Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/188-7.1.5-Inverse-hyperbolic-sine-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [371]. This is test number [188].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.73 (370)	0.27 (1)
Mathematica	98.11 (364)	1.89 (7)
Maple	65.77 (244)	34.23 (127)
Fricas	42.32 (157)	57.68 (214)
Maxima	31.27 (116)	68.73 (255)
Giac	26.42 (98)	73.58 (273)
Sympy	26.15 (97)	73.85 (274)
Mupad	21.02 (78)	78.98 (293)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

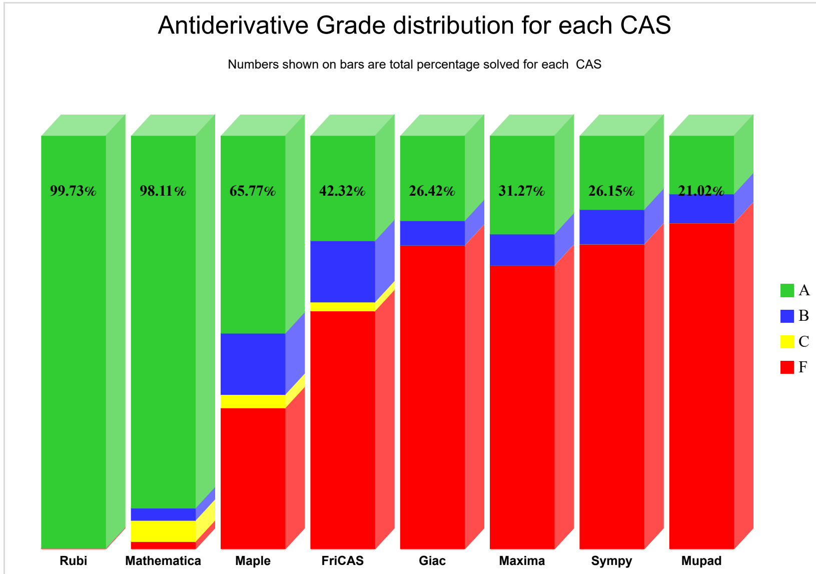
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

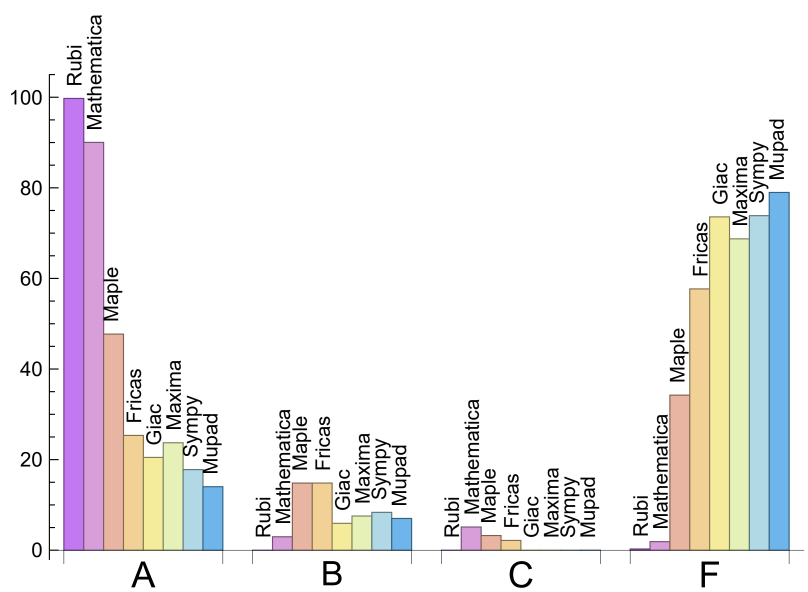
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.73	0.00	0.00	0.27
Mathematica	90.03	2.96	5.12	1.89
Maple	47.71	14.82	3.23	34.23
Fricas	25.34	14.82	2.16	57.68
Maxima	23.72	7.55	0.00	68.73
Giac	20.49	5.93	0.00	73.58
Sympy	17.79	8.36	0.00	73.85
Mupad	N/A	7.01	0.00	78.98

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	7	85.71 %	14.29 %	0.00 %
Maple	127	100.00 %	0.00 %	0.00 %
Fricas	214	59.35 %	0.00 %	40.65 %
Giac	273	81.68 %	1.10 %	17.22 %
Maxima	255	92.55 %	1.96 %	5.49 %
Sympy	274	82.85 %	2.19 %	14.96 %
Mupad	293	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

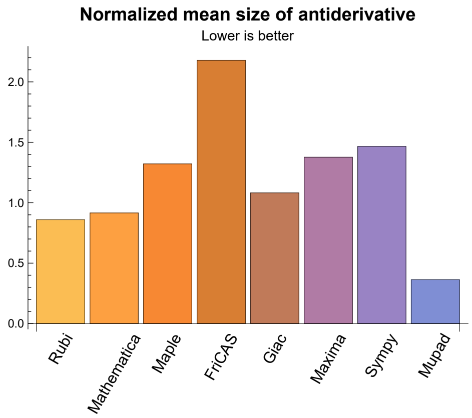
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

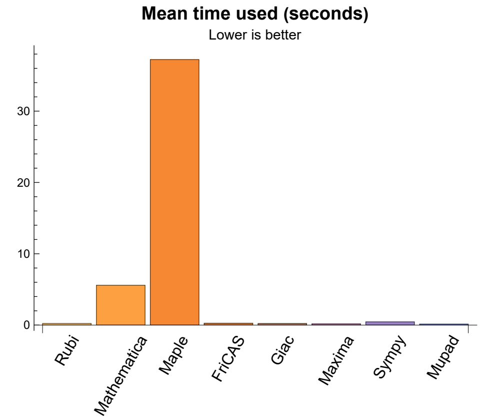
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.22	178.01	0.86	132.50	1.00
Mathematica	5.58	203.97	0.92	119.00	0.93
Maple	37.23	285.48	1.32	108.50	1.09
Maxima	0.17	102.60	1.38	30.00	0.69
Fricas	0.26	309.16	2.18	96.00	1.39
Sympy	0.45	243.89	1.47	26.00	0.87
Giac	0.22	93.05	1.08	0.00	0.00
Mupad	0.14	16.62	0.36	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
10
20
30
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{37, 38, 42, 46, 154, 326, 327\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

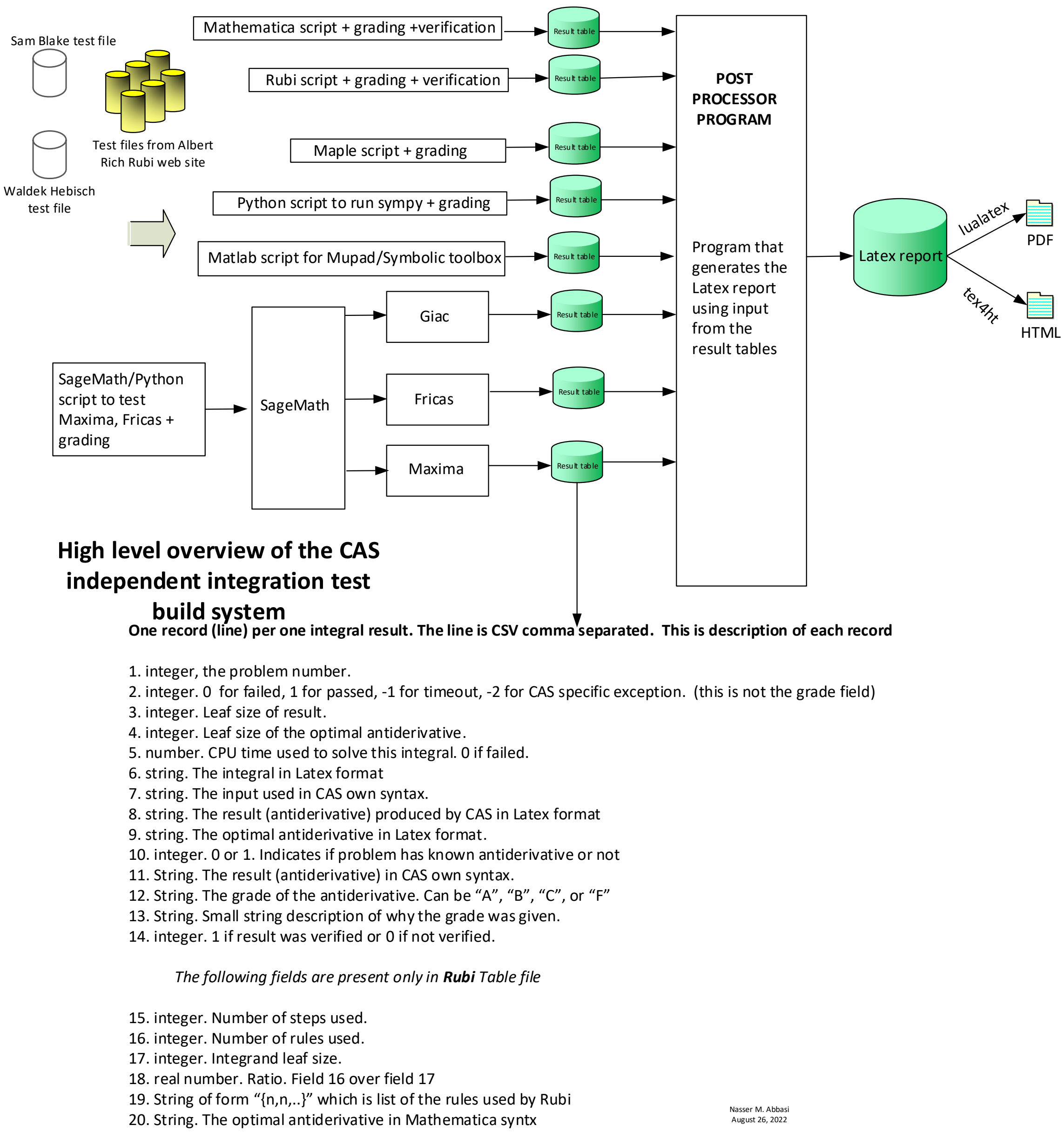
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371 }

B grade: { }

C grade: { }

F grade: { 369 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 287, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371 }

B grade: { 61, 103, 104, 119, 145, 152, 154, 196, 202, 302, 368 }

C grade: { 37, 38, 42, 46, 74, 125, 153, 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 22, 31, 94, 208, 214, 220, 245 }

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 37, 42, 50, 53, 57, 58, 59, 60, 61, 63, 64, 65, 69, 70, 72, 73, 74, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 130, 131, 133, 134, 135, 136, 138, 140, 141, 145, 146, 147, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 171, 172, 173, 177, 178, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 285, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 317, 324, 342, 345, 346, 347, 348, 351, 353, 362, 363, 364, 365, 366, 368 }

B grade: { 9, 10, 11, 18, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 51, 52, 62, 66, 127, 129, 132, 137, 139, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 162, 163, 168, 169, 170, 174, 175, 176, 280, 343, 344, 349, 350, 352, 354, 355, 356, 357 }

C grade: { 228, 229, 230, 231, 232, 233, 234, 235, 284, 286, 288, 290 }

F grade: { 2, 3, 12, 16, 31, 54, 55, 56, 67, 68, 71, 75, 78, 79, 80, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 126, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 236, 237, 238, 239, 240, 241, 242, 243, 266, 287, 291, 307, 308, 309, 310, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 358, 359, 360, 361, 367, 369, 370, 371 }

2.1.4 Maxima

A grade: { 4, 5, 6, 7, 9, 10, 12, 13, 14, 15, 23, 24, 28, 29, 30, 32, 33, 49, 50, 57, 61, 64, 84, 88, 92, 93, 97, 119, 121, 136, 146, 155, 161, 167, 173, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 285, 289, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 304, 306, 317, 324, 342, 346, 347, 350, 353, 354, 362, 363, 366, 367, 369 }

B grade: { 58, 59, 60, 63, 65, 66, 115, 116, 117, 118, 122, 124, 134, 262, 268, 272, 273, 274, 276, 280, 283, 305, 349, 351, 352, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 8, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 263, 264, 265, 266, 267, 269, 270, 271, 275, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.1.5 FriCAS

A grade: { 6, 7, 23, 24, 28, 29, 30, 32, 33, 53, 57, 58, 59, 60, 61, 67, 68, 69, 75, 76, 77, 84, 88, 92, 93, 97, 119, 136, 146, 155, 161, 167, 173, 179, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 281, 282, 283, 284, 285, 288, 292, 293, 294, 296, 297, 298, 299, 301, 304, 305, 306, 315, 316, 317, 322, 323, 324, 342, 346, 347, 349, 350, 351, 353, 355, 356, 357, 362, 363, 366, 369, 371 }

B grade: { 4, 5, 9, 10, 11, 12, 13, 14, 15, 63, 64, 65, 66, 70, 115, 116, 117, 118, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 134, 137, 138, 139, 140, 141, 147, 148, 149, 150, 272, 273, 274, 275, 276, 277, 280, 289, 300, 302, 314, 321, 352, 354, 367, 368, 370 }

C grade: { 228, 229, 230, 231, 232, 233, 234, 235 }

F grade: { 1, 2, 3, 8, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 126, 132, 133, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 269, 270, 271, 278, 279, 286, 287, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 318, 319, 320, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365 }

2.1.6 Sympy

A grade: { 4, 5, 6, 7, 13, 14, 15, 23, 24, 28, 29, 30, 32, 33, 57, 60, 61, 67, 68, 69, 70, 75, 76, 77, 84, 88, 92, 93, 97, 119, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 209, 215, 221, 227, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 258, 259, 281, 282, 283, 285, 294, 304, 346, 362, 363, 366 }

B grade: { 12, 58, 59, 115, 116, 117, 118, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 147, 148, 149, 150, 266, 267, 268, 272, 273, 274, 275, 276, 277, 367 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 142, 143, 144, 145, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 252, 253, 260, 261, 262, 263, 264, 265, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 368, 369, 370, 371 }

2.1.7 Giac

A grade: { 6, 7, 23, 24, 28, 29, 30, 32, 33, 53, 57, 58, 59, 60, 84, 88, 92, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 280, 281, 282, 283, 285, 292, 293, 294, 296, 297, 298, 299, 300, 301, 304, 305, 306, 342, 346, 347, 349, 350, 351, 353, 362, 363, 369 }

B grade: { 9, 15, 61, 63, 64, 65, 66, 115, 116, 117, 118, 119, 121, 289, 302, 352, 354, 355, 356, 357, 366, 367 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 287, 288, 290, 291, 295, 303, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 358, 359, 360, 361, 364, 365, 368, 370, 371 }

2.1.8 Mupad

A grade: { 23, 24, 28, 29, 30, 32, 33, 53, 57, 84, 88, 92, 93, 97, 136, 146, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 281, 282, 342, 346, 347, 362, 363 }

B grade: { 6, 7, 61, 119, 272, 273, 274, 275, 276, 277, 283, 285, 294, 301, 302, 304, 305, 317, 324, 353, 354, 366, 367, 368, 369, 371 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 279, 280, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 303, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 351, 352, 355, 356, 357, 358, 359, 360, 361, 364, 365, 370 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	168	272	0	0	0	0	-1
N.S.	1	1.00	0.99	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.009	3.422	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	240	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.091	1.372	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	322	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.032	1.379	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	166	259	228	525	316	0	-1
N.S.	1	1.00	0.94	1.47	1.30	2.98	1.80	0.00	-0.01
time (sec)	N/A	0.117	0.108	0.681	0.264	0.384	0.284	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	121	189	150	281	190	0	-1
N.S.	1	1.00	0.98	1.52	1.21	2.27	1.53	0.00	-0.01
time (sec)	N/A	0.064	0.067	0.674	0.262	0.401	0.169	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	96	84	121	99	122	78
N.S.	1	1.00	0.94	0.99	0.87	1.25	1.02	1.26	0.80
time (sec)	N/A	0.038	0.032	0.627	0.270	0.346	0.104	0.441	0.289

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	43	26	41	28
N.S.	1	1.00	1.00	1.03	1.00	1.43	0.87	1.37	0.93
time (sec)	N/A	0.010	0.008	0.758	0.266	0.347	0.056	0.398	0.002

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	175	292	0	0	0	0	-1
N.S.	1	1.00	0.94	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.046	3.332	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	188	91	565	0	232	-1
N.S.	1	1.00	0.96	2.29	1.11	6.89	0.00	2.83	-0.01
time (sec)	N/A	0.039	0.070	4.016	0.292	0.378	0.000	0.582	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	166	283	151	2021	0	0	-1
N.S.	1	1.00	1.30	2.21	1.18	15.79	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.262	4.523	0.299	0.446	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	205	520	0	4886	0	0	-1
N.S.	1	1.00	1.12	2.84	0.00	26.70	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.313	4.496	0.000	0.795	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	354	0	586	1185	743	0	-1
N.S.	1	1.00	0.96	0.00	1.59	3.22	2.02	0.00	-0.00
time (sec)	N/A	0.493	0.277	180.000	0.296	0.363	0.474	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	248	361	378	616	454	0	-1
N.S.	1	1.00	1.04	1.51	1.58	2.58	1.90	0.00	-0.00
time (sec)	N/A	0.321	0.194	2.283	0.300	0.342	0.308	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	193	223	257	233	0	-1
N.S.	1	1.00	1.01	1.38	1.59	1.84	1.66	0.00	-0.01
time (sec)	N/A	0.218	0.158	2.863	0.277	0.380	0.180	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	72	72	96	82	111	-1
N.S.	1	1.00	1.61	1.57	1.57	2.09	1.78	2.41	-0.02
time (sec)	N/A	0.040	0.040	0.787	0.258	0.408	0.092	0.452	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	273	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	0.157	0.163	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	191	549	0	0	0	0	-1
N.S.	1	1.00	0.73	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.143	9.322	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	270	1017	0	0	0	0	-1
N.S.	1	1.00	0.77	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.477	12.866	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	305	394	0	0	0	0	-1
N.S.	1	1.00	0.77	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.879	0.405	11.836	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	188	254	0	0	0	0	-1
N.S.	1	1.00	0.77	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	0.254	10.635	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0	-1
N.S.	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.100	8.740	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	0	56	0	0	0	0	-1
N.S.	1	1.00	0.00	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.017	3.167	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.147	180.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.277	180.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	288	616	0	0	0	0	-1
N.S.	1	1.00	0.80	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	1.028	10.270	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	150	272	0	0	0	0	-1
N.S.	1	1.00	0.83	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.471	8.714	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	118	0	0	0	0	-1
N.S.	1	1.00	0.84	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.148	3.281	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	2.017	180.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	3.451	180.000	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.185	3.323	180.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.042	180.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.252	180.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.541	180.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	640	640	413	1321	0	0	0	0	-1
N.S.	1	1.00	0.65	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	0.900	4.816	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	301	921	0	0	0	0	-1
N.S.	1	1.00	0.70	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.601	4.667	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	208	582	0	0	0	0	-1
N.S.	1	1.00	0.92	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.802	3.439	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	1353	992	0	0	0	0	-1
N.S.	1	1.00	2.04	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.159	4.423	2.805	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	1384	1814	0	0	0	0	-1
N.S.	1	1.00	1.77	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.756	7.299	6.063	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	779	2101	0	0	0	0	-1
N.S.	1	1.00	0.85	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	2.642	4.479	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	546	1585	0	0	0	0	-1
N.S.	1	1.00	0.84	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	1.464	5.724	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	392	1065	0	0	0	0	-1
N.S.	1	1.00	1.11	3.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.817	3.165	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	984	984	2864	1838	0	0	0	0	-1
N.S.	1	1.00	2.91	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.352	11.094	2.434	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1228	1228	1667	2916	0	0	0	0	-1
N.S.	1	1.00	1.36	2.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	5.020	4.572	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	901	901	1047	2335	0	0	0	0	-1
N.S.	1	1.00	1.16	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	1.804	4.401	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	656	1679	0	0	0	0	-1
N.S.	1	1.00	1.33	3.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	0.882	3.269	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1536	1536	7163	3928	0	0	0	0	-1
N.S.	1	1.00	4.66	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.783	22.445	2.635	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	304	790	0	0	0	0	-1
N.S.	1	1.00	0.71	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	0.557	6.039	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	233	486	0	0	0	0	-1
N.S.	1	1.00	0.90	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.365	6.143	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	158	227	87	0	0	0	-1
N.S.	1	1.00	1.32	1.89	0.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.139	4.536	0.273	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	77	28	0	0	0	-1
N.S.	1	1.00	1.02	1.64	0.60	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.024	0.405	0.260	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	256	678	0	0	0	0	-1
N.S.	1	1.00	0.79	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.389	2.338	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	448	1770	0	0	0	0	-1
N.S.	1	1.00	1.01	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	1.502	8.370	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.128	0.104	180.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	397	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	0.188	180.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	304	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.396	0.184	180.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	206	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.016	180.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.134	0.208	180.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	200	318	110	255	162	-1
N.S.	1	1.00	0.73	1.53	2.43	0.84	1.95	1.24	-0.01
time (sec)	N/A	0.126	0.062	0.264	0.255	0.353	0.299	0.409	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	130	210	91	170	131	-1
N.S.	1	1.00	0.82	1.44	2.33	1.01	1.89	1.46	-0.01
time (sec)	N/A	0.079	0.043	0.280	0.266	0.342	0.190	0.416	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	60	74	149	75	104	111	-1
N.S.	1	1.00	0.79	0.97	1.96	0.99	1.37	1.46	-0.01
time (sec)	N/A	0.049	0.030	0.260	0.264	0.342	0.116	0.412	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	135	31	30	57	46	92	76
N.S.	1	1.00	3.97	0.91	0.88	1.68	1.35	2.71	2.24
time (sec)	N/A	0.009	0.224	0.265	0.263	0.402	0.069	0.411	0.446

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	153	388	0	0	0	0	-1
N.S.	1	1.00	1.17	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.011	4.372	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	75	111	167	0	110	-1
N.S.	1	1.00	1.00	1.32	1.95	2.93	0.00	1.93	-0.02
time (sec)	N/A	0.055	0.029	2.812	0.262	0.381	0.000	0.447	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	112	146	236	0	199	-1
N.S.	1	1.00	1.20	1.22	1.59	2.57	0.00	2.16	-0.01
time (sec)	N/A	0.072	0.124	2.080	0.260	0.379	0.000	0.460	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	149	213	284	285	0	381	-1
N.S.	1	1.00	1.16	1.65	2.20	2.21	0.00	2.95	-0.01
time (sec)	N/A	0.113	0.159	2.086	0.271	0.414	0.000	0.451	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	179	359	357	343	0	709	-1
N.S.	1	1.00	1.07	2.15	2.14	2.05	0.00	4.25	-0.01
time (sec)	N/A	0.169	0.150	2.030	0.269	0.426	0.000	0.449	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	145	0	0	182	366	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.55	1.11	0.00	-0.00
time (sec)	N/A	0.377	0.117	180.000	0.000	0.365	0.457	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	107	0	0	146	243	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.69	1.15	0.00	-0.00
time (sec)	N/A	0.252	0.092	180.000	0.000	0.353	0.283	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	79	113	0	114	138	0	-1
N.S.	1	1.00	0.63	0.90	0.00	0.90	1.10	0.00	-0.01
time (sec)	N/A	0.157	0.053	2.467	0.000	0.443	0.162	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	46	0	88	63	0	-1
N.S.	1	1.00	1.04	1.02	0.00	1.96	1.40	0.00	-0.02
time (sec)	N/A	0.037	0.017	1.861	0.000	0.385	0.095	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	251	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	0.022	1.674	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	178	206	0	0	0	0	-1
N.S.	1	1.00	1.00	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.081	6.845	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	279	374	0	0	0	0	-1
N.S.	1	1.00	1.19	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.092	8.208	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	1830	764	0	0	0	0	-1
N.S.	1	1.00	3.83	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	10.273	10.971	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	175	0	0	225	432	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.63	1.22	0.00	-0.00
time (sec)	N/A	0.315	0.132	180.000	0.000	0.381	0.452	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	129	169	0	180	248	0	-1
N.S.	1	1.00	0.64	0.83	0.00	0.89	1.22	0.00	-0.00
time (sec)	N/A	0.210	0.087	2.506	0.000	0.354	0.272	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	67	0	139	109	0	-1
N.S.	1	1.00	0.90	0.86	0.00	1.78	1.40	0.00	-0.01
time (sec)	N/A	0.051	0.023	2.005	0.000	0.483	0.152	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	346	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.024	1.668	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	259	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	0.090	4.092	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	524	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	0.171	5.629	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	49	0	0	0	0	-1
N.S.	1	1.00	0.73	0.82	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.385	0.088	3.251	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	0	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.158	0.037	2.694	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	0	0	0	-1
N.S.	1	1.00	1.00	1.09	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.018	0.015	1.600	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	0.148	1.661	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	83	146	0	0	0	0	-1
N.S.	1	1.00	0.54	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.283	4.116	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	62	73	0	0	0	0	-1
N.S.	1	1.00	0.74	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.118	3.237	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	34	0	0	0	0	-1
N.S.	1	1.00	1.11	0.89	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.060	1.921	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.028	1.426	1.220	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	110	213	0	0	0	0	-1
N.S.	1	1.00	0.43	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.239	3.792	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	117	107	0	0	0	0	-1
N.S.	1	1.00	0.80	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.063	3.221	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	51	0	0	0	0	-1
N.S.	1	1.00	0.84	0.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.057	1.928	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.029	1.425	1.221	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	0.312	180.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.836	1.009	180.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	330	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.959	180.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	138	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.013	180.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.041	0.129	180.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	656	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.388	1.173	180.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	251	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	1.212	180.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.042	180.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	582	0	0	0	0	0	-1
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	3.525	180.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.414	180.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	939	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	7.231	180.000	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	1.483	180.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	471	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.731	180.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	217	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	0.676	180.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	111	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.079	180.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	301	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	1.773	180.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	155	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.150	180.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	375	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	1.807	180.000	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	207	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.400	180.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	429	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	1.740	180.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.358	180.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.032	180.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	71	93	1221	1041	527	841	-1
N.S.	1	1.00	0.71	0.93	12.21	10.41	5.27	8.41	-0.01
time (sec)	N/A	0.056	0.069	0.724	0.291	0.363	0.508	0.920	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	83	86	782	663	394	613	-1
N.S.	1	1.00	0.79	0.82	7.45	6.31	3.75	5.84	-0.01
time (sec)	N/A	0.048	0.051	0.703	0.276	0.386	0.420	0.791	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	73	439	352	258	416	-1
N.S.	1	1.00	0.84	0.96	5.78	4.63	3.39	5.47	-0.01
time (sec)	N/A	0.044	0.031	0.718	0.273	0.404	0.209	0.697	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	62	205	167	148	243	-1
N.S.	1	1.00	0.84	0.91	3.01	2.46	2.18	3.57	-0.01
time (sec)	N/A	0.027	0.044	0.675	0.271	0.355	0.151	0.571	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	140	36	35	65	51	99	85
N.S.	1	1.00	3.59	0.92	0.90	1.67	1.31	2.54	2.18
time (sec)	N/A	0.015	0.251	0.628	0.254	0.337	0.097	0.406	0.484

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	145	0	0	0	0	-1
N.S.	1	1.00	0.86	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.018	2.389	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	54	72	210	0	134	-1
N.S.	1	1.00	0.88	1.10	1.47	4.29	0.00	2.73	-0.02
time (sec)	N/A	0.037	0.027	0.692	0.278	0.471	0.000	0.442	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	60	109	211	0	0	-1
N.S.	1	1.00	0.97	1.02	1.85	3.58	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.039	0.700	0.262	0.354	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	74	0	512	0	0	-1
N.S.	1	1.00	0.98	0.88	0.00	6.10	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.059	0.699	0.000	0.431	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	61	80	244	477	0	0	-1
N.S.	1	1.00	0.68	0.89	2.71	5.30	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.054	0.688	0.284	0.422	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	61	94	0	896	0	0	-1
N.S.	1	1.00	0.53	0.82	0.00	7.79	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.056	0.921	0.000	0.419	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	155	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.083	180.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	192	582	0	2430	1268	0	-1
N.S.	1	1.00	0.97	2.95	0.00	12.34	6.44	0.00	-0.01
time (sec)	N/A	0.201	0.159	3.930	0.000	0.378	0.791	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	170	222	0	1477	916	0	-1
N.S.	1	1.00	0.99	1.29	0.00	8.59	5.33	0.00	-0.01
time (sec)	N/A	0.172	0.126	3.543	0.000	0.382	0.579	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	147	308	0	784	610	0	-1
N.S.	1	1.00	1.08	2.26	0.00	5.76	4.49	0.00	-0.01
time (sec)	N/A	0.144	0.107	3.138	0.000	0.385	0.340	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	120	135	0	361	335	0	-1
N.S.	1	1.00	1.17	1.31	0.00	3.50	3.25	0.00	-0.01
time (sec)	N/A	0.099	0.118	2.355	0.000	0.435	0.205	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	87	90	0	141	143	0	-1
N.S.	1	1.00	1.53	1.58	0.00	2.47	2.51	0.00	-0.02
time (sec)	N/A	0.049	0.065	2.173	0.000	0.359	0.113	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	369	0	0	0	0	-1
N.S.	1	1.00	0.86	3.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.034	2.523	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	154	207	0	0	0	0	-1
N.S.	1	1.00	1.54	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.407	3.366	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	120	161	216	412	0	0	-1
N.S.	1	1.00	1.41	1.89	2.54	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.159	4.228	0.281	0.488	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	212	274	0	0	0	0	-1
N.S.	1	1.00	1.25	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	1.132	4.790	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	3.124	180.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	355	1389	0	4411	2518	0	-1
N.S.	1	1.00	1.09	4.26	0.00	13.53	7.72	0.00	-0.00
time (sec)	N/A	0.315	0.292	4.431	0.000	0.579	1.326	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	303	409	0	2632	1828	0	-1
N.S.	1	1.00	1.09	1.47	0.00	9.43	6.55	0.00	-0.00
time (sec)	N/A	0.264	0.243	3.961	0.000	0.449	0.905	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	258	680	0	1395	1173	0	-1
N.S.	1	1.00	1.14	3.00	0.00	6.15	5.17	0.00	-0.00
time (sec)	N/A	0.214	0.199	3.348	0.000	0.404	0.564	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	200	243	0	632	685	0	-1
N.S.	1	1.00	1.24	1.51	0.00	3.93	4.25	0.00	-0.01
time (sec)	N/A	0.156	0.125	2.463	0.000	0.393	0.363	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	147	160	0	239	282	0	-1
N.S.	1	1.00	1.47	1.60	0.00	2.39	2.82	0.00	-0.01
time (sec)	N/A	0.077	0.104	2.289	0.000	0.407	0.176	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	128	674	0	0	0	0	-1
N.S.	1	1.00	0.83	4.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.034	2.671	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	315	436	0	0	0	0	-1
N.S.	1	1.00	1.90	2.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.485	3.474	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	229	367	0	0	0	0	-1
N.S.	1	1.00	1.46	2.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.565	4.583	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	694	581	0	0	0	0	-1
N.S.	1	1.00	2.66	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	7.034	5.532	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.469	180.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	475	639	0	4046	2876	0	-1
N.S.	1	1.00	1.36	1.83	0.00	11.59	8.24	0.00	-0.00
time (sec)	N/A	0.432	0.397	4.323	0.000	0.500	1.480	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	412	1124	0	2103	1889	0	-1
N.S.	1	1.00	1.47	4.00	0.00	7.48	6.72	0.00	-0.00
time (sec)	N/A	0.330	0.280	4.784	0.000	0.462	0.919	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	300	371	0	949	1027	0	-1
N.S.	1	1.00	1.54	1.90	0.00	4.87	5.27	0.00	-0.01
time (sec)	N/A	0.212	0.188	2.562	0.000	0.399	0.544	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	226	245	0	344	444	0	-1
N.S.	1	1.00	1.97	2.13	0.00	2.99	3.86	0.00	-0.01
time (sec)	N/A	0.111	0.168	2.693	0.000	0.381	0.342	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	157	1058	0	0	0	0	-1
N.S.	1	1.00	0.84	5.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.046	2.817	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	501	748	0	0	0	0	-1
N.S.	1	1.00	2.14	3.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	1.078	3.652	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	360	654	0	0	0	0	-1
N.S.	1	1.00	1.94	3.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.856	5.115	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	1182	1087	0	0	0	0	-1
N.S.	1	1.00	3.07	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	7.991	5.812	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	1.461	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	151	194	0	0	0	0	-1
N.S.	1	1.00	0.71	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.255	6.076	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0	-1
N.S.	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.171	7.517	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	130	0	0	0	0	-1
N.S.	1	1.00	0.72	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.141	6.077	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0	-1
N.S.	1	1.00	0.88	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.063	5.092	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0	-1
N.S.	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.063	3.054	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	0.635	180.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	281	602	0	0	0	0	-1
N.S.	1	1.00	1.10	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	0.823	7.062	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	193	388	0	0	0	0	-1
N.S.	1	1.00	1.03	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.633	8.283	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	138	342	0	0	0	0	-1
N.S.	1	1.00	0.75	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.543	6.712	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	97	160	0	0	0	0	-1
N.S.	1	1.00	0.94	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.226	4.547	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	77	128	0	0	0	0	-1
N.S.	1	1.00	0.85	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.073	3.324	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.074	180.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	316	896	0	0	0	0	-1
N.S.	1	1.00	0.99	2.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	0.916	6.743	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	179	579	0	0	0	0	-1
N.S.	1	1.00	0.72	2.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	0.506	8.082	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	216	507	0	0	0	0	-1
N.S.	1	1.00	0.88	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.570	6.562	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	120	239	0	0	0	0	-1
N.S.	1	1.00	0.77	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.243	5.261	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	100	190	0	0	0	0	-1
N.S.	1	1.00	0.80	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.211	3.313	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.778	180.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	410	1244	0	0	0	0	-1
N.S.	1	1.00	1.00	3.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.581	1.395	6.905	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	318	800	0	0	0	0	-1
N.S.	1	1.00	0.94	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	0.814	8.213	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	258	709	0	0	0	0	-1
N.S.	1	1.00	0.78	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	0.652	6.628	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	181	333	0	0	0	0	-1
N.S.	1	1.00	0.89	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	0.591	4.477	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	130	272	0	0	0	0	-1
N.S.	1	1.00	0.81	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.315	3.336	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	2.548	180.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	0.450	180.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.211	180.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	238	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	0.287	180.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	140	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.079	180.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.100	0.140	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	1.441	180.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	343	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.090	0.335	180.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	225	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	0.219	180.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	238	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	0.215	180.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	0.079	180.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	272	0	0	0	0	0	-1
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.172	0.170	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.989	180.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	701	701	342	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.454	0.498	180.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	223	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.016	0.232	180.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	238	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.839	0.303	180.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	126	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.058	180.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	458	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	1.396	0.122	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.749	180.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	835	835	343	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.089	0.364	180.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	225	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.357	0.229	180.000	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	238	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.008	0.237	180.000	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	125	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	0.057	180.000	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	698	0	0	0	0	0	-1
N.S.	1	1.00	3.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	3.120	180.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.734	180.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	320	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.257	180.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.161	180.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	217	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.155	180.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.045	180.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.028	0.132	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.062	0.051	180.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	490	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	0.451	180.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	253	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.301	180.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	327	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	0.228	180.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	147	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.080	180.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.031	0.126	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.068	0.055	180.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	551	0	0	0	0	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.939	2.628	180.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	390	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.743	1.222	180.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	389	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.659	1.053	180.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	227	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	0.467	180.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.030	0.147	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.070	0.056	180.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	701	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.984	1.821	180.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	429	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	1.427	180.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	474	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	1.020	180.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	235	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.678	180.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	238	0	0	0	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.146	0.125	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.077	0.061	180.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	113	238	0	815	0	0	-1
N.S.	1	1.00	0.38	0.80	0.00	2.73	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.153	3.004	0.000	0.140	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	113	212	0	517	0	0	-1
N.S.	1	1.00	0.64	1.20	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.129	3.019	0.000	0.118	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	87	205	0	279	0	0	-1
N.S.	1	1.00	0.33	0.79	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.035	3.059	0.000	0.110	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	87	179	0	159	0	0	-1
N.S.	1	1.00	0.61	1.26	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.023	3.020	0.000	0.101	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	61	161	0	128	0	0	-1
N.S.	1	1.00	0.27	0.72	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.023	3.246	0.000	0.135	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	56	140	0	169	0	0	-1
N.S.	1	1.00	0.53	1.32	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.020	3.237	0.000	0.096	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	58	202	0	283	0	0	-1
N.S.	1	1.00	0.22	0.76	0.00	1.06	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.022	3.018	0.000	0.121	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	176	0	383	0	0	-1
N.S.	1	1.00	0.42	1.21	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.027	3.056	0.000	0.109	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.095	180.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.082	180.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.073	180.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.065	180.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.045	180.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	109	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.071	180.000	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	106	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.057	180.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	110	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.065	180.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	172.819	180.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	180.011	180.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	156.255	180.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	167.703	180.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	131.867	180.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	97.915	180.000	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	78.281	180.000	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	139.333	180.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	129.959	180.000	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	148.422	180.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	115.996	180.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	147.138	180.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	48.846	180.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	74.560	180.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	75.772	180.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	131.953	180.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	127	204	0	199	0	0	-1
N.S.	1	1.00	0.97	1.56	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.079	3.500	0.000	0.394	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	110	167	0	161	0	0	-1
N.S.	1	1.00	1.03	1.56	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.063	3.495	0.000	0.348	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	91	238	98	0	0	-1
N.S.	1	1.00	1.00	1.49	3.90	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.039	3.514	0.293	0.381	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	23	0	0	0	0	-1
N.S.	1	1.00	0.77	0.74	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	0.047	3.851	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	47	44	0	0	0	0	-1
N.S.	1	1.00	1.31	1.22	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.083	0.047	4.946	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	85	61	0	0	0	0	-1
N.S.	1	0.97	1.20	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.190	3.789	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	266	0	0	332	694	0	-1
N.S.	1	1.00	1.13	0.00	0.00	1.41	2.95	0.00	-0.00
time (sec)	N/A	0.227	0.163	180.000	0.000	0.416	1.618	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	211	139	0	259	568	0	-1
N.S.	1	1.00	1.12	0.74	0.00	1.37	3.01	0.00	-0.01
time (sec)	N/A	0.132	0.111	2.926	0.000	0.359	0.987	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	124	82	394	160	298	0	-1
N.S.	1	1.00	1.17	0.77	3.72	1.51	2.81	0.00	-0.01
time (sec)	N/A	0.072	0.065	2.879	0.281	0.363	0.612	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	36	0	0	0	0	-1
N.S.	1	1.00	0.79	0.77	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.098	0.183	3.965	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	70	72	0	0	0	0	-1
N.S.	1	1.00	1.30	1.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.173	3.857	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	108	110	0	0	0	0	-1
N.S.	1	1.00	1.29	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.238	3.879	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	179	32	26	0	13
N.S.	1	1.00	1.00	0.93	11.93	2.13	1.73	0.00	0.87
time (sec)	N/A	0.048	0.015	2.996	0.307	0.374	0.488	0.000	0.225

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	132	32	26	0	13
N.S.	1	1.00	1.00	0.93	8.80	2.13	1.73	0.00	0.87
time (sec)	N/A	0.048	0.013	3.038	0.274	0.342	0.440	0.000	0.195

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	84	32	24	0	13
N.S.	1	1.00	1.00	0.93	5.60	2.13	1.60	0.00	0.87
time (sec)	N/A	0.029	0.012	3.099	0.266	0.400	0.418	0.000	0.199

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	30	22	0	11
N.S.	1	1.00	1.00	1.09	0.00	2.73	2.00	0.00	1.00
time (sec)	N/A	0.050	0.021	3.030	0.000	0.351	0.651	0.000	0.221

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	150	32	26	0	13
N.S.	1	1.00	1.00	1.08	11.54	2.46	2.00	0.00	1.00
time (sec)	N/A	0.048	0.011	3.035	0.284	0.389	0.904	0.000	0.210

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	32	29	0	13
N.S.	1	1.00	1.00	0.93	0.00	2.13	1.93	0.00	0.87
time (sec)	N/A	0.047	0.011	2.925	0.000	0.380	1.008	0.000	0.203

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	203	0	0	0	0	-1
N.S.	1	1.00	1.11	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.393	5.332	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	98	168	0	0	0	0	-1
N.S.	1	1.00	1.14	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.244	5.296	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	62	131	119	115	0	76	-1
N.S.	1	1.00	1.35	2.85	2.59	2.50	0.00	1.65	-0.02
time (sec)	N/A	0.041	0.062	5.238	0.471	0.375	0.000	0.420	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	0.400	180.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	1.852	180.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	71	102	52	42	74	45
N.S.	1	1.00	0.88	1.42	2.04	1.04	0.84	1.48	0.90
time (sec)	N/A	0.027	0.013	0.418	0.271	0.344	0.181	0.403	0.247

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	89	0	67	0	0	-1
N.S.	1	1.00	0.74	0.88	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.086	0.222	0.000	0.089	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	30	42	27	40	28
N.S.	1	1.00	1.00	0.91	0.88	1.24	0.79	1.18	0.82
time (sec)	N/A	0.017	0.011	0.279	0.255	0.446	0.081	0.385	0.250

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	35	77	0	0	0	0	-1
N.S.	1	1.00	0.22	0.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.010	0.150	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.007	0.410	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	42	66	0	75	0	0	-1
N.S.	1	1.00	0.56	0.88	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.027	0.147	0.000	0.179	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	46	106	0	58	-1
N.S.	1	1.00	1.00	0.85	1.39	3.21	0.00	1.76	-0.03
time (sec)	N/A	0.020	0.006	0.414	0.264	0.390	0.000	0.422	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	88	101	0	0	0	0	-1
N.S.	1	1.00	0.45	0.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.109	0.148	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.007	0.334	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	43	47	46	40	0	50	-1
N.S.	1	1.00	0.60	0.65	0.64	0.56	0.00	0.69	-0.01
time (sec)	N/A	0.017	0.016	0.272	0.464	0.359	0.000	0.401	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	37	37	36	35	0	48	-1
N.S.	1	1.00	0.66	0.66	0.64	0.62	0.00	0.86	-0.02
time (sec)	N/A	0.012	0.012	0.282	0.465	0.344	0.000	0.398	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	42	24	23	28	29	40	31
N.S.	1	1.00	1.20	0.69	0.66	0.80	0.83	1.14	0.89
time (sec)	N/A	0.008	0.060	0.279	0.471	0.417	0.096	0.388	0.920

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	78	0	0	0	0	-1
N.S.	1	1.00	1.00	1.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.007	2.044	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	25	0	35	-1
N.S.	1	1.00	1.00	0.81	0.77	0.96	0.00	1.35	-0.04
time (sec)	N/A	0.009	0.008	0.281	0.501	0.411	0.000	0.398	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	30	32	0	52	-1
N.S.	1	1.00	0.74	0.67	0.65	0.70	0.00	1.13	-0.02
time (sec)	N/A	0.013	0.010	0.349	0.520	0.382	0.000	0.399	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	39	41	40	37	0	67	-1
N.S.	1	1.00	0.63	0.66	0.65	0.60	0.00	1.08	-0.02
time (sec)	N/A	0.014	0.012	0.359	0.473	0.345	0.000	0.393	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	44	51	50	42	0	82	-1
N.S.	1	1.00	0.56	0.65	0.64	0.54	0.00	1.05	-0.01
time (sec)	N/A	0.018	0.013	0.280	0.466	0.368	0.000	0.389	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	54	69	122	0	74	-1
N.S.	1	1.00	1.02	0.96	1.23	2.18	0.00	1.32	-0.02
time (sec)	N/A	0.026	0.027	0.882	0.282	0.380	0.000	0.423	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	27	45	0	47	27
N.S.	1	1.00	0.88	1.15	0.82	1.36	0.00	1.42	0.82
time (sec)	N/A	0.012	0.016	0.867	0.307	0.365	0.000	0.421	0.034

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	77	31	43	96	0	49	25
N.S.	1	1.00	3.08	1.24	1.72	3.84	0.00	1.96	1.00
time (sec)	N/A	0.013	0.060	0.881	0.270	0.399	0.000	0.399	0.549

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	114	0	0	0	0	-1
N.S.	1	1.00	1.00	2.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.007	2.372	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	31	30	49	20	39	27
N.S.	1	1.00	1.00	1.07	1.03	1.69	0.69	1.34	0.93
time (sec)	N/A	0.017	0.013	0.280	0.257	0.382	0.413	0.435	0.227

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	46	97	58	0	84	43
N.S.	1	1.00	0.88	0.92	1.94	1.16	0.00	1.68	0.86
time (sec)	N/A	0.024	0.017	0.885	0.264	0.361	0.000	0.426	0.236

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	53	47	62	0	75	-1
N.S.	1	1.00	0.89	0.98	0.87	1.15	0.00	1.39	-0.02
time (sec)	N/A	0.028	0.021	0.875	0.269	0.375	0.000	0.439	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.053	0.209	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.035	0.142	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.034	0.133	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.039	0.131	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	120	0	0	0	0	-1
N.S.	1	1.00	1.00	2.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.007	2.780	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.039	0.136	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.033	0.208	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.086	0.937	0.000	0.380	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	188	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.098	0.941	0.000	0.374	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.020	0.934	0.000	0.360	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	47	44	52	0	0	39
N.S.	1	1.00	0.96	0.94	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.026	0.018	0.257	0.269	0.390	0.000	0.000	0.534

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	150	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.523	0.921	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	197	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.031	0.957	0.925	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	229	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.040	0.468	0.924	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	149	0	0	269	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.086	0.930	0.000	0.390	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	180	0	0	188	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.101	0.928	0.000	0.367	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	114	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.010	0.014	0.906	0.000	0.353	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	48	44	52	0	0	39
N.S.	1	1.00	0.96	0.96	0.88	1.04	0.00	0.00	0.78
time (sec)	N/A	0.026	0.019	0.247	0.268	0.379	0.000	0.000	0.452

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	146	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.476	0.940	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.020	1.023	1.263	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	227	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.036	0.564	0.928	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.079	0.188	1.158	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	258	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.078	0.147	0.920	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	259	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.022	0.037	0.915	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.020	0.005	0.939	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.039	0.253	0.798	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.052	0.550	0.926	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	365	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.061	0.648	0.927	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	337	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	0.193	1.131	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	255	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	0.175	0.951	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	259	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.022	0.036	0.920	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	180	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.023	0.005	0.903	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	291	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.038	0.254	1.134	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	308	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.062	0.560	0.903	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	370	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.645	0.921	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.063	180.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	244	1175	0	0	0	0	-1
N.S.	1	1.00	0.93	4.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.041	4.033	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	187	649	0	0	0	0	-1
N.S.	1	1.00	0.96	3.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.039	2.621	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	127	263	0	0	0	0	-1
N.S.	1	1.00	0.95	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.016	1.682	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.067	180.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.542	180.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	153	0	0	0	0	-1
N.S.	1	1.00	1.00	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.401	2.747	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	119	464	491	138	0	173	-1
N.S.	1	1.00	0.72	2.81	2.98	0.84	0.00	1.05	-0.01
time (sec)	N/A	0.123	0.060	0.218	0.272	0.341	0.000	0.396	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	288	273	117	0	140	-1
N.S.	1	1.00	0.89	2.50	2.37	1.02	0.00	1.22	-0.01
time (sec)	N/A	0.089	0.076	0.224	0.255	0.344	0.000	0.401	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	145	175	93	0	106	-1
N.S.	1	1.00	1.09	2.16	2.61	1.39	0.00	1.58	-0.01
time (sec)	N/A	0.049	0.068	0.223	0.259	0.330	0.000	0.413	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	46	89	141	73	0	80	-1
N.S.	1	1.00	1.48	2.87	4.55	2.35	0.00	2.58	-0.03
time (sec)	N/A	0.012	0.025	0.353	0.288	0.331	0.000	0.392	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	99	126	160	136	0	158	180
N.S.	1	1.00	1.11	1.42	1.80	1.53	0.00	1.78	2.02
time (sec)	N/A	0.084	0.051	0.301	0.263	0.361	0.000	0.431	1.038

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	110	284	170	183	0	234	269
N.S.	1	1.00	1.11	2.87	1.72	1.85	0.00	2.36	2.72
time (sec)	N/A	0.078	0.105	0.269	0.267	0.349	0.000	0.454	1.261

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	129	459	313	181	0	384	-1
N.S.	1	1.00	1.11	3.96	2.70	1.56	0.00	3.31	-0.01
time (sec)	N/A	0.059	0.137	0.224	0.268	0.360	0.000	0.431	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	162	502	352	230	0	715	-1
N.S.	1	1.00	1.04	3.22	2.26	1.47	0.00	4.58	-0.01
time (sec)	N/A	0.079	0.079	0.226	0.260	0.459	0.000	0.444	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	192	1003	594	295	0	1173	-1
N.S.	1	1.00	0.93	4.85	2.87	1.43	0.00	5.67	-0.00
time (sec)	N/A	0.133	0.515	0.233	0.260	0.360	0.000	0.453	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	0.185	0.140	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	138	0	0	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.121	0.141	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.065	0.142	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.027	0.135	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.027	0.092	0.191	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.024	0.316	0.168	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	134	0	0	0	0	-1
N.S.	1	1.00	0.87	2.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.013	2.256	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	0	0	0	-1
N.S.	1	1.00	1.00	1.33	0.00	0.00	0.00	0.00	-0.33
time (sec)	N/A	0.040	0.043	2.085	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	38	37	66	61	105	88
N.S.	1	1.00	0.91	0.84	0.82	1.47	1.36	2.33	1.96
time (sec)	N/A	0.034	0.018	0.292	0.262	0.350	0.200	0.410	0.569

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	0	39	152	76	113	99
N.S.	1	1.00	0.89	0.00	0.85	3.30	1.65	2.46	2.15
time (sec)	N/A	0.035	0.029	0.222	0.260	0.375	21.196	0.436	0.361

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	330	46	0	242	0	0	41
N.S.	1	1.00	6.73	0.94	0.00	4.94	0.00	0.00	0.84
time (sec)	N/A	0.025	0.673	2.143	0.000	0.390	0.000	0.000	1.086

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	A	A	F	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	28	0	1	1	0	1	1
N.S.	1	0.00	1.04	0.00	0.04	0.04	0.00	0.04	0.04
time (sec)	N/A	0.031	0.356	0.818	0.488	0.322	0.000	0.391	0.224

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	108	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	2.92	0.00	0.00	-0.03
time (sec)	N/A	0.049	0.030	180.000	0.000	0.359	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	0	0	33	0	0	23
N.S.	1	1.00	0.83	0.00	0.00	1.14	0.00	0.00	0.79
time (sec)	N/A	0.044	0.017	180.000	0.000	0.347	0.000	0.000	0.245

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [343] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.00	12	0.417
2	A	10	6	1.00	14	0.429
3	A	12	7	1.00	14	0.500
4	A	5	5	1.00	16	0.312
5	A	4	4	1.00	16	0.250
6	A	4	4	1.00	14	0.286
7	A	3	2	1.00	8	0.250
8	A	8	5	1.00	16	0.312
9	A	3	3	1.00	16	0.188
10	A	4	4	1.00	16	0.250
11	A	5	5	1.00	16	0.312
12	A	18	7	1.00	18	0.389
13	A	13	7	1.00	18	0.389
14	A	9	7	1.00	16	0.438
15	A	3	3	1.00	10	0.300
16	A	10	6	1.00	18	0.333
17	A	10	7	1.00	18	0.389
18	A	13	10	1.00	18	0.556
19	A	27	7	1.00	18	0.389
20	A	17	6	1.00	18	0.333
21	A	11	7	1.00	16	0.438
22	A	4	4	1.00	10	0.400
23	A	0	0	0.00	0	0.000
24	A	0	0	0.00	0	0.000
25	A	19	7	1.00	18	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	11	7	1.00	16	0.438
27	A	5	5	1.00	10	0.500
28	A	0	0	0.00	0	0.000
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	3	3	1.00	16	0.188
32	A	0	0	0.00	0	0.000
33	A	0	0	0.00	0	0.000
34	A	16	12	1.00	30	0.400
35	A	13	8	1.00	30	0.267
36	A	8	6	1.00	28	0.214
37	A	22	20	1.00	30	0.667
38	A	35	22	1.00	30	0.733
39	A	24	17	1.00	30	0.567
40	A	20	12	1.00	30	0.400
41	A	12	9	1.00	28	0.321
42	A	29	24	1.00	30	0.800
43	A	30	18	1.00	30	0.600
44	A	26	15	1.00	30	0.500
45	A	14	10	1.00	28	0.357
46	A	37	29	1.00	30	0.967
47	A	13	7	1.00	30	0.233
48	A	9	7	1.00	30	0.233
49	A	6	5	1.00	28	0.179
50	A	1	1	1.00	23	0.043
51	A	10	7	1.00	30	0.233
52	A	13	10	1.00	30	0.333
53	A	0	0	0.00	0	0.000
54	A	13	9	1.00	34	0.265
55	A	11	8	1.00	32	0.250
56	A	9	7	1.00	24	0.292
57	A	0	0	0.00	0	0.000
58	A	6	6	1.00	10	0.600
59	A	5	5	1.00	10	0.500
60	A	5	5	1.00	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	6	0.500
62	A	9	6	1.00	10	0.600
63	A	4	4	1.00	10	0.400
64	A	5	5	1.00	10	0.500
65	A	6	6	1.00	10	0.600
66	A	7	7	1.00	10	0.700
67	A	19	8	1.00	12	0.667
68	A	14	8	1.00	12	0.667
69	A	10	8	1.00	10	0.800
70	A	4	4	1.00	8	0.500
71	A	11	7	1.00	12	0.583
72	A	11	8	1.00	12	0.667
73	A	14	11	1.00	12	0.917
74	A	40	16	1.00	12	1.333
75	A	18	11	1.00	12	0.917
76	A	12	10	1.00	10	1.000
77	A	5	4	1.00	8	0.500
78	A	13	8	1.00	12	0.667
79	A	13	9	1.00	12	0.750
80	A	21	13	1.00	12	1.083
81	A	14	8	1.00	12	0.667
82	A	10	8	1.00	10	0.800
83	A	3	3	1.00	8	0.375
84	A	0	0	0.00	0	0.000
85	A	12	7	1.00	12	0.583
86	A	8	7	1.00	10	0.700
87	A	4	4	1.00	8	0.500
88	A	0	0	0.00	0	0.000
89	A	24	12	1.00	12	1.000
90	A	14	12	1.00	10	1.200
91	A	5	5	1.00	8	0.625
92	A	0	0	0.00	0	0.000
93	A	0	0	0.00	0	0.000
94	A	22	9	1.00	16	0.562
95	A	14	9	1.00	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	12	0.333
97	A	0	0	0.00	0	0.000
98	A	23	12	1.00	18	0.667
99	A	14	10	1.00	16	0.625
100	A	8	7	1.00	14	0.500
101	A	16	10	1.00	16	0.625
102	A	9	8	1.00	14	0.571
103	A	18	10	1.00	16	0.625
104	A	10	8	1.00	14	0.571
105	A	20	9	1.00	18	0.500
106	A	12	8	1.00	16	0.500
107	A	7	6	1.00	14	0.429
108	A	16	10	1.00	16	0.625
109	A	8	7	1.00	14	0.500
110	A	22	15	1.00	16	0.938
111	A	9	8	1.00	14	0.571
112	A	21	13	1.00	16	0.812
113	A	10	8	1.00	14	0.571
114	A	3	3	1.00	21	0.143
115	A	6	5	1.00	21	0.238
116	A	6	5	1.00	21	0.238
117	A	6	5	1.00	21	0.238
118	A	5	5	1.00	19	0.263
119	A	4	3	1.00	10	0.300
120	A	7	7	1.00	21	0.333
121	A	6	6	1.00	21	0.286
122	A	4	4	1.00	21	0.190
123	A	7	7	1.00	21	0.333
124	A	5	5	1.00	21	0.238
125	A	8	7	1.00	21	0.333
126	A	3	3	1.00	23	0.130
127	A	9	7	1.00	23	0.304
128	A	8	6	1.00	23	0.261
129	A	7	7	1.00	23	0.304
130	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	12	0.333
132	A	8	8	1.00	23	0.348
133	A	9	7	1.00	23	0.304
134	A	5	5	1.00	23	0.217
135	A	11	9	1.00	23	0.391
136	A	0	0	0.00	0	0.000
137	A	17	9	1.00	23	0.391
138	A	13	7	1.00	23	0.304
139	A	12	9	1.00	23	0.391
140	A	8	7	1.00	21	0.333
141	A	6	4	1.00	12	0.333
142	A	9	9	1.00	23	0.391
143	A	11	8	1.00	23	0.348
144	A	9	9	1.00	23	0.391
145	A	16	12	1.00	23	0.522
146	A	0	0	0.00	0	0.000
147	A	16	6	1.00	23	0.261
148	A	13	8	1.00	23	0.348
149	A	9	6	1.00	21	0.286
150	A	6	4	1.00	12	0.333
151	A	10	9	1.00	23	0.391
152	A	13	9	1.00	23	0.391
153	A	10	10	1.00	23	0.435
154	A	21	12	1.00	23	0.522
155	A	0	0	0.00	0	0.000
156	A	14	7	1.00	23	0.304
157	A	11	7	1.00	23	0.304
158	A	11	7	1.00	23	0.304
159	A	8	7	1.00	21	0.333
160	A	5	5	1.00	12	0.417
161	A	0	0	0.00	0	0.000
162	A	13	6	1.00	23	0.261
163	A	10	6	1.00	23	0.261
164	A	10	6	1.00	23	0.261
165	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	12	0.500
167	A	0	0	0.00	0	0.000
168	A	26	9	1.00	23	0.391
169	A	20	9	1.00	23	0.391
170	A	18	10	1.00	23	0.435
171	A	11	10	1.00	21	0.476
172	A	7	7	1.00	12	0.583
173	A	0	0	0.00	0	0.000
174	A	24	8	1.00	23	0.348
175	A	17	8	1.00	23	0.348
176	A	18	10	1.00	23	0.435
177	A	9	9	1.00	21	0.429
178	A	8	7	1.00	12	0.583
179	A	0	0	0.00	0	0.000
180	A	21	9	1.00	25	0.360
181	A	16	9	1.00	25	0.360
182	A	16	9	1.00	25	0.360
183	A	11	9	1.00	23	0.391
184	A	8	7	1.00	14	0.500
185	A	0	0	0.00	0	0.000
186	A	43	12	1.00	25	0.480
187	A	27	11	1.00	25	0.440
188	A	24	12	1.00	25	0.480
189	A	13	11	1.00	23	0.478
190	A	9	8	1.00	14	0.571
191	A	0	0	0.00	0	0.000
192	A	46	12	1.00	25	0.480
193	A	29	11	1.00	25	0.440
194	A	26	12	1.00	25	0.480
195	A	14	11	1.00	23	0.478
196	A	10	8	1.00	14	0.571
197	A	0	0	0.00	0	0.000
198	A	77	13	1.00	25	0.520
199	A	42	11	1.00	25	0.440
200	A	35	13	1.00	25	0.520

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	16	11	1.00	23	0.478
202	A	11	8	1.00	14	0.571
203	A	0	0	0.00	0	0.000
204	A	20	8	1.00	25	0.320
205	A	15	8	1.00	25	0.320
206	A	15	8	1.00	25	0.320
207	A	10	8	1.00	23	0.348
208	A	7	6	1.00	14	0.429
209	A	0	0	0.00	0	0.000
210	A	19	7	1.00	25	0.280
211	A	14	7	1.00	25	0.280
212	A	14	7	1.00	25	0.280
213	A	8	7	1.00	23	0.304
214	A	8	7	1.00	14	0.500
215	A	0	0	0.00	0	0.000
216	A	36	10	1.00	25	0.400
217	A	26	10	1.00	25	0.400
218	A	24	11	1.00	25	0.440
219	A	13	11	1.00	23	0.478
220	A	9	8	1.00	14	0.571
221	A	0	0	0.00	0	0.000
222	A	34	9	1.00	25	0.360
223	A	23	9	1.00	25	0.360
224	A	24	11	1.00	25	0.440
225	A	11	10	1.00	23	0.435
226	A	10	8	1.00	14	0.571
227	A	0	0	0.00	0	0.000
228	A	8	7	1.00	23	0.304
229	A	6	5	1.00	23	0.217
230	A	7	7	1.00	23	0.304
231	A	5	5	1.00	23	0.217
232	A	6	6	1.00	23	0.261
233	A	4	4	1.00	23	0.174
234	A	7	7	1.00	23	0.304
235	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	25	0.120
237	A	3	3	1.00	25	0.120
238	A	3	3	1.00	25	0.120
239	A	3	3	1.00	25	0.120
240	A	3	3	1.00	25	0.120
241	A	3	3	1.00	25	0.120
242	A	3	3	1.00	25	0.120
243	A	3	3	1.00	25	0.120
244	A	0	0	0.00	0	0.000
245	A	0	0	0.00	0	0.000
246	A	0	0	0.00	0	0.000
247	A	0	0	0.00	0	0.000
248	A	0	0	0.00	0	0.000
249	A	0	0	0.00	0	0.000
250	A	0	0	0.00	0	0.000
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	0	0	0.00	0	0.000
255	A	0	0	0.00	0	0.000
256	A	0	0	0.00	0	0.000
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	0	0	0.00	0	0.000
260	A	7	6	1.00	30	0.200
261	A	6	6	1.00	30	0.200
262	A	4	4	1.00	28	0.143
263	A	5	4	1.00	30	0.133
264	A	6	6	1.00	30	0.200
265	A	4	4	0.97	30	0.133
266	A	15	9	1.00	30	0.300
267	A	11	9	1.00	30	0.300
268	A	7	6	1.00	28	0.214
269	A	6	4	1.00	30	0.133
270	A	7	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	11	8	1.00	30	0.267
272	A	2	2	1.00	30	0.067
273	A	2	2	1.00	30	0.067
274	A	2	2	1.00	28	0.071
275	A	2	2	1.00	30	0.067
276	A	2	2	1.00	30	0.067
277	A	2	2	1.00	30	0.067
278	A	8	8	1.00	30	0.267
279	A	7	7	1.00	30	0.233
280	A	3	3	1.00	28	0.107
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	5	5	1.00	10	0.500
284	A	4	4	1.00	10	0.400
285	A	3	3	1.00	8	0.375
286	A	5	5	1.00	6	0.833
287	A	5	5	1.00	10	0.500
288	A	3	3	1.00	10	0.300
289	A	5	5	1.00	10	0.500
290	A	6	6	1.00	10	0.600
291	A	5	5	1.00	10	0.500
292	A	7	5	1.00	10	0.500
293	A	6	5	1.00	8	0.625
294	A	6	6	1.00	6	1.000
295	A	5	5	1.00	10	0.500
296	A	3	3	1.00	10	0.300
297	A	4	4	1.00	10	0.400
298	A	5	4	1.00	10	0.400
299	A	6	4	1.00	10	0.400
300	A	6	6	1.00	10	0.600
301	A	3	3	1.00	8	0.375
302	A	5	5	1.00	6	0.833
303	A	5	5	1.00	10	0.500
304	A	3	3	1.00	10	0.300
305	A	5	5	1.00	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	5	4	1.00	10	0.400
307	A	3	3	1.00	10	0.300
308	A	3	3	1.00	10	0.300
309	A	3	3	1.00	8	0.375
310	A	3	3	1.00	6	0.500
311	A	5	5	1.00	10	0.500
312	A	3	3	1.00	10	0.300
313	A	3	3	1.00	10	0.300
314	A	3	2	1.00	20	0.100
315	A	5	4	1.00	20	0.200
316	A	2	2	1.00	20	0.100
317	A	4	3	1.00	18	0.167
318	A	1	1	1.00	20	0.050
319	A	1	1	1.00	20	0.050
320	A	2	2	1.00	20	0.100
321	A	3	2	1.00	20	0.100
322	A	5	4	1.00	20	0.200
323	A	2	2	1.00	20	0.100
324	A	4	3	1.00	18	0.167
325	A	1	1	1.00	20	0.050
326	A	1	1	1.00	20	0.050
327	A	2	2	1.00	20	0.100
328	A	2	2	1.00	22	0.091
329	A	2	2	1.00	22	0.091
330	A	1	1	1.00	22	0.045
331	A	1	1	1.00	22	0.045
332	A	1	1	1.00	22	0.045
333	A	2	2	1.00	22	0.091
334	A	2	2	1.00	22	0.091
335	A	2	2	1.00	22	0.091
336	A	2	2	1.00	22	0.091
337	A	1	1	1.00	22	0.045
338	A	1	1	1.00	22	0.045
339	A	1	1	1.00	22	0.045
340	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	2	2	1.00	22	0.091
342	A	0	0	0.00	0	0.000
343	A	8	8	1.00	40	0.200
344	A	7	7	1.00	40	0.175
345	A	6	7	1.00	38	0.184
346	A	0	0	0.00	0	0.000
347	A	0	0	0.00	0	0.000
348	A	6	6	1.00	10	0.600
349	A	5	4	1.00	12	0.333
350	A	5	4	1.00	12	0.333
351	A	5	4	1.00	10	0.400
352	A	5	4	1.00	8	0.500
353	A	9	8	1.00	12	0.667
354	A	9	8	1.00	12	0.667
355	A	6	5	1.00	12	0.417
356	A	7	6	1.00	12	0.500
357	A	8	7	1.00	12	0.583
358	A	37	8	1.00	14	0.571
359	A	27	8	1.00	14	0.571
360	A	17	8	1.00	12	0.667
361	A	7	4	1.00	10	0.400
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	7	7	1.00	19	0.368
365	A	2	2	1.00	15	0.133
366	A	4	4	1.00	12	0.333
367	A	4	4	1.00	14	0.286
368	A	6	6	1.00	10	0.600
369	F	0	0	N/A	0.	N/A
370	A	2	2	1.00	26	0.077
371	A	2	2	1.00	26	0.077

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\sinh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=170

$$-\frac{\sinh^{-1}(cx)^2}{2e}+\frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\Big(2}{}$$

```
[Out] -1/2*arcsinh(c*x)^2/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2
*d^2+e^2)^(1/2)))/e+arcsinh(c*x)*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d
^2+e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e+polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A]

time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5827, 5680, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\sinh^{-1}(cx)^2}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(cx)}{d+ex}\,dx &= \text{Subst}\left(\int \frac{x\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)\\
&= -\frac{\sinh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd+\cdots}\right.\\
&= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+\cdots}}\right.}{e}\\
&= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+\cdots}}\right.}{e}\\
&= -\frac{\sinh^{-1}(cx)^2}{2e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+\cdots}}\right.}{e}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 168, normalized size = 0.99

$$-\frac{\sinh^{-1}(cx)^2}{2e}+\frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcSinh[c*x]^2/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqr
t[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqr
t[c^2*d^2 + e^2])])/e + PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 + e^2])]/e + PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))
]/e
```

Maple [A]
time = 3.42, size = 272, normalized size = 1.60

method	result
derivativedivides	$-\frac{c\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2 -}\right.}{cd+\sqrt{c}}\right.}{e}$
default	$-\frac{c\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{c\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e\left(cx+\sqrt{c^2x^2 -}\right.}{cd+\sqrt{c}}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*c*arcsinh(c*x)^2/e+1/e*c*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)
^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))+1/e*c*arcsinh(c*x)
*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^
(1/2)))+1/e*c*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*
d+(c^2*d^2+e^2)^(1/2)))+1/e*c*dilog((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^
2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)/(d + e*x),x)
```

```
[Out] int(asinh(c*x)/(d + e*x), x)
```

3.2 $\int \frac{\sinh^{-1}(cx)^2}{d+ex} \, dx$

Optimal. Leaf size=260

$$-\frac{\sinh^{-1}(cx)^3}{3e}+\frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(c$$

```
[Out] -1/3*arcsinh(c*x)^3/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^2*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(
c*d-(c^2*d^2+e^2)^(1/2)))/e+2*arcsinh(c*x)*polylog(2,-e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c
*d-(c^2*d^2+e^2)^(1/2)))/e-2*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2
*d^2+e^2)^(1/2)))/e
```

Rubi [A]
time = 0.29, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5827, 5680, 2221, 2611, 2320, 6724}

$$\frac{2\sinh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2\sinh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)^2\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)^2\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\sinh^{-1}(cx)^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*ArcSinh[c*x]^3/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (2*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSi
nh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*
x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (2*PolyLog[3, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd}\right.$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\text{si}}}{cd+\sqrt{c}}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\text{si}}}{cd+\sqrt{c}}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\text{si}}}{cd+\sqrt{c}}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^3}{3e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\text{si}}}{cd+\sqrt{c}}\right.}{e}$$

Mathematica [A]
time = 0.09, size = 240, normalized size = 0.92

$$-\frac{\sinh^{-1}(cx)^3 - 3\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right) - 3\sinh^{-1}(cx)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) - 6\sinh^{-1}(cx)\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right) - 6\sinh^{-1}(cx)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right) + 6\text{PolyLog}\left(3,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right) + 6\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*(ArcSinh[c*x]^3 - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])] - 3*ArcSinh[c*x]^2*Log[1 + (e*E^ArcSinh[c*x])/(c*d + S
qrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - 6*ArcSinh[c*x]*PolyLog[2, -((e*E^ArcSinh[c*x])/(
c*d + Sqrt[c^2*d^2 + e^2]))] + 6*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sq
rt[c^2*d^2 + e^2])] + 6*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
 + e^2]))])/e
```

Maple [F]
time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(cx)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arcsinh(c*x)^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(c*x)^2/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arcsinh(c*x)^2/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(asinh(c*x)**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c*x)^2/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(c\,x)^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)^2/(d + e*x),x)
[Out] int(asinh(c*x)^2/(d + e*x), x)
```

3.3 $\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx$

Optimal. Leaf size=348

$$-\frac{\sinh^{-1}(cx)^4}{4e}+\frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx$$

```
[Out] -1/4*arcsinh(c*x)^4/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c
^2*d^2+e^2)^(1/2)))/e+arcsinh(c*x)^3*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c
^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))
/(c*d-(c^2*d^2+e^2)^(1/2)))/e+3*arcsinh(c*x)^2*polylog(2,-e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*x+(c^2
*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-6*arcsinh(c*x)*polylog(3,-e*(c*
x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*
x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+6*polylog(4,-e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A]
time = 0.31, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$\frac{3\sinh^{-1}(cx)^2\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{3\sinh^{-1}(cx)^2\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{6\sinh^{-1}(cx)\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\text{Li}_4\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{6\text{Li}_4\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\sinh^{-1}(cx)^3\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\sinh^{-1}(cx)^3\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\sinh^{-1}(cx)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] -1/4*ArcSinh[c*x]^4/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - S
qrt[c^2*d^2 + e^2])])/e + (ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
 Sqrt[c^2*d^2 + e^2])])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x
])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (3*ArcSinh[c*x]^2*PolyLog[2, -((e*E^A
rcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*PolyLog[3,
-((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e - (6*ArcSinh[c*x]*Pol
yLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[
4, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (6*PolyLog[4, -(
(e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(cx)^3}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^3\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \text{Subst}\left(\int \frac{e^x x^3}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{\quad}{cd+\cdots}\right.$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh}\cdots}{cd+\sqrt{c^2}\cdots}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh}\cdots}{cd+\sqrt{c^2}\cdots}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh}\cdots}{cd+\sqrt{c^2}\cdots}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh}\cdots}{cd+\sqrt{c^2}\cdots}\right.}{e}$$

$$= -\frac{\sinh^{-1}(cx)^4}{4e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh}\cdots}{cd+\sqrt{c^2}\cdots}\right.}{e}$$

Mathematica [A]
time = 0.03, size = 322, normalized size = 0.93

$$\frac{-\sinh^{-1}(cx)^4+4\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)+4\sinh^{-1}(cx)^3\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+12\sinh^{-1}(cx)^2\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)+12\sinh^{-1}(cx)^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-24\sinh^{-1}(cx)\text{PolyLog}\left(3,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-24\sinh^{-1}(cx)\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+24\text{PolyLog}\left(4,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)+24\text{PolyLog}\left(4,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*x]^3/(d + e*x),x]
```

```
[Out] (-ArcSinh[c*x]^4 + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[
c^2*d^2 + e^2])] + 4*ArcSinh[c*x]^3*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[
c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
+ Sqrt[c^2*d^2 + e^2])] + 12*ArcSinh[c*x]^2*PolyLog[2, -((e*E^ArcSinh[c*x])
/(c*d + Sqrt[c^2*d^2 + e^2]))] - 24*ArcSinh[c*x]*PolyLog[3, (e*E^ArcSinh[c*
x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 24*ArcSinh[c*x]*PolyLog[3, -((e*E^Arc
Sinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] + 24*PolyLog[4, (e*E^ArcSinh[c*x])
/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 24*PolyLog[4, -((e*E^ArcSinh[c*x])/(c*d
+ Sqrt[c^2*d^2 + e^2]))])/(4*e)
```

Maple [F]

time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(cx)^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c*x)^3/(e*x+d),x)
[Out] int(arcsinh(c*x)^3/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(arcsinh(c*x)^3/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="fricas")
[Out] integral(arcsinh(c*x)^3/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*x)**3/(e*x+d),x)
[Out] Integral(asinh(c*x)**3/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*x)^3/(e*x+d),x, algorithm="giac")
[Out] integrate(arcsinh(c*x)^3/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(c\,x)^3}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*x)^3/(d + e*x),x)
[Out] int(asinh(c*x)^3/(d + e*x), x)
```

3.4 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=176

$$-\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c}-\frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c}-\frac{b(4d(19c^2d^2-16e^2)+e(26c^2d^2-9e^2)\,x)\sqrt{1+c^2x^2}}{96c^3}-$$

```
[Out] -1/32*b*(8*c^4*d^4-24*c^2*d^2*e^2+3*e^4)*arcsinh(c*x)/c^4/e+1/4*(e*x+d)^4*(
a+b*arcsinh(c*x))/e-7/48*b*d*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/16*b*(e*x+d)^3
*(c^2*x^2+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2-16*e^2)+e*(26*c^2*d^2-9*e^2)*x
)*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A]
time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5828, 757, 847, 794, 221}

$$\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)^3}{16c}-\frac{7bd\sqrt{c^2x^2+1}\,(d+ex)^2}{48c}-\frac{b(8c^4d^4-24c^2d^2e^2+3e^4)\sinh^{-1}(cx)}{32c^4e}-\frac{b\sqrt{c^2x^2+1}\,(ex(26c^2d^2-9e^2)+4d(19c^2d^2-16e^2))}{96c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-7*b*d*(d + e*x)^2*Sqrt[1 + c^2*x^2])/(48*c) - (b*(d + e*x)^3*Sqrt[1 + c^2
*x^2])/(16*c) - (b*(4*d*(19*c^2*d^2 - 16*e^2) + e*(26*c^2*d^2 - 9*e^2)*x)*S
qrt[1 + c^2*x^2])/(96*c^3) - (b*(8*c^4*d^4 - 24*c^2*d^2*e^2 + 3*e^4)*ArcSin
h[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*ArcSinh[c*x]))/(4*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{(bc)\int\frac{(d+ex)^4}{\sqrt{1+c^2x^2}}\,dx}{4e}$$
$$= -\frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{4e} - \frac{b\int\frac{(d+ex)^2(4c^2}{\sqrt{1}}}{1}$$
$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} + \frac{(d+ex)^4\left(a+b\,\mathrm{s}\right.}{4e}$$
$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b(4d(19c^2d^2-16}{}$$
$$= -\frac{7bd(d+ex)^2\sqrt{1+c^2x^2}}{48c} - \frac{b(d+ex)^3\sqrt{1+c^2x^2}}{16c} - \frac{b(4d(19c^2d^2-16}{}$$

Mathematica [A]
time = 0.11, size = 166, normalized size = 0.94

$$\frac{24ac^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-bc\sqrt{1+c^2x^2}\left(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3)\right)+3b(24c^2d^2e-3e^3+8c^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3))\sinh^{-1}(cx)}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[1 + c^2*
x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e
^3*x^3)) + 3*b*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2
*x^2 + e^3*x^3))*ArcSinh[c*x])/(96*c^4)
```

Maple [A]
time = 0.68, size = 259, normalized size = 1.47

method	result
derivativedivides	$\frac{(cex+cd)^4 a}{4c^3 e} + \frac{b\left(\frac{\mathrm{arcsinh}(cx)c^4d^4}{4e} + \mathrm{arcsinh}(cx)c^4d^3x + \frac{3e\,\mathrm{arcsinh}(cx)c^4d^2x^2}{2} + e^2\,\mathrm{arcsinh}(cx)c^4d\,x^3 + \frac{e^3\,\mathrm{arcsinh}(cx)c^4x^4}{4} - \frac{c^4d^4\,\mathrm{arcsi}}{}\right.}{}$
default	$\frac{(cex+cd)^4 a}{4c^3 e} + \frac{b\left(\frac{\mathrm{arcsinh}(cx)c^4d^4}{4e} + \mathrm{arcsinh}(cx)c^4d^3x + \frac{3e\,\mathrm{arcsinh}(cx)c^4d^2x^2}{2} + e^2\,\mathrm{arcsinh}(cx)c^4d\,x^3 + \frac{e^3\,\mathrm{arcsinh}(cx)c^4x^4}{4} - \frac{c^4d^4\,\mathrm{arcsi}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4/e*arcsinh(c*x)*c^4*d^4+arcsinh(c*
x)*c^4*d^3*x+3/2*e*arcsinh(c*x)*c^4*d^2*x^2+e^2*arcsinh(c*x)*c^4*d*x^3+1/4*
e^3*arcsinh(c*x)*c^4*x^4-1/4/e*(c^4*d^4*arcsinh(c*x)+4*d^3*c^3*e*(c^2*x^2+1
)^(1/2)+6*d^2*c^2*e^2*(1/2*(c^2*x^2+1)^(1/2)*c*x-1/2*arcsinh(c*x))+4*d*c*e^
3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+e^4*(1/4*(c^2*x^2+1
)^(1/2)*x^3*c^3-3/8*(c^2*x^2+1)^(1/2)*c*x+3/8*arcsinh(c*x)))))
```

Maxima [A]
time = 0.26, size = 228, normalized size = 1.30

$$\frac{1}{4}ax^4e^3 + adx^3e^2 + \frac{3}{2}ad^2x^2e + ad^3x + \frac{3}{4}\left(2x^2\,\mathrm{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2} - \frac{\mathrm{arsinh}(cx)}{c^3}\right)\right)bd^2e + \frac{\left(cx\,\mathrm{arsinh}(cx) - \sqrt{c^2x^2+1}\right)bd^3}{c} + \frac{1}{3}\left(3x^3\,\mathrm{arsinh}(cx) - c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2} - \frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)bde^2 + \frac{1}{32}\left(8x^4\,\mathrm{arsinh}(cx) - \left(\frac{2\sqrt{c^2x^2+1}\,x^3}{c^2} - \frac{3\sqrt{c^2x^2+1}\,x}{c^4} + \frac{3\,\mathrm{arsinh}(cx)}{c^5}\right)c\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x + 3/4*(2*x^2*arcsin
h(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d^2*e + (c*x*arc
sinh(c*x) - sqrt(c^2*x^2 + 1))*b*d^3/c + 1/3*(3*x^3*arcsinh(c*x) - c*(sqrt(
c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*b*d*e^2 + 1/32*(8*x^4*arcs
inh(c*x) - (2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arc
sinh(c*x)/c^5)*c)*b*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(158) = 316$.
time = 0.38, size = 525, normalized size = 2.98

$$\frac{24\,ac^4x^4\cosh(1)^3 + 24\,ac^4x^4\sinh(1)^3 + 96\,ac^4dx^3\cosh(1)^2 + 144\,ac^4d^2x^2\cosh(1) + 96\,ac^4d^3x + 24\,(3\,ac^4x^4\cosh(1) + 4\,ac^4dx^3)\sinh(1)^2 + 3\,(32\,bc^4dx^3\cosh(1)^2 + 32\,bc^4d^3x + (8\,bc^4x^4 - 3\,b)\cosh(1)^3 + (8\,bc^4x^4 - 3\,b)\sinh(1)^3 + (32\,bc^4dx^3 + 3\,(8\,bc^4x^4 - 3\,b)\cosh(1))\sinh(1)^2 + 24\,(2\,bc^4d^2x^2 + bc^2d^2)\cosh(1) + (64\,bc^4dx^3\cosh(1) + 48\,bc^4d^2x^2 + 24\,bc^2d^2 + 3\,(8\,bc^4x^4 - 3\,b)\cosh(1)^2)\sinh(1))\log\left(cx + \sqrt{c^2x^2+1}\right) + 24\,(3\,ac^4x^4\cosh(1)^2 + 8\,ac^4dx^3\cosh(1) + 6\,ac^4d^2x^2)\sinh(1) - (72\,bc^3d^2x\cosh(1) + 96\,bc^3d^3 + 3\,(2\,bc^3x^3 - 3\,bcx)\cosh(1)^3 + 3\,(2\,bc^3x^3 - 3\,bcx)\sinh(1)^3 + 32\,(bc^3dx^2 - 2\,bcd)\cosh(1)^2 + (32\,bc^3dx^2 - 64\,bcd + 9\,(2\,bc^3x^3 - 3\,bcx)\cosh(1))\sinh(1)^2 + (72\,bc^3d^2x + 9\,(2\,bc^3x^3 - 3\,bcx)\cosh(1)^2 + 64\,(bc^3dx^2 - 2\,bcd)\cosh(1))\sinh(1))\sqrt{c^2x^2+1}}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*x^4*cosh(1)^3 + 24*a*c^4*x^4*sinh(1)^3 + 96*a*c^4*d*x^3*cosh
(1)^2 + 144*a*c^4*d^2*x^2*cosh(1) + 96*a*c^4*d^3*x + 24*(3*a*c^4*x^4*cosh(1
) + 4*a*c^4*d*x^3)*sinh(1)^2 + 3*(32*b*c^4*d*x^3*cosh(1)^2 + 32*b*c^4*d^3*x
 + (8*b*c^4*x^4 - 3*b)*cosh(1)^3 + (8*b*c^4*x^4 - 3*b)*sinh(1)^3 + (32*b*c^
4*d*x^3 + 3*(8*b*c^4*x^4 - 3*b)*cosh(1))*sinh(1)^2 + 24*(2*b*c^4*d^2*x^2 +
b*c^2*d^2)*cosh(1) + (64*b*c^4*d*x^3*cosh(1) + 48*b*c^4*d^2*x^2 + 24*b*c^2*
d^2 + 3*(8*b*c^4*x^4 - 3*b)*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)
) + 24*(3*a*c^4*x^4*cosh(1)^2 + 8*a*c^4*d*x^3*cosh(1) + 6*a*c^4*d^2*x^2)*si
nh(1) - (72*b*c^3*d^2*x*cosh(1) + 96*b*c^3*d^3 + 3*(2*b*c^3*x^3 - 3*b*c*x)*
cosh(1)^3 + 3*(2*b*c^3*x^3 - 3*b*c*x)*sinh(1)^3 + 32*(b*c^3*d*x^2 - 2*b*c*d
)*cosh(1)^2 + (32*b*c^3*d*x^2 - 64*b*c*d + 9*(2*b*c^3*x^3 - 3*b*c*x)*cosh(1
))*sinh(1)^2 + (72*b*c^3*d^2*x + 9*(2*b*c^3*x^3 - 3*b*c*x)*cosh(1)^2 + 64*(
b*c^3*d*x^2 - 2*b*c*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [A]
time = 0.28, size = 316, normalized size = 1.80

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{asinh}(cx) + \frac{3bd^2ex^2\operatorname{asinh}(cx)}{2} + bde^2x^3\operatorname{asinh}(cx) + \frac{be^3x^4\operatorname{asinh}(cx)}{4} - \frac{bd^3\sqrt{c^2x^2+1}}{c} - \frac{3bd^2ex\sqrt{c^2x^2+1}}{4c} - \frac{bde^2x^2\sqrt{c^2x^2+1}}{3c} - \frac{be^3x^3\sqrt{c^2x^2+1}}{16c} + \frac{3bd^2e\operatorname{asinh}(cx)}{4c^2} + \frac{2bde^2\sqrt{c^2x^2+1}}{3c^3} + \frac{3be^3x\sqrt{c^2x^2+1}}{32c^3} - \frac{3be^3\operatorname{asinh}(cx)}{32c^4} & \text{for } c \neq 0 \\ a\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*asinh(c*x) + 3*b*d**2*e*x**2*asinh(c*x)/2 + b*d*e**2*x**3*asinh(c*x
) + b*e**3*x**4*asinh(c*x)/4 - b*d**3*sqrt(c**2*x**2 + 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 + 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 + 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 + 1)/(16*c) + 3*b*d**2*e*asinh(c*x)/(4*c**2) + 2*b*d*
e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*b*e**3*x*sqrt(c**2*x**2 + 1)/(32*c**3
) - 3*b*e**3*asinh(c*x)/(32*c**4), Ne(c, 0)), (a*(d**3*x + 3*d**2*e*x**2/2
+ d*e**2*x**3 + e**3*x**4/4), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^3, x)
```

3.5 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=124

$$-\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c}-\frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}+\frac{(d+ex)^3\left(a+ \right.}{3e}$$

```
[Out] -1/6*b*d*(2*d^2-3*e^2/c^2)*arcsinh(c*x)/e+1/3*(e*x+d)^3*(a+b*arcsinh(c*x))/
e-1/9*b*(e*x+d)^2*(c^2*x^2+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*d^2-4*e^2)
*(c^2*x^2+1)^(1/2)/c^3
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5828, 757, 794, 221}

$$\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)}{3e}-\frac{bd\left(2d^2-\frac{3e^2}{c^2}\right)\sinh^{-1}(cx)}{6e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)^2}{9c}-\frac{b\sqrt{c^2x^2+1}\,(4(4c^2d^2-e^2)+5c^2dex)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/9*(b*(d + e*x)^2*Sqrt[1 + c^2*x^2])/c - (b*(4*(4*c^2*d^2 - e^2) + 5*c^2*
d*e*x)*Sqrt[1 + c^2*x^2])/(18*c^3) - (b*d*(2*d^2 - (3*e^2)/c^2)*ArcSinh[c*x
])/(6*e) + ((d + e*x)^3*(a + b*ArcSinh[c*x]))/(3*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{1+c^2x^2}}\,dx}{3e}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} + \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)(3c}{\sqrt{1}}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3} + \frac{(d}{}$$
$$= -\frac{b(d+ex)^2\sqrt{1+c^2x^2}}{9c} - \frac{b(4(4c^2d^2-e^2)+5c^2dex)\sqrt{1+c^2x^2}}{18c^3} - \frac{bd}{}$$

Mathematica [A]
time = 0.07, size = 121, normalized size = 0.98

$$\frac{6ac^3x(3d^2+3dex+e^2x^2) - b\sqrt{1+c^2x^2}\,(-4e^2+c^2(18d^2+9dex+2e^2x^2)) + 3bc(6c^2d^2x+2c^2e^2x^3+3d(e+2c^2ex^2))\sinh^{-1}(cx)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*
(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + 3*b*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*
(e + 2*c^2*e*x^2))*ArcSinh[c*x])/(18*c^3)
```

Maple [A]
time = 0.67, size = 189, normalized size = 1.52

method	result

derivativedivides	$\frac{\frac{(cex+cd)^3 a}{3c^2 e}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e}+\operatorname{arcsinh}(cx)c^3d^2x+e\operatorname{arcsinh}(cx)c^3d\,x^2+\frac{e^2\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^3d^3\operatorname{arcsinh}(cx)+3d^2c^2e\sqrt{c^2x^2}}{}\right.}{c^2}}{c}$
default	$\frac{\frac{(cex+cd)^3 a}{3c^2 e}+\frac{b\left(\frac{\operatorname{arcsinh}(cx)c^3d^3}{3e}+\operatorname{arcsinh}(cx)c^3d^2x+e\operatorname{arcsinh}(cx)c^3d\,x^2+\frac{e^2\operatorname{arcsinh}(cx)c^3x^3}{3}-\frac{c^3d^3\operatorname{arcsinh}(cx)+3d^2c^2e\sqrt{c^2x^2}}{}\right.}{c^2}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3/e*arcsinh(c*x)*c^3*d^3+arcsinh(c*
x)*c^3*d^2*x+e*arcsinh(c*x)*c^3*d*x^2+1/3*e^2*arcsinh(c*x)*c^3*x^3-1/3/e*(c
^3*d^3*arcsinh(c*x)+3*d^2*c^2*e*(c^2*x^2+1)^(1/2)+3*d*c*e^2*(1/2*(c^2*x^2+1
)^(1/2)*c*x-1/2*arcsinh(c*x))+e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x
^2+1)^(1/2)))))
```

Maxima [A]
time = 0.26, size = 150, normalized size = 1.21

$$\frac{1}{3}\,ax^3e^2+adx^2e+ad^2x+\frac{1}{2}\left(2\,x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)bde+\frac{\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)bd^2}{c}+\frac{1}{9}\left(3\,x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\,\sqrt{c^2x^2+1}}{c^4}\right)\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 + a*d*x^2*e + a*d^2*x + 1/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2
*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*b*d*e + (c*x*arcsinh(c*x) - sqrt(c^2*x
^2 + 1))*b*d^2/c + 1/9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 -
 2*sqrt(c^2*x^2 + 1)/c^4))*b*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(108) = 216.
time = 0.40, size = 281, normalized size = 2.27

$$\frac{6\,ac^3x^3\cosh(1)^2+6\,ac^3x^3\sinh(1)^2+18\,ac^3dx^2\cosh(1)+18\,ac^3d^2x+3\left(2\,bc^3x^3\cosh(1)^2+2\,bc^3x^3\sinh(1)^2+6\,bc^3d^2x+3\left(2\,bc^3dx^2+bcd\right)\cosh(1)+\left(4\,bc^3x^3\cosh(1)+6\,bc^3dx^2+3\,bcd\right)\sinh(1)\right)\log\left(cx+\sqrt{c^2x^2+1}\right)+6\left(2\,ac^3x^3\cosh(1)+3\,ac^3dx^2\right)\sinh(1)-\left(9\,bc^2dx\cosh(1)+18\,bc^2d^2+2\left(bc^2x^2-2\,b\right)\cosh(1)^2+2\left(bc^2x^2-2\,b\right)\sinh(1)^2+\left(9\,bc^2dx+4\left(bc^2x^2-2\,b\right)\cosh(1)\right)\sinh(1)\right)\sqrt{c^2x^2+1}}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*x^3*cosh(1)^2 + 6*a*c^3*x^3*sinh(1)^2 + 18*a*c^3*d*x^2*cosh(1
) + 18*a*c^3*d^2*x + 3*(2*b*c^3*x^3*cosh(1)^2 + 2*b*c^3*x^3*sinh(1)^2 + 6*b
```

```
*c^3*d^2*x + 3*(2*b*c^3*d*x^2 + b*c*d)*cosh(1) + (4*b*c^3*x^3*cosh(1) + 6*b
*c^3*d*x^2 + 3*b*c*d)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)) + 6*(2*a*c^3*x^
3*cosh(1) + 3*a*c^3*d*x^2)*sinh(1) - (9*b*c^2*d*x*cosh(1) + 18*b*c^2*d^2 +
2*(b*c^2*x^2 - 2*b)*cosh(1)^2 + 2*(b*c^2*x^2 - 2*b)*sinh(1)^2 + (9*b*c^2*d*
x + 4*(b*c^2*x^2 - 2*b)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.17, size = 190, normalized size = 1.53

$$\begin{cases} ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x \operatorname{asinh}(cx) + bdex^2 \operatorname{asinh}(cx) + \frac{be^2x^3 \operatorname{asinh}(cx)}{3} - \frac{bd^2\sqrt{c^2x^2+1}}{c} - \frac{bdex\sqrt{c^2x^2+1}}{2c} - \frac{be^2x^2\sqrt{c^2x^2+1}}{9c} + \frac{bde \operatorname{asinh}(cx)}{2c^2} + \frac{2be^2\sqrt{c^2x^2+1}}{9c^3} & \text{for}\ c \neq 0 \\ a\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*asinh(c*x) + b*
d*e*x**2*asinh(c*x) + b*e**2*x**3*asinh(c*x)/3 - b*d**2*sqrt(c**2*x**2 + 1)
/c - b*d*e*x*sqrt(c**2*x**2 + 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 + 1)/(9
*c) + b*d*e*asinh(c*x)/(2*c**2) + 2*b*e**2*sqrt(c**2*x**2 + 1)/(9*c**3), Ne
(c, 0)), (a*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c\,x))\,(d + e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))*(d + e*x)^2, x)
```

3.6 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=97

$$-\frac{3bd\sqrt{1+c^2x^2}}{4c}-\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}+\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}$$

```
[Out] -1/4*b*(2*d^2-e^2/c^2)*arcsinh(c*x)/e+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))/e-3/
4*b*d*(c^2*x^2+1)^(1/2)/c-1/4*b*(e*x+d)*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5828, 757, 655, 221}

$$\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}-\frac{b\sqrt{c^2x^2+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{c^2x^2+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[1 + c^2*x^2])/(4*c) - (b*(d + e*x)*Sqrt[1 + c^2*x^2])/(4*c) -
(b*(2*d^2 - e^2/c^2)*ArcSinh[c*x])/(4*e) + ((d + e*x)^2*(a + b*ArcSinh[c*x]
))/(2*e)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{(bc)\int\frac{(d+ex)^2}{\sqrt{1+c^2x^2}}\,dx}{2e}$$

$$=-\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c}+\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}{2e}-\frac{b\int\frac{2c^2d^2-e^2+3c}{\sqrt{1+c^2}}}{4ce}$$

$$=-\frac{3bd\sqrt{1+c^2x^2}}{4c}-\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c}+\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx\right)}{2e}$$

$$=-\frac{3bd\sqrt{1+c^2x^2}}{4c}-\frac{b(d+ex)\sqrt{1+c^2x^2}}{4c}-\frac{b\left(2d^2-\frac{e^2}{c^2}\right)\sinh^{-1}(cx)}{4e}+$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.94

$$adx+\frac{1}{2}aex^2-\frac{bd\sqrt{1+c^2x^2}}{c}-\frac{bex\sqrt{1+c^2x^2}}{4c}+\frac{be\sinh^{-1}(cx)}{4c^2}+bdx\sinh^{-1}(cx)+\frac{1}{2}bex^2\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x]),x]
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[1 + c^2*x^2])/c - (b*e*x*Sqrt[1 + c^2*x^2])
/(4*c) + (b*e*ArcSinh[c*x])/(4*c^2) + b*d*x*ArcSinh[c*x] + (b*e*x^2*ArcSinh
[c*x])/2
```

Maple [A]
time = 0.63, size = 96, normalized size = 0.99

method	result	s
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^2x+\frac{\operatorname{arcsinh}(cx)e\,c^2x^2}{2}-dc\sqrt{c^2x^2+1}-\frac{e\left(\frac{\sqrt{c^2x^2+1}\,cx}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}\right)}{c}}{c}$	9

default	$\frac{\frac{a\left(d\,c^{2}x+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\operatorname{arcsinh}(cx)d\,c^{2}x+\frac{\operatorname{arcsinh}(cx)e\,c^{2}x^{2}}{2}-dc\sqrt{c^{2}x^{2}+1}-\frac{e\left(\frac{\sqrt{c^{2}x^{2}+1}\,cx}{2}-\frac{\operatorname{arcsinh}(cx)}{2}\right)}{2}\right)}{c}}{c}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c*(d*c^2*x+1/2*c^2*e*x^2)+b/c*(arcsinh(c*x)*d*c^2*x+1/2*arcsinh(c*x)
*e*c^2*x^2-d*c*(c^2*x^2+1)^(1/2)-1/2*e*(1/2*(c^2*x^2+1)^(1/2)*c*x-1/2*arcsi
nh(c*x))))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 0.87

$$\frac{1}{2}\,ax^{2}e+adx+\frac{1}{4}\left(2\,x^{2}\operatorname{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^{2}x^{2}+1}\,x}{c^{2}}-\frac{\operatorname{arsinh}\left(cx\right)}{c^{3}}\right)\right)be+\frac{\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^{2}x^{2}+1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*x + 1/4*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2
- arcsinh(c*x)/c^3))*b*e + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b*d/c
```

Fricas [A]
time = 0.35, size = 121, normalized size = 1.25

$$\frac{2\,ac^{2}x^{2}\cosh\left(1\right)+2\,ac^{2}x^{2}\sinh\left(1\right)+4\,ac^{2}dx+\left(4\,bc^{2}dx+\left(2\,bc^{2}x^{2}+b\right)\cosh\left(1\right)+\left(2\,bc^{2}x^{2}+b\right)\sinh\left(1\right)\right)\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)-\sqrt{c^{2}x^{2}+1}\left(bcx\cosh\left(1\right)+bcx\sinh\left(1\right)+4\,bcd\right)}{4\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*x^2*cosh(1) + 2*a*c^2*x^2*sinh(1) + 4*a*c^2*d*x + (4*b*c^2*d*x
 + (2*b*c^2*x^2 + b)*cosh(1) + (2*b*c^2*x^2 + b)*sinh(1))*log(c*x + sqrt(c^
2*x^2 + 1)) - sqrt(c^2*x^2 + 1)*(b*c*x*cosh(1) + b*c*x*sinh(1) + 4*b*c*d))/
c^2
```

Sympy [A]
time = 0.10, size = 99, normalized size = 1.02

$$\begin{cases} adx+\frac{aex^{2}}{2}+bdx\operatorname{asinh}\left(cx\right)+\frac{bex^{2}\operatorname{asinh}\left(cx\right)}{2}-\frac{bd\sqrt{c^{2}x^{2}+1}}{c}-\frac{bex\sqrt{c^{2}x^{2}+1}}{4c}+\frac{be\operatorname{asinh}\left(cx\right)}{4c^{2}} & \text{for}\; c\neq 0\\ a\left(dx+\frac{ex^{2}}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*asinh(c*x) + b*e*x**2*asinh(c*x)/2 -
b*d*sqrt(c**2*x**2 + 1)/c - b*e*x*sqrt(c**2*x**2 + 1)/(4*c) + b*e*asinh(c*x
)/(4*c**2), Ne(c, 0)), (a*(d*x + e*x**2/2), True))
```

Giac [A]
time = 0.44, size = 122, normalized size = 1.26

$$\frac{1}{2}aex^2 + \left(x\log\left(cx + \sqrt{c^2x^2+1}\right) - \frac{\sqrt{c^2x^2+1}}{c}\right)bd + \frac{1}{4}\left(2x^2\log\left(cx + \sqrt{c^2x^2+1}\right) - c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2} + \frac{\log\left(-x|c| + \sqrt{c^2x^2+1}\right)}{c^2|c|}\right)\right)be + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*e*x^2 + (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b*d +
1/4*(2*x^2*log(c*x + sqrt(c^2*x^2 + 1)) - c*(sqrt(c^2*x^2 + 1)*x/c^2 + log(
-x*abs(c) + sqrt(c^2*x^2 + 1))/(c^2*abs(c))))*b*e + a*d*x
```

Mupad [B]
time = 0.29, size = 78, normalized size = 0.80

$$\frac{a\,x\,(2\,d+e\,x)}{2} - \frac{b\,d\left(\sqrt{c^2\,x^2+1} - c\,x\,\mathrm{asinh}(c\,x)\right)}{c} - \frac{b\,e\,x\,\sqrt{c^2\,x^2+1}}{4\,c} + b\,e\,x\,\mathrm{asinh}(c\,x)\,\left(\frac{x}{2} + \frac{1}{4\,c^2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 - (b*d*((c^2*x^2 + 1)^(1/2) - c*x*asinh(c*x)))/c - (b*e
*x*(c^2*x^2 + 1)^(1/2))/(4*c) + b*e*x*asinh(c*x)*(x/2 + 1/(4*c^2*x))
```

3.7 $\int \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=30

$$ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)$$

```
[Out] a*x+b*x*arcsinh(c*x)-b*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5772, 267}

$$ax-\frac{b\sqrt{c^2x^2+1}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^{-1}(cx)\right) \, dx &= ax+b\int \sinh^{-1}(cx) \, dx\\ &= ax+bx\sinh^{-1}(cx)-(bc)\int \frac{x}{\sqrt{1+c^2x^2}} \, dx\\ &= ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$ax-\frac{b\sqrt{1+c^2x^2}}{c}+bx\sinh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c*x],x]
[Out] a*x - (b*Sqrt[1 + c^2*x^2])/c + b*x*ArcSinh[c*x]
```

Maple [A]
time = 0.76, size = 31, normalized size = 1.03

method	result	size
default	$ax+\frac{b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^{2}x^{2}+1}\right)}{c}$	31
derivativedivides	$\frac{acx+b\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^{2}x^{2}+1}\right)}{c}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b/c*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.00

$$ax+\frac{\left(cx\operatorname{arsinh}\left(cx\right)-\sqrt{c^{2}x^{2}+1}\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*b/c
```

Fricas [A]
time = 0.35, size = 43, normalized size = 1.43

$$\frac{bcx\log\left(cx+\sqrt{c^{2}x^{2}+1}\right)+acx-\sqrt{c^{2}x^{2}+1}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="fricas")
[Out] (b*c*x*log(c*x + sqrt(c^2*x^2 + 1)) + a*c*x - sqrt(c^2*x^2 + 1)*b)/c
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.87

$$ax+b\left(\begin{cases}x\operatorname{asinh}\left(cx\right)-\frac{\sqrt{c^{2}x^{2}+1}}{c} & \text{for}\: c\neq 0\\0 & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*asinh(c*x) - sqrt(c**2*x**2 + 1)/c, Ne(c, 0)), (0, Tru
e))
```

Giac [A]
time = 0.40, size = 41, normalized size = 1.37

$$\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(c*x),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*b + a*x
```

Mupad [B]
time = 0.00, size = 28, normalized size = 0.93

$$a\,x-\frac{b\,\sqrt{c^2\,x^2+1}}{c}+b\,x\,\mathrm{asinh}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c*x),x)
```

```
[Out] a*x - (b*(c^2*x^2 + 1)^(1/2))/c + b*x*asinh(c*x)
```

3.8 $\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/b/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1
/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1
)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2)
)/(c*d-(c^2*d^2+e^2)^(1/2)))/e+b*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+
(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5827, 5680, 2221, 2317, 2438}

$$\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be}+\frac{b\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{b\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] -1/2*(a + b*ArcSinh[c*x])^2/(b*e) + ((a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcS
inh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])*Log[1 + (
e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (b*PolyLog[2, -((e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (b*PolyLog[2, -((e*E^ArcSinh
[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{d+ex}\,dx &= \text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right) + \text{Su}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\sin}{}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\sin}{}\\
&= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+b\sin}{}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 175, normalized size = 0.94

$$\frac{-\left(\left(a+b\sinh^{-1}(cx)\right)\left(a+b\sinh^{-1}(cx)-2b\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-2b\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)\right)+2b^2\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)+2b^2\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])*(a + b*ArcSinh[c*x] - 2*b*Log[1 + (e*E^ArcSinh[c*x]
)/(c*d - Sqrt[c^2*d^2 + e^2])] - 2*b*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt
[c^2*d^2 + e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 + e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/(2*b*e)
```

Maple [A]
time = 3.33, size = 292, normalized size = 1.56

method	result
derivativedivides	$\frac{ac\ln(cex+cd)}{e}-\frac{bc\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{bc\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{bc\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e}{}\right.}{}$
default	$\frac{ac\ln(cex+cd)}{e}-\frac{bc\,\mathrm{arcsinh}(cx)^2}{2e}+\frac{bc\,\mathrm{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{bc\,\mathrm{arcsinh}(cx)\ln\left(\frac{cd+e}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c*ln(c*e*x+c*d)/e-1/2*b*c*arcsinh(c*x)^2/e+b*c/e*arcsinh(c*x)*ln((-c
*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)
))+b*c/e*arcsinh(c*x)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2)
)/(c*d+(c^2*d^2+e^2)^(1/2)))+b*c/e*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^
2*d^2+e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))+b*c/e*dilog((-c*d-e*(c*x+(c^2*
x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + b*integrate(log(c*x + sqrt(c^2*x^2 + 1))/(x*e + d),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{(cx)}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{d+e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x), x)
```

3.9 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

```
[Out] (-a-b*arcsinh(c*x))/e/(e*x+d)-b*c*arctanh((-c^2*d*x+e)/(c^2*d^2+e^2)^(1/2)/
(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5828, 739, 212}

$$-\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])/(e*(d + e*x))) - (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^
2*d^2 + e^2]*Sqrt[1 + c^2*x^2])])/(e*Sqrt[c^2*d^2 + e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}\\
&= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{(bc)\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx, x, \frac{e-c^2dx}{\sqrt{1+c^2x^2}}\right)}{e}\\
&= -\frac{a+b\sinh^{-1}(cx)}{e(d+ex)}-\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{e\sqrt{c^2d^2+e^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 79, normalized size = 0.96

$$
-\frac{\frac{a+b\sinh^{-1}(cx)}{d+ex}+\frac{bc\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{\sqrt{c^2d^2+e^2}}}{e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c*x])/(d + e*x) + (b*c*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^
2 + e^2]*Sqrt[1 + c^2*x^2])])/Sqrt[c^2*d^2 + e^2])/e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(78) = 156.
time = 4.02, size = 188, normalized size = 2.29

method	result
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}-\frac{b\,c^2\operatorname{arcsinh}(cx)}{(cex+cd)e}-\frac{b\,c^2\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+c^2 \cdots}}{cx+\frac{cd}{e}}\right)}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}}{c}$

default	$\frac{-\frac{a\,c^2}{(cex+cd)e}-\frac{b\,c^2\,\mathrm{arcsinh}(cx)}{(cex+cd)e}-\frac{b\,c^2 \ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+}}{cx+\frac{cd}{e}}\right.}{e^2\sqrt{\frac{c^2d^2+e^2}{e^2}}}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a*c^2/(c*e*x+c*d)/e-b*c^2/(c*e*x+c*d)/e*arcsinh(c*x)-b*c^2/e^2/((c^2*
d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d/e)+2*((c^2*d^2
+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2
))/(c*x+c*d/e)))
```

<u>Maxima</u> [**A**]
time = 0.29, size = 91, normalized size = 1.11

$$\left(\frac{c\,\mathrm{arsinh}\left(\frac{cdxe}{|xe^2+de|}-\frac{e^2}{c|xe^2+de|}\right)e^{(-2)}}{\sqrt{c^2d^2e^{(-2)}+1}}-\frac{\mathrm{arsinh}\,(cx)}{xe^2+de}\right)b-\frac{a}{xe^2+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (c*arcsinh(c*d*x*e/abs(x*e^2 + d*e) - e^2/(c*abs(x*e^2 + d*e)))*e^(-2)/sqrt
(c^2*d^2*e^(-2) + 1) - arcsinh(c*x)/(x*e^2 + d*e))*b - a/(x*e^2 + d*e)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(76) = 152$.
time = 0.38, size = 565, normalized size = 6.89

$$-\frac{ac^2d^3+ad\cosh(1)^2+2\,ad\cosh(1)\sinh(1)+ad\sinh(1)^2-(bcdx\cosh(1)+bcdx\sinh(1)+bcd^2)\sqrt{\frac{(c^2d^2+1)\cosh(1)-(c^2d^2-1)\sinh(1)}{\cosh(1)-\sinh(1)}}\log\left(-\frac{c^3d^2x-cd\cosh(1)-cd\sinh(1)+\left(c^2d^2+cd\sqrt{\frac{(c^2d^2+1)\cosh(1)-(c^2d^2-1)\sinh(1)}{\cosh(1)-\sinh(1)}}+\cosh(1)^2+2\cosh(1)\sinh(1)+\sinh(1)^2\right)\sqrt{c^2x^2+1}+(c^2dx-\cosh(1)-\sinh(1))\sqrt{\frac{(c^2d^2+1)\cosh(1)-(c^2d^2-1)\sinh(1)}{\cosh(1)-\sinh(1)}}}{x\cosh(1)+x\sinh(1)+d}\right)-(bc^2d^2x\cosh(1)+bx\cosh(1)^3+3\,bx\cosh(1)\sinh(1)^2+bx\sinh(1)^3+(bc^2d^2x+3\,bx\cosh(1)^2)\sinh(1))\log\left(cx+\sqrt{c^2x^2+1}\right)-(bc^2d^2x\cosh(1)+bc^2d^3+bx\cosh(1)^3+bx\sinh(1)^3+bd\cosh(1)^2+(3\,bx\cosh(1)+bd)\sinh(1)^2+(bc^2d^2x+3\,bx\cosh(1)^2+2\,bd\cosh(1))\sinh(1))\log\left(-cx+\sqrt{c^2x^2+1}\right)}{c^2d^3x\cosh(1)^2+c^2d^4\cosh(1)+dx\cosh(1)^4+dx\sinh(1)^4+d^2\cosh(1)^3+(4\,dx\cosh(1)+d^2)\sinh(1)^3+(c^2d^3x+6\,dx\cosh(1)^2+3\,d^2\cosh(1))\sinh(1)^2+(2\,c^2d^3x\cosh(1)+c^2d^4+4\,dx\cosh(1)^3+3\,d^2\cosh(1)^2)\sinh(1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*cosh(1)^2 + 2*a*d*cosh(1)*sinh(1) + a*d*sinh(1)^2 - (b*c*
d*x*cosh(1) + b*c*d*x*sinh(1) + b*c*d^2)*sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2
*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1)))*log(-(c^3*d^2*x - c*d*cosh(1) - c*d
*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2*d^2 - 1)*sinh(
1))/(cosh(1) - sinh(1))) + cosh(1)^2 + 2*cosh(1)*sinh(1) + sinh(1)^2)*sqrt(
c^2*x^2 + 1) + (c^2*d*x - cosh(1) - sinh(1))*sqrt(((c^2*d^2 + 1)*cosh(1) -
(c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*sinh(1) + d)) -
```

```
 (b*c^2*d^2*x*cosh(1) + b*x*cosh(1)^3 + 3*b*x*cosh(1)*sinh(1)^2 + b*x*sinh(
1)^3 + (b*c^2*d^2*x + 3*b*x*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)
) - (b*c^2*d^2*x*cosh(1) + b*c^2*d^3 + b*x*cosh(1)^3 + b*x*sinh(1)^3 + b*d*
cosh(1)^2 + (3*b*x*cosh(1) + b*d)*sinh(1)^2 + (b*c^2*d^2*x + 3*b*x*cosh(1)^
2 + 2*b*d*cosh(1))*sinh(1))*log(-c*x + sqrt(c^2*x^2 + 1)))/(c^2*d^3*x*cosh(
1)^2 + c^2*d^4*cosh(1) + d*x*cosh(1)^4 + d*x*sinh(1)^4 + d^2*cosh(1)^3 + (4
*d*x*cosh(1) + d^2)*sinh(1)^3 + (c^2*d^3*x + 6*d*x*cosh(1)^2 + 3*d^2*cosh(1
))*sinh(1)^2 + (2*c^2*d^3*x*cosh(1) + c^2*d^4 + 4*d*x*cosh(1)^3 + 3*d^2*cos
h(1)^2)*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(cx \right)}}{\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(78) = 156.
time = 0.58, size = 232, normalized size = 2.83

$$\left(\frac{c \log\left(-c^2de + \sqrt{c^2d^2+e^2}\,|c||e|\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}{\sqrt{c^2d^2+e^2}\,|e|} - \frac{\log\left(cx+\sqrt{c^2x^2+1}\right)}{(ex+d)e} - \frac{c\log\left(-c^2de + \sqrt{c^2d^2+e^2}\left(\sqrt{c^2 - \frac{2\,c^2d}{ex+d} + \frac{c^2d^2}{(ex+d)^2} + \frac{e^2}{(ex+d)^2}} + \frac{\sqrt{c^2d^2e^2+e^4}}{(ex+d)e}\right)|e|\right)}{\sqrt{c^2d^2+e^2}\,|e|\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)}\right)b - \frac{a}{(ex+d)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*abs(c)*abs(e))*sgn(1/(e*x + d))*sgn(e
)/(sqrt(c^2*d^2 + e^2)*abs(e)) - log(c*x + sqrt(c^2*x^2 + 1))/((e*x + d)*e)
 - c*log(-c^2*d*e + sqrt(c^2*d^2 + e^2)*(sqrt(c^2 - 2*c^2*d/(e*x + d) + c^2
*d^2/(e*x + d)^2 + e^2/(e*x + d)^2) + sqrt(c^2*d^2*e^2 + e^4)/((e*x + d)*e)
)*abs(e))/(sqrt(c^2*d^2 + e^2)*abs(e)*sgn(1/(e*x + d))*sgn(e)))*b - a/((e*x
 + d)*e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c\,x\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(c*x))/(d + e*x)^2, x)
```

3.10 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=128

$$-\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] 1/2*(-a-b*arcsinh(c*x))/e/(e*x+d)^2-1/2*b*c^3*d*arctanh((-c^2*d*x+e)/(c^2*d
^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(3/2)-1/2*b*c*(c^2*x^2+1)^
(1/2)/(c^2*d^2+e^2)/(e*x+d)
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5828, 745, 739, 212}

$$-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{2e\left(c^2d^2+e^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)) - (a + b*ArcSinh[c
*x])/(2*e*(d + e*x)^2) - (b*c^3*d*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2 + e^2
]*Sqrt[1 + c^2*x^2])])/(2*e*(c^2*d^2 + e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^3}\,dx &= -\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{2e}\\ &= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\int \frac{1}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{2e\left(c^2d^2+e^2\right)}\\ &= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{\left(bc^3d\right)\text{Subst}\left(\int \frac{1}{c^2d^2+e^2-x^2}\,dx,x,\frac{e-}{\sqrt{1-}}\right.}{2e\left(c^2d^2+e^2\right)}\\ &= -\frac{bc\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc^3d\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}}\right.}{2e\left(c^2d^2+e^2\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 166, normalized size = 1.30

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{1+c^2x^2}}{\left(c^2d^2+e^2\right)(d+ex)}-\frac{b\sinh^{-1}(cx)}{e(d+ex)^2}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2+e^2\right)^{3/2}}-\frac{bc^3d\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\sqrt{1+c^2x^2}\right)}{e\left(c^2d^2+e^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x))
 - (b*ArcSinh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*(c^2*d^2 +
e^2)^(3/2)) - (b*c^3*d*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x
^2]])/(e*(c^2*d^2 + e^2)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(117) = 234$.
time = 4.52, size = 283, normalized size = 2.21

method	result
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}-\frac{b\,c^3 \operatorname{arcsinh}(cx)}{2(cex+cd)^2 e}-\frac{b\,c^3\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)}-\frac{b\,c^4 d\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{c}{e}\right.}{e}}{}\right.}{}}{c}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2 e}-\frac{b\,c^3 \operatorname{arcsinh}(cx)}{2(cex+cd)^2 e}-\frac{b\,c^3\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{2e\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)}-\frac{b\,c^4 d\ln\left(\frac{\frac{2c^2d^2+2e^2}{e^2}-\frac{2dc\left(cx+\frac{c}{e}\right.}{e}}{}\right.}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a*c^3/(c*e*x+c*d)^2/e-1/2*b*c^3/(c*e*x+c*d)^2/e*arcsinh(c*x)-1/2*
b*c^3/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d
^2+e^2)/e^2)^(1/2)-1/2*b*c^4/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)*
ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c
*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e)))
```

Maxima [A]
time = 0.30, size = 151, normalized size = 1.18

$$\frac{1}{2}\left(\left(\left(\frac{c^2d\,\operatorname{arsinh}\left(\frac{cdxe^{(-1)}}{|de^{(-1)}+x|}-\frac{1}{c|de^{(-1)}+x|}\right)e^{(-4)}}{\left(c^2d^2e^{(-2)}+1\right)^{\frac{3}{2}}}-\frac{\sqrt{c^2x^2+1}}{c^2d^2xe+c^2d^3+xe^3+de^2}\right)c-\frac{\operatorname{arsinh}(cx)}{x^2e^3+2\,dxe^2+d^2e}\right)b-\frac{a}{2\,(x^2e^3+2\,dxe^2+d^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*((c^2*d*arcsinh(c*d*x*e^(-1)/abs(d*e^(-1) + x) - 1/(c*abs(d*e^(-1) + x)
))*e^(-4)/(c^2*d^2*e^(-2) + 1)^(3/2) - sqrt(c^2*x^2 + 1)/(c^2*d^2*x*e + c^2
*d^3 + x*e^3 + d*e^2))*c - arcsinh(c*x)/(x^2*e^3 + 2*d*x*e^2 + d^2*e))*b -
1/2*a/(x^2*e^3 + 2*d*x*e^2 + d^2*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. 2(114) = 228.
time = 0.45, size = 2021, normalized size = 15.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="fricas")

[Out] -1/2*(2*b*c^4*d^5*x*cosh(1) + (a + b)*c^4*d^6 + 2*b*c^2*d^3*x*cosh(1)^3 + (
b*c^2*d^2*x^2 + a*d^2)*cosh(1)^4 + (b*c^2*d^2*x^2 + a*d^2)*sinh(1)^4 + 2*(b
*c^2*d^3*x + 2*(b*c^2*d^2*x^2 + a*d^2)*cosh(1))*sinh(1)^3 + (b*c^4*d^4*x^2
+ (2*a + b)*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 + 6*b*c^2*d^3*x*cosh(1) + (
2*a + b)*c^2*d^4 + 6*(b*c^2*d^2*x^2 + a*d^2)*cosh(1)^2)*sinh(1)^2 - (b*c^3*
d^3*x^2*cosh(1)^2 + b*c^3*d^3*x^2*sinh(1)^2 + 2*b*c^3*d^4*x*cosh(1) + b*c^3
*d^5 + 2*(b*c^3*d^3*x^2*cosh(1) + b*c^3*d^4*x)*sinh(1))*sqrt(((c^2*d^2 + 1)
*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1)))*log(-(c^3*d^2*x - c*
d*cosh(1) - c*d*sinh(1) + (c^2*d^2 + c*d*sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2
*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))) + cosh(1)^2 + 2*cosh(1)*sinh(1) + s
inh(1)^2)*sqrt(c^2*x^2 + 1) + (c^2*d*x - cosh(1) - sinh(1))*sqrt(((c^2*d^2
+ 1)*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))))/(x*cosh(1) + x*
sinh(1) + d)) - (b*c^4*d^4*x^2*cosh(1)^2 + 2*b*c^4*d^5*x*cosh(1) + 2*b*c^2*
d^2*x^2*cosh(1)^4 + 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*cosh(1)^6 + b*x^2*sinh(
1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*x)*sinh(1)^5 + (2*b*c^2
*d^2*x^2 + 15*b*x^2*cosh(1)^2 + 10*b*d*x*cosh(1))*sinh(1)^4 + 4*(2*b*c^2*d^
2*x^2*cosh(1) + b*c^2*d^3*x + 5*b*x^2*cosh(1)^3 + 5*b*d*x*cosh(1)^2)*sinh(1
)^3 + (b*c^4*d^4*x^2 + 12*b*c^2*d^2*x^2*cosh(1)^2 + 12*b*c^2*d^3*x*cosh(1)
+ 15*b*x^2*cosh(1)^4 + 20*b*d*x*cosh(1)^3)*sinh(1)^2 + 2*(b*c^4*d^4*x^2*cos
h(1) + b*c^4*d^5*x + 4*b*c^2*d^2*x^2*cosh(1)^3 + 6*b*c^2*d^3*x*cosh(1)^2 +
3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(1)^4)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)
) - (2*b*c^4*d^5*x*cosh(1) + b*c^4*d^6 + 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*co
sh(1)^6 + b*x^2*sinh(1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*x)
*sinh(1)^5 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^4 + (2*b*c^2*d^2*x^2 + 15*b*
x^2*cosh(1)^2 + 10*b*d*x*cosh(1) + b*d^2)*sinh(1)^4 + 4*(b*c^2*d^3*x + 5*b*
x^2*cosh(1)^3 + 5*b*d*x*cosh(1)^2 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh
(1)^3 + (b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 + 12*b*c^2
*d^3*x*cosh(1) + 2*b*c^2*d^4 + 15*b*x^2*cosh(1)^4 + 20*b*d*x*cosh(1)^3 + 6*
(2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2)*sinh(1)^2 + 2*(b*c^4*d^5*x + 6*b*c^2*d
^3*x*cosh(1)^2 + 3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(1)^4 + 2*(2*b*c^2*d^2*x^2
 + b*d^2)*cosh(1)^3 + (b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1))*sinh(1))*log(-
c*x + sqrt(c^2*x^2 + 1)) + 2*(b*c^4*d^5*x + 3*b*c^2*d^3*x*cosh(1)^2 + 2*(b*
c^2*d^2*x^2 + a*d^2)*cosh(1)^3 + (b*c^4*d^4*x^2 + (2*a + b)*c^2*d^4)*cosh(1
))*sinh(1) + (b*c^3*d^4*x*cosh(1)^2 + b*c^3*d^5*cosh(1) + b*c*d^2*x*cosh(1)
^4 + b*c*d^2*x*sinh(1)^4 + b*c*d^3*cosh(1)^3 + (4*b*c*d^2*x*cosh(1) + b*c*d
^3)*sinh(1)^3 + (b*c^3*d^4*x + 6*b*c*d^2*x*cosh(1)^2 + 3*b*c*d^3*cosh(1))*s
inh(1)^2 + (2*b*c^3*d^4*x*cosh(1) + b*c^3*d^5 + 4*b*c*d^2*x*cosh(1)^3 + 3*b
*c*d^3*cosh(1)^2)*sinh(1))*sqrt(c^2*x^2 + 1))/(2*c^4*d^7*x*cosh(1)^2 + c^4*
d^8*cosh(1) + 4*c^2*d^5*x*cosh(1)^4 + d^2*x^2*cosh(1)^7 + d^2*x^2*sinh(1)^7
 + 2*d^3*x*cosh(1)^6 + (7*d^2*x^2*cosh(1) + 2*d^3*x)*sinh(1)^6 + (2*c^2*d^4
*x^2 + d^4)*cosh(1)^5 + (2*c^2*d^4*x^2 + 21*d^2*x^2*cosh(1)^2 + 12*d^3*x*co
sh(1) + d^4)*sinh(1)^5 + (4*c^2*d^5*x + 35*d^2*x^2*cosh(1)^3 + 30*d^3*x*cos
h(1)^2 + 5*(2*c^2*d^4*x^2 + d^4)*cosh(1))*sinh(1)^4 + (c^4*d^6*x^2 + 2*c^2*
```

```
d^6)*cosh(1)^3 + (c^4*d^6*x^2 + 16*c^2*d^5*x*cosh(1) + 2*c^2*d^6 + 35*d^2*x
^2*cosh(1)^4 + 40*d^3*x*cosh(1)^3 + 10*(2*c^2*d^4*x^2 + d^4)*cosh(1)^2)*sin
h(1)^3 + (2*c^4*d^7*x + 24*c^2*d^5*x*cosh(1)^2 + 21*d^2*x^2*cosh(1)^5 + 30*
d^3*x*cosh(1)^4 + 10*(2*c^2*d^4*x^2 + d^4)*cosh(1)^3 + 3*(c^4*d^6*x^2 + 2*c
^2*d^6)*cosh(1))*sinh(1)^2 + (4*c^4*d^7*x*cosh(1) + c^4*d^8 + 16*c^2*d^5*x*
cosh(1)^3 + 7*d^2*x^2*cosh(1)^6 + 12*d^3*x*cosh(1)^5 + 5*(2*c^2*d^4*x^2 + d
^4)*cosh(1)^4 + 3*(c^4*d^6*x^2 + 2*c^2*d^6)*cosh(1)^2)*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}(cx)}{(d + ex)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**3,x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}(c\,x)}{(d + e\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^3,x)
[Out] int((a + b*asinh(c*x))/(d + e*x)^3, x)
```

3.11 $\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=183

$$-\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2d^2+e^2}\sqrt{\cdots}}\right.}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

```
[Out] 1/3*(-a-b*arcsinh(c*x))/e/(e*x+d)^3-1/6*b*c^3*(2*c^2*d^2-e^2)*arctanh((-c^2
*d*x+e)/(c^2*d^2+e^2)^(1/2)/(c^2*x^2+1)^(1/2))/e/(c^2*d^2+e^2)^(5/2)-1/6*b*
c*(c^2*x^2+1)^(1/2)/(c^2*d^2+e^2)/(e*x+d)^2-1/2*b*c^3*d*(c^2*x^2+1)^(1/2)/(
c^2*d^2+e^2)^2/(e*x+d)
```

Rubi [A]
time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$,
Rules used = {5828, 759, 821, 739, 212}

$$-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc\sqrt{c^2x^2+1}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{c^2x^2+1}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{bc^3\left(2c^2d^2-e^2\right)\tanh^{-1}\left(\frac{e-c^2dx}{\sqrt{c^2x^2+1}\sqrt{c^2d^2+e^2}}\right)}{6e\left(c^2d^2+e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*(b*c*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*(d + e*x)^2) - (b*c^3*d*Sqrt[
1 + c^2*x^2])/(2*(c^2*d^2 + e^2)^2*(d + e*x)) - (a + b*ArcSinh[c*x])/(3*e*(
d + e*x)^3) - (b*c^3*(2*c^2*d^2 - e^2)*ArcTanh[(e - c^2*d*x)/(Sqrt[c^2*d^2
+ e^2]*Sqrt[1 + c^2*x^2])])/(6*e*(c^2*d^2 + e^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{1}{(d+ex)^3\sqrt{1+c^2x^2}}\,dx}{3e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{\left(bc^3\right)\int \frac{-2d+ex}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{6e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc^3(2c^2 \ldots}{\ldots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{(bc^3(2c^2 \ldots}{\ldots}$$

$$= -\frac{bc\sqrt{1+c^2x^2}}{6\left(c^2d^2+e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{1+c^2x^2}}{2\left(c^2d^2+e^2\right)^2(d+ex)}-\frac{a+b\sinh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3(2c^2d^2 \ldots}{\ldots}$$

Mathematica [A]

time = 0.31, size = 205, normalized size = 1.12

$$\frac{1}{6}\left(-\frac{2a}{e(d+ex)^3}-\frac{bc\sqrt{1+c^2x^2}\,(e^2+c^2d(4d+3ex))}{(c^2d^2+e^2)^2\,(d+ex)^2}-\frac{2b\sinh^{-1}(cx)}{e(d+ex)^3}-\frac{bc^3(-2c^2d^2+e^2)\log(d+ex)}{e\,(c^2d^2+e^2)^{5/2}}+\frac{bc^3(-2c^2d^2+e^2)\log\left(e-c^2dx+\sqrt{c^2d^2+e^2}\,\sqrt{1+c^2x^2}\right)}{e\,(c^2d^2+e^2)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/(d + e*x)^4,x]
```

```
[Out] ((-2*a)/(e*(d + e*x)^3) - (b*c*Sqrt[1 + c^2*x^2]*(e^2 + c^2*d*(4*d + 3*e*x)
))/((c^2*d^2 + e^2)^2*(d + e*x)^2) - (2*b*ArcSinh[c*x])/(e*(d + e*x)^3) - (
b*c^3*(-2*c^2*d^2 + e^2)*Log[d + e*x])/(e*(c^2*d^2 + e^2)^(5/2)) + (b*c^3*(
-2*c^2*d^2 + e^2)*Log[e - c^2*d*x + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + c^2*x^2]])
/(e*(c^2*d^2 + e^2)^(5/2)))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(168) = 336.
time = 4.50, size = 520, normalized size = 2.84

method	result
derivativedivides	$-\frac{a\,c^4}{3(cex+cd)^3e}-\frac{b\,c^4\operatorname{arcsinh}(cx)}{3(cex+cd)^3e}-\frac{b\,c^4\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)^2}-\frac{b\,c^5d\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(c\right.}{}}}{2e\left(c^2d^2+e^2\right)^2\left(cx\right.}$
default	$-\frac{a\,c^4}{3(cex+cd)^3e}-\frac{b\,c^4\operatorname{arcsinh}(cx)}{3(cex+cd)^3e}-\frac{b\,c^4\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(cx+\frac{cd}{e}\right)}{e}+\frac{c^2d^2+e^2}{e^2}}}{6e^2\left(c^2d^2+e^2\right)\left(cx+\frac{cd}{e}\right)^2}-\frac{b\,c^5d\sqrt{\left(cx+\frac{cd}{e}\right)^2-\frac{2dc\left(c\right.}{}}}{2e\left(c^2d^2+e^2\right)^2\left(cx\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/3*a*c^4/(c*e*x+c*d)^3/e-1/3*b*c^4/(c*e*x+c*d)^3/e*arcsinh(c*x)-1/6*
b*c^4/e^2/(c^2*d^2+e^2)/(c*x+c*d/e)^2*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c
^2*d^2+e^2)/e^2)^(1/2)-1/2*b*c^5/e*d/(c^2*d^2+e^2)^2/(c*x+c*d/e)*((c*x+c*d/
e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2)-1/2*b*c^6/e^2*d^2/(c^2*d^
2+e^2)^2/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d
/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2
+e^2)/e^2)^(1/2))/(c*x+c*d/e))+1/6*b*c^4/e^2/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e
^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)
^(1/2)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*
d/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*c*integrate(1/3/(c^3*x^6*e^4 + 3*c^3*d*x^5*e^3 + 3*c*d^2*x^2*e^2 + c
*d^3*x*e + (3*c^3*d^2*e^2 + c*e^4)*x^4 + (c^3*d^3*e + 3*c*d*e^3)*x^3 + (c^2
*x^5*e^4 + 3*c^2*d*x^4*e^3 + (3*c^2*d^2*e^2 + e^4)*x^3 + 3*d^2*x*e^2 + d^3*
e + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(c^6*d^3 - 3*c^4*
d*e^2)*log(x*e + d)/(c^6*d^6*e + 3*c^4*d^4*e^3 + 3*c^2*d^2*e^5 + e^7) + (3*
c^6*d^6 + 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e^2 - c^2*e^6)*x^2 + (5*
c^6*d^5*e + 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x + (c^6*d^6 - 3*c^4*d^4*e^2 + (c^
6*d^3*e^3 - 3*c^4*d*e^5)*x^3 + 3*(c^6*d^4*e^2 - 3*c^4*d^2*e^4)*x^2 + 3*(c^6
*d^5*e - 3*c^4*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*(c^6*d^6 + 3*c^4*d^4*e^2 +
3*c^2*d^2*e^4 + e^6)*log(c*x + sqrt(c^2*x^2 + 1)))/(c^6*d^9*e + 3*c^4*d^7*e
^3 + 3*c^2*d^5*e^5 + (c^6*d^6*e^4 + 3*c^4*d^4*e^6 + 3*c^2*d^2*e^8 + e^10)*x
^3 + d^3*e^7 + 3*(c^6*d^7*e^3 + 3*c^4*d^5*e^5 + 3*c^2*d^3*e^7 + d*e^9)*x^2
+ 3*(c^6*d^8*e^2 + 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 + d^2*e^8)*x) - I*(3*c^6*d
^2 - c^4*e^2)*(log(I*c*x + 1) - log(-I*c*x + 1))/((c^6*d^6 + 3*c^4*d^4*e^2
+ 3*c^2*d^2*e^4 + e^6)*c))*b - 1/3*a/(x^3*e^4 + 3*d*x^2*e^3 + 3*d^2*x*e^2 +
 d^3*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4886 vs. $2(164) = 328$.
time = 0.79, size = 4886, normalized size = 26.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(9*b*c^6*d^8*x*cosh(1) + 3*b*c^4*d^4*x^3*cosh(1)^5 + (2*a + 3*b)*c^6*d
^9 + 2*a*d^3*cosh(1)^6 + 2*a*d^3*sinh(1)^6 + 3*(b*c^4*d^4*x^3 + 4*a*d^3*cos
h(1))*sinh(1)^5 + 3*(3*b*c^4*d^5*x^2 + 2*a*c^2*d^5)*cosh(1)^4 + 3*(5*b*c^4*
d^4*x^3*cosh(1) + 3*b*c^4*d^5*x^2 + 2*a*c^2*d^5 + 10*a*d^3*cosh(1)^2)*sinh(
1)^4 + 3*(b*c^6*d^6*x^3 + 3*b*c^4*d^6*x)*cosh(1)^3 + (3*b*c^6*d^6*x^3 + 30*
b*c^4*d^4*x^3*cosh(1)^2 + 9*b*c^4*d^6*x + 40*a*d^3*cosh(1)^3 + 12*(3*b*c^4*
d^5*x^2 + 2*a*c^2*d^5)*cosh(1))*sinh(1)^3 + 3*(3*b*c^6*d^7*x^2 + (2*a + b)*
c^4*d^7)*cosh(1)^2 + 3*(3*b*c^6*d^7*x^2 + 10*b*c^4*d^4*x^3*cosh(1)^3 + (2*a
 + b)*c^4*d^7 + 10*a*d^3*cosh(1)^4 + 6*(3*b*c^4*d^5*x^2 + 2*a*c^2*d^5)*cosh
(1)^2 + 3*(b*c^6*d^6*x^3 + 3*b*c^4*d^6*x)*cosh(1))*sinh(1)^2 + (6*b*c^5*d^7
*x*cosh(1) - b*c^3*d^3*x^3*cosh(1)^5 - b*c^3*d^3*x^3*sinh(1)^5 + 2*b*c^5*d^
```

8 - 3*b*c^3*d^4*x^2*cosh(1)^4 - (5*b*c^3*d^3*x^3*cosh(1) + 3*b*c^3*d^4*x^2)
*sinh(1)^4 + (2*b*c^5*d^5*x^3 - 3*b*c^3*d^5*x)*cosh(1)^3 + (2*b*c^5*d^5*x^3
 - 10*b*c^3*d^3*x^3*cosh(1)^2 - 12*b*c^3*d^4*x^2*cosh(1) - 3*b*c^3*d^5*x)*s
inh(1)^3 + (6*b*c^5*d^6*x^2 - b*c^3*d^6)*cosh(1)^2 + (6*b*c^5*d^6*x^2 - 10*
b*c^3*d^3*x^3*cosh(1)^3 - 18*b*c^3*d^4*x^2*cosh(1)^2 - b*c^3*d^6 + 3*(2*b*c
^5*d^5*x^3 - 3*b*c^3*d^5*x)*cosh(1))*sinh(1)^2 + (6*b*c^5*d^7*x - 5*b*c^3*d
^3*x^3*cosh(1)^4 - 12*b*c^3*d^4*x^2*cosh(1)^3 + 3*(2*b*c^5*d^5*x^3 - 3*b*c^
3*d^5*x)*cosh(1)^2 + 2*(6*b*c^5*d^6*x^2 - b*c^3*d^6)*cosh(1))*sinh(1))*sqrt
(((c^2*d^2 + 1)*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1)))*log(-
(c^3*d^2*x - c*d*cosh(1) - c*d*sinh(1) + (c^2*d^2 - c*d*sqrt(((c^2*d^2 + 1)
*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))) + cosh(1)^2 + 2*cosh
(1)*sinh(1) + sinh(1)^2)*sqrt(c^2*x^2 + 1) - (c^2*d*x - cosh(1) - sinh(1))*
sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))))/
(x*cosh(1) + x*sinh(1) + d)) - 2*(3*b*c^6*d^7*x^2*cosh(1)^2 + 3*b*c^6*d^8*x
*cosh(1) + 9*b*c^4*d^5*x^2*cosh(1)^4 + 9*b*c^2*d^3*x^2*cosh(1)^6 + b*x^3*co
sh(1)^9 + b*x^3*sinh(1)^9 + 3*b*d*x^2*cosh(1)^8 + 3*(3*b*x^3*cosh(1) + b*d*
x^2)*sinh(1)^8 + 3*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^7 + 3*(b*c^2*d^2*x^3 +
 12*b*x^3*cosh(1)^2 + 8*b*d*x^2*cosh(1) + b*d^2*x)*sinh(1)^7 + 3*(3*b*c^2*d
^3*x^2 + 28*b*x^3*cosh(1)^3 + 28*b*d*x^2*cosh(1)^2 + 7*(b*c^2*d^2*x^3 + b*d
^2*x)*cosh(1))*sinh(1)^6 + 3*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x)*cosh(1)^5 + 3*
(b*c^4*d^4*x^3 + 18*b*c^2*d^3*x^2*cosh(1) + 3*b*c^2*d^4*x + 42*b*x^3*cosh(1
)^4 + 56*b*d*x^2*cosh(1)^3 + 21*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^2)*sinh(1
)^5 + 3*(3*b*c^4*d^5*x^2 + 45*b*c^2*d^3*x^2*cosh(1)^2 + 42*b*x^3*cosh(1)^5
+ 70*b*d*x^2*cosh(1)^4 + 35*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^3 + 5*(b*c^4*
d^4*x^3 + 3*b*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (b*c^6*d^6*x^3 + 9*b*c^4*d^6*
x)*cosh(1)^3 + (b*c^6*d^6*x^3 + 36*b*c^4*d^5*x^2*cosh(1) + 9*b*c^4*d^6*x +
180*b*c^2*d^3*x^2*cosh(1)^3 + 84*b*x^3*cosh(1)^6 + 168*b*d*x^2*cosh(1)^5 +
105*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^4 + 30*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x
)*cosh(1)^2)*sinh(1)^3 + 3*(b*c^6*d^7*x^2 + 18*b*c^4*d^5*x^2*cosh(1)^2 + 45
*b*c^2*d^3*x^2*cosh(1)^4 + 12*b*x^3*cosh(1)^7 + 28*b*d*x^2*cosh(1)^6 + 21*(
b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^5 + 10*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x)*cos
h(1)^3 + (b*c^6*d^6*x^3 + 9*b*c^4*d^6*x)*cosh(1))*sinh(1)^2 + 3*(2*b*c^6*d^
7*x^2*cosh(1) + b*c^6*d^8*x + 12*b*c^4*d^5*x^2*cosh(1)^3 + 18*b*c^2*d^3*x^2
*cosh(1)^5 + 3*b*x^3*cosh(1)^8 + 8*b*d*x^2*cosh(1)^7 + 7*(b*c^2*d^2*x^3 + b
*d^2*x)*cosh(1)^6 + 5*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x)*cosh(1)^4 + (b*c^6*d^
6*x^3 + 9*b*c^4*d^6*x)*cosh(1)^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1)) - 2
*(3*b*c^6*d^8*x*cosh(1) + b*c^6*d^9 + b*x^3*cosh(1)^9 + b*x^3*sinh(1)^9 + 3
*b*d*x^2*cosh(1)^8 + 3*(3*b*x^3*cosh(1) + b*d*x^2)*sinh(1)^8 + 3*(b*c^2*d^2
*x^3 + b*d^2*x)*cosh(1)^7 + 3*(b*c^2*d^2*x^3 + 12*b*x^3*cosh(1)^2 + 8*b*d*x
^2*cosh(1) + b*d^2*x)*sinh(1)^7 + (9*b*c^2*d^3*x^2 + b*d^3)*cosh(1)^6 + (9*
b*c^2*d^3*x^2 + 84*b*x^3*cosh(1)^3 + 84*b*d*x^2*cosh(1)^2 + b*d^3 + 21*(b*c
^2*d^2*x^3 + b*d^2*x)*cosh(1))*sinh(1)^6 + 3*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x
)*cosh(1)^5 + 3*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x + 42*b*x^3*cosh(1)^4 + 56*b*
d*x^2*cosh(1)^3 + 21*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^2 + 2*(9*b*c^2*d^3*x
^2 + b*d^3)*cosh(1))*sinh(1)^5 + 3*(3*b*c^4*d^5*x^2 + b*c^2*d^5)*cosh(1)^4

```
+ 3*(3*b*c^4*d^5*x^2 + 42*b*x^3*cosh(1)^5 + b*c^2*d^5 + 70*b*d*x^2*cosh(1)^
4 + 35*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^3 + 5*(9*b*c^2*d^3*x^2 + b*d^3)*co
sh(1)^2 + 5*(b*c^4*d^4*x^3 + 3*b*c^2*d^4*x)*cosh(1))*sinh(1)^4 + (b*c^6*d^6
*x^3 + 9*b*c^4*d^6*x)*cosh(1)^3 + (b*c^6*d^6*x^3 + 9*b*c^4*d^6*x + 84*b*x^3
*cosh(1)^6 + 168*b*d*x^2*cosh(1)^5 + 105*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^
4 + 20*(9*b*c^2*d^3*x^2 + b*d^3)*cosh(1)^3 + 30*(b*c^4*d^4*x^3 + 3*b*c^2*d^
4*x)*cosh(1)^2 + 12*(3*b*c^4*d^5*x^2 + b*c^2*d^5)*cosh(1))*sinh(1)^3 + 3*(b
*c^6*d^7*x^2 + b*c^4*d^7)*cosh(1)^2 + 3*(b*c^6*d^7*x^2 + b*c^4*d^7 + 12*b*x
^3*cosh(1)^7 + 28*b*d*x^2*cosh(1)^6 + 21*(b*c^2*d^2*x^3 + b*d^2*x)*cosh(1)^
5 + 5*(9*b*c^2*d^3*x^2 + b*d^3)*cosh(1)^4 + 10*(b*c^4*d^4*x^3 + 3*b*c^2*d^4
*x)*cosh(1)^3 + 6*(3*b*c^4*d^5*x^2 + b*c^2*d^5)...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}(cx)}{(d + ex)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(e*x+d)**4,x)
[Out] Integral((a + b*asinh(c*x))/(d + e*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(e*x+d)^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)/(e*x + d)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{asinh}(c\,x)}{(d + e\,x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + e*x)^4,x)
[Out] int((a + b*asinh(c*x))/(d + e*x)^4, x)
```

3.12 $\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=368

$$2b^2d^3x-\frac{4b^2de^2x}{3c^2}+\frac{3}{4}b^2d^2ex^2-\frac{3b^2e^3x^2}{32c^2}+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4-\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4bde^2\sqrt{1+}$$

```
[Out] 2*b^2*d^3*x-4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2-3/32*b^2*e^3*x^2/c^2+2/9*
b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arcsinh(c*x))^2/e+3/4*d^2*e*(a+
b*arcsinh(c*x))^2/c^2-3/32*e^3*(a+b*arcsinh(c*x))^2/c^4+1/4*(e*x+d)^4*(a+b*
arcsinh(c*x))^2/e-2*b*d^3*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/3*b*d*e^
2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-3/2*b*d^2*e*x*(a+b*arcsinh(c*x))
*(c^2*x^2+1)^(1/2)/c+3/16*b*e^3*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-
2/3*b*d*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c-1/8*b*e^3*x^3*(a+b*a
rcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.49, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$-\frac{3e^3\left(a+b\sinh^{-1}(cx)\right)^2}{32c^4}-\frac{2bd^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{3bd^2ex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{3d^2e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{2bde^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c}-\frac{be^3x^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{8c}+\frac{4bde^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{3c^3}+\frac{3be^3x\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{16c^3}-\frac{d^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e}+\frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)^2}{4e}-\frac{4b^2de^2x}{3c^2}-\frac{3b^2e^3x^2}{32c^2}+2b^2d^3x+\frac{3}{4}b^2d^2ex^2+\frac{2}{9}b^2de^2x^3+\frac{1}{32}b^2e^3x^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x - (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 - (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*d*e^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSin
h[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*
c) + (3*b*e^3*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(16*c^3) - (2*b*d*e
^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[1 +
c^2*x^2]*(a + b*ArcSinh[c*x]))/(8*c) - (d^4*(a + b*ArcSinh[c*x])^2)/(4*e) +
 (3*d^2*e*(a + b*ArcSinh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcSinh[c*x])^2)/
(32*c^4) + ((d + e*x)^4*(a + b*ArcSinh[c*x])^2)/(4*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx = \frac{(d+ex)^4 \left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{2e}$$

$$= \frac{(d+ex)^4 \left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{d^4\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}} + \frac{4d^3ex\left(a+b\sinh\right.}{\sqrt{1+c^2}}\right.}{}$$

$$= \frac{(d+ex)^4 \left(a+b\sinh^{-1}(cx)\right)^2}{4e} - \left(2bcd^3\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx - $$

$$= -\frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{3bd^2ex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}\right.}{2c}$$

$$= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{1+c^2x^2}\left(a+b\sin\right.}{c}$$

$$= 2b^2d^3x - \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 - \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2b}{}$$

Mathematica [A]
time = 0.28, size = 354, normalized size = 0.96

$$\frac{c\left(72a^2c^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)-6ab\sqrt{1+c^2x^2}\left(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3)\right)+b^2cx\left(-3e^2(128d+9ex)+c^2(576d^3+216d^2ex+64de^2x^2+9e^3x^3)\right)\right)-6b\left(-3a(24c^2d^2e-3e^3+8c^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3))+bc\sqrt{1+c^2x^2}\left(-e^2(64d+9ex)+c^2(96d^3+72d^2ex+32de^2x^2+6e^3x^3)\right)\right)\sinh^{-1}(cx)+9b^2(24c^2d^2e-3e^3+8c^4x(4d^3+6d^2ex+4de^2x^2+e^3x^3))\sinh^{-1}(cx)^2}{288c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[1
 + c^2*x^2]*(-(e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^
2 + 6*e^3*x^3)) + b^2*c*x*(-3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*
e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*(-3*a*(24*c^2*d^2*e - 3*e^3 + 8*c^4
*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)) + b*c*Sqrt[1 + c^2*x^2]*(-(
e^2*(64*d + 9*e*x)) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))
)*ArcSinh[c*x] + 9*b^2*(24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x +
 4*d*e^2*x^2 + e^3*x^3))*ArcSinh[c*x]^2)/(288*c^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^3 \left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^3*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.30, size = 586, normalized size = 1.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arcsinh(c*x)^2*e^3 + b^2*d*x^3*arcsinh(c*x)^2*e^2 + 3/2*b^2*d^2
*x^2*arcsinh(c*x)^2*e + b^2*d^3*x*arcsinh(c*x)^2 + 1/4*a^2*x^4*e^3 + a^2*d*
x^3*e^2 + 3/2*a^2*d^2*x^2*e + 2*b^2*d^3*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)
/c) + a^2*d^3*x + 3/2*(2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - ar
csinh(c*x)/c^3))*a*b*d^2*e + 3/4*(c^2*(x^2/c^2 - log(c*x + sqrt(c^2*x^2 + 1
))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcsinh(c*x))*
b^2*d^2*e + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^3/c + 2/3*(3*x^3
*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4))*a*
b*d*e^2 - 2/9*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/c^4)*ar
csinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*d*e^2 + 1/16*(8*x^4*arcsinh(c*x) - (2
*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3*arcsinh(c*x)/c^5
)*c)*a*b*e^3 + 1/32*((x^4/c^2 - 3*x^2/c^4 + 3*log(c*x + sqrt(c^2*x^2 + 1))^
2/c^6)*c^2 - 2*(2*sqrt(c^2*x^2 + 1)*x^3/c^2 - 3*sqrt(c^2*x^2 + 1)*x/c^4 + 3
*arcsinh(c*x)/c^5)*c*arcsinh(c*x))*b^2*e^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(321) = 642$.
time = 0.36, size = 1185, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(216*(2*a^2 + b^2)*c^4*d^2*x^2*cosh(1) + 288*(a^2 + 2*b^2)*c^4*d^3*x
 + 9*((8*a^2 + b^2)*c^4*x^4 - 3*b^2*c^2*x^2)*cosh(1)^3 + 9*((8*a^2 + b^2)*c^
4*x^4 - 3*b^2*c^2*x^2)*sinh(1)^3 + 32*((9*a^2 + 2*b^2)*c^4*d*x^3 - 12*b^2*c
^2*d*x)*cosh(1)^2 + 9*(32*b^2*c^4*d*x^3*cosh(1)^2 + 32*b^2*c^4*d^3*x + (8*b
^2*c^4*x^4 - 3*b^2)*cosh(1)^3 + (8*b^2*c^4*x^4 - 3*b^2)*sinh(1)^3 + (32*b^2
*c^4*d*x^3 + 3*(8*b^2*c^4*x^4 - 3*b^2)*cosh(1))*sinh(1)^2 + 24*(2*b^2*c^4*d
^2*x^2 + b^2*c^2*d^2)*cosh(1) + (64*b^2*c^4*d*x^3*cosh(1) + 48*b^2*c^4*d^2*
x^2 + 24*b^2*c^2*d^2 + 3*(8*b^2*c^4*x^4 - 3*b^2)*cosh(1)^2)*sinh(1))*log(c*
x + sqrt(c^2*x^2 + 1))^2 + (32*(9*a^2 + 2*b^2)*c^4*d*x^3 - 384*b^2*c^2*d*x
 + 27*((8*a^2 + b^2)*c^4*x^4 - 3*b^2*c^2*x^2)*cosh(1))*sinh(1)^2 + 6*(96*a*b
```

```
*c^4*d*x^3*cosh(1)^2 + 96*a*b*c^4*d^3*x + 3*(8*a*b*c^4*x^4 - 3*a*b)*cosh(1)
^3 + 3*(8*a*b*c^4*x^4 - 3*a*b)*sinh(1)^3 + 3*(32*a*b*c^4*d*x^3 + 3*(8*a*b*c
^4*x^4 - 3*a*b)*cosh(1))*sinh(1)^2 + 72*(2*a*b*c^4*d^2*x^2 + a*b*c^2*d^2)*c
osh(1) + 3*(64*a*b*c^4*d*x^3*cosh(1) + 48*a*b*c^4*d^2*x^2 + 24*a*b*c^2*d^2
+ 3*(8*a*b*c^4*x^4 - 3*a*b)*cosh(1)^2)*sinh(1) - (72*b^2*c^3*d^2*x*cosh(1)
+ 96*b^2*c^3*d^3 + 3*(2*b^2*c^3*x^3 - 3*b^2*c*x)*cosh(1)^3 + 3*(2*b^2*c^3*x
^3 - 3*b^2*c*x)*sinh(1)^3 + 32*(b^2*c^3*d*x^2 - 2*b^2*c*d)*cosh(1)^2 + (32*
b^2*c^3*d*x^2 - 64*b^2*c*d + 9*(2*b^2*c^3*x^3 - 3*b^2*c*x)*cosh(1))*sinh(1)
^2 + (72*b^2*c^3*d^2*x + 9*(2*b^2*c^3*x^3 - 3*b^2*c*x)*cosh(1)^2 + 64*(b^2*
c^3*d*x^2 - 2*b^2*c*d)*cosh(1))*sinh(1))*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(
c^2*x^2 + 1)) + (216*(2*a^2 + b^2)*c^4*d^2*x^2 + 27*((8*a^2 + b^2)*c^4*x^4
- 3*b^2*c^2*x^2)*cosh(1)^2 + 64*((9*a^2 + 2*b^2)*c^4*d*x^3 - 12*b^2*c^2*d*x
)*cosh(1))*sinh(1) - 6*(72*a*b*c^3*d^2*x*cosh(1) + 96*a*b*c^3*d^3 + 3*(2*a*
b*c^3*x^3 - 3*a*b*c*x)*cosh(1)^3 + 3*(2*a*b*c^3*x^3 - 3*a*b*c*x)*sinh(1)^3
+ 32*(a*b*c^3*d*x^2 - 2*a*b*c*d)*cosh(1)^2 + (32*a*b*c^3*d*x^2 - 64*a*b*c*d
 + 9*(2*a*b*c^3*x^3 - 3*a*b*c*x)*cosh(1))*sinh(1)^2 + (72*a*b*c^3*d^2*x + 9
*(2*a*b*c^3*x^3 - 3*a*b*c*x)*cosh(1)^2 + 64*(a*b*c^3*d*x^2 - 2*a*b*c*d)*cos
h(1))*sinh(1))*sqrt(c^2*x^2 + 1))/c^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(364) = 728$.
time = 0.47, size = 743, normalized size = 2.02

$$\begin{cases} a^2d^3x + \frac{3a^2d^2ex^2}{2} + a^2de^2x^3 + \frac{a^2e^3x^4}{4} + 2abd^3x\operatorname{asinh}(cx) + 3abd^2ex^2\operatorname{asinh}(cx) + 2abde^2x^3\operatorname{asinh}(cx) + \frac{abe^3x^4\operatorname{asinh}(cx)}{2} - \frac{2abd^3\sqrt{c^2x^2+1}}{c} - \frac{3abd^2ex\sqrt{c^2x^2+1}}{2c} - \frac{2abde^2x^2\sqrt{c^2x^2+1}}{3c} - \frac{abe^3x^3\sqrt{c^2x^2+1}}{8c} + \frac{3abd^2e\operatorname{asinh}(cx)}{2c^2} + \frac{4abde^2\sqrt{c^2x^2+1}}{3c^3} + \frac{3abe^3x\sqrt{c^2x^2+1}}{16c^3} - \frac{3abe^3\operatorname{asinh}(cx)}{16c^4} + b^2d^3x\operatorname{asinh}^2(cx) + 2b^2d^3x + \frac{3b^2d^2ex^2\operatorname{asinh}^2(cx)}{2} + \frac{3b^2d^2ex^2}{4} + b^2de^2x^3\operatorname{asinh}^2(cx) + \frac{2b^2de^2x^3}{9} + \frac{b^2e^3x^4\operatorname{asinh}^2(cx)}{4} + \frac{b^2e^3x^4}{32} - \frac{2b^2d^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} - \frac{3b^2d^2ex\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{2c} - \frac{2b^2de^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c} - \frac{b^2e^3x^3\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{8c} + \frac{3b^2d^2e\operatorname{asinh}^2(cx)}{4c^2} - \frac{4b^2de^2x}{3c^2} - \frac{3b^2e^3x^2}{32c^2} + \frac{4b^2de^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{3c^3} + \frac{3b^2e^3x\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{16c^3} - \frac{3b^2e^3\operatorname{asinh}^2(cx)}{32c^4} & \text{for}\: c \neq 0 \\ a^2\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*asinh(c*x) + 3*a*b*d**2*e*x**2*asinh(c*x) + 2*a*b*d
*e**2*x**3*asinh(c*x) + a*b*e**3*x**4*asinh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 + 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 + 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 + 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 + 1)/(8*c) + 3*a*b*
d**2*e*asinh(c*x)/(2*c**2) + 4*a*b*d*e**2*sqrt(c**2*x**2 + 1)/(3*c**3) + 3*
a*b*e**3*x*sqrt(c**2*x**2 + 1)/(16*c**3) - 3*a*b*e**3*asinh(c*x)/(16*c**4)
+ b**2*d**3*x*asinh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*asinh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*asinh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*asinh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 + 1)*as
inh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 + 1)*asinh(c*x)/(8*c) + 3*b**2*d**2*e*asinh(c
*x)**2/(4*c**2) - 4*b**2*d*e**2*x/(3*c**2) - 3*b**2*e**3*x**2/(32*c**2) + 4
*b**2*d*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(3*c**3) + 3*b**2*e**3*x*sqrt(c
**2*x**2 + 1)*asinh(c*x)/(16*c**3) - 3*b**2*e**3*asinh(c*x)**2/(32*c**4), N
e(c, 0)), (a**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), Tru
e))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}(c\,x))^2\,(d + e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^3, x)
```

3.13 $\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=239

$$2b^2d^2x-\frac{4b^2e^2x}{9c^2}+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3-\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{4be^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}$$

```
[Out] 2*b^2*d^2*x-4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+b
*arcsinh(c*x))^2/e+1/2*d*e*(a+b*arcsinh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcs
inh(c*x))^2/e-2*b*d^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c+4/9*b*e^2*(a+b
*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c^3-b*d*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)
^(1/2)/c-2/9*b*e^2*x^2*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.32, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$-\frac{2bd^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bdex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}+\frac{de\left(a+b\sinh^{-1}(cx)\right)^2}{2c^2}-\frac{2be^2x^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c}+\frac{4be^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{9c^3}-\frac{d^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e}+\frac{(d+ex)^3\left(a+b\sinh^{-1}(cx)\right)^2}{3e}-\frac{4b^2e^2x}{9c^2}+2b^2d^2x+\frac{1}{2}b^2dex^2+\frac{2}{27}b^2e^2x^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x - (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
- (2*b*d^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + (4*b*e^2*Sqrt[1 + c^
2*x^2]*(a + b*ArcSinh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[1 + c^2*x^2]*(a + b*Ar
cSinh[c*x]))/c - (2*b*e^2*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(9*c)
 - (d^3*(a + b*ArcSinh[c*x])^2)/(3*e) + (d*e*(a + b*ArcSinh[c*x])^2)/(2*c^2
) + ((d + e*x)^3*(a + b*ArcSinh[c*x])^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \frac{(d+ex)^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}dx}{3e}$$

$$= \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \left(\frac{d^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{3d^2ex(a+b\sin}{\sqrt{1+c}}\right.}{}$$

$$= \frac{(d+ex)^3 \left(a+b\sinh^{-1}(cx)\right)^2}{3e} - \left(2bcd^2\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx - $$

$$= -\frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bdex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(c\right.}{c}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} +$$

$$= 2b^2d^2x - \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-\right.}{c}$$

Mathematica [A]
time = 0.19, size = 248, normalized size = 1.04

$$\frac{18a^2c^3x(3d^2+3dex+e^2x^2) - 6ab\sqrt{1+c^2x^2}\left(-4e^2+c^2(18d^2+9dex+2e^2x^2)\right) + b^2cx\left(-24e^2+c^2(108d^2+27dex+4e^2x^2)\right) - 6b\left(-3a(3cde+2c^3x(3d^2+3dex+e^2x^2)) + b\sqrt{1+c^2x^2}\left(-4e^2+c^2(18d^2+9dex+2e^2x^2)\right)\right)\sinh^{-1}(cx) + 9b^2c(6c^2d^2x+2c^2e^2x^3+3d(e+2c^2ex^2))\sinh^{-1}(cx)^2}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (18*a^2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 6*a*b*Sqrt[1 + c^2*x^2]*(-4*e^2
 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)) + b^2*c*x*(-24*e^2 + c^2*(108*d^2 +
27*d*e*x + 4*e^2*x^2)) - 6*b*(-3*a*(3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^
2*x^2)) + b*Sqrt[1 + c^2*x^2]*(-4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))
)*ArcSinh[c*x] + 9*b^2*c*(6*c^2*d^2*x + 2*c^2*e^2*x^3 + 3*d*(e + 2*c^2*e*x^
2))*ArcSinh[c*x]^2)/(54*c^3)
```

Maple [A]
time = 2.28, size = 361, normalized size = 1.51

$$\frac{\frac{(cex+cd)^3a^2}{3c^2e} + \frac{b^2\left(-\frac{2e^2\,\mathrm{arcsinh}(cx)\left(c^2x^2+1\right)^{\frac{3}{2}}}{9} + \frac{e\left(18\,\mathrm{arcsinh}(cx)^2xce+54\,\mathrm{arcsinh}(cx)^2cd+4cex+27cd\right)\left(c^2x^2+1\right)}{54} - \frac{\mathrm{arcsinh}(cx)\left(3c^2dex+6c^2d^2-2e^2\right)\sqrt{c^2x^2}}{3}\right.}{c^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a^2/c^2/e+b^2/c^2*(-2/9*e^2*arcsinh(c*x)*(c^2*x^2+1)
^(3/2)+1/54*e*(18*arcsinh(c*x)^2*x*c*e+54*arcsinh(c*x)^2*c*d+4*c*e*x+27*c*d
)*(c^2*x^2+1)-1/3*arcsinh(c*x)*(3*c^2*d*e*x+6*c^2*d^2-2*e^2)*(c^2*x^2+1)^(1
/2)+1/27*(27*arcsinh(c*x)^2*c^2*d^2-9*arcsinh(c*x)^2*e^2+54*c^2*d^2-14*e^2)
*x*c-1/4*d*e*c*(2*arcsinh(c*x)^2+1))+2*a*b/c^2*(1/3*e^2*arcsinh(c*x)*x^3*c^
3+e*arcsinh(c*x)*x^2*c^3*d+arcsinh(c*x)*x*c^3*d^2+1/3/e*arcsinh(c*x)*c^3*d^
3-1/3/e*(e^3*(1/3*c^2*x^2*(c^2*x^2+1)^(1/2)-2/3*(c^2*x^2+1)^(1/2))+3*c*d*e^
2*(1/2*(c^2*x^2+1)^(1/2)*c*x-1/2*arcsinh(c*x))+3*d^2*c^2*e*(c^2*x^2+1)^(1/2
)+c^3*d^3*arcsinh(c*x))))
```

Maxima [A]
time = 0.30, size = 378, normalized size = 1.58

$$\frac{1}{3}b^2x^3\operatorname{arsinh}(cx)^2e^2+b^2dx^2\operatorname{arsinh}(cx)^2e+b^2d^2x\operatorname{arsinh}(cx)^2+\frac{1}{3}a^2x^3e^2+a^2dx^2e+2b^2d^2\left(x-\frac{\sqrt{c^2x^2+1}\operatorname{arsinh}(cx)}{c}\right)+a^2d^2x+\left(2x^2\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\right)abde+\frac{1}{2}\left(c^2\left(\frac{x^2}{c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right)-2c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\operatorname{arsinh}(cx)}{c^3}\right)\operatorname{arsinh}(cx)\right)b^2de+\frac{2\left(cx\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)abd^2}{c}+\frac{2}{9}\left(3x^3\operatorname{arsinh}(cx)-c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\right)abe^2-\frac{2}{27}\left(3c\left(\frac{\sqrt{c^2x^2+1}\,x^2}{c^2}-\frac{2\sqrt{c^2x^2+1}}{c^4}\right)\operatorname{arsinh}(cx)-\frac{c^2x^3-6x}{c^2}\right)b^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arcsinh(c*x)^2*e^2 + b^2*d*x^2*arcsinh(c*x)^2*e + b^2*d^2*x*arc
sinh(c*x)^2 + 1/3*a^2*x^3*e^2 + a^2*d*x^2*e + 2*b^2*d^2*(x - sqrt(c^2*x^2 +
 1)*arcsinh(c*x)/c) + a^2*d^2*x + (2*x^2*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1
)*x/c^2 - arcsinh(c*x)/c^3))*a*b*d*e + 1/2*(c^2*(x^2/c^2 - log(c*x + sqrt(c
^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3)*arcs
inh(c*x))*b^2*d*e + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b*d^2/c + 2/
9*(3*x^3*arcsinh(c*x) - c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/
c^4))*a*b*e^2 - 2/27*(3*c*(sqrt(c^2*x^2 + 1)*x^2/c^2 - 2*sqrt(c^2*x^2 + 1)/
c^4)*arcsinh(c*x) - (c^2*x^3 - 6*x)/c^2)*b^2*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(213) = 426$.
time = 0.34, size = 616, normalized size = 2.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(27*(2*a^2 + b^2)*c^3*d*x^2*cosh(1) + 54*(a^2 + 2*b^2)*c^3*d^2*x + 2*(
(9*a^2 + 2*b^2)*c^3*x^3 - 12*b^2*c*x)*cosh(1)^2 + 9*(2*b^2*c^3*x^3*cosh(1)^
2 + 2*b^2*c^3*x^3*sinh(1)^2 + 6*b^2*c^3*d^2*x + 3*(2*b^2*c^3*d*x^2 + b^2*c*
d)*cosh(1) + (4*b^2*c^3*x^3*cosh(1) + 6*b^2*c^3*d*x^2 + 3*b^2*c*d)*sinh(1))
*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*((9*a^2 + 2*b^2)*c^3*x^3 - 12*b^2*c*x)*
sinh(1)^2 + 6*(6*a*b*c^3*x^3*cosh(1)^2 + 6*a*b*c^3*x^3*sinh(1)^2 + 18*a*b*c
^3*d^2*x + 9*(2*a*b*c^3*d*x^2 + a*b*c*d)*cosh(1) + 3*(4*a*b*c^3*x^3*cosh(1)
```

```
 + 6*a*b*c^3*d*x^2 + 3*a*b*c*d)*sinh(1) - (9*b^2*c^2*d*x*cosh(1) + 18*b^2*c
^2*d^2 + 2*(b^2*c^2*x^2 - 2*b^2)*cosh(1)^2 + 2*(b^2*c^2*x^2 - 2*b^2)*sinh(1
)^2 + (9*b^2*c^2*d*x + 4*(b^2*c^2*x^2 - 2*b^2)*cosh(1))*sinh(1))*sqrt(c^2*x
^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (27*(2*a^2 + b^2)*c^3*d*x^2 + 4*((9
*a^2 + 2*b^2)*c^3*x^3 - 12*b^2*c*x)*cosh(1))*sinh(1) - 6*(9*a*b*c^2*d*x*cos
h(1) + 18*a*b*c^2*d^2 + 2*(a*b*c^2*x^2 - 2*a*b)*cosh(1)^2 + 2*(a*b*c^2*x^2
- 2*a*b)*sinh(1)^2 + (9*a*b*c^2*d*x + 4*(a*b*c^2*x^2 - 2*a*b)*cosh(1))*sinh
(1))*sqrt(c^2*x^2 + 1))/c^3
```

Sympy [A]
time = 0.31, size = 454, normalized size = 1.90

$$\begin{cases} a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + 2abd^2x\operatorname{asinh}(cx) + 2abdex^2\operatorname{asinh}(cx) + \frac{2abe^2x^3\operatorname{asinh}(cx)}{3} - \frac{2abd^2\sqrt{c^2x^2+1}}{c} - \frac{abdex\sqrt{c^2x^2+1}}{c} - \frac{2abe^2x^2\sqrt{c^2x^2+1}}{9c} + \frac{abde\operatorname{asinh}(cx)}{c^2} + \frac{4abe^2\sqrt{c^2x^2+1}}{9c^3} + b^2d^2x\operatorname{asinh}^2(cx) + 2b^2d^2x + b^2dex^2\operatorname{asinh}^2(cx) + \frac{b^2dex^2}{2} + \frac{b^2e^2x^3\operatorname{asinh}^2(cx)}{3} + \frac{2b^2e^2x^3}{27} - \frac{2b^2d^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} - \frac{b^2dex\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} - \frac{2b^2e^2x^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c} + \frac{b^2de\operatorname{asinh}^2(cx)}{2c^2} - \frac{4b^2e^2x}{9c^2} + \frac{4b^2e^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{9c^3} & \text{for}\ c \neq 0 \\ a^2\left(d^2x + dex^2 + \frac{e^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*as
inh(c*x) + 2*a*b*d*e*x**2*asinh(c*x) + 2*a*b*e**2*x**3*asinh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 + 1)/c - a*b*d*e*x*sqrt(c**2*x**2 + 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 + 1)/(9*c) + a*b*d*e*asinh(c*x)/c**2 + 4*a*b*e**2*sqrt(
c**2*x**2 + 1)/(9*c**3) + b**2*d**2*x*asinh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*asinh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*asinh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 +
1)*asinh(c*x)/(9*c) + b**2*d*e*asinh(c*x)**2/(2*c**2) - 4*b**2*e**2*x/(9*c*
*2) + 4*b**2*e**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/(9*c**3), Ne(c, 0)), (a**2
*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{asinh}(c\,x))^2\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^2,x)
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^2, x)
```

3.14 $\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=140

$$2b^2dx+\frac{1}{4}b^2ex^2-\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e}$$

```
[Out] 2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arcsinh(c*x))^2/e+1/4*e*(a+b*arcsinh(c
*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arcsinh(c*x))^2/e-2*b*d*(a+b*arcsinh(c*x))*(c
^2*x^2+1)^(1/2)/c-1/2*b*e*x*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5828, 5838, 5783, 5798, 8, 5812, 30}

$$-\frac{2bd\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{c}-\frac{bex\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)}{2c}+\frac{e\left(a+b\sinh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e}+2b^2dx+\frac{1}{4}b^2ex^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/
c - (b*e*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/(2*c) - (d^2*(a + b*ArcS
inh[c*x])^2)/(2*e) + (e*(a + b*ArcSinh[c*x])^2)/(4*c^2) + ((d + e*x)^2*(a +
 b*ArcSinh[c*x])^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)\left(a+b\sinh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \left(\frac{d^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{2dex(a+b\sinh^{-1}}{\sqrt{1+c^2x^2}}\right.}{e}$$

$$= \frac{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx - \frac{(bc}{}$$

$$= -\frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{2c}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{1+c^2x^2}\,(}{2}$$

Mathematica [A]

time = 0.16, size = 142, normalized size = 1.01

$$\frac{c\left(2a^2cx(2d+ex)+b^2cx(8d+ex)-2ab(4d+ex)\sqrt{1+c^2x^2}\right)+2b\left(-bc(4d+ex)\sqrt{1+c^2x^2}+a(e+4c^2dx+2c^2ex^2)\right)\sinh^{-1}(cx)+b^2(e+4c^2dx+2c^2ex^2)\sinh^{-1}(cx)^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) + b^2*c*x*(8*d + e*x) - 2*a*b*(4*d + e*x)*Sqrt[1
+ c^2*x^2]) + 2*b*(-(b*c*(4*d + e*x)*Sqrt[1 + c^2*x^2]) + a*(e + 4*c^2*d*x
+ 2*c^2*e*x^2))*ArcSinh[c*x] + b^2*(e + 4*c^2*d*x + 2*c^2*e*x^2)*ArcSinh[c*
x]^2)/(4*c^2)
```

Maple [A]

time = 2.86, size = 193, normalized size = 1.38

method	result
derivativedivides	$\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(2\operatorname{arcsinh}(cx)^2c^2x^2-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2-}\right.}{4}\right.}{c}$
default	$\frac{a^2\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+\frac{b^2\left(dc\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+\frac{e\left(2\operatorname{arcsinh}(cx)^2c^2x^2-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2-}\right.}{4}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2/c*(d*c^2*x+1/2*c^2*e*x^2)+b^2/c*(d*c*(arcsinh(c*x)^2*c*x-2*arcsinh
(c*x)*(c^2*x^2+1)^(1/2)+2*c*x)+1/4*e*(2*arcsinh(c*x)^2*c^2*x^2-2*arcsinh(c*
x)*(c^2*x^2+1)^(1/2)*c*x+arcsinh(c*x)^2+c^2*x^2+1))+2*a*b/c*(arcsinh(c*x)*d
*c^2*x+1/2*arcsinh(c*x)*e*c^2*x^2-d*c*(c^2*x^2+1)^(1/2)-1/2*e*(1/2*(c^2*x^2
+1)^(1/2)*c*x-1/2*arcsinh(c*x))))
```

Maxima [A]
time = 0.28, size = 223, normalized size = 1.59

$$\frac{1}{2}\,b^2x^2\,\mathrm{arsinh}\left(cx\right)^2e+b^2dx\,\mathrm{arsinh}\left(cx\right)^2+\frac{1}{2}\,a^2x^2e+2\,b^2d\left(x-\frac{\sqrt{c^2x^2+1}\,\mathrm{arsinh}\left(cx\right)}{c}\right)+a^2dx+\frac{1}{2}\left(2\,x^2\,\mathrm{arsinh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\mathrm{arsinh}\left(cx\right)}{c^3}\right)\right)abe+\frac{1}{4}\left(c^2\left(\frac{x^2}{c^2}-\frac{\log\left(cx+\sqrt{c^2x^2+1}\right)^2}{c^4}\right)-2\,c\left(\frac{\sqrt{c^2x^2+1}\,x}{c^2}-\frac{\mathrm{arsinh}\left(cx\right)}{c^3}\right)\mathrm{arsinh}\left(cx\right)\right)b^2e+\frac{2\left(cx\,\mathrm{arsinh}\left(cx\right)-\sqrt{c^2x^2+1}\right)abd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arcsinh(c*x)^2*e + b^2*d*x*arcsinh(c*x)^2 + 1/2*a^2*x^2*e + 2*b
^2*d*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*d*x + 1/2*(2*x^2*arcsinh(
c*x) - c*(sqrt(c^2*x^2 + 1)*x/c^2 - arcsinh(c*x)/c^3))*a*b*e + 1/4*(c^2*(x^
2/c^2 - log(c*x + sqrt(c^2*x^2 + 1))^2/c^4) - 2*c*(sqrt(c^2*x^2 + 1)*x/c^2
- arcsinh(c*x)/c^3)*arcsinh(c*x))*b^2*e + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^
2 + 1))*a*b*d/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(128) = 256$.
time = 0.38, size = 257, normalized size = 1.84

$$\frac{(2\,a^2+b^2)c^2x^2\cosh\left(1\right)+(2\,a^2+b^2)c^2x^2\sinh\left(1\right)+4\,(a^2+2\,b^2)c^2dx+(4\,b^2c^2dx+(2\,b^2c^2x^2+b^2)\cosh\left(1\right)+(2\,b^2c^2x^2+b^2)\sinh\left(1\right))\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\left(4\,abc^2dx+(2\,abc^2x^2+ab)\cosh\left(1\right)+(2\,abc^2x^2+ab)\sinh\left(1\right)-(b^2cx\cosh\left(1\right)+b^2cx\sinh\left(1\right)+4\,b^2cd)\sqrt{c^2x^2+1}\right)\log\left(cx+\sqrt{c^2x^2+1}\right)-2\,(abcx\cosh\left(1\right)+abcx\sinh\left(1\right)+4\,abcd)\sqrt{c^2x^2+1}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*x^2*cosh(1) + (2*a^2 + b^2)*c^2*x^2*sinh(1) + 4*(a^2
 + 2*b^2)*c^2*d*x + (4*b^2*c^2*d*x + (2*b^2*c^2*x^2 + b^2)*cosh(1) + (2*b^2
*c^2*x^2 + b^2)*sinh(1))*log(c*x + sqrt(c^2*x^2 + 1))^2 + 2*(4*a*b*c^2*d*x
+ (2*a*b*c^2*x^2 + a*b)*cosh(1) + (2*a*b*c^2*x^2 + a*b)*sinh(1) - (b^2*c*x*
cosh(1) + b^2*c*x*sinh(1) + 4*b^2*c*d)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^
2*x^2 + 1)) - 2*(a*b*c*x*cosh(1) + a*b*c*x*sinh(1) + 4*a*b*c*d)*sqrt(c^2*x^
2 + 1))/c^2
```

Sympy [A]
time = 0.18, size = 233, normalized size = 1.66

$$\begin{cases} a^2dx+\frac{a^2ex^2}{2}+2abdx\,\mathrm{asinh}\left(cx\right)+abex^2\,\mathrm{asinh}\left(cx\right)-\frac{2abd\sqrt{c^2x^2+1}}{c}-\frac{abex\sqrt{c^2x^2+1}}{2c}+\frac{abe\,\mathrm{asinh}\left(cx\right)}{2c^2}+b^2dx\,\mathrm{asinh}^2\left(cx\right)+2b^2dx+\frac{b^2ex^2\,\mathrm{asinh}^2\left(cx\right)}{2}+\frac{b^2ex^2}{4}-\frac{2b^2d\sqrt{c^2x^2+1}\,\mathrm{asinh}\left(cx\right)}{c}-\frac{b^2ex\sqrt{c^2x^2+1}\,\mathrm{asinh}\left(cx\right)}{2c}+\frac{b^2e\,\mathrm{asinh}^2\left(cx\right)}{4c^2} & \text{for}\: c\neq 0 \\ a^2\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*asinh(c*x) + a*b*e*x**2*asi
nh(c*x) - 2*a*b*d*sqrt(c**2*x**2 + 1)/c - a*b*e*x*sqrt(c**2*x**2 + 1)/(2*c)
 + a*b*e*asinh(c*x)/(2*c**2) + b**2*d*x*asinh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*asinh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 + 1)*asinh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 + 1)*asinh(c*x)/(2*c) + b**2*e*asinh(c*x)*
*2/(4*c**2), Ne(c, 0)), (a**2*(d*x + e*x**2/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c\,x))^2\,(d+e\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x), x)
```

3.15 $\int \left(a+b\sinh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=46

$$2b^2x-\frac{2b\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2$$

```
[Out] 2*b^2*x+x*(a+b*arcsinh(c*x))^2-2*b*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {5772, 5798, 8}

$$-\frac{2b\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/c + x*(a + b*ArcSinh
[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(cx)\right)^2 dx = x\left(a+b\sinh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}dx$$
$$= -\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx$$
$$= 2b^2x-\frac{2b\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{c}+x\left(a+b\sinh^{-1}(cx)\right)^2$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 1.61

$$\left(a^2+2b^2\right)x-\frac{2ab\sqrt{1+c^2x^2}}{c}+\frac{2b\left(acx-b\sqrt{1+c^2x^2}\right)\sinh^{-1}(cx)}{c}+b^2x\sinh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[1 + c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 + c^
2*x^2])*ArcSinh[c*x])/c + b^2*x*ArcSinh[c*x]^2
```

Maple [A]
time = 0.79, size = 72, normalized size = 1.57

method	result	size
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72
default	$\frac{a^2cx+b^2\left(\operatorname{arcsinh}(cx)^2cx-2\operatorname{arcsinh}(cx)\sqrt{c^2x^2+1}+2cx\right)+2ab\left(\operatorname{arcsinh}(cx)cx-\sqrt{c^2x^2+1}\right)}{c}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arcsinh(c*x)^2*c*x-2*arcsinh(c*x)*(c^2*x^2+1)^(1/2)+2*c*x
)+2*a*b*(arcsinh(c*x)*c*x-(c^2*x^2+1)^(1/2)))
```

Maxima [A]
time = 0.26, size = 72, normalized size = 1.57

$$b^2x\,\operatorname{arsinh}(cx)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2+1}\,\operatorname{arsinh}(cx)}{c}\right)+a^2x+\frac{2\left(cx\,\operatorname{arsinh}(cx)-\sqrt{c^2x^2+1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arcsinh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 + 1)*arcsinh(c*x)/c) + a^2*x
 + 2*(c*x*arcsinh(c*x) - sqrt(c^2*x^2 + 1))*a*b/c
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.
time = 0.41, size = 96, normalized size = 2.09

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2+1}\right)^2+(a^2+2\,b^2)cx-2\,\sqrt{c^2x^2+1}\,ab+2\left(abcx-\sqrt{c^2x^2+1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2+1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 + 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 + 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 + 1)*b^2)*log(c*x + sqrt(c^2*x^2 + 1
)))/c
```

Sympy [**A**]
time = 0.09, size = 82, normalized size = 1.78

$$\begin{cases} a^2x+2abx\operatorname{asinh}(cx)-\frac{2ab\sqrt{c^2x^2+1}}{c}+b^2x\operatorname{asinh}^2(cx)+2b^2x-\frac{2b^2\sqrt{c^2x^2+1}\operatorname{asinh}(cx)}{c} & \text{for } c\neq 0\\ a^2x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*asinh(c*x) - 2*a*b*sqrt(c**2*x**2 + 1)/c + b**2
*x*asinh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 + 1)*asinh(c*x)/c, Ne(c
, 0)), (a**2*x, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(44) = 88$.
time = 0.45, size = 111, normalized size = 2.41

$$2\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)-\frac{\sqrt{c^2x^2+1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2+1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2+1}\,\log\left(cx+\sqrt{c^2x^2+1}\right)}{c^2}\right)\right)b^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 + 1)) - sqrt(c^2*x^2 + 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 + 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 + 1)*log(c*x + sqrt(c^2*x^2
 + 1))/c^2))*b^2 + a^2*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b\,\text{asinh}(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2,x)
```

```
[Out] int((a + b*asinh(c*x))^2, x)
```

3.16 $\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=291

$$-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

```
[Out] -1/3*(a+b*arcsinh(c*x))^3/b/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x^2+1)^
(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+(a+b*arcsinh(c*x))^2*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*x))*polylog(2,
-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e+2*b*(a+b*arcsinh(c*
x))*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e-2*b^2
*polylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e-2*b^2*po
lylog(3,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e
```

Rubi [A]
time = 0.30, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5827, 5680, 2221, 2611, 2320, 6724}

$$\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{2b\left(a+b\sinh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e}+\frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e}-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be}-\frac{2b^2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e}-\frac{2b^2\text{Li}_3\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] -1/3*(a + b*ArcSinh[c*x])^3/(b*e) + ((a + b*ArcSinh[c*x])^2*Log[1 + (e*E^Ar
cSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/e + ((a + b*ArcSinh[c*x])^2*Log[1
 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])])/e + (2*b*(a + b*ArcSinh
[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/e + (
2*b*(a + b*ArcSinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^
2 + e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2
 + e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2]))])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\cosh(x)}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2+e^2}+ee^x}\,dx, x, \sinh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e} + \frac{(a+}{}$$

Mathematica [**A**]
time = 0.16, size = 273, normalized size = 0.94

$$\frac{-\frac{\left(a+b\sinh^{-1}(cx)\right)^3}{b}+3\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)+3\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)+6b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)+6b\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)-6b^2\text{PolyLog}\left(3,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-6b^2\text{PolyLog}\left(3,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^3/b) + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*E^ArcSin
h[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] + 3*(a + b*ArcSinh[c*x])^2*Log[1 + (e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*ArcSinh[c*x])*Pol
yLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] + 6*b*(a + b*Arc
Sinh[c*x])*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(3*e)
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(x*e + d) + integrate(b^2*log(c*x + sqrt(c^2*x^2 + 1))^2/(x*e
 + d) + 2*a*b*log(c*x + sqrt(c^2*x^2 + 1))/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{asinh}(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x),x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x), x)
```

3.17 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{(a+b\sinh^{-1}(cx))^2}{e(d+ex)}+\frac{2bc(a+b\sinh^{-1}(cx))\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2bc(a+b\sinh^{-1}(cx))\log\left(1+\frac{}{cd+}\right.}{e\sqrt{c^2d^2+e^2}}$$

```
[Out] -(a+b*arcsinh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x
^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b*c*(a+b*ar
csinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^
2*d^2+e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2
+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)-2*b^2*c*polylog(2,-e*(c*x+(c^2*x^2+1)^(
1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5828, 5843, 3403, 2296, 2221, 2317, 2438}

$$\frac{2bc(a+b\sinh^{-1}(cx))\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2bc(a+b\sinh^{-1}(cx))\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e\sqrt{c^2d^2+e^2}}-\frac{(a+b\sinh^{-1}(cx))^2}{e(d+ex)}+\frac{2b^2c\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcSinh[c*x])*Log[1
 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2])
 - (2*b*c*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d
^2 + e^2])])/(e*Sqrt[c^2*d^2 + e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcSinh[c
*x])/(c*d - Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2 + e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*Sqrt[c^2*d^2
+ e^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\text{Subst}\left(\int \frac{a+bx}{cd+e\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\text{Subst}\left(\int \frac{e^x(a+bx)}{-e+2cde^x+ee^{2x}}\,dx, x, \sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2+e^2}+2ee^x}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2d^2+e^2}}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc}{}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}} - \frac{2bc}{}$$

Mathematica [A]
time = 0.14, size = 191, normalized size = 0.73

$$\frac{-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{d+ex} + \frac{2bc\left(\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)+b\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-b\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{\sqrt{c^2d^2+e^2}}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] (-((a + b*ArcSinh[c*x])^2/(d + e*x)) + (2*b*c*((a + b*ArcSinh[c*x])*(Log[1
+ (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*
x])/(c*d + Sqrt[c^2*d^2 + e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d)
 + Sqrt[c^2*d^2 + e^2])] - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^
2*d^2 + e^2]))]))/Sqrt[c^2*d^2 + e^2])/e
```

Maple [A]
time = 9.32, size = 549, normalized size = 2.09

method	result
derivativedivides	$\frac{-\frac{a^2c^2}{(cex+cd)e}-\frac{b^2c^2\operatorname{arcsinh}(cx)^2}{e(cex+cd)}+\frac{2b^2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c^2\operatorname{arcsinh}(cx)\,\mathrm{l}}{}}{}$
default	$\frac{-\frac{a^2c^2}{(cex+cd)e}-\frac{b^2c^2\operatorname{arcsinh}(cx)^2}{e(cex+cd)}+\frac{2b^2c^2\operatorname{arcsinh}(cx)\ln\left(\frac{-cd-e\left(cx+\sqrt{c^2x^2+1}\right)+\sqrt{c^2d^2+e^2}}{-cd+\sqrt{c^2d^2+e^2}}\right)}{e\sqrt{c^2d^2+e^2}}-\frac{2b^2c^2\operatorname{arcsinh}(cx)\,\mathrm{l}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a^2*c^2/(c*e*x+c*d)/e-b^2*c^2*arcsinh(c*x)^2/e/(c*e*x+c*d)+2*b^2*c^2/
e*arcsinh(c*x)/(c^2*d^2+e^2)^(1/2)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*
d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*b^2*c^2/e*arcsinh(c*x)/(c^2*d
^2+e^2)^(1/2)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(c*d+(
c^2*d^2+e^2)^(1/2)))+2*b^2*c^2/e/(c^2*d^2+e^2)^(1/2)*dilog((-c*d-e*(c*x+(c^
2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-2*b^2*c^2/
e/(c^2*d^2+e^2)^(1/2)*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1
/2))/(c*d+(c^2*d^2+e^2)^(1/2)))-2*a*b*c^2/(c*e*x+c*d)/e*arcsinh(c*x)-2*a*b*
c^2/e^2/((c^2*d^2+e^2)/e^2)^(1/2)*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d/
e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+
e^2)/e^2)^(1/2))/(c*x+c*d/e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 2*(c*arcsinh(c*d*x*e/abs(x*e^2 + d*e) - e^2/(c*abs(x*e^2 + d*e)))*e^(-2)/sq
rt(c^2*d^2*e^(-2) + 1) - arcsinh(c*x)/(x*e^2 + d*e))*a*b - b^2*(log(c*x + s
qrt(c^2*x^2 + 1))^2/(x*e^2 + d*e) - integrate(2*(c^3*x^2 + sqrt(c^2*x^2 + 1
)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*x^4*e^2 + c^3*d*x^3*e + c*x^
2*e^2 + c*d*x*e + (c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*e)*sqrt(c^2*x^2 +
1)), x)) - a^2/(x*e^2 + d*e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(x^2*e^2 + 2*d*x*e
 + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**2,x)
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{(d + e\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^2,x)
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^2, x)
```

