*b
^2)*Cosh[ArcCosh[-1 + d*x^2]/2] + b^2*ArcCosh[-1 + d*x^2]^2*Cosh[ArcCosh[-1
 + d*x^2]/2] - 5*a*b*Sinh[ArcCosh[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*(2
*a*Cosh[ArcCosh[-1 + d*x^2]/2] - 5*b*Sinh[ArcCosh[-1 + d*x^2]/2]))))/(2*d*x
)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(dx^{2} - 1\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(5/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(5/2), x)
```

3.262 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2} dx$

Optimal. Leaf size=239

$$\frac{3b(2x^2-dx^4)\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}+\frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}(-1+$$

```
[Out] x*(a+b*arccosh(d*x^2-1))^(3/2)+3/2*b^(3/2)*cosh(1/2*arccosh(d*x^2-1))*erfi(
1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a
/b))*2^(1/2)*Pi^(1/2)/d/x-3/2*b^(3/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a
+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2
^(1/2)*Pi^(1/2)/d/x+3*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(1/2)/x/(d*x^
2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6001, 6005}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}(dx^2-1)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}(dx^2-1)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}+\frac{3b(2x^2-dx^4)\sqrt{a+b\cosh^{-1}(dx^2-1)}}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

```
[Out] (3*b*(2*x^2 - d*x^4)*Sqrt[a + b*ArcCosh[-1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (3*b^(3/2)*Sqrt[Pi/2]*Cosh
[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 6001

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[2*b*n*(2*c*d*x^2 + d^2*x^4)*((a + b*Ar
cCosh[c + d*x^2])^(n - 1)/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 6005

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
```

```
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx=\frac{3b(2x^2-dx^4)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+d\right.\right.$$

$$=\frac{3b(2x^2-dx^4)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+d\right.\right.$$

Mathematica [A]
time = 0.32, size = 221, normalized size = 0.92

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\left(3b^{3/2}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)-3b^{3/2}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{a+b\cosh^{-1}(-1+dx^2)}\left(a\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)+b\cosh^{-1}(-1+dx^2)\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)-3b\sinh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\right)\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[
-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*(a*Cosh[ArcCosh
[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*Cosh[ArcCosh[-1 + d*x^2]/2] - 3*b*S
inh[ArcCosh[-1 + d*x^2]/2])))/(2*d*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(d x^{2} - 1\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(3/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(3/2), x)
```

3.263 $\int \sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx$

Optimal. Leaf size=206

$$\frac{2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)}{dx}-\frac{\sqrt{b}\,\sqrt{\frac{\pi}{2}}\,\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+}}{}\right.}{dx}$$

```
[Out] -1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/
2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x-1/2*
cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(
1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+2*cosh(1/2
*arccosh(d*x^2-1))^2*(a+b*arccosh(d*x^2-1))^(1/2)/d/x
```

Rubi [A]

time = 0.02, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {6000}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{2\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (
Sqrt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 +
 d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sqrt[
b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 6000

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqr
t[a + b*ArcCosh[-1 + d*x^2]]*(Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2/(d*x)), x]
+ (-Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*ArcC
osh[-1 + d*x^2]]*(Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d*x))
, x] - Simp[Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*A
rcCosh[-1 + d*x^2]]*(Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]]/(d
*x)), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)}{dx} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\,\mathrm{e}}{}$$

Mathematica [A]
time = 0.17, size = 178, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\left(4\sqrt{a+b\cosh^{-1}(-1+dx^2)}\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)+\sqrt{b}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)-\sqrt{b}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\right)}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCos
h[-1 + d*x^2]/2] + Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/
(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) - Sqrt[b]*Sqrt[2*Pi]*Er
f[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[
a/(2*b)])))/(2*d*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{acosh}\left(dx^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\operatorname{acosh}\left(d\,x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(1/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(1/2), x)
```

3.264 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{b}\,dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\right.}{}$$

```
[Out] 1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)-1/2*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6005}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}(dx^2-1)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}(dx^2-1)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]
]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt
[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x)
```

Rule 6005

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}(-1+dx^2)\right)\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(-1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\right.}{\sqrt{b}\,dx}$$

Mathematica [A]
time = 0.18, size = 134, normalized size = 0.81

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{b}\,dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x
^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Erf[Sqrt[a + b*A
rcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])))/(S
qrt[b]*d*x))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{arccosh}\left(d\,x^2-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{acosh}\,(dx^2 - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\,\mathrm{acosh}\,(d\,x^2 - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(1/2), x)
```

3.265 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{\sqrt{dx^2}\,\sqrt{-2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\,\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)(\cos}{b^{3/2}dx}$$

```
[Out] 1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x+1/2*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-(d*x^2)^(1/
2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$,
Rules used = {6007}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{bdx\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

```
[Out] -((Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])) +
 (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (Sqr
t[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sq
rt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x)
```

Rule 6007

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-
Sqrt[d*x^2])*(Sqrt[-2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])), x]
 + (Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2
]/2]*(Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + S
imp[Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*
(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; Fre
eQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx = -\frac{\sqrt{dx^2}\,\sqrt{-2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}} + \frac{\sqrt{\frac{\pi}{2}}\,\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)}{}$$

Mathematica [A]
time = 0.71, size = 209, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\sqrt{2\pi}\,\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\,\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\,\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)-4\sqrt{b}\,\sinh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\right)}{2b^{3/2}dx\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Er
fi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh
[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Erf[Sqrt[a + b*ArcC
osh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) - 4*Sqr
t[b]*Sinh[ArcCosh[-1 + d*x^2]/2]))/(2*b^(3/2)*d*x*Sqrt[a + b*ArcCosh[-1 + d
*x^2]])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^{2} - 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(3/2), x)
```

3.266 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=253

$$\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\right.}{}$$

```
[Out] 1/6*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x-1/6*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/3*(-d*x^4
+2*x^2)/b/x/(a+b*arccosh(d*x^2-1))^(3/2)/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)-1/3*
x/b^2/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6010, 6005}

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}-\frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}+\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{dx^2-2}\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*x^2 - d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(3*b^(5/2)*d*x) - (Sqrt[Pi/2]*Cosh[Arc
Cosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*
(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3*b^(5/2)*d*x)
```

Rule 6005

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] - Simp[Sqrt[Pi
/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(Sqrt[b]*d*x)), x] /; FreeQ[{a, b, d},
 x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx=\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}-\frac{\ }{3b^2\sqrt{a+b\cosh}}$$

$$=\frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}-\frac{\ }{3b^2\sqrt{a+b\cosh}}$$

Mathematica [A]
time = 0.60, size = 238, normalized size = 0.94

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\sqrt{2\pi}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{b}\left(\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)+b\sinh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\right)\right)}{6b^{5/2}dx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

```
[Out] -1/6*(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(
3/2)*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)
] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(3/2)*Erf[Sqrt[
a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
]) + 4*Sqrt[b]*((a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2] + b
*Sinh[ArcCosh[-1 + d*x^2]/2])))/(b^(5/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(3
/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^{2} - 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(5/2),x)
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(5/2), x)
```

3.267 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=302

$$\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}-\frac{\sqrt{dx^2}\sqrt{-}}{15b^3dx\sqrt{a+b\,\mathrm{co}}}$$

```
[Out] -1/15*x/b^2/(a+b*arccosh(d*x^2-1))^(3/2)+1/30*cosh(1/2*arccosh(d*x^2-1))*er
fi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/
2*a/b))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/30*cosh(1/2*arccosh(d*x^2-1))*erf(1/
2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b
))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/5*(-d*x^4+2*x^2)/b/x/(a+b*arccosh(d*x^2-1
))^(5/2)/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)-1/15*(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b
^3/d/x/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6010, 6007}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{15b^3dx\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}}+\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{dx^2-2}\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (2*x^2 - d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[-1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*S
qrt[-2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]
*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*
Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(15*b^(7/2)*d*x) + (Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqr
t[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b^(7/2)*d*x)
```

Rule 6007

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[(-
Sqrt[d*x^2])*(Sqrt[-2 + d*x^2]/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])), x]
 + (Simp[Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2
]/2]*(Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x] + S
imp[Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*
(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]]/(b^(3/2)*d*x)), x]) /; Fre
eQ[{a, b, d}, x]
```

Rule 6010

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[(-
x)*((a + b*ArcCosh[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (Dist
[1/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x]
+ Simp[(2*c*x^2 + d*x^4)*((a + b*ArcCosh[c + d*x^2])^(n + 1)/(2*b*(n + 1)*x
*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2])), x]) /; FreeQ[{a, b, c, d}, x]
&& EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx = \frac{2x^2-dx^4}{5bx\sqrt{dx^2}\,\sqrt{-2+dx^2}\,\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}} - \frac{}{15b^2\left(a+b\cosh\right.}$$

$$= \frac{2x^2-dx^4}{5bx\sqrt{dx^2}\,\sqrt{-2+dx^2}\,\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}} - \frac{}{15b^2\left(a+b\cosh\right.}$$

Mathematica [A]
time = 0.72, size = 260, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\left(\sqrt{2\pi}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{2\pi}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)+4\sqrt{b}\left(-b\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)-\left(3b^2+\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\right)\right)}{30b^{7/2}dx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*
Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*Erf[Sqrt[a + b*
ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4
*Sqrt[b]*(-(b*(a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2]) - (3
*b^2 + (a + b*ArcCosh[-1 + d*x^2])^2)*Sinh[ArcCosh[-1 + d*x^2]/2])))/(30*b^
(7/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
[Out] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^{2} - 1\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(7/2),x)
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(7/2),x)
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(7/2), x)
```

3.268 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
[Out] -integrate((b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.269 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=265

$$-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{3b\left(a+b\cosh\right.}{}$$

```
[Out] -1/4*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+3/2
*b*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+1)^(1/2))^2)/c+3/2*b^2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
))*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)
-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+3/4*b^3*polylog(4,-1
/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Rubi [A]

time = 0.21, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6813, 5882, 3799, 2221, 2611, 6744, 2320, 6724}

$$\frac{3b^2\text{Li}_3\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}+\frac{3b^3\text{Li}_4\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c) - ((a + b*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (3*b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog
[2, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*Ar
cCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^(-2*ArcCosh[Sqrt[1 - c*x]
/Sqrt[1 + c*x]])])/(2*c) + (3*b^3*PolyLog[4, -E^(-2*ArcCosh[Sqrt[1 - c*x]/S
qrt[1 + c*x]])])/(4*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
```

```
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^3\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\right.\right.}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log}{c}$$

Mathematica [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(381) = 762$.
time = 0.23, size = 870, normalized size = 3.28

method	result
default	$-\frac{a^3 \ln(cx-1)}{2c} + \frac{a^3 \ln(cx+1)}{2c} + \frac{b^3 \mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c} - \frac{b^3 \mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 \ln\left(\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + \sqrt{\ldots}\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
[Out] -1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)+1/4*b^3/c*arccosh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^4-b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)^(1/2))^2+1)-3/2*b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*b^3/c*arccosh((-c*x
+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3/
4*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1
/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b^2/c*arccosh((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)
*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2+1)-3*a*b^2/c*arccosh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a*b^
2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-
1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a^2*b*arccosh((-c*x
+1)^(1/2)/(c*x+1)^(1/2))^2/c-3*a^2*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2+1)-3/2*a^2*b/c*polylog(2,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^3/c + integrate(1/8*(((c*x + 1)*sqrt(-c*x
 + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*log(c*x + 1)^3 - 6*((c*x + 1)*sqrt(-c*x +
 1)*a*b^2 - (-c*x + 1)^(3/2)*a*b^2)*log(c*x + 1)^2 - 6*((4*a*b^2 + (b^3*c*x
 + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x + 1))*(c*x + 1)*sqrt(-c*x +
 1) - (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x +
1))*(-c*x + 1)^(3/2) + ((4*a*b^2 + (b^3*c*x - b^3)*log(c*x + 1) - (b^3*c*x
- b^3)*log(-c*x + 1))*(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) -
 (b^3*c*x + b^3)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^3 + (c*x - 1)*b^
3)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
 sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*
log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2 + (((c*x + 1)*b^3 + (c*x - 1)*b^3)*lo
g(c*x + 1)^3 - 6*((c*x + 1)*a*b^2 + (c*x - 1)*a*b^2)*log(c*x + 1)^2 + 12*((
c*x + 1)*a^2*b + (c*x - 1)*a^2*b)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-
c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + 12*((c*x + 1)*sqrt(-c*x +
 1)*a^2*b - (-c*x + 1)^(3/2)*a^2*b)*log(c*x + 1) - 6*(4*(c*x + 1)*sqrt(-c*x
 + 1)*a^2*b - 4*(-c*x + 1)^(3/2)*a^2*b + ((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-
c*x + 1)^(3/2)*b^3)*log(c*x + 1)^2 + (4*(c*x + 1)*a^2*b + 4*(c*x - 1)*a^2*b
 + ((c*x + 1)*b^3 + (c*x - 1)*b^3)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b^2 + (c
*x - 1)*a*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 4*((c*x + 1)*sqrt(-c*x + 1)*a*b^2 - (-c*x +
1)^(3/2)*a*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt
(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x +
1)*sqrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x +
1) + (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccosh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{acosh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acosh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acosh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acosh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\operatorname{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.270 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=196

$$-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{b\left(a+b\cos\right.}{}$$

```
[Out] -1/3*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+b*(
a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(2,-1/((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)^(1/2))^2)/c+1/2*b^2*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c
*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^
2)/c
```

Rubi [A]

time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 5882, 3799, 2221, 2611, 2320, 6724}

$$\frac{b\,\text{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}+\frac{b^2\,\text{Li}_3\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c) - ((a + b*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2,
-E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, -E^(-2*A
rcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\cosh^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\right.\right.}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log}{c}$$

Mathematica [**F**]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

Maple [**A**]
time = 0.06, size = 491, normalized size = 2.51

method	result

default	$-\frac{a^2 \ln(cx-1)}{2c}+\frac{a^2 \ln(cx+1)}{2c}+\frac{b^2 \operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}-\frac{b^2 \operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)+1/3*b^2/c*arccosh((-c*x+1)^(1/2)/(
c*x+1)^(1/2))^3-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(((-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+1)^(1/2))^2+1)-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*po
lylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/
2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/2*b^2/c*polylog(3,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/c-
2*a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)
^(1/2))^2+1)-a*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)
/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2/c + integrate(-1/4*(((c*x + 1)*sqrt(-c*
x + 1)*b^2 - (-c*x + 1)^(3/2)*b^2)*log(c*x + 1)^2 + (((c*x + 1)*b^2 + (c*x
- 1)*b^2)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b + (c*x - 1)*a*b)*log(c*x + 1))*
sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1))
- 4*((c*x + 1)*sqrt(-c*x + 1)*a*b - (-c*x + 1)^(3/2)*a*b)*log(c*x + 1) + 2*
((4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*
x + 1)*sqrt(-c*x + 1) - (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x +
b^2)*log(-c*x + 1))*(-c*x + 1)^(3/2) + ((4*a*b + (b^2*c*x - b^2)*log(c*x +
1) - (b^2*c*x - b^2)*log(-c*x + 1))*(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*lo
g(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^2 +
(c*x - 1)*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^2 - (-c*x + 1)
```

```
^(3/2)*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x + 1)*s
qrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) +
 (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c
*x + 1) + sqrt(-c*x + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccosh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2}{c^2x^2-1}\,dx-\int\frac{b^2\operatorname{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{2ab\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acosh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acosh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.271 $\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{b\mathrm{PolyLog}\left(2\right.}{}$$

```
[Out] -1/2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+1/2*b
*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1
)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {212, 6813, 5882, 3799, 2221, 2317, 2438}

$$-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\,\mathrm{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c) - ((a + b*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (b*PolyLog[2, -E^(-2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/
(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{1-}}\right.\right.}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+\right.}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+\right.}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+\right.}{c}$$

Mathematica [F]
time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

Maple [A]
time = 0.06, size = 207, normalized size = 1.56

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}-\frac{b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx}}{\sqrt{cx-}}}\right.\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/2*b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(
1/2))^2/c-b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1
/2)+1)^(1/2))^2+1)-1/2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+
1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-
c*x + 1)) + sqrt(-c*x + 1)))/c + 8*integrate(1/2*(c*x + 1)*sqrt(-c*x + 1)*(
log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1)*sqrt(-c*x + 1) - (c^
2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) + (c^2*x^2 - 1)*(c*x
 - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x
 + 1))), x) + 8*integrate(-1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))
/((c^2*x^2 - 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 8*integ
rate(1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c
*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(
c*x - 1)/c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b\operatorname{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*acosh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\operatorname{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.272 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccosh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a c^{2} x^{2} - a + b c^{2} x^{2} \operatorname{acosh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)} - b \operatorname{acosh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acosh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*acosh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.273 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 3.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccosh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 2*(2*c*x*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c
*x + 1)) + (c*x + 1)*sqrt(-c*x + 1) - (-c*x + 1)^(3/2))/(2*(c*x + 1)*sqrt(-
c*x + 1)*a*b*c - 2*(-c*x + 1)^(3/2)*a*b*c - ((c*x - 1)*b^2*c*log(c*x + 1) -
 2*(c*x - 1)*a*b*c)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1
) + sqrt(-c*x + 1)) - ((c*x + 1)*sqrt(-c*x + 1)*b^2*c - (-c*x + 1)^(3/2)*b^
2*c)*log(c*x + 1) + 2*((c*x - 1)*b^2*c*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))
*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + (c*x + 1)*sqrt(-c*x + 1)*b^2*c - (
-c*x + 1)^(3/2)*b^2*c)*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(
c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))) - integrate(-2*(2*(c*x + 1)*s
qrt(-c*x + 1)*(sqrt(c*x + 1) + sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x +
 1)) + ((c*x + 1)^2 + 2*(c*x + 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x
 + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/(2*(a*b*c^2*x^2 - a*b)*(c*x +
 1)^2*sqrt(-c*x + 1) - 4*(a*b*c^2*x^2 - a*b)*(c*x + 1)*(-c*x + 1)^(3/2) + 2
*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(5/2) + ((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/
2)*log(c*x + 1) - 2*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(3/2))*(sqrt(c*x + 1) +
sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) + 2*(2*(a*b*c^2*x^2 - a*b)
*(c*x + 1)*(c*x - 1) + 2*(a*b*c^2*x^2 - a*b)*(c*x - 1)^2 - ((b^2*c^2*x^2 -
```