3.18 $\int \frac{(a+b\sinh^{-1}(cx))^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=349

$$-\frac{bc\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{(c^2d^2+e^2)\,(d+ex)}-\frac{(a+b\sinh^{-1}(cx))^2}{2e(d+ex)^2}+\frac{bc^3d(a+b\sinh^{-1}(cx))\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\,(c^2d^2+e^2)^{3/2}}$$

```
[Out] -1/2*(a+b*arcsinh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2+e^2)+b*c
^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(
1/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c^3*d*(a+b*arcsinh(c*x))*ln(1+e*(c*x+(c^2*x^
2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)^(3/2)+b^2*c^3*d*poly
log(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d-(c^2*d^2+e^2)^(1/2)))/e/(c^2*d^2+e^2)
^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c^2*x^2+1)^(1/2))/(c*d+(c^2*d^2+e^2)^(1
/2)))/e/(c^2*d^2+e^2)^(3/2)-b*c*(a+b*arcsinh(c*x))*(c^2*x^2+1)^(1/2)/(c^2*d
^2+e^2)/(e*x+d)
```

Rubi [A]

time = 0.42, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {5828, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{bc\sqrt{c^2x^2+1}\,(a+b\sinh^{-1}(cx))}{(c^2d^2+e^2)\,(d+ex)}+\frac{bc^3d(a+b\sinh^{-1}(cx))\log\left(\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}+1\right)}{e\,(c^2d^2+e^2)^{3/2}}-\frac{bc^3d(a+b\sinh^{-1}(cx))\log\left(\frac{ee^{\sinh^{-1}(cx)}}{\sqrt{c^2d^2+e^2}+cd}+1\right)}{e\,(c^2d^2+e^2)^{3/2}}-\frac{(a+b\sinh^{-1}(cx))^2}{2e(d+ex)^2}+\frac{b^2c^2\log(d+ex)}{e\,(c^2d^2+e^2)}+\frac{b^2c^3d\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)}{e\,(c^2d^2+e^2)^{3/2}}-\frac{b^2c^3d\text{Li}_2\left(-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)}{e\,(c^2d^2+e^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*x)))
 - (a + b*ArcSinh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d*(a + b*ArcSinh[c*x])
*Log[1 + (e*E^ArcSinh[c*x])/(c*d - Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2
)^(3/2)) - (b*c^3*d*(a + b*ArcSinh[c*x])*Log[1 + (e*E^ArcSinh[c*x])/(c*d +
Sqrt[c^2*d^2 + e^2])])/(e*(c^2*d^2 + e^2)^(3/2)) + (b^2*c^2*Log[d + e*x])/(
e*(c^2*d^2 + e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d - Sqrt
[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2)) - (b^2*c^3*d*PolyLog[2, -((e*
E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))])/(e*(c^2*d^2 + e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{(bc)\int \frac{a+b\sinh^{-1}(cx)}{(d+ex)^2\sqrt{1+c^2x^2}}\,dx}{e}$$

$$= -\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{(bc^2)\,\text{Subst}\left(\int \frac{a+bx}{(cd+e\sinh(x))^2}\,dx,x,\sinh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{(b^2c^2)\,\text{Subst}\left(\int \frac{\cos}{cd+e}\right.}{c^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{(b^2c^2)\,\text{Subst}\left(\int \frac{1}{cd+x}\right.}{e\,(c^2d^2+e^2}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{b^2c^2\log(d+ex)}{e\,(c^2d^2+e^2)}+$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx\right.}{e\,(}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx\right.}{e\,(}$$

$$= -\frac{bc\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{2e(d+ex)^2}+\frac{bc^3d\left(a+b\sinh^{-1}(cx\right.}{e\,(}$$

Mathematica [A]
time = 0.48, size = 270, normalized size = 0.77

$$\frac{-\frac{2bce\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}{(c^2d^2+e^2)(d+ex)}-\frac{\left(a+b\sinh^{-1}(cx)\right)^2}{(d+ex)^2}+\frac{2b^2c^2\log(d+ex)}{c^2d^2+e^2}+\frac{2bc^3d\left(\left(a+b\sinh^{-1}(cx)\right)\left(\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd-\sqrt{c^2d^2+e^2}}\right)-\log\left(1+\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)+b\,\text{PolyLog}\left(2,\frac{ee^{\sinh^{-1}(cx)}}{-cd+\sqrt{c^2d^2+e^2}}\right)-b\,\text{PolyLog}\left(2,-\frac{ee^{\sinh^{-1}(cx)}}{cd+\sqrt{c^2d^2+e^2}}\right)\right)}{(c^2d^2+e^2)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] ((-2*b*c*e*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x]))/((c^2*d^2 + e^2)*(d + e*
x)) - (a + b*ArcSinh[c*x])^2/(d + e*x)^2 + (2*b^2*c^2*Log[d + e*x])/(c^2*d^
2 + e^2) + (2*b*c^3*d*((a + b*ArcSinh[c*x])*(Log[1 + (e*E^ArcSinh[c*x])/(c*
d - Sqrt[c^2*d^2 + e^2])] - Log[1 + (e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2
+ e^2])]) + b*PolyLog[2, (e*E^ArcSinh[c*x])/(-(c*d) + Sqrt[c^2*d^2 + e^2])]
```

```
 - b*PolyLog[2, -((e*E^ArcSinh[c*x])/(c*d + Sqrt[c^2*d^2 + e^2]))]))/(c^2*d
^2 + e^2)^(3/2))/(2*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(365) = 730$.
time = 12.87, size = 1017, normalized size = 2.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a^2*c^3/(c*e*x+c*d)^2/e-1/2*b^2*c^5*arcsinh(c*x)^2/e/(c^2*d^2+e^2
)/(c*e*x+c*d)^2*d^2-b^2*c^4*arcsinh(c*x)/(c^2*d^2+e^2)/(c*e*x+c*d)^2*d*(c^2
*x^2+1)^(1/2)-b^2*c^4*arcsinh(c*x)*e/(c^2*d^2+e^2)/(c*e*x+c*d)^2*(c^2*x^2+1
)^(1/2)*x+b^2*c^5*arcsinh(c*x)/e/(c^2*d^2+e^2)/(c*e*x+c*d)^2*d^2+2*b^2*c^5*
arcsinh(c*x)/(c^2*d^2+e^2)/(c*e*x+c*d)^2*d*x+b^2*c^5*arcsinh(c*x)*e/(c^2*d^
2+e^2)/(c*e*x+c*d)^2*x^2-1/2*b^2*c^3*arcsinh(c*x)^2*e/(c^2*d^2+e^2)/(c*e*x+
c*d)^2+b^2*c^3/e/(c^2*d^2+e^2)*ln(2*(c*x+(c^2*x^2+1)^(1/2))*d*c+e*(c*x+(c^2
*x^2+1)^(1/2))^2-e)-2*b^2*c^3/e/(c^2*d^2+e^2)*ln(c*x+(c^2*x^2+1)^(1/2))+b^2
*c^4/e/(c^2*d^2+e^2)^(3/2)*d*arcsinh(c*x)*ln((-c*d-e*(c*x+(c^2*x^2+1)^(1/2)
)+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-b^2*c^4/e/(c^2*d^2+e^2)^
(3/2)*d*arcsinh(c*x)*ln((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))
/(c*d+(c^2*d^2+e^2)^(1/2)))+b^2*c^4/e/(c^2*d^2+e^2)^(3/2)*d*dilog((-c*d-e*(
c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+e^2)^(1/2))/(-c*d+(c^2*d^2+e^2)^(1/2)))-b^2
*c^4/e/(c^2*d^2+e^2)^(3/2)*d*dilog((c*d+e*(c*x+(c^2*x^2+1)^(1/2))+(c^2*d^2+
e^2)^(1/2))/(c*d+(c^2*d^2+e^2)^(1/2)))-a*b*c^3/(c*e*x+c*d)^2/e*arcsinh(c*x)
-a*b*c^3/e/(c^2*d^2+e^2)/(c*x+c*d/e)*((c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^
2*d^2+e^2)/e^2)^(1/2)-a*b*c^4/e^2*d/(c^2*d^2+e^2)/((c^2*d^2+e^2)/e^2)^(1/2)
*ln((2*(c^2*d^2+e^2)/e^2-2*d*c/e*(c*x+c*d/e)+2*((c^2*d^2+e^2)/e^2)^(1/2)*((
c*x+c*d/e)^2-2*d*c/e*(c*x+c*d/e)+(c^2*d^2+e^2)/e^2)^(1/2))/(c*x+c*d/e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] ((c^2*d*arcsinh(c*d*x*e^(-1)/abs(d*e^(-1) + x) - 1/(c*abs(d*e^(-1) + x)))*e
^(-4)/(c^2*d^2*e^(-2) + 1)^(3/2) - sqrt(c^2*x^2 + 1)/(c^2*d^2*x*e + c^2*d^3
 + x*e^3 + d*e^2))*c - arcsinh(c*x)/(x^2*e^3 + 2*d*x*e^2 + d^2*e))*a*b - 1/
2*b^2*(log(c*x + sqrt(c^2*x^2 + 1))^2/(x^2*e^3 + 2*d*x*e^2 + d^2*e) - 2*int
egrate((c^3*x^2 + sqrt(c^2*x^2 + 1)*c^2*x + c)*log(c*x + sqrt(c^2*x^2 + 1))
/(c^3*x^5*e^3 + 2*c^3*d*x^4*e^2 + 2*c*d*x^2*e^2 + c*d^2*x*e + (c^3*d^2*e +
c*e^3)*x^3 + (c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e + e^3)*x^2 + 2*d*x
*e^2 + d^2*e)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/(x^2*e^3 + 2*d*x*e^2 + d^2*
e)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)/(x^3*e^3 + 3*d*x^2
*e^2 + 3*d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))^2}{(d + ex)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2/(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2/(e*x + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{asinh}(c\,x))^2}{(d + e\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(c*x))^2/(d + e*x)^3, x)
```

3.19 $\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)} \, dx$

Optimal. Leaf size=394

$$\frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx\right.}{4bc^3}$$

```
[Out] d^3*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arcsinh(c*x))*cos
h(a/b)/b/c^3+3/4*d*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+3/2*d^2*
e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-1/4*e^3*cosh(2*a/b)*Shi(2*a/b
+2*arcsinh(c*x))/b/c^4+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arcsinh(c*x))/b/c^4-
d^3*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c+3/4*d*e^2*Shi(a/b+arcsinh(c*x))*sin
h(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^2+1/4*e^3*
Chi(2*a/b+2*arcsinh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Shi(3*a/b+3*arcsinh(c
*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arcsinh(c*x))*sinh(4*a/b)/b/c^4
```

Rubi [A]
time = 0.88, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556, 12}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{4bc^4}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^4}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{4bc^4}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\sinh^{-1}(cx)\right)}{8bc^4}-\frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{3de^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{3d^2e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (3*d*e^2*Cosh[a/b]
*CoshIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (3*d*e^2*Cosh[(3*a)/b]*CoshI
ntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*a)/
b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (e^3*CoshIntegral[(2*a)/b +
2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (e^3*CoshIntegral[(4*a)/b + 4*Ar
cSinh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) - (d^3*Sinh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]])/(b*c) + (3*d*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(
4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(
2*b*c^2) - (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(4*b*
c^4) - (3*d*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*
c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcSinh[c*x]])/(8*b*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^3}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\cosh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh(x)\sinh^2(x)}{a+bx} + \frac{e^3\cosh(x)\sinh^3(x)}{a+bx}\right)\,dx}{c^4}$$

$$= \frac{d^3\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c\right.}{c^2}$$

$$= \frac{(3d^2e)\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} + \frac{(3de^2)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} + \frac{\cosh(3x)}{4(a+bx)}\right)\right.}{c^3}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} + \frac{(3d^2e)\,\text{Sub}}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{d^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{(3de^2\cosh}{}$$

$$= \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{3de^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{3de^2\cos}{}$$

Mathematica [A]
time = 0.40, size = 305, normalized size = 0.77

$$\frac{d^3\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{bc}+\frac{3de^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\right)}{4bc^3}+\frac{e^3\left(2\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\right)}{8bc^4}-\frac{3d^2e\left(\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^3*(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcSinh[c*x]]))/(b*c) + (3*d*e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSi
nh[c*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] + Sinh[a/b]*
SinhIntegral[a/b + ArcSinh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcS
inh[c*x])]))/(4*b*c^3) + (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c*x])]*Sinh[
(2*a)/b] - CoshIntegral[4*(a/b + ArcSinh[c*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcSinh[c*x])]))/(8*b*c^4) - (3*d^2*e*(CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]
]))/(2*b*c^2)
```

Maple [A]
time = 11.84, size = 394, normalized size = 1.00

method	result
derivativedivides	$-\frac{e^{3} e^{-\frac{4a}{b}} \operatorname{expIntegral}\left(1,-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^{3}b}+\frac{e^{3} e^{\frac{4a}{b}} \operatorname{expIntegral}\left(1,4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^{3}b}+\frac{3e\, e^{\frac{2a}{b}} \operatorname{expIntegral}\left(1,2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)d^{2}}{4cb}$
default	$-\frac{e^{3} e^{-\frac{4a}{b}} \operatorname{expIntegral}\left(1,-4 \operatorname{arcsinh}(cx)-\frac{4a}{b}\right)}{16c^{3}b}+\frac{e^{3} e^{\frac{4a}{b}} \operatorname{expIntegral}\left(1,4 \operatorname{arcsinh}(cx)+\frac{4a}{b}\right)}{16c^{3}b}+\frac{3e\, e^{\frac{2a}{b}} \operatorname{expIntegral}\left(1,2 \operatorname{arcsinh}(cx)+\frac{2a}{b}\right)d^{2}}{4cb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*a/b+4*arcsinh(c*x))+3/4/c*e/b*exp(2*a/b)*Ei(1,2*a/b+2*ar
csinh(c*x))*d^2-1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*a/b+2*arcsinh(c*x))-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)*d^2+1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(c*x)-2*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*d^3+3/8/c^2
*d/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)
-a/b)*d^3+3/8/c^2*d/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-3/8/c^2*d*e^2/b
*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arcsinh(c*x)+3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x*e + d^3)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d + e*x)**3/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{a+b\,\mathrm{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*asinh(c*x)),x)
```

```
[Out] int((d + e*x)^3/(a + b*asinh(c*x)), x)
```

3.20 $\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}-\frac{de$$

```
[Out] d^2*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c-1/4*e^2*Chi(a/b+arcsinh(c*x))*cosh(
a/b)/b/c^3+1/4*e^2*Chi(3*a/b+3*arcsinh(c*x))*cosh(3*a/b)/b/c^3+d*e*cosh(2*a
/b)*Shi(2*a/b+2*arcsinh(c*x))/b/c^2-d^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c
+1/4*e^2*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arcsinh(c*x)
)*sinh(2*a/b)/b/c^2-1/4*e^2*Shi(3*a/b+3*arcsinh(c*x))*sinh(3*a/b)/b/c^3
```

Rubi [A]
time = 0.52, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556}

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3a}{b}+3\sinh^{-1}(cx)\right)}{4bc^3}-\frac{de\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{bc^2}+\frac{de\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{bc^2}+\frac{d^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e^2*Cosh[a/b]*Cos
hIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) + (e^2*Cosh[(3*a)/b]*CoshIntegral[
(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2*ArcSin
h[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]])/(b*c) + (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(4*b*c^3) +
 (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c*x]])/(b*c^2) - (e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSinh[c*x]])/(4*b*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))^2}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\cosh(x)}{a+bx}+\frac{e^2\cosh(x)\sinh^2(x)}{a+bx}+\frac{cde\sinh(2x)}{a+bx}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c} + \frac{(de)\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(cx)\right)}{c^2} +$$

$$= \frac{e^2\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \sinh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh(}{a-}\right.}{c}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} - \frac{d^2\sinh}{}$$

$$= \frac{d^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc} - \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{4bc^3} + \frac{e^2\cosh\left(\frac{3a}{b}\right.}{}$$

Mathematica [A]
time = 0.25, size = 188, normalized size = 0.77

$$\frac{(4c^2d^2-e^2)\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-4cde\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-4c^2d^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+4cde\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)-e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x]),x]
```

```
[Out] ((4*c^2*d^2 - e^2)*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c*x])] - 4*c*d*e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 4*c^2*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcS
inh[c*x]] + e^2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + 4*c*d*e*Cosh[(
2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] - e^2*Sinh[(3*a)/b]*SinhIntegr
al[3*(a/b + ArcSinh[c*x])])/(4*b*c^3)
```

Maple [A]
time = 10.64, size = 254, normalized size = 1.04

method	result
derivativedivides	$-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \mathrm{expIntegral}\left(1,-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2 b}-\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2 b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right) d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}$
default	$-\frac{e^2 \mathrm{e}^{-\frac{3a}{b}} \mathrm{expIntegral}\left(1,-3\,\mathrm{arcsinh}(cx)-\frac{3a}{b}\right)}{8c^2 b}-\frac{e^2 \mathrm{e}^{\frac{3a}{b}} \mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2 b}-\frac{\mathrm{e}^{\frac{a}{b}} \mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right) d^2}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(c*x)-3*a/b)-1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arcsinh(c*x)+3*a/b)-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
sinh(c*x)-a/b)*d^2+1/8/c^2/b*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
/b+2*arcsinh(c*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{a+b\operatorname{asinh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^2/(a + b*asinh(c*x)), x)
```

3.21 $\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{e\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}+\frac{e\cosh}{}$$

```
[Out] d*Chi(a/b+arcsinh(c*x))*cosh(a/b)/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arcsinh
(c*x))/b/c^2-d*Shi(a/b+arcsinh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arcsin
h(c*x))*sinh(2*a/b)/b/c^2
```

Rubi [A]
time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5830, 6874, 3384, 3379, 3382, 5556, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)}{2bc^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]])/(b*c) - (e*CoshIntegral[(2*a
)/b + 2*ArcSinh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (d*Sinh[a/b]*SinhIntegral[
a/b + ArcSinh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcS
inh[c*x]])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{a+b\sinh^{-1}(cx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)(cd+e\sinh(x))}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{\text{Subst}\left(\int \left(\frac{cd\cosh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)\,dx,x,\sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{d\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c}+\frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{c^2}\\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\sinh^{-1}(cx)\right)}{c^2}+\frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}\right.}{c}\\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}+\frac{e\text{Subst}\left(\int \frac{\sin}{a}\right.}{}\\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}+\frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)}{}\\
&= \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{bc}-\frac{e\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 98, normalized size = 0.84

$$\frac{2cd\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)-e\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2cd\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x]),x]
```

```
[Out] (2*c*d*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c*x]] - e*CoshIntegral[2*(a/b +
 ArcSinh[c*x])]*Sinh[(2*a)/b] - 2*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[
c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])])/(2*b*c^2)
```

Maple [A]
time = 8.74, size = 120, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}-\frac{d\,e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b}-\frac{e\,e^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}+\frac{e\,e^{\frac{2a}{b}}\cdots}{\cdots}}{c}$
default	$\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)d}{2b}-\frac{d\,e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b}-\frac{e\,e^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}(cx)-\frac{2a}{b}\right)}{4cb}+\frac{e\,e^{\frac{2a}{b}}\cdots}{\cdots}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*d/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2*d/b*exp(-a/b)*Ei(1,-arcsi
nh(c*x)-a/b)-1/4*e/c/b*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b)+1/4*e/c/b*ex
p(2*a/b)*Ei(1,2*a/b+2*arcsinh(c*x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((x*e + d)/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+b\operatorname{asinh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{a+b\operatorname{asinh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)/(a + b*asinh(c*x)), x)
```

3.22 $\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] Chi((a+b*arcsinh(c*x))/b)*cosh(a/b)/b/c-Shi((a+b*arcsinh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5774, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c*x])/b])/(b*c) - (Sinh[a/b]*SinhInt
egral[(a + b*ArcSinh[c*x])/b])/(b*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x\right.}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-1),x]
[Out] Integrate[(a + b*ArcSinh[c*x])^(-1), x]
```

Maple [A]
time = 3.17, size = 56, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56
default	$\frac{-\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b}}{c}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c*(-1/2/b*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh(c
*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arcsinh(c*x) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{asinh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x)),x)
[Out] Integral(1/(a + b*asinh(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{asinh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x)),x)
[Out] int(1/(a + b*asinh(c*x)), x)
```

3.23 $\int \frac{1}{(d+ex)(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\,\text{arcsinh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arcsinh(c*x) + a)*(x*e + d)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x*e + a*d + (b*x*e + b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(cx))(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c\,x))\ (d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)), x)
```

3.24 $\int \frac{1}{(d+ex)^2(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

`[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)`

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]`

`[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]`

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])),x]`

`[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])), x]`

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2\left(a+b\,\mathrm{arcsinh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arcsinh(c*x) + a)*(x*e + d)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*x^2*e^2 + 2*a*d*x*e + a*d^2 + (b*x^2*e^2 + 2*b*d*x*e + b*d^2)
*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}\left(cx\right)\right)\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x)),x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))*(d + e*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))*(d + e*x)^2),x)
[Out] int(1/((a + b*asinh(c*x))*(d + e*x)^2), x)
```

3.25 $\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=359

$$-\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-$$

```
[Out] 2*d*e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d^2*cosh(a/b)*Shi((a+
b*arcsinh(c*x))/b)/b^2/c-1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c^
3+3/4*e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(c*x))/b)/b^2/c^3-d^2*Chi((a+b*arcs
inh(c*x))/b)*sinh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^
2/c^3-2*d*e*Shi(2*(a+b*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Chi(3*(
a+b*arcsinh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arc
sinh(c*x))-2*d*e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))-e^2*x^2*(c^2*x^
2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A]
time = 0.39, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5829, 5773, 5819, 3384, 3379, 3382, 5778}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}+\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{2de\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{d^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d^2\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{2dex\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (2*d*e*x*Sqrt[1 + c
^2*x^2])/(b*c*(a + b*ArcSinh[c*x])) - (e^2*x^2*Sqrt[1 + c^2*x^2])/(b*c*(a +
 b*ArcSinh[c*x])) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c*x
]))/b])/(b^2*c^2) - (d^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(b
^2*c) + (e^2*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/b])/(4*b^2*c^3) -
(3*e^2*CoshIntegral[(3*(a + b*ArcSinh[c*x]))/b]*Sinh[(3*a)/b])/(4*b^2*c^3)
+ (d^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) - (e^2*Cosh[
a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(4*b^2*c^3) - (2*d*e*Sinh[(2*a)/
b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/
b]*SinhIntegral[(3*(a + b*ArcSinh[c*x]))/b])/(4*b^2*c^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\sinh^{-1}(cx)\right)^2} + \frac{2dex}{\left(a+b\sinh^{-1}(cx)\right)^2} + \frac{e^2x^2}{\left(a+b\sinh^{-1}(cx)\right)^2}\right) dx$$

$$= d^2 \int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + (2de)\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx + e^2 \int \frac{}{\left(a+b\text{s}\right.}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{(cd^2}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{d^2\text{S}}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2de}{}$$

$$= -\frac{d^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{2dex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} - \frac{e^2x^2\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)} + \frac{2de}{}$$

Mathematica [A]
time = 1.03, size = 288, normalized size = 0.80

$$-\frac{\frac{4bc^2d^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)} + \frac{8bc^2dex\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)} + \frac{4bc^2e^2x^2\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)} - 8cde\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) + (4c^2d^2-e^2)\,\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right) + 3e^2\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)\sinh\left(\frac{3a}{b}\right) - 4c^2d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right) + 8cde\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right) - 3e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -1/4*((4*b*c^2*d^2*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (8*b*c^2*d*e*x
*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (4*b*c^2*e^2*x^2*Sqrt[1 + c^2*x^
2])/(a + b*ArcSinh[c*x]) - 8*c*d*e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcS
inh[c*x])] + (4*c^2*d^2 - e^2)*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] +
 3*e^2*CoshIntegral[3*(a/b + ArcSinh[c*x])]*Sinh[(3*a)/b] - 4*c^2*d^2*Cosh[
a/b]*SinhIntegral[a/b + ArcSinh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + Ar
cSinh[c*x]] + 8*c*d*e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c*x])] -
3*e^2*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c*x])])/(b^2*c^3)
```

Maple [A]
time = 10.27, size = 616, normalized size = 1.72

method	result
derivativedivides	$\frac{\left(-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx\right)e^2}{8c^2b(a+b\operatorname{arcsinh}(cx))}+\frac{3e^2e^{\frac{3a}{b}}\operatorname{expIntegral}\left(1,3\operatorname{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\right.}{8b\,c}$

default	$\frac{\left(-4c^2x^2\sqrt{c^2x^2+1}+4c^3x^3-\sqrt{c^2x^2+1}+3cx\right)e^2}{8c^2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{3e^2e^{\frac{3a}{b}}\,\mathrm{expIntegral}\left(1,3\,\mathrm{arcsinh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3+3cx+4c^2x^2\sqrt{}\right.}{8b\,c^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/8*(-4*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c^3*x^3-(c^2*x^2+1)^(1/2)+3*c*x)*e
^2/c^2/b/(a+b*arcsinh(c*x))+3/8*e^2/c^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh(c*x)+
3*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3+3*c*x+4*c^2*x^2*(c^2*x^2+1)^(1/2)+(c^2*x^2+
1)^(1/2))/(a+b*arcsinh(c*x))-3/8/b^2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arcsinh(c*
x)-3*a/b)+1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d^2/b/(a+b*arcsinh(c*x))+1/2*d^2/b^2
*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/8*(-(c^2*x^2+1)^(1/2)+c*x)*e^2/c^2/b/(a+
b*arcsinh(c*x))-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d^2*(
c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arcsi
nh(c*x)-a/b)+1/8/c^2/b*e^2*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))+1/8/c
^2/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/2*(2*c^2*x^2-2*(c^2*x^2+1)^(
1/2)*c*x+1)*d*e/c/b/(a+b*arcsinh(c*x))-e*d/c/b^2*exp(2*a/b)*Ei(1,2*a/b+2*ar
csinh(c*x))-1/2/b*e*d/c*(2*c^2*x^2+1+2*(c^2*x^2+1)^(1/2)*c*x)/(a+b*arcsinh(
c*x))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^5*e^2 + 2*c^3*d*x^4*e + 2*c*d*x^2*e + c*d^2*x + (c^3*d^2 + c*e^2)*x
^3 + (c^2*x^4*e^2 + 2*c^2*d*x^3*e + (c^2*d^2 + e^2)*x^2 + 2*d*x*e + d^2)*sq
rt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*
c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))
) + integrate((3*c^5*x^6*e^2 + 4*c^5*d*x^5*e + 8*c^3*d*x^3*e + (c^5*d^2 + 6
*c^3*e^2)*x^4 + 4*c*d*x*e + c*d^2 + (2*c^3*d^2 + 3*c*e^2)*x^2 + (3*c^3*x^4*
e^2 + 4*c^3*d*x^3*e - c*d^2 + (c^3*d^2 + c*e^2)*x^2)*(c^2*x^2 + 1) + (6*c^4
*x^5*e^2 + 8*c^4*d*x^4*e + 8*c^2*d*x^2*e + (2*c^4*d^2 + 7*c^2*e^2)*x^3 + (c
^2*d^2 + 2*e^2)*x + 2*d*e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4 + (c^2*x^2 + 1)*
a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*x^2 + 1)*b^2*c^3*
x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*sqrt(c^2*x^2 + 1)
)*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*x)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x^2*e^2 + 2*d*x*e + d^2)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x)
 + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*asinh(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{(a+b\operatorname{asinh}(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)^2/(a + b*asinh(c*x))^2, x)
```

3.26 $\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{d\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\right.}{b^2c}$$

```
[Out] e*Chi(2*(a+b*arcsinh(c*x))/b)*cosh(2*a/b)/b^2/c^2+d*cosh(a/b)*Shi((a+b*arcs
inh(c*x))/b)/b^2/c-d*Chi((a+b*arcsinh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a+b
*arcsinh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(
c*x))-e*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A]
time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5829, 5773, 5819, 3384, 3379, 3382, 5778}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{d\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[1 + c^2*x^2])/(b*c*(a + b*ArcSinh[c*x]))) - (e*x*Sqrt[1 + c^2*x^2
])/(b*c*(a + b*ArcSinh[c*x])) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*Arc
Sinh[c*x]))/b])/(b^2*c^2) - (d*CoshIntegral[(a + b*ArcSinh[c*x])/b]*Sinh[a/
b])/(b^2*c) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*x])/b])/(b^2*c) -
(e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c*x]))/b])/(b^2*c^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx &= \int \left(\frac{d}{\left(a+b\sinh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\sinh^{-1}(cx)\right)^2}\right)dx \\
&= d\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx+e\int \frac{x}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{(cd)\int \frac{x}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx}}{b} \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{d\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}\right.}{bc} \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}\right.}{b^2c^2} \\
&= -\frac{d\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{ex\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}\right.}{b^2c^2}
\end{aligned}
$$

Mathematica [A]

time = 0.47, size = 150, normalized size = 0.83

$$
-\frac{\frac{bcd\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}+\frac{bcex\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}-e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)+cd\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)-cd\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)+e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\right)}{b^2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] -(((b*c*d*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x]) + (b*c*e*x*Sqrt[1 + c^2*x
^2])/(a + b*ArcSinh[c*x]) - e*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c
*x])] + c*d*CoshIntegral[a/b + ArcSinh[c*x]]*Sinh[a/b] - c*d*Cosh[a/b]*Sinh
Integral[a/b + ArcSinh[c*x]] + e*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c*x])]])/(b^2*c^2))
```

Maple [A]

time = 8.71, size = 272, normalized size = 1.51

method	result
derivativedivides	$\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2(a+b\,\mathrm{arcsinh}(cx))b}+\frac{d\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{d\left(cx+\sqrt{c^2x^2+1}\right)}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{d\,\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\right.}{2b^2}$
default	$\frac{\left(-\sqrt{c^2x^2+1}+cx\right)d}{2(a+b\,\mathrm{arcsinh}(cx))b}+\frac{d\,\mathrm{e}^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{d\left(cx+\sqrt{c^2x^2+1}\right)}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{d\,\mathrm{e}^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\right.}{2b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)*d/(a+b*arcsinh(c*x))/b+1/2*d/b^2*exp(a/b)
*Ei(1,arcsinh(c*x)+a/b)-1/2/b*d*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-
1/2/b^2*d*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b)+1/4*(2*c^2*x^2-2*(c^2*x^2+1)^(1
/2)*c*x+1)*e/c/(a+b*arcsinh(c*x))/b-1/2*e/c/b^2*exp(2*a/b)*Ei(1,2*a/b+2*arc
sinh(c*x))-1/4*e/c/b*(2*c^2*x^2+1+2*(c^2*x^2+1)^(1/2)*c*x)/(a+b*arcsinh(c*x
))-1/2*e/c/b^2*exp(-2*a/b)*Ei(1,-2*arcsinh(c*x)-2*a/b))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^4*e + c^3*d*x^3 + c*x^2*e + c*d*x + (c^2*x^3*e + c^2*d*x^2 + x*e +
d)*sqrt(c^2*x^2 + 1))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c +
(b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2
+ 1))) + integrate((2*c^5*x^5*e + c^5*d*x^4 + 4*c^3*x^3*e + 2*c^3*d*x^2 + 2
*c*x*e + (2*c^3*x^3*e + c^3*d*x^2 - c*d)*(c^2*x^2 + 1) + c*d + (4*c^4*x^4*e
 + 2*c^4*d*x^3 + 4*c^2*x^2*e + c^2*d*x + e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^4
 + (c^2*x^2 + 1)*a*b*c^3*x^2 + 2*a*b*c^3*x^2 + a*b*c + (b^2*c^5*x^4 + (c^2*
x^2 + 1)*b^2*c^3*x^2 + 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 + b^2*c^2*x)*
sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^3 + a*b*c^2*
x)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{\left(a + b\operatorname{asinh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*asinh(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*asinh(c*x))^2,x)
```

```
[Out] int((d + e*x)/(a + b*asinh(c*x))^2, x)
```

3.27 $\int \frac{1}{(a+b\sinh^{-1}(cx))^2} dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(c*x))/b)/b^2/c-Chi((a+b*arcsinh(c*x))/b)*sinh(a/
b)/b^2/c-(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
```

Rubi [A]
time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{c^2x^2+1}}{bc\left(a+b\sinh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + c^2*x^2]/(b*c*(a + b*ArcSinh[c*x]))) - (CoshIntegral[(a + b*ArcS
inh[c*x])/b]*Sinh[a/b])/(b^2*c) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c*
x])/b])/(b^2*c)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(cx)\right)}{bc}-\text{sinh}$$

$$= -\frac{\sqrt{1+c^2x^2}}{bc\left(a+b\sinh^{-1}(cx)\right)}-\frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{b^2c}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh}{b^2c}$$

Mathematica [A]
time = 0.15, size = 71, normalized size = 0.84

$$\frac{-\frac{b\sqrt{1+c^2x^2}}{a+b\sinh^{-1}(cx)}-\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(cx)\right)}{b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + c^2*x^2])/(a + b*ArcSinh[c*x])) - CoshIntegral[a/b + ArcSinh
[c*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c*x]])/(b^2*c)
```

Maple [A]
time = 3.28, size = 118, normalized size = 1.39

method	result	si
derivativedivides	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	11
default	$\frac{\frac{-\sqrt{c^2x^2+1}+cx}{2b(a+b\,\mathrm{arcsinh}(cx))}+\frac{e^{\frac{a}{b}}\,\mathrm{expIntegral}\left(1,\mathrm{arcsinh}(cx)+\frac{a}{b}\right)}{2b^2}-\frac{cx+\sqrt{c^2x^2+1}}{2b(a+b\,\mathrm{arcsinh}(cx))}-\frac{e^{-\frac{a}{b}}\,\mathrm{expIntegral}\left(1,-\mathrm{arcsinh}(cx)-\frac{a}{b}\right)}{2b^2}}{c}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-(c^2*x^2+1)^(1/2)+c*x)/b/(a+b*arcsinh(c*x))+1/2/b^2*exp(a/b)*Ei(
1,arcsinh(c*x)+a/b)-1/2/b*(c*x+(c^2*x^2+1)^(1/2))/(a+b*arcsinh(c*x))-1/2/b^
2*exp(-a/b)*Ei(1,-arcsinh(c*x)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^2 + sqrt(c^2*x^2 + 1)*a*b
*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)*b^2*c^2*x + b^2*c)*log(c*
x + sqrt(c^2*x^2 + 1))) + integrate((c^4*x^4 + 2*c^2*x^2 + (c^2*x^2 + 1)*(c
^2*x^2 - 1) + (2*c^3*x^3 + c*x)*sqrt(c^2*x^2 + 1) + 1)/(a*b*c^4*x^4 + (c^2*
x^2 + 1)*a*b*c^2*x^2 + 2*a*b*c^2*x^2 + a*b + (b^2*c^4*x^4 + (c^2*x^2 + 1)*b
^2*c^2*x^2 + 2*b^2*c^2*x^2 + b^2 + 2*(b^2*c^3*x^3 + b^2*c*x)*sqrt(c^2*x^2 +
 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^3*x^3 + a*b*c*x)*sqrt(c^2*x^2
+ 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c*x))^2,x)
```

```
[Out] int(1/(a + b*asinh(c*x))^2, x)
```

3.28 $\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

[In] Int[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]

[Out] Defer[Int][1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2),x]

[Out] Integrate[1/((d + e*x)*(a + b*ArcSinh[c*x])^2), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^3*e + a*b*c^3*d*x^2 + a*b
*c*x*e + a*b*c*d + (b^2*c^3*x^3*e + b^2*c^3*d*x^2 + b^2*c*x*e + b^2*c*d + (
b^2*c^2*x^2*e + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)
) + (a*b*c^2*x^2*e + a*b*c^2*d*x)*sqrt(c^2*x^2 + 1)) + integrate((c^5*d*x^4
 + 2*c^3*d*x^2 + (c^3*d*x^2 - 2*c*x*e - c*d)*(c^2*x^2 + 1) + c*d + (2*c^4*d
*x^3 - 2*c^2*x^2*e + c^2*d*x - e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^6*e^2 + 2*a
*b*c^5*d*x^5*e + 4*a*b*c^3*d*x^3*e + 2*a*b*c*d*x*e + a*b*c*d^2 + (a*b*c^5*d
^2 + 2*a*b*c^3*e^2)*x^4 + (2*a*b*c^3*d^2 + a*b*c*e^2)*x^2 + (a*b*c^3*x^4*e^
2 + 2*a*b*c^3*d*x^3*e + a*b*c^3*d^2*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^6*e^2 +
 2*b^2*c^5*d*x^5*e + 4*b^2*c^3*d*x^3*e + 2*b^2*c*d*x*e + b^2*c*d^2 + (b^2*c
^5*d^2 + 2*b^2*c^3*e^2)*x^4 + (2*b^2*c^3*d^2 + b^2*c*e^2)*x^2 + (b^2*c^3*x^
4*e^2 + 2*b^2*c^3*d*x^3*e + b^2*c^3*d^2*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*x^5
*e^2 + 2*b^2*c^4*d*x^4*e + 2*b^2*c^2*d*x^2*e + b^2*c^2*d^2*x + (b^2*c^4*d^2
 + b^2*c^2*e^2)*x^3)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a
*b*c^4*x^5*e^2 + 2*a*b*c^4*d*x^4*e + 2*a*b*c^2*d*x^2*e + a*b*c^2*d^2*x + (a
*b*c^4*d^2 + a*b*c^2*e^2)*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x*e + a^2*d + (b^2*x*e + b^2*d)*arcsinh(c*x)^2 + 2*(a*b*x*e
 + a*b*d)*arcsinh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{asinh}(cx))^2 (d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,\left(d+e\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)), x)
```

3.29 $\int \frac{1}{(d+ex)^2(a+b\sinh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 3.45, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\sinh^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcSinh[c*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + c*x + (c^2*x^2 + 1)^(3/2))/(a*b*c^3*x^4*e^2 + 2*a*b*c^3*d*x^3*e
 + 2*a*b*c*d*x*e + a*b*c*d^2 + (a*b*c^3*d^2 + a*b*c*e^2)*x^2 + (b^2*c^3*x^4
*e^2 + 2*b^2*c^3*d*x^3*e + 2*b^2*c*d*x*e + b^2*c*d^2 + (b^2*c^3*d^2 + b^2*c
*e^2)*x^2 + (b^2*c^2*x^3*e^2 + 2*b^2*c^2*d*x^2*e + b^2*c^2*d^2*x)*sqrt(c^2*
x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + (a*b*c^2*x^3*e^2 + 2*a*b*c^2*d*x^2
*e + a*b*c^2*d^2*x)*sqrt(c^2*x^2 + 1)) - integrate((c^5*x^5*e - c^5*d*x^4 +
 2*c^3*x^3*e - 2*c^3*d*x^2 + c*x*e + (c^3*x^3*e - c^3*d*x^2 + 3*c*x*e + c*d
)*(c^2*x^2 + 1) - c*d + (2*c^4*x^4*e - 2*c^4*d*x^3 + 5*c^2*x^2*e - c^2*d*x
+ 2*e)*sqrt(c^2*x^2 + 1))/(a*b*c^5*x^7*e^3 + 3*a*b*c^5*d*x^6*e^2 + 3*a*b*c*
d^2*x*e + a*b*c*d^3 + (3*a*b*c^5*d^2*e + 2*a*b*c^3*e^3)*x^5 + (a*b*c^5*d^3
+ 6*a*b*c^3*d*e^2)*x^4 + (6*a*b*c^3*d^2*e + a*b*c*e^3)*x^3 + (2*a*b*c^3*d^3
 + 3*a*b*c*d*e^2)*x^2 + (a*b*c^3*x^5*e^3 + 3*a*b*c^3*d*x^4*e^2 + 3*a*b*c^3*
d^2*x^3*e + a*b*c^3*d^3*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^7*e^3 + 3*b^2*c^5*d
*x^6*e^2 + 3*b^2*c*d^2*x*e + b^2*c*d^3 + (3*b^2*c^5*d^2*e + 2*b^2*c^3*e^3)*
x^5 + (b^2*c^5*d^3 + 6*b^2*c^3*d*e^2)*x^4 + (6*b^2*c^3*d^2*e + b^2*c*e^3)*x
^3 + (2*b^2*c^3*d^3 + 3*b^2*c*d*e^2)*x^2 + (b^2*c^3*x^5*e^3 + 3*b^2*c^3*d*x
^4*e^2 + 3*b^2*c^3*d^2*x^3*e + b^2*c^3*d^3*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*
x^6*e^3 + 3*b^2*c^4*d*x^5*e^2 + 3*b^2*c^2*d^2*x^2*e + b^2*c^2*d^3*x + (3*b^
2*c^4*d^2*e + b^2*c^2*e^3)*x^4 + (b^2*c^4*d^3 + 3*b^2*c^2*d*e^2)*x^3)*sqrt(
c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a*b*c^4*x^6*e^3 + 3*a*b*c^4
*d*x^5*e^2 + 3*a*b*c^2*d^2*x^2*e + a*b*c^2*d^3*x + (3*a*b*c^4*d^2*e + a*b*c
^2*e^3)*x^4 + (a*b*c^4*d^3 + 3*a*b*c^2*d*e^2)*x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*x^2*e^2 + 2*a^2*d*x*e + a^2*d^2 + (b^2*x^2*e^2 + 2*b^2*d*x*
e + b^2*d^2)*arcsinh(c*x)^2 + 2*(a*b*x^2*e^2 + 2*a*b*d*x*e + a*b*d^2)*arcsi
nh(c*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(cx \right)}\right)^{2} \left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*asinh(c*x))**2*(d + e*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arcsinh(c*x) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2),x)
```

```
[Out] int(1/((a + b*asinh(c*x))^2*(d + e*x)^2), x)
```

3.30 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=75

$$\frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)^2}{e(1+m)}-\frac{2bc\text{Int}\left(\frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}},x\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)/e/(1+m)
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/Sqrt[1 + c^2*x^2], x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{e(1+m)}$$

Mathematica [A]
time = 3.32, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (x*e + d)^(m + 1)*a^2*e^(-1)/(m + 1) + (b^2*x*e + b^2*d)*e^(m*log(x*e + d)
- 1)*log(c*x + sqrt(c^2*x^2 + 1))^2/(m + 1) + integrate(-2*((b^2*c^2*d*x -
a*b*(m + 1)*e - (a*b*c^2*(m + 1) - b^2*c^2)*x^2*e)*sqrt(c^2*x^2 + 1)*(x*e +
 d)^m + (b^2*c^3*d*x^2 - (a*b*c^3*(m + 1) - b^2*c^3)*x^3*e + b^2*c*d - (a*b
*c*(m + 1) - b^2*c)*x*e)*(x*e + d)^m)*log(c*x + sqrt(c^2*x^2 + 1))/(c^3*(m
+ 1)*x^3*e + c*(m + 1)*x*e + (c^2*(m + 1)*x^2*e + (m + 1)*e)*sqrt(c^2*x^2 +
 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*(x*e + d)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx))^2 (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x))**2,x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*(d + e*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*(e*x + d)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))^2\,(d + e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))^2*(d + e*x)^m,x)
```

```
[Out] int((a + b*asinh(c*x))^2*(d + e*x)^m, x)
```

3.31 $\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=179

$$-\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}F_1\left(2+m;\frac{1}{2},\frac{1}{2};3+m;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}}+\frac{(d+$$

```
[Out] (e*x+d)^(1+m)*(a+b*arcsinh(c*x))/e/(1+m)-b*c*(e*x+d)^(2+m)*AppellF1(2+m,1/2
,1/2,3+m,(e*x+d)/(d-e/(-c^2)^(1/2)),(e*x+d)/(d+e/(-c^2)^(1/2)))*(1+(-e*x-d)
/(d-e/(-c^2)^(1/2)))^(1/2)*(1+(-e*x-d)/(d+e/(-c^2)^(1/2)))^(1/2)/e^2/(1+m)/
(2+m)/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {5828, 774, 138}

$$\frac{(d+ex)^{m+1}\left(a+b\sinh^{-1}(cx)\right)}{e(m+1)}-\frac{bc\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}(d+ex)^{m+2}F_1\left(m+2;\frac{1}{2},\frac{1}{2};m+3;\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}},\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}\right)}{e^2(m+1)(m+2)\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*c*(d + e*x)^(2 + m)*Sqrt[1 - (d + e*x)/(d - e/Sqrt[-c^2])]*Sqrt[1 - (d
 + e*x)/(d + e/Sqrt[-c^2])]*AppellF1[2 + m, 1/2, 1/2, 3 + m, (d + e*x)/(d -
 e/Sqrt[-c^2]), (d + e*x)/(d + e/Sqrt[-c^2])])/(e^2*(1 + m)*(2 + m)*Sqrt[1
+ c^2*x^2])) + ((d + e*x)^(1 + m)*(a + b*ArcSinh[c*x]))/(e*(1 + m))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{1+c^2x^2}}\, dx}{e(1+m)}$$

$$= \frac{(d+ex)^{1+m}\left(a+b\sinh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(bc\sqrt{1-\frac{d+ex}{d-\frac{\sqrt{-c^2}\,e}{c^2}}}\sqrt{1-\frac{d+ex}{d+\cdots}}\right.}{\cdots}$$

$$= -\frac{bc(d+ex)^{2+m}\sqrt{1-\frac{d+ex}{d-\frac{e}{\sqrt{-c^2}}}}\sqrt{1-\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,F_1\left(2+m;\frac{1}{2},\cdots\right.}{e^2(1+m)(2+m)\sqrt{1+c^2x^2}}$$

Mathematica [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\sinh^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]),x]
[Out] Integrate[(d + e*x)^m*(a + b*ArcSinh[c*x]), x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (ex+d)^m \left(a+b\,\mathrm{arcsinh}\,(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arcsinh(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="maxima")
```

```
[Out] ((x*e + d)*e^(m*log(x*e + d) - 1)*log(c*x + sqrt(c^2*x^2 + 1))/(m + 1) - in
tegrate((c^2*x^2*e + c^2*d*x)*(x*e + d)^m/(c^2*(m + 1)*x^2*e + (m + 1)*e),
x) - integrate((c*x*e + c*d)*(x*e + d)^m/(c^3*(m + 1)*x^3*e + c*(m + 1)*x*e
 + (c^2*(m + 1)*x^2*e + (m + 1)*e)*sqrt(c^2*x^2 + 1)), x))*b + (x*e + d)^(m
 + 1)*a*e^(-1)/(m + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)*(x*e + d)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(cx)) (d + ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(d + e*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arcsinh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*(e*x + d)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c\,x))\,\left(d + e\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))*(d + e*x)^m,x)
[Out] int((a + b*asinh(c*x))*(d + e*x)^m, x)
```

3.32 $\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(d+ex)^m}{a+b\sinh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx = \int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\sinh^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]),x]
```

```
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{a+b\,\mathrm{arcsinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="maxima")
[Out] integrate((x*e + d)^m/(b*arcsinh(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="fricas")
[Out] integral((x*e + d)^m/(b*arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x)),x)
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*asinh(c*x)),x)
[Out] int((d + e*x)^m/(a + b*asinh(c*x)), x)
```

3.33 $\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx = \int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\left(a+b\sinh^{-1}(cx)\right)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2,x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcSinh[c*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arcsinh(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 + 1)^(3/2)*(x*e + d)^m + (c^3*x^3 + c*x)*(x*e + d)^m)/(a*b*c^3*x
^2 + sqrt(c^2*x^2 + 1)*a*b*c^2*x + a*b*c + (b^2*c^3*x^2 + sqrt(c^2*x^2 + 1)
*b^2*c^2*x + b^2*c)*log(c*x + sqrt(c^2*x^2 + 1))) + integrate(((c^3*(m + 1)
*x^3*e + c^3*d*x^2 + c*(m - 1)*x*e - c*d)*(c^2*x^2 + 1)*(x*e + d)^m + (2*c^
4*(m + 1)*x^4*e + 2*c^4*d*x^3 + c^2*(3*m + 1)*x^2*e + c^2*d*x + m*e)*sqrt(c
^2*x^2 + 1)*(x*e + d)^m + (c^5*(m + 1)*x^5*e + c^5*d*x^4 + 2*c^3*(m + 1)*x^
3*e + 2*c^3*d*x^2 + c*(m + 1)*x*e + c*d)*(x*e + d)^m)/(a*b*c^5*x^5*e + a*b*
c^5*d*x^4 + 2*a*b*c^3*x^3*e + 2*a*b*c^3*d*x^2 + a*b*c*x*e + a*b*c*d + (a*b*
c^3*x^3*e + a*b*c^3*d*x^2)*(c^2*x^2 + 1) + (b^2*c^5*x^5*e + b^2*c^5*d*x^4 +
 2*b^2*c^3*x^3*e + 2*b^2*c^3*d*x^2 + b^2*c*x*e + b^2*c*d + (b^2*c^3*x^3*e +
 b^2*c^3*d*x^2)*(c^2*x^2 + 1) + 2*(b^2*c^4*x^4*e + b^2*c^4*d*x^3 + b^2*c^2*
x^2*e + b^2*c^2*d*x)*sqrt(c^2*x^2 + 1))*log(c*x + sqrt(c^2*x^2 + 1)) + 2*(a
*b*c^4*x^4*e + a*b*c^4*d*x^3 + a*b*c^2*x^2*e + a*b*c^2*d*x)*sqrt(c^2*x^2 +
1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + d)^m/(b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(a+b\operatorname{asinh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*asinh(c*x))**2,x)
[Out] Integral((d + e*x)**m/(a + b*asinh(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arcsinh(c*x))^2,x, algorithm="giac")
[Out] integrate((e*x + d)^m/(b*arcsinh(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*asinh(c*x))^2,x)
[Out] int((d + e*x)^m/(a + b*asinh(c*x))^2, x)
```

3.34 $\int (f+gx)^3 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) dx$

Optimal. Leaf size=640

$$-\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{3bfg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{bcf^2gx^3\sqrt{d+c^2dx^2}}{3\sqrt{1+c^2x^2}}$$

```
[Out] 1/2*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/8*f*g^2*x*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)
^(1/2)+f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*g^3
*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(c^2*d*x^2+d)^(1/2)
/c/(c^2*x^2+1)^(1/2)+2/15*b*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)
-1/4*b*c*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/16*b*f*g^2*x^2*(c^
2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/3*b*c*f^2*g*x^3*(c^2*d*x^2+d)^(1/2)/
(c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/
16*b*c*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/25*b*c*g^3*x^5*(c^
2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/16*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d
)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5845, 5838, 5785, 5783, 30, 5798, 5806, 5812, 272, 45, 5804, 12}

$$\frac{1}{2}f^3x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{f^2g(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{c^2}+\frac{3fg^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{3}{4}fg^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{g^3(c^2x^2+1)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^4}-\frac{g^3(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^4}-\frac{3fg^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcf^3x^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}-\frac{bf^2gx\sqrt{c^2dx^2+d}}{c\sqrt{c^2x^2+1}}-\frac{bcf^2gx^3\sqrt{c^2dx^2+d}}{3\sqrt{c^2x^2+1}}-\frac{3bfg^2x^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}}-\frac{3bcfg^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{bcg^3x^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bg^3x^3\sqrt{c^2dx^2+d}}{45c\sqrt{c^2x^2+1}}+\frac{2bg^3x\sqrt{c^2dx^2+d}}{15c^3\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -((b*f^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2])) + (2*b*g^3*x*Sqrt[
d + c^2*d*x^2])/(15*c^3*Sqrt[1 + c^2*x^2]) - (b*c*f^3*x^2*Sqrt[d + c^2*d*x^
2])/(4*Sqrt[1 + c^2*x^2]) - (3*b*f*g^2*x^2*Sqrt[d + c^2*d*x^2])/(16*c*Sqrt[
1 + c^2*x^2]) - (b*c*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(3*Sqrt[1 + c^2*x^2]) -
 (b*g^3*x^3*Sqrt[d + c^2*d*x^2])/(45*c*Sqrt[1 + c^2*x^2]) - (3*b*c*f*g^2*x^
4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*c*g^3*x^5*Sqrt[d + c^2*d
*x^2])/(25*Sqrt[1 + c^2*x^2]) + (f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/2 + (3*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*c^2) + (3
*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (f^2*g*(1 + c^2*x^
2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/c^2 - (g^3*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^4) + (g^3*(1 + c^2*x^2)^2*Sqrt[d
+ c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (f^3*Sqrt[d + c^2*d*x^2]*(a +
b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2]) - (3*f*g^2*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\int \left(f^3\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+3f^2gx\right.}{}$$

$$= \frac{\left(f^3\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}} + \cdots$$

$$= \frac{1}{2}f^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d+c^2dx}$$

$$= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bcf^2gx^3\sqrt{d}}{3\sqrt{1+}}$$

$$= -\frac{bf^2gx\sqrt{d+c^2dx^2}}{c\sqrt{1+c^2x^2}}+\frac{2bg^3x\sqrt{d+c^2dx^2}}{15c^3\sqrt{1+c^2x^2}}-\frac{bcf^3x^2\sqrt{d+}}{4\sqrt{1+c}}$$

Mathematica [A]
time = 0.90, size = 413, normalized size = 0.65

$$\frac{240a\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(-16g^3+c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g^3x^3)\right)-9600bc^2f^2g\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\right)-128bg^3\sqrt{d+c^2dx^2}\left(cx(-30+5c^2x^2+9c^4x^4)-15\sqrt{1+c^2x^2}(-2+c^2x^2+3c^4x^4)\sinh^{-1}(cx)\right)+3600ac\sqrt{d}f(4c^2f^2-3g^2)\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)-3600bc^3f^3\sqrt{d+c^2dx^2}\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)-675bcfg^2\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{28800c^4\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-16*g^3 + c^2*g*(120*f^2 + 45
*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3*x
^3)) - 9600*b*c^2*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
^2)^(3/2)*ArcSinh[c*x]) - 128*b*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x
^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c
*x]) + 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x +
 Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 3600*b*c^3*f^3*Sqrt[d + c^2*d*x^2]*(Cosh[2*
ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 675
*b*c*f*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4
*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(28800*c^4*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. $2(560) = 1120$.
time = 4.82, size = 1321, normalized size = 2.06

method	result	size
default	Expression too large to display	1321

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/5*a*g^3*x^2*(c^2*d*x^2+d)^(3/2)/c^2/d-2/15*a*g^3/d/c^4*(c^2*d*x^2+d)^(3/2
)+3/4*a*f*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-3/8*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(
1/2)-3/8*a*f*g^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d
)^(1/2)+a*f^2*g*(c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f^3*x*(c^2*d*x^2+d)^(1/2)+1
/2*a*f^3*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1
/16*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2*(4*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1
/2)/c^3+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^
5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*
x+1)*g^3*(-1+5*arcsinh(c*x))/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(8
*c^5*x^5+8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)
+4*c*x+(c^2*x^2+1)^(1/2))*f*g^2*(-1+4*arcsinh(c*x))/c^3/(c^2*x^2+1)+1/288*(
d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^
2*x^2+1)^(1/2)*c*x+1)*g*(36*arcsinh(c*x)*c^2*f^2-12*c^2*f^2-3*arcsinh(c*x)*
g^2+g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c
^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f^3*(2*arcsinh(c*x)-1)/c/(c^2*x^2+
1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g*(6*arcsin
h(c*x)*c^2*f^2-6*c^2*f^2-arcsinh(c*x)*g^2+g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2
*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*g*(6*arcsinh(c*x)*c^2*f^2+
6*c^2*f^2-arcsinh(c*x)*g^2-g^2)/c^4/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*
(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^3*(2*arcs
inh(c*x)+1)/c/(c^2*x^2+1)+1/288*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2
+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(36*arcsinh(c*x)*c
^2*f^2+12*c^2*f^2-3*arcsinh(c*x)*g^2-g^2)/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2
+1))^(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2
*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(1+4*arcsinh(c*x))/c^3/(c^2*x^
2+1)+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+2
8*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1
)*g^3*(1+5*arcsinh(c*x))/c^4/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.35 $\int (f+gx)^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=431

$$-\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bg^2x^2\sqrt{d+c^2dx^2}}{16c\sqrt{1+c^2x^2}}-\frac{2bcfgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}-\frac{bcg^2x^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}+$$

```
[Out] 1/2*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*g^2*x*(a+b*arcsinh(c*x
))*(c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)+2/3*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/3*b*f*g
*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-1/4*b*c*f^2*x^2*(c^2*d*x^2+d)^(1
/2)/(c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-2/9*b*c*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c*g^2*x^4*(c
^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2
+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)
^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5845, 5838, 5785, 5783, 30, 5798, 5806, 5812}

$$\frac{1}{2}f^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{2fg(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2}+\frac{g^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}g^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{g^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc^3\sqrt{c^2x^2+1}}-\frac{bcf^2x^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}-\frac{2bfgx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}-\frac{2bcfgx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}-\frac{bg^2x^2\sqrt{c^2dx^2+d}}{16c\sqrt{c^2x^2+1}}-\frac{bcg^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*f*g*x*Sqrt[d + c^2*d*x^2])/(3*c*Sqrt[1 + c^2*x^2]) - (b*c*f^2*x^2*Sqr
t[d + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*g^2*x^2*Sqrt[d + c^2*d*x^2])/(
16*c*Sqrt[1 + c^2*x^2]) - (2*b*c*f*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sqrt[1 + c
^2*x^2]) - (b*c*g^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) + (f^2*
x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g^2*x*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x]))/(8*c^2) + (g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh
[c*x]))/4 + (2*f*g*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/
(3*c^2) + (f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 +
c^2*x^2]) - (g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c^3*Sqrt
[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
```

```
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int (f+gx)^2\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{\sqrt{d+c^2dx^2}\,\int (f+gx)^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{\sqrt{d+c^2dx^2}\,\int \left(f^2\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)+2fgx\sqrt{\cdots}\right.}{\cdots}\\
&= \frac{\left(f^2\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}+\frac{(2\cdots}{\cdots}\\
&= \frac{1}{2}f^2x\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d+c^2dx^2}\,(\cdots\\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{2bcfgx^3\sqrt{d\cdots}}{9\sqrt{1+c\cdots}}\\
&= -\frac{2bfgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcf^2x^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bg^2x^2\sqrt{d+c\cdots}}{16c\sqrt{1+c\cdots}}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 301, normalized size = 0.70

$$\frac{48ac\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,(12c^2f^2x+3g^2x(1+2c^2x^2)+16f(g+c^2gx^2))-256bcfg\sqrt{d+c^2dx^2}\,\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\right)+144a\sqrt{d}\,(2cf-g)(2cf+g)\sqrt{1+c^2x^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)-144bc^2f^2\sqrt{d+c^2dx^2}\,\left(\cosh\left(2\sinh^{-1}(cx)\right)-2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)-9bg^2\sqrt{d+c^2dx^2}\,\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{1152c^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(12*c^2*f^2*x + 3*g^2*x*(1 +
2*c^2*x^2) + 16*f*(g + c^2*g*x^2)) - 256*b*c*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x
 + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]) + 144*a*Sqrt[d]*(2*c*f - g
```

```
)*(2*c*f + g)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] -
144*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(A
rcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 9*b*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSi
nh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]))/(1
152*c^3*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(373) = 746$.
time = 4.67, size = 921, normalized size = 2.14

method	result
default	$\frac{a\,g^{2} x \left(c^{2} d\,x^{2}+d \right)^{\frac{3}{2}}}{4 c^{2} d}-\frac{a\,g^{2} x \sqrt{c^{2} d\,x^{2}+d}}{8 c^{2}}-\frac{a\,g^{2} d \ln \left(\frac{x\,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{8 c^{2} \sqrt{c^{2} d}}+\frac{2 a f g \left(c^{2} d\,x^{2}+d \right)^{\frac{3}{2}}}{3 c^{2} d}+\frac{a\,f^{2} x \sqrt{c^{2} d}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4*a*g^2*x*(c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a*g^2/c^2*x*(c^2*d*x^2+d)^(1/2)-1
/8*a*g^2/c^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+
2/3*a*f*g*(c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f^2*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f
^2*d*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/16*(d
*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2*(4*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3+1/
256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3
+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+4*arcsinh(c*x
))/c^3/(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2
)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(-1+3*arcsinh(c*x))/c^2/
(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/
2)+2*c*x+(c^2*x^2+1)^(1/2))*f^2*(2*arcsinh(c*x)-1)/c/(c^2*x^2+1)+1/4*(d*(c^
2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)-1)/c^2/
(c^2*x^2+1)+1/4*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*f*g
*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c
^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f^2*(2*arcsinh(c*x)+1)/c/
(c^2*x^2+1)+1/36*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c
^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(1+3*arcsinh(c*x))/c^2/(c^2*x^2
+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c
^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+4*arcsin
h(c*x))/c^3/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
2*b*f*g*x + b*f^2)*arcsinh(c*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.36 $\int (f+gx)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=227

$$-\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}}-\frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}}-\frac{bcgx^3\sqrt{d+c^2dx^2}}{9\sqrt{1+c^2x^2}}+\frac{1}{2}fx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{g(1+c^2}{}$$

```
[Out] 1/2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/3*g*(c^2*x^2+1)*(a+b*arcsi
nh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1
)^(1/2)-1/4*b*c*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/9*b*c*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+1/4*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+
d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5845, 5838, 5785, 5783, 30, 5798}

$$\frac{1}{2}fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{f\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc\sqrt{c^2x^2+1}}+\frac{g(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{3c^2}-\frac{bcfx^2\sqrt{c^2dx^2+d}}{4\sqrt{c^2x^2+1}}-\frac{bgx\sqrt{c^2dx^2+d}}{3c\sqrt{c^2x^2+1}}-\frac{bcgx^3\sqrt{c^2dx^2+d}}{9\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/3*(b*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (b*c*f*x^2*Sqrt[d
 + c^2*d*x^2])/(4*Sqrt[1 + c^2*x^2]) - (b*c*g*x^3*Sqrt[d + c^2*d*x^2])/(9*Sq
rt[1 + c^2*x^2]) + (f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (g*(1
 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*c^2) + (f*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c*Sqrt[1 + c^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
```

```
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\sqrt{d+c^2dx^2}\int (f+gx)\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{d+c^2dx^2}\int \left(f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)+gx\sqrt{1+\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(f\sqrt{d+c^2dx^2}\right)\int \sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}} + \frac{(g}{}$$

$$= \frac{1}{2}fx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right) + \frac{g(1+c^2x^2)\sqrt{d+c^2}}{}$$

$$= -\frac{bgx\sqrt{d+c^2dx^2}}{3c\sqrt{1+c^2x^2}} - \frac{bcfx^2\sqrt{d+c^2dx^2}}{4\sqrt{1+c^2x^2}} - \frac{bcgx^3\sqrt{d+c^2d}}{9\sqrt{1+c^2x^2}}$$

Mathematica [**A**]
time = 0.80, size = 208, normalized size = 0.92

$$\frac{1}{6}a\sqrt{d+c^2dx^2}\left(\frac{2g}{c^2}+x(3f+2gx)\right) - \frac{bg\sqrt{d+c^2dx^2}\left(3cx+c^3x^3-3(1+c^2x^2)^{3/2}\sinh^{-1}(cx)\right)}{9c^2\sqrt{1+c^2x^2}} + \frac{a\sqrt{d}\,f\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{2c} + \frac{bf\sqrt{d+c^2dx^2}\left(-\cosh\left(2\sinh^{-1}(cx)\right)+2\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)\right)}{8c\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2]*((2*g)/c^2 + x*(3*f + 2*g*x)))/6 - (b*g*Sqrt[d + c^2
*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*ArcSinh[c*x]))/(9*c^2*Sqrt
[1 + c^2*x^2]) + (a*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*
c) + (b*f*Sqrt[d + c^2*d*x^2]*(-Cosh[2*ArcSinh[c*x]] + 2*ArcSinh[c*x]*(ArcS
inh[c*x] + Sinh[2*ArcSinh[c*x]])))/(8*c*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(195) = 390$.
time = 3.44, size = 582, normalized size = 2.56

method	result
default	$\frac{ag(c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+\frac{afx\sqrt{c^2dx^2+d}}{2}+\frac{afd\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{2\sqrt{c^2d}}+b\left(\frac{\sqrt{d(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{4\sqrt{c^2x^2+1}\,c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*g*(c^2*d*x^2+d)^(3/2)/c^2/d+1/2*a*f*x*(c^2*d*x^2+d)^(1/2)+1/2*a*f*d*l
n(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^2*x
```

```
^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2+1/72*(d*(c^2*x^2+1))^(1/2
)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+
1)*g*(-1+3*arcsinh(c*x))/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*
x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(2*arcsinh(c*x)-
1)/c/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1
)*g*(arcsinh(c*x)-1)/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^
2*x^2+1)^(1/2)*c*x+1)*g*(arcsinh(c*x)+1)/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1
))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*
(2*arcsinh(c*x)+1)/c/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c
^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(1+3*arcsinh
(c*x))/c^2/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\left(c^{2}x^{2}+1\right)}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2), x)
```

3.37 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=664

$$\frac{a\sqrt{d+c^2dx^2}}{g}-\frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}}+\frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g}-\frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}}-\frac{\left(1+\frac{c^2f^2}{g^2}\right)}{}$$

```
[Out] a*(c^2*d*x^2+d)^(1/2)/g+b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g-b*c*x*(c^2*d*x
^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^
(1/2)/b/g/(c^2*x^2+1)^(1/2)-1/2*(1+c^2*f^2/g^2)*(a+b*arcsinh(c*x))^2*(c^2*d
*x^2+d)^(1/2)/b/c/(g*x+f)/(c^2*x^2+1)^(1/2)-a*arctanh((-c^2*f*x+g)/(c^2*f^2
+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/
(c^2*x^2+1)^(1/2)+b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f
^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/
2)-b*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))
*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+b*polylog(2,
-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(
c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2
))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*f^2+g^2)^(1/2)*(c^2*d*x^2+d)^(1/2)/g^2
/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*(c^2*d*x^2+d)
^(1/2)/b/c/(g*x+f)
```

Rubi [A]
time = 1.16, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 20, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5845, 5839, 697, 5835, 6874, 267, 739, 212, 5856, 1668, 12, 5855, 5798, 8, 5843, 3403, 2296, 2221, 2317, 2438}

$$-\frac{\sqrt{c^2dx^2+d}\left(\frac{c^2f^2}{g^2}+1\right)\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2x^2+1}\,(f+gx)}+\frac{\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)}-\frac{cx\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{c^2x^2+1}}-\frac{a\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\tanh^{-1}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}+\frac{a\sqrt{c^2dx^2+d}}{g}+\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\sqrt{c^2f^2+g^2}\,\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{g^2\sqrt{c^2x^2+1}}-\frac{bcx\sqrt{c^2dx^2+d}}{g\sqrt{c^2x^2+1}}+\frac{b\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{g}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*Sqrt[d + c^2*d*x^2])/g - (b*c*x*Sqrt[d + c^2*d*x^2])/(g*Sqrt[1 + c^2*x^2
]) + (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g - (c*x*Sqrt[d + c^2*d*x^2]*(a +
 b*ArcSinh[c*x])^2)/(2*b*g*Sqrt[1 + c^2*x^2]) - ((1 + (c^2*f^2)/g^2)*Sqrt[d
 + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f + g*x)*Sqrt[1 + c^2*x^2]) +
 (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(f +
 g*x)) - (a*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(
Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[
c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g
^2]*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt
```

```
[c^2*f^2 + g^2])])/(g^2*Sqrt[1 + c^2*x^2]) + (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d
+ c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])
/(g^2*Sqrt[1 + c^2*x^2]) - (b*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]*PolyL
og[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^2*Sqrt[1 + c^2
*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
```

```
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
```

```
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d+c^2dx^2}\,\int \frac{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)} - \frac{\sqrt{d+c^2dx^2}\,\int \frac{(-g+\cdots}{2bc\cdots}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\,(a\cdots}{2bc(f+gx)\sqrt{1+\cdots}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\,(a\cdots}{2bc(f+gx)\sqrt{1+\cdots}}$$

$$= -\frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\,(a\cdots}{2bc(f+gx)\sqrt{1+\cdots}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\cdots}{2b\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{cx\sqrt{d+c^2dx^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bg\sqrt{1+c^2x^2}} - \frac{\left(1+\frac{c^2f^2}{g^2}\right)\cdots}{2b\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\sqrt{d+c^2dx^2}\,(a+b\cdots}{2bg\sqrt{1+c^2\cdots}}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\cdots}{\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\cdots}{\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\cdots}{\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\cdots}{\cdots}$$

$$= \frac{a\sqrt{d+c^2dx^2}}{g} - \frac{bcx\sqrt{d+c^2dx^2}}{g\sqrt{1+c^2x^2}} + \frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g} - \frac{cx\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.42, size = 1353, normalized size = 2.04

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d + c^2*d*x^2] + 2*a*Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Log[f + g*x] -
 2*a*c*Sqrt[d]*f*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 2*a*Sqrt[d]*Sqr
t[c^2*f^2 + g^2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d +
 c^2*d*x^2]] + b*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1 + c^2*x^2] + 2*g*Ar
cSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2*(c^2*f^2 + g^2)*((
(-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/Sqrt[
c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*
I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*Ar
cTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]
] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi
+ (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(
c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (Ar
cCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)
*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*
c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt
[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2
])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f
^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2
*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi
 + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi
+ (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2
])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))
/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
)] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^
2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c
^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))])))/Sqrt[-(c^2*f^2) - g^2]
))/Sqrt[1 + c^2*x^2]))/(2*g^2)
```

Maple [A]
time = 2.80, size = 992, normalized size = 1.49

method	result

default	$\frac{a\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^{2} f^{2}+g^{2}\right)}{g^{2}}}}{g}-\frac{a\,c^{2} d f \ln\left(\frac{-\frac{c^{2} d f}{g}+\left(x+\frac{f}{g}\right) c^{2} d}{\sqrt{c^{2} d}}+\sqrt{\left(x+\frac{f}{g}\right)^{2} c^{2} d-\frac{2 c^{2} d f \left(x+\frac{f}{g}\right)}{g}}\right.}{g^{2}\sqrt{c^{2} d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)-a/g^2*c
^2*d*f*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^3*d/(d*(c^2*f^2+
g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2
+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^
(1/2))/(x+f/g))*c^2*f^2-a/g*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+
g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d
-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(d*(c^2*x^2
+1))^(1/2)/(c^2*x^2+1)^(1/2)*f*arcsinh(c*x)^2*c/g^2+b*(d*(c^2*x^2+1))^(1/2)
/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2-b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/
2)/g*x*c+b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)/g*arcsinh(c*x)+b*(d*(c^2*x^2+1
))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x)*ln((-(c*x+(
c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*
(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*arcsinh(c*x
)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)
^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g
^2*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+
g^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] -(c*sqrt(d)*f*arcsinh(c*x)/g^2 - sqrt(c^2*d*f^2/g^2 + d)*arcsinh(c*f*x/abs(
g*x + f) - g/(c*abs(g*x + f)))/g - sqrt(c^2*d*x^2 + d)/g)*a + b*integrate(s
qrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(g*x + f), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\operatorname{asinh}(c\,x)\right)\,\sqrt{d\,c^2\,x^2+d}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x), x)
```

3.38 $\int \frac{\sqrt{d+c^2dx^2}\,(a+b\sinh^{-1}(cx))}{(f+gx)^2}\,dx$

Optimal. Leaf size=781

$$-\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)}-\frac{b\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g(f+gx)}+\frac{ac^3f^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}+\frac{bc^3f^2\sqrt{d+c^2dx^2}\,\sinh^{-1}(cx)^2}{2g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}}$$

```
[Out] -a*(c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g/(g*x+
f)+a*c^3*f^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(c^2*x^2+1)
^(1/2)+1/2*b*c^3*f^2*arcsinh(c*x)^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)/(
c^2*x^2+1)^(1/2)-1/2*(-c^2*f*x+g)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2
)/b/c/(c^2*f^2+g^2)/(g*x+f)^2/(c^2*x^2+1)^(1/2)+b*c*ln(g*x+f)*(c^2*d*x^2+d)
^(1/2)/g^2/(c^2*x^2+1)^(1/2)+a*c^2*f*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/
2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+
1)^(1/2)-b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+
b*c^2*f*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2
)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)-b*c^2*f*p
olylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d
)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+(
c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)+1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2
)*(c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2
```

Rubi [A]

time = 1.76, antiderivative size = 781, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 22, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {5845, 5839, 37, 5834, 12, 1665, 858, 221, 739, 212, 5856, 5855, 5783, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{\sqrt{c^2dx^2+d}\,(g-c^2fx)^2\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2x^2+1}\,(c^2f^2+g^2)\,(f+gx)^2}+\frac{\sqrt{c^2x^2+1}\,\sqrt{c^2dx^2+d}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)^2}+\frac{ac^2f\sqrt{c^2dx^2+d}\,\tanh^{-1}\left(\frac{g-c^2fx}{\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}-\frac{a\sqrt{c^2dx^2+d}}{g(f+gx)}+\frac{ac^3f^2\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{g^2\sqrt{c^2x^2+1}\,(c^2f^2+g^2)}-\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}+\frac{bc^2f\sqrt{c^2dx^2+d}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{g^2\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}-\frac{bc^2f\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{g^2\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}+\frac{bc^2f\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{g^2\sqrt{c^2x^2+1}\,\sqrt{c^2f^2+g^2}}+\frac{bc\sqrt{c^2dx^2+d}\,\log(f+gx)}{g^2\sqrt{c^2x^2+1}}-\frac{b\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)}{g(f+gx)}+\frac{bc^3f^2\sqrt{c^2dx^2+d}\,\sinh^{-1}(cx)^2}{2g^2\sqrt{c^2x^2+1}\,(c^2f^2+g^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d + c^2*d*x^2])/(g*(f + g*x))) - (b*Sqrt[d + c^2*d*x^2]*ArcSinh[c
*x])/(g*(f + g*x)) + (a*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/(g^2*(c^2
*f^2 + g^2)*Sqrt[1 + c^2*x^2]) + (b*c^3*f^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x
]^2)/(2*g^2*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]) - ((g - c^2*f*x)^2*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*(c^2*f^2 + g^2)*(f + g*x)^2*Sqrt[
1 + c^2*x^2]) + (Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])
^2)/(2*b*c*(f + g*x)^2) + (a*c^2*f*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x
)/(Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1
```

```
 + c^2*x^2]) - (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh
[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^
2*x^2]) + (b*c^2*f*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]
*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2
]) + (b*c*Sqrt[d + c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[1 + c^2*x^2]) - (b*c^
2*f*Sqrt[d + c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
 + g^2]))])/(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]) + (b*c^2*f*Sqrt[d +
 c^2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/
(g^2*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5834

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x], x]] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && L
tQ[m + p + 1, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d+c^2dx^2}\int \frac{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{(f+gx)^2}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc(f+gx)^2} - \frac{\sqrt{d+c^2dx^2}\int \frac{(-2g+2}{}}{2bc\sqrt{1+}}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c}}{2bc}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c}}{2bc}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c}}{2bc}$$

$$= -\frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{1+c^2x^2}\sqrt{d+c}}{2bc}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{\left(g-c^2fx\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\left(c^2f^2+g^2\right)(f+gx)^2\sqrt{1+c^2x^2}} + \frac{\sqrt{}}{}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g^2\left(c^2f^2+g^2\right)\sqrt{1+c^2x^2}} + \frac{bc^3f^2\sqrt{d+c^2dx}}{2g^2\left(c^2f^2+g^2\right)}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sin}{g^2\left(c^2f^2+g^2\right)\sqrt{1+}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sin}{g^2\left(c^2f^2+g^2\right)\sqrt{1+}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sin}{g^2\left(c^2f^2+g^2\right)\sqrt{1+}}$$

$$= -\frac{a\sqrt{d+c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)}{g(f+gx)} + \frac{ac^3f^2\sqrt{d+c^2dx^2}\sin}{g^2\left(c^2f^2+g^2\right)\sqrt{1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.30, size = 1384, normalized size = 1.77

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(f + g*x)^2,x]
```

```
[Out] ((-2*a*g*Sqrt[d + c^2*d*x^2])/(f + g*x) - (2*a*c^2*Sqrt[d]*f*Log[f + g*x])/
Sqrt[c^2*f^2 + g^2] + 2*a*c*Sqrt[d]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]
] + (2*a*c^2*Sqrt[d]*f*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sq
rt[d + c^2*d*x^2]])/Sqrt[c^2*f^2 + g^2] + b*c*Sqrt[d + c^2*d*x^2]*((-2*g*Ar
cSinh[c*x])/(c*f + c*g*x) + ArcSinh[c*x]^2/Sqrt[1 + c^2*x^2] + ((2*I)*c*f*P
i*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])/(Sqrt[c^2*f
^2 + g^2]*Sqrt[1 + c^2*x^2]) + (2*Log[1 + (g*x)/f])/Sqrt[1 + c^2*x^2] + (2*
c*f*(2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*
x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*x])*ArcTanh[((c*f -
 I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (ArcCos[(
(-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSi
nh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*Sqrt[-(c^2*f^2) - g^
2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] + (ArcCos[((-I)*c*
f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[
-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])
/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f
^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[((-I)*c*f)/g] + (2*
I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) -
 g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2) - g^2])*(1 + I*Cot
[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Co
t[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[(
(c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[
((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + Cot[(Pi + (2*I)*ArcS
inh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSi
nh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2) - g^2])*(I*c*f + g
 - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f +
g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - PolyLog[
2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqrt[-(c^2*f^2) - g^2]
*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2
]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))])))/(Sqrt[-(c^2*f^2) - g^2]*Sqrt[1 + c
^2*x^2])))/(2*g^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1813 vs. $2(745) = 1490$.
time = 6.06, size = 1814, normalized size = 2.32

method	result	size
default	Expression too large to display	1814

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBOS
E)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g
)+d*(c^2*f^2+g^2)/g^2)^(1/2)+a/g^2*c^4*f^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+(
x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2
+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+a/g^3*c^4*f^3/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g
^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+
g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(
1/2))/(x+f/g))+a/g*c^2*f/(c^2*f^2+g^2)*d/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*
d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f
/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/
(c^2*f^2+g^2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/
2)*x+a*c^2/(c^2*f^2+g^2)*d*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+
f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)+
1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)*arcsinh(c*x)^2*c/g^2+b*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g^2/(g*x+f)*x^3*c^4*f-b*(d*(c^2*x^
2+1))^(1/2)*arcsinh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(d*(c^2*x^2+1))^(1/2)*arcsin
h(c*x)/(c^2*x^2+1)/g/(g*x+f)*x^2*c^2+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(
c^2*x^2+1)^(1/2)/g/(g*x+f)*x*c+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^
2+1)/g^2/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)^(
1/2)/g^2/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/(c^2*x^2+1)/g/(g*
x+f)-b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*
f*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*
f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^2*x^2+1)^(1/2)/g^2/(
c^2*f^2+g^2)^(1/2)*f*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^
2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2)*c^3/(c^2*x
^2+1)^(1/2)/g^2/(c^2*f^2+g^2)*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^
2*x^2+1)^(1/2))-g)*f^2-2*b*(d*(c^2*x^2+1))^(1/2)*c^3/(c^2*x^2+1)^(1/2)/g^2/
(c^2*f^2+g^2)*ln(c*x+(c^2*x^2+1)^(1/2))*f^2-b*(d*(c^2*x^2+1))^(1/2)*c^2/(c^
2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*f*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-
c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/2
)*c^2/(c^2*x^2+1)^(1/2)/g^2/(c^2*f^2+g^2)^(1/2)*f*dilog(((c*x+(c^2*x^2+1)^(
1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1
))^(1/2)*c/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2
*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)-2*b*(d*(c^2*x^2+1))^(1/2)*c/(c^2*x^2+1)^(1/
2)/(c^2*f^2+g^2)*ln(c*x+(c^2*x^2+1)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="ma
xima")
```

```
[Out] -(c^2*d*f*arcsinh(c*f*x/(g*abs(x + f/g)) - 1/(c*abs(x + f/g)))/(sqrt(c^2*d*
f^2/g^2 + d)*g^3) - c*sqrt(d)*arcsinh(c*x)/g^2 + sqrt(c^2*d*x^2 + d)/(g^2*x
 + f*g))*a + b*integrate(sqrt(c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 + 1))/(
g^2*x^2 + 2*f*g*x + f^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(a+b\operatorname{asinh}\left(cx\right)\right)}{\left(f+gx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*(c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(d*(c**2*x**2 + 1))*(a + b*asinh(c*x))/(f + g*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\sqrt{d\,c^2\,x^2+d}}{{(f+g\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

3.39 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=918

$$-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{3bdfg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{2bcdf^2gx^3\sqrt{d}}{5\sqrt{1+c}}$$

```
[Out] 3/8*d*f^3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/16*d*f*g^2*x*(a+b*arcs
inh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d
*x^2+d)^(1/2)+1/4*d*f^3*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2
)+1/2*d*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+3/5*d*
f^2*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-1/5*d*g^3*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*(c^2*x^2+1
)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*(c^2*d*x^2+d
)^(1/2)/c/(c^2*x^2+1)^(1/2)+2/35*b*d*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2
+1)^(1/2)-5/16*b*c*d*f^3*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/32*b*d
*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/5*b*c*d*f^2*g*x^3*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(
c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
-7/32*b*c*d*f*g^2*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/25*b*c^3*d*f^
2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-8/175*b*c*d*g^3*x^5*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/12*b*c^3*d*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^
2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3
/16*d*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-3/
32*d*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200, 5808, 5806, 5812, 272, 45, 5804, 12, 380}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d*f^2*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) + (2*b*d*g^3*x
*Sqrt[d + c^2*d*x^2])/(35*c^3*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^3*x^2*Sqrt[d
+ c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (3*b*d*f*g^2*x^2*Sqrt[d + c^2*d*x^2]
)/(32*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d*f^2*g*x^3*Sqrt[d + c^2*d*x^2])/(5*Sqr
t[1 + c^2*x^2]) - (b*d*g^3*x^3*Sqrt[d + c^2*d*x^2])/(105*c*Sqrt[1 + c^2*x^2
]) - (b*c^3*d*f^3*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (7*b*c*
d*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(32*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d*f^2*g*
```

```
x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (8*b*c*d*g^3*x^5*Sqrt[d +
 c^2*d*x^2])/(175*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*g^2*x^6*Sqrt[d + c^2*d*x^
2])/(12*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^3*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt
[1 + c^2*x^2]) + (3*d*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (
3*d*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(16*c^2) + (3*d*f*g^2
*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f^3*x*(1 + c^2*x^2)*S
qrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*f*g^2*x^3*(1 + c^2*x^2)*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/2 + (3*d*f^2*g*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) - (d*g^3*(1 + c^2*x^2)^2*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^4) + (d*g^3*(1 + c^2*x^2)^3*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (3*d*f^3*Sqrt[d + c^2*d*x^2]
*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2]) - (3*d*f*g^2*Sqrt[d + c
^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
```

```
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
```

```
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^3\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}\right.}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(f^3(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\right.}{}$$

$$=\frac{\left(df^3\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,d}{\sqrt{1+c^2x^2}}$$

$$=\frac{1}{4}df^3x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{2}df$$

$$=\frac{3}{8}df^3x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d+}$$

$$=-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^3x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcd}{}$$

$$=-\frac{3bdf^2gx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}+\frac{2bdg^3x\sqrt{d+c^2dx^2}}{35c^3\sqrt{1+c^2x^2}}-\frac{5bcdf^3}{16}$$

Mathematica [A]
time = 2.64, size = 779, normalized size = 0.85

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (5040*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(-32*g^3 + c^2*g*(336*f^2 +
 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*x + 70*f*g^2*x^2 +
20*g^3*x^3) + 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2*x^2 + 64*g^3*x^3))
 - 940800*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x
^2)^(3/2)*ArcSinh[c*x]) - 37632*b*c^2*d*f^2*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30
 + 5*c^2*x^2 + 9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)
*ArcSinh[c*x]) - 12544*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 +
```

```
9*c^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x])
- 256*b*d*g^3*Sqrt[d + c^2*d*x^2]*(c*x*(840 - 140*c^2*x^2 + 63*c^4*x^4 + 22
5*c^6*x^6) - 105*Sqrt[1 + c^2*x^2]*(8 - 4*c^2*x^2 + 3*c^4*x^4 + 15*c^6*x^6)
*ArcSinh[c*x]) + 529200*a*c*d^(3/2)*f*(2*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*L
og[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] - 352800*b*c^3*d*f^3*Sqrt[d + c^2*d
*x^2]*(Cosh[2*ArcSinh[c*x]] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh
[c*x]])) - 22050*b*c^3*d*f^3*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4
*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) - 66150*b*c*d*f*g^2*S
qrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x
]*Sinh[4*ArcSinh[c*x]]) + 3675*b*c*d*f*g^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[
c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cosh[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSin
h[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*ArcSinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]]
 + Sinh[6*ArcSinh[c*x]])))/(2822400*c^4*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. $2(802) = 1604$.
time = 4.48, size = 2101, normalized size = 2.29

method	result	size
default	Expression too large to display	2101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/7*a*g^3*x^2*(c^2*d*x^2+d)^(5/2)/c^2/d-2/35*a*g^3/d/c^4*(c^2*d*x^2+d)^(5/2
)+1/2*a*f*g^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/8*a*f*g^2/c^2*x*(c^2*d*x^2+d)^(
3/2)-3/16*a*f*g^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-3/16*a*f*g^2/c^2*d^2*ln(x*c^2
*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3/5*a*f^2*g/c^2/d*(c^2*
d*x^2+d)^(5/2)+1/4*a*f^3*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f^3*d*x*(c^2*d*x^2+d)^
(1/2)+3/8*a*f^3*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(
1/2)+b*(3/32*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2*(2*c^2*f^2-g^2)*d/(c^2*
x^2+1)^(1/2)/c^3+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8+64*(c^2*x^2+1)^(1
/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x
^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(-1+7*arcsinh
(c*x))*d/c^4/(c^2*x^2+1)+1/768*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*(c^2*x^
2+1)^(1/2)*c^6*x^6+64*c^5*x^5+48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3+18*c^
2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*f*g^2*(-1+6*arcsinh(c*x))*
d/c^3/(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(
1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+
1)^(1/2)*c*x+1)*g*(60*arcsinh(c*x)*c^2*f^2-12*c^2*f^2+5*arcsinh(c*x)*g^2-g^
2)*d/c^4/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*(c^2*x^2+1)^(
1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2)
)*f*(8*arcsinh(c*x)*c^2*f^2-2*c^2*f^2+12*arcsinh(c*x)*g^2-3*g^2)*d/c^3/(c^2
*x^2+1)+1/384*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+
```

```
5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(36*arcsinh(c*x)*c^2*f^2-12*c^2*f^2-
3*arcsinh(c*x)*g^2+g^2)*d/c^4/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^
3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(32*arcsinh(c*
x)*c^2*f^2-16*c^2*f^2-6*arcsinh(c*x)*g^2+3*g^2)*d/c^3/(c^2*x^2+1)+3/128*(d*
(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g*(8*arcsinh(c*x)*c^2*
f^2-8*c^2*f^2-arcsinh(c*x)*g^2+g^2)*d/c^4/(c^2*x^2+1)+3/128*(d*(c^2*x^2+1))
^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*g*(8*arcsinh(c*x)*c^2*f^2+8*c^2*f^
2-arcsinh(c*x)*g^2-g^2)*d/c^4/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^
3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*(32*arcsinh(c*
x)*c^2*f^2+16*c^2*f^2-6*arcsinh(c*x)*g^2-3*g^2)*d/c^3/(c^2*x^2+1)+1/384*(d*
(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*
x^2+1)^(1/2)*c*x+1)*g*(36*arcsinh(c*x)*c^2*f^2+12*c^2*f^2-3*arcsinh(c*x)*g^
2-g^2)*d/c^4/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*(c^2*x^2+
1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(
1/2))*f*(8*arcsinh(c*x)*c^2*f^2+2*c^2*f^2+12*arcsinh(c*x)*g^2+3*g^2)*d/c^3/
(c^2*x^2+1)+1/3200*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x
^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/
2)*c*x+1)*g*(60*arcsinh(c*x)*c^2*f^2+12*c^2*f^2+5*arcsinh(c*x)*g^2+g^2)*d/c
^4/(c^2*x^2+1)+1/768*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*(c^2*x^2+1)^(1/2)
*c^6*x^6+64*c^5*x^5-48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3-18*c^2*x^2*(c^2
*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(1+6*arcsinh(c*x))*d/c^3/(c^2*
x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*(c^2*x^2+1)^(1/2)*x^7*c^
7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1)^(1/2
)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(1+7*arcsinh(c*x))*d/c^
4/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 + 3*a*d*f^2*g*x + a*d*f^3 +
 (3*a*c^2*d*f^2*g + a*d*g^3)*x^3 + (a*c^2*d*f^3 + 3*a*d*f*g^2)*x^2 + (b*c^2
*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 + 3*b*d*f^2*g*x + b*d*f^3 + (3*b*c^2*d*f^2
*g + b*d*g^3)*x^3 + (b*c^2*d*f^3 + 3*b*d*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2
*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.40 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{3/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=651

$$-\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bdg^2x^2\sqrt{d+c^2dx^2}}{32c\sqrt{1+c^2x^2}}-\frac{4bcdfgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3df^2x^4\sqrt{d+}}{16\sqrt{1+c^2}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/16*d*g^2*x*(a+b*arcsin
h(c*x))*(c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/4*d*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/
6*d*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/5*d*f*g*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*x*(c^2*d
*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-5/16*b*c*d*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c
^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-4/
15*b*c*d*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^2*x^4
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-7/96*b*c*d*g^2*x^4*(c^2*d*x^2+d)^(1/
2)/(c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-1/36*b*c^3*d*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+3/16*d*f^2*
(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-1/32*d*g^2*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200, 5808, 5806, 5812}

$$\frac{3}{8}df^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}df^2x(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{2dfg(c^2x^2+1)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^2}+\frac{dg^2x\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{16c^2}+\frac{1}{8}dg^2x^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dg^2x^3(c^2x^2+1)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)-\frac{dg^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc^3\sqrt{c^2x^2+1}}-\frac{5bcdf^2x^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{2bdfgx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}-\frac{4bcdfgx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bdg^2x^2\sqrt{c^2dx^2+d}}{32c\sqrt{c^2x^2+1}}-\frac{7bcdg^2x^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{bc^3df^2x^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{2bc^3dfgx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}-\frac{bc^3dg^2x^6\sqrt{c^2dx^2+d}}{36\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d*f*g*x*Sqrt[d + c^2*d*x^2])/(5*c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f^2*x
^2*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (b*d*g^2*x^2*Sqrt[d + c^2*
d*x^2])/(32*c*Sqrt[1 + c^2*x^2]) - (4*b*c*d*f*g*x^3*Sqrt[d + c^2*d*x^2])/(1
5*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f^2*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt[1 + c
^2*x^2]) - (7*b*c*d*g^2*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (
2*b*c^3*d*f*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2]) - (b*c^3*d*g^
2*x^6*Sqrt[d + c^2*d*x^2])/(36*Sqrt[1 + c^2*x^2]) + (3*d*f^2*x*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSi
nh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8
 + (d*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*
g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (2*d*f*
g*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*
f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

```
- (d*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c^3*Sqrt[1 + c^2
*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
```

```
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)^2\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int\left(f^2(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+\right.}{}$$

$$= \frac{\left(df^2\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{4}df^2x\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{6}dg^2$$

$$= \frac{3}{8}df^2x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d+c^2d}$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{4bcdfg}{15}$$

$$= -\frac{2bdfgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdf^2x^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bdg^2x^2}{32c}$$

Mathematica [A]
time = 1.46, size = 546, normalized size = 0.84

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (240*a*c*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2]*(96*f*g*(1 + c^2*x^2)^2 +
30*c^2*f^2*x*(5 + 2*c^2*x^2) + 5*g^2*x*(3 + 14*c^2*x^2 + 8*c^4*x^4)) - 1280
0*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(3*c*x + c^3*x^3 - 3*(1 + c^2*x^2)^(3/2)*Ar
cSinh[c*x]) - 512*b*c*d*f*g*Sqrt[d + c^2*d*x^2]*(c*x*(-30 + 5*c^2*x^2 + 9*c
^4*x^4) - 15*Sqrt[1 + c^2*x^2]*(-2 + c^2*x^2 + 3*c^4*x^4)*ArcSinh[c*x]) + 3
```

```
600*a*d^(3/2)*(6*c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[
d + c^2*d*x^2]] - 7200*b*c^2*d*f^2*Sqrt[d + c^2*d*x^2]*(Cosh[2*ArcSinh[c*x]
] - 2*ArcSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]])) - 450*b*c^2*d*f^2
*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c
*x]*Sinh[4*ArcSinh[c*x]]) - 450*b*d*g^2*Sqrt[d + c^2*d*x^2]*(8*ArcSinh[c*x]
^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*ArcSinh[c*x]]) + 25*b*d*g
^2*Sqrt[d + c^2*d*x^2]*(72*ArcSinh[c*x]^2 + 18*Cosh[2*ArcSinh[c*x]] + 9*Cos
h[4*ArcSinh[c*x]] - 2*Cosh[6*ArcSinh[c*x]] + 12*ArcSinh[c*x]*(-3*Sinh[2*Arc
Sinh[c*x]] - 3*Sinh[4*ArcSinh[c*x]] + Sinh[6*ArcSinh[c*x]])))/(57600*c^3*Sq
rt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. $2(567) = 1134$.
time = 5.72, size = 1585, normalized size = 2.43

method	result	size
default	Expression too large to display	1585

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*a*g^2*x*(c^2*d*x^2+d)^(5/2)/c^2/d-1/24*a*g^2/c^2*x*(c^2*d*x^2+d)^(3/2)-
1/16*a*g^2/c^2*d*x*(c^2*d*x^2+d)^(1/2)-1/16*a*g^2/c^2*d^2*ln(x*c^2*d/(c^2*d
)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2/5*a*f*g/c^2/d*(c^2*d*x^2+d)^(5
/2)+1/4*a*f^2*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f^2*d*x*(c^2*d*x^2+d)^(1/2)+3/8*a
*f^2*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/3
2*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2*(6*c^2*f^2-g^2)*d/(c^2*x^2+1)^(1/2)/
c^3+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*(c^2*x^2+1)^(1/2)*c^6*x^6+6
4*c^5*x^5+48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1
/2)+6*c*x+(c^2*x^2+1)^(1/2))*g^2*(-1+6*arcsinh(c*x))*d/c^3/(c^2*x^2+1)+1/40
0*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4
+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(-1
+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8
*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^
2*x^2+1)^(1/2))*(8*arcsinh(c*x)*c^2*f^2-2*c^2*f^2+4*arcsinh(c*x)*g^2-g^2)*d
/c^3/(c^2*x^2+1)+1/48*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*
x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(-1+3*arcsinh(c*x))*d/c^2/
(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1
/2)+2*c*x+(c^2*x^2+1)^(1/2))*(32*arcsinh(c*x)*c^2*f^2-16*c^2*f^2-2*arcsinh(
c*x)*g^2+g^2)*d/c^3/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2
+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)-1)*d/c^2/(c^2*x^2+1)+1/8*(d*(c^2*x^2+1))
^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)+1)*d/c^2/(c^2*x^
2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c
*x-(c^2*x^2+1)^(1/2))*(32*arcsinh(c*x)*c^2*f^2+16*c^2*f^2-2*arcsinh(c*x)*g^
```

```
2-g^2)*d/c^3/(c^2*x^2+1)+1/48*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1
)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(1+3*arcsinh(c*x))
*d/c^2/(c^2*x^2+1)+1/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/
2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*
(8*arcsinh(c*x)*c^2*f^2+2*c^2*f^2+4*arcsinh(c*x)*g^2+g^2)*d/c^3/(c^2*x^2+1)
+1/400*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^
4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*f*
g*(1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7
*x^7-32*(c^2*x^2+1)^(1/2)*c^6*x^6+64*c^5*x^5-48*(c^2*x^2+1)^(1/2)*c^4*x^4+3
8*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+6*ar
csinh(c*x))*d/c^3/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 + 2*a*d*f*g*x + a*d*f^2 + (a*
c^2*d*f^2 + a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 + 2*b*d*f*g
*x + b*d*f^2 + (b*c^2*d*f^2 + b*d*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 +
d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.41 $\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=353

$$-\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcdgx^3\sqrt{d+c^2dx^2}}{15\sqrt{1+c^2x^2}}-\frac{bc^3dfx^4\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{bc^3dgx^5\sqrt{d+c^2dx^2}}{25\sqrt{1+c^2x^2}}$$

```
[Out] 3/8*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(c^2*x^2+1)*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/5*d*g*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))
*(c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2
)-5/16*b*c*d*f*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/15*b*c*d*g*x^3*(
c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(c^2*d*x^2+d)^(1/2)
/(c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)
+3/16*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 5798, 200}

$$\frac{3}{8}dfx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{3df\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{16bc\sqrt{c^2x^2+1}}+\frac{dg\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{5c^2}-\frac{5bcdfx^2\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{bdgx\sqrt{c^2dx^2+d}}{5c\sqrt{c^2x^2+1}}-\frac{2bcdgx^3\sqrt{c^2dx^2+d}}{15\sqrt{c^2x^2+1}}-\frac{bc^3dfx^4\sqrt{c^2dx^2+d}}{16\sqrt{c^2x^2+1}}-\frac{bc^3dgx^5\sqrt{c^2dx^2+d}}{25\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/5*(b*d*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (5*b*c*d*f*x^2*S
qrt[d + c^2*d*x^2])/(16*Sqrt[1 + c^2*x^2]) - (2*b*c*d*g*x^3*Sqrt[d + c^2*d*
x^2])/(15*Sqrt[1 + c^2*x^2]) - (b*c^3*d*f*x^4*Sqrt[d + c^2*d*x^2])/(16*Sqrt
[1 + c^2*x^2]) - (b*c^3*d*g*x^5*Sqrt[d + c^2*d*x^2])/(25*Sqrt[1 + c^2*x^2])
 + (3*d*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (d*f*x*(1 + c^2*x
^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/4 + (d*g*(1 + c^2*x^2)^2*Sqrt
[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*c^2) + (3*d*f*Sqrt[d + c^2*d*x^2]*
(a + b*ArcSinh[c*x])^2)/(16*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\int (f+gx)\left(d+c^2dx^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx &= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx\right.}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(f(1+c^2x^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)+g\right.}{\sqrt{1+c^2x^2}} \\
&= \frac{\left(df\sqrt{d+c^2dx^2}\right)\int \left(1+c^2x^2\right)^{3/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}} + \\
&= \frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{dg(1+}{} \\
&= \frac{3}{8}dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{1}{4}dfx\left(1+c^2x^2\right)\sqrt{} \\
&= -\frac{bdgx\sqrt{d+c^2dx^2}}{5c\sqrt{1+c^2x^2}}-\frac{5bcdfx^2\sqrt{d+c^2dx^2}}{16\sqrt{1+c^2x^2}}-\frac{2bcdgx^3\sqrt{}}{15\sqrt{1+}}
\end{aligned}
$$

Mathematica [A]
time = 0.82, size = 392, normalized size = 1.11

$$
\frac{-640bcdgx(3+c^2x^2)\sqrt{d+c^2dx^2}-128bc^3dgx^3(5+3c^2x^2)\sqrt{d+c^2dx^2}+240ad\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}\left(8g(1+c^2x^2)^2+5c^2fx(5+2c^2x^2)\right)+3200bdg(1+c^2x^2)^{3/2}\sqrt{d+c^2dx^2}\sinh^{-1}(cx)+640bdg(1+c^2x^2)^{3/2}(-2+3c^2x^2)\sqrt{d+c^2dx^2}\sinh^{-1}(cx)-1200bcdf\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)+3600acd^{3/2}f\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+2400bcdf\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(\sinh^{-1}(cx)+\sinh\left(2\sinh^{-1}(cx)\right)\right)-75bcdf\sqrt{d+c^2dx^2}\left(8\sinh^{-1}(cx)^2+\cosh\left(4\sinh^{-1}(cx)\right)-4\sinh^{-1}(cx)\sinh\left(4\sinh^{-1}(cx)\right)\right)}{9600c^2\sqrt{1+c^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-640*b*c*d*g*x*(3 + c^2*x^2)*Sqrt[d + c^2*d*x^2] - 128*b*c^3*d*g*x^3*(5 +
3*c^2*x^2)*Sqrt[d + c^2*d*x^2] + 240*a*d*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x
^2]*(8*g*(1 + c^2*x^2)^2 + 5*c^2*f*x*(5 + 2*c^2*x^2)) + 3200*b*d*g*(1 + c^2
*x^2)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] + 640*b*d*g*(1 + c^2*x^2)^(3/2
)*(-2 + 3*c^2*x^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x] - 1200*b*c*d*f*Sqrt[d +
 c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] + 3600*a*c*d^(3/2)*f*Sqrt[1 + c^2*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 2400*b*c*d*f*Sqrt[d + c^2*d*x^2]*Ar
cSinh[c*x]*(ArcSinh[c*x] + Sinh[2*ArcSinh[c*x]]) - 75*b*c*d*f*Sqrt[d + c^2*
```

```
d*x^2]*(8*ArcSinh[c*x]^2 + Cosh[4*ArcSinh[c*x]] - 4*ArcSinh[c*x]*Sinh[4*Arc
Sinh[c*x]]))/(9600*c^2*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(305) = 610$.
time = 3.16, size = 1065, normalized size = 3.02

method	result
default	$\frac{ag(c^2dx^2+d)^{\frac{5}{2}}}{5c^2d} + \frac{afx(c^2dx^2+d)^{\frac{3}{2}}}{4} + \frac{3afdx\sqrt{c^2dx^2+d}}{8} + \frac{3afd^2\ln\left(\frac{xc^2d}{\sqrt{c^2d}}+\sqrt{c^2dx^2+d}\right)}{8\sqrt{c^2d}} + b\left(\frac{3\sqrt{d(c^2}}{1}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*g/c^2/d*(c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x
*(c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/
2))/(c^2*d)^(1/2)+b*(3/16*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsi
nh(c*x)^2*d+1/800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^
5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2
)*c*x+1)*g*(-1+5*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2
)*(8*c^5*x^5+8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(
1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(-1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/96*(d
*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2
*x^2+1)^(1/2)*c*x+1)*g*(-1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*x
^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2)
)*f*(2*arcsinh(c*x)-1)*d/c/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+
(c^2*x^2+1)^(1/2)*c*x+1)*g*(arcsinh(c*x)-1)*d/c^2/(c^2*x^2+1)+1/16*(d*(c^2*
x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*g*(arcsinh(c*x)+1)*d/c^2/(c
^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)
+2*c*x-(c^2*x^2+1)^(1/2))*f*(2*arcsinh(c*x)+1)*d/c/(c^2*x^2+1)+1/96*(d*(c^2
*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+
1)^(1/2)*c*x+1)*g*(1+3*arcsinh(c*x))*d/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1)
)^(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^
2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh(c*x))*d/c/(c^2*x^2+1)+1/
800*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x
^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*g*(1+
5*arcsinh(c*x))*d/c^2/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*g*x^3 + a*c^2*d*f*x^2 + a*d*g*x + a*d*f + (b*c^2*d*g*x^3
+ b*c^2*d*f*x^2 + b*d*g*x + b*d*f)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^2x^2+1\right)\right)^{\frac{3}{2}}(a+b\operatorname{asinh}(cx))(f+gx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2), x)
```

3.42 $\int \frac{(d+c^2dx^2)^{3/2}(a+b\sinh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=984

$$\frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3}-\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}}-\frac{bcd(c^2f^2+g^2)\,x\sqrt{d+c^2dx^2}}{g^3\sqrt{1+c^2x^2}}+\frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}}-\frac{bc^3dx}{9g}$$

```
[Out] a*d*(c^2*f^2+g^2)*(c^2*d*x^2+d)^(1/2)/g^3+b*d*(c^2*f^2+g^2)*arcsinh(c*x)*(c
^2*d*x^2+d)^(1/2)/g^3-1/2*c^2*d*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/
g^2+1/3*d*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g-1/3*b*c*d*x*
(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-b*c*d*(c^2*f^2+g^2)*x*(c^2*d*x^2+d)
^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2
*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/4*c
*d*f*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)-1/2*c
*d*(c^2*f^2+g^2)*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^3/(c^2*x^2+
1)^(1/2)-1/2*d*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c
/g^4/(g*x+f)/(c^2*x^2+1)^(1/2)-a*d*(c^2*f^2+g^2)^(3/2)*arctanh((-c^2*f*x+g)
/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)
^(1/2)+b*d*(c^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c
^2*f^2+g^2)^(3/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2
+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)+b*d*(c^2*f^2+g^2)^(
3/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d
*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/2)-b*d*(c^2*f^2+g^2)^(3/2)*polylog(2,-(c*x
+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^4/(c
^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2+g^2)*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)*
(c^2*d*x^2+d)^(1/2)/b/c/g^2/(g*x+f)
```

Rubi [A]
time = 1.35, antiderivative size = 984, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 24, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5845, 5840, 5785, 5783, 30, 5798, 5839, 697, 5835, 6874, 267, 739, 212, 5856, 1668, 12, 5855, 8, 5843, 3403, 2296, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2])/g^3 - (b*c*d*x*Sqrt[d + c^2*d*x^2
])/(3*g*Sqrt[1 + c^2*x^2]) - (b*c*d*(c^2*f^2 + g^2)*x*Sqrt[d + c^2*d*x^2])/
(g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d*f*x^2*Sqrt[d + c^2*d*x^2])/(4*g^2*Sqrt[1
 + c^2*x^2]) - (b*c^3*d*x^3*Sqrt[d + c^2*d*x^2])/(9*g*Sqrt[1 + c^2*x^2]) +
```

```
(b*d*(c^2*f^2 + g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x])/g^3 - (c^2*d*f*x*Sqr
t[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g^2) + (d*(1 + c^2*x^2)*Sqrt[d +
c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) - (c*d*f*Sqrt[d + c^2*d*x^2]*(a + b*
ArcSinh[c*x])^2)/(4*b*g^2*Sqrt[1 + c^2*x^2]) - (c*d*(c^2*f^2 + g^2)*x*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^3*Sqrt[1 + c^2*x^2]) - (d*(c^
2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f +
g*x)*Sqrt[1 + c^2*x^2]) + (d*(c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^2*(f + g*x)) - (a*d*(c^2*f^2 + g^2
)^(3/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sqrt
[1 + c^2*x^2])])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[
d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2
+ g^2])])/(g^4*Sqrt[1 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2
*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])
])/(g^4*Sqrt[1 + c^2*x^2]) + (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]
*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1
 + c^2*x^2]) - (b*d*(c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2]*PolyLog[2, -(
(E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^4*Sqrt[1 + c^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
```

```
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5840

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{3/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^{3/2}(a+b\sinh^{-1}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\sqrt{d+c^2dx^2}\right)\int \left(-\frac{c^2f\sqrt{1+c^2x^2}(a+b\sinh^{-1}(cx))}{g^2} + \frac{c^2x\sqrt{1+c^2x}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d\left(1+\frac{c^2f^2}{g^2}\right)\sqrt{d+c^2dx^2}\right)\int \frac{\sqrt{1+c^2x^2}(a+b\sinh^{-1}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}} - \frac{\left(c^2\right.}{}$$

$$= -\frac{c^2dfx\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^2} + \frac{d(1+c^2x^2)\sqrt{d+c^2dx}}{3g}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}$$

$$= -\frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+c^2dx^2}}{4g^2\sqrt{1+c^2x^2}} - \frac{bc^3dx^3\sqrt{d+c^2dx^2}}{9g\sqrt{1+c^2x^2}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+}}{4g^2\sqrt{1+c^2}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+}}{4g^2\sqrt{1+c^2}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} + \frac{bc^3dfx^2\sqrt{d+}}{4g^2\sqrt{1+c^2}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)}{g^3\sqrt{1+}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)}{g^3\sqrt{1+}}$$

$$= \frac{ad(c^2f^2+g^2)\sqrt{d+c^2dx^2}}{g^3} - \frac{bcdx\sqrt{d+c^2dx^2}}{3g\sqrt{1+c^2x^2}} - \frac{bcd(c^2f^2+g^2)}{g^3\sqrt{1+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.09, size = 2864, normalized size = 2.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(3/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d*Sqrt[d + c^2*d*x^2]*(8*g^2 + c^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)))/(6*g^
3) + (a*d^(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*c*d^(3/2)*f*(2
*c^2*f^2 + 3*g^2)*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*g^4) - (a*d^
(3/2)*(c^2*f^2 + g^2)^(3/2)*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^
2]*Sqrt[d + c^2*d*x^2]])/g^4 + (b*d*Sqrt[d + c^2*d*x^2]*((-2*c*g*x)/Sqrt[1
+ c^2*x^2] + 2*g*ArcSinh[c*x] - (c*f*ArcSinh[c*x]^2)/Sqrt[1 + c^2*x^2] + (2
*(c^2*f^2 + g^2)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2
*f^2 + g^2]])/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f +
 I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*
I)*ArcSinh[c*x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqr
t[-(c^2*f^2) - g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*C
ot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((
c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[(
(1/2 - I/2)*Sqrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c
*(f + g*x)])] + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi
 + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*T
an[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)
*E^(ArcSinh[c*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)]
)] - (ArcCos[((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*Arc
Sinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sq
rt[-(c^2*f^2) - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f +
 g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos
[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4
])/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) -
 g^2])*(I + Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f
^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sq
rt[-(c^2*f^2) - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)
*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I
)*ArcSinh[c*x])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f
) + I*g + Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c
*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))/Sqr
t[-(c^2*f^2) - g^2]))/Sqrt[1 + c^2*x^2]))/(2*g^2) + (b*d*Sqrt[d + c^2*d*x^2
]*((-9*(ArcSinh[c*x]*(Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]
)] - Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])]) + PolyLog[2,
(E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - PolyLog[2, -((E^ArcSin
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/Sqrt[c^2*f^2 + g^2] + (-18*c*g*(4
*c^2*f^2 + g^2)*x + 18*g*(4*c^2*f^2 + g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x] -
```

```
 18*c*f*(2*c^2*f^2 + g^2)*ArcSinh[c*x]^2 + 9*c*f*g^2*Cosh[2*ArcSinh[c*x]] +
 6*g^3*ArcSinh[c*x]*Cosh[3*ArcSinh[c*x]] + 9*(8*c^4*f^4 + 8*c^2*f^2*g^2 + g
^4)*(((-I)*Pi*ArcTanh[(-g + c*f*Tanh[ArcSinh[c*x]/2])/Sqrt[c^2*f^2 + g^2]])
/Sqrt[c^2*f^2 + g^2] - (2*ArcCos[((-I)*c*f)/g]*ArcTanh[((c*f + I*g)*Cot[(Pi
 + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + (Pi - (2*I)*ArcSinh[c*
x])*ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2)
- g^2]] + (ArcCos[((-I)*c*f)/g] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I
)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] - (2*I)*ArcTanh[((c*f - I*g)*Ta
n[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]])*Log[((1/2 - I/2)*S
qrt[-(c^2*f^2) - g^2])/(E^(ArcSinh[c*x]/2)*Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])]
 + (ArcCos[((-I)*c*f)/g] + (2*I)*(ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcS
inh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]] + ArcTanh[((c*f - I*g)*Tan[(Pi + (2*I
)*ArcSinh[c*x])/4])/Sqrt[-(c^2*f^2) - g^2]]))*Log[((1/2 + I/2)*E^(ArcSinh[c
*x]/2)*Sqrt[-(c^2*f^2) - g^2])/(Sqrt[(-I)*g]*Sqrt[c*(f + g*x)])] - (ArcCos[
((-I)*c*f)/g] + (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]
)/Sqrt[-(c^2*f^2) - g^2]])*Log[((I*c*f + g)*((-I)*c*f + g + Sqrt[-(c^2*f^2)
 - g^2])*(1 + I*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sqrt[-
(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))] - (ArcCos[((-I)*c*f)/g
] - (2*I)*ArcTanh[((c*f + I*g)*Cot[(Pi + (2*I)*ArcSinh[c*x])/4])/Sqrt[-(c^2
*f^2) - g^2]])*Log[((I*c*f + g)*(I*c*f - g + Sqrt[-(c^2*f^2) - g^2])*(I + C
ot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(c*f - I*g + Sqrt[-(c^2*f^2) - g^2]*Co
t[(Pi + (2*I)*ArcSinh[c*x])/4]))] + I*(PolyLog[2, ((I*c*f + Sqrt[-(c^2*f^2)
 - g^2])*(I*c*f + g - I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x]
)/4]))/(g*(I*c*f + g + I*Sqrt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x
])/4]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) - g^2])*(-(c*f) + I*g + Sqr
t[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))/(g*(I*c*f + g + I*Sq
rt[-(c^2*f^2) - g^2]*Cot[(Pi + (2*I)*ArcSinh[c*x])/4]))]))/Sqrt[-(c^2*f^2)
- g^2]) - 18*c*f*g^2*ArcSinh[c*x]*Sinh[2*ArcSinh[c*x]] - 2*g^3*Sinh[3*ArcSi
nh[c*x]])/g^4))/(72*Sqrt[1 + c^2*x^2])
```

Maple [A]
time = 2.43, size = 1838, normalized size = 1.87

method	result	size
default	Expression too large to display	1838

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c^3*d/(c^2*x^2+1)^(1/2)/g^2*x^2-b*(d*(c^2*x^2
+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g^3*x*c^3*f^2+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^
2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln((-(c*x+(c^2*x^2+1)^(1
/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(c^2*f^2+g^2)
^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*arcsinh(c*x)*ln(((c*x+
(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*
```

```
(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g^3*arcsinh(c*x)*c^2*f^2+1/3*b*(d*(c^2*
x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)*x^4*c^4+5/3*b*(d*(c^2*x^2+1))^(1
/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*f^3*arcsinh(c*x)^2*c^3*d/g^4-3/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x
^2+1)^(1/2)*f*arcsinh(c*x)^2*c*d/g^2+1/3*a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(
x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)-1/2*a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-2*c^2*
d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*x-3/2*a/g^2*c^2*d^2*f*ln((-c^2*d*f
/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2
*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^4*d^2*c^4*f^3*ln((-c^2*d*f/g+(x+f/g
)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)
/g^2)^(1/2))/(c^2*d)^(1/2)+a/g^3*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(
c^2*f^2+g^2)/g^2)^(1/2)*c^2*f^2-2*a/g^3*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln(
(2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((
x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^2
*f^2-a/g*d^2/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*
f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f
/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-a/g^5*d^2/(d*(c^2*f^2+g^2)/g^2)^(1
/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(
1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/
g))*c^4*f^4+a/g*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)
^(1/2)-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^4*d/(c^2*x^2+1)/g^2*arcsinh(c*x)*x^3
-1/2*b*(d*(c^2*x^2+1))^(1/2)*f*c^2*d/(c^2*x^2+1)/g^2*arcsinh(c*x)*x+b*(d*(c
^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g^3*arcsinh(c*x)*x^2*c^4*f^2-1/9*b*(d*(c^2*x
^2+1))^(1/2)*d/(c^2*x^2+1)^(1/2)/g*x^3*c^3-4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c
^2*x^2+1)^(1/2)/g*x*c+1/8*b*(d*(c^2*x^2+1))^(1/2)*f*c*d/(c^2*x^2+1)^(1/2)/g
^2+b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4*dilo
g((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^
(1/2)))-b*(c^2*f^2+g^2)^(3/2)*d*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/g^4
*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^
2)^(1/2)))+4/3*b*(d*(c^2*x^2+1))^(1/2)*d/(c^2*x^2+1)/g*arcsinh(c*x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^2*d*x^2 + a*d + (b*c^2*d*x^2 + b*d)*arcsinh(c*x))*sqrt(c^2*d*
x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(c^2x^2+1))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(3/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(3/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(3/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{asinh}(c\,x))\,{\left(d\,c^2\,x^2+d\right)}^{3/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(3/2))/(f + g*x), x)
```

3.43 $\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=1228

$$-\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}+\frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}}-\frac{25bcd^2f^3x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{15bd^2fg^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{3bcd^2f}{7}$$

```
[Out] 15/64*d^2*f*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f^3*x*(
c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^3*x*(c^2*x^2+1)
^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-1/7*d^2*g^3*(c^2*x^2+1)^3*(a+b*ar
csinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^4+1/9*d^2*g^3*(c^2*x^2+1)^4*(a+b*arcsinh(
c*x))*(c^2*d*x^2+d)^(1/2)/c^4-1/36*b*d^2*f^3*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d
)^(1/2)/c-3/7*b*d^2*f^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-15/256*
b*d^2*f*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-3/7*b*c*d^2*f^2*g*x
^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/256*b*c*d^2*f*g^2*x^4*(c^2*d*x^
2+d)^(1/2)/(c^2*x^2+1)^(1/2)-9/35*b*c^3*d^2*f^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(
c^2*x^2+1)^(1/2)-17/96*b*c^3*d^2*f*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-3/49*b*c^5*d^2*f^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-3/64*b
*c^5*d^2*f*g^2*x^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-15/256*d^2*f*g^2*(
a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)+5/16*d^2*f^
3*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)-25/96*b*c*d^2*f^3*x^2*(c^2*d*x^2
+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/189*b*d^2*g^3*x^3*(c^2*d*x^2+d)^(1/2)/c/(c^2*
x^2+1)^(1/2)-5/96*b*c^3*d^2*f^3*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1
/21*b*c*d^2*g^3*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*
g^3*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/81*b*c^5*d^2*g^3*x^9*(c^2*d
*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^
2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)+15/128*d^2*f*g^2*x*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^
2+d)^(1/2)+3/7*d^2*f^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/
2)/c^2+2/63*b*d^2*g^3*x*(c^2*d*x^2+d)^(1/2)/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 1228, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200, 5808, 5806, 5812, 272, 45, 5804, 12, 380}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-3*b*d^2*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g
^3*x*Sqrt[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f^3*x^2*
```

```
Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (15*b*d^2*f*g^2*x^2*Sqrt[d +
c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d + c^2*d
*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(189*c*S
qrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^3*x^4*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 +
c^2*x^2]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d + c^2*d*x^2])/(256*Sqrt[1 + c^2*x^
2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^2*x^2]) -
(b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*x^2]) - (17*b*c^3*d^
2*f*g^2*x^6*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f^2*
g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (19*b*c^3*d^2*g^3*x^7*S
qrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d
 + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x
^2])/(81*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d
*x^2])/(36*c) + (5*d^2*f^3*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 +
 (15*d^2*f*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(128*c^2) + (15*
d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/64 + (5*d^2*f^3*x*(
1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (5*d^2*f*g^2*x^
3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (d^2*f^3*x*(
1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (3*d^2*f*g^2*x
^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/8 + (3*d^2*f^2
*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) - (d^2
*g^3*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(7*c^4) + (d
^2*g^3*(1 + c^2*x^2)^4*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(9*c^4) +
(5*d^2*f^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2
*x^2]) - (15*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(256*b*c
^3*Sqrt[1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
```

```
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^3 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \int (f+gx)^3 \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right) \int \left(f^3(1+c^2x^2)^{5/2} \left(a+b\sinh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d+c^2dx^2}\right) \int \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{6}d^2f^3x\left(1+c^2x^2\right)^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) + \frac{3}{8}$$

$$= -\frac{bd^2f^3(1+c^2x^2)^{5/2} \sqrt{d+c^2dx^2}}{36c} + \frac{5}{24}d^2f^3x\left(1+c^2x^2\right.$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{3bcd^2f^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} -$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} + \frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{25b}{}$$

$$= -\frac{3bd^2f^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} + \frac{2bd^2g^3x\sqrt{d+c^2dx^2}}{63c^3\sqrt{1+c^2x^2}} - \frac{25b}{}$$

Mathematica [A]
time = 5.02, size = 1667, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (3*a*d^2*f^2*g*Sqrt[d + c^2*d*x^2])/(7*c^2) - (2*a*d^2*g^3*Sqrt[d + c^2*d*x
^2])/(63*c^4) + (11*a*d^2*f^3*x*Sqrt[d + c^2*d*x^2])/16 + (15*a*d^2*f*g^2*x
*Sqrt[d + c^2*d*x^2])/(128*c^2) + (9*a*d^2*f^2*g*x^2*Sqrt[d + c^2*d*x^2])/7
 + (a*d^2*g^3*x^2*Sqrt[d + c^2*d*x^2])/(63*c^2) + (13*a*c^2*d^2*f^3*x^3*Sqr
t[d + c^2*d*x^2])/24 + (59*a*d^2*f*g^2*x^3*Sqrt[d + c^2*d*x^2])/64 + (9*a*c
^2*d^2*f^2*g*x^4*Sqrt[d + c^2*d*x^2])/7 + (5*a*d^2*g^3*x^4*Sqrt[d + c^2*d*x
^2])/21 + (a*c^4*d^2*f^3*x^5*Sqrt[d + c^2*d*x^2])/6 + (17*a*c^2*d^2*f*g^2*x
^5*Sqrt[d + c^2*d*x^2])/16 + (3*a*c^4*d^2*f^2*g*x^6*Sqrt[d + c^2*d*x^2])/7
+ (19*a*c^2*d^2*g^3*x^6*Sqrt[d + c^2*d*x^2])/63 + (3*a*c^4*d^2*f*g^2*x^7*Sq
rt[d + c^2*d*x^2])/8 + (a*c^4*d^2*g^3*x^8*Sqrt[d + c^2*d*x^2])/9 - (3*b*d^2
```

```
*f^2*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) + (2*b*d^2*g^3*x*Sqrt
[d + c^2*d*x^2])/(63*c^3*Sqrt[1 + c^2*x^2]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d +
 c^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (b*d^2*g^3*x^3*Sqrt[d + c^2*d*x^2])/(1
89*c*Sqrt[1 + c^2*x^2]) - (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*S
qrt[1 + c^2*x^2]) - (b*c*d^2*g^3*x^5*Sqrt[d + c^2*d*x^2])/(21*Sqrt[1 + c^2*
x^2]) - (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2])
- (19*b*c^3*d^2*g^3*x^7*Sqrt[d + c^2*d*x^2])/(441*Sqrt[1 + c^2*x^2]) - (b*c
^5*d^2*g^3*x^9*Sqrt[d + c^2*d*x^2])/(81*Sqrt[1 + c^2*x^2]) + (5*b*d^2*f*(8*
c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]^2)/(256*c^3*Sqrt[1 + c^2*
x^2]) - (3*b*d^2*f*(5*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x
]])/(128*c^3*Sqrt[1 + c^2*x^2]) - (3*b*d^2*f^3*Sqrt[d + c^2*d*x^2]*Cosh[4*A
rcSinh[c*x]])/(256*c*Sqrt[1 + c^2*x^2]) - (3*b*d^2*f*g^2*Sqrt[d + c^2*d*x^2
]*Cosh[4*ArcSinh[c*x]])/(512*c^3*Sqrt[1 + c^2*x^2]) - (b*d^2*f^3*Sqrt[d + c
^2*d*x^2]*Cosh[6*ArcSinh[c*x]])/(1152*c*Sqrt[1 + c^2*x^2]) - (b*d^2*f*g^2*S
qrt[d + c^2*d*x^2]*Cosh[6*ArcSinh[c*x]])/(384*c^3*Sqrt[1 + c^2*x^2]) - (3*b
*d^2*f*g^2*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]])/(8192*c^3*Sqrt[1 + c^2
*x^2]) + (5*a*d^(5/2)*f^3*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(16*c)
- (15*a*d^(5/2)*f*g^2*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(128*c^3) +
 (b*d^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(27648*c^2*f^2*g*Sqrt[1 + c^2*x^2]
 - 2048*g^3*Sqrt[1 + c^2*x^2] + 82944*c^4*f^2*g*x^2*Sqrt[1 + c^2*x^2] + 102
4*c^2*g^3*x^2*Sqrt[1 + c^2*x^2] + 82944*c^6*f^2*g*x^4*Sqrt[1 + c^2*x^2] + 1
5360*c^4*g^3*x^4*Sqrt[1 + c^2*x^2] + 27648*c^8*f^2*g*x^6*Sqrt[1 + c^2*x^2]
+ 19456*c^6*g^3*x^6*Sqrt[1 + c^2*x^2] + 7168*c^8*g^3*x^8*Sqrt[1 + c^2*x^2]
+ 3024*c*f*(5*c^2*f^2 - g^2)*Sinh[2*ArcSinh[c*x]] + 1512*c*f*(2*c^2*f^2 + g
^2)*Sinh[4*ArcSinh[c*x]] + 336*c^3*f^3*Sinh[6*ArcSinh[c*x]] + 1008*c*f*g^2*
Sinh[6*ArcSinh[c*x]] + 189*c*f*g^2*Sinh[8*ArcSinh[c*x]]))/(64512*c^4*Sqrt[1
 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2915 vs. $2(1080) = 2160$.
time = 4.57, size = 2916, normalized size = 2.37

method	result	size
default	Expression too large to display	2916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/9*a*g^3*x^2*(c^2*d*x^2+d)^(7/2)/c^2/d-2/63*a*g^3/d/c^4*(c^2*d*x^2+d)^(7/2
)+3/8*a*f*g^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/16*a*f*g^2/c^2*x*(c^2*d*x^2+d)^
(5/2)-5/64*a*f*g^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-15/128*a*f*g^2/c^2*d^2*x*(c^
2*d*x^2+d)^(1/2)-15/128*a*f*g^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2
+d)^(1/2))/(c^2*d)^(1/2)+3/7*a*f^2*g*(c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f^3*x*
(c^2*d*x^2+d)^(5/2)+5/24*a*f^3*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f^3*d^2*x*(c^
```

2*d*x^2+d)^(1/2)+5/16*a*f^3*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2
))/(c^2*d)^(1/2)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2*(8*c^2*f^2
-3*g^2)*d^2/(c^2*x^2+1)^(1/2)/c^3+1/41472*(d*(c^2*x^2+1))^(1/2)*(256*x^10*c
^10+256*(c^2*x^2+1)^(1/2)*x^9*c^9+704*c^8*x^8+576*(c^2*x^2+1)^(1/2)*x^7*c^7
+688*x^6*c^6+432*(c^2*x^2+1)^(1/2)*x^5*c^5+280*c^4*x^4+120*(c^2*x^2+1)^(1/2
)*x^3*c^3+41*c^2*x^2+9*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(-1+9*arcsinh(c*x))*d^2
/c^4/(c^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*(c^2*x^2+1)
^(1/2)*x^8*c^8+320*c^7*x^7+256*(c^2*x^2+1)^(1/2)*c^6*x^6+272*c^5*x^5+160*(c
^2*x^2+1)^(1/2)*c^4*x^4+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*
x^2+1)^(1/2))*f*g^2*(-1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+3/25088*(d*(c^2
x^2+1))^(1/2)(64*c^8*x^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^
2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+
7*(c^2*x^2+1)^(1/2)*c*x+1)*g*(28*arcsinh(c*x)*c^2*f^2-4*c^2*f^2+7*arcsinh(c
*x)*g^2-g^2)*d^2/c^4/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+3
2*(c^2*x^2+1)^(1/2)*c^6*x^6+64*c^5*x^5+48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*
x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1)^(1/2))*f*(6*arcsinh(c*x)
*c^2*f^2-c^2*f^2+18*arcsinh(c*x)*g^2-3*g^2)*d^2/c^3/(c^2*x^2+1)+3/640*(d*(c
^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^
2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*f^2*g*(-1+5*ar
csinh(c*x))*d^2/c^2/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*(
c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*
x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c^2*f^2-2*c^2*f^2+4*arcsinh(c*x)*g^2-g^2)*d
^2/c^3/(c^2*x^2+1)+1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1
/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(81*arcsinh(c*x)*c^2*f^2
-27*c^2*f^2-6*arcsinh(c*x)*g^2+2*g^2)*d^2/c^4/(c^2*x^2+1)+3/256*(d*(c^2*x^2
+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*
f*(10*arcsinh(c*x)*c^2*f^2-5*c^2*f^2-2*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x
^2+1)+3/256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g*(10*a
rcsinh(c*x)*c^2*f^2-10*c^2*f^2-arcsinh(c*x)*g^2+g^2)*d^2/c^4/(c^2*x^2+1)+3/
256*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*g*(10*arcsinh(c
*x)*c^2*f^2+10*c^2*f^2-arcsinh(c*x)*g^2-g^2)*d^2/c^4/(c^2*x^2+1)+3/256*(d*(
c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^
(1/2))*f*(10*arcsinh(c*x)*c^2*f^2+5*c^2*f^2-2*arcsinh(c*x)*g^2-g^2)*d^2/c^3
/(c^2*x^2+1)+1/1152*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^
3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(81*arcsinh(c*x)*c^2*f^2+27*c^
2*f^2-6*arcsinh(c*x)*g^2-2*g^2)*d^2/c^4/(c^2*x^2+1)+3/1024*(d*(c^2*x^2+1))^
(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+
1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*f*(8*arcsinh(c*x)*c^2*f^2+2*c^2*f^2+4*arc
sinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+3/640*(d*(c^2*x^2+1))^(1/2)*(16*x^6*
c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3*c^3+13
*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*f^2*g*(1+5*arcsinh(c*x))*d^2/c^2/(c^2*x
^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*(c^2*x^2+1)^(1/2)*c^6*x^6
+64*c^5*x^5-48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3-18*c^2*x^2*(c^2*x^2+1)^
(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*f*(6*arcsinh(c*x)*c^2*f^2+c^2*f^2+18*arcsinh
(c*x)*g^2+3*g^2)*d^2/c^3/(c^2*x^2+1)+3/25088*(d*(c^2*x^2+1))^(1/2)*(64*c^8*

```
x^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+
104*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x
+1)*g*(28*arcsinh(c*x)*c^2*f^2+4*c^2*f^2+7*arcsinh(c*x)*g^2+g^2)*d^2/c^4/(c
^2*x^2+1)+3/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9-128*(c^2*x^2+1)^(1/2)*
x^8*c^8+320*c^7*x^7-256*(c^2*x^2+1)^(1/2)*c^6*x^6+272*c^5*x^5-160*(c^2*x^2+
1)^(1/2)*c^4*x^4+88*c^3*x^3-32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^
(1/2))*f*g^2*(1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/41472*(d*(c^2*x^2+1))
^(1/2)*(256*x^10*c^10-256*(c^2*x^2+1)^(1/2)*x^9*c^9+704*c^8*x^8-576*(c^2*x^
2+1)^(1/2)*x^7*c^7+688*x^6*c^6-432*(c^2*x^2+1)^(1/2)*x^5*c^5+280*c^4*x^4-12
0*(c^2*x^2+1)^(1/2)*x^3*c^3+41*c^2*x^2-9*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(1+9*
arcsinh(c*x))*d^2/c^4/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g + 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 + 6*a*c
^2*d^2*f*g^2)*x^4 + (6*a*c^2*d^2*f^2*g + a*d^2*g^3)*x^3 + (2*a*c^2*d^2*f^3
+ 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g + 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 + 6*b*c^2*d^2*f*g^2)*x^4 + (6*b*c^2*d^2*f^2*g + b*d^2*g^3)*x^3 + (2*
b*c^2*d^2*f^3 + 3*b*d^2*f*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^3\,(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.44 $\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=901

$$-\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d+c^2dx^2}}{256c\sqrt{1+c^2x^2}}-\frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{5bc^3d^2f^2}{96}$$

```
[Out] -1/36*b*d^2*f^2*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/128*d^2*g^2*x*(a+b*arcsinh(c*x))*(c^2*
d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+
5/24*d^2*f^2*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/48*d^2*
g^2*x^3*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*x*(c
^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(c^2*x^2
+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)+2/7*d^2*f*g*(c^2*x^2+1)^3*(a+b
*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(c^2*d*x^2+d)^(1/2)/
c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(
1/2)-5/256*b*d^2*g^2*x^2*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-2/7*b*c*d^
2*f*g*x^3*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f^2*x^4*(c^2
*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-59/768*b*c*d^2*g^2*x^4*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^
(1/2)-17/288*b*c^3*d^2*g^2*x^6*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-2/49*b
*c^5*d^2*f*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/64*b*c^5*d^2*g^2*x
^8*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f^2*(a+b*arcsinh(c*x))^2*
(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)-5/256*d^2*g^2*(a+b*arcsinh(c*x))^
2*(c^2*d*x^2+d)^(1/2)/b/c^3/(c^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 901, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200, 5808, 5806, 5812, 272, 45}

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (-2*b*d^2*f*g*x*Sqrt[d + c^2*d*x^2])/(7*c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*
f^2*x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (5*b*d^2*g^2*x^2*Sqrt
[d + c^2*d*x^2])/(256*c*Sqrt[1 + c^2*x^2]) - (2*b*c*d^2*f*g*x^3*Sqrt[d + c^
2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f^2*x^4*Sqrt[d + c^2*d*x^2])
/(96*Sqrt[1 + c^2*x^2]) - (59*b*c*d^2*g^2*x^4*Sqrt[d + c^2*d*x^2])/(768*Sqr
t[1 + c^2*x^2]) - (6*b*c^3*d^2*f*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1 + c^
2*x^2]) - (17*b*c^3*d^2*g^2*x^6*Sqrt[d + c^2*d*x^2])/(288*Sqrt[1 + c^2*x^2]
```

```
) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2]) - (b*c
^5*d^2*g^2*x^8*Sqrt[d + c^2*d*x^2])/(64*Sqrt[1 + c^2*x^2]) - (b*d^2*f^2*(1
+ c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f^2*x*Sqrt[d + c^2*d*
x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*g^2*x*Sqrt[d + c^2*d*x^2]*(a + b*Arc
Sinh[c*x]))/(128*c^2) + (5*d^2*g^2*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/64 + (5*d^2*f^2*x*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/24 + (5*d^2*g^2*x^3*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c
*x]))/48 + (d^2*f^2*x*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/6 + (d^2*g^2*x^3*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x]))/8 + (2*d^2*f*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]
))/(7*c^2) + (5*d^2*f^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(32*b*c
*Sqrt[1 + c^2*x^2]) - (5*d^2*g^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2
)/(256*b*c^3*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5808

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^p*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2*p + 1))), x] + (Dist[2*d*(p/(m + 2*p + 1)), Int[(f*x
)^m*(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x], x] - Dist[b*c*(n/(f*(m
+ 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^
2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d
, e, f, m}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)^2 \left(d+c^2dx^2\right)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)^2 \left(1+c^2x^2\right)^{5/2} \left(a+b\sinh^{-1}\right.}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \left(f^2(1+c^2x^2)^{5/2} \left(a+b\sinh^{-1}(cx)\right)\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d+c^2dx^2}\right)\int (1+c^2x^2)^{5/2} \left(a+b\sinh^{-1}(cx)\right) \, d}{\sqrt{1+c^2x^2}}$$

$$= \frac{1}{6}d^2f^2x\left(1+c^2x^2\right)^2 \sqrt{d+c^2dx^2} \left(a+b\sinh^{-1}(cx)\right) + \frac{1}{8}d$$

$$= -\frac{bd^2f^2(1+c^2x^2)^{5/2}\sqrt{d+c^2dx^2}}{36c} + \frac{5}{24}d^2f^2x\left(1+c^2x^2\right)$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{2bcd^2fgx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}} - \frac{6bc^3}{}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{2bc}{}$$

$$= -\frac{2bd^2fgx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}} - \frac{25bcd^2f^2x^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}} - \frac{5bc}{}$$

Mathematica [A]
time = 1.80, size = 1047, normalized size = 1.16

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-5160960*b*c^2*f*g*x*Sqrt[d + c^2*d*x^2] - 5160960*b*c^4*f*g*x^3*Sqrt
[d + c^2*d*x^2] - 3096576*b*c^6*f*g*x^5*Sqrt[d + c^2*d*x^2] - 737280*b*c^8*
f*g*x^7*Sqrt[d + c^2*d*x^2] + 5160960*a*c*f*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^
2*d*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 705
600*a*c*g^2*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 15482880*a*c^3*f*g*x^
2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 9784320*a*c^5*f^2*x^3*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 5550720*a*c^3*g^2*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d
 + c^2*d*x^2] + 15482880*a*c^5*f*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2
] + 3010560*a*c^7*f^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 6397440*a
*c^5*g^2*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 5160960*a*c^7*f*g*x^6*
```

```
Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1 + c^2*
x^2]*Sqrt[d + c^2*d*x^2] + 352800*b*(8*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*A
rcSinh[c*x]^2 - 141120*b*(15*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcS
inh[c*x]] - 211680*b*c^2*f^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 352
80*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 15680*b*c^2*f^2*Sqrt[d
+ c^2*d*x^2]*Cosh[6*ArcSinh[c*x]] - 15680*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[6*
ArcSinh[c*x]] - 2205*b*g^2*Sqrt[d + c^2*d*x^2]*Cosh[8*ArcSinh[c*x]] + 56448
00*a*c^2*Sqrt[d]*f^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x
^2]] - 705600*a*Sqrt[d]*g^2*Sqrt[1 + c^2*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d +
c^2*d*x^2]] + 840*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(6144*c*f*g*Sqrt[1 + c
^2*x^2] + 18432*c^3*f*g*x^2*Sqrt[1 + c^2*x^2] + 18432*c^5*f*g*x^4*Sqrt[1 +
c^2*x^2] + 6144*c^7*f*g*x^6*Sqrt[1 + c^2*x^2] + 336*(15*c^2*f^2 - g^2)*Sinh
[2*ArcSinh[c*x]] + 168*(6*c^2*f^2 + g^2)*Sinh[4*ArcSinh[c*x]] + 112*c^2*f^2
*Sinh[6*ArcSinh[c*x]] + 112*g^2*Sinh[6*ArcSinh[c*x]] + 21*g^2*Sinh[8*ArcSin
h[c*x]])))/(18063360*c^3*Sqrt[1 + c^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2334 vs. $2(791) = 1582$.
time = 4.40, size = 2335, normalized size = 2.59

method	result	size
default	Expression too large to display	2335

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8*a*g^2*x*(c^2*d*x^2+d)^(7/2)/c^2/d-1/48*a*g^2/c^2*x*(c^2*d*x^2+d)^(5/2)-
5/192*a*g^2/c^2*d*x*(c^2*d*x^2+d)^(3/2)-5/128*a*g^2/c^2*d^2*x*(c^2*d*x^2+d)
^(1/2)-5/128*a*g^2/c^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c
^2*d)^(1/2)+2/7*a*f*g*(c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f^2*x*(c^2*d*x^2+d)^(
5/2)+5/24*a*f^2*d*x*(c^2*d*x^2+d)^(3/2)+5/16*a*f^2*d^2*x*(c^2*d*x^2+d)^(1/2
)+5/16*a*f^2*d^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2
)+b*(5/256*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)^2*(8*c^2*f^2-g^2)*d^2/(c^2*x^
2+1)^(1/2)/c^3+1/16384*(d*(c^2*x^2+1))^(1/2)*(128*c^9*x^9+128*(c^2*x^2+1)^(
1/2)*x^8*c^8+320*c^7*x^7+256*(c^2*x^2+1)^(1/2)*c^6*x^6+272*c^5*x^5+160*(c^2
*x^2+1)^(1/2)*c^4*x^4+88*c^3*x^3+32*c^2*x^2*(c^2*x^2+1)^(1/2)+8*c*x+(c^2*x^
2+1)^(1/2))*g^2*(-1+8*arcsinh(c*x))*d^2/c^3/(c^2*x^2+1)+1/3136*(d*(c^2*x^2+
1))^(1/2)*(64*c^8*x^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2
+1)^(1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^
2*x^2+1)^(1/2)*c*x+1)*f*g*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d
*(c^2*x^2+1))^(1/2)*(32*c^7*x^7+32*(c^2*x^2+1)^(1/2)*c^6*x^6+64*c^5*x^5+48*
(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^
2*x^2+1)^(1/2))*(6*arcsinh(c*x)*c^2*f^2-c^2*f^2+6*arcsinh(c*x)*g^2-g^2)*d^2
/c^3/(c^2*x^2+1)+1/320*(d*(c^2*x^2+1))^(1/2)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/
```

```
2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)
^(1/2)*c*x+1)*f*g*(-1+5*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*x^
2+1))^(1/2)*(8*c^5*x^5+8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3+8*c^2*x^2*(c^
2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*(24*arcsinh(c*x)*c^2*f^2-6*c^2*f^2+
4*arcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/64*(d*(c^2*x^2+1))^(1/2)*(4*c
^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*f*g
*(-1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3
*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*(30*arcsinh(c*x)*
c^2*f^2-15*c^2*f^2-2*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+5/64*(d*(c^2
*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)-1)*d^2/c
^2/(c^2*x^2+1)+5/64*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)
*f*g*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+1/256*(d*(c^2*x^2+1))^(1/2)*(2*c^
3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*(30*arcsinh(c*x)
*c^2*f^2+15*c^2*f^2-2*arcsinh(c*x)*g^2-g^2)*d^2/c^3/(c^2*x^2+1)+1/64*(d*(c^
2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2
+1)^(1/2)*c*x+1)*f*g*(1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/1024*(d*(c^2*
x^2+1))^(1/2)*(8*c^5*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(
c^2*x^2+1)^(1/2)+4*c*x-(c^2*x^2+1)^(1/2))*(24*arcsinh(c*x)*c^2*f^2+6*c^2*f^
2+4*arcsinh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+1/320*(d*(c^2*x^2+1))^(1/2)*(
16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x^2+1)^(1/2)*x^3
*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(1+5*arcsinh(c*x))*d^2/c^2/(
c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*(c^2*x^2+1)^(1/2)*c^
6*x^6+64*c^5*x^5-48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3-18*c^2*x^2*(c^2*x^
2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*(6*arcsinh(c*x)*c^2*f^2+c^2*f^2+6*arcsi
nh(c*x)*g^2+g^2)*d^2/c^3/(c^2*x^2+1)+1/3136*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x
^8-64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+1
04*c^4*x^4-56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+
1)*f*g*(1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/16384*(d*(c^2*x^2+1))^(1/2)
*(128*c^9*x^9-128*(c^2*x^2+1)^(1/2)*x^8*c^8+320*c^7*x^7-256*(c^2*x^2+1)^(1/
2)*c^6*x^6+272*c^5*x^5-160*(c^2*x^2+1)^(1/2)*c^4*x^4+88*c^3*x^3-32*c^2*x^2*
(c^2*x^2+1)^(1/2)+8*c*x-(c^2*x^2+1)^(1/2))*g^2*(1+8*arcsinh(c*x))*d^2/c^3/(
c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fr
icas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 + 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 + 2*a*c^2*d^2*g^2)*x^4 + (2*a*c^2
*d^2*f^2 + a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 + 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 + 2*b*c^2*d^2*
g^2)*x^4 + (2*b*c^2*d^2*f^2 + b*d^2*g^2)*x^2)*arcsinh(c*x))*sqrt(c^2*d*x^2
+ d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="gi
ac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)^2\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.45 $\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=494

$$-\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{5bc^3d^2fx^4\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d}}{35\sqrt{1+}}$$

```
[Out] -1/36*b*d^2*f*(c^2*x^2+1)^(5/2)*(c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*arc
sinh(c*x))*(c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(c^2*x^2+1)*(a+b*arcsinh(c*x))*
(c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2
+d)^(1/2)+1/7*d^2*g*(c^2*x^2+1)^3*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/c^
2-1/7*b*d^2*g*x*(c^2*d*x^2+d)^(1/2)/c/(c^2*x^2+1)^(1/2)-25/96*b*c*d^2*f*x^2
*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/7*b*c*d^2*g*x^3*(c^2*d*x^2+d)^(1/2
)/(c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1
/2)-3/35*b*c^3*d^2*g*x^5*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)-1/49*b*c^5*d
^2*g*x^7*(c^2*d*x^2+d)^(1/2)/(c^2*x^2+1)^(1/2)+5/32*d^2*f*(a+b*arcsinh(c*x)
)^2*(c^2*d*x^2+d)^(1/2)/b/c/(c^2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5845, 5838, 5786, 5785, 5783, 30, 14, 267, 5798, 200}

$$\frac{1}{6}d^2fx\left(c^2x^2+1\right)^2\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5}{24}d^2fx\left(c^2x^2+1\right)\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)+\frac{5d^2f\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)^2}{32bc\sqrt{c^2x^2+1}}+\frac{d^2g\left(c^2x^2+1\right)^3\sqrt{c^2dx^2+d}\left(a+b\sinh^{-1}(cx)\right)}{7c^2}-\frac{25bcd^2fx^2\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{bd^2f\left(c^2x^2+1\right)^{5/2}\sqrt{c^2dx^2+d}}{36c}-\frac{bd^2gx\sqrt{c^2dx^2+d}}{7c\sqrt{c^2x^2+1}}-\frac{bcd^2gx^3\sqrt{c^2dx^2+d}}{7\sqrt{c^2x^2+1}}-\frac{bc^5d^2gx^7\sqrt{c^2dx^2+d}}{49\sqrt{c^2x^2+1}}-\frac{5bc^3d^2fx^4\sqrt{c^2dx^2+d}}{96\sqrt{c^2x^2+1}}-\frac{3bc^3d^2gx^5\sqrt{c^2dx^2+d}}{35\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] -1/7*(b*d^2*g*x*Sqrt[d + c^2*d*x^2])/(c*Sqrt[1 + c^2*x^2]) - (25*b*c*d^2*f*
x^2*Sqrt[d + c^2*d*x^2])/(96*Sqrt[1 + c^2*x^2]) - (b*c*d^2*g*x^3*Sqrt[d + c
^2*d*x^2])/(7*Sqrt[1 + c^2*x^2]) - (5*b*c^3*d^2*f*x^4*Sqrt[d + c^2*d*x^2])/
(96*Sqrt[1 + c^2*x^2]) - (3*b*c^3*d^2*g*x^5*Sqrt[d + c^2*d*x^2])/(35*Sqrt[1
 + c^2*x^2]) - (b*c^5*d^2*g*x^7*Sqrt[d + c^2*d*x^2])/(49*Sqrt[1 + c^2*x^2])
 - (b*d^2*f*(1 + c^2*x^2)^(5/2)*Sqrt[d + c^2*d*x^2])/(36*c) + (5*d^2*f*x*Sq
rt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/16 + (5*d^2*f*x*(1 + c^2*x^2)*Sqrt[
d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/24 + (d^2*f*x*(1 + c^2*x^2)^2*Sqrt[d +
 c^2*d*x^2]*(a + b*ArcSinh[c*x]))/6 + (d^2*g*(1 + c^2*x^2)^3*Sqrt[d + c^2*d
*x^2]*(a + b*ArcSinh[c*x]))/(7*c^2) + (5*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*A
rcSinh[c*x])^2)/(32*b*c*Sqrt[1 + c^2*x^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
```

```
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int (f+gx)\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int (f+gx)\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\left(f\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)+\right.}{\sqrt{1+c^2x^2}}$$

$$=\frac{\left(d^2f\sqrt{d+c^2dx^2}\right)\int\left(1+c^2x^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{\sqrt{1+c^2x^2}}$$

$$=\frac{1}{6}d^2fx\left(1+c^2x^2\right)^2\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)+\frac{d^2g(}{}$$

$$=-\frac{bd^2f\left(1+c^2x^2\right)^{5/2}\sqrt{d+c^2dx^2}}{36c}+\frac{5}{24}d^2fx\left(1+c^2x^2\right)\sqrt{d}$$

$$=-\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3\sqrt{d+c^2dx^2}}{7\sqrt{1+c^2x^2}}-\frac{3bc^3d^2gx^5}{35\sqrt{1}}$$

$$=-\frac{bd^2gx\sqrt{d+c^2dx^2}}{7c\sqrt{1+c^2x^2}}-\frac{25bcd^2fx^2\sqrt{d+c^2dx^2}}{96\sqrt{1+c^2x^2}}-\frac{bcd^2gx^3}{7\sqrt{}}$$

Mathematica [A]

time = 0.88, size = 656, normalized size = 1.33

$$\frac{d^2\left(-80640bcgx\sqrt{d+c^2dx^2}-80640bc^3gx^3\sqrt{d+c^2dx^2}-48384bc^5gx^5\sqrt{d+c^2dx^2}-11520bc^7gx^7\sqrt{d+c^2dx^2}+80640ag\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+388080ac^2fx\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+241920ac^2gx^2\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+305760ac^4fx^3\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+241920ac^4gx^4\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+94080ac^6fx^5\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+80640ac^6gx^6\sqrt{1+c^2x^2}\sqrt{d+c^2dx^2}+88200bcf\sqrt{d+c^2dx^2}\sinh^{-1}(cx)^2-66150bcf\sqrt{d+c^2dx^2}\cosh\left(2\sinh^{-1}(cx)\right)-6615bcf\sqrt{d+c^2dx^2}\cosh\left(4\sinh^{-1}(cx)\right)-490bcf\sqrt{d+c^2dx^2}\cosh\left(6\sinh^{-1}(cx)\right)+176400ac\sqrt{d}f\sqrt{1+c^2x^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)+420b\sqrt{d+c^2dx^2}\sinh^{-1}(cx)\left(192g\sqrt{1+c^2x^2}+576c^2gx^2\sqrt{1+c^2x^2}+576c^4gx^4\sqrt{1+c^2x^2}+192c^6gx^6\sqrt{1+c^2x^2}+315cf\sinh\left(2\sinh^{-1}(cx)\right)+63cf\sinh\left(4\sinh^{-1}(cx)\right)+7cf\sinh\left(6\sinh^{-1}(cx)\right)\right)\right)}{564480c^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]),x]
```

```
[Out] (d^2*(-80640*b*c*g*x*Sqrt[d + c^2*d*x^2] - 80640*b*c^3*g*x^3*Sqrt[d + c^2*d
*x^2] - 48384*b*c^5*g*x^5*Sqrt[d + c^2*d*x^2] - 11520*b*c^7*g*x^7*Sqrt[d +
c^2*d*x^2] + 80640*a*g*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 388080*a*c^2
*f*x*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[d + c^2*d*x^2] + 305760*a*c^4*f*x^3*Sqrt[1 + c^2*x^2]*Sqrt[d +
c^2*d*x^2] + 241920*a*c^4*g*x^4*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 940
80*a*c^6*f*x^5*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 80640*a*c^6*g*x^6*Sq
rt[1 + c^2*x^2]*Sqrt[d + c^2*d*x^2] + 88200*b*c*f*Sqrt[d + c^2*d*x^2]*ArcSi
nh[c*x]^2 - 66150*b*c*f*Sqrt[d + c^2*d*x^2]*Cosh[2*ArcSinh[c*x]] - 6615*b*c
*f*Sqrt[d + c^2*d*x^2]*Cosh[4*ArcSinh[c*x]] - 490*b*c*f*Sqrt[d + c^2*d*x^2]
*Cosh[6*ArcSinh[c*x]] + 176400*a*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*Log[c*d*x +
Sqrt[d]*Sqrt[d + c^2*d*x^2]] + 420*b*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]*(192*
g*Sqrt[1 + c^2*x^2] + 576*c^2*g*x^2*Sqrt[1 + c^2*x^2] + 576*c^4*g*x^4*Sqrt[
1 + c^2*x^2] + 192*c^6*g*x^6*Sqrt[1 + c^2*x^2] + 315*c*f*Sinh[2*ArcSinh[c*x
]] + 63*c*f*Sinh[4*ArcSinh[c*x]] + 7*c*f*Sinh[6*ArcSinh[c*x]])))/(564480*c^
2*Sqrt[1 + c^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. $2(430) = 860$.
time = 3.27, size = 1679, normalized size = 3.40

method	result	size
default	Expression too large to display	1679

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a*g*(c^2*d*x^2+d)^(7/2)/c^2/d+1/6*a*f*x*(c^2*d*x^2+d)^(5/2)+5/24*a*f*d*
x*(c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3*ln(x*
c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(5/32*(d*(c^2*x^2+
1))^(1/2)/(c^2*x^2+1)^(1/2)/c*f*arcsinh(c*x)^2*d^2+1/6272*(d*(c^2*x^2+1))^(
1/2)*(64*c^8*x^8+64*(c^2*x^2+1)^(1/2)*x^7*c^7+144*x^6*c^6+112*(c^2*x^2+1)^(
1/2)*x^5*c^5+104*c^4*x^4+56*(c^2*x^2+1)^(1/2)*x^3*c^3+25*c^2*x^2+7*(c^2*x^2
+1)^(1/2)*c*x+1)*g*(-1+7*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x
^2+1))^(1/2)*(32*c^7*x^7+32*(c^2*x^2+1)^(1/2)*c^6*x^6+64*c^5*x^5+48*(c^2*x^
2+1)^(1/2)*c^4*x^4+38*c^3*x^3+18*c^2*x^2*(c^2*x^2+1)^(1/2)+6*c*x+(c^2*x^2+1
)^(1/2))*f*(-1+6*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/640*(d*(c^2*x^2+1))^(1/2
)*(16*x^6*c^6+16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4+20*(c^2*x^2+1)^(1/2)*
```

```
x^3*c^3+13*c^2*x^2+5*(c^2*x^2+1)^(1/2)*c*x+1)*g*(-1+5*arcsinh(c*x))*d^2/c^2
/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5*x^5+8*(c^2*x^2+1)^(1/2)*c^4
*x^4+12*c^3*x^3+8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c*x+(c^2*x^2+1)^(1/2))*f*(-1+
4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*
(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*g*(-1+3*arcs
inh(c*x))*d^2/c^2/(c^2*x^2+1)+15/256*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2
*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^2+1)^(1/2))*f*(2*arcsinh(c*x)-1)*d^2/c/
(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g
*(arcsinh(c*x)-1)*d^2/c^2/(c^2*x^2+1)+5/128*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-
(c^2*x^2+1)^(1/2)*c*x+1)*g*(arcsinh(c*x)+1)*d^2/c^2/(c^2*x^2+1)+15/256*(d*(
c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^
(1/2))*f*(2*arcsinh(c*x)+1)*d^2/c/(c^2*x^2+1)+1/128*(d*(c^2*x^2+1))^(1/2)*(
4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*
g*(1+3*arcsinh(c*x))*d^2/c^2/(c^2*x^2+1)+3/512*(d*(c^2*x^2+1))^(1/2)*(8*c^5
*x^5-8*(c^2*x^2+1)^(1/2)*c^4*x^4+12*c^3*x^3-8*c^2*x^2*(c^2*x^2+1)^(1/2)+4*c
*x-(c^2*x^2+1)^(1/2))*f*(1+4*arcsinh(c*x))*d^2/c/(c^2*x^2+1)+1/640*(d*(c^2*
x^2+1))^(1/2)*(16*x^6*c^6-16*(c^2*x^2+1)^(1/2)*x^5*c^5+28*c^4*x^4-20*(c^2*x
^2+1)^(1/2)*x^3*c^3+13*c^2*x^2-5*(c^2*x^2+1)^(1/2)*c*x+1)*g*(1+5*arcsinh(c*
x))*d^2/c^2/(c^2*x^2+1)+1/2304*(d*(c^2*x^2+1))^(1/2)*(32*c^7*x^7-32*(c^2*x^
2+1)^(1/2)*c^6*x^6+64*c^5*x^5-48*(c^2*x^2+1)^(1/2)*c^4*x^4+38*c^3*x^3-18*c^
2*x^2*(c^2*x^2+1)^(1/2)+6*c*x-(c^2*x^2+1)^(1/2))*f*(1+6*arcsinh(c*x))*d^2/c
/(c^2*x^2+1)+1/6272*(d*(c^2*x^2+1))^(1/2)*(64*c^8*x^8-64*(c^2*x^2+1)^(1/2)*
x^7*c^7+144*x^6*c^6-112*(c^2*x^2+1)^(1/2)*x^5*c^5+104*c^4*x^4-56*(c^2*x^2+1
)^(1/2)*x^3*c^3+25*c^2*x^2-7*(c^2*x^2+1)^(1/2)*c*x+1)*g*(1+7*arcsinh(c*x))*
d^2/c^2/(c^2*x^2+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 + 2*a*c^2*d^2*g*x^3 + 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 + 2*b*c
^2*d^2*g*x^3 + 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arcsinh(c*x))*sqrt(
c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x)),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))*(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x)),x, algorithm="giac
")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f+g\,x)\,\left(a+b\,\mathrm{asinh}(c\,x)\right)\,{\left(d\,c^2\,x^2+d\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)*(a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2), x)
```

3.46 $\int \frac{\left(d+c^2dx^2\right)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1536

$$\frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5}+\frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}}-\frac{bcd^2(c^2f^2+g^2)^2x\sqrt{d+c^2dx^2}}{g^5\sqrt{1+c^2x^2}}-\frac{bcd^2(c^2f^2+2g^2)x\sqrt{d}}{3g^3\sqrt{1+c^2x^2}}$$

```
[Out] 1/4*b*c^3*d^2*f*(c^2*f^2+2*g^2)*x^2*(c^2*d*x^2+d)^(1/2)/g^4/(c^2*x^2+1)^(1/
2)-1/4*c*d^2*f*(c^2*f^2+2*g^2)*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g
^4/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^2*arcsinh(c*x)*(c^2*d*x^2+d)^(1/2)
/g^5+1/3*d^2*(c^2*f^2+2*g^2)*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)/g^3-1/2*c^2*d^2*f*(c^2*f^2+2*g^2)*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(
1/2)/g^4-b*c*d^2*(c^2*f^2+g^2)^2*x*(c^2*d*x^2+d)^(1/2)/g^5/(c^2*x^2+1)^(1/2
)-1/3*b*c*d^2*(c^2*f^2+2*g^2)*x*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1
/16*b*c^3*d^2*f*x^2*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)-1/9*b*c^3*d^2
*(c^2*f^2+2*g^2)*x^3*(c^2*d*x^2+d)^(1/2)/g^3/(c^2*x^2+1)^(1/2)+1/16*b*c^5*d
^2*f*x^4*(c^2*d*x^2+d)^(1/2)/g^2/(c^2*x^2+1)^(1/2)+1/16*c*d^2*f*(a+b*arcsin
h(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^2/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^(
5/2)*arcsinh(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))
*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*arcsin
h(c*x)*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2
+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-1/3*d^2*(c^2*x^2+1)*(a+b*arcsinh(c*x))*(c^2
*d*x^2+d)^(1/2)/g+1/5*d^2*(c^2*x^2+1)^2*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1
/2)/g+a*d^2*(c^2*f^2+g^2)^2*(c^2*d*x^2+d)^(1/2)/g^5-1/2*c*d^2*(c^2*f^2+g^2)
^2*x*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/g^5/(c^2*x^2+1)^(1/2)-1/2*d
^2*(c^2*f^2+g^2)^3*(a+b*arcsinh(c*x))^2*(c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)
/(c^2*x^2+1)^(1/2)+1/2*d^2*(c^2*f^2+g^2)^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)
^(1/2)*(c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)-1/4*c^4*d^2*f*x^3*(a+b*arcsinh(c
*x))*(c^2*d*x^2+d)^(1/2)/g^2+2/15*b*c*d^2*x*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+
1)^(1/2)-1/45*b*c^3*d^2*x^3*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-1/25*b*
c^5*d^2*x^5*(c^2*d*x^2+d)^(1/2)/g/(c^2*x^2+1)^(1/2)-a*d^2*(c^2*f^2+g^2)^(5/
2)*arctanh((-c^2*f*x+g)/(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2))*(c^2*d*x^2+d
)^(1/2)/g^6/(c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2+g^2)^(5/2)*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x
^2+1)^(1/2)-b*d^2*(c^2*f^2+g^2)^(5/2)*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*d*x^2+d)^(1/2)/g^6/(c^2*x^2+1)^(1/2)-1/8*c^
2*d^2*f*x*(a+b*arcsinh(c*x))*(c^2*d*x^2+d)^(1/2)/g^2
```

Rubi [A]

time = 1.78, antiderivative size = 1536, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 29, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.967$, Rules used = {5845, 5840, 5785, 5783, 30, 5798, 5806, 5812, 272, 45, 5804, 12, 5839, 697, 5835, 6874, 267, 739, 212, 5856, 1668, 5855, 8, 5843, 3403, 2296, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] (a*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2])/g^5 + (2*b*c*d^2*x*Sqrt[d + c
^2*d*x^2])/(15*g*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d +
 c^2*d*x^2])/(g^5*Sqrt[1 + c^2*x^2]) - (b*c*d^2*(c^2*f^2 + 2*g^2)*x*Sqrt[d
+ c^2*d*x^2])/(3*g^3*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*x^2*Sqrt[d + c^2*d*x
^2])/(16*g^2*Sqrt[1 + c^2*x^2]) + (b*c^3*d^2*f*(c^2*f^2 + 2*g^2)*x^2*Sqrt[d
 + c^2*d*x^2])/(4*g^4*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*x^3*Sqrt[d + c^2*d*x^
2])/(45*g*Sqrt[1 + c^2*x^2]) - (b*c^3*d^2*(c^2*f^2 + 2*g^2)*x^3*Sqrt[d + c^
2*d*x^2])/(9*g^3*Sqrt[1 + c^2*x^2]) + (b*c^5*d^2*f*x^4*Sqrt[d + c^2*d*x^2])
/(16*g^2*Sqrt[1 + c^2*x^2]) - (b*c^5*d^2*x^5*Sqrt[d + c^2*d*x^2])/(25*g*Sqr
t[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^2*Sqrt[d + c^2*d*x^2]*ArcSinh[c*x]
)/g^5 - (c^2*d^2*f*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(8*g^2) - (c
^2*d^2*f*(c^2*f^2 + 2*g^2)*x*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(2*g
^4) - (c^4*d^2*f*x^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(4*g^2) - (d
^2*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g) + (d^2*(c^
2*f^2 + 2*g^2)*(1 + c^2*x^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(3*g
^3) + (d^2*(1 + c^2*x^2)^2*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x]))/(5*g)
+ (c*d^2*f*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(16*b*g^2*Sqrt[1 + c
^2*x^2]) - (c*d^2*f*(c^2*f^2 + 2*g^2)*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*
x])^2)/(4*b*g^4*Sqrt[1 + c^2*x^2]) - (c*d^2*(c^2*f^2 + g^2)^2*x*Sqrt[d + c^
2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*g^5*Sqrt[1 + c^2*x^2]) - (d^2*(c^2*f^
2 + g^2)^3*Sqrt[d + c^2*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^6*(f + g*x)
*Sqrt[1 + c^2*x^2]) + (d^2*(c^2*f^2 + g^2)^2*Sqrt[1 + c^2*x^2]*Sqrt[d + c^2
*d*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*g^4*(f + g*x)) - (a*d^2*(c^2*f^2 + g
^2)^(5/2)*Sqrt[d + c^2*d*x^2]*ArcTanh[(g - c^2*f*x)/(Sqrt[c^2*f^2 + g^2]*Sq
rt[1 + c^2*x^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*S
qrt[d + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*
f^2 + g^2])])/(g^6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d
 + c^2*d*x^2]*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 +
 g^2])])/(g^6*Sqrt[1 + c^2*x^2]) + (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^
2*d*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(g^
6*Sqrt[1 + c^2*x^2]) - (b*d^2*(c^2*f^2 + g^2)^(5/2)*Sqrt[d + c^2*d*x^2]*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(g^6*Sqrt[1 + c
^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5804

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_
), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSin
h[c*x], u, x] - Dist[b*c*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[Simpl
ifyIntegrand[u/Sqrt[d + e*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && IntegerQ[p - 1/2] && NeQ[p, -2^(-1)] && (IGtQ[(m + 1)/2,
0] || ILtQ[(m + 2*p + 3)/2, 0])
```

Rule 5806

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
 (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Arc
Sinh[c*x])^n/(f*(m + 2))), x] + (Dist[(1/(m + 2))*Simp[Sqrt[d + e*x^2]/Sqrt
[1 + c^2*x^2]], Int[(f*x)^m*((a + b*ArcSinh[c*x])^n/Sqrt[1 + c^2*x^2]), x],
 x] - Dist[b*c*(n/(f*(m + 2)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int
[(f*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d,
e, f, m}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && (IGtQ[m, -2] || EqQ[n, 1])
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
```

```
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5835

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcSinh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[u*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2
]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ
[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5839

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.) + (g_.)*(x_))^(m_)*Sqr
t[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(f + g*x)^m*(d + e*x^2)*((a + b*A
rcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[1/(b*c*Sqrt[d]*(n +
1)), Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcSi
nh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d
] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5840

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(
a + b*ArcSinh[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{
a, b, c, d, e, f, g}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IGtQ[p + 1/2, 0
] && GtQ[d, 0] && IGtQ[n, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
```

```
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rule 5855

```
Int[ArcSinh[(c_.)*(x_)]^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol]
:> With[{u = ExpandIntegrand[(d + e*x^2)^p*ArcSinh[c*x]^n, RFx, x]}, Int[u,
 x] /; SumQ[u]] /; FreeQ[{c, d, e}, x] && RationalFunctionQ[RFx, x] && IGtQ
[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 5856

```
Int[(ArcSinh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(
p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcSinh[c*x
])^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && I
GtQ[n, 0] && EqQ[e, c^2*d] && IntegerQ[p - 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+c^2dx^2)^{5/2}\left(a+b\sinh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int \frac{(1+c^2x^2)^{5/2}(a+b\sinh^{-1}(cx))}{f+gx}\,dx}{\sqrt{1+c^2x^2}}$$

$$= \frac{\left(d^2\sqrt{d+c^2dx^2}\right)\int\left(\frac{(-c^4f^3-2c^2fg^2)\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}{g^4} + \frac{c^2(\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(c^4d^2f\sqrt{d+c^2dx^2}\right)\int x^2\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\,dx}{g^2\sqrt{1+c^2x^2}} + \cdots$$

$$= -\frac{c^2d^2f(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}\left(a+b\sinh^{-1}(cx)\right)}{2g^4} - \frac{c^4d^2fx^3\cdots}{\cdots}$$

$$= -\frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2f(c^2f^2+2g^2)\,x^2\sqrt{d+\cdots}}{4g^4\sqrt{1+c^2x^2}}\cdots$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx\cdots}{16g^2\cdots}$$

$$= \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2+2g^2)\,x\sqrt{d+c^2dx^2}}{3g^3\sqrt{1+c^2x^2}} + \frac{bc^3d^2fx\cdots}{16g^2\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{3g\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{3g\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{3g\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{g\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{g\cdots}$$

$$= \frac{ad^2(c^2f^2+g^2)^2\sqrt{d+c^2dx^2}}{g^5} + \frac{2bcd^2x\sqrt{d+c^2dx^2}}{15g\sqrt{1+c^2x^2}} - \frac{bcd^2(c^2f^2\cdots}{g\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.44, size = 7163, normalized size = 4.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c^2*d*x^2)^(5/2)*(a + b*ArcSinh[c*x]))/(f + g*x),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3927 vs. 2(1420) = 2840.
time = 2.64, size = 3928, normalized size = 2.56

method	result	size
default	Expression too large to display	3928

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x,method=_RETURNVERBOSE)
```

```
[Out] -b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*ln((
(c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)
))*arcsinh(c*x)*c^4*f^4-b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^5*x
*c^5*f^4+b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^5*arcsinh(c*x)*c^4*f^4+b
*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*ln((-(
c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)
))*arcsinh(c*x)-b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)
^(1/2)/g^2*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2
*f^2+g^2)^(1/2)))*arcsinh(c*x)-1/2*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2
)*f^5*arcsinh(c*x)^2*c^5*d^2/g^6-5/4*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1
/2)*f^3*arcsinh(c*x)^2*c^3*d^2/g^4-15/16*b*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1
)^(1/2)*f*arcsinh(c*x)^2*c*d^2/g^2+1/5*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2
+1)/g*arcsinh(c*x)*x^6*c^6+14/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*
arcsinh(c*x)*x^4*c^4+a/g^5*d^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*
f^2+g^2)/g^2)^(1/2)*c^4*f^4+2*a/g^3*d^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g
)+d*(c^2*f^2+g^2)/g^2)^(1/2)*c^2*f^2-3*a/g^3*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2
)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/
2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g)
)*c^2*f^2-3*a/g^5*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2
-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d
*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4-7/8*a/g^2*c^2*d^2
*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*x-15/8*a
/g^2*c^2*d^3*f*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d
-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-1/2*a/g^4*d^
2*c^4*f^3*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2)*x
```

```
-5/2*a/g^4*d^3*c^4*f^3*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)
^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^
6*d^3*c^6*f^5*ln((-c^2*d*f/g+(x+f/g)*c^2*d)/(c^2*d)^(1/2)+((x+f/g)^2*c^2*d-
2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(c^2*d)^(1/2)-a/g^7*d^3/(d*
(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d
*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g
^2)/g^2)^(1/2))/(x+f/g))*c^6*f^6+8/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1
)/g^3*arcsinh(c*x)*x^2*c^4*f^2-1/4*b*(d*(c^2*x^2+1))^(1/2)*f*c^6*d^2/(c^2*x
^2+1)/g^2*arcsinh(c*x)*x^5-11/8*b*(d*(c^2*x^2+1))^(1/2)*f*c^4*d^2/(c^2*x^2+
1)/g^2*arcsinh(c*x)*x^3-9/8*b*(d*(c^2*x^2+1))^(1/2)*f*c^2*d^2/(c^2*x^2+1)/g
^2*arcsinh(c*x)*x-1/4*a/g^2*c^2*d*f*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*
(c^2*f^2+g^2)/g^2)^(3/2)*x-a/g*d^3/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2
*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*
c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))-1/9*b*(d*(c^
2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g^3*x^3*c^5*f^2+1/16*b*(d*(c^2*x^2+1)
)^(1/2)*f*c^5*d^2/(c^2*x^2+1)^(1/2)/g^2*x^4+9/16*b*(d*(c^2*x^2+1))^(1/2)*f*
c^3*d^2/(c^2*x^2+1)^(1/2)/g^2*x^2-7/3*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+
1)^(1/2)/g^3*x*c^3*f^2+1/4*b*(d*(c^2*x^2+1))^(1/2)*f^3*c^5*d^2/(c^2*x^2+1)^
(1/2)/g^4*x^2+1/5*a/g*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/
g^2)^(5/2)+1/3*a/g*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g
^2)^(3/2)-1/25*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*x^5*c^5-b*d^
2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilog(((c
*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))
+b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^2*dilo
g((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^
(1/2)))-11/45*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*x^3*c^3-23/15
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)^(1/2)/g*x*c+33/128*b*(d*(c^2*x^2+1
))^(1/2)*f*c*d^2/(c^2*x^2+1)^(1/2)/g^2+1/8*b*(d*(c^2*x^2+1))^(1/2)*f^3*c^3*
d^2/(c^2*x^2+1)^(1/2)/g^4+2*b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)
/(c^2*x^2+1)^(1/2)/g^4*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/
2))/(-c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*c^2*f^2-2*b*d^2*(d*(c^2*x^2+1)
)^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^4*ln(((c*x+(c^2*x^2+1)^(1/2
))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))*arcsinh(c*x)*c^2*f
^2+34/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*arcsinh(c*x)*x^2*c^2+7/3
*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g^3*arcsinh(c*x)*c^2*f^2+1/3*a/g^3
*d*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(3/2)*c^2*f^2+
b*d^2*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)/(c^2*x^2+1)^(1/2)/g^6*ln((-
(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2
)))*arcsinh(c*x)*c^4*f^4+23/15*b*(d*(c^2*x^2+1))^(1/2)*d^2/(c^2*x^2+1)/g*ar
csinh(c*x)+a/g*d^2*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2
)^(1/2)-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*c^6*d^2/(c^2*x^2+1)/g^4*arcsinh(c*x
)*x^3-1/2*b*(d*(c^2*x^2+1))^(1/2)*f^3*c^4*d^2/(...
```

<u>Maxima</u> [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="fric
as")
```

```
[Out] integral((a*c^4*d^2*x^4 + 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 + 2*b*c^
2*d^2*x^2 + b*d^2)*arcsinh(c*x))*sqrt(c^2*d*x^2 + d)/(g*x + f), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(c^{2}x^{2}+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{asinh}\left(cx\right)\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*d*x**2+d)**(5/2)*(a+b*asinh(c*x))/(g*x+f),x)
```

```
[Out] Integral((d*(c**2*x**2 + 1))**(5/2)*(a + b*asinh(c*x))/(f + g*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*d*x^2+d)^(5/2)*(a+b*arcsinh(c*x))/(g*x+f),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c\,x))\,\left(d\,c^2\,x^2+d\right)^{5/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x),x)
[Out] int(((a + b*asinh(c*x))*(d + c^2*d*x^2)^(5/2))/(f + g*x), x)
```

3.47 $\int \frac{(f+gx)^3(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$-\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}}-\frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}-\frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}}+\frac{3f^2g(1+c^2x^2)\left(a+b\sinh\right.}{c^2\sqrt{d+c^2dx^2}}$$

```
[Out] 3*f^2*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2/3*g^3*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^4/(c^2*d*x^2+d)^(1/2)+3/2*f*g^2*x*(c^2*x^2+1)*
(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/3*g^3*x^2*(c^2*x^2+1)*(a+b*arc
sinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c^2*x^2+1)^(1/2)/c/(c^2*d*x
^2+d)^(1/2)+2/3*b*g^3*x*(c^2*x^2+1)^(1/2)/c^3/(c^2*d*x^2+d)^(1/2)-3/4*b*f*g
^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^
3/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{f^3\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{3f^2g(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{3fg^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{c^2dx^2+d}}+\frac{g^3x^2(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3c^2\sqrt{c^2dx^2+d}}-\frac{2g^3(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{3c^4\sqrt{c^2dx^2+d}}-\frac{3fg^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{3bf^2gx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}-\frac{3bfg^2x^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}-\frac{bg^3x^3\sqrt{c^2x^2+1}}{9c\sqrt{c^2dx^2+d}}+\frac{2bg^3x\sqrt{c^2x^2+1}}{3c^3\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) + (2*b*g^3*x*Sqrt[
1 + c^2*x^2])/(3*c^3*Sqrt[d + c^2*d*x^2]) - (3*b*f*g^2*x^2*Sqrt[1 + c^2*x^2
])/(4*c*Sqrt[d + c^2*d*x^2]) - (b*g^3*x^3*Sqrt[1 + c^2*x^2])/(9*c*Sqrt[d +
c^2*d*x^2]) + (3*f^2*g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^
2*d*x^2]) - (2*g^3*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^4*Sqrt[d + c^2*
d*x^2]) + (3*f*g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(2*c^2*Sqrt[d + c^
2*d*x^2]) + (g^3*x^2*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/(3*c^2*Sqrt[d + c^
2*d*x^2]) + (f^3*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d +
c^2*d*x^2]) - (3*f*g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(4*b*c^3*S
qrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] && !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^3(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{3f^2gx(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{3fg^2x^2(a+b\sinh^{-1}(c\right.}{\sqrt{1+c^2x^2}}}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(3f^2g\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh^{-1}(c}{\sqrt{1+c^2x^2}}}{\sqrt{d+c^2dx^2}}$$

$$= \frac{3f^2g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{3fg^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx\right.}{2c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} - \frac{bg^3x^3\sqrt{1+c^2x^2}}{9c\sqrt{d+c^2dx^2}} + \frac{3f^2}{}$$

$$= -\frac{3bf^2gx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} + \frac{2bg^3x\sqrt{1+c^2x^2}}{3c^3\sqrt{d+c^2dx^2}} - \frac{3bfg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} - \frac{bg^3}{9c}$$

Mathematica [A]
time = 0.56, size = 304, normalized size = 0.71

$$\frac{4\sqrt{d}\,g\left(-2bcx\sqrt{1+c^2x^2}\,(-6g^2+c^2(27f^2+g^2x^2))+3a(1+c^2x^2)(-4g^2+c^2(18f^2+9fgx+2g^2x^2))\right)+12b\sqrt{d}\,g(1+c^2x^2)(-4g^2+c^2(18f^2+9fgx+2g^2x^2))\sinh^{-1}(cx)+18bc\sqrt{d}\,f(2c^2f^2-3g^2)\sqrt{1+c^2x^2}\sinh^{-1}(cx)^2-27bc\sqrt{d}\,fg^2\sqrt{1+c^2x^2}\cosh\left(2\sinh^{-1}(cx)\right)+36acf(2c^2f^2-3g^2)\sqrt{d+c^2dx^2}\log\left(cdx+\sqrt{d}\sqrt{d+c^2dx^2}\right)}{72c^4\sqrt{d}\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*Sqrt[d]*g*(-2*b*c*x*Sqrt[1 + c^2*x^2]*(-6*g^2 + c^2*(27*f^2 + g^2*x^2))
+ 3*a*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))) + 12*b*S
qrt[d]*g*(1 + c^2*x^2)*(-4*g^2 + c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2))*ArcSin
h[c*x] + 18*b*c*Sqrt[d]*f*(2*c^2*f^2 - 3*g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
]^2 - 27*b*c*Sqrt[d]*f*g^2*Sqrt[1 + c^2*x^2]*Cosh[2*ArcSinh[c*x]] + 36*a*c*
f*(2*c^2*f^2 - 3*g^2)*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*
d*x^2]])/(72*c^4*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(382) = 764.
time = 6.04, size = 790, normalized size = 1.84

method	result

default	$\frac{a\,g^{3}x^{2}\sqrt{c^{2}d\,x^{2}+d}}{3c^{2}d}-\frac{2a\,g^{3}\sqrt{c^{2}d\,x^{2}+d}}{3d\,c^{4}}+\frac{3af\,g^{2}x\sqrt{c^{2}d\,x^{2}+d}}{2c^{2}d}-\frac{3af\,g^{2}\ln\left(\frac{x\,c^{2}d}{\sqrt{c^{2}d}}+\sqrt{c^{2}d\,x^{2}+d}\right)}{2c^{2}\sqrt{c^{2}d}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/3*a*g^3*x^2/c^2/d*(c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(c^2*d*x^2+d)^(1/2)
+3/2*a*f*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-3/2*a*f*g^2/c^2*ln(x*c^2*d/(c^2*d)
^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+3*a*f^2*g/c^2/d*(c^2*d*x^2+d)^(1/
2)+a*f^3*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/4
*(d*(c^2*x^2+1))^(1/2)*f*arcsinh(c*x)^2*(2*c^2*f^2-3*g^2)/(c^2*x^2+1)^(1/2)
/c^3/d+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4+4*(c^2*x^2+1)^(1/2)*x^3*c^3+5*
c^2*x^2+3*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(-1+3*arcsinh(c*x))/c^4/d/(c^2*x^2+1
)+3/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(
c^2*x^2+1)^(1/2))*f*g^2*(2*arcsinh(c*x)-1)/c^3/d/(c^2*x^2+1)+3/8*(d*(c^2*x^
2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g*(4*arcsinh(c*x)*c^2*f^2-4*c
^2*f^2-arcsinh(c*x)*g^2+g^2)/c^4/d/(c^2*x^2+1)+3/8*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*g*(4*arcsinh(c*x)*c^2*f^2+4*c^2*f^2-arcsinh
(c*x)*g^2-g^2)/c^4/d/(c^2*x^2+1)+3/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^
2*x^2*(c^2*x^2+1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*f*g^2*(2*arcsinh(c*x)+1)/c
^3/d/(c^2*x^2+1)+1/72*(d*(c^2*x^2+1))^(1/2)*(4*c^4*x^4-4*(c^2*x^2+1)^(1/2)*
x^3*c^3+5*c^2*x^2-3*(c^2*x^2+1)^(1/2)*c*x+1)*g^3*(1+3*arcsinh(c*x))/c^4/d/(
c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(cx))(f + gx)^3}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**3/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^3*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f + g\,x)^3\,(a + b\operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.48 $\int \frac{(f+gx)^2(a+b\sinh^{-1}(cx))}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=258

$$-\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}-\frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}}+\frac{2fg(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{g^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx\right.}{2c^2\sqrt{d+c^2dx^2}}$$

```
[Out] 2*f*g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)+1/2*g^2*x*(c^2
*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(c^2*x^2+1)^(1
/2)/c/(c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c^2*x^2+1)^(1/2)/c/(c^2*d*x^2+d)^(
1/2)+1/2*f^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
-1/4*g^2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c^3/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{f^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{2fg(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}+\frac{g^2x(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{c^2dx^2+d}}-\frac{g^2\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{4bc^3\sqrt{c^2dx^2+d}}-\frac{2bfgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}-\frac{bg^2x^2\sqrt{c^2x^2+1}}{4c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2]) - (b*g^2*x^2*Sqrt[1
+ c^2*x^2])/(4*c*Sqrt[d + c^2*d*x^2]) + (2*f*g*(1 + c^2*x^2)*(a + b*ArcSinh
[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (g^2*x*(1 + c^2*x^2)*(a + b*ArcSinh[c*x
]))/(2*c^2*Sqrt[d + c^2*d*x^2]) + (f^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x
])^2)/(2*b*c*Sqrt[d + c^2*d*x^2]) - (g^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c
*x])^2)/(4*b*c^3*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f^2(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{2fgx(a+b\sinh^{-1}(cx))}{\sqrt{1+c^2x^2}} + \frac{g^2x^2(a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\right.}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} + \frac{\left(2fg\sqrt{1+c^2x^2}\right)\int \frac{x(a+b\sinh^{-1}(c}{\sqrt{1+c^2x^2}}}{\sqrt{d+c^2dx^2}}$$

$$= \frac{2fg(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}} + \frac{g^2x(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{2c^2\sqrt{d+c^2dx^2}}$$

$$= -\frac{2bfgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}} - \frac{bg^2x^2\sqrt{1+c^2x^2}}{4c\sqrt{d+c^2dx^2}} + \frac{2fg(1+c^2x^2)\left(a+b\sinh\right.}{c^2\sqrt{d+c^2dx^2}}$$

Mathematica [A]
time = 0.36, size = 233, normalized size = 0.90

$$\frac{4c\sqrt{d}\,g\left(-4bcfx\sqrt{1+c^2x^2}+a(4f+gx)(1+c^2x^2)\right)+4bc\sqrt{d}\,g(4f+gx)(1+c^2x^2)\sinh^{-1}(cx)+2b\sqrt{d}\,(2c^2f^2-g^2)\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)^2-b\sqrt{d}\,g^2\sqrt{1+c^2x^2}\,\cosh\left(2\sinh^{-1}(cx)\right)+4a(2c^2f^2-g^2)\sqrt{d+c^2dx^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{8c^3\sqrt{d}\,\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (4*c*Sqrt[d]*g*(-4*b*c*f*x*Sqrt[1 + c^2*x^2] + a*(4*f + g*x)*(1 + c^2*x^2))
 + 4*b*c*Sqrt[d]*g*(4*f + g*x)*(1 + c^2*x^2)*ArcSinh[c*x] + 2*b*Sqrt[d]*(2*
c^2*f^2 - g^2)*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 - b*Sqrt[d]*g^2*Sqrt[1 + c^
2*x^2]*Cosh[2*ArcSinh[c*x]] + 4*a*(2*c^2*f^2 - g^2)*Sqrt[d + c^2*d*x^2]*Log
[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(8*c^3*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(230) = 460$.
time = 6.14, size = 486, normalized size = 1.88

method	result
default	$\frac{a\,g^2x\sqrt{c^2d\,x^2+d}}{2c^2d} - \frac{a\,g^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+d}\right)}{2c^2\sqrt{c^2d}} + \frac{2afg\sqrt{c^2d\,x^2+d}}{c^2d} + \frac{a\,f^2\ln\left(\frac{x\,c^2d}{\sqrt{c^2d}}+\sqrt{c^2d\,x^2+\right.}{\sqrt{c^2d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2*a*g^2*x/c^2/d*(c^2*d*x^2+d)^(1/2)-1/2*a*g^2/c^2*ln(x*c^2*d/(c^2*d)^(1/2
)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+2*a*f*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f^
2*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+b*(1/4*(d*(c^
2*x^2+1))^(1/2)*arcsinh(c*x)^2*(2*c^2*f^2-g^2)/(c^2*x^2+1)^(1/2)/c^3/d+1/16
*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3+2*c^2*x^2*(c^2*x^2+1)^(1/2)+2*c*x+(c^2*x^
2+1)^(1/2))*g^2*(2*arcsinh(c*x)-1)/c^3/d/(c^2*x^2+1)+(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)-1)/c^2/d/(c^2*x^2+1)+(d
*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x^2+1)^(1/2)*c*x+1)*f*g*(arcsinh(c*x)+1)/
c^2/d/(c^2*x^2+1)+1/16*(d*(c^2*x^2+1))^(1/2)*(2*c^3*x^3-2*c^2*x^2*(c^2*x^2+
1)^(1/2)+2*c*x-(c^2*x^2+1)^(1/2))*g^2*(2*arcsinh(c*x)+1)/c^3/d/(c^2*x^2+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*a
rcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))(f + gx)^2}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*(f + g*x)**2/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((g*x + f)^2*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)^2\,(a+b\,\mathrm{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.49 $\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/c^2/(c^2*d*x^2+d)^(1/2)-b*g*x*(c^2*x^2+1)^
(1/2)/c/(c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/
c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {5845, 5838, 5783, 5798, 8}

$$\frac{f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}+\frac{g(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{c^2dx^2+d}}-\frac{bgx\sqrt{c^2x^2+1}}{c\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + c^2*d*x^2])) + (g*(1 + c^2*x^2)*(a
+ b*ArcSinh[c*x]))/(c^2*Sqrt[d + c^2*d*x^2]) + (f*Sqrt[1 + c^2*x^2]*(a + b*
ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{(f+gx)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\sqrt{1+c^2x^2}\int \left(\frac{f\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}+\frac{gx\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\right)dx}{\sqrt{d+c^2dx^2}} \\
&= \frac{\left(f\sqrt{1+c^2x^2}\right)\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}+\frac{\left(g\sqrt{1+c^2x^2}\right)\int \frac{x\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{1+c^2x^2}}\,d\cdots}{\sqrt{d+c^2dx^2}} \\
&= \frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}-\cdots \\
&= -\frac{bgx\sqrt{1+c^2x^2}}{c\sqrt{d+c^2dx^2}}+\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{c^2\sqrt{d+c^2dx^2}}+\frac{f\sqrt{1+c^2x^2}\,(a\cdots}{2bc\sqrt{d+\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 158, normalized size = 1.32

$$\frac{2\sqrt{d}\,g\left(a+ac^2x^2-bcx\sqrt{1+c^2x^2}\right)+2b\sqrt{d}\,g(1+c^2x^2)\sinh^{-1}(cx)+bc\sqrt{d}\,f\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)^2+2acf\sqrt{d+c^2dx^2}\,\log\left(cdx+\sqrt{d}\,\sqrt{d+c^2dx^2}\right)}{2c^2\sqrt{d}\,\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcSinh[c*x]))/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (2*Sqrt[d]*g*(a + a*c^2*x^2 - b*c*x*Sqrt[1 + c^2*x^2]) + 2*b*Sqrt[d]*g*(1 +
 c^2*x^2)*ArcSinh[c*x] + b*c*Sqrt[d]*f*Sqrt[1 + c^2*x^2]*ArcSinh[c*x]^2 + 2
*a*c*f*Sqrt[d + c^2*d*x^2]*Log[c*d*x + Sqrt[d]*Sqrt[d + c^2*d*x^2]])/(2*c^2
*Sqrt[d]*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(108) = 216$.
time = 4.54, size = 227, normalized size = 1.89

method	result
default	$\frac{ag\sqrt{c^2 d\,x^2+d}}{c^2 d}+\frac{af\ln\left(\frac{x\,c^2 d}{\sqrt{c^2 d}}+\sqrt{c^2 d\,x^2+d}\right)}{\sqrt{c^2 d}}+b\left(\frac{\sqrt{d\,(c^2x^2+1)}\,f\operatorname{arcsinh}(cx)^2}{2\sqrt{c^2x^2+1}\,cd}+\frac{\sqrt{d\,(c^2x^2+1)}\,(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*g/c^2/d*(c^2*d*x^2+d)^(1/2)+a*f*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1
/2))/(c^2*d)^(1/2)+b*(1/2*(d*(c^2*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*f*arc
sinh(c*x)^2+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+(c^2*x^2+1)^(1/2)*c*x+1)*g*(
arcsinh(c*x)-1)/c^2/d/(c^2*x^2+1)+1/2*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2-(c^2*x
^2+1)^(1/2)*c*x+1)*g*(arcsinh(c*x)+1)/c^2/d/(c^2*x^2+1))
```

Maxima [A]
time = 0.27, size = 87, normalized size = 0.72

$$\frac{bf\operatorname{arsinh}(cx)^2}{2\,c\sqrt{d}}-\frac{bgx}{c\sqrt{d}}+\frac{af\operatorname{arsinh}(cx)}{c\sqrt{d}}+\frac{\sqrt{c^2dx^2+d}\,bg\operatorname{arsinh}(cx)}{c^2d}+\frac{\sqrt{c^2dx^2+d}\,ag}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*b*f*arcsinh(c*x)^2/(c*sqrt(d)) - b*g*x/(c*sqrt(d)) + a*f*arcsinh(c*x)/(
c*sqrt(d)) + sqrt(c^2*d*x^2 + d)*b*g*arcsinh(c*x)/(c^2*d) + sqrt(c^2*d*x^2
+ d)*a*g/(c^2*d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral((a*g*x + a*f + (b*g*x + b*f)*arcsinh(c*x))/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx))(f + gx)}{\sqrt{d(c^2x^2 + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*asinh(c*x))*(f + g*x)/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
[Out] integrate((g*x + f)*(b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f + g\,x)\,(a + b \operatorname{asinh}(c\,x))}{\sqrt{d\,c^2\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2),x)
[Out] int(((f + g*x)*(a + b*asinh(c*x)))/(d + c^2*d*x^2)^(1/2), x)
```