```
b^2)*(c*x + 1)*(c*x - 1) + (b^2*c^2*x^2 - b^2)*(c*x - 1)^2)*log(c*x + 1))*s
qrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) -
 ((b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(-c*x + 1) - 2*(b^2*c^2*x^2 - b^2)*(c
*x + 1)*(-c*x + 1)^(3/2) + (b^2*c^2*x^2 - b^2)*(-c*x + 1)^(5/2))*log(c*x +
1) - 2*((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/2)*(sqrt(c*x + 1) + sqrt(-c*x + 1
))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) - (b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(
-c*x + 1) + 2*(b^2*c^2*x^2 - b^2)*(c*x + 1)*(-c*x + 1)^(3/2) - (b^2*c^2*x^2
 - b^2)*(-c*x + 1)^(5/2) - 2*((b^2*c^2*x^2 - b^2)*(c*x + 1)*(c*x - 1) + (b^
2*c^2*x^2 - b^2)*(c*x - 1)^2)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqr
t(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, choosing root of [1,0,-4,0,%
```

```
%%{4,[2,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0
,-4,0,
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
[Out] -int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.274 $\int \cosh^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=76

$$-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b}+\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] -1/2*arccosh(c*exp(b*x+a))^2/b+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+(c*
exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b+1/2*polylog(2,-(c*exp(b*x+
a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2320, 5882, 3799, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\cosh^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*E^(a + b*x)],x]
```

```
[Out] -1/2*ArcCosh[c*E^(a + b*x)]^2/b + (ArcCosh[c*E^(a + b*x)]*Log[1 + E^(2*ArcC
osh[c*E^(a + b*x)])])/b + PolyLog[2, -E^(2*ArcCosh[c*E^(a + b*x)])]/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}\left(ce^{a+bx}\right) dx &= \frac{\mathrm{Subst}\left(\int \frac{\cosh^{-1}(cx)}{x} dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\mathrm{Subst}\left(\int x\tanh(x)\, dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{2\mathrm{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}} dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\mathrm{Subst}\left(\int \log(1+ \right.}{} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\right.}{} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\mathrm{Li}_2\left(-e^{2\cosh^{-1}\left(ce\right.}\right.}{2b}
\end{aligned}
$$

Mathematica [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int \cosh^{-1}\left(ce^{a+bx}\right) dx$$

Verification is not applicable to the result.

[In] Integrate[ArcCosh[c*E^(a + b*x)],x]

[Out] Integrate[ArcCosh[c*E^(a + b*x)], x]

Maple [A]
time = 0.05, size = 110, normalized size = 1.45

method	result
derivativedivides	$\frac{-\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)^{2}}{2}+\operatorname{arccosh}\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+a}-1}\,\sqrt{c\,e^{bx+a}+1}\right)^{2}\right)+\frac{\operatorname{polylog}\left(2,-\left(c\,e^{bx+a}\right.\right.}{}}{b}$
default	$\frac{-\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)^{2}}{2}+\operatorname{arccosh}\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+a}-1}\,\sqrt{c\,e^{bx+a}+1}\right)^{2}\right)+\frac{\operatorname{polylog}\left(2,-\left(c\,e^{bx+a}\right.\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(c*exp(b*x+a)),x,method=_RETURNVERBOSE)

[Out] 1/b*(-1/2*arccosh(c*exp(b*x+a))^2+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)+1/2*polylog(2,-(c*exp(b*x+a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="maxima")

[Out] b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) - c*e^(b*x + a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(c*e^(b*x + a) - 1))), x) + x*log(c*e^(b*x + a) + sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x + a) - 1)) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)))/b - 1/2*(b*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(ce^{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*exp(b*x+a)),x)
```

```
[Out] Integral(acosh(c*exp(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(c\,\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*exp(a + b*x)),x)
```

```
[Out] int(acosh(c*exp(a + b*x)), x)
```

3.275 $\int e^{\cosh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=165

$$\frac{e^{-3\cosh^{-1}(a+bx)}}{48b^4}-\frac{3ae^{-2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(1+6a^2)\,e^{-\cosh^{-1}(a+bx)}}{8b^4}-\frac{a(3+4a^2)\,e^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(1+6a^2)\,e^{3\cosh^{-1}(a+bx}}{24b^4}$$

```
[Out] 1/48/b^4/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3-3/16*a/b^4/(b*x+a+(b*x+a
-1)^(1/2)*(b*x+a+1)^(1/2))^2+1/8*(6*a^2+1)/b^4/(b*x+a+(b*x+a-1)^(1/2)*(b*x+
a+1)^(1/2))-1/16*a*(4*a^2+3)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2/b^4+
1/24*(6*a^2+1)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^4-3/32*a*(b*x+a+
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^4/b^4+1/80*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^5/b^4+1/8*a*(4*a^2+3)*arccosh(b*x+a)/b^4
```

Rubi [A]
time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6015, 2320, 12, 1642}

$$-\frac{(4a^2+3)\,ae^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(4a^2+3)\,a\cosh^{-1}(a+bx)}{8b^4}+\frac{(6a^2+1)\,e^{-\cosh^{-1}(a+bx)}}{8b^4}+\frac{(6a^2+1)\,e^{3\cosh^{-1}(a+bx)}}{24b^4}-\frac{3ae^{-2\cosh^{-1}(a+bx)}}{16b^4}-\frac{3ae^{4\cosh^{-1}(a+bx)}}{32b^4}+\frac{e^{-3\cosh^{-1}(a+bx)}}{48b^4}+\frac{e^{5\cosh^{-1}(a+bx)}}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcCosh[a + b*x])) - (3*a)/(16*b^4*E^(2*ArcCosh[a + b*x])) +
 (1 + 6*a^2)/(8*b^4*E^ArcCosh[a + b*x]) - (a*(3 + 4*a^2)*E^(2*ArcCosh[a + b
*x]))/(16*b^4) + ((1 + 6*a^2)*E^(3*ArcCosh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcCosh[a + b*x]))/(32*b^4) + E^(5*ArcCosh[a + b*x])/(80*b^4) + (a*(3 + 4*a
^2)*ArcCosh[a + b*x])/(8*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}x^3\,dx = \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)\left(1-2ax+x^2\right)^3}{16b^3x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{(-1+x^2)\left(1-2ax+x^2\right)^3}{x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{16b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{6a}{x^3}-\frac{2(1+6a^2)}{x^2}+\frac{2a(3+4a^2)}{x}-2a(3+4a^2)\,x+2(1+6a^2)\,x^2-6ax^3+\right.\right.}{16b^4}$$

$$= \frac{e^{-3\cosh^{-1}(a+bx)}}{48b^4}-\frac{3ae^{-2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(1+6a^2)\,e^{-\cosh^{-1}(a+bx)}}{8b^4}-\frac{a(3+4a^2)\,e^{2\cosh^{-1}(a-}}{16b^4}$$

Mathematica [A]
time = 0.06, size = 138, normalized size = 0.84

$$\frac{30ab^4x^4+24b^5x^5+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,\left(-16-83a^2-6a^4+a(29+6a^2)\,bx-2(4+3a^2)\,b^2x^2+6ab^3x^3+24b^4x^4\right)+15a(3+4a^2)\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{120b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-16 - 83
*a^2 - 6*a^4 + a*(29 + 6*a^2)*b*x - 2*(4 + 3*a^2)*b^2*x^2 + 6*a*b^3*x^3 + 2
4*b^4*x^4) + 15*a*(3 + 4*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a +
 b*x]])/(120*b^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 376, normalized size = 2.28

method	result
default	$-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-24\,\mathrm{csgn}(b)b^4x^4\sqrt{b^2x^2+2abx+a^2-1}-6\,\mathrm{csgn}(b)a\,b^3x^3\sqrt{b^2x^2+2abx+a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(-24*csgn(b)*b^4*x^4*(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)-6*csgn(b)*a*b^3*x^3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+6*csgn(b)*
a^2*b^2*x^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*c
sgn(b)*a^3*b*x+6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^4+8*(b^2*x^2+2*a*b
*x+a^2-1)^(1/2)*csgn(b)*b^2*x^2-29*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a*
b*x+83*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2-60*ln(((b^2*x^2+2*a*b*x+a^
2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a^3+16*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csg
n(b)-45*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b
)/b^4/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/5*b*x^5+1/4*a*x^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(247) = 494$.
time = 0.30, size = 495, normalized size = 3.00

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4+\frac{(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}x^2}{5\,b^2}-\frac{(a^2-1)a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{5\,b^4}-\frac{7\,(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}ax}{20\,b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}\,(a^2-1)ax}{5\,b^3}+\frac{(a^2-1)^2a\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{5\,b^4}+\frac{7\,(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}a^2}{12\,b^4}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}\,(a^2-1)a^2}{5\,b^4}+\frac{7\,(5\,a^2b^2-(a^2-1)b^2)a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{40\,b^6}-\frac{2\,(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}(a^2-1)}{15\,b^4}-\frac{7\,(5\,a^2b^2-(a^2-1)b^2)\sqrt{b^2x^2+2\,abx+a^2-1}\,ax}{40\,b^5}-\frac{7\,(5\,a^2b^2-(a^2-1)b^2)(a^2-1)a\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{40\,b^6}-\frac{7\,(5\,a^2b^2-(a^2-1)b^2)\sqrt{b^2x^2+2\,abx+a^2-1}\,a^2}{40\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="maxima"
)
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*x^2/b^2 - 1
/5*(a^2 - 1)*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*
b)/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*a*x/b^3 + 1/5*(a^2 - 1)^2*a*log(2*b^2*x +
2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^4 + 7/12*(b^2*x^2 + 2*a*b*
x + a^2 - 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1
)*a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*a^3*log(2*b^2*x + 2*a*b + 2*sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2 - 1)
^(3/2)*(a^2 - 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*(a^2 - 1)*a*log(2
*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^6 - 7/40*(5*a^2*b
^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a^2/b^6
```

Fricas [A]
time = 0.33, size = 133, normalized size = 0.81

$$\frac{24\,b^5x^5+30\,ab^4x^4+(24\,b^4x^4+6\,ab^3x^3-2\,(3\,a^2+4)b^2x^2-6\,a^4+(6\,a^3+29\,a)bx-83\,a^2-16)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-15\,(4\,a^3+3\,a)\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="fricas"
)
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 + 4
)*b^2*x^2 - 6*a^4 + (6*a^3 + 29*a)*b*x - 83*a^2 - 16)*sqrt(b*x + a + 1)*sqr
t(b*x + a - 1) - 15*(4*a^3 + 3*a)*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1) - a))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(247) = 494.
time = 0.43, size = 564, normalized size = 3.42

$$\frac{24 b^2 x^5 + 30 a b x^4 + 5\left((bx+a+1)\left(2(bx+a+1)\left(\frac{3(bx+a+1)}{b^3} - \frac{12ab^{12}+13b^{12}}{b^{15}}\right) + \frac{36a^2b^{12}+84ab^{12}+43b^{12}}{b^{15}}\right) - \frac{3(8a^3b^{12}+36a^2b^{12}+36ab^{12}+13b^{12})}{b^{15}}\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1} + 5\left(\left((bx+a+1)\left(2(bx+a+1)\left(\frac{3(bx+a+1)}{b^3}-\frac{12ab^{12}+13b^{12}}{b^{15}}\right)+\frac{36a^2b^{12}+84ab^{12}+43b^{12}}{b^{15}}\right)-\frac{3(8a^3b^{12}+36a^2b^{12}+36ab^{12}+13b^{12})}{b^{15}}\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1} - \frac{6(8a^3+12a^2+12a+3)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^3}\right)a + \left(\left(\left(2(bx+a+1)\left(3(bx+a+1)\left(\frac{4(bx+a+1)}{b^4}-\frac{20ab^{20}+21b^{20}}{b^{24}}\right)+\frac{120a^2b^{20}+260ab^{20}+133b^{20}}{b^{24}}\right)-\frac{5(48a^3b^{20}+168a^2b^{20}+172ab^{20}+59b^{20})}{b^{24}}\right)(bx+a+1)+\frac{15(8a^4b^{20}+48a^3b^{20}+72a^2b^{20}+52ab^{20}+13b^{20})}{b^{24}}\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+\frac{30(8a^4+16a^3+24a^2+12a+3)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^4}\right)b - \frac{30(8a^3+12a^2+12a+3)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^3}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/120*(24*b^2*x^5 + 30*a*b*x^4 + 5*((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*x
+ a + 1)/b^3 - (12*a*b^12 + 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 43*
b^12)/b^15) - 3*(8*a^3*b^12 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sqrt
(b*x + a + 1)*sqrt(b*x + a - 1) + 5*(((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*
x + a + 1)/b^3 - (12*a*b^12 + 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 4
3*b^12)/b^15) - 3*(8*a^3*b^12 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sq
rt(b*x + a + 1)*sqrt(b*x + a - 1) - 6*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(
b*x + a + 1) - sqrt(b*x + a - 1))/b^3)*a + (((2*(b*x + a + 1)*(3*(b*x + a +
 1)*(4*(b*x + a + 1)/b^4 - (20*a*b^20 + 21*b^20)/b^24) + (120*a^2*b^20 + 26
0*a*b^20 + 133*b^20)/b^24) - 5*(48*a^3*b^20 + 168*a^2*b^20 + 172*a*b^20 + 5
9*b^20)/b^24)*(b*x + a + 1) + 15*(8*a^4*b^20 + 48*a^3*b^20 + 72*a^2*b^20 +
52*a*b^20 + 13*b^20)/b^24)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 30*(8*a^4
+ 16*a^3 + 24*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b^
4)*b - 30*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))/b^3)/b
```

Mupad [B]
time = 69.81, size = 1408, normalized size = 8.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)
```

```
[Out] (a*x^4)/4 - ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((3*a)/2 + 2*a^3))/(b^4
*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*x - 1)^(
1/2))^19*((3*a)/2 + 2*a^3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^19)
- (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((29*a)/2 + (58*a^3)/3))/(b^4*((
a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) - (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^17*((29*a)/2 + (58*a^3)/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))
^17) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(654*a - (4552*a^3)/3 + (35
84*a^5)/5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) - (((a - 1)^(1/2)
 - (a + b*x - 1)^(1/2))^15*(654*a - (4552*a^3)/3 + (3584*a^5)/5))/(b^4*((a
+ 1)^(1/2) - (a + b*x + 1)^(1/2))^15) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/
2))^7*(4622*a - 16024*a^3 + 11776*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)
^(1/2))^7) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^13*(4622*a - 16024*a^3
+ 11776*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^13) - (((a - 1)^(1
/2) - (a + b*x - 1)^(1/2))^9*(11095*a - 48012*a^3 + 39936*a^5))/(b^4*((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^9) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^11*(11095*a - 48012*a^3 + 39936*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^
(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a + 1)
^(1/2)*(64*a^4 - 128*a^2 + 64))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
4) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^16*(a - 1)^(1/2)*(a + 1)^(1/2)*
(64*a^4 - 128*a^2 + 64))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16) + (
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(a - 1)^(1/2)*(a + 1)^(1/2)*(3712*a
^4 - (12544*a^2)/3 + 1408/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6)
 + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^14*(a - 1)^(1/2)*(a + 1)^(1/2)*(3
712*a^4 - (12544*a^2)/3 + 1408/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^14) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/
2)*(25536*a^4 - (56960*a^2)/3 + 4928/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1
)^(1/2))^8) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12*(a - 1)^(1/2)*(a +
1)^(1/2)*(25536*a^4 - (56960*a^2)/3 + 4928/3))/(b^4*((a + 1)^(1/2) - (a + b
*x + 1)^(1/2))^12) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(a - 1)^(1/2
)*(a + 1)^(1/2)*((231168*a^4)/5 - (160256*a^2)/5 + 11008/5))/(b^4*((a + 1)^
(1/2) - (a + b*x + 1)^(1/2))^10))/((45*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)
)^4)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (10*((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (120*((a - 1)^(1
/2) - (a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (21
0*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(
1/2))^8 - (252*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^10 + (210*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12)/((a
```

```
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^12 - (120*((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^14)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^14 + (45*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))^16)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16 - (10*((
a - 1)^(1/2) - (a + b*x - 1)^(1/2))^18)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^18 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^20/((a + 1)^(1/2) - (a + b*x
+ 1)^(1/2))^20 + 1) + (b*x^5)/5 + (a*atanh(((a - 1)^(1/2) - (a + b*x - 1)^(
1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(4*a^2 + 3))/(2*b^4)
```