3.50 $\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}$$

```
[Out] 1/2*(a+b*arcsinh(c*x))^2*(c^2*x^2+1)^(1/2)/b/c/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{5783\}$

$$\frac{\sqrt{c^2x^2+1}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{c^2dx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^2)/(2*b*c*Sqrt[d + c^2*d*x^2])
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{d+c^2dx^2}}\,dx &= \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}\\ &= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)^2}{2bc\sqrt{d+c^2dx^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 1.02

$$\frac{\sqrt{1+c^2x^2}\,\sinh^{-1}(cx)\left(2a+b\sinh^{-1}(cx)\right)}{2c\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/Sqrt[d + c^2*d*x^2],x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*ArcSinh[c*x]*(2*a + b*ArcSinh[c*x]))/(2*c*Sqrt[d + c^2*d
*x^2])
```

Maple [A]
time = 0.40, size = 77, normalized size = 1.64

method	result	size
default	$\frac{a \ln\left(\frac{x\,c^{2} d}{\sqrt{c^{2} d}}+\sqrt{c^{2} d\,x^{2}+d}\right)}{\sqrt{c^{2} d}}+\frac{b \sqrt{d \left(c^{2} x^{2}+1\right)}\, \operatorname{arcsinh}\left(c x\right)^{2}}{2 \sqrt{c^{2} x^{2}+1}\, c d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x*c^2*d/(c^2*d)^(1/2)+(c^2*d*x^2+d)^(1/2))/(c^2*d)^(1/2)+1/2*b*(d*(c^2
*x^2+1))^(1/2)/(c^2*x^2+1)^(1/2)/c/d*arcsinh(c*x)^2
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.60

$$\frac{b \operatorname{arsinh}\left(cx\right)^{2}}{2\,c\sqrt{d}}+\frac{a \operatorname{arsinh}\left(cx\right)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*arcsinh(c*x)^2/(c*sqrt(d)) + a*arcsinh(c*x)/(c*sqrt(d))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b \operatorname{asinh}\left(cx\right)}{\sqrt{d\left(c^{2} x^{2}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/sqrt(d*(c**2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/sqrt(c^2*d*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*asinh(c*x))/(d + c^2*d*x^2)^(1/2), x)
```

3.51 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}-\frac{\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}}{cf+\sqrt{c^2}}\right.}{\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}}$$

```
[Out] (a+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2))
)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)-(a+b*arcsinh(c*
x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(
1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)+b*polylog(2,-(c*x+(c^2*x^2+1)^
(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(
c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2
)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(1/2)/(c^2*d*x^2+d)^(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5845, 5843, 3403, 2296, 2221, 2317, 2438}

$$\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}-\frac{\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}+\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}-\frac{b\sqrt{c^2x^2+1}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\sqrt{c^2f^2+g^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - S
qrt[c^2*f^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (Sqrt[1 +
 c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f
^2 + g^2])])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) + (b*Sqrt[1 + c^2*x^
2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2
*f^2 + g^2]*Sqrt[d + c^2*d*x^2]) - (b*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^Arc
Sinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/(Sqrt[c^2*f^2 + g^2]*Sqrt[d + c
^2*d*x^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\sqrt{1+c^2x^2}\,\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf-g+e^{2x}g}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2+g^2}}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\left(2g\sqrt{\right.}}{}$$

$$= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\,(a+}{}$$

$$= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\,(a+}{}$$

$$= \frac{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}} - \frac{\sqrt{1+c^2x^2}\,(a+}{}$$

Mathematica [A]
time = 0.39, size = 256, normalized size = 0.79

$$\frac{\frac{a\log(f+gx)}{\sqrt{d}} - \frac{a\log\left(d(g-c^2fx)+\sqrt{d}\,\sqrt{c^2f^2+g^2}\,\sqrt{d+c^2dx^2}\right)}{\sqrt{d}} + \frac{b\sqrt{1+c^2x^2}\left(\sinh^{-1}(cx)\left(\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)-\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)+\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right)-\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)}{\sqrt{d+c^2dx^2}}}{\sqrt{c^2f^2+g^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 +
 g^2]*Sqrt[d + c^2*d*x^2]])/Sqrt[d] + (b*Sqrt[1 + c^2*x^2]*(ArcSinh[c*x]*(L
og[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - Log[1 + (E^ArcSinh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])]) + PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c
*f) + Sqrt[c^2*f^2 + g^2])] - PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))]))/Sqrt[d + c^2*d*x^2])/Sqrt[c^2*f^2 + g^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(329) = 658$.

time = 2.34, size = 678, normalized size = 2.09

method	result
default	$-\frac{a\ln\left(\frac{\frac{2d\left(c^2f^2+g^2\right)}{g^2}-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}\sqrt{\left(x+\frac{f}{g}\right)^2c^2d-\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+\frac{d\left(c^2f^2+g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{\frac{d\left(c^2f^2+g^2\right)}{g^2}}}+\frac{b\sqrt{d\left(c^2x^2\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -a/g/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f
/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c
^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2
)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln
((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(
1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4
*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*arcsinh(c*x)*ln(((c*x+(c^2*x^2+1)^(1/2))*
g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))+b*(d*(c^2*x^2+1))^(1/
2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2
+g^2)*dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2
*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*f^2+g^2)^(1/2)*(c^2*x^2+1)^(
1/2)/d/(c^4*f^2*x^2+c^2*g^2*x^2+c^2*f^2+g^2)*dilog(((c*x+(c^2*x^2+1)^(1/2))
*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*x^
2 + d*g*x + d*f), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}\left(cx\right)}{\sqrt{d\left(c^{2}x^{2}+1\right)}\,\left(f+gx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)/(c^2*d*x^2+d)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{\left(f+g\,x\right)\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((f + g*x)*(d + c^2*d*x^2)^(1/2)), x)
```

3.52 $\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx$

Optimal. Leaf size=444

$$-\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}}+\frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\left(c^2f^2+g^2\right)^{3/2}\sqrt{d+c^2dx^2}}-\frac{c^2f\sqrt{1+}}{}$$

```
[Out] -g*(c^2*x^2+1)*(a+b*arcsinh(c*x))/(c^2*f^2+g^2)/(g*x+f)/(c^2*d*x^2+d)^(1/2)
+b*c*ln(g*x+f)*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(c^2*d*x^2+d)^(1/2)+c^2*f*(a
+b*arcsinh(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*
(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*arcsin
h(c*x))*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+
1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+d)^(1/2)+b*c^2*f*polylog(2,-(c*x+(c
^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^
2)^(3/2)/(c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(
c*f+(c^2*f^2+g^2)^(1/2)))*(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)^(3/2)/(c^2*d*x^2+
d)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5845, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$-\frac{g(c^2x^2+1)\left(a+b\sinh^{-1}(cx)\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)(f+gx)}+\frac{c^2f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}}-\frac{c^2f\sqrt{c^2x^2+1}\left(a+b\sinh^{-1}(cx)\right)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}}+\frac{bc^2f\sqrt{c^2x^2+1}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}}-\frac{bc^2f\sqrt{c^2x^2+1}\,\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)^{3/2}}+\frac{bc\sqrt{c^2x^2+1}\log(f+gx)}{\sqrt{c^2dx^2+d}\,(c^2f^2+g^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] -((g*(1 + c^2*x^2)*(a + b*ArcSinh[c*x]))/((c^2*f^2 + g^2)*(f + g*x)*Sqrt[d
+ c^2*d*x^2])) + (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^A
rcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d
+ c^2*d*x^2]) - (c^2*f*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])*Log[1 + (E^Ar
cSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d +
 c^2*d*x^2]) + (b*c*Sqrt[1 + c^2*x^2]*Log[f + g*x])/((c^2*f^2 + g^2)*Sqrt[d
 + c^2*d*x^2]) + (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
 - (b*c^2*f*Sqrt[1 + c^2*x^2]*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c
^2*f^2 + g^2]))])/((c^2*f^2 + g^2)^(3/2)*Sqrt[d + c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
```

```
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5845

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[Simp[(d + e*x^2)^p/(1 + c^2*x^2)
^p], Int[(f + g*x)^m*(1 + c^2*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && IntegerQ[
p - 1/2] &&  !GtQ[d, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{d+c^2dx^2}}\,dx = \frac{\sqrt{1+c^2x^2}\int \frac{a+b\sinh^{-1}(cx)}{(f+gx)^2\sqrt{1+c^2x^2}}\,dx}{\sqrt{d+c^2dx^2}}$$

$$= \frac{\left(c\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\sinh(x))^2}\,dx, x, \sinh^{-1}(cx)\right)}{\sqrt{d+c^2dx^2}}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(c^2f\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\sinh(x)}\right.}{(c^2f^2+g^2)\sqrt{d+c^2d}}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{\left(bc\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{cf+x}\,dx, x,\right.}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{bc\sqrt{1+c^2x^2}\log(f+gx)}{(c^2f^2+g^2)\sqrt{d+c^2dx^2}} + \frac{\left(2c^2fg\sqrt{\right.}{}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{(c^2f^2+g^2)^{3/2}\sqrt{d+}}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{(c^2f^2+g^2)^{3/2}\sqrt{d+}}$$

$$= -\frac{g(1+c^2x^2)\left(a+b\sinh^{-1}(cx)\right)}{(c^2f^2+g^2)(f+gx)\sqrt{d+c^2dx^2}} + \frac{c^2f\sqrt{1+c^2x^2}\left(a+b\sinh^{-1}(cx)\right)\log}{(c^2f^2+g^2)^{3/2}\sqrt{d+}}$$

<u>Mathematica</u> [A]
time = 1.50, size = 448, normalized size = 1.01

$$\frac{-ag\sqrt{c^2f^2+g^2}(d+c^2dx^2)+ac^2\sqrt{d}f(f+gx)\sqrt{d+c^2dx^2}\log(f+gx)-ac^2\sqrt{d}f(f+gx)\sqrt{d+c^2dx^2}\log\left(d(g-c^2fx)+\sqrt{d}\sqrt{c^2f^2+g^2}\sqrt{d+c^2dx^2}\right)-bd\sqrt{1+c^2x^2}\left(g\sqrt{c^2f^2+g^2}\sqrt{1+c^2x^2}\sinh^{-1}(cx)-c^2f(f+gx)\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)+c^2f(f+gx)\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)-c\sqrt{c^2f^2+g^2}(f+gx)\log(c(f+gx))-c^2f(f+gx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right)+c^2f(f+gx)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)}{d(c^2f^2+g^2)^{3/2}(f+gx)\sqrt{d+c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c*x])/((f + g*x)^2*Sqrt[d + c^2*d*x^2]),x]
```

```
[Out] (-(a*g*Sqrt[c^2*f^2 + g^2]*(d + c^2*d*x^2)) + a*c^2*Sqrt[d]*f*(f + g*x)*Sqr
t[d + c^2*d*x^2]*Log[f + g*x] - a*c^2*Sqrt[d]*f*(f + g*x)*Sqrt[d + c^2*d*x^
2]*Log[d*(g - c^2*f*x) + Sqrt[d]*Sqrt[c^2*f^2 + g^2]*Sqrt[d + c^2*d*x^2]] -
 b*d*Sqrt[1 + c^2*x^2]*(g*Sqrt[c^2*f^2 + g^2]*Sqrt[1 + c^2*x^2]*ArcSinh[c*x
] - c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2
```

```
*f^2 + g^2])] + c^2*f*(f + g*x)*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])] - c*Sqrt[c^2*f^2 + g^2]*(f + g*x)*Log[c*(f + g*x)
] - c^2*f*(f + g*x)*PolyLog[2, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 +
g^2])] + c^2*f*(f + g*x)*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))]))/(d*(c^2*f^2 + g^2)^(3/2)*(f + g*x)*Sqrt[d + c^2*d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(442) = 884$.
time = 8.37, size = 1770, normalized size = 3.99

method	result	size
default	Expression too large to display	1770

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -a/d/(c^2*f^2+g^2)/(x+f/g)*((x+f/g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2+g^2)/(d*(c^2*f^2+g^2)/g^2)^(1/2)*ln((2*d
*(c^2*f^2+g^2)/g^2-2*c^2*d*f/g*(x+f/g)+2*(d*(c^2*f^2+g^2)/g^2)^(1/2)*((x+f/
g)^2*c^2*d-2*c^2*d*f/g*(x+f/g)+d*(c^2*f^2+g^2)/g^2)^(1/2))/(x+f/g))+b*(d*(c
^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^3*c^4*f
-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*f^2+g^2)/(g*x+f)*x*c^2*f-b*(d*
(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^2)/(g*x+f)*x^2*c^2
*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)/(c^2*f^2+g^2)/(
g*x+f)*x*c*g+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)/(c^2*f^2+g^
2)/(g*x+f)*x*c^2*f+b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+1)^(1/2)
/(c^2*f^2+g^2)/(g*x+f)*c*f-b*(d*(c^2*x^2+1))^(1/2)*arcsinh(c*x)/d/(c^2*x^2+
1)/(c^2*f^2+g^2)/(g*x+f)*g+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6
*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*arc
sinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g
^2)^(1/2))/(-c*f+(c^2*f^2+g^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^
4)*c^2*f*arcsinh(c*x)*(c^2*f^2+g^2)^(1/2)*ln(((c*x+(c^2*x^2+1)^(1/2))*g+c*f
+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*
(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^
2*f^2*g^2+g^4)*c^3*ln(c*x+(c^2*x^2+1)^(1/2))*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c
^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*
f^2*g^2+g^4)*c^3*ln((c*x+(c^2*x^2+1)^(1/2))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2
))-g)*f^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2)*
dilog((-(c*x+(c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2+g^2)^(1/2))/(-c*f+(c^2*f^2+g
^2)^(1/2)))-b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*
f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c^2*f*(c^2*f^2+g^2)^(1/2
)*dilog(((c*x+(c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2+g^2)^(1/2))/(c*f+(c^2*f^2+g
```

```
^2)^(1/2)))-2*b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^
4*f^2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln(c*x+(c^2*x^2+1)^(
1/2))*g^2+b*(d*(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)/d/(c^6*f^4*x^2+2*c^4*f^
2*g^2*x^2+c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2+g^4)*c*ln((c*x+(c^2*x^2+1)^(1/2
))^2*g+2*c*f*(c*x+(c^2*x^2+1)^(1/2))-g)*g^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(c^2*d*x^2 + d)*(b*arcsinh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*d*
f*g*x^3 + 2*d*f*g*x + d*f^2 + (c^2*d*f^2 + d*g^2)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{asinh}{\left(c x \right)}}{\sqrt{d \left(c^{2} x^{2} + 1\right)} \left(f + g x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))/(g*x+f)**2/(c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))/(sqrt(d*(c**2*x**2 + 1))*(f + g*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))/(g*x+f)^2/(c^2*d*x^2+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)/(sqrt(c^2*d*x^2 + d)*(g*x + f)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c\,x)}{{\left(f+g\,x\right)}^2\,\sqrt{d\,c^2\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asinh(c*x))/((f + g*x)^2*(d + c^2*d*x^2)^(1/2)), x)
```

3.53 $\int \frac{(a+b\sinh^{-1}(cx))^n \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}} \, dx$

Optimal. Leaf size=37

$$\operatorname{Int}\left(\frac{(a+b\sinh^{-1}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
[Out] Defer[Int][((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x
]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx = \int \frac{(a+b\sinh^{-1}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Mathematica [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
[Out] Integrate[((a + b*ArcSinh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^n \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^n*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(n>0)', see `assume?` for more detai
ls)Is n
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**n*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^n*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^n}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^n)/(c^2*x^2 + 1)^(1/2), x)
```

3.54 $\int \frac{(a+b\sinh^{-1}(cx))^2\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=438

$$\frac{m(a+b\sinh^{-1}(cx))^4}{12b^2c}-\frac{m(a+b\sinh^{-1}(cx))^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{3bc}-\frac{m(a+b\sinh^{-1}(cx))^3\log\left(1+\frac{}{cf}\right.}{3bc}$$

```
[Out] 1/12*m*(a+b*arcsinh(c*x))^4/b^2/c+1/3*(a+b*arcsinh(c*x))^3*ln(h*(g*x+f)^m)/
b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/b/c-1/3*m*(a+b*arcsinh(c*x))^3*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g
/(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))^2*polylog(2,-(c*x+(c^2
*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))^2*polylo
g(2,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+2*b*m*(a+b*arcs
inh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c
+2*b*m*(a+b*arcsinh(c*x))*polylog(3,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^
2+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2
+g^2)^(1/2)))/c-2*b^2*m*polylog(4,-(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+
g^2)^(1/2)))/c
```

Rubi [A]
time = 0.49, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {5783, 5846, 5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$\frac{m(a+b\sinh^{-1}(cx))^4}{12b^2c}-\frac{m(a+b\sinh^{-1}(cx))^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m(a+b\sinh^{-1}(cx))^2\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}+\frac{2bm(a+b\sinh^{-1}(cx))\text{Li}_3\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}+\frac{2bm(a+b\sinh^{-1}(cx))\text{Li}_3\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m(a+b\sinh^{-1}(cx))^3\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{3bc}-\frac{m(a+b\sinh^{-1}(cx))^3\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{3bc}+\frac{(a+b\sinh^{-1}(cx))^3\log(h(f+gx)^m)}{3bc}-\frac{2b^2m\text{Li}_4\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{2b^2m\text{Li}_4\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^4)/(12*b^2*c) - (m*(a + b*ArcSinh[c*x])^3*Log[1 + (
E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(3*b*c) - (m*(a + b*ArcSinh
[c*x])^3*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(3*b*c) +
 ((a + b*ArcSinh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c) - (m*(a + b*ArcSinh[c*
x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m
*(a + b*ArcSinh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2
 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)
/(c*f - Sqrt[c^2*f^2 + g^2]))])/c + (2*b*m*(a + b*ArcSinh[c*x])*PolyLog[3,
-((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4,
 -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (2*b^2*m*PolyLog[4
, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5846

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[g*(m/(b*c*S
```

```
qrt[d]*(n + 1))), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\int \frac{(a+b\sinh^{-1}(cx))}{f+gx}}{3bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc} - \frac{(gm)\mathrm{Subst}\left(\int \frac{(a+bx)^3}{cf+g\,\mathrm{s}}\right.}{}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\mathrm{s}}}{cf-\sqrt{}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\mathrm{s}}}{cf-\sqrt{}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\mathrm{s}}}{cf-\sqrt{}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\mathrm{s}}}{cf-\sqrt{}}\right.}{3bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{12b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\mathrm{s}}}{cf-\sqrt{}}\right.}{3bc}$$

Mathematica [A]
time = 0.19, size = 397, normalized size = 0.91

$$-\frac{-\frac{m\left(a+b\sinh^{-1}(cx)\right)^4}{4b} + m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right) + m\left(a+b\sinh^{-1}(cx)\right)^3\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right) - \left(a+b\sinh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right) + 3bm\left(\left(a+b\sinh^{-1}(cx)\right)^2\mathrm{PolyLog}\left(2,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right) - 2b\left(a+b\sinh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right) + 2b^2\mathrm{PolyLog}\left(4,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right)\right) + 3bm\left(\left(a+b\sinh^{-1}(cx)\right)^2\mathrm{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right) - 2b\left(a+b\sinh^{-1}(cx)\right)\mathrm{PolyLog}\left(3,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right) + 2b^2\mathrm{PolyLog}\left(4,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -1/3*(-1/4*(m*(a + b*ArcSinh[c*x])^4)/b + m*(a + b*ArcSinh[c*x])^3*Log[1 +
(E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] + m*(a + b*ArcSinh[c*x])^3*
Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] - (a + b*ArcSinh[c*
x])^3*Log[h*(f + g*x)^m] + 3*b*m*((a + b*ArcSinh[c*x])^2*PolyLog[2, (E^ArcS
inh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] - 2*b*(a + b*ArcSinh[c*x])*Poly
Log[3, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])] + 2*b^2*PolyLog[4
```

```
, (E^ArcSinh[c*x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + 3*b*m*((a + b*ArcSi
nh[c*x])^2*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))] -
2*b*(a + b*ArcSinh[c*x])*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 + g^2]))] + 2*b^2*PolyLog[4, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g
^2]))]))/(b*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(cx))^2 \ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral((b^2*arcsinh(c*x)^2 + 2*a*b*arcsinh(c*x) + a^2)*log((g*x + f)^m*h)
/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{asinh}\,(cx))^2 \log\left(h(f+gx)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))**2*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**2*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))^2*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 3.64Unable to divide
, perhaps due to rounding error%%%{720,[0,4,1,1,1,1,4,0]%%%}+%%%{-1260,[0,4
,1,1,1
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^2}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x))^2)/(c^2*x^2 + 1)^(1/2), x)
```

3.55 $\int \frac{(a+b\sinh^{-1}(cx)) \log(h(f+gx)^m)}{\sqrt{1+c^2x^2}} \, dx$

Optimal. Leaf size=332

$$\frac{m(a+b\sinh^{-1}(cx))^3}{6b^2c}-\frac{m(a+b\sinh^{-1}(cx))^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{2bc}-\frac{m(a+b\sinh^{-1}(cx))^2\log\left(1+\frac{}{cf}\right.}{2bc}$$

```
[Out] 1/6*m*(a+b*arcsinh(c*x))^3/b^2/c+1/2*(a+b*arcsinh(c*x))^2*ln(h*(g*x+f)^m)/b
/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+
g^2)^(1/2)))/b/c-1/2*m*(a+b*arcsinh(c*x))^2*ln(1+(c*x+(c^2*x^2+1)^(1/2))*g/
(c*f+(c^2*f^2+g^2)^(1/2)))/b/c-m*(a+b*arcsinh(c*x))*polylog(2,-(c*x+(c^2*x^
2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*(a+b*arcsinh(c*x))*polylog(2,-
(c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+
(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c+b*m*polylog(3,-(c*x+(c^2*
x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Rubi [A]
time = 0.40, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5783, 5846, 5827, 5680, 2221, 2611, 2320, 6724}

$$\frac{m(a+b\sinh^{-1}(cx))^3}{6b^2c}-\frac{m(a+b\sinh^{-1}(cx))\,\mathrm{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m(a+b\sinh^{-1}(cx))\,\mathrm{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m(a+b\sinh^{-1}(cx))^2\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{2bc}-\frac{m(a+b\sinh^{-1}(cx))^2\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{2bc}+\frac{(a+b\sinh^{-1}(cx))^2\log(h(f+gx)^m)}{2bc}+\frac{bm\,\mathrm{Li}_3\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}+\frac{bm\,\mathrm{Li}_3\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*(a + b*ArcSinh[c*x])^3)/(6*b^2*c) - (m*(a + b*ArcSinh[c*x])^2*Log[1 + (E
^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])])/(2*b*c) - (m*(a + b*ArcSinh[
c*x])^2*Log[1 + (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])])/(2*b*c) +
((a + b*ArcSinh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c) - (m*(a + b*ArcSinh[c*x
])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*(a
 + b*ArcSinh[c*x])*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^
2]))])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])
)])/c + (b*m*PolyLog[3, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])
/c
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5846

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcSinh[(c_.)*(x_)]*(b_
.))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[Log[h*(f + g*x)^m]*
((a + b*ArcSinh[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] - Dist[g*(m/(b*c*S
qrt[d]*(n + 1))), Int[(a + b*ArcSinh[c*x])^(n + 1)/(f + g*x), x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[e, c^2*d] && GtQ[d, 0] && IGtQ[n,
 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\int \frac{\left(a+b\sinh^{-1}(cx)\right)^2}{f+gx}\,d}{2bc}$$

$$= \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} - \frac{(gm)\text{Subst}\left(\int \frac{(a+bx)^2\cos}{cf+g\sinh}\right.}{2b}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} + \frac{\left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc} -$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh}}{cf-\sqrt{c^2}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh}}{cf-\sqrt{c^2}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh}}{cf-\sqrt{c^2}}\right.}{2bc}$$

$$= \frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{6b^2c} - \frac{m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh}}{cf-\sqrt{c^2}}\right.}{2bc}$$

Mathematica [A]
time = 0.18, size = 304, normalized size = 0.92

$$\frac{\frac{m\left(a+b\sinh^{-1}(cx)\right)^3}{3b} - m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right) - m\left(a+b\sinh^{-1}(cx)\right)^2\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right) + \left(a+b\sinh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right) + 2bm\left(-\left(\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right)\right) + b\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(cx)}g}{-cf+\sqrt{c^2f^2+g^2}}\right)\right) + 2bm\left(-\left(\left(a+b\sinh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right) + b\text{PolyLog}\left(3,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcSinh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 + c^2*x^2],x]
```

```
[Out] ((m*(a + b*ArcSinh[c*x])^3)/(3*b) - m*(a + b*ArcSinh[c*x])^2*Log[1 + (E^Arc
Sinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2])] - m*(a + b*ArcSinh[c*x])^2*Log[1
```

```
+ (E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2])] + (a + b*ArcSinh[c*x])^2*
Log[h*(f + g*x)^m] + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, (E^ArcSinh[c*
x]*g)/(-(c*f) + Sqrt[c^2*f^2 + g^2])]) + b*PolyLog[3, (E^ArcSinh[c*x]*g)/(-
(c*f) + Sqrt[c^2*f^2 + g^2])]) + 2*b*m*(-((a + b*ArcSinh[c*x])*PolyLog[2, -
((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]) + b*PolyLog[3, -((E^ArcS
inh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))]))/(2*b*c)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(cx)) \ln(h(gx + f)^m)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
[Out] int((a+b*arcsinh(c*x))*ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
[Out] integral((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(cx)) \log(h(f + gx)^m)}{\sqrt{c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(c*x))*ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))*log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(c*x))*log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)*log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*asinh(c*x)))/(c^2*x^2 + 1)^(1/2), x)
```

3.56 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=197

$$\frac{m\sinh^{-1}(cx)^2}{2c}-\frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}+\frac{\sinh}{}$$

```
[Out] 1/2*m*arcsinh(c*x)^2/c+arcsinh(c*x)*ln(h*(g*x+f)^m)/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*arcsinh(c*x)*ln(1+(
c*x+(c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2+g^2)^(1/2)))/c-m*polylog(2,-(c*x+(c^2*x^2+1
)^(1/2))*g/(c*f+(c^2*f^2+g^2)^(1/2)))/c
```

Rubi [A]
time = 0.22, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {221, 2451, 5827, 5680, 2221, 2317, 2438}

$$-\frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\text{Li}_2\left(-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{cf-\sqrt{c^2f^2+g^2}}+1\right)}{c}-\frac{m\sinh^{-1}(cx)\log\left(\frac{ge^{\sinh^{-1}(cx)}}{\sqrt{c^2f^2+g^2}+cf}+1\right)}{c}+\frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c}+\frac{m\sinh^{-1}(cx)^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*f
- Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (E^ArcSinh[c*x]*g)/(c*
f + Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m])/c - (m*Pol
yLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c - (m*PolyLog[
2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2451

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}}\,dx = \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sinh^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{x\cosh(x)}{c^2f+cg\sinh(x)}\,dx, x, \sinh^{-1}(cx)\right.$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} + \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\text{Subst}\left(\int \frac{e^x x}{c^2f+ce^xg-c\sqrt{}}\right.$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\right.}{}$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\right.}{}$$

$$= \frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(\right.}{}$$

Mathematica [A]
time = 0.02, size = 206, normalized size = 1.05

$$\frac{m\sinh^{-1}(cx)^2}{2c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}g}{c^2f-c\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\sinh^{-1}(cx)\log\left(1+\frac{ce^{\sinh^{-1}(cx)}g}{c^2f+c\sqrt{c^2f^2+g^2}}\right)}{c} + \frac{\sinh^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - \frac{m\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf-\sqrt{c^2f^2+g^2}}\right)}{c} - \frac{m\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(cx)}g}{cf+\sqrt{c^2f^2+g^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] (m*ArcSinh[c*x]^2)/(2*c) - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x]*g)/(c^
2*f - c*Sqrt[c^2*f^2 + g^2])])/c - (m*ArcSinh[c*x]*Log[1 + (c*E^ArcSinh[c*x
]*g)/(c^2*f + c*Sqrt[c^2*f^2 + g^2])])/c + (ArcSinh[c*x]*Log[h*(f + g*x)^m]
)/c - (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f - Sqrt[c^2*f^2 + g^2]))])/c -
 (m*PolyLog[2, -((E^ArcSinh[c*x]*g)/(c*f + Sqrt[c^2*f^2 + g^2]))])/c
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(h(f+gx)^{m}\right)}{\sqrt{c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(c**2*x**2+1)**(1/2),x)
[Out] Integral(log(h*(f + g*x)**m)/sqrt(c**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(h\,{\left(f+g\,x\right)}^{m}\right)}{\sqrt{c^{2}\,x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(c^2*x^2 + 1)^(1/2), x)
```

3.57 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1+c^2x^2}\,(a+b\sinh^{-1}(cx))}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx=\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1+c^2x^2}\,\left(a+b\sinh^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])),x]
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\left(a+b\,\mathrm{arcsinh}\,(cx)\right)\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 + b)*
arcsinh(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(h(f+gx)^m\right)}{(a+b\operatorname{asinh}(cx))\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*asinh(c*x))/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/((a + b*asinh(c*x))*sqrt(c**2*x**2 + 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arcsinh(c*x))/(c^2*x^2+1)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(c^2*x^2 + 1)*(b*arcsinh(c*x) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)\,\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/((a + b*asinh(c*x))*(c^2*x^2 + 1)^(1/2)), x)
```

3.58 $\int x^3 \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=131

$$\frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2}-\frac{x^3\sqrt{1+(a+bx)^2}}{16b}-\frac{(4a(16-19a^2)-(9-26a^2)\,(a+bx))\,\sqrt{1+(a+bx)^2}}{96b^4}-\frac{(3-2$$

```
[Out] -1/32*(8*a^4-24*a^2+3)*arcsinh(b*x+a)/b^4+1/4*x^4*arcsinh(b*x+a)+7/48*a*x^2
*(1+(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1+(b*x+a)^2)^(1/2)/b-1/96*(4*a*(-19*a^2+
16)-(-26*a^2+9)*(b*x+a))*(1+(b*x+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5859, 5828, 757, 847, 794, 221}

$$-\frac{(4a(16-19a^2)-(9-26a^2)\,(a+bx))\,\sqrt{(a+bx)^2+1}}{96b^4}-\frac{(8a^4-24a^2+3)\sinh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{(a+bx)^2+1}}{48b^2}+\frac{1}{4}x^4\sinh^{-1}(a+bx)-\frac{x^3\sqrt{(a+bx)^2+1}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[1 + (a + b*x)^2])/(48*b^2) - (x^3*Sqrt[1 + (a + b*x)^2])/(16*
b) - ((4*a*(16 - 19*a^2) - (9 - 26*a^2)*(a + b*x))*Sqrt[1 + (a + b*x)^2])/(
96*b^4) - ((3 - 24*a^2 + 8*a^4)*ArcSinh[a + b*x])/(32*b^4) + (x^4*ArcSinh[a
 + b*x])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx) - \frac{1}{4}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\sinh^{-1}(a+bx) - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-\frac{3-4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\frac{a}{b}\right.}{\sqrt{1+x^2}}\right.$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} + \frac{1}{4}x^4\sinh^{-1}(a+bx) - \frac{1}{48}\text{Subst}\left(\int\right.$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{(4a(16-19a^2)-(9-26a^2)(a+b}{96b^4}$$

$$= \frac{7ax^2\sqrt{1+(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1+(a+bx)^2}}{16b} - \frac{(4a(16-19a^2)-(9-26a^2)(a+b}{96b^4}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 0.73

$$\frac{\sqrt{1+a^2+2abx+b^2x^2}\,(50a^3+9bx-26a^2bx-6b^3x^3+a(-55+14b^2x^2)) - 3(3-24a^2+8a^4-8b^4x^4)\sinh^{-1}(a+bx)}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x],x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*x^3
 + a*(-55 + 14*b^2*x^2)) - 3*(3 - 24*a^2 + 8*a^4 - 8*b^4*x^4)*ArcSinh[a + b
*x])/(96*b^4)
```

Maple [A]
time = 0.26, size = 200, normalized size = 1.53

method	result
derivativedivides	$\frac{-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}+a^3\sqrt{1+(bx}{}$

default	$\frac{-\operatorname{arcsinh}(bx+a)a^3(bx+a)+\frac{3\operatorname{arcsinh}(bx+a)a^2(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)^3+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^4}{4}+a^3\sqrt{1+(bx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(-arcsinh(b*x+a)*a^3*(b*x+a)+3/2*arcsinh(b*x+a)*a^2*(b*x+a)^2-arcsinh
(b*x+a)*a*(b*x+a)^3+1/4*arcsinh(b*x+a)*(b*x+a)^4+a^3*(1+(b*x+a)^2)^(1/2)-3/
2*a^2*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/2*arcsinh(b*x+a))+a*(1/3*(b*x+a)^2
*(1+(b*x+a)^2)^(1/2)-2/3*(1+(b*x+a)^2)^(1/2))-1/16*(b*x+a)^3*(1+(b*x+a)^2)^
(1/2)+3/32*(b*x+a)*(1+(b*x+a)^2)^(1/2)-3/32*arcsinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(114) = 228$.
time = 0.26, size = 318, normalized size = 2.43

$$\frac{1}{4}x^4\operatorname{arsinh}(bx+a)-\frac{1}{96}\left(\frac{6\sqrt{b^2x^2+2abx+a^2+1}\,x^3}{b^2}-\frac{14\sqrt{b^2x^2+2abx+a^2+1}\,ax^2}{b^3}+\frac{105a^4\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^5}+\frac{35\sqrt{b^2x^2+2abx+a^2+1}\,a^2x}{b^4}-\frac{90(a^2+1)a^2\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^5}-\frac{105\sqrt{b^2x^2+2abx+a^2+1}\,a^3}{b^5}-\frac{9\sqrt{b^2x^2+2abx+a^2+1}\,(a^2+1)x}{b^4}+\frac{9(a^2+1)^2\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^5}+\frac{55\sqrt{b^2x^2+2abx+a^2+1}\,(a^2+1)a}{b^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^3/b^
2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsinh(2*(b^2
*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 35*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)*a^2*x/b^4 - 90*(a^2 + 1)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*
a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3
/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*x/b^4 + 9*(a^2 + 1)^2*
arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 55*sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a/b^5)*b
```

Fricas [A]
time = 0.35, size = 110, normalized size = 0.84

$$\frac{3\left(8b^4x^4-8a^4+24a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)-\left(6b^3x^3-14ab^2x^2-50a^3+(26a^2-9)bx+55a\right)\sqrt{b^2x^2+2abx+a^2+1}}{96b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b*x + 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(117) = 234.
time = 0.30, size = 255, normalized size = 1.95

$$\begin{cases} -\frac{a^4 \operatorname{asinh}(a+bx)}{4b^4} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}}{48b^4} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}}{48b^3} + \frac{3a^2 \operatorname{asinh}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}}{48b^2} - \frac{55a\sqrt{a^2+2abx+b^2x^2+1}}{96b^4} + \frac{x^4 \operatorname{asinh}(a+bx)}{4} - \frac{x^3\sqrt{a^2+2abx+b^2x^2+1}}{16b} + \frac{3x\sqrt{a^2+2abx+b^2x^2+1}}{32b^3} - \frac{3 \operatorname{asinh}(a+bx)}{32b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{asinh}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(48
*b**3) + 3*a**2*asinh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/(48*b**2) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(96*b**4
) + x**4*asinh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(16*b
) + 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(32*b**3) - 3*asinh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

Giac [A]
time = 0.41, size = 162, normalized size = 1.24

$$\frac{1}{4}x^4\log\left(bx+a+\sqrt{(bx+a)^2+1}\right) - \frac{1}{96}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x\left(\frac{3\,x}{b^2}-\frac{7\,a}{b^3}\right)+\frac{26\,a^2b^3-9\,b^3}{b^7}\right)x-\frac{5\,(10\,a^3b^2-11\,ab^2)}{b^7}\right)-\frac{3\,(8\,a^4-24\,a^2+3)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b^4|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 - 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 - 11*a*b^2)/b^7) - 3*(8*a^4 - 24*a^2 + 3)*log(-a*b - (x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^4*abs(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\operatorname{asinh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x),x)
```

```
[Out] int(x^3*asinh(a + b*x), x)
```

3.59 $\int x^2 \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=90

$$-\frac{x^2\sqrt{1+(a+bx)^2}}{9b}+\frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3}-\frac{a(3-2a^2)\sinh^{-1}(a+bx)}{6b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)$$

```
[Out] -1/6*a*(-2*a^2+3)*arcsinh(b*x+a)/b^3+1/3*x^3*arcsinh(b*x+a)-1/9*x^2*(1+(b*x
+a)^2)^(1/2)/b+1/18*(5*a*b*x-11*a^2+4)*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5828, 757, 794, 221}

$$\frac{(-11a^2+5abx+4)\sqrt{(a+bx)^2+1}}{18b^3}-\frac{a(3-2a^2)\sinh^{-1}(a+bx)}{6b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)-\frac{x^2\sqrt{(a+bx)^2+1}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x],x]
```

```
[Out] -1/9*(x^2*Sqrt[1 + (a + b*x)^2])/b + ((4 - 11*a^2 + 5*a*b*x)*Sqrt[1 + (a +
b*x)^2])/(18*b^3) - (a*(3 - 2*a^2)*ArcSinh[a + b*x])/(6*b^3) + (x^3*ArcSinh
[a + b*x])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{1}{3}x^3\sinh^{-1}(a+bx) - \frac{1}{9}\text{Subst}\left(\int \frac{\left(-\frac{2-3a^2}{b^2}-\frac{5ax}{b^2}\right)\left(-\frac{a}{b}+\right.}{\sqrt{1+x^2}}\right.$$

$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3} + \frac{1}{3}x^3\sinh^{-1}(a+bx)$$

$$= -\frac{x^2\sqrt{1+(a+bx)^2}}{9b} + \frac{(4-11a^2+5abx)\sqrt{1+(a+bx)^2}}{18b^3} - \frac{a(3-2a^2)\sinh^{-1}(a}{6b^3}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.82

$$\frac{(4-11a^2+5abx-2b^2x^2)\sqrt{1+a^2+2abx+b^2x^2}+(-9a+6a^3+6b^3x^3)\sinh^{-1}(a+bx)}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x],x]
```

```
[Out] ((4 - 11*a^2 + 5*a*b*x - 2*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-9
*a + 6*a^3 + 6*b^3*x^3)*ArcSinh[a + b*x])/(18*b^3)
```

Maple [A]
time = 0.28, size = 130, normalized size = 1.44

method	result
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-a^2\sqrt{1+(bx+a)^2}+a\left(\frac{(bx+a)\sqrt{1-}}{}\right.}{b^3}$
default	$\frac{\operatorname{arcsinh}(bx+a)a^2(bx+a)-\operatorname{arcsinh}(bx+a)a(bx+a)^2+\frac{\operatorname{arcsinh}(bx+a)(bx+a)^3}{3}-a^2\sqrt{1+(bx+a)^2}+a\left(\frac{(bx+a)\sqrt{1-}}{}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(arcsinh(b*x+a)*a^2*(b*x+a)-arcsinh(b*x+a)*a*(b*x+a)^2+1/3*arcsinh(b*
x+a)*(b*x+a)^3-a^2*(1+(b*x+a)^2)^(1/2)+a*(1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)-1
/2*arcsinh(b*x+a))-1/9*(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+2/9*(1+(b*x+a)^2)^(1/2
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(78) = 156$.
time = 0.27, size = 210, normalized size = 2.33

$$\frac{1}{3}x^3\operatorname{arsinh}(bx+a)-\frac{1}{18}b\left(\frac{2\sqrt{b^2x^2+2abx+a^2+1}\,x^2}{b^2}-\frac{15a^3\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^4}-\frac{5\sqrt{b^2x^2+2abx+a^2+1}\,ax}{b^3}+\frac{9(a^2+1)a\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^4}+\frac{15\sqrt{b^2x^2+2abx+a^2+1}\,a^2}{b^4}-\frac{4\sqrt{b^2x^2+2abx+a^2+1}\,(a^2+1)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x^2/
b^2 - 15*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^
4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b^3 + 9*(a^2 + 1)*a*arcsinh(2*(
b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 15*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)/b^
4)
```

Fricas [A]
time = 0.34, size = 91, normalized size = 1.01

$$\frac{3\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+2\,abx+a^2+1}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(83) = 166$.
time = 0.19, size = 170, normalized size = 1.89

$$\begin{cases} \frac{a^3 \operatorname{asinh}(a+bx)}{3b^3} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}}{18b^2} - \frac{a \operatorname{asinh}(a+bx)}{2b^3} + \frac{x^3 \operatorname{asinh}(a+bx)}{3} - \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}}{9b} + \frac{2\sqrt{a^2+2abx+b^2x^2+1}}{9b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{asinh}(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a),x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(18*b**2
) - a*asinh(a + b*x)/(2*b**3) + x**3*asinh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 + 1)/(9*b) + 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(9*b*
*3), Ne(b, 0)), (x**3*asinh(a)/3, True))
```

Giac [A]
time = 0.42, size = 131, normalized size = 1.46

$$\frac{1}{3}\,x^3\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b-4\,b}{b^5}\right)+\frac{3\,(2\,a^3-3\,a)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b^3|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b - 4*b)/b^5) + 3*(2*a^3 - 3*a
)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*ab
s(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asinh}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x),x)
```

```
[Out] int(x^2*asinh(a + b*x), x)
```

3.60 $\int x \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)$$

```
[Out] 1/4*(-2*a^2+1)*arcsinh(b*x+a)/b^2+1/2*x^2*arcsinh(b*x+a)+3/4*a*(1+(b*x+a)^2
)^(1/2)/b^2-1/4*x*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5828, 757, 655, 221}

$$\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}}{4b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{x\sqrt{(a+bx)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[1 + (a + b*x)^2])/(4*b^2) - (x*Sqrt[1 + (a + b*x)^2])/(4*b) + ((1
 - 2*a^2)*ArcSinh[a + b*x])/(4*b^2) + (x^2*ArcSinh[a + b*x])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
```

```
- Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
- 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$

$$= -\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{-\frac{1-2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{1+x^2}}\,dx,x,a+b$$

$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{1}{2}x^2\sinh^{-1}(a+bx)+\frac{(1-2a^2)\,\text{Subst}\left(\right.}{}$$

$$= \frac{3a\sqrt{1+(a+bx)^2}}{4b^2}-\frac{x\sqrt{1+(a+bx)^2}}{4b}+\frac{(1-2a^2)\sinh^{-1}(a+bx)}{4b^2}+\frac{1}{2}x^2\sinh^{-1}($$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.79

$$\frac{(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}+(1-2a^2+2b^2x^2)\sinh^{-1}(a+bx)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (1 - 2*a^2 + 2*b^2*x^2)*Ar
cSinh[a + b*x])/(4*b^2)
```

Maple [A]
time = 0.26, size = 74, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)+a\sqrt{1+(bx+a)^2}-\frac{(bx+a)\sqrt{1+(bx+a)^2}}{4}+\frac{\operatorname{arcsinh}(bx+a)}{4}}{b^2}$
default	$\frac{\frac{\operatorname{arcsinh}(bx+a)(bx+a)^2}{2}-\operatorname{arcsinh}(bx+a)a(bx+a)+a\sqrt{1+(bx+a)^2}-\frac{(bx+a)\sqrt{1+(bx+a)^2}}{4}+\frac{\operatorname{arcsinh}(bx+a)}{4}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*arcsinh(b*x+a)*(b*x+a)^2-arcsinh(b*x+a)*a*(b*x+a)+a*(1+(b*x+a)^2
)^(1/2)-1/4*(b*x+a)*(1+(b*x+a)^2)^(1/2)+1/4*arcsinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(64) = 128.
time = 0.26, size = 149, normalized size = 1.96

$$\frac{1}{2}x^2\operatorname{arsinh}(bx+a)-\frac{1}{4}b\left(\frac{3a^2\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3}+\frac{\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^2}-\frac{(a^2+1)\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3}-\frac{3\sqrt{b^2x^2+2abx+a^2+1}\,a}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(b*x + a) - 1/4*b*(3*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2
*b^2 + 4*(a^2 + 1)*b^2))/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - (a
^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3
*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)
```

Fricas [A]
time = 0.34, size = 75, normalized size = 0.99

$$\frac{(2b^2x^2-2a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)-\sqrt{b^2x^2+2abx+a^2+1}\,(bx-3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^2
```

Sympy [A]
time = 0.12, size = 104, normalized size = 1.37

$$\begin{cases} -\frac{a^2\operatorname{asinh}(a+bx)}{2b^2}+\frac{3a\sqrt{a^2+2abx+b^2x^2+1}}{4b^2}+\frac{x^2\operatorname{asinh}(a+bx)}{2}-\frac{x\sqrt{a^2+2abx+b^2x^2+1}}{4b}+\frac{\operatorname{asinh}(a+bx)}{4b^2} & \text{for } b\neq 0 \\ \frac{x^2\operatorname{asinh}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 + 1)/(4*b**2) + x**2*asinh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)/(4*b) + asinh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*asinh(a)/2, True)
)
```

Giac [A]
time = 0.41, size = 111, normalized size = 1.46

$$\frac{1}{2}x^2\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)-\frac{1}{4}\left(\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b^2|b|}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 + 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*(x/b^2 - 3*a/b^3) - (2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b)))*b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{asinh}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x),x)
```

```
[Out] int(x*asinh(a + b*x), x)
```

3.61 $\int \sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{1+(a+bx)^2}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)}{b}$$

```
[Out] (b*x+a)*arcsinh(b*x+a)/b-(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5858, 5772, 267}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x],x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/b) + ((a + b*x)*ArcSinh[a + b*x])/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 135 vs. $2(34) = 68$.
time = 0.22, size = 135, normalized size = 3.97

$$x\sinh^{-1}(a+bx) - \frac{2b\sqrt{1+a^2+2abx+b^2x^2} + a\left(b+\sqrt{b^2}\right)\log\left(-a-\sqrt{b^2}\,x+\sqrt{1+a^2+2abx+b^2x^2}\right) + a\left(-b+\sqrt{b^2}\right)\log\left(a-\sqrt{b^2}\,x+\sqrt{1+a^2+2abx+b^2x^2}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x],x]
```

```
[Out] x*ArcSinh[a + b*x] - (2*b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*(b + Sqrt[b
^2])*Log[-a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]] + a*(-b + Sq
rt[b^2])*Log[a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(2*b^2)
```

Maple [A]
time = 0.26, size = 31, normalized size = 0.91

method	result	size
derivativedivides	$\frac{(bx+a)\,\mathrm{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$	31
default	$\frac{(bx+a)\,\mathrm{arcsinh}(bx+a)-\sqrt{1+(bx+a)^2}}{b}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((b*x+a)*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.88

$$\frac{(bx+a)\,\mathrm{arsinh}\,(bx+a) - \sqrt{(bx+a)^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arcsinh(b*x + a) - sqrt((b*x + a)^2 + 1))/b
```

Fricas [A]
time = 0.40, size = 57, normalized size = 1.68

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))/b
```

Sympy [A]
time = 0.07, size = 46, normalized size = 1.35

$$\begin{cases} \frac{a\,\text{asinh}\,(a+bx)}{b}+x\,\text{asinh}\,(a+bx)-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for}\ b\neq 0\\ x\,\text{asinh}\,(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a),x)
```

```
[Out] Piecewise((a*asinh(a + b*x)/b + x*asinh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(32) = 64.
time = 0.41, size = 92, normalized size = 2.71

$$-b\left(\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b*
abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/b^2) + x*log(b*x + a + sqrt((b*
x + a)^2 + 1))
```

Mupad [B]
time = 0.45, size = 76, normalized size = 2.24

$$x\,\mathrm{asinh}(a+b\,x)-\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{b}+\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x),x)
```

```
[Out] x*asinh(a + b*x) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/b + (a*log((a^2 + b^
2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

3.62 $\int \frac{\sinh^{-1}(a+bx)}{x}\,dx$

Optimal. Leaf size=131

$$-\frac{1}{2}\sinh^{-1}(a+bx)^2+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+\text{Poly}$$

```
[Out] -1/2*arcsinh(b*x+a)^2+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a
^2+1)^(1/2)))+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))+polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A]
time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5859, 5827, 5680, 2221, 2317, 2438}

$$\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{2}\sinh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a -
Sqrt[1 + a^2])] + ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 +
 a^2])] + PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + PolyLog[2, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x} dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}} dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}} dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}(}{}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\,\text{lo}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\,\text{lo}$$

$$= -\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)\,\text{lo}$$

Mathematica [A]
time = 0.01, size = 153, normalized size = 1.17

$$-\frac{1}{2}\sinh^{-1}(a+bx)^2 + \sinh^{-1}(a+bx)\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \sinh^{-1}(a+bx)\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \text{PolyLog}\left(2, -\frac{e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right) + \text{PolyLog}\left(2, \frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2 + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((-(a
/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]*Log[1 + E^ArcSinh[a + b*x]/((
-(a/b) + Sqrt[1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcSinh[a + b*x]/(-a + Sqrt[
1 + a^2]))] + PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(153) = 306$.
time = 4.37, size = 388, normalized size = 2.96

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(bx+a)^2}{2}+\frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)+2\ln\left(\right.\right.}{}$
default	$-\frac{\operatorname{arcsinh}(bx+a)^2}{2}+\frac{\left(a^2+1+\sqrt{a^2+1}\,a\right)\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{1+(bx+a)^2}}{-a+\sqrt{a^2+1}}\right)+2\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsinh(b*x+a)^2+(a^2+1+(a^2+1)^(1/2)*a)/(a^2+1)*arcsinh(b*x+a)*(ln(((
a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2*ln(((a^2+1)^(1/
2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a^2+ln(((a^2+1)^(1/2)-b*x-(1
+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)
^(1/2))/(a+(a^2+1)^(1/2)))*(a^2+1)^(1/2)*a)+dilog(((a^2+1)^(1/2)-b*x-(1+(b*
x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1
/2))/(-a+(a^2+1)^(1/2)))+a*arcsinh(b*x+a)/(a^2+1)^(1/2)*ln(((a^2+1)^(1/2)-b
*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-a*arcsinh(b*x+a)/(a^2+1)^(1/2)*l
n(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x,x)
```

```
[Out] Integral(asinh(a + b*x)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x,x)
```

```
[Out] int(asinh(a + b*x)/x, x)
```

3.63 $\int \frac{\sinh^{-1}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{\sinh^{-1}(a+bx)}{x}-\frac{b\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

```
[Out] -arcsinh(b*x+a)/x-b*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b*x+a)^2)^(1/2)
)/(a^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5859, 5828, 739, 212}

$$-\frac{b\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{\sqrt{a^2+1}}-\frac{\sinh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+bx)^2}}\right)$$
$$= -\frac{\sinh^{-1}(a+bx)}{x} - \frac{b\tanh^{-1}\left(\frac{b\left(\frac{1}{b}+\frac{a(a+bx)}{b}\right)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.00

$$-\frac{\sinh^{-1}(a+bx)}{x} - \frac{b\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{\sqrt{1+a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]/x) - (b*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1
+ (a + b*x)^2])])/Sqrt[1 + a^2]
```

Maple [A]
time = 2.81, size = 75, normalized size = 1.32

method	result	size
derivativedivides	$b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx} - \frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$	75

default	$b\left(-\frac{\operatorname{arcsinh}(bx+a)}{bx}-\frac{\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+a^2+1}}{bx}\right)}{\sqrt{a^2+1}}\right)$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/b/x*arcsinh(b*x+a)-1/(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.
time = 0.26, size = 111, normalized size = 1.95

$$-\frac{b\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{\sqrt{a^2+1}}-\frac{\operatorname{arsinh}\left(bx+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] -b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqr
t(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*
b^2)*abs(x)))/sqrt(a^2 + 1) - arcsinh(b*x + a)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(51) = 102$.
time = 0.38, size = 167, normalized size = 2.93

$$\frac{\sqrt{a^2+1}\,bx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\,\left(a^2-\sqrt{a^2+1}\,a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+(a^2+1)x\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(a^2-(a^2+1)x+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{(a^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*
(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + (a^
2 + 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (a^2 - (a^2 +
1)*x + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/((a^2 + 1)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(a+bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.
time = 0.45, size = 110, normalized size = 1.93

$$\frac{b\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}}-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 +
1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1)
))/sqrt(a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 + 1))/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^2,x)
```

```
[Out] int(asinh(a + b*x)/x^2, x)
```

3.64 $\int \frac{\sinh^{-1}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=92

$$-\frac{b\sqrt{1+(a+bx)^2}}{2(1+a^2)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2(1+a^2)^{3/2}}$$

```
[Out] -1/2*arcsinh(b*x+a)/x^2+1/2*a*b^2*arctanh((1+a*(b*x+a))/(a^2+1)^(1/2)/(1+(b
*x+a)^2)^(1/2))/(a^2+1)^(3/2)-1/2*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5859, 5828, 745, 739, 212}

$$\frac{ab^2\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{2(a^2+1)^{3/2}}-\frac{b\sqrt{(a+bx)^2+1}}{2(a^2+1)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x) - ArcSinh[a + b*x]/(2*x^2) + (
a*b^2*ArcTanh[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(2*
(1 + a^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{x^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}-\frac{(ab)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+b\right.}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{(ab)\text{Subst}\left(\int \frac{1}{\frac{1}{b^2}+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{\frac{1}{b}+\frac{a(a+bx)}{b}}{\sqrt{1+(a+b}}\right.}{2\left(1+a^2\right)}\\
&= -\frac{b\sqrt{1+(a+bx)^2}}{2\left(1+a^2\right)x}-\frac{\sinh^{-1}(a+bx)}{2x^2}+\frac{ab^2\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{2\left(1+a^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 110, normalized size = 1.20

$$-\frac{\sinh^{-1}(a+bx)+\frac{bx\left(\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}+abx\log(x)-abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)\right)}{\left(1+a^2\right)^{3/2}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*(ArcSinh[a + b*x] + (b*x*(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x
^2] + a*b*x*Log[x] - a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2
 + 2*a*b*x + b^2*x^2]]))/(1 + a^2)^(3/2))/x^2
```

Maple [A]
time = 2.08, size = 112, normalized size = 1.22

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+}}{bx}\right.}{2(a^2+1)^{\frac{3}{2}}}\right.$
default	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)}{2b^2x^2}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+2abx+2\sqrt{a^2+1}\,\sqrt{b^2x^2+2abx+}}{bx}\right.}{2(a^2+1)^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2/b^2/x^2*arcsinh(b*x+a)-1/2/(a^2+1)/b/x*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2)+1/2*a/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/b/x))
```

Maxima [A]
time = 0.26, size = 146, normalized size = 1.59

$$\frac{1}{2}\left(\frac{ab\operatorname{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{(a^2+1)x}\right)b-\frac{\operatorname{arsinh}(bx+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^
2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2
 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((a
^2 + 1)*x))*b - 1/2*arcsinh(b*x + a)/x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(78) = 156.
time = 0.38, size = 236, normalized size = 2.57

$$\frac{\sqrt{a^2+1}\,ab^2x^2\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\,\left(a^2+\sqrt{a^2+1}\,a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)b^2x^2+(a^4+2\,a^2+1)x^2\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)bx-(a^4-(a^4+2\,a^2+1)x^2+2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,(a^4+2\,a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*a*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a
)/x) - (a^2 + 1)*b^2*x^2 + (a^4 + 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*b*x -
 (a^4 - (a^4 + 2*a^2 + 1)*x^2 + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1)))/((a^4 + 2*a^2 + 1)*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}{(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)/x**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(78) = 156.
time = 0.46, size = 199, normalized size = 2.16

$$-\frac{1}{2}\left(\frac{ab\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{2\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab+a^2|b|+|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)(a^2+1)}\right)b-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqr
t(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(
a^2 + 1)))/(a^2 + 1)^(3/2) - 2*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1))*a*b + a^2*abs(b) + abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^2 - a^2 - 1)*(a^2 + 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 + 1))/
x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^3,x)
```

```
[Out] int(asinh(a + b*x)/x^3, x)
```

3.65 $\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=129

$$-\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2}+\frac{ab^2\sqrt{1+(a+bx)^2}}{2(1+a^2)^2x}-\frac{\sinh^{-1}(a+bx)}{3x^3}+\frac{(1-2a^2)b^3\tanh^{-1}\left(\frac{1+a(a+bx)}{\sqrt{1+a^2}\sqrt{1+(a+bx)^2}}\right)}{6(1+a^2)^{5/2}}$$

```
[Out] -1/3*arcsinh(b*x+a)/x^3+1/6*(-2*a^2+1)*b^3*arctanh((1+a*(b*x+a))/(a^2+1)^(1
/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(5/2)-1/6*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^
2+1/2*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x
```

Rubi [A]
time = 0.11, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5859, 5828, 759, 821, 739, 212}

$$\frac{(1-2a^2)b^3\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{6(a^2+1)^{5/2}}+\frac{ab^2\sqrt{(a+bx)^2+1}}{2(a^2+1)^2x}-\frac{b\sqrt{(a+bx)^2+1}}{6(a^2+1)x^2}-\frac{\sinh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -1/6*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^
2])/(2*(1 + a^2)^2*x) - ArcSinh[a + b*x]/(3*x^3) + ((1 - 2*a^2)*b^3*ArcTanh
[(1 + a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(6*(1 + a^2)^(5/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
```

```
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{b^2\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)}{6(1+a^2)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2(1+a^2)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} - \frac{((1-2a^2)b^2)\,\text{Subst}}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2(1+a^2)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{((1-2a^2)b^2)\,\text{Subst}}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{6(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}}{2(1+a^2)^2 x} - \frac{\sinh^{-1}(a+bx)}{3x^3} + \frac{(1-2a^2)b^3\tanh^{-1}}{6}$$

Mathematica [A]
time = 0.16, size = 149, normalized size = 1.16

$$\frac{-\sqrt{1+a^2}\,bx(1+a^2-3abx)\sqrt{1+a^2+2abx+b^2x^2}-2(1+a^2)^{5/2}\sinh^{-1}(a+bx)+(-1+2a^2)\,b^3x^3\log(x)+(1-2a^2)\,b^3x^3\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{6(1+a^2)^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^4,x]
```

```
[Out] (-(Sqrt[1 + a^2]*b*x*(1 + a^2 - 3*a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
 - 2*(1 + a^2)^(5/2)*ArcSinh[a + b*x] + (-1 + 2*a^2)*b^3*x^3*Log[x] + (1 -
2*a^2)*b^3*x^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x +
 b^2*x^2]])/(6*(1 + a^2)^(5/2)*x^3)
```

Maple [A]
time = 2.09, size = 213, normalized size = 1.65

method	result

derivativedivides	$b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+}{}\right.}{}\right.}{2(}\right.$
default	$b^3\left(-\frac{\operatorname{arcsinh}(bx+a)}{3b^3x^3}-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{6(a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)bx}+\frac{a\ln\left(\frac{2a^2+2+}{}\right.}{}\right.}{2(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3/b^3/x^3*arcsinh(b*x+a)-1/6/(a^2+1)/b^2/x^2*(b^2*x^2+2*a*b*x+a^2+1
)^(1/2)-1/2*a/(a^2+1)*(-1/(a^2+1)/b/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a/(a^2+
1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/b/x))+1/6/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/b/x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(111) = 222$.
time = 0.27, size = 284, normalized size = 2.20

$$-\frac{1}{6}\left(\frac{3a^2b^2\operatorname{arsinh}\left(\frac{2abx}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}+\frac{2a^2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}\right)}{(a^2+1)^{\frac{5}{2}}}-\frac{b^2\operatorname{arsinh}\left(\frac{2abx}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}+\frac{2a^2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4a^2b^2+4(a^2+1)b^2}\,|x|}\right)}{(a^2+1)^{\frac{3}{2}}}-\frac{3\sqrt{b^2x^2+2abx+a^2+1}\,ab}{(a^2+1)^2x}+\frac{\sqrt{b^2x^2+2abx+a^2+1}}{(a^2+1)x^2}\right)b-\frac{\operatorname{arsinh}(bx+a)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x))
 + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) - b^2*arcsinh(2*a*b*x/(sqrt(-4*a
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2)
 - 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x) + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)/((a^2 + 1)*x^2))*b - 1/3*arcsinh(b*x + a)/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(111) = 222$.

time = 0.41, size = 285, normalized size = 2.21

$$\frac{(2a^2-1)\sqrt{a^2+1}\,b^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)+3(a^3+a)b^3x^3+2(a^6+3a^4+3a^2+1)x^3\log\left(-bx-a+\sqrt{b^2x^2+2abx+a^2+1}\right)-2(a^6+3a^4-(a^6+3a^4+3a^2+1)x^3+3a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)+(3(a^3+a)b^2x^2-(a^4+2a^2+1)bx)\sqrt{b^2x^2+2abx+a^2+1}}{6(a^6+3a^4+3a^2+1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^2 - 1)*sqrt(a^2 + 1)*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^
2 + 1) + a)/x) + 3*(a^3 + a)*b^3*x^3 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 2*(a^6 + 3*a^4 - (a^6 + 3*a
^4 + 3*a^2 + 1)*x^3 + 3*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)) + (3*(a^3 + a)*b^2*x^2 - (a^4 + 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))/((a^6 + 3*a^4 + 3*a^2 + 1)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**4,x)
```

```
[Out] Integral(asinh(a + b*x)/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(111) = 222.
time = 0.45, size = 381, normalized size = 2.95

$$\frac{1}{6}b\left(\frac{(2a^2b^2-b^2)\log\left(\frac{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{(a^4+2a^2+1)\sqrt{a^2+1}}-\frac{2\left(2\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)^3a^2b^2-6\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)a^4b^2-4a^5b|b|-\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)^3b^2-7\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)a^2b^2-8a^3b|b|-\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)b^2-4ab|b|\right)}{(a^4+2a^2+1)\left(\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)^2-a^2-1\right)^2}\right)-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b*((2*a^2*b^2 - b^2)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1) + 2*sqrt(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 2*(2*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))*a^4*b^2 - 4*a^5*b*abs(b) - (x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^3*b^2 - 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*a^2*b^2 - 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*b^2 - 4*a*b*abs(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2 - a^2 - 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 + 1)
)/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a + b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^4,x)
```

```
[Out] int(asinh(a + b*x)/x^4, x)
```

3.66 $\int \frac{\sinh^{-1}(a+bx)}{x^5}\,dx$

Optimal. Leaf size=167

$$-\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3}+\frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2}+\frac{(4-11a^2)b^3\sqrt{1+(a+bx)^2}}{24(1+a^2)^3x}-\frac{\sinh^{-1}(a+bx)}{4x^4}-\frac{a(3-2a^2)b^4\tanh^{-1}}{\quad}$$

```
[Out] -1/4*arcsinh(b*x+a)/x^4-1/8*a*(-2*a^2+3)*b^4*arctanh((1+a*(b*x+a))/(a^2+1)^
(1/2)/(1+(b*x+a)^2)^(1/2))/(a^2+1)^(7/2)-1/12*b*(1+(b*x+a)^2)^(1/2)/(a^2+1)
/x^3+5/24*a*b^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)^2/x^2+1/24*(-11*a^2+4)*b^3*(1+(
b*x+a)^2)^(1/2)/(a^2+1)^3/x
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5859, 5828, 759, 849, 821, 739, 212}

$$-\frac{a(3-2a^2)b^4\tanh^{-1}\left(\frac{a(a+bx)+1}{\sqrt{a^2+1}\sqrt{(a+bx)^2+1}}\right)}{8(a^2+1)^{7/2}}+\frac{(4-11a^2)b^3\sqrt{(a+bx)^2+1}}{24(a^2+1)^3x}+\frac{5ab^2\sqrt{(a+bx)^2+1}}{24(a^2+1)^2x^2}-\frac{b\sqrt{(a+bx)^2+1}}{12(a^2+1)x^3}-\frac{\sinh^{-1}(a+bx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -1/12*(b*Sqrt[1 + (a + b*x)^2])/((1 + a^2)*x^3) + (5*a*b^2*Sqrt[1 + (a + b*
x)^2])/(24*(1 + a^2)^2*x^2) + ((4 - 11*a^2)*b^3*Sqrt[1 + (a + b*x)^2])/(24*
(1 + a^2)^3*x) - ArcSinh[a + b*x]/(4*x^4) - (a*(3 - 2*a^2)*b^4*ArcTanh[(1 +
 a*(a + b*x))/(Sqrt[1 + a^2]*Sqrt[1 + (a + b*x)^2])])/(8*(1 + a^2)^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
```

```
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{x^5}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^5}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3} - \frac{\sinh^{-1}(a+bx)}{4x^4} - \frac{b^2\text{Subst}\left(\int \frac{\frac{3a}{b}+\frac{2x}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)}{12(1+a^2)}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2} - \frac{\sinh^{-1}(a+bx)}{4x^4} + \frac{b^4\text{Subst}\left(\int \frac{-\frac{2(2-}{b}}{\left(-\frac{a}{b}+\frac{x}{b}\right)}\right.}{24}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2} + \frac{(4-11a^2)b^3\sqrt{1+(a+bx)^2}}{24(1+a^2)^3x} - \frac{\text{sinh}}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2} + \frac{(4-11a^2)b^3\sqrt{1+(a+bx)^2}}{24(1+a^2)^3x} - \frac{\text{sinh}}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}}{12(1+a^2)x^3} + \frac{5ab^2\sqrt{1+(a+bx)^2}}{24(1+a^2)^2x^2} + \frac{(4-11a^2)b^3\sqrt{1+(a+bx)^2}}{24(1+a^2)^3x} - \frac{\text{sinh}}{}$$

Mathematica [**A**]
time = 0.15, size = 179, normalized size = 1.07

$$\frac{1}{8}\left(-\frac{b\sqrt{1+a^2+2abx+b^2x^2}\,(2+2a^4-5abx-5a^3bx-4b^2x^2+a^2(4+11b^2x^2))}{3(1+a^2)^3x^3} - \frac{2\sinh^{-1}(a+bx)}{x^4} - \frac{a(-3+2a^2)b^4\log(x)}{(1+a^2)^{7/2}} + \frac{a(-3+2a^2)b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/x^5,x]
```

```
[Out] (-1/3*(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 - 5*a*b*x - 5*a^3*b*x
 - 4*b^2*x^2 + a^2*(4 + 11*b^2*x^2)))/((1 + a^2)^3*x^3) - (2*ArcSinh[a + b*
x])/x^4 - (a*(-3 + 2*a^2)*b^4*Log[x])/(1 + a^2)^(7/2) + (a*(-3 + 2*a^2)*b^4
*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1
 + a^2)^(7/2))/8
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(145) = 290$.
time = 2.03, size = 359, normalized size = 2.15

method	result
derivativedivides	$b^4 \left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4} - \frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3} - \frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)b^2x^2} - \frac{3a\left(-\frac{\sqrt{b^2}}{}\right.}{}\right.}{}\right.$
default	$b^4 \left(-\frac{\operatorname{arcsinh}(bx+a)}{4b^4x^4} - \frac{\sqrt{b^2x^2+2abx+a^2+1}}{12(a^2+1)b^3x^3} - \frac{5a\left(-\frac{\sqrt{b^2x^2+2abx+a^2+1}}{2(a^2+1)b^2x^2} - \frac{3a\left(-\frac{\sqrt{b^2}}{}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-1/4/b^4/x^4*arcsinh(b*x+a)-1/12/(a^2+1)/b^3/x^3*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2)-5/12*a/(a^2+1)*(-1/2/(a^2+1)/b^2/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
-3/2*a/(a^2+1)*(-1/(a^2+1)/b/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a/(a^2+1)^(3/2
)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/b/x))+
1/2/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+
1)^(1/2))/b/x))-1/6/(a^2+1)*(-1/(a^2+1)/b/x*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a
/(a^2+1)^(3/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2))/b/x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs.

$2(145) = 290$.
time $= 0.27$, size $= 357$, normalized size $= 2.14$

$$\frac{1}{24}\left(\frac{15\,a^{3}b^{3}\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}+\frac{2\,a^{2}}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}+\frac{2}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}\right)}{(a^{2}+1)^{\frac{7}{2}}}-\frac{9\,ab^{3}\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}+\frac{2\,a^{2}}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}+\frac{2}{\sqrt{-4\,a^{2}b^{2}+4\,(a^{2}+1)b^{2}}\,|x|}\right)}{(a^{2}+1)^{\frac{5}{2}}}-\frac{15\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\,a^{2}b^{2}}{(a^{2}+1)^{3}x}+\frac{4\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\,b^{2}}{(a^{2}+1)^{2}x}+\frac{5\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\,ab}{(a^{2}+1)^{2}x^{2}}-\frac{2\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}}{(a^{2}+1)x^{3}}\right)b-\frac{\mathrm{arsinh}\left(bx+a\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(15*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)
) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2
+ 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 9*a*b^3*arcsinh(2*a*b*x/(sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 +
1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^
(5/2) - 15*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^3*x) + 4*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/((a^2 + 1)^2*x) + 5*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1)*a*b/((a^2 + 1)^2*x^2) - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/
((a^2 + 1)*x^3))*b - 1/4*arcsinh(b*x + a)/x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(145) = 290$.
time $= 0.43$, size $= 343$, normalized size $= 2.05$

$$\frac{3\,(2\,a^{3}-3\,a)\sqrt{a^{2}+1}\,b^{4}x^{4}\log\left(-\frac{a^{2}bx+a^{3}+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\left(a^{2}+\sqrt{a^{2}+1}\,a+1\right)+(abx+a^{2}+1)\sqrt{a^{2}+1}+a}{x}\right)-(11\,a^{4}+7\,a^{2}-4)b^{4}x^{4}+6\,(a^{8}+4\,a^{6}+6\,a^{4}+4\,a^{2}+1)x^{4}\log\left(-bx-a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)-6\,(a^{8}+4\,a^{6}-(a^{8}+4\,a^{6}+6\,a^{4}+4\,a^{2}+1)x^{4}+6\,a^{4}+4\,a^{2}+1)\log\left(bx+a+\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}\right)-((11\,a^{4}+7\,a^{2}-4)b^{3}x^{3}-5\,(a^{5}+2\,a^{3}+a)b^{2}x^{2}+2\,(a^{6}+3\,a^{4}+3\,a^{2}+1)bx)\sqrt{b^{2}x^{2}+2\,abx+a^{2}+1}}{24\,(a^{8}+4\,a^{6}+6\,a^{4}+4\,a^{2}+1)x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(2*a^3 - 3*a)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sq
rt(a^2 + 1) + a)/x) - (11*a^4 + 7*a^2 - 4)*b^4*x^4 + 6*(a^8 + 4*a^6 + 6*a^4
 + 4*a^2 + 1)*x^4*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*(a^
8 + 4*a^6 - (a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4 + 6*a^4 + 4*a^2 + 1)*log(
b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - ((11*a^4 + 7*a^2 - 4)*b^3*x^
3 - 5*(a^5 + 2*a^3 + a)*b^2*x^2 + 2*(a^6 + 3*a^4 + 3*a^2 + 1)*b*x)*sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

Sympy [F]
time $= 0.00$, size $= 0$, normalized size $= 0.00$

$$\int \frac{\operatorname{asinh}\left(a+bx\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/x**5,x)
```

```
[Out] Integral(asinh(a + b*x)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(145) = 290.

time = 0.45, size = 709, normalized size = 4.25

$$-\frac{1}{24}\,b\left(\frac{3\,(2\,a^3b^3-3\,ab^3)\log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{(a^6+3\,a^4+3\,a^2+1)\sqrt{a^2+1}}-\frac{2\left(6\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^5a^3b^3-16\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^5b^3+42\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^7b^3+12\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2a^6b^2|b|+20\,a^8b^2|b|-9\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^5ab^3+8\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3a^3b^3+93\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^5b^3+36\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2a^4b^2|b|+56\,a^6b^2|b|+24\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3ab^3+60\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^3b^3+36\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2a^2b^2|b|+48\,a^4b^2|b|+9\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)ab^3+12\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2b^2|b|+8\,a^2b^2|b|-4\,b^2|b|\right)}{(a^6+3\,a^4+3\,a^2+1)\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)^3}\right)-\frac{\log\left(bx+a+\sqrt{(bx+a)^2+1}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/24*b*(3*(2*a^3*b^3 - 3*a*b^3)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1))
 - 2*(6*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^3*b^3 - 16*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 42*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^2*a^6*b^2*abs(b) + 20*a^8*b^2*abs(b) - 9*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 8*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^3*a^3*b^3 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^
5*b^3 + 36*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b)
+ 56*a^6*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a
*b^3 + 60*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 36*(x*ab
s(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 48*a^4*b^2*abs
(b) + 9*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 12*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*b^2*abs(b) + 8*a^2*b^2*abs(b) - 4*b
^2*abs(b))/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))^2 - a^2 - 1)^3)) - 1/4*log(b*x + a + sqrt((b*x + a)^2 + 1))/x^4
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/x^5,x)
```

```
[Out] int(asinh(a + b*x)/x^5, x)
```

3.67 $\int x^3 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=331

$$\frac{4ax}{3b^3}-\frac{2a^3x}{b^3}-\frac{3(a+bx)^2}{32b^4}+\frac{3a^2(a+bx)^2}{4b^4}-\frac{2a(a+bx)^3}{9b^4}+\frac{(a+bx)^4}{32b^4}-\frac{4a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3b^4}+\frac{2a^3\sqrt{}}{}$$

```
[Out] 4/3*a*x/b^3-2*a^3*x/b^3-3/32*(b*x+a)^2/b^4+3/4*a^2*(b*x+a)^2/b^4-2/9*a*(b*x
+a)^3/b^4+1/32*(b*x+a)^4/b^4-3/32*arcsinh(b*x+a)^2/b^4+3/4*a^2*arcsinh(b*x+
a)^2/b^4-1/4*a^4*arcsinh(b*x+a)^2/b^4+1/4*x^4*arcsinh(b*x+a)^2-4/3*a*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2*a^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
^4+3/16*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4-3/2*a^2*(b*x+a)*arcs
inh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4+2/3*a*(b*x+a)^2*arcsinh(b*x+a)*(1+(b*x+a
)^2)^(1/2)/b^4-1/8*(b*x+a)^3*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.38, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$-\frac{a^4\sinh^{-1}(a+bx)^2}{4b^4}+\frac{2a^3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^4}-\frac{2a^3x}{b^3}+\frac{3a^2(a+bx)^2}{4b^4}+\frac{3a^2\sinh^{-1}(a+bx)^2}{4b^4}-\frac{3a^2(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{2b^4}-\frac{2a(a+bx)^3}{9b^4}+\frac{(a+bx)^4}{32b^4}-\frac{3(a+bx)^2}{32b^4}+\frac{2a(a+bx)^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{3b^4}-\frac{4a\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{3b^4}-\frac{3\sinh^{-1}(a+bx)^2}{32b^4}-\frac{(a+bx)^3\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{8b^4}+\frac{3(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{16b^4}+\frac{4ax}{3b^3}+\frac{1}{4}x^4\sinh^{-1}(a+bx)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (4*a*x)/(3*b^3) - (2*a^3*x)/b^3 - (3*(a + b*x)^2)/(32*b^4) + (3*a^2*(a + b*
x)^2)/(4*b^4) - (2*a*(a + b*x)^3)/(9*b^4) + (a + b*x)^4/(32*b^4) - (4*a*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) + (2*a^3*Sqrt[1 + (a + b*x)^2]
*ArcSinh[a + b*x])/b^4 + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(16*b^4) - (3*a^2*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b
^4) + (2*a*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*b^4) - ((
a + b*x)^3*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(8*b^4) - (3*ArcSinh[a +
 b*x]^2)/(32*b^4) + (3*a^2*ArcSinh[a + b*x]^2)/(4*b^4) - (a^4*ArcSinh[a + b
*x]^2)/(4*b^4) + (x^4*ArcSinh[a + b*x]^2)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^4 \sinh^{-1}(x)}{b^4\sqrt{1+x^2}} - \frac{4a^3 x \sinh^{-1}(x)}{b^4\sqrt{1+x^2}} + \frac{6a^2x^2 \sinh}{b^4\sqrt{1+}}\right.\right.$$

$$= \frac{1}{4}x^4 \sinh^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int \frac{x^4 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^4} + \frac{(2a)\text{Subst}\left(\int \frac{x^3 \sinh^{-}}{\sqrt{1+}}\right.}{b^4}$$

$$= \frac{2a^3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^4} - \frac{3a^2(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx}{2b^4}$$

$$= -\frac{2a^3x}{b^3} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(}{3b^4}$$

$$= \frac{4ax}{3b^3} - \frac{2a^3x}{b^3} - \frac{3(a+bx)^2}{32b^4} + \frac{3a^2(a+bx)^2}{4b^4} - \frac{2a(a+bx)^3}{9b^4} + \frac{(a+bx)^4}{32b^4} - \frac{4a\sqrt{1+}}{}$$

Mathematica [A]
time = 0.12, size = 145, normalized size = 0.44

$$\frac{bx(-300a^3+78a^2bx+a(330-28b^2x^2)+9bx(-3+b^2x^2))+6\sqrt{1+a^2+2abx+b^2x^2}\,(50a^3+9bx-26a^2bx-6b^3x^3+a(-55+14b^2x^2))\sinh^{-1}(a+bx)-9(3-24a^2+8a^4-8b^4x^4)\sinh^{-1}(a+bx)^2}{288b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-300*a^3 + 78*a^2*b*x + a*(330 - 28*b^2*x^2) + 9*b*x*(-3 + b^2*x^2))
+ 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(50*a^3 + 9*b*x - 26*a^2*b*x - 6*b^3*
x^3 + a*(-55 + 14*b^2*x^2))*ArcSinh[a + b*x] - 9*(3 - 24*a^2 + 8*a^4 - 8*b^
4*x^4)*ArcSinh[a + b*x]^2)/(288*b^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arcsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x+a)^2,x)
```

```
[Out] int(x^3*arcsinh(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(1/2*
(b^3*x^6 + 2*a*b^2*x^5 + (a^2*b + b)*x^4 + (b^2*x^5 + a*b*x^4)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

Fricas [A]
time = 0.37, size = 182, normalized size = 0.55

$$\frac{9\,b^4x^4 - 28\,ab^3x^3 + 3\,(26\,a^2-9)b^2x^2 - 30\,(10\,a^3-11\,a)bx + 9\,(8\,b^4x^4 - 8\,a^4 + 24\,a^2 - 3)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^2 - 6\,(6\,b^3x^3 - 14\,ab^2x^2 - 50\,a^3 + (26\,a^2-9)bx + 55\,a)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{288\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*b^4*x^4 - 28*a*b^3*x^3 + 3*(26*a^2 - 9)*b^2*x^2 - 30*(10*a^3 - 11*
a)*b*x + 9*(8*b^4*x^4 - 8*a^4 + 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2 - 6*(6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 - 9)*b
*x + 55*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)))/b^4
```

Sympy [A]
time = 0.46, size = 366, normalized size = 1.11

$$\begin{cases} -\frac{a^4\operatorname{asinh}^2(a+bx)}{4b^4} - \frac{25a^3x}{24b^3} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^4} + \frac{13a^2x^2}{48b^2} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^3} + \frac{3a^2\operatorname{asinh}^2(a+bx)}{4b^4} - \frac{7ax^3}{72b} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{24b^2} + \frac{55ax}{48b^3} - \frac{55a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{48b^4} + \frac{x^4\operatorname{asinh}^2(a+bx)}{4} + \frac{x^4}{32} - \frac{x^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{8b} - \frac{3x^2}{32b^2} + \frac{3x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{16b^3} - \frac{3\operatorname{asinh}^2(a+bx)}{32b^4} & \text{for}\: b \neq 0 \\ \frac{x^4\operatorname{asinh}^2(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)**2/(4*b**4) - 25*a**3*x/(24*b**3) + 25*a**3
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**4) + 13*a**2*x*
*2/(48*b**2) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)/(24*b**3) + 3*a**2*asinh(a + b*x)**2/(4*b**4) - 7*a*x**3/(72*b) + 7*a*x**
2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(24*b**2) + 55*a*x/(4
```

```
8*b**3) - 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(48*b**4
) + x**4*asinh(a + b*x)**2/4 + x**4/32 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 + 1)*asinh(a + b*x)/(8*b) - 3*x**2/(32*b**2) + 3*x*sqrt(a**2 + 2*a*b*x +
 b**2*x**2 + 1)*asinh(a + b*x)/(16*b**3) - 3*asinh(a + b*x)**2/(32*b**4), N
e(b, 0)), (x**4*asinh(a)**2/4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^3*arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\mathrm{asinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x)^2,x)
[Out] int(x^3*asinh(a + b*x)^2, x)
```

3.68 $\int x^2 \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=211

$$-\frac{4x}{9b^2}+\frac{2a^2x}{b^2}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}+\frac{4\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{9b^3}-\frac{2a^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+}{b^3}$$

```
[Out] -4/9*x/b^2+2*a^2*x/b^2-1/2*a*(b*x+a)^2/b^3+2/27*(b*x+a)^3/b^3-1/2*a*arcsinh
(b*x+a)^2/b^3+1/3*a^3*arcsinh(b*x+a)^2/b^3+1/3*x^3*arcsinh(b*x+a)^2+4/9*arc
sinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2*a^2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2
)/b^3+a*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3-2/9*(b*x+a)^2*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.25, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$\frac{a^3\sinh^{-1}(a+bx)^2}{3b^3}-\frac{2a^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^3}+\frac{2a^2x}{b^2}-\frac{a(a+bx)^2}{2b^3}+\frac{2(a+bx)^3}{27b^3}-\frac{a\sinh^{-1}(a+bx)^2}{2b^3}+\frac{a(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^3}-\frac{2(a+bx)^2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{9b^3}+\frac{4\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{9b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)^2-\frac{4x}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-4*x)/(9*b^2) + (2*a^2*x)/b^2 - (a*(a + b*x)^2)/(2*b^3) + (2*(a + b*x)^3)/
(27*b^3) + (4*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(9*b^3) - (2*a^2*Sqrt
[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b^3 + (a*(a + b*x)*Sqrt[1 + (a + b*x)^2
]*ArcSinh[a + b*x])/b^3 - (2*(a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
b*x])/(9*b^3) - (a*ArcSinh[a + b*x]^2)/(2*b^3) + (a^3*ArcSinh[a + b*x]^2)/(
3*b^3) + (x^3*ArcSinh[a + b*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2 \sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{3}x^3 \sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$
$$= \frac{1}{3}x^3 \sinh^{-1}(a+bx)^2 - \frac{2}{3}\text{Subst}\left(\int\left(-\frac{a^3\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} + \frac{3a^2x\sinh^{-1}(x)}{b^3\sqrt{1+x^2}} - \frac{3ax^2 \sin}{b^3\sqrt{1}}\right.\right.$$
$$= \frac{1}{3}x^3 \sinh^{-1}(a+bx)^2 - \frac{2\text{Subst}\left(\int\frac{x^3\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{3b^3} + \frac{(2a)\text{Subst}\left(\int\frac{x^2 \sin}{\sqrt{1}}\right.}{}$$
$$= -\frac{2a^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^3} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+b}{b^3}$$
$$= \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{9b^3} - \frac{2a^2\sqrt{1}}{}$$
$$= -\frac{4x}{9b^2} + \frac{2a^2x}{b^2} - \frac{a(a+bx)^2}{2b^3} + \frac{2(a+bx)^3}{27b^3} + \frac{4\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{9b^3} -$$

Mathematica [A]
time = 0.09, size = 107, normalized size = 0.51

$$\frac{bx(-24+66a^2-15abx+4b^2x^2)-6\sqrt{1+a^2+2abx+b^2x^2}\,(-4+11a^2-5abx+2b^2x^2)\sinh^{-1}(a+bx)+9(-3a+2a^3+2b^3x^3)\sinh^{-1}(a+bx)^2}{54b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-24 + 66*a^2 - 15*a*b*x + 4*b^2*x^2) - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2
*x^2]*(-4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x] + 9*(-3*a + 2*a^
3 + 2*b^3*x^3)*ArcSinh[a + b*x]^2)/(54*b^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsinh}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^2,x)
```

```
[Out] int(x^2*arcsinh(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2/3*
(b^3*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^
3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)
^(3/2) + a), x)
```

Fricas [A]
time = 0.35, size = 146, normalized size = 0.69

$$\frac{4\,b^3x^3 - 15\,ab^2x^2 + 6\,(11\,a^2-4)bx + 9\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2 - 6\,(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{54\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*b^3*x^3 - 15*a*b^2*x^2 + 6*(11*a^2 - 4)*b*x + 9*(2*b^3*x^3 + 2*a^3
- 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^2*x^2 -
5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^3
```

Sympy [A]
time = 0.28, size = 243, normalized size = 1.15

$$\begin{cases} \frac{a^3\operatorname{asinh}^2(a+bx)}{3b^3} + \frac{11a^2x}{9b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^3} - \frac{5ax^2}{18b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^2} - \frac{a\operatorname{asinh}^2(a+bx)}{2b^3} + \frac{x^3\operatorname{asinh}^2(a+bx)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b} - \frac{4x}{9b^2} + \frac{4\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{9b^3} & \text{for}\ b \neq 0 \\ \frac{x^3\operatorname{asinh}^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**2/(3*b**3) + 11*a**2*x/(9*b**2) - 11*a**2*s
qrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3) - 5*a*x**2/(18*
b) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**2) - a
*asinh(a + b*x)**2/(2*b**3) + x**3*asinh(a + b*x)**2/3 + 2*x**3/27 - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b) - 4*x/(9*b**2) +
 4*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(9*b**3), Ne(b, 0)),
 (x**3*asinh(a)**2/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{asinh}(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x)^2,x)
```

```
[Out] int(x^2*asinh(a + b*x)^2, x)
```

3.69 $\int x \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=126

$$-\frac{2ax}{b}+\frac{(a+bx)^2}{4b^2}+\frac{2a\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{b^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}$$

```
[Out] -2*a*x/b+1/4*(b*x+a)^2/b^2+1/4*arcsinh(b*x+a)^2/b^2-1/2*a^2*arcsinh(b*x+a)^
2/b^2+1/2*x^2*arcsinh(b*x+a)^2+2*a*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2-1
/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5859, 5828, 5838, 5783, 5798, 8, 5812, 30}

$$-\frac{a^2\sinh^{-1}(a+bx)^2}{2b^2}+\frac{(a+bx)^2}{4b^2}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b^2}+\frac{\sinh^{-1}(a+bx)^2}{4b^2}+\frac{2a\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)^2-\frac{2ax}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*a*x)/b + (a + b*x)^2/(4*b^2) + (2*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b
*x])/b^2 - ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*b^2) + Arc
Sinh[a + b*x]^2/(4*b^2) - (a^2*ArcSinh[a + b*x]^2)/(2*b^2) + (x^2*ArcSinh[a
 + b*x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5838

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a
+ b*ArcSinh[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
 && EqQ[e, c^2*d] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ[n
, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1] || (EqQ[m, 2]
&& LtQ[p, -2]))
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^2\,dx,x,a+bx\right)}{b}$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2 - \text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2 - \text{Subst}\left(\int\left(\frac{a^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}} - \frac{2ax\sinh^{-1}(x)}{b^2\sqrt{1+x^2}} + \frac{x^2\sinh^{-1}(x)}{b^2\sqrt{1+x^2}}\right)\right.$$
$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^2 - \frac{\text{Subst}\left(\int\frac{x^2\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx,x,a+bx\right)}{b^2} + \frac{(2a)\text{Subst}\left(\int\frac{x\sinh^{-1}(x)}{\sqrt{1+x}}\right.}{b^2}$$
$$= \frac{2a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b^2} - $$
$$= -\frac{2ax}{b} + \frac{(a+bx)^2}{4b^2} + \frac{2a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b^2} - \frac{(a+bx)\sqrt{1+(a+bx}}{2b^2}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.63

$$\frac{bx(-6a+bx)+2(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)+(1-2a^2+2b^2x^2)\sinh^{-1}(a+bx)^2}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^2,x]
```

```
[Out] (b*x*(-6*a + b*x) + 2*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + (1 - 2*a^2 + 2*b^2*x^2)*ArcSinh[a + b*x]^2)/(4*b^2)
```

Maple [A]
time = 2.47, size = 113, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a)^2}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a\right.}{b^2}$
default	$\frac{\frac{\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a)^2}{2}-\frac{\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}\,(bx+a)}{2}-\frac{\operatorname{arcsinh}(bx+a)^2}{4}+\frac{(bx+a)^2}{4}+\frac{1}{4}-a\left(\operatorname{arcsinh}(bx+a\right.}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*(1+(b*x+a)^2)*arcsinh(b*x+a)^2-1/2*arcsinh(b*x+a)*(1+(b*x+a)^2)^
(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^2+1/4*(b*x+a)^2+1/4-a*(arcsinh(b*x+a)^2*(b
*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate((b^3
*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^
3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + a), x)
```

Fricas [A]
time = 0.44, size = 114, normalized size = 0.90

$$\frac{b^2x^2 - 6\,abx + (2\,b^2x^2 - 2\,a^2 + 1)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)^2 - 2\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,(bx - 3\,a)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2 - 6*a*b*x + (2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))^2 - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*
log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b^2
```

Sympy [A]
time = 0.16, size = 138, normalized size = 1.10

$$\begin{cases} -\frac{a^2\operatorname{asinh}^2(a+bx)}{2b^2} - \frac{3ax}{2b} + \frac{3a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b^2} + \frac{x^2\operatorname{asinh}^2(a+bx)}{2} + \frac{x^2}{4} - \frac{x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{2b} + \frac{\operatorname{asinh}^2(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \frac{x^2\operatorname{asinh}^2(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**2/(2*b**2) - 3*a*x/(2*b) + 3*a*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b**2) + x**2*asinh(a + b*x)**2/
2 + x**2/4 - x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(2*b) +
asinh(a + b*x)**2/(4*b**2), Ne(b, 0)), (x**2*asinh(a)**2/2, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{asinh}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x)^2,x)
```

```
[Out] int(x*asinh(a + b*x)^2, x)
```

3.70 $\int \sinh^{-1}(a+bx)^2 \, dx$

Optimal. Leaf size=45

$$2x-\frac{2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}$$

```
[Out] 2*x+(b*x+a)*arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5798, 8}

$$\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}-\frac{2\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/b + ((a + b*x)*ArcSinh[a +
 b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}(a+bx)^2\,dx &= \frac{\text{Subst}\left(\int \sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b} \\
&= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b} - \frac{2\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b} \\
&= -\frac{2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b} + \frac{2\text{Subst}\left(\int 1\,dx, x,\right.}{b} \\
&= 2x - \frac{2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.04

$$\frac{2(a+bx)-2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)+(a+bx)\sinh^{-1}(a+bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) - 2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + (a + b*x)*ArcSinh
[a + b*x]^2)/b
```

Maple [A]
time = 1.86, size = 46, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$	46
default	$\frac{\operatorname{arcsinh}(bx+a)^2(bx+a)-2\operatorname{arcsinh}(bx+a)\sqrt{1+(bx+a)^2}+2bx+2a}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsinh(b*x+a)^2*(b*x+a)-2*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+2*b*x+2*
a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - integrate(2*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^3 + 3*a*
b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a),
 x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(43) = 86$.
time = 0.39, size = 88, normalized size = 1.96

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\,bx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 2*b*x - 2*s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1)))/b
```

Sympy [A]
time = 0.09, size = 63, normalized size = 1.40

$$\begin{cases} \frac{a\operatorname{asinh}^2(a+bx)}{b} + x\operatorname{asinh}^2(a+bx) + 2x - \frac{2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{b} & \text{for } b \neq 0 \\ x\operatorname{asinh}^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**2/b + x*asinh(a + b*x)**2 + 2*x - 2*sqrt(a**2
+ 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/b, Ne(b, 0)), (x*asinh(a)**2, Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}(a + b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2,x)
```

```
[Out] int(asinh(a + b*x)^2, x)
```

3.71 $\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=205

$$-\frac{1}{3}\sinh^{-1}(a+bx)^3+\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+2\,\mathrm{s}$$

```
[Out] -1/3*arcsinh(b*x+a)^3+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)
^(1/2)))+2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^
(1/2)))-2*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))-2*polylo
g(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A]

time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5859, 5827, 5680, 2221, 2611, 2320, 6724}

$$2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{3}\sinh^{-1}(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a
- Sqrt[1 + a^2])] + ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt
[1 + a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1
+ a^2])] + 2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3,
 E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \frac{\text{Subst}\left(\int \frac{e^x x^2}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \text{Subst}$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2$$

$$= -\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^2$$

<u>Mathematica</u> [A]
time = 0.02, size = 251, normalized size = 1.22

$$-\frac{1}{3}\sinh^{-1}(a+bx)^3 + \sinh^{-1}(a+bx)^2\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + \sinh^{-1}(a+bx)^2\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) + 2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right) + 2\sinh^{-1}(a+bx)\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right) - 2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) - 2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a + b*x]^3 + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^2*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 2*ArcSinh[a + b*x]*PolyLog[2, -(E^ArcSi
nh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 2*ArcSinh[a + b*x]*PolyLog[2
, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 2*PolyLog[3, E^Ar
cSinh[a + b*x]/(a - Sqrt[1 + a^2])] - 2*PolyLog[3, E^ArcSinh[a + b*x]/(a +
Sqrt[1 + a^2])]
```

Maple [F]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(bx+a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x,x)
```

```
[Out] int(arcsinh(b*x+a)^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x,x)
```

```
[Out] int(asinh(a + b*x)^2/x, x)
```

3.72 $\int \frac{\sinh^{-1}(a+bx)^2}{x^2} \, dx$

Optimal. Leaf size=178

$$-\frac{\sinh^{-1}(a+bx)^2}{x}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}+\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}-\frac{2b}{}$$

```
[Out] -arcsinh(b*x+a)^2/x-2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+2*b*polylog(2,(b*x+a+(1+(b*x+a)^2)^
(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5828, 5843, 3403, 2296, 2221, 2317, 2438}

$$-\frac{2b\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}-\frac{\sinh^{-1}(a+bx)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^2/x) - (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/
(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*ArcSinh[a + b*x]*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (2*b*PolyLog[2, E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (2*b*PolyLog[2, E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)^2}{x^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b} \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right) \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} + 2\text{Subst}\left(\int \frac{x}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right) \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} + 4\text{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right) \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} + \frac{4\text{Subst}\left(\int \frac{e^x x}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{4\text{Sub}\cdots}{\cdots} \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\cdots}{\sqrt{\cdots}} \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\cdots}{\sqrt{\cdots}} \\
&= -\frac{\sinh^{-1}(a+bx)^2}{x} - \frac{2b\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{2b\sinh^{-1}(a+bx)\cdots}{\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 178, normalized size = 1.00

$$
\frac{-\sinh^{-1}(a+bx)\left(\sqrt{1+a^2}\sinh^{-1}(a+bx)+2bx\left(-\log\left(\frac{a+\sqrt{1+a^2}-e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+\log\left(\frac{-a+\sqrt{1+a^2}+e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right)\right)\right)-2bx\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+2bx\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] (-(ArcSinh[a + b*x]*(Sqrt[1 + a^2]*ArcSinh[a + b*x] + 2*b*x*(-Log[(a + Sqrt
[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + Log[(-a + Sqrt[1 + a
^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])]))) - 2*b*x*PolyLog[2, E^Arc
Sinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*b*x*PolyLog[2, E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x)
```

Maple [A]
time = 6.84, size = 206, normalized size = 1.16

method	result
derivativedivides	$b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{}}{-a+\sqrt{}}\right.\right.}{\sqrt{a^2+1}}\right.$
default	$b\left(-\frac{\operatorname{arcsinh}(bx+a)^2}{bx}+\frac{2\operatorname{arcsinh}(bx+a)\left(\ln\left(\frac{\sqrt{a^2+1}-bx-\sqrt{1+(bx+a)^2}}{a+\sqrt{a^2+1}}\right)-\ln\left(\frac{\sqrt{a^2+1}+bx+\sqrt{}}{-a+\sqrt{}}\right.\right.}{\sqrt{a^2+1}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b*(-arcsinh(b*x+a)^2/b/x+2*arcsinh(b*x+a)*(ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))-ln(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-
a+(a^2+1)^(1/2))))/(a^2+1)^(1/2)+2/(a^2+1)^(1/2)*dilog(((a^2+1)^(1/2)-b*x-(
1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/(a^2+1)^(1/2)*dilog(((a^2+1)^(1/2)
+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x + integrate(2*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^4 + 3*a*b^2*x^3 +
 (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}(a + bx)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**2,x)
[Out] Integral(asinh(a + b*x)**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{asinh}\left(a+b\,x\right)}^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^2,x)
[Out] int(asinh(a + b*x)^2/x^2, x)
```

3.73 $\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=235

$$-\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x}-\frac{\sinh^{-1}(a+bx)^2}{2x^2}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{(1+a^2)^{3/2}}-\frac{ab^2\sinh}{}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/x^2+b^2*ln(x)/(a^2+1)+a*b^2*arcsinh(b*x+a)*ln(1-(b*x+
a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2*arcsinh(b*x+a
)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a*b^2*p
olylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a*b^2
*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-b*a
rcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A]
time = 0.35, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5859, 5828, 5843, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{ab^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{ab^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}+\frac{b^2\log(x)}{a^2+1}+\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{ab^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{(a^2+1)^{3/2}}-\frac{b\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{(a^2+1)\,x}-\frac{\sinh^{-1}(a+bx)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x)) - ArcSinh[a + b
*x]^2/(2*x^2) + (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqr
t[1 + a^2])])/(1 + a^2)^(3/2) - (a*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a
 + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (b^2*Log[x])/(1 + a^2) + (a
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) -
(a*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(3/2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
```

```
- 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \operatorname{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{2x^2} + \operatorname{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b\operatorname{Subst}\left(\int \frac{\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x,\right.}{1+a^2}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{e^x x}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\right.}{1+}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{b^2\log(x)}{1+a^2} - \frac{(2ab)\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1\right.}{(1+a^2)^{3/2}}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{(1+a^2)\,x} - \frac{\sinh^{-1}(a+bx)^2}{2x^2} + \frac{ab^2\sinh^{-1}(a+bx)\log\left(1\right.}{(1+a^2)^{3/2}}$$

Mathematica [A]
time = 0.09, size = 279, normalized size = 1.19

$$-\frac{2\sqrt{1+a^2}\,bx\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)+\sqrt{1+a^2}\,\sinh^{-1}(a+bx)^2+a^2\sqrt{1+a^2}\,\sinh^{-1}(a+bx)^2+2ab^2x^2\sinh^{-1}(a+bx)\log\left(\frac{a+\sqrt{1+a^2}-e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)-2ab^2x^2\sinh^{-1}(a+bx)\log\left(\frac{-a+\sqrt{1+a^2}+e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right)-2\sqrt{1+a^2}\,b^2x^2\log(x)-2ab^2x^2\operatorname{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+2ab^2x^2\operatorname{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^3,x]
```

```
[Out] -1/2*(2*Sqrt[1 + a^2]*b*x*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x] + Sqrt[1 +
 a^2]*ArcSinh[a + b*x]^2 + a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^2 + 2*a*b^2*x
^2*ArcSinh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[
1 + a^2])] - 2*a*b^2*x^2*ArcSinh[a + b*x]*Log[(-a + Sqrt[1 + a^2] + E^ArcSi
nh[a + b*x])/(-a + Sqrt[1 + a^2])] - 2*Sqrt[1 + a^2]*b^2*x^2*Log[x] - 2*a*b
^2*x^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 2*a*b^2*x^2*Pol
yLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/((1 + a^2)^(3/2)*x^2)
```

Maple [A]
time = 8.21, size = 374, normalized size = 1.59

method	result
derivativedivides	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(4(bx+a)a-2a^2-2(bx+a)^2+\operatorname{arcsinh}(bx+a)+2(bx+a)\sqrt{1+(bx+a)^2}+a^2\operatorname{arcsinh}(bx+a\right.}{2(a^2+1)b^2x^2}\right.$
default	$b^2\left(-\frac{\operatorname{arcsinh}(bx+a)\left(4(bx+a)a-2a^2-2(bx+a)^2+\operatorname{arcsinh}(bx+a)+2(bx+a)\sqrt{1+(bx+a)^2}+a^2\operatorname{arcsinh}(bx+a\right.}{2(a^2+1)b^2x^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*arcsinh(b*x+a)*(4*(b*x+a)*a-2*a^2-2*(b*x+a)^2+arcsinh(b*x+a)+2*(b
*x+a)*(1+(b*x+a)^2)^(1/2)+a^2*arcsinh(b*x+a)-2*a*(1+(b*x+a)^2)^(1/2))/(a^2+
1)/b^2/x^2-1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a
)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/(a^2+1)^(3/2)*a*arcsinh(b*x+a)*ln(((a^2+1)
^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/(a^2+1)^(3/2)*dilog((
(a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))*a+1/(a^2+1)^(3/2)
*dilog(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))*a+1/(a^2
+1)*ln(2*a*(b*x+a+(1+(b*x+a)^2)^(1/2))-(b*x+a+(1+(b*x+a)^2)^(1/2))^2+1)-2/(
a^2+1)*ln(b*x+a+(1+(b*x+a)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^2 + integrate((b^
3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b)
 + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^5 + 3*a*b^2*x
^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x
^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}(a + bx)}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a+b\,x\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^3,x)
```

```
[Out] int(asinh(a + b*x)^2/x^3, x)
```

3.74 $\int \frac{\sinh^{-1}(a+bx)^2}{x^4} \, dx$

Optimal. Leaf size=478

$$-\frac{b^2}{3(1+a^2)x}-\frac{b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{3(1+a^2)x^2}+\frac{ab^2\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)}{(1+a^2)^2x}-\frac{\sinh^{-1}(a+bx)^2}{3x^3}-$$

```
[Out] -1/3*b^2/(a^2+1)/x-1/3*arcsinh(b*x+a)^2/x^3-a*b^3*ln(x)/(a^2+1)^2-a^2*b^3*a
rcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(
5/2)+1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/
2)))/(a^2+1)^(3/2)+a^2*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/
(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-a^2*b^3*polylog(2,(b*x+a+(1+
(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(5/2)+1/3*b^3*polylog(2,(b*x+a
+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+a^2*b^3*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(5/2)-1/3*b^3*polylog(
2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-1/3*b*arcsin
h(b*x+a)*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x^2+a*b^2*arcsinh(b*x+a)*(1+(b*x+a)^2)
^(1/2)/(a^2+1)^2/x
```

Rubi [A]

time = 1.16, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 16, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {5859, 5828, 5843, 3406, 3405, 3403, 2296, 2221, 2317, 2438, 2747, 31, 6873, 12, 6874, 32}

$$\frac{b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3(a^2+1)^{3/2}}-\frac{a^2b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{5/2}}-\frac{b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{3(a^2+1)^{3/2}}+\frac{a^2b^3\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{(a^2+1)^{5/2}}-\frac{ab^3\log(x)}{(a^2+1)^2}+\frac{b^3\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{3(a^2+1)^{3/2}}-\frac{a^2b^3\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{5/2}}-\frac{b^3\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{3(a^2+1)^{3/2}}+\frac{a^2b^3\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{(a^2+1)^{5/2}}-\frac{b^2}{3(a^2+1)x}+\frac{ab^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{(a^2+1)^2x}-\frac{b\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)}{3(a^2+1)x^2}-\frac{\sinh^{-1}(a+bx)^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] -1/3*b^2/((1 + a^2)*x) - (b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(3*(1 +
 a^2)*x^2) + (a*b^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)^2*x)
 - ArcSinh[a + b*x]^2/(3*x^3) - (a^2*b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh
[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*ArcSinh[a + b*x]*Log
[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^
3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^
2)^(5/2) - (b^3*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt[1 + a
^2])])/(3*(1 + a^2)^(3/2)) - (a*b^3*Log[x])/(1 + a^2)^2 - (a^2*b^3*PolyLog[
2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(5/2) + (b^3*PolyLog[
2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2)) + (a^2*b^3*
PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2)^(5/2) - (b^3*
PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(3*(1 + a^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 3406

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(-b)*(c + d*x)^m*Cos[e + f*x]*((a + b*Sin[e + f*x])^(n +
 1)/(f*(n + 1)*(a^2 - b^2))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m*(a
+ b*Sin[e + f*x])^(n + 1), x], x] - Dist[b*((n + 2)/((n + 1)*(a^2 - b^2))),
 Int[(c + d*x)^m*Sin[e + f*x]*(a + b*Sin[e + f*x])^(n + 1), x], x] + Dist[b
*d*(m/(f*(n + 1)*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && ILtQ[n, -2] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{2}{3}\text{Subst}\left(\int \frac{x}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} - \frac{\sinh^{-1}(a+bx)^2}{3x^3} + \frac{b\text{Subst}\left(\int \frac{\cosh(x)}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,d\right.}{3(1+}$$

$$= -\frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)^2x} - \text{sinh}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(}{3(1+a^2)^2x}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(}{3(1+a^2)^2x}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{2ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(}{3(1+a^2)^2x}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a}{(1+a^2)^2x}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a}{(1+a^2)^2x}$$

$$= -\frac{b^2}{3(1+a^2)x} - \frac{b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{3(1+a^2)x^2} + \frac{ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a}{(1+a^2)^2x}$$

$$b^2 \quad b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx) \quad ab^2\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.27, size = 1830, normalized size = 3.83

Too large to display

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/x^4,x]
```

```
[Out] (-((b*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/((1 + a^2)*x^2)) - ArcSinh[a
+ b*x]^2/x^3 - (b^2*(1 + a^2 - 3*a*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]))
/((1 + a^2)^2*x) + (I*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSinh[a + b*x]/2])/Sqrt
[1 + a^2]])/(1 + a^2)^(5/2) - ((2*I)*a^2*b^3*Pi*ArcTanh[(-1 - a*Tanh[ArcSin
h[a + b*x]/2])/Sqrt[1 + a^2]])/(1 + a^2)^(5/2) - (3*a*b^3*Log[-((b*x)/a)])/
(1 + a^2)^2 + (b^3*(-2*ArcCos[I*a]*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSin
h[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)*ArcSinh[a + b*x])*ArcTanh[((I
 + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (ArcCos[I*a]
+ (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a
^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1
 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcSinh[a + b*x]/2)*Sqrt[b*x])] +
(ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4
])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])
/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(ArcSinh[a + b*x]/2))/(Sqrt[2]*S
qrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a + I*(-1 + Sqrt[-1 - a^2]))*
(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(I + a - Sqrt[-1 - a^2]*Cot[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a] - (2*I)*ArcTanh[((-I + a)*Co
t[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*(a - I*(1
 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a +
Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] + I*(PolyLog[2, -((((
-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a +
 b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))
] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^2]*Cot[(Pi + (2
*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh
[a + b*x])/4])])))/(-1 - a^2)^(5/2) - (2*a^2*b^3*(-2*ArcCos[I*a]*ArcTanh[((
-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] - (Pi - (2*I)
*ArcSinh[a + b*x])*ArcTanh[((I + a)*Tan[(Pi + (2*I)*ArcSinh[a + b*x])/4])/S
qrt[-1 - a^2]] + (ArcCos[I*a] + (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4])/Sqrt[-1 - a^2]] + (2*I)*ArcTanh[((I + a)*Tan[(Pi + (2*I)
*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[Sqrt[-1 - a^2]/(Sqrt[2]*E^(ArcS
inh[a + b*x]/2)*Sqrt[b*x])] + (ArcCos[I*a] - (2*I)*(ArcTanh[((-I + a)*Cot[(
Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]] + ArcTanh[((I + a)*Tan[(Pi
 + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]]))*Log[(I*Sqrt[-1 - a^2]*E^(A
rcSinh[a + b*x]/2))/(Sqrt[2]*Sqrt[b*x])] - (ArcCos[I*a] + (2*I)*ArcTanh[((-
I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 - a^2]])*Log[((I + a)*
(a + I*(-1 + Sqrt[-1 - a^2]))*(I + Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(
```

```
I + a - Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])] - (ArcCos[I*a
] - (2*I)*ArcTanh[((-I + a)*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])/Sqrt[-1 -
 a^2]])*Log[((I + a)*(a - I*(1 + Sqrt[-1 - a^2]))*(-I + Cot[(Pi + (2*I)*Arc
Sinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*
x])/4])] + I*(PolyLog[2, -((((-I)*a + Sqrt[-1 - a^2])*(I + a + Sqrt[-1 - a^
2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1 - a^2]*Cot[(Pi
+ (2*I)*ArcSinh[a + b*x])/4]))] - PolyLog[2, ((I*a + Sqrt[-1 - a^2])*(I + a
 + Sqrt[-1 - a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4]))/(-I - a + Sqrt[-1
- a^2]*Cot[(Pi + (2*I)*ArcSinh[a + b*x])/4])])))/(-1 - a^2)^(5/2))/3
```

Maple [**A**]
time = 10.97, size = 764, normalized size = 1.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/3*(a^4*arcsinh(b*x+a)^2-4*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^3+7*
arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a^2*(b*x+a)-3*arcsinh(b*x+a)*(1+(b*x+a)^
2)^(1/2)*a*(b*x+a)^2-3*a^4*arcsinh(b*x+a)+9*arcsinh(b*x+a)*a^3*(b*x+a)-9*ar
csinh(b*x+a)*a^2*(b*x+a)^2+3*arcsinh(b*x+a)*a*(b*x+a)^3+2*a^2*arcsinh(b*x+a
)^2+a^4-2*a^3*(b*x+a)+a^2*(b*x+a)^2-arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*a+ar
csinh(b*x+a)*(1+(b*x+a)^2)^(1/2)*(b*x+a)+arcsinh(b*x+a)^2+a^2-2*(b*x+a)*a+(
b*x+a)^2)/b^3/x^3/(a^2+1)^2-1/(a^2+1)^2*a*ln(2*a*(b*x+a+(1+(b*x+a)^2)^(1/2)
)-(b*x+a+(1+(b*x+a)^2)^(1/2))^2+1)+2/(a^2+1)^2*a*ln(b*x+a+(1+(b*x+a)^2)^(1/
2))-1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1
/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1)^(5/2)*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+
b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))-1/3/(a^2+1)^(5/2)*dilog(((a^2+
1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))+1/3/(a^2+1)^(5/2)*dilo
g(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3/(a^2+1)^(
5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+
1)^(1/2)))-2/3/(a^2+1)^(5/2)*a^2*arcsinh(b*x+a)*ln(((a^2+1)^(1/2)+b*x+(1+(b
*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2)))+2/3/(a^2+1)^(5/2)*a^2*dilog(((a^2+1)^(1
/2)-b*x-(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))-2/3/(a^2+1)^(5/2)*a^2*dilog
(((a^2+1)^(1/2)+b*x+(1+(b*x+a)^2)^(1/2))/(-a+(a^2+1)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/x^3 + integrate(2/3
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(b^3*x^6 + 3*a*b
```

```
^2*x^5 + (3*a^2*b + b)*x^4 + (a^3 + a)*x^3 + (b^2*x^5 + 2*a*b*x^4 + (a^2 +
1)*x^3)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^2/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^2(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/x**4,x)
[Out] Integral(asinh(a + b*x)**2/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/x^4,x, algorithm="giac")
[Out] integrate(arcsinh(b*x + a)^2/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/x^4,x)
[Out] int(asinh(a + b*x)^2/x^4, x)
```

3.75 $\int x^2 \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=355

$$\frac{14\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2(1+(a+bx)^2)^{3/2}}{27b^3}-\frac{3a\sinh^{-1}(a}{4b^3}$$

```
[Out] -2/27*(1+(b*x+a)^2)^(3/2)/b^3-3/4*a*arcsinh(b*x+a)/b^3-4/3*(b*x+a)*arcsinh(
b*x+a)/b^3+6*a^2*(b*x+a)*arcsinh(b*x+a)/b^3-3/2*a*(b*x+a)^2*arcsinh(b*x+a)/
b^3+2/9*(b*x+a)^3*arcsinh(b*x+a)/b^3-1/2*a*arcsinh(b*x+a)^3/b^3+1/3*a^3*arc
sinh(b*x+a)^3/b^3+1/3*x^3*arcsinh(b*x+a)^3+14/9*(1+(b*x+a)^2)^(1/2)/b^3-6*a
^2*(1+(b*x+a)^2)^(1/2)/b^3+3/4*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3+2/3*arcsin
h(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-3*a^2*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/
2)/b^3+3/2*a*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3-1/3*(b*x+a)^2
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.32, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5859, 5828, 5843, 3398, 3377, 2718, 3392, 30, 2715, 8, 2713}

$$\frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}-\frac{6a^2\sqrt{(a+bx)^2+1}}{b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}-\frac{3a^2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b^3}+\frac{3a\sqrt{(a+bx)^2+1}(a+bx)}{4b^3}-\frac{2((a+bx)^2+1)^{3/2}}{27b^3}+\frac{14\sqrt{(a+bx)^2+1}}{9b^3}+\frac{2(a+bx)^3\sinh^{-1}(a+bx)}{9b^3}-\frac{\sqrt{(a+bx)^2+1}(a+bx)^2\sinh^{-1}(a+bx)^2}{3b^3}-\frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{(a+bx)^2+1}(a+bx)\sinh^{-1}(a+bx)^2}{2b^3}-\frac{4(a+bx)\sinh^{-1}(a+bx)}{3b^3}-\frac{a\sinh^{-1}(a+bx)^3}{2b^3}+\frac{2\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{3b^3}-\frac{3a\sinh^{-1}(a+bx)}{4b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] (14*Sqrt[1 + (a + b*x)^2])/(9*b^3) - (6*a^2*Sqrt[1 + (a + b*x)^2])/b^3 + (3
*a*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b^3) - (2*(1 + (a + b*x)^2)^(3/2))/(
27*b^3) - (3*a*ArcSinh[a + b*x])/(4*b^3) - (4*(a + b*x)*ArcSinh[a + b*x])/(
3*b^3) + (6*a^2*(a + b*x)*ArcSinh[a + b*x])/b^3 - (3*a*(a + b*x)^2*ArcSinh[
a + b*x])/(2*b^3) + (2*(a + b*x)^3*ArcSinh[a + b*x])/(9*b^3) + (2*Sqrt[1 +
(a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^3) - (3*a^2*Sqrt[1 + (a + b*x)^2]*Arc
Sinh[a + b*x]^2)/b^3 + (3*a*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
]^2)/(2*b^3) - ((a + b*x)^2*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(3*b^
3) - (a*ArcSinh[a + b*x]^3)/(2*b^3) + (a^3*ArcSinh[a + b*x]^3)/(3*b^3) + (x
^3*ArcSinh[a + b*x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh^{-1}(a+bx)^3\,dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)^2 \sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}\\
&= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3 \sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)\\
&= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx, x, \sinh^{-1}(a+bx)\right)\\
&= \frac{1}{3}x^3\sinh^{-1}(a+bx)^3 - \text{Subst}\left(\int \left(-\frac{a^3x^2}{b^3}+\frac{3a^2x^2\sinh(x)}{b^3}-\frac{3ax^2\sinh^2(x)}{b^3}+\frac{x\right.\right.\\
&= \frac{a^3\sinh^{-1}(a+bx)^3}{3b^3}+\frac{1}{3}x^3\sinh^{-1}(a+bx)^3-\frac{\text{Subst}\left(\int x^2\sinh^3(x)\,dx, x, \sinh^{-1}(a\right.}{b^3}\\
&= -\frac{3a(a+bx)^2\sinh^{-1}(a+bx)}{2b^3}+\frac{2(a+bx)^3\sinh^{-1}(a+bx)}{9b^3}-\frac{3a^2\sqrt{1+(a+bx)}}{b}\\
&= \frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}+\frac{6a^2(a+bx)\sinh^{-1}(a+bx)}{b^3}-\frac{3a(a+bx)^2\sinh^{-1}}{2b^3}\\
&= \frac{2\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2(1}{}\\
&= \frac{14\sqrt{1+(a+bx)^2}}{9b^3}-\frac{6a^2\sqrt{1+(a+bx)^2}}{b^3}+\frac{3a(a+bx)\sqrt{1+(a+bx)^2}}{4b^3}-\frac{2(}{}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 175, normalized size = 0.49

$$\frac{(160-575a^2+65abx-8b^2x^2)\sqrt{1+a^2+2abx+b^2x^2}+3(170a^3+132a^2bx+8bx(-6+b^2x^2)-15a(5+2b^2x^2))\sinh^{-1}(a+bx)-18\sqrt{1+a^2+2abx+b^2x^2}(-4+11a^2-5abx+2b^2x^2)\sinh^{-1}(a+bx)^2+18(-3a+2a^3+2b^3x^3)\sinh^{-1}(a+bx)^3}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a + b*x]^3,x]
```

```
[Out] ((160 - 575*a^2 + 65*a*b*x - 8*b^2*x^2)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] +
 3*(170*a^3 + 132*a^2*b*x + 8*b*x*(-6 + b^2*x^2) - 15*a*(5 + 2*b^2*x^2))*Ar
cSinh[a + b*x] - 18*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(-4 + 11*a^2 - 5*a*b*
x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 18*(-3*a + 2*a^3 + 2*b^3*x^3)*ArcSinh[a
 + b*x]^3)/(108*b^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsinh}(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(b*x+a)^3,x)
```

```
[Out] int(x^2*arcsinh(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate((b^3
*x^5 + 2*a*b^2*x^4 + (a^2*b + b)*x^3 + (b^2*x^4 + a*b*x^3)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*
x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(
3/2) + a), x)
```

Fricas [A]
time = 0.38, size = 225, normalized size = 0.63

$$\frac{18\,(2\,b^3x^3+2\,a^3-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-18\,(2\,b^2x^2-5\,abx+11\,a^2-4)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+3\,(8\,b^3x^3-30\,ab^2x^2+170\,a^3+12\,(11\,a^2-4)bx-75\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-(8\,b^2x^2-65\,abx+575\,a^2-160)\sqrt{b^2x^2+2\,abx+a^2+1}}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(18*(2*b^3*x^3 + 2*a^3 - 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))^3 - 18*(2*b^2*x^2 - 5*a*b*x + 11*a^2 - 4)*sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(8*b^3*x^
3 - 30*a*b^2*x^2 + 170*a^3 + 12*(11*a^2 - 4)*b*x - 75*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1)) - (8*b^2*x^2 - 65*a*b*x + 575*a^2 - 160)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [A]
time = 0.45, size = 432, normalized size = 1.22

$$\begin{cases} \frac{a^3 \operatorname{asinh}^3(a+bx)}{3b^3} + \frac{85a^3 \operatorname{asinh}(a+bx)}{18b^3} + \frac{11a^2 x \operatorname{asinh}(a+bx)}{3b^2} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{6b^3} - \frac{575a^2\sqrt{a^2+2abx+b^2x^2+1}}{108b^3} - \frac{5ax^2\operatorname{asinh}(a+bx)}{6b} + \frac{5ax\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{6b^2} + \frac{65ax\sqrt{a^2+2abx+b^2x^2+1}}{108b^2} - \frac{a\operatorname{asinh}^3(a+bx)}{2b^3} - \frac{25a\operatorname{asinh}(a+bx)}{12b^3} + \frac{x^3\operatorname{asinh}^3(a+bx)}{3} + \frac{2x^3\operatorname{asinh}(a+bx)}{9} - \frac{x^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{3b} - \frac{2x^2\sqrt{a^2+2abx+b^2x^2+1}}{27b} - \frac{4x\operatorname{asinh}(a+bx)}{3b^2} + \frac{2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{3b^3} + \frac{40\sqrt{a^2+2abx+b^2x^2+1}}{27b^3} & \text{for}\ b \neq 0 \\ \frac{x^3\operatorname{asinh}^3(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a**3*asinh(a + b*x)**3/(3*b**3) + 85*a**3*asinh(a + b*x)/(18*b**
3) + 11*a**2*x*asinh(a + b*x)/(3*b**2) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)*asinh(a + b*x)**2/(6*b**3) - 575*a**2*sqrt(a**2 + 2*a*b*x + b**2
*x**2 + 1)/(108*b**3) - 5*a*x**2*asinh(a + b*x)/(6*b) + 5*a*x*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(6*b**2) + 65*a*x*sqrt(a**2 + 2*a
*b*x + b**2*x**2 + 1)/(108*b**2) - a*asinh(a + b*x)**3/(2*b**3) - 25*a*asin
h(a + b*x)/(12*b**3) + x**3*asinh(a + b*x)**3/3 + 2*x**3*asinh(a + b*x)/9 -
 x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b) - 2*x**2
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b) - 4*x*asinh(a + b*x)/(3*b**2)
+ 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(3*b**3) + 40*sq
rt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(27*b**3), Ne(b, 0)), (x**3*asinh(a)**3/
3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{asinh}(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a + b*x)^3,x)
```

```
[Out] int(x^2*asinh(a + b*x)^3, x)
```

3.76 $\int x \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=203

$$\frac{6a\sqrt{1+(a+bx)^2}}{b^2}-\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}+\frac{3\sinh^{-1}(a+bx)}{8b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}+\frac{3(a+bx)^2 s}{}$$

```
[Out] 3/8*arcsinh(b*x+a)/b^2-6*a*(b*x+a)*arcsinh(b*x+a)/b^2+3/4*(b*x+a)^2*arcsinh
(b*x+a)/b^2+1/4*arcsinh(b*x+a)^3/b^2-1/2*a^2*arcsinh(b*x+a)^3/b^2+1/2*x^2*a
rcsinh(b*x+a)^3+6*a*(1+(b*x+a)^2)^(1/2)/b^2-3/8*(b*x+a)*(1+(b*x+a)^2)^(1/2)
/b^2+3*a*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^2-3/4*(b*x+a)*arcsinh(b*x+a
)^2*(1+(b*x+a)^2)^(1/2)/b^2
```

Rubi [A]

time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5859, 5828, 5843, 3398, 3377, 2718, 3392, 30, 2715, 8}

$$-\frac{a^2\sinh^{-1}(a+bx)^3}{2b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}}{8b^2}+\frac{6a\sqrt{(a+bx)^2+1}}{b^2}+\frac{\sinh^{-1}(a+bx)^3}{4b^2}-\frac{3(a+bx)\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{4b^2}+\frac{3a\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b^2}+\frac{3(a+bx)^2\sinh^{-1}(a+bx)}{4b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}+\frac{3\sinh^{-1}(a+bx)}{8b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (6*a*Sqrt[1 + (a + b*x)^2])/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(8*b^
2) + (3*ArcSinh[a + b*x])/(8*b^2) - (6*a*(a + b*x)*ArcSinh[a + b*x])/b^2 +
(3*(a + b*x)^2*ArcSinh[a + b*x])/(4*b^2) + (3*a*Sqrt[1 + (a + b*x)^2]*ArcSi
nh[a + b*x]^2)/b^2 - (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)
/(4*b^2) + ArcSinh[a + b*x]^3/(4*b^2) - (a^2*ArcSinh[a + b*x]^3)/(2*b^2) +
(x^2*ArcSinh[a + b*x]^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int x^2\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= \frac{1}{2}x^2\sinh^{-1}(a+bx)^3 - \frac{3}{2}\text{Subst}\left(\int \left(\frac{a^2x^2}{b^2}-\frac{2ax^2\sinh(x)}{b^2}+\frac{x^2\sinh^2(x)}{b^2}\right)\,dx, x, \text{si}$$

$$= -\frac{a^2\sinh^{-1}(a+bx)^3}{2b^2}+\frac{1}{2}x^2\sinh^{-1}(a+bx)^3-\frac{3\text{Subst}\left(\int x^2\sinh^2(x)\,dx, x, \sinh^{-1}(a\right.}{2b^2}$$

$$= \frac{3(a+bx)^2\sinh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b^2}-\frac{3(a+bx)\sqrt{1}}{}$$

$$= -\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}-\frac{6a(a+bx)\sinh^{-1}(a+bx)}{b^2}+\frac{3(a+bx)^2\sinh^{-1}(a+}{4b^2}$$

$$= \frac{6a\sqrt{1+(a+bx)^2}}{b^2}-\frac{3(a+bx)\sqrt{1+(a+bx)^2}}{8b^2}+\frac{3\sinh^{-1}(a+bx)}{8b^2}-\frac{6a(a+bx}{}$$

Mathematica [A]
time = 0.09, size = 129, normalized size = 0.64

$$\frac{3(15a-bx)\sqrt{1+a^2+2abx+b^2x^2}+(3-42a^2-36abx+6b^2x^2)\sinh^{-1}(a+bx)+6(3a-bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2+(2-4a^2+4b^2x^2)\sinh^{-1}(a+bx)^3}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a + b*x]^3,x]
```

```
[Out] (3*(15*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (3 - 42*a^2 - 36*a*b*x
+ 6*b^2*x^2)*ArcSinh[a + b*x] + 6*(3*a - b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*
x^2]*ArcSinh[a + b*x]^2 + (2 - 4*a^2 + 4*b^2*x^2)*ArcSinh[a + b*x]^3)/(8*b^
2)
```

Maple [A]
time = 2.51, size = 169, normalized size = 0.83

method	result

derivativedivides	$\frac{\frac{\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a)^3}{2}-\frac{3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4}-\frac{\operatorname{arcsinh}(bx+a)^3}{4}+\frac{3\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a}{4}}{}$
default	$\frac{\frac{\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a)^3}{2}-\frac{3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}\,(bx+a)}{4}-\frac{\operatorname{arcsinh}(bx+a)^3}{4}+\frac{3\left(1+(bx+a)^2\right)\operatorname{arcsinh}(bx+a}{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*(1+(b*x+a)^2)*arcsinh(b*x+a)^3-3/4*arcsinh(b*x+a)^2*(1+(b*x+a)^2
)^(1/2)*(b*x+a)-1/4*arcsinh(b*x+a)^3+3/4*(1+(b*x+a)^2)*arcsinh(b*x+a)-3/8*(
b*x+a)*(1+(b*x+a)^2)^(1/2)-3/8*arcsinh(b*x+a)-a*(arcsinh(b*x+a)^3*(b*x+a)-3
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*arcsinh(b*x+a)*(b*x+a)-6*(1+(b*x+a)
^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3/2*
(b^3*x^4 + 2*a*b^2*x^3 + (a^2*b + b)*x^2 + (b^2*x^3 + a*b*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(
b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + a), x)
```

Fricas [A]
time = 0.35, size = 180, normalized size = 0.89

$$\frac{2\,(2\,b^2x^2-2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-6\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+3\,(2\,b^2x^2-12\,abx-14\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-3\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-15\,a)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*b^2*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))^3 - 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a)*log(b*x + a + sqrt
(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 3*(2*b^2*x^2 - 12*a*b*x - 14*a^2 + 1)*lo
g(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1)*(b*x - 15*a))/b^2
```

Sympy [A]
time = 0.27, size = 248, normalized size = 1.22

$$\begin{cases} -\frac{a^2 \operatorname{asinh}^3(a+bx)}{2b^2} - \frac{21a^2 \operatorname{asinh}(a+bx)}{4b^2} - \frac{9ax \operatorname{asinh}(a+bx)}{2b} + \frac{9a\sqrt{a^2+2abx+b^2x^2+1} \operatorname{asinh}^2(a+bx)}{4b^2} + \frac{45a\sqrt{a^2+2abx+b^2x^2+1}}{8b^2} + \frac{x^2 \operatorname{asinh}^3(a+bx)}{2} + \frac{3x^2 \operatorname{asinh}(a+bx)}{4} - \frac{3x\sqrt{a^2+2abx+b^2x^2+1} \operatorname{asinh}^2(a+bx)}{4b} - \frac{3x\sqrt{a^2+2abx+b^2x^2+1}}{8b} + \frac{\operatorname{asinh}^3(a+bx)}{4b^2} + \frac{3 \operatorname{asinh}(a+bx)}{8b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{asinh}^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**2*asinh(a + b*x)**3/(2*b**2) - 21*a**2*asinh(a + b*x)/(4*b**
2) - 9*a*x*asinh(a + b*x)/(2*b) + 9*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*
asinh(a + b*x)**2/(4*b**2) + 45*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(8*b
**2) + x**2*asinh(a + b*x)**3/2 + 3*x**2*asinh(a + b*x)/4 - 3*x*sqrt(a**2 +
 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) - 3*x*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)/(8*b) + asinh(a + b*x)**3/(4*b**2) + 3*asinh(a + b*x)/(8*
b**2), Ne(b, 0)), (x**2*asinh(a)**3/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{asinh}(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a + b*x)^3,x)
```

```
[Out] int(x*asinh(a + b*x)^3, x)
```

3.77 $\int \sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=78

$$-\frac{6\sqrt{1+(a+bx)^2}}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}-\frac{3\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{b}+\frac{(a+bx)\sinh^{-1}(a+b$$

```
[Out] 6*(b*x+a)*arcsinh(b*x+a)/b+(b*x+a)*arcsinh(b*x+a)^3/b-6*(1+(b*x+a)^2)^(1/2)
/b-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5858, 5772, 5798, 267}

$$-\frac{6\sqrt{(a+bx)^2+1}}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b}-\frac{3\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{b}+\frac{6(a+bx)\sinh^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + (a + b*x)^2])/b + (6*(a + b*x)*ArcSinh[a + b*x])/b - (3*Sqrt[1
 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/b + ((a + b*x)*ArcSinh[a + b*x]^3)/b
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

n}, x]

Rubi steps

$$\int \sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} - \frac{3\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b} + \frac{6\text{Subst}\left(\int \sinh^{-1}\right.}{}$$

$$= \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a}{b}$$

$$= -\frac{6\sqrt{1+(a+bx)^2}}{b} + \frac{6(a+bx)\sinh^{-1}(a+bx)}{b} - \frac{3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx}{b}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.90

$$\frac{-6\sqrt{1+(a+bx)^2} + 6(a+bx)\sinh^{-1}(a+bx) - 3\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2 + (a+bx)\sinh^{-1}(a+bx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3,x]
[Out] (-6*Sqrt[1 + (a + b*x)^2] + 6*(a + b*x)*ArcSinh[a + b*x] - 3*Sqrt[1 + (a + 
b*x)^2]*ArcSinh[a + b*x]^2 + (a + b*x)*ArcSinh[a + b*x]^3)/b
```

Maple [A]
time = 2.00, size = 67, normalized size = 0.86

method	result	s
derivativedivides	$\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6\operatorname{arcsinh}(bx+a)(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$	6
default	$\frac{\operatorname{arcsinh}(bx+a)^3(bx+a)-3\operatorname{arcsinh}(bx+a)^2\sqrt{1+(bx+a)^2}+6\operatorname{arcsinh}(bx+a)(bx+a)-6\sqrt{1+(bx+a)^2}}{b}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(arcsinh(b*x+a)^3*(b*x+a)-3*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)+6*arcs
inh(b*x+a)*(b*x+a)-6*(1+(b*x+a)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - integrate(3*(b^3*x^3
 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^3 + 3*
a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + a
), x)
```

Fricas [A]
time = 0.48, size = 139, normalized size = 1.78

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\,\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+6\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-6\,\sqrt{b^2x^2+2\,abx+a^2+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 3*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 6*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 6*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))/b
```

Sympy [A]
time = 0.15, size = 109, normalized size = 1.40

$$\begin{cases} \frac{a\operatorname{asinh}^3(a+bx)}{b}+\frac{6a\operatorname{asinh}(a+bx)}{b}+x\operatorname{asinh}^3(a+bx)+6x\operatorname{asinh}(a+bx)-\frac{3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{b}-\frac{6\sqrt{a^2+2abx+b^2x^2+1}}{b} & \text{for } b\neq 0 \\ x\operatorname{asinh}^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((a*asinh(a + b*x)**3/b + 6*a*asinh(a + b*x)/b + x*asinh(a + b*x)*
*3 + 6*x*asinh(a + b*x) - 3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/b - 6*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/b, Ne(b, 0)), (x*asinh(a
)**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a+b\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3,x)
```

```
[Out] int(asinh(a + b*x)^3, x)
```

3.78 $\int \frac{\sinh^{-1}(a+bx)^3}{x}\,dx$

Optimal. Leaf size=275

$$-\frac{1}{4}\sinh^{-1}(a+bx)^4+\sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+\sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+3\,\mathrm{s}$$

```
[Out] -1/4*arcsinh(b*x+a)^4+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-
(a^2+1)^(1/2)))+arcsinh(b*x+a)^3*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))+3*arcsinh(b*x+a)^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2
+1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+
1)^(1/2)))-6*arcsinh(b*x+a)*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1
)^(1/2)))+6*polylog(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))+6*poly
log(4,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))
```

Rubi [A]
time = 0.27, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5827, 5680, 2221, 2611, 6744, 2320, 6724}

$$3\sinh^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+3\sinh^{-1}(a+bx)^2\mathrm{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)-6\sinh^{-1}(a+bx)\mathrm{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)-6\sinh^{-1}(a+bx)\mathrm{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+6\mathrm{Li}_4\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+6\mathrm{Li}_4\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)+\sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)-\frac{1}{4}\sinh^{-1}(a+bx)^4$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a
- Sqrt[1 + a^2])] + ArcSinh[a + b*x]^3*Log[1 - E^ArcSinh[a + b*x]/(a + Sqrt
[1 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[
1 + a^2])] + 3*ArcSinh[a + b*x]^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1
 + a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 +
a^2])] - 6*ArcSinh[a + b*x]*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4, E
^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \frac{\text{Subst}\left(\int \frac{e^x x^3}{-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}}{}$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3$$

$$= -\frac{1}{4}\sinh^{-1}(a+bx)^4 + \sinh^{-1}(a+bx)^3\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right) + \sinh^{-1}(a+bx)^3$$

Mathematica [A]

time = 0.02, size = 346, normalized size = 1.26

$$-\frac{1}{4}\sinh^{-1}(a+bx)^4+\sinh^{-1}(a+bx)^3\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)+\sinh^{-1}(a+bx)^3\log\left(1+\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)+3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)+3\sinh^{-1}(a+bx)^2\text{PolyLog}\left(2,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)-6\sinh^{-1}(a+bx)\text{PolyLog}\left(3,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}-\frac{\sqrt{1+a^2}}{b}\right)b}\right)-6\sinh^{-1}(a+bx)\text{PolyLog}\left(3,-\frac{e^{\sinh^{-1}(a+bx)}}{\left(-\frac{a}{b}+\frac{\sqrt{1+a^2}}{b}\right)b}\right)+6\text{PolyLog}\left(4,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+6\text{PolyLog}\left(4,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a + b*x]^4 + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x]/((-
(a/b) - Sqrt[1 + a^2]/b)*b)] + ArcSinh[a + b*x]^3*Log[1 + E^ArcSinh[a + b*x
]/((-(a/b) + Sqrt[1 + a^2]/b)*b)] + 3*ArcSinh[a + b*x]^2*PolyLog[2, -(E^Arc
Sinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] + 3*ArcSinh[a + b*x]^2*PolyL
og[2, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b)*b))] - 6*ArcSinh[a +
 b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) - Sqrt[1 + a^2]/b)*b))] - 6*
ArcSinh[a + b*x]*PolyLog[3, -(E^ArcSinh[a + b*x]/((-(a/b) + Sqrt[1 + a^2]/b
)*b))] + 6*PolyLog[4, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*PolyLog[4
, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
```

Maple [**F**]
time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(bx+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{asinh}(a+b\,x)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x,x)
```

```
[Out] int(asinh(a + b*x)^3/x, x)
```

3.79 $\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=268

$$-\frac{\sinh^{-1}(a+bx)^3}{x}-\frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}+\frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}}-$$

```
[Out] -arcsinh(b*x+a)^3/x-3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+3*b*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)
^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6*b*arcsinh(b*x+a)*polylog(2,(b
*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6*b*arcsinh(b*x+
a)*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)+6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(1/2)-6
*b*polylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5859, 5828, 5843, 3403, 2296, 2221, 2611, 2320, 6724}

$$-\frac{6b\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{6b\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{6b\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}-\frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{\sqrt{a^2+1}}+\frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{\sqrt{a^2+1}}-\frac{\sinh^{-1}(a+bx)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a + b*x]^3/x) - (3*b*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x
]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (3*b*ArcSinh[a + b*x]^2*Log[1 - E^A
rcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^2] - (6*b*ArcSinh[a + b*x]
*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 + a^2] + (6*b*A
rcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1
+ a^2] + (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/Sqrt[1 +
a^2] - (6*b*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/Sqrt[1 + a^
2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 3\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + 6\text{Subst}\left(\int \frac{e^x x^2}{-\frac{1}{b}-\frac{2ae^x}{b}+\frac{e^{2x}}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} + \frac{6\text{Subst}\left(\int \frac{e^x x^2}{-\frac{2a}{b}-\frac{2\sqrt{1+a^2}}{b}+\frac{2e^x}{b}}\,dx, x, \sinh^{-1}(a+bx)\right)}{\sqrt{1+a^2}} - \frac{6\text{Sub}}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)}{}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{x} - \frac{3b\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}} + \frac{3b\sinh^{-1}(a+bx)}{}$$

Mathematica [A]
time = 0.09, size = 259, normalized size = 0.97

$$-\frac{\sqrt{1+a^2}\sinh^{-1}(a+bx)^3-3bx\sinh^{-1}(a+bx)^2\log\left(\frac{a+\sqrt{1+a^2}-e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+3bx\sinh^{-1}(a+bx)^2\log\left(\frac{-a+\sqrt{1+a^2}+e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right)+6bx\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)-6bx\sinh^{-1}(a+bx)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)-6bx\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+6bx\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{\sqrt{1+a^2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^2,x]
```

```
[Out] -((Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 - 3*b*x*ArcSinh[a + b*x]^2*Log[(a + Sqr
t[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2])] + 3*b*x*ArcSinh[a + b
*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] +
 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])]
- 6*b*x*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])]
 - 6*b*x*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b*x*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(Sqrt[1 + a^2]*x))
```

Maple [F]
time = 4.09, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(bx+a\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^2,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x + integrate(3*(b^3*x^
2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b
)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^4 + 3*a*b^2*x^3
 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*b*x^2 + (a^2 + 1)*x)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3{(a + bx)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**2,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a+b\,x\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x^2,x)
```

```
[Out] int(asinh(a + b*x)^3/x^2, x)
```

3.80 $\int \frac{\sinh^{-1}(a+bx)^3}{x^3} \, dx$

Optimal. Leaf size=514

$$-\frac{3b^2\sinh^{-1}(a+bx)^2}{2(1+a^2)}-\frac{3b\sqrt{1+(a+bx)^2}\sinh^{-1}(a+bx)^2}{2(1+a^2)x}-\frac{\sinh^{-1}(a+bx)^3}{2x^2}+\frac{3b^2\sinh^{-1}(a+bx)\log\left(1-\right.}{1+a^2}$$

```
[Out] -3/2*b^2*arcsinh(b*x+a)^2/(a^2+1)-1/2*arcsinh(b*x+a)^3/x^2+3*b^2*arcsinh(b*
x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)+3/2*a*b^2*
arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1
)^(3/2)+3*b^2*arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1
/2)))/(a^2+1)-3/2*a*b^2*arcsinh(b*x+a)^2*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))/(
a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))
/(a-(a^2+1)^(1/2)))/(a^2+1)+3*a*b^2*arcsinh(b*x+a)*polylog(2,(b*x+a+(1+(b*x
+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*b^2*polylog(2,(b*x+a+(1+(b
*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)-3*a*b^2*arcsinh(b*x+a)*polylog(2
,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3*a*b^2*polyl
og(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a-(a^2+1)^(1/2)))/(a^2+1)^(3/2)+3*a*b^2*p
olylog(3,(b*x+a+(1+(b*x+a)^2)^(1/2))/(a+(a^2+1)^(1/2)))/(a^2+1)^(3/2)-3/2*b
*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/(a^2+1)/x
```

Rubi [A]
time = 0.59, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5859, 5828, 5843, 3405, 3403, 2296, 2221, 2611, 2320, 6724, 5680, 2317, 2438}

$$\frac{3ab^2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}+\frac{3b^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{a^2+1}+\frac{3b^2\text{Li}_2\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{a^2+1}-\frac{3ab^2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}+\frac{3ab^2\text{Li}_3\left(\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{a^2+1}}\right)}{(a^2+1)^{3/2}}-\frac{3b^2\sinh^{-1}(a+bx)^2}{2(a^2+1)}+\frac{3ab^2\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{2(a^2+1)^{3/2}}-\frac{3ab^2\sinh^{-1}(a+bx)^2\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{2(a^2+1)^{3/2}}+\frac{3b^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{a^2+1}}\right)}{a^2+1}+\frac{3b^2\sinh^{-1}(a+bx)\log\left(1-\frac{e^{\sinh^{-1}(a+bx)}}{\sqrt{a^2+1}+a}\right)}{a^2+1}-\frac{3b\sqrt{(a+bx)^2+1}\sinh^{-1}(a+bx)^2}{2(a^2+1)x}-\frac{\sinh^{-1}(a+bx)^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*b^2*ArcSinh[a + b*x]^2)/(2*(1 + a^2)) - (3*b*Sqrt[1 + (a + b*x)^2]*ArcS
inh[a + b*x]^2)/(2*(1 + a^2)*x) - ArcSinh[a + b*x]^3/(2*x^2) + (3*b^2*ArcSi
nh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3
*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/
(2*(1 + a^2)^(3/2)) + (3*b^2*ArcSinh[a + b*x]*Log[1 - E^ArcSinh[a + b*x]/(a
 + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b^2*ArcSinh[a + b*x]^2*Log[1 - E^ArcSi
nh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)) + (3*b^2*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2) + (3*a*b^2*ArcSinh[a + b*
x]*PolyLog[2, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])/(1 + a^2)^(3/2) + (3
*b^2*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1 + a^2) - (3*a*b
^2*ArcSinh[a + b*x]*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(1
+ a^2)^(3/2) - (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])])
```

```
/(1 + a^2)^(3/2) + (3*a*b^2*PolyLog[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2
])])/(1 + a^2)^(3/2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3403

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*(E^((-I)*e + f*fz*x)/((-
I)*b + 2*a*E^((-I)*e + f*fz*x) + I*b*E^(2*((-I)*e + f*fz*x)))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 5843

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/S
qrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[In
t[(a + b*x)^n*(c*f + g*Sinh[x])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b,
 c, d, e, f, g, n}, x] && EqQ[e, c^2*d] && IntegerQ[m] && GtQ[d, 0] && (GtQ
[m, 0] || IGtQ[n, 0])
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2\sqrt{1+x^2}}\,dx, x, a+bx\right)$$

$$= -\frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{x^2}{\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}\,dx, x, \sinh^{-1}(a+bx)\right)$$

$$= -\frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} + \frac{(3b)\text{Subst}\left(\int \frac{x\cosh(x)}{-\frac{a}{b}+\frac{\sinh(x)}{b}}\right.}{1 -}$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

$$= -\frac{3b^2\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)} - \frac{3b\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2\left(1+a^2\right)x} - \frac{\sinh^{-1}(a+bx)^3}{2x^2} +$$

Mathematica [**A**]
time = 0.17, size = 524, normalized size = 1.02

$$\frac{-3\sqrt{1+a^2}\,b^2x^2\sinh^{-1}(a+bx)^2-3\sqrt{1+a^2}\,bx\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2-\sqrt{1+a^2}\,\sinh^{-1}(a+bx)^3-a^2\sqrt{1+a^2}\,\sinh^{-1}(a+bx)^3+6\sqrt{1+a^2}\,b^2x^2\sinh^{-1}(a+bx)\log\left(\frac{a+\sqrt{1+a^2}-e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)-3ab^2x^2\sinh^{-1}(a+bx)^2\log\left(\frac{a+\sqrt{1+a^2}-e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)+6\sqrt{1+a^2}\,b^2x^2\sinh^{-1}(a+bx)\log\left(\frac{-a+\sqrt{1+a^2}+e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right)+3ab^2x^2\sinh^{-1}(a+bx)^2\log\left(\frac{-a+\sqrt{1+a^2}+e^{\sinh^{-1}(a+bx)}}{-a+\sqrt{1+a^2}}\right)+6b^2x^2\left(\sqrt{1+a^2}+a\sinh^{-1}(a+bx)\right)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+6b^2x^2\left(\sqrt{1+a^2}-a\sinh^{-1}(a+bx)\right)\text{PolyLog}\left(2,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)-6ab^2x^2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a-\sqrt{1+a^2}}\right)+6ab^2x^2\text{PolyLog}\left(3,\frac{e^{\sinh^{-1}(a+bx)}}{a+\sqrt{1+a^2}}\right)}{2\left(1+a^2\right)^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/x^3,x]
```

```
[Out] (-3*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]^2 - 3*Sqrt[1 + a^2]*b*x*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2 - Sqrt[1 + a^2]*ArcSinh[a + b*
x]^3 - a^2*Sqrt[1 + a^2]*ArcSinh[a + b*x]^3 + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSi
nh[a + b*x]*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a + b*x])/(a + Sqrt[1 + a^2]
)] - 3*a*b^2*x^2*ArcSinh[a + b*x]^2*Log[(a + Sqrt[1 + a^2] - E^ArcSinh[a +
b*x])/(a + Sqrt[1 + a^2])] + 6*Sqrt[1 + a^2]*b^2*x^2*ArcSinh[a + b*x]*Log[(
-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sqrt[1 + a^2])] + 3*a*b^2*x^
2*ArcSinh[a + b*x]^2*Log[(-a + Sqrt[1 + a^2] + E^ArcSinh[a + b*x])/(-a + Sq
rt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] + a*ArcSinh[a + b*x])*PolyLog[2, E
^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*b^2*x^2*(Sqrt[1 + a^2] - a*ArcSi
nh[a + b*x])*PolyLog[2, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])] - 6*a*b^2*x
^2*PolyLog[3, E^ArcSinh[a + b*x]/(a - Sqrt[1 + a^2])] + 6*a*b^2*x^2*PolyLog
[3, E^ArcSinh[a + b*x]/(a + Sqrt[1 + a^2])])/(2*(1 + a^2)^(3/2)*x^2)
```

Maple [**F**]
time = 5.63, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(bx+a\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/x^3,x)
```

```
[Out] int(arcsinh(b*x+a)^3/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/x^2 + integrate(3/2
*(b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x +
a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/(b^3*x^5 + 3*a
*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2
+ 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^3/x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{3}(a+bx)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/x**3,x)
```

```
[Out] Integral(asinh(a + b*x)**3/x**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^3/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{asinh}\left(a+b\,x\right)}^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/x^3,x)
```

```
[Out] int(asinh(a + b*x)^3/x^3, x)
```

3.81 $\int \frac{x^2}{\sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=60

$$-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

```
[Out] -1/4*Chi(arcsinh(b*x+a))/b^3+a^2*Chi(arcsinh(b*x+a))/b^3+1/4*Chi(3*arcsinh(
b*x+a))/b^3-a*Shi(2*arcsinh(b*x+a))/b^3
```

Rubi [A]
time = 0.38, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5830, 6873, 12, 6874, 3382, 5556, 3379}

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}-\frac{a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x],x]
```

```
[Out] -1/4*CoshIntegral[ArcSinh[a + b*x]]/b^3 + (a^2*CoshIntegral[ArcSinh[a + b*x
]])/b^3 + CoshIntegral[3*ArcSinh[a + b*x]]/(4*b^3) - (a*SinhIntegral[2*ArcS
inh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{b^2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)(a-\sinh(x))^2}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2\cosh(x)}{x}-\frac{2a\cosh(x)\sinh(x)}{x}+\frac{\cosh(x)\sinh^2(x)}{x}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}\right.}{b^3}$$

$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2}{}$$

$$= \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{4b^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\right.}{}$$

$$= -\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{4b^3} + \frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^3} + \frac{\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3} - \frac{a\text{Shi}\left(2\right.}{}$$

Mathematica [A]
time = 0.09, size = 44, normalized size = 0.73

$$\frac{(-1+4a^2)\,\text{Chi}\left(\sinh^{-1}(a+bx)\right)+\text{Chi}\left(3\sinh^{-1}(a+bx)\right)-4a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x],x]
```

```
[Out] ((-1 + 4*a^2)*CoshIntegral[ArcSinh[a + b*x]] + CoshIntegral[3*ArcSinh[a + b
*x]] - 4*a*SinhIntegral[2*ArcSinh[a + b*x]])/(4*b^3)
```

Maple [A]
time = 3.25, size = 49, normalized size = 0.82

method	result

derivativedivides	$\frac{a^2 \text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a)) - a \text{hyperbolicSineIntegral}(2 \text{arcsinh}(bx+a)) - \frac{\text{hyperbolicCosineIntegral}(\text{arcsi}}{4}}{b^3}$
default	$\frac{a^2 \text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a)) - a \text{hyperbolicSineIntegral}(2 \text{arcsinh}(bx+a)) - \frac{\text{hyperbolicCosineIntegral}(\text{arcsi}}{4}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(a^2*Chi(arcsinh(b*x+a))-a*Shi(2*arcsinh(b*x+a))-1/4*Chi(arcsinh(b*x+
a))+1/4*Chi(3*arcsinh(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(x^2/arcsinh(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a),x)
[Out] Integral(x**2/asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x),x)
```

```
[Out] int(x^2/asinh(a + b*x), x)
```

3.82 $\int \frac{x}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=30

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}$$

```
[Out] -a*Chi(arcsinh(b*x+a))/b^2+1/2*Shi(2*arcsinh(b*x+a))/b^2
```

Rubi [A]
time = 0.16, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5859, 5830, 6873, 12, 6874, 3382, 5556, 3379}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{bx}\,dx, x, \sinh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)(-a+\sinh(x))}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a\cosh(x)}{x}+\frac{\cosh(x)\sinh(x)}{x}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+b
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b^2} \\
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b^2} \\
&= -\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2} + \frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b^2} + \frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x],x]
```

```
[Out] -((a*CoshIntegral[ArcSinh[a + b*x]])/b^2) + SinhIntegral[2*ArcSinh[a + b*x]
]/(2*b^2)
```

Maple [A]
time = 2.69, size = 27, normalized size = 0.90

method	result	size

derivativedivides	$\frac{\frac{\text{hyperbolicSineIntegral}(2\,\text{arcsinh}(bx+a))}{2}-a\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a))}{b^2}$	27
default	$\frac{\frac{\text{hyperbolicSineIntegral}(2\,\text{arcsinh}(bx+a))}{2}-a\,\text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a))}{b^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b^2*(1/2*Shi(2*arcsinh(b*x+a))-a*Chi(arcsinh(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(x/arcsinh(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(x/arcsinh(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}\left(a + bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a),x)
[Out] Integral(x/asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x),x)
```

```
[Out] int(x/asinh(a + b*x), x)
```

3.83 $\int \frac{1}{\sinh^{-1}(a+bx)} dx$

Optimal. Leaf size=11

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

[Out] Chi(arcsinh(b*x+a))/b

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5858, 5774, 3382}

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a + b*x]^(-1),x]

[Out] CoshIntegral[ArcSinh[a + b*x]]/b

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-1),x]
[Out] CoshIntegral[ArcSinh[a + b*x]]/b
```

Maple [A]
time = 1.60, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a))}{b}$	12
default	$\frac{\text{hyperbolicCosineIntegral}(\text{arcsinh}(bx+a))}{b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] Chi(arcsinh(b*x+a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="maxima")
[Out] integrate(1/arcsinh(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="fricas")
[Out] integral(1/arcsinh(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a),x)
[Out] Integral(1/asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a),x, algorithm="giac")
[Out] integrate(1/arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\operatorname{asinh}(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x),x)
[Out] int(1/asinh(a + b*x), x)
```

3.84 $\int \frac{1}{x\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x\sinh^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sinh^{-1}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]),x]
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)}\,dx,x,a+bx\right)}{b}$$

Mathematica [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sinh^{-1}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]), x]
```

Maple [A]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{arcsinh}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a),x)
```

```
[Out] int(1/x/arcsinh(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a),x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)),x)
[Out] int(1/(x*asinh(a + b*x)), x)
```

3.85 $\int \frac{x^2}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)}+\frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)}-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-$$

```
[Out] -2*a*Chi(2*arcsinh(b*x+a))/b^3-1/4*Shi(arcsinh(b*x+a))/b^3+a^2*Shi(arcsinh(
b*x+a))/b^3+3/4*Shi(3*arcsinh(b*x+a))/b^3-a^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsi
nh(b*x+a)+2*a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)-(b*x+a)^2*(1+(
b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)
```

Rubi [A]

time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5859, 5829, 5773, 5819, 3379, 5778, 3382}

$$\frac{a^2\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^3}-\frac{a^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{2a\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{3\text{Shi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}+\frac{2a(a+bx)\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}-\frac{(a+bx)^2\sqrt{(a+bx)^2+1}}{b^3\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x])) + (2*a*(a + b*x)*Sqrt
[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]) - ((a + b*x)^2*Sqrt[1 + (a + b*x)
^2])/(b^3*ArcSinh[a + b*x]) - (2*a*CoshIntegral[2*ArcSinh[a + b*x]])/b^3 -
SinhIntegral[ArcSinh[a + b*x]]/(4*b^3) + (a^2*SinhIntegral[ArcSinh[a + b*x]
])/b^3 + (3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^2} - \frac{2ax}{b^2\sinh^{-1}(x)^2} + \frac{x^2}{b^2\sinh^{-1}(x)^2}\right) dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\text{Subst}}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} -$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} + \frac{2a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)} -$$

Mathematica [A]
time = 0.28, size = 83, normalized size = 0.54

$$\frac{-\frac{4b^2x^2\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)} - 8a\text{Chi}\left(2\sinh^{-1}(a+bx)\right) + \left(-1+4a^2\right)\text{Shi}\left(\sinh^{-1}(a+bx)\right) + 3\text{Shi}\left(3\sinh^{-1}(a+bx)\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^2,x]
```

```
[Out] ((-4*b^2*x^2*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/ArcSinh[a + b*x] - 8*a*Cosh
Integral[2*ArcSinh[a + b*x]] + (-1 + 4*a^2)*SinhIntegral[ArcSinh[a + b*x]]
+ 3*SinhIntegral[3*ArcSinh[a + b*x]])/(4*b^3)
```

Maple [A]
time = 4.12, size = 146, normalized size = 0.95

method	result
derivativedivides	$-\frac{a(2\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)-\sinh(2\,\text{arcsinh}(bx+a)))}{\text{arcsinh}(bx+a)}+\frac{\sqrt{1+(bx+a)^2}}{4\,\text{arcsinh}(bx+a)}-\text{hyperbolicSineIn}$
default	$-\frac{a(2\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)-\sinh(2\,\text{arcsinh}(bx+a)))}{\text{arcsinh}(bx+a)}+\frac{\sqrt{1+(bx+a)^2}}{4\,\text{arcsinh}(bx+a)}-\text{hyperbolicSineIn}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/b^3*(-a*(2*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)+1/4/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)-1/4*Shi(arcsinh(b*x+a))-1/4/arcsinh(b*x+a)*cosh(3*arcsinh(b*x+a))+3/4*Shi(3*arcsinh(b*x+a))+a^2*(Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="maxima")

[Out] -(b^3*x^5 + 3*a*b^2*x^4 + (3*a^2*b + b)*x^3 + (a^3 + a)*x^2 + (b^2*x^4 + 2*a*b*x^3 + (a^2 + 1)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((3*b^5*x^6 + 14*a*b^4*x^5 + 2*(13*a^2*b^3 + 3*b^3)*x^4 + 8*(3*a^3*b^2 + 2*a*b^2)*x^3 + (11*a^4*b + 14*a^2*b + 3*b)*x^2 + (3*b^3*x^4 + 8*a*b^2*x^3 + (7*a^2*b + b)*x^2 + 2*(a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^5 + 2*a^3 + a)*x + (6*b^4*x^5 + 22*a*b^3*x^4 + (30*a^2*b^2 + 7*b^2)*x^3 + (18*a^3*b + 13*a*b)*x^2 + 2*(2*a^4 + 3*a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="fricas")

[Out] integral(x^2/arcsinh(b*x + a)^2, x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x**2/asinh(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\mathrm{asinh}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x)^2,x)
```

```
[Out] int(x^2/asinh(a + b*x)^2, x)
```

3.86 $\int \frac{x}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=84

$$\frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)}+\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}$$

```
[Out] Chi(2*arcsinh(b*x+a))/b^2-a*Shi(arcsinh(b*x+a))/b^2+a*(1+(b*x+a)^2)^(1/2)/b
^2/arcsinh(b*x+a)-(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5859, 5829, 5773, 5819, 3379, 5778, 3382}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{a\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2}+\frac{a\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)\sqrt{(a+bx)^2+1}}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^2,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(b^2*ArcSinh[a + b*x]) - ((a + b*x)*Sqrt[1 + (a +
 b*x)^2])/(b^2*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b^2 - (
a*SinhIntegral[ArcSinh[a + b*x]])/b^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^2}+\frac{x}{b\sinh^{-1}(x)^2}\right)\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a\right.}{b^2}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Subs}}{}$$
$$= \frac{a\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b^2\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Shi}(\text{s}}{}$$

Mathematica [A]
time = 0.12, size = 62, normalized size = 0.74

$$-\frac{bx\sqrt{1+(a+bx)^2} - \sinh^{-1}(a+bx)\text{Chi}\left(2\sinh^{-1}(a+bx)\right) + a\sinh^{-1}(a+bx)\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b^2\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^2,x]
[Out] -((b*x*Sqrt[1 + (a + b*x)^2] - ArcSinh[a + b*x]*CoshIntegral[2*ArcSinh[a +
b*x]] + a*ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/(b^2*ArcSinh[a +
 b*x]))
```

Maple [A]
time = 3.24, size = 73, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{\sinh(2\,\text{arcsinh}(bx+a))}{2\,\text{arcsinh}(bx+a)}+\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))-\frac{a\left(\text{hyperbolicSineIntegral}(\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a\right.}{\text{arcsinh}(bx+a)}}{b^2}$
default	$\frac{-\frac{\sinh(2\,\text{arcsinh}(bx+a))}{2\,\text{arcsinh}(bx+a)}+\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))-\frac{a\left(\text{hyperbolicSineIntegral}(\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a\right.}{\text{arcsinh}(bx+a)}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/2/arcsinh(b*x+a)*sinh(2*arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a))-a*(
Shi(arcsinh(b*x+a))*arcsinh(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^4 + 3*a*b^2*x^3 + (3*a^2*b + b)*x^2 + (a^3 + a)*x + (b^2*x^3 + 2*a*
b*x^2 + (a^2 + 1)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2
*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate((2*b^5*x^5 + 9*a*b^4*x
^4 + a^5 + 4*(4*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2 + 5*a*b^2)*x^2 +
(2*b^3*x^3 + 5*a*b^2*x^2 + 4*a^2*b*x + a^3 + a)*(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*(3*a^4*b + 4*a^2*b + b)*x + (4*b^4*x^4 + 14*a*b^3*x^3 + 2*a^4 + 2*(9
*a^2*b^2 + 2*b^2)*x^2 + 3*a^2 + (10*a^3*b + 7*a*b)*x + 1)*sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1) + a)/((b^5*x^4 + 4*a*b^4*x^3 + a^4*b + 2*a^2*b + 2*(3*a^2*
b^3 + b^3)*x^2 + (b^3*x^2 + 2*a*b^2*x + a^2*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + 4*(a^3*b^2 + a*b^2)*x + 2*(b^4*x^3 + 3*a*b^3*x^2 + a^3*b + a*b + (3*a^2
*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{2}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(x/asinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x)^2,x)
```

```
[Out] int(x/asinh(a + b*x)^2, x)
```

3.87 $\int \frac{1}{\sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)}+\frac{\mathrm{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] Shi(arcsinh(b*x+a))/b-(1+(b*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3379}

$$\frac{\mathrm{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x])) + SinhIntegral[ArcSinh[a + b*
x]]/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sinh^{-1}(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 1.11

$$\frac{-\sqrt{1+(a+bx)^2}+\sinh^{-1}(a+bx)\text{Shi}\left(\sinh^{-1}(a+bx)\right)}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-2),x]
[Out] (-Sqrt[1 + (a + b*x)^2] + ArcSinh[a + b*x]*SinhIntegral[ArcSinh[a + b*x]])/
(b*ArcSinh[a + b*x])
```

Maple [A]
time = 1.92, size = 34, normalized size = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\text{hyperbolicSineIntegral}(\operatorname{arcsinh}(bx+a))}{b}$	34
default	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{\operatorname{arcsinh}(bx+a)}+\text{hyperbolicSineIntegral}(\operatorname{arcsinh}(bx+a))}{b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)+Shi(arcsinh(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^2 + 2*a*b^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
 + integrate((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*
x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 - 1) + 2*a^2 + 4*(a^3*b +
 a*b)*x + (2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2
+ b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2) + 2*a^
2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(b*x + a)^(-2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^2(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2,x)
```

```
[Out] Integral(asinh(a + b*x)**(-2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^(-2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\operatorname{asinh}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x)^2,x)
```

```
[Out] int(1/asinh(a + b*x)^2, x)
```

3.88 $\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(a+bx)^2}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]^2),x]
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^2), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right)\sinh^{-1}(x)^2} dx, x, a+bx\right)}{b}$$

Mathematica [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^2),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^2), x]
```

Maple [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a)^2,x)
```

```
[Out] int(1/x/arcsinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/((b^3*x^3 + 2*a*b^2*x^2 + (a^2*b + b)*x + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1)*(b^2*x^2 + a*b*x))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))) - integrate((a*b^4*x^4 + 4*a^2*b^3*x^3 + a^5 + 2*a^3 + 2*(3*a^3*
b^2 + a*b^2)*x^2 + (a*b^2*x^2 + a^3 + 2*(a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 4*(a^4*b + a^2*b)*x + (2*a*b^3*x^3 + 2*a^4 + 2*(3*a^2*b^2 +
 b^2)*x^2 + 3*a^2 + (6*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1) + a)/((b^5*x^6 + 4*a*b^4*x^5 + 2*(3*a^2*b^3 + b^3)*x^4 + 4*(a^3*b^2 + a
*b^2)*x^3 + (a^4*b + 2*a^2*b + b)*x^2 + (b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(b^4*x^5 + 3*a*b^3*x^4 + (3*a^2*b^2 + b^
2)*x^3 + (a^3*b + a*b)*x^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a
+ sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^{2}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\,\mathrm{asinh}\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)^2),x)
```

```
[Out] int(1/(x*asinh(a + b*x)^2), x)
```

3.89 $\int \frac{x^2}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=257

$$-\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2}+\frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2}-\frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2}+\frac{a}{b^3\sinh^{-1}(a+bx)}-\frac{a}{b^3\sinh}$$

```
[Out] a/b^3/arcsinh(b*x+a)+(-b*x-a)/b^3/arcsinh(b*x+a)-1/2*a^2*(b*x+a)/b^3/arcsin
h(b*x+a)+2*a*(b*x+a)^2/b^3/arcsinh(b*x+a)-3/2*(b*x+a)^3/b^3/arcsinh(b*x+a)-
1/8*Chi(arcsinh(b*x+a))/b^3+1/2*a^2*Chi(arcsinh(b*x+a))/b^3+9/8*Chi(3*arcsi
nh(b*x+a))/b^3-2*a*Shi(2*arcsinh(b*x+a))/b^3-1/2*a^2*(1+(b*x+a)^2)^(1/2)/b^
3/arcsinh(b*x+a)^2+a*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2-1/2*(
b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b^3/arcsinh(b*x+a)^2
```

Rubi [A]
time = 0.34, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5859, 5829, 5773, 5818, 5774, 3382, 5779, 5780, 5556, 12, 3379, 5783}

$$\frac{a^2\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^3}-\frac{a^2(a+bx)}{2b^3\sinh^{-1}(a+bx)}-\frac{a^2\sqrt{(a+bx)^2+1}}{2b^3\sinh^{-1}(a+bx)^2}-\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{8b^3}+\frac{9\text{Chi}\left(3\sinh^{-1}(a+bx)\right)}{8b^3}-\frac{2a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^3}-\frac{3(a+bx)^3}{2b^3\sinh^{-1}(a+bx)}+\frac{2a(a+bx)^2}{b^3\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)^2}{2b^3\sinh^{-1}(a+bx)^2}-\frac{a+bx}{b^3\sinh^{-1}(a+bx)}+\frac{a\sqrt{(a+bx)^2+1}\,(a+bx)}{b^3\sinh^{-1}(a+bx)^2}+\frac{a}{b^3\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(a^2*Sqrt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) + (a*(a + b*x)*Sq
rt[1 + (a + b*x)^2])/(b^3*ArcSinh[a + b*x]^2) - ((a + b*x)^2*Sqrt[1 + (a +
b*x)^2])/(2*b^3*ArcSinh[a + b*x]^2) + a/(b^3*ArcSinh[a + b*x]) - (a + b*x)/
(b^3*ArcSinh[a + b*x]) - (a^2*(a + b*x))/(2*b^3*ArcSinh[a + b*x]) + (2*a*(a
 + b*x)^2)/(b^3*ArcSinh[a + b*x]) - (3*(a + b*x)^3)/(2*b^3*ArcSinh[a + b*x]
) - CoshIntegral[ArcSinh[a + b*x]]/(8*b^3) + (a^2*CoshIntegral[ArcSinh[a +
b*x]])/(2*b^3) + (9*CoshIntegral[3*ArcSinh[a + b*x]])/(8*b^3) - (2*a*SinhIn
tegral[2*ArcSinh[a + b*x]])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x^2}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sinh^{-1}(x)^3} - \frac{2ax}{b^2\sinh^{-1}(x)^3} + \frac{x^2}{b^2\sinh^{-1}(x)^3}\right)\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^3} + \frac{a^2\text{Subst}}{}$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

$$= -\frac{a^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} + \frac{a(a+bx)\sqrt{1+(a+bx)^2}}{b^3\sinh^{-1}(a+bx)^2} - \frac{(a+bx)^2\sqrt{1+(a+bx)^2}}{2b^3\sinh^{-1}(a+bx)^2} +$$

Mathematica [A]
time = 0.24, size = 110, normalized size = 0.43

$$\frac{-\frac{4bx\left(bx\sqrt{1+a^2+2abx+b^2x^2}+(2+2a^2+5abx+3b^2x^2)\sinh^{-1}(a+bx)\right)}{\sinh^{-1}(a+bx)^2} + (-1+4a^2)\,\text{Chi}\left(\sinh^{-1}(a+bx)\right) + 9\text{Chi}\left(3\sinh^{-1}(a+bx)\right) - 16a\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcSinh[a + b*x]^3,x]
```

```
[Out] ((-4*b*x*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (2 + 2*a^2 + 5*a*b*x + 3*
b^2*x^2)*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2 + (-1 + 4*a^2)*CoshIntegral[
ArcSinh[a + b*x]] + 9*CoshIntegral[3*ArcSinh[a + b*x]] - 16*a*SinhIntegral[
2*ArcSinh[a + b*x]])/(8*b^3)
```

Maple [A]
time = 3.79, size = 213, normalized size = 0.83

method	result
derivativedivides	$\frac{a\left(-4\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)^2+2\cosh(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)+\sinh(2\,\mathrm{arcsinh}(bx+a))\right)}{2\,\mathrm{arcsinh}(bx+a)^2}+\frac{\sqrt{}}{}$
default	$\frac{a\left(-4\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)^2+2\cosh(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)+\sinh(2\,\mathrm{arcsinh}(bx+a))\right)}{2\,\mathrm{arcsinh}(bx+a)^2}+\frac{\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/2*a*(-4*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+2*cosh(2*arcsinh(b*
x+a))*arcsinh(b*x+a)+sinh(2*arcsinh(b*x+a)))/arcsinh(b*x+a)^2+1/8/arcsinh(b
*x+a)^2*(1+(b*x+a)^2)^(1/2)+1/8/arcsinh(b*x+a)*(b*x+a)-1/8*Chi(arcsinh(b*x+
a))-1/8/arcsinh(b*x+a)^2*cosh(3*arcsinh(b*x+a))-3/8/arcsinh(b*x+a)*sinh(3*a
rcsinh(b*x+a))+9/8*Chi(3*arcsinh(b*x+a))+1/2*a^2*(Chi(arcsinh(b*x+a))*arcsi
nh(b*x+a)^2-arcsinh(b*x+a)*(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^9 + 7*a*b^7*x^8 + 3*(7*a^2*b^6 + b^6)*x^7 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^6 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^5 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^4 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^3 + (a^7*b
+ 3*a^5*b + 3*a^3*b + a*b)*x^2 + (b^5*x^6 + 4*a*b^4*x^5 + (6*a^2*b^3 + b^3)
*x^4 + 2*(2*a^3*b^2 + a*b^2)*x^3 + (a^4*b + a^2*b)*x^2)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1)^(3/2) + (3*b^6*x^7 + 15*a*b^5*x^6 + 5*(6*a^2*b^4 + b^4)*x^5 + 15
*(2*a^3*b^3 + a*b^3)*x^4 + (15*a^4*b^2 + 15*a^2*b^2 + 2*b^2)*x^3 + (3*a^5*b
 + 5*a^3*b + 2*a*b)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (3*b^8*x^9 + 23*a*
b^7*x^8 + (77*a^2*b^6 + 9*b^6)*x^7 + 3*(49*a^3*b^5 + 17*a*b^5)*x^6 + (175*a
^4*b^4 + 120*a^2*b^4 + 9*b^4)*x^5 + (133*a^5*b^3 + 150*a^3*b^3 + 33*a*b^3)*
x^4 + 3*(21*a^6*b^2 + 35*a^4*b^2 + 15*a^2*b^2 + b^2)*x^3 + (17*a^7*b + 39*a
^5*b + 27*a^3*b + 5*a*b)*x^2 + (3*b^5*x^6 + 14*a*b^4*x^5 + 2*(13*a^2*b^3 +
2*b^3)*x^4 + 12*(2*a^3*b^2 + a*b^2)*x^3 + (11*a^4*b + 12*a^2*b + b)*x^2 + 2
*(a^5 + 2*a^3 + a)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (9*b^6*x^7 + 51
*a*b^5*x^6 + (120*a^2*b^4 + 17*b^4)*x^5 + 5*(30*a^3*b^3 + 13*a*b^3)*x^4 + (
105*a^4*b^2 + 93*a^2*b^2 + 10*b^2)*x^3 + (39*a^5*b + 59*a^3*b + 20*a*b)*x^2
 + 2*(3*a^6 + 7*a^4 + 5*a^2 + 1)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*(a^8
```

```
+ 3*a^6 + 3*a^4 + a^2)*x + (9*b^7*x^8 + 60*a*b^6*x^7 + (171*a^2*b^5 + 22*b^
5)*x^6 + 2*(135*a^3*b^4 + 52*a*b^4)*x^5 + (255*a^4*b^3 + 196*a^2*b^3 + 18*b
^3)*x^4 + 2*(72*a^5*b^2 + 92*a^3*b^2 + 25*a*b^2)*x^3 + (45*a^6*b + 86*a^4*b
 + 46*a^2*b + 5*b)*x^2 + 2*(3*a^7 + 8*a^5 + 7*a^3 + 2*a)*x)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^7*x^8 + 18*a*b^6*x^7 + (45*a^2*b^5 + 7*b^5)*x^6 + 4*(15*a^3*b^4 + 7*a*b^4)
*x^5 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^4 + 2*(9*a^5*b^2 + 14*a^3*b^2 +
5*a*b^2)*x^3 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x^2)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(5*a^2*b^6
+ b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b^4 + 6*a^2
*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)*(b^2*x^2
+ 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 + a^2*b^2 +
 (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2
+ 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a*b^6*x^4 +
 a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3*b^4 + 3*a
*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integrate(1/2*(9
*b^10*x^10 + 82*a*b^9*x^9 + 2*a^10 + 2*(167*a^2*b^8 + 18*b^8)*x^8 + 8*a^8 +
 32*(25*a^3*b^7 + 8*a*b^7)*x^7 + 2*(623*a^4*b^6 + 394*a^2*b^6 + 27*b^6)*x^6
 + 12*a^6 + 4*(329*a^5*b^5 + 342*a^3*b^5 + 69*a*b^5)*x^5 + 4*(238*a^6*b^4 +
 365*a^4*b^4 + 144*a^2*b^4 + 9*b^4)*x^4 + 8*a^4 + 16*(29*a^7*b^3 + 61*a^5*b
^3 + 39*a^3*b^3 + 7*a*b^3)*x^3 + (9*b^6*x^6 + 46*a*b^5*x^5 + 2*a^6 + 4*(24*
a^2*b^4 + b^4)*x^4 + 4*a^4 + 8*(13*a^3*b^3 + 2*a*b^3)*x^3 + (61*a^4*b^2 + 2
4*a^2*b^2 - b^2)*x^2 + 2*a^2 + 2*(9*a^5*b + 8*a^3*b - a*b)*x)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + (145*a^8*b^2 + 396*a^6*b^2 + 366*a^4*b^2 + 124*a^2*b^2
 + 9*b^2)*x^2 + (36*b^7*x^7 + 220*a*b^6*x^6 + 8*a^7 + 8*(71*a^2*b^5 + 6*b^5
)*x^5 + 20*a^5 + 16*(50*a^3*b^4 + 13*a*b^4)*x^4 + (660*a^4*b^3 + 356*a^2*b^
3 + 13*b^3)*x^3 + 16*a^3 + (316*a^5*b^2 + 300*a^3*b^2 + 39*a*b^2)*x^2 + 2*(
40*a^6*b + 62*a^4*b + 21*a^2*b - b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^
(3/2) + (54*b^8*x^8 + 384*a*b^7*x^7 + 12*a^8 + 6*(197*a^2*b^6 + 20*b^6)*x^6
 + 36*a^6 + 12*(171*a^3*b^5 + 52*a*b^5)*x^5 + (2190*a^4*b^4 + 1332*a^2*b^4
+ 83*b^4)*x^4 + 38*a^4 + 4*(366*a^5*b^3 + 372*a^3*b^3 + 71*a*b^3)*x^3 + (59
4*a^6*b^2 + 912*a^4*b^2 + 357*a^2*b^2 + 19*b^2)*x^2 + 16*a^2 + 2*(66*a^7*b
+ 144*a^5*b + 97*a^3*b + 19*a*b)*x + 2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*a
^2 + 2*(13*a^9*b + 44*a^7*b + 54*a^5*b + 28*a^3*b + 5*a*b)*x + (36*b^9*x^9
+ 292*a*b^8*x^8 + 8*a^9 + 4*(261*a^2*b^7 + 28*b^7)*x^7 + 28*a^7 + 4*(539*a^
3*b^6 + 172*a*b^6)*x^6 + (2828*a^4*b^5 + 1788*a^2*b^5 + 123*b^5)*x^5 + 36*a
^5 + (2436*a^5*b^4 + 2540*a^3*b^4 + 519*a*b^4)*x^4 + (1372*a^6*b^3 + 2120*a
^4*b^3 + 855*a^2*b^3 + 57*b^3)*x^3 + 20*a^3 + (484*a^7*b^2 + 1032*a^5*b^2 +
 681*a^3*b^2 + 133*a*b^2)*x^2 + 2*(48*a^8*b + 134*a^6*b + 129*a^4*b + 48*a^
2*b + 5*b)*x + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^8 + 8*a*b^9
*x^7 + a^8*b^2 + 4*a^6*b^2 + 4*(7*a^2*b^8 + b^8)*x^6 + 6*a^4*b^2 + 8*(7*a^3
*b^7 + 3*a*b^7)*x^5 + 2*(35*a^4*b^6 + 30*a^2*b^6 + 3*b^6)*x^4 + 4*a^2*b^2 +
 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^3 + (b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*
b^4*x^2 + 4*a^3*b^3*x + a^4*b^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6
```

```
*b^4 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^2 + 4*(b^7*x^5 + 5*a*b^6*x^4 + a^5*b
^2 + a^3*b^2 + (10*a^2*b^5 + b^5)*x^3 + (10*a^3*b^4 + 3*a*b^4)*x^2 + (5*a^4
*b^3 + 3*a^2*b^3)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^6 + 6*a
*b^7*x^5 + a^6*b^2 + 2*a^4*b^2 + (15*a^2*b^6 + 2*b^6)*x^4 + a^2*b^2 + 4*(5*
a^3*b^5 + 2*a*b^5)*x^3 + (15*a^4*b^4 + 12*a^2*b...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x^2/arcsinh(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/asinh(b*x+a)**3,x)
[Out] Integral(x**2/asinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arcsinh(b*x+a)^3,x, algorithm="giac")
[Out] integrate(x^2/arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\mathrm{asinh}\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/asinh(a + b*x)^3,x)
[Out] int(x^2/asinh(a + b*x)^3, x)
```

3.90 $\int \frac{x}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=147

$$\frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2}-\frac{1}{2b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}-$$

```
[Out] -1/2/b^2/arcsinh(b*x+a)+1/2*a*(b*x+a)/b^2/arcsinh(b*x+a)-(b*x+a)^2/b^2/arcs
inh(b*x+a)-1/2*a*Chi(arcsinh(b*x+a))/b^2+Shi(2*arcsinh(b*x+a))/b^2+1/2*a*(1
+(b*x+a)^2)^(1/2)/b^2/arcsinh(b*x+a)^2-1/2*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b^2/
arcsinh(b*x+a)^2
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5859, 5829, 5773, 5818, 5774, 3382, 5779, 5780, 5556, 12, 3379, 5783}

$$-\frac{a\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b^2}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b^2}-\frac{(a+bx)^2}{b^2\sinh^{-1}(a+bx)}+\frac{a(a+bx)}{2b^2\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)}{2b^2\sinh^{-1}(a+bx)^2}-\frac{1}{2b^2\sinh^{-1}(a+bx)}+\frac{a\sqrt{(a+bx)^2+1}}{2b^2\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] (a*Sqrt[1 + (a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 +
(a + b*x)^2])/(2*b^2*ArcSinh[a + b*x]^2) - 1/(2*b^2*ArcSinh[a + b*x]) + (a*
(a + b*x))/(2*b^2*ArcSinh[a + b*x]) - (a + b*x)^2/(b^2*ArcSinh[a + b*x]) -
(a*CoshIntegral[ArcSinh[a + b*x]])/(2*b^2) + SinhIntegral[2*ArcSinh[a + b*x
]]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
```

```
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{a}{b}+\frac{x}{b}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{b\sinh^{-1}(x)^3}+\frac{x}{b\sinh^{-1}(x)^3}\right)\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, \ldots\right.}{2b^2}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+\ldots}{2b^2\sinh^{-1}\ldots}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+\ldots}{2b^2\sinh^{-1}\ldots}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+\ldots}{2b^2\sinh^{-1}\ldots}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+\ldots}{2b^2\sinh^{-1}\ldots}$$

$$= \frac{a\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{2b^2\sinh^{-1}(a+bx)^2} - \frac{1}{2b^2\sinh^{-1}(a+bx)} + \frac{a(a+\ldots}{2b^2\sinh^{-1}\ldots}$$

Mathematica [A]
time = 0.06, size = 117, normalized size = 0.80

$$-\frac{bx\sqrt{1+a^2+2abx+b^2x^2}+\sinh^{-1}(a+bx)+a^2\sinh^{-1}(a+bx)+3abx\sinh^{-1}(a+bx)+2b^2x^2\sinh^{-1}(a+bx)+a\sinh^{-1}(a+bx)^2\text{Chi}(\sinh^{-1}(a+bx))-2\sinh^{-1}(a+bx)^2\text{Shi}(2\sinh^{-1}(a+bx))}{2b^2\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(b*x*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + ArcSinh[a + b*x] + a^2*ArcSin
h[a + b*x] + 3*a*b*x*ArcSinh[a + b*x] + 2*b^2*x^2*ArcSinh[a + b*x] + a*ArcS
inh[a + b*x]^2*CoshIntegral[ArcSinh[a + b*x]] - 2*ArcSinh[a + b*x]^2*SinhIn
tegral[2*ArcSinh[a + b*x]])/(b^2*ArcSinh[a + b*x]^2)
```

Maple [A]
time = 3.22, size = 107, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{\sinh(2\,\text{arcsinh}(bx+a))}{4\,\text{arcsinh}(bx+a)^2}-\frac{\cosh(2\,\text{arcsinh}(bx+a))}{2\,\text{arcsinh}(bx+a)}+\text{hyperbolicSineIntegral}(2\,\text{arcsinh}(bx+a))-\frac{a\left(\text{hyperbolicCosineIntegral}(\text{arcsi}\right.}{}}{b^2}$
default	$\frac{-\frac{\sinh(2\,\text{arcsinh}(bx+a))}{4\,\text{arcsinh}(bx+a)^2}-\frac{\cosh(2\,\text{arcsinh}(bx+a))}{2\,\text{arcsinh}(bx+a)}+\text{hyperbolicSineIntegral}(2\,\text{arcsinh}(bx+a))-\frac{a\left(\text{hyperbolicCosineIntegral}(\text{arcsi}\right.}{}}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-1/4/arcsinh(b*x+a)^2*sinh(2*arcsinh(b*x+a))-1/2/arcsinh(b*x+a)*cosh
(2*arcsinh(b*x+a))+Shi(2*arcsinh(b*x+a))-1/2*a*(Chi(arcsinh(b*x+a))*arcsinh
(b*x+a)^2-arcsinh(b*x+a)*(b*x+a)-(1+(b*x+a)^2)^(1/2))/arcsinh(b*x+a)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
```

^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (2*b^8*x^8 + 15*a*b^7*x^
7 + a^8 + (49*a^2*b^6 + 6*b^6)*x^6 + 3*a^6 + (91*a^3*b^5 + 33*a*b^5)*x^5 +
3*(35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 3*a^4 + (77*a^5*b^3 + 90*a^3*b^3
+ 21*a*b^3)*x^3 + (35*a^6*b^2 + 60*a^4*b^2 + 27*a^2*b^2 + 2*b^2)*x^2 + (2*b
^5*x^5 + 9*a*b^4*x^4 + a^5 + 2*(8*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(7*a^3*b^2
 + 3*a*b^2)*x^2 + 6*(a^4*b + a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + (6*b^6*x^6 + 33*a*b^5*x^5 + 3*a^6 + 5*(15*a^2*b^4 + 2*b^4)*x^4 + 7*a^
4 + (90*a^3*b^3 + 37*a*b^3)*x^3 + (60*a^4*b^2 + 51*a^2*b^2 + 5*b^2)*x^2 + 5
*a^2 + (21*a^5*b + 31*a^3*b + 10*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ a^2 + 3*(3*a^7*b + 7*a^5*b + 5*a^3*b + a*b)*x + (6*b^7*x^7 + 39*a*b^6*x^6
 + 3*a^7 + 2*(54*a^2*b^5 + 7*b^5)*x^5 + 8*a^5 + (165*a^3*b^4 + 64*a*b^4)*x^
4 + (150*a^4*b^3 + 116*a^2*b^3 + 11*b^3)*x^3 + 7*a^3 + (81*a^5*b^2 + 104*a^
3*b^2 + 29*a*b^2)*x^2 + (24*a^6*b + 46*a^4*b + 25*a^2*b + 3*b)*x + 2*a)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)) + (3*b^7*x^7 + 18*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^
4 + 7*a*b^4)*x^4 + (45*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 1
4*a^3*b^2 + 5*a*b^2)*x^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))/((b^8*x^6 + 6*a*b^7*x^5 + a^6*b^2 + 3*a^4*b^2 + 3*(
5*a^2*b^6 + b^6)*x^4 + 3*a^2*b^2 + 4*(5*a^3*b^5 + 3*a*b^5)*x^3 + 3*(5*a^4*b
^4 + 6*a^2*b^4 + b^4)*x^2 + (b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^4 + 4*a*b^5*x^3 + a^4*b^2 +
 a^2*b^2 + (6*a^2*b^4 + b^4)*x^2 + 2*(2*a^3*b^3 + a*b^3)*x)*(b^2*x^2 + 2*a*
b*x + a^2 + 1) + b^2 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b^3)*x + 3*(b^7*x^5 + 5*a
*b^6*x^4 + a^5*b^2 + 2*a^3*b^2 + 2*(5*a^2*b^5 + b^5)*x^3 + a*b^2 + 2*(5*a^3
*b^4 + 3*a*b^4)*x^2 + (5*a^4*b^3 + 6*a^2*b^3 + b^3)*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2) + integr
ate(1/2*(4*b^9*x^9 + 35*a*b^8*x^8 + 3*a^9 + 8*(17*a^2*b^7 + 2*b^7)*x^7 + 12
a^7 + 4(77*a^3*b^6 + 27*a*b^6)*x^6 + 8*(56*a^4*b^5 + 39*a^2*b^5 + 3*b^5)*
x^5 + 18*a^5 + 2*(217*a^5*b^4 + 250*a^3*b^4 + 57*a*b^4)*x^4 + 8*(35*a^6*b^3
 + 60*a^4*b^3 + 27*a^2*b^3 + 2*b^3)*x^3 + (4*b^5*x^5 + 19*a*b^4*x^4 + 36*a^
2*b^3*x^3 + 34*a^3*b^2*x^2 + 16*a^4*b*x + 3*a^5 - 3*a)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1)^2 + 12*a^3 + 4*(29*a^7*b^2 + 69*a^5*b^2 + 51*a^3*b^2 + 11*a*b^2)*
x^2 + (16*b^6*x^6 + 92*a*b^5*x^5 + 12*a^6 + 4*(55*a^2*b^4 + 4*b^4)*x^4 + 12
a^4 + 20(14*a^3*b^3 + 3*a*b^3)*x^3 + 4*(50*a^4*b^2 + 21*a^2*b^2)*x^2 - 3*
a^2 + (76*a^5*b + 52*a^3*b - 3*a*b)*x - 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2) + 3*(8*b^7*x^7 + 54*a*b^6*x^6 + 6*a^7 + 4*(39*a^2*b^5 + 4*b^5)*x^5 + 12
a^5 + 2(125*a^3*b^4 + 38*a*b^4)*x^4 + 8*(30*a^4*b^3 + 18*a^2*b^3 + b^3)*x
^3 + 7*a^3 + (138*a^5*b^2 + 136*a^3*b^2 + 23*a*b^2)*x^2 + 2*(22*a^6*b + 32*
a^4*b + 11*a^2*b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(7*a^8*b + 22*a^
6*b + 24*a^4*b + 10*a^2*b + b)*x + (16*b^8*x^8 + 124*a*b^7*x^7 + 12*a^8 + 1
2*(35*a^2*b^6 + 4*b^6)*x^6 + 36*a^6 + 4*(203*a^3*b^5 + 69*a*b^5)*x^5 + 4*(2
45*a^4*b^4 + 165*a^2*b^4 + 12*b^4)*x^4 + 39*a^4 + 3*(252*a^5*b^3 + 280*a^3*
b^3 + 61*a*b^3)*x^3 + (364*a^6*b^2 + 600*a^4*b^2 + 261*a^2*b^2 + 19*b^2)*x^

```
2 + 18*a^2 + (100*a^7*b + 228*a^5*b + 165*a^3*b + 37*a*b)*x + 3)*sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1) + 3*a)/((b^9*x^8 + 8*a*b^8*x^7 + a^8*b + 4*a^6*b +
4*(7*a^2*b^7 + b^7)*x^6 + 8*(7*a^3*b^6 + 3*a*b^6)*x^5 + 6*a^4*b + 2*(35*a^4
*b^5 + 30*a^2*b^5 + 3*b^5)*x^4 + 8*(7*a^5*b^4 + 10*a^3*b^4 + 3*a*b^4)*x^3 +
 (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 4*a^2*b + 4*(7*a^6*b^3 + 15*a^4*b^3 + 9*a^2*b^3 + b^3
)*x^2 + 4*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + a^3*b + (10*a^2*b^4 + b^4)*x^3 +
 (10*a^3*b^3 + 3*a*b^3)*x^2 + (5*a^4*b^2 + 3*a^2*b^2)*x)*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2) + 6*(b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 2*a^4*b + (15*a^2*b^
5 + 2*b^5)*x^4 + 4*(5*a^3*b^4 + 2*a*b^4)*x^3 + a^2*b + (15*a^4*b^3 + 12*a^2
*b^3 + b^3)*x^2 + 2*(3*a^5*b^2 + 4*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x +
 a^2 + 1) + 8*(a^7*b^2 + 3*a^5*b^2 + 3*a^3*b^2 + a*b^2)*x + 4*(b^8*x^7 + 7*
a*b^7*x^6 + a^7*b + 3*a^5*b + 3*(7*a^2*b^6 + b^6)*x^5 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^4 + 3*a^3*b + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^3 + 3*(7*a^5*b^3 +
 10*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^
2)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(x/arcsinh(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}^{3}\left(a + bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(b*x+a)**3,x)
[Out] Integral(x/asinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\operatorname{asinh}\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(a + b*x)^3,x)
```

```
[Out] int(x/asinh(a + b*x)^3, x)
```

3.91 $\int \frac{1}{\sinh^{-1}(a+bx)^3} \, dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}+\frac{\operatorname{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}$$

```
[Out] 1/2*(-b*x-a)/b/arcsinh(b*x+a)+1/2*Chi(arcsinh(b*x+a))/b-1/2*(1+(b*x+a)^2)^(
1/2)/b/arcsinh(b*x+a)^2
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5858, 5773, 5818, 5774, 3382}

$$\frac{\operatorname{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}-\frac{\sqrt{(a+bx)^2+1}}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^(-3),x]
```

```
[Out] -1/2*Sqrt[1 + (a + b*x)^2]/(b*ArcSinh[a + b*x]^2) - (a + b*x)/(2*b*ArcSinh[
a + b*x]) + CoshIntegral[ArcSinh[a + b*x]]/(2*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sinh^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2}+\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}\\
&= -\frac{\sqrt{1+(a+bx)^2}}{2b\sinh^{-1}(a+bx)^2}-\frac{a+bx}{2b\sinh^{-1}(a+bx)}+\frac{\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 0.84

$$
\frac{-\frac{\sqrt{1+(a+bx)^2}}{\sinh^{-1}(a+bx)^2}-\frac{a+bx}{\sinh^{-1}(a+bx)}+\text{Chi}\left(\sinh^{-1}(a+bx)\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^(-3),x]
[Out] (-(Sqrt[1 + (a + b*x)^2]/ArcSinh[a + b*x]^2) - (a + b*x)/ArcSinh[a + b*x] +
 CoshIntegral[ArcSinh[a + b*x]])/(2*b)
```

Maple [A]
time = 1.93, size = 51, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\operatorname{arcsinh}(bx+a)^2}-\frac{bx+a}{2\operatorname{arcsinh}(bx+a)}+\frac{\text{hyperbolicCosineIntegral}(\operatorname{arcsinh}(bx+a))}{2}}{b}$	51
default	$\frac{-\frac{\sqrt{1+(bx+a)^2}}{2\operatorname{arcsinh}(bx+a)^2}-\frac{bx+a}{2\operatorname{arcsinh}(bx+a)}+\frac{\text{hyperbolicCosineIntegral}(\operatorname{arcsinh}(bx+a))}{2}}{b}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2/arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)-1/2/arcsinh(b*x+a)*(b*x+a)+1
/2*Chi(arcsinh(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^7*x^
7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*
a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 +
 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3
*b*x + a^4 - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^5 + 5*a*b^4*
x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*
a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9
*a^2*b + b)*x + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + 3*(15*a^2*b^4 + 2*b^4)*
x^4 + 6*a^4 + 12*(5*a^3*b^3 + 2*a*b^3)*x^3 + (45*a^4*b^2 + 36*a^2*b^2 + 4*b
^2)*x^2 + 4*a^2 + 2*(9*a^5*b + 12*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 + 1) + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/((
b^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^
3*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4
```

```
*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/
2) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(
2*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^
2 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 +
b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2
)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^2) + integrate(1/2*(b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^
2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30
*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 +
(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4 + 3)*(b^2*x^2 + 2*
a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (4*
b^5*x^5 + 20*a*b^4*x^4 + 4*a^5 + 4*(10*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4*(10*a
^3*b^2 + 3*a*b^2)*x^2 + (20*a^4*b + 12*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + 3*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6 + 2*(15*a^2*b^4
 + 2*b^4)*x^4 + 4*a^4 + 8*(5*a^3*b^3 + 2*a*b^3)*x^3 + (30*a^4*b^2 + 24*a^2*
b^2 + b^2)*x^2 + a^2 + 2*(6*a^5*b + 8*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*
x + a^2 + 1) + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + (4*b^7*x^7 +
 28*a*b^6*x^6 + 4*a^7 + 12*(7*a^2*b^5 + b^5)*x^5 + 12*a^5 + 20*(7*a^3*b^4 +
 3*a*b^4)*x^4 + (140*a^4*b^3 + 120*a^2*b^3 + 9*b^3)*x^3 + 9*a^3 + 3*(28*a^5
*b^2 + 40*a^3*b^2 + 9*a*b^2)*x^2 + (28*a^6*b + 60*a^4*b + 27*a^2*b + b)*x +
 a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)/((b^8*x^8 + 8*a*b^7*x^7 + a^8 +
4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b
^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)
*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 +
4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2
 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*a^2 + 8*(a^7*b
 + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b
^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*
b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^
6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)
*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="fricas")
[Out] integral(arcsinh(b*x + a)^(-3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3,x)
```

```
[Out] Integral(asinh(a + b*x)**(-3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)^(-3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\operatorname{asinh}\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/asinh(a + b*x)^3,x)
```

```
[Out] int(1/asinh(a + b*x)^3, x)
```

3.92 $\int \frac{1}{x \sinh^{-1}(a+bx)^3} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x \sinh^{-1}(a+bx)^3}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(b*x+a)^3,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \sinh^{-1}(a+bx)^3} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcSinh[a + b*x]^3),x]
[Out] Defer[Subst][Defer[Int][1/((-(a/b) + x/b)*ArcSinh[x]^3), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{x \sinh^{-1}(a+bx)^3} dx = \frac{\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x}{b}\right) \sinh^{-1}(x)^3} dx, x, a+bx\right)}{b}$$

Mathematica [A]
time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sinh^{-1}(a+bx)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcSinh[a + b*x]^3),x]
[Out] Integrate[1/(x*ArcSinh[a + b*x]^3), x]
```

Maple [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arcsinh}(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arcsinh(b*x+a)^3,x)
```

```
[Out] int(1/x/arcsinh(b*x+a)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^8*x^8 + 7*a*b^7*x^7 + 3*(7*a^2*b^6 + b^6)*x^6 + 5*(7*a^3*b^5 + 3*a*
b^5)*x^5 + (35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 3*(7*a^5*b^3 + 10*a^3*b^
3 + 3*a*b^3)*x^3 + (7*a^6*b^2 + 15*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + (b^5*x^
5 + 4*a*b^4*x^4 + (6*a^2*b^3 + b^3)*x^3 + 2*(2*a^3*b^2 + a*b^2)*x^2 + (a^4*
b + a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^6*x^6 + 15*a*b^5*x
^5 + 5*(6*a^2*b^4 + b^4)*x^4 + 15*(2*a^3*b^3 + a*b^3)*x^3 + (15*a^4*b^2 + 1
5*a^2*b^2 + 2*b^2)*x^2 + (3*a^5*b + 5*a^3*b + 2*a*b)*x)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x - (a*b^7*x^7 + 7*a^2*b^6*x
^6 + a^8 + 3*a^6 + 3*(7*a^3*b^5 + a*b^5)*x^5 + 5*(7*a^4*b^4 + 3*a^2*b^4)*x^
4 + 3*a^4 + (35*a^5*b^3 + 30*a^3*b^3 + 3*a*b^3)*x^3 + 3*(7*a^6*b^2 + 10*a^4
*b^2 + 3*a^2*b^2)*x^2 + (a*b^4*x^4 + a^5 + 2*(2*a^2*b^3 + b^3)*x^3 + 2*a^3
+ 6*(a^3*b^2 + a*b^2)*x^2 + 2*(2*a^4*b + 3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2) + (3*a*b^5*x^5 + 3*a^6 + (15*a^2*b^4 + 4*b^4)*x^4 + 7
*a^4 + (30*a^3*b^3 + 19*a*b^3)*x^3 + (30*a^4*b^2 + 33*a^2*b^2 + 5*b^2)*x^2
+ 5*a^2 + 5*(3*a^5*b + 5*a^3*b + 2*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1
) + a^2 + (7*a^7*b + 15*a^5*b + 9*a^3*b + a*b)*x + (3*a*b^6*x^6 + 3*a^7 + 2
*(9*a^2*b^5 + b^5)*x^5 + 8*a^5 + (45*a^3*b^4 + 16*a*b^4)*x^4 + (60*a^4*b^3
+ 44*a^2*b^3 + 3*b^3)*x^3 + 7*a^3 + (45*a^5*b^2 + 56*a^3*b^2 + 13*a*b^2)*x^
2 + (18*a^6*b + 34*a^4*b + 17*a^2*b + b)*x + 2*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^7*x^7 + 1
8*a*b^6*x^6 + (45*a^2*b^5 + 7*b^5)*x^5 + 4*(15*a^3*b^4 + 7*a*b^4)*x^4 + (45
*a^4*b^3 + 42*a^2*b^3 + 5*b^3)*x^3 + 2*(9*a^5*b^2 + 14*a^3*b^2 + 5*a*b^2)*x
^2 + (3*a^6*b + 7*a^4*b + 5*a^2*b + b)*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/((b^8*x^8 + 6*a*b^7*x^7 + 3*(5*a^2*b^6 + b^6)*x^6 + 4*(5*a^3*b^5 + 3*a*b^
5)*x^5 + 3*(5*a^4*b^4 + 6*a^2*b^4 + b^4)*x^4 + 6*(a^5*b^3 + 2*a^3*b^3 + a*b
^3)*x^3 + (a^6*b^2 + 3*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + (b^5*x^5 + 3*a*b^4*
x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*
(b^6*x^6 + 4*a*b^5*x^5 + (6*a^2*b^4 + b^4)*x^4 + 2*(2*a^3*b^3 + a*b^3)*x^3
+ (a^4*b^2 + a^2*b^2)*x^2)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 3*(b^7*x^7 + 5*a
*b^6*x^6 + 2*(5*a^2*b^5 + b^5)*x^5 + 2*(5*a^3*b^4 + 3*a*b^4)*x^4 + (5*a^4*b
^3 + 6*a^2*b^3 + b^3)*x^3 + (a^5*b^2 + 2*a^3*b^2 + a*b^2)*x^2)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*(a*b^9*x^9 + 10*a^2*b^8*x^8 + 2*a^10 + 8*a^8 + 4*(11*a^3*b^
7 + a*b^7)*x^7 + 16*(7*a^4*b^6 + 2*a^2*b^6)*x^6 + 12*a^6 + 2*(91*a^5*b^5 +
```

```
54*a^3*b^5 + 3*a*b^5)*x^5 + 4*(49*a^6*b^4 + 50*a^4*b^4 + 9*a^2*b^4)*x^4 + 8
*a^4 + 4*(35*a^7*b^3 + 55*a^5*b^3 + 21*a^3*b^3 + a*b^3)*x^3 + (a*b^5*x^5 +
2*a^6 + 2*(3*a^2*b^4 + 2*b^4)*x^4 + 4*a^4 + 2*(7*a^3*b^3 + 8*a*b^3)*x^3 + 8
*(2*a^4*b^2 + 3*a^2*b^2 + b^2)*x^2 + 2*a^2 + (9*a^5*b + 16*a^3*b + 7*a*b)*x
)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 16*(4*a^8*b^2 + 9*a^6*b^2 + 6*a^4*b^2 +
 a^2*b^2)*x^2 + (4*a*b^6*x^6 + 8*a^7 + 4*(7*a^2*b^5 + 3*b^5)*x^5 + 20*a^5 +
 16*(5*a^3*b^4 + 4*a*b^4)*x^4 + 2*(60*a^4*b^3 + 70*a^2*b^3 + 11*b^3)*x^3 +
16*a^3 + (100*a^5*b^2 + 156*a^3*b^2 + 57*a*b^2)*x^2 + (44*a^6*b + 88*a^4*b
+ 51*a^2*b + 7*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*a*b^7*x
^7 + 12*a^8 + 12*(4*a^2*b^6 + b^6)*x^6 + 36*a^6 + 6*(27*a^3*b^5 + 14*a*b^5)
*x^5 + 4*(75*a^4*b^4 + 63*a^2*b^4 + 5*b^4)*x^4 + 38*a^4 + (330*a^5*b^3 + 40
8*a^3*b^3 + 95*a*b^3)*x^3 + 2*(108*a^6*b^2 + 186*a^4*b^2 + 84*a^2*b^2 + 5*b
^2)*x^2 + 16*a^2 + (78*a^7*b + 180*a^5*b + 131*a^3*b + 29*a*b)*x + 2)*(b^2*
x^2 + 2*a*b*x + a^2 + 1) + 2*a^2 + (17*a^9*b + 52*a^7*b + 54*a^5*b + 20*a^3
*b + a*b)*x + (4*a*b^8*x^8 + 8*a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 28*a^7 + 20*
(7*a^3*b^6 + 2*a*b^6)*x^6 + 2*(154*a^4*b^5 + 84*a^2*b^5 + 3*b^5)*x^5 + 36*a
^5 + (420*a^5*b^4 + 380*a^3*b^4 + 51*a*b^4)*x^4 + (364*a^6*b^3 + 500*a^4*b^
3 + 153*a^2*b^3 + 3*b^3)*x^3 + 20*a^3 + (196*a^7*b^2 + 384*a^5*b^2 + 213*a^
3*b^2 + 25*a*b^2)*x^2 + (60*a^8*b + 160*a^6*b + 141*a^4*b + 42*a^2*b + b)*x
 + 4*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^10*x^11 + 8*a*b^9*x^10 + 4*(
7*a^2*b^8 + b^8)*x^9 + 8*(7*a^3*b^7 + 3*a*b^7)*x^8 + 2*(35*a^4*b^6 + 30*a^2
*b^6 + 3*b^6)*x^7 + 8*(7*a^5*b^5 + 10*a^3*b^5 + 3*a*b^5)*x^6 + 4*(7*a^6*b^4
 + 15*a^4*b^4 + 9*a^2*b^4 + b^4)*x^5 + 8*(a^7*b^3 + 3*a^5*b^3 + 3*a^3*b^3 +
 a*b^3)*x^4 + (a^8*b^2 + 4*a^6*b^2 + 6*a^4*b^2 + 4*a^2*b^2 + b^2)*x^3 + (b^
6*x^7 + 4*a*b^5*x^6 + 6*a^2*b^4*x^5 + 4*a^3*b^3*x^4 + a^4*b^2*x^3)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^2 + 4*(b^7*x^8 + 5*a*b^6*x^7 + (10*a^2*b^5 + b^5)*x^6
 + (10*a^3*b^4 + 3*a*b^4)*x^5 + (5*a^4*b^3 + 3*a^2*b^3)*x^4 + (a^5*b^2 + a^
3*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 6*(b^8*x^9 + 6*a*b^7*x^8
+ (15*a^2*b^6 + 2*b^6)*x^7 + 4*(5*a^3*b^5 + 2*a*b^5)*x^6 + (15*a^4*b^4 + 12
*a^2*b^4 + b^4)*x^5 + 2*(3*a^5*b^3 + 4*a^3*b^3 + a*b^3)*x^4 + (a^6*b^2 + 2*
a^4*b^2 + a^2*b^2)*x^3)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 4*(b^9*x^10 + 7*a*b
^8*x^9 + 3*(7*a^2*b^7 + b^7)*x^8 + 5*(7*a^3*b^6 + 3*a*b^6)*x^7 + (35*a^4*b^
5 + 30*a^2*b^5 + 3*b^5)*x^6 + 3*(7*a^5*b^4 + 10...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arcsinh(b*x + a)^3), x)
```

Sympy [A]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{asinh}^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/asinh(b*x+a)**3,x)
```

```
[Out] Integral(1/(x*asinh(a + b*x)**3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arcsinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(b*x + a)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{asinh}{(a+b\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*asinh(a + b*x)^3),x)
```

```
[Out] int(1/(x*asinh(a + b*x)^3), x)
```

3.93 $\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(x^m \left(a+b\sinh^{-1}(c+dx)\right)^n,x\right)$$

```
[Out] Unintegrable(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Defer[Subst][Defer[Int][(-(c/d) + x/d)^m*(a + b*ArcSinh[x])^n, x], x, c + d
*x]/d
```

Rubi steps

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n \, dx = \frac{\text{Subst}\left(\int \left(-\frac{c}{d}+\frac{x}{d}\right)^m \left(a+b\sinh^{-1}(x)\right)^n \, dx,x,c+dx\right)}{d}$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int x^m \left(a+b\sinh^{-1}(c+dx)\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] Integrate[x^m*(a + b*ArcSinh[c + d*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^m (a+b\,\text{arcsinh}\,(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x^m*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^m, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x**m*(a + b*asinh(c + d*x))**n, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] Timed out
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.05

$$\int x^m\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*asinh(c + d*x))^n,x)
[Out] int(x^m*(a + b*asinh(c + d*x))^n, x)
```

3.94 $\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=545

$$\frac{3^{-1-n}e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{8d^3}-\frac{2^{-2-n}ce^{-\frac{2a}{b}}\left(a+b\sinh\cdots\right.}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-3*(a+b*arcsinh(d*x+c))/b)/d^
3/exp(3*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*(a+b*arcsinh(d*x+c))^
n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^3/exp(2*a/b)/(((-a-b*arcsinh(d*x+c
))/b)^n)-1/8*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/
exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*c^2*(a+b*arcsinh(d*x+c))^n*GAMMA
(1+n,(-a-b*arcsinh(d*x+c))/b)/d^3/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/
8*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^3/(((
a+b*arcsinh(d*x+c))/b)^n)-1/2*c^2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n
,(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)-2^(-2-n)*c*exp(2*
a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,2*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*
arcsinh(d*x+c))/b)^n)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,3*(a+b*arcsinh(d*x+c))/b)/d^3/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A]
time = 0.84, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5859, 5830, 6873, 12, 6874, 3388, 2212, 5556, 3389}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (3^(-1 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-3*(a + b*ArcSinh[c +
d*x]))/b])/(8*d^3*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (2^(-2 -
 n)*c*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))
/b])/(d^3*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - ((a + b*ArcSinh[
c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(8*d^3*E^(a/b)*(-(
(a + b*ArcSinh[c + d*x])/b))^n) + (c^2*(a + b*ArcSinh[c + d*x])^n*Gamma[1 +
 n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^3*E^(a/b)*(-((a + b*ArcSinh[c + d*
x])/b))^n) + (E^(a/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSin
h[c + d*x])/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)^n) - (c^2*E^(a/b)*(a +
b*ArcSinh[c + d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^3*((a
+ b*ArcSinh[c + d*x])/b)^n) - (2^(-2 - n)*c*E^((2*a)/b)*(a + b*ArcSinh[c +
d*x])^n*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^3*((a + b*ArcSinh[
c + d*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[
1 + n, (3*(a + b*ArcSinh[c + d*x]))/b])/(8*d^3*((a + b*ArcSinh[c + d*x])/b)
^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^2\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^n\cosh(x)(c-\sinh(x))^2}{d^2}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)(c-\sinh(x))^2\,dx,x,\sinh^{-1}(c+dx)\right)}{d^3}$$

$$=\frac{\text{Subst}\left(\int\left(c^2(a+bx)^n\cosh(x)-2c(a+bx)^n\cosh(x)\sinh(x)+(a+bx)^n\right.\right.}{d^3}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^2(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^3}-\frac{(2c)\text{Subst}\left(\right.}{}$$

$$=\frac{\text{Subst}\left(\int\left(-\frac{1}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)\,dx,x,\sinh^{-1}(c+\right.}{d^3}$$

$$=\frac{c^2e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(}{b}\right.}{2d^3}$$

$$=\frac{c^2e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(}{b}\right.}{2d^3}$$

$$=\frac{3^{-1-n}e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+}{}\right.}{8d^3}$$

Mathematica [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcSinh[c + d*x])^n,x]
[Out] Integrate[x^2*(a + b*ArcSinh[c + d*x])^n, x]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x^2*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x**2*(a + b*asinh(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{asinh}(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(c + d*x))^n,x)
[Out] int(x^2*(a + b*asinh(c + d*x))^n, x)
```

3.95 $\int x\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=267

$$\frac{2^{-3-n}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+\right.}{}$$

```
[Out] 2^(-3-n)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,-2*(a+b*arcsinh(d*x+c))/b)/d^2/ex
p(2*a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)-1/2*c*(a+b*arcsinh(d*x+c))^n*GAMMA(1
+n,(-a-b*arcsinh(d*x+c))/b)/d^2/exp(a/b)/(((-a-b*arcsinh(d*x+c))/b)^n)+1/2*
c*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(a+b*arcsinh(d*x+c))/b)/d^2/(((
a+b*arcsinh(d*x+c))/b)^n)+2^(-3-n)*exp(2*a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(
1+n,2*(a+b*arcsinh(d*x+c))/b)/d^2/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A]
time = 0.34, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$,
Rules used = {5859, 5830, 6873, 12, 6874, 3388, 2212, 5556, 3389}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^2}+\frac{ce^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d^2}+\frac{2^{-n-3}e^{\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c +
d*x]))/b])/(d^2*E^((2*a)/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (c*(a + b*
ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2*d^2*E^(
a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) + (c*E^(a/b)*(a + b*ArcSinh[c + d*x
])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d^2*((a + b*ArcSinh[c + d
*x])/b)^n) + (2^(-3 - n)*E^((2*a)/b)*(a + b*ArcSinh[c + d*x])^n*Gamma[1 + n
, (2*(a + b*ArcSinh[c + d*x]))/b])/(d^2*((a + b*ArcSinh[c + d*x])/b)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^n\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^n\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\frac{(a+bx)^n\cosh(x)(-c+\sinh(x))}{d}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)(-c+\sinh(x))\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}$$

$$=\frac{\text{Subst}\left(\int\left(-c(a+bx)^n\cosh(x)+(a+bx)^n\cosh(x)\sinh(x)\right)\,dx,x,\sinh^{-1}(c$$

$$=\frac{\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\text{Subst}\left(\int(a+$$

$$=\frac{\text{Subst}\left(\int\frac{1}{2}(a+bx)^n\sinh(2x)\,dx,x,\sinh^{-1}(c+dx)\right)}{d^2}-\frac{c\text{Subst}\left(\int e^{-x}(a+b$$

$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c-}{b}\right.}{2d^2}$$

$$=-\frac{ce^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c-}{b}\right.}{2d^2}$$

$$=\frac{2^{-3-n}e^{-\frac{2a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{2(a+b\sin}{}\right.}{d^2}$$

<u>Mathematica</u> [A]

time = 0.96, size = 330, normalized size = 1.24

$$\frac{2^{-3-n}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(-2^{2+n}c\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\cosh\left(\frac{a}{b}\right)\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^n\cosh\left(\frac{2a}{b}\right)\Gamma\left(1+n,\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+2^{2+n}c\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)+2^{2+n}c\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\sinh^{-1}(c+dx)\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\Gamma\left(1+n,-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)+\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^n\Gamma\left(1+n,\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] (2^(-3 - n)*(a + b*ArcSinh[c + d*x])^n*(-(2^(2 + n)*c*(a/b + ArcSinh[c + d*
x])^n*Cosh[a/b]*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)]) + (-((a + b*Ar
cSinh[c + d*x])/b))^n*Cosh[(2*a)/b]*Gamma[1 + n, (2*(a + b*ArcSinh[c + d*x]
))/b] + 2^(2 + n)*c*(a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSin
h[c + d*x])/b)]*Sinh[a/b] + 2^(2 + n)*c*(-((a + b*ArcSinh[c + d*x])/b))^n*G
amma[1 + n, a/b + ArcSinh[c + d*x]]*(Cosh[a/b] + Sinh[a/b]) + (a/b + ArcSin
h[c + d*x])^n*Gamma[1 + n, (-2*(a + b*ArcSinh[c + d*x]))/b]*(Cosh[(2*a)/b]
- Sinh[(2*a)/b]) + (-((a + b*ArcSinh[c + d*x])/b))^n*Gamma[1 + n, (2*(a + b
```

```
*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b]))/(d^2*(-((a + b*ArcSinh[c + d*x])^2/b
^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{arcsinh}(dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^n,x)
[Out] int(x*(a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**n,x)
[Out] Integral(x*(a + b*asinh(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{asinh}(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^n,x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^n, x)
```

3.96 $\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx$

Optimal. Leaf size=128

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\right.}{}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(-a-b*arcsinh(d*x+c))/b)/d/exp(a/b)/((
(-a-b*arcsinh(d*x+c))/b)^n)-1/2*exp(a/b)*(a+b*arcsinh(d*x+c))^n*GAMMA(1+n,(
a+b*arcsinh(d*x+c))/b)/d/(((a+b*arcsinh(d*x+c))/b)^n)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5774, 3388, 2212}

$$\frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}-\frac{e^{a/b}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\text{Gamma}\left(n+1,\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*Gamma[1 + n, -((a + b*ArcSinh[c + d*x])/b)])/(2
*d*E^(a/b)*(-((a + b*ArcSinh[c + d*x])/b))^n) - (E^(a/b)*(a + b*ArcSinh[c +
 d*x])^n*Gamma[1 + n, (a + b*ArcSinh[c + d*x])/b])/(2*d*((a + b*ArcSinh[c +
 d*x])/b)^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
```

```
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^n\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^n\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int x^n\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}x^n\,dx\right.}{}$$

$$= \frac{e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2d}$$

Mathematica [A]
time = 0.01, size = 138, normalized size = 1.08

$$\frac{\left(a+b\sinh^{-1}(c+dx)\right)^n\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)^n\Gamma\left(1+n,-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)-\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^n\Gamma\left(1+n,\frac{a}{b}+\sinh^{-1}(c+dx)\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n,x]
```

```
[Out] ((a + b*ArcSinh[c + d*x])^n*((a/b + ArcSinh[c + d*x])^n*Gamma[1 + n, -((a +
 b*ArcSinh[c + d*x])/b)]*(Cosh[a/b] - Sinh[a/b]) - (-((a + b*ArcSinh[c + d*
x])/b))^n*Gamma[1 + n, a/b + ArcSinh[c + d*x]]*(Cosh[a/b] + Sinh[a/b])))/(2
*d*(-((a + b*ArcSinh[c + d*x])^2/b^2))^n)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n,x)
[Out] Integral((a + b*asinh(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{asinh}(c + d\,x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^n,x)
[Out] int((a + b*asinh(c + d*x))^n, x)
```

3.97 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^n/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^n/x,x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^n/(-(c/d) + x/d), x], x, c + d*x
]/d
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^n}{-\frac{c}{d}+\frac{x}{d}}\,dx,x,c+dx\right)}{d}$$

Mathematica [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^n/x,x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^n/x, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^n/x,x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^n/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)^n/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**n/x,x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**n/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^n/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^n/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^n/x,x)
```

```
[Out] int((a + b*asinh(c + d*x))^n/x, x)
```

3.98 $\int x^2 \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=496

$$\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\sqrt{a+b\sinh^{-1}(c+dx)}\,\cosh\left(2\sin\right.}{2d^3}$$

```
[Out] 1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^
(1/2)*Pi^(1/2)/d^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b
^(1/2)*3^(1/2)*Pi^(1/2)/d^3/exp(3*a/b)+1/16*c*exp(2*a/b)*erf(2^(1/2)*(a+b*a
rcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3+1/16*c*erfi(2^(1
/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^3/exp(2*
a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)
/d^3+1/4*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1
/2)/d^3+1/16*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^3/
exp(a/b)-1/4*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/
d^3/exp(a/b)+c^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d^3+1/3*(d*x+c)^3*(a+b*
arcsinh(d*x+c))^(1/2)/d^3-1/2*c*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))
^(1/2)/d^3
```

Rubi [A]
time = 1.39, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5859, 5830, 6873, 6874, 5433, 5406, 2236, 2235, 5432, 5407, 5480, 5408}

$$\frac{\sqrt{\pi}\sqrt{b}c^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}-\frac{\sqrt{\pi}\sqrt{b}c^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^3}+\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^3}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ce^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^3}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d^3}+\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^3}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ce^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-\frac{c\cosh\left(2\sinh^{-1}(c+dx)\right)\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (c^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^3 + ((c + d*x)^3*Sqrt[a + b*
ArcSinh[c + d*x]])/(3*d^3) - (c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh
[c + d*x]])/(2*d^3) - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d^3) + (Sqrt[b]*c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(4*d^3) + (Sqrt[b]*c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*d^3) + (Sqrt[b]*E^((3*a)/
b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3)
 + (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^3*E^
(a/b)) - (Sqrt[b]*c^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/
(4*d^3*E^(a/b)) + (Sqrt[b]*c*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(8*d^3*E^((2*a)/b)) - (Sqrt[b]*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(48*d^3*E^((3*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5408

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5480

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
```

```
 1))), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2dx,x,\sqrt{a+b\sinh^{-1}(c+d}\right.}{bd^3}$$

$$=\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2dx,x,\sqrt{a+b\sinh^{-1}(c+}\right.}{bd^3}$$

$$=\frac{2\text{Subst}\left(\int\left(c^2x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+cx^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right.\right.}{bd^3}$$

$$=\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^3}$$

$$=\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-$$

$$=\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-$$

$$=\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-$$

$$=\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-$$

$$=\frac{c^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^3}+\frac{(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d^3}-$$

Mathematica [A]
time = 1.17, size = 656, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-36*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]] + 144*c^2*(c + d*x)*Sqrt[a + b*
ArcSinh[c + d*x]] - 72*c*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*
x]] + Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]] + 9*Sqrt[b]*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]] - 36*Sqrt[b]*c^2*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[
c + d*x]]/Sqrt[b]] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[3*Pi]*Cosh[(3*a)/b]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] - 9*Sqrt[b]*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 36*Sqrt[b]*c^2*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 9*Sqrt[b]*(-1 + 4*c^2
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]
) - 9*Sqrt[b]*c*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*Sinh[(2*a)/b] + 9*Sqrt[b]*c*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[b]*Sqrt[3*Pi]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] + Sqrt[b]*
Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/
b] + 12*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[3*ArcSinh[c + d*x]])/(144*d^3)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a +b \operatorname{arcsinh}\left(d x +c \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asinh(d*x+c))**(1/2),x)

[Out] Integral(x**2*sqrt(a + b*asinh(c + d*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, \sqrt{a + b \operatorname{asinh}(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*asinh(c + d*x))^(1/2),x)

[Out] int(x^2*(a + b*asinh(c + d*x))^(1/2), x)

3.99 $\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2}+\frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c+dx)\right)}{4d^2}-\frac{\sqrt{b}\,ce^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{}}{}\right.}{}$$

```
[Out] -1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*2^
(1/2)*Pi^(1/2)/d^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-1/4*c*exp(a/b)*erf((a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2+1/4*c*erfi((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*b^(1/2)*Pi^(1/2)/d^2/exp(a/b)-c*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/
2)/d^2+1/4*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

Rubi [A]
time = 0.56, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5406, 2236, 2235, 5432, 5407}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}+\frac{\sqrt{\pi}\sqrt{b}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2}+\frac{\cosh\left(2\sinh^{-1}(c+dx)\right)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -((c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d^2) + (Sqrt[a + b*ArcSinh[c +
 d*x]]*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (Sqrt[b]*c*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2) - (Sqrt[b]*E^((2*a)/b)*Sqrt[
Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d^2) + (Sqrt
[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d^2*E^(a/b))
- (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(16*d^2*E^((2*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\sqrt{a+bx}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c+ \cdots}}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int x^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)dx,x,\sqrt{a+b\sinh^{-1}(c \cdots}}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int\left(cx^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)dx,x,\sqrt{a+b\,\text{si}\cdots}}{bd^2}$$

$$= -\frac{\text{Subst}\left(\int x^2\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sin\cdots\right.}{4d^2\cdots}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sin\cdots\right.}{4d^2\cdots}$$

$$= -\frac{c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d^2} + \frac{\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sin\cdots\right.}{4d^2\cdots}$$

Mathematica [A]

time = 1.21, size = 251, normalized size = 0.97

$$\frac{8\sqrt{a+b\sinh^{-1}(c+dx)}\cosh\left(2\sinh^{-1}(c+dx)\right)-16ce^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)+\sqrt{b}\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)-\sqrt{b}\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)}{32d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - (16*c*Sqrt[a + b
*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*x]])/Sqrt
[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)]/Sqrt[
-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]) -
Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cos
h[(2*a)/b] + Sinh[(2*a)/b]))/(32*d^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\,\mathrm{arcsinh}\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(1/2), x)
```

3.100 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\,\mathrm{si}}}{}\right.}{4d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5819, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,d\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{b\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx\right.}{4d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{4d}$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b\,\mathrm{asinh}\,(c + d\,x)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

3.101 $\int x\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=326

$$\frac{3bc\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/}}{4}$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(3/2)*co
sh(2*arcsinh(d*x+c))/d^2-3/128*b^(3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2+3/128*b^(3/2)*erfi(2^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-3/8*b^(3/2)
*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2-3/8*b^(3/2
)*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)-3/16*b*s
inh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/2)/d^2+3/2*b*c*(1+(d*x+c)^2)^
(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d^2
```

Rubi [A]
time = 0.73, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5432, 5407, 2236, 2235, 5406}

$$-\frac{3\sqrt{\pi}\,b^{3/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3\sqrt{\pi}\,b^{3/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d^2}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d^2}-\frac{3b\sinh\left(2\sinh^{-1}(c+dx)\right)\sqrt{a+b\sinh^{-1}(c+dx)}}{16d^2}+\frac{3bc\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+\frac{\cosh\left(2\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (3*b*c*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d^2) - (c*(c
+ d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/d^2 + ((a + b*ArcSinh[c + d*x])^(3/2
)*Cosh[2*ArcSinh[c + d*x]])/(4*d^2) - (3*b^(3/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2) - (3*b^(3/2)*E^((2*a)/b)*Sqrt[P
i/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d^2) - (3*b^(
3/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d^2*E^(a/b))
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(64*d^2*E^((2*a)/b)) - (3*b*Sqrt[a + b*ArcSinh[c + d*x]]*Sinh[2*ArcSinh
[c + d*x]])/(16*d^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{\text{Subst}\left(\int(a+bx)^{3/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$=-\frac{2\text{Subst}\left(\int x^4\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\sinh^{-1}(c\cdots}\right.}{bd^2}$$

$$=-\frac{2\text{Subst}\left(\int x^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\sinh^{-1}(\cdots}\right.}{bd^2}$$

$$=-\frac{2\text{Subst}\left(\int\left(cx^4\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\cdots}\right.}{bd^2}$$

$$=-\frac{\text{Subst}\left(\int x^4\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2}-\frac{(2c)\text{S}\cdots}{\cdots}$$

$$=-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d^2}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\cosh\,(\cdots}{4d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(\cdots\right.}{d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(\cdots\right.}{d^2}$$

$$=\frac{3bc\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(\cdots\right.}{d^2}$$

Mathematica [A]
time = 3.53, size = 582, normalized size = 1.79

[illegible]

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(3/2),x]

[Out] ((-64*a*c*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) - 16*Sqrt[b]*c*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 4*a*(8*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*a)/b]) - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])) + Sqrt[b]*((4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(4*ArcSinh[c + d*x]*Cosh[2*ArcSinh[c + d*x]] - 3*Sinh[2*ArcSinh[c + d*x]])))/(128*d^2)

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)

[Out] int(x*(a+b*arcsinh(d*x+c))^(3/2),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{asinh}{(c+dx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(3/2), x)
```

3.102 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+}}{}\right.}{8d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx\right.}{2d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$=-\frac{3b\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

<u>Mathematica</u> [A]
time = 0.41, size = 272, normalized size = 1.81

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)}{2d}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\sinh^{-1}(c+dx)\right)+(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(-2a+3b)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b \,\mathrm{arcsinh}\left(d x +c \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{asinh}{\left(c + d x \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

3.103 $\int x(a+b\sinh^{-1}(c+dx))^{5/2}\,dx$

Optimal. Leaf size=389

$$-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{5bc\sqrt{1+(c+dx)^2}\,(a+b\sinh^{-1}(c+dx))^{3/2}}{2d^2}-\frac{c(c+dx)\,(a+b\sinh^{-1}\ldots}{d^2\ldots}$$

```
[Out] -c*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d^2+1/4*(a+b*arcsinh(d*x+c))^(5/2)*co
sh(2*arcsinh(d*x+c))/d^2-5/16*b*(a+b*arcsinh(d*x+c))^(3/2)*sinh(2*arcsinh(d
*x+c))/d^2-15/512*b^(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^2/exp(2*a/b)-15/16*b^(5/2)*c*exp(a/b
)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2+15/16*b^(5/2)*c*erfi
((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^2/exp(a/b)+5/2*b*c*(a+b*arc
sinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d^2-15/4*b^2*c*(d*x+c)*(a+b*arcsinh(
d*x+c))^(1/2)/d^2+15/64*b^2*cosh(2*arcsinh(d*x+c))*(a+b*arcsinh(d*x+c))^(1/
2)/d^2
```

Rubi [A]

time = 0.83, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5830, 6873, 6874, 5433, 5432, 5406, 2236, 2235, 5407}

$$-\frac{15\sqrt{\pi}b^{5/2}ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2}+\frac{15\sqrt{\pi}b^{5/2}ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d^2}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d^2}-\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2}+\frac{15b^2\cosh\left(2\sinh^{-1}(c+dx)\right)\sqrt{a+b\sinh^{-1}(c+dx)}}{64d^2}+\frac{5bc\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2}-\frac{5b\sinh\left(2\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{16d^2}-\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d^2}+\frac{\cosh\left(2\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-15*b^2*c*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d^2) + (5*b*c*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d^2) - (c*(c + d*x)*(a +
b*ArcSinh[c + d*x])^(5/2))/d^2 + (15*b^2*Sqrt[a + b*ArcSinh[c + d*x]]*Cosh[
2*ArcSinh[c + d*x]])/(64*d^2) + ((a + b*ArcSinh[c + d*x])^(5/2)*Cosh[2*ArcS
inh[c + d*x]])/(4*d^2) - (15*b^(5/2)*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcS
inh[c + d*x]]/Sqrt[b]])/(16*d^2) - (15*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(
Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d^2) + (15*b^(5/2)*c*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d^2*E^(a/b)) - (15*
b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2
56*d^2*E^((2*a)/b)) - (5*b*(a + b*ArcSinh[c + d*x])^(3/2)*Sinh[2*ArcSinh[c
+ d*x]])/(16*d^2)
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5432

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Cosh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int\left(-\frac{c}{d}+\frac{x}{d}\right)\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int(a+bx)^{5/2}\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{2\text{Subst}\left(\int x^6\cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\sinh^{-1}(c+}\right.}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int x^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\sinh^{-1}(c}\right.}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int\left(cx^6\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx,x,\sqrt{a+b\,\text{si}}\right.}{bd^2}$$

$$= -\frac{\text{Subst}\left(\int x^6\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx,x,\sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\text{Sul}}{}$$

$$= -\frac{c(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d^2} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\cosh\left(2\right.}{4d^2}$$

$$= \frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d^2} - \frac{c(c+dx)\left(a+b\sinh^{-1}(\right.}{d^2}$$

$$= -\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2} + \frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh\right.}{2d^2}$$

$$= -\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2} + \frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh\right.}{2d^2}$$

$$= -\frac{15b^2c(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d^2} + \frac{5bc\sqrt{1+(c+dx)^2}\left(a+b\sinh\right.}{2d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 939 vs. 2(389) = 778.
time = 7.23, size = 939, normalized size = 2.41

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-1920*b^2*c^2*Sqrt[a + b*ArcSinh[c + d*x]] - 1920*b^2*c*d*x*Sqrt[a + b*Arc
Sinh[c + d*x]] + 1280*a*b*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*Sqrt[a + b*Ar
cSinh[c + d*x]] - 1024*a*b*c^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]
] - 1024*a*b*c*d*x*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c + d*x]] + 1280*b^2
*c*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]*ArcSinh[c + d*x]*Sqrt[a + b*ArcSinh[c
+ d*x]] - 512*b^2*c^2*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] - 512
*b^2*c*d*x*ArcSinh[c + d*x]^2*Sqrt[a + b*ArcSinh[c + d*x]] + 128*a^2*Sqrt[a
 + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 120*b^2*Sqrt[a + b*ArcSin
h[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 256*a*b*ArcSinh[c + d*x]*Sqrt[a + b*
ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] + 128*b^2*ArcSinh[c + d*x]^2*Sqr
t[a + b*ArcSinh[c + d*x]]*Cosh[2*ArcSinh[c + d*x]] - 128*a^2*Sqrt[b]*c*Sqrt
[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 480*b^(5/2)*c*S
qrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 15*b^(5/2)*S
qrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
] + (256*a^2*b*c*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]] + (256*a^2*b*c*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)])/(E^(a/b)*Sqr
t[a + b*ArcSinh[c + d*x]]) + 128*a^2*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 480*b^(5/2)*c*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 32*Sqrt[b]*(4*a^2 - 15*b^2)*c*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 15*b
^(5/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh
[(2*a)/b] - 15*b^(5/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 160*a*b*Sqrt[a + b*ArcSinh[c +
 d*x]]*Sinh[2*ArcSinh[c + d*x]] - 160*b^2*ArcSinh[c + d*x]*Sqrt[a + b*ArcSi
nh[c + d*x]]*Sinh[2*ArcSinh[c + d*x]])/(512*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c + d*x))^(5/2), x)
```

3.104 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}\right.}{d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.25, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5819, 3389, 2211, 2236, 2235}

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\, dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2}\, dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\, dx, x, \cdots\right)}{2d}$$

$$= -\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(\cdots\right.}{d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\cdots\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\cdots\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\cdots\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\cdots\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(\cdots\right.}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.
time = 1.48, size = 458, normalized size = 2.56

$$\frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)+4a\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\sinh^{-1}(c+dx)\right)+(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(-2a+3b)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)+\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(2\sqrt{1+(c+dx)^2}\left(a-5b\sinh^{-1}(c+dx)\right)+b(c+dx)\left(15+4\sinh^{-1}(c+dx)^2\right)\right)+\left(4a^2+12ab+15b^2\right)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+\left(4a^2-12ab+15b^2\right)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
```

```
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{asinh}\,(c + dx))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

3.105 $\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=411

$$-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}+\frac{c^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{\ }}{}\right.}{2}$$

```
[Out] 1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^
(1/2)/d^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/d^3/exp(3*a/b)/b^(1/2)+1/4*c*exp(2*a/b)*erf(2^(1/2)*(a+b*arcs
inh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/b^(1/2)-1/4*c*erfi(2^(1/2)*
(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d^3/exp(2*a/b)/b^(1/2)
-1/8*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/2)+
1/2*c^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/b^(1/
2)-1/8*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b^(1/
2)+1/2*c^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d^3/exp(a/b)/b
^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5830, 6873, 6874, 5407, 2236, 2235, 5406, 5737}

$$\frac{\sqrt{\pi}\,c^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\pi}\,c^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{\sqrt{\frac{\pi}{2}}\,ce^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}+\frac{\sqrt{\frac{\pi}{3}}\,e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}-\frac{\sqrt{\frac{\pi}{2}}\,ce^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}-\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}+\frac{\sqrt{\frac{\pi}{3}}\,e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/8*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt[b]*
d^3) + (c^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*
Sqrt[b]*d^3) + (c*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(2*Sqrt[b]*d^3) + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d^3) - (Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b]*d^3*E^(a/b)) + (c^2*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d^3*E^(a/b)) - (c*S
qrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*Sqrt[b]*
d^3*E^((2*a)/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(8*Sqrt[b]*d^3*E^((3*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5737

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(-\frac{c}{d}+\frac{x}{d}\right)^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)^2}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}$$

$$= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)^2\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^3}$$

$$= \frac{2\text{Subst}\left(\int \left(c^2\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+c\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)+\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}\right.\right.\right.}{bd^3}$$

$$= \frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^3}+$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{1}{4}\cosh\left(\frac{3a}{b}-\frac{3x^2}{b}\right)-\frac{1}{4}\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\sinh^{-1}(c+\right.}{bd^3}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{2\sqrt{b}\,d^3}$$

$$= \frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^3}+\frac{ce^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh}}{\sqrt{b}}\right.}{2\sqrt{b}\,d^3}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d^3}+\frac{c^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(}{\sqrt{b}}\right.}{2\sqrt{b}\,d^3}$$

Mathematica [A]
time = 0.73, size = 471, normalized size = 1.15

$$\frac{\sqrt{\pi}\left(\sqrt{3}\cosh\left(\frac{3a}{b}\right)\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-3\cosh\left(\frac{a}{b}\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+12c^2\cosh\left(\frac{a}{b}\right)\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)-6\sqrt{2}\,c\cosh\left(\frac{2a}{b}\right)\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{3}\cosh\left(\frac{3a}{b}\right)\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)+3\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{a}{b}\right)-12c^2\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{a}{b}\right)+3(-1+4c^2)\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+6\sqrt{2}\,c\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{2a}{b}\right)+6\sqrt{2}\,c\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+\sqrt{3}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{3a}{b}\right)-\sqrt{3}\,\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\sinh\left(\frac{3a}{b}\right)\right)}{24\sqrt{b}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[Pi]*(Sqrt[3]*Cosh[(3*a)/b]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]] - 3*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] + 12*c^2
*Cosh[a/b]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]] - 6*Sqrt[2]*c*Cosh[(2
*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + Sqrt[3]*Cosh[
(3*a)/b]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]] + 3*Erfi[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 12*c^2*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 3*(-1 + 4*c^2)*Erf[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 6*Sqrt[2]*c*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] + 6*Sqrt[2]*c*Erf[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + Sqrt[
3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b] - Sqrt
[3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*Sinh[(3*a)/b]))/(2
4*Sqrt[b]*d^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a + b \operatorname{asinh}(c + d\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c + d*x))^(1/2), x)
```

3.106 $\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{ce^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}-\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2}-\frac{ce^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\Bigg(}{}$$

```
[Out] -1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/d^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/d^2/exp(2*a/b)/b^(1/2)-1/2*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d^2/b^(1/2)-1/2*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d^2/exp(a/b)/b^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5859, 5830, 6873, 6874, 5407, 2236, 2235, 5406}

$$-\frac{\sqrt{\pi}\,ce^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}-\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/2*(c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt[b
]*d^2) - (E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(4*Sqrt[b]*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/
Sqrt[b]])/(2*Sqrt[b]*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(4*Sqrt[b]*d^2*E^((2*a)/b))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5830

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]*(c*d + e*Sinh[x
])^m, x], x, ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(x)\left(-\frac{c}{d}+\frac{\sinh(x)}{d}\right)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{2\text{Subst}\left(\int \cosh\left(\frac{a-x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right.}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int \cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)\left(c+\sinh\left(\frac{a-x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\sinh^{-1}(c+d}\right.}{bd^2}$$

$$= -\frac{2\text{Subst}\left(\int \left(c\cosh\left(\frac{a}{b}-\frac{x^2}{b}\right)+\frac{1}{2}\sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\right)\,dx, x, \sqrt{a+b\sinh^{-1}(}\right.}{bd^2}$$

$$= -\frac{\text{Subst}\left(\int \sinh\left(\frac{2a}{b}-\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd^2} - \frac{(2c)\text{Subst}\left(\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd^2} + \frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,d\right.}{}$$

$$= -\frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d^2} - \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sin}}{\sqrt{b}}\right.}{4\sqrt{b}\,d^2}$$

<u>Mathematica</u> [**A**]
time = 0.68, size = 217, normalized size = 1.06

$$\frac{\frac{e^{-\frac{a}{b}}\left(4ce^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-4c\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{\sqrt{2\pi}\left(\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)\right)}{\sqrt{b}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((4*c*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c +
 d*x]] - 4*c*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSin
h[c + d*x])/b)])/(E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]]) - (Sqrt[2*Pi]*(Erfi
```

```
[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(-Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b]
 + Sinh[(2*a)/b])))/Sqrt[b])/(8*d^2)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int(x/(a + b*asinh(c + d*x))^(1/2), x)
```

3.107 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5858, 5774, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x\right.}{2}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{}$$

$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c}}{\sqrt{b}}\right.}{2\sqrt{b}\,d}$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 1.21

$$\frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]]) + Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\text{asinh}\,(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

3.108 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{ce^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}}{}$$

```
[Out] 1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/d^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/d^2/exp(2*a/b)+c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2
)/b^(1/2))*Pi^(1/2)/b^(3/2)/d^2-c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/b^(3/2)/d^2/exp(a/b)+2*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*
x+c))^(1/2)-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5859, 5829, 5773, 5819, 3389, 2211, 2236, 2235, 5778, 3388}

$$\frac{\sqrt{\pi}\,ce^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}-\frac{\sqrt{\pi}\,ce^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{\sqrt{\frac{\pi}{2}}\,e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d^2}+\frac{2c\sqrt{(c+dx)^2+1}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2(c+dx)\sqrt{(c+dx)^2+1}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(b*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (c*E^(a/
b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d^2) + (E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(b
^(3/2)*d^2) - (c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3
/2)*d^2*E^(a/b)) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/
Sqrt[b]])/(b^(3/2)*d^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{3/2}}\right)dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, \ldots\right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int e \ldots\right.}{\ldots}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\ldots}{\ldots}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ce^{a/b}\sqrt{\pi}\,\text{erf}\ldots}{\ldots}$$

Mathematica [A]
time = 1.77, size = 301, normalized size = 1.12

$$-\frac{ce^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\sinh^{-1}(c+dx)}\right)+e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{bd^2\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)+\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)-\frac{2\sqrt{b}\sinh\left(2\sinh^{-1}(c+dx)\right)}{\sqrt{a+b\sinh^{-1}(c+dx)}}}{2b^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -((c*(-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*
x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^Arc
Sinh[c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSi
nh[c + d*x])/b)]))/(b*d^2*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh
[c + d*x]])) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*Sin
h[2*ArcSinh[c + d*x]])/Sqrt[a + b*ArcSinh[c + d*x]])/(2*b^(3/2)*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(3/2), x)
```

3.109 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\sinh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,\right.}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+\right.}{b^2d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} +$$

Mathematica [A]
time = 0.15, size = 155, normalized size = 1.27

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(1+e^{2\sinh^{-1}(c+dx)}\right)+e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+e^{\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c + d*x]))) + E^((2*a)/b + ArcSinh[c + d*x])*S
qrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] + E^ArcSinh[
c + d*x]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c
+ d*x])/b)])/(b*d*E^((a + b*ArcSinh[c + d*x])/b)*Sqrt[a + b*ArcSinh[c + d*x
]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(c + d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

3.110 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=365

$$\frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4}{3b^2d^2\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{4c}{3b^2d^2\sqrt{a+}}$$

```
[Out] -2/3*c*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^
2-2/3*c*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d^2/exp(a
/b)-2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/d^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2
^(1/2)*Pi^(1/2)/b^(5/2)/d^2/exp(2*a/b)+2/3*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b
*arcsinh(d*x+c))^(3/2)-2/3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d
*x+c))^(3/2)-4/3/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)+4/3*c*(d*x+c)/b^2/d^2/(
a+b*arcsinh(d*x+c))^(1/2)-8/3*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {5859, 5829, 5773, 5818, 5774, 3388, 2211, 2236, 2235, 5779, 5780, 5556, 12, 3389, 5783}

$$-\frac{2\sqrt{\pi}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{2\pi}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{2\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}+\frac{2\sqrt{2\pi}\,e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d^2}-\frac{8(c+dx)^2}{3b^2d^2\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{4c(c+dx)}{3b^2d^2\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4}{3b^2d^2\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}(c+dx)}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}+\frac{2c\sqrt{(c+dx)^2+1}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) -
4/(3*b^2*d^2*Sqrt[a + b*ArcSinh[c + d*x]]) + (4*c*(c + d*x))/(3*b^2*d^2*Sqr
t[a + b*ArcSinh[c + d*x]]) - (8*(c + d*x)^2)/(3*b^2*d^2*Sqrt[a + b*ArcSinh[
c + d*x]]) - (2*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(3*b^(5/2)*d^2) - (2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2) - (2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d^2*E^(a/b)) + (2*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{5/2}}\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x\right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\text{Subs}}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{3bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2d^2}$$

Mathematica [A]
time = 1.81, size = 375, normalized size = 1.03

$$\frac{\frac{\sqrt{b}ce^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(e^{a/b}\left(-2a+b+2ae^{2\sinh^{-1}(c+dx)}+be^{2\sinh^{-1}(c+dx)}+2b\left(-1+e^{2\sinh^{-1}(c+dx)}\right)\sinh^{-1}(c+dx)\right)+2e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+2be^{\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}+2\sqrt{2\pi}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)-2\sqrt{2\pi}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)-\frac{\sqrt{b}\left(4\left(a+b\sinh^{-1}(c+dx)\right)\cosh\left(2\sinh^{-1}(c+dx)\right)+b\sinh\left(2\sinh^{-1}(c+dx)\right)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}}{3b^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((Sqrt[b]*c*(E^(a/b)*(-2*a + b + 2*a*E^(2*ArcSinh[c + d*x]) + b*E^(2*ArcSin
h[c + d*x]) + 2*b*(-1 + E^(2*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 2*E^((2
*a)/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d
*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] + 2*b*E^ArcSinh[c + d*x]*(-((a + b*
ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/(E^
((a + b*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2)) + 2*Sqrt[2*Pi]
*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh
[(2*a)/b]) - 2*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - (Sqrt[b]*(4*(a + b*ArcSinh[c + d*x])*C
osh[2*ArcSinh[c + d*x]] + b*Sinh[2*ArcSinh[c + d*x]]))/(a + b*ArcSinh[c + d
*x])^(3/2))/(3*b^(5/2)*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asinh}{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(5/2), x)
```

3.111 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\cdots$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5773, 5818, 5774, 3388, 2211, 2236, 2235}

$$\frac{2\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x\right.}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\right.}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}}{}$$

Mathematica [A]
time = 0.40, size = 207, normalized size = 1.31

$$\frac{e^{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\sinh^{-1}(c+dx)}\right)-2b\sinh^{-1}(c+dx)+be^{2\sinh^{-1}(c+dx)}\left(1+2\sinh^{-1}(c+dx)\right)\right)-2e^{\frac{2a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-2be^{\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c + d*x])) - 2*b*ArcSinh[c + d*x] +
b*E^(2*ArcSinh[c + d*x])*(1 + 2*ArcSinh[c + d*x]))) - 2*E^((2*a)/b + ArcSin
h[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2
```

```
, a/b + ArcSinh[c + d*x]] - 2*b*E^ArcSinh[c + d*x]*(-((a + b*ArcSinh[c + d*
x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)])/(3*b^2*d*E^((a + b
*ArcSinh[c + d*x])/b)*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{asinh}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

3.112 $\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}} \, dx$

Optimal. Leaf size=445

$$\frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{4}{15b^2d^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}+\frac{}{15b^2d^2\left(\right.}$$

```
[Out] -4/15/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*(d*x+c)/b^2/d^2/(a+b*arcsin
h(d*x+c))^(3/2)-8/15*(d*x+c)^2/b^2/d^2/(a+b*arcsinh(d*x+c))^(3/2)+4/15*c*ex
p(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2-4/15*c*
erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d^2/exp(a/b)+8/15
*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2
)/b^(7/2)/d^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(7/2)/d^2/exp(2*a/b)+2/5*c*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsi
nh(d*x+c))^(5/2)-2/5*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d^2/(a+b*arcsinh(d*x+c))
^(5/2)+8/15*c*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)-32/15*
(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d^2/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {5859, 5829, 5773, 5818, 5819, 3389, 2211, 2236, 2235, 5779, 5778, 3388, 5783}

$$\frac{4\sqrt{\pi}\,ce^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}\,e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{4\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}+\frac{8\sqrt{2\pi}\,e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d^2}-\frac{32\sqrt{(c+dx)^2+1}\,(c+dx)}{15b^3d^2\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{8c\sqrt{(c+dx)^2+1}}{15b^3d^2\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{8(c+dx)^2}{15b^2d^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}+\frac{4c(c+dx)}{15b^2d^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4}{15b^2d^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2\sqrt{(c+dx)^2+1}\,(c+dx)}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}+\frac{2c\sqrt{(c+dx)^2+1}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (2*c*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) - (2*(
c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) -
4/(15*b^2*d^2*(a + b*ArcSinh[c + d*x])^(3/2)) + (4*c*(c + d*x))/(15*b^2*d^2
*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*(c + d*x)^2)/(15*b^2*d^2*(a + b*ArcSi
nh[c + d*x])^(3/2)) + (8*c*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a + b*Ar
cSinh[c + d*x]]) - (32*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(15*b^3*d^2*Sqrt[a
+ b*ArcSinh[c + d*x]]) + (4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(15*b^(7/2)*d^2) + (8*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2) - (4*c*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d^2*E^(a/b)) + (8*Sqrt[
2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d^2
*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5829

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcSinh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{x}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{-\frac{c}{d}+\frac{x}{d}}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}+\frac{x}{d\left(a+b\sinh^{-1}(x)\right)^{7/2}}\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d^2} - \frac{c\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x,\right.}{d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\text{Subst}}{}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

$$= \frac{2c\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{2(c+dx)\sqrt{1+(c+dx)^2}}{5bd^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{}{15b^2d^2}$$

Mathematica [A]

time = 1.74, size = 429, normalized size = 0.96

$$-\frac{c\left(-6b^2e^{\sinh^{-1}(c+dx)}-2e^{-\sinh^{-1}(c+dx)}\left(4a^2+2ab\left(-1+4\sinh^{-1}(c+dx)\right)+b^2\left(3-2\sinh^{-1}(c+dx)+4\sinh^{-1}(c+dx)^2\right)\right)+8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(2a+b+2b\sinh^{-1}(c+dx)\right)+2b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)\right)}{30b^3d^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}+\frac{8\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)-\sinh\left(\frac{2a}{b}\right)\right)+8\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{2a}{b}\right)+\sinh\left(\frac{2a}{b}\right)\right)+\frac{\sqrt{b}\left(-4b\left(a+b\sinh^{-1}(c+dx)\right)\cosh\left(2\sinh^{-1}(c+dx)\right)-\left(3b^2+16\left(a+b\sinh^{-1}(c+dx)\right)^2\right)\sinh\left(2\sinh^{-1}(c+dx)\right)\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}}{15b^{7/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/30*(c*(-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c +
 d*x]) + b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c
+ d*x] + 8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*
Gamma[1/2, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b +
ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[
c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b)))/(
b^3*d^2*(a + b*ArcSinh[c + d*x])^(5/2)) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + 8*Sqrt[
2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] +
Sinh[(2*a)/b]) + (Sqrt[b]*(-4*b*(a + b*ArcSinh[c + d*x])*Cosh[2*ArcSinh[c +
 d*x]] - (3*b^2 + 16*(a + b*ArcSinh[c + d*x])^2)*Sinh[2*ArcSinh[c + d*x]]))
/(a + b*ArcSinh[c + d*x])^(5/2))/(15*b^(7/2)*d^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{asinh}(c + dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c + d*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(a + b \operatorname{asinh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(x/(a + b*asinh(c + d*x))^(7/2), x)
```

3.113 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^{a/b}\sqrt{\pi}}{}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5773, 5818, 5819, 3389, 2211, 2236, 2235}

$$-\frac{4\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2\sqrt{(c+dx)^2+1}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx\right.}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\text{Su}}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}$$

Mathematica [A]
time = 0.36, size = 238, normalized size = 1.22

$$\frac{-6b^2e^{\sinh^{-1}(c+dx)} - 2e^{-\sinh^{-1}(c+dx)}\left(4a^2+2ab\left(-1+4\sinh^{-1}(c+dx)\right)+b^2\left(3-2\sinh^{-1}(c+dx)+4\sinh^{-1}(c+dx)^2\right)\right) + 8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right) - 4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(2a+b+2b\sinh^{-1}(c+dx)\right)+2b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{30b^3d\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

3.114 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=91

$$\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)}-\frac{b(e(c+dx))^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-(c+dx)^2\right)}{de^2(1+m)(2+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*hyperg
eom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+m)/(2+m)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5859, 5776, 371}

$$\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)}{de^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2])/
(d*e^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{1+x^2}}\,d\right.}{de(1+m}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b(e(c+dx))^{2+m}\,{}_2F_1\left(\right.}{de^2(1+}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.87

$$-\frac{(c+dx)(e(c+dx))^m\left(-\left((2+m)\left(a+b\sinh^{-1}(c+dx)\right)\right)+b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-(c+dx)^2\right)\right)}{d(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -(((c + d*x)*(e*(c + d*x))^m*(-((2 + m)*(a + b*ArcSinh[c + d*x])) + b*(c +
d*x)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*x)^2]))/(d*(1 + m
)*(2 + m)))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^m \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*((d*x*e^m + c*e^m)*(d*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d*(m + 1)) - integrate((d^2*x^2*e^m + 2*c*d*x*e^m + c^2*e^m)*(d*x +
```

```
 c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1), x) - integ
rate((d*x*e^m + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 +
 c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x
^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)), x)) + (d*x*e + c*e)^(m + 1)*a*e^(-1)/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
[Out] integral((b*arcsinh(d*x + c) + a)*((d*x + c)*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x)), x)
```

3.115 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=100

$$-\frac{be^4\sqrt{1+(c+dx)^2}}{5d}+\frac{2be^4(1+(c+dx)^2)^{3/2}}{15d}-\frac{be^4(1+(c+dx)^2)^{5/2}}{25d}+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}$$

```
[Out] 2/15*b*e^4*(1+(d*x+c)^2)^(3/2)/d-1/25*b*e^4*(1+(d*x+c)^2)^(5/2)/d+1/5*e^4*(
d*x+c)^5*(a+b*arcsinh(d*x+c))/d-1/5*b*e^4*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 272, 45}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{5d}-\frac{be^4((c+dx)^2+1)^{5/2}}{25d}+\frac{2be^4((c+dx)^2+1)^{3/2}}{15d}-\frac{be^4\sqrt{(c+dx)^2+1}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/5*(b*e^4*Sqrt[1 + (c + d*x)^2])/d + (2*b*e^4*(1 + (c + d*x)^2)^(3/2))/(1
5*d) - (b*e^4*(1 + (c + d*x)^2)^(5/2))/(25*d) + (e^4*(c + d*x)^5*(a + b*Arc
Sinh[c + d*x]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{1+x^2}}\, dx\right.}{5d}$$
$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x}}\, dx,\right.}{10d}$$
$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1+x}} -\right.\right.}{}$$
$$= -\frac{be^4\sqrt{1+(c+dx)^2}}{5d} + \frac{2be^4(1+(c+dx)^2)^{3/2}}{15d} - \frac{be^4(1+(c+d}{25d}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 0.71

$$\frac{e^4\left(-\frac{1}{75}b\sqrt{1+(c+dx)^2}\left(5-10(c+dx)^2+3(1+(c+dx)^2)^2\right)+\frac{1}{5}(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x]),x]
[Out] (e^4*(-1/75*(b*Sqrt[1 + (c + d*x)^2]*(5 - 10*(c + d*x)^2 + 3*(1 + (c + d*x)
^2)^2)) + ((c + d*x)^5*(a + b*ArcSinh[c + d*x]))/5))/d
```

Maple [A]
time = 0.72, size = 93, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{e^4(dx+c)^5a}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+}}{}\right.}{d}$
default	$\frac{\frac{e^4(dx+c)^5a}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arcsinh}(dx+c)}{5}-\frac{(dx+c)^4\sqrt{1+(dx+c)^2}}{25}+\frac{4(dx+c)^2\sqrt{1+(dx+c)^2}}{75}-\frac{8\sqrt{1+}}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*e^4*(d*x+c)^5*a+e^4*b*(1/5*(d*x+c)^5*arcsinh(d*x+c)-1/25*(d*x+c)^4
*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75*(1+(d*x+c)^2)^
(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(82) = 164$.
time = 0.29, size = 1221, normalized size = 12.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d^4*x^5*e^4 + a*c*d^3*x^4*e^4 + 2*a*c^2*d^2*x^3*e^4 + 2*a*c^3*d*x^2*e
^4 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2
*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c
^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*arcsinh
(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh
(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2
/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c^2*d^2*e^4 +
1/24*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^
2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2
*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3
/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*c*d^3*e^4 + 1/600*(120*x^
```

```
5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^4/d^2 - 54*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3*x/d^5 - 32*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^2 + 1)*c^3*arcsi
nh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 + 945*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^
2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c
^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d^6)*d)*b*d^4*e^4
 + a*c^4*x*e^4 + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^4
*e^4/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(82) = 164$.

time = 0.36, size = 1041, normalized size = 10.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^2 +
 5*a*c^4*d*x)*cosh(1)^4 + 60*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3
+ 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*cosh(1)^3*sinh(1) + 90*(a*d^5*x^5 + 5*a*c
*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*cosh(1)^2*sin
h(1)^2 + 60*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c^2*d^3*x^3 + 10*a*c^3*d^2*x^
2 + 5*a*c^4*d*x)*cosh(1)*sinh(1)^3 + 15*(a*d^5*x^5 + 5*a*c*d^4*x^4 + 10*a*c
^2*d^3*x^3 + 10*a*c^3*d^2*x^2 + 5*a*c^4*d*x)*sinh(1)^4 + 15*((b*d^5*x^5 + 5
*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*c
osh(1)^4 + 4*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x
^2 + 5*b*c^4*d*x + b*c^5)*cosh(1)^3*sinh(1) + 6*(b*d^5*x^5 + 5*b*c*d^4*x^4
+ 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*cosh(1)^2*sinh
(1)^2 + 4*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2
+ 5*b*c^4*d*x + b*c^5)*cosh(1)*sinh(1)^3 + (b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*
b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*sinh(1)^4)*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - ((3*b*d^4*x^4 + 12*b*c*d^3*x^3 +
 3*b*c^4 + 2*(9*b*c^2 - 2*b)*d^2*x^2 - 4*b*c^2 + 4*(3*b*c^3 - 2*b*c)*d*x +
8*b)*cosh(1)^4 + 4*(3*b*d^4*x^4 + 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 - 2
*b)*d^2*x^2 - 4*b*c^2 + 4*(3*b*c^3 - 2*b*c)*d*x + 8*b)*cosh(1)^3*sinh(1) +
6*(3*b*d^4*x^4 + 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 - 2*b)*d^2*x^2 - 4*b
*c^2 + 4*(3*b*c^3 - 2*b*c)*d*x + 8*b)*cosh(1)^2*sinh(1)^2 + 4*(3*b*d^4*x^4
+ 12*b*c*d^3*x^3 + 3*b*c^4 + 2*(9*b*c^2 - 2*b)*d^2*x^2 - 4*b*c^2 + 4*(3*b*c
^3 - 2*b*c)*d*x + 8*b)*cosh(1)*sinh(1)^3 + (3*b*d^4*x^4 + 12*b*c*d^3*x^3 +
3*b*c^4 + 2*(9*b*c^2 - 2*b)*d^2*x^2 - 4*b*c^2 + 4*(3*b*c^3 - 2*b*c)*d*x + 8
*b)*sinh(1)^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(85) = 170$.
time = 0.51, size = 527, normalized size = 5.27

$$\begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{asinh}(c+dx)}{5d} + bc^4e^4x\operatorname{asinh}(c+dx) - \frac{bc^4e^4\sqrt{c^2+2cdx+d^2x^2+1}}{25d} + 2bc^3de^4x^2\operatorname{asinh}(c+dx) - \frac{4bc^3e^4x\sqrt{c^2+2cdx+d^2x^2+1}}{25} + 2bc^2d^2e^4x^3\operatorname{asinh}(c+dx) - \frac{6bc^2de^4x^2\sqrt{c^2+2cdx+d^2x^2+1}}{25} + \frac{4bc^2e^4\sqrt{c^2+2cdx+d^2x^2+1}}{75d} + bcd^3e^4x^4\operatorname{asinh}(c+dx) - \frac{4bcd^2e^4x^3\sqrt{c^2+2cdx+d^2x^2+1}}{25} + \frac{8bce^4x\sqrt{c^2+2cdx+d^2x^2+1}}{75} + \frac{bd^4e^4x^5\operatorname{asinh}(c+dx)}{5} - \frac{bd^3e^4x^4\sqrt{c^2+2cdx+d^2x^2+1}}{25} + \frac{4bde^4x^2\sqrt{c^2+2cdx+d^2x^2+1}}{75} - \frac{8be^4\sqrt{c^2+2cdx+d^2x^2+1}}{75d} & \text{for}\: d \neq 0 \\ c^4e^4x(a+b\operatorname{asinh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*asinh(c + d*x)/(5*d)
 + b*c**4*e**4*x*asinh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)/(25*d) + 2*b*c**3*d*e**4*x**2*asinh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 2*b*c**2*d**2*e**4*x**3*asinh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(75*d) + b*c*d**3*e**4*x**4*asin
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 + b*d**4*e**4*x**5*as
inh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 +
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(
c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(86) = 172$.
time = 0.92, size = 841, normalized size = 8.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d)
)/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^4*e^4 + (2*x^2*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c
/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))*abs(d))/(d^2*abs(d)))*d)*b*c^3*d*e^4 + 1/3*(6*x^3*log(d*x + c + sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2
 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d)
- sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c^2*d^2*e^4
 + 1/24*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)
/d^7)*x - 5*(10*c^3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d
```

```
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b
*c*d^3*e^4 + 1/600*(120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*(3*x*(4*x/d^2 - 9*c/d^3) + (47*c
^2*d^5 - 16*d^5)/d^9)*x - 7*(22*c^3*d^4 - 23*c*d^4)/d^9)*x + (274*c^4*d^3 -
 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 - 40*c^3 + 15*c)*log(-c*d - (x*abs(
d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^5*abs(d)))*d)*b*d^4*e^4
+ a*c^4*e^4*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x)), x)
```

3.116 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=105

$$\frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d}-\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh\right.}{4d}$$

```
[Out] -3/32*b*e^3*arcsinh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d+3/32*
b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2
)/d
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 327, 221}

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{16d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)}{32d}-\frac{3be^3\sinh^{-1}(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(32*d) - (b*e^3*(c + d*x)^3*Sqrt[
1 + (c + d*x)^2])/(16*d) - (3*b*e^3*ArcSinh[c + d*x])/(32*d) + (e^3*(c + d*
x)^4*(a + b*ArcSinh[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}}\, dx\right.}{4d}$$
$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} + \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+d\right.}{4d}$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} +$$
$$= \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}}{32d} - \frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}}{16d} -$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.79

$$\frac{e^3\left(3b(c+dx)\sqrt{1+(c+dx)^2}-2b(c+dx)^3\sqrt{1+(c+dx)^2}-3b\sinh^{-1}(c+dx)+8(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (c + d
*x)^2] - 3*b*ArcSinh[c + d*x] + 8*(c + d*x)^4*(a + b*ArcSinh[c + d*x])))/(3
2*d)
```

Maple [A]
time = 0.70, size = 86, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{e^3(dx+c)^4a}{4}+b\,e^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3\operatorname{arcsinh}(dx\cdots}{32}\right.}{d}$
default	$\frac{\frac{e^3(dx+c)^4a}{4}+b\,e^3\left(\frac{(dx+c)^4\operatorname{arcsinh}(dx+c)}{4}-\frac{(dx+c)^3\sqrt{1+(dx+c)^2}}{16}+\frac{3(dx+c)\sqrt{1+(dx+c)^2}}{32}-\frac{3\operatorname{arcsinh}(dx\cdots}{32}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e^3*(d*x+c)^4*a+b*e^3*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3
*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(89) = 178$.

time = 0.28, size = 782, normalized size = 7.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4*e^3 + a*c*d^2*x^3*e^3 + 3/2*a*c^2*d*x^2*e^3 + 3/4*(2*x^2*arcs
inh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*b*c*d^2*e^3 + 1/96*(24*x^4*arcsi
nh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*
x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*(c^2 + 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*x*e^3 + ((d*x + c)*arcsinh(
d*x + c) - sqrt((d*x + c)^2 + 1))*b*c^3*e^3/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(89) = 178$.
time = 0.39, size = 663, normalized size = 6.31

$$\frac{8\left(ad^{4}x^{4}+4acd^{3}x^{3}+6ac^{2}d^{2}x^{2}+4ac^{3}dx\right)\cosh\left(1\right)^{3}+24\left(ad^{4}x^{4}+4acd^{3}x^{3}+6ac^{2}d^{2}x^{2}+4ac^{3}dx\right)\cosh\left(1\right)^{2}\sinh\left(1\right)+24\left(ad^{4}x^{4}+4acd^{3}x^{3}+6ac^{2}d^{2}x^{2}+4ac^{3}dx\right)\cosh\left(1\right)\sinh\left(1\right)^{2}+8\left(ad^{4}x^{4}+4acd^{3}x^{3}+6ac^{2}d^{2}x^{2}+4ac^{3}dx\right)\sinh\left(1\right)^{3}+\left(\left(8bd^{4}x^{4}+32bcd^{3}x^{3}+48bc^{2}d^{2}x^{2}+32bc^{3}dx+8bc^{4}-3b\right)\cosh\left(1\right)^{3}+3\left(8bd^{4}x^{4}+32bcd^{3}x^{3}+48bc^{2}d^{2}x^{2}+32bc^{3}dx+8bc^{4}-3b\right)\cosh\left(1\right)^{2}\sinh\left(1\right)+3\left(8bd^{4}x^{4}+32bcd^{3}x^{3}+48bc^{2}d^{2}x^{2}+32bc^{3}dx+8bc^{4}-3b\right)\cosh\left(1\right)\sinh\left(1\right)^{2}+\left(8bd^{4}x^{4}+32bcd^{3}x^{3}+48bc^{2}d^{2}x^{2}+32bc^{3}dx+8bc^{4}-3b\right)\sinh\left(1\right)^{3}\right)\log\left(dx+c+\sqrt{d^{2}x^{2}+2cdx+c^{2}+1}\right)-\sqrt{d^{2}x^{2}+2cdx+c^{2}+1}\left(\left(2bd^{3}x^{3}+6bcd^{2}x^{2}+2bc^{3}+3\left(2bc^{2}-b\right)dx-3bc\right)\cosh\left(1\right)^{3}+3\left(2bd^{3}x^{3}+6bcd^{2}x^{2}+2bc^{3}+3\left(2bc^{2}-b\right)dx-3bc\right)\cosh\left(1\right)^{2}\sinh\left(1\right)+3\left(2bd^{3}x^{3}+6bcd^{2}x^{2}+2bc^{3}+3\left(2bc^{2}-b\right)dx-3bc\right)\cosh\left(1\right)\sinh\left(1\right)^{2}+\left(2bd^{3}x^{3}+6bcd^{2}x^{2}+2bc^{3}+3\left(2bc^{2}-b\right)dx-3bc\right)\sinh\left(1\right)^{3}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)*cosh(1)
^3 + 24*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)*cosh(1)
^2*sinh(1) + 24*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c^3*d*x)
*cosh(1)*sinh(1)^2 + 8*(a*d^4*x^4 + 4*a*c*d^3*x^3 + 6*a*c^2*d^2*x^2 + 4*a*c
^3*d*x)*sinh(1)^3 + ((8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*
b*c^3*d*x + 8*b*c^4 - 3*b)*cosh(1)^3 + 3*(8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48
*b*c^2*d^2*x^2 + 32*b*c^3*d*x + 8*b*c^4 - 3*b)*cosh(1)^2*sinh(1) + 3*(8*b*d
^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*b*c^3*d*x + 8*b*c^4 - 3*b)*
cosh(1)*sinh(1)^2 + (8*b*d^4*x^4 + 32*b*c*d^3*x^3 + 48*b*c^2*d^2*x^2 + 32*b
*c^3*d*x + 8*b*c^4 - 3*b)*sinh(1)^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*b*d^3*x^3 + 6*b*c*d^2*x^
2 + 2*b*c^3 + 3*(2*b*c^2 - b)*d*x - 3*b*c)*cosh(1)^3 + 3*(2*b*d^3*x^3 + 6*b
*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 - b)*d*x - 3*b*c)*cosh(1)^2*sinh(1) + 3*(
2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 - b)*d*x - 3*b*c)*cosh(1
)*sinh(1)^2 + (2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 2*b*c^3 + 3*(2*b*c^2 - b)*d*x
- 3*b*c)*sinh(1)^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(94) = 188$.
time = 0.42, size = 394, normalized size = 3.75

$$\begin{cases} ac^{3}e^{3}x+\frac{3ac^{2}de^{3}x^{2}}{2}+acd^{2}e^{3}x^{3}+\frac{ad^{3}e^{3}x^{4}}{4}+\frac{bc^{4}e^{3}\operatorname{asinh}\left(c+dx\right)}{4d}+bc^{3}e^{3}x\operatorname{asinh}\left(c+dx\right)-\frac{bc^{3}e^{3}\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{16d}+\frac{3bc^{2}de^{3}x^{2}\operatorname{asinh}\left(c+dx\right)}{2}-\frac{3bc^{2}e^{3}x\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{16}+bcd^{2}e^{3}x^{3}\operatorname{asinh}\left(c+dx\right)-\frac{3bcde^{3}x^{2}\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{16}+\frac{3bce^{3}\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{32d}+\frac{bd^{3}e^{3}x^{4}\operatorname{asinh}\left(c+dx\right)}{4}-\frac{bd^{2}e^{3}x^{3}\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{16}+\frac{3be^{3}x\sqrt{c^{2}+2cdx+d^{2}x^{2}+1}}{32}-\frac{3be^{3}\operatorname{asinh}\left(c+dx\right)}{32d} & \text{for}\: d\neq 0 \\ c^{3}e^{3}x\left(a+b\operatorname{asinh}\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*asinh(c + d*x)/(4*d) + b*c**3*e**3*x*asinh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*asinh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/16 + b*c*d**2*e**3*x**3*asinh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(32*d) + b*d**3*e**3*x**4*asinh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/32 - 3*b*e**3*asinh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*asinh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(93) = 186.
time = 0.79, size = 613, normalized size = 5.84

$$\frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2 - \left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right)\right)bc^3e^3 + \frac{3}{4}\left(2x^2\log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right) - \left(\sqrt{d^2x^2 + 2cdx + c^2 + 1}\left(\frac{x}{d^2} - \frac{3c}{d^3}\right) - \frac{(2c^2 - 1)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right)|d|\right)}{d^2|d|}\right)d\right)bc^2de^3 + \frac{1}{6}\left(6x^3\log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right) - \left(\sqrt{d^2x^2 + 2cdx + c^2 + 1}\left(x\left(\frac{2x}{d^2} - \frac{5c}{d^3}\right) + \frac{11c^2d - 4d}{d^5}\right) + \frac{3(2c^3 - 3c)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right)|d|\right)}{d^3|d|}\right)d\right)bcd^2e^3 + \frac{1}{96}\left(24x^4\log\left(dx + c + \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right) - \left(\sqrt{d^2x^2 + 2cdx + c^2 + 1}\left(\left(2x\left(\frac{3x}{d^2} - \frac{7c}{d^3}\right) + \frac{26c^2d^3 - 9d^3}{d^7}\right)x - \frac{5(10c^3d^2 - 11cd^2)}{d^7}\right) - \frac{3(8c^4 - 24c^2 + 3)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2cdx + c^2 + 1}\right)|d|\right)}{d^4|d|}\right)d\right)bd^3e^3 + ac^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 - (d*(c*log(-c*d
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)))*b*c^3*e^3 + 3/4*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 -
 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*
abs(d)))*d)*b*c^2*d*e^3 + 1/6*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (1
1*c^2*d - 4*d)/d^5) + 3*(2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*abs(d))/(d^3*abs(d)))*d)*b*c*d^2*e^3 + 1/96*(24*x^4*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 - 9*d^3)/d^7)*x - 5*(10*c^
3*d^2 - 11*c*d^2)/d^7) - 3*(8*c^4 - 24*c^2 + 3)*log(-c*d - (x*abs(d) - sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^4*abs(d)))*d)*b*d^3*e^3 + a*c^3*e
^3*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^3\,(a + b\,\mathrm{asinh}(c + d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x)), x)
```

3.117 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=76

$$\frac{be^2\sqrt{1+(c+dx)^2}}{3d}-\frac{be^2\left(1+(c+dx)^2\right)^{3/2}}{9d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}$$

```
[Out] -1/9*b*e^2*(1+(d*x+c)^2)^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1
/3*b*e^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]

time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 272, 45}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d}-\frac{be^2\left((c+dx)^2+1\right)^{3/2}}{9d}+\frac{be^2\sqrt{(c+dx)^2+1}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2])/(3*d) - (b*e^2*(1 + (c + d*x)^2)^(3/2))/(9*d)
 + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}}\,d$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx$$
$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{3d} - \frac{(be^2)\,\text{Subst}\left(\int\left(-\frac{1}{\sqrt{1+x}}$$
$$= \frac{be^2\sqrt{1+(c+dx)^2}}{3d} - \frac{be^2(1+(c+dx)^2)^{3/2}}{9d} + \frac{e^2(c+dx)^3\,(a+$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.84

$$\frac{e^2\left(-\frac{1}{9}b(-2+c^2+2cdx+d^2x^2)\sqrt{1+(c+dx)^2}+\frac{1}{3}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-1/9*(b*(-2 + c^2 + 2*c*d*x + d^2*x^2)*Sqrt[1 + (c + d*x)^2]) + ((c +
 d*x)^3*(a + b*ArcSinh[c + d*x]))/3))/d
```

Maple [A]
time = 0.72, size = 73, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{e^2(dx+c)^3a}{3}+b\,e^2\left(\frac{(dx+c)^3\,\mathrm{arcsinh}(dx+c)}{3}-\frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9}+\frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$	73
default	$\frac{\frac{e^2(dx+c)^3a}{3}+b\,e^2\left(\frac{(dx+c)^3\,\mathrm{arcsinh}(dx+c)}{3}-\frac{(dx+c)^2\sqrt{1+(dx+c)^2}}{9}+\frac{2\sqrt{1+(dx+c)^2}}{9}\right)}{d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*e^2*(d*x+c)^3*a+b*e^2*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*
(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(63) = 126$.
time = 0.27, size = 439, normalized size = 5.78

$$\frac{1}{3}ad^2x^3e^2+acdx^2e^2+\frac{1}{2}\left(2x^2\operatorname{arsinh}(dx+c)-d\left(\frac{3c^2\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3}+\frac{\sqrt{d^2x^2+2cdx+c^2+1}\,x}{d^2}-\frac{(c^2+1)\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^3}-\frac{3\sqrt{d^2x^2+2cdx+c^2+1}\,c}{d^3}\right)\right)bcde^2+\frac{1}{18}\left(6x^3\operatorname{arsinh}(dx+c)-d\left(\frac{2\sqrt{d^2x^2+2cdx+c^2+1}\,x^2}{d^2}-\frac{15c^3\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^4}-\frac{5\sqrt{d^2x^2+2cdx+c^2+1}\,cx}{d^3}+\frac{9(c^2+1)c\operatorname{arsinh}\left(\frac{2(d^2x+cd)}{\sqrt{-4c^2d^2+4(c^2+1)d^2}}\right)}{d^4}+\frac{15\sqrt{d^2x^2+2cdx+c^2+1}\,c^2}{d^4}-\frac{4\sqrt{d^2x^2+2cdx+c^2+1}\,(c^2+1)}{d^4}\right)\right)bd^2e^2+ac^2xe^2+\frac{\left((dx+c)\operatorname{arsinh}(dx+c)-\sqrt{(dx+c)^2+1}\right)bc^2e^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3*e^2 + a*c*d*x^2*e^2 + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*b*c*d*e^2 + 1/18*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2
 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)
*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)*(c^2 + 1)/d^4))*b*d^2*e^2 + a*c^2*x*e^2 + ((d*x + c)*arcsinh(d*x + c) - s
qrt((d*x + c)^2 + 1))*b*c^2*e^2/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(63) = 126$.
time = 0.40, size = 352, normalized size = 4.63

$$\frac{3\left(ad^3x^3+3acd^2x^2+3ac^2dx\right)\cosh(1)^2+6\left(ad^3x^3+3acd^2x^2+3ac^2dx\right)\cosh(1)\sinh(1)+3\left(ad^3x^3+3acd^2x^2+3ac^2dx\right)\sinh(1)^2+3\left(\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3\right)\cosh(1)^2+2\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3\right)\cosh(1)\sinh(1)+\left(bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3\right)\sinh(1)^2\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)-\sqrt{d^2x^2+2cdx+c^2+1}\left(\left(bd^2x^2+2bcdx+bc^2-2b\right)\cosh(1)^2+2\left(bd^2x^2+2bcdx+bc^2-2b\right)\cosh(1)\sinh(1)+\left(bd^2x^2+2bcdx+bc^2-2b\right)\sinh(1)^2\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x)*cosh(1)^2 + 6*(a*d^3*x^3 +
 3*a*c*d^2*x^2 + 3*a*c^2*d*x)*cosh(1)*sinh(1) + 3*(a*d^3*x^3 + 3*a*c*d^2*x^
2 + 3*a*c^2*d*x)*sinh(1)^2 + 3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3)*cosh(1)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cosh
(1)*sinh(1) + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sinh(1)^2)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*b)*cosh(1)^2 + 2*(b*d^2*x^2 +
 2*b*c*d*x + b*c^2 - 2*b)*cosh(1)*sinh(1) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
- 2*b)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(63) = 126$.
time = 0.21, size = 258, normalized size = 3.39

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{asinh}(c+dx)}{3d} + bc^2e^2x\operatorname{asinh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + bcde^2x^2\operatorname{asinh}(c+dx) - \frac{2bce^2x\sqrt{c^2+2cdx+d^2x^2+1}}{9} + \frac{bd^2e^2x^3\operatorname{asinh}(c+dx)}{3} - \frac{bde^2x^2\sqrt{c^2+2cdx+d^2x^2+1}}{9} + \frac{2be^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} & \text{for } d \neq 0 \\ c^2e^2x(a+b\operatorname{asinh}(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*asinh(c + d*x)/(3*d) + b*c**2*e**2*x*asinh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + b*c*d*e**2*x**2*asinh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + b*d**2*e**2*x**3*asinh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*asinh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(66) = 132$.
time = 0.70, size = 416, normalized size = 5.47

$$\frac{1}{3}ad^2e^2x^3 + acde^2x^2 - \left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2+1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2cdx+c^2+1}}{d^2}\right) - x\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)\right)bc^2e^2 + \frac{1}{2}\left(2x^2\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2+1}\left(\frac{x}{d^2} - \frac{3c}{d^3}\right) - \frac{(2c^2-1)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2+1}\right)|d|\right)}{d^2|d|}\right)d\right)bcde^2 + \frac{1}{18}\left(6x^3\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right) - \left(\sqrt{d^2x^2+2cdx+c^2+1}\left(x\left(\frac{2x}{d^2} - \frac{5c}{d^3}\right) + \frac{11c^2d-4d}{d^5}\right) + \frac{3(2c^3-3c)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2cdx+c^2+1}\right)|d|\right)}{d^3|d|}\right)d\right)bd^2e^2 + ac^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c^2*e^2 +
 1/2*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*(x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d)
 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*c*d*e^2 +
1/18*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d - 4*d)/d^5) + 3*(
```

```
2*c^3 - 3*c)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(
d))/(d^3*abs(d)))*d)*b*d^2*e^2 + a*c^2*e^2*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x)), x)
```

3.118 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=68

$$-\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$

```
[Out] 1/4*b*e*arcsinh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d-1/4*b*e*(d*
x+c)*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5859, 12, 5776, 327, 221}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)}{4d}+\frac{be\sinh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/4*(b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/d + (b*e*ArcSinh[c + d*x])/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d} - \frac{(be)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx,x,\right.}{2d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{2d}$$
$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}}{4d} + \frac{be\sinh^{-1}(c+dx)}{4d} + \frac{e(c+dx)^2\left(a\right.}{}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.84

$$\frac{e\left(-b(c+dx)\sqrt{1+(c+dx)^2}+b\sinh^{-1}(c+dx)+2(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e*(-(b*(c + d*x)*Sqrt[1 + (c + d*x)^2]) + b*ArcSinh[c + d*x] + 2*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])))/(4*d)
```

Maple [A]
time = 0.68, size = 62, normalized size = 0.91

method	result	size

derivativedivides	$\frac{\frac{e(dx+c)^2a}{2}+be\left(\frac{(dx+c)^2\,\mathrm{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\mathrm{arcsinh}(dx+c)}{4}\right)}{d}$	62
default	$\frac{\frac{e(dx+c)^2a}{2}+be\left(\frac{(dx+c)^2\,\mathrm{arcsinh}(dx+c)}{2}-\frac{(dx+c)\sqrt{1+(dx+c)^2}}{4}+\frac{\mathrm{arcsinh}(dx+c)}{4}\right)}{d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a+b*e*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(63) = 126$.
time = 0.27, size = 205, normalized size = 3.01

$$\frac{1}{2}\,adx^2e+\frac{1}{4}\left(2\,x^2\,\mathrm{arsinh}\left(dx+c\right)-d\left(\frac{3\,c^2\,\mathrm{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^3}+\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,x}{d^2}-\frac{(c^2+1)\,\mathrm{arsinh}\left(\frac{2\,(d^2x+cd)}{\sqrt{-4\,c^2d^2+4\,(c^2+1)d^2}}\right)}{d^3}-\frac{3\,\sqrt{d^2x^2+2\,cdx+c^2+1}\,c}{d^3}\right)\right)bde+acxe+\frac{\left((dx+c)\,\mathrm{arsinh}\left(dx+c\right)-\sqrt{(dx+c)^2+1}\right)bce}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2*e + 1/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1
)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*b*d*e + a*c*x*e +
 ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b*c*e/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(63) = 126$.
time = 0.35, size = 167, normalized size = 2.46

$$\frac{2\,(ad^2x^2+2\,acdx)\cosh\left(1\right)+\left((2\,bd^2x^2+4\,bcdx+2\,bc^2+b)\cosh\left(1\right)+(2\,bd^2x^2+4\,bcdx+2\,bc^2+b)\sinh\left(1\right)\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+2\,(ad^2x^2+2\,acdx)\sinh\left(1\right)-\sqrt{d^2x^2+2\,cdx+c^2+1}\,\left((bdx+bc)\cosh\left(1\right)+(bdx+bc)\sinh\left(1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*d^2*x^2 + 2*a*c*d*x)*cosh(1) + ((2*b*d^2*x^2 + 4*b*c*d*x + 2*b*c^
2 + b)*cosh(1) + (2*b*d^2*x^2 + 4*b*c*d*x + 2*b*c^2 + b)*sinh(1))*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*d^2*x^2 + 2*a*c*d*x)*sinh(1)
 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b*d*x + b*c)*cosh(1) + (b*d*x + b*c)
*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(58) = 116.
time = 0.15, size = 148, normalized size = 2.18

$$\begin{cases} acex + \frac{adex^2}{2} + \frac{bc^2e\,\mathrm{asinh}\,(c+dx)}{2d} + bcex\,\mathrm{asinh}\,(c+dx) - \frac{bce\sqrt{c^2+2cdx+d^2x^2+1}}{4d} + \frac{bdex^2\,\mathrm{asinh}\,(c+dx)}{2} - \frac{bex\sqrt{c^2+2cdx+d^2x^2+1}}{4} + \frac{be\,\mathrm{asinh}\,(c+dx)}{4d} & \text{for } d \neq 0 \\ cex(a+b\,\mathrm{asinh}\,(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*asinh(c + d*x)/(2*d) + b*c*e*x
*asinh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(4*d) + b*d*e*
x**2*asinh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + b*e*
asinh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(60) = 120.
time = 0.57, size = 243, normalized size = 3.57

$$\frac{1}{2}\,adex^2 - \left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2+2\,cdx+c^2+1}}{d^2}\right) - x\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\right)bce + \frac{1}{4}\left(2\,x^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right) - \left(\sqrt{d^2x^2+2\,cdx+c^2+1}\left(\frac{x}{d^2} - \frac{3\,c}{d^3}\right) - \frac{(2\,c^2-1)\log\left(-cd - \left(x|d| - \sqrt{d^2x^2+2\,cdx+c^2+1}\right)|d|\right)}{d^2|d|}\right)d\right)bde + acex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*e*x^2 - (d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*abs(d))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))*b*c*e + 1/4*(2*x^2*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
x/d^2 - 3*c/d^3) - (2*c^2 - 1)*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))*abs(d))/(d^2*abs(d)))*d)*b*d*e + a*c*e*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{asinh}(c + d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x)), x)
```

3.119 $\int \left(a+b\sinh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$ax-\frac{b\sqrt{1+(c+dx)^2}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arcsinh(d*x+c)/d-b*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5858, 5772, 267}

$$ax-\frac{b\sqrt{(c+dx)^2+1}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[1 + (c + d*x)^2])/d + (b*(c + d*x)*ArcSinh[c + d*x])/d
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\sinh^{-1}(c+dx)\right)\,dx &= ax+b\int \sinh^{-1}(c+dx)\,dx\\
&= ax+\frac{b\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,c+dx\right)}{d}\\
&= ax+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,c+dx\right)}{d}\\
&= ax-\frac{b\sqrt{1+(c+dx)^2}}{d}+\frac{b(c+dx)\sinh^{-1}(c+dx)}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(39) = 78.
time = 0.25, size = 140, normalized size = 3.59

$$ax+bx\sinh^{-1}(c+dx)-\frac{b\left(2d\sqrt{1+c^2+2cdx+d^2x^2}+c\left(d+\sqrt{d^2}\right)\log\left(-c-\sqrt{d^2}\,x+\sqrt{1+c^2+2cdx+d^2x^2}\right)+c\left(-d+\sqrt{d^2}\right)\log\left(c-\sqrt{d^2}\,x+\sqrt{1+c^2+2cdx+d^2x^2}\right)\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcSinh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcSinh[c + d*x] - (b*(2*d*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] + c*
(d + Sqrt[d^2])*Log[-c - Sqrt[d^2]*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] +
 c*(-d + Sqrt[d^2])*Log[c - Sqrt[d^2]*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]
]))/(2*d^2)
```

Maple [A]
time = 0.63, size = 36, normalized size = 0.92

method	result	size
default	$ax+\frac{b\left((dx+c)\,\mathrm{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$	36
derivativedivides	$\frac{(dx+c)a+b\left((dx+c)\,\mathrm{arcsinh}(dx+c)-\sqrt{1+(dx+c)^2}\right)}{d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b/d*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2))
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.90

$$ax + \frac{\left((dx + c)\operatorname{arsinh}(dx + c) - \sqrt{(dx + c)^2 + 1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*b/d
```

Fricas [A]
time = 0.34, size = 65, normalized size = 1.67

$$\frac{adx + (bdx + bc)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b)/d
```

Sympy [A]
time = 0.10, size = 51, normalized size = 1.31

$$ax + b\left(\begin{cases} \frac{c\operatorname{asinh}(c+dx)}{d} + x\operatorname{asinh}(c + dx) - \frac{\sqrt{c^2 + 2cdx + d^2x^2 + 1}}{d} & \text{for}\ d \neq 0 \\ x\operatorname{asinh}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*asinh(c + d*x)/d + x*asinh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*asinh(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(37) = 74$.
time = 0.41, size = 99, normalized size = 2.54

$$-\left(d\left(\frac{c\log\left(-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right)|d|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{(dx + c)^2 + 1}\right)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*abs(d))/(d
*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)/d^2) - x*log(d*x + c + sqrt((d
*x + c)^2 + 1)))*b + a*x
```

Mupad [B]
time = 0.48, size = 85, normalized size = 2.18

$$a\,x+b\,x\,\mathrm{asinh}(c+d\,x)-\frac{b\,\sqrt{c^2+2\,c\,d\,x+d^2\,x^2+1}}{d}+\frac{b\,c\,d^2\,\ln\left(\sqrt{c^2+2\,c\,d\,x+d^2\,x^2+1}+\frac{x\,d^2+c\,d}{\sqrt{d^2}}\right)}{{\left(d^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(c + d*x),x)
```

```
[Out] a*x + b*x*asinh(c + d*x) - (b*(c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2))/d + (b*c
*d^2*log((c^2 + d^2*x^2 + 2*c*d*x + 1)^(1/2) + (c*d + d^2*x)/(d^2)^(1/2)))/
(d^2)^(3/2)
```

3.120 $\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)}{de}-\frac{b\mathrm{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)}{2de}$$

```
[Out] 1/2*(a+b*arcsinh(d*x+c))^2/b/d/e+(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x
+c)^2)^(1/2))^2)/d/e-1/2*b*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5859, 12, 5775, 3797, 2221, 2317, 2438}

$$\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{b\mathrm{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcSinh[c + d*x])*Log[1 - E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*
d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \ldots$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} - \ldots$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de} + \ldots$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.86

$$\frac{-\left(\left(a+b\sinh^{-1}(c+dx)\right)\left(a+b\sinh^{-1}(c+dx)-2b\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\right)\right)+b^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)}{2bde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])*(a + b*ArcSinh[c + d*x] - 2*b*Log[1 - E^(2*ArcS
inh[c + d*x])])) + b^2*PolyLog[2, E^(2*ArcSinh[c + d*x])])/(2*b*d*e)
```

Maple [A]
time = 2.39, size = 145, normalized size = 1.79

method	result
derivativedivides	$\frac{\frac{a\ln(dx+c)}{e}-\frac{b\,\mathrm{arcsinh}(dx+c)^2}{2e}+\frac{b\,\mathrm{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{b\,\mathrm{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)}{e}$ … $}{d}$
default	$\frac{\frac{a\ln(dx+c)}{e}-\frac{b\,\mathrm{arcsinh}(dx+c)^2}{2e}+\frac{b\,\mathrm{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{b\,\mathrm{polylog}\left(2,-dx-c-\sqrt{1+(dx+c)^2}\right)}{e}$ … $}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/e*ln(d*x+c)-1/2*b/e*arcsinh(d*x+c)^2+b/e*arcsinh(d*x+c)*ln(1+d*x+c+(
1+(d*x+c)^2)^(1/2))+b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+b/e*arcsinh(d
*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] b*integrate(log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*x*e + c*e), x) + a*e^(-
1)*log(d*x*e + c*e)/d
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arcsinh(d*x + c) + a)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c+dx}\,dx + \int \frac{b\,\mathrm{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x), x)
```

3.121 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}$$

```
[Out] (-a-b*arcsinh(d*x+c))/d/e^2/(d*x+c)-b*arctanh((1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5776, 272, 65, 213}

$$-\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])/(d*e^2*(c + d*x))) - (b*ArcTanh[Sqrt[1 + (c + d*
x)^2]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,(c+dx)^2\right)}{2de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}+\frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+(c+dx)^2}\right)}{de^2}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{de^2(c+dx)}-\frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.88

$$-\frac{\frac{a+b\sinh^{-1}(c+dx)}{c+dx}+b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -(((a + b*ArcSinh[c + d*x])/(c + d*x) + b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(
d*e^2))
```

Maple [A]
time = 0.69, size = 54, normalized size = 1.10

method	result	size
derivativedivides	$\frac{-\frac{a}{e^{2}(dx+c)}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^{2}}}\right)\right)}{e^{2}}}{d}$	54
default	$\frac{-\frac{a}{e^{2}(dx+c)}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{dx+c}-\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^{2}}}\right)\right)}{e^{2}}}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/e^2/(d*x+c)+b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2
)^(1/2))))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 1.47

$$-b\left(\frac{\operatorname{arsinh}\left(\frac{de^{2}}{|d^{2}xe^{2}+cde^{2}|}\right)e^{(-2)}}{d}+\frac{\operatorname{arsinh}\left(dx+c\right)}{d^{2}xe^{2}+cde^{2}}\right)-\frac{a}{d^{2}xe^{2}+cde^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b*(arcsinh(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arcsinh(d*x + c)/(d^
2*x*e^2 + c*d*e^2)) - a/(d^2*x*e^2 + c*d*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(45) = 90$.
time = 0.47, size = 210, normalized size = 4.29

$$\frac{bdx\log\left(dx+c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\right)-ac-(bcdx+bc^{2})\log\left(-dx-c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}+1\right)+(bdx+bc)\log\left(-dx-c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}\right)+(bcdx+bc^{2})\log\left(-dx-c+\sqrt{d^{2}x^{2}+2\,cdx+c^{2}+1}-1\right)}{(cd^{2}x+c^{2}d)\cosh\left(1\right)^{2}+2\,(cd^{2}x+c^{2}d)\cosh\left(1\right)\sinh\left(1\right)+(cd^{2}x+c^{2}d)\sinh\left(1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - a*c - (b*c*d*x +
b*c^2)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) + (b*d*x + b*c
)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (b*c*d*x + b*c^2)*log
(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1))/((c*d^2*x + c^2*d)*cosh
(1)^2 + 2*(c*d^2*x + c^2*d)*cosh(1)*sinh(1) + (c*d^2*x + c^2*d)*sinh(1)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b\,\mathrm{asinh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*asinh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(47) = 94.
time = 0.44, size = 134, normalized size = 2.73

$$-b\left(\frac{\log\left(dx+c+\sqrt{(dx+c)^2+1}\right)}{(dex+ce)de} + \frac{d\log\left(\sqrt{\frac{e^2}{(dex+ce)^2}+1}+\frac{\sqrt{d^2e^4}}{(dex+ce)de}\right)}{e^2|d|^2\mathrm{sgn}\left(\frac{1}{dex+ce}\right)\mathrm{sgn}\,(d)\,\mathrm{sgn}\,(e)}\right) - \frac{a}{(dex+ce)de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] -b*(log(d*x + c + sqrt((d*x + c)^2 + 1))/((d*e*x + c*e)*d*e) + d*log(sqrt(e
^2/(d*e*x + c*e)^2 + 1) + sqrt(d^2*e^4)/((d*e*x + c*e)*d*e))/(e^2*abs(d)^2*
sgn(1/(d*e*x + c*e))*sgn(d)*sgn(e))) - a/((d*e*x + c*e)*d*e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{{(c\,e+d\,e\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^2, x)
```

3.122 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] 1/2*(-a-b*arcsinh(d*x+c))/d/e^3/(d*x+c)^2-1/2*b*(1+(d*x+c)^2)^(1/2)/d/e^3/(
d*x+c)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5859, 12, 5776, 270}

$$-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{2de^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(b*Sqrt[1 + (c + d*x)^2])/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])
/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{2de^3}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{2de^3(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{2de^3(c+dx)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.97

$$
\frac{-\frac{b\sqrt{1+(c+dx)^2}}{2(c+dx)}+\frac{-a-b\sinh^{-1}(c+dx)}{2(c+dx)^2}}{de^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^3,x]
[Out] (-1/2*(b*Sqrt[1 + (c + d*x)^2])/(c + d*x) + (-a - b*ArcSinh[c + d*x])/(2*(c
 + d*x)^2))/(d*e^3)
```

Maple [A]
time = 0.70, size = 60, normalized size = 1.02

method	result	size
derivativedivides	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$	60
default	$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{2(dx+c)^2}-\frac{\sqrt{1+(dx+c)^2}}{2(dx+c)}\right)}{e^3}}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*
(1+(d*x+c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.
time = 0.26, size = 109, normalized size = 1.85

$$-\frac{1}{2}\,b\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d}{d^3xe^3+cd^2e^3}+\frac{\operatorname{arsinh}\left(dx+c\right)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)-\frac{a}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*x*e^3 + c*d^2*e^3) + arcsi
nh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 1/2*a/(d^3*x^2*e^3
 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(51) = 102$.
time = 0.35, size = 211, normalized size = 3.58

$$\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^2dx+bc^3\right)\sqrt{d^2x^2+2\,cdx+c^2+1}}{2\left(\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^3+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^2\sinh\left(1\right)+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)\sinh\left(1\right)^2+\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\sinh\left(1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) - (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/((c^2*d^3
*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^3 + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*
d)*cosh(1)^2*sinh(1) + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)*sinh(1
)^2 + (c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*sinh(1)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b\operatorname{asinh}\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{{(c\,e+d\,e\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^3, x)
```

3.123 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4}\,dx$

Optimal. Leaf size=84

$$-\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}$$

```
[Out] 1/3*(-a-b*arcsinh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctanh((1+(d*x+c)^2)^(1/2)
)/d/e^4-1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5776, 272, 44, 65, 213}

$$-\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3}-\frac{b\sqrt{(c+dx)^2+1}}{6de^4(c+dx)^2}+\frac{b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -1/6*(b*Sqrt[1 + (c + d*x)^2])/(d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x
])/(3*d*e^4*(c + d*x)^3) + (b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(6*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^4x^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{de^4}$$
$$= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4}$$
$$= -\frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{6de^4}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, (c+$$
$$\frac{}{12de^4}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3} - \frac{b\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+(c\right.}{6de^4}$$
$$= -\frac{b\sqrt{1+(c+dx)^2}}{6de^4(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4}$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.98

$$-\frac{2a+b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}+2b\sinh^{-1}(c+dx)-b(c+dx)^3\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{6de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] -1/6*(2*a + b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2] + 2*b*ArcSinh[c +
 d*x] - b*(c + d*x)^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4*(c + d*x)^3)
```

Maple [A]
time = 0.70, size = 74, normalized size = 0.88

method	result	size

derivativedivides	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$	74
default	$\frac{-\frac{a}{3e^4(dx+c)^3}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{3(dx+c)^3}-\frac{\sqrt{1+(dx+c)^2}}{6(dx+c)^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right)}{6}\right)}{e^4}}{d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a/e^4/(d*x+c)^3+b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^
2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(-I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))*e^(-4
)/d + 2*(d^2*x^2 + 2*c*d*x + c^2 + log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)))/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 6
*integrate(1/3/(d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4
+ 4*(5*c^3*d^3 + c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*
c^5*d + 2*c^3*d)*x*e^4 + (c^6 + c^4)*e^4 + (d^5*x^5*e^4 + 5*c*d^4*x^4*e^4 +
 (10*c^2*d^3 + d^3)*x^3*e^4 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^4 + (5*c^4*d + 3
*c^2*d)*x*e^4 + (c^5 + c^3)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)) -
1/3*a/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(71) = 142$.
time = 0.43, size = 512, normalized size = 6.10

$$-\frac{2\,ac^3-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx+bc^6\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}+1\right)-2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx+bc^6\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}-1\right)+\left(bc^3dx+bc^4\right)\sqrt{d^2x^2+2\,cdx+c^2+1}}{6\left(\left(c^3d^4x^3+3\,c^4d^3x^2+3\,c^5d^2x+c^6d\right)\cosh\left(1\right)^4+4\left(c^3d^4x^3+3\,c^4d^3x^2+3\,c^5d^2x+c^6d\right)\cosh\left(1\right)^3\sinh\left(1\right)+6\left(c^3d^4x^3+3\,c^4d^3x^2+3\,c^5d^2x+c^6d\right)\cosh\left(1\right)^2\sinh\left(1\right)^2+4\left(c^3d^4x^3+3\,c^4d^3x^2+3\,c^5d^2x+c^6d\right)\cosh\left(1\right)\sinh\left(1\right)^3+\left(c^3d^4x^3+3\,c^4d^3x^2+3\,c^5d^2x+c^6d\right)\sinh\left(1\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*
c^5*d*x + b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1) - 2*
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)) + (b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x +
 b*c^6)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) + (b*c^3*d*x
+ b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/((c^3*d^4*x^3 + 3*c^4*d^3*x^2 +
 3*c^5*d^2*x + c^6*d)*cosh(1)^4 + 4*(c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^
2*x + c^6*d)*cosh(1)^3*sinh(1) + 6*(c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2
*x + c^6*d)*cosh(1)^2*sinh(1)^2 + 4*(c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^
2*x + c^6*d)*cosh(1)*sinh(1)^3 + (c^3*d^4*x^3 + 3*c^4*d^3*x^2 + 3*c^5*d^2*x
 + c^6*d)*sinh(1)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^4, x)
```

3.124 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5} dx$

Optimal. Leaf size=90

$$-\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}+\frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}$$

```
[Out] 1/4*(-a-b*arcsinh(d*x+c))/d/e^5/(d*x+c)^4-1/12*b*(1+(d*x+c)^2)^(1/2)/d/e^5/
(d*x+c)^3+1/6*b*(1+(d*x+c)^2)^(1/2)/d/e^5/(d*x+c)
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5859, 12, 5776, 277, 270}

$$-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{(c+dx)^2+1}}{6de^5(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(b*Sqrt[1 + (c + d*x)^2])/(d*e^5*(c + d*x)^3) + (b*Sqrt[1 + (c + d*x)
^2])/(6*d*e^5*(c + d*x)) - (a + b*ArcSinh[c + d*x])/(4*d*e^5*(c + d*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^5}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^5x^5}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^5}\,dx,x,c+dx\right)}{de^5}\\
&= -\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx,x,c+dx\right)}{4de^5}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}-\frac{b\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx,x,c+\right.}{6de^5}\\
&= -\frac{b\sqrt{1+(c+dx)^2}}{12de^5(c+dx)^3}+\frac{b\sqrt{1+(c+dx)^2}}{6de^5(c+dx)}-\frac{a+b\sinh^{-1}(c+dx)}{4de^5(c+dx)^4}
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 61, normalized size = 0.68

$$
-\frac{b(c+dx)\left(1-2(c+dx)^2\right)\sqrt{1+(c+dx)^2}+3\left(a+b\sinh^{-1}(c+dx)\right)}{12de^5(c+dx)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] -1/12*(b*(c + d*x)*(1 - 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] + 3*(a + b*Arc
Sinh[c + d*x]))/(d*e^5*(c + d*x)^4)
```

Maple [**A**]
time = 0.69, size = 80, normalized size = 0.89

method	result	size
derivativedivides	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$	80
default	$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{4(dx+c)^4}-\frac{\sqrt{1+(dx+c)^2}}{12(dx+c)^3}+\frac{\sqrt{1+(dx+c)^2}}{6dx+6c}\right)}{e^5}}{d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arcsinh(d*x+c)-1/12/(d*x+c)
^3*(1+(d*x+c)^2)^(1/2)+1/6/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.
time = 0.28, size = 244, normalized size = 2.71

$$\frac{1}{12}b\left(\frac{(2d^4x^4+8cd^3x^3+2c^4+(12c^2d^2+d^2)x^2+c^2+2(4c^3d+cd)x-1)d}{(d^5x^3e^5+3cd^4x^2e^5+3c^2d^3xe^5+c^3d^2e^5)\sqrt{d^2x^2+2cdx+c^2+1}}-\frac{3\operatorname{arsinh}(dx+c)}{d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5}\right)-\frac{a}{4(d^5x^4e^5+4cd^4x^3e^5+6c^2d^3x^2e^5+4c^3d^2xe^5+c^4de^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 + d^2)*x^2 + c^2 + 2
*(4*c^3*d + c*d)*x - 1)*d/((d^5*x^3*e^5 + 3*c*d^4*x^2*e^5 + 3*c^2*d^3*x*e^5
 + c^3*d^2*e^5)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*arcsinh(d*x + c)/(d^
5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2*x*e^5 + c^4*d*e
^5)) - 1/4*a/(d^5*x^4*e^5 + 4*c*d^4*x^3*e^5 + 6*c^2*d^3*x^2*e^5 + 4*c^3*d^2
*x*e^5 + c^4*d*e^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(77) = 154$.
time = 0.42, size = 477, normalized size = 5.30

$$\frac{3ad^4x^4+12acd^3x^3+18ac^2d^2x^2+12ac^3dx-3bc^4\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)+(2bc^4d^3x^3+6bc^5d^2x^2+2bc^7-bc^5+(6bc^6-bc^4)dx)\sqrt{d^2x^2+2cdx+c^2+1}}{12\left((c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\cosh(1)^5+5(c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\cosh(1)^4\sinh(1)+10(c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\cosh(1)^3\sinh(1)^2+10(c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\cosh(1)^2\sinh(1)^3+5(c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\cosh(1)\sinh(1)^4+(c^4d^5x^4+4c^5d^4x^3+6c^6d^3x^2+4c^7d^2x+c^8d)\sinh(1)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (2*b*c^4*d^3*x^3 + 6
```

```
*b*c^5*d^2*x^2 + 2*b*c^7 - b*c^5 + (6*b*c^6 - b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))/((c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2
*x + c^8*d)*cosh(1)^5 + 5*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*
c^7*d^2*x + c^8*d)*cosh(1)^4*sinh(1) + 10*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*
c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(1)^3*sinh(1)^2 + 10*(c^4*d^5*x^4 +
4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(1)^2*sinh(1)^3 +
5*(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)*cosh(
1)*sinh(1)^4 + (c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*c^6*d^3*x^2 + 4*c^7*d^2*x +
 c^8*d)*sinh(1)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx+\int \frac{b\operatorname{asinh}(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*asinh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asinh}(c+d\,x)}{{(c\,e+d\,e\,x)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^5, x)
```

3.125 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4}+\frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2}-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}-\frac{3b\tanh^{-1}\left(\sqrt{1+(c+dx)^2}\right)}{40de^6}$$

```
[Out] 1/5*(-a-b*arcsinh(d*x+c))/d/e^6/(d*x+c)^5-3/40*b*arctanh((1+(d*x+c)^2)^(1/2
))/d/e^6-1/20*b*(1+(d*x+c)^2)^(1/2)/d/e^6/(d*x+c)^4+3/40*b*(1+(d*x+c)^2)^(1
/2)/d/e^6/(d*x+c)^2
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5776, 272, 44, 65, 213}

$$-\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{(c+dx)^2+1}}{40de^6(c+dx)^2}-\frac{b\sqrt{(c+dx)^2+1}}{20de^6(c+dx)^4}-\frac{3b\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{40de^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -1/20*(b*Sqrt[1 + (c + d*x)^2])/(d*e^6*(c + d*x)^4) + (3*b*Sqrt[1 + (c + d*
x)^2])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])/(5*d*e^6*(c + d*x)
^5) - (3*b*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(40*d*e^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^6}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{e^6x^6}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^6}\,dx, x, c+dx\right)}{de^6}$$

$$= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\text{Subst}\left(\int \frac{1}{x^5\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de^6}$$

$$= -\frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+x}}\,dx, x, (c+dx)^2\right)}{10de^6}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} - \frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} - \frac{(3b)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, (c\right.}{40de^6}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)\text{Subst}}{}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)\text{Subst}}{}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}}{20de^6(c+dx)^4} + \frac{3b\sqrt{1+(c+dx)^2}}{40de^6(c+dx)^2} - \frac{a+b\sinh^{-1}(c+dx)}{5de^6(c+dx)^5} - \frac{3b\tanh^{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 61, normalized size = 0.53

$$-\frac{\frac{a+b\sinh^{-1}(c+dx)}{(c+dx)^5} + b\sqrt{1+(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1+(c+dx)^2\right)}{5de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] -1/5*((a + b*ArcSinh[c + d*x])/(c + d*x)^5 + b*Sqrt[1 + (c + d*x)^2]*Hyperg
eometric2F1[1/2, 3, 3/2, 1 + (c + d*x)^2])/(d*e^6)
```

Maple [A]
time = 0.92, size = 94, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right.}{40}\right.}{e^6}}{d}$
default	$\frac{-\frac{a}{5e^6(dx+c)^5}+\frac{b\left(-\frac{\operatorname{arcsinh}(dx+c)}{5(dx+c)^5}-\frac{\sqrt{1+(dx+c)^2}}{20(dx+c)^4}+\frac{3\sqrt{1+(dx+c)^2}}{40(dx+c)^2}-\frac{3\operatorname{arctanh}\left(\frac{1}{\sqrt{1+(dx+c)^2}}\right.}{40}\right.}{e^6}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a/e^6/(d*x+c)^5+b/e^6*(-1/5/(d*x+c)^5*arcsinh(d*x+c)-1/20/(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)+3/40/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-3/40*arctanh(1/(1
+(d*x+c)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] -1/30*b*(3*I*(log(I*(d^2*x + c*d)/d + 1) - log(-I*(d^2*x + c*d)/d + 1))*e^(
-6)/d - 2*(3*d^4*x^4 + 12*c*d^3*x^3 + 3*c^4 + (18*c^2*d^2 - d^2)*x^2 - c^2
+ 2*(6*c^3*d - c*d)*x - 3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))
/(d^6*x^5*e^6 + 5*c*d^5*x^4*e^6 + 10*c^2*d^4*x^3*e^6 + 10*c^3*d^3*x^2*e^6 +
 5*c^4*d^2*x*e^6 + c^5*d*e^6) - 30*integrate(1/5/(d^8*x^8*e^6 + 8*c*d^7*x^7
*e^6 + (28*c^2*d^6 + d^6)*x^6*e^6 + 2*(28*c^3*d^5 + 3*c*d^5)*x^5*e^6 + 5*(1
4*c^4*d^4 + 3*c^2*d^4)*x^4*e^6 + 4*(14*c^5*d^3 + 5*c^3*d^3)*x^3*e^6 + (28*c
^6*d^2 + 15*c^4*d^2)*x^2*e^6 + 2*(4*c^7*d + 3*c^5*d)*x*e^6 + (c^8 + c^6)*e^
6 + (d^7*x^7*e^6 + 7*c*d^6*x^6*e^6 + (21*c^2*d^5 + d^5)*x^5*e^6 + 5*(7*c^3*
d^4 + c*d^4)*x^4*e^6 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^6 + (21*c^5*d^2 + 10
*c^3*d^2)*x^2*e^6 + (7*c^6*d + 5*c^4*d)*x*e^6 + (c^7 + c^5)*e^6)*sqrt(d^2*x
```

```
^2 + 2*c*d*x + c^2 + 1)), x)) - 1/5*a/(d^6*x^5*e^6 + 5*c*d^5*x^4*e^6 + 10*c
^2*d^4*x^3*e^6 + 10*c^3*d^3*x^2*e^6 + 5*c^4*d^2*x*e^6 + c^5*d*e^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(97) = 194$.
time = 0.42, size = 896, normalized size = 7.79

$$-\frac{8\,ac^5 - 8\left(bd^5x^5 + 5\,bcd^4x^4 + 10\,bc^2d^3x^3 + 10\,bc^3d^2x^2 + 5\,bc^4dx\right)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) + 3\left(bc^5d^5x^5 + 5\,bc^6d^4x^4 + 10\,bc^7d^3x^3 + 10\,bc^8d^2x^2 + 5\,bc^9dx + bc^{10}\right)\log\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1} + 1\right) - 8\left(bd^5x^5 + 5\,bcd^4x^4 + 10\,bc^2d^3x^3 + 10\,bc^3d^2x^2 + 5\,bc^4dx + bc^5\right)\log\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1}\right) - 3\left(bc^5d^5x^5 + 5\,bc^6d^4x^4 + 10\,bc^7d^3x^3 + 10\,bc^8d^2x^2 + 5\,bc^9dx + bc^{10}\right)\log\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx + c^2 + 1} - 1\right) - \left(3\,bc^5d^3x^3 + 9\,bc^6d^2x^2 + 3\,bc^8 - 2\,bc^6 + \left(9\,bc^7 - 2\,bc^5\right)dx\right)\sqrt{d^2x^2 + 2\,cdx + c^2 + 1}}{40\left(\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)^6 + 6\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)^5\sinh\left(1\right) + 15\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)^4\sinh\left(1\right)^2 + 20\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)^3\sinh\left(1\right)^3 + 15\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)^2\sinh\left(1\right)^4 + 6\left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\cosh\left(1\right)\sinh\left(1\right)^5 + \left(c^5d^6x^5 + 5\,c^6d^5x^4 + 10\,c^7d^4x^3 + 10\,c^8d^3x^2 + 5\,c^9d^2x + c^{10}d\right)\sinh\left(1\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3
*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
3*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 +
5*b*c^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) + 1)
 - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b
*c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*(b*
c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10*b*c^8*d^2*x^2 + 5*b*c
^9*d*x + b*c^10)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1) - 1) - (3
*b*c^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 - 2*b*c^6 + (9*b*c^7 - 2*b*c^5)*
d*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/((c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*
c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^6 + 6*(c^5*d^6
*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10
*d)*cosh(1)^5*sinh(1) + 15*(c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 +
10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^4*sinh(1)^2 + 20*(c^5*d^6*x^
5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)
*cosh(1)^3*sinh(1)^3 + 15*(c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 1
0*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*cosh(1)^2*sinh(1)^4 + 6*(c^5*d^6*x^5
+ 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d^3*x^2 + 5*c^9*d^2*x + c^10*d)*c
osh(1)*sinh(1)^5 + (c^5*d^6*x^5 + 5*c^6*d^5*x^4 + 10*c^7*d^4*x^3 + 10*c^8*d
^3*x^2 + 5*c^9*d^2*x + c^10*d)*sinh(1)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx + \int \frac{b\operatorname{asinh}\left(c+dx\right)}{c^6+6c^5dx+15c^4d^2x^2+20c^3d^3x^3+15c^2d^4x^4+6cd^5x^5+d^6x^6}\,dx}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*asinh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}(c + d\,x)}{(c\,e + d\,e\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6,x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^6, x)
```

3.126 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=187

$$\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)}-\frac{2b(e(c+dx))^{2+m}\left(a+b\sinh^{-1}(c+dx)\right)\, {}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-(c+dx\right.}{de^2(1+m)(2+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/d/e/(1+m)-2*b*(e*(d*x+c))^(2+m)*(a
+b*arcsinh(d*x+c))*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-(d*x+c)^2)/d/e^2/(1+
m)/(2+m)+2*b^2*(e*(d*x+c))^(3+m)*hypergeom([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2
*m, 5/2+1/2*m],-(d*x+c)^2)/d/e^3/(3+m)/(m^2+3*m+2)
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5859, 5776, 5817}

$$\frac{2b^2(e(c+dx))^{m+3}\, {}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b(e(c+dx))^{m+2}\, {}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2(m+1)(m+2)}+\frac{(e(c+dx))^{m+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/
2, (4 + m)/2, -(c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)) + (2*b^2*(e*(c + d*x))
^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 -(c + d*x)^2])/(d*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m} \left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{}} \right.}{d}$$

$$= \frac{(e(c+dx))^{1+m} \left(a+b\sinh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{2b(e(c+dx))^{2+m} (a}{}$$

Mathematica [A]
time = 0.08, size = 155, normalized size = 0.83

$$\frac{(c+dx)(e(c+dx))^m \left(\left(a+b\sinh^{-1}(c+dx)\right)^2 - \frac{2b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right) {}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};-(c+dx)^2\right)}{2+m} + \frac{2b^2(c+dx)^2 {}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};-(c+dx)^2\right)}{(2+m)(3+m)}\right)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcSinh[c + d*x])^2 - (2*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, -(c + d*
x)^2])/(2 + m) + (2*b^2*(c + d*x)^2*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 +
m/2}, {2 + m/2, 5/2 + m/2}, -(c + d*x)^2])/((2 + m)*(3 + m))))/(d*(1 + m))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dex+ce\right)^m \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (d*x*e + c*e)^(m + 1)*a^2*e^(-1)/(d*(m + 1)) + (b^2*d*x*e^m + b^2*c*e^m)*(d
*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d*(m + 1)) +
integrate(-2*((b^2*c^2*e^m - ((m*e^m + e^m)*c^2 + m*e^m + e^m)*a*b - ((m*e^
m + e^m)*a*b*d^2 - b^2*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b*c*d - b^2*c*d*e^
m)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - (((m*e^m + e^m)*a*b*d
^3 - b^2*d^3*e^m)*x^3 + ((m*e^m + e^m)*c^3 + (m*e^m + e^m)*c)*a*b - (c^3*e^
m + c*e^m)*b^2 + 3*((m*e^m + e^m)*a*b*c*d^2 - b^2*c*d^2*e^m)*x^2 + ((3*(m*e
^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a*b - (3*c^2*d*e^m + d*e^m)*b^2)*x)*(d*x
 + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*(m + 1)*x^3
+ 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m +
 1))*x + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*((d*x + c)
*e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^2, x)
```

3.127 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=197

$$\frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{8be^4(c+dx)^2\sqrt{1}}{}$$

[Out] 16/75*b^2*e^4*x-8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^2/d-16/75*b*e^4*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d+8/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d

Rubi [A]
time = 0.20, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5812, 5798, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^2}{5d} - \frac{2be^4\sqrt{(c+dx)^2+1}\,(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{25d} + \frac{8be^4\sqrt{(c+dx)^2+1}\,(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{75d} - \frac{16be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{16}{75}b^2e^4x$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]

[Out] (16*b^2*e^4*x)/75 - (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^5)/(125*d) - (16*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*d) + (8*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(75*d) - (2*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^2)/(5*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^2 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)^2}{5d} - \frac{(2be^4)\,\text{Subst}\left(\int \frac{x^5(a+b\sinh^{-1}}{\sqrt{1+x}}\right.}{5d}$$

$$= -\frac{2be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{25d} + \frac{e^4(c+}{}$$

$$= \frac{2b^2e^4(c+dx)^5}{125d} + \frac{8be^4(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{75d}$$

$$= -\frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+dx)^2}\,(a+}{75d}$$

$$= \frac{16}{75}b^2e^4x - \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{1+(c+}}{}$$

Mathematica [A]
time = 0.16, size = 192, normalized size = 0.97

$$\frac{e^4\left(240b^2(c+dx)-40b^2(c+dx)^3+9(25a^2+2b^2)(c+dx)^5+30ab\sqrt{1+(c+dx)^2}\left(-8+4(c+dx)^2-3(c+dx)^4\right)+30b\left(15a(c+dx)^5-8b\sqrt{1+(c+dx)^2}+4b(c+dx)^2\sqrt{1+(c+dx)^2}-3b(c+dx)^4\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+225b^2(c+dx)^5\sinh^{-1}(c+dx)^2\right)}{1125d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[1 + (c + d*x)^2]*(-8 + 4*(c + d*x)^2 - 3*(c + d*x)^4) + 30
*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[1 + (c + d*x)^2] + 4*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2] - 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 2
25*b^2*(c + d*x)^5*ArcSinh[c + d*x]^2))/(1125*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(177) = 354$.
time = 3.93, size = 582, normalized size = 2.95

method	result

default	$\frac{e^4(dx+c)^5a^2}{5d} + \frac{e^4b^2\left(18x^5d^5+240c-540\operatorname{arcsinh}(dx+c)\sqrt{d^2x^2+2cdx+c^2+1}\,x^2c^2d^2+240\operatorname{arcsinh}(dx+c)\sqrt{d^2x^2+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^4*(d*x+c)^5*a^2/d+1/1125*e^4*b^2*(-240*arcsinh(d*x+c)*(d^2*x^2+2*c*d*
x+c^2+1)^(1/2)+225*arcsinh(d*x+c)^2*c^5+18*x^5*d^5+240*c+90*x*c^4*d-40*c^3-
40*d^3*x^3+18*c^5+240*d*x+180*x^2*c^3*d^2-90*arcsinh(d*x+c)*(d^2*x^2+2*c*d*
x+c^2+1)^(1/2)*c^4+120*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2+225
*arcsinh(d*x+c)^2*x^5*d^5-120*x^2*c*d^2-540*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x
+c^2+1)^(1/2)*x^2*c^2*d^2-360*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*
x*c^3*d+240*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d-360*arcsinh(
d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^3*c*d^3-90*arcsinh(d*x+c)*(d^2*x^2+2
*c*d*x+c^2+1)^(1/2)*x^4*d^4+120*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2
)*x^2*d^2+1125*arcsinh(d*x+c)^2*x^4*c*d^4+2250*arcsinh(d*x+c)^2*x^3*c^2*d^3
+2250*arcsinh(d*x+c)^2*x^2*c^3*d^2+1125*arcsinh(d*x+c)^2*x*c^4*d-120*x*c^2*
d+180*x^3*c^2*d^3+90*x^4*c*d^4)/d+2*e^4*a*b/d*(1/5*(d*x+c)^5*arcsinh(d*x+c)
-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/75
*(1+(d*x+c)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*d^4*x^5*e^4 + a^2*c*d^3*x^4*e^4 + 2*a^2*c^2*d^2*x^3*e^4 + 2*a^2*c^3
*d*x^2*e^4 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^3*d*e^4 + 2/3*(6
*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15
*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*
c^2*d^2*e^4 + 1/12*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d
```

```
^2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 +
9*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d
^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*c*d^3*e^4
 + 1/300*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-4
*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^
3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c^
2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
+ 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)/
sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/d
^6)*d)*a*b*d^4*e^4 + a^2*c^4*x*e^4 + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((
d*x + c)^2 + 1))*a*b*c^4*e^4/d + 1/5*(b^2*d^4*x^5*e^4 + 5*b^2*c*d^3*x^4*e^4
 + 10*b^2*c^2*d^2*x^3*e^4 + 10*b^2*c^3*d*x^2*e^4 + 5*b^2*c^4*x*e^4)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2/5*(b^2*d^7*x^7*e^
4 + 7*b^2*c*d^6*x^6*e^4 + (21*b^2*c^2*d^5 + b^2*d^5)*x^5*e^4 + 5*(7*b^2*c^3
*d^4 + b^2*c*d^4)*x^4*e^4 + 5*(7*b^2*c^4*d^3 + 2*b^2*c^2*d^3)*x^3*e^4 + 10*
(2*b^2*c^5*d^2 + b^2*c^3*d^2)*x^2*e^4 + 5*(b^2*c^6*d + b^2*c^4*d)*x*e^4 + (
b^2*d^6*x^6*e^4 + 6*b^2*c*d^5*x^5*e^4 + 15*b^2*c^2*d^4*x^4*e^4 + 20*b^2*c^3
*d^3*x^3*e^4 + 15*b^2*c^4*d^2*x^2*e^4 + 5*b^2*c^5*d*x*e^4)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2430 vs. $2(170) = 340$.
time = 0.38, size = 2430, normalized size = 12.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*((9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 + 10*(9
*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 - 4*b^2
*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^2)*d*x)*cosh(1)
^4 + 4*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 + 10*(9*
(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 - 4*b^2*
c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^2)*d*x)*cosh(1)^
3*sinh(1) + 6*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*x^4 +
 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b^2)*c^3 -
```

```
4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^2)*d*x)*c
osh(1)^2*sinh(1)^2 + 4*(9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^2)*c*
d^4*x^4 + 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*x^3 + 30*(3*(25*a^2 + 2*b
^2)*c^3 - 4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 + 16*b^
2)*d*x)*cosh(1)*sinh(1)^3 + (9*(25*a^2 + 2*b^2)*d^5*x^5 + 45*(25*a^2 + 2*b^
2)*c*d^4*x^4 + 10*(9*(25*a^2 + 2*b^2)*c^2 - 4*b^2)*d^3*x^3 + 30*(3*(25*a^2
+ 2*b^2)*c^3 - 4*b^2*c)*d^2*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 - 8*b^2*c^2 +
16*b^2)*d*x)*sinh(1)^4 + 225*((b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d
^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)^4 + 4*(b^2*d
^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*
c^4*d*x + b^2*c^5)*cosh(1)^3*sinh(1) + 6*(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 1
0*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)^2
*sinh(1)^2 + 4*(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2
*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*cosh(1)*sinh(1)^3 + (b^2*d^5*x^5 +
5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x +
 b^2*c^5)*sinh(1)^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3
0*(15*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*
x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh(1)^4 + 60*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^
4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh
(1)^3*sinh(1) + 90*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10
*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*cosh(1)^2*sinh(1)^2 + 60*(a*b*d
^5*x^5 + 5*a*b*c*d^4*x^4 + 10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*
c^4*d*x + a*b*c^5)*cosh(1)*sinh(1)^3 + 15*(a*b*d^5*x^5 + 5*a*b*c*d^4*x^4 +
10*a*b*c^2*d^3*x^3 + 10*a*b*c^3*d^2*x^2 + 5*a*b*c^4*d*x + a*b*c^5)*sinh(1)^
4 - ((3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 - 2*b^2)*
d^2*x^2 - 4*b^2*c^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*x + 8*b^2)*cosh(1)^4 + 4*(3
*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 - 2*b^2)*d^2*x^2
 - 4*b^2*c^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*x + 8*b^2)*cosh(1)^3*sinh(1) + 6*(
3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 - 2*b^2)*d^2*x^
2 - 4*b^2*c^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*x + 8*b^2)*cosh(1)^2*sinh(1)^2 +
4*(3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 - 2*b^2)*d^2
*x^2 - 4*b^2*c^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*x + 8*b^2)*cosh(1)*sinh(1)^3 +
 (3*b^2*d^4*x^4 + 12*b^2*c*d^3*x^3 + 3*b^2*c^4 + 2*(9*b^2*c^2 - 2*b^2)*d^2*
x^2 - 4*b^2*c^2 + 4*(3*b^2*c^3 - 2*b^2*c)*d*x + 8*b^2)*sinh(1)^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
- 30*((3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 - 2*a*b)
*d^2*x^2 - 4*a*b*c^2 + 4*(3*a*b*c^3 - 2*a*b*c)*d*x + 8*a*b)*cosh(1)^4 + 4*(
3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 - 2*a*b)*d^2*x^
2 - 4*a*b*c^2 + 4*(3*a*b*c^3 - 2*a*b*c)*d*x + 8*a*b)*cosh(1)^3*sinh(1) + 6*
(3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 - 2*a*b)*d^2*x
^2 - 4*a*b*c^2 + 4*(3*a*b*c^3 - 2*a*b*c)*d*x + 8*a*b)*cosh(1)^2*sinh(1)^2 +
 4*(3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 - 2*a*b)*d^
2*x^2 - 4*a*b*c^2 + 4*(3*a*b*c^3 - 2*a*b*c)*d*x + 8*a*b)*cosh(1)*sinh(1)^3
+ (3*a*b*d^4*x^4 + 12*a*b*c*d^3*x^3 + 3*a*b*c^4 + 2*(9*a*b*c^2 - 2*a*b)*d^2
*x^2 - 4*a*b*c^2 + 4*(3*a*b*c^3 - 2*a*b*c)*d*x + 8*a*b)*sinh(1)^4)*sqrt(d^2
```

```
*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(184) = 368$.
time = 0.79, size = 1268, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
sinh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*asinh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*asinh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*asinh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/25 + 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*asinh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/75 + 2*a*b*d**4*e**4*x**5*asinh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 + 1)/(75*d) + b**2*c**5*e**4*asinh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*asinh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*asinh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*asinh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*c**2
*e**4*x/75 + 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)/25 - 8*b**2*c*d*e**4*x**2/75 + 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + b**2*d**4*e**4*x**5*asinh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*b**2*d**2*e**4*x**3/225 + 8*b**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^2, x)
```

3.128 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=172

$$-\frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}+\frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{16d}-\frac{be^3(c+dx)^3\sqrt{1+(}$$

```
[Out] -3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arcsinh(d*
x+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d+3/16*b*e^3*(d*x+c)*(a+
b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*
x+c))*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{8d}+\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{16d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}-\frac{3b^2e^3(c+dx)^2}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (-3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) + (3*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(16*d) - (b*e^3*(c
 + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(8*d) - (3*e^3*(a
 + b*ArcSinh[c + d*x])^2)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x]
)^2)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\sinh}{\sqrt{1+}}\right.}{2d}$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{8d} + \frac{e^3(c+}{}$$

$$= \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{16d}$$

$$= -\frac{3b^2e^3(c+dx)^2}{32d} + \frac{b^2e^3(c+dx)^4}{32d} + \frac{3be^3(c+dx)\sqrt{1+(c+dx}}{16}$$

Mathematica [A]

time = 0.13, size = 170, normalized size = 0.99

$$\frac{e^3\left(-3b^2(c+dx)^2+(8a^2+b^2)(c+dx)^4+2ab(c+dx)(3-2(c+dx)^2)\sqrt{1+(c+dx)^2}-6ab\sinh^{-1}(c+dx)+2b(c+dx)\left(8a(c+dx)^3+3b\sqrt{1+(c+dx)^2}-2b(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+b^2(-3+8(c+dx)^4)\sinh^{-1}(c+dx)^2\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^3*(-3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*(c + d*x)*(3 -
 2*(c + d*x)^2)*Sqrt[1 + (c + d*x)^2] - 6*a*b*ArcSinh[c + d*x] + 2*b*(c + d
*x)*(8*a*(c + d*x)^3 + 3*b*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*x]
^2))/(32*d)
```

Maple [A]
time = 3.54, size = 222, normalized size = 1.29

method	result
default	$\frac{e^3(dx+c)^4a^2}{4d} - \frac{e^3b^2\left(-16(\cosh^2(2\operatorname{arcsinh}(dx+c)))\operatorname{arcsinh}(dx+c)^2+8\sinh(2\operatorname{arcsinh}(dx+c))\cosh(2\operatorname{arcsinh}(dx+c))\operatorname{arcsinh}(dx+c)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^2/d-1/256*e^3*b^2*(-16*cosh(2*arcsinh(d*x+c))^2*arcsinh
(d*x+c)^2+8*sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)+32
*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^2-32*arcsinh(d*x+c)*sinh(2*arcsinh(d
*x+c))-2*cosh(2*arcsinh(d*x+c))^2+8*arcsinh(d*x+c)^2+16*cosh(2*arcsinh(d*x+
c))+1)/d+2*e^3*a*b/d*(1/4*(d*x+c)^4*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/32*arcsinh(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4*e^3 + a^2*c*d^2*x^3*e^3 + 3/2*a^2*c^2*d*x^2*e^3 + 3/2*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arcsinh(d*x + c
```

```
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a*b*c*d^2*e^3 + 1/48*(24
*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
+ 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 - 9
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsinh(
2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2*c^3*x*e^3 + 2*((d
*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c^3*e^3/d + 1/4*(b^2*
d^3*x^4*e^3 + 4*b^2*c*d^2*x^3*e^3 + 6*b^2*c^2*d*x^2*e^3 + 4*b^2*c^3*x*e^3)*
log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(1/2*(b^2*d^6
*x^6*e^3 + 6*b^2*c*d^5*x^5*e^3 + (15*b^2*c^2*d^4 + b^2*d^4)*x^4*e^3 + 4*(5*
b^2*c^3*d^3 + b^2*c*d^3)*x^3*e^3 + 2*(7*b^2*c^4*d^2 + 3*b^2*c^2*d^2)*x^2*e^
3 + 4*(b^2*c^5*d + b^2*c^3*d)*x*e^3 + (b^2*d^5*x^5*e^3 + 5*b^2*c*d^4*x^4*e^
3 + 10*b^2*c^2*d^3*x^3*e^3 + 10*b^2*c^3*d^2*x^2*e^3 + 4*b^2*c^4*d*x*e^3)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x
+ c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(150) = 300$.
time = 0.38, size = 1477, normalized size = 8.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^
2)*c^2 - b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*x)*cosh(1)^3 +
3*((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^2)*c
^2 - b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*x)*cosh(1)^2*sinh(1
) + 3*((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 + b^
2)*c^2 - b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*x)*cosh(1)*sinh
(1)^2 + ((8*a^2 + b^2)*d^4*x^4 + 4*(8*a^2 + b^2)*c*d^3*x^3 + 3*(2*(8*a^2 +
b^2)*c^2 - b^2)*d^2*x^2 + 2*(2*(8*a^2 + b^2)*c^3 - 3*b^2*c)*d*x)*sinh(1)^3
+ ((8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x
+ 8*b^2*c^4 - 3*b^2)*cosh(1)^3 + 3*(8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b
^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8*b^2*c^4 - 3*b^2)*cosh(1)^2*sinh(1) + 3*
(8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 48*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8
```

```
*b^2*c^4 - 3*b^2)*cosh(1)*sinh(1)^2 + (8*b^2*d^4*x^4 + 32*b^2*c*d^3*x^3 + 4
8*b^2*c^2*d^2*x^2 + 32*b^2*c^3*d*x + 8*b^2*c^4 - 3*b^2)*sinh(1)^3)*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*((8*a*b*d^4*x^4 + 32*a*b*c*d
^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*x + 8*a*b*c^4 - 3*a*b)*cosh(1)^3
 + 3*(8*a*b*d^4*x^4 + 32*a*b*c*d^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*
x + 8*a*b*c^4 - 3*a*b)*cosh(1)^2*sinh(1) + 3*(8*a*b*d^4*x^4 + 32*a*b*c*d^3*
x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3*d*x + 8*a*b*c^4 - 3*a*b)*cosh(1)*sinh
(1)^2 + (8*a*b*d^4*x^4 + 32*a*b*c*d^3*x^3 + 48*a*b*c^2*d^2*x^2 + 32*a*b*c^3
*d*x + 8*a*b*c^4 - 3*a*b)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2
*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 - 3*b^2*c + 3*(2*b^2*c^2 - b^2)*
d*x)*cosh(1)^3 + 3*(2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 - 3*b^2*c +
 3*(2*b^2*c^2 - b^2)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^2*d^3*x^3 + 6*b^2*c*d^
2*x^2 + 2*b^2*c^3 - 3*b^2*c + 3*(2*b^2*c^2 - b^2)*d*x)*cosh(1)*sinh(1)^2 +
(2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 2*b^2*c^3 - 3*b^2*c + 3*(2*b^2*c^2 - b^2
)*d*x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*c^
3 - 3*a*b*c + 3*(2*a*b*c^2 - a*b)*d*x)*cosh(1)^3 + 3*(2*a*b*d^3*x^3 + 6*a*b
*c*d^2*x^2 + 2*a*b*c^3 - 3*a*b*c + 3*(2*a*b*c^2 - a*b)*d*x)*cosh(1)^2*sinh(
1) + 3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*c^3 - 3*a*b*c + 3*(2*a*b*c^
2 - a*b)*d*x)*cosh(1)*sinh(1)^2 + (2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 2*a*b*
c^3 - 3*a*b*c + 3*(2*a*b*c^2 - a*b)*d*x)*sinh(1)^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(155) = 310$.
time = 0.58, size = 916, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*asinh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*asinh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 2*a*b*c*d**2*e**3*x**3*asinh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(16*d) + a*b*d**3*e**3*x**4*asinh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 - 3*a*b*e**3*asinh(c + d*x)/
(16*d) + b**2*c**4*e**3*asinh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*asinh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)*asinh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
```

```
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/8 - 3*b**2*c*e**3*x/16 + 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*asinh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 3*b**2*d*e**3*x**2/32 + 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/16 - 3*b**2*e**3*a
sinh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**2, True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^3\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^2, x)
```

3.129 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=136

$$-\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}+\frac{4be^2\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}\,(a+$$

[Out] -4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^2/d+4/9*b*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-2/9*b*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d

Rubi [A]
time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5812, 5798, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{(c+dx)^2+1}\,(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{4be^2\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)}{9d}+\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4}{9}b^2e^2x$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]

[Out] (-4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) + (4*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) - (2*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^2)/(3*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d} - \frac{(2be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\sin}{\sqrt{1+}}\right.}{3d}$$

$$= -\frac{2be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{e^2(c}{}$$

$$= \frac{2b^2e^2(c+dx)^3}{27d} + \frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{9d}$$

$$= -\frac{4}{9}b^2e^2x + \frac{2b^2e^2(c+dx)^3}{27d} + \frac{4be^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-}\right.}{9d}$$

Mathematica [A]
time = 0.11, size = 147, normalized size = 1.08

$$\frac{e^2\left(-12b^2(c+dx)+(9a^2+2b^2)(c+dx)^3+6ab(2-(c+dx)^2)\sqrt{1+(c+dx)^2}+6b\left(3a(c+dx)^3+2b\sqrt{1+(c+dx)^2}-b(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+9b^2(c+dx)^3\sinh^{-1}(c+dx)^2\right)}{27d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*(-12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*(2 - (c + d*x
)^2)*Sqrt[1 + (c + d*x)^2] + 6*b*(3*a*(c + d*x)^3 + 2*b*Sqrt[1 + (c + d*x)^
2] - b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 9*b^2*(c + d*x
)^3*ArcSinh[c + d*x]^2))/(27*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(122) = 244.
time = 3.14, size = 308, normalized size = 2.26

method	result
default	$\frac{e^2(dx+c)^3a^2}{3d}+\frac{e^2b^2\left(9\,\mathrm{arcsinh}(dx+c)^2x^3d^3+27\,\mathrm{arcsinh}(dx+c)^2x^2c\,d^2-6\,\mathrm{arcsinh}(dx+c)\sqrt{d^2x^2+2cdx+c^2+1}\,x^2d^2+27\,\mathrm{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^2/d+1/27*e^2*b^2*(9*arcsinh(d*x+c)^2*x^3*d^3+27*arcsinh
(d*x+c)^2*x^2*c*d^2-6*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+
27*arcsinh(d*x+c)^2*x*c^2*d+2*d^3*x^3-12*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^
2+1)^(1/2)*x*c*d+9*arcsinh(d*x+c)^2*c^3+6*x^2*c*d^2-6*arcsinh(d*x+c)*(d^2*x
^2+2*c*d*x+c^2+1)^(1/2)*c^2+6*x*c^2*d+2*c^3+12*arcsinh(d*x+c)*(d^2*x^2+2*c*
d*x+c^2+1)^(1/2)-12*d*x-12*c)/d+2*a*b*e^2/d*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1
/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3*e^2 + a^2*c*d*x^2*e^2 + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*
arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*
```

```
c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3
))*a*b*c*d*e^2 + 1/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1
)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*(c^2 + 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*x*e^2 + 2*((d*x + c)*arcsinh(d*x +
 c) - sqrt((d*x + c)^2 + 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*x^3*e^2 + 3*b^2*c
*d*x^2*e^2 + 3*b^2*c^2*x*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^2 - integrate(2/3*(b^2*d^5*x^5*e^2 + 5*b^2*c*d^4*x^4*e^2 + (10*b^2*c^2*
d^3 + b^2*d^3)*x^3*e^2 + 3*(3*b^2*c^3*d^2 + b^2*c*d^2)*x^2*e^2 + 3*(b^2*c^4
*d + b^2*c^2*d)*x*e^2 + (b^2*d^4*x^4*e^2 + 4*b^2*c*d^3*x^3*e^2 + 6*b^2*c^2*
d^2*x^2*e^2 + 3*b^2*c^3*d*x*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3
*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(117) = 234$.
time = 0.39, size = 784, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(((9*a^2 + 2*b^2)*d^3*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*a^2 +
2*b^2)*c^2 - 4*b^2)*d*x)*cosh(1)^2 + 9*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(1)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cosh(1)*sinh(1) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*sinh(1)^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))^2 + 2*((9*a^2 + 2*b^2)*d^3*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*
a^2 + 2*b^2)*c^2 - 4*b^2)*d*x)*cosh(1)*sinh(1) + ((9*a^2 + 2*b^2)*d^3*x^3 +
 3*(9*a^2 + 2*b^2)*c*d^2*x^2 + 3*((9*a^2 + 2*b^2)*c^2 - 4*b^2)*d*x)*sinh(1)
^2 + 6*(3*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*cosh(1)
^2 + 6*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*cosh(1)*si
nh(1) + 3*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^3)*sinh(1)
^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 - 2*b^2)*cosh(1)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b^2)*cosh
(1)*sinh(1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b^2)*sinh(1)^2))*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 6*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*((a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - 2*a*b)*cosh(1)^2 + 2*(a*b*
d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - 2*a*b)*cosh(1)*sinh(1) + (a*b*d^2*x^2 + 2
*a*b*c*d*x + a*b*c^2 - 2*a*b)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(126) = 252$.

time = 0.34, size = 610, normalized size = 4.49

$$\begin{cases} a^2c^2e^2x + a^2cde^2x^2 + \frac{a^2d^2e^2x^3}{3} + \frac{2abc^3e^2\operatorname{asinh}(c+dx)}{3d} + 2abc^2e^2x\operatorname{asinh}(c+dx) - \frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + 2abcde^2x^2\operatorname{asinh}(c+dx) - \frac{4abce^2x\sqrt{c^2+2cdx+d^2x^2+1}}{9} + \frac{2abd^2e^2x^3\operatorname{asinh}(c+dx)}{3} - \frac{2abde^2x^2\sqrt{c^2+2cdx+d^2x^2+1}}{9} + \frac{4abe^2\sqrt{c^2+2cdx+d^2x^2+1}}{9d} + \frac{b^2c^3e^2\operatorname{asinh}^2(c+dx)}{3d} + b^2c^2e^2x\operatorname{asinh}^2(c+dx) + \frac{2b^2c^2e^2x}{9} - \frac{2b^2c^2e^2\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{9d} + b^2cde^2x^2\operatorname{asinh}^2(c+dx) + \frac{2b^2cde^2x^2}{9} - \frac{4b^2ce^2x\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{9} + \frac{b^2d^2e^2x^3\operatorname{asinh}^2(c+dx)}{3} + \frac{2b^2d^2e^2x^3}{27} - \frac{2b^2de^2x^2\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{9} - \frac{4b^2e^2x}{9} + \frac{4b^2e^2\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{9d} & \text{for}\ d \neq 0 \\ c^2e^2x(a+b\operatorname{asinh}(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*asinh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*asinh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a*b*c*d*e**2*
x**2*asinh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9
 + 2*a*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/9 + 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
(9*d) + b**2*c**3*e**2*asinh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*asinh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*asinh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/9 + b**2*d**2*e**2*x**3*asinh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asi
nh(c + d*x)/9 - 4*b**2*e**2*x/9 + 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**2,
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^2, x)
```

3.130 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$\frac{b^2e(c+dx)^2}{4d}-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a\right.}{}$$

```
[Out] 1/4*b^2*e*(d*x+c)^2/d+1/4*e*(a+b*arcsinh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*a
rcsinh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)
/d
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{b^2e(c+dx)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x]))/(2*d) + (e*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(c + d*x)
^2*(a + b*ArcSinh[c + d*x])^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}\\ &= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}\\ &= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\right.}{d}\\ &= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{e(c+dx)^2}{}\\ &= \frac{b^2e(c+dx)^2}{4d}-\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 120, normalized size = 1.17

$$\frac{e\left((2a^2+b^2)(c+dx)^2-2ab(c+dx)\sqrt{1+(c+dx)^2}+2ab\sinh^{-1}(c+dx)+2b(c+dx)\left(2a(c+dx)-b\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+b^2(1+2(c+dx)^2)\sinh^{-1}(c+dx)^2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*((2*a^2 + b^2)*(c + d*x)^2 - 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] + 2*a
*b*ArcSinh[c + d*x] + 2*b*(c + d*x)*(2*a*(c + d*x) - b*Sqrt[1 + (c + d*x)^2
])*ArcSinh[c + d*x] + b^2*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^2))/(4*d)
```

Maple [A]
time = 2.36, size = 135, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a^2}{2}+e\,b^2\left(\frac{\operatorname{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\operatorname{arcsinh}(dx+c)(dx+c)\sqrt{1+(dx+c)^2}}{2}-\frac{\operatorname{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\right.}{d}$
default	$\frac{\frac{e(dx+c)^2a^2}{2}+e\,b^2\left(\frac{\operatorname{arcsinh}(dx+c)^2\left(1+(dx+c)^2\right)}{2}-\frac{\operatorname{arcsinh}(dx+c)(dx+c)\sqrt{1+(dx+c)^2}}{2}-\frac{\operatorname{arcsinh}(dx+c)^2}{4}+\frac{(dx+c)^2}{4}+\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^2+e*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcs
inh(d*x+c)*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1
/4)+2*e*a*b*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1
/4*arcsinh(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2*e + 1/2*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a*b*d*e + a^2*c
*x*e + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a*b*c*e/d + 1
/2*(b^2*d*x^2*e + 2*b^2*c*x*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2 - integrate((b^2*d^4*x^4*e + 4*b^2*c*d^3*x^3*e + (5*b^2*c^2*d^2 + b^
2*d^2)*x^2*e + 2*(b^2*c^3*d + b^2*c*d)*x*e + (b^2*d^3*x^3*e + 3*b^2*c*d^2*x
^2*e + 2*b^2*c^2*d*x*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(97) = 194$.
time = 0.43, size = 361, normalized size = 3.50

$$\frac{\left((2b^2d^2x^2+4b^2cdx+2b^2c^2+b^2)\cosh(1)+(2b^2d^2x^2+4b^2cdx+2b^2c^2+b^2)\sinh(1)\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)^2+\left((2a^2+b^2)d^2x^2+2(2a^2+b^2)cdx\right)\cosh(1)+2\left((2abd^2x^2+4abcdx+2abc^2+ab)\cosh(1)+(2abd^2x^2+4abcdx+2abc^2+ab)\sinh(1)-\sqrt{d^2x^2+2cdx+c^2+1}\left((b^2dx+b^2c)\cosh(1)+(b^2dx+b^2c)\sinh(1)\right)\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)+\left((2a^2+b^2)d^2x^2+2(2a^2+b^2)cdx\right)\sinh(1)-2\sqrt{d^2x^2+2cdx+c^2+1}\left((abdx+abc)\cosh(1)+(abdx+abc)\sinh(1)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + b^2)*cosh(1) + (2*b^2*d^2*
x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + b^2)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))^2 + ((2*a^2 + b^2)*d^2*x^2 + 2*(2*a^2 + b^2)*c*d*x)*cos
h(1) + 2*((2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + a*b)*cosh(1) + (2*a*b*
d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + a*b)*sinh(1) - sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*((b^2*d*x + b^2*c)*cosh(1) + (b^2*d*x + b^2*c)*sinh(1)))*log(d*x
+ c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + ((2*a^2 + b^2)*d^2*x^2 + 2*(2*a^
2 + b^2)*c*d*x)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b*d*x + a
*b*c)*cosh(1) + (a*b*d*x + a*b*c)*sinh(1)))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(88) = 176$.
time = 0.20, size = 335, normalized size = 3.25

$$\begin{cases} a^2cex+\frac{a^2dex^2}{2}+\frac{abc^2e\,\mathrm{asinh}(c+dx)}{d}+2abcex\,\mathrm{asinh}(c+dx)-\frac{abce\sqrt{c^2+2cdx+d^2x^2+1}}{2d}+abdex^2\,\mathrm{asinh}(c+dx)-\frac{abex\sqrt{c^2+2cdx+d^2x^2+1}}{2}+\frac{abe\,\mathrm{asinh}(c+dx)}{2d}+\frac{b^2c^2e\,\mathrm{asinh}^2(c+dx)}{2d}+b^2cex\,\mathrm{asinh}^2(c+dx)+\frac{b^2cex}{2}-\frac{b^2ce\sqrt{c^2+2cdx+d^2x^2+1}\,\mathrm{asinh}(c+dx)}{2d}+\frac{b^2dex^2\,\mathrm{asinh}^2(c+dx)}{2}+\frac{b^2dex^2}{4}-\frac{b^2ex\sqrt{c^2+2cdx+d^2x^2+1}\,\mathrm{asinh}(c+dx)}{2}+\frac{b^2e\,\mathrm{asinh}^2(c+dx)}{4d} & \text{for } d\neq 0 \\ cex(a+b\,\mathrm{asinh}(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*asinh(c + d*x)/d + 2*a
*b*c*e*x*asinh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d
) + a*b*d*e*x**2*asinh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/2 + a*b*e*asinh(c + d*x)/(2*d) + b**2*c**2*e*asinh(c + d*x)**2/(2*d) +
b**2*c*e*x*asinh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(2*d) + b**2*d*e*x**2*asinh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/2 + b**2*e*asinh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))
**2, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^2, x)
```

3.131 $\int \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=57

$$2b^2x-\frac{2b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}$$

```
[Out] 2*b^2*x+(d*x+c)*(a+b*arcsinh(d*x+c))^2/d-2*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c
)^2)^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5858, 5772, 5798, 8}

$$-\frac{2b\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + ((c + d*
x)*(a + b*ArcSinh[c + d*x])^2)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)}{\sqrt{1+x^2}}\,dx, x, c+d\right.}{d}$$
$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+c\right.}{d}$$
$$= 2b^2x - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}\right.}{d}$$

Mathematica [A]
time = 0.07, size = 87, normalized size = 1.53

$$\frac{(a^2+2b^2)(c+dx)-2ab\sqrt{1+(c+dx)^2}+2b\left(ac+adx-b\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+b^2(c+dx)\sinh^{-1}(c+dx)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2] + 2*b*(a*c + a*d*x -
 b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + b^2*(c + d*x)*ArcSinh[c + d*x]
^2)/d
```

Maple [A]
time = 2.17, size = 90, normalized size = 1.58

method	result
derivativedivides	$\frac{(dx+c)a^2+b^2\left(\text{arcsinh}(dx+c)^2(dx+c)-2\,\text{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\,\text{arcsinh}(dx+\right.}{d}$
default	$\frac{(dx+c)a^2+b^2\left(\text{arcsinh}(dx+c)^2(dx+c)-2\,\text{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}+2dx+2c\right)+2ab\left((dx+c)\,\text{arcsinh}(dx+\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^2+b^2*(arcsinh(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^
2)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - integrate(2*(d^3*x^
3 + 2*c*d^2*x^2 + (c^2*d + d)*x + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d^2*x^
2 + c*d*x))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c
*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c)
, x))*b^2 + a^2*x + 2*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*
a*b/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(55) = 110.
time = 0.36, size = 141, normalized size = 2.47

$$\frac{(a^2+2\,b^2)dx+(b^2dx+b^2c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2-2\,\sqrt{d^2x^2+2\,cdx+c^2+1}\,ab+2\left(abdx+abc-\sqrt{d^2x^2+2\,cdx+c^2+1}\,b^2\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(51) = 102.
time = 0.11, size = 143, normalized size = 2.51

$$\begin{cases} a^2x+\frac{2abc\operatorname{asinh}(c+dx)}{d}+2abx\operatorname{asinh}(c+dx)-\frac{2ab\sqrt{c^2+2cdx+d^2x^2+1}}{d}+\frac{b^2c\operatorname{asinh}^2(c+dx)}{d}+b^2x\operatorname{asinh}^2(c+dx)+2b^2x-\frac{2b^2\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\operatorname{asinh}(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*asinh(c + d*x)/d + 2*a*b*x*asinh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + b**2*c*asinh(c + d*x)**2/d + b*
*2*x*asinh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**2, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a + b\,\mathrm{asinh}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2,x)
[Out] int((a + b*asinh(c + d*x))^2, x)
```

3.132 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{ce+dex}\,dx$

Optimal. Leaf size=116

$$\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde}+\frac{(a+b\sinh^{-1}(c+dx))^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)}{de}-\frac{b(a+b\sinh^{-1}(c+dx))\,\mathrm{PolyLog}}{de}$$

```
[Out] 1/3*(a+b*arcsinh(d*x+c))^3/b/d/e+(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-b*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)
^2)^(1/2))^2)/d/e-1/2*b^2*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e
```

Rubi [A]
time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5775, 3797, 2221, 2611, 2320, 6724}

$$-\frac{b\,\mathrm{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{de}+\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^2}{de}-\frac{b^2\mathrm{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcSinh[c + d*x])^2*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (b*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^
(-2*ArcSinh[c + d*x])])/(d*e) - (b^2*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(
2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(a+b\sinh^{-1}(c+dx))^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{ex}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{x}\,dx, x, c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \sinh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde} + \frac{(a+b\sinh^{-1}(c+dx))^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}$$
$$= -\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde} + \frac{(a+b\sinh^{-1}(c+dx))^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}$$
$$= -\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde} + \frac{(a+b\sinh^{-1}(c+dx))^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}$$
$$= -\frac{(a+b\sinh^{-1}(c+dx))^3}{3bde} + \frac{(a+b\sinh^{-1}(c+dx))^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}$$

Mathematica [A]
time = 0.03, size = 100, normalized size = 0.86

$$\frac{-2(a+b\sinh^{-1}(c+dx))^2\left(a+b\sinh^{-1}(c+dx)-3b\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)\right)+6b^2(a+b\sinh^{-1}(c+dx))\,\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)-3b^3\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)}{6bde}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2*(a + b*ArcSinh[c + d*x] - 3*b*Log[1 - E^(2*A
rcSinh[c + d*x])]) + 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(2*ArcSinh
[c + d*x])] - 3*b^3*PolyLog[3, E^(2*ArcSinh[c + d*x])])/(6*b*d*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(136) = 272$.
time = 2.52, size = 369, normalized size = 3.18

method	result
derivativedivides	$\frac{\frac{a^2\ln(dx+c)}{e}-\frac{b^2\operatorname{arcsinh}(dx+c)^3}{3e}+\frac{b^2\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{2b^2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-c\right.}{e}}{}$

default	$\frac{a^2\ln(dx+c)}{e}-\frac{b^2\operatorname{arcsinh}(dx+c)^3}{3e}+\frac{b^2\operatorname{arcsinh}(dx+c)^2\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{2b^2\operatorname{arcsinh}(dx+c)\operatorname{polylog}\left(2,-dx-\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/e*ln(d*x+c)-1/3*b^2/e*arcsinh(d*x+c)^3+b^2/e*arcsinh(d*x+c)^2*ln(1
+d*x+c+(1+(d*x+c)^2)^(1/2))+2*b^2/e*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x
+c)^2)^(1/2))-2*b^2/e*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+b^2/e*arcsinh(d
*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*b^2/e*arcsinh(d*x+c)*polylog(2,d*
x+c+(1+(d*x+c)^2)^(1/2))-2*b^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-a*b/e
*arcsinh(d*x+c)^2+2*a*b/e*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*
a*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+2*a*b/e*arcsinh(d*x+c)*ln(1-d*x
-c-(1+(d*x+c)^2)^(1/2))+2*a*b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*e^(-1)*log(d*x*e + c*e)/d + integrate(b^2*log(d*x + c + sqrt((d*x + c)^
2 + 1))^2/(d*x*e + c*e) + 2*a*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*x*e
 + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*e^(-1)/(d*
x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx+\int \frac{b^2\operatorname{asinh}^2(c+dx)}{c+dx}\,dx+\int \frac{2ab\operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*asinh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x), x)
```

3.133 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{(a+b\sinh^{-1}(c+dx))^2}{de^2(c+dx)}-\frac{4b(a+b\sinh^{-1}(c+dx))\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{2b^2\text{PolyLog}\left(2,-e^{\sinh^{-1}(c+dx)}\right)}{de^2}$$

```
[Out] -(a+b*arcsinh(d*x+c))^2/d/e^2/(d*x+c)-4*b*(a+b*arcsinh(d*x+c))*arctanh(d*x+
c+(1+(d*x+c)^2)^(1/2))/d/e^2-2*b^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/
e^2+2*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5816, 4267, 2317, 2438}

$$-\frac{(a+b\sinh^{-1}(c+dx))^2}{de^2(c+dx)}-\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{de^2}-\frac{2b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{2b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^2/(d*e^2*(c + d*x))) - (4*b*(a + b*ArcSinh[c + d
*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (2*b^2*PolyLog[2, -E^ArcSinh[c
+ d*x]])/(d*e^2) + (2*b^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^2x^2}\,dx,x,c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^2} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\sinh^{-1}(c+\right.}{de^2} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2(c+dx)} - \frac{4b\left(a+b\sinh^{-1}(c+dx)\right)\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 154, normalized size = 1.54

$$
\frac{-\frac{a^2}{c+dx}+ab\left(-\frac{2\sinh^{-1}(c+dx)}{c+dx}+2\log\left(\frac{2\sinh^2\left(\frac{1}{2}\sinh^{-1}(c+dx)\right)}{c+dx}\right)\right)+b^2\left(\sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)}{c+dx}+2\log\left(1-e^{-\sinh^{-1}(c+dx)}\right)-2\log\left(1+e^{-\sinh^{-1}(c+dx)}\right)\right)+2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)\right)}{de^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + a*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*Sinh[Ar
cSinh[c + d*x]/2]^2)/(c + d*x)]) + b^2*(ArcSinh[c + d*x]*(-(ArcSinh[c + d*x
]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^(-ArcSinh[c +
 d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2, E^(-ArcSinh[
c + d*x])]))/(d*e^2)
```

Maple [A]
time = 3.37, size = 207, normalized size = 2.07

method	result

derivativedivides	$\frac{-\frac{a^2}{e^2(dx+c)}-\frac{b^2\operatorname{arcsinh}(dx+c)^2}{e^2(dx+c)}-\frac{2b^2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e^2}-\frac{2b^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(d}\right.}{e^2}}{}$
default	$\frac{-\frac{a^2}{e^2(dx+c)}-\frac{b^2\operatorname{arcsinh}(dx+c)^2}{e^2(dx+c)}-\frac{2b^2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e^2}-\frac{2b^2\operatorname{polylog}\left(2,-dx-c-\sqrt{1+(d}\right.}{e^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/e^2/(d*x+c)-b^2/e^2*arcsinh(d*x+c)^2/(d*x+c)-2*b^2/e^2*arcsinh(d*
x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*b^2/e^2*polylog(2,-d*x-c-(1+(d*x+c)^
2)^(1/2))+2*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*b^2/e^
2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2*a*b/e^2*(-1/(d*x+c)*arcsinh(d*x+c)
-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*(arcsinh(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arcsinh(d*x + c)
/(d^2*x*e^2 + c*d*e^2)) - b^2*(log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2/(d^2*x*e^2 + c*d*e^2) - integrate(2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c) + 1)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))/(d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*e^
2 + 2*(2*c^3*d + c*d)*x*e^2 + (c^4 + c^2)*e^2 + (d^3*x^3*e^2 + 3*c*d^2*x^2*
e^2 + (3*c^2*d + d)*x*e^2 + (c^3 + c)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
)), x)) - a^2/(d^2*x*e^2 + c*d*e^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*e^(-2)/(d^
2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^2\operatorname{asinh}^2\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2ab\operatorname{asinh}\left(c+dx\right)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*asinh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*asinh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

3.134 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{(ce+dex)^3}\,dx$

Optimal. Leaf size=85

$$-\frac{b\sqrt{1+(c+dx)^2}\,(a+b\sinh^{-1}(c+dx))}{de^3(c+dx)}-\frac{(a+b\sinh^{-1}(c+dx))^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -1/2*(a+b*arcsinh(d*x+c))^2/d/e^3/(d*x+c)^2+b^2*ln(d*x+c)/d/e^3-b*(a+b*arcs
inh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 12, 5776, 5800, 29}

$$-\frac{b\sqrt{(c+dx)^2+1}\,(a+b\sinh^{-1}(c+dx))}{de^3(c+dx)}-\frac{(a+b\sinh^{-1}(c+dx))^2}{2de^3(c+dx)^2}+\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -((b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(d*e^3*(c + d*x))) - (
a + b*ArcSinh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) + (b^2*Log[c + d*x])/(d*e^3
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
```

```
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^3x^3}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^3}\\
&= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2}{}\\
&= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}+\frac{b^2}{}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 120, normalized size = 1.41

$$-\frac{a\left(a+2b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)+2b\left(a+b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)\sinh^{-1}(c+dx)+b^2\sinh^{-1}(c+dx)^2-2b^2(c+dx)^2\log(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a*(a + 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) + 2*b*(a + b*
(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])*ArcSinh[c + d*x] + b^2*ArcSinh
[c + d*x]^2 - 2*b^2*(c + d*x)^2*Log[c + d*x])/(d*e^3*(c + d*x)^2)
```

Maple [A]
time = 4.23, size = 161, normalized size = 1.89

method	result
derivativedivides	$\frac{-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2 \operatorname{arcsinh}(dx+c)}{e^3}-\frac{b^2 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{e^3(dx+c)}-\frac{b^2 \operatorname{arcsinh}(dx+c)^2}{2e^3(dx+c)^2}+\frac{b^2 \ln\left(\left(dx+c+\sqrt{1+(dx \cdots}{e^3}}{d}$
default	$\frac{-\frac{a^2}{2e^3(dx+c)^2}-\frac{b^2 \operatorname{arcsinh}(dx+c)}{e^3}-\frac{b^2 \operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{e^3(dx+c)}-\frac{b^2 \operatorname{arcsinh}(dx+c)^2}{2e^3(dx+c)^2}+\frac{b^2 \ln\left(\left(dx+c+\sqrt{1+(dx \cdots}{e^3}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^2/e^3/(d*x+c)^2-b^2/e^3*arcsinh(d*x+c)-b^2/e^3*arcsinh(d*x+c)/(
d*x+c)*(1+(d*x+c)^2)^(1/2)-1/2*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+b^2/e^3*l
n((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)+2*a*b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)
-1/2/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(78) = 156$.
time = 0.28, size = 216, normalized size = 2.54

$$-\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d\operatorname{arsinh}\left(dx+c\right)}{d^3xe^3+cd^2e^3}-\frac{e^{(-3)}\log\left(dx+c\right)}{d}\right)b^2-ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2+1}\,d}{d^3xe^3+cd^2e^3}+\frac{\operatorname{arsinh}\left(dx+c\right)}{d^3x^2e^3+2\,cd^2xe^3+c^2de^3}\right)-\frac{b^2\operatorname{arsinh}\left(dx+c\right)^2}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}-\frac{a^2}{2\left(d^3x^2e^3+2\,cd^2xe^3+c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*x*e^3 + c*d^2*e
^3) - e^(-3)*log(d*x + c)/d)*b^2 - a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d
/(d^3*x*e^3 + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 +
c^2*d*e^3)) - 1/2*b^2*arcsinh(d*x + c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2
*d*e^3) - 1/2*a^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(78) = 156$.
time = 0.49, size = 412, normalized size = 4.85

$$-\frac{2\,abc^2d^2x^2+4\,abc^3dx+2\,abc^4+b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+a^2c^2-2\left(abd^2x^2+2\,abcdx-\left(b^2c^2dx+b^2c^3\right)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-2\left(b^2c^2d^2x^2+2\,b^2c^3dx+b^2c^4\right)\log\left(dx+c\right)-2\left(abd^2x^2+2\,abcdx+abc^2\right)\log\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+2\left(abc^2dx+abc^3\right)\sqrt{d^2x^2+2\,cdx+c^2+1}}{2\left(\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^3+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)^2\sinh\left(1\right)+3\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\cosh\left(1\right)\sinh\left(1\right)^2+\left(c^2d^3x^2+2\,c^3d^2x+c^4d\right)\sinh\left(1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 + b^2*c^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + a^2*c^2 - 2*(a*b*d^2*x^2 + 2*a*b*c*
```

```
d*x - (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
 + b^2*c^4)*log(d*x + c) - 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
 - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/((c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^
3 + 3*(c^2*d^3*x^2 + 2*c^3*d^2*x + c^4*d)*cosh(1)^2*sinh(1) + 3*(c^2*d^3*x^
2 + 2*c^3*d^2*x + c^4*d)*cosh(1)*sinh(1)^2 + (c^2*d^3*x^2 + 2*c^3*d^2*x + c
^4*d)*sinh(1)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ab\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

3.135 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{(ce+dex)^4}\,dx$

Optimal. Leaf size=169

$$-\frac{b^2}{3de^4(c+dx)}-\frac{b\sqrt{1+(c+dx)^2}\,(a+b\sinh^{-1}(c+dx))}{3de^4(c+dx)^2}-\frac{(a+b\sinh^{-1}(c+dx))^2}{3de^4(c+dx)^3}+\frac{2b(a+b\sinh^{-1}(c+dx}{3d}$$

```
[Out] -1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+
b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+1/3*b^2*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-1/3*b^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(
1/2))/d/e^4-1/3*b*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5809, 5816, 4267, 2317, 2438, 30}

$$-\frac{b\sqrt{(c+dx)^2+1}\,(a+b\sinh^{-1}(c+dx))}{3de^4(c+dx)^2}-\frac{(a+b\sinh^{-1}(c+dx))^2}{3de^4(c+dx)^3}+\frac{2b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{3de^4}+\frac{b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)}{3de^4}-\frac{b^2}{3de^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*b^2/(d*e^4*(c + d*x)) - (b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*
x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^2/(3*d*e^4*(c + d*x)^
3) + (2*b*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) +
 (b^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(3*d*e^4) - (b^2*PolyLog[2, E^ArcSin
h[c + d*x]])/(3*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} - \frac{b\text{Su}}{}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(\right.}{3de^4(c+}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(\right.}{3de^4(c+}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(\right.}{3de^4(c+}$$

$$= -\frac{b^2}{3de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(\right.}{3de^4(c+}$$

Mathematica [A]
time = 1.13, size = 212, normalized size = 1.25

$$-\frac{4a^2+ab\left(8\sinh^{-1}(c+dx)+2\sinh\left(2\sinh^{-1}(c+dx)\right)+\log\left(\tanh\left(\frac{1}{2}\sinh^{-1}(c+dx)\right)\right)\left(-3(c+dx)+\sinh\left(3\sinh^{-1}(c+dx)\right)\right)\right)+b^2\left(4(c+dx)^2+4\sinh^{-1}(c+dx)^2+4(c+dx)^3\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-4(c+dx)^3\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)+\sinh^{-1}(c+dx)\left(2\sinh\left(2\sinh^{-1}(c+dx)\right)+\left(\log\left(1-e^{-\sinh^{-1}(c+dx)}\right)-\log\left(1+e^{-\sinh^{-1}(c+dx)}\right)\right)\left(-3(c+dx)+\sinh\left(3\sinh^{-1}(c+dx)\right)\right)\right)\right)}{12de^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] -1/12*(4*a^2 + a*b*(8*ArcSinh[c + d*x] + 2*Sinh[2*ArcSinh[c + d*x]] + Log[T
anh[ArcSinh[c + d*x]/2]]*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]])) + b^2*(
4*(c + d*x)^2 + 4*ArcSinh[c + d*x]^2 + 4*(c + d*x)^3*PolyLog[2, -E^(-ArcSin
h[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + ArcSinh[c
+ d*x]*(2*Sinh[2*ArcSinh[c + d*x]] + (Log[1 - E^(-ArcSinh[c + d*x])] - Log[
1 + E^(-ArcSinh[c + d*x])])*(-3*(c + d*x) + Sinh[3*ArcSinh[c + d*x]]))))/(d
*e^4*(c + d*x)^3)
```

Maple [A]
time = 4.79, size = 274, normalized size = 1.62

method	result
derivativedivides	$-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{3e^4(dx+c)^2}-\frac{b^2\operatorname{arcsinh}(dx+c)^2}{3e^4(dx+c)^3}-\frac{b^2}{3e^4(dx+c)}+\frac{b^2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1}\right.}{3e^4}$
default	$-\frac{a^2}{3e^4(dx+c)^3}-\frac{b^2\operatorname{arcsinh}(dx+c)\sqrt{1+(dx+c)^2}}{3e^4(dx+c)^2}-\frac{b^2\operatorname{arcsinh}(dx+c)^2}{3e^4(dx+c)^3}-\frac{b^2}{3e^4(dx+c)}+\frac{b^2\operatorname{arcsinh}(dx+c)\ln\left(1+dx+c+\sqrt{1}\right.}{3e^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/e^4/(d*x+c)^3-1/3*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1+(d*x+c)
^2)^(1/2)-1/3*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-1/3*b^2/e^4/(d*x+c)+1/3*b^
2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+1/3*b^2/e^4*polylog(2,
-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*b^2/e^4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c
)^2)^(1/2))-1/3*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2*a*b/e^4*(-1/
3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/
(1+(d*x+c)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^4*x^3*e^4 +
3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^2/(d^4*x^3*e^4 + 3*c
*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 + b
^2*d^3)*x^3 + 3*(c^3 + c)*a*b + (c^3 + c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^2)
*x^2 + (3*(3*c^2*d + d)*a*b + (3*c^2*d + d)*b^2)*x + (b^2*c^2 + 3*(c^2 + 1)
*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*c*d + b^2*c*d)*x)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^7
*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4 + 5*(7*c^3*d^4 + c*
```

```
d^4)*x^4*e^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 + 10*c^3*d^2
)*x^2*e^4 + (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4 + (d^6*x^6*e^4 + 6*
c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d^3 + c*d^3)*x^3*e^4
+ 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^3*d)*x*e^4 + (c^6 +
c^4)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*e^(-4)/(d^
4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**4,x)
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*asinh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4,x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

3.136 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=87

$$\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(1+m)}-\frac{3b\text{Int}\left(\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de(1+m)} - \frac{(3b)\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{\cdots}}\right.}{d\cdots}$$

Mathematica [A]
time = 3.12, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dex + ce\right)^{m} \left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
[Out] (d*x*e + c*e)^(m + 1)*a^3*e^(-1)/(d*(m + 1)) + (b^3*d*x*e^m + b^3*c*e^m)*(d
*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d*(m + 1)) +
integrate(-3*(((b^3*c^2*e^m - ((m*e^m + e^m)*c^2 + m*e^m + e^m)*a*b^2 - ((m
*e^m + e^m)*a*b^2*d^2 - b^3*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b^2*c*d - b^3
*c*d*e^m)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - (((m*e^m + e^m
)*c^3 + (m*e^m + e^m)*c)*a*b^2 - (c^3*e^m + c*e^m)*b^3 + ((m*e^m + e^m)*a*b
^2*d^3 - b^3*d^3*e^m)*x^3 + 3*((m*e^m + e^m)*a*b^2*c*d^2 - b^3*c*d^2*e^m)*x
^2 + ((3*(m*e^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a*b^2 - (3*c^2*d*e^m + d*e^
m)*b^3)*x)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
- (((m*e^m + e^m)*a^2*b*d^2*x^2 + 2*(m*e^m + e^m)*a^2*b*c*d*x + ((m*e^m + e
^m)*c^2 + m*e^m + e^m)*a^2*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m
 + ((m*e^m + e^m)*a^2*b*d^3*x^3 + 3*(m*e^m + e^m)*a^2*b*c*d^2*x^2 + (3*(m*e
^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a^2*b*x + ((m*e^m + e^m)*c^3 + (m*e^m +
e^m)*c)*a^2*b)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + c*(m + 1) + (3*c^
2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m +
1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*((d*x + c)*e)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b\operatorname{asinh}(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^3, x)
```