3.276 $\int e^{\cosh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}+\frac{(1+4a^2)\,e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}-\frac{(1+4a^2)\cosh^{-1}(a}{8b^3}$$

```
[Out] 1/16/b^3/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2-1/2*a/b^3/(b*x+a+(b*x+a-
1)^(1/2)*(b*x+a+1)^(1/2))+1/16*(4*a^2+1)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(
1/2))^2/b^3-1/6*a*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^3+1/32*(b*x+a
+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^4/b^3-1/8*(4*a^2+1)*arccosh(b*x+a)/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6015, 2320, 12, 1642}

$$\frac{(4a^2+1)\,e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{(4a^2+1)\cosh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] 1/(16*b^3*E^(2*ArcCosh[a + b*x])) - a/(2*b^3*E^ArcCosh[a + b*x]) + ((1 + 4*
a^2)*E^(2*ArcCosh[a + b*x]))/(16*b^3) - (a*E^(3*ArcCosh[a + b*x]))/(6*b^3)
+ E^(4*ArcCosh[a + b*x])/(32*b^3) - ((1 + 4*a^2)*ArcCosh[a + b*x])/(8*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\cosh^{-1}(a+bx)}x^2\,dx &= \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1-2ax+x^2)^2}{8b^2x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(1-2ax+x^2)^2}{x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}\\ &= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1-4a^2}{x}+(1+4a^2)\,x-4ax^2+x^3\right)\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}\\ &= \frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}+\frac{(1+4a^2)\,e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\cosh}}{3}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 119, normalized size = 1.03

$$\frac{8ab^3x^3+6b^4x^4+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(2a^3-3bx-2a^2bx+6b^3x^3+a(13+2b^2x^2))-3(1+4a^2)\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(2*a^3 - 3*
b*x - 2*a^2*b*x + 6*b^3*x^3 + a*(13 + 2*b^2*x^2)) - 3*(1 + 4*a^2)*Log[a + b
*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(24*b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 288, normalized size = 2.50

method	result
default	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(6\,\text{csgn}(b)b^3x^3\sqrt{b^2x^2+2abx+a^2-1}+2\,\text{csgn}(b)a\,b^2x^2\sqrt{b^2x^2+2abx+a^2-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(6*csgn(b)*b^3*x^3*(b^2*x^2+2*a*b*x+a^
2-1)^(1/2)+2*csgn(b)*a*b^2*x^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*csgn(b)*(b^2
*x^2+2*a*b*x+a^2-1)^(1/2)*a^2*b*x+2*csgn(b)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a
^3-3*csgn(b)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*b*x+13*csgn(b)*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*a-12*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))
*a^2-3*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b)))*csgn(b)/b
^3/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/4*b*x^4+1/3*a*x^3
```

Maxima [A]
time = 0.28, size = 275, normalized size = 2.39

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}x}{4\,b^2}-\frac{5\,(b^2x^2+2\,abx+a^2-1)^{\frac{3}{2}}a}{12\,b^3}-\frac{(5\,a^2b^2-(a^2-1)b^2)a^2\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{8\,b^5}+\frac{(5\,a^2b^2-(a^2-1)b^2)\sqrt{b^2x^2+2\,abx+a^2-1}\,x}{8\,b^4}+\frac{(5\,a^2b^2-(a^2-1)b^2)(a^2-1)\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{8\,b^5}+\frac{(5\,a^2b^2-(a^2-1)b^2)\sqrt{b^2x^2+2\,abx+a^2-1}\,a}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*x/b^2 - 5/1
2*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 - 1)*b^
2)*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 1
/8*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x/b^4 + 1/
8*(5*a^2*b^2 - (a^2 - 1)*b^2)*(a^2 - 1)*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 1/8*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*a/b^5
```

Fricas [A]
time = 0.33, size = 112, normalized size = 0.97

$$\frac{6\,b^4x^4+8\,ab^3x^3+(6\,b^3x^3+2\,ab^2x^2+2\,a^3-(2\,a^2+3)bx+13\,a)\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,+3\,(4\,a^2+1)\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,-a\right)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="fricas"
)
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 +
 3)*b*x + 13*a)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 3*(4*a^2 + 1)*log(-b*
x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. 2(173) = 346.
time = 0.42, size = 417, normalized size = 3.63

$$\frac{6\,b^2x^4+8\,abx^3+4\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left((bx+a+1)\left(\frac{2(bx+a+1)}{b^2}-\frac{6ab^6+7b^6}{b^8}\right)+\frac{3(2a^2b^6+6ab^6+3b^6)}{b^8}\right)+4\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left((bx+a+1)\left(\frac{2(bx+a+1)}{b^2}-\frac{6ab^6+7b^6}{b^8}\right)+\frac{3(2a^2b^6+6ab^6+3b^6)}{b^8}\right)+\frac{6(2a^2+2a+1)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^2}\right)a+\left(\left((bx+a+1)\left(2(bx+a+1)\left(\frac{3(bx+a+1)}{b^3}-\frac{12ab^{12}+13b^{12}}{b^{15}}\right)+\frac{36a^2b^{12}+84ab^{12}+43b^{12}}{b^{15}}\right)-\frac{3(8a^3b^{12}+36a^2b^{12}+36ab^{12}+13b^{12})}{b^{15}}\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-\frac{6(8a^3+12a^2+12a+3)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^3}\right)b+\frac{24(2a^2+2a+1)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*b^2*x^4 + 8*a*b*x^3 + 4*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x +
 a + 1)*(2*(b*x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*
b^6 + 3*b^6)/b^8) + 4*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x + a + 1)*(
2*(b*x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*b^6 + 3*b
^6)/b^8) + 6*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b
^2)*a + (((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*x + a + 1)/b^3 - (12*a*b^12
+ 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 43*b^12)/b^15) - 3*(8*a^3*b^1
2 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1) - 6*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))/b^3)*b + 24*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a -
1))/b^2)/b
```

Mupad [B]
time = 33.48, size = 1067, normalized size = 9.28

$$\frac{a\,x^3}{3}+\frac{b\,x^4}{4}+\frac{\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)\left(2\,a^2+\frac{1}{2}\right)}{b^3\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{15}\left(2\,a^2+\frac{1}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{15}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^3\left(\frac{64\,a^4}{3}-58\,a^2+\frac{35}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^3}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{13}\left(\frac{64\,a^4}{3}-58\,a^2+\frac{35}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{13}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^5\left(\frac{2368\,a^4}{3}-862\,a^2+\frac{273}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^5}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{11}\left(\frac{2368\,a^4}{3}-862\,a^2+\frac{273}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{11}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^7\left(\frac{9856\,a^4}{3}-3178\,a^2+\frac{715}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^7}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^9\left(\frac{9856\,a^4}{3}-3178\,a^2+\frac{715}{2}\right)}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^9}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^4\left(192\,a-192\,a^3\right)\sqrt{a-1}\,\sqrt{a+1}}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^4}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{12}\left(192\,a-192\,a^3\right)\sqrt{a-1}\,\sqrt{a+1}}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{12}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^6\left(\frac{2816\,a}{3}-\frac{5888\,a^3}{3}\right)\sqrt{a-1}\,\sqrt{a+1}}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^6}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{10}\left(\frac{2816\,a}{3}-\frac{5888\,a^3}{3}\right)\sqrt{a-1}\,\sqrt{a+1}}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{10}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^8\left(\frac{5504\,a}{3}-\frac{13648\,a^3}{3}\right)\sqrt{a-1}\,\sqrt{a+1}}{b^3\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^8}}{\frac{28\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^4}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^4}-\frac{8\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^2}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^2}-\frac{56\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^6}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^6}+\frac{70\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^8}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^8}-\frac{56\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{10}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{10}}+\frac{28\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{12}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{12}}-\frac{8\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{14}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{14}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{16}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{16}}+1}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x-1}}{\sqrt{a+1}-\sqrt{a+b\,x+1}}\right)\left(a^2+\frac{1}{4}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(2*a^2 + 1/
2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^15*(2*a^2 + 1/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
15) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((64*a^4)/3 - 58*a^2 + 35/2)
)/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) + (((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^13*((64*a^4)/3 - 58*a^2 + 35/2))/(b^3*((a + 1)^(1/2) - (a + b
*x + 1)^(1/2))^13) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*((2368*a^4)/3
 - 862*a^2 + 273/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a -
 1)^(1/2) - (a + b*x - 1)^(1/2))^11*((2368*a^4)/3 - 862*a^2 + 273/2))/(b^3*
((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^7*((9856*a^4)/3 - 3178*a^2 + 715/2))/(b^3*((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^7) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9*((9856*a^4)/3 -
3178*a^2 + 715/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^9) + (((a - 1
```

```
)^(1/2) - (a + b*x - 1)^(1/2))^4*(192*a - 192*a^3)*(a - 1)^(1/2)*(a + 1)^(1
/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) + (((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^12*(192*a - 192*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^12) + (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^6*((2816*a)/3 - (5888*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^6) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^10*((2816*a)/3 - (5888*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a + 1)^
(1/2) - (a + b*x + 1)^(1/2))^10) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8
*((5504*a)/3 - (11648*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^8))/((28*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4
)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (8*((a - 1)^(1/2) - (a + b*x -
1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (56*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (70*((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
8 - (56*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^10 + (28*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12)/((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^12 - (8*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^14
)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^14 + ((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^16/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16 + 1) - (2*atanh(((a - 1
)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(a^2
+ 1/4))/b^3
```

3.277 $\int e^{\cosh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=67

$$\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}$$

```
[Out] 1/4/b^2/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))-1/4*a*(b*x+a+(b*x+a-1)^(1/2
)*(b*x+a+1)^(1/2))^2/b^2+1/12*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^2
+1/2*a*arccosh(b*x+a)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6015, 2320, 12, 1642}

$$-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}+\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcCosh[a + b*x]) - (a*E^(2*ArcCosh[a + b*x]))/(4*b^2) + E^(3*Ar
cCosh[a + b*x])/(12*b^2) + (a*ArcCosh[a + b*x])/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
```

```
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{\cosh^{-1}(a+bx)}x\,dx &= \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{4bx^2}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{x^2}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{4b^2}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2a}{x}-2ax+x^2\right)\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{4b^2}\\
&= \frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 93, normalized size = 1.39

$$\frac{1}{6}\left(3ax^2+2bx^3+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(-2-a^2+abx+2b^2x^2)}{b^2}+\frac{3a\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - a^2 + a*b*
x + 2*b^2*x^2))/b^2 + (3*a*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]])/b^2)/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 194, normalized size = 2.90

method	result
default	$-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-2\sqrt{b^2x^2+2abx+a^2-1}\,\operatorname{csgn}(b)b^2x^2-\sqrt{b^2x^2+2abx+a^2-1}\,\operatorname{csgn}(b)\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x,method=_RETURNVERBOSE)
```

[Out] -1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*b^2*x^2-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a*b*x+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)-3*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b)/b^2/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/3*b*x^3+1/2*a*x^2

Maxima [A]
time = 0.29, size = 177, normalized size = 2.64

$$\frac{1}{3}bx^3+\frac{1}{2}ax^2+\frac{a^3\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{2b^2}-\frac{\sqrt{b^2x^2+2abx+a^2-1}\,ax}{2b}-\frac{(a^2-1)a\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{2b^2}-\frac{\sqrt{b^2x^2+2abx+a^2-1}\,a^2}{2b^2}+\frac{(b^2x^2+2abx+a^2-1)^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="maxima")

[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x/b - 1/2*(a^2 - 1)*a*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)/b^2

Fricas [A]
time = 0.34, size = 88, normalized size = 1.31

$$\frac{2b^3x^3+3ab^2x^2+(2b^2x^2+abx-a^2-2)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-3a\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="fricas")

[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + (2*b^2*x^2 + a*b*x - a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - 3*a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b^2

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x,x)

[Out] Integral(x*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(101) = 202.
time = 0.41, size = 280, normalized size = 4.18

$$\frac{2b^2x^3+\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left((bx+a+1)\left(\frac{2(bx+a+1)}{b^2}-\frac{6ab^6+7b^6}{b^8}\right)+\frac{3(2a^2b^6+6ab^6+3b^6)}{b^8}\right)+\frac{6(2a^2+2a+1)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)}{b^2}\right)b+\frac{3\left((bx+a+1)^2-2(bx+a+1)a-2bx-2a-2\right)a}{b}+\frac{3\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,(bx-a-2)-2(2a+1)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)\right)a}{b}+\frac{3\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,(bx-a-2)-2(2a+1)\log\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)\right)}{b}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="giac")

[Out] 1/6*(2*b^2*x^3 + (sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x + a + 1)*(2*(b*x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*b^6 + 3*b^6)/b^8) + 6*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b^2)*b + 3*((b*x + a + 1)^2 - 2*(b*x + a + 1)*a - 2*b*x - 2*a - 2)*a/b + 3*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*(b*x - a - 2) - 2*(2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1)))*a/b + 3*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*(b*x - a - 2) - 2*(2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1)))/b)/b

Mupad [B]
time = 16.72, size = 852, normalized size = 12.72

$$\frac{a\,x^2}{2}-\frac{\frac{2\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}{b^2\,\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^3\,\left(42\,a-\frac{160\,a^3}{3}\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^3}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^9\,\left(42\,a-\frac{160\,a^3}{3}\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^9}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^5\,\left(212\,a-288\,a^3\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^5}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^7\,\left(212\,a-288\,a^3\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^7}+\frac{2\,a\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{11}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{11}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^2\,\left(8\,a^2-8\right)\,\sqrt{a-1}\,\sqrt{a+1}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^2}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{10}\,\left(8\,a^2-8\right)\,\sqrt{a-1}\,\sqrt{a+1}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{10}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^4\,\left(160\,a^2-32\right)\,\sqrt{a-1}\,\sqrt{a+1}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^4}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^8\,\left(160\,a^2-32\right)\,\sqrt{a-1}\,\sqrt{a+1}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^8}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^6\,\left(\frac{1040\,a^2}{3}-\frac{272}{3}\right)\,\sqrt{a-1}\,\sqrt{a+1}}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^6}}{\frac{15\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^4}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^4}-\frac{6\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^2}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^2}-\frac{20\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^6}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^6}+\frac{15\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^8}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^8}-\frac{6\,{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{10}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{10}}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^{12}}{{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^{12}}+1}+\frac{b\,x^3}{3}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x-1}}{\sqrt{a+1}-\sqrt{a+b\,x+1}}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)

[Out] (a*x^2)/2 - ((2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(42*a - (160*a^3)/3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9*(42*a - (160*a^3)/3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^9) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(212*a - 288*a^3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^7*(212*a - 288*a^3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^7) + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11)/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*(8*a^2 - 8)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(8*a^2 - 8)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^10) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(160*a^2 - 32)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(160*a^2 - 32)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*((1040*a^2)/3 - 272/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6))/((15*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (6*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (20*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (15*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8 - (6*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^10 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^12 + 1) + (b*x^3)/3 + (2*a*atanh(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))))/b^2

3.278 $\int e^{\cosh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

```
[Out] 1/4*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2/b-1/2*arccosh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6014, 2320, 12, 14}

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x],x]
```

```
[Out] E^(2*ArcCosh[a + b*x])/(4*b) - ArcCosh[a + b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6014

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int e^x\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{e^{2\cosh^{-1}(a+bx)}}{4b} - \frac{\cosh^{-1}(a+bx)}{2b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(31) = 62.
time = 0.02, size = 69, normalized size = 2.23

$$\frac{(a+bx)\left(a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)-\log\left(a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x],x]
[Out] ((a + b*x)*(a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) - Log[a + b*x +
 Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(41) = 82.
time = 0.09, size = 147, normalized size = 4.74

method	result
default	$ax+\frac{b\,x^2}{2}+\frac{\sqrt{bx+a-1}\,(bx+a+1)^{\frac{3}{2}}}{2b}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{2b}-\frac{\sqrt{(bx+a-1)\,(bx+a+1)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+1/2/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(3/2)-1/2*(b*x+a-1)^(1/2)*(b*
x+a+1)^(1/2)/b-1/2*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a+1)^(1/2)/(b*x+a-1)^(1
/2)*ln((1/2*b*(1+a)+1/2*(-1+a)*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(b*(1+a)+(-1+a
)*b)*x+(1+a)*(-1+a))^(1/2))/(b^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(41) = 82$.
time = 0.29, size = 143, normalized size = 4.61

$$\frac{1}{2}\,bx^2 + ax - \frac{a^2\log\left(2\,b^2x + 2\,ab + 2\,\sqrt{b^2x^2 + 2\,abx + a^2 - 1}\,b\right)}{2\,b} + \frac{1}{2}\,\sqrt{b^2x^2 + 2\,abx + a^2 - 1}\,x + \frac{(a^2 - 1)\log\left(2\,b^2x + 2\,ab + 2\,\sqrt{b^2x^2 + 2\,abx + a^2 - 1}\,b\right)}{2\,b} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 - 1}\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x - 1/2*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x +
a^2 - 1)*b)/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x + 1/2*(a^2 - 1)*log
(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b + 1/2*sqrt(b^2*
x^2 + 2*a*b*x + a^2 - 1)*a/b
```

Fricas [**A**]
time = 0.35, size = 66, normalized size = 2.13

$$\frac{b^2x^2 + 2\,abx + \sqrt{bx + a + 1}\,(bx + a)\sqrt{bx + a - 1} + \log\left(-bx + \sqrt{bx + a + 1}\,\sqrt{bx + a - 1} - a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b*x + a + 1)*(b*x + a)*sqrt(b*x + a - 1) + lo
g(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(41) = 82$.
time = 0.40, size = 151, normalized size = 4.87

$$\frac{1}{2}\,bx^2 + ax + \frac{\sqrt{bx + a + 1}\,\sqrt{bx + a - 1}\,(bx - a - 2) + 2\left(\sqrt{bx + a + 1}\,\sqrt{bx + a - 1} + 2\,\log\left(\sqrt{bx + a + 1} - \sqrt{bx + a - 1}\right)\right)a - 2\,(2\,a + 1)\log\left(\sqrt{bx + a + 1} - \sqrt{bx + a - 1}\right) + 2\,\sqrt{bx + a + 1}\,\sqrt{bx + a - 1} + 4\,\log\left(\sqrt{bx + a + 1} - \sqrt{bx + a - 1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*(b*x - a - 2) +
2*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*log(sqrt(b*x + a + 1) - sqrt(b*x
 + a - 1)))*a - 2*(2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1)) + 2*
sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*log(sqrt(b*x + a + 1) - sqrt(b*x +
a - 1)))/b
```