3.137 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=326

$$\frac{16}{25}ab^2e^4x-\frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d}+\frac{76b^3e^4(1+(c+dx)^2)^{3/2}}{1125d}-\frac{6b^3e^4(1+(c+dx)^2)^{5/2}}{625d}+\frac{16b^3e^4(c+dx)\sinh}{25d}$$

```
[Out] 16/25*a*b^2*e^4*x+76/1125*b^3*e^4*(1+(d*x+c)^2)^(3/2)/d-6/625*b^3*e^4*(1+(d
*x+c)^2)^(5/2)/d+16/25*b^3*e^4*(d*x+c)*arcsinh(d*x+c)/d-8/75*b^2*e^4*(d*x+c
)^3*(a+b*arcsinh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))/d+1
/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^3/d-298/375*b^3*e^4*(1+(d*x+c)^2)^(1/
2)/d-8/25*b*e^4*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d+4/25*b*e^4*(d*
x+c)^2*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/25*b*e^4*(d*x+c)^4*(a
+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 267, 272, 45}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d}-\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^3}{5d}-\frac{3be^4\sqrt{(c+dx)^2+1}(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d}+\frac{4be^4\sqrt{(c+dx)^2+1}(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d}-\frac{8be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d}-\frac{6b^3e^4((c+dx)^2+1)^{5/2}}{625d}+\frac{76b^3e^4((c+dx)^2+1)^{3/2}}{1125d}-\frac{298b^3e^4\sqrt{(c+dx)^2+1}}{375d}+\frac{16b^3e^4(c+dx)\sinh^{-1}(c+dx)}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (16*a*b^2*e^4*x)/25 - (298*b^3*e^4*Sqrt[1 + (c + d*x)^2])/(375*d) + (76*b^3
*e^4*(1 + (c + d*x)^2)^(3/2))/(1125*d) - (6*b^3*e^4*(1 + (c + d*x)^2)^(5/2)
)/(625*d) + (16*b^3*e^4*(c + d*x)*ArcSinh[c + d*x])/(25*d) - (8*b^2*e^4*(c
+ d*x)^3*(a + b*ArcSinh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*A
rcSinh[c + d*x]))/(125*d) - (8*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c
 + d*x])^2)/(25*d) + (4*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcS
inh[c + d*x])^2)/(25*d) - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b
*ArcSinh[c + d*x])^2)/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^3)
/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^3}{5d} - \frac{(3be^4)\,\text{Subst}\left(\int \frac{x^5(a+b\sinh}{\sqrt{1+}}\right.}{5d} \\
&= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{25d} + \frac{e^4(c}{} \\
&= \frac{6b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)}{125d} + \frac{4be^4(c+dx)^2\sqrt{1+(c}}{} \\
&= -\frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+dx)^5\left(a+b\right.}{125d} \\
&= \frac{16}{25}ab^2e^4x - \frac{8b^2e^4(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{75d} + \frac{6b^2e^4(c+d}{} \\
&= \frac{16}{25}ab^2e^4x - \frac{6b^3e^4\sqrt{1+(c+dx)^2}}{125d} + \frac{4b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{125d} - \\
&= \frac{16}{25}ab^2e^4x - \frac{298b^3e^4\sqrt{1+(c+dx)^2}}{375d} + \frac{76b^3e^4\left(1+(c+dx)^2\right)^{3/2}}{1125d}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 355, normalized size = 1.09

$$\frac{e^4\left(240ab^2(c+dx)-40ab^2(c+dx)^3+3a(25a^2+6b^2)(c+dx)^5+\frac{1}{15}b\sqrt{1+(c+dx)^2}\left(-8(225a^2+518b^2)+4(225a^2+68b^2)(c+dx)^2-27(25a^2+2b^2)(c+dx)^4\right)-b\left(-240b^2(c+dx)+40b^2(c+dx)^3-225a^2(c+dx)^5-18b^2(c+dx)^5+240ab\sqrt{1+(c+dx)^2}-120ab(c+dx)^2\sqrt{1+(c+dx)^2}+90ab(c+dx)^4\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)-15b^2\left(-15a(c+dx)^5+8b\sqrt{1+(c+dx)^2}-4b(c+dx)^2\sqrt{1+(c+dx)^2}+3b(c+dx)^4\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2+75b^3(c+dx)^5\sinh^{-1}(c+dx)^3\right)}{375d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) - 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[1 + (c + d*x)^2]*(-8*(225*a^2 + 518*b^2) + 4*(225*a^2 +
68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 - b*(-240*b^2*(c
 + d*x) + 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*(c + d*x)^5 + 2
40*a*b*Sqrt[1 + (c + d*x)^2] - 120*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] +
90*a*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 15*b^2*(-15*a*
(c + d*x)^5 + 8*b*Sqrt[1 + (c + d*x)^2] - 4*b*(c + d*x)^2*Sqrt[1 + (c + d*x
)^2] + 3*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 75*b^3*(
c + d*x)^5*ArcSinh[c + d*x]^3))/(375*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1388 vs. $2(292) = 584$.
time = 4.43, size = 1389, normalized size = 4.26

method	result	size
default	Expression too large to display	1389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*e^4*(d*x+c)^5*a^3/d+1/5625*e^4*b^3*(-1800*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)
*arcsinh(d*x+c)^2+1125*arcsinh(d*x+c)^3*c^5+270*arcsinh(d*x+c)*c^5-600*arcs
inh(d*x+c)*c^3+3600*arcsinh(d*x+c)*c-54*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^4+2
72*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2-4144*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)-675
*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^4-675*arcsinh(d*x+c)^2*(d
^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^4*d^4+900*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c
^2+1)^(1/2)*x^2*d^2+5625*arcsinh(d*x+c)^3*x^4*c*d^4+11250*arcsinh(d*x+c)^3*
x^3*c^2*d^3+11250*arcsinh(d*x+c)^3*x^2*c^3*d^2+1350*arcsinh(d*x+c)*x^4*c*d^
4+5625*arcsinh(d*x+c)^3*x*c^4*d+2700*arcsinh(d*x+c)*x^3*c^2*d^3+2700*arcsin
h(d*x+c)*x^2*c^3*d^2+1350*arcsinh(d*x+c)*x*c^4*d-1800*arcsinh(d*x+c)*x^2*c*
d^2-1800*arcsinh(d*x+c)*x*c^2*d+900*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1
)^(1/2)*c^2+1125*arcsinh(d*x+c)^3*x^5*d^5+270*arcsinh(d*x+c)*x^5*d^5-600*ar
csinh(d*x+c)*x^3*d^3+3600*arcsinh(d*x+c)*x*d-54*(d^2*x^2+2*c*d*x+c^2+1)^(1/
2)*x^4*d^4+272*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2-2700*arcsinh(d*x+c)^2*
(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c^3*d+1800*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*
x+c^2+1)^(1/2)*x*c*d-2700*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^
3*c*d^3-4050*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*c^2*d^2-216
*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c^3*d+544*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*
c*d-216*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^3*c*d^3-324*(d^2*x^2+2*c*d*x+c^2+1)
^(1/2)*x^2*c^2*d^2)/d+1/375*e^4*a*b^2*(-240*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x
+c^2+1)^(1/2)+225*arcsinh(d*x+c)^2*c^5+18*x^5*d^5+240*c+90*x*c^4*d-40*c^3-4
0*d^3*x^3+18*c^5+240*d*x+180*x^2*c^3*d^2-90*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x
+c^2+1)^(1/2)*c^4+120*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2+225*
arcsinh(d*x+c)^2*x^5*d^5-120*x^2*c*d^2-540*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+
c^2+1)^(1/2)*x^2*c^2*d^2-360*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x
```

```
*c^3*d+240*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d-360*arcsinh(d
*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^3*c*d^3-90*arcsinh(d*x+c)*(d^2*x^2+2*
c*d*x+c^2+1)^(1/2)*x^4*d^4+120*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)
*x^2*d^2+1125*arcsinh(d*x+c)^2*x^4*c*d^4+2250*arcsinh(d*x+c)^2*x^3*c^2*d^3+
2250*arcsinh(d*x+c)^2*x^2*c^3*d^2+1125*arcsinh(d*x+c)^2*x*c^4*d-120*x*c^2*d
+180*x^3*c^2*d^3+90*x^4*c*d^4)/d+3*e^4*a^2*b/d*(1/5*(d*x+c)^5*arcsinh(d*x+c
)-1/25*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+4/75*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-8/7
5*(1+(d*x+c)^2)^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*d^4*x^5*e^4 + a^3*c*d^3*x^4*e^4 + 2*a^3*c^2*d^2*x^3*e^4 + 2*a^3*c^3
*d*x^2*e^4 + 3*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/s
qrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*
x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2
))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^3*d*e^4 + (6*x
^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c
^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*
d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*
c^2*d^2*e^4 + 1/8*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*a
rcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x +
c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9
*(c^2 + 1)^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^
5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*c*d^3*e^
4 + 1/200*(120*x^5*arcsinh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^3/d^3 + 126*sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)*c^2*x^2/d^4 - 945*c^5*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x^2/d^4 + 1050*(c
^2 + 1)*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6
 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*(c^2 + 1)*c*x/d^5 - 225*(c^2 + 1)^2*c*arcsinh(2*(d^2*x + c*d)
/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*(c^2 + 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)^2/
```

```
d^6)*d)*a^2*b*d^4*e^4 + a^3*c^4*x*e^4 + 3*((d*x + c)*arcsinh(d*x + c) - sqr
t((d*x + c)^2 + 1))*a^2*b*c^4*e^4/d + 1/5*(b^3*d^4*x^5*e^4 + 5*b^3*c*d^3*x^
4*e^4 + 10*b^3*c^2*d^2*x^3*e^4 + 10*b^3*c^3*d*x^2*e^4 + 5*b^3*c^4*x*e^4)*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/5*((5*a*b^2*
d^7 - b^3*d^7)*x^7*e^4 + 7*(5*a*b^2*c*d^6 - b^3*c*d^6)*x^6*e^4 + (5*(21*c^2
*d^5 + d^5)*a*b^2 - (21*c^2*d^5 + d^5)*b^3)*x^5*e^4 + 5*(5*(7*c^3*d^4 + c*d
^4)*a*b^2 - (7*c^3*d^4 + c*d^4)*b^3)*x^4*e^4 + 5*(c^7 + c^5)*a*b^2*e^4 + 5*
(5*(7*c^4*d^3 + 2*c^2*d^3)*a*b^2 - (7*c^4*d^3 + 2*c^2*d^3)*b^3)*x^3*e^4 + 5
*((21*c^5*d^2 + 10*c^3*d^2)*a*b^2 - 2*(2*c^5*d^2 + c^3*d^2)*b^3)*x^2*e^4 +
5*((7*c^6*d + 5*c^4*d)*a*b^2 - (c^6*d + c^4*d)*b^3)*x*e^4 + ((5*a*b^2*d^6 -
 b^3*d^6)*x^6*e^4 + 6*(5*a*b^2*c*d^5 - b^3*c*d^5)*x^5*e^4 - 5*(3*b^3*c^2*d^
4 - (15*c^2*d^4 + d^4)*a*b^2)*x^4*e^4 + 5*(c^6 + c^4)*a*b^2*e^4 - 20*(b^3*c
^3*d^3 - (5*c^3*d^3 + c*d^3)*a*b^2)*x^3*e^4 - 15*(b^3*c^4*d^2 - (5*c^4*d^2
+ 2*c^2*d^2)*a*b^2)*x^2*e^4 - 5*(b^3*c^5*d - 2*(3*c^5*d + 2*c^3*d)*a*b^2)*x
*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2
+ 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4411 vs. $2(281) = 562$.
time = 0.58, size = 4411, normalized size = 13.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(15*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4
- 10*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 - 30*(4*a*b^2*c - (25*a^3
 + 6*a*b^2)*c^3)*d^2*x^2 - 15*(8*a*b^2*c^2 - (25*a^3 + 6*a*b^2)*c^4 - 16*a*
b^2)*d*x)*cosh(1)^4 + 60*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b
^2)*c*d^4*x^4 - 10*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 - 30*(4*a*b
^2*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 - 15*(8*a*b^2*c^2 - (25*a^3 + 6*a*b^
2)*c^4 - 16*a*b^2)*d*x)*cosh(1)^3*sinh(1) + 90*(3*(25*a^3 + 6*a*b^2)*d^5*x^
5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4 - 10*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^
2)*d^3*x^3 - 30*(4*a*b^2*c - (25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 - 15*(8*a*b^2*
c^2 - (25*a^3 + 6*a*b^2)*c^4 - 16*a*b^2)*d*x)*cosh(1)^2*sinh(1)^2 + 60*(3*(
25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d^4*x^4 - 10*(4*a*b^2 -
 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 - 30*(4*a*b^2*c - (25*a^3 + 6*a*b^2)*c^3
)*d^2*x^2 - 15*(8*a*b^2*c^2 - (25*a^3 + 6*a*b^2)*c^4 - 16*a*b^2)*d*x)*cosh(
1)*sinh(1)^3 + 15*(3*(25*a^3 + 6*a*b^2)*d^5*x^5 + 15*(25*a^3 + 6*a*b^2)*c*d
^4*x^4 - 10*(4*a*b^2 - 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*x^3 - 30*(4*a*b^2*c -
(25*a^3 + 6*a*b^2)*c^3)*d^2*x^2 - 15*(8*a*b^2*c^2 - (25*a^3 + 6*a*b^2)*c^4
- 16*a*b^2)*d*x)*sinh(1)^4 + 1125*((b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*
c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh(1)^4 + 4*(
```

b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5
*b^3*c^4*d*x + b^3*c^5)*cosh(1)^3*sinh(1) + 6*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^
4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh
(1)^2*sinh(1)^2 + 4*(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 1
0*b^3*c^3*d^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)*cosh(1)*sinh(1)^3 + (b^3*d^5*x
^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*c^4*
d*x + b^3*c^5)*sinh(1)^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^
3 + 225*(15*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*
a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*cosh(1)^4 + 60*(a*b^2*d^5*
x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a
*b^2*c^4*d*x + a*b^2*c^5)*cosh(1)^3*sinh(1) + 90*(a*b^2*d^5*x^5 + 5*a*b^2*c
*d^4*x^4 + 10*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x +
a*b^2*c^5)*cosh(1)^2*sinh(1)^2 + 60*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10
*a*b^2*c^2*d^3*x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*co
sh(1)*sinh(1)^3 + 15*(a*b^2*d^5*x^5 + 5*a*b^2*c*d^4*x^4 + 10*a*b^2*c^2*d^3*
x^3 + 10*a*b^2*c^3*d^2*x^2 + 5*a*b^2*c^4*d*x + a*b^2*c^5)*sinh(1)^4 - ((3*b
^3*d^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 - 4*b^3*c^2 + 2*(9*b^3*c^2 - 2*b^
3)*d^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 - 2*b^3*c)*d*x)*cosh(1)^4 + 4*(3*b^3*d^4*
x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 - 4*b^3*c^2 + 2*(9*b^3*c^2 - 2*b^3)*d^2*
x^2 + 8*b^3 + 4*(3*b^3*c^3 - 2*b^3*c)*d*x)*cosh(1)^3*sinh(1) + 6*(3*b^3*d^4
*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 - 4*b^3*c^2 + 2*(9*b^3*c^2 - 2*b^3)*d^2
*x^2 + 8*b^3 + 4*(3*b^3*c^3 - 2*b^3*c)*d*x)*cosh(1)^2*sinh(1)^2 + 4*(3*b^3*
d^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 - 4*b^3*c^2 + 2*(9*b^3*c^2 - 2*b^3)*
d^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 - 2*b^3*c)*d*x)*cosh(1)*sinh(1)^3 + (3*b^3*d
^4*x^4 + 12*b^3*c*d^3*x^3 + 3*b^3*c^4 - 4*b^3*c^2 + 2*(9*b^3*c^2 - 2*b^3)*d
^2*x^2 + 8*b^3 + 4*(3*b^3*c^3 - 2*b^3*c)*d*x)*sinh(1)^4)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 15*((
9*(25*a^2*b + 2*b^3)*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 - 10*(4*b^3
- 9*(25*a^2*b + 2*b^3)*c^2)*d^3*x^3 - 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5
 - 30*(4*b^3*c - 3*(25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c - 15*(8*b^3*
c^2 - 3*(25*a^2*b + 2*b^3)*c^4 - 16*b^3)*d*x)*cosh(1)^4 + 4*(9*(25*a^2*b +
2*b^3)*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 - 10*(4*b^3 - 9*(25*a^2*b
+ 2*b^3)*c^2)*d^3*x^3 - 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 - 30*(4*b^3*c
 - 3*(25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c - 15*(8*b^3*c^2 - 3*(25*a^
2*b + 2*b^3)*c^4 - 16*b^3)*d*x)*cosh(1)^3*sinh(1) + 6*(9*(25*a^2*b + 2*b^3)
*d^5*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^
3)*c^2)*d^3*x^3 - 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 - 30*(4*b^3*c - 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c - 15*(8*b^3*c^2 - 3*(25*a^2*b +
2*b^3)*c^4 - 16*b^3)*d*x)*cosh(1)^2*sinh(1)^2 + 4*(9*(25*a^2*b + 2*b^3)*d^5
x^5 + 45(25*a^2*b + 2*b^3)*c*d^4*x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c
^2)*d^3*x^3 - 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 - 30*(4*b^3*c - 3*(25*a
^2*b + 2*b^3)*c^3)*d^2*x^2 + 240*b^3*c - 15*(8*b^3*c^2 - 3*(25*a^2*b + 2*b^
3)*c^4 - 16*b^3)*d*x)*cosh(1)*sinh(1)^3 + (9*(25*a^2*b + 2*b^3)*d^5*x^5 + 4
5*(25*a^2*b + 2*b^3)*c*d^4*x^4 - 10*(4*b^3 - 9*(25*a^2*b + 2*b^3)*c^2)*d^3*
x^3 - 40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 - 30*(4*b^3*c - 3*(25*a^2*b + 2

```
*b^3)*c^3)*d^2*x^2 + 240*b^3*c - 15*(8*b^3*c^2 - 3*(25*a^2*b + 2*b^3)*c^4 -
 16*b^3)*d*x)*sinh(1)^4 - 30*((3*a*b^2*d^4*x^4 + 12*a*b^2*c*d^3*x^3 + 3*a*b
^2*c^4 - 4*a*b^2*c^2 + 2*(9*a*b^2*c^2 - 2*a*b^2)*d^2*x^2 + 8*a*b^2 + 4*(3*a
*b^2*c^3 - 2*a*b^2*c)*d*x)*cosh(1)^4 + 4*(3*a*b...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(306) = 612$.
time = 1.33, size = 2518, normalized size = 7.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*asinh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*asinh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*asinh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*asinh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*asinh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25
+ 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/25 + 3*a**2*b*d**4
*e**4*x**5*asinh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/25 + 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(25*d) + 3*a*b*
*2*c**5*e**4*asinh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*asinh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)*asinh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*asinh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*asi
nh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c**2*
e**4*x/25 + 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*asinh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*c*d*e**4*x**2/25 + 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*asinh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/25 - 8*a*b**2*d*
*2*e**4*x**3/75 + 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/(25*d) + b**3*c**5*e**4*asinh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*asinh(c + d*x)/(125*d) + b**3*c**4*e**4*x*asinh(
```

```
c + d*x)**3 + 6*b**3*c**4*e**4*x*asinh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*asi
nh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*asinh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**3*e**4*asi
nh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*asinh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*asinh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/625 - 8*b**3*c**2*e**4*x*asinh(c + d*
x)/25 + 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x
)**2/(25*d) + 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*asinh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*asi
nh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
 1)*asinh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*c*d*e**4*x**2*asinh(c + d*x)/25 + 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*c*e**4*asinh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*asinh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*asinh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)*asinh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
**2*x**2 + 1)/625 - 8*b**3*d**2*e**4*x**3*asinh(c + d*x)/75 + 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/25 + 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/5625 + 16*b**3*e**4*x*asin
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*asinh(c))**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^3, x)
```

3.138 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=279

$$\frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d}-\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d}-\frac{45b^3e^3\sinh^{-1}(c+dx)}{256d}-\frac{9b^2e^3(c+dx)^2\,(}{}$$

```
[Out] -45/256*b^3*e^3*arcsinh(d*x+c)/d-9/32*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c)
)/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))/d-3/32*e^3*(a+b*arcsinh(d*x
+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^3/d+45/256*b^3*e^3*(d*x+c)*
(1+(d*x+c)^2)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/d+9/32*b*
e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-3/16*b*e^3*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5776, 5812, 5783, 327, 221}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d}-\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d}+\frac{9be^3\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{32d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^3}{32d}-\frac{3b^3e^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{128d}+\frac{45b^3e^3\sqrt{(c+dx)^2+1}\,(c+dx)}{256d}-\frac{45b^3e^3\sinh^{-1}(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (45*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(256*d) - (3*b^3*e^3*(c + d*x)
^3*Sqrt[1 + (c + d*x)^2])/(128*d) - (45*b^3*e^3*ArcSinh[c + d*x])/(256*d) -
 (9*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(32*d) + (3*b^2*e^3*(c +
d*x)^4*(a + b*ArcSinh[c + d*x]))/(32*d) + (9*b*e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(32*d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 +
(c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^3)/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^3)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\sin}{\sqrt{1+}}\right.}{4d}$$

$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{e^3(c}{}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)}{32d} + \frac{9be^3(c+dx)\sqrt{1+(c}}{}$$

$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{128d} - \frac{9b^2e^3(c+dx)^2\left(a+b\sinh\right.}{32d}$$

$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx}}{128d}$$

$$= \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}}{256d} - \frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx}}{128d}$$

Mathematica [A]
time = 0.24, size = 303, normalized size = 1.09

$$\frac{e^3\left(-72ab^2(c+dx)^2+8a(8a^2+3b^2)(c+dx)^4+3b(c+dx)\sqrt{1+(c+dx)^2}\left(3(8a^2+5b^2)-2(8a^2+b^2)(c+dx)^2\right)-9b(8a^2+5b^2)\sinh^{-1}(c+dx)-24b(c+dx)\left(3b^2(c+dx)-8a^2(c+dx)^3-b^2(c+dx)^3-6ab\sqrt{1+(c+dx)^2}+4ab(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+24b^2\left(-3a+8a(c+dx)^4+3b(c+dx)\sqrt{1+(c+dx)^2}-2b(c+dx)^3\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2+8b^3(-3+8(c+dx)^4)\sinh^{-1}(c+dx)^3\right)}{256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^3*(-72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*(c + d*
x)*Sqrt[1 + (c + d*x)^2]*(3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2)*(c + d*x)^2)
- 9*b*(8*a^2 + 5*b^2)*ArcSinh[c + d*x] - 24*b*(c + d*x)*(3*b^2*(c + d*x) -
8*a^2*(c + d*x)^3 - b^2*(c + d*x)^3 - 6*a*b*Sqrt[1 + (c + d*x)^2] + 4*a*b*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 24*b^2*(-3*a + 8*a*(c
+ d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sqrt[1 + (
c + d*x)^2])*ArcSinh[c + d*x]^2 + 8*b^3*(-3 + 8*(c + d*x)^4)*ArcSinh[c + d*
x]^3))/(256*d)
```

Maple [A]
time = 3.96, size = 409, normalized size = 1.47

method	result
default	$\frac{e^3(dx+c)^4a^3}{4d} - \frac{b^3e^3\left(-32(\cosh^2(2\operatorname{arcsinh}(dx+c)))\operatorname{arcsinh}(dx+c)^3+24\sinh(2\operatorname{arcsinh}(dx+c))\cosh(2\operatorname{arcsinh}(dx+c))\operatorname{arcsinh}(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^3/d-1/512*b^3*e^3*(-32*cosh(2*arcsinh(d*x+c))^2*arcsinh
(d*x+c)^3+24*sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^2
+64*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^3-96*arcsinh(d*x+c)^2*sinh(2*arcs
inh(d*x+c))-12*arcsinh(d*x+c)*cosh(2*arcsinh(d*x+c))^2+16*arcsinh(d*x+c)^3+
3*sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))+96*cosh(2*arcsinh(d*x+c))*a
rcsinh(d*x+c)-48*sinh(2*arcsinh(d*x+c))+6*arcsinh(d*x+c))/d-3/256*a*b^2*e^3
*(-16*cosh(2*arcsinh(d*x+c))^2*arcsinh(d*x+c)^2+8*sinh(2*arcsinh(d*x+c))*co
sh(2*arcsinh(d*x+c))*arcsinh(d*x+c)+32*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c
)^2-32*arcsinh(d*x+c)*sinh(2*arcsinh(d*x+c))-2*cosh(2*arcsinh(d*x+c))^2+8*a
rcsinh(d*x+c)^2+16*cosh(2*arcsinh(d*x+c))+1)/d+3*a^2*b*e^3/d*(1/4*(d*x+c)^4
*arcsinh(d*x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^
2)^(1/2)-3/32*arcsinh(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*x^4*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + 9/4*(2*x^
2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(
c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*ar
csinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arcsinh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^
2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d
^2 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 -
 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*c*d^2*e^3 + 1/32
*(24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 -
14*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*
```

```
d^2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5
 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^2*b*d^3*e^3 + a^3*c^3*x*e^3 +
 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^3*e^3/d + 1
/4*(b^3*d^3*x^4*e^3 + 4*b^3*c*d^2*x^3*e^3 + 6*b^3*c^2*d*x^2*e^3 + 4*b^3*c^3
*x*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + integrate(3/4*
((4*a*b^2*d^6 - b^3*d^6)*x^6*e^3 + 6*(4*a*b^2*c*d^5 - b^3*c*d^5)*x^5*e^3 +
(4*(15*c^2*d^4 + d^4)*a*b^2 - (15*c^2*d^4 + d^4)*b^3)*x^4*e^3 + 4*(c^6 + c^
4)*a*b^2*e^3 + 4*(4*(5*c^3*d^3 + c*d^3)*a*b^2 - (5*c^3*d^3 + c*d^3)*b^3)*x^
3*e^3 + 2*(6*(5*c^4*d^2 + 2*c^2*d^2)*a*b^2 - (7*c^4*d^2 + 3*c^2*d^2)*b^3)*x
^2*e^3 + 4*(2*(3*c^5*d + 2*c^3*d)*a*b^2 - (c^5*d + c^3*d)*b^3)*x*e^3 + ((4*
a*b^2*d^5 - b^3*d^5)*x^5*e^3 + 5*(4*a*b^2*c*d^4 - b^3*c*d^4)*x^4*e^3 + 4*(c
^5 + c^3)*a*b^2*e^3 - 2*(5*b^3*c^2*d^3 - 2*(10*c^2*d^3 + d^3)*a*b^2)*x^3*e^
3 - 2*(5*b^3*c^3*d^2 - 2*(10*c^3*d^2 + 3*c*d^2)*a*b^2)*x^2*e^3 - 4*(b^3*c^4
*d - (5*c^4*d + 3*c^2*d)*a*b^2)*x*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(244) = 488$.
time = 0.45, size = 2632, normalized size = 9.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*((8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 - 3*(3*
a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 - 2*(9*a*b^2*c - 2*(8*a^3 + 3*a*b^
2)*c^3)*d*x)*cosh(1)^3 + 8*((8*b^3*d^4*x^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*
d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^3)*cosh(1)^3 + 3*(8*b^3*d^4*x^4
+ 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^
3)*cosh(1)^2*sinh(1) + 3*(8*b^3*d^4*x^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2
*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3*b^3)*cosh(1)*sinh(1)^2 + (8*b^3*d^4*x
^4 + 32*b^3*c*d^3*x^3 + 48*b^3*c^2*d^2*x^2 + 32*b^3*c^3*d*x + 8*b^3*c^4 - 3
*b^3)*sinh(1)^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 24*((
8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 - 3*(3*a*b^2 - 2*(
8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 - 2*(9*a*b^2*c - 2*(8*a^3 + 3*a*b^2)*c^3)*d*x
)*cosh(1)^2*sinh(1) + 24*((8*a^3 + 3*a*b^2)*d^4*x^4 + 4*(8*a^3 + 3*a*b^2)*c
*d^3*x^3 - 3*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*x^2 - 2*(9*a*b^2*c - 2
*(8*a^3 + 3*a*b^2)*c^3)*d*x)*cosh(1)*sinh(1)^2 + 8*((8*a^3 + 3*a*b^2)*d^4*x
^4 + 4*(8*a^3 + 3*a*b^2)*c*d^3*x^3 - 3*(3*a*b^2 - 2*(8*a^3 + 3*a*b^2)*c^2)*
d^2*x^2 - 2*(9*a*b^2*c - 2*(8*a^3 + 3*a*b^2)*c^3)*d*x)*sinh(1)^3 + 24*((8*a
*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x
```

```
 + 8*a*b^2*c^4 - 3*a*b^2)*cosh(1)^3 + 3*(8*a*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x
^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x + 8*a*b^2*c^4 - 3*a*b^2)*cosh(
1)^2*sinh(1) + 3*(8*a*b^2*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x
^2 + 32*a*b^2*c^3*d*x + 8*a*b^2*c^4 - 3*a*b^2)*cosh(1)*sinh(1)^2 + (8*a*b^2
*d^4*x^4 + 32*a*b^2*c*d^3*x^3 + 48*a*b^2*c^2*d^2*x^2 + 32*a*b^2*c^3*d*x + 8
*a*b^2*c^4 - 3*a*b^2)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b^3*c + 3*(2*b^3*c^2 - b^3)*d*x)
*cosh(1)^3 + 3*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b^3*c + 3*(
2*b^3*c^2 - b^3)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^
2 + 2*b^3*c^3 - 3*b^3*c + 3*(2*b^3*c^2 - b^3)*d*x)*cosh(1)*sinh(1)^2 + (2*b
^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b^3*c + 3*(2*b^3*c^2 - b^3)*d*
x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((8*(
8*a^2*b + b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 - 24*b^3*c^2 + 8*(8*a
^2*b + b^3)*c^4 - 24*(b^3 - 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*
b^3 - 16*(3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^2*b +
 b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 - 24*b^3*c^2 + 8*(8*a^2*b + b^
3)*c^4 - 24*(b^3 - 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 - 16*
(3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*(8*(8*a^2*b +
b^3)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 - 24*b^3*c^2 + 8*(8*a^2*b + b^3
)*c^4 - 24*(b^3 - 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 - 16*(
3*b^3*c - 2*(8*a^2*b + b^3)*c^3)*d*x)*cosh(1)*sinh(1)^2 + (8*(8*a^2*b + b^3
)*d^4*x^4 + 32*(8*a^2*b + b^3)*c*d^3*x^3 - 24*b^3*c^2 + 8*(8*a^2*b + b^3)*c
^4 - 24*(b^3 - 2*(8*a^2*b + b^3)*c^2)*d^2*x^2 - 24*a^2*b - 15*b^3 - 16*(3*b
^3*c - 2*(8*a^2*b + b^3)*c^3)*d*x)*sinh(1)^3 - 16*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*((2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2*x^2 + 2*a*b^2*c^3 - 3*a*b^2*c +
3*(2*a*b^2*c^2 - a*b^2)*d*x)*cosh(1)^3 + 3*(2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2
*x^2 + 2*a*b^2*c^3 - 3*a*b^2*c + 3*(2*a*b^2*c^2 - a*b^2)*d*x)*cosh(1)^2*sin
h(1) + 3*(2*a*b^2*d^3*x^3 + 6*a*b^2*c*d^2*x^2 + 2*a*b^2*c^3 - 3*a*b^2*c + 3
*(2*a*b^2*c^2 - a*b^2)*d*x)*cosh(1)*sinh(1)^2 + (2*a*b^2*d^3*x^3 + 6*a*b^2*
c*d^2*x^2 + 2*a*b^2*c^3 - 3*a*b^2*c + 3*(2*a*b^2*c^2 - a*b^2)*d*x)*sinh(1)^
3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*((2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 +
 2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*b + b^3)*c^2)*d*x -
3*(8*a^2*b + 5*b^3)*c)*cosh(1)^3 + 3*(2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*
b + b^3)*c*d^2*x^2 + 2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*
b + b^3)*c^2)*d*x - 3*(8*a^2*b + 5*b^3)*c)*cosh(1)^2*sinh(1) + 3*(2*(8*a^2*
b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 + 2*(8*a^2*b + b^3)*c^3 - 3*
(8*a^2*b + 5*b^3 - 2*(8*a^2*b + b^3)*c^2)*d*x - 3*(8*a^2*b + 5*b^3)*c)*cosh
(1)*sinh(1)^2 + (2*(8*a^2*b + b^3)*d^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*x^2 +
2*(8*a^2*b + b^3)*c^3 - 3*(8*a^2*b + 5*b^3 - 2*(8*a^2*b + b^3)*c^2)*d*x - 3
*(8*a^2*b + 5*b^3)*c)*sinh(1)^3))/d
```

<u>**Sympy**</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 1828 vs. $2(260) = 520$.

time = 0.90, size = 1828, normalized size = 6.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*asinh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*asinh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*asinh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*asinh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 + 1)/16 + 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*asinh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/16 + 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)/32 - 9*a**2*b*e**3*asinh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*asinh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)
*asinh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*c*e**3*x/16 + 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*asinh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a*b**2*d
*e**3*x**2/32 + 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)/16 - 9*a*b**2*e**3*asinh(c + d*x)**2/(32*d) + b**3*c**4*e**3*asinh
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*asinh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*asinh(c + d*x)**3 + 3*b**3*c**3*e**3*x*asinh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*asinh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*asinh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c**2*
e**3*asinh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*asinh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*c*e**3*x*asinh(c + d*x)/16 + 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(32*d) + 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(256*d) + b**3*d**3*e**3*
x**4*asinh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*asinh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/128 - 9*b**3*d*
```

```
e**3*x**2*asinh(c + d*x)/32 + 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**2/32 + 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/256 - 3*b**3*e**3*asinh(c + d*x)**3/(32*d) - 45*b**3*e**3*asinh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*asinh(c))**3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^3, x)
```

3.139 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=227

$$-\frac{4}{3}ab^2e^2x+\frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d}-\frac{2b^3e^2\left(1+(c+dx)^2\right)^{3/2}}{27d}-\frac{4b^3e^2(c+dx)\sinh^{-1}(c+dx)}{3d}+\frac{2b^2e^2(c+dx)}{}$$

```
[Out] -4/3*a*b^2*e^2*x-2/27*b^3*e^2*(1+(d*x+c)^2)^(3/2)/d-4/3*b^3*e^2*(d*x+c)*arc
sinh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))/d+1/3*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^3/d+14/9*b^3*e^2*(1+(d*x+c)^2)^(1/2)/d+2/3*b*e^2*(a+
b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 267, 272, 45}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d}-\frac{4}{3}ab^2e^2x+\frac{2be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}-\frac{be^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^3}{3d}-\frac{2b^3e^2\left((c+dx)^2+1\right)^{3/2}}{27d}+\frac{14b^3e^2\sqrt{(c+dx)^2+1}}{9d}-\frac{4b^3e^2(c+dx)\sinh^{-1}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-4*a*b^2*e^2*x)/3 + (14*b^3*e^2*Sqrt[1 + (c + d*x)^2])/(9*d) - (2*b^3*e^2*
(1 + (c + d*x)^2)^(3/2))/(27*d) - (4*b^3*e^2*(c + d*x)*ArcSinh[c + d*x])/(3
*d) + (2*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x]))/(9*d) + (2*b*e^2*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) - (b*e^2*(c + d*x)^2*S
qrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(
a + b*ArcSinh[c + d*x])^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^3}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\sinh}{\sqrt{1+}}\right.}{d}$$

$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d} + \frac{e^2(c}{}$$

$$= \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1+(c+dx)^2}\,(}{3}$$

$$= -\frac{4}{3}ab^2e^2x + \frac{2b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)}{9d} + \frac{2be^2\sqrt{1}}{}$$

$$= -\frac{4}{3}ab^2e^2x - \frac{4b^3e^2(c+dx)\sinh^{-1}(c+dx)}{3d} + \frac{2b^2e^2(c+dx)^3\,(a}{}$$

$$= -\frac{4}{3}ab^2e^2x + \frac{14b^3e^2\sqrt{1+(c+dx)^2}}{9d} - \frac{2b^3e^2\left(1+(c+dx)^2\right)^{3/2}}{27d}$$

Mathematica [A]
time = 0.20, size = 258, normalized size = 1.14

$$\frac{e^2\left(-12ab^2(c+dx)+a(3a^2+2b^2)(c+dx)^3+\frac{1}{3}b\sqrt{1+(c+dx)^2}\left(18a^2+40b^2-(9a^2+2b^2)(c+dx)^2\right)-b\left(12b^2(c+dx)-9a^2(c+dx)^3-2b^2(c+dx)^3-12ab\sqrt{1+(c+dx)^2}+6ab(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)-3b^2\left(-3a(c+dx)^3-2b\sqrt{1+(c+dx)^2}+b(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2+3b^3(c+dx)^3\sinh^{-1}(c+dx)^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^2*(-12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[1 + (c
+ d*x)^2]*(18*a^2 + 40*b^2 - (9*a^2 + 2*b^2)*(c + d*x)^2))/3 - b*(12*b^2*(c
 + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 - 12*a*b*Sqrt[1 + (c + d*x)
^2] + 6*a*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] - 3*b^2*(-3
*a*(c + d*x)^3 - 2*b*Sqrt[1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*
x)^2])*ArcSinh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcSinh[c + d*x]^3))/(9*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(203) = 406$.
time = 3.35, size = 680, normalized size = 3.00

method	result
default	$\frac{e^2(dx+c)^3a^3}{3d}+\frac{b^3e^2\left(9\operatorname{arcsinh}(dx+c)^3x^3d^3+27\operatorname{arcsinh}(dx+c)^3x^2c\,d^2+27\operatorname{arcsinh}(dx+c)^3x\,c^2d-9\operatorname{arcsinh}(dx+c)^2\sqrt{d^2x^2+2cd}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^3/d+1/27*b^3*e^2*(9*arcsinh(d*x+c)^3*x^3*d^3+27*arcsinh
(d*x+c)^3*x^2*c*d^2+27*arcsinh(d*x+c)^3*x*c^2*d-9*arcsinh(d*x+c)^2*(d^2*x^2
+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+6*arcsinh(d*x+c)*x^3*d^3+9*arcsinh(d*x+c)^3*c
^3-18*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d+18*arcsinh(d*x+c
)*x^2*c*d^2-9*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2+18*arcsinh
(d*x+c)*x*c^2*d-2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+6*arcsinh(d*x+c)*c^
3-4*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d+18*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*ar
csinh(d*x+c)^2-36*arcsinh(d*x+c)*x*d-2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2-36
*arcsinh(d*x+c)*c+40*(d^2*x^2+2*c*d*x+c^2+1)^(1/2))/d+1/9*a*b^2*e^2*(9*arcs
inh(d*x+c)^2*x^3*d^3+27*arcsinh(d*x+c)^2*x^2*c*d^2+27*arcsinh(d*x+c)^2*x*c^
2*d-6*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+2*d^3*x^3+9*arcs
inh(d*x+c)^2*c^3-12*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d+6*x^
2*c*d^2-6*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2+6*x*c^2*d+2*c^3+
12*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)-12*d*x-12*c)/d+3*a^2*b*e^2/
d*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+2/9*(1+(d
*x+c)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*x^3*e^2 + a^3*c*d*x^2*e^2 + 3/2*(2*x^2*arcsinh(d*x + c) - d*(3*
c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt
(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c
/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 +
 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c
^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 +
 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)*(c^2 + 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*x*e^2 + 3*((d*x + c)*arcsi
```

```
nh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*d^2*x^3*e^2
 + 3*b^3*c*d*x^2*e^2 + 3*b^3*c^2*x*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))^3 + integrate(((3*a*b^2*d^5 - b^3*d^5)*x^5*e^2 + 5*(3*a*b^2*c
*d^4 - b^3*c*d^4)*x^4*e^2 + 3*(c^5 + c^3)*a*b^2*e^2 + (3*(10*c^2*d^3 + d^3)
*a*b^2 - (10*c^2*d^3 + d^3)*b^3)*x^3*e^2 + 3*((10*c^3*d^2 + 3*c*d^2)*a*b^2
- (3*c^3*d^2 + c*d^2)*b^3)*x^2*e^2 + 3*((5*c^4*d + 3*c^2*d)*a*b^2 - (c^4*d
+ c^2*d)*b^3)*x*e^2 + ((3*a*b^2*d^4 - b^3*d^4)*x^4*e^2 + 3*(c^4 + c^2)*a*b^
2*e^2 + 4*(3*a*b^2*c*d^3 - b^3*c*d^3)*x^3*e^2 - 3*(2*b^3*c^2*d^2 - (6*c^2*d
^2 + d^2)*a*b^2)*x^2*e^2 - 3*(b^3*c^3*d - 2*(2*c^3*d + c*d)*a*b^2)*x*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d
*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(195) = 390$.
time = 0.40, size = 1395, normalized size = 6.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cosh(1)^
2 + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cosh(1)*sin
h(1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sinh(1)^2)
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*((3*a^3 + 2*a*b^2)*
d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2 - 3*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^
2)*d*x)*cosh(1)^2 + 9*(3*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d
*x + a*b^2*c^3)*cosh(1)^2 + 6*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*
c^2*d*x + a*b^2*c^3)*cosh(1)*sinh(1) + 3*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2
 + 3*a*b^2*c^2*d*x + a*b^2*c^3)*sinh(1)^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*((b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2 - 2*b^3)*cosh(1)^2 + 2*(b^3*d^2*x^
2 + 2*b^3*c*d*x + b^3*c^2 - 2*b^3)*cosh(1)*sinh(1) + (b^3*d^2*x^2 + 2*b^3*c
*d*x + b^3*c^2 - 2*b^3)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1))^2 + 6*((3*a^3 + 2*a*b^2)*d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2
- 3*(4*a*b^2 - (3*a^3 + 2*a*b^2)*c^2)*d*x)*cosh(1)*sinh(1) + 3*((3*a^3 + 2*
a*b^2)*d^3*x^3 + 3*(3*a^3 + 2*a*b^2)*c*d^2*x^2 - 3*(4*a*b^2 - (3*a^3 + 2*a*
b^2)*c^2)*d*x)*sinh(1)^2 + 3*(((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2*b
^3)*c*d^2*x^2 - 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 - 3*(4*b^3 - (9*a^2*b + 2*
b^3)*c^2)*d*x)*cosh(1)^2 + 2*((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2*b^
3)*c*d^2*x^2 - 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 - 3*(4*b^3 - (9*a^2*b + 2*b
^3)*c^2)*d*x)*cosh(1)*sinh(1) + ((9*a^2*b + 2*b^3)*d^3*x^3 + 3*(9*a^2*b + 2
*b^3)*c*d^2*x^2 - 12*b^3*c + (9*a^2*b + 2*b^3)*c^3 - 3*(4*b^3 - (9*a^2*b +
2*b^3)*c^2)*d*x)*sinh(1)^2 - 6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b^2*d^
2*x^2 + 2*a*b^2*c*d*x + a*b^2*c^2 - 2*a*b^2)*cosh(1)^2 + 2*(a*b^2*d^2*x^2 +
```

```
 2*a*b^2*c*d*x + a*b^2*c^2 - 2*a*b^2)*cosh(1)*sinh(1) + (a*b^2*d^2*x^2 + 2*
a*b^2*c*d*x + a*b^2*c^2 - 2*a*b^2)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(((9*a^2*b + 2*b^3
)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*x - 18*a^2*b - 40*b^3 + (9*a^2*b + 2*b^
3)*c^2)*cosh(1)^2 + 2*((9*a^2*b + 2*b^3)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*
x - 18*a^2*b - 40*b^3 + (9*a^2*b + 2*b^3)*c^2)*cosh(1)*sinh(1) + ((9*a^2*b
+ 2*b^3)*d^2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*x - 18*a^2*b - 40*b^3 + (9*a^2*b
 + 2*b^3)*c^2)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(211) = 422$.
time = 0.56, size = 1173, normalized size = 5.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*asinh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*asinh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(3*d) + 3*a**2*b*c*d*e
**2*x**2*asinh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)/3 + a**2*b*d**2*e**2*x**3*asinh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 + 1)/3 + 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)/(3*d) + a*b**2*c**3*e**2*asinh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*asinh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
asinh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*asinh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 4*a*b**2*e**2*x/3 + 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + b**3*c**3*e**
2*asinh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*asinh(c + d*x)**3 + 2*b**3*c**2*e**2*x*asinh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**3*c*d*e**2*x*
*2*asinh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 4*b**3*c*e**2*asinh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*asi
nh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27
 - 4*b**3*e**2*x*asinh(c + d*x)/3 + 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 + 1)*asinh(c + d*x)**2/(3*d) + 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asinh(c))**3, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^3, x)
```

3.140 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=161

$$-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d}+\frac{3b^3e\sinh^{-1}(c+dx)}{8d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{}}{}$$

```
[Out] 3/8*b^3*e*arcsinh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))/d+1/4*e
*(a+b*arcsinh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^3/d-3/8*b^3*
e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^2*(1+(
d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5776, 5812, 5783, 327, 221}

$$\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d}-\frac{3be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^3}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d}-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}}{8d}+\frac{3b^3e\sinh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(8*d) + (3*b^3*e*ArcSinh[c + d*x
])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x]))/(4*d) - (3*b*e*(c
 + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/(4*d) + (e*(a + b
*ArcSinh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^3)/(2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2(a+b\sinh^{-1}(x}{\sqrt{1+x^2}}\right.}{2d}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d} + \frac{e(c+dx}{}$$

$$= \frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)}{4d} - \frac{3be(c+dx)\sqrt{1+(c+dx}}{4}$$

$$= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d} + \frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+\right.}{4d}$$

$$= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}}{8d} + \frac{3b^3e\sinh^{-1}(c+dx)}{8d} + \frac{3b^2e(c+}{}$$

Mathematica [A]
time = 0.12, size = 200, normalized size = 1.24

$$\frac{e\left(2a(2a^2+3b^2)(c+dx)^2-3b(2a^2+b^2)(c+dx)\sqrt{1+(c+dx)^2}+3b(2a^2+b^2)\sinh^{-1}(c+dx)-6b(c+dx)\left(-2a^2(c+dx)-b^2(c+dx)+2ab\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+6b^2\left(a+2a(c+dx)^2-b(c+dx)\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2+2b^3(1+2(c+dx)^2)\sinh^{-1}(c+dx)^3\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*(c + d*x)*Sqrt[1 +
(c + d*x)^2] + 3*b*(2*a^2 + b^2)*ArcSinh[c + d*x] - 6*b*(c + d*x)*(-2*a^2*(
c + d*x) - b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] +
6*b^2*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c + d*x)^2])*ArcSinh[c +
 d*x]^2 + 2*b^3*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^3))/(8*d)
```

Maple [A]
time = 2.46, size = 243, normalized size = 1.51

method	result

derivativedivides	$\frac{\frac{e(dx+c)^2a^3}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2(dx+c)\sqrt{1+(dx+c)^2}}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx\right.}{}\right.}{}$
default	$\frac{\frac{e(dx+c)^2a^3}{2}+e\,b^3\left(\frac{\operatorname{arcsinh}(dx+c)^3\left(1+(dx+c)^2\right)}{2}-\frac{3\operatorname{arcsinh}(dx+c)^2(dx+c)\sqrt{1+(dx+c)^2}}{4}-\frac{\operatorname{arcsinh}(dx+c)^3}{4}+\frac{3\left(1+(dx\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^3+e*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)-3/4*arcs
inh(d*x+c)^2*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^3+3/4*(1+(d*x+c
)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*x+c))+3*e
*a*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(d*x+c)*(1+(d
*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+3*e*a^2*b*(1/2*(d*x+
c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2*e + 3/4*(2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x +
 c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 +
 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^2*b*d*e + a^3
*c*x*e + 3*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b*c*e/d
 + 1/2*(b^3*d*x^2*e + 2*b^3*c*x*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))^3 + integrate(3/2*((2*a*b^2*d^4 - b^3*d^4)*x^4*e + 2*(c^4 + c^2)*a
*b^2*e + 4*(2*a*b^2*c*d^3 - b^3*c*d^3)*x^3*e + (2*(6*c^2*d^2 + d^2)*a*b^2 -
 (5*c^2*d^2 + d^2)*b^3)*x^2*e + 2*(2*(2*c^3*d + c*d)*a*b^2 - (c^3*d + c*d)*
b^3)*x*e + (2*(c^3 + c)*a*b^2*e + (2*a*b^2*d^3 - b^3*d^3)*x^3*e + 3*(2*a*b^
2*c*d^2 - b^3*c*d^2)*x^2*e - 2*(b^3*c^2*d - (3*c^2*d + d)*a*b^2)*x*e)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x +
 c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(151) = 302$.

time = 0.39, size = 632, normalized size = 3.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*((2*b^3*d^2*x^2 + 4*b^3*c*d*x + 2*b^3*c^2 + b^3)*cosh(1) + (2*b^3*d^
2*x^2 + 4*b^3*c*d*x + 2*b^3*c^2 + b^3)*sinh(1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^3 + 6*((2*a*b^2*d^2*x^2 + 4*a*b^2*c*d*x + 2*a*b^2*c^2
 + a*b^2)*cosh(1) + (2*a*b^2*d^2*x^2 + 4*a*b^2*c*d*x + 2*a*b^2*c^2 + a*b^2)
*sinh(1) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^3*d*x + b^3*c)*cosh(1) + (
b^3*d*x + b^3*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
^2 + 2*((2*a^3 + 3*a*b^2)*d^2*x^2 + 2*(2*a^3 + 3*a*b^2)*c*d*x)*cosh(1) + 3*
((2*(2*a^2*b + b^3)*d^2*x^2 + 4*(2*a^2*b + b^3)*c*d*x + 2*a^2*b + b^3 + 2*(
2*a^2*b + b^3)*c^2)*cosh(1) + (2*(2*a^2*b + b^3)*d^2*x^2 + 4*(2*a^2*b + b^3
)*c*d*x + 2*a^2*b + b^3 + 2*(2*a^2*b + b^3)*c^2)*sinh(1) - 4*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1)*((a*b^2*d*x + a*b^2*c)*cosh(1) + (a*b^2*d*x + a*b^2*c)*
sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*((2*a^3 + 3*
a*b^2)*d^2*x^2 + 2*(2*a^3 + 3*a*b^2)*c*d*x)*sinh(1) - 3*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)*(((2*a^2*b + b^3)*d*x + (2*a^2*b + b^3)*c)*cosh(1) + ((2*a^2
*b + b^3)*d*x + (2*a^2*b + b^3)*c)*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(148) = 296$.

time = 0.36, size = 685, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*asinh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*asinh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/(4*d) + 3*a**2*b*d*e*x**2*asinh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 3*a**2*b*e*asinh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*asinh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*asinh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*asinh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/2 + 3*a*
b**2*e*asinh(c + d*x)**2/(4*d) + b**3*c**2*e*asinh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*asinh(c + d*x)/(4*d) + b**3*c*e*x*asinh(c + d*x)**3 + 3*b**3*c*e*
x*asinh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) +
 b**3*d*e*x**2*asinh(c + d*x)**3/2 + 3*b**3*d*e*x**2*asinh(c + d*x)/4 - 3*b
```

```
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 + b**3*e*asinh(c + d*x)**3/(4*d)
+ 3*b**3*e*asinh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**3, Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^3, x)
```

3.141 $\int \left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=100

$$6ab^2x-\frac{6b^3\sqrt{1+(c+dx)^2}}{d}+\frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d}-\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+\frac{(c+$$

```
[Out] 6*a*b^2*x+6*b^3*(d*x+c)*arcsinh(d*x+c)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^3/d-6
*b^3*(1+(d*x+c)^2)^(1/2)/d-3*b*(a+b*arcsinh(d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5772, 5798, 267}

$$6ab^2x-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{(c+dx)^2+1}}{d}+\frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[1 + (c + d*x)^2])/d + (6*b^3*(c + d*x)*ArcSinh[c +
d*x])/d - (3*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^2)/d + ((c +
d*x)*(a + b*ArcSinh[c + d*x])^3)/d
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} - \frac{(3b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^2}{\sqrt{1+x^2}}\,dx, x, c+ \right.}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= 6ab^2x - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\sinh\right.}{d}$$

$$= 6ab^2x + \frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= 6ab^2x - \frac{6b^3\sqrt{1+(c+dx)^2}}{d} + \frac{6b^3(c+dx)\sinh^{-1}(c+dx)}{d} - \frac{3b\sqrt{1+(c+}{}$$

Mathematica [A]
time = 0.10, size = 147, normalized size = 1.47

$$\frac{a(a^2+6b^2)(c+dx) - 3b(a^2+2b^2)\sqrt{1+(c+dx)^2} - 3b\left(-a^2(c+dx) - 2b^2(c+dx) + 2ab\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx) - 3b^2\left(-a(c+dx) + b\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2 + b^3(c+dx)\sinh^{-1}(c+dx)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[1 + (c + d*x)^2] - 3*b*
(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[
c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x
]^2 + b^3*(c + d*x)*ArcSinh[c + d*x]^3)/d
```

Maple [A]
time = 2.29, size = 160, normalized size = 1.60

method	result

derivativedivides	$\frac{(dx+c)a^3+b^3\left(\operatorname{arcsinh}(dx+c)^3(dx+c)-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(d\right.}{}$
default	$\frac{(dx+c)a^3+b^3\left(\operatorname{arcsinh}(dx+c)^3(dx+c)-3\operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}+6(dx+c)\operatorname{arcsinh}(dx+c)-6\sqrt{1+(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*(arcsinh(d*x+c)^3*(d*x+c)-3*arcsinh(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+3*a*b^2*(arcsinh
(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+3*a^2*b*(
(d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + a^3*x + 3*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^2*b/d + integrate(3*((c^3
+ c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2
+ ((3*c^2*d + d)*a*b^2 - (c^2*d + d)*b^3)*x + ((c^2 + 1)*a*b^2 + (a*b^2*d^2
 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^3*x^3 + 3*c*d^2*
x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(96) = 192.
time = 0.41, size = 239, normalized size = 2.39

$$\frac{(b^3dx+b^3c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3+(a^3+6\,ab^2)dx+3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,cdx+c^2+1}\,b^3\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2-3\left(2\sqrt{d^2x^2+2\,cdx+c^2+1}\,ab^2-(a^2b+2\,b^3)dx-(a^2b+2\,b^3)c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-3\sqrt{d^2x^2+2\,cdx+c^2+1}\,(a^2b+2\,b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
```

```
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*(a^2*b + 2*b^3))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.
time = 0.18, size = 282, normalized size = 2.82

$$\begin{cases} a^3x + \frac{3a^2bc\,\mathrm{asinh}\,(c+dx)}{d} + 3a^2bx\,\mathrm{asinh}\,(c+dx) - \frac{3a^2b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{3ab^2c\,\mathrm{asinh}^2\,(c+dx)}{d} + 3ab^2x\,\mathrm{asinh}^2\,(c+dx) + 6ab^2x - \frac{6ab^2\sqrt{c^2+2cdx+d^2x^2+1}\,\mathrm{asinh}\,(c+dx)}{d} + \frac{b^3c\,\mathrm{asinh}^3\,(c+dx)}{d} + \frac{6b^3c\,\mathrm{asinh}\,(c+dx)}{d} + b^3x\,\mathrm{asinh}^3\,(c+dx) + 6b^3x\,\mathrm{asinh}\,(c+dx) - \frac{3b^3\sqrt{c^2+2cdx+d^2x^2+1}\,\mathrm{asinh}^2\,(c+dx)}{d} - \frac{6b^3\sqrt{c^2+2cdx+d^2x^2+1}}{d} & \text{for}\ d \neq 0 \\ x(a+b\,\mathrm{asinh}\,(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*asinh(c + d*x)/d + 3*a**2*b*x*asinh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 3*a*b**2*c*asinh(c + d
*x)**2/d + 3*a*b**2*x*asinh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + b**3*c*asinh(c + d*x)**3/d + 6
*b**3*c*asinh(c + d*x)/d + b**3*x*asinh(c + d*x)**3 + 6*b**3*x*asinh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d, Ne(d, 0)), (x*(a + b*asinh(c))**
3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c + d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3, x)
```

3.142 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{ce+dex}\,dx$

Optimal. Leaf size=155

$$\frac{(a+b\sinh^{-1}(c+dx))^4}{4bde}+\frac{(a+b\sinh^{-1}(c+dx))^3\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)}{de}-\frac{3b(a+b\sinh^{-1}(c+dx))^2\,\mathrm{PolyL}}{2de}$$

```
[Out] 1/4*(a+b*arcsinh(d*x+c))^4/b/d/e+(a+b*arcsinh(d*x+c))^3*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3/2*b*(a+b*arcsinh(d*x+c))^2*polylog(2,1/(d*x+c+(1+(
d*x+c)^2)^(1/2))^2)/d/e-3/2*b^2*(a+b*arcsinh(d*x+c))*polylog(3,1/(d*x+c+(1+
(d*x+c)^2)^(1/2))^2)/d/e-3/4*b^3*polylog(4,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)
/d/e
```

Rubi [A]

time = 0.21, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3b^2\mathrm{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{2de}-\frac{3b\mathrm{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^2}{2de}+\frac{(a+b\sinh^{-1}(c+dx))^4}{4bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^3}{de}-\frac{3b^3\mathrm{Li}_4\left(e^{-2\sinh^{-1}(c+dx)}\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcSinh[c + d*x])^3*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(2*d*e) - (3*b^3*PolyLog[4, E^(-2*ArcSinh[c
 + d*x])])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{ex}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x}\,dx,x,c+dx\right)}{de}\\
&= \frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}-\frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx,x,\sinh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}\\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{4bde}+\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)}{de}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 128, normalized size = 0.83

$$\frac{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{b}+4\left(a+b\sinh^{-1}(c+dx)\right)^3\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)+6b\left(a+b\sinh^{-1}(c+dx)\right)^2\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)-6b^2\left(a+b\sinh^{-1}(c+dx)\right)\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)+3b^3\text{PolyLog}\left(4,e^{2\sinh^{-1}(c+dx)}\right)}{4de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (-((a + b*ArcSinh[c + d*x])^4/b) + 4*(a + b*ArcSinh[c + d*x])^3*Log[1 - E^(
2*ArcSinh[c + d*x])] + 6*b*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 6*b^2*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^(2*ArcSinh[c +
d*x])] + 3*b^3*PolyLog[4, E^(2*ArcSinh[c + d*x])])/(4*d*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(179) = 358$.
time = 2.67, size = 674, normalized size = 4.35

method	result
derivativedivides	$\frac{\frac{a^3\ln(dx+c)}{e}-\frac{b^3\operatorname{arcsinh}(dx+c)^4}{4e}+\frac{b^3\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{3b^3\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx\right.}{e}}{}$
default	$\frac{\frac{a^3\ln(dx+c)}{e}-\frac{b^3\operatorname{arcsinh}(dx+c)^4}{4e}+\frac{b^3\operatorname{arcsinh}(dx+c)^3\ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e}+\frac{3b^3\operatorname{arcsinh}(dx+c)^2\operatorname{polylog}\left(2,-dx\right.}{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/e*ln(d*x+c)-1/4*b^3/e*arcsinh(d*x+c)^4+b^3/e*arcsinh(d*x+c)^3*ln(1
+d*x+c+(1+(d*x+c)^2)^(1/2))+3*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d
*x+c)^2)^(1/2))-6*b^3/e*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2)
)+6*b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+b^3/e*arcsinh(d*x+c)^3*ln(1
-d*x-c-(1+(d*x+c)^2)^(1/2))+3*b^3/e*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*
x+c)^2)^(1/2))-6*b^3/e*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+
6*b^3/e*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-a*b^2/e*arcsinh(d*x+c)^3+3*a*b
^2/e*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+6*a*b^2/e*arcsinh(d*x
+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-6*a*b^2/e*polylog(3,-d*x-c-(1+(d*
x+c)^2)^(1/2))+3*a*b^2/e*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6
*a*b^2/e*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*a*b^2/e*poly
log(3,d*x+c+(1+(d*x+c)^2)^(1/2))-3/2*a^2*b/e*arcsinh(d*x+c)^2+3*a^2*b/e*arc
sinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*a^2*b/e*polylog(2,-d*x-c-(1+(
d*x+c)^2)^(1/2))+3*a^2*b/e*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+3
*a^2*b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*e^(-1)*log(d*x*e + c*e)/d + integrate(b^3*log(d*x + c + sqrt((d*x + c)^
2 + 1))^3/(d*x*e + c*e) + 3*a*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d
*x*e + c*e) + 3*a^2*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*x*e + c*e), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx+\int \frac{b^3\operatorname{asinh}^3\left(c+dx\right)}{c+dx}\,dx+\int \frac{3ab^2\operatorname{asinh}^2\left(c+dx\right)}{c+dx}\,dx+\int \frac{3a^2b\operatorname{asinh}\left(c+dx\right)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*asinh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*asinh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
sinh(c + d*x)/(c + d*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{c\,e+d\,e\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x), x)
```

3.143 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)}-\frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{6b^2\left(a+b\sinh^{-1}(c+dx)\right)\text{Po}}{de^2}$$

```
[Out] -(a+b*arcsinh(d*x+c))^3/d/e^2/(d*x+c)-6*b*(a+b*arcsinh(d*x+c))^2*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d*x-c-
(1+(d*x+c)^2)^(1/2))/d/e^2+6*b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1+(d
*x+c)^2)^(1/2))/d/e^2+6*b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2-6*b
^3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A]
time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5776, 5816, 4267, 2611, 2320, 6724}

$$-\frac{6b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}+\frac{6b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)}-\frac{6b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^2}+\frac{6b^3\text{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{6b^3\text{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^3/(d*e^2*(c + d*x))) - (6*b*(a + b*ArcSinh[c + d
*x])^2*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^2*(a + b*ArcSinh[c + d*x
])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (6*b^3*PolyLog[3, -E^ArcSinh[c
 + d*x]])/(d*e^2) - (6*b^3*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\text{Subst}\left(\int (a+bx)^2\text{csch}(x)\,dx, x, \sinh^{-1}(c-\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^2(c+dx)} - \frac{6b\left(a+b\sinh^{-1}(c+dx)\right)^2\tanh^{-1}\left(e^{\sinh^{-1}(c+dx}\right.}{de^2}$$

Mathematica [A]
time = 0.48, size = 315, normalized size = 1.90

$$\frac{-\frac{a^3}{c+dx}-\frac{3a^2b\sinh^{-1}(c+dx)}{c+dx}+3a^2b\log(c+dx)-3a^2b\log\left(1+\sqrt{1+c^2+2cdx+d^2x^2}\right)+3ab^2\left(\sinh^{-1}(c+dx)\left(-\frac{\sinh^{-1}(c+dx)}{c+dx}+2\log\left(1-e^{-\sinh^{-1}(c+dx)}\right)-2\log\left(1+e^{-\sinh^{-1}(c+dx)}\right)\right)+2\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-2\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)\right)+b^3\left(-\frac{\sinh^{-1}(c+dx)^3}{c+dx}+3\sinh^{-1}(c+dx)^2\log\left(1-e^{-\sinh^{-1}(c+dx)}\right)-3\sinh^{-1}(c+dx)^2\log\left(1+e^{-\sinh^{-1}(c+dx)}\right)+6\sinh^{-1}(c+dx)\text{PolyLog}\left(2,-e^{-\sinh^{-1}(c+dx)}\right)-6\sinh^{-1}(c+dx)\text{PolyLog}\left(2,e^{-\sinh^{-1}(c+dx)}\right)+6\text{PolyLog}\left(3,-e^{-\sinh^{-1}(c+dx)}\right)-6\text{PolyLog}\left(3,e^{-\sinh^{-1}(c+dx)}\right)\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^3/(c + d*x)) - (3*a^2*b*ArcSinh[c + d*x])/(c + d*x) + 3*a^2*b*Log[c +
d*x] - 3*a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]] + 3*a*b^2*(ArcSin
h[c + d*x]*(-(ArcSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])
] - 2*Log[1 + E^(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])
] - 2*PolyLog[2, E^(-ArcSinh[c + d*x])]) + b^3*(-(ArcSinh[c + d*x]^3/(c + d
*x)) + 3*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c +
d*x]^2*Log[1 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-
ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] +
 6*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]
))/(d*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(212) = 424$.

time = 3.47, size = 436, normalized size = 2.63

method	result
derivativedivides	$-\frac{a^3}{e^2(dx+c)}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{e^2(dx+c)}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e^2}-\frac{6b^3 \operatorname{arcsinh}(dx+c) \operatorname{polylog}\left(2,-dx-c\right.}{e^2}$
default	$-\frac{a^3}{e^2(dx+c)}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{e^2(dx+c)}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2 \ln\left(1+dx+c+\sqrt{1+(dx+c)^2}\right)}{e^2}-\frac{6b^3 \operatorname{arcsinh}(dx+c) \operatorname{polylog}\left(2,-dx-c\right.}{e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/e^2/(d*x+c)-b^3/e^2*arcsinh(d*x+c)^3/(d*x+c)-3*b^3/e^2*arcsinh(d*
x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*b^3/e^2*arcsinh(d*x+c)*polylog(2,-
d*x-c-(1+(d*x+c)^2)^(1/2))+6*b^3/e^2*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+
3*b^3/e^2*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*b^3/e^2*arcsin
h(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-6*b^3/e^2*polylog(3,d*x+c+(1+
(d*x+c)^2)^(1/2))-3*a*b^2/e^2*arcsinh(d*x+c)^2/(d*x+c)-6*a*b^2/e^2*arcsinh(
d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-6*a*b^2/e^2*polylog(2,-d*x-c-(1+(d*x
+c)^2)^(1/2))+6*a*b^2/e^2*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+6*
a*b^2/e^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+3*a^2*b/e^2*(-1/(d*x+c)*arcs
inh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^2*x*e^2 + c*d*e^
2) - 3*a^2*b*(arcsinh(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arcsinh(d*
x + c)/(d^2*x*e^2 + c*d*e^2)) - a^3/(d^2*x*e^2 + c*d*e^2) + integrate(3*((c
^3 + c)*a*b^2 + (c^3 + c)*b^3 + (a*b^2*d^3 + b^3*d^3)*x^3 + 3*(a*b^2*c*d^2
+ b^3*c*d^2)*x^2 + ((3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 +
 (c^2 + 1)*a*b^2 + (a*b^2*d^2 + b^3*d^2)*x^2 + 2*(a*b^2*c*d + b^3*c*d)*x)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))^2/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 + d^3)*x^3*e^2 + (10
```

```
*c^3*d^2 + 3*c*d^2)*x^2*e^2 + (5*c^4*d + 3*c^2*d)*x*e^2 + (c^5 + c^3)*e^2 +
 (d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*e^2 + 2*(2*c^3*d +
c*d)*x*e^2 + (c^4 + c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*e^(-2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**2,x)
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*asinh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*asinh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^3}{\left(c\,e+d\,e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2,x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

3.144 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=157

$$\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3}-\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}+\frac{3b^2\left(a+b\right.}{}$$

```
[Out] 3/2*b*(a+b*arcsinh(d*x+c))^2/d/e^3-1/2*(a+b*arcsinh(d*x+c))^3/d/e^3/(d*x+c)
^2+3*b^2*(a+b*arcsinh(d*x+c))*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3
/2*b^3*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-3/2*b*(a+b*arcsinh(
d*x+c))^2*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.21, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$,
Rules used = {5859, 12, 5776, 5800, 5775, 3797, 2221, 2317, 2438}

$$\frac{3b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}-\frac{3b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}+\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2}-\frac{3b^3\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)}{2de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (3*b*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^3) - (3*b*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^3/(
2*d*e^3*(c + d*x)^2) + (3*b^2*(a + b*ArcSinh[c + d*x])*Log[1 - E^(-2*ArcSin
h[c + d*x])])/(d*e^3) - (3*b^3*PolyLog[2, E^(-2*ArcSinh[c + d*x])])/(2*d*e^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^2\sqrt{1+x^2}}\,dx, x, c+dx\right)}{2de^3}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(}{}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(}{}$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} -$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} -$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} -$$

$$= -\frac{3b\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3} - \frac{3b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} -$$

Mathematica [A]
time = 0.56, size = 229, normalized size = 1.46

$$-\frac{3b^2\left(a+b(c+dx)\left(-c-dx+\sqrt{1+c^2+2cdx+d^2x^2}\right)\right)\sinh^{-1}(c+dx)^2+b^3\sinh^{-1}(c+dx)^3+3b\sinh^{-1}(c+dx)\left(a\left(a+2b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)-2b^2(c+dx)^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\right)+a\left(a\left(a+3b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}\right)-6b^2(c+dx)^2\log(c+dx)\right)+3b^3(c+dx)^2\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(3*b^2*(a + b*(c + d*x)*(-c - d*x + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])
)*ArcSinh[c + d*x]^2 + b^3*ArcSinh[c + d*x]^3 + 3*b*ArcSinh[c + d*x]*(a*(a
+ 2*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]) - 2*b^2*(c + d*x)^2*Log[
1 - E^(-2*ArcSinh[c + d*x])]) + a*(a*(a + 3*b*(c + d*x)*Sqrt[1 + c^2 + 2*c*
d*x + d^2*x^2]) - 6*b^2*(c + d*x)^2*Log[c + d*x]) + 3*b^3*(c + d*x)^2*PolyL
og[2, E^(-2*ArcSinh[c + d*x])])/(d*e^3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(163) = 326$.
time = 4.58, size = 367, normalized size = 2.34

method	result
derivativedivides	$-\frac{a^3}{2e^3(dx+c)^2}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{2e^3(dx+c)}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2}{2e^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{2e^3(dx+c)^2}+\frac{3b^3 \operatorname{arcsinh}(dx+c)\ln\left(1+\right.}{}$
default	$-\frac{a^3}{2e^3(dx+c)^2}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2\sqrt{1+(dx+c)^2}}{2e^3(dx+c)}-\frac{3b^3 \operatorname{arcsinh}(dx+c)^2}{2e^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{2e^3(dx+c)^2}+\frac{3b^3 \operatorname{arcsinh}(dx+c)\ln\left(1+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^3/e^3/(d*x+c)^2-3/2*b^3/e^3*arcsinh(d*x+c)^2/(d*x+c)*(1+(d*x+c)
^2)^(1/2)-3/2*b^3/e^3*arcsinh(d*x+c)^2-1/2*b^3/e^3*arcsinh(d*x+c)^3/(d*x+c)
^2+3*b^3/e^3*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+3*b^3/e^3*polyl
og(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*
x+c)^2)^(1/2))+3*b^3/e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-3*a*b^2/e^3*a
rcsinh(d*x+c)-3*a*b^2/e^3*arcsinh(d*x+c)/(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/2*a*
b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+3*a*b^2/e^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2
))^2-1)+3*a^2*b/e^3*(-1/2/(d*x+c)^2*arcsinh(d*x+c)-1/2/(d*x+c)*(1+(d*x+c)^2
)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh(d*x + c)/(d^3*x*e^3 + c*d^2
*e^3) - e^(-3)*log(d*x + c)/d)*a*b^2 - 1/2*(log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2*integrate
(3/2*(d^2*x^2 + 2*c*d*x + c^2 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)
 + 1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/(d^5*x^5*e^3 + 5*c
*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)*x^3*e^3 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^3
+ (5*c^4*d + 3*c^2*d)*x*e^3 + (c^5 + c^3)*e^3 + (d^4*x^4*e^3 + 4*c*d^3*x^3*
e^3 + (6*c^2*d^2 + d^2)*x^2*e^3 + 2*(2*c^3*d + c*d)*x*e^3 + (c^4 + c^2)*e^3
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x))*b^3 - 3/2*a^2*b*(sqrt(d^2*x^2 + 2
```

```
*c*d*x + c^2 + 1)*d/(d^3*x*e^3 + c*d^2*e^3) + arcsinh(d*x + c)/(d^3*x^2*e^3
 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 3/2*a*b^2*arcsinh(d*x + c)^2/(d^3*x^2*e^3
+ 2*c*d^2*x*e^3 + c^2*d*e^3) - 1/2*a^3/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d
*e^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*e^(-3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**3,x)
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*asinh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3,x)
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

3.145 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=261

$$-\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}+\frac{b\left(a+b\,\mathrm{si}\right.}{}$$

```
[Out] -b^2*(a+b*arcsinh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^3/d/e^4/(d
*x+c)^3+b*(a+b*arcsinh(d*x+c))^2*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-b
^3*arctanh((1+(d*x+c)^2)^(1/2))/d/e^4+b^2*(a+b*arcsinh(d*x+c))*polylog(2,-d
*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-b^2*(a+b*arcsinh(d*x+c))*polylog(2,d*x+c+(1
+(d*x+c)^2)^(1/2))/d/e^4-b^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+b^
3*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-1/2*b*(a+b*arcsinh(d*x+c))^2*(
1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A]
time = 0.27, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5859, 12, 5776, 5809, 5816, 4267, 2611, 2320, 6724, 272, 65, 213}

$$\frac{b^2\mathrm{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\mathrm{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)}-\frac{b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}+\frac{b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4}-\frac{b^3\mathrm{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^4}+\frac{b^3\mathrm{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)}{de^4}-\frac{b^3\tanh^{-1}\left(\sqrt{(c+dx)^2+1}\right)}{de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -((b^2*(a + b*ArcSinh[c + d*x]))/(d*e^4*(c + d*x))) - (b*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c +
d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcSinh[c + d*x])^2*ArcTanh[E^Arc
Sinh[c + d*x]])/(d*e^4) - (b^3*ArcTanh[Sqrt[1 + (c + d*x)^2]])/(d*e^4) + (b
^2*(a + b*ArcSinh[c + d*x])*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) - (b^2
*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (b^3*Po
lyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (b^3*PolyLog[3, E^ArcSinh[c + d*x]
])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} - \frac{bS}{}$$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \Big($$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \Big($$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \Big($$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \Big($$

$$= -\frac{b^2\left(a+b\sinh^{-1}(c+dx)\right)}{de^4(c+dx)} - \frac{b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \Big($$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 694 vs. $2(261) = 522$.
time = 7.03, size = 694, normalized size = 2.66

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^3/(d*e^4*(c + d*x)^3) - (a^2*b*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(2
*d*e^4*(c + d*x)^2) - (a^2*b*ArcSinh[c + d*x])/(d*e^4*(c + d*x)^3) - (a^2*b
*Log[c + d*x])/(2*d*e^4) + (a^2*b*Log[1 + Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2]
])/(2*d*e^4) + (a*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x])] - (2*(-2 + 4*A
```

```
rcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d*x)*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c + d*x]*Log[1 + E^
(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSinh[c + d*x])] + 2*
ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d*x]*Log[1 - E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d*x]*Log[1 + E^(-Ar
cSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))/(8*d*e^4) + (b^3*
(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcSinh[c + d*x]^3*Coth[
ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh[c + d*x]/2]^2 - (c
+ d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 - 24*ArcSinh[c + d*x]^
2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^2*Log[1 + E^(-ArcSin
h[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*ArcSinh[c + d*x]*PolyL
og[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[
c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[3, E^(-ArcS
inh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c + d*x]/2]^2 - (16*ArcS
inh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x
]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[ArcSinh[c + d*x]/2])
)/(48*d*e^4)
```

Maple [A]
time = 5.53, size = 581, normalized size = 2.23

method	result
derivativedivides	$-\frac{a^3}{3e^4(dx+c)^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{2e^4(dx+c)^2}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{3e^4(dx+c)^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)}{e^4(dx+c)}-\frac{b^3 \operatorname{arcsinh}(dx+c)^2 \ln\left(1-dx-\right.}{2}$
default	$-\frac{a^3}{3e^4(dx+c)^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)^2 \sqrt{1+(dx+c)^2}}{2e^4(dx+c)^2}-\frac{b^3 \operatorname{arcsinh}(dx+c)^3}{3e^4(dx+c)^3}-\frac{b^3 \operatorname{arcsinh}(dx+c)}{e^4(dx+c)}-\frac{b^3 \operatorname{arcsinh}(dx+c)^2 \ln\left(1-dx-\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/e^4/(d*x+c)^3-1/2*b^3/e^4/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)-1/3*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-b^3/e^4/(d*x+c)*arcsinh(
d*x+c)-1/2*b^3/e^4*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-b^3/e^4
*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+b^3/e^4*polylog(3,d*x+
```

```
c+(1+(d*x+c)^2)^(1/2))+1/2*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2
)^(1/2))+b^3/e^4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-b^3/e
^4*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*b^3/e^4*arctanh(d*x+c+(1+(d*x+c)
^2)^(1/2))-a*b^2/e^4/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*arcsinh(d*x+c)-a*b^2/e^4
/(d*x+c)^3*arcsinh(d*x+c)^2-a*b^2/e^4/(d*x+c)-a*b^2/e^4*arcsinh(d*x+c)*ln(1
-d*x-c-(1+(d*x+c)^2)^(1/2))-a*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+
a*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+a*b^2/e^4*polylog(
2,-d*x-c-(1+(d*x+c)^2)^(1/2))+3*a^2*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/
6/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^4*x^3*e^4 +
3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^3/(d^4*x^3*e^4 + 3*c
*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(((3*(c^3 + c)*a*b^2
 + (c^3 + c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c*d
^2)*x^2 + (3*(3*c^2*d + d)*a*b^2 + (3*c^2*d + d)*b^3)*x + (b^3*c^2 + 3*(c^2
 + 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*(3*a*b^2*c*d + b^3*c*d)*x)*sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2*b*x + (
c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
)/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4 + 5*(7*c^3*d^
4 + c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 + 10*c
^3*d^2)*x^2*e^4 + (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4 + (d^6*x^6*e^
4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d^3 + c*d^3)*x^
3*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^3*d)*x*e^4 + (
c^6 + c^4)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*e^(-4)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*
d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^3\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3ab^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3a^2b\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*asinh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*asinh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*asinh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}{{\left(c\,e+d\,e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

3.146 $\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=87

$$\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(1+m)}-\frac{4b\text{Int}\left(\frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{1+(c+dx)^2}},x\right)}{e(1+m)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arcsinh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e/(1+m)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcSinh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x]
, x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\sinh^{-1}(c+dx)\right)^4}{de(1+m)}-\frac{(4b)\text{Subst}\left(\int \frac{(ex)^{1+m}(\cdots}{\sqrt{\cdots}}\right.}{de\cdots}$$

Mathematica [A]
time = 1.47, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^m (a + b \operatorname{arcsinh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] (d*x*e + c*e)^(m + 1)*a^4*e^(-1)/(d*(m + 1)) + (b^4*d*x*e^m + b^4*c*e^m)*(d
*x + c)^m*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d*(m + 1)) +
integrate(-2*(2*((b^4*c^2*e^m - ((m*e^m + e^m)*c^2 + m*e^m + e^m)*a*b^3 - (
(m*e^m + e^m)*a*b^3*d^2 - b^4*d^2*e^m)*x^2 - 2*((m*e^m + e^m)*a*b^3*c*d - b
^4*c*d*e^m)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(d*x + c)^m - (((m*e^m + e
^m)*c^3 + (m*e^m + e^m)*c)*a*b^3 - (c^3*e^m + c*e^m)*b^4 + ((m*e^m + e^m)*a
*b^3*d^3 - b^4*d^3*e^m)*x^3 + 3*((m*e^m + e^m)*a*b^3*c*d^2 - b^4*c*d^2*e^m)
*x^2 + ((3*(m*e^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a*b^3 - (3*c^2*d*e^m + d*
e^m)*b^4)*x)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^
3 - 3*(((m*e^m + e^m)*a^2*b^2*d^2*x^2 + 2*(m*e^m + e^m)*a^2*b^2*c*d*x + ((m
*e^m + e^m)*c^2 + m*e^m + e^m)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
d*x + c)^m + ((m*e^m + e^m)*a^2*b^2*d^3*x^3 + 3*(m*e^m + e^m)*a^2*b^2*c*d^2
*x^2 + (3*(m*e^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a^2*b^2*x + ((m*e^m + e^m)
*c^3 + (m*e^m + e^m)*c)*a^2*b^2)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))^2 - 2*(((m*e^m + e^m)*a^3*b*d^2*x^2 + 2*(m*e^m + e^m)*a
^3*b*c*d*x + ((m*e^m + e^m)*c^2 + m*e^m + e^m)*a^3*b)*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*(d*x + c)^m + ((m*e^m + e^m)*a^3*b*d^3*x^3 + 3*(m*e^m + e^m)*a
^3*b*c*d^2*x^2 + (3*(m*e^m + e^m)*c^2*d + (m*e^m + e^m)*d)*a^3*b*x + ((m*e^
m + e^m)*c^3 + (m*e^m + e^m)*c)*a^3*b)*(d*x + c)^m)*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)))/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2 + c^3*(m
+ 1) + c*(m + 1) + (3*c^2*d*(m + 1) + d*(m + 1))*x + (d^2*(m + 1)*x^2 + 2*c
*d*(m + 1)*x + c^2*(m + 1) + m + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*((d*x + c)*e)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m \left(a+b\operatorname{asinh}(c+dx)\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*asinh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*asinh(c + d*x))^4, x)
```

3.147 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=349

$$-\frac{45b^4e^3(c+dx)^2}{128d}+\frac{3b^4e^3(c+dx)^4}{128d}+\frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}-\frac{3b^3e^3(c+dx)^3}{}$$

```
[Out] -45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*a
rcsinh(d*x+c))^2/d-9/16*b^2*e^3*(d*x+c)^2*(a+b*arcsinh(d*x+c))^2/d+3/16*b^2
*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^2/d-3/32*e^3*(a+b*arcsinh(d*x+c))^4/d+1
/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^4/d+45/64*b^3*e^3*(d*x+c)*(a+b*arcsin
h(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))
*(1+(d*x+c)^2)^(1/2)/d+3/8*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^
2)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.43, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$-\frac{3b^3e^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{32d}+\frac{45b^3e^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{64d}+\frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d}-\frac{45b^2e^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{128d}-\frac{9b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d}-\frac{be^3(c+dx)^3\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d}+\frac{3be^3(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{8d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^4}{4d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^4}{32d}+\frac{3b^4e^3(c+dx)^4}{128d}-\frac{45b^4e^3(c+dx)^2}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) + (45*b
^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(64*d) - (
3*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(32*d
) - (45*b^2*e^3*(a + b*ArcSinh[c + d*x])^2)/(128*d) - (9*b^2*e^3*(c + d*x)^
2*(a + b*ArcSinh[c + d*x])^2)/(16*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcSin
h[c + d*x])^2)/(16*d) + (3*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*Arc
Sinh[c + d*x])^3)/(8*d) - (b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*A
rcSinh[c + d*x])^3)/(4*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^4)/(32*d) + (e^
3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^4}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{x^4(a+b\sinh}{\sqrt{1+}}\right.}{d}$$

$$= -\frac{be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{4d} + \frac{e^3(c}{}$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^2}{16d} + \frac{3be^3(c+dx)\sqrt{1+(c}}{}$$

$$= -\frac{3b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{32d} - \frac{9b^2e}{}$$

$$= \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}\right.}{64d}$$

$$= -\frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} + \frac{45b^3e^3(c+dx)\sqrt{1+(c}}{}$$

Mathematica [A]
time = 0.40, size = 475, normalized size = 1.36

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*(-9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
 + d*x)^4 + 2*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(24*a^2 + 45*b^2 - 2*(8*a
^2 + 3*b^2)*(c + d*x)^2) - 6*a*b*(8*a^2 + 15*b^2)*ArcSinh[c + d*x] + 2*b*(c
 + d*x)*(-72*a*b^2*(c + d*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 +
72*a^2*b*Sqrt[1 + (c + d*x)^2] + 45*b^3*Sqrt[1 + (c + d*x)^2] - 48*a^2*b*(c
 + d*x)^2*Sqrt[1 + (c + d*x)^2] - 6*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*
ArcSinh[c + d*x] + 3*b^2*(-24*a^2 - 15*b^2 - 24*b^2*(c + d*x)^2 + 64*a^2*(c
 + d*x)^4 + 8*b^2*(c + d*x)^4 + 48*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 32
*a*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 + 16*b^3*(-3*a +
```

```
 8*a*(c + d*x)^4 + 3*b*(c + d*x)*Sqrt[1 + (c + d*x)^2] - 2*b*(c + d*x)^3*Sq
rt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + 4*b^4*(-3 + 8*(c + d*x)^4)*ArcSin
h[c + d*x]^4))/(128*d)
```

Maple [A]
time = 4.32, size = 639, normalized size = 1.83

method	result	size
default	Expression too large to display	639

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^3*(d*x+c)^4*a^4/d-1/1024*e^3*b^4*(-64*cosh(2*arcsinh(d*x+c))^2*arcsin
h(d*x+c)^4+64*sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^
3+128*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^4-256*sinh(2*arcsinh(d*x+c))*ar
csinh(d*x+c)^3-48*cosh(2*arcsinh(d*x+c))^2*arcsinh(d*x+c)^2+32*arcsinh(d*x+
c)^4+24*sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)+384*co
sh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^2-384*arcsinh(d*x+c)*sinh(2*arcsinh(d*x
+c))-6*cosh(2*arcsinh(d*x+c))^2+24*arcsinh(d*x+c)^2+192*cosh(2*arcsinh(d*x+
c))+3)/d-1/128*e^3*a*b^3*(-32*cosh(2*arcsinh(d*x+c))^2*arcsinh(d*x+c)^3+24*
sinh(2*arcsinh(d*x+c))*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^2+64*cosh(2*ar
csinh(d*x+c))*arcsinh(d*x+c)^3-96*arcsinh(d*x+c)^2*sinh(2*arcsinh(d*x+c))-1
2*arcsinh(d*x+c)*cosh(2*arcsinh(d*x+c))^2+16*arcsinh(d*x+c)^3+3*sinh(2*arcs
inh(d*x+c))*cosh(2*arcsinh(d*x+c))+96*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)
-48*sinh(2*arcsinh(d*x+c))+6*arcsinh(d*x+c))/d-3/128*e^3*a^2*b^2*(-16*cosh(
2*arcsinh(d*x+c))^2*arcsinh(d*x+c)^2+8*sinh(2*arcsinh(d*x+c))*cosh(2*arcsin
h(d*x+c))*arcsinh(d*x+c)+32*cosh(2*arcsinh(d*x+c))*arcsinh(d*x+c)^2-32*arcs
inh(d*x+c)*sinh(2*arcsinh(d*x+c))-2*cosh(2*arcsinh(d*x+c))^2+8*arcsinh(d*x+
c)^2+16*cosh(2*arcsinh(d*x+c))+1)/d+4*e^3*a^3*b/d*(1/4*(d*x+c)^4*arcsinh(d*
x+c)-1/16*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+3/32*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/
32*arcsinh(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*d^3*x^4*e^3 + a^4*c*d^2*x^3*e^3 + 3/2*a^4*c^2*d*x^2*e^3 + 3*(2*x^2*
arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^
2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcs
inh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^
```

```
2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arcsinh(d*x + c
) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*
x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*c*x/d^3 + 9*(c^2 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2
 + 4*(c^2 + 1)*d^2))/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)/d^4))*a^3*b*c*d^2*e^3 + 1/24*(
24*x^4*arcsinh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*x^3/d^2 - 14
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x^2/d^3 + 105*c^4*arcsinh(2*(d^2*x + c
*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*c^2*x/d^4 - 90*(c^2 + 1)*c^2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^
2 + 4*(c^2 + 1)*d^2))/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^3/d^5 -
 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*x/d^4 + 9*(c^2 + 1)^2*arcsin
h(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^5 + 55*sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)*(c^2 + 1)*c/d^5)*d)*a^3*b*d^3*e^3 + a^4*c^3*x*e^3 + 4
*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^3*e^3/d + 1/4
*(b^4*d^3*x^4*e^3 + 4*b^4*c*d^2*x^3*e^3 + 6*b^4*c^2*d*x^2*e^3 + 4*b^4*c^3*x
*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + integrate((((4*a
*b^3*d^6 - b^4*d^6)*x^6*e^3 + 6*(4*a*b^3*c*d^5 - b^4*c*d^5)*x^5*e^3 + 4*(c^
6 + c^4)*a*b^3*e^3 + (4*(15*c^2*d^4 + d^4)*a*b^3 - (15*c^2*d^4 + d^4)*b^4)*
x^4*e^3 + 4*(4*(5*c^3*d^3 + c*d^3)*a*b^3 - (5*c^3*d^3 + c*d^3)*b^4)*x^3*e^3
 + 2*(6*(5*c^4*d^2 + 2*c^2*d^2)*a*b^3 - (7*c^4*d^2 + 3*c^2*d^2)*b^4)*x^2*e^
3 + 4*(2*(3*c^5*d + 2*c^3*d)*a*b^3 - (c^5*d + c^3*d)*b^4)*x*e^3 + ((4*a*b^3
*d^5 - b^4*d^5)*x^5*e^3 + 4*(c^5 + c^3)*a*b^3*e^3 + 5*(4*a*b^3*c*d^4 - b^4*
c*d^4)*x^4*e^3 - 2*(5*b^4*c^2*d^3 - 2*(10*c^2*d^3 + d^3)*a*b^3)*x^3*e^3 - 2
*(5*b^4*c^3*d^2 - 2*(10*c^3*d^2 + 3*c*d^2)*a*b^3)*x^2*e^3 - 4*(b^4*c^4*d -
(5*c^4*d + 3*c^2*d)*a*b^3)*x*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 6*(a^2*b^2*d^6*x^6*e^3 + 6*a
^2*b^2*c*d^5*x^5*e^3 + (15*c^2*d^4 + d^4)*a^2*b^2*x^4*e^3 + 4*(5*c^3*d^3 +
c*d^3)*a^2*b^2*x^3*e^3 + 3*(5*c^4*d^2 + 2*c^2*d^2)*a^2*b^2*x^2*e^3 + 2*(3*c
^5*d + 2*c^3*d)*a^2*b^2*x*e^3 + (c^6 + c^4)*a^2*b^2*e^3 + (a^2*b^2*d^5*x^5*
e^3 + 5*a^2*b^2*c*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)*a^2*b^2*x^3*e^3 + (10*c^
3*d^2 + 3*c*d^2)*a^2*b^2*x^2*e^3 + (5*c^4*d + 3*c^2*d)*a^2*b^2*x*e^3 + (c^5
 + c^3)*a^2*b^2*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4046 vs. $2(308) = 616$.
time = 0.50, size = 4046, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

[Out] 1/128*(4*((8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)^3 + 3*(8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)^2*sinh(1) + 3*(8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*cosh(1)*sinh(1)^2 + (8*b^4*d^4*x^4 + 32*b^4*c*d^3*x^3 + 48*b^4*c^2*d^2*x^2 + 32*b^4*c^3*d*x + 8*b^4*c^4 - 3*b^4)*sinh(1)^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + ((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*cosh(1)^3 + 16*((8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*cosh(1)^3 + 3*(8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*cosh(1)^2*sinh(1) + 3*(8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*cosh(1)*sinh(1)^2 + (8*a*b^3*d^4*x^4 + 32*a*b^3*c*d^3*x^3 + 48*a*b^3*c^2*d^2*x^2 + 32*a*b^3*c^3*d*x + 8*a*b^3*c^4 - 3*a*b^3)*sinh(1)^3 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 - 3*b^4*c + 3*(2*b^4*c^2 - b^4)*d*x)*cosh(1)^3 + 3*(2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 - 3*b^4*c + 3*(2*b^4*c^2 - b^4)*d*x)*cosh(1)^2*sinh(1) + 3*(2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 - 3*b^4*c + 3*(2*b^4*c^2 - b^4)*d*x)*cosh(1)*sinh(1)^2 + (2*b^4*d^3*x^3 + 6*b^4*c*d^2*x^2 + 2*b^4*c^3 - 3*b^4*c + 3*(2*b^4*c^2 - b^4)*d*x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*cosh(1)^2*sinh(1) + 3*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*cosh(1)*sinh(1)^2 + ((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*x^4 + 4*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c*d^3*x^3 - 3*(24*a^2*b^2 + 15*b^4 - 2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^2)*d^2*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 - 9*(8*a^2*b^2 + 5*b^4)*c)*d*x)*sinh(1)^3 + 3*((8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 - 24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*(b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 - 15*b^4 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 - 24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*(b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 - 15*b^4 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*(8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 - 24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*(b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 - 15*b^4 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*cosh(1)*sinh(1)^2 + (8*(8*a^2*b^2 + b^4)*d^4*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*x^3 - 24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*(b^4 - 2*(8*a^2*b^2 + b^4)*c^2)*d^2*x^2 - 24*a^2*b^2 - 15*b^4 - 16*(3*b^4*c - 2*(8*a^2*b^2 + b^4)*c^3)*d*x)*si

```
nh(1)^3 - 16*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((2*a*b^3*d^3*x^3 + 6*a*b^3*
c*d^2*x^2 + 2*a*b^3*c^3 - 3*a*b^3*c + 3*(2*a*b^3*c^2 - a*b^3)*d*x)*cosh(1)^
3 + 3*(2*a*b^3*d^3*x^3 + 6*a*b^3*c*d^2*x^2 + 2*a*b^3*c^3 - 3*a*b^3*c + 3*(2
*a*b^3*c^2 - a*b^3)*d*x)*cosh(1)^2*sinh(1) + 3*(2*a*b^3*d^3*x^3 + 6*a*b^3*c
*d^2*x^2 + 2*a*b^3*c^3 - 3*a*b^3*c + 3*(2*a*b^3*c^2 - a*b^3)*d*x)*cosh(1)*s
inh(1)^2 + (2*a*b^3*d^3*x^3 + 6*a*b^3*c*d^2*x^2 + 2*a*b^3*c^3 - 3*a*b^3*c +
 3*(2*a*b^3*c^2 - a*b^3)*d*x)*sinh(1)^3))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))^2 + 2*((8*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a*b
^3)*c*d^3*x^3 - 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 - 24*(3*a*b^3 - 2*
(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3 - 16*(9*a*b^3*c - 2*
(8*a^3*b + 3*a*b^3)*c^3)*d*x)*cosh(1)^3 + 3*(8*(8*a^3*b + 3*a*b^3)*d^4*x^4
+ 32*(8*a^3*b + 3*a*b^3)*c*d^3*x^3 - 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c
^4 - 24*(3*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3
 - 16*(9*a*b^3*c - 2*(8*a^3*b + 3*a*b^3)*c^3)*d*x)*cosh(1)^2*sinh(1) + 3*(8
*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*x^3 - 72*a*b^3*
c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 - 24*(3*a*b^3 - 2*(8*a^3*b + 3*a*b^3)*c^2)*
d^2*x^2 - 24*a^3*b - 45*a*b^3 - 16*(9*a*b^3*c - 2*(8*a^3*b + 3*a*b^3)*c^3)*
d*x)*cosh(1)*sinh(1)^2 + (8*(8*a^3*b + 3*a*b^3)*d^4*x^4 + 32*(8*a^3*b + 3*a
*b^3)*c*d^3*x^3 - 72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 - 24*(3*a*b^3 -
2*(8*a^3*b + 3*a*b^3)*c^2)*d^2*x^2 - 24*a^3*b - 45*a*b^3 - 16*(9*a*b^3*c -
2*(8*a^3*b + 3*a*b^3)*c^3)*d*x)*sinh(1)^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*((2*(8*a^2*b^2 + b^4)*d^3*x^3 + 6*(8*a^2*b...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2876 vs. $2(325) = 650$.

time = 1.48, size = 2876, normalized size = 8.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*asinh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*asinh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*asinh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*as
inh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/
4 + 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(8*d) + a**3*b*d**
3*e**3*x**4*asinh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)/4 + 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/8 - 3
*a**3*b*e**3*asinh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*asinh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*asinh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*asinh(c + d*x)**2 + 9*a**2*b**2*c
```

```
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*asinh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 + 1)*asinh(c + d*x)/4 - 9*a**2*b**2*c*e**3*x/8 + 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*asinh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*
x)/4 - 9*a**2*b**2*d*e**3*x**2/16 + 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/8 - 9*a**2*b**2*e**3*asinh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*asinh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*asinh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*asinh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*as
inh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*as
inh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*asinh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*asinh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)/32 - 9*a*b**3*c**2*e**3*asinh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*asinh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*asinh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32
 - 9*a*b**3*c*e**3*x*asinh(c + d*x)/4 + 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 + 1)*asinh(c + d*x)**2/(8*d) + 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 + 1)/(64*d) + a*b**3*d**3*e**3*x**4*asinh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*asinh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/32 - 9*a*b**3*d*e**3*x**2*asinh(c + d*x
)/8 + 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**
2/8 + 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/64 - 3*a*b**3*e
**3*asinh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*asinh(c + d*x)/(64*d) + b**4*c
**4*e**3*asinh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*asinh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*asinh(c + d*x)**4 + 3*b**4*c**3*e**3*x*asinh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 + 1)*asinh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*asinh(c + d*x)
**4/2 + 9*b**4*c**2*d*e**3*x**2*asinh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)/32 - 9*b**4*c**2*e**3*asinh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*asinh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*asinh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 + 1)*asinh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*c*e**3*x*asinh(c + d*x)**2/8 - 45*b**
4*c*e**3*x/64 + 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c
+ d*x)**3/(8*d) + 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*asinh(c + d*x)**4/4 + 3*b**4*d**3*e*
```

```
*3*x**4*asinh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/32 - 9*b**4*d
*e**3*x**2*asinh(c + d*x)**2/16 - 45*b**4*d*e**3*x**2/128 + 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/8 + 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/64 - 3*b**4*e**3*asinh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*asinh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*asinh(c))**4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^4 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^4, x)
```

3.148 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=281

$$-\frac{160}{27}b^4e^2x+\frac{8b^4e^2(c+dx)^3}{81d}+\frac{160b^3e^2\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)}}{27d}$$

```
[Out] -160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d-8/3*b^2*e^2*(d*x+c)*(a+b*arcsinh
(d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^2/d+1/3*e^2*(d*x+c)
^3*(a+b*arcsinh(d*x+c))^4/d+160/27*b^3*e^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^
2)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/
d+8/9*b*e^2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d-4/9*b*e^2*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5776, 5812, 5798, 5772, 8, 30}

$$\frac{160b^3e^2\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)}{27d}-\frac{8b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)}{27d}+\frac{4b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{9d}-\frac{8b^2e^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{3d}+\frac{8be^2\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)^3}{9d}-\frac{4be^2(c+dx)^2\sqrt{(c+dx)^2+1}\,\left(a+b\sinh^{-1}(c+dx)\right)^3}{9d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^4}{3d}+\frac{8b^4e^2(c+dx)^3}{81d}-\frac{160}{27}b^4e^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (-160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) + (160*b^3*e^2*Sqrt[1
+ (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^3*e^2*(c + d*x)^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/(27*d) - (8*b^2*e^2*(c + d*x)
*(a + b*ArcSinh[c + d*x])^2)/(3*d) + (4*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[
c + d*x])^2)/(9*d) + (8*b*e^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]
)^3)/(9*d) - (4*b*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c +
d*x])^3)/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^4)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^4}{3d} - \frac{(4be^2)\,\text{Subst}\left(\int \frac{x^3(a+b\sinh^{-1}}{\sqrt{1+}}\right.}{3d}$$

$$= -\frac{4be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9d} + \frac{e^2(c-}{}$$

$$= \frac{4b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^2}{9d} + \frac{8be^2\sqrt{1+(c+dx)^2}\,(}{9}$$

$$= -\frac{8b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{27d} - \frac{8b^2e^2}{}$$

$$= \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{27d}$$

$$= -\frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} + \frac{160b^3e^2\sqrt{1+(c+dx)^2}\,(a+b\,\text{si}}{27d}$$

Mathematica [A]
time = 0.28, size = 412, normalized size = 1.47

$$\frac{e^2\left(-24b^2(9a^2+20b^2)(c+dx)+(27a^4+36a^2b^2+8b^4)(c+dx)^3+12ab\sqrt{1+(c+dx)^2}\left(6a^2+40b^2-(3a^2+2b^2)(c+dx)^2\right)+12b\left(-36ab^2(c+dx)+9a^3(c+dx)^3+6ab^2(c+dx)^3+18a^2b\sqrt{1+(c+dx)^2}+40b^3\sqrt{1+(c+dx)^2}-9a^2b(c+dx)^2\sqrt{1+(c+dx)^2}-2b^3(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+18b^2\left(-12b^2(c+dx)+9a^2(c+dx)^3+2b^2(c+dx)^3+12ab\sqrt{1+(c+dx)^2}-6ab(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2-36b^3\left(-3a(c+dx)^3-2b\sqrt{1+(c+dx)^2}+b(c+dx)^2\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^3+27b^4(c+dx)^3\sinh^{-1}(c+dx)^4\right)}{81d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*(-24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
 + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2]*(6*a^2 + 40*b^2 - (3*a^2 + 2*b^2)*
(c + d*x)^2) + 12*b*(-36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3 + 6*a*b^2*(c +
 d*x)^3 + 18*a^2*b*Sqrt[1 + (c + d*x)^2] + 40*b^3*Sqrt[1 + (c + d*x)^2] - 9
*a^2*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] - 2*b^3*(c + d*x)^2*Sqrt[1 + (c +
d*x)^2])*ArcSinh[c + d*x] + 18*b^2*(-12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 +
 2*b^2*(c + d*x)^3 + 12*a*b*Sqrt[1 + (c + d*x)^2] - 6*a*b*(c + d*x)^2*Sqrt[
1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 - 36*b^3*(-3*a*(c + d*x)^3 - 2*b*Sqrt[
1 + (c + d*x)^2] + b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3
+ 27*b^4*(c + d*x)^3*ArcSinh[c + d*x]^4))/(81*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. $2(255) = 510$.
time = 4.78, size = 1124, normalized size = 4.00

method	result	size
default	Expression too large to display	1124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*(d*x+c)^3*a^4/d+1/81*e^2*b^4*(27*arcsinh(d*x+c)^4*x^3*d^3+81*arcsin
h(d*x+c)^4*x^2*c*d^2+81*arcsinh(d*x+c)^4*x*c^2*d-36*arcsinh(d*x+c)^3*(d^2*x
^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+36*arcsinh(d*x+c)^2*x^3*d^3+27*arcsinh(d*x+
c)^4*c^3-72*arcsinh(d*x+c)^3*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d+108*arcsin
h(d*x+c)^2*x^2*c*d^2-36*arcsinh(d*x+c)^3*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*c^2+
108*arcsinh(d*x+c)^2*x*c^2*d-24*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2
)*x^2*d^2+8*d^3*x^3+36*arcsinh(d*x+c)^2*c^3-48*arcsinh(d*x+c)*(d^2*x^2+2*c*
d*x+c^2+1)^(1/2)*x*c*d+24*x^2*c*d^2+72*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*arcsin
h(d*x+c)^3-216*arcsinh(d*x+c)^2*x*d-24*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+
1)^(1/2)*c^2+24*x*c^2*d-216*arcsinh(d*x+c)^2*c+8*c^3+480*arcsinh(d*x+c)*(d^
2*x^2+2*c*d*x+c^2+1)^(1/2)-480*d*x-480*c)/d+4/27*e^2*a*b^3*(9*arcsinh(d*x+c
)^3*x^3*d^3+27*arcsinh(d*x+c)^3*x^2*c*d^2+27*arcsinh(d*x+c)^3*x*c^2*d-9*arc
sinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d^2+6*arcsinh(d*x+c)*x^3*d^
3+9*arcsinh(d*x+c)^3*c^3-18*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*
x*c*d+18*arcsinh(d*x+c)*x^2*c*d^2-9*arcsinh(d*x+c)^2*(d^2*x^2+2*c*d*x+c^2+1
)^(1/2)*c^2+18*arcsinh(d*x+c)*x*c^2*d-2*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x^2*d
^2+6*arcsinh(d*x+c)*c^3-4*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*x*c*d+18*(d^2*x^2+2
*c*d*x+c^2+1)^(1/2)*arcsinh(d*x+c)^2-36*arcsinh(d*x+c)*x*d-2*(d^2*x^2+2*c*d
*x+c^2+1)^(1/2)*c^2-36*arcsinh(d*x+c)*c+40*(d^2*x^2+2*c*d*x+c^2+1)^(1/2))/d
+2/9*e^2*a^2*b^2*(9*arcsinh(d*x+c)^2*x^3*d^3+27*arcsinh(d*x+c)^2*x^2*c*d^2+
27*arcsinh(d*x+c)^2*x*c^2*d-6*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)*
x^2*d^2+2*d^3*x^3+9*arcsinh(d*x+c)^2*c^3-12*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x
+c^2+1)^(1/2)*x*c*d+6*x^2*c*d^2-6*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1
/2)*c^2+6*x*c^2*d+2*c^3+12*arcsinh(d*x+c)*(d^2*x^2+2*c*d*x+c^2+1)^(1/2)-12*
d*x-12*c)/d+4*e^2*a^3*b/d*(1/3*(d*x+c)^3*arcsinh(d*x+c)-1/9*(d*x+c)^2*(1+(d
*x+c)^2)^(1/2)+2/9*(1+(d*x+c)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*d^2*x^3*e^2 + a^4*c*d*x^2*e^2 + 2*(2*x^2*arcsinh(d*x + c) - d*(3*c^
2*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-
4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d
^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arcsinh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)*x^2/d^2 - 15*c^3*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4
*(c^2 + 1)*d^2))/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c*x/d^3 + 9*(c^2
 + 1)*c*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^4 + 1
5*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1)*(c^2 + 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*x*e^2 + 4*((d*x + c)*arcsinh
(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^2*x^3*e^2 +
 3*b^4*c*d*x^2*e^2 + 3*b^4*c^2*x*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^4 + integrate(2/3*(2*((3*a*b^3*d^5 - b^4*d^5)*x^5*e^2 + 3*(c^5
+ c^3)*a*b^3*e^2 + 5*(3*a*b^3*c*d^4 - b^4*c*d^4)*x^4*e^2 + (3*(10*c^2*d^3 +
 d^3)*a*b^3 - (10*c^2*d^3 + d^3)*b^4)*x^3*e^2 + 3*((10*c^3*d^2 + 3*c*d^2)*a
*b^3 - (3*c^3*d^2 + c*d^2)*b^4)*x^2*e^2 + 3*((5*c^4*d + 3*c^2*d)*a*b^3 - (c
^4*d + c^2*d)*b^4)*x*e^2 + (3*(c^4 + c^2)*a*b^3*e^2 + (3*a*b^3*d^4 - b^4*d^
4)*x^4*e^2 + 4*(3*a*b^3*c*d^3 - b^4*c*d^3)*x^3*e^2 - 3*(2*b^4*c^2*d^2 - (6*
c^2*d^2 + d^2)*a*b^3)*x^2*e^2 - 3*(b^4*c^3*d - 2*(2*c^3*d + c*d)*a*b^3)*x*e
^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3 + 9*(a^2*b^2*d^5*x^5*e^2 + 5*a^2*b^2*c*d^4*x^4*e^2 + (10*c^2
*d^3 + d^3)*a^2*b^2*x^3*e^2 + (10*c^3*d^2 + 3*c*d^2)*a^2*b^2*x^2*e^2 + (5*c
^4*d + 3*c^2*d)*a^2*b^2*x*e^2 + (c^5 + c^3)*a^2*b^2*e^2 + (a^2*b^2*d^4*x^4*
e^2 + 4*a^2*b^2*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*a^2*b^2*x^2*e^2 + 2*(2*c^
3*d + c*d)*a^2*b^2*x*e^2 + (c^4 + c^2)*a^2*b^2*e^2)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^3 +
3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
 c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(246) = 492$.
time = 0.46, size = 2103, normalized size = 7.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*((b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(1)
^2 + 2*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(1)*si
nh(1) + (b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*sinh(1)^2
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 36*(3*(a*b^3*d^3*x^3
 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*cosh(1)^2 + 6*(a*b^3*d^
3*x^3 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*cosh(1)*sinh(1) +
3*(a*b^3*d^3*x^3 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x + a*b^3*c^3)*sinh(1)
```

```
^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^
2 - 2*b^4)*cosh(1)^2 + 2*(b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2 - 2*b^4)*cosh
(1)*sinh(1) + (b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2 - 2*b^4)*sinh(1)^2))*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + ((27*a^4 + 36*a^2*b^2 + 8
*b^4)*d^3*x^3 + 3*(27*a^4 + 36*a^2*b^2 + 8*b^4)*c*d^2*x^2 - 3*(72*a^2*b^2 +
 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8*b^4)*c^2)*d*x)*cosh(1)^2 + 18*(((9*a^2*
b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 - 12*b^4*c + (9*a^2*
b^2 + 2*b^4)*c^3 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*x)*cosh(1)^2 + 2*(
(9*a^2*b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 - 12*b^4*c +
(9*a^2*b^2 + 2*b^4)*c^3 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*x)*cosh(1)*
sinh(1) + ((9*a^2*b^2 + 2*b^4)*d^3*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d^2*x^2 -
12*b^4*c + (9*a^2*b^2 + 2*b^4)*c^3 - 3*(4*b^4 - (9*a^2*b^2 + 2*b^4)*c^2)*d*
x)*sinh(1)^2 - 6*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b^3*d^2*x^2 + 2*a*b^
3*c*d*x + a*b^3*c^2 - 2*a*b^3)*cosh(1)^2 + 2*(a*b^3*d^2*x^2 + 2*a*b^3*c*d*x
 + a*b^3*c^2 - 2*a*b^3)*cosh(1)*sinh(1) + (a*b^3*d^2*x^2 + 2*a*b^3*c*d*x +
a*b^3*c^2 - 2*a*b^3)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))^2 + 2*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*x^3 + 3*(27*a^4 + 36*a^2*b^
2 + 8*b^4)*c*d^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*x)*cosh(1)*sinh(1) + ((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*x^3 + 3*
(27*a^4 + 36*a^2*b^2 + 8*b^4)*c*d^2*x^2 - 3*(72*a^2*b^2 + 160*b^4 - (27*a^4
 + 36*a^2*b^2 + 8*b^4)*c^2)*d*x)*sinh(1)^2 + 12*(3*((3*a^3*b + 2*a*b^3)*d^3
*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 - 12*a*b^3*c + (3*a^3*b + 2*a*b^3)*c
^3 - 3*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)*d*x)*cosh(1)^2 + 6*((3*a^3*b + 2
*a*b^3)*d^3*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 - 12*a*b^3*c + (3*a^3*b +
 2*a*b^3)*c^3 - 3*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)*d*x)*cosh(1)*sinh(1)
+ 3*((3*a^3*b + 2*a*b^3)*d^3*x^3 + 3*(3*a^3*b + 2*a*b^3)*c*d^2*x^2 - 12*a*b
^3*c + (3*a^3*b + 2*a*b^3)*c^3 - 3*(4*a*b^3 - (3*a^3*b + 2*a*b^3)*c^2)*d*x)
*sinh(1)^2 - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(((9*a^2*b^2 + 2*b^4)*d^2*x^
2 - 18*a^2*b^2 - 40*b^4 + 2*(9*a^2*b^2 + 2*b^4)*c*d*x + (9*a^2*b^2 + 2*b^4)
*c^2)*cosh(1)^2 + 2*((9*a^2*b^2 + 2*b^4)*d^2*x^2 - 18*a^2*b^2 - 40*b^4 + 2*
(9*a^2*b^2 + 2*b^4)*c*d*x + (9*a^2*b^2 + 2*b^4)*c^2)*cosh(1)*sinh(1) + ((9*
a^2*b^2 + 2*b^4)*d^2*x^2 - 18*a^2*b^2 - 40*b^4 + 2*(9*a^2*b^2 + 2*b^4)*c*d*
x + (9*a^2*b^2 + 2*b^4)*c^2)*sinh(1)^2))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)) - 12*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(((3*a^3*b + 2*a*b^3)
*d^2*x^2 - 6*a^3*b - 40*a*b^3 + 2*(3*a^3*b + 2*a*b^3)*c*d*x + (3*a^3*b + 2*
a*b^3)*c^2)*cosh(1)^2 + 2*((3*a^3*b + 2*a*b^3)*d^2*x^2 - 6*a^3*b - 40*a*b^3
 + 2*(3*a^3*b + 2*a*b^3)*c*d*x + (3*a^3*b + 2*a*b^3)*c^2)*cosh(1)*sinh(1) +
 ((3*a^3*b + 2*a*b^3)*d^2*x^2 - 6*a^3*b - 40*a*b^3 + 2*(3*a^3*b + 2*a*b^3)*
c*d*x + (3*a^3*b + 2*a*b^3)*c^2)*sinh(1)^2))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(264) = 528$.
time = 0.92, size = 1889, normalized size = 6.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**4,x)

[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 + 4*a**3*b*c**3*e**2*asinh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*asinh(c + d*x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 4*a**3*b*c*d*e**2*x**2*asinh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 4*a**3*b*d**2*e**2*x**3*asinh(c + d*x)/3 - 4*a**3*b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/9 + 8*a**3*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(9*d) + 2*a**2*b**2*c**3*e**2*asinh(c + d*x)**2/d + 6*a**2*b**2*c**2*e**2*x*asinh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 6*a**2*b**2*c*d*e**2*x**2*asinh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*asinh(c + d*x)**2 + 4*a**2*b**2*d**2*e**2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/3 - 8*a**2*b**2*e**2*x/3 + 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*asinh(c + d*x)**3/(3*d) + 8*a*b**3*c**3*e**2*asinh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*asinh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*asinh(c + d*x)/3 - 4*a*b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) - 8*a*b**3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + 4*a*b**3*c*d*e**2*x**2*asinh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*asinh(c + d*x)/3 - 8*a*b**3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 16*a*b**3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 16*a*b**3*c*e**2*asinh(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*asinh(c + d*x)**3/3 + 8*a*b**3*d**2*e**2*x**3*asinh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/27 - 16*a*b**3*e**2*x*asinh(c + d*x)/3 + 8*a*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2/(3*d) + 160*a*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(27*d) + b**4*c**3*e**2*asinh(c + d*x)**4/(3*d) + 4*b**4*c**3*e**2*asinh(c + d*x)**2/(9*d) + b**4*c**2*e**2*x*asinh(c + d*x)**4 + 4*b**4*c**2*e**2*x*asinh(c + d*x)**2/3 + 8*b**4*c**2*e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*asinh(c + d*x)**4 + 4*b**4*c*d*e**2*x**2*asinh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*c*e**2*asinh(c + d*x)**2/(3*d) + b**4*d**2*e**2*x**3*asinh(c + d*x)**4/3 + 4*b**4*d**2*e**2*x**3*asinh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/9 - 8*b**4*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/27 - 8*b**4*e**2*x*asinh(c + d*x)**2/3 - 160*b**4*e**2*x/27 + 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +

```
d**2*x**2 + 1)*asinh(c + d*x)**3/(9*d) + 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 + 1)*asinh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*asin
h(c))**4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{asinh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^4, x)
```

3.149 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=195

$$\frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}+\frac{3b^2e(c+$$

```
[Out] 3/4*b^4*e*(d*x+c)^2/d+3/4*b^2*e*(a+b*arcsinh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arcsinh(d*x+c))^2/d+1/4*e*(a+b*arcsinh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(
a+b*arcsinh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2
)^(1/2)/d-b*e*(d*x+c)*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5776, 5812, 5783, 30}

$$-\frac{3b^3e(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d}+\frac{3b^2e\left(a+b\sinh^{-1}(c+dx)\right)^2}{4d}-\frac{be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^4}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^4}{4d}+\frac{3b^4e(c+dx)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(a +
 b*ArcSinh[c + d*x]))/(2*d) + (3*b^2*e*(a + b*ArcSinh[c + d*x])^2)/(4*d) +
(3*b^2*e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^2)/(2*d) - (b*e*(c + d*x)*Sqr
t[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/d + (e*(a + b*ArcSinh[c + d*
x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^4)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^4}{2d} - \frac{(2be)\text{Subst}\left(\int \frac{x^2(a+b\sinh^{-1}(x}{\sqrt{1+x^2}}\right.}{d}$$

$$= -\frac{be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d} + \frac{e(c+dx)}{}$$

$$= \frac{3b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{2d} - \frac{be(c+dx)\sqrt{1+(c+dx}}{}$$

$$= -\frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{2d} + \frac{3b^2e(c+}{}$$

$$= \frac{3b^4e(c+dx)^2}{4d} - \frac{3b^3e(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+d\right.}{2d}$$

Mathematica [A]
time = 0.19, size = 300, normalized size = 1.54

$$\frac{e\left((2a^4+6a^2b^2+3b^4)(c+dx)^2-2ab(2a^2+3b^2)(c+dx)\sqrt{1+(c+dx)^2}+2ab(2a^2+3b^2)\sinh^{-1}(c+dx)-2b(c+dx)\left(-4a^3(c+dx)-6ab^2(c+dx)+6a^2b\sqrt{1+(c+dx)^2}+3b^3\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+3b^2\left(2a^2+b^2+4a^2(c+dx)^2+2b^2(c+dx)^2-4ab(c+dx)\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2+4b^3\left(a+2a(c+dx)^2-b(c+dx)\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^3+b^4(1+2(c+dx)^2)\sinh^{-1}(c+dx)^4\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*(c + d*
x)*Sqrt[1 + (c + d*x)^2] + 2*a*b*(2*a^2 + 3*b^2)*ArcSinh[c + d*x] - 2*b*(c
+ d*x)*(-4*a^3*(c + d*x) - 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[1 + (c + d*x)^2
] + 3*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 3*b^2*(2*a^2 + b^2 + 4*
a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*(c + d*x)*Sqrt[1 + (c + d*x)^2]
)*ArcSinh[c + d*x]^2 + 4*b^3*(a + 2*a*(c + d*x)^2 - b*(c + d*x)*Sqrt[1 + (c
 + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(1 + 2*(c + d*x)^2)*ArcSinh[c + d*x]^4
))/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(179) = 358.
time = 2.56, size = 371, normalized size = 1.90

method	result
derivativedivides	$\frac{\frac{e(dx+c)^2a^4}{2}+e\,b^4\left(\frac{\operatorname{arcsinh}(dx+c)^4\left(1+(dx+c)^2\right)}{2}-\operatorname{arcsinh}(dx+c)^3(dx+c)\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinl}}{}\right.}{}$
default	$\frac{\frac{e(dx+c)^2a^4}{2}+e\,b^4\left(\frac{\operatorname{arcsinh}(dx+c)^4\left(1+(dx+c)^2\right)}{2}-\operatorname{arcsinh}(dx+c)^3(dx+c)\sqrt{1+(dx+c)^2}-\frac{\operatorname{arcsinh}(dx+c)^4}{4}+\frac{3\operatorname{arcsinl}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*e*(d*x+c)^2*a^4+e*b^4*(1/2*arcsinh(d*x+c)^4*(1+(d*x+c)^2)-arcsinh(
d*x+c)^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^4+3/2*arcsinh(d*x+c
)^2*(1+(d*x+c)^2)-3/2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)*(d*x+c)-3/4*arcsin
h(d*x+c)^2+3/4*(d*x+c)^2+3/4)+4*e*a*b^3*(1/2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)
-3/4*arcsinh(d*x+c)^2*(d*x+c)*(1+(d*x+c)^2)^(1/2)-1/4*arcsinh(d*x+c)^3+3/4*
(1+(d*x+c)^2)*arcsinh(d*x+c)-3/8*(d*x+c)*(1+(d*x+c)^2)^(1/2)-3/8*arcsinh(d*
x+c))+6*e*a^2*b^2*(1/2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)-1/2*arcsinh(d*x+c)*(1
+(d*x+c)^2)^(1/2)*(d*x+c)-1/4*arcsinh(d*x+c)^2+1/4*(d*x+c)^2+1/4)+4*e*a^3*b
*(1/2*(d*x+c)^2*arcsinh(d*x+c)-1/4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1/4*arcsinh(
d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*a^4*d*x^2*e + (2*x^2*arcsinh(d*x + c) - d*(3*c^2*arcsinh(2*(d^2*x + c*d
)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*d^2))/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*x/d^2 - (c^2 + 1)*arcsinh(2*(d^2*x + c*d)/sqrt(-4*c^2*d^2 + 4*(c^2 + 1)*
d^2))/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*c/d^3))*a^3*b*d*e + a^4*c*x
*e + 4*((d*x + c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b*c*e/d + 1
/2*(b^4*d*x^2*e + 2*b^4*c*x*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4 + integrate(2*((2*(c^4 + c^2)*a*b^3*e + (2*a*b^3*d^4 - b^4*d^4)*x^4*
e + 4*(2*a*b^3*c*d^3 - b^4*c*d^3)*x^3*e + (2*(6*c^2*d^2 + d^2)*a*b^3 - (5*c
^2*d^2 + d^2)*b^4)*x^2*e + 2*(2*(2*c^3*d + c*d)*a*b^3 - (c^3*d + c*d)*b^4)*
x*e + (2*(c^3 + c)*a*b^3*e + (2*a*b^3*d^3 - b^4*d^3)*x^3*e + 3*(2*a*b^3*c*d
^2 - b^4*c*d^2)*x^2*e - 2*(b^4*c^2*d - (3*c^2*d + d)*a*b^3)*x*e)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3
```

```
 + 3*(a^2*b^2*d^4*x^4*e + 4*a^2*b^2*c*d^3*x^3*e + (6*c^2*d^2 + d^2)*a^2*b^2
*x^2*e + 2*(2*c^3*d + c*d)*a^2*b^2*x*e + (c^4 + c^2)*a^2*b^2*e + (a^2*b^2*d
^3*x^3*e + 3*a^2*b^2*c*d^2*x^2*e + (3*c^2*d + d)*a^2*b^2*x*e + (c^3 + c)*a^
2*b^2*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*
x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(186) = 372$.
time = 0.40, size = 949, normalized size = 4.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(((2*b^4*d^2*x^2 + 4*b^4*c*d*x + 2*b^4*c^2 + b^4)*cosh(1) + (2*b^4*d^2*
x^2 + 4*b^4*c*d*x + 2*b^4*c^2 + b^4)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1))^4 + 4*((2*a*b^3*d^2*x^2 + 4*a*b^3*c*d*x + 2*a*b^3*c^2 +
 a*b^3)*cosh(1) + (2*a*b^3*d^2*x^2 + 4*a*b^3*c*d*x + 2*a*b^3*c^2 + a*b^3)*s
inh(1) - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((b^4*d*x + b^4*c)*cosh(1) + (b^
4*d*x + b^4*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3
 + 3*((2*(2*a^2*b^2 + b^4)*d^2*x^2 + 2*a^2*b^2 + b^4 + 4*(2*a^2*b^2 + b^4)*
c*d*x + 2*(2*a^2*b^2 + b^4)*c^2)*cosh(1) + (2*(2*a^2*b^2 + b^4)*d^2*x^2 + 2
*a^2*b^2 + b^4 + 4*(2*a^2*b^2 + b^4)*c*d*x + 2*(2*a^2*b^2 + b^4)*c^2)*sinh(
1) - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b^3*d*x + a*b^3*c)*cosh(1) + (
a*b^3*d*x + a*b^3*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2 + ((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^
4)*c*d*x)*cosh(1) + 2*((2*(2*a^3*b + 3*a*b^3)*d^2*x^2 + 2*a^3*b + 3*a*b^3 +
 4*(2*a^3*b + 3*a*b^3)*c*d*x + 2*(2*a^3*b + 3*a*b^3)*c^2)*cosh(1) + (2*(2*a
^3*b + 3*a*b^3)*d^2*x^2 + 2*a^3*b + 3*a*b^3 + 4*(2*a^3*b + 3*a*b^3)*c*d*x +
 2*(2*a^3*b + 3*a*b^3)*c^2)*sinh(1) - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
((2*a^2*b^2 + b^4)*d*x + (2*a^2*b^2 + b^4)*c)*cosh(1) + ((2*a^2*b^2 + b^4)*
d*x + (2*a^2*b^2 + b^4)*c)*sinh(1)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)) + ((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*x^2 + 2*(2*a^4 + 6*a^2*b^2 +
3*b^4)*c*d*x)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(((2*a^3*b + 3*
a*b^3)*d*x + (2*a^3*b + 3*a*b^3)*c)*cosh(1) + ((2*a^3*b + 3*a*b^3)*d*x + (2
*a^3*b + 3*a*b^3)*c)*sinh(1)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(178) = 356$.
time = 0.54, size = 1027, normalized size = 5.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*asinh(c + d*x)/d
+ 4*a**3*b*c*e*x*asinh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 + 1)/d + 2*a**3*b*d*e*x**2*asinh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1) + a**3*b*e*asinh(c + d*x)/d + 3*a**2*b**2*c**2*e*asinh(c
 + d*x)**2/d + 6*a**2*b**2*c*e*x*asinh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*asinh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x) + 3*a**2*b**2*e*asinh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*asinh(c + d*x)**3/d + 3*a*b**3*c**2*e*
asinh(c + d*x)/d + 4*a*b**3*c*e*x*asinh(c + d*x)**3 + 6*a*b**3*c*e*x*asinh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/(2*d) + 2*a*b**3*
d*e*x**2*asinh(c + d*x)**3 + 3*a*b**3*d*e*x**2*asinh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 + 1)/2 + a*b**3*e*asinh(c + d*x)**3/d + 3*a*b*
*3*e*asinh(c + d*x)/(2*d) + b**4*c**2*e*asinh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*asinh(c + d*x)**2/(2*d) + b**4*c*e*x*asinh(c + d*x)**4 + 3*b**4*c*e*x*
asinh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 + 1)*asinh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 +
1)*asinh(c + d*x)/(2*d) + b**4*d*e*x**2*asinh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*asinh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 + 1)*asinh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 + 1)*asinh(c + d*x)/2 + b**4*e*asinh(c + d*x)**4/(4*d) + 3*b**4*e*asinh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*asinh(c))**4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^4, x)
```

3.150 $\int \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=115

$$24b^4x - \frac{24b^3\sqrt{1+(c+dx)^2}\,\left(a+b\sinh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{1+(c+dx}{}$$

```
[Out] 24*b^4*x+12*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^2/d+(d*x+c)*(a+b*arcsinh(d*x+c
))^4/d-24*b^3*(a+b*arcsinh(d*x+c))*(1+(d*x+c)^2)^(1/2)/d-4*b*(a+b*arcsinh(d
*x+c))^3*(1+(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5858, 5772, 5798, 8}

$$-\frac{24b^3\sqrt{(c+dx)^2+1}\,(a+b\sinh^{-1}(c+dx))}{d} + \frac{12b^2(c+dx)\,(a+b\sinh^{-1}(c+dx))^2}{d} - \frac{4b\sqrt{(c+dx)^2+1}\,(a+b\sinh^{-1}(c+dx))^3}{d} + \frac{(c+dx)\,(a+b\sinh^{-1}(c+dx))^4}{d} + 24b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x]))/d + (12*
b^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^2)/d - (4*b*Sqrt[1 + (c + d*x)^2]*(a
 + b*ArcSinh[c + d*x])^3)/d + ((c + d*x)*(a + b*ArcSinh[c + d*x])^4)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4}{d}-\frac{(4b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{1+x^2}}\,dx,x,c+\right.}{d}$$

$$=-\frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$=\frac{12b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{d}$$

$$=-\frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\sinh^{-}\right.}{d}$$

$$=24b^4x-\frac{24b^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+\right.}{d}$$

Mathematica [A]
time = 0.17, size = 226, normalized size = 1.97

$$\frac{\left(a^4+12a^2b^2+24b^4\right)(c+dx)-4ab\left(a^2+6b^2\right)\sqrt{1+(c+dx)^2}-4b\left(-a^3(c+dx)-6ab^2(c+dx)+3a^2b\sqrt{1+(c+dx)^2}+6b^3\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)+6b^2\left(a^2(c+dx)+2b^2(c+dx)-2ab\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^2-4b^3\left(-a(c+dx)+b\sqrt{1+(c+dx)^2}\right)\sinh^{-1}(c+dx)^3+b^4(c+dx)\sinh^{-1}(c+dx)^4}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[1 + (c +
d*x)^2] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*b*Sqrt[1 + (c +
 d*x)^2] + 6*b^3*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x] + 6*b^2*(a^2*(c +
d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^2 -
4*b^3*(-(a*(c + d*x)) + b*Sqrt[1 + (c + d*x)^2])*ArcSinh[c + d*x]^3 + b^4*(
c + d*x)*ArcSinh[c + d*x]^4)/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(111) = 222$.
time = 2.69, size = 245, normalized size = 2.13

method	result

derivativedivides	$\frac{(dx+c)a^4+b^4\left(\operatorname{arcsinh}(dx+c)^4(dx+c)-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12\operatorname{arcsinh}(dx+c)^2(dx+c)-24\operatorname{arcsinh}(\right.}{}$
default	$\frac{(dx+c)a^4+b^4\left(\operatorname{arcsinh}(dx+c)^4(dx+c)-4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}+12\operatorname{arcsinh}(dx+c)^2(dx+c)-24\operatorname{arcsinh}(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*(arcsinh(d*x+c)^4*(d*x+c)-4*arcsinh(d*x+c)^3*(1+(d*x+c
)^2)^(1/2)+12*arcsinh(d*x+c)^2*(d*x+c)-24*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2
)+24*d*x+24*c)+4*a*b^3*(arcsinh(d*x+c)^3*(d*x+c)-3*arcsinh(d*x+c)^2*(1+(d*x
+c)^2)^(1/2)+6*(d*x+c)*arcsinh(d*x+c)-6*(1+(d*x+c)^2)^(1/2))+6*a^2*b^2*(arc
sinh(d*x+c)^2*(d*x+c)-2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*d*x+2*c)+4*a^3
*b*((d*x+c)*arcsinh(d*x+c)-(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + a^4*x + 4*((d*x
+ c)*arcsinh(d*x + c) - sqrt((d*x + c)^2 + 1))*a^3*b/d + integrate(2*(2*((c
^3 + c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2)*x
^2 + ((3*c^2*d + d)*a*b^3 - (c^2*d + d)*b^4)*x + ((c^2 + 1)*a*b^3 + (a*b^3*
d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*(a^2*b^2*d^3
*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 + c)*a^2*b^2 +
(a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2)/(d^3*x^
3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(111) = 222$.
time = 0.38, size = 344, normalized size = 2.99

$$\frac{(b^4dx+b^4c)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^4+4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2+1}\,b^4\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^3+(a^4+12\,a^2b^2+24\,b^4)dx-6\left(2\sqrt{d^2x^2+2\,cdx+c^2+1}\,ab^3-(a^2b^2+2\,b^4)dx-(a^2b^2+2\,b^4)c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)^2+4\left((a^3b+6\,ab^3)dx+(a^3b+6\,ab^3)c-3\,(a^2b^2+2\,b^4)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-4\,(a^3b+6\,ab^3)\sqrt{d^2x^2+2\,cdx+c^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(105) = 210$.
time = 0.34, size = 444, normalized size = 3.86

$$\begin{cases} a^4x + \frac{4a^3bc\operatorname{asinh}(c+dx)}{d} + 4a^3bx\operatorname{asinh}(c+dx) - \frac{4a^3b\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{6a^2b^2c\operatorname{asinh}^2(c+dx)}{d} + 6a^2b^2x\operatorname{asinh}^2(c+dx) + 12a^2b^2x - \frac{12a^2b^2\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{d} + \frac{4ab^3c\operatorname{asinh}^3(c+dx)}{d} + \frac{24ab^3c\operatorname{asinh}(c+dx)}{d} + 4ab^3x\operatorname{asinh}^3(c+dx) + 24ab^3x\operatorname{asinh}(c+dx) - \frac{12ab^3\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}^2(c+dx)}{d} - \frac{24ab^3\sqrt{c^2+2cdx+d^2x^2+1}}{d} + \frac{b^4c\operatorname{asinh}^4(c+dx)}{d} + \frac{12b^4c\operatorname{asinh}^2(c+dx)}{d} + b^4x\operatorname{asinh}^4(c+dx) + 12b^4x\operatorname{asinh}^2(c+dx) + 24b^4x - \frac{4b^4\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}^3(c+dx)}{d} - \frac{24b^4\sqrt{c^2+2cdx+d^2x^2+1}\operatorname{asinh}(c+dx)}{d} & \text{for}\ d \neq 0 \\ x(a+b\operatorname{asinh}(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*asinh(c + d*x)/d + 4*a**3*b*x*asinh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d + 6*a**2*b**2*c*asinh(c
+ d*x)**2/d + 6*a**2*b**2*x*asinh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)/d + 4*a*b**3*c*asinh
(c + d*x)**3/d + 24*a*b**3*c*asinh(c + d*x)/d + 4*a*b**3*x*asinh(c + d*x)**
3 + 24*a*b**3*x*asinh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
+ 1)*asinh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 + 1)/d
 + b**4*c*asinh(c + d*x)**4/d + 12*b**4*c*asinh(c + d*x)**2/d + b**4*x*asin
h(c + d*x)**4 + 12*b**4*x*asinh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 + 1)*asinh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 + 1)*asinh(c + d*x)/d, Ne(d, 0)), (x*(a + b*asinh(c))**4, Tru
e))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{asinh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4,x)
[Out] int((a + b*asinh(c + d*x))^4, x)
```

3.151 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{ce+dex}\,dx$

Optimal. Leaf size=186

$$\frac{(a+b\sinh^{-1}(c+dx))^5}{5bde}+\frac{(a+b\sinh^{-1}(c+dx))^4\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)}{de}-\frac{2b(a+b\sinh^{-1}(c+dx))^3\,\mathrm{Poly}}{de}$$

```
[Out] 1/5*(a+b*arcsinh(d*x+c))^5/b/d/e+(a+b*arcsinh(d*x+c))^4*ln(1-1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-2*b*(a+b*arcsinh(d*x+c))^3*polylog(2,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3*b^2*(a+b*arcsinh(d*x+c))^2*polylog(3,1/(d*x+c+(1+(d
*x+c)^2)^(1/2))^2)/d/e-3*b^3*(a+b*arcsinh(d*x+c))*polylog(4,1/(d*x+c+(1+(d*
x+c)^2)^(1/2))^2)/d/e-3/2*b^4*polylog(5,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/
e
```

Rubi [A]
time = 0.22, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3b^3\mathrm{Li}_4\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{de}-\frac{3b^2\mathrm{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^2}{de}-\frac{2b\mathrm{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^3}{de}+\frac{(a+b\sinh^{-1}(c+dx))^5}{5bde}+\frac{\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^4}{de}-\frac{3b^4\mathrm{Li}_5\left(e^{-2\sinh^{-1}(c+dx)}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (a + b*ArcSinh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcSinh[c + d*x])^4*Log[1 -
E^(-2*ArcSinh[c + d*x])])/(d*e) - (2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2
, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyL
og[3, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^3*(a + b*ArcSinh[c + d*x])*Pol
yLog[4, E^(-2*ArcSinh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, E^(-2*ArcSinh[c
 + d*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{ex}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x}\,dx, x, c+dx\right)}{de} \\
&= \frac{\text{Subst}\left(\int (a+bx)^4\coth(x)\,dx, x, \sinh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} - \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1-e^{2x}}\,dx, x, \sinh^{-1}(c+dx)\right)}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de} \\
&= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right.}{de}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 157, normalized size = 0.84

$$\frac{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^5}{5b} + \left(a+b\sinh^{-1}(c+dx)\right)^4\log\left(1-e^{2\sinh^{-1}(c+dx)}\right) + 2b\left(a+b\sinh^{-1}(c+dx)\right)^3\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right) - 3b^2\left(a+b\sinh^{-1}(c+dx)\right)^2\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right) + 3b^3\left(a+b\sinh^{-1}(c+dx)\right)\text{PolyLog}\left(4,e^{2\sinh^{-1}(c+dx)}\right) - \frac{3}{2}b^4\text{PolyLog}\left(5,e^{2\sinh^{-1}(c+dx)}\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (-1/5*(a + b*ArcSinh[c + d*x])^5/b + (a + b*ArcSinh[c + d*x])^4*Log[1 - E^(
2*ArcSinh[c + d*x])] + 2*b*(a + b*ArcSinh[c + d*x])^3*PolyLog[2, E^(2*ArcSi
nh[c + d*x])] - 3*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[3, E^(2*ArcSinh[c
```

```
+ d*x])] + 3*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[4, E^(2*ArcSinh[c + d*x])
] - (3*b^4*PolyLog[5, E^(2*ArcSinh[c + d*x])])/2)/(d*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(222) = 444$.
time = 2.82, size = 1058, normalized size = 5.69

method	result	size
derivativedivides	Expression too large to display	1058
default	Expression too large to display	1058

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(6*a^2*b^2/e*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*a^2*b^
2/e*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+6*a^2*b^2/e*arcsinh
(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+24*b^4/e*arcsinh(d*x+c)*polylog(4
,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*a^3*b/e*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))
+4*a^3*b/e*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12*b^4/e*arcsinh(d*x+c)^2*
polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*a^2*b^2/e*arcsinh(d*x+c)^3-12*a^2*b
^2/e*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-12*a^2*b^2/e*polylog(3,-d*x-c-(1+
(d*x+c)^2)^(1/2))+b^4/e*arcsinh(d*x+c)^4*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-a*
b^3/e*arcsinh(d*x+c)^4+24*a*b^3/e*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))+24*a
*b^3/e*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))-12*b^4/e*arcsinh(d*x+c)^2*poly
log(3,d*x+c+(1+(d*x+c)^2)^(1/2))+24*b^4/e*arcsinh(d*x+c)*polylog(4,d*x+c+(1
+(d*x+c)^2)^(1/2))+b^4/e*arcsinh(d*x+c)^4*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4
*b^4/e*arcsinh(d*x+c)^3*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*b^4/e*arcsi
nh(d*x+c)^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-2*a^3*b/e*arcsinh(d*x+c)^2
-24*a*b^3/e*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*a^3*b/e*
arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*a^2*b^2/e*arcsinh(d*x+c)*
polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-24*a*b^3/e*arcsinh(d*x+c)*polylog(3,d
*x+c+(1+(d*x+c)^2)^(1/2))+4*a*b^3/e*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^
2)^(1/2))+12*a*b^3/e*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))
+4*a^3*b/e*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4*a*b^3/e*arcsinh
(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*a*b^3/e*arcsinh(d*x+c)^2*polyl
og(2,d*x+c+(1+(d*x+c)^2)^(1/2))+a^4/e*ln(d*x+c)-1/5*b^4/e*arcsinh(d*x+c)^5-
24*b^4/e*polylog(5,d*x+c+(1+(d*x+c)^2)^(1/2))-24*b^4/e*polylog(5,-d*x-c-(1+
(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^4*e^(-1)*log(d*x*e + c*e)/d + integrate(b^4*log(d*x + c + sqrt((d*x + c)^
2 + 1))^4/(d*x*e + c*e) + 4*a*b^3*log(d*x + c + sqrt((d*x + c)^2 + 1))^3/(d
*x*e + c*e) + 6*a^2*b^2*log(d*x + c + sqrt((d*x + c)^2 + 1))^2/(d*x*e + c*e
) + 4*a^3*b*log(d*x + c + sqrt((d*x + c)^2 + 1))/(d*x*e + c*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*e^(-1)/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{asinh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{asinh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{asinh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3 b \operatorname{asinh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*asinh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*asinh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*asinh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*asinh(c + d*x)/(c + d
*x), x))/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x), x)
```

3.152 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{(ce+dex)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{(a+b\sinh^{-1}(c+dx))^4}{de^2(c+dx)}-\frac{8b(a+b\sinh^{-1}(c+dx))^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}-\frac{12b^2(a+b\sinh^{-1}(c+dx))^2\,\mathrm{Po}}{de^2}$$

```
[Out] -(a+b*arcsinh(d*x+c))^4/d/e^2/(d*x+c)-8*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*
x+c+(1+(d*x+c)^2)^(1/2))/d/e^2-12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,-d*x
-c-(1+(d*x+c)^2)^(1/2))/d/e^2+12*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c
+(1+(d*x+c)^2)^(1/2))/d/e^2+24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1
+(d*x+c)^2)^(1/2))/d/e^2-24*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*
x+c)^2)^(1/2))/d/e^2-24*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^2+24*
b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^2
```

Rubi [A]
time = 0.21, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5776, 5816, 4267, 2611, 6744, 2320, 6724}

$$\frac{24b^3\mathrm{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{de^2}-\frac{24b^3\mathrm{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))}{de^2}-\frac{12b^2\mathrm{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^2}{de^2}+\frac{12b^2\mathrm{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^2}{de^2}-\frac{(a+b\sinh^{-1}(c+dx))^4}{de^2(c+dx)}-\frac{8b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)(a+b\sinh^{-1}(c+dx))^3}{de^2}-\frac{24b^4\mathrm{Li}_4\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^2}+\frac{24b^4\mathrm{Li}_4\left(e^{\sinh^{-1}(c+dx)}\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcSinh[c + d*x])^4/(d*e^2*(c + d*x))) - (8*b*(a + b*ArcSinh[c + d
*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^2) - (12*b^2*(a + b*ArcSinh[c + d*
x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^2) + (12*b^2*(a + b*ArcSinh[c +
 d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^2) + (24*b^3*(a + b*ArcSinh[c
 + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^3*(a + b*ArcSinh[
c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^2) - (24*b^4*PolyLog[4, -E^A
rcSinh[c + d*x]])/(d*e^2) + (24*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\text{Subst}\left(\int (a+bx)^3\text{csch}(x)\,dx, x, \sinh^{-1}(c+\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{de^2(c+dx)} - \frac{8b\left(a+b\sinh^{-1}(c+dx)\right)^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right.}{de^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 501 vs. 2(234) = 468.
time = 1.08, size = 501, normalized size = 2.14

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] ((-2*a^4)/(c + d*x) + 4*a^3*b*((-2*ArcSinh[c + d*x])/(c + d*x) + 2*Log[(2*S
inh[ArcSinh[c + d*x]/2]^2)/(c + d*x)]) + 12*a^2*b^2*(ArcSinh[c + d*x]*(-(Ar
cSinh[c + d*x]/(c + d*x)) + 2*Log[1 - E^(-ArcSinh[c + d*x])] - 2*Log[1 + E^
(-ArcSinh[c + d*x])]) + 2*PolyLog[2, -E^(-ArcSinh[c + d*x])] - 2*PolyLog[2,
 E^(-ArcSinh[c + d*x])]) + 8*a*b^3*(-(ArcSinh[c + d*x]^3/(c + d*x)) + 3*Arc
Sinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] - 3*ArcSinh[c + d*x]^2*Log[1
 + E^(-ArcSinh[c + d*x])] + 6*ArcSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c +
d*x])] - 6*ArcSinh[c + d*x]*PolyLog[2, E^(-ArcSinh[c + d*x])] + 6*PolyLog[3
, -E^(-ArcSinh[c + d*x])] - 6*PolyLog[3, E^(-ArcSinh[c + d*x])]) + b^4*(Pi^
4 - 2*ArcSinh[c + d*x]^4 - (2*ArcSinh[c + d*x]^4)/(c + d*x) - 8*ArcSinh[c +
 d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] + 8*ArcSinh[c + d*x]^3*Log[1 - E^Arc
Sinh[c + d*x]] + 24*ArcSinh[c + d*x]^2*PolyLog[2, -E^(-ArcSinh[c + d*x])] +
 24*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] + 48*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] - 48*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] + 48*PolyLog[4, -E^(-ArcSinh[c + d*x])] + 48*PolyLog[4, E^Arc
Sinh[c + d*x]]))/(2*d*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(299) = 598$.
time = 3.65, size = 748, normalized size = 3.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4/e^2/(d*x+c)-b^4/e^2*arcsinh(d*x+c)^4/(d*x+c)+4*b^4/e^2*arcsinh(d*
x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*b^4/e^2*arcsinh(d*x+c)^2*polylog(
2,d*x+c+(1+(d*x+c)^2)^(1/2))-24*b^4/e^2*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(
d*x+c)^2)^(1/2))+24*b^4/e^2*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-4*b^4/e^2*
arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-12*b^4/e^2*arcsinh(d*x+c)^
2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24*b^4/e^2*arcsinh(d*x+c)*polylog(3
,-d*x-c-(1+(d*x+c)^2)^(1/2))-24*b^4/e^2*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2
))-4*a*b^3/e^2*arcsinh(d*x+c)^3/(d*x+c)+12*a*b^3/e^2*arcsinh(d*x+c)^2*ln(1-
d*x-c-(1+(d*x+c)^2)^(1/2))+24*a*b^3/e^2*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(
d*x+c)^2)^(1/2))-24*a*b^3/e^2*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-12*a*b^3
/e^2*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-24*a*b^3/e^2*arcsinh(
d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+24*a*b^3/e^2*polylog(3,-d*x-c-
(1+(d*x+c)^2)^(1/2))-6*a^2*b^2/e^2*arcsinh(d*x+c)^2/(d*x+c)+12*a^2*b^2/e^2*
arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*a^2*b^2/e^2*polylog(2,d*x
+c+(1+(d*x+c)^2)^(1/2))-12*a^2*b^2/e^2*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)
^2)^(1/2))-12*a^2*b^2/e^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*a^3*b/e^2
*(-1/(d*x+c)*arcsinh(d*x+c)-arctanh(1/(1+(d*x+c)^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^2*x*e^2 + c*d*e^
2) - 4*a^3*b*(arcsinh(d*e^2/abs(d^2*x*e^2 + c*d*e^2))*e^(-2)/d + arcsinh(d*
x + c)/(d^2*x*e^2 + c*d*e^2)) - a^4/(d^2*x*e^2 + c*d*e^2) + integrate(2*(2*
((c^3 + c)*a*b^3 + (c^3 + c)*b^4 + (a*b^3*d^3 + b^4*d^3)*x^3 + 3*(a*b^3*c*d
^2 + b^4*c*d^2)*x^2 + ((3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^
2 + (c^2 + 1)*a*b^3 + (a*b^3*d^2 + b^4*d^2)*x^2 + 2*(a*b^3*c*d + b^4*c*d)*x
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))^3 + 3*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2
*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)
*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1))^2)/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 + d^3)*x
^3*e^2 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^2 + (5*c^4*d + 3*c^2*d)*x*e^2 + (c^5
+ c^3)*e^2 + (d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*e^2 + 2
*(2*c^3*d + c*d)*x*e^2 + (c^4 + c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*e^(-2)/(d^2*x^2 + 2*c*d
*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4\operatorname{asinh}^4(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3\operatorname{asinh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2\operatorname{asinh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b\operatorname{asinh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*asinh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*asinh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*asinh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2,x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

3.153 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=186

$$\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3}-\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2}+\frac{6b^2\left(a+b\right.}{}$$

```
[Out] 2*b*(a+b*arcsinh(d*x+c))^3/d/e^3-1/2*(a+b*arcsinh(d*x+c))^4/d/e^3/(d*x+c)^2
+6*b^2*(a+b*arcsinh(d*x+c))^2*ln(1-1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-6
*b^3*(a+b*arcsinh(d*x+c))*polylog(2,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-
3*b^4*polylog(3,1/(d*x+c+(1+(d*x+c)^2)^(1/2))^2)/d/e^3-2*b*(a+b*arcsinh(d*x
+c))^3*(1+(d*x+c)^2)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A]
time = 0.27, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5776, 5800, 5775, 3797, 2221, 2611, 2320, 6724}

$$-\frac{6b^3\text{Li}_2\left(e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^3}+\frac{6b^2\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^3}-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)}+\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2}-\frac{3b^4\text{Li}_3\left(e^{-2\sinh^{-1}(c+dx)}\right)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (2*b*(a + b*ArcSinh[c + d*x])^3)/(d*e^3) - (2*b*Sqrt[1 + (c + d*x)^2]*(a +
b*ArcSinh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcSinh[c + d*x])^4/(2*d*
e^3*(c + d*x)^2) + (6*b^2*(a + b*ArcSinh[c + d*x])^2*Log[1 - E^(-2*ArcSinh[
c + d*x])])/(d*e^3) - (6*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[2, E^(-2*ArcS
inh[c + d*x])])/(d*e^3) - (3*b^4*PolyLog[3, E^(-2*ArcSinh[c + d*x])])/(d*e^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{e^3x^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{x^3}\,dx,x,c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{x^2\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de^3}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \cdots$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \cdots$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \cdots$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \cdots$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \cdots$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \cdots$$

$$= -\frac{2b\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.86, size = 360, normalized size = 1.94

$$\frac{-\frac{2a^4}{(c+dx)^2}-\frac{8a^3b\sqrt{1+(c+dx)^2}}{c+dx}-\frac{8a^3b\sinh^{-1}(c+dx)}{(c+dx)^2}-\frac{2b^4\sinh^{-1}(c+dx)^4}{(c+dx)^2}+24a^2b^2\left(-\frac{\sqrt{1+(c+dx)^2}\sinh^{-1}(c+dx)}{c+dx}-\frac{\sinh^{-1}(c+dx)^2}{2(c+dx)^2}+\log(c+dx)\right)+8ab^3\left(\sinh^{-1}(c+dx)\left(3\sinh^{-1}(c+dx)-\frac{3\sqrt{1+(c+dx)^2}\sinh^{-1}(c+dx)}{c+dx}-\frac{\sinh^{-1}(c+dx)^2}{(c+dx)^2}+6\log\left(1-e^{-2\sinh^{-1}(c+dx)}\right)\right)-3\text{PolyLog}\left(2,e^{-2\sinh^{-1}(c+dx)}\right)\right)+b^4\left(i\pi^3-8\sinh^{-1}(c+dx)^3-\frac{8\sqrt{1+(c+dx)^2}\sinh^{-1}(c+dx)^3}{c+dx}+24\sinh^{-1}(c+dx)^2\log\left(1-e^{2\sinh^{-1}(c+dx)}\right)+24\sinh^{-1}(c+dx)\text{PolyLog}\left(2,e^{2\sinh^{-1}(c+dx)}\right)-12\text{PolyLog}\left(3,e^{2\sinh^{-1}(c+dx)}\right)\right)}{4de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] ((-2*a^4)/(c + d*x)^2 - (8*a^3*b*Sqrt[1 + (c + d*x)^2])/(c + d*x) - (8*a^3*
b*ArcSinh[c + d*x])/(c + d*x)^2 - (2*b^4*ArcSinh[c + d*x]^4)/(c + d*x)^2 +
24*a^2*b^2*(-((Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x)) - ArcSinh
[c + d*x]^2/(2*(c + d*x)^2) + Log[c + d*x]) + 8*a*b^3*(ArcSinh[c + d*x]*(3*
ArcSinh[c + d*x] - (3*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x])/(c + d*x) - A
rcSinh[c + d*x]^2/(c + d*x)^2 + 6*Log[1 - E^(-2*ArcSinh[c + d*x])]) - 3*Pol
yLog[2, E^(-2*ArcSinh[c + d*x])]) + b^4*(I*Pi^3 - 8*ArcSinh[c + d*x]^3 - (8
*Sqrt[1 + (c + d*x)^2]*ArcSinh[c + d*x]^3)/(c + d*x) + 24*ArcSinh[c + d*x]^
2*Log[1 - E^(2*ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]*PolyLog[2, E^(2*Arc
Sinh[c + d*x])] - 12*PolyLog[3, E^(2*ArcSinh[c + d*x])]))/(4*d*e^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(206) = 412$.

time = 5.12, size = 654, normalized size = 3.52

method	result
derivativedivides	$-\frac{a^4}{2e^3(dx+c)^2}-\frac{2b^4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{e^3(dx+c)}-\frac{2b^4\operatorname{arcsinh}(dx+c)^3}{e^3}-\frac{b^4\operatorname{arcsinh}(dx+c)^4}{2e^3(dx+c)^2}+\frac{6b^4\operatorname{arcsinh}(dx+c)^2\ln\left(1-\right.}{}$
default	$-\frac{a^4}{2e^3(dx+c)^2}-\frac{2b^4\operatorname{arcsinh}(dx+c)^3\sqrt{1+(dx+c)^2}}{e^3(dx+c)}-\frac{2b^4\operatorname{arcsinh}(dx+c)^3}{e^3}-\frac{b^4\operatorname{arcsinh}(dx+c)^4}{2e^3(dx+c)^2}+\frac{6b^4\operatorname{arcsinh}(dx+c)^2\ln\left(1-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*a^4/e^3/(d*x+c)^2-2*b^4/e^3*arcsinh(d*x+c)^3/(d*x+c)*(1+(d*x+c)^2
)^(1/2)-2*b^4/e^3*arcsinh(d*x+c)^3-1/2*b^4/e^3*arcsinh(d*x+c)^4/(d*x+c)^2+6
*b^4/e^3*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+12*b^4/e^3*arcsin
h(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-12*b^4/e^3*polylog(3,d*x+c+(1
+(d*x+c)^2)^(1/2))+6*b^4/e^3*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2
))+12*b^4/e^3*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-12*b^4/e
^3*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-6*a*b^3/e^3*arcsinh(d*x+c)^2/(d*x+
c)*(1+(d*x+c)^2)^(1/2)-6*a*b^3/e^3*arcsinh(d*x+c)^2-2*a*b^3/e^3*arcsinh(d*x
+c)^3/(d*x+c)^2+12*a*b^3/e^3*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))
```

```
+12*a*b^3/e^3*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+12*a*b^3/e^3*arcsinh(d*x
+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+12*a*b^3/e^3*polylog(2,-d*x-c-(1+(d*x+c
)^2)^(1/2))-6*a^2*b^2/e^3*arcsinh(d*x+c)-6*a^2*b^2/e^3*arcsinh(d*x+c)/(d*x+
c)*(1+(d*x+c)^2)^(1/2)-3*a^2*b^2/e^3*arcsinh(d*x+c)^2/(d*x+c)^2+6*a^2*b^2/e
^3*ln((d*x+c+(1+(d*x+c)^2)^(1/2))^2-1)+4*a^3*b/e^3*(-1/2/(d*x+c)^2*arcsinh(
d*x+c)-1/2/(d*x+c)*(1+(d*x+c)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^3*x^2*e^3 +
2*c*d^2*x*e^3 + c^2*d*e^3) - 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d*arcsinh
(d*x + c)/(d^3*x*e^3 + c*d^2*e^3) - e^(-3)*log(d*x + c)/d)*a^2*b^2 - 2*a^3*
b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*d/(d^3*x*e^3 + c*d^2*e^3) + arcsinh(d*
x + c)/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3)) - 3*a^2*b^2*arcsinh(d*x +
 c)^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 1/2*a^4/(d^3*x^2*e^3 + 2*
c*d^2*x*e^3 + c^2*d*e^3) + integrate(2*(2*(c^3 + c)*a*b^3 + (c^3 + c)*b^4 +
 (2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (2*(3*c^
2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 + 2*(c^2 + 1)*a*b^3 + (2*a
*b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(d^6*x^6*
e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 + d^4)*x^4*e^3 + 4*(5*c^3*d^3 + c*d^3)*
x^3*e^3 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d + 2*c^3*d)*x*e^3 +
 (c^6 + c^4)*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)*x^3*
e^3 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^3 + (5*c^4*d + 3*c^2*d)*x*e^3 + (c^5 + c
^3)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*e^(-3)/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^4\operatorname{asinh}^4(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4ab^3\operatorname{asinh}^3(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{6a^2b^2\operatorname{asinh}^2(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4a^3b\operatorname{asinh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*asinh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*asinh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*asinh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

3.154 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{(ce+dex)^4}\,dx$

Optimal. Leaf size=385

$$-\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)}-\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3}-\frac{8b^3(a+$$

[Out] -2*b^2*(a+b*arcsinh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arcsinh(d*x+c))^4/d/e^4/(d*x+c)^3-8*b^3*(a+b*arcsinh(d*x+c))*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+4/3*b*(a+b*arcsinh(d*x+c))^3*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+2*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2*b^2*(a+b*arcsinh(d*x+c))^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^3*(a+b*arcsinh(d*x+c))*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4+4*b^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))/d/e^4-4*b^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))/d/e^4-2/3*b*(a+b*arcsinh(d*x+c))^3*(1+(d*x+c)^2)^(1/2)/d/e^4/(d*x+c)^2

Rubi [A]

time = 0.38, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5859, 12, 5776, 5809, 5816, 4267, 2611, 6744, 2320, 6724, 2317, 2438}

$$-\frac{4b^3\text{Li}_3\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{4b^3\text{Li}_3\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}-\frac{8b^3\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)}{de^4}+\frac{2b^2\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4}-\frac{2b^2\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4}-\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)}-\frac{2b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3}+\frac{4b\tanh^{-1}\left(e^{\sinh^{-1}(c+dx)}\right)\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4}-\frac{4b^4\text{Li}_2\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^4}+\frac{4b^4\text{Li}_2\left(e^{\sinh^{-1}(c+dx)}\right)}{de^4}+\frac{4b^4\text{Li}_4\left(-e^{\sinh^{-1}(c+dx)}\right)}{de^4}-\frac{4b^4\text{Li}_4\left(e^{\sinh^{-1}(c+dx)}\right)}{de^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]

[Out] (-2*b^2*(a + b*ArcSinh[c + d*x])^2)/(d*e^4*(c + d*x)) - (2*b*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (a + b*ArcSinh[c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcSinh[c + d*x])*ArcTanh[E^ArcSinh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcSinh[c + d*x])^3*ArcTanh[E^ArcSinh[c + d*x]])/(3*d*e^4) - (4*b^4*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) + (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (2*b^2*(a + b*ArcSinh[c + d*x])^2*PolyLog[2, E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[3, -E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^3*(a + b*ArcSinh[c + d*x])*PolyLog[3, E^ArcSinh[c + d*x]])/(d*e^4) + (4*b^4*PolyLog[4, -E^ArcSinh[c + d*x]])/(d*e^4) - (4*b^4*PolyLog[4, E^ArcSinh[c + d*x]])/(d*e^4)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^4}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^4}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{x^3\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de^4}$$

$$= -\frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} -$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

$$= -\frac{2b^2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de^4(c+dx)} - \frac{2b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1182 vs. $2(385) = 770$.
time = 7.99, size = 1182, normalized size = 3.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^4/(d*e^4*(c + d*x)^3) + (a^2*b^2*(-8*PolyLog[2, -E^(-ArcSinh[c + d*x
])] - (2*(-2 + 4*ArcSinh[c + d*x]^2 + 2*Cosh[2*ArcSinh[c + d*x]] - 3*(c + d
```

```
*x)*ArcSinh[c + d*x]*Log[1 - E^(-ArcSinh[c + d*x])] + 3*(c + d*x)*ArcSinh[c
 + d*x]*Log[1 + E^(-ArcSinh[c + d*x])] - 4*(c + d*x)^3*PolyLog[2, E^(-ArcSi
nh[c + d*x])] + 2*ArcSinh[c + d*x]*Sinh[2*ArcSinh[c + d*x]] + ArcSinh[c + d
*x]*Log[1 - E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]] - ArcSinh[c + d
*x]*Log[1 + E^(-ArcSinh[c + d*x])]*Sinh[3*ArcSinh[c + d*x]]))/(c + d*x)^3))
/(4*d*e^4) + (a*b^3*(-24*ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2] + 4*ArcS
inh[c + d*x]^3*Coth[ArcSinh[c + d*x]/2] - 6*ArcSinh[c + d*x]^2*Csch[ArcSinh
[c + d*x]/2]^2 - (c + d*x)*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^4 -
24*ArcSinh[c + d*x]^2*Log[1 - E^(-ArcSinh[c + d*x])] + 24*ArcSinh[c + d*x]^
2*Log[1 + E^(-ArcSinh[c + d*x])] + 48*Log[Tanh[ArcSinh[c + d*x]/2]] - 48*Ar
cSinh[c + d*x]*PolyLog[2, -E^(-ArcSinh[c + d*x])] + 48*ArcSinh[c + d*x]*Pol
yLog[2, E^(-ArcSinh[c + d*x])] - 48*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 48
*PolyLog[3, E^(-ArcSinh[c + d*x])] - 6*ArcSinh[c + d*x]^2*Sech[ArcSinh[c +
d*x]/2]^2 - (16*ArcSinh[c + d*x]^3*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3
+ 24*ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Tanh[
ArcSinh[c + d*x]/2]))/(12*d*e^4) + (b^4*(-2*Pi^4 + 4*ArcSinh[c + d*x]^4 - 2
4*ArcSinh[c + d*x]^2*Coth[ArcSinh[c + d*x]/2] + 2*ArcSinh[c + d*x]^4*Coth[A
rcSinh[c + d*x]/2] - 4*ArcSinh[c + d*x]^3*Csch[ArcSinh[c + d*x]/2]^2 - ((c
+ d*x)*ArcSinh[c + d*x]^4*Csch[ArcSinh[c + d*x]/2]^4)/2 + 96*ArcSinh[c + d*
x]*Log[1 - E^(-ArcSinh[c + d*x])] - 96*ArcSinh[c + d*x]*Log[1 + E^(-ArcSinh
[c + d*x])] + 16*ArcSinh[c + d*x]^3*Log[1 + E^(-ArcSinh[c + d*x])] - 16*Arc
Sinh[c + d*x]^3*Log[1 - E^ArcSinh[c + d*x]] - 48*(-2 + ArcSinh[c + d*x]^2)*
PolyLog[2, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[2, E^(-ArcSinh[c + d*x])] -
 48*ArcSinh[c + d*x]^2*PolyLog[2, E^ArcSinh[c + d*x]] - 96*ArcSinh[c + d*x]
*PolyLog[3, -E^(-ArcSinh[c + d*x])] + 96*ArcSinh[c + d*x]*PolyLog[3, E^ArcS
inh[c + d*x]] - 96*PolyLog[4, -E^(-ArcSinh[c + d*x])] - 96*PolyLog[4, E^Arc
Sinh[c + d*x]] - 4*ArcSinh[c + d*x]^3*Sech[ArcSinh[c + d*x]/2]^2 - (8*ArcSi
nh[c + d*x]^4*Sinh[ArcSinh[c + d*x]/2]^4)/(c + d*x)^3 + 24*ArcSinh[c + d*x]
^2*Tanh[ArcSinh[c + d*x]/2] - 2*ArcSinh[c + d*x]^4*Tanh[ArcSinh[c + d*x]/2]
))/(24*d*e^4) + (4*a^3*b*((ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2])/12 -
Csch[ArcSinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Coth[ArcSinh[c + d*x]/2]*
Csch[ArcSinh[c + d*x]/2]^2)/24 - Log[Tanh[ArcSinh[c + d*x]/2]]/6 - Sech[Arc
Sinh[c + d*x]/2]^2/24 - (ArcSinh[c + d*x]*Tanh[ArcSinh[c + d*x]/2])/12 - (A
rcSinh[c + d*x]*Sech[ArcSinh[c + d*x]/2]^2*Tanh[ArcSinh[c + d*x]/2])/24))/(
d*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(469) = 938$.
time = 5.81, size = 1087, normalized size = 2.82

method	result	size
derivativedivides	Expression too large to display	1087
default	Expression too large to display	1087

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*a^2*b^2/e^4/(d*x+c)^2*arcsinh(d*x+c)*(1+(d*x+c)^2)^(1/2)-4*b^4/e^4*polylog(4,d*x+c+(1+(d*x+c)^2)^(1/2))-1/3*a^4/e^4/(d*x+c)^3+4*b^4/e^4*polylog(4,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*b^4/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))-4*b^4/e^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2*a*b^3/e^4/(d*x+c)^2*arcsinh(d*x+c)^2*(1+(d*x+c)^2)^(1/2)-2*a^2*b^2/e^4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+2*a^2*b^2/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))-2*a*b^3/e^4*arcsinh(d*x+c)^2*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-4*a*b^3/e^4*arcsinh(d*x+c)*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2*a*b^3/e^4*arcsinh(d*x+c)^2*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4*a*b^3/e^4*arcsinh(d*x+c)*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-2/3*b^4/e^4/(d*x+c)^2*arcsinh(d*x+c)^3*(1+(d*x+c)^2)^(1/2)-4/3*a*b^3/e^4/(d*x+c)^3*arcsinh(d*x+c)^3-4*a*b^3/e^4/(d*x+c)*arcsinh(d*x+c)-2*a^2*b^2/e^4/(d*x+c)^3*arcsinh(d*x+c)^2-2*b^4/e^4*arcsinh(d*x+c)^2*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+4*b^4/e^4*arcsinh(d*x+c)*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))+2/3*b^4/e^4*arcsinh(d*x+c)^3*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+2*b^4/e^4*arcsinh(d*x+c)^2*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-4*b^4/e^4*arcsinh(d*x+c)*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))+4*b^4/e^4*arcsinh(d*x+c)*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))-4*b^4/e^4*arcsinh(d*x+c)*ln(1+d*x+c+(1+(d*x+c)^2)^(1/2))+4*a*b^3/e^4*polylog(3,d*x+c+(1+(d*x+c)^2)^(1/2))-4*a*b^3/e^4*polylog(3,-d*x-c-(1+(d*x+c)^2)^(1/2))-8*a*b^3/e^4*arctanh(d*x+c+(1+(d*x+c)^2)^(1/2))-2*a^2*b^2/e^4/(d*x+c)-2*a^2*b^2/e^4*polylog(2,d*x+c+(1+(d*x+c)^2)^(1/2))+2*a^2*b^2/e^4*polylog(2,-d*x-c-(1+(d*x+c)^2)^(1/2))-1/3*b^4/e^4/(d*x+c)^3*arcsinh(d*x+c)^4-2*b^4/e^4/(d*x+c)*arcsinh(d*x+c)^2-2/3*b^4/e^4*arcsinh(d*x+c)^3*ln(1-d*x-c-(1+(d*x+c)^2)^(1/2))+4*a^3*b/e^4*(-1/3/(d*x+c)^3*arcsinh(d*x+c)-1/6/(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+1/6*arctanh(1/(1+(d*x+c)^2)^(1/2))))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")

[Out] -1/3*b^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 1/3*a^4/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 + c)*a*b^3 + (c^3 + c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (3*(3*c^2*d + d)*a*b^3 + (3*c^2*d + d)*b^4)*x + (b^4*c^2 + 3*(c^2 + 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(3*a*b^3*c*d + b^4*c*d)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d + d)*a^2*b^2*x + (c^3 + c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 + 1)*a^2*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 +

```
 2*c*d*x + c^2 + 1))^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (3*c^2*d +
d)*a^3*b*x + (c^3 + c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 + 1)*a
^3*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)))/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4
 + 5*(7*c^3*d^4 + c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*
c^5*d^2 + 10*c^3*d^2)*x^2*e^4 + (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4
 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d
^3 + c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^
3*d)*x*e^4 + (c^6 + c^4)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*e^(-4)/(d^4*x^4 + 4*c*d
^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^4\operatorname{asinh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4ab^3\operatorname{asinh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{6a^2b^2\operatorname{asinh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{4a^3b\operatorname{asinh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*asinh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*asinh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*asinh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*asinh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^4}{(c\,e+d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4,x)
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

3.155 $\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e(c+dx))^m}{a+b\sinh^{-1}(c+dx)},x\right)$$

```
[Out] Unintegrable((e*(d*x+c))^m/(a+b*arcsinh(d*x+c)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcSinh[x]), x], x, c + d*x]/d
```

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

Mathematica [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{(ce+dex)^m}{a+b\sinh^{-1}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]),x]
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcSinh[c + d*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^m}{a+b\,\mathrm{arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(((d*x + c)*e)^m/(b*arcsinh(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e(c+dx))^m}{a+b\operatorname{asinh}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m/(a + b*asinh(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^m/(b*arcsinh(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,e+d\,e\,x)^m}{a+b\,\mathrm{asinh}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m/(a + b*asinh(c + d*x)), x)
```

3.156 $\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}$$

```
[Out] 1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-3/16*e^4*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/16*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*cosh(5
*a/b)/b/d-1/8*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b/d+3/16*e^4*Shi(3*
(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Shi(5*(a+b*arcsinh(d*x+c))
/b)*sinh(5*a/b)/b/d
```

Rubi [A]
time = 0.32, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}+\frac{e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8bd}+\frac{3e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) - (3*e^4*C
osh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) + (e^4*
Cosh[(5*a)/b]*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4
*Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b*d) + (3*e^4*Sinh[
(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d) - (e^4*Sinh
[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^4}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{x^4}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)} - \frac{3\cosh(3x)}{16(a+bx)} + \frac{\cosh(5x)}{16(a+bx)}\right)\,dx, x, \sinh^{-1}(c+dx)\right)}{d} \\
&= \frac{e^4\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \sinh^{-1}(c\right.}{8d} \\
&= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Sub}}{} \\
&= \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{8bd} - \frac{3e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+d\right.}{16bd}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 151, normalized size = 0.71

$$
\frac{e^4\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{16bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^4*(2*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b
+ ArcSinh[c + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3
*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*Sin
hIntegral[5*(a/b + ArcSinh[c + d*x])]))/(16*b*d)
```

Maple [A]
time = 6.08, size = 194, normalized size = 0.91

method	result
derivativedivides	$-\frac{e^4 e^{\frac{5a}{b}}\text{expIntegral}\left(1,5\,\text{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}}\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 e^{\frac{a}{b}}\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\right.}{16b}$
default	$-\frac{e^4 e^{\frac{5a}{b}}\text{expIntegral}\left(1,5\,\text{arcsinh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}}\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\frac{3a}{b}\right)}{32b}-\frac{e^4 e^{\frac{a}{b}}\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\right.}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32*e^4/b*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a
/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/16*e^4/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/
b)-1/16*e^4/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32*e^4/b*exp(-3*a/b)*Ei
(1,-3*arcsinh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5
*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b*a
rcsinh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{d^{4} x^{4}}{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{4 c d^{3} x^{3}}{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{6 c^{2} d^{2} x^{2}}{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{4 c^{3} d x}{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**4*(Integral(c**4/(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/(a + b*
asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*asinh(c + d*x)), x) + I
ntegral(6*c**2*d**2*x**2/(a + b*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(
a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x)), x)
```

3.157 $\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{e^3\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{4a}{b}\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{4bd}$$

```
[Out] -1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*
Shi(4*(a+b*arcsinh(d*x+c))/b)/b/d+1/4*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*sin
h(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b/d
```

Rubi [A]
time = 0.24, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{8bd}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{4bd}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(4*b*d) -
(e^3*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(4*a)/b])/(8*b*d) -
(e^3*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) +
(e^3*Cosh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(8*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^3}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{8d}-\frac{e^3\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\sinh^{-1}(c\right.}{4d}\\
&= -\frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{S}}{}\\
&= \frac{e^3\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4a}{b}+4\sinh^{-1}(c+dx)\right)\sinh\left(\right.}{8bd}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 109, normalized size = 0.75

$$\frac{e^3\left(2\operatorname{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-\operatorname{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)-2\cosh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\operatorname{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^3*(2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - CoshIntegr
al[4*(a/b + ArcSinh[c + d*x])]*Sinh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral
[2*(a/b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[
c + d*x])]))/(8*b*d)
```

Maple [A]
time = 7.52, size = 134, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{e^{3}\mathrm{e}^{\frac{4a}{b}}\operatorname{expIntegral}\left(1,4\operatorname{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b}-\frac{e^{3}\mathrm{e}^{\frac{2a}{b}}\operatorname{expIntegral}\left(1,2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b}+\frac{e^{3}\mathrm{e}^{-\frac{2a}{b}}\operatorname{expIntegral}\left(1,-2\operatorname{arcsinh}(dx+c)-\right.}{8b}}{d}$
default	$\frac{\frac{e^{3}\mathrm{e}^{\frac{4a}{b}}\operatorname{expIntegral}\left(1,4\operatorname{arcsinh}(dx+c)+\frac{4a}{b}\right)}{16b}-\frac{e^{3}\mathrm{e}^{\frac{2a}{b}}\operatorname{expIntegral}\left(1,2\operatorname{arcsinh}(dx+c)+\frac{2a}{b}\right)}{8b}+\frac{e^{3}\mathrm{e}^{-\frac{2a}{b}}\operatorname{expIntegral}\left(1,-2\operatorname{arcsinh}(dx+c)-\right.}{8b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b*arcsinh(d*x + c)
+ a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int \frac{d^{3}x^{3}}{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int \frac{3cd^{2}x^{2}}{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int \frac{3c^{2}dx}{a+b\operatorname{asinh}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*asinh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{a+b\,\mathrm{asinh}\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x)), x)
```

3.158 $\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2}{}$$

```
[Out] -1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d+1/4*e^2*Chi(3*(a+b*arcsi
nh(d*x+c))/b)*cosh(3*a/b)/b/d+1/4*e^2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)
/b/d-1/4*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b/d
```

Rubi [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/4*(e^2*Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) + (e^2*
Cosh[(3*a)/b]*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d) + (e^2*
Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b*d) - (e^2*Sinh[(3*
a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^2}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)\,dx,x,\sinh^{-1}(c+dx)\right)}{d}\\
&= -\frac{e^2\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}+\frac{e^2\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\sinh^{-1}(\right.}{4d}\\
&= -\frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Sub}}{}\\
&= -\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\sinh^{-1}(c+\right.}{4bd}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 102, normalized size = 0.72

$$\frac{e^2\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (e^2*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*Cos
hIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSin
h[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b
*d)
```

Maple [A]
time = 6.08, size = 130, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{e^2 e^{\frac{3a}{b}}\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}}\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 e^{-\frac{a}{b}}\text{expIntegral}\left(1,-\text{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}}{d}$
default	$\frac{-\frac{e^2 e^{\frac{3a}{b}}\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\frac{3a}{b}\right)}{8b}+\frac{e^2 e^{\frac{a}{b}}\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\frac{a}{b}\right)}{8b}+\frac{e^2 e^{-\frac{a}{b}}\text{expIntegral}\left(1,-\text{arcsinh}(dx+c)-\frac{a}{b}\right)}{8b}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*e^2/b*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/8*e^2/b*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/8*
e^2/b*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b*arcsinh(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{d^2x^2}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{2cdx}{a+b\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c)),x)
```

```
[Out] e**2*(Integral(c**2/(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
asinh(c + d*x)), x) + Integral(2*c*d*x/(a + b*asinh(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x)), x)
```

3.159 $\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{e\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\sinh\left(\frac{2a}{b}\right)}{2bd}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{2bd}$$

```
[Out] 1/2*e*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arcsin
h(d*x+c))/b)*sinh(2*a/b)/b/d
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5859, 12, 5780, 5556, 3384, 3379, 3382}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/2*(e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/b])/(b*d) +
 (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{ce+dex}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int \frac{x}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}\\
&= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}\\
&= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subs}}{}\\
&= -\frac{e\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+\right.}{2bd}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 61, normalized size = 0.88

$$-\frac{e\left(\mathrm{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] -1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSinh[c + d*x]]))/(b*d)
```

Maple [A]
time = 5.09, size = 66, normalized size = 0.96

method	result	size
derivativedivides	$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66
default	$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,-2\,\mathrm{arcsinh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a+b\operatorname{asinh}(c+dx)}\,dx+\int \frac{dx}{a+b\operatorname{asinh}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
[Out] e*(Integral(c/(a + b*asinh(c + d*x)), x) + Integral(d*x/(a + b*asinh(c + d*
x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{a+b\operatorname{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x)), x)
```

3.160 $\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b/d-Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5858, 5774, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d) - (Sinh[a/b]*Sin
hIntegral[(a + b*ArcSinh[c + d*x])/b])/(b*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sinh^{-1}(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\sinh\ldots}{x}\right.}{}\\
&= \frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{bd}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.84

$$\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-1),x]
```

```
[Out] (Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/
b + ArcSinh[c + d*x]])/(b*d)
```

Maple [A]
time = 3.05, size = 60, normalized size = 1.03

method	result	size
derivativedivides	$\frac{-\frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{expIntegral}\left(1,-\text{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

default	$\frac{-\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\operatorname{expIntegral}\left(1,-\operatorname{arcsinh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/b*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arcsinh
(d*x+c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arcsinh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\operatorname{asinh}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x)),x)
[Out] int(1/(a + b*asinh(c + d*x)), x)
```

3.161 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c)),x)/e
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*d*x + b*c)*arcsinh(d*x + c)*e + (a*d*x + a*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac+adx+bc\,\mathrm{asinh}\,(c+dx)+bdx\,\mathrm{asinh}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral(1/(a*c + a*d*x + b*c*asinh(c + d*x) + b*d*x*asinh(c + d*x)), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))), x)
```

3.162 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=256

$$-\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^4\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{8b^2d}+\frac{9e^4\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{16b^2d}-\frac{5e^4}{}$$

```
[Out] 1/8*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-9/16*e^4*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+5/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arcsinh(d
*x+c))/b)/b^2/d-1/8*e^4*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^2/d+9/16*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Chi(5*(a+b*arcsi
nh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*
arcsinh(d*x+c))
```

Rubi [A]
time = 0.27, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-\frac{5e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^2d}-\frac{9e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-\frac{e^4(c+dx)^4\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^4*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(8*b^2*d) + (9*e^
4*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(16*b^2*d) -
(5*e^4*CoshIntegral[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(16*b^2*
d) + (e^4*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(8*b^2*d) - (
9*e^4*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(16*b^2*d
) + (5*e^4*Cosh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(16*
b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^4\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)} - \frac{9\sinh(3x)}{16(a+bx)} + \frac{5\sinh(5x)}{16(a+bx)}\right.\right.}{bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{e^4\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{8bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sin\right.}{8bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e^4\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2d} + \frac{9e}{}$$

Mathematica [A]
time = 0.82, size = 281, normalized size = 1.10

$$\frac{e^4\left(-\frac{16b(c+dx)^4\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)}+16\left(3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)-5\left(10\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-5\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{5a}{b}\right)-10\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) +
16*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - CoshIntegral[3*(a/b
+ ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh
[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) - 5*(1
0*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b +
ArcSinh[c + d*x])]*Sinh[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]
*Sinh[(5*a)/b] - 10*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcSinh[c + d*x])])))/(16*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(242) = 484$.
time = 7.06, size = 602, normalized size = 2.35

method	result

derivativedivides	$\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx\right.}{32b(a+b\operatorname{arcsinh}(dx+c))}$
default	$\frac{\left(-16(dx+c)^4\sqrt{1+(dx+c)^2}+16(dx+c)^5-12(dx+c)^2\sqrt{1+(dx+c)^2}+20(dx+c)^3-\sqrt{1+(dx+c)^2}+5dx\right.}{32b(a+b\operatorname{arcsinh}(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+(
d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4/b/(a+b*arcs
inh(d*x+c))+5/32*e^4/b^2*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-3/32*(-4*(
d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4
/b/(a+b*arcsinh(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b
)+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4/b/(a+b*arcsinh(d*x+c))+1/16*e^4/b^2
*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(
a+b*arcsinh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+3/32/b
*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(
1/2))/(a+b*arcsinh(d*x+c))+9/32/b^2*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-
3*a/b)-1/32/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/
2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arc
sinh(d*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4 + 5*(7*c^3*d^4
 + c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 + 10*c^
3*d^2)*x^2*e^4 + (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4 + (d^6*x^6*e^4
 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d^3 + c*d^3)*x^3
*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^3*d)*x*e^4 + (c
^6 + c^4)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^
2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b
^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)) + integrate((5*d^8*x^8*e^4 + 40*c*d^7*x^7*e^4 + 10*(14*c^2*d^6
 + d^6)*x^6*e^4 + 20*(14*c^3*d^5 + 3*c*d^5)*x^5*e^4 + 5*(70*c^4*d^4 + 30*c^
```

```
2*d^4 + d^4)*x^4*e^4 + 20*(14*c^5*d^3 + 10*c^3*d^3 + c*d^3)*x^3*e^4 + 10*(1
4*c^6*d^2 + 15*c^4*d^2 + 3*c^2*d^2)*x^2*e^4 + 20*(2*c^7*d + 3*c^5*d + c^3*d
)*x*e^4 + (5*d^6*x^6*e^4 + 30*c*d^5*x^5*e^4 + 3*(25*c^2*d^4 + d^4)*x^4*e^4
+ 4*(25*c^3*d^3 + 3*c*d^3)*x^3*e^4 + 3*(25*c^4*d^2 + 6*c^2*d^2)*x^2*e^4 + 6
*(5*c^5*d + 2*c^3*d)*x*e^4 + (5*c^6 + 3*c^4)*e^4)*(d^2*x^2 + 2*c*d*x + c^2
+ 1) + 5*(c^8 + 2*c^6 + c^4)*e^4 + (10*d^7*x^7*e^4 + 70*c*d^6*x^6*e^4 + (21
0*c^2*d^5 + 13*d^5)*x^5*e^4 + 5*(70*c^3*d^4 + 13*c*d^4)*x^4*e^4 + 2*(175*c^
4*d^3 + 65*c^2*d^3 + 2*d^3)*x^3*e^4 + 2*(105*c^5*d^2 + 65*c^3*d^2 + 6*c*d^2
)*x^2*e^4 + (70*c^6*d + 65*c^4*d + 12*c^2*d)*x*e^4 + (10*c^7 + 13*c^5 + 4*c
^3)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3
+ 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a
*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c
*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(
d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d
+ d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (
3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^2
*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{d^{4}x^{4}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{4cd^{3}x^{3}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{6c^{2}d^{2}x^{2}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{4c^{3}dx}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x)
+ b**2*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*asinh(c +
 d*x) + b**2*asinh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^2, x)
```

3.163 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+$$

```
[Out] -1/2*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+b
*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^2/d+1/2*e^3*Shi(2*(a+b*arcsinh(d*x+c))/b)
*sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b^2/d-
e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3(c+dx)^3\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) -
 (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*d)
 + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*b^2*
d) + (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^
2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(2*
b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^3\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{2(a+bx)}\right)\,dx,x,\sin\cdots\right.}{bd}\\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^3\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{2bd}\cdots\\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\text{si}\cdots\right.}{2bd}\\
&= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{2b^2d}+\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.63, size = 193, normalized size = 1.03

$$-\frac{e^3\left(\frac{2b(c+dx)^3\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-3\log\left(a+b\sinh^{-1}(c+dx)\right)-4\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+3\left(\log\left(a+b\sinh^{-1}(c+dx)\right)+\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)+\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -1/2*(e^3*((2*b*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])
 + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Cosh[(4*a)/b]*C
oshIntegral[4*(a/b + ArcSinh[c + d*x])] - 3*Log[a + b*ArcSinh[c + d*x]] - 4
*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])] + 3*(Log[a + b*ArcS
inh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]) + S
inh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(178) = 356$.
time = 8.28, size = 388, normalized size = 2.06

method	result
derivativedivides	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arc}\right.}{4b^2}$
default	$\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3}{16b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e^3 e^{\frac{4a}{b}}\,\mathrm{expIntegral}\left(1,4\,\mathrm{arc}\right.}{4b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+c
)^2)^(1/2)+8*(d*x+c)^2+1)*e^3/b/(a+b*arcsinh(d*x+c))-1/4*e^3/b^2*exp(4*a/b)
*Ei(1,4*arcsinh(d*x+c)+4*a/b)-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)
^2+1)*e^3/b/(a+b*arcsinh(d*x+c))+1/4*e^3/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+
c)+2*a/b)+1/8/b*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcs
inh(d*x+c))+1/4/b^2*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/16/b*e^
3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*
x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4*arcsi
nh(d*x+c)-4*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 + d^4)*x^4*e^3 + 4*(5*c^3*d^3
 + c*d^3)*x^3*e^3 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d + 2*c^3*
d)*x*e^3 + (c^6 + c^4)*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 +
 d^3)*x^3*e^3 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^3 + (5*c^4*d + 3*c^2*d)*x*e^3
+ (c^5 + c^3)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*
c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2
+ (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)) + integrate((4*d^7*x^7*e^3 + 28*c*d^6*x^6*e^3 + 4*(21*c^2*
d^5 + 2*d^5)*x^5*e^3 + 20*(7*c^3*d^4 + 2*c*d^4)*x^4*e^3 + 4*(35*c^4*d^3 + 2
0*c^2*d^3 + d^3)*x^3*e^3 + 4*(21*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*x^2*e^3 +
4*(7*c^6*d + 10*c^4*d + 3*c^2*d)*x*e^3 + 2*(2*d^5*x^5*e^3 + 10*c*d^4*x^4*e^
3 + (20*c^2*d^3 + d^3)*x^3*e^3 + (20*c^3*d^2 + 3*c*d^2)*x^2*e^3 + (10*c^4*d
 + 3*c^2*d)*x*e^3 + (2*c^5 + c^3)*e^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 4*(c
^7 + 2*c^5 + c^3)*e^3 + (8*d^6*x^6*e^3 + 48*c*d^5*x^5*e^3 + 10*(12*c^2*d^4
+ d^4)*x^4*e^3 + 40*(4*c^3*d^3 + c*d^3)*x^3*e^3 + 3*(40*c^4*d^2 + 20*c^2*d^
2 + d^2)*x^2*e^3 + 2*(24*c^5*d + 20*c^3*d + 3*c*d)*x*e^3 + (8*c^6 + 10*c^4
+ 3*c^2)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3
*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 +
 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 +
1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*
d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c
^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^
2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^2*arcsinh(d*x + c
)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^2 + 2ab\operatorname{asinh}(c+dx) + b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{d^3x^3}{a^2 + 2ab\operatorname{asinh}(c+dx) + b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{3cd^2x^2}{a^2 + 2ab\operatorname{asinh}(c+dx) + b^2\operatorname{asinh}^2(c+dx)}\,dx + \int \frac{3c^2dx}{a^2 + 2ab\operatorname{asinh}(c+dx) + b^2\operatorname{asinh}^2(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2
*asinh(c + d*x)**2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^2, x)
```

3.164 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=184

$$-\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^2\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{4b^2d}-\frac{3e^2\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\sinh\left(\frac{3a}{b}\right)}{4b^2d}-\frac{e^2\cosh\ldots}{}$$

```
[Out] -1/4*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d+3/4*e^2*cosh(3*a/b)*Sh
i(3*(a+b*arcsinh(d*x+c))/b)/b^2/d+1/4*e^2*Chi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^2/d-3/4*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(d*x
+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2(c+dx)^2\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) +
 (e^2*CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(4*b^2*d) - (3*e^
2*CoshIntegral[(3*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(4*b^2*d) - (
e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(4*b^2*d) + (3*e^2*
Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(4*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^2\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4(a+bx)}+\frac{3\sinh(3x)}{4(a+bx)}\right)\,dx,x,\text{s}\right.}{bd}\\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\sinh^{-1}(c+dx)\right)}{4bd}\\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\text{sin}\right.}{4bd}\\
&= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^2\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2d}-\frac{3e}{}
\end{aligned}
$$

Mathematica [A]

time = 0.54, size = 138, normalized size = 0.75

$$\frac{e^2\left(-\frac{4b(c+dx)^2\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)}+\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-3\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*(a/b + Arc
Sinh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*
x]] + 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]))/(4*b^2*d)
```

Maple [A]
time = 6.71, size = 342, normalized size = 1.86

method	result
derivativedivides	$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\,\text{arcsinh}(dx+c))}+\frac{3e^2 e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\right.}{8b^2}}{}$
default	$\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2}{8b(a+b\,\text{arcsinh}(dx+c))}+\frac{3e^2 e^{\frac{3a}{b}}\,\text{expIntegral}\left(1,3\,\text{arcsinh}(dx+c)+\right.}{8b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+
3*d*x+3*c)*e^2/b/(a+b*arcsinh(d*x+c))+3/8*e^2/b^2*exp(3*a/b)*Ei(1,3*arcsinh
(d*x+c)+3*a/b)-1/8*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2/b/(a+b*arcsinh(d*x+c))-
1/8*e^2/b^2*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/8/b*e^2*(d*x+c+(1+(d*x+c)^2
)^(1/2))/(a+b*arcsinh(d*x+c))+1/8/b^2*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/
b)-1/8/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x
+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(
d*x+c)-3*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 + d^3)*x^3*e^2 + (10*c^3*d^2
+ 3*c*d^2)*x^2*e^2 + (5*c^4*d + 3*c^2*d)*x*e^2 + (c^5 + c^3)*e^2 + (d^4*x^4
```

```
*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*e^2 + 2*(2*c^3*d + c*d)*x*e^
2 + (c^4 + c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*
b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^
2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)) + integrate((3*d^6*x^6*e^2 + 18*c*d^5*x^5*e^2 + 3*(15*c^
2*d^4 + 2*d^4)*x^4*e^2 + 12*(5*c^3*d^3 + 2*c*d^3)*x^3*e^2 + 3*(15*c^4*d^2 +
 12*c^2*d^2 + d^2)*x^2*e^2 + 6*(3*c^5*d + 4*c^3*d + c*d)*x*e^2 + (3*d^4*x^4
*e^2 + 12*c*d^3*x^3*e^2 + (18*c^2*d^2 + d^2)*x^2*e^2 + 2*(6*c^3*d + c*d)*x*
e^2 + (3*c^4 + c^2)*e^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(c^6 + 2*c^4 + c
^2)*e^2 + (6*d^5*x^5*e^2 + 30*c*d^4*x^4*e^2 + (60*c^2*d^3 + 7*d^3)*x^3*e^2
+ 3*(20*c^3*d^2 + 7*c*d^2)*x^2*e^2 + (30*c^4*d + 21*c^2*d + 2*d)*x*e^2 + (6
*c^5 + 7*c^3 + 2*c)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^4*x^4 +
4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c^3*d + c*d)*a*b*x + (c^
4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^
2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a
*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcs
inh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{d^{2}x^{2}}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{2cdx}{a^{2} + 2ab\operatorname{asinh}{\left(c + dx \right)} + b^{2}\operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d
*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{asinh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^2, x)
```

3.165 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}$$