Mupad [B]
time = 0.17, size = 79, normalized size = 2.55

$$a\,x+\frac{b\,x^2}{2}-\frac{\ln\left(a+\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}+b\,x\right)}{2\,b}+\frac{x\,\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}}{2}+\frac{a\,\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x,x)
```

```
[Out] a*x + (b*x^2)/2 - log(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/(2
*b) + (x*(a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2))/2 + (a*(a + b*x - 1)^(1/2
)*(a + b*x + 1)^(1/2))/(2*b)
```

3.279 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=100

$$bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+2a\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)+2\sqrt{1-a^2}\,\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+b}}{\sqrt{1+a}\,\sqrt{-1+a+}}\right.$$

```
[Out] b*x+2*a*arcsinh(1/2*(b*x+a-1)^(1/2)*2^(1/2))+a*ln(x)+2*arctan((1-a)^(1/2)*(
b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))*(-a^2+1)^(1/2)+(b*x+a-1)^(1/2)*
(b*x+a+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6020, 14, 103, 163, 65, 221, 95, 211}

$$2\sqrt{1-a^2}\,\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)+\sqrt{a+bx-1}\,\sqrt{a+bx+1}+2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*a*ArcSinh[Sqrt[-1 + a + b*x]
/Sqrt[2]] + 2*Sqrt[1 - a^2]*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
+ a]*Sqrt[-1 + a + b*x])] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6020

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x} dx = \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x} dx$$
$$= \int \left(b+\frac{a}{x}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x}\right) dx$$
$$= bx+a\log(x)+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x} dx$$
$$= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)-\int \frac{1-a^2-abx}{x\sqrt{-1+a+bx}\sqrt{1+a+bx}} dx$$
$$= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)-\left(1-a^2\right)\int \frac{1}{x\sqrt{-1+a+bx}\sqrt{1+a+}$$
$$= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)+(2a)\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}} dx, x, \sqrt{-1+}\right.$$
$$= bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+2a\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)+2\sqrt{1-a^2}\tan^{-1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 141, normalized size = 1.41

$$bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}+a\log(x)+a\log\left(a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}\right)+i\sqrt{1-a^2}\log\left(\frac{2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{(-1+a^2)x}+\frac{2i(-1+a^2+abx)}{\sqrt{1-a^2}(-1+a^2)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + a*Log[x] + a*Log[a + b*x + Sqr
t[-1 + a + b*x]*Sqrt[1 + a + b*x]] + I*Sqrt[1 - a^2]*Log[(2*Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x])/((-1 + a^2)*x) + ((2*I)*(-1 + a^2 + a*b*x))/(Sqrt[1
- a^2]*(-1 + a^2)*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 156, normalized size = 1.56

method	result
default	$\frac{\left(-\mathrm{csgn}(b)\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)\sqrt{a^2-1}+\sqrt{b^2x^2+2abx+a^2-1}\,\mathrm{csgn}(\right.}{\sqrt{b^2x^2+2abx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(b)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)
*(a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+ln(((b^2*x^2+2*a*b*x+a
^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn
(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+b*x+a*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.38, size = 246, normalized size = 2.46

$$\left[bx - a\log\left(-bx + \sqrt{bx+a+1}\,\sqrt{bx+a-1} - a\right) + a\log\left(x\right) + \sqrt{a^2-1}\,\log\left(\frac{a^2bx + a^3 + \left(a^2 - \sqrt{a^2-1}\,a - 1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1} - \left(abx + a^2 - 1\right)\sqrt{a^2-1} - a}{x}\right) + \sqrt{bx+a+1}\,\sqrt{bx+a-1},\ bx - a\log\left(-bx + \sqrt{bx+a+1}\,\sqrt{bx+a-1} - a\right) + a\log\left(x\right) + 2\,\sqrt{-a^2+1}\,\arctan\left(-\frac{\sqrt{-a^2+1}\,bx - \sqrt{-a^2+1}\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}}{a^2-1}\right) + \sqrt{bx+a+1}\,\sqrt{bx+a-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] [b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + a*log(x) + s
qrt(a^2 - 1)*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt(b*x + a
+ 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + sqrt(b*x
 + a + 1)*sqrt(b*x + a - 1), b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x
+ a - 1) - a) + a*log(x) + 2*sqrt(-a^2 + 1)*arctan(-(sqrt(-a^2 + 1)*b*x - s
qrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + sqrt(b*x +
a + 1)*sqrt(b*x + a - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x, x)
```

Giac [A]
time = 0.40, size = 103, normalized size = 1.03

$$bx - a\log\left(\left(\sqrt{bx+a+1} - \sqrt{bx+a-1}\right)^2\right) + a\log\left(|bx|\right) + 2\sqrt{-a^2+1}\arctan\left(\frac{\left(\sqrt{bx+a+1} - \sqrt{bx+a-1}\right)^2 - 2a}{2\sqrt{-a^2+1}}\right) + \sqrt{bx+a+1}\sqrt{bx+a-1} + a + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2) + a*log(abs(b*x)) +
2*sqrt(-a^2 + 1)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*
a)/sqrt(-a^2 + 1)) + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + a + 1
```

Mupad [B]
time = 23.41, size = 2500, normalized size = 25.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x,x)
```

```
[Out] b*x + a*log(x) - a*atan(-(a*(2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) - 2*a*
(a - 1)^(1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) - 68*a^5*(
a - 1)^(1/2)*(a + 1)^(1/2) - 4*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) + 92*a^7*(a
- 1)^(1/2)*(a + 1)^(1/2) - 22*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) - 58*a^9*(a -
 1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a - 1)^(5/2)*(a + 1)^(5/2) + 52*a^7*(a - 1
)^(3/2)*(a + 1)^(3/2) + 14*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 12*a^5*(a - 1
)^(5/2)*(a + 1)^(5/2) - 27*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 9*a^7*(a - 1)^
(5/2)*(a + 1)^(5/2) + 4*a^5*(a - 1)^(7/2)*(a + 1)^(7/2)))/(5*a^2 - 10*a^4 +
 10*a^6 - 5*a^8 + a^10 - 1) - 2*a*((32*(2*a - 10*a^3 + 20*a^5 - 20*a^7 + 10
*a^9 - 2*a^11 - 2*a*(a - 1)*(a + 1) + 2*a^3*(a - 1)^2*(a + 1)^2 - 6*a^5*(a
- 1)^2*(a + 1)^2 + 4*a^7*(a - 1)^2*(a + 1)^2 + 8*a^3*(a - 1)*(a + 1) - 12*a
^5*(a - 1)*(a + 1) + 8*a^7*(a - 1)*(a + 1) - 2*a^9*(a - 1)*(a + 1)))/(5*a^2
 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(1/2)*(a + 1)^
(1/2) - 4*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 6*a^5*(a - 1)^(1/2)*(a + 1)^(1/
2) - 5*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 4*a^7*(a - 1)^(1/2)*(a + 1)^(1/2)
+ 10*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*
a^7*(a - 1)^(3/2)*(a + 1)^(3/2) + 4*a^5*(a - 1)^(5/2)*(a + 1)^(5/2)))/(5*a^
2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - (32*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))*(60*a^2 - 150*a^4 + 200*a^6 - 150*a^8 + 60*a^10 - 10*a^12 - 39*a^4
*(a - 1)^2*(a + 1)^2 + 78*a^6*(a - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1
)^3 - 39*a^8*(a - 1)^2*(a + 1)^2 + 33*a^2*(a - 1)*(a + 1) - 132*a^4*(a - 1)
*(a + 1) + 198*a^6*(a - 1)*(a + 1) - 132*a^8*(a - 1)*(a + 1) + 33*a^10*(a -
 1)*(a + 1) - 10))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 -
 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1
```

)^(1/2))*(17*(a - 1)^(1/2)*(a + 1)^(1/2) - 106*a^2*(a - 1)^(1/2)*(a + 1)^(1/2) + 275*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 48*a^2*(a - 1)^(3/2)*(a + 1)^(3/2) - 380*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) + 196*a^4*(a - 1)^(3/2)*(a + 1)^(3/2) + 295*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) - 300*a^6*(a - 1)^(3/2)*(a + 1)^(3/2) - 122*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 47*a^4*(a - 1)^(5/2)*(a + 1)^(5/2) + 204*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) + 21*a^12*(a - 1)^(1/2)*(a + 1)^(1/2) - 94*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) - 52*a^10*(a - 1)^(3/2)*(a + 1)^(3/2) + 47*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 16*a^6*(a - 1)^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(36*a^2 - 216*a^4 + 540*a^6 - 720*a^8 + 540*a^10 - 216*a^12 + 36*a^14 - 6*(a - 1)*(a + 1) + 15*a^2*(a - 1)^2*(a + 1)^2 - 64*a^4*(a - 1)^2*(a + 1)^2 - 9*a^4*(a - 1)^3*(a + 1)^3 + 262*a^6*(a - 1)^2*(a + 1)^2 + 18*a^6*(a - 1)^3*(a + 1)^3 - 392*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 179*a^10*(a - 1)^2*(a + 1)^2 + 40*a^2*(a - 1)*(a + 1) - 242*a^4*(a - 1)*(a + 1) + 688*a^6*(a - 1)*(a + 1) - 922*a^8*(a - 1)*(a + 1) + 584*a^10*(a - 1)*(a + 1) - 142*a^12*(a - 1)*(a + 1)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (32*(2*a*(a - 1)^2*(a + 1)^2 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a - 1)^3*(a + 1)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3 - 8*a^7*(a - 1)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(a + 1)^2 + 10*a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)*(a + 1) - 10*a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((a - 1)^(3/2)*(a + 1)^(3/2) + 64*a^2*(a - 1)^(1/2)*(a + 1)^(1/2) - 384*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 10*a^2*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) - 157*a^4*(a - 1)^(3/2)*(a + 1)^(3/2) - 1280*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) + 708*a^6*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^4*(a - 1)^(5/2)*(a + 1)^(5/2) - 1097*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^12*(a - 1)^(1/2)*(a + 1)^(1/2) + 180*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) + 742*a^10*(a - 1)^(3/2)*(a + 1)^(3/2) + 64*a^14*(a - 1)^(1/2)*(a + 1)^(1/2) - a^4*(a - 1)^(7/2)*(a + 1)^(7/2) - 372*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 187*a^12*(a - 1)^(3/2)*(a + 1)^(3/2) + 2*a^6*(a - 1)^(7/2)*(a + 1)^(7/2) + 188*a^10*(a - 1)^(5/2)*(a + 1)^(5/2) - 65*a^8*(a - 1)^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1)))*2i - a*((32*(2*a*(a - 1)^2*(a + 1)^2 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a - 1)^3*(a + 1)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3 - 8*a^7*(a - 1)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(a + 1)^2 + 10*a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)*(a + 1) - 10*a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) - 2*a*(a - 1)^(1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1...

3.280 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2} \, dx$

Optimal. Leaf size=109

$$-\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)-\frac{2ab\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}$$

```
[Out] -a/x+2*b*arcsinh(1/2*(b*x+a-1)^(1/2)*2^(1/2))+b*ln(x)-2*a*b*arctan((1-a)^(1
/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(1/2)-(b*x+a-1)^(
1/2)*(b*x+a+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6020, 14, 99, 163, 65, 221, 95, 211}

$$-\frac{2ab\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + 2*b*ArcSinh[Sqrt[-1 + a
 + b*x]/Sqrt[2]] - (2*a*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 +
a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2] + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6020

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\,dx \\
&= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\right)dx \\
&= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\,dx \\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+\int \frac{ab+b^2x}{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx \\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{-1+a+bx}\,\sqrt{1+a+ba} \\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+(2b)\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\sqrt{-1} \right. \\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)-\frac{2ab\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{1}}\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 140, normalized size = 1.28

$$
-\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+b\log\left(a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)-\frac{iab\log\left(\frac{2\left(\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}\right)}{abx}\right)}{\sqrt{1-a^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + b*Log[x] + b*Log[a + b*
x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]] - (I*a*b*Log[(2*(Sqrt[-1 + a + b*
x]*Sqrt[1 + a + b*x] + (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(a*b*x)])/Sqr
t[1 - a^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 237, normalized size = 2.17

method	result
default	$\frac{\left(-\text{csgn}(b)\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)abx+\ln\left(\left(\sqrt{b^2x^2+2abx+a^2-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(b)*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)+a^2-1)/x)*a*b*x+ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(
b))*a^2*b*x-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2-ln(((b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*b*x+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn
(b))*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/
(a^2-1)/x-a/x+b*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.35, size = 334, normalized size = 3.06

$$\left[\frac{\sqrt{a^2-1}\,abx\log\left(\frac{a^2bx+a^3+\left(a^2-\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-(a^2-1)bx\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)+(a^2-1)bx\log(x)-a^3-(a^2-1)bx-(a^2-1)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+a}{(a^2-1)x}, \frac{2\sqrt{-a^2+1}\,abx\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{-a^2+1}\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}}{a^2-1}\right)-(a^2-1)bx\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)+(a^2-1)bx\log(x)-a^3-(a^2-1)bx-(a^2-1)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+a}{(a^2-1)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="fricas"
)
```

```
[Out] [(sqrt(a^2 - 1)*a*b*x*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt
(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) -
 (a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 -
 1)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x
 + a - 1) + a)/((a^2 - 1)*x), (2*sqrt(-a^2 + 1)*a*b*x*arctan(-(sqrt(-a^2 +
1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) - (
a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 - 1
)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x +
 a - 1) + a)/((a^2 - 1)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(92) = 184.
time = 0.42, size = 200, normalized size = 1.83

$$-\frac{\frac{2\,ab^2\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}}+b^2\log\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2\right)-b^2\log\left(|bx|\right)-\frac{4\left(ab^2\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,b^2\right)}{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4\,a\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2+4}+\frac{(bx+a+1)b^2-b^2}{bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -(2*a*b^2*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqrt
(-a^2 + 1))/sqrt(-a^2 + 1) + b^2*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1)
)^2) - b^2*log(abs(b*x)) - 4*(a*b^2*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))
^2 - 2*b^2)/((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a
+ 1) - sqrt(b*x + a - 1))^2 + 4) + ((b*x + a + 1)*b^2 - b^2)/(b*x))/b
```

Mupad [B]
time = 15.82, size = 2500, normalized size = 22.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^2,x)
```

```
[Out] ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((5*b)/4 - (a^2*b)/4))/((a^2 - 1)
*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2) - b/4 + (a*b*((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))*(a - 1)^(1/2)*(a + 1)^(1/2))/(2*(a^2 - 1)*((a + 1)^(1/2)
- (a + b*x + 1)^(1/2))))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1
/2) - (a + b*x + 1)^(1/2)) + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3/((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^3 - (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/
2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1
)^(1/2))^2)) - a/x + b*log(x) - b*atan((((14*a^2 - 42*a^4 + 70*a^6 - 70*a^8
 + 42*a^10 - 14*a^12 + 2*a^14 + 4*a^4*(a - 1)^2*(a + 1)^2 - 12*a^6*(a - 1)^
2*(a + 1)^2 + 12*a^8*(a - 1)^2*(a + 1)^2 - 4*a^10*(a - 1)^2*(a + 1)^2 - 2*a
```

^2*(a - 1)*(a + 1) + 10*a^4*(a - 1)*(a + 1) - 20*a^6*(a - 1)*(a + 1) + 20*a
^8*(a - 1)*(a + 1) - 10*a^10*(a - 1)*(a + 1) + 2*a^12*(a - 1)*(a + 1) - 2)*
512i)/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + ((
2*a^4*(a - 1)^2*(a + 1)^2 - 6*a^6*(a - 1)^2*(a + 1)^2 - 4*a^6*(a - 1)^3*(a
+ 1)^3 + 6*a^8*(a - 1)^2*(a + 1)^2 + 4*a^8*(a - 1)^3*(a + 1)^3 - 2*a^10*(a
- 1)^2*(a + 1)^2 + 2*a^2*(a - 1)*(a + 1) - 10*a^4*(a - 1)*(a + 1) + 20*a^6*
(a - 1)*(a + 1) - 20*a^8*(a - 1)*(a + 1) + 10*a^10*(a - 1)*(a + 1) - 2*a^12
(a - 1)(a + 1))*128i)/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^1
2 + a^14 - 1) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(21*a*(a - 1)^(1/2)*
(a + 1)^(1/2) - 126*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 315*a^5*(a - 1)^(1/2)
*(a + 1)^(1/2) - 52*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 420*a^7*(a - 1)^(1/2)
*(a + 1)^(1/2) + 208*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 315*a^9*(a - 1)^(1/2
)*(a + 1)^(1/2) - 312*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) - 126*a^11*(a - 1)^(1
/2)*(a + 1)^(1/2) + 47*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) + 208*a^9*(a - 1)^(3
/2)*(a + 1)^(3/2) + 21*a^13*(a - 1)^(1/2)*(a + 1)^(1/2) - 94*a^7*(a - 1)^(5
/2)*(a + 1)^(5/2) - 52*a^11*(a - 1)^(3/2)*(a + 1)^(3/2) + 47*a^9*(a - 1)^(5
/2)*(a + 1)^(5/2) - 16*a^7*(a - 1)^(7/2)*(a + 1)^(7/2))*512i)/(((a + 1)^(1/
2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a
^12 + a^14 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(64*a*(a - 1)^(1/
2)*(a + 1)^(1/2) - 384*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 960*a^5*(a - 1)^(1
/2)*(a + 1)^(1/2) - 187*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 1280*a^7*(a - 1)^
(1/2)*(a + 1)^(1/2) + 748*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^9*(a - 1)
^(1/2)*(a + 1)^(1/2) - 1122*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^11*(a -
 1)^(1/2)*(a + 1)^(1/2) + 188*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) + 748*a^9*(a
- 1)^(3/2)*(a + 1)^(3/2) + 64*a^13*(a - 1)^(1/2)*(a + 1)^(1/2) - 376*a^7*(a
 - 1)^(5/2)*(a + 1)^(5/2) - 187*a^11*(a - 1)^(3/2)*(a + 1)^(3/2) + 188*a^9*
(a - 1)^(5/2)*(a + 1)^(5/2) - 65*a^7*(a - 1)^(7/2)*(a + 1)^(7/2))*128i)/(((
a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*
a^10 - 7*a^12 + a^14 - 1)))/((1024*(a*(a - 1)^(1/2)*(a + 1)^(1/2) - 6*a^3*(
a - 1)^(1/2)*(a + 1)^(1/2) + 15*a^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a
- 1)^(3/2)*(a + 1)^(3/2) - 20*a^7*(a - 1)^(1/2)*(a + 1)^(1/2) + 20*a^5*(a -
 1)^(3/2)*(a + 1)^(3/2) + 15*a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 30*a^7*(a -
1)^(3/2)*(a + 1)^(3/2) - 6*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^5*(a - 1)
^(5/2)*(a + 1)^(5/2) + 20*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + a^13*(a - 1)^(1
/2)*(a + 1)^(1/2) - 8*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) - 5*a^11*(a - 1)^(3/2
)*(a + 1)^(3/2) + 4*a^9*(a - 1)^(5/2)*(a + 1)^(5/2)))/(7*a^2 - 21*a^4 + 35*
a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + (64*(14*a^3*(a - 1)^(3/2)*(a
+ 1)^(3/2) - 56*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 84*a^7*(a - 1)^(3/2)*(a +
 1)^(3/2) - 28*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) - 56*a^9*(a - 1)^(3/2)*(a +
1)^(3/2) + 56*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) + 14*a^11*(a - 1)^(3/2)*(a +
1)^(3/2) - 28*a^9*(a - 1)^(5/2)*(a + 1)^(5/2) + 14*a^7*(a - 1)^(7/2)*(a + 1
)^(7/2)))/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1)
- (256*(14*a*(a - 1)^(1/2)*(a + 1)^(1/2) - 84*a^3*(a - 1)^(1/2)*(a + 1)^(1/
2) + 210*a^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 27*a^3*(a - 1)^(3/2)*(a + 1)^(3/
2) - 280*a^7*(a - 1)^(1/2)*(a + 1)^(1/2) + 108*a^5*(a - 1)^(3/2)*(a + 1)^(3

```
/2) + 210*a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 162*a^7*(a - 1)^(3/2)*(a + 1)^(
3/2) - 84*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 9*a^5*(a - 1)^(5/2)*(a + 1)^(5
/2) + 108*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 14*a^13*(a - 1)^(1/2)*(a + 1)^(
1/2) - 18*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) - 27*a^11*(a - 1)^(3/2)*(a + 1)^(
3/2) + 9*a^9*(a - 1)^(5/2)*(a + 1)^(5/2) + 4*a^7*(a - 1)^(7/2)*(a + 1)^(7/2
)))/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + (102
4*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(70*a^2 - 210*a^4 + 350*a^6 - 350*a
^8 + 210*a^10 - 70*a^12 + 10*a^14 - 39*a^4*(a - 1)^2*(a + 1)^2 + 117*a^6*(a
 - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1)^3 - 117*a^8*(a - 1)^2*(a + 1)^
2 - 16*a^8*(a - 1)^3*(a + 1)^3 + 39*a^10*(a - 1)^2*(a + 1)^2 + 33*a^2*(a -
1)*(a + 1) - 165*a^4*(a - 1)*(a + 1) + 330*a^6*(a - 1)*(a + 1) - 330*a^8*(a
 - 1)*(a + 1) + 165*a^10*(a - 1)*(a + 1) - 33*a...
```

3.281 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}-\frac{b^2\mathrm{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+bx}}{\sqrt{1+a}\,\sqrt{-1+a+bx}}\right)}{\left(1-a^2\right)^{3/2}}$$

```
[Out] -1/2*a/x^2-b/x-b^2*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)
^(1/2))/(-a^2+1)^(3/2)-1/2*(b*x+a+1)^(3/2)*(b*x+a-1)^(1/2)/(1+a)/x^2+1/2*b*
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6020, 14, 96, 95, 211}

$$-\frac{b^2\mathrm{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\sqrt{a+bx-1}\,(a+bx+1)^{3/2}}{2(a+1)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] -1/2*a/x^2 - b/x + (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x)
 - (Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 + a)*x^2) - (b^2*ArcTan[(
Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)
^(3/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 6020

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\,dx \\
&= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\right)dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{-1+a+bx}}\,dx}{2(1+a)} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\int \cdots}{\cdots} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\text{Sub}\cdots}{\cdots} \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}-\frac{b^2\tan\cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 142, normalized size = 1.03

$$\frac{1}{2}\left(-\frac{a}{x^2}-\frac{2b}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,\left(-1+a^2+abx\right)}{\left(-1+a^2\right)x^2}-\frac{ib^2\log\left(\frac{4i\sqrt{1-a^2}\left(-1+a^2+abx-i\sqrt{1-a^2}\,\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{b^2x}\right)}{\left(1-a^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-1 + a^2 + a*b
*x))/((-1 + a^2)*x^2) - (I*b^2*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x -
 I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(b^2*x)])/(1 - a^2)
^(3/2))/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.
time = 0.07, size = 236, normalized size = 1.71

method	result
default	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)b^2x^2-a^3bx\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*((a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*b^2*x^2-a^3*b*x*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)-a^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2
)*a*b*x+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a^2-(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/
(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)^2/x^2-b/x-1/2*a/x^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [**A**]
time = 0.37, size = 338, normalized size = 2.45

$$\left[\frac{\sqrt{a^2-1}\,b^2x^2\log\left(\frac{a^2bx+a^3+\left(a^2+\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+(abx+a^2-1)\sqrt{a^2-1}-a}{x}\right)-a^5-(a^3-a)b^2x^2+2\,a^3-2\,(a^4-2\,a^2+1)bx-(a^4+(a^3-a)bx-2\,a^2+1)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a}{2\,(a^4-2\,a^2+1)x^2}, -\frac{2\,\sqrt{-a^2+1}\,b^2x^2\arctan\left(-\frac{\sqrt{-a^2+1}\,bx-\sqrt{-a^2+1}\,\sqrt{bx+a+1}\,\sqrt{bx+a-1}}{a^2-1}\right)+a^5+(a^3-a)b^2x^2-2\,a^3+2\,(a^4-2\,a^2+1)bx+(a^4+(a^3-a)bx-2\,a^2+1)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+a}{2\,(a^4-2\,a^2+1)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + (a^2 + sqrt(a^2 - 1)*a - 1
)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
)/x) - a^5 - (a^3 - a)*b^2*x^2 + 2*a^3 - 2*(a^4 - 2*a^2 + 1)*b*x - (a^4 + (
a^3 - a)*b*x - 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a)/((a^4 -
2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)*b*x
 - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + a^5 + (
a^3 - a)*b^2*x^2 - 2*a^3 + 2*(a^4 - 2*a^2 + 1)*b*x + (a^4 + (a^3 - a)*b*x -
 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + a)/((a^4 - 2*a^2 + 1)*x^2
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(112) = 224.
time = 0.47, size = 326, normalized size = 2.36

$$\frac{\frac{2b^3\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2a}{2\sqrt{-a^2+1}}\right)}{(a^2-1)\sqrt{-a^2+1}}+\frac{4\left(2a^2b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-4a^3b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-2ab^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4+8a^2b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2+4b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-8ab^3\right)}{\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4a\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2+4\right)^2(a^2-1)}-\frac{2(bx+a+1)b^3-ab^3-2b^3}{b^2x^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqr
t(-a^2 + 1))/((a^2 - 1)*sqrt(-a^2 + 1)) + 4*(2*a^2*b^3*(sqrt(b*x + a + 1) -
 sqrt(b*x + a - 1))^6 - 4*a^3*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4
 - b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 2*a*b^3*(sqrt(b*x + a +
1) - sqrt(b*x + a - 1))^4 + 8*a^2*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 + 4*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 8*a*b^3)/(((sqrt(b
*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))^2 + 4)^2*(a^2 - 1)) - (2*(b*x + a + 1)*b^3 - a*b^3 - 2*b^3)/(b^2*x^2)
)/b
```

Mupad [**B**]
time = 14.15, size = 958, normalized size = 6.94

$$\frac{b^2\ln\left(\frac{\sqrt{a-1}-\sqrt{a+bx-1}}{\sqrt{a+1}-\sqrt{a+bx+1}}\right)\sqrt{a-1}\sqrt{a+1}}{2a^4-4a^2+2}-\frac{\frac{ab^2\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^5}{8(a^2-1)\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^5}-\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^3\left(\frac{3ab^2}{8}-\frac{7a^3b^2}{8}\right)}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^3(a^4-2a^2+1)}-\frac{b^2\sqrt{a-1}\sqrt{a+1}}{32(a^2-1)}+\frac{ab^2\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)}{4(a^2-1)\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)}+\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^2\left(\frac{b^2}{16}-\frac{11a^2b^2}{16}\right)\sqrt{a-1}\sqrt{a+1}}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^2(a^4-2a^2+1)}+\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^4\sqrt{a-1}\sqrt{a+1}\left(-\frac{17a^4b^2}{32}+\frac{9a^2b^2}{16}+\frac{15b^2}{32}\right)}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^4(a^6-3a^4+3a^2-1)}}{\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^2}+\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^6}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^6}+\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^4(6a^2-2)}{(a^2-1)\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^4}-\frac{4a\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^3\sqrt{a-1}\sqrt{a+1}}{(a^2-1)\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^3}-\frac{4a\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^5\sqrt{a-1}\sqrt{a+1}}{(a^2-1)\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^5}}-\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)\left(\frac{ab^2}{2(a-1)(a+1)}-\frac{3ab^2(a^2-1)^2}{8(a-1)^3(a+1)^3}\right)}{\sqrt{a+1}-\sqrt{a+bx+1}}-\frac{b^2\ln\left(\frac{\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^2}-a^2-\frac{a^2\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^2}+\frac{2a\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)\sqrt{a-1}\sqrt{a+1}}{\sqrt{a+1}-\sqrt{a+bx+1}}+1\right)\sqrt{a-1}\sqrt{a+1}}{2a^4-4a^2+2}-\frac{\frac{a}{2}+bx}{x^2}+\frac{b^2(a^2-1)\left(\sqrt{a-1}-\sqrt{a+bx-1}\right)^2}{32\left(\sqrt{a+1}-\sqrt{a+bx+1}\right)^2(a-1)^{3/2}(a+1)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^3,x)
```

```
[Out] (b^2*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x +
1)^(1/2)))*(a - 1)^(1/2)*(a + 1)^(1/2))/(2*a^4 - 4*a^2 + 2) - ((a*b^2*((a -
 1)^(1/2) - (a + b*x - 1)^(1/2))^5)/(8*(a^2 - 1)*((a + 1)^(1/2) - (a + b*x
+ 1)^(1/2))^5) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((3*a*b^2)/8 - (7
*a^3*b^2)/8))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3*(a^4 - 2*a^2 + 1)) -
 (b^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(32*(a^2 - 1)) + (a*b^2*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2)))/(4*(a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))
+ (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*(b^2/16 - (11*a^2*b^2)/16)*(a -
1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(a^4 - 2*a
^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a + 1)^(
1/2)*((15*b^2)/32 + (9*a^2*b^2)/16 - (17*a^4*b^2)/32))/(((a + 1)^(1/2) - (a
 + b*x + 1)^(1/2))^4*(3*a^2 - 3*a^4 + a^6 - 1)))/(((a - 1)^(1/2) - (a + b*x
 - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + ((a - 1)^(1/2) - (
a + b*x - 1)^(1/2))^6/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (((a - 1)^(
1/2) - (a + b*x - 1)^(1/2))^4*(6*a^2 - 2))/((a^2 - 1)*((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^4) - (4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(a - 1)^
(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) -
(4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2))/(
(a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5)) - (((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))*((a*b^2)/(2*(a - 1)*(a + 1)) - (3*a*b^2*(a^2 - 1)^2)/(8*(
a - 1)^3*(a + 1)^3)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) - (b^2*log(((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2
 - a^2 - (a^2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a
+ b*x + 1)^(1/2))^2 + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a - 1)^(1
/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(a - 1)^(1/2)
*(a + 1)^(1/2))/(2*a^4 - 4*a^2 + 2) - (a/2 + b*x)/x^2 + (b^2*(a^2 - 1)*((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/(32*((a + 1)^(1/2) - (a + b*x + 1)^(1/
2))^2*(a - 1)^(3/2)*(a + 1)^(3/2))
```

3.282 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1+a}{3\left(1-a^2\right)x^3}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2+1/3*(b*x+a-1)^(3/2)*(b*x+a+1)^(3/2)/(-a^2+1)/x^3-a*b^3
*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(
5/2)-1/2*a*b*(b*x+a+1)^(3/2)*(b*x+a-1)^(1/2)/(1-a)/(1+a)^2/x^2+1/2*a*b^2*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/x
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6020, 14, 98, 96, 95, 211}

$$-\frac{ab^3\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{5/2}}+\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)^2x}+\frac{(a+bx-1)^{3/2}(a+bx+1)^{3/2}}{3\left(1-a^2\right)x^3}-\frac{ab\sqrt{a+bx-1}\,(a+bx+1)^{3/2}}{2(1-a)(a+1)^2x^2}-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2) + (a*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1
 - a^2)^2*x) - (a*b*Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a)*(1 +
 a)^2*x^2) + ((-1 + a + b*x)^(3/2)*(1 + a + b*x)^(3/2))/(3*(1 - a^2)*x^3) -
 (a*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b
*x])])/(1 - a^2)^(5/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 6020

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^4}\,dx$$

$$= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^4}\right)dx$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^4}\,dx$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\,(1-a^2)\,x^3}+\frac{(ab)\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3} \ldots}{1-a^2}$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{ab\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\,(1-a^2)\,x^3}+\ldots$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\,(1-a^2)^2\,x}-\frac{ab\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\ldots$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\,(1-a^2)^2\,x}-\frac{ab\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\ldots$$

$$= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\,(1-a^2)^2\,x}-\frac{ab\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 179, normalized size = 0.95

$$\frac{1}{6}\left(-\frac{2a}{x^3}-\frac{3b}{x^2}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,(-2-2a^4+abx-a^3bx+2b^2x^2+a^2(4+b^2x^2))}{(-1+a^2)^2\,x^3}-\frac{3iab^3\log\left(\frac{4(1-a^2)^{3/2}\left(-i+ia^2+iabx+\sqrt{1-a^2}\,\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{ab^3x}\right)}{(1-a^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - 2*a^4
 + a*b*x - a^3*b*x + 2*b^2*x^2 + a^2*(4 + b^2*x^2)))/((-1 + a^2)^2*x^3) - (
(3*I)*a*b^3*Log[(4*(1 - a^2)^(3/2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sq
rt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(a*b^3*x)])/(1 - a^2)^(5/2))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(157) = 314$.
time = 0.07, size = 374, normalized size = 1.98

method	result
default	$-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(3\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)a\,b^3x^3-a^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(3*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a*b^3*x^3-a^4*b^2*x^2*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)+a^5*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+2*a^6*(b^2*x^2+
2*a*b*x+a^2-1)^(1/2)-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a^2*b^2*x^2-2*a^3*b*x*(b
^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*a^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+2*(b^2*x^2+
2*a*b*x+a^2-1)^(1/2)*b^2*x^2+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a*b*x+6*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*a^2-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x
+a^2-1)^(1/2)/(a^2-1)^3/x^3-1/3*a/x^3-1/2*b/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.40, size = 431, normalized size = 2.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(3*sqrt(a^2 - 1)*a*b^3*x^3*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a
 - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1)
 - a)/x) - 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4
 + 3*a^2 - 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3
```

```
 + a)*b*x + 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3
*a^4 + 3*a^2 - 1)*x^3), 1/6*(6*sqrt(-a^2 + 1)*a*b^3*x^3*arctan(-(sqrt(-a^2
+ 1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) -
 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4 + 3*a^2 -
 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3 + a)*b*x
+ 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3*a^4 + 3*a
^2 - 1)*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(151) = 302.
time = 0.54, size = 487, normalized size = 2.58

$$-\frac{\frac{6ab^4\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2a}{2\sqrt{-a^2+1}}\right)}{(a^4-2a^2+1)\sqrt{-a^2+1}}-\frac{4\left(12a^4b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^8-16a^5b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-3ab^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^{10}+6a^2b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^8-56a^3b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6+48a^4b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4+12b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^8-48ab^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6+192a^2b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-96a^3b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-144ab^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2+32a^2b^4+64b^4\right)}{(a^4-2a^2+1)\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4a\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2+4\right)^3}+\frac{3(bx+a+1)b^4-ab^4-3b^4}{b^3x^3}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*b^4*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/
sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - 4*(12*a^4*b^4*(sqrt(b*
x + a + 1) - sqrt(b*x + a - 1))^8 - 16*a^5*b^4*(sqrt(b*x + a + 1) - sqrt(b*
x + a - 1))^6 - 3*a*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 6*a^2*
b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 56*a^3*b^4*(sqrt(b*x + a +
1) - sqrt(b*x + a - 1))^6 + 48*a^4*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a -
1))^4 + 12*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 48*a*b^4*(sqrt(b
*x + a + 1) - sqrt(b*x + a - 1))^6 + 192*a^2*b^4*(sqrt(b*x + a + 1) - sqrt(
b*x + a - 1))^4 - 96*a^3*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 14
4*a*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 + 32*a^2*b^4 + 64*b^4)/((
a^4 - 2*a^2 + 1)*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x
 + a + 1) - sqrt(b*x + a - 1))^2 + 4)^3) + (3*(b*x + a + 1)*b^4 - a*b^4 - 3
*b^4)/(b^3*x^3))/b
```