```
[Out] e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arcsinh(d*x+
c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x
+c))
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5859, 12, 5778, 3384, 3379, 3382}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] -((e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x]))) + (e*
Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d) - (e*Si
nh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}\\ &= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}\\ &= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \sinh\right.}{bd}\\ &= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\sinh^{-1}(c+dx)\right)}{b^2d}-\frac{e\,s}{}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 97, normalized size = 0.94

$$\frac{e\left(-\frac{b(c+dx)\sqrt{1+c^2+2cdx+d^2x^2}}{a+b\sinh^{-1}(c+dx)}+\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*(c + d*x)*Sqrt[1 + c^2 + 2*c*d*x + d^2*x^2])/(a + b*ArcSinh[c + d*
x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSinh[c + d*x])] - Sinh[(2*a)/
b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(b^2*d)
```

Maple [A]
time = 4.55, size = 160, normalized size = 1.55

method	result
derivativedivides	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1}\right.}{4b(a+b\,\mathrm{arcsinh}(}}{d}$
default	$\frac{\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e}{4b(a+b\,\mathrm{arcsinh}(dx+c))}-\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2+1+2(dx+c)\sqrt{1}\right.}{4b(a+b\,\mathrm{arcsinh}(}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e/b/(a+b*arcsinh(d*
x+c))-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/4/b*e*(2*(d*x+c)^
2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^2*e*exp(-2*a/
b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*x^4*e + 4*c*d^3*x^3*e + (6*c^2*d^2 + d^2)*x^2*e + 2*(2*c^3*d + c*d)*x
*e + (c^4 + c^2)*e + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d + d)*x*e + (c^3
+ c)*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (
c^2*d + d)*a*b + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*
x + b^2*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)) + (a*b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)) + integrate((2*d^5*x^5*e + 10*c*d^4*x^4*e + 4*(5*c^2*d^3 + d^3)*x^3*e
 + 4*(5*c^3*d^2 + 3*c*d^2)*x^2*e + 2*(5*c^4*d + 6*c^2*d + d)*x*e + 2*(d^3*x
^3*e + 3*c*d^2*x^2*e + 3*c^2*d*x*e + c^3*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) +
 2*(c^5 + 2*c^3 + c)*e + (4*d^4*x^4*e + 16*c*d^3*x^3*e + 4*(6*c^2*d^2 + d^2
```

```
)*x^2*e + 8*(2*c^3*d + c*d)*x*e + (4*c^4 + 4*c^2 + 1)*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x
^2 + 4*(c^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c
*d*x + a*b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 2*(3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 +
1)*b^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1
) + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 + 1)) + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 +
 c)*a*b)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^{2} + 2ab \operatorname{asinh}{\left(c + dx \right)} + b^{2} \operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx + \int \frac{dx}{a^{2} + 2ab \operatorname{asinh}{\left(c + dx \right)} + b^{2} \operatorname{asinh}^{2}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*asinh(c + d*x) + b**2*asinh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^2, x)
```

3.166 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}$$

```
[Out] cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^2/d-Chi((a+b*arcsinh(d*x+c))/b)*sin
h(a/b)/b^2/d-(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{(c+dx)^2+1}}{bd\left(a+b\sinh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] -(Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x]))) - (CoshIntegral[(a
+ b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(b^2*d) + (Cosh[a/b]*SinhIntegral[(a +
b*ArcSinh[c + d*x])/b])/(b^2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)}\,dx, x, c+dx\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \sinh^{-1}(c+\right.}{bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{bd\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{b^2d} + \frac{\cosh\left(\right.}{}$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 0.85

$$\frac{-\frac{b\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)} - \text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right) + \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])) - CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x
]])/(b^2*d)
```

Maple [A]
time = 3.32, size = 128, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\operatorname{arcsinh}(dx+c))}+\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\operatorname{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\operatorname{expIntegral}\left(1,-\right.}{2b^2}}{d}$
default	$\frac{\frac{-\sqrt{1+(dx+c)^2}+dx+c}{2b(a+b\operatorname{arcsinh}(dx+c))}+\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{2b^2}-\frac{dx+c+\sqrt{1+(dx+c)^2}}{2b(a+b\operatorname{arcsinh}(dx+c))}-\frac{e^{-\frac{a}{b}}\operatorname{expIntegral}\left(1,-\right.}{2b^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-(1+(d*x+c)^2)^(1/2)+d*x+c)/b/(a+b*arcsinh(d*x+c))+1/2/b^2*exp(a/
b)*Ei(1,arcsinh(d*x+c)+a/b)-1/2/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(
d*x+c))-1/2/b^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d + d)*a*b + (b^2*d^3*x
^2 + 2*b^2*c*d^2*x + (c^2*d + d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (a*
b*d^2*x + a*b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + integrate((d^4*x^4
+ 4*c*d^3*x^3 + c^4 + 2*(3*c^2*d^2 + d^2)*x^2 + (d^2*x^2 + 2*c*d*x + c^2 +
1)*(d^2*x^2 + 2*c*d*x + c^2 - 1) + 2*c^2 + 4*(c^3*d + c*d)*x + (2*d^3*x^3 +
 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d + d)*x + c)*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1) + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 + d^2)*a*b*x^2 + 4*(c
^3*d + c*d)*a*b*x + (c^4 + 2*c^2 + 1)*a*b + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*
b*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(
3*c^2*d^2 + d^2)*b^2*x^2 + 4*(c^3*d + c*d)*b^2*x + (c^4 + 2*c^2 + 1)*b^2 +
```

```
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d + d)*b^2*x + (c^3 + c)*b^2)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d + d)*a*b*x + (c^3 + c)*a*b)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**2,x)
[Out] Integral((a + b*asinh(c + d*x))**(-2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^2,x)
[Out] int(1/(a + b*asinh(c + d*x))^2, x)
```

3.167 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^2},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^2,x)/e
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(a + b\,\mathrm{arcsinh}\,(dx + c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2
+ 1)^(3/2) + c)/(a*b*d^4*x^3*e + 3*a*b*c*d^3*x^2*e + (3*c^2*d^2 + d^2)*a*b*
x*e + (c^3*d + c*d)*a*b*e + (b^2*d^4*x^3*e + 3*b^2*c*d^3*x^2*e + (3*c^2*d^2
 + d^2)*b^2*x*e + (c^3*d + c*d)*b^2*e + (b^2*d^3*x^2*e + 2*b^2*c*d^2*x*e +
b^2*c^2*d*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1)) + (a*b*d^3*x^2*e + 2*a*b*c*d^2*x*e + a*b*c^2*d*e)*sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1)) - integrate((2*(d^2*x^2 + 2*c*d*x + c^2 + 1
)*(d*x + c) + (2*d^2*x^2 + 4*c*d*x + 2*c^2 + 1)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/(a*b*d^6*x^6*e + 6*a*b*c*d^5*x^5*e + (15*c^2*d^4 + 2*d^4)*a*b*x^4*e
 + 4*(5*c^3*d^3 + 2*c*d^3)*a*b*x^3*e + (15*c^4*d^2 + 12*c^2*d^2 + d^2)*a*b*
x^2*e + 2*(3*c^5*d + 4*c^3*d + c*d)*a*b*x*e + (c^6 + 2*c^4 + c^2)*a*b*e + (
a*b*d^4*x^4*e + 4*a*b*c*d^3*x^3*e + 6*a*b*c^2*d^2*x^2*e + 4*a*b*c^3*d*x*e +
 a*b*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (b^2*d^6*x^6*e + 6*b^2*c*d^5*x^
5*e + (15*c^2*d^4 + 2*d^4)*b^2*x^4*e + 4*(5*c^3*d^3 + 2*c*d^3)*b^2*x^3*e +
(15*c^4*d^2 + 12*c^2*d^2 + d^2)*b^2*x^2*e + 2*(3*c^5*d + 4*c^3*d + c*d)*b^2
*x*e + (c^6 + 2*c^4 + c^2)*b^2*e + (b^2*d^4*x^4*e + 4*b^2*c*d^3*x^3*e + 6*b
^2*c^2*d^2*x^2*e + 4*b^2*c^3*d*x*e + b^2*c^4*e)*(d^2*x^2 + 2*c*d*x + c^2 +
1) + 2*(b^2*d^5*x^5*e + 5*b^2*c*d^4*x^4*e + (10*c^2*d^3 + d^3)*b^2*x^3*e +
(10*c^3*d^2 + 3*c*d^2)*b^2*x^2*e + (5*c^4*d + 3*c^2*d)*b^2*x*e + (c^5 + c^3
)*b^2*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)) + 2*(a*b*d^5*x^5*e + 5*a*b*c*d^4*x^4*e + (10*c^2*d^3 + d^
3)*a*b*x^3*e + (10*c^3*d^2 + 3*c*d^2)*a*b*x^2*e + (5*c^4*d + 3*c^2*d)*a*b*x
*e + (c^5 + c^3)*a*b*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*d*x + b^2*c)*arcsinh(d*x + c)^2*e + 2*(a*b*d*x + a*b*c)*ar
csinh(d*x + c)*e + (a^2*d*x + a^2*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{asinh}(c+dx)+2abdx\operatorname{asinh}(c+dx)+b^2c\operatorname{asinh}^2(c+dx)+b^2dx\operatorname{asinh}^2(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*asinh(c + d*x) + 2*a*b*d*x*asinh(c
+ d*x) + b**2*c*asinh(c + d*x)**2 + b**2*d*x*asinh(c + d*x)**2), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\operatorname{asinh}\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^2), x)
```

3.168 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\Big(}{16b}$$

```
[Out] -2*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*ar
csinh(d*x+c))+1/16*e^4*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d-27/32*e^
4*Chi(3*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+25/32*e^4*Chi(5*(a+b*arcs
inh(d*x+c))/b)*cosh(5*a/b)/b^3/d-1/16*e^4*Shi((a+b*arcsinh(d*x+c))/b)*sinh(
a/b)/b^3/d+27/32*e^4*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*
e^4*Shi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4*(1+(d
*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A]
time = 0.58, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{32b^3d}+\frac{25e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)}{32b^3d}-\frac{e^4\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{16b^3d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\mathrm{Shi}\left(\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)}{32b^3d}-\frac{5e^4(c+dx)^5}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^4\sqrt{(c+dx)^2+1}\,(c+dx)^4}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcSinh[c + d*x])) - (5*e^4*(c + d*x
)^5)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[(a +
b*ArcSinh[c + d*x])/b])/(16*b^3*d) - (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*
(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*CoshIntegr
al[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (e^4*Sinh[a/b]*SinhIntegra
l[(a + b*ArcSinh[c + d*x])/b])/(16*b^3*d) + (27*e^4*Sinh[(3*a)/b]*SinhInteg
ral[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d) - (25*e^4*Sinh[(5*a)/b]*Sin
hIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(32*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{(a+b\sinh^{-1}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{(a+b\sinh^{-1}(x))^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\,(a+b\sinh^{-1}(x))^2}\,dx \right.}{bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2e^4(c+dx)^3}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{5e^4(}{2b^2d\,(a+b}$$

Mathematica [A]
time = 0.92, size = 316, normalized size = 0.99

$$\frac{e^4\left(-\frac{16b^2(c+dx)^4\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^2}+\frac{16b\left(-4(c+dx)^3-5(c+dx)^5\right)}{a+b\sinh^{-1}(c+dx)}+48\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)+25\left(2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^
2 + (16*b*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x]) + 48*(
-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c +
d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 25*(2*Cos
h[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 3*Cosh[(3*a)/b]*CoshIntegral[
3*(a/b + ArcSinh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcSinh[c
 + d*x])] - 2*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 3*Sinh[(3*a)
/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])] - Sinh[(5*a)/b]*SinhIntegral[5
*(a/b + ArcSinh[c + d*x])])))/(32*b^3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(302) = 604$.
time = 6.74, size = 896, normalized size = 2.80

method	result	size
derivativedivides	Expression too large to display	896
default	Expression too large to display	896

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/64*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(5*b*arcsi
nh(d*x+c)+5*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-25/64*
e^4/b^3*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)+3/64*(-4*(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(3*b*arcsinh(d*x
+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+27/64*e^4/b^
3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)-1/32*(-(1+(d*x+c)^2)^(1/2)+d*x+c)
*e^4*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+
a^2)-1/32*e^4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)-1/32/b*e^4*(d*x+c+(1+(d
*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1
/2))/(a+b*arcsinh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)+
3/64/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c
)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+9/64/b^2*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64
/b^3*e^4*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/64/b*e^4*(16*(d*x+c)^5
+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d
*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-5/64/b^2*e^4*(16
*(d*x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+
c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-25/64/b^
3*e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((5*a*d^11 + b*d^11)*x^11*e^4 + 11*(5*a*c*d^10 + b*c*d^10)*x^10*e^4 +
(5*(55*c^2*d^9 + 3*d^9)*a + (55*c^2*d^9 + 3*d^9)*b)*x^9*e^4 + 3*(5*(55*c^3*
d^8 + 9*c*d^8)*a + (55*c^3*d^8 + 9*c*d^8)*b)*x^8*e^4 + 3*(5*(110*c^4*d^7 +
36*c^2*d^7 + d^7)*a + (110*c^4*d^7 + 36*c^2*d^7 + d^7)*b)*x^7*e^4 + 21*(5*(
22*c^5*d^6 + 12*c^3*d^6 + c*d^6)*a + (22*c^5*d^6 + 12*c^3*d^6 + c*d^6)*b)*x
^6*e^4 + (5*(462*c^6*d^5 + 378*c^4*d^5 + 63*c^2*d^5 + d^5)*a + (462*c^6*d^5
 + 378*c^4*d^5 + 63*c^2*d^5 + d^5)*b)*x^5*e^4 + (5*(330*c^7*d^4 + 378*c^5*d
^4 + 105*c^3*d^4 + 5*c*d^4)*a + (330*c^7*d^4 + 378*c^5*d^4 + 105*c^3*d^4 +
5*c*d^4)*b)*x^4*e^4 + (5*(165*c^8*d^3 + 252*c^6*d^3 + 105*c^4*d^3 + 10*c^2*
d^3)*a + (165*c^8*d^3 + 252*c^6*d^3 + 105*c^4*d^3 + 10*c^2*d^3)*b)*x^3*e^4
+ (5*(55*c^9*d^2 + 108*c^7*d^2 + 63*c^5*d^2 + 10*c^3*d^2)*a + (55*c^9*d^2 +
 108*c^7*d^2 + 63*c^5*d^2 + 10*c^3*d^2)*b)*x^2*e^4 + (5*(11*c^10*d + 27*c^8
*d + 21*c^6*d + 5*c^4*d)*a + (11*c^10*d + 27*c^8*d + 21*c^6*d + 5*c^4*d)*b)
*x*e^4 + ((5*a*d^8 + b*d^8)*x^8*e^4 + 8*(5*a*c*d^7 + b*c*d^7)*x^7*e^4 + (4*
(35*c^2*d^6 + 2*d^6)*a + (28*c^2*d^6 + d^6)*b)*x^6*e^4 + 2*(4*(35*c^3*d^5 +
 6*c*d^5)*a + (28*c^3*d^5 + 3*c*d^5)*b)*x^5*e^4 + ((350*c^4*d^4 + 120*c^2*d
^4 + 3*d^4)*a + 5*(14*c^4*d^4 + 3*c^2*d^4)*b)*x^4*e^4 + 4*((70*c^5*d^3 + 40
*c^3*d^3 + 3*c*d^3)*a + (14*c^5*d^3 + 5*c^3*d^3)*b)*x^3*e^4 + (2*(70*c^6*d^
2 + 60*c^4*d^2 + 9*c^2*d^2)*a + (28*c^6*d^2 + 15*c^4*d^2)*b)*x^2*e^4 + 2*(2
*(10*c^7*d + 12*c^5*d + 3*c^3*d)*a + (4*c^7*d + 3*c^5*d)*b)*x*e^4 + ((5*c^8
 + 8*c^6 + 3*c^4)*a + (c^8 + c^6)*b)*e^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/
2) + (3*(5*a*d^9 + b*d^9)*x^9*e^4 + 27*(5*a*c*d^8 + b*c*d^8)*x^8*e^4 + ((54
0*c^2*d^7 + 31*d^7)*a + (108*c^2*d^7 + 5*d^7)*b)*x^7*e^4 + 7*((180*c^3*d^6
+ 31*c*d^6)*a + (36*c^3*d^6 + 5*c*d^6)*b)*x^6*e^4 + ((1890*c^4*d^5 + 651*c^
2*d^5 + 20*d^5)*a + (378*c^4*d^5 + 105*c^2*d^5 + 2*d^5)*b)*x^5*e^4 + (5*(37
8*c^5*d^4 + 217*c^3*d^4 + 20*c*d^4)*a + (378*c^5*d^4 + 175*c^3*d^4 + 10*c*d
^4)*b)*x^4*e^4 + ((1260*c^6*d^3 + 1085*c^4*d^3 + 200*c^2*d^3 + 4*d^3)*a + (
252*c^6*d^3 + 175*c^4*d^3 + 20*c^2*d^3)*b)*x^3*e^4 + ((540*c^7*d^2 + 651*c^
5*d^2 + 200*c^3*d^2 + 12*c*d^2)*a + (108*c^7*d^2 + 105*c^5*d^2 + 20*c^3*d^2
)*b)*x^2*e^4 + ((135*c^8*d + 217*c^6*d + 100*c^4*d + 12*c^2*d)*a + (27*c^8*
d + 35*c^6*d + 10*c^4*d)*b)*x*e^4 + ((15*c^9 + 31*c^7 + 20*c^5 + 4*c^3)*a +
 (3*c^9 + 5*c^7 + 2*c^5)*b)*e^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (5*(c^11 +
 3*c^9 + 3*c^7 + c^5)*a + (c^11 + 3*c^9 + 3*c^7 + c^5)*b)*e^4 + (5*b*d^11*x
^11*e^4 + 55*b*c*d^10*x^10*e^4 + 5*(55*c^2*d^9 + 3*d^9)*b*x^9*e^4 + 15*(55*
c^3*d^8 + 9*c*d^8)*b*x^8*e^4 + 15*(110*c^4*d^7 + 36*c^2*d^7 + d^7)*b*x^7*e^
4 + 105*(22*c^5*d^6 + 12*c^3*d^6 + c*d^6)*b*x^6*e^4 + 5*(462*c^6*d^5 + 378*
c^4*d^5 + 63*c^2*d^5 + d^5)*b*x^5*e^4 + 5*(330*c^7*d^4 + 378*c^5*d^4 + 105*
```

```
c^3*d^4 + 5*c*d^4)*b*x^4*e^4 + 5*(165*c^8*d^3 + 252*c^6*d^3 + 105*c^4*d^3 +
 10*c^2*d^3)*b*x^3*e^4 + 5*(55*c^9*d^2 + 108*c^7*d^2 + 63*c^5*d^2 + 10*c^3*
d^2)*b*x^2*e^4 + 5*(11*c^10*d + 27*c^8*d + 21*c^6*d + 5*c^4*d)*b*x*e^4 + 5*
(c^11 + 3*c^9 + 3*c^7 + c^5)*b*e^4 + (5*b*d^8*x^8*e^4 + 40*b*c*d^7*x^7*e^4
+ 4*(35*c^2*d^6 + 2*d^6)*b*x^6*e^4 + 8*(35*c^3*d^5 + 6*c*d^5)*b*x^5*e^4 + (
350*c^4*d^4 + 120*c^2*d^4 + 3*d^4)*b*x^4*e^4 + 4*(70*c^5*d^3 + 40*c^3*d^3 +
 3*c*d^3)*b*x^3*e^4 + 2*(70*c^6*d^2 + 60*c^4*d^2 + 9*c^2*d^2)*b*x^2*e^4 + 4
*(10*c^7*d + 12*c^5*d + 3*c^3*d)*b*x*e^4 + (5*c^8 + 8*c^6 + 3*c^4)*b*e^4)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (15*b*d^9*x^9*e^4 + 135*b*c*d^8*x^8*e^
4 + (540*c^2*d^7 + 31*d^7)*b*x^7*e^4 + 7*(180*c^3*d^6 + 31*c*d^6)*b*x^6*e^4
 + (1890*c^4*d^5 + 651*c^2*d^5 + 20*d^5)*b*x^5*e^4 + 5*(378*c^5*d^4 + 217*c
^3*d^4 + 20*c*d^4)*b*x^4*e^4 + (1260*c^6*d^3 + 1085*c^4*d^3 + 200*c^2*d^3 +
 4*d^3)*b*x^3*e^4 + (540*c^7*d^2 + 651*c^5*d^2 + 200*c^3*d^2 + 12*c*d^2)*b*
x^2*e^4 + (135*c^8*d + 217*c^6*d + 100*c^4*d + 12*c^2*d)*b*x*e^4 + (15*c^9
+ 31*c^7 + 20*c^5 + 4*c^3)*b*e^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (15*b*d^1
0*x^10*e^4 + 150*b*c*d^9*x^9*e^4 + (675*c^2*d^8 + 38*d^8)*b*x^8*e^4 + 8*(22
5*c^3*d^7 + 38*c*d^7)*b*x^7*e^4 + 2*(1575*c^4*d^6 + 532*c^2*d^6 + 16*d^6)*b
*x^6*e^4 + 4*(945*c^5*d^5 + 532*c^3*d^5 + 48*c*d^5)*b*x^5*e^4 + (3150*c^6*d
^4 + 2660*c^4*d^4 + 480*c^2*d^4 + 9*d^4)*b*x^4*e^4 + 4*(450*c^7*d^3 + 532*c
^5*d^3 + 160*c^3*d^3 + 9*c*d^3)*b*x^3*e^4 + (675*c^8*d^2 + 1064*c^6*d^2 + 4
80*c^4*d^2 + 54*c^2*d^2)*b*x^2*e^4 + 2*(75*c^9*d + 152*c^7*d + 96*c^5*d + 1
8*c^3*d)*b*x*e^4 + (15*c^10 + 38*c^8 + 32*c^6 + 9*c^4)*b*e^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3
*(5*a*d^10 + b*d^10)*x^10*e^4 + 30*(5*a*c*d^9 + b*c*d^9)*x^9*e^4 + ((675*c^
2*d^8 + 38*d^8)*a + (135*c^2*d^8 + 7*d^8)*b)*x^8*e^4 + 8*((225*c^3*d^7 + 38
*c*d^7)*a + (45*c^3*d^7 + 7*c*d^7)*b)*x^7*e^4 + (2*(1575*c^4*d^6 + 532*c^2*
d^6 + 16*d^6)*a + (630*c^4*d^6 + 196*c^2*d^6 + 5*d^6)*b)*x^6*e^4 + 2*(2*(94
5*c^5*d^5 + 532*c^3*d^5 + 48*c*d^5)*a + (378*c^5*d^5 + 196*c^3*d^5 + 15*c*d
^5)*b)*x^5*e^4 + ((3150*c^6*d^4 + 2660*c^4*d^4 + 480*c^2*d^4 + 9*d^4)*a + (
630*c^6*d^4 + 490*c^4*d^4 + 75*c^2*d^4 + d^4)*b...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^3
*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c)
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^3 + 3a^2b\operatorname{asinh}(c+dx) + 3ab^2\operatorname{asinh}^2(c+dx) + b^3\operatorname{asinh}^3(c+dx)}\,dx + \int \frac{d^4x^4}{a^3 + 3a^2b\operatorname{asinh}(c+dx) + 3ab^2\operatorname{asinh}^2(c+dx) + b^3\operatorname{asinh}^3(c+dx)}\,dx + \int \frac{4cd^3x^3}{a^3 + 3a^2b\operatorname{asinh}(c+dx) + 3ab^2\operatorname{asinh}^2(c+dx) + b^3\operatorname{asinh}^3(c+dx)}\,dx + \int \frac{6c^2d^2x^2}{a^3 + 3a^2b\operatorname{asinh}(c+dx) + 3ab^2\operatorname{asinh}^2(c+dx) + b^3\operatorname{asinh}^3(c+dx)}\,dx + \int \frac{4c^3dx}{a^3 + 3a^2b\operatorname{asinh}(c+dx) + 3ab^2\operatorname{asinh}^2(c+dx) + b^3\operatorname{asinh}^3(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**4*(Integral(c**4/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a**3 +
3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)*
*3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*as
inh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^3, x)
```

3.169 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=247

$$-\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{e^3\text{Chi}\left(\frac{2(a+b\sinh^{-1}(\cdot}{b}\right.}{2b^3}$$

```
[Out] -3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*ar
csinh(d*x+c))-1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d+e^3*c
osh(4*a/b)*Shi(4*(a+b*arcsinh(d*x+c))/b)/b^3/d+1/2*e^3*Chi(2*(a+b*arcsinh(d
*x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*sinh(4*a/b)/b
^3/d-1/2*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A]

time = 0.45, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382}

$$\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{b^3d}-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{2b^3d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{b^3d}-\frac{2e^3(c+dx)^4}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (2*e^3*(c + d
*x)^4)/(b^2*d*(a + b*ArcSinh[c + d*x])) + (e^3*CoshIntegral[(2*(a + b*ArcSi
nh[c + d*x]))/b]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4*(a + b*Arc
Sinh[c + d*x]))/b]*Sinh[(4*a)/b])/(b^3*d) - (e^3*Cosh[(2*a)/b]*SinhIntegral
[(2*(a + b*ArcSinh[c + d*x]))/b])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*SinhIntegr
al[(4*(a + b*ArcSinh[c + d*x]))/b])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{(3e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx,x\right.}{2bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{2e^3(c}{b^2d\left(a+b\,\mathrm{si}\right.}$$

Mathematica [A]
time = 0.51, size = 179, normalized size = 0.72

$$\frac{e^3\left(-\frac{b^2(c+dx)^3\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^2}+\frac{b\left(-3(c+dx)^2-4(c+dx)^4\right)}{a+b\sinh^{-1}(c+dx)}+\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)-2\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{4a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^3,x]
[Out] (e^3*(-((b^2*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2)
 + (b*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x]) + CoshInte
```

```
gral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegral[4*(a/b + Ar
cSinh[c + d*x])]*Sinh[(4*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSin
h[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x])]))/(
2*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(237) = 474$.
time = 8.08, size = 579, normalized size = 2.34

method	result
derivativedivides	$-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-$
default	$-\frac{\left(-8(dx+c)^3\sqrt{1+(dx+c)^2}+8(dx+c)^4-4(dx+c)\sqrt{1+(dx+c)^2}+8(dx+c)^2+1\right)e^3(4b\operatorname{arcsinh}(dx+c)+4a-b)}{32b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/32*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+
c)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(4*b*arcsinh(d*x+c)+4*a-b)/b^2/(b^2*arcsinh(
d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e^3/b^3*exp(4*a/b)*Ei(1,4*arcsinh(d*
x+c)+4*a/b)+1/16*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e^3*(2*b*ar
csinh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4
*e^3/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/16/b*e^3*(2*(d*x+c)^2+1+
2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/8/b^2*e^3*(2*(d*x+c
)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/4/b^3*e^3*exp(-
2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*
(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh
(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(
1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-1/2/b^3*e^3*exp(
-4*a/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*d^10 + b*d^10)*x^10*e^3 + 10*(4*a*c*d^9 + b*c*d^9)*x^9*e^3 + 3*(
4*(15*c^2*d^8 + d^8)*a + (15*c^2*d^8 + d^8)*b)*x^8*e^3 + 24*(4*(5*c^3*d^7 +
```

c*d^7)*a + (5*c^3*d^7 + c*d^7)*b)*x^7*e^3 + 3*(4*(70*c^4*d^6 + 28*c^2*d^6
+ d^6)*a + (70*c^4*d^6 + 28*c^2*d^6 + d^6)*b)*x^6*e^3 + 6*(4*(42*c^5*d^5 +
28*c^3*d^5 + 3*c*d^5)*a + (42*c^5*d^5 + 28*c^3*d^5 + 3*c*d^5)*b)*x^5*e^3 +
(4*(210*c^6*d^4 + 210*c^4*d^4 + 45*c^2*d^4 + d^4)*a + (210*c^6*d^4 + 210*c^
4*d^4 + 45*c^2*d^4 + d^4)*b)*x^4*e^3 + 4*(4*(30*c^7*d^3 + 42*c^5*d^3 + 15*c
^3*d^3 + c*d^3)*a + (30*c^7*d^3 + 42*c^5*d^3 + 15*c^3*d^3 + c*d^3)*b)*x^3*e
^3 + 3*(4*(15*c^8*d^2 + 28*c^6*d^2 + 15*c^4*d^2 + 2*c^2*d^2)*a + (15*c^8*d^
2 + 28*c^6*d^2 + 15*c^4*d^2 + 2*c^2*d^2)*b)*x^2*e^3 + 2*(4*(5*c^9*d + 12*c^
7*d + 9*c^5*d + 2*c^3*d)*a + (5*c^9*d + 12*c^7*d + 9*c^5*d + 2*c^3*d)*b)*x*
e^3 + ((4*a*d^7 + b*d^7)*x^7*e^3 + 7*(4*a*c*d^6 + b*c*d^6)*x^6*e^3 + (6*(14
*c^2*d^5 + d^5)*a + (21*c^2*d^5 + d^5)*b)*x^5*e^3 + 5*(2*(14*c^3*d^4 + 3*c*
d^4)*a + (7*c^3*d^4 + c*d^4)*b)*x^4*e^3 + (2*(70*c^4*d^3 + 30*c^2*d^3 + d^3
)*a + 5*(7*c^4*d^3 + 2*c^2*d^3)*b)*x^3*e^3 + (6*(14*c^5*d^2 + 10*c^3*d^2 +
c*d^2)*a + (21*c^5*d^2 + 10*c^3*d^2)*b)*x^2*e^3 + (2*(14*c^6*d + 15*c^4*d +
3*c^2*d)*a + (7*c^6*d + 5*c^4*d)*b)*x*e^3 + (2*(2*c^7 + 3*c^5 + c^3)*a + (
c^7 + c^5)*b)*e^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(4*a*d^8 + b*d^
8)*x^8*e^3 + 24*(4*a*c*d^7 + b*c*d^7)*x^7*e^3 + (24*(14*c^2*d^6 + d^6)*a +
(84*c^2*d^6 + 5*d^6)*b)*x^6*e^3 + 6*(8*(14*c^3*d^5 + 3*c*d^5)*a + (28*c^3*d
^5 + 5*c*d^5)*b)*x^5*e^3 + (15*(56*c^4*d^4 + 24*c^2*d^4 + d^4)*a + (210*c^4
*d^4 + 75*c^2*d^4 + 2*d^4)*b)*x^4*e^3 + 4*(3*(56*c^5*d^3 + 40*c^3*d^3 + 5*c
*d^3)*a + (42*c^5*d^3 + 25*c^3*d^3 + 2*c*d^3)*b)*x^3*e^3 + 3*((112*c^6*d^2
+ 120*c^4*d^2 + 30*c^2*d^2 + d^2)*a + (28*c^6*d^2 + 25*c^4*d^2 + 4*c^2*d^2)
*b)*x^2*e^3 + 2*(3*(16*c^7*d + 24*c^5*d + 10*c^3*d + c*d)*a + (12*c^7*d + 1
5*c^5*d + 4*c^3*d)*b)*x*e^3 + (3*(4*c^8 + 8*c^6 + 5*c^4 + c^2)*a + (3*c^8 +
5*c^6 + 2*c^4)*b)*e^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (4*(c^10 + 3*c^8 +
3*c^6 + c^4)*a + (c^10 + 3*c^8 + 3*c^6 + c^4)*b)*e^3 + (4*b*d^10*x^10*e^3 +
40*b*c*d^9*x^9*e^3 + 12*(15*c^2*d^8 + d^8)*b*x^8*e^3 + 96*(5*c^3*d^7 + c*d
^7)*b*x^7*e^3 + 12*(70*c^4*d^6 + 28*c^2*d^6 + d^6)*b*x^6*e^3 + 24*(42*c^5*d
^5 + 28*c^3*d^5 + 3*c*d^5)*b*x^5*e^3 + 4*(210*c^6*d^4 + 210*c^4*d^4 + 45*c^
2*d^4 + d^4)*b*x^4*e^3 + 16*(30*c^7*d^3 + 42*c^5*d^3 + 15*c^3*d^3 + c*d^3)*
b*x^3*e^3 + 12*(15*c^8*d^2 + 28*c^6*d^2 + 15*c^4*d^2 + 2*c^2*d^2)*b*x^2*e^3
+ 8*(5*c^9*d + 12*c^7*d + 9*c^5*d + 2*c^3*d)*b*x*e^3 + 4*(c^10 + 3*c^8 + 3
*c^6 + c^4)*b*e^3 + 2*(2*b*d^7*x^7*e^3 + 14*b*c*d^6*x^6*e^3 + 3*(14*c^2*d^5
+ d^5)*b*x^5*e^3 + 5*(14*c^3*d^4 + 3*c*d^4)*b*x^4*e^3 + (70*c^4*d^3 + 30*c
^2*d^3 + d^3)*b*x^3*e^3 + 3*(14*c^5*d^2 + 10*c^3*d^2 + c*d^2)*b*x^2*e^3 + (
14*c^6*d + 15*c^4*d + 3*c^2*d)*b*x*e^3 + (2*c^7 + 3*c^5 + c^3)*b*e^3)*(d^2*
x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(4*b*d^8*x^8*e^3 + 32*b*c*d^7*x^7*e^3 +
8*(14*c^2*d^6 + d^6)*b*x^6*e^3 + 16*(14*c^3*d^5 + 3*c*d^5)*b*x^5*e^3 + 5*(5
6*c^4*d^4 + 24*c^2*d^4 + d^4)*b*x^4*e^3 + 4*(56*c^5*d^3 + 40*c^3*d^3 + 5*c*
d^3)*b*x^3*e^3 + (112*c^6*d^2 + 120*c^4*d^2 + 30*c^2*d^2 + d^2)*b*x^2*e^3 +
2*(16*c^7*d + 24*c^5*d + 10*c^3*d + c*d)*b*x*e^3 + (4*c^8 + 8*c^6 + 5*c^4
+ c^2)*b*e^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (12*b*d^9*x^9*e^3 + 108*b*c*d
^8*x^8*e^3 + 6*(72*c^2*d^7 + 5*d^7)*b*x^7*e^3 + 42*(24*c^3*d^6 + 5*c*d^6)*b
*x^6*e^3 + (1512*c^4*d^5 + 630*c^2*d^5 + 25*d^5)*b*x^5*e^3 + (1512*c^5*d^4
+ 1050*c^3*d^4 + 125*c*d^4)*b*x^4*e^3 + (1008*c^6*d^3 + 1050*c^4*d^3 + 250*

```
c^2*d^3 + 7*d^3)*b*x^3*e^3 + (432*c^7*d^2 + 630*c^5*d^2 + 250*c^3*d^2 + 21*
c*d^2)*b*x^2*e^3 + (108*c^8*d + 210*c^6*d + 125*c^4*d + 21*c^2*d)*b*x*e^3 +
 (12*c^9 + 30*c^7 + 25*c^5 + 7*c^3)*b*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(4*a*d^9 + b*d^9)*
x^9*e^3 + 27*(4*a*c*d^8 + b*c*d^8)*x^8*e^3 + (6*(72*c^2*d^7 + 5*d^7)*a + (1
08*c^2*d^7 + 7*d^7)*b)*x^7*e^3 + 7*(6*(24*c^3*d^6 + 5*c*d^6)*a + (36*c^3*d^
6 + 7*c*d^6)*b)*x^6*e^3 + ((1512*c^4*d^5 + 630*c^2*d^5 + 25*d^5)*a + (378*c
^4*d^5 + 147*c^2*d^5 + 5*d^5)*b)*x^5*e^3 + ((1512*c^5*d^4 + 1050*c^3*d^4 +
125*c*d^4)*a + (378*c^5*d^4 + 245*c^3*d^4 + 25*c*d^4)*b)*x^4*e^3 + ((1008*c
^6*d^3 + 1050*c^4*d^3 + 250*c^2*d^3 + 7*d^3)*a + (252*c^6*d^3 + 245*c^4*d^3
 + 50*c^2*d^3 + d^3)*b)*x^3*e^3 + ((432*c^7*d^2 + 630*c^5*d^2 + 250*c^3*d^2
 + 21*c*d^2)*a + (108*c^7*d^2 + 147*c^5*d^2 + 50*c^3*d^2 + 3*c*d^2)*b)*x^2*
e^3 + ((108*c^8*d + 210*c^6*d + 125*c^4*d + 21*c^2*d)*a + (27*c^8*d + 49*c^
6*d + 25*c^4*d + 3*c^2*d)*b)*x*e^3 + ((12*c^9 + 30*c^7 + 25*c^5 + 7*c^3)*a
+ (3*c^9 + 7*c^7 + 5*c^5 + c^3)*b)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/
(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 +
4*(5*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b
^2*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c
^2*d + d)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^
4*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*...
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^3*arcsinh(d*x + c
)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{d^{3}x^{3}}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{3cd^{2}x^{2}}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{3c^{2}dx}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c +
 d*x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
```

```
*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x
))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\left(c+d\,x\right))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^3, x)
```

3.170 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\Big(}{8b^3}$$

```
[Out] -e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arcsin
h(d*x+c))-1/8*e^2*Chi((a+b*arcsinh(d*x+c))/b)*cosh(a/b)/b^3/d+9/8*e^2*Chi(3
*(a+b*arcsinh(d*x+c))/b)*cosh(3*a/b)/b^3/d+1/8*e^2*Shi((a+b*arcsinh(d*x+c))
/b)*sinh(a/b)/b^3/d-9/8*e^2*Shi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^3/d
-1/2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A]
time = 0.41, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382, 5774}

$$-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{8b^3d}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{8b^3d}-\frac{3e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2\sqrt{(c+dx)^2+1}\,(c+dx)^2}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
2) - (e^2*(c + d*x))/(b^2*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^3)
/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e^2*Cosh[a/b]*CoshIntegral[(a + b*Ar
cSinh[c + d*x])/b])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*(a + b
*ArcSinh[c + d*x]))/b])/(8*b^3*d) + (e^2*Sinh[a/b]*SinhIntegral[(a + b*ArcS
inh[c + d*x])/b])/(8*b^3*d) - (9*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c + d*x]))/b])/(8*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
```

```
c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e^2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x,\right.}{bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e^2(c+dx)}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{3e^2(}{2b^2d\left(a+b\right.}$$

Mathematica [A]

time = 0.57, size = 216, normalized size = 0.88

$$\frac{e^2\left(-\frac{4b^2(c+dx)^2\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^2}+\frac{4b\left(-2(c+dx)-3(c+dx)^3\right)}{a+b\sinh^{-1}(c+dx)}+8\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-8\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+9\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{8b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^3,x]

[Out] (e^2*((-4*b^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (4*b*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x]) + 8*Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]] - 8*Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 9*(-(Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]) + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSinh[c + d*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])])))/(8*b^3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(232) = 464$.
time = 6.56, size = 507, normalized size = 2.06

method	result
derivativedivides	$-\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\operatorname{expIn}}{}}{}$
default	$-\frac{\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2(3b\operatorname{arcsinh}(dx+c)+3a-b)}{16b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{9e^2e^{\frac{3a}{b}}\operatorname{expIn}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/16*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^2*(3*b*arcsinh(d*x+c)+3*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-9/16*e^2/b^3*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/16*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/16*e^2/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)+a/b)+1/16/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/16/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/16/b^3*e^2*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16/b^3*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((3*a*d^9 + b*d^9)*x^9*e^2 + 9*(3*a*c*d^8 + b*c*d^8)*x^8*e^2 + 3*(3*(12*c^2*d^7 + d^7)*a + (12*c^2*d^7 + d^7)*b)*x^7*e^2 + 21*(3*(4*c^3*d^6 + c*d^6)*a + (4*c^3*d^6 + c*d^6)*b)*x^6*e^2 + 3*(3*(42*c^4*d^5 + 21*c^2*d^5 + d^5)*a + (42*c^4*d^5 + 21*c^2*d^5 + d^5)*b)*x^5*e^2 + 3*(3*(42*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*a + (42*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b)*x^4*e^2 + (3*(84*c^6*d^3 + 105*c^4*d^3 + 30*c^2*d^3 + d^3)*a + (84*c^6*d^3 + 105*c^4*d^3 + 30*c^2*d^3 + d^3)*b)*x^3*e^2 + 3*(3*(12*c^7*d^2 + 21*c^5*d^2 + 10*c^3*d^2 + c*d^2)*a + (12*c^7*d^2 + 21*c^5*d^2 + 10*c^3*d^2 + c*d^2)*b)*x^2*e^2 + 3*(3*(3*c^8*d + 7*c^6*d + 5*c^4*d + c^2*d)*a + (3*c^8*d + 7*c^6*d + 5*c^4*d + c^2*d)*b)*x*e^2 + ((3*a*d^6 + b*d^6)*x^6*e^2 + 6*(3*a*c*d^5 + b*c*d^5)*x^5*e^2 + ((45*c^2*d^4 + 4*d^4)*a + (15*c^2*d^4 + d^4)*b)*x^4*e^2 + 4*((15*c^3*d^3 + 4*c*d^3)*a + (5*c^3*d^3 + c*d^3)*b)*x^3*e^2 + ((45*c^4*d^2 + 24*c^2*d^2 + d^2)*a + 3*(5*c^4*d^2 + 2*c^2*d^2)*b)*x^2*e^2 + 2*((9*c^5*d + 8*c^3*d + c*d)*a + (3*c^5*d + 2*c^3*d)*b)*x*e^2 + ((3*c^6 + 4*c^4 + c^2)*a + (c^6 + c^4)*b)*e^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(3*a*d^7 + b*d^7)*x^7*e^2 + 21*(3*a*c*d^6 + b*c*d^6)*x^6*e^2 + ((189*c^2*d^5 + 17*d^5)*a + (63*c^2*d^5 + 5*d^5)*b)*x^5*e^2 + 5*((63*c^3*d^4 + 17*c*d^4)*a + (21*c^3*d^4 + 5*c*d^4)*b)*x^4*e^2 + (5*(63*c^4*d^3 + 34*c^2*d^3 + 2*d^3)*a + (105*c^4*d^3 + 50*c^2*d^3 + 2*d^3)*b)*x^3*e^2 + ((189*c^5*d^2 + 170*c^3*d^2 + 30*c*d^2)*a + (63*c^5*d^2 + 50*c^3*d^2 + 6*c*d^2)*b)*x^2*e^2 + ((63*c^6*d + 85*c^4*d + 30*c^2*d + 2*d)*a + (21*c^6*d + 25*c^4*d + 6*c^2*d)*b)*x*e^2 + ((9*c^7 + 17*c^5 + 10*c^3 + 2*c)*a + (3*c^7 + 5*c^5 + 2*c^3)*b)*e^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (3*(c^9 + 3*c^7 + 3*c^5 + c^3)*a + (c^9 + 3*c^7 + 3*c^5 + c^3)*b)*e^2 + (3*b*d^9*x^9*e^2 + 27*b*c*d^8*x^8*e^2 + 9*(12*c^2*d^7 + d^7)*b*x^7*e^2 + 63*(4*c^3*d^6 + c*d^6)*b*x^6*e^2 + 9*(42*c^4*d^5 + 21*c^2*d^5 + d^5)*b*x^5*e^2 + 9*(42*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b*x^4*e^2 + 3*(84*c^6*d^3 + 105*c^4*d^3 + 30*c^2*d^3 + d^3)*b*x^3*e^2 + 9*(12*c^7*d^2 + 21*c^5*d^2 + 10*c^3*d^2 + c*d^2)*b*x^2*e^2 + 9*(3*c^8*d + 7*c^6*d + 5*c^4*d + c^2*d)*b*x*e^2 + 3*(c^9 + 3*c^7 + 3*c^5 + c^3)*b*e^2 + (3*b*d^6*x^6*e^2 + 18*b*c*d^5*x^5*e^2 + (45*c^2*d^4 + 4*d^4)*b*x^4*e^2 + 4*(15*c^3*d^3 + 4*c*d^3)*b*x^3*e^2 + (45*c^4*d^2 + 24*c^2*d^2 + d^2)*b*x^2*e^2 + 2*(9*c^5*d + 8*c^3*d + c*d)*b*x*e^2 + (3*c^6 + 4*c^4 + c^2)*b*e^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (9*b*d^7*x^7*e^2 + 63*b*c*d^6*x^6*e^2 + (189*c^2*d^5 + 17*d^5)*b*x^5*e^2 + 5*(63*c^3*d^4 + 17*c*d^4)*b*x^4*e^2 + 5*(63*c^4*d^3 + 34*c^2*d^3 + 2*d^3)*b*x^3*e^2 + (189*c^5*d^2 + 170*c^3*d^2 + 30*c*d^2)*b*x^2*e^2 + (63*c^6*d + 85*c^4*d + 30*c^2*d + 2*d)*b*x*e^2 + (9*c^7 + 17*c^5 + 10*c^3 + 2*c)*b*e^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (9*b*d^8*x^8*e^2 + 72*b*c*d^7*x^7*e^2 + 2*(126*c^2*d^6 + 11*d^6)*b*x^6*e^2 + 12*(42*c^3*d^5 + 11*c*d^5)*b*x^5*e^2 + 6*(105*c^4*d^4 + 55*c^2*d^4 + 3*d^4)*b*x^4*e^2 + 8*(63*c^5*d^3 + 55*c^3*d^3 + 9*c*d^3)*b*x^3*e^2 + (252*c^6*d^2 + 330*c^4*d^2 + 108*c^2*d^2 + 5*d^2)*b*x^2*e^2 + 2*(36*c^7*d + 66*c^5*d + 36*c^3*d + 5*c*d)*b*x*e^2 +

```
(9*c^8 + 22*c^6 + 18*c^4 + 5*c^2)*b*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(3*a*d^8 + b*d^8)*x^
8*e^2 + 24*(3*a*c*d^7 + b*c*d^7)*x^7*e^2 + (2*(126*c^2*d^6 + 11*d^6)*a + 7*
(12*c^2*d^6 + d^6)*b)*x^6*e^2 + 6*(2*(42*c^3*d^5 + 11*c*d^5)*a + 7*(4*c^3*d
^5 + c*d^5)*b)*x^5*e^2 + (6*(105*c^4*d^4 + 55*c^2*d^4 + 3*d^4)*a + 5*(42*c^
4*d^4 + 21*c^2*d^4 + d^4)*b)*x^4*e^2 + 4*(2*(63*c^5*d^3 + 55*c^3*d^3 + 9*c*
d^3)*a + (42*c^5*d^3 + 35*c^3*d^3 + 5*c*d^3)*b)*x^3*e^2 + ((252*c^6*d^2 + 3
30*c^4*d^2 + 108*c^2*d^2 + 5*d^2)*a + (84*c^6*d^2 + 105*c^4*d^2 + 30*c^2*d^
2 + d^2)*b)*x^2*e^2 + 2*((36*c^7*d + 66*c^5*d + 36*c^3*d + 5*c*d)*a + (12*c
^7*d + 21*c^5*d + 10*c^3*d + c*d)*b)*x*e^2 + ((9*c^8 + 22*c^6 + 18*c^4 + 5*
c^2)*a + (3*c^8 + 7*c^6 + 5*c^4 + c^2)*b)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2
*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3
)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*
d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 +
d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 +
 d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d +
3*c^2*d + d)*b^4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c
^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^
3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*
d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 +
2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2
+ 6*c^2*d^2 + d^2)*b^4*x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b
^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^
2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a^2*b^2*d...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*ar
csinh(d*x + c)^2 + 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{d^{2}x^{2}}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx + \int \frac{2cdx}{a^{3} + 3a^{2}b\operatorname{asinh}{\left(c + dx \right)} + 3ab^{2}\operatorname{asinh}^{2}{\left(c + dx \right)} + b^{3}\operatorname{asinh}^{3}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*
x)**2 + b**3*asinh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
sinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)*
*2 + b**3*asinh(c + d*x)**3), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^3, x)
```

3.171 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=156

$$-\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+\dots}{b}\right.}{b^3d}$$

```
[Out] -1/2*e/b^2/d/(a+b*arcsinh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))+e*
cosh(2*a/b)*Shi(2*(a+b*arcsinh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arcsinh(d*x+c)
)/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^2
```

Rubi [A]

time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3384, 3379, 3382, 5783}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e\sqrt{(c+dx)^2+1}\,(c+dx)}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] -1/2*(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^2) -
 e/(2*b^2*d*(a + b*ArcSinh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d*(a + b*ArcSi
nh[c + d*x])) - (e*CoshIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(2*a)/
b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b]
)/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+\right.}{2bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} - \frac{e(c-}{b^2d\left(a+b\sin\right.}$$

Mathematica [A]
time = 0.24, size = 120, normalized size = 0.77

$$\frac{e\left(-\frac{b^2(c+dx)\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^2}+\frac{b\left(-1-2(c+dx)^2\right)}{a+b\sinh^{-1}(c+dx)}-2\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{2a}{b}\right)+2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)}{2b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^3,x]

[Out] (e*(-((b^2*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2) + (b*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 2*CoshIntegral[2*(a/b + ArcSinh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))/(2*b^3*d)

Maple [A]
time = 5.26, size = 239, normalized size = 1.53

method	result
derivativedivides	$-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\,\mathrm{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\,\mathrm{arcsinh}(dx+c)^2+2ab\,\mathrm{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(\right.}{}$
default	$-\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e(2b\,\mathrm{arcsinh}(dx+c)+2a-b)}{8b^2\left(b^2\,\mathrm{arcsinh}(dx+c)^2+2ab\,\mathrm{arcsinh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\,\mathrm{expIntegral}\left(1,2\,\mathrm{arcsinh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/8*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b*arcsinh(d*x+c)+2*a-b)/b^2/(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)+1/2*e/b^3*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*e*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((2*a*d^8 + b*d^8)*x^8*e + 8*(2*a*c*d^7 + b*c*d^7)*x^7*e + (2*(28*c^2*d^6 + 3*d^6)*a + (28*c^2*d^6 + 3*d^6)*b)*x^6*e + 2*(2*(28*c^3*d^5 + 9*c*d^5)*a + (28*c^3*d^5 + 9*c*d^5)*b)*x^5*e + (2*(70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*a + (70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*b)*x^4*e + 4*(2*(14*c^5*d^3 + 15*c^3*d^3 + 3*c*d^3)*a + (14*c^5*d^3 + 15*c^3*d^3 + 3*c*d^3)*b)*x^3*e + (2*(28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d^2)*a + (28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d^2)*b)*x^2*e + 2*(2*(4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*a + (4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*b)*x*e + ((2*a*d^5 + b*d^5)*x^5*e + 5*(2*a*

```
c*d^4 + b*c*d^4)*x^4*e + (2*(10*c^2*d^3 + d^3)*a + (10*c^2*d^3 + d^3)*b)*x^
3*e + (2*(10*c^3*d^2 + 3*c*d^2)*a + (10*c^3*d^2 + 3*c*d^2)*b)*x^2*e + (2*(5
*c^4*d + 3*c^2*d)*a + (5*c^4*d + 3*c^2*d)*b)*x*e + (2*(c^5 + c^3)*a + (c^5
+ c^3)*b)*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*(2*a*d^6 + b*d^6)*x^6
*e + 18*(2*a*c*d^5 + b*c*d^5)*x^5*e + 5*(2*(9*c^2*d^4 + d^4)*a + (9*c^2*d^4
 + d^4)*b)*x^4*e + 20*(2*(3*c^3*d^3 + c*d^3)*a + (3*c^3*d^3 + c*d^3)*b)*x^3
*e + (5*(18*c^4*d^2 + 12*c^2*d^2 + d^2)*a + (45*c^4*d^2 + 30*c^2*d^2 + 2*d^
2)*b)*x^2*e + 2*((18*c^5*d + 20*c^3*d + 5*c*d)*a + (9*c^5*d + 10*c^3*d + 2*
c*d)*b)*x*e + ((6*c^6 + 10*c^4 + 5*c^2 + 1)*a + (3*c^6 + 5*c^4 + 2*c^2)*b)*
e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*(c^8 + 3*c^6 + 3*c^4 + c^2)*a + (c^8
+ 3*c^6 + 3*c^4 + c^2)*b)*e + (2*b*d^8*x^8*e + 16*b*c*d^7*x^7*e + 2*(28*c^2
*d^6 + 3*d^6)*b*x^6*e + 4*(28*c^3*d^5 + 9*c*d^5)*b*x^5*e + 2*(70*c^4*d^4 +
45*c^2*d^4 + 3*d^4)*b*x^4*e + 8*(14*c^5*d^3 + 15*c^3*d^3 + 3*c*d^3)*b*x^3*e
 + 2*(28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d^2)*b*x^2*e + 4*(4*c^7*d + 9*
c^5*d + 6*c^3*d + c*d)*b*x*e + 2*(c^8 + 3*c^6 + 3*c^4 + c^2)*b*e + 2*(b*d^5
*x^5*e + 5*b*c*d^4*x^4*e + (10*c^2*d^3 + d^3)*b*x^3*e + (10*c^3*d^2 + 3*c*d
^2)*b*x^2*e + (5*c^4*d + 3*c^2*d)*b*x*e + (c^5 + c^3)*b*e)*(d^2*x^2 + 2*c*d
*x + c^2 + 1)^(3/2) + (6*b*d^6*x^6*e + 36*b*c*d^5*x^5*e + 10*(9*c^2*d^4 + d
^4)*b*x^4*e + 40*(3*c^3*d^3 + c*d^3)*b*x^3*e + 5*(18*c^4*d^2 + 12*c^2*d^2 +
 d^2)*b*x^2*e + 2*(18*c^5*d + 20*c^3*d + 5*c*d)*b*x*e + (6*c^6 + 10*c^4 + 5
*c^2 + 1)*b*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (6*b*d^7*x^7*e + 42*b*c*d^6*
x^6*e + 14*(9*c^2*d^5 + d^5)*b*x^5*e + 70*(3*c^3*d^4 + c*d^4)*b*x^4*e + (21
0*c^4*d^3 + 140*c^2*d^3 + 11*d^3)*b*x^3*e + (126*c^5*d^2 + 140*c^3*d^2 + 33
*c*d^2)*b*x^2*e + (42*c^6*d + 70*c^4*d + 33*c^2*d + 3*d)*b*x*e + (6*c^7 + 1
4*c^5 + 11*c^3 + 3*c)*b*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(2*a*d^7 + b*d^7)*x^7*e + 21*(2*a*
c*d^6 + b*c*d^6)*x^6*e + 7*(2*(9*c^2*d^5 + d^5)*a + (9*c^2*d^5 + d^5)*b)*x^
5*e + 35*(2*(3*c^3*d^4 + c*d^4)*a + (3*c^3*d^4 + c*d^4)*b)*x^4*e + ((210*c^
4*d^3 + 140*c^2*d^3 + 11*d^3)*a + 5*(21*c^4*d^3 + 14*c^2*d^3 + d^3)*b)*x^3*
e + ((126*c^5*d^2 + 140*c^3*d^2 + 33*c*d^2)*a + (63*c^5*d^2 + 70*c^3*d^2 +
15*c*d^2)*b)*x^2*e + ((42*c^6*d + 70*c^4*d + 33*c^2*d + 3*d)*a + (21*c^6*d
+ 35*c^4*d + 15*c^2*d + d)*b)*x*e + ((6*c^7 + 14*c^5 + 11*c^3 + 3*c)*a + (3
*c^7 + 7*c^5 + 5*c^3 + c)*b)*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2
*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3
*d^4 + 3*c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 +
 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d
)*a^2*b^2 + (b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 +
4*(5*c^3*d^4 + 3*c*d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 +
 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^
4 + (b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2
+ 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3
+ d^3)*b^4*x^2 + 2*(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^
2 + 2*c*d*x + c^2 + 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 +
d^4)*b^4*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d
^2)*b^4*x + (c^5*d + 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))
```

```
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*
a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x
 + c^2 + 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 +
 d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b
^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 +
3*(5*c^2*d^5 + d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^
4*d^3 + 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*
x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*
x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2
*c^3*d^2 + c*d^2)*a*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2
 + 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^
3 + 2*(5*c^3*d^3 + 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3
*x + (c^5*d + 2*c^3*d + c*d)*a*b^3)*sqrt(d^2*x^...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 +
 3*a^2*b*arcsinh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int\frac{c}{a^3+3a^2b\operatorname{asinh}(c+dx)+3ab^2\operatorname{asinh}^2(c+dx)+b^3\operatorname{asinh}^3(c+dx)}\,dx+\int\frac{dx}{a^3+3a^2b\operatorname{asinh}(c+dx)+3ab^2\operatorname{asinh}^2(c+dx)+b^3\operatorname{asinh}^3(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*asinh(c + d*x) + 3*a*b**2*asinh(c + d*x)**2
+ b**3*asinh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*asinh(c + d*x
) + 3*a*b**2*asinh(c + d*x)**2 + b**3*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^3, x)
```

3.172 $\int \frac{1}{(a+b\sinh^{-1}(c+dx))^3} dx$

Optimal. Leaf size=125

$$-\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}$$

```
[Out] 1/2*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))+1/2*Chi((a+b*arcsinh(d*x+c))/b)*cos
h(a/b)/b^3/d-1/2*Shi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^2
```

Rubi [A]

time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5858, 5773, 5818, 5774, 3384, 3379, 3382}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\sqrt{(c+dx)^2+1}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -1/2*Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x])^2) - (c + d*x)/(2*
b^2*d*(a + b*ArcSinh[c + d*x])) + (Cosh[a/b]*CoshIntegral[(a + b*ArcSinh[c
+ d*x])/b])/(2*b^3*d) - (Sinh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b]
)/(2*b^3*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^2}\,dx, x, c+\right.}{2bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{}{a-}\right.}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\text{co}}{}\right.}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Su}}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{2bd\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{C}}{}$$

Mathematica [A]

time = 0.21, size = 100, normalized size = 0.80

$$-\frac{\frac{b^2\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{b(c+dx)}{a+b\sinh^{-1}(c+dx)} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3),x]
```

```
[Out] -1/2*((b^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^2 + (b*(c + d*x)
)/(a + b*ArcSinh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + ArcSinh[c + d*x]]
 + Sinh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]])/(b^3*d)
```

Maple [A]

time = 3.31, size = 190, normalized size = 1.52

method	result
derivativedivides	$\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\,\text{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\,\text{arcsinh}(dx+c)^2+2ab\,\text{arcsinh}(dx+c)+a^2\right)} - \frac{e^{\frac{a}{b}}\,\text{expIntegral}\left(1,\text{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3} - \frac{dx+c+\sqrt{1+(dx+}}{4b(a+b\,\text{arcsinh}(dx+c))}}{d}$

default	$\frac{-\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)(b\operatorname{arcsinh}(dx+c)+a-b)}{4b^2\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)+a^2\right)}-\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcsinh}(dx+c)+\frac{a}{b}\right)}{4b^3}-\frac{dx+c+\sqrt{1+(dx+c}}{4b(a+b\operatorname{arcsinh}(dx+c))^2}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b*arcsinh(d*x+c)+a-b)/b^2/(b^2*arcs
inh(d*x+c)^2+2*a*b*arcsinh(d*x+c)+a^2)-1/4/b^3*exp(a/b)*Ei(1,arcsinh(d*x+c)
+a/b)-1/4/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/4/b^2*(d*x
+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arcsin
h(d*x+c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 + d^5
)*a + (7*c^2*d^5 + d^5)*b)*x^5 + 5*((7*c^3*d^4 + 3*c*d^4)*a + (7*c^3*d^4 +
3*c*d^4)*b)*x^4 + ((35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 + 30*c
^2*d^3 + 3*d^3)*b)*x^3 + 3*((7*c^5*d^2 + 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d
^2 + 10*c^3*d^2 + 3*c*d^2)*b)*x^2 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c
*d^3)*x^3 + (6*a*c^2*d^2 + (6*c^2*d^2 + d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 +
c^2)*b + 2*(2*a*c^3*d + (2*c^3*d + c*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d
^3 + d^3)*a + 5*(6*c^2*d^3 + d^3)*b)*x^3 + 3*((10*c^3*d^2 + 3*c*d^2)*a + 5*
(2*c^3*d^2 + c*d^2)*b)*x^2 + 3*(c^5 + c^3)*a + (3*c^5 + 5*c^3 + 2*c)*b + (3
*(5*c^4*d + 3*c^2*d)*a + (15*c^4*d + 15*c^2*d + 2*d)*b)*x)*(d^2*x^2 + 2*c*d
*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*a + (c^7 + 3*c^5 + 3*c^3 + c)*b +
 ((7*c^6*d + 15*c^4*d + 9*c^2*d + d)*a + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)
*b)*x + (b*d^7*x^7 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 + d^5)*b*x^5 + 5*(7*c^3*d
^4 + 3*c*d^4)*b*x^4 + (35*c^4*d^3 + 30*c^2*d^3 + 3*d^3)*b*x^3 + 3*(7*c^5*d^
2 + 10*c^3*d^2 + 3*c*d^2)*b*x^2 + (7*c^6*d + 15*c^4*d + 9*c^2*d + d)*b*x +
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(
d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b*d^5*x^5 + 5*b*c*d^4*x^4 + (10*c^2
*d^3 + d^3)*b*x^3 + (10*c^3*d^2 + 3*c*d^2)*b*x^2 + (5*c^4*d + 3*c^2*d)*b*x
+ (c^5 + c^3)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (c^7 + 3*c^5 + 3*c^3 + c)*
b + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*d^4 + 2*d^4)*b*x^4 + 12*(5*c^
3*d^3 + 2*c*d^3)*b*x^3 + (45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*b*x^2 + 2*(9*c^5
*d + 12*c^3*d + 4*c*d)*b*x + (3*c^6 + 6*c^4 + 4*c^2 + 1)*b)*sqrt(d^2*x^2 +
```

```
2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + (3*(
a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*(15*c^2*d^4 + 2*d^4)*a
 + (45*c^2*d^4 + 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 + 2*c*d^3)*a + (15*c^3*d^3
 + 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 + 36*c^2*d^2 + 4*d^2)*a + (45*c^4*d^2 + 4
2*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 + 6*c^4 + 4*c^2 + 1)*a + (3*c^6 + 7*c^4
+ 5*c^2 + 1)*b + 2*((9*c^5*d + 12*c^3*d + 4*c*d)*a + (9*c^5*d + 14*c^3*d +
5*c*d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^7*x^6 + 6*a^2*b^
2*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a^2
*b^2*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*a^2*b^2*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*a^2*b^2 + (b^4*d^
7*x^6 + 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)*b^4*x^4 + 4*(5*c^3*d^4 + 3*c*
d^4)*b^4*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 + 2*c^3
*d^2 + c*d^2)*b^4*x + (c^6*d + 3*c^4*d + 3*c^2*d + d)*b^4 + (b^4*d^4*x^3 +
3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1
)^(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 + d^3)*b^4*x^2 + 2*
(2*c^3*d^2 + c*d^2)*b^4*x + (c^4*d + c^2*d)*b^4)*(d^2*x^2 + 2*c*d*x + c^2 +
 1) + 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*b^4*x^3 + 2*(5
*c^3*d^3 + 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*b^4*x + (c^5*d
+ 2*c^3*d + c*d)*b^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 +
 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
 3*(a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a^2*b^2*x^2 +
 2*(2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^4*d + c^2*d)*a^2*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 3*(5*c^2*d^5 + d^5)
*a*b^3*x^4 + 4*(5*c^3*d^4 + 3*c*d^4)*a*b^3*x^3 + 3*(5*c^4*d^3 + 6*c^2*d^3 +
 d^3)*a*b^3*x^2 + 6*(c^5*d^2 + 2*c^3*d^2 + c*d^2)*a*b^3*x + (c^6*d + 3*c^4*
d + 3*c^2*d + d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^3*x^2 + 3*a*b^3*c^2*d
^2*x + a*b^3*c^3*d)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(a*b^3*d^5*x^4
+ 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 + d^3)*a*b^3*x^2 + 2*(2*c^3*d^2 + c*d^2)*a
*b^3*x + (c^4*d + c^2*d)*a*b^3)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(a*b^3*d^
6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 + d^4)*a*b^3*x^3 + 2*(5*c^3*d^3 +
3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 + 6*c^2*d^2 + d^2)*a*b^3*x + (c^5*d + 2*c^3
*d + c*d)*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*
x^2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4 + 2*(5
*c^2*d^4 + d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 + 3*c*d^3)*a^2*b^2*x^2 + (5*c^4*
d^2 + 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d + 2*c^3*d + c*d)*a^2*b^2)*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + integrate(1/2*(d^8*x^8 + 8*c*d^7*x^7 + c^8 +
4*(7*c^2*d^6 + d^6)*x^6 + 4*c^6 + 8*(7*c^3*d^5 + 3*c*d^5)*x^5 + 2*(35*c^4*d
^4 + 30*c^2*d^4 + 3*d^4)*x^4 + 6*c^4 + 8*(7*c^5*d^3 + 10*c^3*d^3 + 3*c*d^3)
*x^3 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3)*(d^2*x
^2 + 2*c*d*x + c^2 + 1)^2 + 4*(7*c^6*d^2 + 15*c^4*d^2 + 9*c^2*d^2 + d^2)*x^
2 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 + d^3)*x^3 + 4*c^3 +
4*(10*c^3*d^2 + 3*c*d^2)*x^2 + (20*c^4*d + 12*c^2*d + 3*d)*x + 3*c)*(d^2*x^
2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(2*d^6*x^6 + 1...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*a
rcsinh(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}\left(c + dx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**3,x)
[Out] Integral((a + b*asinh(c + d*x))**(-3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^3,x)
[Out] int(1/(a + b*asinh(c + d*x))^3, x)
```

3.173 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^3},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^3,x)/e

Rubi [A]

time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^3), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]

time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^3}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^3), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*d^8*x^8 + 8*b*c*d^7*x^7 + (28*c^2*d^6 + 3*d^6)*b*x^6 + 2*(28*c^3*d^
5 + 9*c*d^5)*b*x^5 + (70*c^4*d^4 + 45*c^2*d^4 + 3*d^4)*b*x^4 + 4*(14*c^5*d^
3 + 15*c^3*d^3 + 3*c*d^3)*b*x^3 + (28*c^6*d^2 + 45*c^4*d^2 + 18*c^2*d^2 + d
^2)*b*x^2 + 2*(4*c^7*d + 9*c^5*d + 6*c^3*d + c*d)*b*x + (b*d^5*x^5 + 5*b*c*
d^4*x^4 - (2*a*d^3 - (10*c^2*d^3 + d^3)*b)*x^3 - (6*a*c*d^2 - (10*c^3*d^2 +
 3*c*d^2)*b)*x^2 - 2*(c^3 + c)*a + (c^5 + c^3)*b - (2*(3*c^2*d + d)*a - (5*
c^4*d + 3*c^2*d)*b)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + (3*b*d^6*x^6 +
 18*b*c*d^5*x^5 - (4*a*d^4 - 5*(9*c^2*d^4 + d^4)*b)*x^4 - 4*(4*a*c*d^3 - 5*
(3*c^3*d^3 + c*d^3)*b)*x^3 - ((24*c^2*d^2 + 5*d^2)*a - (45*c^4*d^2 + 30*c^2
*d^2 + 2*d^2)*b)*x^2 - (4*c^4 + 5*c^2 + 1)*a + (3*c^6 + 5*c^4 + 2*c^2)*b -
2*((8*c^3*d + 5*c*d)*a - (9*c^5*d + 10*c^3*d + 2*c*d)*b)*x)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + (c^8 + 3*c^6 + 3*c^4 + c^2)*b - (2*(b*d^3*x^3 + 3*b*c*d^2*
x^2 + (3*c^2*d + d)*b*x + (c^3 + c)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2)
+ (4*b*d^4*x^4 + 16*b*c*d^3*x^3 + (24*c^2*d^2 + 5*d^2)*b*x^2 + 2*(8*c^3*d +
 5*c*d)*b*x + (4*c^4 + 5*c^2 + 1)*b)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + (2*b*d
^5*x^5 + 10*b*c*d^4*x^4 + (20*c^2*d^3 + 3*d^3)*b*x^3 + (20*c^3*d^2 + 9*c*d^
2)*b*x^2 + (10*c^4*d + 9*c^2*d + d)*b*x + (2*c^5 + 3*c^3 + c)*b)*sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) +
 (3*b*d^7*x^7 + 21*b*c*d^6*x^6 - (2*a*d^5 - 7*(9*c^2*d^5 + d^5)*b)*x^5 - 5*
(2*a*c*d^4 - 7*(3*c^3*d^4 + c*d^4)*b)*x^4 - ((20*c^2*d^3 + 3*d^3)*a - 5*(21
*c^4*d^3 + 14*c^2*d^3 + d^3)*b)*x^3 - ((20*c^3*d^2 + 9*c*d^2)*a - (63*c^5*d
^2 + 70*c^3*d^2 + 15*c*d^2)*b)*x^2 - (2*c^5 + 3*c^3 + c)*a + (3*c^7 + 7*c^5
 + 5*c^3 + c)*b - ((10*c^4*d + 9*c^2*d + d)*a - (21*c^6*d + 35*c^4*d + 15*c
^2*d + d)*b)*x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(a^2*b^2*d^9*x^8*e + 8*a
^2*b^2*c*d^8*x^7*e + (28*c^2*d^7 + 3*d^7)*a^2*b^2*x^6*e + 2*(28*c^3*d^6 + 9
*c*d^6)*a^2*b^2*x^5*e + (70*c^4*d^5 + 45*c^2*d^5 + 3*d^5)*a^2*b^2*x^4*e + 4
```

```
*(14*c^5*d^4 + 15*c^3*d^4 + 3*c*d^4)*a^2*b^2*x^3*e + (28*c^6*d^3 + 45*c^4*d
^3 + 18*c^2*d^3 + d^3)*a^2*b^2*x^2*e + 2*(4*c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2
 + c*d^2)*a^2*b^2*x*e + (c^8*d + 3*c^6*d + 3*c^4*d + c^2*d)*a^2*b^2*e + (b^
4*d^9*x^8*e + 8*b^4*c*d^8*x^7*e + (28*c^2*d^7 + 3*d^7)*b^4*x^6*e + 2*(28*c^
3*d^6 + 9*c*d^6)*b^4*x^5*e + (70*c^4*d^5 + 45*c^2*d^5 + 3*d^5)*b^4*x^4*e +
4*(14*c^5*d^4 + 15*c^3*d^4 + 3*c*d^4)*b^4*x^3*e + (28*c^6*d^3 + 45*c^4*d^3
+ 18*c^2*d^3 + d^3)*b^4*x^2*e + 2*(4*c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2 + c*d^
2)*b^4*x*e + (c^8*d + 3*c^6*d + 3*c^4*d + c^2*d)*b^4*e + (b^4*d^6*x^5*e + 5
*b^4*c*d^5*x^4*e + 10*b^4*c^2*d^4*x^3*e + 10*b^4*c^3*d^3*x^2*e + 5*b^4*c^4*
d^2*x*e + b^4*c^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(b^4*d^7*x^6
*e + 6*b^4*c*d^6*x^5*e + (15*c^2*d^5 + d^5)*b^4*x^4*e + 4*(5*c^3*d^4 + c*d^
4)*b^4*x^3*e + 3*(5*c^4*d^3 + 2*c^2*d^3)*b^4*x^2*e + 2*(3*c^5*d^2 + 2*c^3*d
^2)*b^4*x*e + (c^6*d + c^4*d)*b^4*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 3*(b^4
*d^8*x^7*e + 7*b^4*c*d^7*x^6*e + (21*c^2*d^6 + 2*d^6)*b^4*x^5*e + 5*(7*c^3*
d^5 + 2*c*d^5)*b^4*x^4*e + (35*c^4*d^4 + 20*c^2*d^4 + d^4)*b^4*x^3*e + (21*
c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*b^4*x^2*e + (7*c^6*d^2 + 10*c^4*d^2 + 3*c^2
*d^2)*b^4*x*e + (c^7*d + 2*c^5*d + c^3*d)*b^4*e)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + (a^2*b^2*d^6*
x^5*e + 5*a^2*b^2*c*d^5*x^4*e + 10*a^2*b^2*c^2*d^4*x^3*e + 10*a^2*b^2*c^3*d
^3*x^2*e + 5*a^2*b^2*c^4*d^2*x*e + a^2*b^2*c^5*d*e)*(d^2*x^2 + 2*c*d*x + c^
2 + 1)^(3/2) + 3*(a^2*b^2*d^7*x^6*e + 6*a^2*b^2*c*d^6*x^5*e + (15*c^2*d^5 +
 d^5)*a^2*b^2*x^4*e + 4*(5*c^3*d^4 + c*d^4)*a^2*b^2*x^3*e + 3*(5*c^4*d^3 +
2*c^2*d^3)*a^2*b^2*x^2*e + 2*(3*c^5*d^2 + 2*c^3*d^2)*a^2*b^2*x*e + (c^6*d +
 c^4*d)*a^2*b^2*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1) + 2*(a*b^3*d^9*x^8*e + 8*a
*b^3*c*d^8*x^7*e + (28*c^2*d^7 + 3*d^7)*a*b^3*x^6*e + 2*(28*c^3*d^6 + 9*c*d
^6)*a*b^3*x^5*e + (70*c^4*d^5 + 45*c^2*d^5 + 3*d^5)*a*b^3*x^4*e + 4*(14*c^5
*d^4 + 15*c^3*d^4 + 3*c*d^4)*a*b^3*x^3*e + (28*c^6*d^3 + 45*c^4*d^3 + 18*c^
2*d^3 + d^3)*a*b^3*x^2*e + 2*(4*c^7*d^2 + 9*c^5*d^2 + 6*c^3*d^2 + c*d^2)*a*
b^3*x*e + (c^8*d + 3*c^6*d + 3*c^4*d + c^2*d)*a*b^3*e + (a*b^3*d^6*x^5*e +
5*a*b^3*c*d^5*x^4*e + 10*a*b^3*c^2*d^4*x^3*e + 10*a*b^3*c^3*d^3*x^2*e + 5*a
*b^3*c^4*d^2*x*e + a*b^3*c^5*d*e)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + 3*(
a*b^3*d^7*x^6*e + 6*a*b^3*c*d^6*x^5*e + (15*c^2*d^5 + d^5)*a*b^3*x^4*e + 4*
(5*c^3*d^4 + c*d^4)*a*b^3*x^3*e + 3*(5*c^4*d^3 + 2*c^2*d^3)*a*b^3*x^2*e + 2
*(3*c^5*d^2 + 2*c^3*d^2)*a*b^3*x*e + (c^6*d + c^4*d)*a*b^3*e)*(d^2*x^2 + 2*
c*d*x + c^2 + 1) + 3*(a*b^3*d^8*x^7*e + 7*a*b^3*c*d^7*x^6*e + (21*c^2*d^6 +
 2*d^6)*a*b^3*x^5*e + 5*(7*c^3*d^5 + 2*c*d^5)*a*b^3*x^4*e + (35*c^4*d^4 + 2
0*c^2*d^4 + d^4)*a*b^3*x^3*e + (21*c^5*d^3 + 20*c^3*d^3 + 3*c*d^3)*a*b^3*x^
2*e + (7*c^6*d^2 + 10*c^4*d^2 + 3*c^2*d^2)*a*b^3*x*e + (c^7*d + 2*c^5*d + c
^3*d)*a*b^3*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1)) + 3*(a^2*b^2*d^8*x^7*e + 7*a^2*b^2*c*d^7*x^6*e + (2
1*c^2*d^6 + 2*d^6)*a^2*b^2*x^5*e + 5*(7*c^3*d^5...
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/((b^3*d*x + b^3*c)*arcsinh(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c
)*arcsinh(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arcsinh(d*x + c)*e + (a^3*
d*x + a^3*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{3}c+a^{3}dx+3a^{2}bc\operatorname{asinh}(c+dx)+3a^{2}bdx\operatorname{asinh}(c+dx)+3ab^{2}c\operatorname{asinh}^{2}(c+dx)+3ab^{2}dx\operatorname{asinh}^{2}(c+dx)+b^{3}c\operatorname{asinh}^{3}(c+dx)+b^{3}dx\operatorname{asinh}^{3}(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*asinh(c + d*x) + 3*a**2*b*d*x*as
inh(c + d*x) + 3*a*b**2*c*asinh(c + d*x)**2 + 3*a*b**2*d*x*asinh(c + d*x)**
2 + b**3*c*asinh(c + d*x)**3 + b**3*d*x*asinh(c + d*x)**3), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(c\,e+d\,e\,x\right)\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^3), x)
```

3.174 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=410

$$-\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e^4(c+dx)^2\sqrt{}}{b^3d\left(a+b\sin\right.}$$

```
[Out] -2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a+
b*arcsinh(d*x+c))^2+1/48*e^4*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d-27
/32*e^4*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+125/96*e^4*cosh(5*a
/b)*Shi(5*(a+b*arcsinh(d*x+c))/b)/b^4/d-1/48*e^4*Chi((a+b*arcsinh(d*x+c))/b
)*sinh(a/b)/b^4/d+27/32*e^4*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*a/b)/b^4/d
-125/96*e^4*Chi(5*(a+b*arcsinh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(d*x+c)
^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-2*e^4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-25/6*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2
)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.58, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}-\frac{125e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}+\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{48b^4d}-\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{96b^4d}-\frac{25e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{6b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^2}{b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{5e^4(c+dx)^5}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (5*e^4*(c +
 d*x)^5)/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^4*(c + d*x)^2*Sqrt[1 +
 (c + d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (25*e^4*(c + d*x)^4*Sqrt[
1 + (c + d*x)^2])/(6*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^4*CoshIntegral[(a
 + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(48*b^4*d) + (27*e^4*CoshIntegral[(3*(
a + b*ArcSinh[c + d*x]))/b]*Sinh[(3*a)/b])/(32*b^4*d) - (125*e^4*CoshIntegr
al[(5*(a + b*ArcSinh[c + d*x]))/b]*Sinh[(5*a)/b])/(96*b^4*d) + (e^4*Cosh[a/
b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(48*b^4*d) - (27*e^4*Cosh[(3*a
)/b]*SinhIntegral[(3*(a + b*ArcSinh[c + d*x]))/b])/(32*b^4*d) + (125*e^4*Co
sh[(5*a)/b]*SinhIntegral[(5*(a + b*ArcSinh[c + d*x]))/b])/(96*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\left(4e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx\right.}{3bd}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5}{6b^2d\,(a+}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5}{6b^2d\,(a+}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5}{6b^2d\,(a+}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5}{6b^2d\,(a+}$$

$$= -\frac{e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{5}{6b^2d\,(a+}$$

Mathematica [A]
time = 1.39, size = 410, normalized size = 1.00

$$-\frac{e^4\left(\frac{32b^3(c+dx)^4\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{16b^2\left(-4(c+dx)^3-5(c+dx)^5\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2}+\frac{16b\sqrt{1+(c+dx)^2}\left(12(c+dx)^2+25(c+dx)^4\right)}{a+b\sinh^{-1}(c+dx)}+384\left(\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+544\left(-3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)+\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)-\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)+125\left(10\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-5\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)+\text{Chi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{5a}{b}\right)-10\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+5\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\text{Shi}\left(5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/96*(e^4*((32*b^3*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d
*x])^3 - (16*b^2*(-4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcSinh[c + d*x])
```

```
^2 + (16*b*Sqrt[1 + (c + d*x)^2]*(12*(c + d*x)^2 + 25*(c + d*x)^4))/(a + b*
ArcSinh[c + d*x]) + 384*(CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - C
osh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]]) + 544*(-3*CoshIntegral[a/b +
 ArcSinh[c + d*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + 3*Cosh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] - Cosh[(3*a)/
b]*SinhIntegral[3*(a/b + ArcSinh[c + d*x])]) + 125*(10*CoshIntegral[a/b + A
rcSinh[c + d*x]]*Sinh[a/b] - 5*CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcSinh[c + d*x])]*Sinh[(5*a)/b] - 10*Co
sh[a/b]*SinhIntegral[a/b + ArcSinh[c + d*x]] + 5*Cosh[(3*a)/b]*SinhIntegral
[3*(a/b + ArcSinh[c + d*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcSinh[
c + d*x])])))/(b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(384) = 768$.
time = 6.90, size = 1244, normalized size = 3.03

method	result	size
derivativedivides	Expression too large to display	1244
default	Expression too large to display	1244

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/192*(-16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+16*(d*x+c)^5-12*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)+20*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+5*d*x+5*c)*e^4*(25*b^2*ar
csinh(d*x+c)^2+50*a*b*arcsinh(d*x+c)-5*b^2*arcsinh(d*x+c)+25*a^2-5*a*b+2*b^
2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c
)+a^3)+125/192*e^4/b^4*exp(5*a/b)*Ei(1,5*arcsinh(d*x+c)+5*a/b)-1/64*(-4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)+3*d*x+3*c)*e^4*(
9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsinh(d*x+c)+9*a^2-3*a
*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsin
h(d*x+c)+a^3)-27/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c)+3*a/b)+1/96*(-
(1+(d*x+c)^2)^(1/2)+d*x+c)*e^4*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(d*x+c)-b
^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(
d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/96*e^4/b^4*exp(a/b)*Ei(1,arcsinh(d*x
+c)+a/b)-1/48/b*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/96
/b^2*e^4*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/96/b^3*e^4*(d
*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,
-arcsinh(d*x+c)-a/b)+1/32/b*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+
c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+3/64/b^2*e^4*(4*(d*
x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*
arcsinh(d*x+c))^2+9/64/b^3*e^4*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c
)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+27/64/b^4*e^4*exp(-3*a
/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b)-1/96/b*e^4*(16*(d*x+c)^5+20*(d*x+c)^3+16*
(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1
```

```
+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-5/192/b^2*e^4*(16*(d*x+c)^5+20*(d
*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2*(1+(d*x+c)^
2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-25/192/b^3*e^4*(16*(d*
x+c)^5+20*(d*x+c)^3+16*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)+5*d*x+5*c+12*(d*x+c)^2
*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-125/192/b^4*
e^4*exp(-5*a/b)*Ei(1,-5*arcsinh(d*x+c)-5*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*e^4/(b^4
*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x +
c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int\frac{c^4}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx+\int\frac{d^4x^4}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx+\int\frac{4cd^3x^3}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx+\int\frac{6c^2d^2x^2}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx+\int\frac{4c^3dx}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**4*(Integral(c**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**4*x**4/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*
c*d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(6*c*
*2*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**
2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(4*c
```

```
**3*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4
*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^4, x)
```

3.175 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=340

$$-\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^3(c+dx)\sqrt{1}}{b^3d\left(a+b\sin\right.}$$

```
[Out] -1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a+
b*arcsinh(d*x+c))^2-1/3*e^3*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d
+4/3*e^3*Chi(4*(a+b*arcsinh(d*x+c))/b)*cosh(4*a/b)/b^4/d+1/3*e^3*Shi(2*(a+b
*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arcsinh(d*x+c))/b)
*sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d
*x+c))^3-e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-8/3*e^3
*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.44, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382}

$$-\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{3b^4d}+\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)}{3b^4d}-\frac{4e^3\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)}{3b^4d}-\frac{8e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{3b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^3\sqrt{(c+dx)^2+1}(c+dx)}{b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{2e^3(c+dx)^4}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (2*e^3*(c + d
*x)^4)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^3*(c + d*x)*Sqrt[1 + (c +
d*x)^2])/(b^3*d*(a + b*ArcSinh[c + d*x])) - (8*e^3*(c + d*x)^3*Sqrt[1 + (c
+ d*x)^2])/(3*b^3*d*(a + b*ArcSinh[c + d*x])) - (e^3*Cosh[(2*a)/b]*CoshInte
gral[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*Cosh
Integral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) + (e^3*Sinh[(2*a)/b]*Si
nhIntegral[(2*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b
]*SinhIntegral[(4*(a + b*ArcSinh[c + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e^3\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx,x,\right.}{bd}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2}{3b^2d\left(a+\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2}{3b^2d\left(a+\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2}{3b^2d\left(a+\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2}{3b^2d\left(a+\right.}$$

$$= -\frac{e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^3(c+dx)^2}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{2}{3b^2d\left(a+\right.}$$

Mathematica [A]
time = 0.81, size = 318, normalized size = 0.94

$$\frac{e^3\left(-\frac{2b^3(c+dx)^3\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^3}+\frac{b^2\left(-3(c+dx)^2-4(c+dx)^4\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2b\sqrt{1+(c+dx)^2}\left(3(c+dx)+8(c+dx)^3\right)}{a+b\sinh^{-1}(c+dx)}+6\log\left(a+b\sinh^{-1}(c+dx)\right)+30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\log\left(a+b\sinh^{-1}(c+dx)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)+8\left(-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+3\log\left(a+b\sinh^{-1}(c+dx)\right)+4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (b^2*(-3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcSinh[c + d*x])^2 - (2*b
*Sqrt[1 + (c + d*x)^2]*(3*(c + d*x) + 8*(c + d*x)^3))/(a + b*ArcSinh[c + d*
x]) + 6*Log[a + b*ArcSinh[c + d*x]] + 30*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b
```

```
 + ArcSinh[c + d*x])] - Log[a + b*ArcSinh[c + d*x]] - Sinh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcSinh[c + d*x])]) + 8*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/
b + ArcSinh[c + d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcSinh[c + d*
x])] + 3*Log[a + b*ArcSinh[c + d*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b
+ ArcSinh[c + d*x])] - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcSinh[c + d*x]
)])))/(6*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(318) = 636$.
time = 8.21, size = 800, normalized size = 2.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+8*(d*x+c)^4-4*(d*x+c)*(1+(d*x+c
)^2)^(1/2)+8*(d*x+c)^2+1)*e^3*(8*b^2*arcsinh(d*x+c)^2+16*a*b*arcsinh(d*x+c)
-2*b^2*arcsinh(d*x+c)+8*a^2-2*a*b+b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*ar
csinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-2/3*e^3/b^4*exp(4*a/b)*Ei(1,4*ar
csinh(d*x+c)+4*a/b)-1/24*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e^3
*(2*b^2*arcsinh(d*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^2-a*b+
b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x
+c)+a^3)+1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)+1/24/b*e^3*(2*
(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3+1/24/b^2*
e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2+1/
12/b^3*e^3*(2*(d*x+c)^2+1+2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c
))+1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arcsinh(d*x+c)-2*a/b)-1/48/b*e^3*(8*(d*x
+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(
1/2)+1)/(a+b*arcsinh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x
+c)^3*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x
+c))^2-1/6/b^3*e^3*(8*(d*x+c)^4+8*(d*x+c)^2+8*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)
+4*(d*x+c)*(1+(d*x+c)^2)^(1/2)+1)/(a+b*arcsinh(d*x+c))-2/3/b^4*e^3*exp(-4*a
/b)*Ei(1,-4*arcsinh(d*x+c)-4*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*e^3/(b^4*arcsinh(d*x + c
)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*a
rcsinh(d*x + c) + a^4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{4}+4a^{3}b\operatorname{asinh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{asinh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{asinh}^{3}\left(c+dx\right)+b^{4}\operatorname{asinh}^{4}\left(c+dx\right)}\,dx+\int \frac{d^{3}x^{3}}{a^{4}+4a^{3}b\operatorname{asinh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{asinh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{asinh}^{3}\left(c+dx\right)+b^{4}\operatorname{asinh}^{4}\left(c+dx\right)}\,dx+\int \frac{3cd^{2}x^{2}}{a^{4}+4a^{3}b\operatorname{asinh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{asinh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{asinh}^{3}\left(c+dx\right)+b^{4}\operatorname{asinh}^{4}\left(c+dx\right)}\,dx+\int \frac{3c^{2}dx}{a^{4}+4a^{3}b\operatorname{asinh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{asinh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{asinh}^{3}\left(c+dx\right)+b^{4}\operatorname{asinh}^{4}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**3*(Integral(c**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**3*x**3/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*
c*d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2
 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(3*c*
*2*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*
a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^4, x)
```

3.176 $\int \frac{(ce+dex)^2}{(a+b\sinh^{-1}(c+dx))^4}\,dx$

Optimal. Leaf size=331

$$-\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^2\sqrt{1+(c}}{3b^3d\left(a+b\sinh\right.}$$

```
[Out] -1/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b*
arcsinh(d*x+c))^2-1/24*e^2*cosh(a/b)*Shi((a+b*arcsinh(d*x+c))/b)/b^4/d+9/8*
e^2*cosh(3*a/b)*Shi(3*(a+b*arcsinh(d*x+c))/b)/b^4/d+1/24*e^2*Chi((a+b*arcsi
nh(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Chi(3*(a+b*arcsinh(d*x+c))/b)*sinh(3*
a/b)/b^4/d-1/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3
-1/3*e^2*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))-3/2*e^2*(d*x+c)^2*(
1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
```

Rubi [A]
time = 0.48, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382, 5773, 5819}

$$\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{8b^4d}-\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)}{8b^4d}-\frac{3e^2\sqrt{(c+dx)^2+1}\,(c+dx)^2}{2b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2\sqrt{(c+dx)^2+1}}{3b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e^2(c+dx)^3}{2b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e^2\sqrt{(c+dx)^2+1}\,(c+dx)^2}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^
3) - (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*(c + d*x)^
3)/(2*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e^2*Sqrt[1 + (c + d*x)^2])/(3*b^
3*d*(a + b*ArcSinh[c + d*x])) - (3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(
2*b^3*d*(a + b*ArcSinh[c + d*x])) + (e^2*CoshIntegral[(a + b*ArcSinh[c + d*
x])/b]*Sinh[a/b])/(24*b^4*d) - (9*e^2*CoshIntegral[(3*(a + b*ArcSinh[c + d*
x]))/b]*Sinh[(3*a)/b])/(8*b^4*d) - (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcSi
nh[c + d*x])/b])/(24*b^4*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*(a + b*A
rcSinh[c + d*x]))/b])/(8*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
```

```
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{(2e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx\right.}{3bd}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\,(a+}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\,(a+}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\,(a+}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\,(a+}$$

$$= -\frac{e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e^2(c+dx)}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e}{2b^2d\,(a+}$$

Mathematica [A]
time = 0.65, size = 258, normalized size = 0.78

$$\frac{e^2\left(-\frac{8b^3(c+dx)^2\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^3}+\frac{4b^2(-2(c+dx)-3(c+dx)^3)}{\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{4b\sqrt{1+(c+dx)^2}\,(2+9(c+dx)^2)}{a+b\sinh^{-1}(c+dx)}-80\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)+80\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+27\left(3\text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)-\text{Chi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\sinh\left(\frac{3a}{b}\right)-3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3
 + (4*b^2*(-2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcSinh[c + d*x])^2 - (4*b
*Sqrt[1 + (c + d*x)^2]*(2 + 9*(c + d*x)^2))/(a + b*ArcSinh[c + d*x]) - 80*C
oshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] + 80*Cosh[a/b]*SinhIntegral[a
/b + ArcSinh[c + d*x]] + 27*(3*CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/
b] - CoshIntegral[3*(a/b + ArcSinh[c + d*x])]*Sinh[(3*a)/b] - 3*Cosh[a/b]*S
inhIntegral[a/b + ArcSinh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + A
rcSinh[c + d*x])])))/(24*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(307) = 614$.
time = 6.63, size = 709, normalized size = 2.14

method	result
derivativedivides	$\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$
default	$\frac{\left(-4(dx+c)^2\sqrt{1+(dx+c)^2}+4(dx+c)^3-\sqrt{1+(dx+c)^2}+3dx+3c\right)e^2\left(9b^2\operatorname{arcsinh}(dx+c)^2+18ab\operatorname{arcsinh}(dx+c)-3\right.}{48b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+4*(d*x+c)^3-(1+(d*x+c)^2)^(1/2)
+3*d*x+3*c)*e^2*(9*b^2*arcsinh(d*x+c)^2+18*a*b*arcsinh(d*x+c)-3*b^2*arcsinh
(d*x+c)+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)
^2+3*a^2*b*arcsinh(d*x+c)+a^3)+9/16*e^2/b^4*exp(3*a/b)*Ei(1,3*arcsinh(d*x+c
)+3*a/b)-1/48*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*e^2*(b^2*arcsinh(d*x+c)^2+2*a*b*
arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+
3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*
Ei(1,arcsinh(d*x+c)+a/b)+1/24/b*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsin
h(d*x+c))^3+1/48/b^2*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2
+1/48/b^3*e^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))+1/48/b^4*e^2
*exp(-a/b)*Ei(1,-arcsinh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d
*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/1
6/b^2*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)
^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-3/16/b^3*e^2*(4*(d*x+c)^3+3*d*x+3*c+4*(d*
x+c)^2*(1+(d*x+c)^2)^(1/2)+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-9/16/b
^4*e^2*exp(-3*a/b)*Ei(1,-3*arcsinh(d*x+c)-3*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*e^2/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*ar
csinh(d*x + c)^3 + 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c)
+ a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2} \left(\int \frac{c^{2}}{a^{4} + 4a^{3}b \operatorname{asinh}{\left(c + dx \right)} + 6a^{2}b^{2} \operatorname{asinh}^{2}{\left(c + dx \right)} + 4ab^{3} \operatorname{asinh}^{3}{\left(c + dx \right)} + b^{4} \operatorname{asinh}^{4}{\left(c + dx \right)}}\, dx + \int \frac{d^{2}x^{2}}{a^{4} + 4a^{3}b \operatorname{asinh}{\left(c + dx \right)} + 6a^{2}b^{2} \operatorname{asinh}^{2}{\left(c + dx \right)} + 4ab^{3} \operatorname{asinh}^{3}{\left(c + dx \right)} + b^{4} \operatorname{asinh}^{4}{\left(c + dx \right)}}\, dx + \int \frac{2cdx}{a^{4} + 4a^{3}b \operatorname{asinh}{\left(c + dx \right)} + 6a^{2}b^{2} \operatorname{asinh}^{2}{\left(c + dx \right)} + 4ab^{3} \operatorname{asinh}^{3}{\left(c + dx \right)} + b^{4} \operatorname{asinh}^{4}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e**2*(Integral(c**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c +
 d*x)**2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integ
ral(d**2*x**2/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(2*
c*d*x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a
*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^4, x)
```

3.177 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=204

$$-\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{2e(c+dx)\sqrt{1+}}{3b^3d\left(a+b\sinh\right.}$$

```
[Out] -1/6*e/b^2/d/(a+b*arcsinh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+
c))^2+2/3*e*Chi(2*(a+b*arcsinh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(a+
b*arcsinh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/
d/(a+b*arcsinh(d*x+c))^3-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsi
nh(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5859, 12, 5779, 5818, 5778, 3384, 3379, 3382, 5783}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{3b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{e(c+dx)^2}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{e\sqrt{(c+dx)^2+1}\,(c+dx)}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] -1/3*(e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*(a + b*ArcSinh[c + d*x])^3) -
 e/(6*b^2*d*(a + b*ArcSinh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b^2*d*(a + b*A
rcSinh[c + d*x])^2) - (2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*(a + b
*ArcSinh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIntegral[(2*(a + b*ArcSinh[c +
 d*x]))/b])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIntegral[(2*(a + b*ArcSinh[c
 + d*x]))/b])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^3}\,dx, x, c+\right.}{3bd}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(}{3b^2d\left(a+b\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(}{3b^2d\left(a+b\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(}{3b^2d\left(a+b\right.}$$

$$= -\frac{e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{e}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{e(}{3b^2d\left(a+b\right.}$$

Mathematica [A]
time = 0.59, size = 181, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3(c+dx)\sqrt{1+(c+dx)^2}}{\left(a+b\sinh^{-1}(c+dx)\right)^3}+\frac{b^2\left(-1-2(c+dx)^2\right)}{\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{4b(c+dx)\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)}+4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+4\log\left(a+b\sinh^{-1}(c+dx)\right)-4\left(\log\left(a+b\sinh^{-1}(c+dx)\right)+\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)\right)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (
b^2*(-1 - 2*(c + d*x)^2))/(a + b*ArcSinh[c + d*x])^2 - (4*b*(c + d*x)*Sqrt[
1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2
*(a/b + ArcSinh[c + d*x])] + 4*Log[a + b*ArcSinh[c + d*x]] - 4*(Log[a + b*A
rcSinh[c + d*x]] + Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSinh[c + d*x])]))
)/(6*b^4*d)
```

Maple [A]
time = 4.48, size = 333, normalized size = 1.63

method	result
derivativedivides	$\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e}{}$
default	$\frac{\left(-2(dx+c)\sqrt{1+(dx+c)^2}+2(dx+c)^2+1\right)e\left(2b^2\operatorname{arcsinh}(dx+c)^2+4ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+2a^2-ab+b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}-\frac{e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-2*(d*x+c)*(1+(d*x+c)^2)^(1/2)+2*(d*x+c)^2+1)*e*(2*b^2*arcsinh(d
*x+c)^2+4*a*b*arcsinh(d*x+c)-b^2*arcsinh(d*x+c)+2*a^2-a*b+b^2)/b^3/(b^3*arc
sinh(d*x+c)^3+3*a*b^2*arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)-1/3*e/b^
4*exp(2*a/b)*Ei(1,2*arcsinh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2+1+2*(d*x+c)
*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/b^2*e*(2*(d*x+c)^2+1+2*(d
*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2+1+
2*(d*x+c)*(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*E
i(1,-2*arcsinh(d*x+c)-2*a/b))
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*e/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 +
 6*a^2*b^2*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int\frac{c}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx+\int\frac{dx}{a^4+4a^3b\operatorname{asinh}(c+dx)+6a^2b^2\operatorname{asinh}^2(c+dx)+4ab^3\operatorname{asinh}^3(c+dx)+b^4\operatorname{asinh}^4(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] e*(Integral(c/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)*
*2 + 4*a*b**3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x) + Integral(d*
x/(a**4 + 4*a**3*b*asinh(c + d*x) + 6*a**2*b**2*asinh(c + d*x)**2 + 4*a*b**
3*asinh(c + d*x)**3 + b**4*asinh(c + d*x)**4), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^4, x)
```

3.178 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{\sqrt{1+(c+dx)^2}}{6b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(c}{b}\right.}{6b^4}$$

```
[Out] 1/6*(-d*x-c)/b^2/d/(a+b*arcsinh(d*x+c))^2+1/6*cosh(a/b)*Shi((a+b*arcsinh(d*
x+c))/b)/b^4/d-1/6*Chi((a+b*arcsinh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^3-1/6*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*a
rcsinh(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5858, 5773, 5818, 5819, 3384, 3379, 3382}

$$-\frac{\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}+\frac{\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sqrt{(c+dx)^2+1}}{6b^3d\left(a+b\sinh^{-1}(c+dx)\right)}-\frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2}-\frac{\sqrt{(c+dx)^2+1}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -1/3*Sqrt[1 + (c + d*x)^2]/(b*d*(a + b*ArcSinh[c + d*x])^3) - (c + d*x)/(6*
b^2*d*(a + b*ArcSinh[c + d*x])^2) - Sqrt[1 + (c + d*x)^2]/(6*b^3*d*(a + b*A
rcSinh[c + d*x])) - (CoshIntegral[(a + b*ArcSinh[c + d*x])/b]*Sinh[a/b])/(6
*b^4*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcSinh[c + d*x])/b])/(6*b^4*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\sinh^{-1}(x))^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\,(a+b\sinh^{-1}(x))^3}\,dx, x, c+\right.}{3bd}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1}}{6b^3d\,(a+}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1}}{6b^3d\,(a+}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1}}{6b^3d\,(a+}$$

$$= -\frac{\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^2} - \frac{\sqrt{1}}{6b^3d\,(a+}$$

Mathematica [A]
time = 0.31, size = 130, normalized size = 0.81

$$-\frac{\frac{2b^3\sqrt{1+(c+dx)^2}}{(a+b\sinh^{-1}(c+dx))^3} + \frac{b^2(c+dx)}{(a+b\sinh^{-1}(c+dx))^2} + \frac{b\sqrt{1+(c+dx)^2}}{a+b\sinh^{-1}(c+dx)} + \text{Chi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-4),x]
```

```
[Out] -1/6*((2*b^3*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh[c + d*x])^3 + (b^2*(c +
d*x))/(a + b*ArcSinh[c + d*x])^2 + (b*Sqrt[1 + (c + d*x)^2])/(a + b*ArcSinh
[c + d*x]) + CoshIntegral[a/b + ArcSinh[c + d*x]]*Sinh[a/b] - Cosh[a/b]*Sin
hIntegral[a/b + ArcSinh[c + d*x]])/(b^4*d)
```

Maple [A]
time = 3.34, size = 272, normalized size = 1.70

method	result

derivativedivides	$\frac{\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcs}\right.}{12b^4}}{}$
default	$\frac{\frac{\left(-\sqrt{1+(dx+c)^2}+dx+c\right)\left(b^2\operatorname{arcsinh}(dx+c)^2+2ab\operatorname{arcsinh}(dx+c)-b^2\operatorname{arcsinh}(dx+c)+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arcsinh}(dx+c)^3+3a\,b^2\operatorname{arcsinh}(dx+c)^2+3a^2b\operatorname{arcsinh}(dx+c)+a^3\right)}+\frac{e^{\frac{a}{b}}\operatorname{expIntegral}\left(1,\operatorname{arcs}\right.}{12b^4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/12*(-(1+(d*x+c)^2)^(1/2)+d*x+c)*(b^2*arcsinh(d*x+c)^2+2*a*b*arcsinh(
d*x+c)-b^2*arcsinh(d*x+c)+a^2-a*b+2*b^2)/b^3/(b^3*arcsinh(d*x+c)^3+3*a*b^2*
arcsinh(d*x+c)^2+3*a^2*b*arcsinh(d*x+c)+a^3)+1/12/b^4*exp(a/b)*Ei(1,arcsinh
(d*x+c)+a/b)-1/6/b*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^3-1/12/
b^2*(d*x+c+(1+(d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))^2-1/12/b^3*(d*x+c+(1+(
d*x+c)^2)^(1/2))/(a+b*arcsinh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arcsinh(d*x+
c)-a/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*((a^2*d^11 + a*b*d^11 + 2*b^2*d^11)*x^11 + 11*(a^2*c*d^10 + a*b*c*d^10
 + 2*b^2*c*d^10)*x^10 + 5*((11*c^2*d^9 + d^9)*a^2 + (11*c^2*d^9 + d^9)*a*b
+ 2*(11*c^2*d^9 + d^9)*b^2)*x^9 + 15*((11*c^3*d^8 + 3*c*d^8)*a^2 + (11*c^3*
d^8 + 3*c*d^8)*a*b + 2*(11*c^3*d^8 + 3*c*d^8)*b^2)*x^8 + 10*((33*c^4*d^7 +
18*c^2*d^7 + d^7)*a^2 + (33*c^4*d^7 + 18*c^2*d^7 + d^7)*a*b + 2*(33*c^4*d^7
 + 18*c^2*d^7 + d^7)*b^2)*x^7 + 14*((33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a^2
 + (33*c^5*d^6 + 30*c^3*d^6 + 5*c*d^6)*a*b + 2*(33*c^5*d^6 + 30*c^3*d^6 + 5
*c*d^6)*b^2)*x^6 + 2*((231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a^2
 + (231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*a*b + 2*(231*c^6*d^5 +
 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2)*x^5 + 10*((33*c^7*d^4 + 63*c^5*d^4
 + 35*c^3*d^4 + 5*c*d^4)*a^2 + (33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*
d^4)*a*b + 2*(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b^2)*x^4 + 5*
((33*c^8*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a^2 + (33*c^8*d^
3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*a*b + 2*(33*c^8*d^3 + 84*c^
6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2)*x^3 + ((a^2*d^6 + a*b*d^6 + 2*b
^2*d^6)*x^6 + 6*(a^2*c*d^5 + a*b*c*d^5 + 2*b^2*c*d^5)*x^5 + (15*a*b*c^2*d^4
```

+ (15*c^2*d^4 + d^4)*a^2 + 2*(15*c^2*d^4 + d^4)*b^2)*x^4 + 4*(5*a*b*c^3*d^
3 + (5*c^3*d^3 + c*d^3)*a^2 + 2*(5*c^3*d^3 + c*d^3)*b^2)*x^3 + (c^6 + c^4 +
3*c^2 + 3)*a^2 + (c^6 - c^2)*a*b + 2*(c^6 + c^4)*b^2 + (3*(5*c^4*d^2 + 2*c
^2*d^2 + d^2)*a^2 + (15*c^4*d^2 - d^2)*a*b + 6*(5*c^4*d^2 + 2*c^2*d^2)*b^2)
x^2 + 2((3*c^5*d + 2*c^3*d + 3*c*d)*a^2 + (3*c^5*d - c*d)*a*b + 2*(3*c^5*
d + 2*c^3*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + (5*(a^2*d^7 + a*
b*d^7 + 2*b^2*d^7)*x^7 + 35*(a^2*c*d^6 + a*b*c*d^6 + 2*b^2*c*d^6)*x^6 + (3*
(35*c^2*d^5 + 3*d^5)*a^2 + 5*(21*c^2*d^5 + d^5)*a*b + 6*(35*c^2*d^5 + 3*d^5
)*b^2)*x^5 + 5*((35*c^3*d^4 + 9*c*d^4)*a^2 + 5*(7*c^3*d^4 + c*d^4)*a*b + 2*
(35*c^3*d^4 + 9*c*d^4)*b^2)*x^4 + (5*(35*c^4*d^3 + 18*c^2*d^3 + 2*d^3)*a^2
+ (175*c^4*d^3 + 50*c^2*d^3 - 2*d^3)*a*b + 2*(175*c^4*d^3 + 90*c^2*d^3 + 4*
d^3)*b^2)*x^3 + (5*c^7 + 9*c^5 + 10*c^3 + 6*c)*a^2 + (5*c^7 + 5*c^5 - 2*c^3
- 2*c)*a*b + 2*(5*c^7 + 9*c^5 + 4*c^3)*b^2 + (15*(7*c^5*d^2 + 6*c^3*d^2 +
2*c*d^2)*a^2 + (105*c^5*d^2 + 50*c^3*d^2 - 6*c*d^2)*a*b + 6*(35*c^5*d^2 + 3
0*c^3*d^2 + 4*c*d^2)*b^2)*x^2 + ((35*c^6*d + 45*c^4*d + 30*c^2*d + 6*d)*a^2
+ (35*c^6*d + 25*c^4*d - 6*c^2*d - 2*d)*a*b + 2*(35*c^6*d + 45*c^4*d + 12*
c^2*d)*b^2)*x)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^2 + (c^11 + 5*c^9 + 10*c^7 + 1
0*c^5 + 5*c^3 + c)*a^2 + (c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*a*b +
2*(c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*b^2 + 5*((11*c^9*d^2 + 36*c
^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*a^2 + (11*c^9*d^2 + 36*c^7*d^2
+ 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*a*b + 2*(11*c^9*d^2 + 36*c^7*d^2 + 42*
c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*b^2)*x^2 + (b^2*d^11*x^11 + 11*b^2*c*d^10*x
^10 + 5*(11*c^2*d^9 + d^9)*b^2*x^9 + 15*(11*c^3*d^8 + 3*c*d^8)*b^2*x^8 + 10
*(33*c^4*d^7 + 18*c^2*d^7 + d^7)*b^2*x^7 + 14*(33*c^5*d^6 + 30*c^3*d^6 + 5*
c*d^6)*b^2*x^6 + 2*(231*c^6*d^5 + 315*c^4*d^5 + 105*c^2*d^5 + 5*d^5)*b^2*x^
5 + 10*(33*c^7*d^4 + 63*c^5*d^4 + 35*c^3*d^4 + 5*c*d^4)*b^2*x^4 + 5*(33*c^8
*d^3 + 84*c^6*d^3 + 70*c^4*d^3 + 20*c^2*d^3 + d^3)*b^2*x^3 + 5*(11*c^9*d^2
+ 36*c^7*d^2 + 42*c^5*d^2 + 20*c^3*d^2 + 3*c*d^2)*b^2*x^2 + (11*c^10*d + 45
*c^8*d + 70*c^6*d + 50*c^4*d + 15*c^2*d + d)*b^2*x + (b^2*d^6*x^6 + 6*b^2*c
*d^5*x^5 + (15*c^2*d^4 + d^4)*b^2*x^4 + 4*(5*c^3*d^3 + c*d^3)*b^2*x^3 + 3*(
5*c^4*d^2 + 2*c^2*d^2 + d^2)*b^2*x^2 + 2*(3*c^5*d + 2*c^3*d + 3*c*d)*b^2*x
+ (c^6 + c^4 + 3*c^2 + 3)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)^(5/2) + (5*b^2
*d^7*x^7 + 35*b^2*c*d^6*x^6 + 3*(35*c^2*d^5 + 3*d^5)*b^2*x^5 + 5*(35*c^3*d^
4 + 9*c*d^4)*b^2*x^4 + 5*(35*c^4*d^3 + 18*c^2*d^3 + 2*d^3)*b^2*x^3 + 15*(7*
c^5*d^2 + 6*c^3*d^2 + 2*c*d^2)*b^2*x^2 + (35*c^6*d + 45*c^4*d + 30*c^2*d +
6*d)*b^2*x + (5*c^7 + 9*c^5 + 10*c^3 + 6*c)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 +
1)^2 + (c^11 + 5*c^9 + 10*c^7 + 10*c^5 + 5*c^3 + c)*b^2 + (10*b^2*d^8*x^8
+ 80*b^2*c*d^7*x^7 + 2*(140*c^2*d^6 + 13*d^6)*b^2*x^6 + 4*(140*c^3*d^5 + 39
*c*d^5)*b^2*x^5 + 2*(350*c^4*d^4 + 195*c^2*d^4 + 11*d^4)*b^2*x^4 + 8*(70*c^
5*d^3 + 65*c^3*d^3 + 11*c*d^3)*b^2*x^3 + (280*c^6*d^2 + 390*c^4*d^2 + 132*c
^2*d^2 + 3*d^2)*b^2*x^2 + 2*(40*c^7*d + 78*c^5*d + 44*c^3*d + 3*c*d)*b^2*x
+ (10*c^8 + 26*c^6 + 22*c^4 + 3*c^2 - 3)*b^2)*(d^2*x^2 + 2*c*d*x + c^2 + 1)
^(3/2) + 2*(5*b^2*d^9*x^9 + 45*b^2*c*d^8*x^8 + (180*c^2*d^7 + 17*d^7)*b^2*x
^7 + 7*(60*c^3*d^6 + 17*c*d^6)*b^2*x^6 + 3*(210*c^4*d^5 + 119*c^2*d^5 + 6*d
^5)*b^2*x^5 + 5*(126*c^5*d^4 + 119*c^3*d^4 + 18*c*d^4)*b^2*x^4 + 5*(84*c^6*

```
d^3 + 119*c^4*d^3 + 36*c^2*d^3 + d^3)*b^2*x^3 + 3*(60*c^7*d^2 + 119*c^5*d^2
 + 60*c^3*d^2 + 5*c*d^2)*b^2*x^2 + (45*c^8*d + 119*c^6*d + 90*c^4*d + 15*c^
2*d - d)*b^2*x + (5*c^9 + 17*c^7 + 18*c^5 + 5*c^3 - c)*b^2)*(d^2*x^2 + 2*c*
d*x + c^2 + 1) + (5*b^2*d^10*x^10 + 50*b^2*c*d^9*x^9 + 3*(75*c^2*d^8 + 7*d^
8)*b^2*x^8 + 24*(25*c^3*d^7 + 7*c*d^7)*b^2*x^7 + (1050*c^4*d^6 + 588*c^2*d^
6 + 31*d^6)*b^2*x^6 + 6*(210*c^5*d^5 + 196*c^3*...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral(1/(b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2
*arcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{asinh}\left(c + dx\right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**4,x)
[Out] Integral((a + b*asinh(c + d*x))**(-4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^4,x)
[Out] int(1/(a + b*asinh(c + d*x))^4, x)
```

3.179 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^4},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^4,x)/e
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^4), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 2.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^4), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\left(a + b\,\mathrm{arcsinh}\left(dx + c\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/((b^4*d*x + b^4*c)*arcsinh(d*x + c)^4*e + 4*(a*b^3*d*x + a*b^3*c
)*arcsinh(d*x + c)^3*e + 6*(a^2*b^2*d*x + a^2*b^2*c)*arcsinh(d*x + c)^2*e +
 4*(a^3*b*d*x + a^3*b*c)*arcsinh(d*x + c)*e + (a^4*d*x + a^4*c)*e), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4c+a^4dx+4a^3bc\,\mathrm{asinh}\,(c+dx)+4a^3bdx\,\mathrm{asinh}\,(c+dx)+6a^2b^2c\,\mathrm{asinh}^2\,(c+dx)+6a^2b^2dx\,\mathrm{asinh}^2\,(c+dx)+4ab^3c\,\mathrm{asinh}^3\,(c+dx)+4ab^3dx\,\mathrm{asinh}^3\,(c+dx)+b^4c\,\mathrm{asinh}^4\,(c+dx)+b^4dx\,\mathrm{asinh}^4\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*asinh(c + d*x) + 4*a**3*b*d*x*as
inh(c + d*x) + 6*a**2*b**2*c*asinh(c + d*x)**2 + 6*a**2*b**2*d*x*asinh(c +
d*x)**2 + 4*a*b**3*c*asinh(c + d*x)**3 + 4*a*b**3*d*x*asinh(c + d*x)**3 + b
**4*c*asinh(c + d*x)**4 + b**4*d*x*asinh(c + d*x)**4), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^4), x)
```

3.180 $\int (ce+dex)^4 \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=361

$$\frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d}+\frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{b}\,e^4e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\cdots}{\cdots}\right.}{\cdots}$$

```
[Out] 1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^(1/
2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d+1/192*e^4
*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/
d/exp(3*a/b)+1/32*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi
^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.62, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}-\frac{\sqrt{\pi}\sqrt{b}\,e^4e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^4e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}\,e^4e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}+\frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) + (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(64*d) + (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Sinh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}\,\sqrt{}}\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{a+bx}}\,d\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{(ibe^4)\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{a+bx}}\right.\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,d\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{(be^4)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,d\right.}{320d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{e^4\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\right.}{160}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} + \frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b}}{}\right.}{32d}$$

Mathematica [A]

time = 0.45, size = 342, normalized size = 0.95

$$\frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)-25\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+150e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+25\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-3\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{2400d\sqrt{-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(-150*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcS
```

```
inh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 25*Sqrt[3]*E^((
2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]
))/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*Ar
cSinh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[5]*E^((10*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcSinh[c + d*x]))/
b]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 \sqrt{a + b \operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4 \left(\int c^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int d^4 x^4 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 4cd^3x^3 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 6c^2d^2x^2 \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx + \int 4c^3dx \sqrt{a + b \operatorname{asinh}(c + dx)}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(1/2), x)
```

3.181 $\int (ce+dex)^3\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}+\frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{\sqrt{b}\,e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+}}{\sqrt{b}}\right.}{256d}$$

```
[Out] 1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)
*2^(1/2)*Pi^(1/2)/d+1/64*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b*ar
csinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*arcsi
nh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.45, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3e^{-\frac{4a}{b}}\,\mathrm{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3e^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcSinh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d) + (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) + (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}}\right.}{}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{(be^3)\,\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+b}}\right.\right.}{}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+}}{4d}$$

$$= -\frac{3e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+}}{4d}$$

Mathematica [A]
time = 0.21, size = 223, normalized size = 0.82

$$\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-4\sqrt{2}\,\Gamma\left(\frac{3}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{128d\sqrt{-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcSinh[c + d*x]],x]
[Out] (e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcSinh[c + d*x]))/b] - 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh
```

```
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcSinh[c + d*x])/b)]*(-4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcSinh[c +
d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(128*
d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{3} \sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3} \left(\int c^{3} \sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\, dx + \int d^{3}x^{3} \sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\, dx + \int 3cd^{2}x^{2} \sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\, dx + \int 3c^{2}dx \sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(1/2), x)
```

3.182 $\int (ce+dex)^2 \sqrt{a+b\sinh^{-1}(c+dx)}\, dx$

Optimal. Leaf size=245

$$\frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d}-\frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{b}\,e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\cdots}{\cdots}\right)}{\cdots}$$

```
[Out] 1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d+1/16*e^2*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b*arcs
inh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.39, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5777, 5819, 3393, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{\sqrt{\pi}\sqrt{b}\,e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}+\frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*d) + (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(48*d) + (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\sqrt{a+b\sinh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\Big(\int \frac{}{\sqrt{1+x^2}\,\sqrt{}}}{}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{a+bx}}\,d\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(ibe^2)\,\text{Subst}\Big(\int \Big(\frac{3i\sinh(x)}{4\sqrt{a+b}}}{}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,d\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{(be^2)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,d\right.}{48d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} + \frac{e^2\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\right.}{24}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\text{erf}\Bigg(\frac{\sqrt{a+}}{}}{16d}$$

Mathematica [A]
time = 0.29, size = 238, normalized size = 0.97

$$\frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-9e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{72d\sqrt{-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c
```

```
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 9*E^((2*a)/b)*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[3/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcSin
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)]
)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 \sqrt{a + b\operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int c^2 \sqrt{a + b\operatorname{asinh}(c + dx)}\, dx + \int d^2x^2 \sqrt{a + b\operatorname{asinh}(c + dx)}\, dx + \int 2cdx \sqrt{a + b\operatorname{asinh}(c + dx)}\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*asinh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(1/2), x)
```

3.183 $\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$\frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}-\frac{\sqrt{b}\,ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+d}}{\sqrt{b}}\right.}{16d}$$

```
[Out] -1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)+1/4*e*(a+b*arcsinh(d*x+c))^(1/2)/d+1/
2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5859, 12, 5777, 5819, 3393, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcSinh[
c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int ex\sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \frac{\cdots}{\sqrt{1+x^2}\sqrt{a\cdots}}\right.}{\cdots}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \frac{(be)\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx,\cdots\right.}{4d}$$

$$= \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(be)\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}\cdots\right.\right.}{\cdots}$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \cdots$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \cdots$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \cdots$$

$$= \frac{e\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} - \cdots$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.85

$$\frac{ee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{8\sqrt{2}\,d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*Sqrt[a + b*ArcSinh[c + d*x]]*(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[3/2, (-
2*(a + b*ArcSinh[c + d*x]))/b] + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)/b)]*Gamma[3/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(8*Sqrt[2]*d*E^((2*a)/b)
*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{a + b\operatorname{arcsinh}(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int c\sqrt{a + b\operatorname{asinh}(c + dx)}\, dx + \int dx\sqrt{a + b\operatorname{asinh}(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x*sqrt(a + b*asin
h(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2), x)
```

3.184 $\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}+\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/4
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x+
c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5772, 5819, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\sqrt{b}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/d + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sqrt{a+b\sinh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sinh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,d\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} - \frac{b\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{b\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx\right.}{4d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+\right.}{2d}$$

$$= \frac{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{d} + \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{4d}$$

Mathematica [A]
time = 0.10, size = 111, normalized size = 0.97

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c +
d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{arcsinh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{asinh}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^(1/2), x)
```

3.185 $\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

[Out] Unintegrable((a+b*arcsinh(d*x+c))^(1/2)/(d*x+c),x)/e

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcSinh[x]]/x, x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\sinh^{-1}(x)}}{x}\,dx, x, c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x),x]
```

```
[Out] Integrate[Sqrt[a + b*ArcSinh[c + d*x]]/(c*e + d*e*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arcsinh}\left(dx+c\right)}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(1/2)/(d*e*x+c*e),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c + d*x))/(c + d*x), x)/e
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(d*x + c) + a)/(d*e*x + c*e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

3.186 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=601

$$-\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d}+\frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d}-\frac{3be$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3/2)/d+3/16000*b^(3/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d+3/160
00*b^(3/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-1/384*b^(3/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-1/384*b^(3/2)*e^4*erfi(3^(1/2)*(a+b
*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+3/64*b^(3/2)*
e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/64*b^(3/2
)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/25*b*e
^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+2/25*b*e^4*(d*x+c)^2*(1
+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/50*b*e^4*(d*x+c)^4*(1+(d*x
+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.09, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\frac{3\sqrt{\pi}b^{3/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{200d}+\frac{3\sqrt{\frac{\pi}{5}}b^{3/2}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}+\frac{3\sqrt{\pi}b^{3/2}e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{3\sqrt{3\pi}b^{3/2}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{200d}+\frac{3\sqrt{\frac{\pi}{5}}b^{3/2}e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3200d}+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{5d}-\frac{3be^4\sqrt{(c+dx)^2+1}(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{50d}+\frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{25d}-\frac{4be^4\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d) + (2*b
*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(25*d)
 - (3*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(5*d) + (3*b^(3/
2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) -
 (b^(3/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(200*d) - (3*b^(3/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d) + (3*b^(3/2)*e^4*E^((5*a)
/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d
) + (3*b^(3/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64
*d*E^(a/b)) - (b^(3/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(200*d*E^((3*a)/b)) - (3*b^(3/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3200*d*E^((3*a)/b)) + (3*b^(3/
```

```
2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32
00*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{5d} - \frac{(3be^4)\,\text{Subst}\left(\int \frac{x^5\sqrt{}}{}\right.}{}$$

$$= -\frac{3be^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{50d} + \frac{e^4}{}$$

$$= \frac{2be^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} - \frac{3be^4}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

$$= -\frac{4be^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{25d} + \frac{2be^4(c+dx}{}$$

Mathematica [A]

time = 0.33, size = 343, normalized size = 0.57

$$-\frac{be^4e^{-\frac{5a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(2250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+9\sqrt{5}\,\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)-125\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+2250e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-125\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+9\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{36000d\sqrt{-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -1/36000*(b*e^4*Sqrt[a + b*ArcSinh[c + d*x]]*(2250*E^((6*a)/b)*Sqrt[-((a +
b*ArcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + 9*Sqrt[5]*Sqrt
[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 125*
Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcS
inh[c + d*x]))/b] + 2250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, -((a + b*ArcSinh[c + d*x])/b)] - 125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 9*Sqrt[5]*
E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (5*(a + b*ArcSi
nh[c + d*x]))/b]))/(d*E^((5*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4 \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int ac^4\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int ad^4x^4\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int bc^4\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 4acd^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 6ac^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 4ac^3dx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int bd^4x^4\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 4bcd^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 6bc^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 4bc^3dx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(a*c**4*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**4*x**4
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**4*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(4*a*c*d**3*x**3*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(6*a*c**2*d**2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral
(4*a*c**3*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**4*x**4*sqrt(a
+ b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c*d**3*x**3*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(6*b*c**2*d**2*x**2*sqrt(a +
 b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(4*b*c**3*d*x*sqrt(a + b*as
inh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(3/2), x)
```

3.187 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} \, dx$

<u>**Optimal**</u>. Leaf size=360

$$\frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{64d}-\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(3/2)/d+3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^3*ex
p(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b^(3/2
)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)+9/64
*b*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-3/32*b*e^3*
(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

<u>**Rubi**</u> [A]
time = 0.71, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}\,b^{3/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}e^3e^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{3be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}+\frac{9be^3\sqrt{(c+dx)^2+1}\,(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{64d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (9*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(64*
d) - (3*b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]
])/(32*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) + (e^3*(c + d*x)^
4*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (3*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[P
i]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d) + (3*b^(3/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(2048*d*E^((4*a)/b)) - (3*b^(3/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqr
t[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(128*d*E^((2*a)/b))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 2211</u>

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4\sqrt{}}{}\right.}{}$$

$$= -\frac{3be^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} + \frac{e^3}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

$$= \frac{9be^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} - \frac{3be^3(}{}$$

Mathematica [A]

time = 0.22, size = 225, normalized size = 0.62

$$\frac{be^3e^{-\frac{4a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)+8\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-8\sqrt{2}\,\Gamma\left(\frac{5}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{5}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{512d\sqrt{-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5
/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 8*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[5/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcSinh[c + d*x])/b)]*(-8*Sqrt[2]*Gamma[5/2, (2*(a + b*ArcSinh
[c + d*x]))/b] + E^((2*a)/b)*Gamma[5/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/
(512*d*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\,(a+b\operatorname{arcsinh}(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int ac^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int ad^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int bc^3\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 3acd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int 3ac^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int bd^3x^3\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 3bcd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 3bc^2dx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*asinh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(3/2), x)
```

3.188 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=328

$$\frac{be^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d}-\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{e^2(c+$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/288*b^(3/2)*e^2*exp(3*a/b)
*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288*b
^(3/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2
)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d-3/32*b^(3/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/d/exp(a/b)+1/3*b*e^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-1/6*b*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.58, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}-\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{3d}-\frac{be^2\sqrt{(c+dx)^2+1}(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{be^2\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(3*d) - (b*e^2*(
c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (e^2
*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(3*d) - (3*b^(3/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(32*d) + (b^(3/2)*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(96*d) - (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]
])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSin
h[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3\sqrt{a}}{}\right.}{}$$

$$= -\frac{be^2(c+dx)^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{e^2(}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

$$= \frac{be^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{3d} - \frac{be^2(c+dx)^2\sqrt{}}{}$$

Mathematica [A]
time = 0.22, size = 238, normalized size = 0.73

$$-\frac{be^2e^{-\frac{3a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-27e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{216d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] -1/216*(b*e^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*A
rcSinh[c + d*x])/b)]*Gamma[5/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b
+ ArcSinh[c + d*x]]*Gamma[5/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 27*E^((2*
a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2, -((a + b*ArcSinh[c + d*x])/b)
] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[5/2, (3*(
a + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x]
)^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dex + ce\right)^{2} \left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2} \left(\int ac^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int ad^{2} x^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int bc^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}\, dx + \int 2acdx \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int bd^{2} x^{2} \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}\, dx + \int 2bcdx \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*asinh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*asinh(c + d*x)
)*asinh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*asinh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(3/2), x)
```

3.189 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$-\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}\right.}{2d}$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/
2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(1+(d*x+c)^2)^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$,
Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}\,b^{3/2}ee^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}-\frac{3be\sqrt{(c+dx)^2+1}\,(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d)
 + (e*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh
[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*e*Sqrt[Pi/2]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
```

```
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\text{Subst}\left(\int \frac{x^2\sqrt{a+\cdots}}{\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(c+\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

$$= -\frac{3be(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d} + \frac{e(a-\cdots}{\cdots}$$

Mathematica [A]
time = 0.08, size = 142, normalized size = 0.69

$$\frac{bee^{-\frac{2a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{5}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{5}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{16\sqrt{2}\,d\sqrt{-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (b*e*Sqrt[a + b*ArcSinh[c + d*x]]*(-(Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[5/2
, (-2*(a + b*ArcSinh[c + d*x]))/b]) + E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c +
 d*x])/b)]*Gamma[5/2, (2*(a + b*ArcSinh[c + d*x]))/b]))/(16*Sqrt[2]*d*E^((2
*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])^2/b^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int ac\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int adx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,dx+\int bc\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx+\int bdx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(a*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
asinh(c + d*x)), x) + Integral(b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2), x)
```

3.190 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$-\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right.}{8d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arcsinh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+
c))^(1/2)/d
```

Rubi [A]

time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(2*d) + ((c + d*x
)*(a + b*ArcSinh[c + d*x])^(3/2))/d + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{d} - \frac{(3b)\text{Subst}\left(\int \frac{x\sqrt{a+b\sinh^{-1}(x)}}{\sqrt{1+x^2}}\,dx\right.}{2d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

$$= -\frac{3b\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c+\right.}{d}$$

Mathematica [A]
time = 0.17, size = 272, normalized size = 1.81

$$\frac{ae^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)}{2d}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\sinh^{-1}(c+dx)\right)+(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(-2a+3b)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c
+ d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c + d*
x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/(2*d*E^(a/b)) + (Sqrt[b]*(4*S
qrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*x)
*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(8*d)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{asinh}(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2),x)
[Out] Integral((a + b*asinh(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{asinh}(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(3/2), x)
```

3.191 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(3/2)/(d*x+c),x)/e
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
[Out] int((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}{c+dx}\,dx+\int \frac{b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(3/2)/(d*e*x+c*e),x)
[Out] (Integral(a*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

3.192 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=701

$$\frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d}-\frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}+\frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(\ldots}}{100d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(5/2)/d+3/32000*b^(5/2)*e^4*exp(5*a/
b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/d-3/320
00*b^(5/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^
(1/2)/d/exp(5*a/b)-5/2304*b^(5/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+5/2304*b^(5/2)*e^4*erfi(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)+15/128*b^(5
/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/128*
b^(5/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-4/
15*b*e^4*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/15*b*e^4*(d*x+c
)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-1/10*b*e^4*(d*x+c)^4*(
a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+2/5*b^2*e^4*(d*x+c)*(a+b*ar
csinh(d*x+c))^(1/2)/d-1/15*b^2*e^4*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(1/2)/d+3
/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.45, antiderivative size = 701, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5819, 3389, 2211, 2236, 2235, 3393}

$$\frac{15\sqrt{\pi}b^{5/2}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{\sqrt{3\pi}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}-\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{240d}+\frac{3\sqrt{\frac{\pi}{5}}b^{5/2}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6400d}-\frac{15\sqrt{\pi}b^{5/2}e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{\sqrt{3\pi}b^{5/2}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1280d}+\frac{\sqrt{\frac{\pi}{3}}b^{5/2}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{240d}-\frac{3\sqrt{\frac{\pi}{5}}b^{5/2}e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6400d}+\frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{100d}-\frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d}+\frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d}+\frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{5d}-\frac{be^4\sqrt{(c+dx)^2+1}(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{10d}+\frac{2be^4\sqrt{(c+dx)^2+1}(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{15d}-\frac{4be^4\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*e^4*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(5*d) - (b^2*e^4*(c + d*
x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(15*d) + (3*b^2*e^4*(c + d*x)^5*Sqrt[a +
 b*ArcSinh[c + d*x]])/(100*d) - (4*b*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSi
nh[c + d*x])^(3/2))/(15*d) + (2*b*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a
+ b*ArcSinh[c + d*x])^(3/2))/(15*d) - (b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)
^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(10*d) + (e^4*(c + d*x)^5*(a + b*ArcSin
h[c + d*x])^(5/2))/(5*d) + (15*b^(5/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) - (b^(5/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*E
rf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d) - (b^(5/2)*e^4*
E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])
/(1280*d) + (3*b^(5/2)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(6400*d) - (15*b^(5/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a
```

```
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d*E^(a/b)) + (b^(5/2)*e^4*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(240*d*E^((3*a)/b)) +
 (b^(5/2)*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(1280*d*E^((3*a)/b)) - (3*b^(5/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(6400*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{5d} - \frac{(be^4)\,\text{Subst}\Big(\int \frac{x^5(a+b}{\sqrt{}} \cdots}{}$$

$$= -\frac{be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{10d} + \frac{e^4 \cdots}{}$$

$$= \frac{3b^2e^4(c+dx)^5\sqrt{a+b\sinh^{-1}(c+dx)}}{100d} + \frac{2be^4(c+dx)^2\sqrt{1+ \cdots}}{}$$

$$= -\frac{b^2e^4(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{15d} + \frac{3b^2e^4(c+dx)^5\sqrt{a \cdots}}{10 \cdots}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

$$= \frac{2b^2e^4(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{5d} - \frac{b^2e^4(c+dx)^3\sqrt{a+b \cdots}}{15d}$$

Mathematica [A]
time = 0.50, size = 342, normalized size = 0.49

$$-\frac{e^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-33750e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+27\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-625\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)+33750e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+625\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-27\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},\frac{5\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{540000d\left(-\frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/540000*(e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(-33750*E^((6*a)/b)*Sqrt[-((a
 + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + 27*Sqrt[5]*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (-5*(a + b*ArcSinh[c + d*x]))/b] -
625*Sqrt[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (-3*(a + b*
ArcSinh[c + d*x]))/b] + 33750*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[7/2, -((a + b*ArcSinh[c + d*x])/b)] + 625*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a +
 b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (3*(a + b*ArcSinh[c + d*x]))/b] - 27*Sq
rt[5]*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (5*(a + b
*ArcSinh[c + d*x]))/b]))/(d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))
^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dex+ce\right)^{4}\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(5/2), x)
```

3.193 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} \, dx$

Optimal. Leaf size=455

$$-\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d}-\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d}+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(}}{256d}$$

```
[Out] -3/32*e^3*(a+b*arcsinh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c
))^(5/2)/d+15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5/2)*
e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/1638
4*b^(5/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a
/b)+15/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-5/
32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d-225/204
8*b^2*e^3*(a+b*arcsinh(d*x+c))^(1/2)/d-45/256*b^2*e^3*(d*x+c)^2*(a+b*arcsin
h(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.02, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5819, 3393, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\pi}b^{5/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}b^{5/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}+\frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{256d}-\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d}-\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d}-\frac{5be^3\sqrt{(c+dx)^2+1}(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d}+\frac{15be^3\sqrt{(c+dx)^2+1}(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{64d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcSinh[c + d*x]])/(2048*d) - (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcSinh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcSinh[c + d*x]])/(256*d) + (15*b*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*
(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*(c + d*x)^3*Sqrt[1 + (c +
 d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (3*e^3*(a + b*ArcSinh[c +
 d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(5/2))/(4*
d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x
]])/Sqrt[b]])/(16384*d) + (15*b^(5/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(512*d) - (15*b^(5/2)*e^3*Sqrt[P
i]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16384*d*E^((4*a)/b)) +
(15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(512*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{(5be^3)\,\text{Subst}\left(\int \frac{x^4(a+\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{5be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d} + \cdots$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15be^3(c+dx)\sqrt{1+\cdots}}{\cdots}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\cdots}{\cdots}$$

$$= -\frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\cdots}{\cdots}$$

$$= -\frac{45b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\sinh\cdots}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\cdots}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\cdots}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\cdots}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} - \frac{45b^2e^3(c+dx)^2\sqrt{a+b\,\cdots}}{256d}$$

Mathematica [A]

time = 0.23, size = 223, normalized size = 0.49

$$-\frac{e^3 e^{-\frac{4a}{b}} \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} \left(\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-16\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-16\sqrt{2}\,\Gamma\left(\frac{7}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{7}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{2048d\left(-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/2048*(e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(Sqrt[a/b + ArcSinh[c + d*x]]*G
amma[7/2, (-4*(a + b*ArcSinh[c + d*x]))/b] - 16*Sqrt[2]*E^((2*a)/b)*Sqrt[a/
b + ArcSinh[c + d*x]]*Gamma[7/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a
)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-16*Sqrt[2]*Gamma[7/2, (2*(a + b*
ArcSinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[7/2, (4*(a + b*ArcSinh[c + d*x]))
/b])))/(d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 \left(a+b\,\mathrm{arcsinh}\,(dx+c)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int a^{2}c^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int a^{2}d^{3}x^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int b^{2}c^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}^{2}\left(c+dx\right)dx+\int 2abc^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)dx+\int 3a^{2}cd^{2}x^{2}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int 3a^{2}c^{2}dx\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\,dx+\int b^{2}d^{3}x^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}^{2}\left(c+dx\right)dx+\int 2abd^{3}x^{3}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)dx+\int 3b^{2}cd^{2}x^{2}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}^{2}\left(c+dx\right)dx+\int 3b^{2}c^{2}dx\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}^{2}\left(c+dx\right)dx+\int 6abcd^{2}x^{2}\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)dx+\int 6abc^{2}dx\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(a**2*c**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d**
3*x**3*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**3*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**3*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x), x) + Integral(3*a**2*c*d**2*x**2*sqrt(a + b*asinh(c
+ d*x)), x) + Integral(3*a**2*c**2*d*x*sqrt(a + b*asinh(c + d*x)), x) + Int
egral(b**2*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) + Int
egral(2*a*b*d**3*x**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) + Integ
ral(3*b**2*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) + I
ntegral(3*b**2*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x) +
Integral(6*a*b*c*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x) +
Integral(6*a*b*c**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^3\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(5/2), x)
```

3.194 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=394

$$-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{36d}+\frac{5be^2\sqrt{1+(c+dx)^2}\,(a+}{9d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)/d+5/1728*b^(5/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/1728
*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d+15/64*b^(5/2)*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/d/exp(a/b)+5/9*b*e^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^
(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)
/d-5/6*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d+5/36*b^2*e^2*(d*x+c)^3*
(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.84, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5819, 3389, 2211, 2236, 2235, 3393}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}+\frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{36d}-\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{3d}-\frac{5be^2\sqrt{(c+dx)^2+1}\,(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{18d}+\frac{5be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
 d*x)^3*Sqrt[a + b*ArcSinh[c + d*x]])/(36*d) + (5*b*e^2*Sqrt[1 + (c + d*x)^
2]*(a + b*ArcSinh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*(c + d*x)^2*Sqrt[1 + (c
 + d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (e^2*(c + d*x)^3*(a + b
*ArcSinh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt
[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d) + (5*b^(5/2)*e^2*E^((3*a)/b)*Sqrt
[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(576*d) + (15*b
^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(64*d*E^(a/
b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])
/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{3d} - \frac{(5be^2)\,\text{Subst}\left(\int \frac{x^3(a+\cdots}{\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{5be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{18d} + \cdots$$

$$= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{36d} + \frac{5be^2\sqrt{1+(c+dx)^2}\cdots}{\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

$$= -\frac{5b^2e^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a\cdots}}{3\cdots}$$

Mathematica [A]

time = 0.30, size = 238, normalized size = 0.60

$$-\frac{e^2 e^{-\frac{3a}{b}} (a+b\sinh^{-1}(c+dx))^{5/2} \left(81e^{\frac{4a}{b}} \sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\, \Gamma\left(\frac{7}{2}, \frac{a}{b}+\sinh^{-1}(c+dx)\right) + \sqrt{3}\, \sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\, \Gamma\left(\frac{7}{2}, -\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right) - 81e^{\frac{2a}{b}} \sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\, \Gamma\left(\frac{7}{2}, -\frac{a+b\sinh^{-1}(c+dx)}{b}\right) - \sqrt{3}\, e^{\frac{6a}{b}} \sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\, \Gamma\left(\frac{7}{2}, \frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{648d \left(-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] -1/648*(e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(81*E^((4*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[7/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[7/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 81*E^((2*a
)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, -((a + b*ArcSinh[c + d*x])/b)]
 - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (3*(a
 + b*ArcSinh[c + d*x]))/b]))/(d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b
^2))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2 (a+b\operatorname{arcsinh}(dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int a^2c^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int a^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int b^2c^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx+\int 2abc^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 2a^2cdx\sqrt{a+b\operatorname{asinh}(c+dx)}\,dx+\int b^2d^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx+\int 2abd^2x^2\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx+\int 2b^2cdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^2(c+dx)\,dx+\int 4abcdx\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(a**2*c**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d**
2*x**2*sqrt(a + b*asinh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*asinh(c + d*x)
), x) + Integral(b**2*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x),
 x) + Integral(2*b**2*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(4*a*b*c*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(5/2), x)
```

3.195 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}-\frac{5be(c+dx)\sqrt{1+(c+dx)^2}\,(a+b\si$$

```
[Out] 1/4*e*(a+b*arcsinh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/
2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arcsinh(
d*x+c))^(3/2)*(1+(d*x+c)^2)^(1/2)/d+15/64*b^2*e*(a+b*arcsinh(d*x+c))^(1/2)/
d+15/32*b^2*e*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.48, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5859, 12, 5777, 5812, 5783, 5819, 3393, 3388, 2211, 2236, 2235}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d}+\frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d}-\frac{5be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{8d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*e*Sqrt[a + b*ArcSinh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqrt
[a + b*ArcSinh[c + d*x]])/(32*d) - (5*b*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*(
a + b*ArcSinh[c + d*x])^(3/2))/(8*d) + (e*(a + b*ArcSinh[c + d*x])^(5/2))/(
4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) - (15*b^(5/2)*e
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d} - \frac{(5be)\text{Subst}\left(\int \frac{x^2(a+b\sin}{\sqrt{1}}\right.}{4}$$

$$= -\frac{5be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{8d} + \frac{e(c}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c}}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c}}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}(c+dx)}}{32d} - \frac{5be(c+dx)\sqrt{1+(c}}{}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

$$= \frac{15b^2e\sqrt{a+b\sinh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\sinh^{-1}}}{32d}$$

Mathematica [A]
time = 0.06, size = 126, normalized size = 0.48

$$\frac{ee^{-\frac{2a}{b}}\left(-b^3\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{7}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{7}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{32\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-(b^3*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh
[c + d*x]))/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[7/2, (
2*(a + b*ArcSinh[c + d*x]))/b]))/(32*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSi
nh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)\,(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{5}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^{2}c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int a^{2}dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,dx + \int b^{2}c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{2}\,(c + dx)\,dx + \int 2abc\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c + dx)\,dx + \int b^{2}dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{2}\,(c + dx)\,dx + \int 2abdx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c + dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(a**2*c*sqrt(a + b*asinh(c + d*x)), x) + Integral(a**2*d*x*sqrt(
a + b*asinh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x), x) + Integral(b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2, x
) + Integral(2*a*b*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2), x)
```

3.196 $\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=179

$$\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c\right.}{d}$$

```
[Out] (d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)/d+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arcsinh(d*x+c))^(3/2)*(1+(d*
x+c)^2)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5819, 3389, 2211, 2236, 2235}

$$\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d}-\frac{5b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcSinh[c + d*x]])/(4*d) - (5*b*Sqrt[1 + (c +
d*x)^2]*(a + b*ArcSinh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c
 + d*x])^(5/2))/d + (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}{\sqrt{1+x^2}}\,dx,x,c\right.}{2d}$$

$$= -\frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(a+b\sinh^{-1}(c\right.}{d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c\right.}{2d}$$

<u>Mathematica</u> [**B**] Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(179) = 358.
time = 1.40, size = 458, normalized size = 2.56

$$\frac{8a^2e^{-\frac{a}{b}}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}}\right)+4a\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(-3\sqrt{1+(c+dx)^2}+2(c+dx)\sinh^{-1}(c+dx)\right)+(2a+3b)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)+(-2a+3b)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)+\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\sinh^{-1}(c+dx)}\left(2\sqrt{1+(c+dx)^2}\left(a-5b\sinh^{-1}(c+dx)\right)+b(c+dx)\left(15+4\sinh^{-1}(c+dx)^2\right)\right)+(4a^2+12ab+15b^2)\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(-\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)+(4a^2-12ab+15b^2)\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)\left(\cosh\left(\frac{a}{b}\right)+\sinh\left(\frac{a}{b}\right)\right)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] ((8*a^2*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSi
nh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[c
 + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 4*a*Sqrt[b]*(4
*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c + d*
x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x
```

```
]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*
ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sq
rt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSinh[c + d*
x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b + 15*b^2)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqr
t[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{asinh}\,(c+dx))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{asinh}(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(5/2), x)
```

3.197 $\int \frac{(a+b\sinh^{-1}(c+dx))^{5/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arcsinh(d*x+c))^(5/2)/(d*x+c),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{5/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sinh^{-1}(c+dx))^{5/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{5/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{5/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{5/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(d x+c\right)\right)^{\frac{5}{2}}}{d e x+c e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^{2} \sqrt{a+b \operatorname{asinh}\left(c+d x\right)}}{c+d x}\, dx+\int \frac{b^{2} \sqrt{a+b \operatorname{asinh}\left(c+d x\right)} \operatorname{asinh}^{2}\left(c+d x\right)}{c+d x}\, dx+\int \frac{2 a b \sqrt{a+b \operatorname{asinh}\left(c+d x\right)} \operatorname{asinh}\left(c+d x\right)}{c+d x}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2*sqrt(a + b*asinh(c + d*x))/(c + d*x), x) + Integral(b**2*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)/(c + d*x), x))/e
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{5/2}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

3.198 $\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=835

$$-\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d}+\frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx}}{1125d}$$

```
[Out] 14/15*b^2*e^4*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d-7/45*b^2*e^4*(d*x+c)^3*(
a+b*arcsinh(d*x+c))^(3/2)/d+7/100*b^2*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(3
/2)/d+1/5*e^4*(d*x+c)^5*(a+b*arcsinh(d*x+c))^(7/2)/d+21/320000*b^(7/2)*e^4*
exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)
/d+21/320000*b^(7/2)*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5
^(1/2)*Pi^(1/2)/d/exp(5*a/b)-35/13824*b^(7/2)*e^4*exp(3*a/b)*erf(3^(1/2)*(a
+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-35/13824*b^(7/2)*e^4*e
rfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/
b)+105/256*b^(7/2)*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/256*b^(7/2)*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/
2)/d/exp(a/b)-28/75*b*e^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+
14/75*b*e^4*(d*x+c)^2*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/50
*b*e^4*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-1813/1125
*b^3*e^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d+119/1125*b^3*e^4*
(d*x+c)^2*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d-21/1000*b^3*e^4*
(d*x+c)^4*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 2.09, antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-1813*b^3*e^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(1125*d)
 + (119*b^3*e^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*
x]])/(1125*d) - (21*b^3*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*Ar
cSinh[c + d*x]])/(1000*d) + (14*b^2*e^4*(c + d*x)*(a + b*ArcSinh[c + d*x])^
(3/2))/(15*d) - (7*b^2*e^4*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(45*
d) + (7*b^2*e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(3/2))/(100*d) - (28*b
*e^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d) + (14*b*e
^4*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(75*d)
 - (7*b*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2
```

```
))/(50*d) + (e^4*(c + d*x)^5*(a + b*ArcSinh[c + d*x])^(7/2))/(5*d) + (105*b
^(7/2)*e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(256
*d) - (119*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(18000*d) - (21*b^(7/2)*e^4*E^((3*a)/b)*Sqrt[3*Pi]*
Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000*d) + (21*b^(7/2
)*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(64000*d) + (105*b^(7/2)*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(256*d*E^(a/b)) - (119*b^(7/2)*e^4*Sqrt[Pi/3]*Erfi[(Sqrt[3]*S
qrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(18000*d*E^((3*a)/b)) - (21*b^(7/2)*
e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(64000
*d*E^((3*a)/b)) + (21*b^(7/2)*e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(64000*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int x^4\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{5d} - \frac{(7be^4)\,\text{Subst}\left(\int \frac{x^5(a+b}{\sqrt{}}\right.}{}$$

$$= -\frac{7be^4(c+dx)^4\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{50d} + \frac{e^4}{}$$

$$= \frac{7b^2e^4(c+dx)^5\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{100d} + \frac{14be^4(c+dx)^2\sqrt{1}}{}$$

$$= -\frac{21b^3e^4(c+dx)^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1000d} - \frac{7}{}$$

$$= \frac{119b^3e^4(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} - \frac{21}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

$$= -\frac{1813b^3e^4\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{1125d} + \frac{119b^3e^4(}{}$$

Mathematica [A]
time = 0.36, size = 343, normalized size = 0.41

$$\frac{be^4e^{-\frac{5a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(506250e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+81\sqrt{5}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)-3125\sqrt{3}e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+506250e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-3125\sqrt{3}e^{\frac{8a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+81\sqrt{5}e^{\frac{10a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{8100000d\left(-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^4*(a + b*ArcSinh[c + d*x])^(5/2)*(506250*E^((6*a)/b)*Sqrt[-((a + b*Arc
Sinh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + 81*Sqrt[5]*Sqrt[a/b
 + ArcSinh[c + d*x]]*Gamma[9/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3125*Sqr
t[3]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh
[c + d*x]))/b] + 506250*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2,
 -((a + b*ArcSinh[c + d*x])/b)] - 3125*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*Ar
cSinh[c + d*x])/b)]*Gamma[9/2, (3*(a + b*ArcSinh[c + d*x]))/b] + 81*Sqrt[5]
*E^((10*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (5*(a + b*ArcS
inh[c + d*x]))/b]))/(8100000*d*E^((5*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^
2))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7315 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{asinh}(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*asinh(c + d*x))^(7/2), x)
```

3.199 $\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=547

$$\frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d}-\frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-}}}{2048d}$$

```
[Out] -525/2048*b^2*e^3*(a+b*arcsinh(d*x+c))^(3/2)/d-105/256*b^2*e^3*(d*x+c)^2*(a
+b*arcsinh(d*x+c))^(3/2)/d+35/256*b^2*e^3*(d*x+c)^4*(a+b*arcsinh(d*x+c))^(3
/2)/d-3/32*e^3*(a+b*arcsinh(d*x+c))^(7/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arcsinh(
d*x+c))^(7/2)/d+105/4096*b^(7/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-105/4096*b^(7/2)*e^3*erfi(2^(1/2)*(
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-105/131072
*b^(7/2)*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/
d+105/131072*b^(7/2)*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2
)/d/exp(4*a/b)+21/64*b*e^3*(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)
^(1/2)/d-7/32*b*e^3*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2
)/d+1575/4096*b^3*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2
)/d-105/2048*b^3*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 1.36, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}+\frac{105\sqrt{\pi}\,b^{7/2}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{131072d}-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{105b^3e^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d}+\frac{1575b^3e^3\sqrt{(c+dx)^2+1}\,(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d}+\frac{35b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{256d}-\frac{105b^2e^3(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{256d}-\frac{525b^2e^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{2048d}+\frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{4d}-\frac{7be^3\sqrt{(c+dx)^2+1}\,(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{32d}+\frac{21be^3\sqrt{(c+dx)^2+1}\,(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{64d}-\frac{3e^3\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (1575*b^3*e^3*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])
/(4096*d) - (105*b^3*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/(2048*d) - (525*b^2*e^3*(a + b*ArcSinh[c + d*x])^(3/2))/(2048
*d) - (105*b^2*e^3*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (3
5*b^2*e^3*(c + d*x)^4*(a + b*ArcSinh[c + d*x])^(3/2))/(256*d) + (21*b*e^3*(
c + d*x)*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(64*d) - (7*
b*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(32
*d) - (3*e^3*(a + b*ArcSinh[c + d*x])^(7/2))/(32*d) + (e^3*(c + d*x)^4*(a +
 b*ArcSinh[c + d*x])^(7/2))/(4*d) - (105*b^(7/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*E
rf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(131072*d) + (105*b^(7/2)*e^3
*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(2048*d) + (105*b^(7/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]]
```

```
)/Sqrt[b]])/(131072*d*E^((4*a)/b)) - (105*b^(7/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt
[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2048*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int x^3\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{4d} - \frac{(7be^3)\,\text{Subst}\left(\int \frac{x^4(a+b}{\sqrt{}}\right.}{}$$

$$= -\frac{7be^3(c+dx)^3\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{32d} + \frac{e^3}{}$$

$$= \frac{35b^2e^3(c+dx)^4\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{256d} + \frac{21be^3(c+dx)\sqrt{1}}{}$$

$$= -\frac{105b^3e^3(c+dx)^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{2048d} -$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

$$= \frac{1575b^3e^3(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{4096d} - \frac{10}{}$$

Mathematica [A]
time = 0.23, size = 225, normalized size = 0.41

$$-\frac{be^3e^{-\frac{4a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)+32\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\left(-32\sqrt{2}\,\Gamma\left(\frac{9}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{9}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{8192d\left(-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/8192*(b*e^3*(a + b*ArcSinh[c + d*x])^(5/2)*(-(Sqrt[a/b + ArcSinh[c + d*x
]]*Gamma[9/2, (-4*(a + b*ArcSinh[c + d*x]))/b]) + 32*Sqrt[2]*E^((2*a)/b)*Sq
rt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^
((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*(-32*Sqrt[2]*Gamma[9/2, (2*(a
 + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*Gamma[9/2, (4*(a + b*ArcSinh[c + d
*x]))/b])))/(d*E^((4*a)/b)*(-((a + b*ArcSinh[c + d*x])^2/b^2))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{asinh}(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*asinh(c + d*x))^(7/2), x)
```

3.200 $\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=481

$$\frac{175b^3e^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{54d}-\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{216d}$$

```
[Out] -35/18*b^2*e^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)^
3*(a+b*arcsinh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arcsinh(d*x+c))^(7/2)
/d+35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arcsinh(d
*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^2*exp
(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-105/128*b^(7/2)*e^
2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)+7/9*b*e^2*(a
+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-7/18*b*e^2*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d+175/54*b^3*e^2*(1+(d*x+c)^2)^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/d-35/216*b^3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 1.01, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {5859, 12, 5777, 5812, 5798, 5772, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$-\frac{105\sqrt{\pi}\,b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}\,b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}-\frac{105\sqrt{\pi}\,b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{35\sqrt{\frac{\pi}{3}}\,b^{7/2}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{175b^3e^2\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{54d}-\frac{35b^3e^2(c+dx)^2\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{216d}+\frac{35b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{108d}-\frac{35b^2e^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{18d}+\frac{7be^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{9d}-\frac{7be^2(c+dx)^2\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{18d}+\frac{e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (175*b^3*e^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(54*d) - (
35*b^3*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/
(216*d) - (35*b^2*e^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(18*d) + (3
5*b^2*e^2*(c + d*x)^3*(a + b*ArcSinh[c + d*x])^(3/2))/(108*d) + (7*b*e^2*Sq
rt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*(c + d
*x)^2*Sqrt[1 + (c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(18*d) + (e^2*(
c + d*x)^3*(a + b*ArcSinh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(128*d) + (35*b^(7/2)*
e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(3456*d) - (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]
/Sqrt[b]])/(128*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int x^2\left(a+b\sinh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{(7be^2)\,\text{Subst}\Big(\int \frac{x^3(a+b}{\sqrt{}}$$

$$= -\frac{7be^2(c+dx)^2\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{18d} + \frac{e^2}{}$$

$$= \frac{35b^2e^2(c+dx)^3\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{108d} + \frac{7be^2\sqrt{1+(c+dx}}{}$$

$$= -\frac{35b^3e^2(c+dx)^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{216d} - \frac{3}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

$$= \frac{175b^3e^2\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{54d} - \frac{35b^3e^2(c+}{}$$

Mathematica [A]
time = 0.24, size = 238, normalized size = 0.49

$$\frac{be^2e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}\left(-243e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-243e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{1944d\left(-\frac{(a+b\sinh^{-1}(c+dx))^2}{b^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (b*e^2*(a + b*ArcSinh[c + d*x])^(5/2)*(-243*E^((4*a)/b)*Sqrt[-((a + b*ArcSi
nh[c + d*x])/b)]*Gamma[9/2, a/b + ArcSinh[c + d*x]] + Sqrt[3]*Sqrt[a/b + Ar
cSinh[c + d*x]]*Gamma[9/2, (-3*(a + b*ArcSinh[c + d*x]))/b] - 243*E^((2*a)/
b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (3*(a +
 b*ArcSinh[c + d*x]))/b]))/(1944*d*E^((3*a)/b)*(-((a + b*ArcSinh[c + d*x])^
2/b^2))^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2\,(a + b\,\mathrm{asinh}(c + d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*asinh(c + d*x))^(7/2), x)
```

3.201 $\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=305

$$-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b^2e\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{64d}+\frac{35b^2e(c+dx)^2}{}$$

```
[Out] 35/64*b^2*e*(a+b*arcsinh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arcsinh
(d*x+c))^(3/2)/d+1/4*e*(a+b*arcsinh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*ar
csinh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsin
h(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8*b*e*
(d*x+c)*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/128*b^3*e*(d*x
+c)*(1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.53, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5859, 12, 5777, 5812, 5783, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}b^{7/2}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e(c+dx)\sqrt{(c+dx)^2+1}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d}+\frac{35b^2e\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{64d}-\frac{7be(c+dx)\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{8d}+\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{2d}+\frac{e\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*e*(c + d*x)*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(
128*d) + (35*b^2*e*(a + b*ArcSinh[c + d*x])^(3/2))/(64*d) + (35*b^2*e*(c +
d*x)^2*(a + b*ArcSinh[c + d*x])^(3/2))/(32*d) - (7*b*e*(c + d*x)*Sqrt[1 + (
c + d*x)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(8*d) + (e*(a + b*ArcSinh[c + d
*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcSinh[c + d*x])^(7/2))/(2*d) -
(105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcSinh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\text{Subst}\left(\int x\left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{2d}-\frac{(7be)\text{Subst}\left(\int \frac{x^2(a+b\sinh}{\sqrt{1+}}\right.}{4d}$$

$$=-\frac{7be(c+dx)\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{8d}+\frac{e(c+}{}$$

$$=\frac{35b^2e(c+dx)^2\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{32d}-\frac{7be(c+dx)\sqrt{1+(c}}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

$$=-\frac{105b^3e(c+dx)\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{128d}+\frac{35b}{}$$

Mathematica [A]

time = 0.06, size = 125, normalized size = 0.41

$$\frac{ee^{-\frac{2a}{b}}\left(b^4\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{9}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+b^4e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{9}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{64\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e*(b^4*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[9/2, (-2*(a + b*ArcSinh[c
 + d*x]))/b] + b^4*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[9/2, (2*(
a + b*ArcSinh[c + d*x]))/b]))/(64*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[
c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*asinh(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2), x)
```

3.202 $\int \left(a+b \sinh^{-1}(c+dx)\right)^{7/2} \, dx$

Optimal. Leaf size=216

$$-\frac{105b^3\sqrt{1+(c+dx)^2}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{1+(c+dx}$$

```
[Out] 35/4*b^2*(d*x+c)*(a+b*arcsinh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arcsinh(d*x+c))^
(7/2)/d+105/32*b^(7/2)*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/32*b^(7/2)*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/
exp(a/b)-7/2*b*(a+b*arcsinh(d*x+c))^(5/2)*(1+(d*x+c)^2)^(1/2)/d-105/8*b^3*(
1+(d*x+c)^2)^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\frac{105\sqrt{\pi}\,b^{7/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\,b^{7/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{(c+dx)^2+1}\,\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{(c+dx)^2+1}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*Sqrt[1 + (c + d*x)^2]*Sqrt[a + b*ArcSinh[c + d*x]])/(8*d) + (35*b
^2*(c + d*x)*(a + b*ArcSinh[c + d*x])^(3/2))/(4*d) - (7*b*Sqrt[1 + (c + d*x
)^2]*(a + b*ArcSinh[c + d*x])^(5/2))/(2*d) + ((c + d*x)*(a + b*ArcSinh[c +
d*x])^(7/2))/d + (105*b^(7/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\sinh^{-1}(c+dx)\right)^{7/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}(x)\right)^{7/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}{d}-\frac{(7b)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}{\sqrt{1+x^2}}\,dx,x,\right.}{2d}$$

$$=-\frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}{2d}+\frac{(c+dx)\left(a+b\sinh^{-1}(\right.}{d}$$

$$=\frac{35b^2(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}{4d}-\frac{7b\sqrt{1+(c+dx)^2}\left(a+b\sinh^{-}\right.}{2d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\,\text{si}\right.}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\,\text{si}\right.}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\,\text{si}\right.}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\,\text{si}\right.}{4d}$$

$$=-\frac{105b^3\sqrt{1+(c+dx)^2}\sqrt{a+b\sinh^{-1}(c+dx)}}{8d}+\frac{35b^2(c+dx)\left(a+b\,\text{si}\right.}{4d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 698 vs. 2(216) = 432.

time = 3.12, size = 698, normalized size = 3.23

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2),x]
[Out] ((16*a^3*Sqrt[a + b*ArcSinh[c + d*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcS
inh[c + d*x]])/Sqrt[a/b + ArcSinh[c + d*x]]) + Gamma[3/2, -((a + b*ArcSinh[
```

```
c + d*x])/b)]/Sqrt[-((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b) + 12*a^2*Sqrt[b
]*(4*Sqrt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(-3*Sqrt[1 + (c + d*x)^2] + 2*(c
+ d*x)*ArcSinh[c + d*x]) + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
 d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a + 3*b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + 6*a*Sqrt[b]*(4*Sq
rt[b]*Sqrt[a + b*ArcSinh[c + d*x]]*(2*Sqrt[1 + (c + d*x)^2]*(a - 5*b*ArcSin
h[c + d*x]) + b*(c + d*x)*(15 + 4*ArcSinh[c + d*x]^2)) + (4*a^2 + 12*a*b +
15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + S
inh[a/b]) + (4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d
*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*ArcS
inh[c + d*x]]*(2*b*(c + d*x)*(-10*a + 35*b*ArcSinh[c + d*x] + 4*b*ArcSinh[c
 + d*x]^3) + Sqrt[1 + (c + d*x)^2]*(-4*a^2 + 4*a*b*ArcSinh[c + d*x] - 7*b^2
*(15 + 4*ArcSinh[c + d*x]^2))) + (8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sq
rt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
 (-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(32*d)
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{arcsinh}(dx+c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{asinh}(c + d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^(7/2), x)
```

3.203 $\int \frac{(a+b\sinh^{-1}(c+dx))^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^{7/2}}{c+dx},x\right)}{e}$$

[Out] Unintegrable((a+b*arcsinh(d*x+c))^(7/2)/(d*x+c),x)/e

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{7/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^(7/2)/x, x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sinh^{-1}(c+dx))^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{7/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^{7/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(c+dx))^{7/2}}{ce+dex}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x),x]

[Out] Integrate[(a + b*ArcSinh[c + d*x])^(7/2)/(c*e + d*e*x), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{7}{2}}}{dex + ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*x*e + c*e), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**(7/2)/(d*e*x+c*e),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^{7/2}}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x),x)
[Out] int((a + b*asinh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

3.204 $\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$\frac{e^4e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{e^4e^{\frac{5a}{b}}\sqrt{\frac{\pi}{5}}\,\text{Erf}}{}$$

```
[Out] 1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)+1/16*e^4*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arcsinh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*erf(3^(
1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)-1/32*e^
4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3
*a/b)/b^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d}-\frac{\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{5}}e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b
]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]
])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt
[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) - (e^4*Sqrt[3*Pi
]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((3
*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3\cosh(3x)}{16\sqrt{a+bx}} + \frac{\cosh(5x)}{16\sqrt{a+bx}}\right) dx, x, \sinh^{-1}(c+d}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^4\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x\right.}{8d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{32d} + \frac{e^4\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx, x\right.}{32d}$$

$$= \frac{e^4\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{16bd} + \frac{e^4\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\right.}{}$$

$$= \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d} - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\,\text{si}}}{\sqrt{}}\right.}{32\sqrt{b}\,d}$$

<u>Mathematica</u> [A]
time = 0.26, size = 320, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(-10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)-5\sqrt{3}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+10e^{\frac{4a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+5\sqrt{3}\,e^{\frac{8a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-\sqrt{5}\,e^{\frac{10a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{160d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^4*(-10*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh
[c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcSinh[c + d*x]))/b] - 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c +
```

```
d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqr
t[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]
+ 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*A
rcSinh[c + d*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Ga
mma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Ar
cSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{\sqrt{a + b \operatorname{arcsinh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4 \left(\int \frac{c^4}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx + \int \frac{d^4 x^4}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx + \int \frac{4 c d^3 x^3}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx + \int \frac{6 c^2 d^2 x^2}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx + \int \frac{4 c^3 dx}{\sqrt{a + b \operatorname{asinh}(c + dx)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*asinh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*asinh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(1/2), x)
```

3.205 $\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{e^3e^{-\frac{4a}{b}}\sqrt{\pi}\,\text{Er}}{}$$

```
[Out] 1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/d/b^(1/2)-1/16*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\,\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\,\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{-\frac{2a}{b}}\,\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/32*(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b
]])/(Sqrt[b]*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSi
nh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^3\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x,\right.}{4d}$$

$$= -\frac{e^3\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{16d} + \frac{e^3\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx,\right.}{16d}$$

$$= -\frac{e^3\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x}{b}}\right.}{}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d} + \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\text{s}}}{}\right.}{8\sqrt{b}\,d}$$

Mathematica [A]
time = 0.16, size = 205, normalized size = 0.94

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-2\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(-2\sqrt{2}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{32d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcSinh[c
```

```
+ d*x]]*(-2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b
)*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a +
b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{\sqrt{a + b\,\mathrm{arcsinh}\,(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x*e + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{d^3x^3}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{3cd^2x^2}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx + \int \frac{3c^2dx}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(1/2),x)
[Out] e**3*(Integral(c**3/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*asinh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(1/2), x)
```

3.206 $\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{e^2e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Er}}{}$$

```
[Out] 1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2
))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arcsinh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)-1/8*e^2*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {5859, 12, 5780, 5556, 3388, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/8*(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(Sqrt
[b]*d) + (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*
x]])/Sqrt[b]])/(8*Sqrt[b]*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*
x]]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a
+ b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= -\frac{e^2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{e^2\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx \ldots\right.}{4d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{8d} - \frac{e^2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, \ldots\right.}{8d}$$

$$= \frac{e^2\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{4bd} - \frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}} \ldots\right.}{}$$

$$= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} + \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b \ldots}}{}\right.}{8\sqrt{b}\,d}$$

Mathematica [A]
time = 0.16, size = 217, normalized size = 1.01

$$\frac{e^2e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)-3e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-\sqrt{3}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{24d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]
```

```
*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcSinh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{\sqrt{a+b\operatorname{arcsinh}(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx + \int \frac{d^2x^2}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx + \int \frac{2cdx}{\sqrt{a+b\operatorname{asinh}(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*asinh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*asinh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*asinh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(1/2), x)
```

3.207 $\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$-\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

```
[Out] -1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5859, 12, 5780, 5556, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] -1/4*(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/S
qrt[b]])/(Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d
*x]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{2d}$$

$$= -\frac{e\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{4d} + \frac{e\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x,\right.}{4d}$$

$$= -\frac{e\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{2bd} + \frac{e\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\right.}{}$$

$$= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a}}{}\right.}{4\sqrt{b}}$$

Mathematica [A]
time = 0.04, size = 119, normalized size = 1.05

$$\frac{ee^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{4\sqrt{2}\,d\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (e*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c + d
*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*Ar
```

```
cSinh[c + d*x]))/b]))/(4*Sqrt[2]*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]]
)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{a + b\,\mathrm{arcsinh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\,dx + \int \frac{dx}{\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*asinh(c + d*x)), x) + Integral(d*x/sqrt(a + b*asin
h(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(1/2), x)
```

3.208 $\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/2
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5858, 5774, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
+ (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/b
))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sinh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\sinh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, \ldots}{2bd}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \ldots}{bd}$$

$$= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+\ldots}}{\sqrt{b}}\ldots\right)}{2\sqrt{b}\,d}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]],x]
[Out] Integrate[1/Sqrt[a + b*ArcSinh[c + d*x]], x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
[Out] int(1/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(1/2), x)
```

3.209 $\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\sinh^{-1}(c+dx)}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(1/2),x)/e
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcSinh[x]]), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\sinh^{-1}(x)}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\sinh^{-1}(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcSinh[c + d*x]]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,\sqrt{a+b\,\mathrm{arcsinh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*asinh(c + d*x)) + d*x*sqrt(a + b*asinh(c + d*x))),
 x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arcsinh(d*x + c) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\sqrt{a+b\,\mathrm{asinh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(1/2)), x)
```

3.210 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{e^4e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}$$

```
[Out] -1/8*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/
d+1/8*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a
/b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(3/2)/d-3/16*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-1/16*e^4*exp(5*a/b)*erf(5^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16*e^4*e
rfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d/
exp(5*a/b)-2*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.44, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5859, 12, 5778, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{\sqrt{5\pi}e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}-\frac{3\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}e^4e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}-\frac{2e^4(c+dx)^4\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^
(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
+ d*x]])/Sqrt[b]])/(16*b^(3/2)*d) - (e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/b)) - (3*e^4*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*
d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e^4)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} - \frac{9\sinh(3x)}{16\sqrt{a+b}\right.\right.}{}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^4\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{4bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{8bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(\right.}{4b^2d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{8b^{3/2}d}$$

Mathematica [A]

time = 0.45, size = 490, normalized size = 1.34

$$\frac{e^4e^{-5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-e^{\frac{5a}{b}}+3e^{\frac{5a}{b}+2\sinh^{-1}(c+dx)}-2e^{\frac{5a}{b}+4\sinh^{-1}(c+dx)}-2e^{\frac{5a}{b}+6\sinh^{-1}(c+dx)}+3e^{\frac{5a}{b}+8\sinh^{-1}(c+dx)}-e^{\frac{5a}{b}+10\sinh^{-1}(c+dx)}+2e^{\frac{6a}{b}+5\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{5}e^{5\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)-3\sqrt{3}e^{\frac{2a}{b}+5\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+2e^{\frac{4a}{b}+5\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-3\sqrt{3}e^{\frac{8a}{b}+5\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+\sqrt{5}e^{5\left(\frac{2a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)}{16bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-E^((5*a)/b) + 3*E^((5*a)/b + 2*ArcSinh[c + d*x]) - 2*E^((5*a)/b + 4*
ArcSinh[c + d*x]) - 2*E^((5*a)/b + 6*ArcSinh[c + d*x]) + 3*E^((5*a)/b + 8*A
rcSinh[c + d*x]) - E^((5*a)/b + 10*ArcSinh[c + d*x]) + 2*E^((6*a)/b + 5*Arc
Sinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*
x]] + Sqrt[5]*E^(5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Ga
```

```
mma[1/2, (-5*(a + b*ArcSinh[c + d*x]))/b] - 3*Sqrt[3]*E^((2*a)/b + 5*ArcSin
h[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSi
nh[c + d*x]))/b] + 2*E^((4*a)/b + 5*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh
[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - 3*Sqrt[3]*E^((8*
a)/b + 5*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (3*(a +
b*ArcSinh[c + d*x]))/b] + Sqrt[5]*E^(5*((2*a)/b + ArcSinh[c + d*x]))*Sqrt[a
/b + ArcSinh[c + d*x]]*Gamma[1/2, (5*(a + b*ArcSinh[c + d*x]))/b]))/(16*b*d
*E^(5*(a/b + ArcSinh[c + d*x]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{d^4x^4}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{4cd^3x^3}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{6c^2d^2x^2}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx+\int \frac{4c^3dx}{a\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt
(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(3/2), x)
```

3.211 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}$$

```
[Out] -1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)
*Pi^(1/2)/b^(3/2)/d-1/4*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arcsinh(d*
x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5859, 12, 5778, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}-\frac{2e^3(c+dx)^3\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]
])/(4*b^(3/2)*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcS
inh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*A
rcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) - (e^3*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d*E^((2*a)/b)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e^3)\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(}{2\sqrt{a-}}\right.\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^3\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{2bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\sinh}\right.}{b^2d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right.}{4b^{3/2}d}$$

Mathematica [A]
time = 0.30, size = 253, normalized size = 0.97

$$\frac{e^3e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)-e^{\frac{4a}{b}}\left(-\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)-2\sinh\left(2\sinh^{-1}(c+dx)\right)+\sinh\left(4\sinh^{-1}(c+dx)\right)\right)\right)}{4bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcSinh[c +
 d*x]))/b] - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] - E^((4*a)/b)*(-(Sqrt[2]*E^((2*a)/b)*
Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcSinh[c + d
```

```
*x]))/b] - 2*Sinh[2*ArcSinh[c + d*x]] + Sinh[4*ArcSinh[c + d*x]])))/(4*b*d*
E^((4*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3} \left(\int \frac{c^{3}}{a \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{d^{3} x^{3}}{a \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{3 c d^{2} x^{2}}{a \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}}\, dx + \int \frac{3 c^{2} d x}{a \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} + b \sqrt{a + b \operatorname{asinh}{\left(c + dx \right)}} \operatorname{asinh}{\left(c + dx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*asinh(c + d*x)
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(3*c*d**2*x*
```

```
*2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b
*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{asinh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(3/2), x)
```

3.212 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$-\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{e^2e^{a/b}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{e^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}$$

```
[Out] 1/4*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
-1/4*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)-1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/
2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/
2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5859, 12, 5778, 3389, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}-\frac{2e^2(c+dx)^2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(4*b^
(3/2)*d) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c +
d*x]])/Sqrt[b]])/(4*b^(3/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c +
d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[
a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e^2)\,\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4\sqrt{a+bx}} + \frac{3\sinh(3}{4\sqrt{a+}}\right.\right.}{bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^2\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{2bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{4bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(}\right.}{2b^2d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}$$

Mathematica [A]
time = 0.23, size = 327, normalized size = 1.28

$$\frac{e^2e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-e^{\frac{3a}{b}}+e^{\frac{3a}{b}+2\sinh^{-1}(c+dx)}+e^{\frac{3a}{b}+4\sinh^{-1}(c+dx)}-e^{\frac{3a}{b}+6\sinh^{-1}(c+dx)}-e^{\frac{4a}{b}+3\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)+\sqrt{3}\,e^{3\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)-e^{\frac{2a}{b}+3\sinh^{-1}(c+dx)}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+\sqrt{3}\,e^{\frac{6a}{b}+3\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)}{4bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c + d*x]) + E^((3*a)/b + 4*ArcS
inh[c + d*x]) - E^((3*a)/b + 6*ArcSinh[c + d*x]) - E^((4*a)/b + 3*ArcSinh[c
 + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1/2, a/b + ArcSinh[c + d*x]] +
Sqrt[3]*E^(3*ArcSinh[c + d*x])*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/
2, (-3*(a + b*ArcSinh[c + d*x]))/b] - E^((2*a)/b + 3*ArcSinh[c + d*x])*Sqrt
```

```
[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] +
 Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*Gamm
a[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/(4*b*d*E^(3*(a/b + ArcSinh[c + d*x
]))*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^{2}}{(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a\sqrt{a + b\,\mathrm{asinh}\,(c + dx)} + b\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c + dx)}\,dx + \int \frac{d^{2}x^{2}}{a\sqrt{a + b\,\mathrm{asinh}\,(c + dx)} + b\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c + dx)}\,dx + \int \frac{2cdx}{a\sqrt{a + b\,\mathrm{asinh}\,(c + dx)} + b\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*asinh(c + d*x)
```

```
) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(3/2), x)
```

3.213 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+}}{}\right.}{b^{3/2}d}$$

```
[Out] 1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(a+b
*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 12, 5778, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) +
 (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[
b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]]
)/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+\right.}{bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{e\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+d\right.}{bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{(2e)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\sinh\right.}{b^2d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+d}}{\sqrt{b}}\right.}{b^{3/2}d}$$

Mathematica [A]
time = 0.08, size = 147, normalized size = 0.99

$$\frac{ee^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\sinh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)-\sqrt{2}\,e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)-2e^{\frac{2a}{b}}\sinh\left(2\sinh^{-1}(c+dx)\right)\right)}{2bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(3/2),x]
```

```
[Out] (e*(Sqrt[2]*Sqrt[-((a + b*ArcSinh[c + d*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSi
nh[c + d*x]))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*Gamma[1
/2, (2*(a + b*ArcSinh[c + d*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c + d*x]
]))/(2*b*d*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c + d*x]])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}+b\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)}\,dx+\int \frac{dx}{a\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}+b\sqrt{a+b\operatorname{asinh}\left(c+dx\right)}\operatorname{asinh}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a + b*asinh(c + d*x))*
asinh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*asinh(c + d*x)) + b*sqrt(a
 + b*asinh(c + d*x))*asinh(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(3/2), x)
```

3.214 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((
a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(1+(d*x+c)
^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5858, 5773, 5819, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}\,e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{(c+dx)^2+1}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(b*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (E^(a/b)*Sq
rt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d) + (Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\sinh^{-1}(c+dx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\sinh^{-1}(x))^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+b\sinh^{-1}(x)}}\,dx,\right.}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \sinh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^{-1}(c+\right.}{b^2d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{bd\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} +$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\sinh^{-1}(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(-3/2), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\operatorname{asinh}(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(3/2), x)
```

3.215 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(3/2),x)/e
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(3/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(3/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+adx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+bc\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+bdx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/(a*c*sqrt(a + b*asinh(c + d*x)) + a*d*x*sqrt(a + b*asinh(c + d*x
)) + b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b*d*x*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(3/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(3/2)), x)
```

3.216 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=437

$$-\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{e^4e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\,(\cdots}{\cdots}$$

```
[Out] 1/12*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d+1/12*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(
a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(
1/2)*Pi^(1/2)/b^(5/2)/d-3/8*e^4*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)+5/24*e^4*exp(5*a/b)*erf(5^(1/2)
*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*e^4*er
fi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d/e
xp(5*a/b)-2/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3
/2)-16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-20/3*e^4*(d*x+c)^5/
b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3388, 2211, 2236, 2235}

$$\frac{\sqrt{\pi}e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}+\frac{5\sqrt{5\pi}e^4e^{-\frac{5a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}-\frac{20e^4(c+dx)^5}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (20*
e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^4*E^(a/b)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d) - (3*e^4*E^(
(3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8
*b^(5/2)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcSinh
[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSin
h[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) - (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt
[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^((3*a)/b)) + (5*
e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(24*b^
(5/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(8e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{3/}}\right.}{3bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

Mathematica [A]

time = 2.63, size = 551, normalized size = 1.26

$$\frac{e^4\left(-2e^{\sinh^{-1}(c+dx)}\left(2a+b+2b\sinh^{-1}(c+dx)\right)+e^{-\sinh^{-1}(c+dx)}\left(4a-2b+4b\sinh^{-1}(c+dx)-4e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)+e^{-\frac{5a}{b}}\left(-e^{5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(10a+b+10b\sinh^{-1}(c+dx)\right)-10\sqrt{5}\,b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)+e^{-\frac{3a}{b}}\left(3e^{3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(6a+b+6b\sinh^{-1}(c+dx)\right)+18\sqrt{3}\,b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)-4be^{-\frac{a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)+3e^{-3\sinh^{-1}(c+dx)}\left(-6a+b-6b\sinh^{-1}(c+dx)+6\sqrt{3}\,e^{3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)+e^{-5\sinh^{-1}(c+dx)}\left(10a-b+10b\sinh^{-1}(c+dx)-10\sqrt{5}\,e^{5\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{5(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{48b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-2*E^ArcSinh[c + d*x]*(2*a + b + 2*b*ArcSinh[c + d*x]) + (4*a - 2*b +
 4*b*ArcSinh[c + d*x] - 4*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c +
 d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSi
nh[c + d*x] + (-(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcSinh[c
+ d*x])) - 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
5*(a + b*ArcSinh[c + d*x]))/b])/E^((5*a)/b) + (3*E^(3*(a/b + ArcSinh[c + d*
x]))*(6*a + b + 6*b*ArcSinh[c + d*x]) + 18*Sqrt[3]*b*(-((a + b*ArcSinh[c +
d*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b])/E^((3*a)/b) -
(4*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c +
d*x])/b)])/E^(a/b) + (3*(-6*a + b - 6*b*ArcSinh[c + d*x] + 6*Sqrt[3]*E^(3*(
a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*
x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b]))/E^(3*ArcSinh[c + d*x]) + (
10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x])
)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a +
b*ArcSinh[c + d*x]))/b])/E^(5*ArcSinh[c + d*x])))/(48*b^2*d*(a + b*ArcSinh[
c + d*x])^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{d^{4}x^{4}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{4cd^{3}x^{3}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{6c^{2}d^{2}x^{2}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{4c^{3}dx}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b
**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(5/2), x)
```

3.217 $\int \frac{(ce+dex)^3}{(a+b\sinh^{-1}(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{Erf}\left(\right.}{}$$

```
[Out] -2/3*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5
/2)/d+2/3*e^3*erfi(2*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
/exp(4*a/b)+1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d-1/3*e^3*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(1+(d*
x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4*e^3*(d*x+c)^2/b^2/d/(a+b*arc
sinh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3389, 2211, 2236, 2235}

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\mathrm{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\mathrm{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{-\frac{2a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{16e^3(c+dx)^4}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2e^3\sqrt{(c+dx)^2+1}\,(c+dx)^3}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (16*e^3
*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e^3*E^((4*a)/b)*S
qrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (e^3
*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]]
)/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(3*b^(5/2)*d*E^((4*a)/b)) - (e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b
*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(2e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{3/}}\right.}{bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{a}}$$

Mathematica [A]

time = 1.22, size = 390, normalized size = 1.20

$$\frac{e^3e^{-4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-8be^{4\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)+4\sqrt{2}\,be^{\frac{2a}{b}+4\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+\frac{1}{2}e^{\frac{4a}{b}}\left(-\left(-1+e^{2\sinh^{-1}(c+dx)}\right)^2\left(b\left(-1+e^{4\sinh^{-1}(c+dx)}\right)+8a\left(1+e^{2\sinh^{-1}(c+dx)}+e^{4\sinh^{-1}(c+dx)}\right)+8b\left(1+e^{2\sinh^{-1}(c+dx)}+e^{4\sinh^{-1}(c+dx)}\right)\sinh^{-1}(c+dx)\right)-8\sqrt{2}\,e^{\frac{2a}{b}+4\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+16e^{4\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{12b^2d\,(a+b\sinh^{-1}(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-8*b*E^(4*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gam
ma[1/2, (-4*(a + b*ArcSinh[c + d*x]))/b] + 4*Sqrt[2]*b*E^((2*a)/b + 4*ArcSi
nh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*Ar
cSinh[c + d*x]))/b] + (E^((4*a)/b)*(-((-1 + E^(2*ArcSinh[c + d*x]))^2*(b*(-
1 + E^(4*ArcSinh[c + d*x])) + 8*a*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSin
h[c + d*x])) + 8*b*(1 + E^(2*ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x]))*Ar
cSinh[c + d*x])) - 8*Sqrt[2]*E^((2*a)/b + 4*ArcSinh[c + d*x])*Sqrt[a/b + Ar
cSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d
*x]))/b] + 16*E^(4*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(
a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]))/2))/(1
2*b^2*d*E^(4*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{d^{3}x^{3}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{3cd^{2}x^{2}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{3c^{2}dx}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*
x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(5/2), x)
```

3.218 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{e^2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\Big(}{}$$

```
[Out] -1/6*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d-1/6*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a
/b)+1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(1+(d*x+c)^2)^
(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-8/3*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x
+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3388, 2211, 2236, 2235, 5774}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}\,e^2e^{-\frac{3a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}-\frac{4e^2(c+dx)^3}{b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2e^2\sqrt{(c+dx)^2+1}\,(c+dx)^2}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^
(3/2)) - (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*e^2*
(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) + (e^2*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*
d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d
*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqr
t[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
```

```
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(4e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{3/}}\right.}{3bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{b^2d\sqrt{a}}$$

Mathematica [A]
time = 1.05, size = 389, normalized size = 1.21

$$\frac{e^2 e^{-3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(2e^{\frac{4a}{b}+3\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)-6\sqrt{3}\,be^{3\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)+2be^{\frac{2a}{b}+3\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)-e^{\frac{3a}{b}}\left(\left(-1+e^{2\sinh^{-1}(c+dx)}\right)\left(b\left(-1+e^{4\sinh^{-1}(c+dx)}\right)+a\left(6+4e^{2\sinh^{-1}(c+dx)}+6e^{4\sinh^{-1}(c+dx)}\right)+2b\left(3+2e^{2\sinh^{-1}(c+dx)}+3e^{4\sinh^{-1}(c+dx)}\right)\sinh^{-1}(c+dx)\right)+6\sqrt{3}\,e^{3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{3(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{12b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a +
b*ArcSinh[c + d*x])*Gamma[1/2, a/b + ArcSinh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcSinh[c + d*x]))/b] + 2*b*E^((2*a)/b + 3*ArcSinh[c + d*x])*(-((a + b*Ar
cSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)] - E^((3
*a)/b)*((-1 + E^(2*ArcSinh[c + d*x]))*(b*(-1 + E^(4*ArcSinh[c + d*x])) + a*
(6 + 4*E^(2*ArcSinh[c + d*x]) + 6*E^(4*ArcSinh[c + d*x])) + 2*b*(3 + 2*E^(2
*ArcSinh[c + d*x]) + 3*E^(4*ArcSinh[c + d*x]))*ArcSinh[c + d*x]) + 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSin
h[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/(12*b^2*d*E^(3*(a
/b + ArcSinh[c + d*x]))*(a + b*ArcSinh[c + d*x])^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{d^{2}x^{2}}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{2cdx}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asi
nh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqr
t(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*as
inh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*asinh(c + d*x)) +
2*a*b*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x)**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\operatorname{asinh}(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(5/2), x)
```

3.219 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2ee^{\frac{2a}{b}}\sqrt{2\pi}\,\mathrm{Erf}$$

```
[Out] -2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b/d/(
a+b*arcsinh(d*x+c))^(3/2)-4/3*e/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)-8/3*e*(d*x
+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5859, 12, 5779, 5818, 5780, 5556, 3389, 2211, 2236, 2235, 5783}

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2
)) - (4*e)/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (8*e*(c + d*x)^2)/(3*b^
2*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2
]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e*Sqrt[2*Pi]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/
b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{3/2}}\right.}{3bd}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4e}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

Mathematica [A]

time = 0.47, size = 227, normalized size = 1.09

$$\frac{ee^{-2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(-4\sqrt{2}\,be^{2\sinh^{-1}(c+dx)}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+e^{\frac{2a}{b}}\left(-4a+b-4ae^{4\sinh^{-1}(c+dx)}-be^{4\sinh^{-1}(c+dx)}-4b\left(1+e^{4\sinh^{-1}(c+dx)}\right)\sinh^{-1}(c+dx)+4\sqrt{2}\,e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,(a+b\sinh^{-1}(c+dx))\,\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)\right)}{6b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(5/2),x]
```

```
[Out] (e*(-4*Sqrt[2]*b*E^(2*ArcSinh[c + d*x])*(-((a + b*ArcSinh[c + d*x])/b))^(3/
2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*
a*E^(4*ArcSinh[c + d*x]) - b*E^(4*ArcSinh[c + d*x]) - 4*b*(1 + E^(4*ArcSinh
[c + d*x]))*ArcSinh[c + d*x] + 4*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*Arc
Sinh[c + d*x]))/b])))/(6*b^2*d*E^(2*(a/b + ArcSinh[c + d*x]))*(a + b*ArcSin
h[c + d*x])^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{\left(a+b\operatorname{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx+\int \frac{dx}{a^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}+2ab\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+b^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asinh(c +
 d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*asinh(c + d*x)) + 2*a*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(5/2), x)
```

3.220 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}+\frac{2e^{a/b}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2e}{}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3
*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(
1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*ar
csinh(d*x+c))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5773, 5818, 5774, 3388, 2211, 2236, 2235}

$$\frac{2\sqrt{\pi}\,e^{a/b}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}\,e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{2\sqrt{(c+dx)^2+1}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(3*b*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*(c +
 d*x))/(3*b^2*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcSinh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+b\sinh^{-1}(x))^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\,(a+b\sinh^{-1}(x))^{3/2}}\,dx, x \ldots\right.}{3bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{4\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{3bd\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\sinh^{-1}(c+dx)}} + \frac{2e^{a/b}\sqrt{\pi}\ldots}{\ldots}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^(-5/2), x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{arcsinh}\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b \operatorname{asinh}(c + d x))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c + d*x))^(5/2), x)
```