Mupad [**B**]
time = 18.89, size = 1537, normalized size = 8.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^4,x)
[Out] ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((3*b^3)/32 - (a^2*b^3)/32))/(((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(a^4 - 2*a^2 + 1)) - b^3/(192*(a^2 - 1
)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*((9*a^2*b^3)/8 - b^3/2 + (5*a
^4*b^3)/8))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4*(3*a^2 - 3*a^4 + a^6 -
 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*((a^2*b^3)/32 - (21*b^3)/64
 + (3*a^4*b^3)/64))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8*(3*a^2 - 3*a^4
 + a^6 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*((103*b^3)/96 - (12
1*a^2*b^3)/32 + (11*a^4*b^3)/32 + (67*a^6*b^3)/96))/(((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^6*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) - (((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))^3*(a - 1)^(1/2)*(a + 1)^(1/2)*((17*a*b^3)/32 + (17*a^3*b
^3)/96))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3*(3*a^2 - 3*a^4 + a^6 - 1)
) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^7*(a - 1)^(1/2)*(a + 1)^(1/2)*((
3*a^3*b^3)/16 - (63*a*b^3)/32 + (9*a^5*b^3)/32))/(((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^7*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) - (((a - 1)^(1/2) - (a +
b*x - 1)^(1/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2)*((17*a^3*b^3)/16 - (79*a*b^3)
/32 + (29*a^5*b^3)/32))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5*(6*a^4 - 4
*a^2 - 4*a^6 + a^8 + 1)) + (a*b^3*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a
- 1)^(1/2)*(a + 1)^(1/2))/(32*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(a^4 -
2*a^2 + 1)))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3/((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^3 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9/((a + 1)^(1/2)
 - (a + b*x + 1)^(1/2))^9 + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(15*a^
2 - 3))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a - 1)^(1/
2) - (a + b*x - 1)^(1/2))^7*(15*a^2 - 3))/((a^2 - 1)*((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^7) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(12*a - 20*a^
3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6*(a
^4 - 2*a^2 + 1)) - (6*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/
2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) - (6*
a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^
2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8)) - (a/3 + (b*x)/2)/x^3 + ((
(a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((b^3*(1792*a^4 - 2048*a^2 + 256))/(40
96*(a^2 - 1)^3) - (b^3*(25*a^2 - 9))/(64*(a^2 - 1)^2) + (8*a*(a - 1)^(3/2)*
(a + 1)^(3/2)*((a*b^3*(a - 1)^(1/2)*(a + 1)^(1/2))/(8*(a^2 - 1)^2) - (a*b^3
*(a - 1)^(3/2)*(a + 1)^(3/2))/(8*(a^2 - 1)^3)))/(a^2 - 1)^2))/((a + 1)^(1/2
) - (a + b*x + 1)^(1/2)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((a*b^3
*(a - 1)^(1/2)*(a + 1)^(1/2))/(16*(a^2 - 1)^2) - (a*b^3*(a - 1)^(3/2)*(a +
1)^(3/2))/(16*(a^2 - 1)^3)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (b^3
*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3)/(192*(a^2 - 1)*((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^3) + (a*b^3*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2
/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - a^2 - (a^2*((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + (2*a*((a - 1)
^(1/2) - (a + b*x - 1)^(1/2))*(a - 1)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) -
 (a + b*x + 1)^(1/2)) + 1)*(a - 1)^(1/2)*(a + 1)^(1/2))/(6*a^2 - 6*a^4 + 2*
a^6 - 2) - (a*b^3*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2)
```

```
- (a + b*x + 1)^(1/2)))*(a - 1)^(1/2)*(a + 1)^(1/2))/(6*a^2 - 6*a^4 + 2*a^6
 - 2)
```

3.283 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5}\,dx$

Optimal. Leaf size=238

$$-\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2\sqrt{-1+a+bx}\,\sqrt{1}}{24\left(1-a^2\right)^2x^2}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3-1/4*(4*a^2+1)*b^4*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(
1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(7/2)-1/4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2)/x^4+1/12*a*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x^3+1/24*(2*a^2+3
)*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/x^2+1/24*a*(2*a^2+13)*b^3*
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^3/x
```

Rubi [A]
time = 0.15, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6020, 14, 99, 156, 12, 95, 211}

$$-\frac{\left(4a^2+1\right)b^4\text{ArcTan}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{4\left(1-a^2\right)^{7/2}}+\frac{a\left(2a^2+13\right)b^3\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{24\left(1-a^2\right)^3x}+\frac{\left(2a^2+3\right)b^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{24\left(1-a^2\right)^2x^2}+\frac{ab\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{12\left(1-a^2\right)x^3}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4x^4}-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*x^4) + (
a*b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(12*(1 - a^2)*x^3) + ((3 + 2*a^2)
*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^2*x^2) + (a*(13 +
2*a^2)*b^3*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^3*x) - ((1 +
 4*a^2)*b^4*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a
 + b*x])])/(4*(1 - a^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 6020

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\,dx \\
&= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\right)dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\,dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{1}{4}\int \frac{ab+b^2x}{x^4\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\int}{x^3\cdots} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{(3+\cdots}{\cdots} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{(3+\cdots}{\cdots} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{(3+\cdots}{\cdots} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{(3+\cdots}{\cdots} \\
&= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{(3+\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 198, normalized size = 0.83

$$
\frac{1}{24}\left(-\frac{6a}{x^4}-\frac{8b}{x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\,\left(6+\frac{2abx}{-1+a^2}-\frac{(3+2a^2)b^2x^2}{(-1+a^2)^2}+\frac{a(13+2a^2)b^3x^3}{(-1+a^2)^3}\right)}{x^4}-\frac{3i(1+4a^2)\,b^4\log\left(\frac{16i(1-a^2)^{5/2}\left(-1+a^2+abx-i\sqrt{1-a^2}\,\sqrt{-1+a+bx}\,\sqrt{1+a+bx}\right)}{b^4(x+4a^2x)}\right)}{(1-a^2)^{7/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(6 + (2*a*b
*x)/(-1 + a^2) - ((3 + 2*a^2)*b^2*x^2)/(-1 + a^2)^2 + (a*(13 + 2*a^2)*b^3*x
```

```
^3)/(-1 + a^2)^3))/x^4 - ((3*I)*(1 + 4*a^2)*b^4*Log[((16*I)*(1 - a^2)^(5/2)
*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))
/(b^4*(x + 4*a^2*x))])/(1 - a^2)^(7/2))/24
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(198) = 396$.
time = 0.06, size = 603, normalized size = 2.53

method	result
default	$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(12\sqrt{a^2-1}\,\ln\left(\frac{2a^2-2+2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}}{x}\right)a^2b^4x^4-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(12*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*a^2*b^4*x^4-2*a^5*b^3*x^3*(b^
2*x^2+2*a*b*x+a^2-1)^(1/2)+2*a^6*b^2*x^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+3*(a
^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/
x)*b^4*x^4-2*a^7*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-11*(b^2*x^2+2*a*b*x+a^2-
1)^(1/2)*a^3*b^3*x^3-6*a^8*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-a^4*b^2*x^2*(b^2*x
^2+2*a*b*x+a^2-1)^(1/2)+6*a^5*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+13*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*a*b^3*x^3+24*a^6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-4*(b^2
*x^2+2*a*b*x+a^2-1)^(1/2)*a^2*b^2*x^2-6*a^3*b*x*(b^2*x^2+2*a*b*x+a^2-1)^(1/
2)-36*a^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*b^2
*x^2+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a*b*x+24*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
*a^2-6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)
^4/x^4-1/3*b/x^3-1/4*a/x^4
```

Maple [**F(-2)**]

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see 'assume?' for more det
ails)Is
```

Fricas [**A**]
time = 0.44, size = 569, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/24*(3*(4*a^2 + 1)*sqrt(a^2 - 1)*b^4*x^4*log((a^2*b*x + a^3 + (a^2 + sqrt
(a^2 - 1)*a - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sq
rt(a^2 - 1) - a)/x) - 6*a^9 - (2*a^5 + 11*a^3 - 13*a)*b^4*x^4 + 24*a^7 - 36
*a^5 + 24*a^3 - 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 +
 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4 - 4*a^2 + 3)*b^2*x^2 + 36*a
^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 + 6)*sqrt(b*x + a + 1)*sqrt(b
*x + a - 1) - 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*x^4), -1/24*(6*(4*a^2
 + 1)*sqrt(-a^2 + 1)*b^4*x^4*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(-a^2 + 1)*s
qrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + 6*a^9 + (2*a^5 + 11*a^3 -
13*a)*b^4*x^4 - 24*a^7 + 36*a^5 - 24*a^3 + 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 +
 1)*b*x + (6*a^8 + (2*a^5 + 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4
- 4*a^2 + 3)*b^2*x^2 + 36*a^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 +
6)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2
 + 1)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(192) = 384.
time = 0.57, size = 817, normalized size = 3.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(4*a^2*b^5 + b^5)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 - 2*a)/sqrt(-a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*sqrt(-a^2 + 1)) + 2*
(128*a^6*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 12*a^2*b^5*(sqrt(
b*x + a + 1) - sqrt(b*x + a - 1))^14 - 128*a^7*b^5*(sqrt(b*x + a + 1) - sqr
t(b*x + a - 1))^8 - 168*a^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12
+ 448*a^4*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 3*b^5*(sqrt(b*x
+ a + 1) - sqrt(b*x + a - 1))^14 - 1216*a^5*b^5*(sqrt(b*x + a + 1) - sqrt(b
```

```
*x + a - 1))^8 - 42*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12 + 512*
a^6*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 + 768*a^2*b^5*(sqrt(b*x +
 a + 1) - sqrt(b*x + a - 1))^10 - 2544*a^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*
x + a - 1))^8 + 5632*a^4*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 84
*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 - 1536*a^5*b^5*(sqrt(b*x +
a + 1) - sqrt(b*x + a - 1))^4 - 312*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a
 - 1))^8 + 1920*a^2*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 7552*a^
3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 1024*a^4*b^5*(sqrt(b*x +
a + 1) - sqrt(b*x + a - 1))^2 + 336*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a -
 1))^6 - 992*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 5888*a^2*b^5
*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 256*a^3*b^5 - 192*b^5*(sqrt(b*
x + a + 1) - sqrt(b*x + a - 1))^2 - 1664*a*b^5)/((a^6 - 3*a^4 + 3*a^2 - 1)*
((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(
b*x + a - 1))^2 + 4)^4) - (4*(b*x + a + 1)*b^5 - a*b^5 - 4*b^5)/(b^4*x^4))/
b
```

Mupad [B]
time = 28.03, size = 2500, normalized size = 10.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^5,x)
```

```
[Out] (log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(
1/2)))*(b^4*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^2*b^4*(a - 1)^(1/2)*(a + 1)^(
1/2)))/(48*a^4 - 32*a^2 - 32*a^6 + 8*a^8 + 8) - (a/4 + (b*x)/3)/x^4 - (log(
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^2 - a^2 - (a^2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) -
 (a + b*x + 1)^(1/2))^2 + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a - 1
)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(b^4*(a -
 1)^(1/2)*(a + 1)^(1/2) + 4*a^2*b^4*(a - 1)^(1/2)*(a + 1)^(1/2)))/(48*a^4 -
 32*a^2 - 32*a^6 + 8*a^8 + 8) - ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(
(17*a*b^4)/192 - (5*a^3*b^4)/192))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3
*(3*a^2 - 3*a^4 + a^6 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11*((7
*a^3*b^4)/64 - (81*a*b^4)/128 + (3*a^5*b^4)/128))/(((a + 1)^(1/2) - (a + b*
x + 1)^(1/2))^11*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) + (((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))^5*((229*a^3*b^4)/64 - (119*a*b^4)/128 + (119*a^5*b^4)/384
))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5*(6*a^4 - 4*a^2 - 4*a^6 + a^8 +
1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9*((1025*a*b^4)/384 - (1745*a^
3*b^4)/128 + (385*a^5*b^4)/128 + (239*a^7*b^4)/384))/(((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^9*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) + (((a - 1
)^(1/2) - (a + b*x - 1)^(1/2))^7*((1103*a*b^4)/384 - (2199*a^3*b^4)/128 + (
1039*a^5*b^4)/128 + (521*a^7*b^4)/384))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/
2))^7*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) - (b^4*(a - 1)^(1/2)*(a
```

 + 1)^(1/2))/(1024*(a^4 - 2*a^2 + 1)) + (a*b^4*((a - 1)^(1/2) - (a + b*x -
1)^(1/2)))/(192*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(a^4 - 2*a^2 + 1)) +
(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*(b^4/128 - (a^2*b^4)/384)*(a - 1)^
(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(3*a^2 - 3*a^
4 + a^6 - 1)) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a +
 1)^(1/2)*((11*b^4)/512 + (249*a^2*b^4)/256 + (a^4*b^4)/1536))/(((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^4*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) + (((a - 1)
^(1/2) - (a + b*x - 1)^(1/2))^10*(a - 1)^(1/2)*(a + 1)^(1/2)*(b^4/256 + (11
67*a^2*b^4)/256 - (225*a^4*b^4)/256 - (47*a^6*b^4)/256))/(((a + 1)^(1/2) -
(a + b*x + 1)^(1/2))^10*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) + (((
a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(a - 1)^(1/2)*(a + 1)^(1/2)*((7*b^4)/
256 + (1665*a^2*b^4)/256 - (5365*a^4*b^4)/768 - (707*a^6*b^4)/768))/(((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^6*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 -
 1)) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/2)
*((8765*a^2*b^4)/768 - (239*b^4)/1024 - (11789*a^4*b^4)/512 + (1471*a^6*b^4
)/256 + (3635*a^8*b^4)/3072))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8*(15*
a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1)))/(((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^4/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 + ((a - 1)^(1/2)
 - (a + b*x - 1)^(1/2))^12/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^12 + (((a
 - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(28*a^2 - 4))/((a^2 - 1)*((a + 1)^(1/2)
 - (a + b*x + 1)^(1/2))^6) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(28*
a^2 - 4))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^10) + (((a - 1)^
(1/2) - (a + b*x - 1)^(1/2))^8*(70*a^4 - 60*a^2 + 6))/(((a + 1)^(1/2) - (a
+ b*x + 1)^(1/2))^8*(a^4 - 2*a^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^7*(24*a - 56*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^7*(a^4 - 2*a^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2
))^9*(24*a - 56*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*
x + 1)^(1/2))^9*(a^4 - 2*a^2 + 1)) - (8*a*((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x +
1)^(1/2))^5) - (8*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11*(a - 1)^(1/2)*
(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^11)) - (((a
 - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((5*a*(a^2 - 1)^2*((a*b^4)/(32*(a - 1)
^2*(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(128*(a - 1)^5*(a + 1)^5)))/((a - 1)^
(5/2)*(a + 1)^(5/2)) - (b^4*(23*a^2 - 7))/(512*(a - 1)^(5/2)*(a + 1)^(5/2))
 + (5*b^4*(a^2 - 1)*(9*a^2 - 1))/(512*(a - 1)^(7/2)*(a + 1)^(7/2))))/((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^
3*((a*b^4)/(96*(a - 1)^2*(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(384*(a - 1)^5*
(a + 1)^5)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3 + (((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))*((5*((a*b^4)/(32*(a - 1)^2*(a + 1)^2) - (5*a*b^4*(a^2 -
1)^3)/(128*(a - 1)^5*(a + 1)^5))*(9*a^2 - 1))/((a - 1)*(a + 1)) - (a*b^4)/(
16*(a - 1)^2*(a + 1)^2) - (10*a*(a^2 - 1)^2*((10*a*(a^2 - 1)^2*((a*b^4)/(32
(a - 1)^2(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(128*(a - 1)^5*(a + 1)^5)))/(
(a - 1)^(5/2)*(a + 1)^(5/2)) - (b^4*(23*a^2 - 7))/(256*(a - 1)^(5/2)*(a + 1
)^(5/2)) + (5*b^4*(a^2 - 1)*(9*a^2 - 1))/(256*(a - 1)^(7/2)*(a + 1)^(7/2)))
)/((a - 1)^(5/2)*(a + 1)^(5/2)) + (5*a*b^4*(a^2 - 1)*(3*a^4 - 4*a^2 + 1))/(

```
32*(a - 1)^5*(a + 1)^5)))/((a + 1)^(1/2) - (a +...
```

3.284 $\int e^{\cosh^{-1}(a+bx)^2} x^3 \, dx$

Optimal. Leaf size=359

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{16b^4e}+\frac{3a^2\sqrt{\pi}\,\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^4e}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2-\cosh^{-1}(a+bx)\right)}{32b^4e^4}+\frac{\sqrt{\pi}\,\text{Erfi}(}{}$$

```
[Out] -1/32*erfi(-2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(4)-1/16*erfi(-1+arccosh(b*x+
a))*Pi^(1/2)/b^4/exp(1)-3/8*a^2*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1)
+1/16*erfi(1+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1)+3/8*a^2*erfi(1+arccosh(b*x
+a))*Pi^(1/2)/b^4/exp(1)+1/32*erfi(2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(4)+3/
16*a*erfi(-3/2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+arcco
sh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)+1/4*a^3*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)
/b^4/exp(1/4)-3/16*a*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3
*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arccosh(b*x
+a))*Pi^(1/2)/b^4/exp(9/4)
```

Rubi [A]
time = 0.54, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6015, 6873, 12, 6874, 5623, 2266, 2235, 5625}

$$\frac{\sqrt{\pi}\,a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^4}-\frac{\sqrt{\pi}\,a^3\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^4}+\frac{3\sqrt{\pi}\,a^2\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}\,a^2\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^4}+\frac{3\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-3\right)\right)}{16e^{9/4}b^4}+\frac{3\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{16\sqrt[4]{e}\,b^4}-\frac{3\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{16\sqrt[4]{e}\,b^4}-\frac{3\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+3\right)\right)}{16e^{9/4}b^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{16eb^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(2-\cosh^{-1}(a+bx)\right)}{32e^4b^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{16eb^4}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\cosh^{-1}(a+bx)+2\right)}{32e^4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1 -
 ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcCosh[a + b*x]])/(32*b
^4*E^4) + (Sqrt[Pi]*Erfi[1 + ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi
]*Erfi[1 + ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcCosh[a + b*
x]])/(32*b^4*E^4) + (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/(16*b^
4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/
4)) + (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3
*a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt[
Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi[
(3 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2} x^3 \, dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3 \sinh(x) \, dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))^3 \sinh(x)}{b^3} \, dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))^3 \sinh(x) \, dx, x, \cosh^{-1}(a+bx)\right)}{b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-a^3 e^{x^2} \sinh(x) + 3a^2 e^{x^2} \cosh(x)\sinh(x) - 3ae^{x^2}\cosh^2(x)\sinh(x) + e^{x^2} \cos\right.\right.}{b^4}$$

$$= \frac{\text{Subst}\left(\int e^{x^2} \cosh^3(x)\sinh(x) \, dx, x, \cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\text{Subst}\left(\int e^{x^2} \cosh^2(x) \sinh\right.}{b^4}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2} - \frac{1}{8}e^{-2x+x^2} + \frac{1}{8}e^{2x+x^2} + \frac{1}{16}e^{4x+x^2}\right) dx, x, \cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3}{}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x+x^2} \, dx, x, \cosh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Subst}\left(\int e^{4x+x^2} \, dx, x, \cosh^{-1}(a+bx)\right)}{16b^4} -$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2} \, dx, x, \cosh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2} \, dx, x, \cosh^{-1}(a+b\right.}{16b^4e^4}$$

$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{16b^4e} + \frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^4e} + \frac{\sqrt{\pi}\,\text{erfi}\left(2-\cos\right.}{32b^4e}$$

Mathematica [A]
time = 0.23, size = 198, normalized size = 0.55

$$\frac{\sqrt{\pi}\left(-2a(3+4a^2)e^{15/4}\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+2(1+6a^2)e^3\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)-6ae^{7/4}\text{Erfi}\left(\frac{3}{2}-\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(2-\cosh^{-1}(a+bx)\right)-6ae^{15/4}\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)-8a^3e^{15/4}\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)+2e^3\text{Erfi}\left(1+\cosh^{-1}(a+bx)\right)+12a^2e^3\text{Erfi}\left(1+\cosh^{-1}(a+bx)\right)-6ae^{7/4}\text{Erfi}\left(\frac{3}{2}+\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(2+\cosh^{-1}(a+bx)\right)\right)}{32b^4e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*(3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcCosh[a + b*x]] + 2*(1 +
6*a^2)*E^3*Erfi[1 - ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 - ArcCosh[a +
b*x]] + Erfi[2 - ArcCosh[a + b*x]] - 6*a*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*
x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*x]] + 2*E^3*Erfi[1 + ArcCosh[
a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
ArcCosh[a + b*x]] + Erfi[2 + ArcCosh[a + b*x]]))/(32*b^4*E^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arccosh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(acosh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{e}^{\mathrm{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(acosh(a + b*x)^2),x)
[Out] int(x^3*exp(acosh(a + b*x)^2), x)
```

3.285 $\int e^{\cosh^{-1}(a+bx)^2} x^2 \, dx$

Optimal. Leaf size=251

$$-\frac{a\sqrt{\pi}\,\mathrm{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4b^3e}-\frac{a\sqrt{\pi}\,\mathrm{Erfi}\left(1+\cosh^{-1}(a+bx)\right)}{4b^3e}-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(-3+2\cosh^{-1}(a+bx)\right)\right)}{16b^3e^{9/4}}-$$

```
[Out] 1/4*a*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1)-1/4*a*erfi(1+arccosh(b*x+
a))*Pi^(1/2)/b^3/exp(1)-1/16*erfi(-3/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(9/4
)-1/16*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)-1/4*a^2*erfi(-1/2+ar
ccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/
b^3/exp(1/4)+1/4*a^2*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*er
fi(3/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)
```

Rubi [A]
time = 0.38, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6015, 6873, 12, 6874, 5623, 2266, 2235, 5625}

$$-\frac{\sqrt{\pi}a^2\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}a^2\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^3}-\frac{\sqrt{\pi}a\mathrm{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}a\mathrm{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{4eb^3}-\frac{\sqrt{\pi}\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-3\right)\right)}{16e^{9/4}b^3}-\frac{\sqrt{\pi}\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{16\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{16\sqrt[4]{e}b^3}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+3\right)\right)}{16e^{9/4}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x^2,x]
```

```
[Out] -1/4*(a*Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(b^3*E) - (a*Sqrt[Pi]*Erfi[1 +
 ArcCosh[a + b*x]])/(4*b^3*E) - (Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2]
)/(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E
^(1/4)) - (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4))
+ (Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[
Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 +
 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2}x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(a-\cosh(x))^2\sinh(x)}{b^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}(a-\cosh(x))^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(a^2e^{x^2}\sinh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh^2(x)\sinh(x)\right)\,dx,x,\text{cosh}}{b^3}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh^2(x)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int e^{x^2}\cosh(x)\,\text{sin}\right.}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}+\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\text{S}}{}$$

$$= -\frac{\text{Subst}\left(\int e^{-3x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx,x,\cosh^{-1}(a+\right.}{8b^3e^{9/4}}$$

$$= -\frac{a\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4b^3e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{4b^3e} - \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-3\right.}{}$$

Mathematica [A]
time = 0.16, size = 136, normalized size = 0.54

$$\frac{\sqrt{\pi}\left(\left(1+4a^2\right)e^2\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)-4ae^{5/4}\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(\frac{3}{2}-\cosh^{-1}(a+bx)\right)+e^2\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)+4a^2e^2\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)-4ae^{5/4}\text{Erfi}\left(1+\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(\frac{3}{2}+\cosh^{-1}(a+bx)\right)\right)}{16b^3e^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^2,x]
```

```
[Out] (Sqrt[Pi]*((1 + 4*a^2)*E^2*Erfi[1/2 - ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[
1 - ArcCosh[a + b*x]] + Erfi[3/2 - ArcCosh[a + b*x]] + E^2*Erfi[1/2 + ArcCo
sh[a + b*x]] + 4*a^2*E^2*Erfi[1/2 + ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[1
+ ArcCosh[a + b*x]] + Erfi[3/2 + ArcCosh[a + b*x]]))/(16*b^3*E^(9/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int {\mathrm e}^{\operatorname{arccosh}\left(b x +a \right)^{2}} x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arccosh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**2,x)
```

```
[Out] Integral(x**2*exp(acosh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \, e^{\operatorname{acosh}(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(acosh(a + b*x)^2),x)
```

```
[Out] int(x^2*exp(acosh(a + b*x)^2), x)
```

3.286 $\int e^{\cosh^{-1}(a+bx)^2} x\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^2e}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(1+\cosh^{-1}(a+bx)\right)}{8b^2e}+\frac{a\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx)\right)\right)}{4b^2\sqrt[4]{e}}-\frac{a\sqrt{\pi}$$

```
[Out] -1/8*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^2/exp(1)+1/8*erfi(1+arccosh(b*x+a))
*Pi^(1/2)/b^2/exp(1)+1/4*a*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)-
1/4*a*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

Rubi [A]
time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6015, 6873, 12, 6874, 5623, 2266, 2235, 5625}

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^2}+\frac{\sqrt{\pi}\,a\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\mathrm{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcCos
h[a + b*x]])/(8*b^2*E) + (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 6015

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCosh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2}x\,dx=\frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$=\frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))\sinh(x)}{b}\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$=\frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^2}$$
$$=\frac{\text{Subst}\left(\int \left(-ae^{x^2}\sinh(x)+e^{x^2}\cosh(x)\sinh(x)\right)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^2}$$
$$=\frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Subst}\left(\int e^{x^2}\sinh(x)\,dx,x,\cosh\right.}{b^2}$$
$$=\frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Subst}\left(\int \left(-\frac{1}{2}e^{-x+x^2}+\right.\right.}{}$$
$$=-\frac{\text{Subst}\left(\int e^{-2x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{4b^2}+\frac{\text{Subst}\left(\int e^{2x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{4b^2}+$$
$$=-\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{4b^2e}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx,x,\cosh^{-1}(a+bx\right.}{4b^2e}$$
$$=\frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^2e}+\frac{\sqrt{\pi}\,\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{8b^2e}+\frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-1+2\right.}{4b^2\sqrt[4]{e}}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(-2ae^{3/4}\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)-2ae^{3/4}\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(1+\cosh^{-1}(a+bx)\right)\right)}{8b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x,x]
[Out] (Sqrt[Pi]*(-2*a*E^(3/4)*Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1 - ArcCosh[a +
 b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcCosh[a + b*x]] + Erfi[1 + ArcCosh[a + b*
x]]))/(8*b^2*E)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arccosh(b*x + a)^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\operatorname{acosh}^{2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x,x)
```

```
[Out] Integral(x*exp(acosh(a + b*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{e}^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(acosh(a + b*x)^2),x)
```

```
[Out] int(x*exp(acosh(a + b*x)^2), x)
```

3.287 $\int e^{\cosh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}$$

```
[Out] -1/4*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arccosh(b*x
+a))*Pi^(1/2)/b/exp(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6014, 5623, 2266, 2235}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2,x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(b*E^(1/4)) + (Sqrt[Pi]*E
rfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 6014

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\cosh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int e^{-x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int e^{x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}\\
&= -\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 42, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(\text{Erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\text{Erfi}\left(\frac{1}{2}+\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*(Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1/2 + ArcCosh[a + b*x]]))/(4
*b*E^(1/4))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2),x)
```

```
[Out] int(exp(arccosh(b*x+a)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2),x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2),x)
```

```
[Out] int(exp(acosh(a + b*x)^2), x)
```

3.288 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x}, x\right)$$

```
[Out] CannotIntegrate(exp(arccosh(b*x+a)^2)/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x,x]
```

```
[Out] Integrate[E^ArcCosh[a + b*x]^2/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arccosh}(bx+a)^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acosh}^{2}(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x,x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\mathrm{acosh}(a+b\,x)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2)/x,x)
```

```
[Out] int(exp(acosh(a + b*x)^2)/x, x)
```

3.289 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x^2},x\right)$$

```
[Out] CannotIntegrate(exp(arccosh(b*x+a)^2)/x^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2}\,dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2}\,dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[E^ArcCosh[a + b*x]^2/x^2, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\operatorname{arccosh}(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x^2,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arccosh(b*x + a)^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acosh}^{2}(a+bx)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x**2,x)
```

```
[Out] Integral(exp(acosh(a + b*x)**2)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{e^{\operatorname{acosh}(a+b\,x)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2)/x^2,x)
[Out] int(exp(acosh(a + b*x)^2)/x^2, x)
```

3.290 $\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=60

$$-\frac{\cosh^{-1}(a+bx)^2}{2d}+\frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d}+\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(a+bx)}\right)}{2d}$$

```
[Out] -1/2*arccosh(b*x+a)^2/d+arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^2)/d+1/2*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5996, 12, 5882, 3799, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}(a+bx)}\right)}{2d}-\frac{\cosh^{-1}(a+bx)^2}{2d}+\frac{\cosh^{-1}(a+bx)\log\left(e^{2\cosh^{-1}(a+bx)}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -1/2*ArcCosh[a + b*x]^2/d + (ArcCosh[a + b*x]*Log[1 + E^(2*ArcCosh[a + b*x]
)])/d + PolyLog[2, -E^(2*ArcCosh[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5882

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Tanh[-a/b + x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5996

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\cosh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \text{Subst}\left(\int \log\left(1+e^{2}\right.\right.$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.}{2d}$$

$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(a+bx)}\right.}{2d}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.88

$$\frac{\cosh^{-1}(a+bx)\left(\cosh^{-1}(a+bx)+2\log\left(1+e^{-2\cosh^{-1}(a+bx)}\right)\right)-\text{PolyLog}\left(2,-e^{-2\cosh^{-1}(a+bx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (ArcCosh[a + b*x]*(ArcCosh[a + b*x] + 2*Log[1 + E^(-2*ArcCosh[a + b*x])]) -
 PolyLog[2, -E^(-2*ArcCosh[a + b*x])])/(2*d)
```

Maple [A]
time = 0.06, size = 92, normalized size = 1.53

method	result
derivativedivides	$\frac{-\frac{b\,\mathrm{arccosh}(bx+a)^2}{2d}+\frac{b\,\mathrm{arccosh}(bx+a)\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(bx+a+\sqrt{bx+\right.\right.}{}}{b}$
default	$\frac{-\frac{b\,\mathrm{arccosh}(bx+a)^2}{2d}+\frac{b\,\mathrm{arccosh}(bx+a)\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{d}+\frac{b\,\mathrm{polylog}\left(2,-\left(bx+a+\sqrt{bx+\right.\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*b/d*arccosh(b*x+a)^2+b/d*arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/
2)*(b*x+a+1)^(1/2))^2)+1/2*b/d*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^
(1/2))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccosh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int\frac{\operatorname{acosh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acosh(a + b*x)/(a + b*x), x)/d
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{acosh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(acosh(a + b*x)/(d*x + (a*d)/b), x)
```

3.291 $\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

```
[Out] Chi(arccosh(x))
```

Rubi [A]
time = 0.08, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5953, 3382}

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

```
[Out] CoshIntegral[ArcCosh[x]]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5953

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*
Simp[(d1 + e1*x)^p/(1 + c*x)^p]*Simp[(d2 + e2*x)^p/(-1 + c*x)^p], Subst[Int
[x^n*Cosh[-a/b + x/b]^m*Sinh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcCosh[c*
x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e
2, (-c)*d2] && IGtQ[p + 3/2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx &= \text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(x)\right)\\
&= \mathrm{Chi}\left(\cosh^{-1}(x)\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(3) = 6.
time = 0.04, size = 19, normalized size = 6.33

$$\frac{1}{2}(-1+x)\mathrm{Chi}\left(\cosh^{-1}(x)\right)\operatorname{csch}^2\left(\frac{1}{2}\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

```
[Out] ((-1 + x)*CoshIntegral[ArcCosh[x]]*Csch[ArcCosh[x]/2]^2)/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(3) = 6$.
time = 0.22, size = 78, normalized size = 26.00

method	result
default	$-\frac{\sqrt{-2+2x}\,\sqrt{2+2x}\,\sqrt{-1+x}\,\sqrt{1+x}\,\mathrm{expIntegral}(1,\mathrm{arccosh}(x))}{4(x^2-1)}-\frac{\sqrt{-2+2x}\,\sqrt{2+2x}\,\sqrt{-1+x}}{4(x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2+2*x)^(1/2)*(2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,arccosh(x))
/(x^2-1)-1/4*(-2+2*x)^(1/2)*(2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,-ar
ccosh(x))/(x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x + 1)*sqrt(x - 1)*x/((x^2 - 1)*arccosh(x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x-1}\,\sqrt{x+1}\,\operatorname{acosh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(x)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x - 1)*sqrt(x + 1)*acosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.33

$$\int \frac{x}{\mathrm{acosh}\left(x\right)\,\sqrt{x-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

3.292 $\int x^3 \cosh^{-1}(a+bx^4)\,dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{-1+a+bx^4}\,\sqrt{1+a+bx^4}}{4b}+\frac{(a+bx^4)\cosh^{-1}(a+bx^4)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arccosh(b*x^4+a)/b-1/4*(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 5995, 5879, 75}

$$\frac{(a+bx^4)\cosh^{-1}(a+bx^4)}{4b}-\frac{\sqrt{a+bx^4-1}\,\sqrt{a+bx^4+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x^4],x]
```

```
[Out] -1/4*(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4])/b + ((a + b*x^4)*ArcCosh[a
+ b*x^4])/(4*b)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^3\cosh^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \cosh^{-1}(a+bx)\,dx,x,x^4\right)\\ &= \frac{\text{Subst}\left(\int \cosh^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\ &= \frac{(a+bx^4)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,a+bx^4\right)}{4b}\\ &= -\frac{\sqrt{-1+a+bx^4}\,\sqrt{1+a+bx^4}}{4b}+\frac{(a+bx^4)\cosh^{-1}\left(a+bx^4\right)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.93

$$\frac{-\sqrt{-1+a+bx^4}\,\sqrt{1+a+bx^4}+(a+bx^4)\cosh^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x^4],x]
[Out] (-(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4]) + (a + b*x^4)*ArcCosh[a + b*x^
4])/(4*b)
```