3.221 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(5/2),x)/e

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(5/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(5/2)), x]

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b\,\mathrm{arcsinh}\,(dx + c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+a^2dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}+2abc\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+2abdx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}\,(c+dx)+b^2c\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2\,(c+dx)+b^2dx\sqrt{a+b\,\mathrm{asinh}\,(c+dx)}\,\mathrm{asinh}^2\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(a**2*c*sqrt(a + b*asinh(c + d*x)) + a**2*d*x*sqrt(a + b*asinh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*asinh(c + d*x))*asinh(c + d*x) + b**2*c*sqrt(a + b*asinh(c + d*x))
*asinh(c + d*x)**2 + b**2*d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2)
, x)/e
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(5/2)), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(5/2)), x)
```

3.222 $\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^4(c+dx)^5}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{32e^4(c+}{5b^3d\sqrt{a}}$$

```
[Out] -16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)-1/30*e^4*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1
/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arc
sinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d-9/20*e^4*erfi(3^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)
-5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*5^(1/2
)*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(
1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/5*e^4*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/
b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*(1+(d*x+c)^2)^(1/2)/b^3
/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.98, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5779, 5818, 5778, 3389, 2211, 2236, 2235}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}e^4e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}-\frac{5\sqrt{5\pi}e^4e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\frac{\sqrt{\pi}e^4e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}-\frac{9\sqrt{3\pi}e^4e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}e^4e^{-\frac{5a}{b}}\text{Erfi}\left(\frac{\sqrt{5}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}-\frac{40e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{3b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{32e^4\sqrt{(c+dx)^2+1}(c+dx)^2}{5b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^4(c+dx)^5}{3b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^4\sqrt{(c+dx)^2+1}(c+dx)^4}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (
4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e^4*(c +
d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (40*
e^4*(c + d*x)^4*Sqrt[1 + (c + d*x)^2])/(3*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) - (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b
^(7/2)*d) + (9*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c
 + d*x]])/Sqrt[b]])/(20*b^(7/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqr
t[5]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) - (9*e^
4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(20*b^(7
```

```
/2)*d*E^((3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcSinh[c +
 d*x]])/Sqrt[b]])/(12*b^(7/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
```

```
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\text{Subst}\left(\int \frac{x^4}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(8e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{5/}}\right.}{5bd}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

$$= -\frac{2e^4(c+dx)^4\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{3b^2}$$

Mathematica [A]
time = 1.82, size = 701, normalized size = 1.32

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-6*b^2*E^ArcSinh[c + d*x] - 3*b^2*E^(5*ArcSinh[c + d*x]) + (-8*a^2 +
4*a*b - 6*b^2 - 4*(4*a - b)*b*ArcSinh[c + d*x] - 8*b^2*ArcSinh[c + d*x]^2 +
 8*E^(a/b + ArcSinh[c + d*x])*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c
 + d*x])^2*Gamma[1/2, a/b + ArcSinh[c + d*x]])/E^ArcSinh[c + d*x] - (10*(a
+ b*ArcSinh[c + d*x])*(E^(5*(a/b + ArcSinh[c + d*x]))*(10*a + b + 10*b*ArcS
inh[c + d*x]) + 10*Sqrt[5]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-5*(a + b*ArcSinh[c + d*x]))/b]))/E^((5*a)/b) + 9*(b^2*E^(3*ArcSinh[c +
 d*x]) + (2*(a + b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a +
 b + 6*b*ArcSinh[c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3
/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) - (4*(a + b*
ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*
x]) + 2*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh
[c + d*x])/b)]))/E^(a/b) + (9*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b +
6*b*ArcSinh[c + d*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b +
ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c +
 d*x]))/b])))/E^(3*ArcSinh[c + d*x]) - (3*b^2 + 10*(a + b*ArcSinh[c + d*x])
*(10*a - b + 10*b*ArcSinh[c + d*x] - 10*Sqrt[5]*E^(5*(a/b + ArcSinh[c + d*x
]))*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (5*(a
+ b*ArcSinh[c + d*x]))/b]))/E^(5*ArcSinh[c + d*x])))/(240*b^3*d*(a + b*ArcS
inh[c + d*x])^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{4}\left(\int \frac{c^{4}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{d^{4}x^{4}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{4cd^{3}x^{3}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{6c^{2}d^{2}x^{2}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{4c^{3}dx}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**4*x**4/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
4*c*d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(6
*c**2*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asin
h(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c +
d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral
(4*c**3*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c
+ d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)
**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*asinh(c + d*x))^(7/2), x)
```

3.223 $\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=420

$$-\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^3(c+dx)^4}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^3(c+}{5b^3d\sqrt{a}}$$

```
[Out] -4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^2
/d/(a+b*arcsinh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arcsinh(d*x+c
))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)-4/15*e^3*exp(2*a/b)*erf(2^(1/2
)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d-4/15*e^3*e
rfi(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/
exp(2*a/b)-2/5*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(
5/2)-16/5*e^3*(d*x+c)*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-
128/15*e^3*(d*x+c)^3*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5859, 12, 5779, 5818, 5778, 3388, 2211, 2236, 2235}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4\sqrt{2\pi}e^3e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{128e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{16e^3\sqrt{(c+dx)^2+1}(c+dx)}{5b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{16e^3(c+dx)^4}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^3\sqrt{(c+dx)^2+1}(c+dx)^3}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16
*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^3*(c +
d*x)*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (128*e
^3*(c + d*x)^3*Sqrt[1 + (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]
]) + (16*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(15*b^(7/2)*d) - (4*e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sq
rt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) - (4*e^3*S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)
*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\text{Subst}\left(\int \frac{x^3}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(6e^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{5/}}\right.}{5bd}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

$$= -\frac{2e^3(c+dx)^3\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^2}$$

<u>Mathematica</u> [A]
time = 1.43, size = 429, normalized size = 1.02

$$\frac{e^3\left(4\left(a+b\sinh^{-1}(c+dx)\right)\left(-4ae^{-2\sinh^{-1}(c+dx)}+be^{-2\sinh^{-1}(c+dx)}-4be^{-2\sinh^{-1}(c+dx)}\sinh^{-1}(c+dx)+e^{2\sinh^{-1}(c+dx)}\left(4a+b+4b\sinh^{-1}(c+dx)\right)+4\sqrt{2}\,be^{-\frac{2a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)+4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{2(a+b\sinh^{-1}(c+dx))}{b}\right)\right)-4\left(a+b\sinh^{-1}(c+dx)\right)\left(-8ae^{-4\sinh^{-1}(c+dx)}+be^{-4\sinh^{-1}(c+dx)}\left(1-8\sinh^{-1}(c+dx)\right)+e^{4\sinh^{-1}(c+dx)}\left(8a+b+8b\sinh^{-1}(c+dx)\right)+16be^{-\frac{4a}{b}}\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)+16e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{4(a+b\sinh^{-1}(c+dx))}{b}\right)\right)+6b^2\sinh\left(2\sinh^{-1}(c+dx)\right)-3b^2\sinh\left(4\sinh^{-1}(c+dx)\right)\right)}{60b^3d\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*(4*(a + b*ArcSinh[c + d*x])*((-4*a)/E^(2*ArcSinh[c + d*x]) + b/E^(2*Ar
cSinh[c + d*x]) - (4*b*ArcSinh[c + d*x])/E^(2*ArcSinh[c + d*x]) + E^(2*ArcS
inh[c + d*x])*(4*a + b + 4*b*ArcSinh[c + d*x]) + (4*Sqrt[2]*b*(-((a + b*Arc
Sinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2
*a)/b) + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[
c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[c + d*x]))/b]) - 4*(a + b*ArcSinh[c
+ d*x])*((-8*a)/E^(4*ArcSinh[c + d*x]) + (b*(1 - 8*ArcSinh[c + d*x]))/E^(4*
ArcSinh[c + d*x]) + E^(4*ArcSinh[c + d*x])*(8*a + b + 8*b*ArcSinh[c + d*x])
 + (16*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcSin
h[c + d*x]))/b])/E^((4*a)/b) + 16*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c + d*x]]*
(a + b*ArcSinh[c + d*x])*Gamma[1/2, (4*(a + b*ArcSinh[c + d*x]))/b]) + 6*b^
2*Sinh[2*ArcSinh[c + d*x]] - 3*b^2*Sinh[4*ArcSinh[c + d*x]]))/(60*b^3*d*(a
+ b*ArcSinh[c + d*x])^(5/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{3}\left(\int \frac{c^{3}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{d^{3}x^{3}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{3cd^{2}x^{2}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{3c^{2}dx}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**3*x**3/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
3*c*d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(3
*c**2*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c +
d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**
2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\operatorname{asinh}(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*asinh(c + d*x))^(7/2), x)
```

3.224 $\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=410

$$-\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e^2(c+dx)^3}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{16e^2}{15b^3d\sqrt{a}}$$

```
[Out] -8/15*e^2*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d/
(a+b*arcsinh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arcsinh(d*x+c))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(7/2)/d-1/15*e^2*erfi((a+b*arcsinh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh
(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2)*(a+
b*arcsinh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)-2/5*
e^2*(d*x+c)^2*(1+(d*x+c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-16/15*e^2*
(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)-24/5*e^2*(d*x+c)^2*(1+
(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5859, 12, 5779, 5818, 5778, 3389, 2211, 2236, 2235, 5773, 5819}

$$\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{3\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}e^2e^{-\frac{3a}{b}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}-\frac{24e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{5b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{16e^2\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^2(c+dx)^3}{5b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2e^2\sqrt{(c+dx)^2+1}(c+dx)^2}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^2*(c + d*x)^2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^
(5/2)) - (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (4*e
^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (16*e^2*Sqrt[1 +
 (c + d*x)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (24*e^2*(c + d*x)^
2*Sqrt[1 + (c + d*x)^2])/(5*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (e^2*E^(a
/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) - (3
*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt
[b]])/(5*b^(7/2)*d) - (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c + d*x]]/Sqrt[
b]])/(15*b^(7/2)*d*E^(a/b)) + (3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*Ar
cSinh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
```

```
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{x^2}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(4e^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\,\left(a+b\sinh^{-1}(x)\right)^{5/\ldots}} \ldots\right)}{5bd}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

$$= -\frac{2e^2(c+dx)^2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{8e^2(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{\ldots}{5b^2\ldots}$$

Mathematica [A]
time = 1.02, size = 474, normalized size = 1.16

$$\frac{e^2\left(3b^2e^{\sinh^{-1}(c+dx)}+e^{-\sinh^{-1}(c+dx)}\left(4a^2-2ab+3b^2+2(4a-b)b\sinh^{-1}(c+dx)+4b^2\sinh^{-1}(c+dx)^2-4e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right)\right)-3\left(b^2e^{3\sinh^{-1}(c+dx)}+2e^{-\frac{3a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(6a+b+6b\sinh^{-1}(c+dx)\right)+6\sqrt{3}\,b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)\right)+2e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(2a+b+2b\sinh^{-1}(c+dx)\right)+2b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)-3e^{-3\sinh^{-1}(c+dx)}\left(b^2+2\left(a+b\sinh^{-1}(c+dx)\right)\left(6a-b+6b\sinh^{-1}(c+dx)-6\sqrt{3}\,e^{3\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{3\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)\right)\right)}{60b^3d\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(3*b^2*E^ArcSinh[c + d*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*Arc
Sinh[c + d*x] + 4*b^2*ArcSinh[c + d*x]^2 - 4*E^(a/b + ArcSinh[c + d*x])*Sqr
t[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2, a/b + ArcSi
nh[c + d*x]])/E^ArcSinh[c + d*x] - 3*(b^2*E^(3*ArcSinh[c + d*x]) + (2*(a +
b*ArcSinh[c + d*x])*(E^(3*(a/b + ArcSinh[c + d*x]))*(6*a + b + 6*b*ArcSinh[
c + d*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, (-
3*(a + b*ArcSinh[c + d*x]))/b]))/E^((3*a)/b)) + (2*(a + b*ArcSinh[c + d*x])
*(E^(a/b + ArcSinh[c + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a +
 b*ArcSinh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/
E^(a/b) - (3*(b^2 + 2*(a + b*ArcSinh[c + d*x])*(6*a - b + 6*b*ArcSinh[c + d
*x] - 6*Sqrt[3]*E^(3*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b + ArcSinh[c + d*x]]
*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (3*(a + b*ArcSinh[c + d*x]))/b])))/E^(
3*ArcSinh[c + d*x])))/(60*b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arcsinh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e^{2}\left(\int \frac{c^{2}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{d^{2}x^{2}}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx+\int \frac{2cdx}{a^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}+3a^{2}b\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}(c+dx)+3ab^{2}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{2}(c+dx)+b^{3}\sqrt{a+b\operatorname{asinh}(c+dx)}\operatorname{asinh}^{3}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*
asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(
c + d*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Inte
gral(d**2*x**2/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d
*x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(
2*c*d*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*
x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2
+ b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*asinh(c + d*x))^(7/2), x)
```

3.225 $\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{32e(c+}{15b^3d\sqrt{}}$$

```
[Out] -4/15*e/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arcsin
h(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arcsinh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(1+(d*x+
c)^2)^(1/2)/b/d/(a+b*arcsinh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(1+(d*x+c)^2)^(1
/2)/b^3/d/(a+b*arcsinh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$,
Rules used = {5859, 12, 5779, 5818, 5778, 3388, 2211, 2236, 2235, 5783}

$$\frac{8\sqrt{2\pi}\,e e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\,e e^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{32e\sqrt{(c+dx)^2+1}\,(c+dx)}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2e\sqrt{(c+dx)^2+1}\,(c+dx)}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*(c + d*x)*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2
)) - (4*e)/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*e*(c + d*x)^2)/(1
5*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (32*e*(c + d*x)*Sqrt[1 + (c + d*x
)^2])/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) + (8*e*E^((2*a)/b)*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (8*e
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c + d*x]])/Sqrt[b]])/(15*b^(7/
2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ce+dex}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\text{Subst}\left(\int \frac{x}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{(2e)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\right.}{5bd} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b} \\
&= -\frac{2e(c+dx)\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4e}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b}
\end{aligned}
$$

<u>Mathematica</u> **[A]**
time = 0.68, size = 235, normalized size = 0.93

$$
-\frac{e\left(\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\left(4a+b+4b\sinh^{-1}(c+dx)\right)+8\sqrt{2}\,b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)+e^{-2\sinh^{-1}(c+dx)}\left(-8a+2b-8b\sinh^{-1}(c+dx)+8\sqrt{2}\,e^{2\left(\frac{a}{b}+\sinh^{-1}(c+dx)\right)}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{b}\right)\right)\right)+3b^2\sinh\left(2\sinh^{-1}(c+dx)\right)\right)}{15b^3d\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcSinh[c + d*x])^(7/2),x]
```

```
[Out] -1/15*(e*((a + b*ArcSinh[c + d*x])*((2*E^(2*(a/b + ArcSinh[c + d*x]))*(4*a
+ b + 4*b*ArcSinh[c + d*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c + d*x])/b))^(
```

```
3/2)*Gamma[1/2, (-2*(a + b*ArcSinh[c + d*x]))/b])/E^((2*a)/b) + (-8*a + 2*b
 - 8*b*ArcSinh[c + d*x] + 8*Sqrt[2]*E^(2*(a/b + ArcSinh[c + d*x]))*Sqrt[a/b
 + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])*Gamma[1/2, (2*(a + b*ArcSinh[
c + d*x]))/b])/E^(2*ArcSinh[c + d*x])) + 3*b^2*Sinh[2*ArcSinh[c + d*x]]))/(
b^3*d*(a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\left(a + b \operatorname{arcsinh}\left(dx + c\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x*e + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^3\sqrt{a + b\operatorname{asinh}(c + dx)} + 3a^2b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx) + 3ab^2\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^2(c + dx) + b^3\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^3(c + dx)}\, dx + \int \frac{dx}{a^3\sqrt{a + b\operatorname{asinh}(c + dx)} + 3a^2b\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}(c + dx) + 3ab^2\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^2(c + dx) + b^3\sqrt{a + b\operatorname{asinh}(c + dx)}\operatorname{asinh}^3(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] e*(Integral(c/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(
c + d*x))*asinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*
x)**2 + b**3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x) + Integral(d
*x/(a**3*sqrt(a + b*asinh(c + d*x)) + 3*a**2*b*sqrt(a + b*asinh(c + d*x))*a
sinh(c + d*x) + 3*a*b**2*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**2 + b**
3*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arcsinh(d*x + c) + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*asinh(c + d*x))^(7/2), x)
```

3.226 $\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{1+(c+dx)^2}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4e^{a/b}\sqrt{\pi}\,\mathrm{E}}{}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arcsinh(d*x+c))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsi
nh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arcsinh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(1+(d*x+c)^2)^(1/2)/b/d/(a+
b*arcsinh(d*x+c))^(5/2)-8/15*(1+(d*x+c)^2)^(1/2)/b^3/d/(a+b*arcsinh(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5858, 5773, 5818, 5819, 3389, 2211, 2236, 2235}

$$-\frac{4\sqrt{\pi}e^{a/b}\,\mathrm{Erf}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\sinh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{(c+dx)^2+1}}{15b^3d\sqrt{a+b\sinh^{-1}(c+dx)}}-\frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}}-\frac{2\sqrt{(c+dx)^2+1}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + (c + d*x)^2])/(5*b*d*(a + b*ArcSinh[c + d*x])^(5/2)) - (4*(c +
 d*x))/(15*b^2*d*(a + b*ArcSinh[c + d*x])^(3/2)) - (8*Sqrt[1 + (c + d*x)^2]
)/(15*b^3*d*Sqrt[a + b*ArcSinh[c + d*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (4*Sqrt[Pi]*Erfi[Sqrt[a +
b*ArcSinh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} + \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\left(a+b\sinh^{-1}(x)\right)^{5/2}}\,dx, x\right.}{5bd}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} + \frac{4\text{Sub}}{}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^3d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^3d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^3d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^3d}$$

$$= -\frac{2\sqrt{1+(c+dx)^2}}{5bd\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\sinh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15b^3d}$$

<u>Mathematica</u> [A]
time = 0.15, size = 238, normalized size = 1.22

$$\frac{-6b^2e^{\sinh^{-1}(c+dx)} - 2e^{-\sinh^{-1}(c+dx)}\left(4a^2+2ab\left(-1+4\sinh^{-1}(c+dx)\right)+b^2\left(3-2\sinh^{-1}(c+dx)+4\sinh^{-1}(c+dx)^2\right)\right) + 8e^{a/b}\sqrt{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)^2\Gamma\left(\frac{1}{2},\frac{a}{b}+\sinh^{-1}(c+dx)\right) - 4e^{-\frac{a}{b}}\left(a+b\sinh^{-1}(c+dx)\right)\left(e^{\frac{a}{b}+\sinh^{-1}(c+dx)}\left(2a+b+2b\sinh^{-1}(c+dx)\right)+2b\left(-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\sinh^{-1}(c+dx)}{b}\right)\right)}{30b^3d\left(a+b\sinh^{-1}(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c + d*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c + d*x]) +
b^2*(3 - 2*ArcSinh[c + d*x] + 4*ArcSinh[c + d*x]^2)))/E^ArcSinh[c + d*x] +
8*E^(a/b)*Sqrt[a/b + ArcSinh[c + d*x]]*(a + b*ArcSinh[c + d*x])^2*Gamma[1/2
, a/b + ArcSinh[c + d*x]] - (4*(a + b*ArcSinh[c + d*x])*(E^(a/b + ArcSinh[c
 + d*x])*(2*a + b + 2*b*ArcSinh[c + d*x]) + 2*b*(-((a + b*ArcSinh[c + d*x])
/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c + d*x])/b)]))/E^(a/b))/(30*b^3*d*(
a + b*ArcSinh[c + d*x])^(5/2))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{asinh}\,(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**(-7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(c + d*x))^(7/2), x)
```

3.227 $\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arcsinh(d*x+c))^(7/2),x)/e
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcSinh[x])^(7/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\sinh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\sinh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcSinh[c + d*x])^(7/2)), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\left(a + b\,\mathrm{arcsinh}\,(dx + c)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x*e + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{3}c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)} + a^{3}dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)} + 3a^{2}bc\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c+dx) + 3a^{2}bdx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}\,(c+dx) + 3ab^{2}c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{2}\,(c+dx) + 3ab^{2}dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{2}\,(c+dx) + b^{3}c\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{3}\,(c+dx) + b^{3}dx\sqrt{a + b\,\mathrm{asinh}\,(c + dx)}\,\mathrm{asinh}^{3}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*asinh(d*x+c))**(7/2),x)
```

```
[Out] Integral(1/(a**3*c*sqrt(a + b*asinh(c + d*x)) + a**3*d*x*sqrt(a + b*asinh(c
 + d*x)) + 3*a**2*b*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a**2*b*
d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x) + 3*a*b**2*c*sqrt(a + b*asinh
(c + d*x))*asinh(c + d*x)**2 + 3*a*b**2*d*x*sqrt(a + b*asinh(c + d*x))*asin
h(c + d*x)**2 + b**3*c*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3 + b**3*
d*x*sqrt(a + b*asinh(c + d*x))*asinh(c + d*x)**3), x)/e
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arcsinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^(7/2)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*asinh(c + d*x))^(7/2)), x)
```

3.228 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=298

$$\frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d}-\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d}-\frac{28be^3\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{135d(1+c+dx)}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))/d/e+28/405*b*e^2*(e*(d*x+c))^(3/
2)*(1+(d*x+c)^2)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(1+(d*x+c)^2)^(1/2)/d-28/
135*b*e^3*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c+1)+28/135*b*e^(7/2
)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan
((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/
2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c)^2)^(1/2)-1
4/135*b*e^(7/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2
)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c
))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x
+c)^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 327, 335, 311, 226, 1210}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de}-\frac{14be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}+\frac{28be^{7/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{135d\sqrt{(c+dx)^2+1}}-\frac{28be^3\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{135d(c+dx+1)}+\frac{28be^2\sqrt{(c+dx)^2+1}(e(c+dx))^{3/2}}{405d}-\frac{4b\sqrt{(c+dx)^2+1}(e(c+dx))^{7/2}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (28*b*e^2*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(405*d) - (4*b*(e*(c +
 d*x))^(7/2)*Sqrt[1 + (c + d*x)^2])/(81*d) - (28*b*e^3*Sqrt[e*(c + d*x)]*Sq
rt[1 + (c + d*x)^2])/(135*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(9/2)*(a + b*
ArcSinh[c + d*x]))/(9*d*e) + (28*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x
)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/
(135*d*Sqrt[1 + (c + d*x)^2]) - (14*b*e^(7/2)*(1 + c + d*x)*Sqrt[(1 + (c +
d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2
])/(135*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{1+x^2}}\right.}{9de}$$

$$= -\frac{4b(e(c+dx))^{7/2}\sqrt{1+(c+dx)^2}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\sin\right.}{9de}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c}{81d}$$

$$= \frac{28be^2(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{405d} - \frac{4b(e(c+dx))^{7/2}\sqrt{1+(c}{81d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.15, size = 113, normalized size = 0.38

$$\frac{2(e(c+dx))^{7/2}\left(45a(c+dx)^3+14b\sqrt{1+(c+dx)^2}-10b(c+dx)^2\sqrt{1+(c+dx)^2}+45b(c+dx)^3\sinh^{-1}(c+dx)-14b\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\right)}{405d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(45*a*(c + d*x)^3 + 14*b*Sqrt[1 + (c + d*x)^2] - 10*
b*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 45*b*(c + d*x)^3*ArcSinh[c + d*x] - 1
4*b*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(405*d*(c + d*x)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 3.00, size = 238, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+}}{45}\right.}{}\right.}{}$
default	$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9}+2b\left(\frac{(dex+ce)^{\frac{9}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{9}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{7}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{9}-\frac{7e^4(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+}}{45}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arcsinh((d*e*x+c*e)
/e)-2/9/e*(1/9*e^2*(d*e*x+c*e)^(7/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-7/45*e^4*(
d*e*x+c*e)^(3/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)+7/15*I*e^5/(I/e)^(1/2)*(1-I/e*
(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^(1/2)*(E
llipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(I/e)
^(1/2),I)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/9*(d*x*e + c*e)^(9/2)*a*e^(-1)/d + 1/810*(180*(d^4*x^4*e^(7/2) + 4*c*d^3*
x^3*e^(7/2) + 6*c^2*d^2*x^2*e^(7/2) + 4*c^3*d*x*e^(7/2) + c^4*e^(7/2))*sqrt
(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d - (45*(I*sqrt(
2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)
*(sqrt(2) + 2*sqrt(d*x + c)) + 1))*e^3 - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt
(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)
) + 1))*e^3 - sqrt(2)*e^3*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1) + sqrt(2
)*e^3*log(d*x - sqrt(2)*sqrt(d*x + c) + c + 1))*e^(1/2) + 40*e^(9/2*log(d*x
```

```
 + c) + 7/2) - 72*e^(5/2*log(d*x + c) + 7/2) + 360*e^(1/2*log(d*x + c) + 7/
2))/d - 810*integrate(2/9*(d^4*x^4*e^(7/2) + 4*c*d^3*x^3*e^(7/2) + 6*c^2*d^
2*x^2*e^(7/2) + 4*c^3*d*x*e^(7/2) + c^4*e^(7/2))*sqrt(d*x + c)/(d^3*x^3 + 3
*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) +
c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 815, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/405*(45*((b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^3*d^2*x + b
*c^4*d)*cosh(1)^3 + 3*(b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^
3*d^2*x + b*c^4*d)*cosh(1)^2*sinh(1) + 3*(b*d^5*x^4 + 4*b*c*d^4*x^3 + 6*b*c
^2*d^3*x^2 + 4*b*c^3*d^2*x + b*c^4*d)*cosh(1)*sinh(1)^2 + (b*d^5*x^4 + 4*b*
c*d^4*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c^3*d^2*x + b*c^4*d)*sinh(1)^3)*sqrt((d*x
 + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)) + 42*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*cosh(1)^3 + 3*b*cosh(1)^2*
sinh(1) + 3*b*cosh(1)*sinh(1)^2 + b*sinh(1)^3)*weierstrassZeta(-4/d^2, 0, w
eierstrassPInverse(-4/d^2, 0, (d*x + c)/d)) + (45*(a*d^5*x^4 + 4*a*c*d^4*x^
3 + 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c^4*d)*cosh(1)^3 + 135*(a*d^5*x^4 +
 4*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c^4*d)*cosh(1)^2*sinh(
1) + 135*(a*d^5*x^4 + 4*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 + 4*a*c^3*d^2*x + a*c
^4*d)*cosh(1)*sinh(1)^2 + 45*(a*d^5*x^4 + 4*a*c*d^4*x^3 + 6*a*c^2*d^3*x^2 +
 4*a*c^3*d^2*x + a*c^4*d)*sinh(1)^3 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*(
(5*b*d^4*x^3 + 15*b*c*d^3*x^2 + (15*b*c^2 - 7*b)*d^2*x + (5*b*c^3 - 7*b*c)*
d)*cosh(1)^3 + 3*(5*b*d^4*x^3 + 15*b*c*d^3*x^2 + (15*b*c^2 - 7*b)*d^2*x + (
5*b*c^3 - 7*b*c)*d)*cosh(1)^2*sinh(1) + 3*(5*b*d^4*x^3 + 15*b*c*d^3*x^2 + (
15*b*c^2 - 7*b)*d^2*x + (5*b*c^3 - 7*b*c)*d)*cosh(1)*sinh(1)^2 + (5*b*d^4*x
^3 + 15*b*c*d^3*x^2 + (15*b*c^2 - 7*b)*d^2*x + (5*b*c^3 - 7*b*c)*d)*sinh(1)
^3))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/d^2
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x)), x)
```

3.229 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=177

$$\frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d}-\frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d}+\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx\right.}{7de}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))/d/e-4/49*b*(e*(d*x+c))^(5/2)*(1+
(d*x+c)^2)^(1/2)/d+20/147*b*e^2*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d-10/
147*b*e^(5/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+(d*x+c
)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 327, 335, 226}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}-\frac{10be^{5/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{147d\sqrt{(c+dx)^2+1}}+\frac{20be^2\sqrt{(c+dx)^2+1}\sqrt{e(c+dx)}}{147d}-\frac{4b\sqrt{(c+dx)^2+1}(e(c+dx))^{5/2}}{49d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (20*b*e^2*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(147*d) - (4*b*(e*(c + d
*x))^(5/2)*Sqrt[1 + (c + d*x)^2])/(49*d) + (2*(e*(c + d*x))^(7/2)*(a + b*Ar
cSinh[c + d*x]))/(7*d*e) - (10*b*e^(5/2)*(1 + c + d*x)*Sqrt[(1 + (c + d*x)^
2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2])/(1
47*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)\,dx=\frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)}{7de}-\frac{(2b)\text{Subst}\left(\int\frac{(ex)^{7/}}{\sqrt{1+}}\right.}{7de}$$

$$=-\frac{4b(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}}{49d}+\frac{2(e(c+dx))^{7/2}\left(a+b\sinh\right.}{7de}$$

$$=\frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d}-\frac{4b(e(c+dx))^{5/2}\sqrt{1+(}}{49d}$$

$$=\frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d}-\frac{4b(e(c+dx))^{5/2}\sqrt{1+(}}{49d}$$

$$=\frac{20be^2\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{147d}-\frac{4b(e(c+dx))^{5/2}\sqrt{1+(}}{49d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.13, size = 113, normalized size = 0.64

$$\frac{2(e(c+dx))^{5/2}\left(21a(c+dx)^3+10b\sqrt{1+(c+dx)^2}-6b(c+dx)^2\sqrt{1+(c+dx)^2}+21b(c+dx)^3\sinh^{-1}(c+dx)-10b\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\right)}{147d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(21*a*(c + d*x)^3 + 10*b*Sqrt[1 + (c + d*x)^2] - 6*b
*(c + d*x)^2*Sqrt[1 + (c + d*x)^2] + 21*b*(c + d*x)^3*ArcSinh[c + d*x] - 10
*b*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2]))/(147*d*(c + d*x)^2)
```

Maple [C] Result contains complex when optimal does not.
time = 3.02, size = 212, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}}}{21}\right.}{}\right)}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7}+2b\left(\frac{(dex+ce)^{\frac{7}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{7}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{5}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{7}-\frac{5e^4\sqrt{dex+ce}\sqrt{\frac{(dex+ce)^2}{e^2}}}{21}\right.}{}\right)}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arcsinh((d*e*x+c*e)
/e)-2/7/e*(1/7*e^2*(d*e*x+c*e)^(5/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-5/21*e^4*(
d*e*x+c*e)^(1/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)+5/21*e^4/(I/e)^(1/2)*(1-I/e*(d
*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^(1/2)*Elli
pticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a*e^(-1)/d + 1/294*(84*(d^3*x^3*e^(5/2) + 3*c*d^2*x
^2*e^(5/2) + 3*c^2*d*x*e^(5/2) + c^3*e^(5/2))*sqrt(d*x + c)*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d + (21*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(sq
rt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x +
c)) + 1)) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) -
 log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)) + sqrt(2)*log(d*x + s
qrt(2)*sqrt(d*x + c) + c + 1) - sqrt(2)*log(d*x - sqrt(2)*sqrt(d*x + c) + c
 + 1))*e^(5/2) - 24*e^(7/2*log(d*x + c) + 5/2) + 56*e^(3/2*log(d*x + c) + 5
/2))/d - 294*integrate(2/7*(d^3*x^3*e^(5/2) + 3*c*d^2*x^2*e^(5/2) + 3*c^2*d
*x*e^(5/2) + c^3*e^(5/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c
^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 517, normalized size = 2.92

$$\frac{2\left(21\left(\left(bd^5x^3+3bcd^4x^2+3bc^2d^3x+bc^3d^2\right)\cosh(1)^2+2\left(bd^5x^3+3bcd^4x^2+3bc^2d^3x+bc^3d^2\right)\cosh(1)\sinh(1)+\left(bd^5x^3+3bcd^4x^2+3bc^2d^3x+bc^3d^2\right)\sinh(1)^2\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\log\left(dx+c+\sqrt{d^2x^2+2cdx+c^2+1}\right)-10\sqrt{d^3\cosh(1)+d^3\sinh(1)}\left(b\cosh(1)^2+2b\cosh(1)\sinh(1)+b\sinh(1)^2\right)\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)+\left(21\left(ad^5x^3+3acd^4x^2+3ac^2d^3x+ac^3d^2\right)\cosh(1)^2+42\left(ad^5x^3+3acd^4x^2+3ac^2d^3x+ac^3d^2\right)\cosh(1)\sinh(1)+21\left(ad^5x^3+3acd^4x^2+3ac^2d^3x+ac^3d^2\right)\sinh(1)^2-2\sqrt{d^2x^2+2cdx+c^2+1}\left(\left(3bd^4x^2+6bcd^3x+(3bc^2-5b)d^2\right)\cosh(1)^2+2\left(3bd^4x^2+6bcd^3x+(3bc^2-5b)d^2\right)\cosh(1)\sinh(1)+\left(3bd^4x^2+6bcd^3x+(3bc^2-5b)d^2\right)\sinh(1)^2\right)\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{147d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/147*(21*((b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cosh(1)^
2 + 2*(b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*cosh(1)*sinh(
1) + (b*d^5*x^3 + 3*b*c*d^4*x^2 + 3*b*c^2*d^3*x + b*c^3*d^2)*sinh(1)^2)*sqr
t((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)) - 10*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*cosh(1)^2 + 2*b*cosh
(1)*sinh(1) + b*sinh(1)^2)*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) + (2
1*(a*d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*cosh(1)^2 + 42*(a
*d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*cosh(1)*sinh(1) + 21*
(a*d^5*x^3 + 3*a*c*d^4*x^2 + 3*a*c^2*d^3*x + a*c^3*d^2)*sinh(1)^2 - 2*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 - 5*b)*
d^2)*cosh(1)^2 + 2*(3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 - 5*b)*d^2)*cosh(1
)*sinh(1) + (3*b*d^4*x^2 + 6*b*c*d^3*x + (3*b*c^2 - 5*b)*d^2)*sinh(1)^2))*s
qrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/d^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{asinh}(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x)), x)
```

3.230 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=261

$$-\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d}+\frac{12be\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{25d(1+c+dx)}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))/d/e-4/25*b*(e*(d*x+c))^(3/2)*(1+
(d*x+c)^2)^(1/2)/d+12/25*b*e*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/(d*x+c
+1)-12/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^
(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d
*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/(1+
(d*x+c)^2)^(1/2)+6/25*b*e^(3/2)*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e
^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*
arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 327, 335, 311, 226, 1210}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de}+\frac{6be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}-\frac{12be^{3/2}(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{25d\sqrt{(c+dx)^2+1}}-\frac{4b\sqrt{(c+dx)^2+1}\,(e(c+dx))^{3/2}}{25d}+\frac{12be\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{25d(c+dx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*(e*(c + d*x))^(3/2)*Sqrt[1 + (c + d*x)^2])/(25*d) + (12*b*e*Sqrt[e*(c
 + d*x)]*Sqrt[1 + (c + d*x)^2])/(25*d*(1 + c + d*x)) + (2*(e*(c + d*x))^(5/
2)*(a + b*ArcSinh[c + d*x]))/(5*d*e) - (12*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
+ (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e
]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2]) + (6*b*e^(3/2)*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(25*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)}{5de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{5/}}{\sqrt{1+}}\right.}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\,\text{si}\right.}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\,\text{si}\right.}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\,\text{si}\right.}{5de}$$

$$= -\frac{4b(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}}{25d} + \frac{12be\sqrt{e(c+dx)}\,\sqrt{1+}}{25d(1+c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.03, size = 87, normalized size = 0.33

$$\frac{2(e(c+dx))^{3/2}\left(5ac+5adx-2b\sqrt{1+(c+dx)^2}+5bc\sinh^{-1}(c+dx)+5bdx\sinh^{-1}(c+dx)+2b\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\right)}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*a*c + 5*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 5*b*c
*ArcSinh[c + d*x] + 5*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/2, 3
/4, 7/4, -(c + d*x)^2]))/(25*d)
```

Maple [C] Result contains complex when optimal does not.
time = 3.06, size = 205, normalized size = 0.79

method	result

derivativedivides	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+}}{}\right.}{}\right.}{de}$
default	$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\frac{e^2(dex+ce)^{\frac{3}{2}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{5}-\frac{3ie^3\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+}}{}\right.}{}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arcsinh((d*e*x+c*e)
/e)-2/5/e*(1/5*e^2*(d*e*x+c*e)^(3/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-3/5*I*e^3/
(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e
)^2/e^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e
*x+c*e)^(1/2)*(I/e)^(1/2),I)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a*e^(-1)/d + 1/50*(20*(d^2*x^2*e^(3/2) + 2*c*d*x*e^
(3/2) + c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1))/d + (5*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) +
 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1))*e - I*sqrt(2)*(l
og(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqr
t(2) - 2*sqrt(d*x + c)) + 1))*e - sqrt(2)*e*log(d*x + sqrt(2)*sqrt(d*x + c)
 + c + 1) + sqrt(2)*e*log(d*x - sqrt(2)*sqrt(d*x + c) + c + 1))*e^(1/2) - 8
*e^(5/2*log(d*x + c) + 3/2) + 40*e^(1/2*log(d*x + c) + 3/2))/d - 50*integra
te(2/5*(d^2*x^2*e^(3/2) + 2*c*d*x*e^(3/2) + c^2*e^(3/2))*sqrt(d*x + c)/(d^3
```

```
*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^
(3/2) + c), x))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 279, normalized size = 1.07

$$\frac{2\left(5\left((bd^3x^2+2\,bcd^2x+bc^2d)\cosh(1)+(bd^3x^2+2\,bcd^2x+bc^2d)\sinh(1)\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)-6\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,(b\cosh(1)+b\sinh(1))\mathrm{weierstrassZeta}\left(-\frac{4}{d^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+\left(5\,(ad^3x^2+2\,acd^2x+ac^2d)\cosh(1)+5\,(ad^3x^2+2\,acd^2x+ac^2d)\sinh(1)-2\sqrt{d^2x^2+2\,cdx+c^2+1}\,((bd^2x+bcd)\cosh(1)+(bd^2x+bcd)\sinh(1))\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{25\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/25*(5*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cosh(1) + (b*d^3*x^2 + 2*b*c*d
^2*x + b*c^2*d)*sinh(1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) - 6*sqrt(d^3*cosh(1) + d^3*sinh(
1))*(b*cosh(1) + b*sinh(1))*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(
-4/d^2, 0, (d*x + c)/d)) + (5*(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)*cosh(1) +
 5*(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)*sinh(1) - 2*sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1)*((b*d^2*x + b*c*d)*cosh(1) + (b*d^2*x + b*c*d)*sinh(1)))*sqrt((d*
x + c)*cosh(1) + (d*x + c)*sinh(1)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}}\,(a+b\operatorname{asinh}(c+dx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x)), x)
```

3.231 $\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right) \, dx$

Optimal. Leaf size=142

$$-\frac{4b\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{9d}+\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}+\frac{2b\sqrt{e}\,(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\ldots}{9d\sqrt{1+\ldots}}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))/d/e-4/9*b*(e*(d*x+c))^(1/2)*(1+(
d*x+c)^2)^(1/2)/d+2/9*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))
^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*e^(1/2)*((1+(d*x+c)^2)/(d*x+c+1)^2)
^(1/2)/d/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 327, 335, 226}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de}+\frac{2b\sqrt{e}\,(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{9d\sqrt{(c+dx)^2+1}}-\frac{4b\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(9*d) + (2*(e*(c + d*x))^(3/
2)*(a + b*ArcSinh[c + d*x]))/(3*d*e) + (2*b*Sqrt[e]*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(9*d*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\sinh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\sinh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)}{3de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{1+x^2}}\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh\right.}{3de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\sinh\right.}{3de}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 87, normalized size = 0.61

$$\frac{2\sqrt{e(c+dx)}\left(3ac+3adx-2b\sqrt{1+(c+dx)^2}+3bc\sinh^{-1}(c+dx)+3bdx\sinh^{-1}(c+dx)+2b\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x]),x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(3*a*c + 3*a*d*x - 2*b*Sqrt[1 + (c + d*x)^2] + 3*b*c*A
rcSinh[c + d*x] + 3*b*d*x*ArcSinh[c + d*x] + 2*b*Hypergeometric2F1[1/4, 1/2
, 5/4, -(c + d*x)^2]))/(9*d)
```

Maple [C] Result contains complex when optimal does not.
time = 3.02, size = 179, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{2a(dex+ce)^{\frac{3}{2}}}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{\cdots}}{3\cdots}\right)}{3e}\right)}{de}$
default	$\frac{\frac{2a(dex+ce)^{\frac{3}{2}}}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\frac{e^2\sqrt{dex+ce}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3}-\frac{e^2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{\cdots}}{3\cdots}\right)}{3e}\right)}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(1/3*a*(d*e*x+c*e)^(3/2)+b*(1/3*(d*e*x+c*e)^(3/2)*arcsinh((d*e*x+c*e)
/e)-2/3/e*(1/3*e^2*(d*e*x+c*e)^(1/2)*((d*e*x+c*e)^2/e^2+1)^(1/2)-1/3*e^2/(I
/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^
2/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a*e^(-1)/d + 1/18*(12*(d*x*e^(1/2) + c*e^(1/2))*sqr
t(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/d - (3*(I*sqrt(
```

```
2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)
*(sqrt(2) + 2*sqrt(d*x + c)) + 1)) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2)
- 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) +
1)) + sqrt(2)*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1) - sqrt(2)*log(d*x -
sqrt(2)*sqrt(d*x + c) + c + 1))*e^(1/2) + 8*e^(3/2*log(d*x + c) + 1/2))/d -
 18*integrate(2/3*(d*x*e^(1/2) + c*e^(1/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^
2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x
))*b
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 159, normalized size = 1.12

$$\frac{2\left(3\left(bd^3x+bcd^2\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+2\,\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,b\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)+\left(3\,ad^3x+3\,acd^2-2\,\sqrt{d^2x^2+2\,cdx+c^2+1}\,bd^2\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*(b*d^3*x + b*c*d^2)*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*log(
d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*sqrt(d^3*cosh(1) + d^3*sin
h(1))*b*weierstrassPInverse(-4/d^2, 0, (d*x + c)/d) + (3*a*d^3*x + 3*a*c*d^
2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*b*d^2)*sqrt((d*x + c)*cosh(1) + (d*
x + c)*sinh(1)))/d^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{e\,(c+dx)}\,(a+b\operatorname{asinh}(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x)), x)
```

3.232 $\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=223

$$-\frac{4b\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{de(1+c+dx)}+\frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de}+\frac{4b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\ }{d\sqrt{e}\,\sqrt{1+}}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*(e*(d*x+c))^(1/2)*(1+(d*x+
c)^2)^(1/2)/d/e/(d*x+c+1)+4*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(
1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*ar
ctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(
1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)-2*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^
(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF
(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+
c+1)^2)^(1/2)/d/e^(1/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5859, 5776, 335, 311, 226, 1210}

$$\frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\,\sqrt{(c+dx)^2+1}}+\frac{4b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{d\sqrt{e}\,\sqrt{(c+dx)^2+1}}-\frac{4b\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{de(c+dx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-4*b*Sqrt[e*(c + d*x)]*Sqrt[1 + (c + d*x)^2])/(d*e*(1 + c + d*x)) + (2*Sqr
t[e*(c + d*x)]*(a + b*ArcSinh[c + d*x]))/(d*e) + (4*b*(1 + c + d*x)*Sqrt[(1
 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[
e]], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2]) - (2*b*(1 + c + d*x)*Sqrt[(1 +
 (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]
], 1/2])/(d*Sqrt[e]*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(2b)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+\right.}{de^2}$$

$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+\right.}{de}$$

$$= -\frac{4b\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{de(1+c+dx)} + \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)}{de} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 61, normalized size = 0.27

$$-\frac{2\sqrt{e(c+dx)}\,\left(-3\left(a+b\sinh^{-1}(c+dx)\right)+2b(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (-2*Sqrt[e*(c + d*x)]*(-3*(a + b*ArcSinh[c + d*x]) + 2*b*(c + d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(c + d*x)^2]))/(3*d*e)
```

Maple [C] Result contains complex when optimal does not.
time = 3.25, size = 161, normalized size = 0.72

method	result

derivativedivides	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\left(\operatorname{EllipticF}\left(\sqrt{d\right.\right.}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(de}{}}}\right.}{de}$
default	$\frac{2\sqrt{dex+ce}\,a+2b\left(\sqrt{dex+ce}\,\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)-\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\left(\operatorname{EllipticF}\left(\sqrt{d\right.\right.}{\sqrt{\frac{i}{e}}\,\sqrt{\frac{(de}{}}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*((d*e*x+c*e)^(1/2)*a+b*((d*e*x+c*e)^(1/2)*arcsinh((d*e*x+c*e)/e)-2*I/
(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e
)^2/e^2+1)^(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e
*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x*e^(1/2) + c*e^(1/2))*e^(-1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1))/(sqrt(d*x + c)*d) - ((I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) +
 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1
))*e^(1/2) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1)
 - log(-1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1))*e^(1/2) - sqrt(2)*e^
(1/2)*log(d*x + sqrt(2)*sqrt(d*x + c) + c + 1) + sqrt(2)*e^(1/2)*log(d*x -
sqrt(2)*sqrt(d*x + c) + c + 1))*e^(-1) + 8*e^(1/2*log(d*x + c) - 1/2))/d -
2*integrate(2*(d*x*e^(1/2) + c*e^(1/2))/((d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^
2*d + d)*x*e + (c^3 + c)*e + (d^2*x^2*e + 2*c*d*x*e + (c^2 + 1)*e)*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1))*sqrt(d*x + c)), x))*b + 2*sqrt(d*x*e + c*e)*a*e^
(-1)/d
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 128, normalized size = 0.57

$$\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,ad+2\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,b\mathrm{weierstrassZeta}\left(-\frac{4}{d^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)\right)}{d^2\cosh(1)+d^2\sinh(1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)) + sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*a*d
+ 2*sqrt(d^3*cosh(1) + d^3*sinh(1))*b*weierstrassZeta(-4/d^2, 0, weierstras
sPInverse(-4/d^2, 0, (d*x + c)/d)))/(d^2*cosh(1) + d^2*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\operatorname{asinh}\left(c + dx\right)}{\sqrt{e\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/sqrt(e*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\operatorname{asinh}(c + d\,x)}{\sqrt{c\,e + d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

3.233 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{2(a+b\sinh^{-1}(c+dx))}{de\sqrt{e(c+dx)}}+\frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(1/2)+2*b*(d*x+c+1)*(cos(2*arctan((
e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))
)*EllipticF(sin(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+
c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(3/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5859, 5776, 335, 226}

$$\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{de^{3/2}\sqrt{(c+dx)^2+1}}-\frac{2(a+b\sinh^{-1}(c+dx))}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (2*b*(1 + c + d*x)*
Sqrt[(1 + (c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)
]/Sqrt[e]], 1/2])/(d*e^(3/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx, x, \sqrt{e(c+dx)}\right)}{de^2}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt{e}\ldots\right.\right.}{de^{3/2}\sqrt{1+(c+dx)^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 56, normalized size = 0.53

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)-2b(c+dx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] - 2*b*(c + d*x)*Hypergeometric2F1[1/4, 1/2, 5/4
, -(c + d*x)^2]))/(d*e*Sqrt[e*(c + d*x)])
```

Maple [C] Result contains complex when optimal does not.
time = 3.24, size = 140, normalized size = 1.32

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}}\right.}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{de}$
default	$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\,\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\frac{i}{e}}\right.}{e\sqrt{\frac{i}{e}}\,\sqrt{\frac{(dex+ce)^2}{e^2}+1}}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arcsinh((d*e*x+c*e)/e)+
2/e/(I/e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x
+c*e)^2/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(4*e^(-3/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(sqrt(d
*x + c)*d) + (I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1)
 - log(-1/2*I*sqrt(2)*(sqrt(2) + 2*sqrt(d*x + c)) + 1))*e^(-3/2) - I*sqrt(2
)*(log(1/2*I*sqrt(2)*(sqrt(2) - 2*sqrt(d*x + c)) + 1) - log(-1/2*I*sqrt(2)*
(sqrt(2) - 2*sqrt(d*x + c)) + 1))*e^(-3/2) + sqrt(2)*e^(-3/2)*log(d*x + sqr
t(2)*sqrt(d*x + c) + c + 1) - sqrt(2)*e^(-3/2)*log(d*x - sqrt(2)*sqrt(d*x +
 c) + c + 1))/d - 4*integrate(1/((d^2*x^2*e^(3/2) + 2*c*d*x*e^(3/2) + (c^2
+ 1)*e^(3/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (d^3*x^3*e^
(3/2) + 3*c*d^2*x^2*e^(3/2) + (3*c^2*d + d)*x*e^(3/2) + (c^3 + c)*e^(3/2))*
sqrt(d*x + c)), x)) - 2*a*e^(-1)/(sqrt(d*x*e + c*e)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 169, normalized size = 1.59

$$-\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,ad^2-2\,\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,(bdx+bc)\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)}{(d^4x+cd^3)\cosh(1)^2+2\,(d^4x+cd^3)\cosh(1)\sinh(1)+(d^4x+cd^3)\sinh(1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d^2*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*a
*d^2 - 2*sqrt(d^3*cosh(1) + d^3*sinh(1))*(b*d*x + b*c)*weierstrassPInverse(
-4/d^2, 0, (d*x + c)/d))/((d^4*x + c*d^3)*cosh(1)^2 + 2*(d^4*x + c*d^3)*cos
h(1)*sinh(1) + (d^4*x + c*d^3)*sinh(1)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{asinh}\left(c + dx\right)}{\left(e\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

3.234 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}}+\frac{4b\sqrt{e(c+dx)}\,\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b(1+c+dx)\sqrt{\frac{1+(c+}{(1+c+}}}{3de^5}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*(1+(d*x+c)^2)^(1/2)/d
/e^2/(e*(d*x+c))^(1/2)+4/3*b*(e*(d*x+c))^(1/2)*(1+(d*x+c)^2)^(1/2)/d/e^3/(d
*x+c+1)-4/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))^2)^(1/2)/
cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticE(sin(2*arctan((e*(d*x+c))
^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/d/e^(5/2)/(
1+(d*x+c)^2)^(1/2)+2/3*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2))
)^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*arctan(
(e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^(1/2)/
d/e^(5/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5859, 5776, 331, 335, 311, 226, 1210}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}}-\frac{4b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{3de^{5/2}\sqrt{(c+dx)^2+1}}+\frac{4b\sqrt{(c+dx)^2+1}\,\sqrt{e(c+dx)}}{3de^3(c+dx+1)}-\frac{4b\sqrt{(c+dx)^2+1}}{3de^2\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[e*(c +
 d*x)]*Sqrt[1 + (c + d*x)^2])/(3*d*e^3*(1 + c + d*x)) - (2*(a + b*ArcSinh[c
 + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticE[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2]) + (2*b*(1 + c + d*x)*Sqrt[(1 + (c + d
*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]], 1/2]
)/(3*d*e^(5/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{(ex)^{3/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{3de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{1+x^2}}\,dx,\right.}{3de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{e^2}}}\,dx\right.}{3de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\,dx\right.}{3de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{3de^2\sqrt{e(c+dx)}} + \frac{4b\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}}{3de^3(1+c+dx)} - \frac{2\left(a+b\sinh^{-1}(c+\right.}{3de(e(c+dx))^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.02, size = 58, normalized size = 0.22

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)+2b(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x] + 2*b*(c + d*x)*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c + d*x)^2]))/(3*d*e*(e*(c + d*x))^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 3.02, size = 202, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellipti}\right.}{3e\sqrt{\frac{i}{e}}}}{e}\right.}{de}$
default	$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{3\sqrt{dex+ce}}+\frac{2i\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\left(\text{Ellipti}\right.}{3e\sqrt{\frac{i}{e}}}}{e}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arcsinh((d*e*x+c*
e)/e)+2/3/e*(-((d*e*x+c*e)^2/e^2+1)^(1/2)/(d*e*x+c*e)^(1/2)+I/e/(I/e)^(1/2)
*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2/e^2+1)^
(1/2)*(EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/
2)*(I/e)^(1/2),I)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(12*e^(1/2)*integrate(1/3/((d^4*x^4*e^3 + 4*c*d^3*x^3*e^3 + (6*c^2*d^2
+ d^2)*x^2*e^3 + 2*(2*c^3*d + c*d)*x*e^3 + (c^4 + c^2)*e^3 + (d^3*x^3*e^3 +
 3*c*d^2*x^2*e^3 + (3*c^2*d + d)*x*e^3 + (c^3 + c)*e^3)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))*sqrt(d*x + c)), x) - 4*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))/((d^2*x*e^3 + c*d*e^3)*sqrt(d*x + c)) - (I*sqrt(2)*(lo
g(1/2*I*sqrt(2)*(sqrt(2)*e^(1/2) + 2*e^(1/2*log(d*x + c) + 1/2))*e^(-1/2) +
 1) - log(-1/2*I*sqrt(2)*(sqrt(2)*e^(1/2) + 2*e^(1/2*log(d*x + c) + 1/2))*e
^(-1/2) + 1))*e^(-5/2) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2)*e^(1/2) - 2*
e^(1/2*log(d*x + c) + 1/2))*e^(-1/2) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2)*e^(
```

```
1/2) - 2*e^(1/2*log(d*x + c) + 1/2))*e^(-1/2) + 1))*e^(-5/2) - sqrt(2)*e^(-
5/2)*log(sqrt(2)*e^(1/2*log(d*x + c) + 1) + e + e^(log(d*x + c) + 1)) + sqr
t(2)*e^(-5/2)*log(-sqrt(2)*e^(1/2*log(d*x + c) + 1) + e + e^(log(d*x + c) +
 1)))/d)*b - 2/3*a*e^(-1)/((d*x*e + c*e)^(3/2)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 283, normalized size = 1.06

$$-\frac{2\left(\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,\mathrm{weierstrassZeta}\left(-\frac{4}{d^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)\right)+\left(ad+2\left(bd^2x+bcd\right)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{3\left(\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)^3+3\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)^2\sinh(1)+3\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh(1)\sinh(1)^2+\left(d^4x^2+2\,cd^3x+c^2d^2\right)\sinh(1)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(d^3*co
sh(1) + d^3*sinh(1))*weierstrassZeta(-4/d^2, 0, weierstrassPInverse(-4/d^2,
 0, (d*x + c)/d)) + (a*d + 2*(b*d^2*x + b*c*d)*sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/((d^4*x^2 + 2*c*d^3*x +
 c^2*d^2)*cosh(1)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(1)^2*sinh(1) +
 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(1)*sinh(1)^2 + (d^4*x^2 + 2*c*d^3*x
 + c^2*d^2)*sinh(1)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

3.235 $\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{2b(1+c+dx)\sqrt{\frac{1+(c+dx)^2}{(1+c+dx)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\right.}{15de^{7/2}\sqrt{1+(c+dx)^2}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))/d/e/(e*(d*x+c))^(5/2)-4/15*b*(1+(d*x+c)^2)^(1/2)/
d/e^2/(e*(d*x+c))^(3/2)-2/15*b*(d*x+c+1)*(cos(2*arctan((e*(d*x+c))^(1/2)/e^
(1/2)))^2)^(1/2)/cos(2*arctan((e*(d*x+c))^(1/2)/e^(1/2)))*EllipticF(sin(2*a
rctan((e*(d*x+c))^(1/2)/e^(1/2))),1/2*2^(1/2))*((1+(d*x+c)^2)/(d*x+c+1)^2)^
(1/2)/d/e^(7/2)/(1+(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5859, 5776, 331, 335, 226}

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}-\frac{2b(c+dx+1)\sqrt{\frac{(c+dx)^2+1}{(c+dx+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)|\frac{1}{2}\right)}{15de^{7/2}\sqrt{(c+dx)^2+1}}-\frac{4b\sqrt{(c+dx)^2+1}}{15de^2(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-4*b*Sqrt[1 + (c + d*x)^2])/(15*d*e^2*(e*(c + d*x))^(3/2)) - (2*(a + b*Arc
Sinh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) - (2*b*(1 + c + d*x)*Sqrt[(1 +
(c + d*x)^2)/(1 + c + d*x)^2]*EllipticF[2*ArcTan[Sqrt[e*(c + d*x)]/Sqrt[e]]
, 1/2])/(15*d*e^(7/2)*Sqrt[1 + (c + d*x)^2])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{(ex)^{5/2}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{5de}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{ex}\sqrt{1+}}\right.}{15de^3}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{e^2}}}\right.}{15de^4}$$

$$= -\frac{4b\sqrt{1+(c+dx)^2}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\sinh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} - \frac{2b(1+c+dx)\sqrt{\frac{1+(c+}{(1+c+}}}{15de^7}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 0.03, size = 61, normalized size = 0.42

$$\frac{-6\left(a+b\sinh^{-1}(c+dx)\right)-4b(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-6*(a + b*ArcSinh[c + d*x]) - 4*b*(c + d*x)*Hypergeometric2F1[-3/4, 1/2, 1
/4, -(c + d*x)^2])/(15*d*e*(e*(c + d*x))^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 3.06, size = 176, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\right.}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right.}{de}$
default	$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arcsinh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{-\frac{2\sqrt{\frac{(dex+ce)^2}{e^2}+1}}{15(dex+ce)^{\frac{3}{2}}}-\frac{2\sqrt{1-\frac{i(dex+ce)}{e}}\sqrt{1+\frac{i(dex+ce)}{e}}\,\operatorname{EllipticF}\left(\right.}{15e^2\sqrt{\frac{i}{e}}\sqrt{\frac{(dex+ce)^2}{e^2}+1}}}{e}\right.}{de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arcsinh((d*e*x+c*
e)/e)+2/5/e*(-1/3*((d*e*x+c*e)^2/e^2+1)^(1/2)/(d*e*x+c*e)^(3/2)-1/3/e^2/(I/
e)^(1/2)*(1-I/e*(d*e*x+c*e))^(1/2)*(1+I/e*(d*e*x+c*e))^(1/2)/((d*e*x+c*e)^2
/e^2+1)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(I/e)^(1/2),I))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/10*(20*e^(1/2)*integrate(1/5/((d^5*x^5*e^4 + 5*c*d^4*x^4*e^4 + (10*c^2*d^
3 + d^3)*x^3*e^4 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^4 + (5*c^4*d + 3*c^2*d)*x*e
^4 + (c^5 + c^3)*e^4 + (d^4*x^4*e^4 + 4*c*d^3*x^3*e^4 + (6*c^2*d^2 + d^2)*x
^2*e^4 + 2*(2*c^3*d + c*d)*x*e^4 + (c^4 + c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))*sqrt(d*x + c)), x) - 4*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1))/((d^3*x^2*e^4 + 2*c*d^2*x*e^4 + c^2*d*e^4)*sqrt(d*x + c))
+ ((I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqrt(2)*e^(1/2) + 2*e^(1/2*log(d*x + c) +
 1/2))*e^(-1/2) + 1) - log(-1/2*I*sqrt(2)*(sqrt(2)*e^(1/2) + 2*e^(1/2*log(d
*x + c) + 1/2))*e^(-1/2) + 1))*e^(-1/2) - I*sqrt(2)*(log(1/2*I*sqrt(2)*(sqr
t(2)*e^(1/2) - 2*e^(1/2*log(d*x + c) + 1/2))*e^(-1/2) + 1) - log(-1/2*I*sqr
t(2)*(sqrt(2)*e^(1/2) - 2*e^(1/2*log(d*x + c) + 1/2))*e^(-1/2) + 1))*e^(-1/
2) + sqrt(2)*e^(-1/2)*log(sqrt(2)*e^(1/2*log(d*x + c) + 1) + e + e^(log(d*x
 + c) + 1)) - sqrt(2)*e^(-1/2)*log(-sqrt(2)*e^(1/2*log(d*x + c) + 1) + e +
e^(log(d*x + c) + 1)))*e^(-3) - 8*e^(-1/2*log(d*x + c) - 7/2))/d)*b - 2/5*a
*e^(-1)/((d*x*e + c*e)^(5/2)*d)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 383, normalized size = 2.64

$$-\frac{2\left(3\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\,bd^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2+1}\right)+2\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\sqrt{d^3\cosh(1)+d^3\sinh(1)}\,\mathrm{weierstrassPInverse}\left(-\frac{4}{d^2},0,\frac{dx+c}{d}\right)+\left(3\,ad^2+2\left(bd^3x+bcd^2\right)\sqrt{d^2x^2+2\,cdx+c^2+1}\right)\sqrt{(dx+c)\cosh(1)+(dx+c)\sinh(1)}\right)}{15\left(\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)\cosh(1)^4+4\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)\cosh(1)^3\sinh(1)+6\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)\cosh(1)^2\sinh(1)^2+4\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)\cosh(1)\sinh(1)^3+\left(d^6x^3+3\,cd^5x^2+3\,c^2d^4x+c^3d^3\right)\sinh(1)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1))*b*d^2*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)) + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3)*sqrt(d^3*cosh(1) + d^3*sinh(1))*weierstrassPInverse(-4/d^2, 0,
(d*x + c)/d) + (3*a*d^2 + 2*(b*d^3*x + b*c*d^2)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))*sqrt((d*x + c)*cosh(1) + (d*x + c)*sinh(1)))/((d^6*x^3 + 3*c*d^5*x^
2 + 3*c^2*d^4*x + c^3*d^3)*cosh(1)^4 + 4*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cosh(1)^3*sinh(1) + 6*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
c^3*d^3)*cosh(1)^2*sinh(1)^2 + 4*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3
*d^3)*cosh(1)*sinh(1)^3 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*s
inh(1)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{asinh}\,(c+dx)}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))/(e*(c + d*x))**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}(c+d\,x)}{{\left(c\,e+d\,e\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*asinh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

3.236 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=134

$$\frac{2(e(c+dx))^{9/2} \left(a+b\sinh^{-1}(c+dx)\right)^2}{9de} - \frac{8b(e(c+dx))^{11/2} \left(a+b\sinh^{-1}(c+dx)\right) \, {}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)}{99de^2}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/99*b*(e*(d*x+c))^(11/2)*
(a+b*arcsinh(d*x+c))*hypergeom([1/2, 11/4],[15/4],-(d*x+c)^2)/d/e^2+16/1287
*b^2*(e*(d*x+c))^(13/2)*hypergeom([1, 13/4, 13/4],[15/4, 17/4],-(d*x+c)^2)/
d/e^3
```

Rubi [A]

time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2(e(c+dx))^{13/2} \, {}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)}{1287de^3} - \frac{8b(e(c+dx))^{11/2} \, {}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right) \left(a+b\sinh^{-1}(c+dx)\right)}{99de^2} + \frac{2(e(c+dx))^{9/2} \left(a+b\sinh^{-1}(c+dx)\right)^2}{9de}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c
 + d*x)^2])/(99*d*e^2) + (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1,
 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)^2])/(1287*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de}-\frac{(4b)\text{Subst}\left(\int \frac{(ex)^{9/2}(}{\sqrt{}}\right.}{}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{9de}-\frac{8b(e(c+dx))^{11/2}\,(a}{}$$

Mathematica [A]
time = 0.09, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{9/2}\left(143\left(a+b\sinh^{-1}(c+dx)\right)^2-52b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right){}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};-(c+dx)^2\right)+8b^2(c+dx)^2{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};-(c+dx)^2\right)\right)}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^2,x]
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcSinh[c + d*x])^2 - 52*b*(c + d*x)*(a
+ b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 11/4, 15/4, -(c + d*x)^2] + 8*
b^2*(c + d*x)^2*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, -(c + d*x)
^2]))/(1287*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\,\mathrm{arcsinh}\,(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/9*(d*x*e + c*e)^(9/2)*a^2*e^(-1)/d + 2/9*(b^2*d^4*x^4*e^(7/2) + 4*b^2*c*d
^3*x^3*e^(7/2) + 6*b^2*c^2*d^2*x^2*e^(7/2) + 4*b^2*c^3*d*x*e^(7/2) + b^2*c^
4*e^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2
/d + integrate(2/9*(((9*a*b*d^5 - 2*b^2*d^5)*x^5*e^(7/2) + 5*(9*a*b*c*d^4 -
 2*b^2*c*d^4)*x^4*e^(7/2) - (20*b^2*c^2*d^3 - 9*(10*c^2*d^3 + d^3)*a*b)*x^3
*e^(7/2) - (20*b^2*c^3*d^2 - 9*(10*c^3*d^2 + 3*c*d^2)*a*b)*x^2*e^(7/2) - (1
0*b^2*c^4*d - 9*(5*c^4*d + 3*c^2*d)*a*b)*x*e^(7/2) - (2*b^2*c^5 - 9*(c^5 +
c^3)*a*b)*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((9*a*
b*d^6 - 2*b^2*d^6)*x^6*e^(7/2) + 6*(9*a*b*c*d^5 - 2*b^2*c*d^5)*x^5*e^(7/2)
+ (9*(15*c^2*d^4 + d^4)*a*b - 2*(15*c^2*d^4 + d^4)*b^2)*x^4*e^(7/2) + 4*(9*
(5*c^3*d^3 + c*d^3)*a*b - 2*(5*c^3*d^3 + c*d^3)*b^2)*x^3*e^(7/2) + 3*(9*(5*
c^4*d^2 + 2*c^2*d^2)*a*b - 2*(5*c^4*d^2 + 2*c^2*d^2)*b^2)*x^2*e^(7/2) + 2*(
9*(3*c^5*d + 2*c^3*d)*a*b - 2*(3*c^5*d + 2*c^3*d)*b^2)*x*e^(7/2) + (9*(c^6
+ c^4)*a*b - 2*(c^6 + c^4)*b^2)*e^(7/2))*sqrt(d*x + c))*log(d*x + c + sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*
x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*arcsinh
(d*x + c)^2*e^3 + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + 3*a*b*c^2*d*x + a*b*c^
3)*arcsinh(d*x + c)*e^3 + (a^2*d^3*x^3 + 3*a^2*c*d^2*x^2 + 3*a^2*c^2*d*x +
a^2*c^3)*e^3)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4845 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^2, x)
```

3.237 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=134

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}-\frac{8b(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)}{63de^2}-$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/63*b*(e*(d*x+c))^(9/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 9/4],[13/4],-(d*x+c)^2)/d/e^2+16/693*b^
2*(e*(d*x+c))^(11/2)*hypergeom([1, 11/4, 11/4],[13/4, 15/4],-(d*x+c)^2)/d/e
^3
```

Rubi [A]
time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)}{693de^3}-\frac{8b(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{63de^2}+\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c +
 d*x)^2])/(63*d*e^2) + (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 1
1/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2])/(693*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}-\frac{(4b)\text{Subst}\left(\int \frac{(ex)^{7/2}(}{\sqrt{}}\right.}{}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{7de}-\frac{8b(e(c+dx))^{9/2}\left(a\right.}{}$$

Mathematica [A]
time = 0.08, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{7/2}\left(99\left(a+b\sinh^{-1}(c+dx)\right)^2-44b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};-(c+dx)^2\right)+8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};-(c+dx)^2\right)\right)}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcSinh[c + d*x])^2 - 44*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 9/4, 13/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, -(c + d*x)^2
]))/(693*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a^2*e^(-1)/d + 2/7*(b^2*d^3*x^3*e^(5/2) + 3*b^2*c*d
^2*x^2*e^(5/2) + 3*b^2*c^2*d*x*e^(5/2) + b^2*c^3*e^(5/2))*sqrt(d*x + c)*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/d + integrate(2/7*(((7*a*b*
d^4 - 2*b^2*d^4)*x^4*e^(5/2) + 4*(7*a*b*c*d^3 - 2*b^2*c*d^3)*x^3*e^(5/2) -
(12*b^2*c^2*d^2 - 7*(6*c^2*d^2 + d^2)*a*b)*x^2*e^(5/2) - 2*(4*b^2*c^3*d - 7
*(2*c^3*d + c*d)*a*b)*x*e^(5/2) - (2*b^2*c^4 - 7*(c^4 + c^2)*a*b)*e^(5/2))*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((7*a*b*d^5 - 2*b^2*d^5)*
x^5*e^(5/2) + 5*(7*a*b*c*d^4 - 2*b^2*c*d^4)*x^4*e^(5/2) + (7*(10*c^2*d^3 +
d^3)*a*b - 2*(10*c^2*d^3 + d^3)*b^2)*x^3*e^(5/2) + (7*(10*c^3*d^2 + 3*c*d^2
)*a*b - 2*(10*c^3*d^2 + 3*c*d^2)*b^2)*x^2*e^(5/2) + (7*(5*c^4*d + 3*c^2*d)*
a*b - 2*(5*c^4*d + 3*c^2*d)*b^2)*x*e^(5/2) + (7*(c^5 + c^3)*a*b - 2*(c^5 +
c^3)*b^2)*e^(5/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x
 + c^2 + 1)^(3/2) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*arcsinh(d*x + c)^2*e^2 + 2*
(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*arcsinh(d*x + c)*e^2 + (a^2*d^2*x^2 +
 2*a^2*c*d*x + a^2*c^2)*e^2)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{5}{2}} (a+b\operatorname{asinh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(5/2)*(a + b*asinh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^2, x)
```

3.238 $\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=134

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de}-\frac{8b(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)}{35de^2}-$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/35*b*(e*(d*x+c))^(7/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 7/4],[11/4],-(d*x+c)^2)/d/e^2+16/315*b^
2*(e*(d*x+c))^(9/2)*hypergeom([1, 9/4, 9/4],[11/4, 13/4],-(d*x+c)^2)/d/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)}{315de^3}-\frac{8b(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{35de^2}+\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c +
 d*x)^2])/(35*d*e^2) + (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])/(315*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de}-\frac{(4b)\text{Subst}\left(\int \frac{(ex)^{5/2}(}{\sqrt{}}\right.}{}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de}-\frac{8b(e(c+dx))^{7/2}(a-}{}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{5/2}\left(63\left(a+b\sinh^{-1}(c+dx)\right)^2-36b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};-(c+dx)^2\right)+8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};-(c+dx)^2\right)\right)}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^2,x]
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcSinh[c + d*x])^2 - 36*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 11/4, -(c + d*x)^2] + 8*b^
2*(c + d*x)^2*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, -(c + d*x)^2])
)/(315*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^2*e^(-1)/d + 2/5*(b^2*d^2*x^2*e^(3/2) + 2*b^2*c*d
*x*e^(3/2) + b^2*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^2/d + integrate(2/5*(((5*a*b*d^3 - 2*b^2*d^3)*x^3*e^(3/2)
 + 3*(5*a*b*c*d^2 - 2*b^2*c*d^2)*x^2*e^(3/2) - (6*b^2*c^2*d - 5*(3*c^2*d +
d)*a*b)*x*e^(3/2) - (2*b^2*c^3 - 5*(c^3 + c)*a*b)*e^(3/2))*sqrt(d^2*x^2 + 2
*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((5*a*b*d^4 - 2*b^2*d^4)*x^4*e^(3/2) + 4*
(5*a*b*c*d^3 - 2*b^2*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2 + d^2)*a*b - 2*(6*c
^2*d^2 + d^2)*b^2)*x^2*e^(3/2) + 2*(5*(2*c^3*d + c*d)*a*b - 2*(2*c^3*d + c*
d)*b^2)*x*e^(3/2) + (5*(c^4 + c^2)*a*b - 2*(c^4 + c^2)*b^2)*e^(3/2))*sqrt(d
*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^
2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(((b^2*d*x + b^2*c)*arcsinh(d*x + c)^2*e + 2*(a*b*d*x + a*b*c)*arcs
inh(d*x + c)*e + (a^2*d*x + a^2*c)*e)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{asinh}(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^{3/2}\,(a + b\,\mathrm{asinh}(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^2, x)
```

3.239 $\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=134

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}-\frac{8b(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)}{15de^2}+$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^2/d/e-8/15*b*(e*(d*x+c))^(5/2)*(
a+b*arcsinh(d*x+c))*hypergeom([1/2, 5/4],[9/4],-(d*x+c)^2)/d/e^2+16/105*b^2
*(e*(d*x+c))^(7/2)*hypergeom([1, 7/4, 7/4],[9/4, 11/4],-(d*x+c)^2)/d/e^3
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)}{105de^3}-\frac{8b(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2}+\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c +
d*x)^2])/(15*d*e^2) + (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4
, 7/4}, {9/4, 11/4}, -(c + d*x)^2])/(105*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
```

```
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\sqrt{ex}\,\left(a+b\sinh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}-\frac{(4b)\text{Subst}\left(\int\frac{(ex)^{3/2}(a+}{\sqrt{1}}\right.}{}$$

$$=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de}-\frac{8b(e(c+dx))^{5/2}\,(a+}{}$$

Mathematica [A]
time = 0.06, size = 110, normalized size = 0.82

$$\frac{2(e(c+dx))^{3/2}\left(35\left(a+b\sinh^{-1}(c+dx)\right)^2-28b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-(c+dx)^2\right)+8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};-(c+dx)^2\right)\right)}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcSinh[c + d*x])^2 - 28*b*(c + d*x)*(a +
 b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9/4, -(c + d*x)^2] + 8*b^2
*(c + d*x)^2*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, -(c + d*x)^2]))/
(105*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a^2*e^(-1)/d + 2/3*(b^2*d*x*e^(1/2) + b^2*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2/d + inte
grate(2/3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*a*b*d^2 - 2*b^2*d^2)*x^2*e
^(1/2) + 2*(3*a*b*c*d - 2*b^2*c*d)*x*e^(1/2) - (2*b^2*c^2 - 3*(c^2 + 1)*a*b
)*e^(1/2))*sqrt(d*x + c) + ((3*a*b*d^3 - 2*b^2*d^3)*x^3*e^(1/2) + 3*(3*a*b*
c*d^2 - 2*b^2*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d + d)*a*b - 2*(3*c^2*d + d)*b
^2)*x*e^(1/2) + (3*(c^3 + c)*a*b - 2*(c^3 + c)*b^2)*e^(1/2))*sqrt(d*x + c))
*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^3*x^3 + 3*c*d^2*x^2 +
c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(1/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2,x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^2, x)
```

3.240 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^2}{de}-\frac{8b(e(c+dx))^{3/2}\,(a+b\sinh^{-1}(c+dx))\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)}{3de^2}+\frac{1}{}$$

```
[Out] -8/3*b*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))*hypergeom([1/2, 3/4],[7/4],-(
d*x+c)^2)/d/e^2+16/15*b^2*(e*(d*x+c))^(5/2)*hypergeom([1, 5/4, 5/4],[7/4, 9
/4],-(d*x+c)^2)/d/e^3+2*(a+b*arcsinh(d*x+c))^2*(e*(d*x+c))^(1/2)/d/e
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)}{15de^3}-\frac{8b(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)(a+b\sinh^{-1}(c+dx))}{3de^2}+\frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^2}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)
^2])/(3*d*e^2) + (16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4
}, {7/4, 9/4}, -(c + d*x)^2])/(15*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$
$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\text{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\sinh^{-1}(x))}{\sqrt{1+x^2}}\,dx\right.}{de}$$
$$= \frac{2\sqrt{e(c+dx)}\,\left(a+b\sinh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\,\left(a+b\sinh^{-1}(c+\right.}{3de^2}$$

Mathematica [A]
time = 0.05, size = 110, normalized size = 0.83

$$\frac{2\sqrt{e(c+dx)}\left(15\left(a+b\sinh^{-1}(c+dx)\right)^2-20b(c+dx)\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-(c+dx)^2\right)+8b^2(c+dx)^2\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};-(c+dx)^2\right)\right)}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcSinh[c + d*x])^2 - 20*b*(c + d*x)*(a + b
*ArcSinh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, -(c + d*x)^2] + 8*b^2*(
c + d*x)^2*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, -(c + d*x)^2]))/(15
*d*e)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^2*e^(-1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^2/d + 2*sqrt(d*x*e + c*e)*a^2*e^(-1)/d + integrate(2*(sqrt(d^2*x^2 + 2*c
*d*x + c^2 + 1)*((a*b*d^2 - 2*b^2*d^2)*x^2*e^(1/2) + 2*(a*b*c*d - 2*b^2*c*d
)*x*e^(1/2) - (2*b^2*c^2 - (c^2 + 1)*a*b)*e^(1/2))*sqrt(d*x + c) + ((a*b*d^
3 - 2*b^2*d^3)*x^3*e^(1/2) + 3*(a*b*c*d^2 - 2*b^2*c*d^2)*x^2*e^(1/2) + ((3*
c^2*d + d)*a*b - 2*(3*c^2*d + d)*b^2)*x*e^(1/2) + ((c^3 + c)*a*b - 2*(c^3 +
 c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1))/(d^4*x^4*e + 4*c*d^3*x^3*e + (6*c^2*d^2 + d^2)*x^2*e + 2*(2*c^3*d +
c*d)*x*e + (c^4 + c^2)*e + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d + d)*x*e +
 (c^3 + c)*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*e^(-1/2)/s
qrt(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^2}{\sqrt{e(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

3.241 $\int \frac{(a+b\sinh^{-1}(c+dx))^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2(a+b\sinh^{-1}(c+dx))^2}{de\sqrt{e(c+dx)}}+\frac{8b\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)}{de^2}-\frac{16b^2(e(c+dx}{}$$

```
[Out] -16/3*b^2*(e*(d*x+c))^(3/2)*hypergeom([3/4, 3/4, 1],[5/4, 7/4],-(d*x+c)^2)/
d/e^3-2*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arcsinh(d*x+c
))*hypergeom([1/4, 1/2],[5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^2
```

Rubi [A]
time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$-\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{e(c+dx)}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)(a+b\sinh^{-1}(c+dx))}{de^2}-\frac{2(a+b\sinh^{-1}(c+dx))^2}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^
2])/(d*e^2) - (16*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1},
{5/4, 7/4}, -(c + d*x)^2])/(3*d*e^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{\sqrt{ex}\sqrt{1+x^2}}\,dx, x, c+dx\right)}{de}$$
$$= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}\left(a+b\sinh^{-1}(c+dx)\right)\ {}_2F_1\left(\frac{1}{4},\right.}{de^2}$$

Mathematica [A]
time = 0.07, size = 109, normalized size = 0.84

$$\frac{2\left(-3\left(a+b\sinh^{-1}(c+dx)\right)^2-4b(c+dx)\left(-3\left(a+b\sinh^{-1}(c+dx)\right)\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-(c+dx)^2\right)+2b(c+dx)\ {}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};-(c+dx)^2\right)\right)\right)}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcSinh[c + d*x])^2 - 4*b*(c + d*x)*(-3*(a + b*ArcSinh[c + d*
x])*Hypergeometric2F1[1/4, 1/2, 5/4, -(c + d*x)^2] + 2*b*(c + d*x)*Hypergeo
metricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, -(c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*
x)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2}{\left(dex+ce\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^2/(d^2*x*e^2 + c*d*e^2) - 2*a^2*e^(-1)/(sqrt(d*x*e + c*e)*d) + integrate
(2*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b*d^2 + 2*b^2*d^2)*x^2*e^(1/2) +
2*(a*b*c*d + 2*b^2*c*d)*x*e^(1/2) + (2*b^2*c^2 + (c^2 + 1)*a*b)*e^(1/2))*sq
rt(d*x + c) + ((a*b*d^3 + 2*b^2*d^3)*x^3*e^(1/2) + 3*(a*b*c*d^2 + 2*b^2*c*d
^2)*x^2*e^(1/2) + ((3*c^2*d + d)*a*b + 2*(3*c^2*d + d)*b^2)*x*e^(1/2) + ((c
^3 + c)*a*b + 2*(c^3 + c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d
^2*x^2 + 2*c*d*x + c^2 + 1))/(d^5*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 +
 d^3)*x^3*e^2 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^2 + (5*c^4*d + 3*c^2*d)*x*e^2
+ (c^5 + c^3)*e^2 + (d^4*x^4*e^2 + 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*
e^2 + 2*(2*c^3*d + c*d)*x*e^2 + (c^4 + c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-3/2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^2}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

3.242 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}-\frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\ {}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)}{3de^2\sqrt{e(c+dx)}}+\frac{16b^2\sqrt{e(c+dx)}\ {}_3F_2\left(\frac{1}{4},\right.}{3de}$$

[Out] -2/3*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arcsinh(d*x+c))*hypergeom([-1/4, 1/2],[3/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(1/2)+16/3*b^2*hypergeom([1/4, 1/4, 1],[3/4, 5/4],-(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^3

Rubi [A]
time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$\frac{16b^2\sqrt{e(c+dx)}\ {}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)}{3de^3}-\frac{8b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{3de^2\sqrt{e(c+dx)}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]

[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*(a + b*ArcSinh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2])/(3*d*e^2*Sqrt[e*(c + d*x)]) + (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])/(3*d*e^3)

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}+\frac{(4b)\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}$$

$$=-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}}-\frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+\right.}{3de^2\sqrt{e(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 0.79

$$-\frac{2\left(\left(a+b\sinh^{-1}(c+dx)\right)^2+4b(c+dx)\left(\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-(c+dx)^2\right)-2b(c+dx)\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};-(c+dx)^2\right)\right)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])^2 + 4*b*(c + d*x)*((a + b*ArcSinh[c + d*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, -(c + d*x)^2] - 2*b*(c + d*x)*Hypergeometr
icPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, -(c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2
))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2}{\left(dex+ce\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2/3*a^2*e^(-1)/((d*x*e +
 c*e)^(3/2)*d) + integrate(2/3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*a*b*d
^2 + 2*b^2*d^2)*x^2*e^(1/2) + 2*(3*a*b*c*d + 2*b^2*c*d)*x*e^(1/2) + (2*b^2*
c^2 + 3*(c^2 + 1)*a*b)*e^(1/2))*sqrt(d*x + c) + ((3*a*b*d^3 + 2*b^2*d^3)*x^
3*e^(1/2) + 3*(3*a*b*c*d^2 + 2*b^2*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d + d)*a*
b + 2*(3*c^2*d + d)*b^2)*x*e^(1/2) + (3*(c^3 + c)*a*b + 2*(c^3 + c)*b^2)*e^
(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))/(d^6
*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*c^2*d^4 + d^4)*x^4*e^3 + 4*(5*c^3*d^3 + c*
d^3)*x^3*e^3 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^3 + 2*(3*c^5*d + 2*c^3*d)*x*
e^3 + (c^6 + c^4)*e^3 + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)
*x^3*e^3 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^3 + (5*c^4*d + 3*c^2*d)*x*e^3 + (c^
5 + c^3)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-5/2)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}{\left(c + d x \right)}\right)^{2}}{\left(e \left(c + d x\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

3.243 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}-\frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)}{15de^2(e(c+dx))^{3/2}}-\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};\right.}{15de^3\sqrt{e(c+d}}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arcsinh(d*x+c
))*hypergeom([-3/4, 1/2],[1/4],-(d*x+c)^2)/d/e^2/(e*(d*x+c))^(3/2)-16/15*b^
2*hypergeom([-1/4, -1/4, 1],[1/4, 3/4],-(d*x+c)^2)/d/e^3/(e*(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5859, 5776, 5817}

$$-\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)\left(a+b\sinh^{-1}(c+dx)\right)}{15de^2(e(c+dx))^{3/2}}-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*(a + b*A
rcSinh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2])/(15*d*e^2
*(e*(c + d*x))^(3/2)) - (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/
4}, -(c + d*x)^2])/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^2}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}$$
$$=-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}+\frac{(4b)\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}$$
$$=-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}-\frac{8b\left(a+b\sinh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+\right.}{15de^2(e(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.82

$$-\frac{2\left(\left(a+b\sinh^{-1}(c+dx)\right)\left(3\left(a+b\sinh^{-1}(c+dx)\right)+4b(c+dx)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-(c+dx)^2\right)\right)+8b^2(c+dx)^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};-(c+dx)^2\right)\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*((a + b*ArcSinh[c + d*x])*(3*(a + b*ArcSinh[c + d*x]) + 4*b*(c + d*x)*H
ypergeometric2F1[-3/4, 1/2, 1/4, -(c + d*x)^2]) + 8*b^2*(c + d*x)^2*Hyperge
ometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, -(c + d*x)^2]))/(15*d*e*(e*(c + d*x
))^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^2}{\left(dex+ce\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(d*x + c)*b^2*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^2/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 2/5*
a^2*e^(-1)/((d*x*e + c*e)^(5/2)*d) + integrate(2/5*(sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)*((5*a*b*d^2 + 2*b^2*d^2)*x^2*e^(1/2) + 2*(5*a*b*c*d + 2*b^2*c*d)
*x*e^(1/2) + (2*b^2*c^2 + 5*(c^2 + 1)*a*b)*e^(1/2))*sqrt(d*x + c) + ((5*a*b
*d^3 + 2*b^2*d^3)*x^3*e^(1/2) + 3*(5*a*b*c*d^2 + 2*b^2*c*d^2)*x^2*e^(1/2) +
 (5*(3*c^2*d + d)*a*b + 2*(3*c^2*d + d)*b^2)*x*e^(1/2) + (5*(c^3 + c)*a*b +
 2*(c^3 + c)*b^2)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1))/(d^7*x^7*e^4 + 7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4
 + 5*(7*c^3*d^4 + c*d^4)*x^4*e^4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*
c^5*d^2 + 10*c^3*d^2)*x^2*e^4 + (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4
 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d
^3 + c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^
3*d)*x*e^4 + (c^6 + c^4)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arcsinh(d*x + c)^2 + 2*a*b*arcsinh(d*x + c) + a^2)*sqrt(d*x +
 c)*e^(-7/2)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^2}{(e(c + dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**2/(e*(c + d*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{asinh}(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)
```

3.244 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/d/e-2/3*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^3)/(9*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{9de} - \frac{(2b)\text{Subst}\left(\int \frac{(ex)^{9/}}{}\right.}{}$$

Mathematica [A]
time = 172.82, size = 0, normalized size = 0.00

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{7}{2}} (a + b\operatorname{arcsinh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/9*(d*x*e + c*e)^(9/2)*a^3*e^(-1)/d + 2/9*(b^3*d^4*x^4*e^(7/2) + 4*b^3*c*d
^3*x^3*e^(7/2) + 6*b^3*c^2*d^2*x^2*e^(7/2) + 4*b^3*c^3*d*x*e^(7/2) + b^3*c^
4*e^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3
/d + integrate(1/3*((((9*a*b^2*d^5 - 2*b^3*d^5)*x^5*e^(7/2) + 5*(9*a*b^2*c*
d^4 - 2*b^3*c*d^4)*x^4*e^(7/2) - (20*b^3*c^2*d^3 - 9*(10*c^2*d^3 + d^3)*a*b
^2)*x^3*e^(7/2) - (20*b^3*c^3*d^2 - 9*(10*c^3*d^2 + 3*c*d^2)*a*b^2)*x^2*e^(
7/2) - (10*b^3*c^4*d - 9*(5*c^4*d + 3*c^2*d)*a*b^2)*x*e^(7/2) - (2*b^3*c^5
- 9*(c^5 + c^3)*a*b^2)*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x
+ c) + ((9*a*b^2*d^6 - 2*b^3*d^6)*x^6*e^(7/2) + 6*(9*a*b^2*c*d^5 - 2*b^3*c*
d^5)*x^5*e^(7/2) + (9*(15*c^2*d^4 + d^4)*a*b^2 - 2*(15*c^2*d^4 + d^4)*b^3)*
x^4*e^(7/2) + 4*(9*(5*c^3*d^3 + c*d^3)*a*b^2 - 2*(5*c^3*d^3 + c*d^3)*b^3)*x
^3*e^(7/2) + 3*(9*(5*c^4*d^2 + 2*c^2*d^2)*a*b^2 - 2*(5*c^4*d^2 + 2*c^2*d^2)
*b^3)*x^2*e^(7/2) + 2*(9*(3*c^5*d + 2*c^3*d)*a*b^2 - 2*(3*c^5*d + 2*c^3*d)*
b^3)*x*e^(7/2) + (9*(c^6 + c^4)*a*b^2 - 2*(c^6 + c^4)*b^3)*e^(7/2))*sqrt(d*
x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 9*((a^2*b*d^5*
x^5*e^(7/2) + 5*a^2*b*c*d^4*x^4*e^(7/2) + (10*c^2*d^3 + d^3)*a^2*b*x^3*e^(7
/2) + (10*c^3*d^2 + 3*c*d^2)*a^2*b*x^2*e^(7/2) + (5*c^4*d + 3*c^2*d)*a^2*b*
x*e^(7/2) + (c^5 + c^3)*a^2*b*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sq
rt(d*x + c) + (a^2*b*d^6*x^6*e^(7/2) + 6*a^2*b*c*d^5*x^5*e^(7/2) + (15*c^2*
d^4 + d^4)*a^2*b*x^4*e^(7/2) + 4*(5*c^3*d^3 + c*d^3)*a^2*b*x^3*e^(7/2) + 3*
(5*c^4*d^2 + 2*c^2*d^2)*a^2*b*x^2*e^(7/2) + 2*(3*c^5*d + 2*c^3*d)*a^2*b*x*e
^(7/2) + (c^6 + c^4)*a^2*b*e^(7/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x
^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x +
(d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*arcsinh
(d*x + c)^3*e^3 + 3*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x +
a*b^2*c^3)*arcsinh(d*x + c)^2*e^3 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 +
3*a^2*b*c^2*d*x + a^2*b*c^3)*arcsinh(d*x + c)*e^3 + (a^3*d^3*x^3 + 3*a^3*c*
d^2*x^2 + 3*a^3*c^2*d*x + a^3*c^3)*e^3)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,(a+b\,\mathrm{asinh}(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^3, x)
```

3.245 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^3)/(7*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\sinh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{7de} - \frac{(6b)\text{Subst}\left(\int \frac{(ex)^{7/2}(}{\sqrt{}}\right.}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

[Out] $Aborted

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{5}{2}} (a + b \operatorname{arcsinh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a^3*e^(-1)/d + 2/7*(b^3*d^3*x^3*e^(5/2) + 3*b^3*c*d
^2*x^2*e^(5/2) + 3*b^3*c^2*d*x*e^(5/2) + b^3*c^3*e^(5/2))*sqrt(d*x + c)*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/d + integrate(3/7*((((7*a*b
^2*d^4 - 2*b^3*d^4)*x^4*e^(5/2) + 4*(7*a*b^2*c*d^3 - 2*b^3*c*d^3)*x^3*e^(5/
2) - (12*b^3*c^2*d^2 - 7*(6*c^2*d^2 + d^2)*a*b^2)*x^2*e^(5/2) - 2*(4*b^3*c^
3*d - 7*(2*c^3*d + c*d)*a*b^2)*x*e^(5/2) - (2*b^3*c^4 - 7*(c^4 + c^2)*a*b^2
)*e^(5/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((7*a*b^2*d^5
- 2*b^3*d^5)*x^5*e^(5/2) + 5*(7*a*b^2*c*d^4 - 2*b^3*c*d^4)*x^4*e^(5/2) + (7
*(10*c^2*d^3 + d^3)*a*b^2 - 2*(10*c^2*d^3 + d^3)*b^3)*x^3*e^(5/2) + (7*(10*
c^3*d^2 + 3*c*d^2)*a*b^2 - 2*(10*c^3*d^2 + 3*c*d^2)*b^3)*x^2*e^(5/2) + (7*(
5*c^4*d + 3*c^2*d)*a*b^2 - 2*(5*c^4*d + 3*c^2*d)*b^3)*x*e^(5/2) + (7*(c^5 +
 c^3)*a*b^2 - 2*(c^5 + c^3)*b^3)*e^(5/2))*sqrt(d*x + c))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 7*((a^2*b*d^4*x^4*e^(5/2) + 4*a^2*b*c*d^
3*x^3*e^(5/2) + (6*c^2*d^2 + d^2)*a^2*b*x^2*e^(5/2) + 2*(2*c^3*d + c*d)*a^2
*b*x*e^(5/2) + (c^4 + c^2)*a^2*b*e^(5/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
*sqrt(d*x + c) + (a^2*b*d^5*x^5*e^(5/2) + 5*a^2*b*c*d^4*x^4*e^(5/2) + (10*c
^2*d^3 + d^3)*a^2*b*x^3*e^(5/2) + (10*c^3*d^2 + 3*c*d^2)*a^2*b*x^2*e^(5/2)
+ (5*c^4*d + 3*c^2*d)*a^2*b*x*e^(5/2) + (c^5 + c^3)*a^2*b*e^(5/2))*sqrt(d*x
 + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2
*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(((b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*arcsinh(d*x + c)^3*e^2 + 3*
(a*b^2*d^2*x^2 + 2*a*b^2*c*d*x + a*b^2*c^2)*arcsinh(d*x + c)^2*e^2 + 3*(a^2
*b*d^2*x^2 + 2*a^2*b*c*d*x + a^2*b*c^2)*arcsinh(d*x + c)*e^2 + (a^3*d^2*x^2
 + 2*a^3*c*d*x + a^3*c^2)*e^2)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{5/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^3, x)
```

3.246 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^3 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de} - \frac{(6b)\text{Subst}\left(\int \frac{(ex)^{5/}}{}\right.}{}$$

Mathematica [A]
time = 156.26, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}} (a+b\operatorname{arcsinh}(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^3*e^(-1)/d + 2/5*(b^3*d^2*x^2*e^(3/2) + 2*b^3*c*d
*x*e^(3/2) + b^3*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^3/d + integrate(3/5*((((5*a*b^2*d^3 - 2*b^3*d^3)*x^3*e^(3
/2) + 3*(5*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2*e^(3/2) - (6*b^3*c^2*d - 5*(3*c^2
*d + d)*a*b^2)*x*e^(3/2) - (2*b^3*c^3 - 5*(c^3 + c)*a*b^2)*e^(3/2))*sqrt(d^
2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((5*a*b^2*d^4 - 2*b^3*d^4)*x^4*e
^(3/2) + 4*(5*a*b^2*c*d^3 - 2*b^3*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2 + d^2)
*a*b^2 - 2*(6*c^2*d^2 + d^2)*b^3)*x^2*e^(3/2) + 2*(5*(2*c^3*d + c*d)*a*b^2
- 2*(2*c^3*d + c*d)*b^3)*x*e^(3/2) + (5*(c^4 + c^2)*a*b^2 - 2*(c^4 + c^2)*b
^3)*e^(3/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)
)^2 + 5*((a^2*b*d^3*x^3*e^(3/2) + 3*a^2*b*c*d^2*x^2*e^(3/2) + (3*c^2*d + d)
*a^2*b*x*e^(3/2) + (c^3 + c)*a^2*b*e^(3/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)*sqrt(d*x + c) + (a^2*b*d^4*x^4*e^(3/2) + 4*a^2*b*c*d^3*x^3*e^(3/2) + (6*
c^2*d^2 + d^2)*a^2*b*x^2*e^(3/2) + 2*(2*c^3*d + c*d)*a^2*b*x*e^(3/2) + (c^4
 + c^2)*a^2*b*e^(3/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*
c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(((b^3*d*x + b^3*c)*arcsinh(d*x + c)^3*e + 3*(a*b^2*d*x + a*b^2*c)*
arcsinh(d*x + c)^2*e + 3*(a^2*b*d*x + a^2*b*c)*arcsinh(d*x + c)*e + (a^3*d*
x + a^3*c)*e)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{asinh}(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^3, x)
```

3.247 $\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=80

$$\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+c
))^(3/2)*(a+b*arcsinh(d*x+c))^2/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c +
 d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx&=\frac{\text{Subst}\left(\int \sqrt{ex}\,\left(a+b\sinh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}\\&=\frac{2(e(c+dx))^{3/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{3de}-\frac{(2b)\text{Subst}\left(\int \frac{(ex)^{3/2}(a+}{\sqrt{1}}\right.}{}\end{aligned}$$

Mathematica [A]
time = 167.70, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex}\,\left(a+b\sinh^{-1}(c+dx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^3, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arcsinh}\,(dx+c))^3\,\sqrt{dex+ce}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a^3*e^(-1)/d + 2/3*(b^3*d*x*e^(1/2) + b^3*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/d + inte
grate(((sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*a*b^2*d^2 - 2*b^3*d^2)*x^2*e^
(1/2) + 2*(3*a*b^2*c*d - 2*b^3*c*d)*x*e^(1/2) - (2*b^3*c^2 - 3*(c^2 + 1)*a*
b^2)*e^(1/2))*sqrt(d*x + c) + ((3*a*b^2*d^3 - 2*b^3*d^3)*x^3*e^(1/2) + 3*(3
*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d + d)*a*b^2 - 2*(3*c^2
*d + d)*b^3)*x*e^(1/2) + (3*(c^3 + c)*a*b^2 - 2*(c^3 + c)*b^3)*e^(1/2))*sqr
t(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*((a^2*b*
d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b*e^(1/2))*sqrt(d^2
*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b*d^3*x^3*e^(1/2) + 3*a^2*b*
c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^2*b*x*e^(1/2) + (c^3 + c)*a^2*b*e^(1/2)
)*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3
 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2
) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\left(c+dx\right)}\left(a+b\operatorname{asinh}\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3,x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^3, x)
```

3.248 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^3}{de}-\frac{6b\mathrm{Int}\left(\frac{\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^2}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arcsin
h(d*x+c))^2*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^2)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\sinh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\
&= \frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^3}{de}-\frac{(6b)\mathrm{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\sinh^{-1}(x))^2}{\sqrt{1+x^2}}\,d\right.}{de}
\end{aligned}$$

Mathematica [A]
time = 131.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(c+dx))^3}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsinh}(dx + c))^3}{\sqrt{dex + ce}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(d*x*e + c*e)*a^3*e^(-1)/d + 2*(b^3*d*x*e^(1/2) + b^3*c*e^(1/2))*e^(-
1)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(sqrt(d*x + c)*d) + i
ntegrate(3*(((a*b^2*d^3 - 2*b^3*d^3)*x^3*e^(1/2) + 3*(a*b^2*c*d^2 - 2*b^3*c
*d^2)*x^2*e^(1/2) + ((3*c^2*d + d)*a*b^2 - 2*(3*c^2*d + d)*b^3)*x*e^(1/2) +
 ((c^3 + c)*a*b^2 - 2*(c^3 + c)*b^3)*e^(1/2) + sqrt(d^2*x^2 + 2*c*d*x + c^2
 + 1)*((a*b^2*d^2 - 2*b^3*d^2)*x^2*e^(1/2) + 2*(a*b^2*c*d - 2*b^3*c*d)*x*e^
(1/2) - (2*b^3*c^2 - (c^2 + 1)*a*b^2)*e^(1/2)))*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 + 1))^2 + (a^2*b*d^3*x^3*e^(1/2) + 3*a^2*b*c*d^2*x^2*e^(1/2
) + (3*c^2*d + d)*a^2*b*x*e^(1/2) + (c^3 + c)*a^2*b*e^(1/2) + (a^2*b*d^2*x^
2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b*e^(1/2))*sqrt(d^2*x^2 +
 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/((d^
2*x^2*e + 2*c*d*x*e + (c^2 + 1)*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d
*x + c) + (d^3*x^3*e + 3*c*d^2*x^2*e + (3*c^2*d + d)*x*e + (c^3 + c)*e)*sqr
t(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*e^(-1/2)/sqrt(d*x + c), x)

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^3}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c + d\,x))^3}{\sqrt{c\,e + d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

3.249 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{6b\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^2}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcsi
nh(d*x+c))^2/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcSinh[x])^2/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x]
)/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 97.92, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arcsinh}\,(dx + c))^3}{(dex + ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*e^(-3/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/(sqrt(d*
x + c)*d) - 2*a^3*e^(-1)/(sqrt(d*x*e + c*e)*d) + integrate(3*(((c^3 + c)*a*
b^2 + 2*(c^3 + c)*b^3 + (a*b^2*d^3 + 2*b^3*d^3)*x^3 + 3*(a*b^2*c*d^2 + 2*b^
3*c*d^2)*x^2 + ((3*c^2*d + d)*a*b^2 + 2*(3*c^2*d + d)*b^3)*x + (2*b^3*c^2 +
 (c^2 + 1)*a*b^2 + (a*b^2*d^2 + 2*b^3*d^2)*x^2 + 2*(a*b^2*c*d + 2*b^3*c*d)*
x)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 + 1))^2 + (a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d + d)*a^2*b*x
+ (c^3 + c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 + 1)*a^2*b)*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1)))/((d^3*x^3*e^(3/2) + 3*c*d^2*x^2*e^(3/2) + (3*c^2*d + d)*x*e^(3/2) + (c
^3 + c)*e^(3/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (d^4*x^4
*e^(3/2) + 4*c*d^3*x^3*e^(3/2) + (6*c^2*d^2 + d^2)*x^2*e^(3/2) + 2*(2*c^3*d
 + c*d)*x*e^(3/2) + (c^4 + c^2)*e^(3/2))*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-3/2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^3}{(e(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

3.250 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=80

$$-\frac{2(a+b\sinh^{-1}(c+dx))^3}{3de(e(c+dx))^{3/2}}+\frac{2b\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^2}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*arc
sinh(d*x+c))^2/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2(a+b\sinh^{-1}(c+dx))^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 78.28, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arcsinh}(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*b^3*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/((d^2*x
*e^3 + c*d*e^3)*sqrt(d*x + c)) - 2/3*a^3*e^(-1)/((d*x*e + c*e)^(3/2)*d) + i
ntegrate((((3*a*b^2*d^3 + 2*b^3*d^3)*x^3*e^(1/2) + 3*(3*a*b^2*c*d^2 + 2*b^3
*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d + d)*a*b^2 + 2*(3*c^2*d + d)*b^3)*x*e^(1/
2) + (3*(c^3 + c)*a*b^2 + 2*(c^3 + c)*b^3)*e^(1/2) + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1)*((3*a*b^2*d^2 + 2*b^3*d^2)*x^2*e^(1/2) + 2*(3*a*b^2*c*d + 2*b^3
*c*d)*x*e^(1/2) + (2*b^3*c^2 + 3*(c^2 + 1)*a*b^2)*e^(1/2)))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 3*(a^2*b*d^3*x^3*e^(1/2) + 3*a^2*b*c*
d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^2*b*x*e^(1/2) + (c^3 + c)*a^2*b*e^(1/2) +
 (a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b*e^(1/2))*
sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)))/((d^4*x^4*e^3 + 4*c*d^3*x^3*e^3 + (6*c^2*d^2 + d^2)*x^2*e^3 + 2*(
2*c^3*d + c*d)*x*e^3 + (c^4 + c^2)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*s
qrt(d*x + c) + (d^5*x^5*e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)*x^3*e^3
+ (10*c^3*d^2 + 3*c*d^2)*x^2*e^3 + (5*c^4*d + 3*c^2*d)*x*e^3 + (c^5 + c^3)*
e^3)*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-5/2)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
d*x + c^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^3}{(e(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

3.251 $\int \frac{(a+b\sinh^{-1}(c+dx))^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{6b\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^2}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^2/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^2/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A]
time = 139.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^{3}}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
[Out] -2/5*b^3*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3/((d^3*x
^2*e^4 + 2*c*d^2*x*e^4 + c^2*d*e^4)*sqrt(d*x + c)) - 2/5*a^3*e^(-1)/((d*x*e
 + c*e)^(5/2)*d) + integrate(3/5*(((5*a*b^2*d^3 + 2*b^3*d^3)*x^3*e^(1/2) +
3*(5*a*b^2*c*d^2 + 2*b^3*c*d^2)*x^2*e^(1/2) + (5*(3*c^2*d + d)*a*b^2 + 2*(3
*c^2*d + d)*b^3)*x*e^(1/2) + (5*(c^3 + c)*a*b^2 + 2*(c^3 + c)*b^3)*e^(1/2)
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((5*a*b^2*d^2 + 2*b^3*d^2)*x^2*e^(1/2)
+ 2*(5*a*b^2*c*d + 2*b^3*c*d)*x*e^(1/2) + (2*b^3*c^2 + 5*(c^2 + 1)*a*b^2)*e
^(1/2)))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 5*(a^2*b*d^3*
x^3*e^(1/2) + 3*a^2*b*c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^2*b*x*e^(1/2) + (
c^3 + c)*a^2*b*e^(1/2) + (a^2*b*d^2*x^2*e^(1/2) + 2*a^2*b*c*d*x*e^(1/2) + (
c^2 + 1)*a^2*b*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/((d^5*x^5*e^4 + 5*c*d^4*x^4*e^4 + (10*c^2
*d^3 + d^3)*x^3*e^4 + (10*c^3*d^2 + 3*c*d^2)*x^2*e^4 + (5*c^4*d + 3*c^2*d)*
x*e^4 + (c^5 + c^3)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) +
(d^6*x^6*e^4 + 6*c*d^5*x^5*e^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d^3
+ c*d^3)*x^3*e^4 + 3*(5*c^4*d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^3*d
)*x*e^4 + (c^6 + c^4)*e^4)*sqrt(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsinh(d*x + c)^3 + 3*a*b^2*arcsinh(d*x + c)^2 + 3*a^2*b*arc
sinh(d*x + c) + a^3)*sqrt(d*x + c)*e^(-7/2)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*
d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{asinh}(c + dx))^3}{(e(c + dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**3/(e*(c + d*x))**(7/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{asinh}(c + d\,x))^3}{(c\,e + d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

3.252 $\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{1+(c+dx)^2}},x\right)}{9e}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/9*b*Unintegrable((e*(d*x
+c))^(9/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{7/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcSinh[c + d*x])^4)/(9*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(9/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(9*d*e)
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx=\frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\sinh^{-1}(x)\right)^4 dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{9/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{9de}-\frac{(8b)\text{Subst}\left(\int \frac{(ex)^{9/2}}{}\right.}{}$$

Mathematica [A]
time = 129.96, size = 0, normalized size = 0.00

$$\int (ce+dex)^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{7}{2}} (a + b \operatorname{arcsinh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] 2/9*(d*x*e + c*e)^(9/2)*a^4*e^(-1)/d + 2/9*(b^4*d^4*x^4*e^(7/2) + 4*b^4*c*d
^3*x^3*e^(7/2) + 6*b^4*c^2*d^2*x^2*e^(7/2) + 4*b^4*c^3*d*x*e^(7/2) + b^4*c^
4*e^(7/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4
/d + integrate(2/9*(2*(((9*a*b^3*d^5 - 2*b^4*d^5)*x^5*e^(7/2) + 5*(9*a*b^3*
c*d^4 - 2*b^4*c*d^4)*x^4*e^(7/2) - (20*b^4*c^2*d^3 - 9*(10*c^2*d^3 + d^3)*a
*b^3)*x^3*e^(7/2) - (20*b^4*c^3*d^2 - 9*(10*c^3*d^2 + 3*c*d^2)*a*b^3)*x^2*e
^(7/2) - (10*b^4*c^4*d - 9*(5*c^4*d + 3*c^2*d)*a*b^3)*x*e^(7/2) - (2*b^4*c^
5 - 9*(c^5 + c^3)*a*b^3)*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*
x + c) + ((9*a*b^3*d^6 - 2*b^4*d^6)*x^6*e^(7/2) + 6*(9*a*b^3*c*d^5 - 2*b^4*
c*d^5)*x^5*e^(7/2) + (9*(15*c^2*d^4 + d^4)*a*b^3 - 2*(15*c^2*d^4 + d^4)*b^4
)*x^4*e^(7/2) + 4*(9*(5*c^3*d^3 + c*d^3)*a*b^3 - 2*(5*c^3*d^3 + c*d^3)*b^4)
*x^3*e^(7/2) + 3*(9*(5*c^4*d^2 + 2*c^2*d^2)*a*b^3 - 2*(5*c^4*d^2 + 2*c^2*d^
2)*b^4)*x^2*e^(7/2) + 2*(9*(3*c^5*d + 2*c^3*d)*a*b^3 - 2*(3*c^5*d + 2*c^3*d
)*b^4)*x*e^(7/2) + (9*(c^6 + c^4)*a*b^3 - 2*(c^6 + c^4)*b^4)*e^(7/2))*sqrt(
d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 27*((a^2*b^2
*d^5*x^5*e^(7/2) + 5*a^2*b^2*c*d^4*x^4*e^(7/2) + (10*c^2*d^3 + d^3)*a^2*b^2
*x^3*e^(7/2) + (10*c^3*d^2 + 3*c*d^2)*a^2*b^2*x^2*e^(7/2) + (5*c^4*d + 3*c^
2*d)*a^2*b^2*x*e^(7/2) + (c^5 + c^3)*a^2*b^2*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*
x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^6*x^6*e^(7/2) + 6*a^2*b^2*c*d^5*x^5
*e^(7/2) + (15*c^2*d^4 + d^4)*a^2*b^2*x^4*e^(7/2) + 4*(5*c^3*d^3 + c*d^3)*a
^2*b^2*x^3*e^(7/2) + 3*(5*c^4*d^2 + 2*c^2*d^2)*a^2*b^2*x^2*e^(7/2) + 2*(3*c
^5*d + 2*c^3*d)*a^2*b^2*x*e^(7/2) + (c^6 + c^4)*a^2*b^2*e^(7/2))*sqrt(d*x +
 c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 18*((a^3*b*d^5*x^
5*e^(7/2) + 5*a^3*b*c*d^4*x^4*e^(7/2) + (10*c^2*d^3 + d^3)*a^3*b*x^3*e^(7/2
) + (10*c^3*d^2 + 3*c*d^2)*a^3*b*x^2*e^(7/2) + (5*c^4*d + 3*c^2*d)*a^3*b*x*
```

```
e^(7/2) + (c^5 + c^3)*a^3*b*e^(7/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt
(d*x + c) + (a^3*b*d^6*x^6*e^(7/2) + 6*a^3*b*c*d^5*x^5*e^(7/2) + (15*c^2*d^
4 + d^4)*a^3*b*x^4*e^(7/2) + 4*(5*c^3*d^3 + c*d^3)*a^3*b*x^3*e^(7/2) + 3*(5
*c^4*d^2 + 2*c^2*d^2)*a^3*b*x^2*e^(7/2) + 2*(3*c^5*d + 2*c^3*d)*a^3*b*x*e^(
7/2) + (c^6 + c^4)*a^3*b*e^(7/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2
 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d
^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(((b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*arcsinh
(d*x + c)^4*e^3 + 4*(a*b^3*d^3*x^3 + 3*a*b^3*c*d^2*x^2 + 3*a*b^3*c^2*d*x +
a*b^3*c^3)*arcsinh(d*x + c)^3*e^3 + 6*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^
2 + 3*a^2*b^2*c^2*d*x + a^2*b^2*c^3)*arcsinh(d*x + c)^2*e^3 + 4*(a^3*b*d^3*
x^3 + 3*a^3*b*c*d^2*x^2 + 3*a^3*b*c^2*d*x + a^3*b*c^3)*arcsinh(d*x + c)*e^3
 + (a^4*d^3*x^3 + 3*a^4*c*d^2*x^2 + 3*a^4*c^2*d*x + a^4*c^3)*e^3)*sqrt(d*x
+ c)*e^(1/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7315 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{7/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^(7/2)*(a + b*asinh(c + d*x))^4, x)
```

3.253 $\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{1+(c+dx)^2}},x\right)}{7e}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/7*b*Unintegrable((e*(d*x
+c))^(7/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcSinh[c + d*x])^4)/(7*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(7/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(7*d*e)
```

Rubi steps

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int (ex)^{5/2} \left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{7/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{7de} - \frac{(8b)\text{Subst}\left(\int \frac{(ex)^{7/2}}{}\right.}{}$$

Mathematica [A]
time = 148.42, size = 0, normalized size = 0.00

$$\int (ce+dex)^{5/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}} (a+b \operatorname{arcsinh}(dx+c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/7*(d*x*e + c*e)^(7/2)*a^4*e^(-1)/d + 2/7*(b^4*d^3*x^3*e^(5/2) + 3*b^4*c*d
^2*x^2*e^(5/2) + 3*b^4*c^2*d*x*e^(5/2) + b^4*c^3*e^(5/2))*sqrt(d*x + c)*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/d + integrate(2/7*(2*(((7*a
*b^3*d^4 - 2*b^4*d^4)*x^4*e^(5/2) + 4*(7*a*b^3*c*d^3 - 2*b^4*c*d^3)*x^3*e^(
5/2) - (12*b^4*c^2*d^2 - 7*(6*c^2*d^2 + d^2)*a*b^3)*x^2*e^(5/2) - 2*(4*b^4*
c^3*d - 7*(2*c^3*d + c*d)*a*b^3)*x*e^(5/2) - (2*b^4*c^4 - 7*(c^4 + c^2)*a*b
^3)*e^(5/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((7*a*b^3*d^
5 - 2*b^4*d^5)*x^5*e^(5/2) + 5*(7*a*b^3*c*d^4 - 2*b^4*c*d^4)*x^4*e^(5/2) +
(7*(10*c^2*d^3 + d^3)*a*b^3 - 2*(10*c^2*d^3 + d^3)*b^4)*x^3*e^(5/2) + (7*(1
0*c^3*d^2 + 3*c*d^2)*a*b^3 - 2*(10*c^3*d^2 + 3*c*d^2)*b^4)*x^2*e^(5/2) + (7
*(5*c^4*d + 3*c^2*d)*a*b^3 - 2*(5*c^4*d + 3*c^2*d)*b^4)*x*e^(5/2) + (7*(c^5
 + c^3)*a*b^3 - 2*(c^5 + c^3)*b^4)*e^(5/2))*sqrt(d*x + c))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 21*((a^2*b^2*d^4*x^4*e^(5/2) + 4*a^2*b
^2*c*d^3*x^3*e^(5/2) + (6*c^2*d^2 + d^2)*a^2*b^2*x^2*e^(5/2) + 2*(2*c^3*d +
 c*d)*a^2*b^2*x*e^(5/2) + (c^4 + c^2)*a^2*b^2*e^(5/2))*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^5*x^5*e^(5/2) + 5*a^2*b^2*c*d^4*x^
4*e^(5/2) + (10*c^2*d^3 + d^3)*a^2*b^2*x^3*e^(5/2) + (10*c^3*d^2 + 3*c*d^2)
*a^2*b^2*x^2*e^(5/2) + (5*c^4*d + 3*c^2*d)*a^2*b^2*x*e^(5/2) + (c^5 + c^3)*
a^2*b^2*e^(5/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1))^2 + 14*((a^3*b*d^4*x^4*e^(5/2) + 4*a^3*b*c*d^3*x^3*e^(5/2) + (6*c^2*d
^2 + d^2)*a^3*b*x^2*e^(5/2) + 2*(2*c^3*d + c*d)*a^3*b*x*e^(5/2) + (c^4 + c^
2)*a^3*b*e^(5/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*
d^5*x^5*e^(5/2) + 5*a^3*b*c*d^4*x^4*e^(5/2) + (10*c^2*d^3 + d^3)*a^3*b*x^3*
e^(5/2) + (10*c^3*d^2 + 3*c*d^2)*a^3*b*x^2*e^(5/2) + (5*c^4*d + 3*c^2*d)*a^
3*b*x*e^(5/2) + (c^5 + c^3)*a^3*b*e^(5/2))*sqrt(d*x + c))*log(d*x + c + sqr
```

```
t(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d +
d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
[Out] integral(((b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2)*arcsinh(d*x + c)^4*e^2 + 4*
(a*b^3*d^2*x^2 + 2*a*b^3*c*d*x + a*b^3*c^2)*arcsinh(d*x + c)^3*e^2 + 6*(a^2
*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + a^2*b^2*c^2)*arcsinh(d*x + c)^2*e^2 + 4*(a
^3*b*d^2*x^2 + 2*a^3*b*c*d*x + a^3*b*c^2)*arcsinh(d*x + c)*e^2 + (a^4*d^2*x
^2 + 2*a^4*c*d*x + a^4*c^2)*e^2)*sqrt(d*x + c)*e^(1/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*asinh(d*x+c))**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,\left(a+b\,\mathrm{asinh}(c+d\,x)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4,x)
[Out] int((c*e + d*e*x)^(5/2)*(a + b*asinh(c + d*x))^4, x)
```

3.254 $\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcSinh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx &= \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\sinh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}\\ &= \frac{2(e(c+dx))^{5/2}\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\text{Subst}\left(\int \frac{(ex)^{5/2}(}{\sqrt{}}\right.}{}\end{aligned}$$

Mathematica [A]
time = 116.00, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (a + b\operatorname{arcsinh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="maxima")
[Out] 2/5*(d*x*e + c*e)^(5/2)*a^4*e^(-1)/d + 2/5*(b^4*d^2*x^2*e^(3/2) + 2*b^4*c*d
*x*e^(3/2) + b^4*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1))^4/d + integrate(2/5*(2*(((5*a*b^3*d^3 - 2*b^4*d^3)*x^3*e^
(3/2) + 3*(5*a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2*e^(3/2) - (6*b^4*c^2*d - 5*(3*c
^2*d + d)*a*b^3)*x*e^(3/2) - (2*b^4*c^3 - 5*(c^3 + c)*a*b^3)*e^(3/2))*sqrt(
d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + ((5*a*b^3*d^4 - 2*b^4*d^4)*x^4
*e^(3/2) + 4*(5*a*b^3*c*d^3 - 2*b^4*c*d^3)*x^3*e^(3/2) + (5*(6*c^2*d^2 + d^
2)*a*b^3 - 2*(6*c^2*d^2 + d^2)*b^4)*x^2*e^(3/2) + 2*(5*(2*c^3*d + c*d)*a*b^
3 - 2*(2*c^3*d + c*d)*b^4)*x*e^(3/2) + (5*(c^4 + c^2)*a*b^3 - 2*(c^4 + c^2)
*b^4)*e^(3/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
1))^3 + 15*((a^2*b^2*d^3*x^3*e^(3/2) + 3*a^2*b^2*c*d^2*x^2*e^(3/2) + (3*c^2
*d + d)*a^2*b^2*x*e^(3/2) + (c^3 + c)*a^2*b^2*e^(3/2))*sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^4*x^4*e^(3/2) + 4*a^2*b^2*c*d^3*x^
3*e^(3/2) + (6*c^2*d^2 + d^2)*a^2*b^2*x^2*e^(3/2) + 2*(2*c^3*d + c*d)*a^2*b
^2*x*e^(3/2) + (c^4 + c^2)*a^2*b^2*e^(3/2))*sqrt(d*x + c))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 10*((a^3*b*d^3*x^3*e^(3/2) + 3*a^3*b*c
*d^2*x^2*e^(3/2) + (3*c^2*d + d)*a^3*b*x*e^(3/2) + (c^3 + c)*a^3*b*e^(3/2))
*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^4*x^4*e^(3/2) +
 4*a^3*b*c*d^3*x^3*e^(3/2) + (6*c^2*d^2 + d^2)*a^3*b*x^2*e^(3/2) + 2*(2*c^3
*d + c*d)*a^3*b*x*e^(3/2) + (c^4 + c^2)*a^3*b*e^(3/2))*sqrt(d*x + c))*log(d
*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 +
 (3*c^2*d + d)*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(((b^4*d*x + b^4*c)*arcsinh(d*x + c)^4*e + 4*(a*b^3*d*x + a*b^3*c)*
arcsinh(d*x + c)^3*e + 6*(a^2*b^2*d*x + a^2*b^2*c)*arcsinh(d*x + c)^2*e + 4
*(a^3*b*d*x + a^3*b*c)*arcsinh(d*x + c)*e + (a^4*d*x + a^4*c)*e)*sqrt(d*x +
 c)*e^(1/2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{asinh}(c+dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*asinh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*asinh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arcsinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*asinh(c + d*x))^4, x)
```

3.255 $\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=82

$$\frac{2(e(c+dx))^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4}{3de} - \frac{8b\text{Int}\left(\frac{(e(c+dx))^{3/2}(a+b\sinh^{-1}(c+dx))^3}{\sqrt{1+(c+dx)^2}}, x\right)}{3e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arcsinh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*x
+c))^(3/2)*(a+b*arcsinh(d*x+c))^3/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcSinh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c +
 d*x])/(3*d*e)
```

Rubi steps

$$\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int \sqrt{ex} \left(a+b\sinh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2} \left(a+b\sinh^{-1}(c+dx)\right)^4}{3de} - \frac{(8b)\text{Subst}\left(\int \frac{(ex)^{3/2}(\ldots}{\sqrt{\ldots}}\right.}{\ldots}$$

Mathematica [A]
time = 147.14, size = 0, normalized size = 0.00

$$\int \sqrt{ce+dex} \left(a+b\sinh^{-1}(c+dx)\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4,x]
```

```
[Out] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcSinh[c + d*x])^4, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(dx+c\right)\right)^{4} \sqrt{dex+ce}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*x*e + c*e)^(3/2)*a^4*e^(-1)/d + 2/3*(b^4*d*x*e^(1/2) + b^4*c*e^(1/2)
)*sqrt(d*x + c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^4/d + inte
grate(2/3*(2*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*a*b^3*d^2 - 2*b^4*d^2)*
x^2*e^(1/2) + 2*(3*a*b^3*c*d - 2*b^4*c*d)*x*e^(1/2) - (2*b^4*c^2 - 3*(c^2 +
 1)*a*b^3)*e^(1/2))*sqrt(d*x + c) + ((3*a*b^3*d^3 - 2*b^4*d^3)*x^3*e^(1/2)
+ 3*(3*a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2*e^(1/2) + (3*(3*c^2*d + d)*a*b^3 - 2*
(3*c^2*d + d)*b^4)*x*e^(1/2) + (3*(c^3 + c)*a*b^3 - 2*(c^3 + c)*b^4)*e^(1/2
))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 9*((
a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b^2*e^(1/
2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^3*x^3*e^(1
/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^2*b^2*x*e^(1/2) + (c^3
+ c)*a^2*b^2*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
 c^2 + 1))^2 + 6*((a^3*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 + 1
)*a^3*b*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d
^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^3*b*x*e^(1/2)
+ (c^3 + c)*a^3*b*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (3*c^2*d + d)*x + (d^2*x^2
+ 2*c*d*x + c^2 + 1)^(3/2) + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*x + c)*e^(1/2),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,(a+b\operatorname{asinh}{(c+dx)})^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*asinh(c + d*x))**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arcsinh(d*x + c) + a)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*asinh(c + d*x))^4, x)
```

3.256 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=78

$$\frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^4}{de}-\frac{8b\text{Int}\left(\frac{\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^3}{\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] 2*(a+b*arcsinh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arcsin
h(d*x+c))^3*(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sinh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcSinh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcSinh[x])^3)/Sqrt[1 + x^2], x], x, c + d*x])
/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sinh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^4}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\ &= \frac{2\sqrt{e(c+dx)}\,(a+b\sinh^{-1}(c+dx))^4}{de}-\frac{(8b)\text{Subst}\left(\int \frac{\sqrt{ex}\,(a+b\sinh^{-1}(x))^3}{\sqrt{1+x^2}}\,dx\right.}{de}\end{aligned}$$

Mathematica [A]
time = 48.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sinh^{-1}(c+dx))^4}{\sqrt{ce+dex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{arcsinh}\left(d x+c\right)\right)^{4}}{\sqrt{d e x+c e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(d*x + c)*b^4*e^(-1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^4/d + 2*sqrt(d*x*e + c*e)*a^4*e^(-1)/d + integrate(2*(2*(sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*((a*b^3*d^2 - 2*b^4*d^2)*x^2*e^(1/2) + 2*(a*b^3*c*d - 2*
b^4*c*d)*x*e^(1/2) - (2*b^4*c^2 - (c^2 + 1)*a*b^3)*e^(1/2))*sqrt(d*x + c) +
 ((a*b^3*d^3 - 2*b^4*d^3)*x^3*e^(1/2) + 3*(a*b^3*c*d^2 - 2*b^4*c*d^2)*x^2*e
^(1/2) + ((3*c^2*d + d)*a*b^3 - 2*(3*c^2*d + d)*b^4)*x*e^(1/2) + ((c^3 + c)
*a*b^3 - 2*(c^3 + c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 + 1))^3 + 3*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c*d*x*e
^(1/2) + (c^2 + 1)*a^2*b^2*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(
d*x + c) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) + (3*c^2*
d + d)*a^2*b^2*x*e^(1/2) + (c^3 + c)*a^2*b^2*e^(1/2))*sqrt(d*x + c))*log(d*
x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*((a^3*b*d^2*x^2*e^(1/2) +
2*a^3*b*c*d*x*e^(1/2) + (c^2 + 1)*a^3*b*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c
^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2)
+ (3*c^2*d + d)*a^3*b*x*e^(1/2) + (c^3 + c)*a^3*b*e^(1/2))*sqrt(d*x + c))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^4*x^4*e + 4*c*d^3*x^3*e
 + (6*c^2*d^2 + d^2)*x^2*e + 2*(2*c^3*d + c*d)*x*e + (c^4 + c^2)*e + (d^3*x
^3*e + 3*c*d^2*x^2*e + (3*c^2*d + d)*x*e + (c^3 + c)*e)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*e^(-1/2)/sqrt(d*x + c),
 x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^4}{\sqrt{e(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^4}{\sqrt{c\,e + d\,e\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

3.257 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{8b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{\sqrt{e(c+dx)}\sqrt{1+(c+dx)^2}},x\right)}{e}$$

```
[Out] -2*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcsinh(d*x+c))^3/(e*(d*x+c))^(1/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst][Defer[Int][(a + b*ArcSinh[x])^3/(Sqrt[e*x]*Sqrt[1 + x^2]), x], x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{\sqrt{ex}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A]
time = 74.56, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1
))^4/(d^2*x*e^2 + c*d*e^2) - 2*a^4*e^(-1)/(sqrt(d*x*e + c*e)*d) + integrate
(2*(2*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((a*b^3*d^2 + 2*b^4*d^2)*x^2*e^(1/
2) + 2*(a*b^3*c*d + 2*b^4*c*d)*x*e^(1/2) + (2*b^4*c^2 + (c^2 + 1)*a*b^3)*e^
(1/2))*sqrt(d*x + c) + ((a*b^3*d^3 + 2*b^4*d^3)*x^3*e^(1/2) + 3*(a*b^3*c*d^
2 + 2*b^4*c*d^2)*x^2*e^(1/2) + ((3*c^2*d + d)*a*b^3 + 2*(3*c^2*d + d)*b^4)*
x*e^(1/2) + ((c^3 + c)*a*b^3 + 2*(c^3 + c)*b^4)*e^(1/2))*sqrt(d*x + c))*log
(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 3*((a^2*b^2*d^2*x^2*e^(1/
2) + 2*a^2*b^2*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b^2*e^(1/2))*sqrt(d^2*x^2 + 2*
c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2
*x^2*e^(1/2) + (3*c^2*d + d)*a^2*b^2*x*e^(1/2) + (c^3 + c)*a^2*b^2*e^(1/2))
*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 2*((a^
3*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 + 1)*a^3*b*e^(1/2))*sqrt
(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^
3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^3*b*x*e^(1/2) + (c^3 + c)*a^3*b*e^(
1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^5
*x^5*e^2 + 5*c*d^4*x^4*e^2 + (10*c^2*d^3 + d^3)*x^3*e^2 + (10*c^3*d^2 + 3*c
*d^2)*x^2*e^2 + (5*c^4*d + 3*c^2*d)*x*e^2 + (c^5 + c^3)*e^2 + (d^4*x^4*e^2
+ 4*c*d^3*x^3*e^2 + (6*c^2*d^2 + d^2)*x^2*e^2 + 2*(2*c^3*d + c*d)*x*e^2 + (
c^4 + c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-3/2)/
(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(c + dx\right)\right)^{4}}{\left(e \left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

3.258 $\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{8b\text{Int}\left(\frac{\left(a+b\sinh^{-1}(c+dx)\right)^3}{(e(c+dx))^{3/2}\sqrt{1+(c+dx)^2}},x\right)}{3e}$$

```
[Out] -2/3*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(3/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(3/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\text{Subst}\left(\int \frac{\left(a+b\sinh^{-1}(x)\right)^3}{(ex)^{3/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{3de}\end{aligned}$$

Mathematica [A]
time = 75.77, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4/(d^3*x^2*e^3 + 2*c*d^2*x*e^3 + c^2*d*e^3) - 2/3*a^4*e^(-1)/((d*x*e +
 c*e)^(3/2)*d) + integrate(2/3*(2*(sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*((3*a*
b^3*d^2 + 2*b^4*d^2)*x^2*e^(1/2) + 2*(3*a*b^3*c*d + 2*b^4*c*d)*x*e^(1/2) +
(2*b^4*c^2 + 3*(c^2 + 1)*a*b^3)*e^(1/2))*sqrt(d*x + c) + ((3*a*b^3*d^3 + 2*
b^4*d^3)*x^3*e^(1/2) + 3*(3*a*b^3*c*d^2 + 2*b^4*c*d^2)*x^2*e^(1/2) + (3*(3*
c^2*d + d)*a*b^3 + 2*(3*c^2*d + d)*b^4)*x*e^(1/2) + (3*(c^3 + c)*a*b^3 + 2*
(c^3 + c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 + 1))^3 + 9*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b^2*c*d*x*e^(1/2) + (c
^2 + 1)*a^2*b^2*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqrt(d*x + c) +
(a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2) + (3*c^2*d + d)*a^2*
b^2*x*e^(1/2) + (c^3 + c)*a^2*b^2*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 6*((a^3*b*d^2*x^2*e^(1/2) + 2*a^3*b*c*d
*x*e^(1/2) + (c^2 + 1)*a^3*b*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)*sqr
t(d*x + c) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x^2*e^(1/2) + (3*c^2*d
+ d)*a^3*b*x*e^(1/2) + (c^3 + c)*a^3*b*e^(1/2))*sqrt(d*x + c))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^6*x^6*e^3 + 6*c*d^5*x^5*e^3 + (15*
c^2*d^4 + d^4)*x^4*e^3 + 4*(5*c^3*d^3 + c*d^3)*x^3*e^3 + 3*(5*c^4*d^2 + 2*c
^2*d^2)*x^2*e^3 + 2*(3*c^5*d + 2*c^3*d)*x*e^3 + (c^6 + c^4)*e^3 + (d^5*x^5*
e^3 + 5*c*d^4*x^4*e^3 + (10*c^2*d^3 + d^3)*x^3*e^3 + (10*c^3*d^2 + 3*c*d^2)
*x^2*e^3 + (5*c^4*d + 3*c^2*d)*x*e^3 + (c^5 + c^3)*e^3)*sqrt(d^2*x^2 + 2*c*
d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-5/2)/
(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^4}{(e(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^4}{(c\,e + d\,e\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

3.259 $\int \frac{(a+b\sinh^{-1}(c+dx))^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{8b\text{Int}\left(\frac{(a+b\sinh^{-1}(c+dx))^3}{(e(c+dx))^{5/2}\sqrt{1+(c+dx)^2}},x\right)}{5e}$$

```
[Out] -2/5*(a+b*arcsinh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*a
rcsinh(d*x+c))^3/(e*(d*x+c))^(5/2)/(1+(d*x+c)^2)^(1/2),x)/e
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcSinh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcSinh[x])^3/((e*x)^(5/2)*Sqrt[1 + x^2]), x], x, c
+ d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\sinh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{(ex)^{5/2}\sqrt{1+x^2}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A]
time = 131.95, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
[Out] Integrate[(a + b*ArcSinh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arcsinh}\,(dx+c))^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
[Out] int((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
[Out] -2/5*sqrt(d*x + c)*b^4*e^(1/2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 +
 1))^4/(d^4*x^3*e^4 + 3*c*d^3*x^2*e^4 + 3*c^2*d^2*x*e^4 + c^3*d*e^4) - 2/5*
a^4*e^(-1)/((d*x*e + c*e)^(5/2)*d) + integrate(2/5*(2*(sqrt(d^2*x^2 + 2*c*d
*x + c^2 + 1)*((5*a*b^3*d^2 + 2*b^4*d^2)*x^2*e^(1/2) + 2*(5*a*b^3*c*d + 2*b
^4*c*d)*x*e^(1/2) + (2*b^4*c^2 + 5*(c^2 + 1)*a*b^3)*e^(1/2))*sqrt(d*x + c)
+ ((5*a*b^3*d^3 + 2*b^4*d^3)*x^3*e^(1/2) + 3*(5*a*b^3*c*d^2 + 2*b^4*c*d^2)*
x^2*e^(1/2) + (5*(3*c^2*d + d)*a*b^3 + 2*(3*c^2*d + d)*b^4)*x*e^(1/2) + (5*
(c^3 + c)*a*b^3 + 2*(c^3 + c)*b^4)*e^(1/2))*sqrt(d*x + c))*log(d*x + c + sq
rt(d^2*x^2 + 2*c*d*x + c^2 + 1))^3 + 15*((a^2*b^2*d^2*x^2*e^(1/2) + 2*a^2*b
^2*c*d*x*e^(1/2) + (c^2 + 1)*a^2*b^2*e^(1/2))*sqrt(d^2*x^2 + 2*c*d*x + c^2
+ 1)*sqrt(d*x + c) + (a^2*b^2*d^3*x^3*e^(1/2) + 3*a^2*b^2*c*d^2*x^2*e^(1/2)
 + (3*c^2*d + d)*a^2*b^2*x*e^(1/2) + (c^3 + c)*a^2*b^2*e^(1/2))*sqrt(d*x +
c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1))^2 + 10*((a^3*b*d^2*x^2
*e^(1/2) + 2*a^3*b*c*d*x*e^(1/2) + (c^2 + 1)*a^3*b*e^(1/2))*sqrt(d^2*x^2 +
2*c*d*x + c^2 + 1)*sqrt(d*x + c) + (a^3*b*d^3*x^3*e^(1/2) + 3*a^3*b*c*d^2*x
^2*e^(1/2) + (3*c^2*d + d)*a^3*b*x*e^(1/2) + (c^3 + c)*a^3*b*e^(1/2))*sqrt(
d*x + c))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 + 1)))/(d^7*x^7*e^4 +
7*c*d^6*x^6*e^4 + (21*c^2*d^5 + d^5)*x^5*e^4 + 5*(7*c^3*d^4 + c*d^4)*x^4*e^
4 + 5*(7*c^4*d^3 + 2*c^2*d^3)*x^3*e^4 + (21*c^5*d^2 + 10*c^3*d^2)*x^2*e^4 +
 (7*c^6*d + 5*c^4*d)*x*e^4 + (c^7 + c^5)*e^4 + (d^6*x^6*e^4 + 6*c*d^5*x^5*e
^4 + (15*c^2*d^4 + d^4)*x^4*e^4 + 4*(5*c^3*d^3 + c*d^3)*x^3*e^4 + 3*(5*c^4*
d^2 + 2*c^2*d^2)*x^2*e^4 + 2*(3*c^5*d + 2*c^3*d)*x*e^4 + (c^6 + c^4)*e^4)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arcsinh(d*x + c)^4 + 4*a*b^3*arcsinh(d*x + c)^3 + 6*a^2*b^2*a
rcsinh(d*x + c)^2 + 4*a^3*b*arcsinh(d*x + c) + a^4)*sqrt(d*x + c)*e^(-7/2)/
(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{asinh}(c + dx))^4}{(e(c + dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c + d*x))**4/(e*(c + d*x))**(7/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{asinh}(c + d\,x))^4}{(c\,e + d\,e\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

3.260 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3\,dx$

Optimal. Leaf size=131

$$-\frac{3(a+bx)^2}{8b}+\frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{4b}-\frac{3\sinh^{-1}(a+bx)^2}{8b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}+(a$$

```
[Out] -3/8*(b*x+a)^2/b-3/8*arcsinh(b*x+a)^2/b-3/4*(b*x+a)^2*arcsinh(b*x+a)^2/b+1/
8*arcsinh(b*x+a)^4/b+3/4*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(
b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5785, 5783, 5776, 5812, 30}

$$-\frac{3(a+bx)^2}{8b}+\frac{\sinh^{-1}(a+bx)^4}{8b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^3}{2b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b}-\frac{3\sinh^{-1}(a+bx)^2}{8b}+\frac{3(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*(a + b*x)^2)/(8*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x
])/(4*b) - (3*ArcSinh[a + b*x]^2)/(8*b) - (3*(a + b*x)^2*ArcSinh[a + b*x]^2
)/(4*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(2*b) + ArcS
inh[a + b*x]^4/(8*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^3}{2b} + \frac{\text{Subst}\left(\int \frac{\sinh}{\sqrt{1}}\right.}{}$$

$$= -\frac{3(a+bx)^2\sinh^{-1}(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{s}}{2b}$$

$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{4b} - \frac{3(a+bx)^2\sin}{4}$$

$$= -\frac{3(a+bx)^2}{8b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{4b}$$

Mathematica [A]

time = 0.08, size = 127, normalized size = 0.97

$$\frac{-3bx(2a+bx)+6(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)-3(1+2a^2+4abx+2b^2x^2)\sinh^{-1}(a+bx)^2+4(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3+\sinh^{-1}(a+bx)^4}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-3*b*x*(2*a + b*x) + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] - 3*(1 + 2*a^2 + 4*a*b*x + 2*b^2*x^2)*ArcSinh[a + b*x]^2 + 4*(a +
 b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3 + ArcSinh[a + b*
x]^4)/(8*b)
```

Maple [A]
time = 3.50, size = 204, normalized size = 1.56

method	result
default	$4\operatorname{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,bx-6\operatorname{arcsinh}(bx+a)^2b^2x^2+4\operatorname{arcsinh}(bx+a)^3\sqrt{b^2x^2+2abx+a^2+1}\,a-1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a)^
2*b^2*x^2+4*arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x
+a)^2*a*b*x+arcsinh(b*x+a)^4-6*arcsinh(b*x+a)^2*a^2+6*arcsinh(b*x+a)*(b^2*x
^2+2*a*b*x+a^2+1)^(1/2)*b*x-3*b^2*x^2+6*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)*a-6*a*b*x-3*arcsinh(b*x+a)^2-3*a^2-3)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

Fricas [A]
time = 0.39, size = 199, normalized size = 1.52

$$\frac{4\sqrt{b^2x^2+2abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)^3-3b^2x^2+\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)^4-6abx-3(2b^2x^2+4abx+2a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)^2+6\sqrt{b^2x^2+2abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/8*(4*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))^3 - 3*b^2*x^2 + log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1))^4 - 6*a*b*x - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 + 6*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx + b^2x^2 + 1} \operatorname{asinh}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a + b\,x)^3\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.261 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=107

$$\frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b}-\frac{\sinh^{-1}(a+bx)}{4b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+}{2b}$$

```
[Out] -1/4*arcsinh(b*x+a)/b-1/2*(b*x+a)^2*arcsinh(b*x+a)/b+1/6*arcsinh(b*x+a)^3/b
+1/4*(b*x+a)*(1+(b*x+a)^2)^(1/2)/b+1/2*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^
2)^(1/2)/b
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5785, 5783, 5776, 327, 221}

$$\frac{(a+bx)\sqrt{(a+bx)^2+1}}{4b}+\frac{\sinh^{-1}(a+bx)^3}{6b}+\frac{(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{2b}-\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b}-\frac{\sinh^{-1}(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] ((a + b*x)*Sqrt[1 + (a + b*x)^2])/(4*b) - ArcSinh[a + b*x]/(4*b) - ((a + b*
x)^2*ArcSinh[a + b*x])/(2*b) + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a +
 b*x]^2)/(2*b) + ArcSinh[a + b*x]^3/(6*b)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)^2}{2b} + \frac{\text{Subst}\left(\int \frac{\text{sinh}}{\sqrt{1}}\right.}{}$$

$$= -\frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\text{sin}}{2b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{(a+bx)^2\sinh^{-1}(a+bx)}{2b} + \frac{(a}{}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}}{4b} - \frac{\sinh^{-1}(a+bx)}{4b} - \frac{(a+bx)^2\,\text{si}}{}$$

Mathematica [A]

time = 0.06, size = 110, normalized size = 1.03

$$\frac{3(a+bx)\sqrt{1+a^2+2abx+b^2x^2} - 3(1+2a^2+4abx+2b^2x^2)\sinh^{-1}(a+bx) + 6(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2 + 2\sinh^{-1}(a+bx)^3}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2,x]
```

```
[Out] (3*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*(1 + 2*a^2 + 4*a*b*x + 2
*b^2*x^2)*ArcSinh[a + b*x] + 6*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*
ArcSinh[a + b*x]^2 + 2*ArcSinh[a + b*x]^3)/(12*b)
```

Maple [A]
time = 3.50, size = 167, normalized size = 1.56

method	result
default	$\frac{6 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2abx+a^2+1}\, bx - 6 \operatorname{arcsinh}(bx+a) b^2x^2 + 6 \operatorname{arcsinh}(bx+a)^2 \sqrt{b^2x^2+2abx+a^2+1}\, a - 12}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-6*arcsinh(b*x+a)
*b^2*x^2+6*arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-12*arcsinh(b*x+
a)*a*b*x+2*arcsinh(b*x+a)^3-6*a^2*arcsinh(b*x+a)+3*(b^2*x^2+2*a*b*x+a^2+1)^
(1/2)*b*x+3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-3*arcsinh(b*x+a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

Fricas [A]
time = 0.35, size = 161, normalized size = 1.50

$$\frac{6\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+2\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-3\,(2\,b^2x^2+4\,abx+2\,a^2+1)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+3\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/12*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1))^2 + 2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))^3 - 3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)) + 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + 2abx + b^2x^2 + 1} \operatorname{asinh}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a + b\,x)^2\,\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.262 $\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^2}{4b}+\frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/4*arcsinh(b*x+a)^2/b+1/2*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5860, 5785, 5783, 30}

$$-\frac{(a+bx)^2}{4b}+\frac{\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{2b}+\frac{\sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] -1/4*(a + b*x)^2/b + ((a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x])/(2*
b) + ArcSinh[a + b*x]^2/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \sqrt{1+x^2}\,\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b} - \frac{\text{Subst}(\int x\,dx, x}{2b}$$

$$= -\frac{(a+bx)^2}{4b} + \frac{(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{2b} + $$

Mathematica [A]
time = 0.04, size = 61, normalized size = 1.00

$$\frac{-bx(2a+bx)+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)+\sinh^{-1}(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(2*a + b*x)) + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh
[a + b*x] + ArcSinh[a + b*x]^2)/(4*b)
```

Maple [A]
time = 3.51, size = 91, normalized size = 1.49

method	result
default	$\frac{2\,\mathrm{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,bx-b^2x^2+2\,\mathrm{arcsinh}(bx+a)\sqrt{b^2x^2+2abx+a^2+1}\,a-2abx+\mathrm{arcsinh}(bx+}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x-b^2*x^2+2*arcsinh(b
*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*a-2*a*b*x+arcsinh(b*x+a)^2-a^2-1)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.

time = 0.29, size = 238, normalized size = 3.90

$$-\frac{1}{4}\left(x^2+\frac{2\,ax}{b}+\frac{2\,\operatorname{arsinh}\left(bx+a\right)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b^2}-\frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{b^2}\right)b-\frac{1}{2}\left(\frac{a^2\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\sqrt{b^2x^2+2\,abx+a^2+1}\,x-\frac{(a^2+1)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,a}{b}\right)\operatorname{arsinh}\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(x^2 + 2*a*x/b + 2*arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^
2*b^2 + 4*(a^2 + 1)*b^2))/b^2 - arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))^2/b^2)*b - 1/2*(a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^
2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x - (a^2 + 1)*a
rcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b - sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*a/b)*arcsinh(b*x + a)
```

Fricas [A]
time = 0.38, size = 98, normalized size = 1.61

$$-\frac{b^2x^2+2\,abx-2\,\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2+2abx+b^2x^2+1}\,\operatorname{asinh}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)*(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}(a+b\,x)\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2), x)
```

3.263 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/2*ln(arcsinh(b*x+a))/b
```

Rubi [A]
time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5860, 5791, 3393, 3382}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + Log[ArcSinh[a + b*x]]/(2*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
```

```
n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}\\
&= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\log\left(\sinh^{-1}(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 24, normalized size = 0.77

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\log\left(\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x],x]
[Out] (CoshIntegral[2*ArcSinh[a + b*x]] + Log[ArcSinh[a + b*x]])/(2*b)
```

Maple [A]
time = 3.85, size = 23, normalized size = 0.74

method	result	size
default	$\frac{\ln(\text{arcsinh}(bx+a))+\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))}{2b}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(ln(arcsinh(b*x+a))+Chi(2*arcsinh(b*x+a)))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}}{\operatorname{asinh}(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{\mathrm{asinh}\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x),x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x), x)
```

3.264 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] (-1-(b*x+a)^2)/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b
```

Rubi [A]
time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5860, 5790, 5780, 5556, 12, 3379}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{(a+bx)^2+1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*x]
]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
```

```
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{1+(a+bx)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.31

$$-\frac{1+a^2+2abx+b^2x^2-\sinh^{-1}(a+bx)\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + a^2 + 2*a*b*x + b^2*x^2 - ArcSinh[a + b*x]*SinhIntegral[2*ArcSinh[a
+ b*x]])/(b*ArcSinh[a + b*x]))
```

Maple [**A**]
time = 4.95, size = 44, normalized size = 1.22

method	result	size
default	$\frac{2\,\mathrm{hyperbolicSineIntegral}(2\,\mathrm{arcsinh}(bx+a))\,\mathrm{arcsinh}(bx+a)-\cosh(2\,\mathrm{arcsinh}(bx+a))-1}{2b\,\mathrm{arcsinh}(bx+a)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(2*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)-cosh(2*arcsinh(b*x+a))-1)/arc
sinh(b*x+a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
 + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b^2*x + a^2
*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((2*b^2*x^2 + 4*a*b*x + 2*a^2
- 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a
^3 + (6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^4*x^4 + 8*a*
b^3*x^3 + 2*a^4 + 3*(4*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(4*a^3*b + 3*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3
*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^
2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b
+ b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}}{\operatorname{asinh}^2(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}}{{\operatorname{asinh}\left(a + b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^2, x)
```

3.265 $\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=71

$$\frac{-1-(a+bx)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)}+\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] 1/2*(-1-(b*x+a)^2)/b/arcsinh(b*x+a)^2+Chi(2*arcsinh(b*x+a))/b-(b*x+a)*(1+(b
*x+a)^2)^(1/2)/b/arcsinh(b*x+a)
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5860, 5790, 5778, 3382}

$$\frac{\mathrm{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}-\frac{\sqrt{(a+bx)^2+1}\,(a+bx)}{b\sinh^{-1}(a+bx)}-\frac{(a+bx)^2+1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + (a + b*x)^2)/(b*ArcSinh[a + b*x]^2) - ((a + b*x)*Sqrt[1 + (a + b*
x)^2])/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
```

```
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{\sinh^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{s}\right.}{b}\\
&= -\frac{1+(a+bx)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{(a+bx)\sqrt{1+(a+bx)^2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 85, normalized size = 1.20

$$
-\frac{1+a^2+2abx+b^2x^2+2(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)-2\sinh^{-1}(a+bx)^2\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b\sinh^{-1}(a+bx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + a^2 + 2*a*b*x + b^2*x^2 + 2*(a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2]*ArcSinh[a + b*x] - 2*ArcSinh[a + b*x]^2*CoshIntegral[2*ArcSinh[a + b
*x]])/(b*ArcSinh[a + b*x]^2)
```

Maple [A]
time = 3.79, size = 61, normalized size = 0.86

method	result

default	$-\frac{-4\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)^2+2\sinh(2\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)+\cosh(2\,\text{arcsinh}(bx+a))+}{4b\,\text{arcsinh}(bx+a)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(-4*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+2*sinh(2*arcsinh(b*x+a))*
arcsinh(b*x+a)+cosh(2*arcsinh(b*x+a))+1)/arcsinh(b*x+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3
*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^5*x^5 + 15*a*b^4*x^4 +
3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*
a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b
^6*x^6 + 18*a*b^5*x^5 + 3*a^6 + (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^
3*b^3 + 7*a*b^3)*x^3 + (45*a^4*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9
*a^5*b + 14*a^3*b + 5*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + ((2*b^4*x
^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(4*a^3*b + a*b)
*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (6*b^5*x^5 + 30*a*b^4*x^4 + 6*a^5
 + (60*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + 3*(20*a^3*b^2 + 7*a*b^2)*x^2 + (30*a^
4*b + 21*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (6*b^6*x^6
 + 36*a*b^5*x^5 + 6*a^6 + (90*a^2*b^4 + 11*b^4)*x^4 + 11*a^4 + 4*(30*a^3*b^
3 + 11*a*b^3)*x^3 + 6*(15*a^4*b^2 + 11*a^2*b^2 + b^2)*x^2 + 6*a^2 + 4*(9*a^
5*b + 11*a^3*b + 3*a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (2*b^7*x^7 +
 14*a*b^6*x^6 + 2*a^7 + (42*a^2*b^5 + 5*b^5)*x^5 + 5*a^5 + 5*(14*a^3*b^4 +
5*a*b^4)*x^4 + 2*(35*a^4*b^3 + 25*a^2*b^3 + 2*b^3)*x^3 + 4*a^3 + 2*(21*a^5*
b^2 + 25*a^3*b^2 + 6*a*b^2)*x^2 + (14*a^6*b + 25*a^4*b + 12*a^2*b + b)*x +
a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x
+ a^2 + 1)) + (b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^
5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3
*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a
^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^7*x^6 + 6*a*b^6*x^5
 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3
+ 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*x^3 + 3*a*b^3*x^2 +
3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^5*x^4 + 4*a
*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)
```

```
*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2 + a*b^2)*x + 3*(b^6
*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b
^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)*x)*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
+ integrate(1/2*((4*b^4*x^4 + 16*a*b^3*x^3 + 4*a^4 + 2*(12*a^2*b^2 - b^2)*x
^2 - 2*a^2 + 4*(4*a^3*b - a*b)*x + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) +
 2*(8*b^5*x^5 + 40*a*b^4*x^4 + 8*a^5 + 4*(20*a^2*b^3 + b^3)*x^3 + 4*a^3 + 4
*(20*a^3*b^2 + 3*a*b^2)*x^2 + (40*a^4*b + 12*a^2*b + b)*x + a)*(b^2*x^2 + 2
*a*b*x + a^2 + 1)^2 + 2*(12*b^6*x^6 + 72*a*b^5*x^5 + 12*a^6 + 18*(10*a^2*b^
4 + b^4)*x^4 + 18*a^4 + 24*(10*a^3*b^3 + 3*a*b^3)*x^3 + 6*(30*a^4*b^2 + 18*
a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(6*a^5*b + 6*a^3*b + a*b)*x - 1)*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2) + 2*(8*b^7*x^7 + 56*a*b^6*x^6 + 8*a^7 + 4*(42*a^2
*b^5 + 5*b^5)*x^5 + 20*a^5 + 20*(14*a^3*b^4 + 5*a*b^4)*x^4 + 5*(56*a^4*b^3
+ 40*a^2*b^3 + 3*b^3)*x^3 + 15*a^3 + (168*a^5*b^2 + 200*a^3*b^2 + 45*a*b^2)
*x^2 + (56*a^6*b + 100*a^4*b + 45*a^2*b + 3*b)*x + 3*a)*(b^2*x^2 + 2*a*b*x
+ a^2 + 1) + (4*b^8*x^8 + 32*a*b^7*x^7 + 4*a^8 + 14*(8*a^2*b^6 + b^6)*x^6 +
 14*a^6 + 28*(8*a^3*b^5 + 3*a*b^5)*x^5 + (280*a^4*b^4 + 210*a^2*b^4 + 17*b^
4)*x^4 + 17*a^4 + 4*(56*a^5*b^3 + 70*a^3*b^3 + 17*a*b^3)*x^3 + 2*(56*a^6*b^
2 + 105*a^4*b^2 + 51*a^2*b^2 + 4*b^2)*x^2 + 8*a^2 + 4*(8*a^7*b + 21*a^5*b +
 17*a^3*b + 4*a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^8*x^8 + 8*
a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*(7*a^3*b^5 + 3*a*b^5)
*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4 + 8*(7*a^5*b^3 + 10*
a^3*b^3 + 3*a*b^3)*x^3 + (b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x
 + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9*a^2
*b^2 + b^2)*x^2 + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 +
 a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b
*x + a^2 + 1)^(3/2) + 6*(b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)
*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2
)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)
+ 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 4*(b^7*x^7 + 7*a*b^6*x^6
+ a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35
*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*
a*b^2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 + 1) + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")

[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1}}{\operatorname{asinh}^3(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(1/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/asinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(1/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + 2\,a\,b\,x + b^2\,x^2 + 1}}{{\operatorname{asinh}\left(a + b\,x\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/asinh(a + b*x)^3, x)
```

3.266 $\int (1+a^2+2abx+b^2x^2)^{3/2} \sinh^{-1}(a+bx)^3 \, dx$

Optimal. Leaf size=235

$$-\frac{51(a+bx)^2}{128b}-\frac{3(a+bx)^4}{128b}+\frac{45(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{64b}+\frac{3(a+bx)\,(1+(a+bx)^2)^{3/2}\sinh^{-1}(a+bx)}{32b}$$

```
[Out] -51/128*(b*x+a)^2/b-3/128*(b*x+a)^4/b+3/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcs
inh(b*x+a)/b-27/128*arcsinh(b*x+a)^2/b-9/16*(b*x+a)^2*arcsinh(b*x+a)^2/b-3/
16*(1+(b*x+a)^2)^2*arcsinh(b*x+a)^2/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsi
nh(b*x+a)^3/b+3/32*arcsinh(b*x+a)^4/b+45/64*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+
a)^2)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^3*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]

time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5860, 5786, 5785, 5783, 5776, 5812, 30, 5798, 14}

$$-\frac{3(a+bx)^4}{128b}-\frac{51(a+bx)^2}{128b}-\frac{9(a+bx)^2\sinh^{-1}(a+bx)^2}{16b}+\frac{((a+bx)^2+1)^{3/2}(a+bx)\sinh^{-1}(a+bx)^3}{4b}+\frac{3\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)^3}{8b}+\frac{3((a+bx)^2+1)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{32b}+\frac{45\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{64b}+\frac{3\sinh^{-1}(a+bx)^4}{32b}-\frac{3((a+bx)^2+1)^2\sinh^{-1}(a+bx)^2}{16b}-\frac{27\sinh^{-1}(a+bx)^2}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] (-51*(a + b*x)^2)/(128*b) - (3*(a + b*x)^4)/(128*b) + (45*(a + b*x)*Sqrt[1
+ (a + b*x)^2]*ArcSinh[a + b*x])/(64*b) + (3*(a + b*x)*(1 + (a + b*x)^2)^(3
/2)*ArcSinh[a + b*x])/(32*b) - (27*ArcSinh[a + b*x]^2)/(128*b) - (9*(a + b*
x)^2*ArcSinh[a + b*x]^2)/(16*b) - (3*(1 + (a + b*x)^2)^2*ArcSinh[a + b*x]^2
)/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b*x)^2]*ArcSinh[a + b*x]^3)/(8*b) + (
(a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh[a + b*x]^3)/(4*b) + (3*ArcSinh[a
+ b*x]^4)/(32*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^3\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)^3\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)^3}{4b} - \frac{3\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{3\left(1+(a+bx)^2\right)^2\sinh^{-1}(a+bx)^2}{16b} + \frac{3(a+bx)\sqrt{1+(a}}{}$$

$$= \frac{3(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{32b} - \frac{9(a+bx)^2}{}$$

$$= \frac{45(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{64b} + \frac{3(a+bx)\,(1}{}$$

$$= -\frac{51(a+bx)^2}{128b} - \frac{3(a+bx)^4}{128b} + \frac{45(a+bx)\sqrt{1+(a+bx)^2}}{64b}$$

Mathematica [A]
time = 0.16, size = 266, normalized size = 1.13

$$-\frac{6a(17+2a^2)\,bx+3(17+6a^2)\,b^2x^2+12ab^3x^3+3b^4x^4-6\sqrt{1+a^2+2abx+b^2x^2}\,(17a+2a^3+17bx+6a^2bx+6ab^2x^2+2b^3x^3)\sinh^{-1}(a+bx)+3(17+8a^4+32a^3bx+40b^2x^2+8b^4x^4+16abx(5+2b^2x^2)+8a^2(5+6b^2x^2))\sinh^{-1}(a+bx)^2-16\sqrt{1+a^2+2abx+b^2x^2}\,(5a+2a^3+5bx+6a^2bx+6ab^2x^2+2b^3x^3)\sinh^{-1}(a+bx)^3-12\sinh^{-1}(a+bx)^4}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/128*(6*a*(17 + 2*a^2)*b*x + 3*(17 + 6*a^2)*b^2*x^2 + 12*a*b^3*x^3 + 3*b^
4*x^4 - 6*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*
b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*(17 + 8*a^4 + 32*a^3*b*
x + 40*b^2*x^2 + 8*b^4*x^4 + 16*a*b*x*(5 + 2*b^2*x^2) + 8*a^2*(5 + 6*b^2*x^
2))*ArcSinh[a + b*x]^2 - 16*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3
```

```
+ 5*b*x + 6*a^2*b*x + 6*a*b^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^3 - 12*ArcS
inh[a + b*x]^4)/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(bx+a\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x)
```

```
[Out] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

Fricas [A]
time = 0.42, size = 332, normalized size = 1.41

$$-\frac{3\,b^4x^4+12\,ab^3x^3+3\,(6\,a^2+17)b^2x^2-16\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-12\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4+6\,(2\,a^3+17\,a)bx+3\,(8\,b^4x^4+32\,ab^3x^3+8\,(6\,a^2+5)b^2x^2+8\,a^4+16\,(2\,a^3+5\,a)bx+40\,a^2+17)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-6\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+17)bx+17\,a)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] -1/128*(3*b^4*x^4 + 12*a*b^3*x^3 + 3*(6*a^2 + 17)*b^2*x^2 - 16*(2*b^3*x^3 +
 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - 12*log(b*x + a +
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4 + 6*(2*a^3 + 17*a)*b*x + 3*(8*b^4*x^4
+ 32*a*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*
a^2 + 17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - 6*(2*b^3*x^3
 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(223) = 446$.
time = 1.62, size = 694, normalized size = 2.95

$$\begin{cases} -\frac{3a^4\operatorname{asinh}^2(a+bx)}{16b} - \frac{3a^3x\operatorname{asinh}^2(a+bx)}{4} - \frac{3a^3x}{32} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{4b} + \frac{3a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{32b} - \frac{9a^2bx^2\operatorname{asinh}^2(a+bx)}{8} - \frac{9a^2bx^2}{64} + \frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{4} + \text{[illegible]} - \frac{51bx^2}{128} + \frac{5x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^3(a+bx)}{8} + \frac{51x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{64} + \frac{3\operatorname{asinh}^4(a+bx)}{32b} - \frac{51\operatorname{asinh}^2(a+bx)}{128b} & \text{for}\: b \neq 0 \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**3,x)
```

```
[Out] Piecewise((-3*a**4*asinh(a + b*x)**2/(16*b) - 3*a**3*x*asinh(a + b*x)**2/4
- 3*a**3*x/32 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3
/(4*b) + 3*a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(32*b)
- 9*a**2*b*x**2*asinh(a + b*x)**2/8 - 9*a**2*b*x**2/64 + 3*a**2*x*sqrt(a**2
 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a**2*x*sqrt(a**2 + 2*a*
b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a**2*asinh(a + b*x)**2/(16*b) -
 3*a*b**2*x**3*asinh(a + b*x)**2/4 - 3*a*b**2*x**3/32 + 3*a*b*x**2*sqrt(a**
2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 9*a*b*x**2*sqrt(a**2 + 2
*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/32 - 15*a*x*asinh(a + b*x)**2/8 - 51
*a*x/64 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/(8*b)
+ 51*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/(64*b) - 3*b**3*
x**4*asinh(a + b*x)**2/16 - 3*b**3*x**4/128 + b**2*x**3*sqrt(a**2 + 2*a*b*x
 + b**2*x**2 + 1)*asinh(a + b*x)**3/4 + 3*b**2*x**3*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/32 - 15*b*x**2*asinh(a + b*x)**2/16 - 51*b*x**
2/128 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**3/8 + 51*x
*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)/64 + 3*asinh(a + b*x)*
*4/(32*b) - 51*asinh(a + b*x)**2/(128*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*a
sinh(a)**3, True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{asinh}(a+b\,x)^3\,{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.267 $\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2\,dx$

Optimal. Leaf size=189

$$\frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b}+\frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b}-\frac{9\sinh^{-1}(a+bx)}{64b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)}{8b}-\frac{(1$$

```
[Out] 1/32*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b-9/64*arcsinh(b*x+a)/b-3/8*(b*x+a)^2*arcs
inh(b*x+a)/b-1/8*(1+(b*x+a)^2)^2*arcsinh(b*x+a)/b+1/4*(b*x+a)*(1+(b*x+a)^2)
^(3/2)*arcsinh(b*x+a)^2/b+1/8*arcsinh(b*x+a)^3/b+15/64*(b*x+a)*(1+(b*x+a)^2
)^(1/2)/b+3/8*(b*x+a)*arcsinh(b*x+a)^2*(1+(b*x+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {5860, 5786, 5785, 5783, 5776, 327, 221, 5798, 201}

$$\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}}{32b}+\frac{15(a+bx)\sqrt{(a+bx)^2+1}}{64b}+\frac{\sinh^{-1}(a+bx)^3}{8b}+\frac{(a+bx)\left((a+bx)^2+1\right)^{3/2}\sinh^{-1}(a+bx)^2}{4b}+\frac{3(a+bx)\sqrt{(a+bx)^2+1}\,\sinh^{-1}(a+bx)^2}{8b}-\frac{3(a+bx)^2\sinh^{-1}(a+bx)}{8b}-\frac{\left((a+bx)^2+1\right)^2\sinh^{-1}(a+bx)}{8b}-\frac{9\sinh^{-1}(a+bx)}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (15*(a + b*x)*Sqrt[1 + (a + b*x)^2])/(64*b) + ((a + b*x)*(1 + (a + b*x)^2)^
(3/2))/(32*b) - (9*ArcSinh[a + b*x])/(64*b) - (3*(a + b*x)^2*ArcSinh[a + b*
x])/(8*b) - ((1 + (a + b*x)^2)^2*ArcSinh[a + b*x])/(8*b) + (3*(a + b*x)*Sqr
t[1 + (a + b*x)^2]*ArcSinh[a + b*x]^2)/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)
^(3/2)*ArcSinh[a + b*x]^2)/(4*b) + ArcSinh[a + b*x]^3/(8*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)^2\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)^2}{4b} - \frac{\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{\left(1+(a+bx)^2\right)^2\sinh^{-1}(a+bx)}{8b} + \frac{3(a+bx)\sqrt{1+(a-}}{}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{32b} - \frac{3(a+bx)^2\sinh^{-1}(a+bx)}{8b}$$

$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/}}{32b}$$

$$= \frac{15(a+bx)\sqrt{1+(a+bx)^2}}{64b} + \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/}}{32b}$$

Mathematica [A]
time = 0.11, size = 211, normalized size = 1.12

$$\frac{\sqrt{1+a^2+2abx+b^2x^2}\,(17a+2a^3+17bx+6a^2bx+6ab^2x^2+2b^3x^3)-(17+40a^2+8a^4)\sinh^{-1}(a+bx)-8bx(10a+4a^3+5bx+6a^2bx+4ab^2x^2+b^3x^3)\sinh^{-1}(a+bx)+8\sqrt{1+a^2+2abx+b^2x^2}\,(5a+2a^3+5bx+6a^2bx+6ab^2x^2+2b^3x^3)\sinh^{-1}(a+bx)^2+8\sinh^{-1}(a+bx)^3}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(17*a + 2*a^3 + 17*b*x + 6*a^2*b*x + 6*a
*b^2*x^2 + 2*b^3*x^3) - (17 + 40*a^2 + 8*a^4)*ArcSinh[a + b*x] - 8*b*x*(10*
a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)*ArcSinh[a + b*x] + 8
*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b
^2*x^2 + 2*b^3*x^3)*ArcSinh[a + b*x]^2 + 8*ArcSinh[a + b*x]^3)/(64*b)
```

Maple [A]
time = 2.93, size = 139, normalized size = 0.74

$$-\frac{-8\,\text{arcsinh}\left(bx+a\right)^2\cosh\left(2\,\text{arcsinh}\left(bx+a\right)\right)\sinh\left(2\,\text{arcsinh}\left(bx+a\right)\right)+4\,\text{arcsinh}\left(bx+a\right)\left(\cosh^2\left(2\,\text{arcsi}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x)
```

```
[Out] -1/128*(-8*arcsinh(b*x+a)^2*cosh(2*arcsinh(b*x+a))*sinh(2*arcsinh(b*x+a))+4
*arcsinh(b*x+a)*cosh(2*arcsinh(b*x+a))^2-16*arcsinh(b*x+a)^3-32*arcsinh(b*x
+a)^2*sinh(2*arcsinh(b*x+a))+32*arcsinh(b*x+a)*cosh(2*arcsinh(b*x+a))-sinh(
2*arcsinh(b*x+a))*cosh(2*arcsinh(b*x+a))-2*arcsinh(b*x+a)-16*sinh(2*arcsinh
(b*x+a)))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

Fricas [A]
time = 0.36, size = 259, normalized size = 1.37

$$\frac{8\,(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+5)bx+5\,a)\sqrt{b^2x^2+2\,abx+a^2+1}\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2+8\,\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3-(8\,b^4x^4+32\,ab^3x^3+8\,(6\,a^2+5)b^2x^2+8\,a^4+16\,(2\,a^3+5\,a)bx+40\,a^2+17)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+(2\,b^3x^3+6\,ab^2x^2+2\,a^3+(6\,a^2+17)bx+17\,a)\sqrt{b^2x^2+2\,abx+a^2+1}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] 1/64*(8*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*
x^2 + 2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
 + 8*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3 - (8*b^4*x^4 + 32*a
*b^3*x^3 + 8*(6*a^2 + 5)*b^2*x^2 + 8*a^4 + 16*(2*a^3 + 5*a)*b*x + 40*a^2 +
17)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (2*b^3*x^3 + 6*a*b^2
*x^2 + 2*a^3 + (6*a^2 + 17)*b*x + 17*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/
b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(173) = 346$.
time = 0.99, size = 568, normalized size = 3.01

$$\begin{cases} -\frac{a^4\operatorname{asinh}(a+bx)}{8b}-\frac{a^3x\operatorname{asinh}(a+bx)}{2}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4b}+\frac{a^3\sqrt{a^2+2abx+b^2x^2+1}}{32b}-\frac{3a^2bx^2\operatorname{asinh}(a+bx)}{4}+\frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4}+\frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}}{32}-\frac{5a^2\operatorname{asinh}(a+bx)}{8b}-\frac{ab^2x^3\operatorname{asinh}(a+bx)}{2}+\frac{3abx^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4}+\frac{3abx^2\sqrt{a^2+2abx+b^2x^2+1}}{32}-\frac{5ax\operatorname{asinh}(a+bx)}{4}+\frac{5a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{8b}+\frac{17a\sqrt{a^2+2abx+b^2x^2+1}}{64b}-\frac{b^3x^4\operatorname{asinh}(a+bx)}{8}+\frac{b^2x^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{4}+\frac{b^2x^3\sqrt{a^2+2abx+b^2x^2+1}}{32}-\frac{5bx^2\operatorname{asinh}(a+bx)}{8}+\frac{5x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}^2(a+bx)}{8}+\frac{17x\sqrt{a^2+2abx+b^2x^2+1}}{64}+\frac{\operatorname{asinh}^3(a+bx)}{8b}-\frac{17\operatorname{asinh}(a+bx)}{64b} & \text{for}\: b \neq 0 \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**4*asinh(a + b*x)/(8*b) - a**3*x*asinh(a + b*x)/2 + a**3*sqrt
(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/(4*b) + a**3*sqrt(a**2 +
```

```
 2*a*b*x + b**2*x**2 + 1)/(32*b) - 3*a**2*b*x**2*asinh(a + b*x)/4 + 3*a**2*
x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2/4 + 3*a**2*x*sqrt(
a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a**2*asinh(a + b*x)/(8*b) - a*b**2*x
**3*asinh(a + b*x)/2 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asin
h(a + b*x)**2/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*a*
x*asinh(a + b*x)/4 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x
)**2/(8*b) + 17*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/(64*b) - b**3*x**4*a
sinh(a + b*x)/8 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)**2/4 + b**2*x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/32 - 5*b*x**2*as
inh(a + b*x)/8 + 5*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a + b*x)**2
/8 + 17*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)/64 + asinh(a + b*x)**3/(8*b)
 - 17*asinh(a + b*x)/(64*b), Ne(b, 0)), (x*(a**2 + 1)**(3/2)*asinh(a)**2, T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a)^2,x, algorithm="giac
")
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{asinh}(a+b\,x)^2\,{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
[Out] int(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.268 $\int (1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)\,dx$

Optimal. Leaf size=106

$$-\frac{5(a+bx)^2}{16b}-\frac{(a+bx)^4}{16b}+\frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{8b}+\frac{(a+bx)\,(1+(a+bx)^2)^{3/2}\sinh^{-1}(a+$$

```
[Out] -5/16*(b*x+a)^2/b-1/16*(b*x+a)^4/b+1/4*(b*x+a)*(1+(b*x+a)^2)^(3/2)*arcsinh(
b*x+a)/b+3/16*arcsinh(b*x+a)^2/b+3/8*(b*x+a)*arcsinh(b*x+a)*(1+(b*x+a)^2)^(
1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5860, 5786, 5785, 5783, 30, 14}

$$-\frac{(a+bx)^4}{16b}-\frac{5(a+bx)^2}{16b}+\frac{((a+bx)^2+1)^{3/2}(a+bx)\sinh^{-1}(a+bx)}{4b}+\frac{3\sqrt{(a+bx)^2+1}\,(a+bx)\sinh^{-1}(a+bx)}{8b}+\frac{3\sinh^{-1}(a+bx)^2}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-5*(a + b*x)^2)/(16*b) - (a + b*x)^4/(16*b) + (3*(a + b*x)*Sqrt[1 + (a + b
*x)^2]*ArcSinh[a + b*x])/(8*b) + ((a + b*x)*(1 + (a + b*x)^2)^(3/2)*ArcSinh
[a + b*x])/(4*b) + (3*ArcSinh[a + b*x]^2)/(16*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5785

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcSinh[c*x])^n/2), x] + (Dist[(1
```

```
/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x^2]], Int[(a + b*ArcSinh[c*x])^n/Sqr
t[1 + c^2*x^2], x], x] - Dist[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 + c^2*x
^2]], Int[x*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e},
 x] && EqQ[e, c^2*d] && GtQ[n, 0]
```

Rule 5786

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[x*(d + e*x^2)^p*((a + b*ArcSinh[c*x])^n/(2*p + 1)), x] +
(Dist[2*d*(p/(2*p + 1)), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSinh[c*x])^n, x]
, x] - Dist[b*c*(n/(2*p + 1))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[x*(1
 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && GtQ[p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\sinh^{-1}(x)\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\left(1+(a+bx)^2\right)^{3/2}\sinh^{-1}(a+bx)}{4b} - \frac{\text{Subst}\left(\int x(1\right.}{}$$

$$= \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\sinh^{-1}(a+bx)}{8b} + \frac{(a+bx)\,(1-}{}$$

$$= -\frac{5(a+bx)^2}{16b} - \frac{(a+bx)^4}{16b} + \frac{3(a+bx)\sqrt{1+(a+bx)^2}\,\text{si}}{8b}$$

Mathematica [A]

time = 0.07, size = 124, normalized size = 1.17

$$\frac{-bx(10a+4a^3+5bx+6a^2bx+4ab^2x^2+b^3x^3)+2\sqrt{1+a^2+2abx+b^2x^2}\,(5a+2a^3+5bx+6a^2bx+6ab^2x^2+2b^3x^3)\sinh^{-1}(a+bx)+3\sinh^{-1}(a+bx)^2}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x],x]
```

```
[Out] (-(b*x*(10*a + 4*a^3 + 5*b*x + 6*a^2*b*x + 4*a*b^2*x^2 + b^3*x^3)) + 2*Sqrt
[1 + a^2 + 2*a*b*x + b^2*x^2]*(5*a + 2*a^3 + 5*b*x + 6*a^2*b*x + 6*a*b^2*x^
2 + 2*b^3*x^3)*ArcSinh[a + b*x] + 3*ArcSinh[a + b*x]^2)/(16*b)
```

Maple [A]
time = 2.88, size = 82, normalized size = 0.77

$$-\frac{-4 \operatorname{arcsinh}(bx+a)\cosh(2\operatorname{arcsinh}(bx+a))\sinh(2\operatorname{arcsinh}(bx+a))+\cosh^2(2\operatorname{arcsinh}(bx+a))-16\sinh}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x)
```

```
[Out] -1/64*(-4*arcsinh(b*x+a)*cosh(2*arcsinh(b*x+a))*sinh(2*arcsinh(b*x+a))+cosh
(2*arcsinh(b*x+a))^2-16*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-12*arcsinh(b*
x+a)^2+8*cosh(2*arcsinh(b*x+a)))/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(92) = 184.
time = 0.28, size = 394, normalized size = 3.72

$$-\frac{1}{16}\left(b^2x^4+4abx^3+6a^2x^2+\frac{4a^3x}{b}+5x^2+\frac{10ax}{b}+\frac{6\,\mathrm{arsinh}(bx+a)\,\mathrm{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^2}-\frac{3\,\mathrm{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)^2}{b^2}\right)b+\frac{1}{8}\left(2(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}x+\frac{2(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}a}{b}+\frac{3(a^2b^2-(a^2+1)b^2)a^2\,\mathrm{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3}-\frac{3(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2abx+a^2+1}\,x}{b^2}-\frac{3(a^2b^2-(a^2+1)b^2)(a^2+1)\,\mathrm{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b^3}-\frac{3(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2abx+a^2+1}\,a}{b^3}\right)\mathrm{arsinh}(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] -1/16*(b^2*x^4 + 4*a*b*x^3 + 6*a^2*x^2 + 4*a^3*x/b + 5*x^2 + 10*a*x/b + 6*a
rcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))
/b^2 - 3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b^2)
*b + 1/8*(2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x + 2*(b^2*x^2 + 2*a*b*x +
a^2 + 1)^(3/2)*a/b + 3*(a^2*b^2 - (a^2 + 1)*b^2)*a^2*arcsinh(2*(b^2*x + a*b
)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 3*(a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1
)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3*(a^2*
b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a/b^3)*arcsinh(b*x +
 a)
```

Fricas [A]
time = 0.36, size = 160, normalized size = 1.51

$$-\frac{b^4x^4+4ab^3x^3+(6a^2+5)b^2x^2+2(2a^3+5a)bx-2(2b^3x^3+6ab^2x^2+2a^3+(6a^2+5)bx+5a)\sqrt{b^2x^2+2abx+a^2+1}\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)-3\log\left(bx+a+\sqrt{b^2x^2+2abx+a^2+1}\right)^2}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] -1/16*(b^4*x^4 + 4*a*b^3*x^3 + (6*a^2 + 5)*b^2*x^2 + 2*(2*a^3 + 5*a)*b*x -
2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2 + 5)*b*x + 5*a)*sqrt(b^2*x^2 +
2*a*b*x + a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - 3*log
(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(95) = 190$.
time = 0.61, size = 298, normalized size = 2.81

$$\begin{cases} -\frac{a^3x}{4} + \frac{a^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4b} - \frac{3a^2bx^2}{8} + \frac{3a^2x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \frac{ab^2x^3}{4} + \frac{3abx^2\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \frac{5ax}{8} + \frac{5a\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{8b} - \frac{b^3x^4}{16} + \frac{b^2x^3\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{4} - \frac{5bx^2}{16} + \frac{5x\sqrt{a^2+2abx+b^2x^2+1}\operatorname{asinh}(a+bx)}{8} + \frac{3\operatorname{asinh}^2(a+bx)}{16b} & \text{for } b \neq 0 \\ x(a^2+1)^{\frac{3}{2}}\operatorname{asinh}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)*asinh(b*x+a),x)
```

```
[Out] Piecewise((-a**3*x/4 + a**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)*asinh(a +
b*x)/(4*b) - 3*a**2*b*x**2/8 + 3*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1
)*asinh(a + b*x)/4 - a*b**2*x**3/4 + 3*a*b*x**2*sqrt(a**2 + 2*a*b*x + b**2*
x**2 + 1)*asinh(a + b*x)/4 - 5*a*x/8 + 5*a*sqrt(a**2 + 2*a*b*x + b**2*x**2
+ 1)*asinh(a + b*x)/(8*b) - b**3*x**4/16 + b**2*x**3*sqrt(a**2 + 2*a*b*x +
b**2*x**2 + 1)*asinh(a + b*x)/4 - 5*b*x**2/16 + 5*x*sqrt(a**2 + 2*a*b*x + b
**2*x**2 + 1)*asinh(a + b*x)/8 + 3*asinh(a + b*x)**2/(16*b), Ne(b, 0)), (x*
(a**2 + 1)**(3/2)*asinh(a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)*arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.269 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

```
[Out] 1/2*Chi(2*arcsinh(b*x+a))/b+1/8*Chi(4*arcsinh(b*x+a))/b+3/8*ln(arcsinh(b*x+
a))/b
```

Rubi [A]
time = 0.10, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5860, 5791, 3393, 3382}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b}+\frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
```

```
[Out] CoshIntegral[2*ArcSinh[a + b*x]]/(2*b) + CoshIntegral[4*ArcSinh[a + b*x]]/(
8*b) + (3*Log[ArcSinh[a + b*x]])/(8*b)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
```

```
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{8b} + \text{Subs}$$
$$= \frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{8b} + \frac{3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Mathematica [A]
time = 0.18, size = 37, normalized size = 0.79

$$\frac{4\text{Chi}\left(2\sinh^{-1}(a+bx)\right)+\text{Chi}\left(4\sinh^{-1}(a+bx)\right)+3\log\left(\sinh^{-1}(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x],x]
[Out] (4*CoshIntegral[2*ArcSinh[a + b*x]] + CoshIntegral[4*ArcSinh[a + b*x]] + 3*
Log[ArcSinh[a + b*x]])/(8*b)
```

Maple [A]
time = 3.96, size = 36, normalized size = 0.77

method	result	size
default	$\frac{3\ln(\text{arcsinh}(bx+a))+4\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))+\text{hyperbolicCosineIntegral}(4\,\text{arcsinh}(bx+a))}{8b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/8*(3*ln(arcsinh(b*x+a))+4*Chi(2*arcsinh(b*x+a))+Chi(4*arcsinh(b*x+a)))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxima
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fricas
")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2 + 2abx + b^2x^2 + 1)^{\frac{3}{2}}}{\operatorname{asinh}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2 + 2\,a\,b\,x + b^2\,x^2 + 1\right)}^{3/2}}{\operatorname{asinh}\left(a + b\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x),x)
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x), x)
```

3.270 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\left(1+(a+bx)^2\right)^2}{b\sinh^{-1}(a+bx)}+\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}$$

```
[Out] -(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)+Shi(2*arcsinh(b*x+a))/b+1/2*Shi(4*arcsinh
(b*x+a))/b
```

Rubi [A]
time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5860, 5790, 5819, 5556, 3379}

$$\frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}-\frac{\left((a+bx)^2+1\right)^2}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] -((1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x])) + SinhIntegral[2*ArcSinh[a + b*
x]]/b + SinhIntegral[4*ArcSinh[a + b*x]]/(2*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}\\
&= -\frac{(1+(a+bx)^2)^2}{b\sinh^{-1}(a+bx)} + \frac{4\text{Subst}\left(\int \frac{x(1+x^2)}{\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\
&= -\frac{(1+(a+bx)^2)^2}{b\sinh^{-1}(a+bx)} + \frac{4\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{(1+(a+bx)^2)^2}{b\sinh^{-1}(a+bx)} + \frac{4\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= -\frac{(1+(a+bx)^2)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int\right.}{}\\
&= -\frac{(1+(a+bx)^2)^2}{b\sinh^{-1}(a+bx)} + \frac{\text{Shi}\left(2\sinh^{-1}(a+bx)\right)}{b} + \frac{\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 70, normalized size = 1.30

$$\frac{-2(1+a^2+2abx+b^2x^2)^2+2\sinh^{-1}(a+bx)\text{Shi}\left(2\sinh^{-1}(a+bx)\right)+\sinh^{-1}(a+bx)\text{Shi}\left(4\sinh^{-1}(a+bx)\right)}{2b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^2,x]
```

```
[Out] (-2*(1 + a^2 + 2*a*b*x + b^2*x^2)^2 + 2*ArcSinh[a + b*x]*SinhIntegral[2*Arc
Sinh[a + b*x]] + ArcSinh[a + b*x]*SinhIntegral[4*ArcSinh[a + b*x]])/(2*b*Ar
cSinh[a + b*x])
```

Maple [A]
time = 3.86, size = 72, normalized size = 1.33

method	result
default	$\frac{8\,\mathrm{hyperbolicSineIntegral}(2\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)+4\,\mathrm{hyperbolicSineIntegral}(4\operatorname{arcsinh}(bx+a))\operatorname{arcsinh}(bx+a)-4\cosh(2\operatorname{arcsinh}}{8b\operatorname{arcsinh}(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(8*Shi(2*arcsinh(b*x+a))*arcsinh(b*x+a)+4*Shi(4*arcsinh(b*x+a))*arcsi
nh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/arcsinh(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="maxi
ma")
```

```
[Out] -((b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2 + 4*(a^3*b
 + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^5*x^5 + 5*a*b^4*x^4 + a^5
 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b
 + 6*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^3*x^2 + 2*a*b
^2*x + a^2*b + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + b)*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) + integrate(((4*b^4*x^4 + 16*a*b
^3*x^3 + 4*a^4 + 3*(8*a^2*b^2 + b^2)*x^2 + 3*a^2 + 2*(8*a^3*b + 3*a*b)*x -
1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(2*b^5*x^5 + 10*a*b^4*x^4 + 2*a^
5 + (20*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + (20*a^3*b^2 + 9*a*b^2)*x^2 + (10*a^4
*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^6*x^6 + 24*a*
b^5*x^5 + 4*a^6 + 3*(20*a^2*b^4 + 3*b^4)*x^4 + 9*a^4 + 4*(20*a^3*b^3 + 9*a*
b^3)*x^3 + 6*(10*a^4*b^2 + 9*a^2*b^2 + b^2)*x^2 + 6*a^2 + 12*(2*a^5*b + 3*a
^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b^4*x^4 + 4*a*b^3*x
^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + (b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x^2
 + 2*a*b*x + a^2) + 2*a^2 + 4*(a^3*b + a*b)*x + 2*(b^3*x^3 + 3*a*b^2*x^2 +
a^3 + (3*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 1)*log(b*x +
 a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2 + 2abx + b^2x^2 + 1)^{\frac{3}{2}}}{\operatorname{asinh}^2(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a^2 + 2\,a\,b\,x + b^2\,x^2 + 1)^{3/2}}{\operatorname{asinh}(a + b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^2, x)
```

3.271 $\int \frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{\sinh^{-1}(a+bx)^3} \, dx$

Optimal. Leaf size=84

$$-\frac{\left(1+(a+bx)^2\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left(1+(a+bx)^2\right)^{3/2}}{b\sinh^{-1}(a+bx)}+\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}$$

```
[Out] -1/2*(1+(b*x+a)^2)^2/b/arcsinh(b*x+a)^2-2*(b*x+a)*(1+(b*x+a)^2)^(3/2)/b/arc
sinh(b*x+a)+Chi(2*arcsinh(b*x+a))/b+Chi(4*arcsinh(b*x+a))/b
```

Rubi [A]
time = 0.20, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5860, 5790, 5814, 5791, 3393, 3382, 5819, 5556}

$$\frac{\text{Chi}\left(2\sinh^{-1}(a+bx)\right)}{b}+\frac{\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{b}-\frac{\left((a+bx)^2+1\right)^2}{2b\sinh^{-1}(a+bx)^2}-\frac{2(a+bx)\left((a+bx)^2+1\right)^{3/2}}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] -1/2*(1 + (a + b*x)^2)^2/(b*ArcSinh[a + b*x]^2) - (2*(a + b*x)*(1 + (a + b*
x)^2)^(3/2))/(b*ArcSinh[a + b*x]) + CoshIntegral[2*ArcSinh[a + b*x]]/b + Co
shIntegral[4*ArcSinh[a + b*x]]/b
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5790

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[Simp[Sqrt[1 + c^2*x^2]*(d + e*x^2)^p]*((a + b*ArcSinh[c*x]
)^(n + 1)/(b*c*(n + 1))), x] - Dist[c*((2*p + 1)/(b*(n + 1)))*Simp[(d + e*x
^2)^p/(1 + c^2*x^2)^p], Int[x*(1 + c^2*x^2)^(p - 1/2)*(a + b*ArcSinh[c*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && LtQ[n,
-1]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(1/(b*c))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Subst[Int[
x^n*Cosh[-a/b + x/b]^(2*p + 1), x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c, d, e, n}, x] && EqQ[e, c^2*d] && IGtQ[2*p, 0]
```