Maple [A]
time = 0.01, size = 45, normalized size = 0.83

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arccosh}(b\,x^4+a)-\sqrt{b\,x^4+a-1}\,\sqrt{b\,x^4+a+1}}{4b}$	45
default	$\frac{(b\,x^4+a)\,\mathrm{arccosh}(b\,x^4+a)-\sqrt{b\,x^4+a-1}\,\sqrt{b\,x^4+a+1}}{4b}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arccosh(b*x^4+a)-(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2))
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.69

$$\frac{(bx^4+a)\,\mathrm{arcosh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arccosh(b*x^4 + a) - sqrt((b*x^4 + a)^2 - 1))/b
```

Fricas [A]
time = 0.33, size = 66, normalized size = 1.22

$$\frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))/b
```

Sympy [A]
time = 0.21, size = 61, normalized size = 1.13

$$\begin{cases} \frac{a\operatorname{acosh}\left(a+bx^4\right)}{4b}+\frac{x^4\operatorname{acosh}\left(a+bx^4\right)}{4}-\frac{\sqrt{a^2+2abx^4+b^2x^8-1}}{4b} & \text{for}\ b\neq 0 \\ \frac{x^4\operatorname{acosh}\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x**4+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x**4)/(4*b) + x**4*acosh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 - 1)/(4*b), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(46) = 92.
time = 0.41, size = 106, normalized size = 1.96

$$\frac{1}{4}\,x^4\log\left(bx^4+a+\sqrt{(bx^4+a)^2-1}\right)-\frac{1}{4}\,b\left(\frac{a\log\left(\left|-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,abx^4+a^2-1}}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 - 1)) - 1/4*b*(a*log(abs(-a*b -
(x^4*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))*abs(b)))/(b*abs(b)) + sq
rt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/b^2)
```

Mupad [B]

time = 4.68, size = 295, normalized size = 5.46

$$\frac{x^4\,\mathrm{acosh}(b\,x^4+a)}{4}-\frac{\frac{4\,a\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)}+\frac{4\,a\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^3}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^3}-\frac{8\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2\sqrt{a-1}\,\sqrt{a+1}}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}}{4\left(\frac{\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^4}-\frac{2\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}+1\right)}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{b\,x^4+a-1}}{\sqrt{a+1}-\sqrt{b\,x^4+a+1}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a + b*x^4),x)
```

```
[Out] (x^4*acosh(a + b*x^4))/4 - ((4*a*((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2)))/(
b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b*x
^4 - 1)^(1/2))^3)/(b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^3) - (8*((a -
1)^(1/2) - (a + b*x^4 - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1
)^(1/2) - (a + b*x^4 + 1)^(1/2))^2))/(4*(((a - 1)^(1/2) - (a + b*x^4 - 1)^(
1/2))^4/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a
+ b*x^4 - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^2 + 1)) + (a
*atanh(((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x^4
+ 1)^(1/2))))/b
```

3.293 $\int x^{-1+n} \cosh^{-1}(a+bx^n)\, dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{-1+a+bx^n}\,\sqrt{1+a+bx^n}}{bn}+\frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arccosh(a+b*x^n)/b/n-(-1+a+b*x^n)^(1/2)*(1+a+b*x^n)^(1/2)/b/n
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5995, 5879, 75}

$$\frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}-\frac{\sqrt{a+bx^n-1}\,\sqrt{a+bx^n+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

```
[Out] -((Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n])/(b*n)) + ((a + b*x^n)*ArcCosh[
a + b*x^n])/(b*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5995

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+n}\cosh^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \cosh^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \cosh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx^n\right)}{bn}\\
&= -\frac{\sqrt{-1+a+bx^n}\sqrt{1+a+bx^n}}{bn} + \frac{(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 50, normalized size = 0.91

$$
\frac{-\sqrt{-1+a+bx^n}\sqrt{1+a+bx^n}+(a+bx^n)\cosh^{-1}(a+bx^n)}{bn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCosh[a + b*x^n],x]
[Out] (-(Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n]) + (a + b*x^n)*ArcCosh[a + b*x^
n])/(b*n)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$
\int x^{-1+n}\operatorname{arccosh}(a+b\,x^n)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arccosh(a+b*x^n),x)
[Out] int(x^(-1+n)*arccosh(a+b*x^n),x)
```

Maxima [**A**]
time = 0.29, size = 39, normalized size = 0.71

$$
\frac{(bx^n+a)\operatorname{arcosh}(bx^n+a)-\sqrt{(bx^n+a)^2-1}}{bn}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arccosh(b*x^n + a) - sqrt((b*x^n + a)^2 - 1))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(51) = 102$.
time = 0.39, size = 152, normalized size = 2.76

$$\frac{(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a)\log\left(b\cosh(n\log(x)) + b\sinh(n\log(x)) + a + \sqrt{\frac{2\,ab + (a^2+b^2-1)\cosh(n\log(x)) - (a^2-b^2-1)\sinh(n\log(x))}{\cosh(n\log(x)) - \sinh(n\log(x))}}\right) - \sqrt{\frac{2\,ab + (a^2+b^2-1)\cosh(n\log(x)) - (a^2-b^2-1)\sinh(n\log(x))}{\cosh(n\log(x)) - \sinh(n\log(x))}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

Sympy [A]
time = 22.80, size = 76, normalized size = 1.38

$$\begin{cases} \log(x)\operatorname{acosh}(a) & \text{for}\: b = 0 \wedge n = 0 \\ \log(x)\operatorname{acosh}(a + b) & \text{for}\: n = 0 \\ \frac{x^n \operatorname{acosh}(a)}{n} & \text{for}\: b = 0 \\ \frac{a\operatorname{acosh}(a+bx^n)}{bn} + \frac{x^n\operatorname{acosh}(a+bx^n)}{n} - \frac{\sqrt{a^2 + 2abx^n + b^2x^{2n} - 1}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acosh(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*acosh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*acosh(a + b), Eq(
n, 0)), (x**n*acosh(a)/n, Eq(b, 0)), (a*acosh(a + b*x**n)/(b*n) + x**n*acos
h(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) - 1)/(b*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.
time = 0.42, size = 124, normalized size = 2.25

$$-\frac{b\left(\frac{a\log\left(\left|-ab - \left(x^n|b| - \sqrt{b^2x^{2n} + 2\,abx^n + a^2 - 1}\right)|b|\right|\right)}{b|b|} + \frac{\sqrt{b^2x^{2n} + 2\,abx^n + a^2 - 1}}{b^2}\right) - x^n\log\left(bx^n + a + \sqrt{b^2x^{2n} + 2\,abx^n + a^2 - 1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(abs(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
)*abs(b)))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)/b^2) - x^n*
log(b*x^n + a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)))/n
```

Mupad [B]
time = 1.07, size = 303, normalized size = 5.51

$$\frac{x^n\,\mathrm{acosh}(a+b\,x^n)}{n}-\frac{\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^3}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^3}+\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}-\frac{8\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^2\,\sqrt{a-1}\,\sqrt{a+1}}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^2}}{n\,\left(\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^4}-\frac{2\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)^2}+1\right)}+\frac{4\,a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x^n-1}}{\sqrt{a+1}-\sqrt{a+b\,x^n+1}}\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acosh(a + b*x^n),x)
```

```
[Out] (x^n*acosh(a + b*x^n))/n - ((4*a*((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))^3)
/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^3) + (4*a*((a - 1)^(1/2) - (a +
 b*x^n - 1)^(1/2)))/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))) - (8*((a -
1)^(1/2) - (a + b*x^n - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1
)^(1/2) - (a + b*x^n + 1)^(1/2))^2))/(n*(((a - 1)^(1/2) - (a + b*x^n - 1)^(
1/2))^4/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a
+ b*x^n - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^2 + 1)) + (4
*a*atanh(((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x^
n + 1)^(1/2))))/(b*n)
```

3.294 $\int \cosh^{-1}\left(\frac{c}{a+bx}\right)\,dx$

Optimal. Leaf size=58

$$\frac{(a+bx)\text{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\text{ArcTan}\left(\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}\right)}{b}$$

[Out] (b*x+a)*arcsech(a/c+b*x/c)/b-2*c*arctan((((1-a/c)*c-b*x)/(b*x+a+c))^(1/2))/b

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6012, 6448, 1983, 12, 209}

$$\frac{(a+bx)\text{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\text{ArcTan}\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcSech[a/c + (b*x)/c])/b - (2*c*ArcTan[Sqrt[((1 - a/c)*c - b*x)
/(a + c + b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
```

```
gerQ[r]
```

Rule 6012

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6448

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSech[c + d*
x]/d), x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cosh^{-1}\left(\frac{c}{a+bx}\right) dx &= \int \operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx \\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \int \frac{\sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}}{1-\frac{a}{c}-\frac{bx}{c}} dx \\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{(4b)\operatorname{Subst}\left(\int \frac{c^2}{2b^2(1+x^2)} dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{c} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{b} \\
&= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{2c\tan^{-1}\left(\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}\right)}{b}
\end{aligned}
$$

Mathematica [**A**]
time = 0.24, size = 116, normalized size = 2.00

$$
x\cosh^{-1}\left(\frac{c}{a+bx}\right)+\frac{2\sqrt{-\frac{a-c+bx}{a+c+bx}}\sqrt{a+c+bx}\left(a\mathrm{ArcTan}\left(\frac{\sqrt{a-c+bx}}{\sqrt{a+c+bx}}\right)-c\tanh^{-1}\left(\frac{\sqrt{a-c+bx}}{\sqrt{a+c+bx}}\right)\right)}{b\sqrt{a-c+bx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c/(a + b*x)],x]
```

```
[Out] x*ArcCosh[c/(a + b*x)] + (2*Sqrt[-((a - c + b*x)/(a + c + b*x))]*Sqrt[a + c
 + b*x]*(a*ArcTan[Sqrt[a - c + b*x]/Sqrt[a + c + b*x]] - c*ArcTanh[Sqrt[a -
 c + b*x]/Sqrt[a + c + b*x]]))/(b*Sqrt[a - c + b*x])
```

Maple [A]
time = 0.16, size = 87, normalized size = 1.50

method	result	size
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arccosh}\left(\frac{c}{bx+a}\right)}{c}-\frac{\sqrt{\frac{c}{bx+a}-1}\,\sqrt{\frac{c}{bx+a}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{b}$	87
default	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arccosh}\left(\frac{c}{bx+a}\right)}{c}-\frac{\sqrt{\frac{c}{bx+a}-1}\,\sqrt{\frac{c}{bx+a}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{b}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*c*(-1/c*(b*x+a)*arccosh(c/(b*x+a))-(c/(b*x+a)-1)^(1/2)*(c/(b*x+a)+1)^(
1/2)/(c^2/(b*x+a)^2-1)^(1/2)*arctan(1/(c^2/(b*x+a)^2-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*log(sqrt(b*x + a + c)*sqrt(-b*x - a + c)*b*x + sqrt(b*x + a + c)
*sqrt(-b*x - a + c)*a + (b*x + a)*c) - 2*b*x*log(b*x + a) + (a + c)*log(b*x
 + a + c) - 2*(b*x + a)*log(b*x + a) + (a - c)*log(-b*x - a + c))/b + integ
rate((b^2*c*x^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c - c^3 + (b^2*x^2
+ 2*a*b*x + a^2 - c^2)*e^(1/2*log(b*x + a + c) + 1/2*log(-b*x - a + c))), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(55) = 110$.

time = 0.43, size = 276, normalized size = 4.76

$$\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)-2\,c\arctan\left(\frac{(b^2x^2+2\,abx+a^2)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-c^2}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{x}\right)-a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}-c}{x}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 +
2*a*b*x + a^2)) + c)/(b*x + a)) - 2*c*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt
(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a
*b*x + a^2 - c^2)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)
/(b^2*x^2 + 2*a*b*x + a^2)) + c)/x) - a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - c)/x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c/(b*x+a)),x)
```

```
[Out] Integral(acosh(c/(a + b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(55) = 110.
time = 2.29, size = 119, normalized size = 2.05

$$\frac{c\arcsin\left(-\frac{bx+a}{c}\right)\operatorname{sgn}(b)\operatorname{sgn}(c)}{|b|}+x\log\left(\sqrt{\frac{c}{bx+a}+1}\sqrt{\frac{c}{bx+a}-1}+\frac{c}{bx+a}\right)-\frac{a\log\left(\frac{\left|-2\,bc-2\sqrt{-b^2x^2-2\,abx-a^2+c^2}\,|b|\right|}{|-2\,b^2x-2\,ab|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] c*arcsin(-(b*x + a)/c)*sgn(b)*sgn(c)/abs(b) + x*log(sqrt(c/(b*x + a) + 1)*s
qrt(c/(b*x + a) - 1) + c/(b*x + a)) - a*log(abs(-2*b*c - 2*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + c^2)*abs(b))/abs(-2*b^2*x - 2*a*b))/abs(b)
```

Mupad [B]
time = 1.51, size = 53, normalized size = 0.91

$$\frac{\operatorname{acosh}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b}+\frac{c\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{c}{a+b\,x}-1}\,\sqrt{\frac{c}{a+b\,x}+1}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c/(a + b*x)),x)
```

```
[Out] (acosh(c/(a + b*x))*(a + b*x))/b + (c*atan(1/((c/(a + b*x) - 1)^(1/2)*(c/(a
 + b*x) + 1)^(1/2))))/b
```

3.295 $\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{-1+\sqrt{1+bx^2}}\,\sqrt{1+\sqrt{1+bx^2}}\,\cosh^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

```
[Out] arccosh((b*x^2+1)^(1/2))^(1+n)*(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2
))^(1/2)/b/(1+n)/x
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6013, 5893}

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\,\sqrt{\sqrt{bx^2+1}+1}\,\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5893

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 +
 c*x]/Sqrt[d1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*(a + b*ArcCosh[
c*x])^(n + 1), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1, c*d1
] && EqQ[e2, (-c)*d2] && NeQ[n, -1]
```

Rule 6013

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[Sqrt[-1 + Sqrt[1 + b*x^2]]*(Sqrt[1 + Sqrt[1 + b*x^2]]/(b*x)), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\,\sqrt{1+\sqrt{1+bx^2}}\right)\mathrm{Subst}\left(\int \frac{\cosh^{-1}(x)^n}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x,\right.}{bx}$$
$$= \frac{\sqrt{-1+\sqrt{1+bx^2}}\,\sqrt{1+\sqrt{1+bx^2}}\,\cosh^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 1.00

$$\frac{\sqrt{-1+\sqrt{1+bx^2}}\,\sqrt{1+\sqrt{1+bx^2}}\,\cosh^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
[Out] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.
time = 0.37, size = 108, normalized size = 1.74

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\sinh\left(n\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 + 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 + 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty}x & \text{for } b=0 \wedge n=-1 \\ 0^n x & \text{for } b=0 \\ \int \frac{1}{\sqrt{bx^2+1}\operatorname{acosh}\left(\sqrt{bx^2+1}\right)}\,dx & \text{for } n=-1 \\ \frac{\sqrt{bx^2}\operatorname{acosh}\left(\sqrt{bx^2+1}\right)\operatorname{acosh}^n\left(\sqrt{bx^2+1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (Integral(1/(s
qrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x), Eq(n, -1)), (sqrt(b*x**2)*aco
sh(sqrt(b*x**2 + 1))*acosh(sqrt(b*x**2 + 1))**n/(b*n*x + b*x), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\operatorname{acosh}\left(\sqrt{b\,x^2+1}\right)}^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2),x)
[Out] int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2), x)
```

3.296 $\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{-1+\sqrt{1+bx^2}}\,\sqrt{1+\sqrt{1+bx^2}}\,\log\left(\cosh^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

```
[Out] ln(arccosh((b*x^2+1)^(1/2)))*(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2))
^(1/2)/b/x
```

Rubi [A]
time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6013, 5891}

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\,\sqrt{\sqrt{bx^2+1}+1}\,\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

Rule 5891

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c*x]/Sqrt[d
1 + e1*x]]*Simp[Sqrt[-1 + c*x]/Sqrt[d2 + e2*x]]*Log[a + b*ArcCosh[c*x]], x]
 /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, (-c)*d2
]
```

Rule 6013

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[Sqrt[-1 + Sqrt[1 + b*x^2]]*(Sqrt[1 + Sqrt[1 + b*x^2]]/(b*x)), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+}}\right.}{bx}$$

$$= \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\cosh^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 1.00

$$\frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\cosh^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 + b*x^2]]])/(b*x)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arccosh}\left(\sqrt{b\,x^2+1}\right)\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

```
[Out] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccosh(sqrt(b*x^2 + 1))), x)
```

Fricas [A]
time = 0.38, size = 33, normalized size = 0.61

$$\frac{\sqrt{bx^2}\,\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^2+1}\,\operatorname{acosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\operatorname{acosh}\left(\sqrt{b\,x^2+1}\right)\,\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)),x)
[Out] int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```