Rule 5814

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(f*x)^m*Sqrt[1 + c^2*x^2]*(d + e*x^2)^p
*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-Dist[f*(m/(b*c*(n + 1
)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m - 1)*(1 + c^2*x^2)^(p
 - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x] - Dist[c*((m + 2*p + 1)/(b*f*(
n + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^
2)^(p - 1/2)*(a + b*ArcSinh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e,
 f}, x] && EqQ[e, c^2*d] && LtQ[n, -1] && IGtQ[2*p, 0] && NeQ[m + 2*p + 1,
0] && IGtQ[m, -3]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{(1+a^2+2abx+b^2x^2)^{3/2}}{\sinh^{-1}(a+bx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} + \frac{2\text{Subst}\left(\int \frac{x(1+x^2)}{\sinh^{-1}(x)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\,(1+(a+bx)^2)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sinh^{-1}(x)}\right.}{b}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\,(1+(a+bx)^2)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,d\right.}{}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\,(1+(a+bx)^2)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{2\text{Subst}\left(\int \left(\frac{1}{2x}+\right.\right.}{}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\,(1+(a+bx)^2)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx\right.}{}$$
$$= -\frac{(1+(a+bx)^2)^2}{2b\sinh^{-1}(a+bx)^2} - \frac{2(a+bx)\,(1+(a+bx)^2)^{3/2}}{b\sinh^{-1}(a+bx)} + \frac{\text{Chi}\left(2\sinh^{-1}(a+b\right.}{b}$$

Mathematica [A]

time = 0.24, size = 108, normalized size = 1.29

$$\frac{-\frac{(1+a^2+2abx+b^2x^2)\left(1+a^2+2abx+b^2x^2+4(a+bx)\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)\right)}{\sinh^{-1}(a+bx)^2} + 2\text{Chi}\left(2\sinh^{-1}(a+bx)\right) + 2\text{Chi}\left(4\sinh^{-1}(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/ArcSinh[a + b*x]^3,x]
```

```
[Out] (-(((1 + a^2 + 2*a*b*x + b^2*x^2)*(1 + a^2 + 2*a*b*x + b^2*x^2 + 4*(a + b*x
)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]))/ArcSinh[a + b*x]^2)
+ 2*CoshIntegral[2*ArcSinh[a + b*x]] + 2*CoshIntegral[4*ArcSinh[a + b*x]])/
(2*b)
```

Maple [A]

time = 3.88, size = 110, normalized size = 1.31

method	result
default	$\frac{16\,\text{hyperbolicCosineIntegral}(2\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)^2+16\,\text{hyperbolicCosineIntegral}(4\,\text{arcsinh}(bx+a))\,\text{arcsinh}(bx+a)^2-8\,\text{sinh}}{16b\,\text{arcsinh}(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(16*Chi(2*arcsinh(b*x+a))*arcsinh(b*x+a)^2+16*Chi(4*arcsinh(b*x+a))*
arcsinh(b*x+a)^2-8*sinh(2*arcsinh(b*x+a))*arcsinh(b*x+a)-4*sinh(4*arcsinh(b
*x+a))*arcsinh(b*x+a)-4*cosh(2*arcsinh(b*x+a))-cosh(4*arcsinh(b*x+a))-3)/ar
csinh(b*x+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/2*((b^6*x^6 + 6*a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(
5*a^3*b^3 + 2*a*b^3)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3
*a^5*b + 4*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + (3*b^7*x^7 + 2
1*a*b^6*x^6 + 3*a^7 + (63*a^2*b^5 + 8*b^5)*x^5 + 8*a^5 + 5*(21*a^3*b^4 + 8*
a*b^4)*x^4 + (105*a^4*b^3 + 80*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (63*a^5*b^2 +
 80*a^3*b^2 + 21*a*b^2)*x^2 + (21*a^6*b + 40*a^4*b + 21*a^2*b + 2*b)*x + 2*
a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (3*b^8*x^8 + 24*a*b^7*x^7 + 3*a^8
+ 2*(42*a^2*b^6 + 5*b^6)*x^6 + 10*a^6 + 12*(14*a^3*b^5 + 5*a*b^5)*x^5 + 6*(
35*a^4*b^4 + 25*a^2*b^4 + 2*b^4)*x^4 + 12*a^4 + 8*(21*a^5*b^3 + 25*a^3*b^3
+ 6*a*b^3)*x^3 + 6*(14*a^6*b^2 + 25*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + 6*a^2
 + 12*(2*a^7*b + 5*a^5*b + 4*a^3*b + a*b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 +
 1) + ((4*b^6*x^6 + 24*a*b^5*x^5 + 4*a^6 + (60*a^2*b^4 + 7*b^4)*x^4 + 7*a^4
 + 4*(20*a^3*b^3 + 7*a*b^3)*x^3 + 2*(30*a^4*b^2 + 21*a^2*b^2 + b^2)*x^2 + 2
*a^2 + 4*(6*a^5*b + 7*a^3*b + a*b)*x - 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 +
 3*(4*b^7*x^7 + 28*a*b^6*x^6 + 4*a^7 + 3*(28*a^2*b^5 + 3*b^5)*x^5 + 9*a^5 +
 5*(28*a^3*b^4 + 9*a*b^4)*x^4 + 2*(70*a^4*b^3 + 45*a^2*b^3 + 3*b^3)*x^3 + 6
*a^3 + 6*(14*a^5*b^2 + 15*a^3*b^2 + 3*a*b^2)*x^2 + (28*a^6*b + 45*a^4*b + 1
8*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + (12*b^8*x^8 + 96*
a*b^7*x^7 + 12*a^8 + 3*(112*a^2*b^6 + 11*b^6)*x^6 + 33*a^6 + 6*(112*a^3*b^5
 + 33*a*b^5)*x^5 + (840*a^4*b^4 + 495*a^2*b^4 + 31*b^4)*x^4 + 31*a^4 + 4*(1
68*a^5*b^3 + 165*a^3*b^3 + 31*a*b^3)*x^3 + (336*a^6*b^2 + 495*a^4*b^2 + 186
*a^2*b^2 + 11*b^2)*x^2 + 11*a^2 + 2*(48*a^7*b + 99*a^5*b + 62*a^3*b + 11*a*
b)*x + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (4*b^9*x^9 + 36*a*b^8*x^8 + 4*a^9
 + (144*a^2*b^7 + 13*b^7)*x^7 + 13*a^7 + 7*(48*a^3*b^6 + 13*a*b^6)*x^6 + 3*
(168*a^4*b^5 + 91*a^2*b^5 + 5*b^5)*x^5 + 15*a^5 + (504*a^5*b^4 + 455*a^3*b^
4 + 75*a*b^4)*x^4 + (336*a^6*b^3 + 455*a^4*b^3 + 150*a^2*b^3 + 7*b^3)*x^3 +
```

```
 7*a^3 + 3*(48*a^7*b^2 + 91*a^5*b^2 + 50*a^3*b^2 + 7*a*b^2)*x^2 + (36*a^8*b
 + 91*a^6*b + 75*a^4*b + 21*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (b^9*x^9 + 9*a*b^8
*x^8 + a^9 + 4*(9*a^2*b^7 + b^7)*x^7 + 4*a^7 + 28*(3*a^3*b^6 + a*b^6)*x^6 +
 6*(21*a^4*b^5 + 14*a^2*b^5 + b^5)*x^5 + 6*a^5 + 2*(63*a^5*b^4 + 70*a^3*b^4
 + 15*a*b^4)*x^4 + 4*(21*a^6*b^3 + 35*a^4*b^3 + 15*a^2*b^3 + b^3)*x^3 + 4*a
^3 + 12*(3*a^7*b^2 + 7*a^5*b^2 + 5*a^3*b^2 + a*b^2)*x^2 + (9*a^8*b + 28*a^6
*b + 30*a^4*b + 12*a^2*b + b)*x + a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/((b
^7*x^6 + 6*a*b^6*x^5 + a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3
*b^4 + 3*a*b^4)*x^3 + 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + (b^4*
x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2
) + 3*(b^5*x^4 + 4*a*b^4*x^3 + a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2
*a^3*b^2 + a*b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + 6*(a^5*b^2 + 2*a^3*b^2
 + a*b^2)*x + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b + 2*(5*a^2*b^4 + b
^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 + 6*a^2*b^2 + b^2)
*x)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + b)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2) + integrate(1/2*((16*b^6*x^6 + 96*a*b^5*x^5 + 16*a^6 +
10*(24*a^2*b^4 + b^4)*x^4 + 10*a^4 + 40*(8*a^3*b^3 + a*b^3)*x^3 + 3*(80*a^4
*b^2 + 20*a^2*b^2 - b^2)*x^2 - 3*a^2 + 2*(48*a^5*b + 20*a^3*b - 3*a*b)*x +
3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(16*b^7*x^7 + 112*a*b^6*x^6 + 16
*a^7 + (336*a^2*b^5 + 23*b^5)*x^5 + 23*a^5 + 5*(112*a^3*b^4 + 23*a*b^4)*x^4
 + (560*a^4*b^3 + 230*a^2*b^3 + 7*b^3)*x^3 + 7*a^3 + (336*a^5*b^2 + 230*a^3
*b^2 + 21*a*b^2)*x^2 + (112*a^6*b + 115*a^4*b + 21*a^2*b)*x)*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^2 + 12*(8*b^8*x^8 + 64*a*b^7*x^7 + 8*a^8 + 2*(112*a^2*b^6 +
 9*b^6)*x^6 + 18*a^6 + 4*(112*a^3*b^5 + 27*a*b^5)*x^5 + (560*a^4*b^4 + 270*
a^2*b^4 + 13*b^4)*x^4 + 13*a^4 + 4*(112*a^5*b^3 + 90*a^3*b^3 + 13*a*b^3)*x^
3 + (224*a^6*b^2 + 270*a^4*b^2 + 78*a^2*b^2 + 3*b^2)*x^2 + 3*a^2 + 2*(32*a^
7*b + 54*a^5*b + 26*a^3*b + 3*a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) +
 4*(16*b^9*x^9 + 144*a*b^8*x^8 + 16*a^9 + (576*a^2*b^7 + 49*b^7)*x^7 + 49*a
^7 + 7*(192*a^3*b^6 + 49*a*b^6)*x^6 + 3*(672*a^4*b^5 + 343*a^2*b^5 + 18*b^5
)*x^5 + 54*a^5 + (2016*a^5*b^4 + 1715*a^3*b^4 + 270*a*b^4)*x^4 + (1344*a^6*
b^3 + 1715*a^4*b^3 + 540*a^2*b^3 + 25*b^3)*x^3 + 25*a^3 + 3*(192*a^7*b^2 +
343*a^5*b^2 + 180*a^3*b^2 + 25*a*b^2)*x^2 + (144*a^8*b + 343*a^6*b + 270*a^
4*b + 75*a^2*b + 4*b)*x + 4*a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (16*b^10*x^1
0 + 160*a*b^9*x^9 + 16*a^10 + 2*(360*a^2*b^8 + 31*b^8)*x^8 + 62*a^8 + 16*(1
20*a^3*b^7 + 31*a*b^7)*x^7 + 7*(480*a^4*b^6 + 248*a^2*b^6 + 13*b^6)*x^6 + 9
1*a^6 + 14*(288*a^5*b^5 + 248*a^3*b^5 + 39*a*b^5)*x^5 + (3360*a^6*b^4 + 434
0*a^4*b^4 + 1365*a^2*b^4 + 61*b^4)*x^4 + 61*a^4 + 4*(480*a^7*b^3 + 868*a^5*
b^3 + 455*a^3*b^3 + 61*a*b^3)*x^3 + (720*a^8*b^2 + 1736*a^6*b^2 + 1365*a^4*
b^2 + 366*a^2*b^2 + 17*b^2)*x^2 + 17*a^2 + 2*(80*a^9*b + 248*a^7*b + 273*a^
5*b + 122*a^3*b + 17*a*b)*x + 1)*sqrt(b^2*x^2 +...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="fric
as")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a^2 + 2abx + b^2x^2 + 1)^{\frac{3}{2}}}{\operatorname{asinh}^3(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**3,x)
```

```
[Out] Integral((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)/asinh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^3,x, algorithm="giac
")
```

```
[Out] integrate((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)/arcsinh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}}{{\mathrm{asinh}\left(a+b\,x\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3,x)
```

```
[Out] int((a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)/asinh(a + b*x)^3, x)
```

3.272 $\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

Rubi [A]

time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^3}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^4/(4*b)
```

Maple [A]
time = 3.00, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^4}{4b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arcsinh(b*x+a)^4/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(13) = 26$.
time = 0.31, size = 179, normalized size = 11.93

$$\frac{\operatorname{arsinh}(bx+a)^3 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b} - \frac{3\,\operatorname{arsinh}(bx+a)^2 \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{2\,b} + \frac{\operatorname{arsinh}(bx+a) \operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^3}{b} - \frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - 3/2*arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2))^2/b + arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))^3/b - 1/4*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2))^4/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.
time = 0.37, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/4*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.49, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^{4}\left(a+bx\right)}{4b} & \text{for}\: b \neq 0 \\ \frac{x \operatorname{asinh}^{3}\left(a\right)}{\sqrt{a^{2}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**4/(4*b), Ne(b, 0)), (x*asinh(a)**3/sqrt(a**2 + 1
), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [B]
time = 0.23, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a+b\,x)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^4/(4*b)
```

3.273 $\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

```
[Out] 1/3*arcsinh(b*x+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^3/(3*b)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(a+bx)^2}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\ &= \frac{\sinh^{-1}(a+bx)^3}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^3/(3*b)
```

Maple [A]
time = 3.04, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^3}{3b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsinh(b*x+a)^3/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(13) = 26$.
time = 0.27, size = 132, normalized size = 8.80

$$\frac{\operatorname{arsinh}(bx+a)^2 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{b} - \frac{\operatorname{arsinh}(bx+a) \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)^2}{b} + \frac{\operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] arcsinh(b*x + a)^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2))/b - arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 +
 1)*b^2))^2/b + 1/3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b
^2))^3/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.
time = 0.34, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.44, size = 26, normalized size = 1.73

$$\begin{cases} \frac{\operatorname{asinh}^3(a+bx)}{3b} & \text{for } b \neq 0 \\ \frac{x \operatorname{asinh}^2(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**3/(3*b), Ne(b, 0)), (x*asinh(a)**2/sqrt(a**2 + 1
), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^2/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [B]
time = 0.19, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a + b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^3/(3*b)
```

3.274 $\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

[Out] 1/2*arcsinh(b*x+a)^2/b

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5860, 5783}

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]

[Out] ArcSinh[a + b*x]^2/(2*b)

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b}\\
&= \frac{\sinh^{-1}(a+bx)^2}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sinh^{-1}(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2],x]
```

```
[Out] ArcSinh[a + b*x]^2/(2*b)
```

Maple [A]
time = 3.10, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\operatorname{arcsinh}(bx+a)^2}{2b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(b*x+a)^2/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.
time = 0.27, size = 84, normalized size = 5.60

$$\frac{\operatorname{arsinh}(bx+a)\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{b}-\frac{\operatorname{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] arcsinh(b*x + a)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b - 1/2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))^2/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.
time = 0.40, size = 32, normalized size = 2.13

$$\frac{\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.42, size = 24, normalized size = 1.60

$$\begin{cases} \frac{\operatorname{asinh}^{2}(a+bx)}{2b} & \text{for}\ b \neq 0 \\ \frac{x \operatorname{asinh}(a)}{\sqrt{a^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((asinh(a + b*x)**2/(2*b), Ne(b, 0)), (x*asinh(a)/sqrt(a**2 + 1),
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1), x)
```

Mupad [B]
time = 0.20, size = 13, normalized size = 0.87

$$\frac{\operatorname{asinh}(a + b\,x)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2),x)
```

```
[Out] asinh(a + b*x)^2/(2*b)
```

3.275 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=11

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

[Out] ln(arcsinh(b*x+a))/b

Rubi [A]

time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5782}

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]

[Out] Log[ArcSinh[a + b*x]]/b

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}\\ &= \frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{\log\left(\sinh^{-1}(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]),x]
[Out] Log[ArcSinh[a + b*x]]/b
```

Maple [A]
time = 3.03, size = 12, normalized size = 1.09

method	result	size
default	$\frac{\ln(\operatorname{arcsinh}(bx+a))}{b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(arcsinh(b*x+a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.
time = 0.35, size = 30, normalized size = 2.73

$$\frac{\log\left(\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fric
as")
[Out] log(log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.
time = 0.65, size = 22, normalized size = 2.00

$$\begin{cases} \frac{\log(\operatorname{asinh}(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((log(asinh(a + b*x))/b, Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)),
True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)), x)
```

Mupad [B]
time = 0.22, size = 11, normalized size = 1.00

$$\frac{\ln(\operatorname{asinh}(a+b\,x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] log(asinh(a + b*x))/b
```

3.276 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

```
[Out] -1/b/arcsinh(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sinh^{-1}(x)^2}\,dx,x,a+bx\right)}{b}\\ &= -\frac{1}{b\sinh^{-1}(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{b\sinh^{-1}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*ArcSinh[a + b*x]))
```

Maple [A]
time = 3.04, size = 14, normalized size = 1.08

method	result	size
default	$-\frac{1}{b\,\mathrm{arcsinh}(bx+a)}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/b/arcsinh(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(13) = 26.
time = 0.28, size = 150, normalized size = 11.54

$$-\frac{b^3x^3+3\,ab^2x^2+a^3+(3\,a^2b+b)x+(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}+a}{\left((b^2x^2+2\,abx+a^2+1)(b^2x+ab)+(b^3x^2+2\,ab^2x+a^2b+b)\sqrt{b^2x^2+2\,abx+a^2+1}\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + (b^2*x^2 + 2*a*b*x + a^2
+ 1)^(3/2) + a)/(((b^2*x^2 + 2*a*b*x + a^2 + 1)*(b^2*x + a*b) + (b^3*x^2 +
2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqr
t(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.
time = 0.39, size = 32, normalized size = 2.46

$$-\frac{1}{b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.90, size = 26, normalized size = 2.00

$$\begin{cases} -\frac{1}{b \operatorname{asinh}\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^2\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*asinh(a + b*x)), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(a)**2
), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2), x)
```

Mupad [B]
time = 0.21, size = 13, normalized size = 1.00

$$-\frac{1}{b \operatorname{asinh}\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(b*asinh(a + b*x))
```

3.277 $\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

```
[Out] -1/2/b/arcsinh(b*x+a)^2
```

Rubi [A]
time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5860, 5783}

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

```
[Out] -1/2*1/(b*ArcSinh[a + b*x]^2)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+a^2+2abx+b^2x^2}\,\sinh^{-1}(a+bx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sinh^{-1}(x)^3}\,dx, x, a+bx\right)}{b}\\
&= -\frac{1}{2b\sinh^{-1}(a+bx)^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{2b\sinh^{-1}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*ArcSinh[a + b*x]^3),x]
```

```
[Out] -1/2*1/(b*ArcSinh[a + b*x]^2)
```

Maple [A]
time = 2.92, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{1}{2b\,\mathrm{arcsinh}(bx+a)^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2/b/arcsinh(b*x+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*(b^7*x^7 + 7*a*b^6*x^6 + a^7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*
a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*
(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^2)*x^2 + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*
a^2*b^2 + b^2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2) + (3*b^5*x^5 + 15*a*b^4*x^4 + 3*a^5 + 5*(6*a^2*b^3 + b^3)*x^3 + 5*
a^3 + 15*(2*a^3*b^2 + a*b^2)*x^2 + (15*a^4*b + 15*a^2*b + 2*b)*x + 2*a)*(b^
2*x^2 + 2*a*b*x + a^2 + 1) + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + (b^5*x^
5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*b^2 + 3*
a*b^2)*x^2 - (b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) - (b^3*x^3 + 3*a*b^2*x^2 +
 a^3 + (3*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (5*a^4*b + 6*a^
2*b + b)*x + (b^4*x^4 + 4*a*b^3*x^3 + a^4 + 2*(3*a^2*b^2 + b^2)*x^2 + 2*a^2
 + 4*(a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)*log(b*x +
a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (3*b^6*x^6 + 18*a*b^5*x^5 + 3*a^6
```

```
+ (45*a^2*b^4 + 7*b^4)*x^4 + 7*a^4 + 4*(15*a^3*b^3 + 7*a*b^3)*x^3 + (45*a^4
*b^2 + 42*a^2*b^2 + 5*b^2)*x^2 + 5*a^2 + 2*(9*a^5*b + 14*a^3*b + 5*a*b)*x +
 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + a)/(((b^4*x^3 + 3*a*b^3*x^2 + 3*a^2
*b^2*x + a^3*b)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 3*(b^5*x^4 + 4*a*b^4*x^3
+ a^4*b + a^2*b + (6*a^2*b^3 + b^3)*x^2 + 2*(2*a^3*b^2 + a*b^2)*x)*(b^2*x^2
 + 2*a*b*x + a^2 + 1)^(3/2) + 3*(b^6*x^5 + 5*a*b^5*x^4 + a^5*b + 2*a^3*b +
2*(5*a^2*b^4 + b^4)*x^3 + 2*(5*a^3*b^3 + 3*a*b^3)*x^2 + a*b + (5*a^4*b^2 +
6*a^2*b^2 + b^2)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + (b^7*x^6 + 6*a*b^6*x^5
+ a^6*b + 3*a^4*b + 3*(5*a^2*b^5 + b^5)*x^4 + 4*(5*a^3*b^4 + 3*a*b^4)*x^3 +
 3*a^2*b + 3*(5*a^4*b^3 + 6*a^2*b^3 + b^3)*x^2 + 6*(a^5*b^2 + 2*a^3*b^2 + a
*b^2)*x + b)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))^2) + integrate(-1/2*(2*b^6*x^6 + 12*a*b^5*x^5 + 2*a^6
 + 3*(10*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4*(10*a^3*b^3 + 3*a*b^3)*x^3 - (2*b^2
*x^2 + 4*a*b*x + 2*a^2 + 3)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(5*a^4*b^2
+ 3*a^2*b^2)*x^2 - 4*(b^3*x^3 + 3*a*b^2*x^2 + a^3 + (3*a^2*b + b)*x + a)*(b
^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 12*
(a^5*b + a^3*b)*x + 4*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^
3 + 2*a^3 + 2*(5*a^3*b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(5/2) + 4*(b^5*x^5 + 5*
a*b^4*x^4 + a^5 + (10*a^2*b^3 + b^3)*x^3 + a^3 + (10*a^3*b^2 + 3*a*b^2)*x^2
 + (5*a^4*b + 3*a^2*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^2 + 6*(b^6*x^6 + 6*
a*b^5*x^5 + a^6 + (15*a^2*b^4 + 2*b^4)*x^4 + 2*a^4 + 4*(5*a^3*b^3 + 2*a*b^3
)*x^3 + (15*a^4*b^2 + 12*a^2*b^2 + b^2)*x^2 + a^2 + 2*(3*a^5*b + 4*a^3*b +
a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 4*(b^7*x^7 + 7*a*b^6*x^6 + a^
7 + 3*(7*a^2*b^5 + b^5)*x^5 + 3*a^5 + 5*(7*a^3*b^4 + 3*a*b^4)*x^4 + (35*a^4
*b^3 + 30*a^2*b^3 + 3*b^3)*x^3 + 3*a^3 + 3*(7*a^5*b^2 + 10*a^3*b^2 + 3*a*b^
2)*x^2 + (7*a^6*b + 15*a^4*b + 9*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^2
 + 1) + (b^8*x^8 + 8*a*b^7*x^7 + a^8 + 4*(7*a^2*b^6 + b^6)*x^6 + 4*a^6 + 8*
(7*a^3*b^5 + 3*a*b^5)*x^5 + 2*(35*a^4*b^4 + 30*a^2*b^4 + 3*b^4)*x^4 + 6*a^4
 + 8*(7*a^5*b^3 + 10*a^3*b^3 + 3*a*b^3)*x^3 + 4*(7*a^6*b^2 + 15*a^4*b^2 + 9
*a^2*b^2 + b^2)*x^2 + 4*a^2 + 8*(a^7*b + 3*a^5*b + 3*a^3*b + a*b)*x + 1)*sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

time = 0.38, size = 32, normalized size = 2.13

$$-\frac{1}{2\,b\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2/(b*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 1.01, size = 29, normalized size = 1.93

$$\begin{cases} -\frac{1}{2b\operatorname{asinh}^2(a+bx)} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^2+1}\operatorname{asinh}^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(1/2),x)
```

```
[Out] Piecewise((-1/(2*b*asinh(a + b*x)**2), Ne(b, 0)), (x/(sqrt(a**2 + 1)*asinh(
a)**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3), x)
```

Mupad [B]
time = 0.20, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\operatorname{asinh}\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^3*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)),x)
```

```
[Out] -1/(2*b*asinh(a + b*x)^2)
```

3.278 $\int \frac{\sinh^{-1}(a+bx)^3}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{\sinh^{-1}(a+bx)^3}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}}-\frac{3\sinh^{-1}(a+bx)^2\log\left(1+e^{2\sinh^{-1}(a+bx)}\right)}{b}-\frac{3\sinh^{-1}(a+bx)\mathrm{Po}}{}$$

```
[Out] arcsinh(b*x+a)^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b
-3*arcsinh(b*x+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3
,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^3/b/(1+(b*x+a)^2)
^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5860, 5787, 5797, 3799, 2221, 2611, 2320, 6724}

$$-\frac{3\sinh^{-1}(a+bx)\text{Li}_2\left(-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{3\text{Li}_3\left(-e^{2\sinh^{-1}(a+bx)}\right)}{2b}+\frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^3}{b}-\frac{3\sinh^{-1}(a+bx)^2\log\left(e^{2\sinh^{-1}(a+bx)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^3/b + ((a + b*x)*ArcSinh[a + b*x]^3)/(b*Sqrt[1 + (a + b*x)
^2]) - (3*ArcSinh[a + b*x]^2*Log[1 + E^(2*ArcSinh[a + b*x])])/b - (3*ArcSin
h[a + b*x]*PolyLog[2, -E^(2*ArcSinh[a + b*x])])/b + (3*PolyLog[3, -E^(2*Arc
Sinh[a + b*x])])/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^3}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^3}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\text{Subst}\left(\int \frac{x\sinh^{-1}(x)^2}{1+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{6\text{Subst}\left(\int \frac{e^{2x}x^2}{1+e^{2x}}\,dx, x, \text{sin}\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^3}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^3}{b\sqrt{1+(a+bx)^2}} - \frac{3\sinh^{-1}(a+bx)^2\log\left(1\right.}{b}$$

Mathematica [A]
time = 0.39, size = 128, normalized size = 1.11

$$\frac{2\sinh^{-1}(a+bx)^2\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 3\log\left(1+e^{-2\sinh^{-1}(a+bx)}\right)\right) + 6\sinh^{-1}(a+bx)\text{PolyLog}\left(2, -e^{-2\sinh^{-1}(a+bx)}\right) + 3\text{PolyLog}\left(3, -e^{-2\sinh^{-1}(a+bx)}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^3/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] (2*ArcSinh[a + b*x]^2*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSi
nh[a + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 3*Log[1 + E^(-2*ArcSinh[a
+ b*x])]) + 6*ArcSinh[a + b*x]*PolyLog[2, -E^(-2*ArcSinh[a + b*x])] + 3*Pol
yLog[3, -E^(-2*ArcSinh[a + b*x])])/(2*b)
```

Maple [A]
time = 5.33, size = 203, normalized size = 1.77

method	result
default	$-\frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-\sqrt{b^2x^2+2abx+a^2+1}\,a+a^2+1\right)\operatorname{arcsinh}(bx+a)^3}{b(b^2x^2+2abx+a^2+1)}+\frac{2\operatorname{arcsinh}(bx+}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^3+2*arcsinh(b*x+a)
^3/b-3*arcsinh(b*x+a)^2*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-3*arcsinh(b*x
+a)*polylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+3/2*polylog(3,-(b*x+a+(1+(b
*x+a)^2)^(1/2))^2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^3/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^3(a+bx)}{(a^2+2abx+b^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**3/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**3/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^3/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(arcsinh(b*x + a)^3/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}(a+b\,x)^3}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
```

```
[Out] int(asinh(a + b*x)^3/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.279 $\int \frac{\sinh^{-1}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sinh^{-1}(a+bx)^2}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}}-\frac{2\sinh^{-1}(a+bx)\log\left(1+e^{2\sinh^{-1}(a+bx)}\right)}{b}-\frac{\text{PolyLog}\left(2,-e^{2\sinh^{-1}(a+bx)}\right)}{b}$$

```
[Out] arcsinh(b*x+a)^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-p
olylog(2,-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b+(b*x+a)*arcsinh(b*x+a)^2/b/(1+(b
*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {5860, 5787, 5797, 3799, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-e^{2\sinh^{-1}(a+bx)}\right)}{b}+\frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{(a+bx)^2+1}}+\frac{\sinh^{-1}(a+bx)^2}{b}-\frac{2\sinh^{-1}(a+bx)\log\left(e^{2\sinh^{-1}(a+bx)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ArcSinh[a + b*x]^2/b + ((a + b*x)*ArcSinh[a + b*x]^2)/(b*Sqrt[1 + (a + b*x)
^2]) - (2*ArcSinh[a + b*x]*Log[1 + E^(2*ArcSinh[a + b*x])])/b - PolyLog[2,
-E^(2*ArcSinh[a + b*x])]/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5797

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Tanh[x], x], x, ArcSinh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)^2}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)^2}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\text{Subst}\left(\int \frac{x\sinh^{-1}(x)}{1+x^2}\,dx, x, a+bx\right)}{b}$$

$$= \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\text{Subst}\left(\int x\tanh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{4\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \sinh\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e\right.}{b}$$

$$= \frac{\sinh^{-1}(a+bx)^2}{b} + \frac{(a+bx)\sinh^{-1}(a+bx)^2}{b\sqrt{1+(a+bx)^2}} - \frac{2\sinh^{-1}(a+bx)\log\left(1+e\right.}{b}$$

Mathematica [A]
time = 0.24, size = 98, normalized size = 1.14

$$\frac{\sinh^{-1}(a+bx)\left(\frac{\left(a+bx-\sqrt{1+a^2+2abx+b^2x^2}\right)\sinh^{-1}(a+bx)}{\sqrt{1+a^2+2abx+b^2x^2}} - 2\log\left(1+e^{-2\sinh^{-1}(a+bx)}\right)\right) + \text{PolyLog}\left(2, -e^{-2\sinh^{-1}(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]^2/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
[Out] (ArcSinh[a + b*x]*(((a + b*x - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])*ArcSinh[a
 + b*x])/Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] - 2*Log[1 + E^(-2*ArcSinh[a + b*
x])]) + PolyLog[2, -E^(-2*ArcSinh[a + b*x])])/b
```

Maple [A]
time = 5.30, size = 168, normalized size = 1.95

method	result

default	$-\frac{\left(b^{2}x^{2}-\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\,bx+2abx-\sqrt{b^{2}x^{2}+2abx+a^{2}+1}\,a+a^{2}+1\right)\operatorname{arcsinh}(bx+a)^{2}}{b(b^{2}x^{2}+2abx+a^{2}+1)}+\frac{2\operatorname{arcsinh}(bx}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-(b^2*x^2+2*a*b*x+a^2+1)
^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)^2+2*arcsinh(b*x+a)
^2/b-2*arcsinh(b*x+a)*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/b-polylog(2,-(b*x
+a+(1+(b*x+a)^2)^(1/2))^2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*arcsinh(b*x + a)^2/(b^4*x^4 + 4*
a*b^3*x^3 + 2*(3*a^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}^{2}(a+bx)}{(a^{2}+2abx+b^{2}x^{2}+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)**2/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)**2/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)^2/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac
")
[Out] integrate(arcsinh(b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a+b\,x)^2}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
[Out] int(asinh(a + b*x)^2/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.280 $\int \frac{\sinh^{-1}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}} dx$

Optimal. Leaf size=46

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

```
[Out] -1/2*ln(1+(b*x+a)^2)/b+(b*x+a)*arcsinh(b*x+a)/b/(1+(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {5860, 5787, 266}

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{(a+bx)^2+1}} - \frac{\log\left((a+bx)^2+1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + (a + b*x)^2]) - Log[1 + (a + b*x)^
2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2),
x_Symbol] :> Simp[x*((a + b*ArcSinh[c*x])^n/(d*Sqrt[d + e*x^2])), x] - Dist
[b*c*(n/d)*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[x*((a + b*ArcSinh[c
*x])^(n - 1)/(1 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[n, 0]
```

Rule 5860

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((A_.) + (B_.)*(x_) +
(C_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/d, Subst[Int[(C/d^2 + (C/d^2)*x^2)
^p*(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, A, B, C,
 n, p}, x] && EqQ[B*(1 + c^2) - 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\sinh^{-1}(a+bx)}{(1+a^2+2abx+b^2x^2)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{(1+x^2)^{3/2}}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, a+bx\right)}{b}$$
$$= \frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+(a+bx)^2}} - \frac{\log\left(1+(a+bx)^2\right)}{2b}$$

Mathematica [A]
time = 0.06, size = 62, normalized size = 1.35

$$\frac{(a+bx)\sinh^{-1}(a+bx)}{b\sqrt{1+a^2+2abx+b^2x^2}} - \frac{\log\left(1+a^2+2abx+b^2x^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2),x]
```

```
[Out] ((a + b*x)*ArcSinh[a + b*x])/(b*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) - Log[1
+ a^2 + 2*a*b*x + b^2*x^2]/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(42) = 84.
time = 5.24, size = 131, normalized size = 2.85

method	result
default	$\frac{2\,\text{arcsinh}(bx+a)}{b} - \frac{\left(b^2x^2-\sqrt{b^2x^2+2abx+a^2+1}\,bx+2abx-\sqrt{b^2x^2+2abx+a^2+1}\,a+a^2+1\right)\text{arcsinh}(bx+a)}{b(b^2x^2+2abx+a^2+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arcsinh(b*x+a)/b-(b^2*x^2-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)*b*x+2*a*b*x-(b^2*
x^2+2*a*b*x+a^2+1)^(1/2)*a+a^2+1)/b/(b^2*x^2+2*a*b*x+a^2+1)*arcsinh(b*x+a)-
1/b*ln(1+(b*x+a+(1+(b*x+a)^2)^(1/2))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(42) = 84.
time = 0.47, size = 119, normalized size = 2.59

$$-\left(\frac{b^2x}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}} + \frac{ab}{(a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2\,abx+a^2+1}}\right)\text{arsinh}\,(bx+a) - \frac{\log\left(b^2x^2+2\,abx+a^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="maxima
")
```

```
[Out] -(b^2*x/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*b
/((a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))*arcsinh(b*x
 + a) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(42) = 84$.
time = 0.37, size = 115, normalized size = 2.50

$$\frac{2\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,(bx + a)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) - (b^2x^2 + 2\,abx + a^2 + 1)\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\,(b^3x^2 + 2\,ab^2x + (a^2 + 1)b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a)*log(b*x + a + sqrt(b^2*x
^2 + 2*a*b*x + a^2 + 1)) - (b^2*x^2 + 2*a*b*x + a^2 + 1)*log(b^2*x^2 + 2*a*
b*x + a^2 + 1))/(b^3*x^2 + 2*a*b^2*x + (a^2 + 1)*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(a + bx)}{(a^2 + 2abx + b^2x^2 + 1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(b**2*x**2+2*a*b*x+a**2+1)**(3/2),x)
```

```
[Out] Integral(asinh(a + b*x)/(a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2), x)
```

Giac [A]
time = 0.42, size = 76, normalized size = 1.65

$$\frac{\left(x + \frac{a}{b}\right)\log\left(bx + a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)}{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}} - \frac{\log\left(b^2x^2 + 2\,abx + a^2 + 1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(b^2*x^2+2*a*b*x+a^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] (x + a/b)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + 1)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a+b\,x)}{{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2),x)
[Out] int(asinh(a + b*x)/(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2), x)
```

3.281 $\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(1+(a+bx)^2)^{3/2}\sinh^{-1}(a+bx)},x\right)$$

```
[Out] Unintegrable(1/(1+(b*x+a)^2)^(3/2)/arcsinh(b*x+a),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
[Out] Defer[Subst][Defer[Int][1/((1 + x^2)^(3/2)*ArcSinh[x]), x], x, a + b*x]/b
```

Rubi steps

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}\sinh^{-1}(x)}\,dx, x, a+bx\right)}{b}$$

Mathematica [A]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+a^2+2abx+b^2x^2)^{3/2}\sinh^{-1}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]),x]
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}\,\text{arcsinh}\,(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

```
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2} + 2abx + b^{2}x^{2} + 1\right)^{\frac{3}{2}} \operatorname{asinh}{\left(a + bx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a),x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a),x, algorithm="giac
")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{asinh}(a+b\,x)\,\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

```
[Out] int(1/(asinh(a + b*x)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

3.282 $\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{1}{b\left(1+(a+bx)^2\right)\sinh^{-1}(a+bx)}-2\text{Int}\left(\frac{a+bx}{\left(1+(a+bx)^2\right)^2\sinh^{-1}(a+bx)},x\right)$$

```
[Out] -1/b/(1+(b*x+a)^2)/arcsinh(b*x+a)-2*Unintegrable((b*x+a)/(1+(b*x+a)^2)^2/ar
csinh(b*x+a),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] -(1/(b*(1 + (a + b*x)^2)*ArcSinh[a + b*x])) - (2*Defer[Subst][Defer[Int][x/
((1 + x^2)^2*ArcSinh[x]), x], x, a + b*x])/b
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sinh^{-1}(x)^2}\,dx,x,a+bx\right)}{b}\\ &= -\frac{1}{b\left(1+(a+bx)^2\right)\sinh^{-1}(a+bx)}-\frac{2\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^2\sinh}\right.}{b}\end{aligned}$$

Mathematica [A]
time = 1.85, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+a^2+2abx+b^2x^2\right)^{3/2}\sinh^{-1}(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2),x]
```

```
[Out] Integrate[1/((1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)*ArcSinh[a + b*x]^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{2}x^{2}+2abx+a^{2}+1\right)^{\frac{3}{2}} \operatorname{arcsinh}\left(bx+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
[Out] int(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="ma
xima")
[Out] -(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/(((b^2*x^2 + 2*a*b*x + a^2 +
 1)*(b^2*x + a*b) + (b^3*x^2 + 2*a*b^2*x + a^2*b + b)*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))) - integrate
((2*b^4*x^4 + 8*a*b^3*x^3 + 2*a^4 + (12*a^2*b^2 + b^2)*x^2 + (2*b^2*x^2 + 4
*a*b*x + 2*a^2 + 1)*(b^2*x^2 + 2*a*b*x + a^2 + 1) + a^2 + 2*(4*a^3*b + a*b)
*x + 2*(2*b^3*x^3 + 6*a*b^2*x^2 + 2*a^3 + (6*a^2*b + b)*x + a)*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1) - 1)/(((b^4*x^4 + 4*a*b^3*x^3 + a^4 + (6*a^2*b^2 + b^
2)*x^2 + a^2 + 2*(2*a^3*b + a*b)*x)*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2) + 2
*(b^5*x^5 + 5*a*b^4*x^4 + a^5 + 2*(5*a^2*b^3 + b^3)*x^3 + 2*a^3 + 2*(5*a^3*
b^2 + 3*a*b^2)*x^2 + (5*a^4*b + 6*a^2*b + b)*x + a)*(b^2*x^2 + 2*a*b*x + a^
2 + 1) + (b^6*x^6 + 6*a*b^5*x^5 + a^6 + 3*(5*a^2*b^4 + b^4)*x^4 + 3*a^4 + 4
*(5*a^3*b^3 + 3*a*b^3)*x^3 + 3*(5*a^4*b^2 + 6*a^2*b^2 + b^2)*x^2 + 3*a^2 +
6*(a^5*b + 2*a^3*b + a*b)*x + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*log(b*x
 + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="fr
icas")
[Out] integral(sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/((b^4*x^4 + 4*a*b^3*x^3 + 2*(3*a
^2 + 1)*b^2*x^2 + a^4 + 4*(a^3 + a)*b*x + 2*a^2 + 1)*arcsinh(b*x + a)^2), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2abx+b^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{asinh}^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+2*a*b*x+a**2+1)**(3/2)/asinh(b*x+a)**2,x)
```

```
[Out] Integral(1/((a**2 + 2*a*b*x + b**2*x**2 + 1)**(3/2)*asinh(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/arcsinh(b*x+a)^2,x, algorithm="gi
ac")
```

```
[Out] integrate(1/((b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*arcsinh(b*x + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\mathrm{asinh}\left(a+b\,x\right)}^2\,{\left(a^2+2\,a\,b\,x+b^2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)),x)
```

```
[Out] int(1/(asinh(a + b*x)^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(3/2)), x)
```

3.283 $\int x^3 \sinh^{-1}(ax^2)\, dx$

Optimal. Leaf size=50

$$-\frac{x^2\sqrt{1+a^2x^4}}{8a}+\frac{\sinh^{-1}(ax^2)}{8a^2}+\frac{1}{4}x^4\sinh^{-1}(ax^2)$$

```
[Out] 1/8*arcsinh(a*x^2)/a^2+1/4*x^4*arcsinh(a*x^2)-1/8*x^2*(a^2*x^4+1)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 281, 327, 221}

$$\frac{\sinh^{-1}(ax^2)}{8a^2}-\frac{x^2\sqrt{a^2x^4+1}}{8a}+\frac{1}{4}x^4\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a*x^2],x]
```

```
[Out] -1/8*(x^2*Sqrt[1 + a^2*x^4])/a + ArcSinh[a*x^2]/(8*a^2) + (x^4*ArcSinh[a*x^
2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\sinh^{-1}\left(ax^2\right)\,dx &= \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{4}\int\frac{2ax^5}{\sqrt{1+a^2x^4}}\,dx\\
&= \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{2}a\int\frac{x^5}{\sqrt{1+a^2x^4}}\,dx\\
&= \frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)-\frac{1}{4}a\text{Subst}\left(\int\frac{x^2}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{1+a^2x^4}}{8a}+\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)}{8a}\\
&= -\frac{x^2\sqrt{1+a^2x^4}}{8a}+\frac{\sinh^{-1}\left(ax^2\right)}{8a^2}+\frac{1}{4}x^4\sinh^{-1}\left(ax^2\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 44, normalized size = 0.88

$$\frac{-ax^2\sqrt{1+a^2x^4}+\left(1+2a^2x^4\right)\sinh^{-1}\left(ax^2\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a*x^2],x]
[Out] (-(a*x^2*Sqrt[1 + a^2*x^4]) + (1 + 2*a^2*x^4)*ArcSinh[a*x^2])/(8*a^2)
```

Maple [**A**]
time = 0.42, size = 71, normalized size = 1.42

method	result	size

default	$\frac{x^{4} \operatorname{arcsinh}\left(a \,x^{2}\right)}{4}-\frac{a \left(\frac{x^{2} \sqrt{a^{2} x^{4}+1}}{4 a^{2}}-\frac{\ln \left(\frac{a^{2} x^{2}}{\sqrt{a^{2}}}+\sqrt{a^{2} x^{4}+1}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*arcsinh(a*x^2)-1/2*a*(1/4*x^2*(a^2*x^4+1)^(1/2)/a^2-1/4/a^2*ln(a^2*
x^2/(a^2)^(1/2)+(a^2*x^4+1)^(1/2))/(a^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(42) = 84$.
time = 0.27, size = 102, normalized size = 2.04

$$\frac{1}{4}\,x^4 \operatorname{arsinh}\left(ax^2\right) + \frac{1}{16}\,a\left(\frac{\log\left(a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} - \frac{\log\left(-a + \frac{\sqrt{a^2x^4+1}}{x^2}\right)}{a^3} + \frac{2\sqrt{a^2x^4+1}}{\left(a^4 - \frac{(a^2x^4+1)a^2}{x^4}\right)x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x^2) + 1/16*a*(log(a + sqrt(a^2*x^4 + 1)/x^2)/a^3 - log(-
a + sqrt(a^2*x^4 + 1)/x^2)/a^3 + 2*sqrt(a^2*x^4 + 1)/((a^4 - (a^2*x^4 + 1)*
a^2/x^4)*x^2))
```

Fricas [A]
time = 0.34, size = 52, normalized size = 1.04

$$-\frac{\sqrt{a^2x^4+1}\,ax^2 - (2\,a^2x^4+1)\log\left(ax^2 + \sqrt{a^2x^4+1}\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 + 1)*log(a*x^2 + sqrt(a^2*x^4 +
1)))/a^2
```

Sympy [A]
time = 0.18, size = 42, normalized size = 0.84

$$\begin{cases} \frac{x^4 \operatorname{asinh}\left(ax^2\right)}{4} - \frac{x^2\sqrt{a^2x^4+1}}{8a} + \frac{\operatorname{asinh}\left(ax^2\right)}{8a^2} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(a*x**2),x)
```

```
[Out] Piecewise((x**4*asinh(a*x**2)/4 - x**2*sqrt(a**2*x**4 + 1)/(8*a) + asinh(a*
x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.40, size = 74, normalized size = 1.48

$$\frac{1}{4}x^4\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{1}{8}a\left(\frac{\sqrt{a^2x^4+1}\,x^2}{a^2}+\frac{\log\left(-x^2|a|+\sqrt{a^2x^4+1}\right)}{a^2|a|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/8*a*(sqrt(a^2*x^4 + 1)*x^2/a^2 +
 log(-x^2*abs(a) + sqrt(a^2*x^4 + 1))/(a^2*abs(a)))
```

Mupad [B]
time = 0.25, size = 45, normalized size = 0.90

$$\frac{x^2\,\mathrm{asinh}(a\,x^2)\,\left(\frac{x^2}{2}+\frac{1}{4\,a^2\,x^2}\right)}{2}-\frac{x^2\,\sqrt{a^2\,x^4+1}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a*x^2),x)
```

```
[Out] (x^2*asinh(a*x^2)*(x^2/2 + 1/(4*a^2*x^2)))/2 - (x^2*(a^2*x^4 + 1)^(1/2))/(8
*a)
```

3.284 $\int x^2 \sinh^{-1}(ax^2)\,dx$

Optimal. Leaf size=101

$$-\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}(ax^2)+\frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}$$

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/9*x*(a^2*x^4+1)^(1/2)/a+1/9*(a*x^2+1)*(cos(2*arcta
n(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^
(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(3/2)/(a^2*x^4+1)^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 327, 226}

$$-\frac{2x\sqrt{a^2x^4+1}}{9a}+\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)\middle|\frac{1}{2}\right)}{9a^{3/2}\sqrt{a^2x^4+1}}+\frac{1}{3}x^3\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(9*a) + (x^3*ArcSinh[a*x^2])/3 + ((1 + a*x^2)*Sqrt
[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/(9*a^(3/
2)*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \sinh^{-1}\left(ax^2\right)\,dx &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}\int \frac{2ax^4}{\sqrt{1+a^2x^4}}\,dx\\ &= \frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)-\frac{1}{3}(2a)\int \frac{x^4}{\sqrt{1+a^2x^4}}\,dx\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx}{9a}\\ &= -\frac{2x\sqrt{1+a^2x^4}}{9a}+\frac{1}{3}x^3\sinh^{-1}\left(ax^2\right)+\frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{9a^{3/2}\sqrt{1+a^2x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 75, normalized size = 0.74

$$\frac{1}{9}\left(-\frac{2(x+a^2x^5)}{a\sqrt{1+a^2x^4}}+3x^3\sinh^{-1}\left(ax^2\right)-\frac{2\sqrt{ia}\,F\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^2],x]
```

```
[Out] ((-2*(x + a^2*x^5))/(a*Sqrt[1 + a^2*x^4]) + 3*x^3*ArcSinh[a*x^2] - (2*Sqrt[
I*a]*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/a^2)/9
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 89, normalized size = 0.88

method	result	size
default	$\frac{x^{3} \operatorname{arcsinh}\left(a\,x^{2}\right)}{3}-\frac{2 a \left(\frac{x \sqrt{a^{2} x^{4}+1}}{3 a^{2}}-\frac{\sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\,\operatorname{EllipticF}\left(x \sqrt{i a}, i\right)}{3 a^{2} \sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}\right)}{3}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*arcsinh(a*x^2)-2/3*a*(1/3/a^2*x*(a^2*x^4+1)^(1/2)-1/3/a^2/(I*a)^(1/
2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*EllipticF(x*(I*a)^
(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 2/9*x^3 - 2*a*integrate(1/3*x^4/(a
^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(
2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqr
t(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*
a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1))/a^(3/2) - 1/12*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x
+ 1)/a^(3/2) + 1/12*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/a^(3/2)
```

Fricas [A]
time = 0.09, size = 67, normalized size = 0.66

$$\frac{3\,ax^{3}\log\left(ax^{2}+\sqrt{a^{2}x^{4}+1}\right)+2\,a\left(-\frac{1}{a^{2}}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{1}{a^{2}}\right)^{\frac{1}{4}}}{x},-1\right)-2\,\sqrt{a^{2}x^{4}+1}\,x}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*x^3*log(a*x^2 + sqrt(a^2*x^4 + 1)) + 2*a*(-1/a^2)^(3/4)*ellipticF(
(-1/a^2)^(1/4)/x, -1) - 2*sqrt(a^2*x^4 + 1)*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{asinh}\left(ax^{2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**2),x)
```

```
[Out] Integral(x**2*asinh(a*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{asinh}\left(a\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x^2),x)
```

```
[Out] int(x^2*asinh(a*x^2), x)
```

3.285 $\int x \sinh^{-1}(ax^2) \, dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)$$

```
[Out] 1/2*x^2*arcsinh(a*x^2)-1/2*(a^2*x^4+1)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6847, 5772, 267}

$$\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{\sqrt{a^2x^4+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^2],x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x\sinh^{-1}\left(ax^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sinh^{-1}(ax)\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)-\frac{1}{2}a\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{1+a^2x^4}}{2a}+\frac{1}{2}x^2\sinh^{-1}\left(ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^2],x]
[Out] -1/2*Sqrt[1 + a^2*x^4]/a + (x^2*ArcSinh[a*x^2])/2
```

Maple [A]
time = 0.28, size = 31, normalized size = 0.91

method	result	size
derivativedivides	$\frac{a\,x^2\,\text{arcsinh}\left(a\,x^2\right)-\sqrt{a^2x^4+1}}{2a}$	31
default	$\frac{a\,x^2\,\text{arcsinh}\left(a\,x^2\right)-\sqrt{a^2x^4+1}}{2a}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(a*x^2*arcsinh(a*x^2)-(a^2*x^4+1)^(1/2))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.88

$$\frac{ax^2\,\text{arsinh}\left(ax^2\right)-\sqrt{a^2x^4+1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="maxima")
[Out] 1/2*(a*x^2*arcsinh(a*x^2) - sqrt(a^2*x^4 + 1))/a
```

Fricas [A]
time = 0.45, size = 42, normalized size = 1.24

$$\frac{ax^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\sqrt{a^2x^4+1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="fricas")
[Out] 1/2*(a*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - sqrt(a^2*x^4 + 1))/a
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.79

$$\begin{cases}\frac{x^2\operatorname{asinh}\left(ax^2\right)}{2}-\frac{\sqrt{a^2x^4+1}}{2a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**2),x)
[Out] Piecewise((x**2*asinh(a*x**2)/2 - sqrt(a**2*x**4 + 1)/(2*a), Ne(a, 0)), (0,
 True))
```

Giac [A]
time = 0.38, size = 40, normalized size = 1.18

$$\frac{1}{2}\,x^2\log\left(ax^2+\sqrt{a^2x^4+1}\right)-\frac{\sqrt{a^2x^4+1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^2),x, algorithm="giac")
[Out] 1/2*x^2*log(a*x^2 + sqrt(a^2*x^4 + 1)) - 1/2*sqrt(a^2*x^4 + 1)/a
```

Mupad [B]
time = 0.25, size = 28, normalized size = 0.82

$$\frac{x^2\operatorname{asinh}(a\,x^2)}{2}-\frac{\sqrt{a^2\,x^4+1}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x^2),x)
[Out] (x^2*asinh(a*x^2))/2 - (a^2*x^4 + 1)^(1/2)/(2*a)
```

3.286 $\int \sinh^{-1}(ax^2)\, dx$

Optimal. Leaf size=162

$$-\frac{2x\sqrt{1+a^2x^4}}{1+ax^2}+x\sinh^{-1}(ax^2)+\frac{2(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,E\left(2\text{ArcTan}(\sqrt{a}\,x)\,|\,\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{1+a^2x^4}}-\frac{(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}}{\sqrt{a}\,\sqrt{1}}$$

```
[Out] x*arcsinh(a*x^2)-2*x*(a^2*x^4+1)^(1/2)/(a*x^2+1)+2*(a*x^2+1)*(cos(2*arctan(
x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1
/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/a^(1/2)/(a^2*x^4+1)^(1/2
)-(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*Ell
ipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2
)/a^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5874, 12, 311, 226, 1210}

$$-\frac{(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\text{ArcTan}(\sqrt{a}\,x)\,|\,\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}+\frac{2(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,E\left(2\text{ArcTan}(\sqrt{a}\,x)\,|\,\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{a^2x^4+1}}-\frac{2x\sqrt{a^2x^4+1}}{ax^2+1}+x\sinh^{-1}(ax^2)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2],x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^4])/(1 + a*x^2) + x*ArcSinh[a*x^2] + (2*(1 + a*x^2)*Sq
rt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[
a]*Sqrt[1 + a^2*x^4]) - ((1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*Elli
pticF[2*ArcTan[Sqrt[a]*x], 1/2])/(Sqrt[a]*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}\left(ax^2\right)\,dx &= x\sinh^{-1}\left(ax^2\right) - \int \frac{2ax^2}{\sqrt{1+a^2x^4}}\,dx \\
&= x\sinh^{-1}\left(ax^2\right) - (2a)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx \\
&= x\sinh^{-1}\left(ax^2\right) - 2\int \frac{1}{\sqrt{1+a^2x^4}}\,dx + 2\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^4}}{1+ax^2} + x\sinh^{-1}\left(ax^2\right) + \frac{2(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a}\,\sqrt{1+a^2x^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 35, normalized size = 0.22

$$x\sinh^{-1}\left(ax^2\right) - \frac{2}{3}ax^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-a^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2],x]
[Out] x*ArcSinh[a*x^2] - (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(a^2*x^4)])/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 77, normalized size = 0.48

method	result	size
default	$x \operatorname{arcsinh}\left(a\,x^{2}\right)-\frac{2 i \sqrt{-i a\,x^{2}+1}\,\sqrt{i a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{i a},i\right)-\operatorname{EllipticE}\left(x \sqrt{i a},i\right)\right)}{\sqrt{i a}\,\sqrt{a^{2} x^{4}+1}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsinh(a*x^2)-2*I/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x
^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-EllipticE(x*(I*a)^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x^2/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2)), x) + x*log(a*x^
2 + sqrt(a^2*x^4 + 1)) - 2*x - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sq
rt(2)*sqrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/
sqrt(a) + 1))/sqrt(a) - 1/4*I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*s
qrt(a))/sqrt(a) + 1) - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a)
 + 1))/sqrt(a) + 1/4*sqrt(2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1)/sqrt(a) - 1
/4*sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a)*x + 1)/sqrt(a)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}\left(ax^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2),x)
```

```
[Out] Integral(asinh(a*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}\left(a\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2),x)
```

```
[Out] int(asinh(a*x^2), x)
```

3.287 $\int \frac{\sinh^{-1}(ax^2)}{x} \, dx$

Optimal. Leaf size=54

$$-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)+\frac{1}{4}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^2)}\right)$$

```
[Out] -1/4*arcsinh(a*x^2)^2+1/2*arcsinh(a*x^2)*ln(1-(a*x^2+(a^2*x^4+1)^(1/2))^2)+
1/4*polylog(2,(a*x^2+(a^2*x^4+1)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\frac{1}{4}\text{Li}_2\left(e^{2\sinh^{-1}(ax^2)}\right)-\frac{1}{4}\sinh^{-1}\left(ax^2\right)^2+\frac{1}{2}\sinh^{-1}\left(ax^2\right)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x,x]
```

```
[Out] -1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 +
PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax^2)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^2)\right)\\ &= -\frac{1}{4}\sinh^{-1}(ax^2)^2 - \text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^2)\right)\\ &= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) - \frac{1}{2}\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.\\ &= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx\right.\\ &= -\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) + \frac{1}{4}\text{Li}_2\left(e^{2\sinh^{-1}(ax^2)}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$-\frac{1}{4}\sinh^{-1}(ax^2)^2 + \frac{1}{2}\sinh^{-1}(ax^2)\log\left(1-e^{2\sinh^{-1}(ax^2)}\right) + \frac{1}{4}\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x,x]
```

```
[Out] -1/4*ArcSinh[a*x^2]^2 + (ArcSinh[a*x^2]*Log[1 - E^(2*ArcSinh[a*x^2])])/2 +
PolyLog[2, E^(2*ArcSinh[a*x^2])]/4
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(a\,x^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x,x)
```

```
[Out] int(arcsinh(a*x^2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x^2)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="fricas")
[Out] integral(arcsinh(a*x^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^{2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x,x)
[Out] Integral(asinh(a*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}(a\,x^2)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x,x)
[Out] int(asinh(a*x^2)/x, x)
```

3.288 $\int \frac{\sinh^{-1}(ax^2)}{x^2} \, dx$

Optimal. Leaf size=75

$$-\frac{\sinh^{-1}(ax^2)}{x}+\frac{\sqrt{a}\,(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}$$

```
[Out] -arcsinh(a*x^2)/x+(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan
(x*a^(1/2)))*EllipticF(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*a^(1/2)*((a^2*
x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 226}

$$\frac{\sqrt{a}\,(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}(ax^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^2]/x) + (Sqrt[a]*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2
]*EllipticF[2*ArcTan[Sqrt[a]*x], 1/2])/Sqrt[1 + a^2*x^4]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
```

```
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax^2)}{x^2}\,dx &= -\frac{\sinh^{-1}(ax^2)}{x}+\int \frac{2a}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}(ax^2)}{x}+(2a)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}(ax^2)}{x}+\frac{\sqrt{a}\,(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,F\left(2\tan^{-1}\left(\sqrt{a}\,x\right)\mid\frac{1}{2}\right)}{\sqrt{1+a^2x^4}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 42, normalized size = 0.56

$$-\frac{\sinh^{-1}(ax^2)+2\sqrt{ia}\,xF\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^2,x]
[Out] -((ArcSinh[a*x^2] + 2*Sqrt[I*a]*x*EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1])/x)
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 66, normalized size = 0.88

method	result	size
default	$-\frac{\operatorname{arcsinh}(a\,x^2)}{x}+\frac{2a\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ia}\,,i\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsinh(a*x^2)/x+2*a/(I*a)^(1/2)*(1-I*a*x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*
x^4+1)^(1/2)*EllipticF(x*(I*a)^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(I*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) +
1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + I
*sqrt(2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1) - log(-1
/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1))/a^(3/2) + sqrt(2)*log(
a*x^2 + sqrt(2)*sqrt(a)*x + 1)/a^(3/2) - sqrt(2)*log(a*x^2 - sqrt(2)*sqrt(a
)*x + 1)/a^(3/2)) + 2*a*integrate(1/(a^3*x^6 + a*x^2 + (a^2*x^4 + 1)^(3/2))
, x) - log(a*x^2 + sqrt(a^2*x^4 + 1))/x
```

Fricas [A]
time = 0.18, size = 75, normalized size = 1.00

$$\frac{2\,a^2x\left(-\frac{1}{a^2}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{1}{a^2}\right)^{\frac{1}{4}}}{x},-1\right)+(x-1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)+x\log\left(ax^2-\sqrt{a^2x^4+1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x*(-1/a^2)^(3/4)*ellipticF((-1/a^2)^(1/4)/x, -1) + (x - 1)*log(a*x^2
 + sqrt(a^2*x^4 + 1)) + x*log(a*x^2 - sqrt(a^2*x^4 + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**2,x)
```

```
[Out] Integral(asinh(a*x**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^2,x)
```

```
[Out] int(asinh(a*x^2)/x^2, x)
```

3.289 $\int \frac{\sinh^{-1}(ax^2)}{x^3} \, dx$

Optimal. Leaf size=33

$$-\frac{\sinh^{-1}(ax^2)}{2x^2} - \frac{1}{2}a\tanh^{-1}\left(\sqrt{1+a^2x^4}\right)$$

```
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh((a^2*x^4+1)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 272, 65, 214}

$$-\frac{1}{2}a\tanh^{-1}\left(\sqrt{a^2x^4+1}\right) - \frac{\sinh^{-1}(ax^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax^2)}{x^3}\,dx &= -\frac{\sinh^{-1}(ax^2)}{2x^2}+\frac{1}{2}\int \frac{2a}{x\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}(ax^2)}{2x^2}+a\int \frac{1}{x\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}(ax^2)}{2x^2}+\frac{1}{4}a\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^4\right)\\
&= -\frac{\sinh^{-1}(ax^2)}{2x^2}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^4}\right)}{2a}\\
&= -\frac{\sinh^{-1}(ax^2)}{2x^2}-\frac{1}{2}a\tanh^{-1}\left(\sqrt{1+a^2x^4}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-\frac{\sinh^{-1}(ax^2)}{2x^2}-\frac{1}{2}a\tanh^{-1}\left(\sqrt{1+a^2x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^2]/x^2 - (a*ArcTanh[Sqrt[1 + a^2*x^4]])/2
```

Maple [A]
time = 0.41, size = 28, normalized size = 0.85

method	result	size
default	$-\frac{\operatorname{arcsinh}(a\,x^2)}{2x^2}-\frac{a\,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^4+1}}\right)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsinh(a*x^2)/x^2-1/2*a*arctanh(1/(a^2*x^4+1)^(1/2))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 1.39

$$-\frac{1}{4}\,a\Big(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\Big)-\frac{\operatorname{arsinh}\left(ax^2\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*arcs
inh(a*x^2)/x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(27) = 54.
time = 0.39, size = 106, normalized size = 3.21

$$-\frac{ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}+1\right)-ax^2\log\left(-ax^2+\sqrt{a^2x^4+1}-1\right)-x^2\log\left(-ax^2+\sqrt{a^2x^4+1}\right)-(x^2-1)\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1) + 1) - a*x^2*log(-a*x^2 + sqrt(a
^2*x^4 + 1) - 1) - x^2*log(-a*x^2 + sqrt(a^2*x^4 + 1)) - (x^2 - 1)*log(a*x^
2 + sqrt(a^2*x^4 + 1)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(ax^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**3,x)
```

```
[Out] Integral(asinh(a*x**2)/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.
time = 0.42, size = 58, normalized size = 1.76

$$-\frac{1}{4}\,a\Big(\log\left(\sqrt{a^2x^4+1}+1\right)-\log\left(\sqrt{a^2x^4+1}-1\right)\Big)-\frac{\log\left(ax^2+\sqrt{a^2x^4+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*a*(log(sqrt(a^2*x^4 + 1) + 1) - log(sqrt(a^2*x^4 + 1) - 1)) - 1/2*log(
a*x^2 + sqrt(a^2*x^4 + 1))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}(a\,x^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^3,x)
```

```
[Out] int(asinh(a*x^2)/x^3, x)
```

3.290 $\int \frac{\sinh^{-1}(ax^2)}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{2a\sqrt{1+a^2x^4}}{3x}+\frac{2a^2x\sqrt{1+a^2x^4}}{3\left(1+ax^2\right)}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}-\frac{2a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{1+a^2x^4}}+\cdots$$

```
[Out] -1/3*arcsinh(a*x^2)/x^3-2/3*a*(a^2*x^4+1)^(1/2)/x+2/3*a^2*x*(a^2*x^4+1)^(1/
2)/(a*x^2+1)-2/3*a^(3/2)*(a*x^2+1)*(cos(2*arctan(x*a^(1/2)))^2)^(1/2)/cos(2
*arctan(x*a^(1/2)))*EllipticE(sin(2*arctan(x*a^(1/2))),1/2*2^(1/2))*((a^2*x
^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)+1/3*a^(3/2)*(a*x^2+1)*(cos(2*arc
tan(x*a^(1/2)))^2)^(1/2)/cos(2*arctan(x*a^(1/2)))*EllipticF(sin(2*arctan(x*
a^(1/2))),1/2*2^(1/2))*((a^2*x^4+1)/(a*x^2+1)^2)^(1/2)/(a^2*x^4+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5875, 12, 331, 311, 226, 1210}

$$-\frac{2a\sqrt{a^2x^4+1}}{3x}+\frac{2a^2x\sqrt{a^2x^4+1}}{3\left(ax^2+1\right)}+\frac{a^{3/2}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}-\frac{2a^{3/2}(ax^2+1)\sqrt{\frac{a^2x^4+1}{(ax^2+1)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{a}\,x\right)|\frac{1}{2}\right)}{3\sqrt{a^2x^4+1}}-\frac{\sinh^{-1}\left(ax^2\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 + a^2*x^4])/(3*x) + (2*a^2*x*Sqrt[1 + a^2*x^4])/(3*(1 + a*x^2)
) - ArcSinh[a*x^2]/(3*x^3) - (2*a^(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 +
 a*x^2)^2]*EllipticE[2*ArcTan[Sqrt[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4]) + (a^
(3/2)*(1 + a*x^2)*Sqrt[(1 + a^2*x^4)/(1 + a*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[a]*x], 1/2])/(3*Sqrt[1 + a^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax^2)}{x^4}\,dx &= -\frac{\sinh^{-1}(ax^2)}{3x^3}+\frac{1}{3}\int \frac{2a}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{\sinh^{-1}(ax^2)}{3x^3}+\frac{1}{3}(2a)\int \frac{1}{x^2\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}(ax^2)}{3x^3}+\frac{1}{3}\left(2a^3\right)\int \frac{x^2}{\sqrt{1+a^2x^4}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}-\frac{\sinh^{-1}(ax^2)}{3x^3}+\frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{1+a^2x^4}}\,dx-\frac{1}{3}\left(2a^2\right)\int \frac{1-ax^2}{\sqrt{1+a^2x^4}}\,d\\
&= -\frac{2a\sqrt{1+a^2x^4}}{3x}+\frac{2a^2x\sqrt{1+a^2x^4}}{3\,(1+ax^2)}-\frac{\sinh^{-1}(ax^2)}{3x^3}-\frac{2a^{3/2}(1+ax^2)\sqrt{\frac{1+a^2x^4}{(1+ax^2)^2}}\,E(}{3\sqrt{1+a^2x^4}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 88, normalized size = 0.45

$$\frac{1}{3}\left(-\frac{2a\sqrt{1+a^2x^4}}{x}-\frac{\sinh^{-1}(ax^2)}{x^3}+\frac{2a^2\left(E\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{ia}\,x\right)\middle|-1\right)\right)}{\sqrt{ia}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^2]/x^4,x]
```

```
[Out] ((-2*a*Sqrt[1 + a^2*x^4])/x - ArcSinh[a*x^2]/x^3 + (2*a^2*(EllipticE[I*ArcS
inh[Sqrt[I*a]*x], -1] - EllipticF[I*ArcSinh[Sqrt[I*a]*x], -1]))/Sqrt[I*a])/
3
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 101, normalized size = 0.51

method	result
default	$-\frac{\operatorname{arcsinh}(a\,x^2)}{3x^3}+\frac{2a\left(-\frac{\sqrt{a^2x^4+1}}{x}+\frac{ia\sqrt{-ia\,x^2+1}\,\sqrt{ia\,x^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{ia},i\right)-\operatorname{EllipticE}\left(x\sqrt{ia},i\right)\right)}{\sqrt{ia}\,\sqrt{a^2x^4+1}}\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(a*x^2)/x^3+2/3*a*(-(a^2*x^4+1)^(1/2)/x+I*a/(I*a)^(1/2)*(1-I*a*
x^2)^(1/2)*(1+I*a*x^2)^(1/2)/(a^2*x^4+1)^(1/2)*(EllipticF(x*(I*a)^(1/2),I)-
EllipticE(x*(I*a)^(1/2),I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a
) + 1) - log(-1/2*I*sqrt(2)*(2*a*x + sqrt(2)*sqrt(a))/sqrt(a) + 1)) - 1/12*
I*sqrt(2)*a^(3/2)*(log(1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)
 - log(-1/2*I*sqrt(2)*(2*a*x - sqrt(2)*sqrt(a))/sqrt(a) + 1)) + 1/12*sqrt(2
)*a^(3/2)*log(a*x^2 + sqrt(2)*sqrt(a)*x + 1) - 1/12*sqrt(2)*a^(3/2)*log(a*x
^2 - sqrt(2)*sqrt(a)*x + 1) + 2*a*integrate(1/3/(a^3*x^8 + a*x^4 + (a^2*x^6
 + x^2)*sqrt(a^2*x^4 + 1)), x) - 1/3*log(a*x^2 + sqrt(a^2*x^4 + 1))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**2)/x**4,x)
```

```
[Out] Integral(asinh(a*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}(a\,x^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^2)/x^4,x)
```

```
[Out] int(asinh(a*x^2)/x^4, x)
```

3.291 $\int \frac{\sinh^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=54

$$-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)+\frac{1}{10}\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^5)}\right)$$

```
[Out] -1/10*arcsinh(a*x^5)^2+1/5*arcsinh(a*x^5)*ln(1-(a*x^5+(a^2*x^10+1)^(1/2))^2
)+1/10*polylog(2,(a*x^5+(a^2*x^10+1)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\frac{1}{10}\text{Li}_2\left(e^{2\sinh^{-1}(ax^5)}\right)-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}(ax^5)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^5]/x,x]
```

```
[Out] -1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 +
 PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2-\frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}\left(ax^5\right)}\right)-\frac{1}{5}\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.\\
&= -\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}\left(ax^5\right)}\right)-\frac{1}{10}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,d\right.\\
&= -\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}\left(ax^5\right)}\right)+\frac{1}{10}\text{Li}_2\left(e^{2\sinh^{-1}\left(ax^5\right)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$-\frac{1}{10}\sinh^{-1}\left(ax^5\right)^2+\frac{1}{5}\sinh^{-1}\left(ax^5\right)\log\left(1-e^{2\sinh^{-1}\left(ax^5\right)}\right)+\frac{1}{10}\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(ax^5\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^5]/x,x]
```

```
[Out] -1/10*ArcSinh[a*x^5]^2 + (ArcSinh[a*x^5]*Log[1 - E^(2*ArcSinh[a*x^5])])/5 +
 PolyLog[2, E^(2*ArcSinh[a*x^5])]/10
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^5)/x,x)
```

```
[Out] int(arcsinh(a*x^5)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="fricas")
[Out] integral(arcsinh(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**5)/x,x)
[Out] Integral(asinh(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^5)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^5)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^5)/x,x)
[Out] int(asinh(a*x^5)/x, x)
```

3.292 $\int x^2 \sinh^{-1}\left(\sqrt{x}\right)\, dx$

Optimal. Leaf size=72

$$-\frac{5}{48}\sqrt{x}\,\sqrt{1+x}+\frac{5}{72}x^{3/2}\sqrt{1+x}-\frac{1}{18}x^{5/2}\sqrt{1+x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5875, 12, 52, 56, 221}

$$-\frac{1}{18}\sqrt{x+1}\,x^{5/2}+\frac{5}{72}\sqrt{x+1}\,x^{3/2}+\frac{1}{3}x^3\sinh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{x+1}\,\sqrt{x}+\frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[1 + x])/48 + (5*x^(3/2)*Sqrt[1 + x])/72 - (x^(5/2)*Sqrt[1
+ x])/18 + (5*ArcSinh[Sqrt[x]])/48 + (x^3*ArcSinh[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh^{-1}\left(\sqrt{x}\right) dx &= \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1+x}}\,dx \\
&= \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{36}\int \frac{x^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) - \frac{5}{48}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{96}\int \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{1}{3}x^3 \sinh^{-1}\left(\sqrt{x}\right) + \frac{5}{48}\mathrm{S} \\
&= -\frac{5}{48}\sqrt{x}\sqrt{1+x} + \frac{5}{72}x^{3/2}\sqrt{1+x} - \frac{1}{18}x^{5/2}\sqrt{1+x} + \frac{5}{48}\sinh^{-1}\left(\sqrt{x}\right) + \frac{1}{3}x^3 \mathrm{s}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 0.60

$$\frac{1}{144}\left(\sqrt{x}\sqrt{1+x}\left(-15+10x-8x^2\right)+3\left(5+16x^3\right)\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-15 + 10*x - 8*x^2) + 3*(5 + 16*x^3)*ArcSinh[Sqrt[x]]
)/144
```

Maple [**A**]
time = 0.27, size = 47, normalized size = 0.65

method	result	size
derivativedivides	$\frac{5\,\mathrm{arcsinh}\left(\sqrt{x}\right)}{48}+\frac{x^{3}\,\mathrm{arcsinh}\left(\sqrt{x}\right)}{3}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}$	47
default	$\frac{5\,\mathrm{arcsinh}\left(\sqrt{x}\right)}{48}+\frac{x^{3}\,\mathrm{arcsinh}\left(\sqrt{x}\right)}{3}+\frac{5x^{\frac{3}{2}}\sqrt{1+x}}{72}-\frac{x^{\frac{5}{2}}\sqrt{1+x}}{18}-\frac{5\sqrt{x}\,\sqrt{1+x}}{48}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/48*arcsinh(x^(1/2))+1/3*x^3*arcsinh(x^(1/2))+5/72*x^(3/2)*(1+x)^(1/2)-1/1
8*x^(5/2)*(1+x)^(1/2)-5/48*x^(1/2)*(1+x)^(1/2)
```

Maxima [A]
time = 0.46, size = 46, normalized size = 0.64

$$\frac{1}{3}\,x^{3}\,\mathrm{arsinh}\left(\sqrt{x}\right)-\frac{1}{18}\,\sqrt{x+1}\,x^{\frac{5}{2}}+\frac{5}{72}\,\sqrt{x+1}\,x^{\frac{3}{2}}-\frac{5}{48}\,\sqrt{x+1}\,\sqrt{x}+\frac{5}{48}\,\mathrm{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(sqrt(x)) - 1/18*sqrt(x + 1)*x^(5/2) + 5/72*sqrt(x + 1)*x^(3
/2) - 5/48*sqrt(x + 1)*sqrt(x) + 5/48*arcsinh(sqrt(x))
```

Fricas [A]
time = 0.36, size = 40, normalized size = 0.56

$$-\frac{1}{144}\left(8\,x^{2}-10\,x+15\right)\sqrt{x+1}\,\sqrt{x}+\frac{1}{48}\left(16\,x^{3}+5\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 - 10*x + 15)*sqrt(x + 1)*sqrt(x) + 1/48*(16*x^3 + 5)*log(sqrt
(x + 1) + sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\,\mathrm{asinh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(x**(1/2)),x)
```

```
[Out] Integral(x**2*asinh(sqrt(x)), x)
```

Giac [A]
time = 0.40, size = 50, normalized size = 0.69

$$\frac{1}{3}x^3\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{144}\left(2\left(4x-5\right)x+15\right)\sqrt{x+1}\sqrt{x}-\frac{5}{48}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(x + 1) + sqrt(x)) - 1/144*(2*(4*x - 5)*x + 15)*sqrt(x + 1)
*sqrt(x) - 5/48*log(sqrt(x + 1) - sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{asinh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(x^(1/2)),x)
```

```
[Out] int(x^2*asinh(x^(1/2)), x)
```

3.293 $\int x \sinh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=56

$$\frac{3}{16}\sqrt{x}\,\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5875, 12, 52, 56, 221}

$$-\frac{1}{8}\sqrt{x+1}\,x^{3/2}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\sqrt{x+1}\,\sqrt{x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[Sqrt[x]],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[1 + x])/16 - (x^(3/2)*Sqrt[1 + x])/8 - (3*ArcSinh[Sqrt[x]])
/16 + (x^2*ArcSinh[Sqrt[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int x\sinh^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{1+x}}\,dx\\
&= \frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)+\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\int\frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,\right.\\
&= \frac{3}{16}\sqrt{x}\sqrt{1+x}-\frac{1}{8}x^{3/2}\sqrt{1+x}-\frac{3}{16}\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.66

$$\frac{1}{16}\left((3-2x)\sqrt{x}\sqrt{1+x}+\left(-3+8x^2\right)\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[Sqrt[x]],x]
[Out] ((3 - 2*x)*Sqrt[x]*Sqrt[1 + x] + (-3 + 8*x^2)*ArcSinh[Sqrt[x]])/16
```

Maple [A]
time = 0.28, size = 37, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{3\,\text{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^2\,\text{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}}\sqrt{1+x}}{8}+\frac{3\sqrt{x}\,\sqrt{1+x}}{16}$	37

default	$-\frac{3 \operatorname{arcsinh}\left(\sqrt{x}\right)}{16}+\frac{x^{2} \operatorname{arcsinh}\left(\sqrt{x}\right)}{2}-\frac{x^{\frac{3}{2}} \sqrt{1+x}}{8}+\frac{3 \sqrt{x}\, \sqrt{1+x}}{16}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*arcsinh(x^(1/2))+1/2*x^2*arcsinh(x^(1/2))-1/8*x^(3/2)*(1+x)^(1/2)+3/1
6*x^(1/2)*(1+x)^(1/2)
```

Maxima [A]
time = 0.47, size = 36, normalized size = 0.64

$$\frac{1}{2}\, x^{2} \operatorname{arsinh}\left(\sqrt{x}\right)-\frac{1}{8}\, \sqrt{x+1}\, x^{\frac{3}{2}}+\frac{3}{16}\, \sqrt{x+1}\, \sqrt{x}-\frac{3}{16}\, \operatorname{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(sqrt(x)) - 1/8*sqrt(x + 1)*x^(3/2) + 3/16*sqrt(x + 1)*sqrt(
x) - 3/16*arcsinh(sqrt(x))
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.62

$$-\frac{1}{16}\,(2\,x-3)\sqrt{x+1}\,\sqrt{x}+\frac{1}{16}\,\left(8\,x^{2}-3\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x - 3)*sqrt(x + 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x + 1) + sq
rt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}\left(\sqrt{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(x**(1/2)),x)
```

```
[Out] Integral(x*asinh(sqrt(x)), x)
```

Giac [A]
time = 0.40, size = 48, normalized size = 0.86

$$\frac{1}{2}\, x^{2} \log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{16}\, \sqrt{x^{2}+x}\,(2\,x-3)+\frac{3}{32}\, \log\left(\left|-2\,x+2\,\sqrt{x^{2}+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(x + 1) + sqrt(x)) - 1/16*sqrt(x^2 + x)*(2*x - 3) + 3/32*lo
g(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{asinh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(x^(1/2)),x)
```

```
[Out] int(x*asinh(x^(1/2)), x)
```

3.294 $\int \sinh^{-1}(\sqrt{x})\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{x}\,\sqrt{1+x}+\frac{1}{2}\sinh^{-1}(\sqrt{x})+x\sinh^{-1}(\sqrt{x})$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5874, 12, 1978, 52, 56, 221}

$$-\frac{1}{2}\sqrt{x}\,\sqrt{x+1}+x\sinh^{-1}(\sqrt{x})+\frac{1}{2}\sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*ArcSinh[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}\left(\sqrt{x}\right)\,dx &= x\sinh^{-1}\left(\sqrt{x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\sinh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\,\sqrt{1+x}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\,\sqrt{1+x}+x\sinh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\,\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 1.20

$$\frac{1}{2}\left(-\sqrt{\frac{x}{1+x}}\,(1+x)+2x\sinh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[x/(1 + x)]*(1 + x)) + 2*x*ArcSinh[Sqrt[x]] + ArcTanh[Sqrt[x/(1 + x)
]])/2
```

Maple [A]
time = 0.28, size = 24, normalized size = 0.69

method	result	size
derivativedivides	$\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\operatorname{arcsinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}$	24
default	$\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}+x\operatorname{arcsinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x^(1/2))+x*arcsinh(x^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Maxima [A]
time = 0.47, size = 23, normalized size = 0.66

$$x\operatorname{arsinh}\left(\sqrt{x}\right)-\frac{1}{2}\sqrt{x+1}\,\sqrt{x}+\frac{1}{2}\operatorname{arsinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="maxima")
[Out] x*arcsinh(sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x) + 1/2*arcsinh(sqrt(x))
```

Fricas [A]
time = 0.42, size = 28, normalized size = 0.80

$$\frac{1}{2}(2x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x+1}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [A]
time = 0.10, size = 29, normalized size = 0.83

$$-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\operatorname{asinh}\left(\sqrt{x}\right)+\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2)),x)
[Out] -sqrt(x)*sqrt(x + 1)/2 + x*asinh(sqrt(x)) + asinh(sqrt(x))/2
```

Giac [A]
time = 0.39, size = 40, normalized size = 1.14

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x^2+x}-\frac{1}{4}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

Mupad [B]
time = 0.92, size = 31, normalized size = 0.89

$$\operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)+x\operatorname{asinh}\left(\sqrt{x}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2)),x)
```

```
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) + x*asinh(x^(1/2)) - (x^(1/2)*(x + 1)^(1
/2))/2
```

3.295 $\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=46

$$-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)+\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1-(x^(1/2)+(1+x)^(1/2))^2)+polylo
g(2,(x^(1/2)+(1+x)^(1/2))^2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)-\sinh^{-1}\left(\sqrt{x}\right)^2+2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)\\ &= -\sinh^{-1}\left(\sqrt{x}\right)^2 - 4\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\sqrt{x}\right)\right)\\ &= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - 2\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.\\ &= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx\right.\\ &= -\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$-\sinh^{-1}\left(\sqrt{x}\right)^2 + 2\sinh^{-1}\left(\sqrt{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right) + \text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\sqrt{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x,x]
```

```
[Out] -ArcSinh[Sqrt[x]]^2 + 2*ArcSinh[Sqrt[x]]*Log[1 - E^(2*ArcSinh[Sqrt[x]])] +
PolyLog[2, E^(2*ArcSinh[Sqrt[x]])]
```

Maple [A]
time = 2.04, size = 78, normalized size = 1.70

method	result
derivativedivides	$-\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\,\text{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\text{polylog}\left(2, -\sqrt{x}\,-\right.$
default	$-\text{arcsinh}\left(\sqrt{x}\right)^2 + 2\,\text{arcsinh}\left(\sqrt{x}\right)\ln\left(1+\sqrt{x}+\sqrt{1+x}\right) + 2\,\text{polylog}\left(2, -\sqrt{x}\,-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(x^(1/2))^2+2*arcsinh(x^(1/2))*ln(1+x^(1/2)+(1+x)^(1/2))+2*polylog(
2,-x^(1/2)-(1+x)^(1/2))+2*arcsinh(x^(1/2))*ln(1-x^(1/2)-(1+x)^(1/2))+2*poly
log(2,x^(1/2)+(1+x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x,x)
```

```
[Out] Integral(asinh(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arcsinh(sqrt(x))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x,x)
[Out] int(asinh(x^(1/2))/x, x)
```

3.296 $\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{1+x}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 37}

$$-\frac{\sqrt{x+1}}{\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^2,x]
```

```
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\int \frac{\sinh^{-1}(\sqrt{x})}{x^2} dx = -\frac{\sinh^{-1}(\sqrt{x})}{x} + \int \frac{1}{2x^{3/2}\sqrt{1+x}} dx$$
$$= -\frac{\sinh^{-1}(\sqrt{x})}{x} + \frac{1}{2}\int \frac{1}{x^{3/2}\sqrt{1+x}} dx$$
$$= -\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\sinh^{-1}(\sqrt{x})}{x}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\sqrt{1+x}}{\sqrt{x}} - \frac{\sinh^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^2,x]
[Out] -(Sqrt[1 + x]/Sqrt[x]) - ArcSinh[Sqrt[x]]/x
```

Maple [A]
time = 0.28, size = 21, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}(\sqrt{x})}{x} - \frac{\sqrt{1+x}}{\sqrt{x}}$	21
default	$-\frac{\operatorname{arcsinh}(\sqrt{x})}{x} - \frac{\sqrt{1+x}}{\sqrt{x}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsinh(x^(1/2))/x-(1+x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.50, size = 20, normalized size = 0.77

$$-\frac{\sqrt{x+1}}{\sqrt{x}} - \frac{\operatorname{arsinh}(\sqrt{x})}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x + 1)/sqrt(x) - arcsinh(sqrt(x))/x
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.96

$$-\frac{\sqrt{x+1}\,\sqrt{x}+\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -(sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) + sqrt(x)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**2,x)
[Out] Integral(asinh(sqrt(x))/x**2, x)
```

Giac [A]
time = 0.40, size = 35, normalized size = 1.35

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{2}{\left(\sqrt{x+1}-\sqrt{x}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^2,x, algorithm="giac")
[Out] -log(sqrt(x + 1) + sqrt(x))/x + 2/((sqrt(x + 1) - sqrt(x))^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^2,x)
[Out] int(asinh(x^(1/2))/x^2, x)
```

3.297 $\int \frac{\sinh^{-1}(\sqrt{x})}{x^3} dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{1+x}}{6x^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{x}}-\frac{\sinh^{-1}(\sqrt{x})}{2x^2}$$

```
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5875, 12, 47, 37}

$$-\frac{\sqrt{x+1}}{6x^{3/2}}-\frac{\sinh^{-1}(\sqrt{x})}{2x^2}+\frac{\sqrt{x+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] -1/6*Sqrt[1 + x]/x^(3/2) + Sqrt[1 + x]/(3*Sqrt[x]) - ArcSinh[Sqrt[x]]/(2*x^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\int \frac{1}{2x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{6}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{1+x}}{6x^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}\,\sqrt{1+x}\,(-1+2x)-3\sinh^{-1}\left(\sqrt{x}\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-1 + 2*x) - 3*ArcSinh[Sqrt[x]])/(6*x^2)
```

Maple [A]
time = 0.35, size = 31, normalized size = 0.67

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$	31
default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{2x^2}-\frac{\sqrt{1+x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arcsinh(x^(1/2))/x^2-1/6*(1+x)^(1/2)/x^(3/2)+1/3*(1+x)^(1/2)/x^(1/2)
```

Maxima [**A**]
time = 0.52, size = 30, normalized size = 0.65

$$\frac{\sqrt{x+1}}{3\sqrt{x}}-\frac{\sqrt{x+1}}{6\,x^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="maxima")
[Out] 1/3*sqrt(x + 1)/sqrt(x) - 1/6*sqrt(x + 1)/x^(3/2) - 1/2*arcsinh(sqrt(x))/x^
2
```

Fricas [**A**]
time = 0.38, size = 32, normalized size = 0.70

$$\frac{(2\,x-1)\sqrt{x+1}\,\sqrt{x}\,-3\,\log\left(\sqrt{x+1}\,+\sqrt{x}\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*((2*x - 1)*sqrt(x + 1)*sqrt(x) - 3*log(sqrt(x + 1) + sqrt(x)))/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**3,x)
[Out] Integral(asinh(sqrt(x))/x**3, x)
```

Giac [**A**]
time = 0.40, size = 52, normalized size = 1.13

$$-\frac{\log\left(\sqrt{x+1}\,+\sqrt{x}\right)}{2\,x^2}+\frac{2\left(3\left(\sqrt{x+1}\,-\sqrt{x}\right)^2-1\right)}{3\left(\left(\sqrt{x+1}\,-\sqrt{x}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(x + 1) - sqrt(x))^2 - 1)
/((sqrt(x + 1) - sqrt(x))^2 - 1)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^3,x)
```

```
[Out] int(asinh(x^(1/2))/x^3, x)
```

3.298 $\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{1+x}}{15x^{5/2}}+\frac{4\sqrt{1+x}}{45x^{3/2}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}$$

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5875, 12, 47, 37}

$$\frac{4\sqrt{x+1}}{45x^{3/2}}-\frac{\sqrt{x+1}}{15x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{8\sqrt{x+1}}{45\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] -1/15*Sqrt[1 + x]/x^(5/2) + (4*Sqrt[1 + x])/(45*x^(3/2)) - (8*Sqrt[1 + x])/
(45*Sqrt[x]) - ArcSinh[Sqrt[x]]/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^4}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{3}\int \frac{1}{2x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{1}{6}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}-\frac{2}{15}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}+\frac{4\sqrt{1+x}}{45x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}+\frac{4}{45}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{15x^{5/2}}+\frac{4\sqrt{1+x}}{45x^{3/2}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.63

$$\frac{\sqrt{x}\,\sqrt{1+x}\,(-3+4x-8x^2)-15\sinh^{-1}\left(\sqrt{x}\right)}{45x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^4,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-3 + 4*x - 8*x^2) - 15*ArcSinh[Sqrt[x]])/(45*x^3)
```

Maple [A]
time = 0.36, size = 41, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^3}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$	41

default	$-\frac{\operatorname{arcsinh}\left(\sqrt{x}\right)}{3x^{3}}-\frac{\sqrt{1+x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1+x}}{45x^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{45\sqrt{x}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(x^(1/2))/x^3-1/15*(1+x)^(1/2)/x^(5/2)+4/45*(1+x)^(1/2)/x^(3/2)
-8/45*(1+x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.47, size = 40, normalized size = 0.65

$$-\frac{8\sqrt{x+1}}{45\sqrt{x}}+\frac{4\sqrt{x+1}}{45x^{\frac{3}{2}}}-\frac{\sqrt{x+1}}{15x^{\frac{5}{2}}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -8/45*sqrt(x + 1)/sqrt(x) + 4/45*sqrt(x + 1)/x^(3/2) - 1/15*sqrt(x + 1)/x^(
5/2) - 1/3*arcsinh(sqrt(x))/x^3
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.60

$$-\frac{(8x^{2}-4x+3)\sqrt{x+1}\sqrt{x}+15\log\left(\sqrt{x+1}+\sqrt{x}\right)}{45x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/45*((8*x^2 - 4*x + 3)*sqrt(x + 1)*sqrt(x) + 15*log(sqrt(x + 1) + sqrt(x)
))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**4,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**4, x)
```

Giac [A]
time = 0.39, size = 67, normalized size = 1.08

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{3\,x^{3}}+\frac{16\left(10\left(\sqrt{x+1}-\sqrt{x}\right)^{4}-5\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)}{45\left(\left(\sqrt{x+1}-\sqrt{x}\right)^{2}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(x + 1) + sqrt(x))/x^3 + 16/45*(10*(sqrt(x + 1) - sqrt(x))^4 -
 5*(sqrt(x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 - 1)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^4,x)
```

```
[Out] int(asinh(x^(1/2))/x^4, x)
```

3.299 $\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^5}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{2\sqrt{1+x}}{35x^{3/2}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}$$

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5875, 12, 47, 37}

$$-\frac{2\sqrt{x+1}}{35x^{3/2}}+\frac{3\sqrt{x+1}}{70x^{5/2}}-\frac{\sqrt{x+1}}{28x^{7/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{4\sqrt{x+1}}{35\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] -1/28*Sqrt[1 + x]/x^(7/2) + (3*Sqrt[1 + x])/(70*x^(5/2)) - (2*Sqrt[1 + x])/
(35*x^(3/2)) + (4*Sqrt[1 + x])/(35*Sqrt[x]) - ArcSinh[Sqrt[x]]/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{x}\right)}{x^5}\,dx &= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{1}{4}\int \frac{1}{2x^{9/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{1}{8}\int \frac{1}{x^{9/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{3}{28}\int \frac{1}{x^{7/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}+\frac{3}{35}\int \frac{1}{x^{5/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{2\sqrt{1+x}}{35x^{3/2}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}-\frac{2}{35}\int \frac{1}{x^{3/2}\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{1+x}}{28x^{7/2}}+\frac{3\sqrt{1+x}}{70x^{5/2}}-\frac{2\sqrt{1+x}}{35x^{3/2}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}-\frac{\sinh^{-1}\left(\sqrt{x}\right)}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.56

$$\frac{\sqrt{x}\,\sqrt{1+x}\,(-5+6x-8x^2+16x^3)-35\sinh^{-1}\left(\sqrt{x}\right)}{140x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[x]]/x^5,x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(-5 + 6*x - 8*x^2 + 16*x^3) - 35*ArcSinh[Sqrt[x]])/(14
0*x^4)
```

Maple [A]
time = 0.28, size = 51, normalized size = 0.65

method	result	size

derivativedivides	$-\frac{\operatorname{arcsinh}(\sqrt{x})}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$	51
default	$-\frac{\operatorname{arcsinh}(\sqrt{x})}{4x^4}-\frac{\sqrt{1+x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70x^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1+x}}{35\sqrt{x}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsinh(x^(1/2))/x^4-1/28*(1+x)^(1/2)/x^(7/2)+3/70*(1+x)^(1/2)/x^(5/2)
-2/35*(1+x)^(1/2)/x^(3/2)+4/35*(1+x)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.47, size = 50, normalized size = 0.64

$$\frac{4\sqrt{x+1}}{35\sqrt{x}}-\frac{2\sqrt{x+1}}{35\,x^{\frac{3}{2}}}+\frac{3\sqrt{x+1}}{70\,x^{\frac{5}{2}}}-\frac{\sqrt{x+1}}{28\,x^{\frac{7}{2}}}-\frac{\operatorname{arsinh}\left(\sqrt{x}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 4/35*sqrt(x + 1)/sqrt(x) - 2/35*sqrt(x + 1)/x^(3/2) + 3/70*sqrt(x + 1)/x^(5
/2) - 1/28*sqrt(x + 1)/x^(7/2) - 1/4*arcsinh(sqrt(x))/x^4
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.54

$$\frac{(16\,x^3-8\,x^2+6\,x-5)\sqrt{x+1}\,\sqrt{x}-35\,\log\left(\sqrt{x+1}+\sqrt{x}\right)}{140\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/140*((16*x^3 - 8*x^2 + 6*x - 5)*sqrt(x + 1)*sqrt(x) - 35*log(sqrt(x + 1)
+ sqrt(x)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x**(1/2))/x**5,x)
```

```
[Out] Integral(asinh(sqrt(x))/x**5, x)
```

Giac [**A**]
time = 0.39, size = 82, normalized size = 1.05

$$-\frac{\log\left(\sqrt{x+1}+\sqrt{x}\right)}{4\,x^4}+\frac{8\left(35\left(\sqrt{x+1}-\sqrt{x}\right)^6-21\left(\sqrt{x+1}-\sqrt{x}\right)^4+7\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)}{35\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(sqrt(x + 1) + sqrt(x))/x^4 + 8/35*(35*(sqrt(x + 1) - sqrt(x))^6 -
21*(sqrt(x + 1) - sqrt(x))^4 + 7*(sqrt(x + 1) - sqrt(x))^2 - 1)/((sqrt(x +
1) - sqrt(x))^2 - 1)^7
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{asinh}\left(\sqrt{x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x^(1/2))/x^5,x)
```

```
[Out] int(asinh(x^(1/2))/x^5, x)
```

3.300 $\int x^2 \sinh^{-1}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=56

$$\frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\mathrm{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

```
[Out] 1/3*x^3*arccsch(x/a)-1/6*a^3*arctanh((1+a^2/x^2)^(1/2))+1/6*a*x^2*(1+a^2/x^
2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5870, 6419, 272, 44, 65, 214}

$$\frac{1}{6}ax^2\sqrt{\frac{a^2}{x^2}+1}-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+\frac{1}{3}x^3\mathrm{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2)/6 + (x^3*ArcCsch[x/a])/3 - (a^3*ArcTanh[Sqrt[1 +
a^2/x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\sinh^{-1}\left(\frac{a}{x}\right)\,dx &= \int x^2\text{csch}^{-1}\left(\frac{x}{a}\right)\,dx\\ &= \frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}a\int \frac{x}{\sqrt{1+\frac{a^2}{x^2}}}\,dx\\ &= \frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,\frac{1}{x^2}\right)\\ &= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{12}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,\frac{1}{x^2}\right)\\ &= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{6}a\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+\frac{a^2}{x^2}}\right)\\ &= \frac{1}{6}a\sqrt{1+\frac{a^2}{x^2}}\,x^2+\frac{1}{3}x^3\text{csch}^{-1}\left(\frac{x}{a}\right)-\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.02

$$\frac{1}{6}\left(a\sqrt{1+\frac{a^2}{x^2}}\,x^2+2x^3\sinh^{-1}\left(\frac{a}{x}\right)-a^3\log\left(\left(1+\sqrt{1+\frac{a^2}{x^2}}\right)x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x^2 + 2*x^3*ArcSinh[a/x] - a^3*Log[(1 + Sqrt[1 + a^2/x
^2])*x])/6
```

Maple [A]
time = 0.88, size = 54, normalized size = 0.96

method	result	size
derivativedivides	$-a^{3}\left(-\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3a^{3}}-\frac{x^{2}\sqrt{1+\frac{a^{2}}{x^{2}}}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^{2}}{x^{2}}}}\right)}{6}\right)$	54
default	$-a^{3}\left(-\frac{x^{3}\operatorname{arcsinh}\left(\frac{a}{x}\right)}{3a^{3}}-\frac{x^{2}\sqrt{1+\frac{a^{2}}{x^{2}}}}{6a^{2}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^{2}}{x^{2}}}}\right)}{6}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*(-1/3*x^3/a^3*arcsinh(a/x)-1/6*x^2/a^2*(1+a^2/x^2)^(1/2)+1/6*arctanh(1
/(1+a^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 1.23

$$\frac{1}{3}\,x^{3}\operatorname{arsinh}\left(\frac{a}{x}\right)-\frac{1}{12}\left(a^{2}\log\left(\sqrt{\frac{a^{2}}{x^{2}}+1}+1\right)-a^{2}\log\left(\sqrt{\frac{a^{2}}{x^{2}}+1}-1\right)-2\,x^{2}\sqrt{\frac{a^{2}}{x^{2}}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a/x) - 1/12*(a^2*log(sqrt(a^2/x^2 + 1) + 1) - a^2*log(sqrt(
a^2/x^2 + 1) - 1) - 2*x^2*sqrt(a^2/x^2 + 1))*a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(46) = 92.
time = 0.38, size = 122, normalized size = 2.18

$$\frac{1}{6}\,a^{3}\log\left(x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}-x\right)+\frac{1}{6}\,ax^{2}\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}+\frac{1}{3}\left(x^{3}-1\right)\log\left(\frac{x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}+a}{x}\right)+\frac{1}{3}\log\left(x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}+a-x\right)-\frac{1}{3}\log\left(x\sqrt{\frac{a^{2}+x^{2}}{x^{2}}}-a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/6*a^3*log(x*sqrt((a^2 + x^2)/x^2) - x) + 1/6*a*x^2*sqrt((a^2 + x^2)/x^2)
+ 1/3*(x^3 - 1)*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/3*log(x*sqrt((a^2
+ x^2)/x^2) + a - x) - 1/3*log(x*sqrt((a^2 + x^2)/x^2) - a - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}\left(\frac{a}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a/x),x)
```

```
[Out] Integral(x**2*asinh(a/x), x)
```

Giac [A]
time = 0.42, size = 74, normalized size = 1.32

$$-\frac{1}{6} a^3 \log(|a|) \operatorname{sgn}(x) + \frac{1}{3} x^3 \log\left(\sqrt{\frac{a^2}{x^2}+1} + \frac{a}{x}\right) + \frac{a^3 \log\left(-x+\sqrt{a^2+x^2}\right)}{6 \operatorname{sgn}(x)} + \frac{\sqrt{a^2+x^2}\, ax}{6 \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] -1/6*a^3*log(abs(a))*sgn(x) + 1/3*x^3*log(sqrt(a^2/x^2 + 1) + a/x) + 1/6*a^
3*log(-x + sqrt(a^2 + x^2))/sgn(x) + 1/6*sqrt(a^2 + x^2)*a*x/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{asinh}\left(\frac{a}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a/x),x)
```

```
[Out] int(x^2*asinh(a/x), x)
```

3.301 $\int x \sinh^{-1}\left(\frac{a}{x}\right) dx$

Optimal. Leaf size=33

$$\frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}\,x+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

```
[Out] 1/2*x^2*arccsch(x/a)+1/2*a*x*(1+a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5870, 6419, 197}

$$\frac{1}{2}ax\sqrt{\frac{a^2}{x^2}+1}+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a/x],x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x)/2 + (x^2*ArcCsch[x/a])/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sinh^{-1}\left(\frac{a}{x}\right)\,dx &= \int x\text{csch}^{-1}\left(\frac{x}{a}\right)\,dx\\ &= \frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}a\int\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\,dx\\ &= \frac{1}{2}a\sqrt{1+\frac{a^2}{x^2}}\,x+\frac{1}{2}x^2\text{csch}^{-1}\left(\frac{x}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.88

$$\frac{1}{2}x\left(a\sqrt{1+\frac{a^2}{x^2}}+x\sinh^{-1}\left(\frac{a}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a/x],x]
```

```
[Out] (x*(a*Sqrt[1 + a^2/x^2] + x*ArcSinh[a/x]))/2
```

Maple [A]
time = 0.87, size = 38, normalized size = 1.15

method	result	size
derivativedivides	$-a^2\left(-\frac{x^2\,\text{arcsinh}\left(\frac{a}{x}\right)}{2a^2}-\frac{x\sqrt{1+\frac{a^2}{x^2}}}{2a}\right)$	38
default	$-a^2\left(-\frac{x^2\,\text{arcsinh}\left(\frac{a}{x}\right)}{2a^2}-\frac{x\sqrt{1+\frac{a^2}{x^2}}}{2a}\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a/x),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*(-1/2*x^2/a^2*arcsinh(a/x)-1/2*x/a*(1+a^2/x^2)^(1/2))
```

Maxima [A]
time = 0.31, size = 27, normalized size = 0.82

$$\frac{1}{2}\,x^2\,\text{arsinh}\left(\frac{a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2}{x^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a/x) + 1/2*a*x*sqrt(a^2/x^2 + 1)
```

Fricas [A]
time = 0.37, size = 45, normalized size = 1.36

$$\frac{1}{2}\,x^2\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\frac{1}{2}\,ax\sqrt{\frac{a^2+x^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) + 1/2*a*x*sqrt((a^2 + x^2)/x^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{asinh}\left(\frac{a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a/x),x)
```

```
[Out] Integral(x*asinh(a/x), x)
```

Giac [A]
time = 0.42, size = 47, normalized size = 1.42

$$\frac{1}{2}\,x^2\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)-\frac{1}{2}\,a|a|\mathrm{sgn}(x)+\frac{\sqrt{a^2+x^2}\,a}{2\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a/x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(a^2/x^2 + 1) + a/x) - 1/2*a*abs(a)*sgn(x) + 1/2*sqrt(a^2 +
 x^2)*a/sgn(x)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.82

$$\frac{x^2\,\mathrm{asinh}\left(\frac{a}{x}\right)}{2}+\frac{a\,x\,\sqrt{\frac{a^2}{x^2}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a/x),x)
[Out] (x^2*asinh(a/x))/2 + (a*x*(a^2/x^2 + 1)^(1/2))/2
```

3.302 $\int \sinh^{-1}\left(\frac{a}{x}\right)\,dx$

Optimal. Leaf size=25

$$x\text{csch}^{-1}\left(\frac{x}{a}\right)+a\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)$$

```
[Out] x*arccsch(x/a)+a*arctanh((1+a^2/x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.833$, Rules used = {5870, 6413, 272, 65, 214}

$$a\tanh^{-1}\left(\sqrt{\frac{a^2}{x^2}+1}\right)+x\text{csch}^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x],x]
```

```
[Out] x*ArcCsch[x/a] + a*ArcTanh[Sqrt[1 + a^2/x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh^{-1}\left(\frac{a}{x}\right) dx &= \int \operatorname{csch}^{-1}\left(\frac{x}{a}\right) dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x} dx \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}} dx, x, \frac{1}{x^2}\right) \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) - \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}} dx, x, \sqrt{1+\frac{a^2}{x^2}}\right)}{a} \\
&= x\operatorname{csch}^{-1}\left(\frac{x}{a}\right) + a\tanh^{-1}\left(\sqrt{1+\frac{a^2}{x^2}}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(25) = 50.
time = 0.06, size = 77, normalized size = 3.08

$$x\sinh^{-1}\left(\frac{a}{x}\right) + \frac{a\sqrt{a^2+x^2}\left(-\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)\right)}{2\sqrt{1+\frac{a^2}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x],x]
```

```
[Out] x*ArcSinh[a/x] + (a*Sqrt[a^2 + x^2]*(-Log[1 - x/Sqrt[a^2 + x^2]] + Log[1 +
x/Sqrt[a^2 + x^2]]))/(2*Sqrt[1 + a^2/x^2]*x)
```

Maple [A]
time = 0.88, size = 31, normalized size = 1.24

method	result	size

derivativedivides	$-a\left(-\frac{x\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{a}-\mathrm{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\right)\right)$	31
default	$-a\left(-\frac{x\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{a}-\mathrm{arctanh}\left(\frac{1}{\sqrt{1+\frac{a^2}{x^2}}}\right)\right)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x),x,method=_RETURNVERBOSE)
[Out] -a*(-x/a*arcsinh(a/x)-arctanh(1/(1+a^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.72

$$\frac{1}{2}\,a\left(\log\left(\sqrt{\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{\frac{a^2}{x^2}+1}-1\right)\right)+x\,\mathrm{arsinh}\left(\frac{a}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="maxima")
[Out] 1/2*a*(log(sqrt(a^2/x^2 + 1) + 1) - log(sqrt(a^2/x^2 + 1) - 1)) + x*arcsinh
(a/x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(23) = 46.
time = 0.40, size = 96, normalized size = 3.84

$$-a\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-x\right)+(x-1)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)+\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}+a-x\right)-\log\left(x\sqrt{\frac{a^2+x^2}{x^2}}-a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="fricas")
[Out] -a*log(x*sqrt((a^2 + x^2)/x^2) - x) + (x - 1)*log((x*sqrt((a^2 + x^2)/x^2)
+ a)/x) + log(x*sqrt((a^2 + x^2)/x^2) + a - x) - log(x*sqrt((a^2 + x^2)/x^2
) - a - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{asinh}\left(\frac{a}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x),x)
```

```
[Out] Integral(asinh(a/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.
time = 0.40, size = 49, normalized size = 1.96

$$a \log (|a|) \operatorname{sgn}(x) + x \log \left(\sqrt{\frac{a^2}{x^2} + 1} + \frac{a}{x} \right) - \frac{a \log \left(-x + \sqrt{a^2 + x^2} \right)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x),x, algorithm="giac")
```

```
[Out] a*log(abs(a))*sgn(x) + x*log(sqrt(a^2/x^2 + 1) + a/x) - a*log(-x + sqrt(a^2
 + x^2))/sgn(x)
```

Mupad [B]
time = 0.55, size = 25, normalized size = 1.00

$$x \operatorname{asinh}\left(\frac{a}{x}\right) + a \ln \left(x + \sqrt{a^2 + x^2}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x),x)
```

```
[Out] x*asinh(a/x) + a*log(x + (a^2 + x^2)^(1/2))*sign(x)
```

3.303 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x} dx$

Optimal. Leaf size=52

$$\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1 - e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1-(a/x+(1+a^2/x^2)^(1/2))^2)-1/2*polylog
(2,(a/x+(1+a^2/x^2)^(1/2))^2)
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$-\frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1 - e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}\,dx &= -\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 + 2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{a}{x}\right)\right)\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \sinh^-\right.\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^-}\right.\\
&= \frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\text{Li}_2\left(e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.00

$$\frac{1}{2}\sinh^{-1}\left(\frac{a}{x}\right)^2 - \sinh^{-1}\left(\frac{a}{x}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{a}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x,x]
```

```
[Out] ArcSinh[a/x]^2/2 - ArcSinh[a/x]*Log[1 - E^(2*ArcSinh[a/x])] - PolyLog[2, E^
(2*ArcSinh[a/x])]/2
```

Maple [A]
time = 2.37, size = 114, normalized size = 2.19

method	result
derivativedivides	$\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^2}{2} - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{polylog}\left(2, -\frac{a}{x}-\sqrt{1+\frac{a^2}{x^2}}\right) - \text{arcs}$

default	$\frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)^2}{2} - \operatorname{arcsinh}\left(\frac{a}{x}\right)\ln\left(1+\frac{a}{x}+\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{polylog}\left(2, -\frac{a}{x}-\sqrt{1+\frac{a^2}{x^2}}\right) - \operatorname{ar}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(a/x)^2-arcsinh(a/x)*ln(1+a/x+(1+a^2/x^2)^(1/2))-polylog(2,-a/x-
(1+a^2/x^2)^(1/2))-arcsinh(a/x)*ln(1-a/x-(1+a^2/x^2)^(1/2))-polylog(2,a/x+(
1+a^2/x^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="maxima")
```

```
[Out] a*integrate(x*log(x)/(a^3 + a*x^2 + (a^2 + x^2)^(3/2)), x) + log(a + sqrt(a
^2 + x^2))*log(x) - 1/2*log(x)^2 - 1/2*log(x)*log(x^2/a^2 + 1) - 1/4*dilog(
-x^2/a^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a/x)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x,x)
```

```
[Out] Integral(asinh(a/x)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a/x)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x,x)
```

```
[Out] int(asinh(a/x)/x, x)
```

3.304 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2} dx$

Optimal. Leaf size=29

$$\frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

```
[Out] -arccsch(x/a)/x+(1+a^2/x^2)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5870, 6419, 267}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^2,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/a - ArcCsch[x/a]/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^2}\,dx = \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^2}\,dx$$
$$= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x} - a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^3}\,dx$$
$$= \frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{\sqrt{1+\frac{a^2}{x^2}}}{a} - \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^2,x]
[Out] Sqrt[1 + a^2/x^2]/a - ArcSinh[a/x]/x
```

Maple [A]
time = 0.28, size = 31, normalized size = 1.07

method	result	size
derivativedivides	$-\frac{\frac{a \operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{1+\frac{a^2}{x^2}}}{a}$	31
default	$-\frac{\frac{a \operatorname{arcsinh}\left(\frac{a}{x}\right)}{x}-\sqrt{1+\frac{a^2}{x^2}}}{a}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/a*(a/x*arcsinh(a/x)-(1+a^2/x^2)^(1/2))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 1.03

$$-\frac{\frac{a \operatorname{arsinh}\left(\frac{a}{x}\right)}{x} - \sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="maxima")
[Out] -(a*arcsinh(a/x)/x - sqrt(a^2/x^2 + 1))/a
```

Fricas [A]
time = 0.38, size = 49, normalized size = 1.69

$$-\frac{a\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-x\sqrt{\frac{a^2+x^2}{x^2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="fricas")
[Out] -(a*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - x*sqrt((a^2 + x^2)/x^2))/(a*x)
```

Sympy [A]
time = 0.41, size = 20, normalized size = 0.69

$$\begin{cases} -\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}+\frac{\sqrt{\frac{a^2}{x^2}+1}}{a} & \text{for}\ a\neq 0\\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**2,x)
[Out] Piecewise((-asinh(a/x)/x + sqrt(a**2/x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Giac [A]
time = 0.44, size = 39, normalized size = 1.34

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{x}+\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^2,x, algorithm="giac")
[Out] -log(sqrt(a^2/x^2 + 1) + a/x)/x + sqrt(a^2/x^2 + 1)/a
```

Mupad [B]
time = 0.23, size = 27, normalized size = 0.93

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{a}-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^2,x)
[Out] (a^2/x^2 + 1)^(1/2)/a - asinh(a/x)/x
```

3.305 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

```
[Out] -1/4*arccsch(x/a)/a^2-1/2*arccsch(x/a)/x^2+1/4*(1+a^2/x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5870, 6419, 342, 327, 221}

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4ax}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^3,x]
```

```
[Out] Sqrt[1 + a^2/x^2]/(4*a*x) - ArcCsch[x/a]/(4*a^2) - ArcCsch[x/a]/(2*x^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^3}\,dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^3}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{1}{2}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^4}\,dx \\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} + \frac{1}{2}a\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a^2x^2}}\,dx, x, \frac{1}{x}\right)}{4a} \\
&= \frac{\sqrt{1+\frac{a^2}{x^2}}}{4ax} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{4a^2} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.88

$$\frac{a\sqrt{1+\frac{a^2}{x^2}}\,x - (2a^2+x^2)\sinh^{-1}\left(\frac{a}{x}\right)}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^3,x]
```

```
[Out] (a*Sqrt[1 + a^2/x^2]*x - (2*a^2 + x^2)*ArcSinh[a/x])/(4*a^2*x^2)
```

Maple [A]
time = 0.88, size = 46, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{\frac{a^2 \operatorname{arcsinh}\left(\frac{a}{x}\right)}{2x^2} - \frac{a\sqrt{1+\frac{a^2}{x^2}}}{4x} + \frac{\operatorname{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$	46

default	$-\frac{\frac{a^2\,\mathrm{arcsinh}\left(\frac{a}{x}\right)}{2x^2}-\frac{a\sqrt{1+\frac{a^2}{x^2}}}{4x}+\frac{\mathrm{arcsinh}\left(\frac{a}{x}\right)}{4}}{a^2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(1/2/x^2*a^2*arcsinh(a/x)-1/4/x*a*(1+a^2/x^2)^(1/2)+1/4*arcsinh(a/x)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(42) = 84$.
time = 0.26, size = 97, normalized size = 1.94

$$\frac{1}{8}\,a\left(\frac{2\,x\sqrt{\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}+1\right)-a^4}-\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}+a\right)}{a^3}+\frac{\log\left(x\sqrt{\frac{a^2}{x^2}+1}-a\right)}{a^3}\right)-\frac{\mathrm{arsinh}\left(\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(2*x*sqrt(a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 + 1) - a^4) - log(x*sqrt(a^2
/x^2 + 1) + a)/a^3 + log(x*sqrt(a^2/x^2 + 1) - a)/a^3) - 1/2*arcsinh(a/x)/x
^2
```

Fricas [A]
time = 0.36, size = 58, normalized size = 1.16

$$\frac{ax\sqrt{\frac{a^2+x^2}{x^2}}-\left(2\,a^2+x^2\right)\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)}{4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*x*sqrt((a^2 + x^2)/x^2) - (2*a^2 + x^2)*log((x*sqrt((a^2 + x^2)/x^2)
 + a)/x))/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{asinh}\left(\frac{a}{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**3,x)
```

```
[Out] Integral(asinh(a/x)/x**3, x)
```

Giac [A]
time = 0.43, size = 84, normalized size = 1.68

$$-\frac{a\left(\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{a^3}-\frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{a^3}-\frac{2\sqrt{a^2+x^2}}{a^2x^2}\right)}{8\,\mathrm{sgn}\left(x\right)}-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*a*(log(a + sqrt(a^2 + x^2))/a^3 - log(-a + sqrt(a^2 + x^2))/a^3 - 2*sq
rt(a^2 + x^2)/(a^2*x^2))/sgn(x) - 1/2*log(sqrt(a^2/x^2 + 1) + a/x)/x^2
```

Mupad [B]
time = 0.24, size = 43, normalized size = 0.86

$$\frac{\sqrt{\frac{a^2}{x^2}+1}}{4\,a\,x}-\frac{\mathrm{asinh}\left(\frac{a}{x}\right)\,\left(\frac{x}{4\,a^2}+\frac{1}{2\,x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^3,x)
```

```
[Out] (a^2/x^2 + 1)^(1/2)/(4*a*x) - (asinh(a/x)*(x/(4*a^2) + 1/(2*x)))/x
```

3.306 $\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4} dx$

Optimal. Leaf size=54

$$-\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3}+\frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

```
[Out] 1/9*(1+a^2/x^2)^(3/2)/a^3-1/3*arccsch(x/a)/x^3-1/3*(1+a^2/x^2)^(1/2)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5870, 6419, 272, 45}

$$\frac{\left(\frac{a^2}{x^2}+1\right)^{3/2}}{9a^3}-\frac{\sqrt{\frac{a^2}{x^2}+1}}{3a^3}-\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a/x]/x^4,x]
```

```
[Out] -1/3*Sqrt[1 + a^2/x^2]/a^3 + (1 + a^2/x^2)^(3/2)/(9*a^3) - ArcCsch[x/a]/(3*
x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
```

```
1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\frac{a}{x}\right)}{x^4}\,dx &= \int \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{x^4}\,dx\\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} - \frac{1}{3}a\int \frac{1}{\sqrt{1+\frac{a^2}{x^2}}\,x^5}\,dx\\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3} + \frac{1}{6}a\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, \frac{1}{x^2}\right)\\
&= -\frac{\sqrt{1+\frac{a^2}{x^2}}}{3a^3} + \frac{\left(1+\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\operatorname{csch}^{-1}\left(\frac{x}{a}\right)}{3x^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.89

$$\left(-\frac{2}{9a^3}+\frac{1}{9ax^2}\right)\sqrt{\frac{a^2+x^2}{x^2}}-\frac{\sinh^{-1}\left(\frac{a}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a/x]/x^4,x]
[Out] (-2/(9*a^3) + 1/(9*a*x^2))*Sqrt[(a^2 + x^2)/x^2] - ArcSinh[a/x]/(3*x^3)
```

Maple [A]
time = 0.88, size = 53, normalized size = 0.98

method	result	size
derivativedivides	$-\frac{\frac{a^3 \operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1+\frac{a^2}{x^2}}}{9x^2}+\frac{2\sqrt{1+\frac{a^2}{x^2}}}{9}}{a^3}$	53
default	$-\frac{\frac{a^3 \operatorname{arcsinh}\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1+\frac{a^2}{x^2}}}{9x^2}+\frac{2\sqrt{1+\frac{a^2}{x^2}}}{9}}{a^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(1/3/x^3*a^3*arcsinh(a/x)-1/9/x^2*a^2*(1+a^2/x^2)^(1/2)+2/9*(1+a^2/x
^2)^(1/2))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.87

$$\frac{1}{9}\,a\left(\frac{\left(\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4}-\frac{3\,\sqrt{\frac{a^2}{x^2}+1}}{a^4}\right)-\frac{\operatorname{arsinh}\left(\frac{a}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a*((a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(a^2/x^2 + 1)/a^4) - 1/3*arcsinh(a/x
)/x^3
```

Fricas [A]
time = 0.38, size = 62, normalized size = 1.15

$$-\frac{3\,a^3\log\left(\frac{x\sqrt{\frac{a^2+x^2}{x^2}}+a}{x}\right)-(a^2x-2\,x^3)\sqrt{\frac{a^2+x^2}{x^2}}}{9\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*a^3*log((x*sqrt((a^2 + x^2)/x^2) + a)/x) - (a^2*x - 2*x^3)*sqrt((a^
2 + x^2)/x^2))/(a^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a/x)/x**4,x)
```

```
[Out] Integral(asinh(a/x)/x**4, x)
```

Giac [A]
time = 0.44, size = 75, normalized size = 1.39

$$-\frac{\log\left(\sqrt{\frac{a^2}{x^2}+1}+\frac{a}{x}\right)}{3\,x^3}-\frac{4\left(a^2-3\left(x-\sqrt{a^2+x^2}\right)^2\right)a}{9\left(a^2-\left(x-\sqrt{a^2+x^2}\right)^2\right)^3\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(a^2/x^2 + 1) + a/x)/x^3 - 4/9*(a^2 - 3*(x - sqrt(a^2 + x^2))^
2)*a/((a^2 - (x - sqrt(a^2 + x^2))^2)^3*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a/x)/x^4,x)
```

```
[Out] int(asinh(a/x)/x^4, x)
```

3.307 $\int x^m \sinh^{-1}(ax^n)\, dx$

Optimal. Leaf size=77

$$\frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m}-\frac{anx^{1+m+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{2n};\frac{1+m+3n}{2n};-a^2x^{2n}\right)}{(1+m)(1+m+n)}$$

```
[Out] x^(1+m)*arcsinh(a*x^n)/(1+m)-a*n*x^(1+m+n)*hypergeom([1/2, 1/2*(1+m+n)/n],[
1/2*(1+m+3*n)/n],-a^2*x^(2*n))/(1+m)/(1+m+n)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\frac{x^{m+1}\sinh^{-1}(ax^n)}{m+1}-\frac{anx^{m+n+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+n+1}{2n};\frac{m+3n+1}{2n};-a^2x^{2n}\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x^n])/(1 + m) - (a*n*x^(1 + m + n)*Hypergeometric2F1[1
/2, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))])/((1 + m)*(1 +
m + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x^m \sinh^{-1}(ax^n)\,dx &= \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{\int \frac{anx^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}\\ &= \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{(an)\int \frac{x^{m+n}}{\sqrt{1+a^2x^{2n}}}\,dx}{1+m}\\ &= \frac{x^{1+m}\sinh^{-1}(ax^n)}{1+m} - \frac{anx^{1+m+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{2n};\frac{1+m+3n}{2n};-a^2x^{2n}\right)}{(1+m)(1+m+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.96

$$\frac{x^{1+m}\left((1+m+n)\sinh^{-1}(ax^n) - anx^n\,{}_2F_1\left(\frac{1}{2},\frac{1+m+n}{2n};\frac{1+m+3n}{2n};-a^2x^{2n}\right)\right)}{(1+m)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcSinh[a*x^n],x]
```

```
[Out] (x^(1 + m)*((1 + m + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, (1
+ m + n)/(2*n), (1 + m + 3*n)/(2*n), -(a^2*x^(2*n))]))/((1 + m)*(1 + m + n)
)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcsinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arcsinh(a*x^n),x)
```

```
[Out] int(x^m*arcsinh(a*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -a*n*integrate(e^(m*log(x) + n*log(x))/(a^3*(m + 1)*x^(3*n) + a*(m + 1)*x^n
 + (a^2*(m + 1)*x^(2*n) + m + 1)*sqrt(a^2*x^(2*n) + 1)), x) + n*integrate(x
^m/(a^2*(m + 1)*x^(2*n) + m + 1), x) + ((m + 1)*x*x^m*log(a*x^n + sqrt(a^2*
x^(2*n) + 1)) - n*x*x^m)/(m^2 + 2*m + 1)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*asinh(a*x**n),x)
```

```
[Out] Integral(x**m*asinh(a*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{asinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*asinh(a*x^n),x)
```

```
[Out] int(x^m*asinh(a*x^n), x)
```

3.308 $\int x^2 \sinh^{-1}(ax^n)\, dx$

Optimal. Leaf size=64

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2n};\frac{3(1+n)}{2n};-a^2x^{2n}\right)}{3(3+n)}$$

```
[Out] 1/3*x^3*arcsinh(a*x^n)-1/3*a*n*x^(3+n)*hypergeom([1/2, 1/2*(3+n)/n],[3/2*(1
+n)/n],-a^2*x^(2*n))/(3+n)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5875, 12, 371}

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2n};\frac{3(n+1)}{2n};-a^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSinh[a*x^n],x]
```

```
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), (3*(1 + n))/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \sinh^{-1}(ax^n)\,dx &= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}\int \frac{anx^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{1}{3}(an)\int \frac{x^{2+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= \frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2n};\frac{3(1+n)}{2n};-a^2x^{2n}\right)}{3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 1.03

$$\frac{1}{3}x^3\sinh^{-1}(ax^n) - \frac{anx^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2n};1+\frac{3+n}{2n};-a^2x^{2n}\right)}{3(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSinh[a*x^n],x]
[Out] (x^3*ArcSinh[a*x^n])/3 - (a*n*x^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/(2*n
), 1 + (3 + n)/(2*n), -(a^2*x^(2*n))])/(3*(3 + n))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcsinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsinh(a*x^n),x)
[Out] int(x^2*arcsinh(a*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="maxima")
[Out] -1/9*n*x^3 + 1/3*x^3*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) - a*n*integrate(1/3
*x^2*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) + n*integrate(
1/3*x^2/(a^2*x^(2*n) + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asinh(a*x**n),x)
```

```
[Out] Integral(x**2*asinh(a*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*arcsinh(a*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \operatorname{asinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asinh(a*x^n),x)
```

```
[Out] int(x^2*asinh(a*x^n), x)
```

3.309 $\int x \sinh^{-1}(ax^n)\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2\sinh^{-1}(ax^n)-\frac{anx^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2+n)}$$

```
[Out] 1/2*x^2*arcsinh(a*x^n)-1/2*a*n*x^(2+n)*hypergeom([1/2, 1/2*(2+n)/n],[3/2+1/
n],-a^2*x^(2*n))/(2+n)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5875, 12, 371}

$$\frac{1}{2}x^2\sinh^{-1}(ax^n)-\frac{anx^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);-a^2x^{2n}\right)}{2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*ArcSinh[a*x^n])/2 - (a*n*x^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/(2*n
), (3 + 2/n)/2, -(a^2*x^(2*n))])/(2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int x\sinh^{-1}(ax^n)\,dx &= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{1}{2}\int \frac{anx^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{1}{2}(an)\int \frac{x^{1+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= \frac{1}{2}x^2\sinh^{-1}(ax^n) - \frac{anx^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2n};\frac{1}{2}\left(3+\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.89

$$
\frac{x^2\left((2+n)\sinh^{-1}(ax^n) - anx^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-a^2x^{2n}\right)\right)}{2(2+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSinh[a*x^n],x]
```

```
[Out] (x^2*((2 + n)*ArcSinh[a*x^n] - a*n*x^n*Hypergeometric2F1[1/2, 1/2 + n^(-1),
 3/2 + n^(-1), -(a^2*x^(2*n))]))/(2*(2 + n))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$
\int x\,\mathrm{arcsinh}\left(a\,x^n\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsinh(a*x^n),x)
```

```
[Out] int(x*arcsinh(a*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="maxima")
```

```
[Out] -1/4*n*x^2 - a*n*integrate(1/2*x*x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) +
1)^(3/2)), x) + 1/2*x^2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)) + n*integrate(1/
2*x/(a^2*x^(2*n) + 1), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{asinh}(ax^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asinh(a*x**n),x)
[Out] Integral(x*asinh(a*x**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsinh(a*x^n),x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{asinh}(a\, x^n)\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*asinh(a*x^n),x)
[Out] int(x*asinh(a*x^n), x)
```

3.310 $\int \sinh^{-1}(ax^n) \, dx$

Optimal. Leaf size=56

$$x \sinh^{-1}(ax^n) - \frac{anx^{1+n} \, _2F_1\left(\frac{1}{2}, \frac{1+n}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -a^2x^{2n}\right)}{1+n}$$

```
[Out] x*arcsinh(a*x^n)-a*n*x^(1+n)*hypergeom([1/2, 1/2*(1+n)/n],[3/2+1/2/n],-a^2*
x^(2*n))/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5874, 12, 371}

$$x \sinh^{-1}(ax^n) - \frac{anx^{n+1} \, _2F_1\left(\frac{1}{2}, \frac{n+1}{2n}; \frac{1}{2}\left(3+\frac{1}{n}\right); -a^2x^{2n}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n],x]
```

```
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5874

```
Int[ArcSinh[u_], x_Symbol] :> Simp[x*ArcSinh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/Sqrt[1 + u^2]), x], x] /; InverseFunctionFreeQ[u, x] && !Funct
ionOfExponentialQ[u, x]
```

Rubi steps

$$\int \sinh^{-1}(ax^n)\,dx = x\sinh^{-1}(ax^n) - \int \frac{anx^n}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= x\sinh^{-1}(ax^n) - (an)\int \frac{x^n}{\sqrt{1+a^2x^{2n}}}\,dx$$
$$= x\sinh^{-1}(ax^n) - \frac{anx^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{1+n}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 1.00

$$x\sinh^{-1}(ax^n) - \frac{anx^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2n};\frac{1}{2}\left(3+\frac{1}{n}\right);-a^2x^{2n}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n],x]
[Out] x*ArcSinh[a*x^n] - (a*n*x^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/(2*n), (3
+ n^(-1))/2, -(a^2*x^(2*n))])/(1 + n)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \operatorname{arcsinh}(a\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n),x)
[Out] int(arcsinh(a*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="maxima")
[Out] -a*n*integrate(x^n/(a^3*x^(3*n) + a*x^n + (a^2*x^(2*n) + 1)^(3/2)), x) - n*
x + n*integrate(1/(a^2*x^(2*n) + 1), x) + x*log(a*x^n + sqrt(a^2*x^(2*n) +
1))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}(ax^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n),x)
```

```
[Out] Integral(asinh(a*x**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asinh}(a\,x^n)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n),x)
```

```
[Out] int(asinh(a*x^n), x)
```

3.311 $\int \frac{\sinh^{-1}(ax^n)}{x} \, dx$

Optimal. Leaf size=60

$$-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}+\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$

```
[Out] -1/2*arcsinh(a*x^n)^2/n+arcsinh(a*x^n)*ln(1-(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2
)/n+1/2*polylog(2,(a*x^n+(1+a^2*(x^n)^2)^(1/2))^2)/n
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5869, 3797, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n}-\frac{\sinh^{-1}(ax^n)^2}{2n}+\frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]^2/n + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5869

```
Int[ArcSinh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Coth[x], x], x, ArcSinh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}(ax^n)}{x}\,dx &= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}(ax^n)\right)}{n}\\
&= -\frac{\sinh^{-1}(ax^n)^2}{2n} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}(ax^n)\right)}{n}\\
&= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \text{si}\right.}{n}\\
&= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\sinh^{-1}}\right.}{2n}\\
&= -\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}(ax^n)}\right)}{2n}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 60, normalized size = 1.00

$$-\frac{\sinh^{-1}(ax^n)^2}{2n} + \frac{\sinh^{-1}(ax^n)\log\left(1-e^{2\sinh^{-1}(ax^n)}\right)}{n} + \frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}(ax^n)}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]^2/n + (ArcSinh[a*x^n]*Log[1 - E^(2*ArcSinh[a*x^n])])/n
+ PolyLog[2, E^(2*ArcSinh[a*x^n])]/(2*n)
```

Maple [**A**]
time = 2.78, size = 120, normalized size = 2.00

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsinh}(a\,x^n)^2}{2}+\operatorname{arcsinh}(a\,x^n)\ln\left(1+a\,x^n+\sqrt{1+a^2x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^n-\sqrt{1+a^2x^{2n}}\right)+\operatorname{arcsinh}(a\,x^n)}{n}$
default	$\frac{-\frac{\operatorname{arcsinh}(a\,x^n)^2}{2}+\operatorname{arcsinh}(a\,x^n)\ln\left(1+a\,x^n+\sqrt{1+a^2x^{2n}}\right)+\operatorname{polylog}\left(2,-a\,x^n-\sqrt{1+a^2x^{2n}}\right)+\operatorname{arcsinh}(a\,x^n)}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/2*arcsinh(a*x^n)^2+arcsinh(a*x^n)*ln(1+a*x^n+(1+a^2*(x^n)^2)^(1/2))
+polylog(2,-a*x^n-(1+a^2*(x^n)^2)^(1/2))+arcsinh(a*x^n)*ln(1-a*x^n-(1+a^2*(
x^n)^2)^(1/2))+polylog(2,a*x^n+(1+a^2*(x^n)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) + a*x*x^n + (a^2*x*x^(2*n) + x)*sq
rt(a^2*x^(2*n) + 1)), x) - 1/2*n*log(x)^2 + n*integrate(log(x)/(a^2*x*x^(2*
n) + x), x) + log(a*x^n + sqrt(a^2*x^(2*n) + 1))*log(x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x,x)
```

```
[Out] Integral(asinh(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x,x)
[Out] int(asinh(a*x^n)/x, x)
```

3.312 $\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\sinh^{-1}(ax^n)}{x}-\frac{anx^{-1+n}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}$$

```
[Out] -arcsinh(a*x^n)/x-a*n*x^(-1+n)*hypergeom([1/2, 1/2*(-1+n)/n],[3/2-1/2/n],-a
^2*x^(2*n))/(1-n)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {5875, 12, 371}

$$-\frac{anx^{n-1}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}-\frac{\sinh^{-1}(ax^n)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) - (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, -1/2*(1 - n)/n
, (3 - n^(-1))/2, -(a^2*x^(2*n))])/(1 - n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(ax^n)}{x^2}\,dx &= -\frac{\sinh^{-1}(ax^n)}{x} + \int \frac{anx^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= -\frac{\sinh^{-1}(ax^n)}{x} + (an)\int \frac{x^{-2+n}}{\sqrt{1+a^2x^{2n}}}\,dx \\
&= -\frac{\sinh^{-1}(ax^n)}{x} - \frac{anx^{-1+n}\,{}_2F_1\left(\frac{1}{2},-\frac{1-n}{2n};\frac{1}{2}\left(3-\frac{1}{n}\right);-a^2x^{2n}\right)}{1-n}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.94

$$
-\frac{\sinh^{-1}(ax^n)}{x} + \frac{anx^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{-1+n}{2n};1+\frac{-1+n}{2n};-a^2x^{2n}\right)}{-1+n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^2,x]
```

```
[Out] -(ArcSinh[a*x^n]/x) + (a*n*x^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/(2*n)
, 1 + (-1 + n)/(2*n), -(a^2*x^(2*n))])/(-1 + n)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$
\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^2,x)
```

```
[Out] int(arcsinh(a*x^n)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n/(a^3*x^2*x^(3*n) + a*x^2*x^n + (a^2*x^2*x^(2*n) + x^2)*sq
rt(a^2*x^(2*n) + 1)), x) - n*integrate(1/(a^2*x^2*x^(2*n) + x^2), x) - (n +
 log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^{n}\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**2,x)
[Out] Integral(asinh(a*x**n)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x^n)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{asinh}\left(a\,x^n\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x^2,x)
[Out] int(asinh(a*x^n)/x^2, x)
```

3.313 $\int \frac{\sinh^{-1}(ax^n)}{x^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\sinh^{-1}(ax^n)}{2x^2}-\frac{anx^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}$$

```
[Out] -1/2*arcsinh(a*x^n)/x^2-1/2*a*n*x^(-2+n)*hypergeom([1/2, 1/2-1/n],[3/2-1/n]
,-a^2*x^(2*n))/(2-n)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5875, 12, 371}

$$-\frac{anx^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}-\frac{\sinh^{-1}(ax^n)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x^n]/x^2 - (a*n*x^(-2 + n)*Hypergeometric2F1[1/2, (1 - 2/n)/
2, (3 - 2/n)/2, -(a^2*x^(2*n))])/(2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5875

```
Int[((a_.) + ArcSinh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcSinh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 + u^2]), x], x
], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
 x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^{-1}(ax^n)}{x^3}\,dx &= -\frac{\sinh^{-1}(ax^n)}{2x^2}+\frac{1}{2}\int \frac{anx^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= -\frac{\sinh^{-1}(ax^n)}{2x^2}+\frac{1}{2}(an)\int \frac{x^{-3+n}}{\sqrt{1+a^2x^{2n}}}\,dx\\ &= -\frac{\sinh^{-1}(ax^n)}{2x^2}-\frac{anx^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1-\frac{2}{n}\right);\frac{1}{2}\left(3-\frac{2}{n}\right);-a^2x^{2n}\right)}{2(2-n)}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 62, normalized size = 0.91

$$\frac{-\left((-2+n)\sinh^{-1}(ax^n)\right)+anx^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{1}{n};\frac{3}{2}-\frac{1}{n};-a^2x^{2n}\right)}{2(-2+n)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a*x^n]/x^3,x]
```

```
[Out] (-((-2 + n)*ArcSinh[a*x^n]) + a*n*x^n*Hypergeometric2F1[1/2, 1/2 - n^(-1),
3/2 - n^(-1), -(a^2*x^(2*n))])/(2*(-2 + n)*x^2)
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}(a\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(a*x^n)/x^3,x)
```

```
[Out] int(arcsinh(a*x^n)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] a*n*integrate(1/2*x^n/(a^3*x^3*x^(3*n) + a*x^3*x^n + (a^2*x^3*x^(2*n) + x^3
)*sqrt(a^2*x^(2*n) + 1)), x) - n*integrate(1/2/(a^2*x^3*x^(2*n) + x^3), x)
- 1/4*(n + 2*log(a*x^n + sqrt(a^2*x^(2*n) + 1)))/x^2
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asinh}\left(ax^{n}\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(a*x**n)/x**3,x)
```

```
[Out] Integral(asinh(a*x**n)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(a*x^n)/x^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x^n)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{asinh}(a\,x^n)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a*x^n)/x^3,x)
```

```
[Out] int(asinh(a*x^n)/x^3, x)
```

3.314 $\int \left(a+ib\textbf{ArcSin}(1-idx^2)\right)^4 \, dx$

Optimal. Leaf size=153

$$384b^4x-\frac{192b^3\sqrt{2idx^2+d^2x^4}\,(a+ib\text{ArcSin}(1-idx^2))}{dx}+48b^2x\big(a+ib\text{ArcSin}\big(1-idx^2\big)\big)^2-\frac{8b\sqrt{2idx^2+d^2x}}{}$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^2+x*(a-I*b*arcsin(-1+I*d*x^2)
)^4-192*b^3*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a
-I*b*arcsin(-1+I*d*x^2))^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]

time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$-\frac{192b^3\sqrt{d^2x^4+2idx^2}\,(a+ib\text{ArcSin}(1-idx^2))}{dx}+48b^2x\big(a+ib\text{ArcSin}\big(1-idx^2\big)\big)^2-\frac{8b\sqrt{d^2x^4+2idx^2}\,(a+ib\text{ArcSin}(1-idx^2))^3}{dx}+x\big(a+ib\text{ArcSin}\big(1-idx^2\big)\big)^4+384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^
2]))/(d*x) + 48*b^2*x*(a + I*b*ArcSin[1 - I*d*x^2])^2 - (8*b*Sqrt[(2*I)*d*x
^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*(a + I*b*ArcSin[1
- I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^4\,dx &= -\frac{8b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}{dx}+x\big(a+ib\sin^{-1}\left(1-idx^2\right.\\ &= -\frac{192b^3\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\big(a+ib\sin^{-1}\left(1\right.\\ &= 384b^4x-\frac{192b^3\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{dx}+48b^2x\big(a+ib\,s\end{aligned}$$

Mathematica [A]
time = 0.09, size = 149, normalized size = 0.97

$$-\frac{8b\sqrt{dx^2(2i+dx^2)}\,(a+ib\text{ArcSin}(1-idx^2))^3}{dx}+x(a+ib\text{ArcSin}(1-idx^2))^4+48b^2\left(8b^2x-\frac{4b\sqrt{dx^2(2i+dx^2)}\,(a+ib\text{ArcSin}(1-idx^2))}{dx}+x(a+ib\text{ArcSin}(1-idx^2))^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^3)/(d*x) + x*
(a + I*b*ArcSin[1 - I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(2*I +
d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2]))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^
2])^2)
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{arcsinh}\left(d\,x^2+i\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^4 + 4*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^3*b + a^4*x +
integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (-3*I*a*b^3*d + 4*I
*b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(-I*a*b^3*sqrt(d) +
I*b^4*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*
x + I)^3 + 3*(a^2*b^2*d^2*x^4 + 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*b^2*d^
(3/2)*x^3 + 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^
2 + 2*I)*sqrt(d)*x + I)^2)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d
)*x)*sqrt(d*x^2 + 2*I) - 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.
time = 0.38, size = 269, normalized size = 1.76

$$\frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2+2i\,d}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^3+(a^4+48a^2b^2+384b^4)dx-6\left(4\sqrt{d^2x^2+2i\,d}\,ab^3-(a^2b^2+8b^4)dx\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)^2+4\left((a^3b+24ab^3)dx-6(a^2b^2+8b^4)\sqrt{d^2x^2+2i\,d}\right)\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)-8(a^3b+24ab^3)\sqrt{d^2x^2+2i\,d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 + 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^4 +
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 + 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 + 2*I*d))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^4,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^4, x)
```

3.315 $\int \left(a+ib\mathbf{ArcSin}(1-idx^2)\right)^3\,dx$

Optimal. Leaf size=129

$$24ab^2x-\frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx}+24ib^3x\text{ArcSin}\left(1-idx^2\right)-\frac{6b\sqrt{2idx^2+d^2x^4}\left(a+ib\text{ArcSin}(1-idx^2)\right)^2}{dx}+x\left(a+\right.$$

[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(-1+I*d*x^2)+x*(a-I*b*arcsin(-1+I*d*x^2))^3-48*b^3*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(-1+I*d*x^2))^2*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x

Rubi [A]
time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {4898, 4924, 12, 1602}

$$-\frac{6b\sqrt{d^2x^4+2idx^2}\left(a+ib\text{ArcSin}(1-idx^2)\right)^2}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^3+24ab^2x+24ib^3x\text{ArcSin}\left(1-idx^2\right)-\frac{48b^3\sqrt{d^2x^4+2idx^2}}{dx}$$

Antiderivative was successfully verified.

[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]

[Out] 24*a*b^2*x - (48*b^3*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + (24*I)*b^3*x*ArcSin[1 - I*d*x^2] - (6*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2])^2)/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^3

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3\,dx &= -\frac{6b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\big(a+ib\sin^{-1}\left(1-idx^2\right.\\ &= 24ab^2x-\frac{6b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}{dx}+x\big(a+ib\sin^{-1}\left(\right.\\ &= 24ab^2x+24ib^3x\sin^{-1}\left(1-idx^2\right)-\frac{6b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-\right.\right.}{dx}\\ &= 24ab^2x+24ib^3x\sin^{-1}\left(1-idx^2\right)-\frac{6b\sqrt{2idx^2+d^2x^4}\,\left(a+ib\sin^{-1}\left(1-\right.\right.}{dx}\\ &= 24ab^2x-\frac{48b^3\sqrt{2idx^2+d^2x^4}}{dx}+24ib^3x\sin^{-1}\left(1-idx^2\right)-\frac{6b\sqrt{2idx^2+}}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 180, normalized size = 1.40

$$\frac{a(a^2+24b^2)\,dx^2-6b(a^2+8b^2)\,\sqrt{dx^2\,(2i+dx^2)}+3ib\left(a^2dx^2+8b^2dx^2-4ab\sqrt{dx^2\,(2i+dx^2)}\right)\text{ArcSin}(1-idx^2)+3b^2\left(-adx^2+2b\sqrt{dx^2\,(2i+dx^2)}\right)\text{ArcSin}(1-idx^2)^2-ib^3dx^2\text{ArcSin}(1-idx^2)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(2*I + d*x^2)] + (3*
I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 -
 I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(2*I + d*x^2)])*ArcSin[1 - I
*d*x^2]^2 - I*b^3*d*x^2*ArcSin[1 - I*d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{arcsinh}\left(d\,x^2+i\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^3 + 3*(x*arcsinh(d*x^2 +
 I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d))*a^2*b + a^3*x +
integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (-3*I*a*b^2*d + 4*I*b^
3*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(-I*a*b^2*sqrt(d) + I*b
^3*sqrt(d))*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x +
 I)^2/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I
) - 2), x)
```

Fricas [A]
time = 0.37, size = 188, normalized size = 1.46

$$\frac{b^3 dx \log\left(dx^2 + \sqrt{d^2x^2 + 2i\,d}\,x + i\right)^3 + (a^3 + 24\,ab^2)dx + 3\left(ab^2dx - 2\sqrt{d^2x^2 + 2i\,d}\,b^3\right)\log\left(dx^2 + \sqrt{d^2x^2 + 2i\,d}\,x + i\right)^2 - 3\left(4\sqrt{d^2x^2 + 2i\,d}\,ab^2 - (a^2b + 8\,b^3)dx\right)\log\left(dx^2 + \sqrt{d^2x^2 + 2i\,d}\,x + i\right) - 6\sqrt{d^2x^2 + 2i\,d}\,(a^2b + 8\,b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 + 2*
I*d)*x + I)^2 - 3*(4*sqrt(d^2*x^2 + 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) - 6*sqrt(d^2*x^2 + 2*I*d)*(a^2*b + 8*
b^3))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^3,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^3, x)
```

3.316 $\int (a+ib\textbf{ArcSin}(1-idx^2))^2\,dx$

Optimal. Leaf size=76

$$8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\,(a+ib\text{ArcSin}(1-idx^2))}{dx}+x\big(a+ib\text{ArcSin}\big(1-idx^2\big)\big)^2$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(-1+I*d*x^2))^2-4*b*(a-I*b*arcsin(-1+I*d*x^2))*(2*I*
d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {4898, 8}

$$-\frac{4b\sqrt{d^2x^4+2idx^2}\,(a+ib\text{ArcSin}(1-idx^2))}{dx}+x\big(a+ib\text{ArcSin}\big(1-idx^2\big)\big)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2\,dx &= -\frac{4b\sqrt{2idx^2+d^2x^4}\,(a+ib\sin^{-1}(1-idx^2))}{dx}+x\big(a+ib\sin^{-1}\left(1-idx^2\right)\big)^2\\ &= 8b^2x-\frac{4b\sqrt{2idx^2+d^2x^4}\,(a+ib\sin^{-1}(1-idx^2))}{dx}+x\big(a+ib\sin^{-1}\left(1-i\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 1.00

$$8b^2x - \frac{4b\sqrt{2idx^2 + d^2x^4}\,(a + ib\text{ArcSin}(1 - idx^2))}{dx} + x\left(a + ib\text{ArcSin}\left(1 - idx^2\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*(a + I*b*ArcSin[1 - I*d*x^2]))/(
d*x) + x*(a + I*b*ArcSin[1 - I*d*x^2])^2
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{arcsinh}\left(d\,x^2 + i\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 - integrate(4*(
d^2*x^4 + 2*I*d*x^2 + (d^(3/2)*x^3 + I*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*
x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)/(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3
+ 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I) - 2), x))*b^2 + a^2*x
```

Fricas [A]
time = 0.36, size = 114, normalized size = 1.50

$$\frac{b^2dx\log\left(dx^2 + \sqrt{d^2x^2 + 2i\,d}\,x + i\right)^2 + (a^2 + 8\,b^2)dx - 4\,\sqrt{d^2x^2 + 2i\,d}\,ab + 2\left(abdx - 2\,\sqrt{d^2x^2 + 2i\,d}\,b^2\right)\log\left(dx^2 + \sqrt{d^2x^2 + 2i\,d}\,x + i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 + 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 + 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^2, x)
```

3.317 $\int (a+ib\textbf{ArcSin}(1-idx^2))\,dx$

Optimal. Leaf size=50

$$ax-\frac{2b\sqrt{2idx^2+d^2x^4}}{dx}+ibx\text{ArcSin}\left(1-idx^2\right)$$

```
[Out] a*x-I*b*x*arcsin(-1+I*d*x^2)-2*b*(2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4924, 12, 1602}

$$ax+ibx\text{ArcSin}\left(1-idx^2\right)-\frac{2b\sqrt{d^2x^4+2idx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[a + I*b*ArcSin[1 - I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(2*I)*d*x^2 + d^2*x^4])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)\,dx &= ax+(ib)\int \sin^{-1}\left(1-idx^2\right)\,dx\\ &= ax+ibx\sin^{-1}\left(1-idx^2\right)-(ib)\int -\frac{2idx^2}{\sqrt{2idx^2+d^2x^4}}\,dx\\ &= ax+ibx\sin^{-1}\left(1-idx^2\right)-(2bd)\int \frac{x^2}{\sqrt{2idx^2+d^2x^4}}\,dx\\ &= ax-\frac{2b\sqrt{2idx^2+d^2x^4}}{dx}+ibx\sin^{-1}\left(1-idx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.96

$$ax-\frac{2b\sqrt{dx^2\left(2i+dx^2\right)}}{dx}+ibx\text{ArcSin}\left(1-idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*b*ArcSin[1 - I*d*x^2],x]
[Out] a*x - (2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x) + I*b*x*ArcSin[1 - I*d*x^2]
```

Maple [A]
time = 0.26, size = 47, normalized size = 0.94

method	result	size
default	$ax+b\left(x\,\mathrm{arcsinh}\left(d\,x^2+i\right)-\frac{2x\left(d\,x^2+2i\right)}{\sqrt{d^2x^4+2id\,x^2}}\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(I+d*x^2),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*arcsinh(I+d*x^2)-2/(2*I*d*x^2+d^2*x^4)^(1/2)*x*(d*x^2+2*I))
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.88

$$\left(x\,\mathrm{arsinh}\left(dx^2+i\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2i\sqrt{d}\right)}{\sqrt{dx^2+2i}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 + I) - 2*(d^(3/2)*x^2 + 2*I*sqrt(d))/(sqrt(d*x^2 + 2*I)*d)
)*b + a*x
```

Fricas [**A**]
time = 0.39, size = 52, normalized size = 1.04

$$\frac{bdx\log\left(dx^2+\sqrt{d^2x^2+2i\,d}\,x+i\right)+adx-2\sqrt{d^2x^2+2i\,d}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 2*sqrt(d^2*x^2 +
2*I*d)*b)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(I+d*x**2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**B**]
time = 0.53, size = 39, normalized size = 0.78

$$a\,x+b\,x\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)-\frac{2\,b\,\sqrt{{\left(d\,x^2+1\mathrm{i}\right)}^2+1}}{d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(d*x^2 + 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 + 1i) - (2*b*((d*x^2 + 1i)^2 + 1)^(1/2))/(d*x)
```

3.318 $\int \frac{1}{a+ib\textbf{ArcSin}(1-idx^2)}\,dx$

Optimal. Leaf size=194

$$\frac{x\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcS}\right.}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcS}\right.\right.}$$

```
[Out] 1/2*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b
))/b/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*x*Si(1/2
*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*a
rcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))
```

Rubi [A]
time = 0.04, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4900}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcSin}(1-idx^2)\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
```

```
[Out] (x*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)]
 - Sinh[a/(2*b)]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^
2]/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Ar
cSin[1 - I*d*x^2]/2])/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d
*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx=\frac{x\text{Ci}\left(-\frac{i\left(a+ib\sin^{-1}(1-idx^2)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)\right)}-\frac{x\left(i\cosh\left(\frac{a}{2b}\right)+\sin\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}(1-\right.\right.}$$

Mathematica [A]
time = 0.52, size = 150, normalized size = 0.77

$$\frac{x\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\text{ArcSin}(1-idx^2)\right)\right)\left(i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-1),x]
[Out] (x*(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(I*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] - Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b
 - ArcSin[1 - I*d*x^2]/2]))/(2*b*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1
 - I*d*x^2]/2]))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\text{arcsinh}\left(d\,x^2+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="fricas")
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i)),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i)), x)
```

3.319 $\int \frac{1}{\left(a+ib\text{ArcSin}(1-idx^2)\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{x\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}+\frac{x(\text{co}}{4b^2(\text{c}}$$

```
[Out] 1/4*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d*x^2))/b)*(cosh(1/2*a/b)-I*sinh(1/2*a/b
))/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+1/4*x*Si(1
/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^2/(cos(1
/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/2*(2*I*d*x^2+d^2*x^4)
^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))
```

Rubi [A]
time = 0.03, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4909}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{d^2x^4+2idx^2}}{2bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] -1/2*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])) + (x
*CosIntegral[((-1/2*I)*(a + I*b*ArcSin[1 - I*d*x^2]))/b]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^
2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Ar
cSin[1 - I*d*x^2]/2])/(4*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I
*d*x^2]/2]))
```

Rule 4909

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{2bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{x\operatorname{Ci}\left(-\frac{i\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right) \cdots\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\sin \cdots\right.\right.}$$

Mathematica [A]
time = 0.96, size = 197, normalized size = 0.80

$$\frac{-\frac{2b\sqrt{dx^2\left(2i+dx^2\right)}}{d\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{x^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\text{ArcSin}\left(1-idx^2\right)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}}{4b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])) + (x^2*
(CosIntegral[(((-I)*a)/b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)]) + (Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - Arc
Sin[1 - I*d*x^2]/2]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]
/2]))/(4*b^2*x)
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 + 3*I*d*x^2 + (d^(3/2)*x^3 + 2*I*sqrt(d)*x)*sqrt(d*x^2 + 2*I)
 - 2)/(a*b*d^2*x^3 + 2*I*a*b*d*x + (b^2*d^2*x^3 + 2*I*b^2*d*x + (b^2*d^(3/2
)*x^2 + I*b^2*sqrt(d))*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqr
t(d)*x + I) + (a*b*d^(3/2)*x^2 + I*a*b*sqrt(d))*sqrt(d*x^2 + 2*I)) + integr
ate(1/2*(d^3*x^6 + 3*I*d^2*x^4 + (d^2*x^4 + I*d*x^2 - 2)*(d*x^2 + 2*I) + (2
```

```
*d^(5/2)*x^5 + 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 + 2*I) + 4*I)/(a*b*d
^3*x^6 + 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 + 2*I*a*b*d*x^2 - a*b
)*(d*x^2 + 2*I) + (b^2*d^3*x^6 + 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b^2*d^2*x
^4 + 2*I*b^2*d*x^2 - b^2)*(d*x^2 + 2*I) + 2*(b^2*d^(5/2)*x^5 + 3*I*b^2*d^(3
/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + 2*(a*b*d^(5/2)*x^5 + 3*I*a*b*d^(3/2)*x^3 - 2*a*b*sqrt(d)*
x)*sqrt(d*x^2 + 2*I)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)*integral(1/
2*sqrt(d^2*x^2 + 2*I*d)*x/(a*b*d*x^2 + 2*I*a*b + (b^2*d*x^2 + 2*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)), x) - sqrt(d^2*x^2 + 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^2,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^2, x)
```

3.320 $\int \frac{1}{\left(a+ib\text{ArcSin}(1-idx^2)\right)^3}\,dx$

Optimal. Leaf size=275

$$-\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^2}-\frac{x}{8b^2\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{x\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\right.\right.}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))+1/16*x*Ci(-1/2*I*(a-I*b*arcsin(-1+I*d
*x^2))/b)*(I*cosh(1/2*a/b)-sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+
sin(1/2*arcsin(-1+I*d*x^2)))-1/16*x*Si(1/2*I*a/b+1/2*arcsin(-1+I*d*x^2))*(I
*cosh(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcs
in(-1+I*d*x^2)))-1/4*(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x
^2))^2
```

Rubi [A]
time = 0.04, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4912, 4900}

$$\frac{x\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(-\frac{i\left(a+ib\text{ArcSin}(1-idx^2)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{ia}{2b}-\frac{1}{2}\text{ArcSin}(1-idx^2)\right)}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x}{8b^2\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)}-\frac{\sqrt{d^2x^4+2idx^2}}{4bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] -1/4*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^2) -
x/(8*b^2*(a + I*b*ArcSin[1 - I*d*x^2])) + (x*CosIntegral[((-1/2*I)*(a + I*b
*ArcSin[1 - I*d*x^2]))/b]*(I*Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(16*b^3*(Cos[A
rcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (x*(I*Cosh[a/(2*b)]
+ Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2])/(16*b^3*
(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^3}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{\int \frac{}{a+ib\sin}}{}$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{4bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^2} - \frac{x}{8b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)} + \frac{x\mathrm{Ci}\Big(}{16b^3\,(\mathrm{co}}$$

Mathematica [A]
time = 0.47, size = 229, normalized size = 0.83

$$\frac{-\frac{8b^2\sqrt{dx^2\left(2i+dx^2\right)}}{d\left(a+ib\mathrm{ArcSin}\left(1-idx^2\right)\right)^2}-\frac{4bx^2}{a+ib\mathrm{ArcSin}\left(1-idx^2\right)}+\frac{2ix^2\left(\mathrm{CosIntegral}\left(\frac{1}{2}\left(-\frac{ia}{b}+\mathrm{ArcSin}\left(1-idx^2\right)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{Si}\left(\frac{ia}{2b}-\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)\right)}{\cos\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)}}{32b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3),x]
```

```
[Out] ((-8*b^2*Sqrt[d*x^2*(2*I + d*x^2)])/(d*(a + I*b*ArcSin[1 - I*d*x^2])^2) - (
4*b*x^2)/(a + I*b*ArcSin[1 - I*d*x^2]) + ((2*I)*x^2*(CosIntegral[(((-I)*a)/
b + ArcSin[1 - I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]) - (Cosh[a/(2*
b)] - I*Sinh[a/(2*b)])*SinIntegral[((I/2)*a)/b - ArcSin[1 - I*d*x^2]/2]))/(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(32*b^3*x)
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(-3*I*a*d^(9/2) - 7*I*b*d^(9/2))
*x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(I*a*d^(5/2) + 20*I*b*d^(5/2))
*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (-3*I*a
*d^3 - 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(I*a*d + I*b*d)*x)*(d
*x^2 + 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(-2*I*a*d^(7/2) -
5*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(I*a*d^(3/2) +
 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 + 2*I) + (b*d^(11/2
)*x^10 + 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 - 2*I*b*d^(5/2)*x^4 - 12*b*d^
(3/2)*x^2 + (b*d^4*x^7 + 3*I*b*d^3*x^5 - 4*b*d^2*x^3 - 4*I*b*d*x)*(d*x^2 +
2*I)^(3/2) + (3*b*d^(9/2)*x^8 + 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)*x^4 - 10*
I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2*I) + (3*b*d^5*x^9 + 15*I*b*d^4*x^
7 - 23*b*d^3*x^5 - 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2*I) - 8*I*b*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(-5*I*a*d^4 - 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^5 - (7*I*a*d^2
 + 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2*I) - 8*I*a*sqrt(d)
)/(a^2*b^2*d^(11/2)*x^9 + 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*b^2*d^(7/2)*x^5
- 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*I*b^4*d^(9/2)*x^7 - 12*b^
4*d^(7/2)*x^5 - 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*I*b^4*d^3*x^4 - 3*b^
4*d^2*x^2 - I*b^4*d)*(d*x^2 + 2*I)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*I*b^4*d^(
7/2)*x^5 - 5*b^4*d^(5/2)*x^3 - 2*I*b^4*d^(3/2)*x)*(d*x^2 + 2*I) + 3*(b^4*d^
5*x^8 + 5*I*b^4*d^4*x^6 - 8*b^4*d^3*x^4 - 4*I*b^4*d^2*x^2)*sqrt(d*x^2 + 2*I
))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I)^2 + (a^2*b^2*d^4*x^6 + 3*I*
a^2*b^2*d^3*x^4 - 3*a^2*b^2*d^2*x^2 - I*a^2*b^2*d)*(d*x^2 + 2*I)^(3/2) + 3*
(a^2*b^2*d^(9/2)*x^7 + 4*I*a^2*b^2*d^(7/2)*x^5 - 5*a^2*b^2*d^(5/2)*x^3 - 2*
I*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2*I) + 2*(a*b^3*d^(11/2)*x^9 + 6*I*a*b^3*d^(9
/2)*x^7 - 12*a*b^3*d^(7/2)*x^5 - 8*I*a*b^3*d^(5/2)*x^3 + (a*b^3*d^4*x^6 + 3
*I*a*b^3*d^3*x^4 - 3*a*b^3*d^2*x^2 - I*a*b^3*d)*(d*x^2 + 2*I)^(3/2) + 3*(a*
b^3*d^(9/2)*x^7 + 4*I*a*b^3*d^(7/2)*x^5 - 5*a*b^3*d^(5/2)*x^3 - 2*I*a*b^3*d
^(3/2)*x)*(d*x^2 + 2*I) + 3*(a*b^3*d^5*x^8 + 5*I*a*b^3*d^4*x^6 - 8*a*b^3*d^
3*x^4 - 4*I*a*b^3*d^2*x^2)*sqrt(d*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)
*sqrt(d)*x + I) + 3*(a^2*b^2*d^5*x^8 + 5*I*a^2*b^2*d^4*x^6 - 8*a^2*b^2*d^3*
x^4 - 4*I*a^2*b^2*d^2*x^2)*sqrt(d*x^2 + 2*I)) + integrate(1/8*(d^6*x^12 + 8
*I*d^5*x^10 - 27*d^4*x^8 - 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*I*d^3*x
^6 - 3*d^2*x^4 + 8*I*d*x^2 + 4)*(d*x^2 + 2*I)^2 + 96*I*d*x^2 + 2*(2*d^(9/2)
*x^9 + 10*I*d^(7/2)*x^7 - 15*d^(5/2)*x^5 + I*d^(3/2)*x^3 - 11*sqrt(d)*x)*(d
*x^2 + 2*I)^(3/2) + 3*(2*d^5*x^10 + 12*I*d^4*x^8 - 26*d^3*x^6 - 24*I*d^2*x^
4 + 3*d*x^2 - 10*I)*(d*x^2 + 2*I) + 2*(2*d^(11/2)*x^11 + 14*I*d^(9/2)*x^9 -
 39*d^(7/2)*x^7 - 61*I*d^(5/2)*x^5 + 61*d^(3/2)*x^3 + 30*I*sqrt(d)*x)*sqrt(
d*x^2 + 2*I) - 48)/(a*b^2*d^6*x^12 + 8*I*a*b^2*d^5*x^10 - 24*a*b^2*d^4*x^8
- 32*I*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b^2*d^4*x^8 + 4*I*a*b^2*d^3*x^
6 - 6*a*b^2*d^2*x^4 - 4*I*a*b^2*d*x^2 + a*b^2)*(d*x^2 + 2*I)^2 + 4*(a*b^2*d
```

```
^(9/2)*x^9 + 5*I*a*b^2*d^(7/2)*x^7 - 9*a*b^2*d^(5/2)*x^5 - 7*I*a*b^2*d^(3/2
)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 + 2*I)^(3/2) + 6*(a*b^2*d^5*x^10 + 6*I*a*
b^2*d^4*x^8 - 13*a*b^2*d^3*x^6 - 12*I*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2)*(d*x^2
 + 2*I) + (b^3*d^6*x^12 + 8*I*b^3*d^5*x^10 - 24*b^3*d^4*x^8 - 32*I*b^3*d^3*
x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 + 4*I*b^3*d^3*x^6 - 6*b^3*d^2*x^4 - 4*I
*b^3*d*x^2 + b^3)*(d*x^2 + 2*I)^2 + 4*(b^3*d^(9/2)*x^9 + 5*I*b^3*d^(7/2)*x^
7 - 9*b^3*d^(5/2)*x^5 - 7*I*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 + 2*I
)^(3/2) + 6*(b^3*d^5*x^10 + 6*I*b^3*d^4*x^8 - 13*b^3*d^3*x^6 - 12*I*b^3*d^2
*x^4 + 4*b^3*d*x^2)*(d*x^2 + 2*I) + 4*(b^3*d^(11/2)*x^11 + 7*I*b^3*d^(9/2)*
x^9 - 18*b^3*d^(7/2)*x^7 - 20*I*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d
*x^2 + 2*I))*log(d*x^2 + sqrt(d*x^2 + 2*I)*sqrt(d)*x + I) + 4*(a*b^2*d^(11/
2)*x^11 + 7*I*a*b^2*d^(9/2)*x^9 - 18*a*b^2*d^(7/2)*x^7 - 20*I*a*b^2*d^(5/2)
*x^5 + 8*a*b^2*d^(3/2)*x^3)*sqrt(d*x^2 + 2*I)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
2 + 2*I*d)*x + I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 +
 2*I*d)*x + I) + a*b^2), x) + 2*sqrt(d^2*x^2 + 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 + 2*I*d)*x + I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 + 2*I*
d)*x + I) + a^2*b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^3,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^3, x)
```

3.321 $\int \left(a - ib\textbf{ArcSin}(1 + idx^2)\right)^4 \, dx$

Optimal. Leaf size=153

$$384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}\,(a - ib\text{ArcSin}(1 + idx^2))}{dx} + 48b^2x\left(a - ib\text{ArcSin}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{-2idx^2 + \cdots}}{\cdots}$$

```
[Out] 384*b^4*x+48*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^2+x*(a-I*b*arcsin(1+I*d*x^2))^
4-192*b^3*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-8*b*(a-I
*b*arcsin(1+I*d*x^2))^3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]

time = 0.02, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$-\frac{192b^3\sqrt{d^2x^4 - 2idx^2}\,(a - ib\text{ArcSin}(1 + idx^2))}{dx} + 48b^2x\left(a - ib\text{ArcSin}\left(1 + idx^2\right)\right)^2 - \frac{8b\sqrt{d^2x^4 - 2idx^2}\,(a - ib\text{ArcSin}(1 + idx^2))^3}{dx} + x\left(a - ib\text{ArcSin}\left(1 + idx^2\right)\right)^4 + 384b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x
^2]))/(d*x) + 48*b^2*x*(a - I*b*ArcSin[1 + I*d*x^2])^2 - (8*b*Sqrt[(-2*I)*d
*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x*(a - I*b*ArcSin[
1 + I*d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^4 \, dx &= -\frac{8b\sqrt{-2idx^2 + d^2x^4}\,\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2 \cdots\right.\right.\\
&= -\frac{192b^3\sqrt{-2idx^2 + d^2x^4}\,\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib\sin^{-1}\left(1 \cdots\right.\right.\\
&= 384b^4x - \frac{192b^3\sqrt{-2idx^2 + d^2x^4}\,\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)}{dx} + 48b^2x\left(a - ib \cdots\right.
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 149, normalized size = 0.97

$$-\frac{8b\sqrt{dx^2(-2i+dx^2)}\,(a-ib\text{ArcSin}(1+idx^2))^3}{dx}+x\left(a-ib\text{ArcSin}(1+idx^2)\right)^4+48b^2\left(8b^2x-\frac{4b\sqrt{dx^2(-2i+dx^2)}\,(a-ib\text{ArcSin}(1+idx^2))}{dx}+x\left(a-ib\text{ArcSin}(1+idx^2)\right)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^4,x]
```

```
[Out] (-8*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^3)/(d*x) + x
*(a - I*b*ArcSin[1 + I*d*x^2])^4 + 48*b^2*(8*b^2*x - (4*b*Sqrt[d*x^2*(-2*I
+ d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2]))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^2)
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^4 + 4*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^3*b + a^4*x +
integrate(2*(2*((a*b^3*d^2 - 2*b^4*d^2)*x^4 - 2*a*b^3 - (3*I*a*b^3*d - 4*I*
b^4*d)*x^2 + ((a*b^3*d^(3/2) - 2*b^4*d^(3/2))*x^3 - 2*(I*a*b^3*sqrt(d) - I*
b^4*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x
- I)^3 + 3*(a^2*b^2*d^2*x^4 - 3*I*a^2*b^2*d*x^2 - 2*a^2*b^2 + (a^2*b^2*d^(3
/2)*x^3 - 2*I*a^2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2
- 2*I)*sqrt(d)*x - I)^2)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*
x)*sqrt(d*x^2 - 2*I) - 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(129) = 258.
time = 0.39, size = 269, normalized size = 1.76

$$\frac{b^4dx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^4+4\left(ab^3dx-2\sqrt{d^2x^2-2i\,d}\,b^4\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^3+(a^4+48\,a^2b^2+384\,b^4)dx-6\left(4\sqrt{d^2x^2-2i\,d}\,ab^3-(a^2b^2+8\,b^4)dx\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^2+4\left((a^3b+24\,ab^3)dx-6\,(a^2b^2+8\,b^4)\sqrt{d^2x^2-2i\,d}\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)-8\,(a^3b+24\,ab^3)\sqrt{d^2x^2-2i\,d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^4 + 4*(a*b^3*d*x - 2*sqrt
(d^2*x^2 - 2*I*d)*b^4)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^4 +
48*a^2*b^2 + 384*b^4)*d*x - 6*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^3 - (a^2*b^2 + 8
*b^4)*d*x)*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 4*((a^3*b + 24*a*b^
3)*d*x - 6*(a^2*b^2 + 8*b^4)*sqrt(d^2*x^2 - 2*I*d))*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) - 8*(a^3*b + 24*a*b^3)*sqrt(d^2*x^2 - 2*I*d))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**4,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^4,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^4, x)
```

3.322 $\int (a - ib\textbf{ArcSin}(1 + idx^2))^3 \, dx$

Optimal. Leaf size=129

$$24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x\text{ArcSin}(1 + idx^2) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\,(a - ib\text{ArcSin}(1 + idx^2))^2}{dx} +$$

```
[Out] 24*a*b^2*x-24*I*b^3*x*arcsin(1+I*d*x^2)+x*(a-I*b*arcsin(1+I*d*x^2))^3-48*b^
3*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x-6*b*(a-I*b*arcsin(1+I*d*x^2))^2*(-2*I*d*x^
2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]

time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4898, 4924, 12, 1602}

$$-\frac{6b\sqrt{d^2x^4 - 2idx^2}\,(a - ib\text{ArcSin}(1 + idx^2))^2}{dx} + x(a - ib\text{ArcSin}(1 + idx^2))^3 + 24ab^2x - 24ib^3x\text{ArcSin}(1 + idx^2) - \frac{48b^3\sqrt{d^2x^4 - 2idx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - (24*I)*b^3*x*Arc
Sin[1 + I*d*x^2] - (6*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*
d*x^2])^2)/(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^3 \, dx = -\frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x\left(a - ib\sin^{-1}\left(1 + idx^2\right.\right.$$

$$= 24ab^2x - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)^2}{dx} + x\left(a - ib\sin^{-1}\left(\right.\right.$$

$$= 24ab^2x - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + \right.\right.}{dx}$$

$$= 24ab^2x - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2 + d^2x^4}\left(a - ib\sin^{-1}\left(1 + \right.\right.}{dx}$$

$$= 24ab^2x - \frac{48b^3\sqrt{-2idx^2 + d^2x^4}}{dx} - 24ib^3x\sin^{-1}\left(1 + idx^2\right) - \frac{6b\sqrt{-2idx^2}}{}$$

Mathematica [A]
time = 0.10, size = 180, normalized size = 1.40

$$\frac{a(a^2 + 24b^2)\,dx^2 - 6b(a^2 + 8b^2)\sqrt{dx^2(-2i + dx^2)} - 3ib\left(a^2dx^2 + 8b^2dx^2 - 4ab\sqrt{dx^2(-2i + dx^2)}\right)\text{ArcSin}(1 + idx^2) + 3b^2\left(-adx^2 + 2b\sqrt{dx^2(-2i + dx^2)}\right)\text{ArcSin}(1 + idx^2)^2 + ib^3dx^2\text{ArcSin}(1 + idx^2)^3}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^3,x]
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2*(-2*I + d*x^2)] - (3
*I)*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
 + I*d*x^2] + 3*b^2*(-(a*d*x^2) + 2*b*Sqrt[d*x^2*(-2*I + d*x^2)])*ArcSin[1
+ I*d*x^2]^2 + I*b^3*d*x^2*ArcSin[1 + I*d*x^2]^3)/(d*x)
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{arcsinh}\left(d\,x^2 - i\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^3,x)
[Out] int((a+b*arcsinh(-I+d*x^2))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^3 + 3*(x*arcsinh(d*x^2 -
 I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d))*a^2*b + a^3*x +
integrate(3*((a*b^2*d^2 - 2*b^3*d^2)*x^4 - 2*a*b^2 - (3*I*a*b^2*d - 4*I*b^3
*d)*x^2 + ((a*b^2*d^(3/2) - 2*b^3*d^(3/2))*x^3 - 2*(I*a*b^2*sqrt(d) - I*b^3
*sqrt(d))*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I
)^2/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I)
- 2), x)
```

Fricas [A]
time = 0.37, size = 188, normalized size = 1.46

$$\frac{b^3dx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^3+(a^3+24\,ab^2)dx+3\left(ab^2dx-2\sqrt{d^2x^2-2i\,d}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)^2-3\left(4\sqrt{d^2x^2-2i\,d}\,ab^2-(a^2b+8\,b^3)dx\right)\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)-6\sqrt{d^2x^2-2i\,d}\,(a^2b+8\,b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^3 + (a^3 + 24*a*b^2)*d*x
+ 3*(a*b^2*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^3)*log(d*x^2 + sqrt(d^2*x^2 - 2*
I*d)*x - I)^2 - 3*(4*sqrt(d^2*x^2 - 2*I*d)*a*b^2 - (a^2*b + 8*b^3)*d*x)*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) - 6*sqrt(d^2*x^2 - 2*I*d)*(a^2*b + 8*
b^3))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^3,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^3, x)
```

3.323 $\int (a - ib\mathbf{ArcSin}(1 + idx^2))^2 \, dx$

Optimal. Leaf size=76

$$8b^2x - \frac{4b\sqrt{-2idx^2 + d^2x^4}\,(a - ib\text{ArcSin}(1 + idx^2))}{dx} + x\big(a - ib\text{ArcSin}\big(1 + idx^2\big)\big)^2$$

```
[Out] 8*b^2*x+x*(a-I*b*arcsin(1+I*d*x^2))^2-4*b*(a-I*b*arcsin(1+I*d*x^2))*(-2*I*d
*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4898, 8}

$$-\frac{4b\sqrt{d^2x^4 - 2idx^2}\,(a - ib\text{ArcSin}(1 + idx^2))}{dx} + x\big(a - ib\text{ArcSin}\big(1 + idx^2\big)\big)^2 + 8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \big(a - ib\sin^{-1}\big(1 + idx^2\big)\big)^2 \, dx &= -\frac{4b\sqrt{-2idx^2 + d^2x^4}\,(a - ib\sin^{-1}(1 + idx^2))}{dx} + x\big(a - ib\sin^{-1}\big(1 + idx\\ &= 8b^2x - \frac{4b\sqrt{-2idx^2 + d^2x^4}\,(a - ib\sin^{-1}(1 + idx^2))}{dx} + x\big(a - ib\sin^{-1}\big(1\end{aligned}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 1.00

$$8b^2x - \frac{4b\sqrt{-2idx^2 + d^2x^4}\,(a - ib\text{ArcSin}(1 + idx^2))}{dx} + x\left(a - ib\text{ArcSin}\left(1 + idx^2\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*(a - I*b*ArcSin[1 + I*d*x^2]))/
(d*x) + x*(a - I*b*ArcSin[1 + I*d*x^2])^2
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int \left(a + b\,\text{arcsinh}\left(d\,x^2 - i\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] 2*(x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*
d))*a*b + (x*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 - integrate(4*(
d^2*x^4 - 2*I*d*x^2 + (d^(3/2)*x^3 - I*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*
x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)/(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3
- 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 2), x))*b^2 + a^2*x
```

Fricas [A]
time = 0.35, size = 114, normalized size = 1.50

$$\frac{b^2dx\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}\;x - i\right)^2 + (a^2 + 8\,b^2)dx - 4\,\sqrt{d^2x^2 - 2i\,d}\;ab + 2\left(abdx - 2\,\sqrt{d^2x^2 - 2i\,d}\;b^2\right)\log\left(dx^2 + \sqrt{d^2x^2 - 2i\,d}\;x - i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + (a^2 + 8*b^2)*d*x - 4
*sqrt(d^2*x^2 - 2*I*d)*a*b + 2*(a*b*d*x - 2*sqrt(d^2*x^2 - 2*I*d)*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^2,x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^2, x)
```

3.324 $\int \left(a - ib\,\mathbf{ArcSin}(1 + idx^2)\right) dx$

Optimal. Leaf size=50

$$ax - \frac{2b\sqrt{-2idx^2 + d^2x^4}}{dx} - ibx\text{ArcSin}\left(1 + idx^2\right)$$

```
[Out] a*x-I*b*x*arcsin(1+I*d*x^2)-2*b*(-2*I*d*x^2+d^2*x^4)^(1/2)/d/x
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4924, 12, 1602}

$$ax - ibx\text{ArcSin}\left(1 + idx^2\right) - \frac{2b\sqrt{d^2x^4 - 2idx^2}}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[a - I*b*ArcSin[1 + I*d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a - ib\sin^{-1}\left(1 + idx^2\right)\right)\,dx &= ax - (ib)\int \sin^{-1}\left(1 + idx^2\right)\,dx \\ &= ax - ibx\sin^{-1}\left(1 + idx^2\right) + (ib)\int \frac{2idx^2}{\sqrt{-2idx^2 + d^2x^4}}\,dx \\ &= ax - ibx\sin^{-1}\left(1 + idx^2\right) - (2bd)\int \frac{x^2}{\sqrt{-2idx^2 + d^2x^4}}\,dx \\ &= ax - \frac{2b\sqrt{-2idx^2 + d^2x^4}}{dx} - ibx\sin^{-1}\left(1 + idx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.96

$$ax - \frac{2b\sqrt{dx^2\left(-2i + dx^2\right)}}{dx} - ibx\text{ArcSin}\left(1 + idx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - I*b*ArcSin[1 + I*d*x^2],x]
[Out] a*x - (2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x) - I*b*x*ArcSin[1 + I*d*x^2]
```

Maple [A]
time = 0.25, size = 48, normalized size = 0.96

method	result	size
default	$ax + b\left(x\,\mathrm{arcsinh}\left(d\,x^2 - i\right) + \frac{2x\left(-d\,x^2 + 2i\right)}{\sqrt{d^2x^4 - 2id\,x^2}}\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arcsinh(-I+d*x^2),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*arcsinh(-I+d*x^2)+2/(-2*I*d*x^2+d^2*x^4)^(1/2)*x*(-d*x^2+2*I))
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.88

$$\left(x\,\mathrm{arsinh}\left(dx^2 - i\right) - \frac{2\left(d^{\frac{3}{2}}x^2 - 2i\sqrt{d}\right)}{\sqrt{dx^2 - 2i}\,d}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="maxima")
```

```
[Out] (x*arcsinh(d*x^2 - I) - 2*(d^(3/2)*x^2 - 2*I*sqrt(d))/(sqrt(d*x^2 - 2*I)*d)
)*b + a*x
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.04

$$\frac{bdx\log\left(dx^2+\sqrt{d^2x^2-2i\,d}\,x-i\right)+adx-2\sqrt{d^2x^2-2i\,d}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 2*sqrt(d^2*x^2 -
2*I*d)*b)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*asinh(-I+d*x**2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arcsinh(-I+d*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.45, size = 39, normalized size = 0.78

$$a\,x+b\,x\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)-\frac{2\,b\,\sqrt{{\left(d\,x^2-\mathrm{i}\right)}^2+1}}{d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(d*x^2 - 1i),x)
```

```
[Out] a*x + b*x*asinh(d*x^2 - 1i) - (2*b*((d*x^2 - 1i)^2 + 1)^(1/2))/(d*x)
```

3.325 $\int \frac{1}{a-ib\text{ArcSin}(1+idx^2)}\,dx$

Optimal. Leaf size=191

$$-\frac{x\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\text{Arc}}{}\right.}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{Arc}\right.\right.}$$

```
[Out] 1/2*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/
b/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*x*Ci(1/2*I*(a
-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b/(cos(1/2*arcsi
n(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))
```

Rubi [A]
time = 0.02, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4900\}$

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\text{ArcSin}(idx^2+1)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
```

```
[Out] -1/2*(x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b
)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^
2]/2])) + (x*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin
[1 + I*d*x^2])/(2*b)])/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{a-ib\sin^{-1}\left(1+idx^2\right)}\,dx=-\frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}(1+idx^2)\right)}{2b}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)\right.}{2b\left(\cos\left(\frac{1}{2}\sin^{-1}\right.\right.}$$

Mathematica [A]
time = 0.48, size = 146, normalized size = 0.76

$$\frac{x\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\left(-i\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)+\left(-i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\right)}{2b\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-1),x]
[Out] (x*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*((-I)*Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + ((-I)*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinIntegral[((I*a)/b +
ArcSin[1 + I*d*x^2])/2]))/(2*b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 +
 I*d*x^2]/2]))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arcsinh}\left(d\,x^{2}-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2)),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="maxima")
[Out] integrate(1/(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="fricas")
[Out] integral(1/(b*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2)),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i)),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i)), x)
```

3.326 $\int \frac{1}{\left(a-ib\mathbf{ArcSin}(1+idx^2)\right)^2}\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)}+\frac{x\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x\left(i\,\text{co}\right.}{4b^2\left(\cos\left(\right.\right.}$$

```
[Out] 1/4*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))
/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/4*x*Shi(1/2*
(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh(1/2*a/b)+sinh(1/2*a/b))/b^2/(cos(1/2*a
rcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/2*(-2*I*d*x^2+d^2*x^4)^(1/2
)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))
```

Rubi [A]

time = 0.02, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4909}

$$\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\text{ArcSin}(idx^2+1)}{2b}\right)}{4b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{d^2x^4-2idx^2}}{2bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] -1/2*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])) + (
x*CosIntegral[((I/2)*(a - I*b*ArcSin[1 + I*d*x^2]))/b]*(Cosh[a/(2*b)] + I*S
inh[a/(2*b)]))/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2])) - (x*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1
 + I*d*x^2])/(2*b)])/(4*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2]))
```

Rule 4909

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[-Sq
rt[-2*c*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcSin[c + d*x^2])), x] + (-Simp[x
*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c + d
*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x
] + Simp[x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Ar
cSin[c + d*x^2])]/(4*b^2*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2
]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{2bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)} + \frac{x\text{Ci}\left(\frac{i\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}{2b}\right)\left(\cosh\left(\frac{a}{2b}\right)\right.}{4b^2\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{s}\right.\right.}$$

Mathematica [A]
time = 1.02, size = 196, normalized size = 0.80

$$\frac{-\frac{2b\sqrt{dx^2(-2i+dx^2)}}{d\left(a-ib\text{ArcSin}(1+idx^2)\right)}+\frac{x^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)-\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)}}{4b^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-2),x]
```

```
[Out] ((-2*b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*(a - I*b*ArcSin[1 + I*d*x^2])) + (x^2
*(CosIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]) - (Cosh[a/(2*b)] - I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1
 + I*d*x^2])/2]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
)/(4*b^2*x)
```

Maple [F]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\text{arcsinh}\left(d\,x^2-i\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 - 3*I*d*x^2 + (d^(3/2)*x^3 - 2*I*sqrt(d)*x)*sqrt(d*x^2 - 2*I)
 - 2)/(a*b*d^2*x^3 - 2*I*a*b*d*x + (b^2*d^2*x^3 - 2*I*b^2*d*x + (b^2*d^(3/2
)*x^2 - I*b^2*sqrt(d))*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqr
t(d)*x - I) + (a*b*d^(3/2)*x^2 - I*a*b*sqrt(d))*sqrt(d*x^2 - 2*I)) + integr
ate(1/2*(d^3*x^6 - 3*I*d^2*x^4 + (d^2*x^4 - I*d*x^2 - 2)*(d*x^2 - 2*I) + (2
```

```
*d^(5/2)*x^5 - 4*I*d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2*I) - 4*I)/(a*b*d
^3*x^6 - 4*I*a*b*d^2*x^4 - 4*a*b*d*x^2 + (a*b*d^2*x^4 - 2*I*a*b*d*x^2 - a*b
)*(d*x^2 - 2*I) + (b^2*d^3*x^6 - 4*I*b^2*d^2*x^4 - 4*b^2*d*x^2 + (b^2*d^2*x
^4 - 2*I*b^2*d*x^2 - b^2)*(d*x^2 - 2*I) + 2*(b^2*d^(5/2)*x^5 - 3*I*b^2*d^(3
/2)*x^3 - 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + 2*(a*b*d^(5/2)*x^5 - 3*I*a*b*d^(3/2)*x^3 - 2*a*b*sqrt(d)*
x)*sqrt(d*x^2 - 2*I)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^2*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)*integral(1/
2*sqrt(d^2*x^2 - 2*I*d)*x/(a*b*d*x^2 - 2*I*a*b + (b^2*d*x^2 - 2*I*b^2)*log(
d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)), x) - sqrt(d^2*x^2 - 2*I*d))/(b^2*d*l
og(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**2,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^2,x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^2, x)
```

3.327 $\int \frac{1}{\left(a-ib\mathbf{ArcSin}(1+idx^2)\right)^3}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^2}-\frac{x}{8b^2\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)}-\frac{x\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)(i\,\text{co}}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\right.\right.}$$

```
[Out] -1/8*x/b^2/(a-I*b*arcsin(1+I*d*x^2))+1/16*x*Shi(1/2*(a-I*b*arcsin(1+I*d*x^2
))/b)*(cosh(1/2*a/b)+I*sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1
/2*arcsin(1+I*d*x^2)))-1/16*x*Ci(1/2*I*(a-I*b*arcsin(1+I*d*x^2))/b)*(I*cosh
(1/2*a/b)+sinh(1/2*a/b))/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I
*d*x^2)))-1/4*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^2
```

Rubi [A]
time = 0.04, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4912, 4900}

$$-\frac{x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{CosIntegral}\left(\frac{i\left(a-ib\text{ArcSin}(1+idx^2)\right)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}+\frac{x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Shi}\left(\frac{a-ib\text{ArcSin}(idx^2+1)}{2b}\right)}{16b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x}{8b^2\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)}-\frac{\sqrt{d^2x^4-2idx^2}}{4bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] -1/4*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^2) -
 x/(8*b^2*(a - I*b*ArcSin[1 + I*d*x^2])) - (x*CosIntegral[((I/2)*(a - I*b*A
rcSin[1 + I*d*x^2]))/b]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(16*b^3*(Cos[Arc
Sin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) + (x*(Cosh[a/(2*b)] + I*
Sinh[a/(2*b)])*SinhIntegral[(a - I*b*ArcSin[1 + I*d*x^2])/(2*b)])/(16*b^3*(
Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4900

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(-x
)*(c*Cos[a/(2*b)] - Sin[a/(2*b)])*(CosIntegral[(c/(2*b))*(a + b*ArcSin[c +
d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x]
 - Simp[x*(c*Cos[a/(2*b)] + Sin[a/(2*b)])*(SinIntegral[(c/(2*b))*(a + b*Arc
Sin[c + d*x^2])]/(2*b*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2
]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
```

```
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^3}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}-\frac{x}{8b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}+\frac{\int \frac{}{a-ib}}{}$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{4bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^2}-\frac{x}{8b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)}-\frac{xC}{16b^3\,(}$$

Mathematica [A]
time = 0.56, size = 227, normalized size = 0.83

$$\frac{-\frac{4bx^2}{a-ib\text{ArcSin}(1+idx^2)}+\frac{8b^2\sqrt{dx^2\left(-2i+dx^2\right)}}{d\left(ia+b\text{ArcSin}(1+idx^2)\right)^2}-\frac{2ix^2\left(\text{CosIntegral}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{Si}\left(\frac{1}{2}\left(\frac{ia}{b}+\text{ArcSin}(1+idx^2)\right)\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)}}{32b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3),x]
```

```
[Out] ((-4*b*x^2)/(a - I*b*ArcSin[1 + I*d*x^2]) + (8*b^2*Sqrt[d*x^2*(-2*I + d*x^2
)])/(d*(I*a + b*ArcSin[1 + I*d*x^2])^2) - ((2*I)*x^2*(CosIntegral[((I*a)/b
+ ArcSin[1 + I*d*x^2])/2]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + (Cosh[a/(2*b)
] + I*Sinh[a/(2*b)])*SinIntegral[((I*a)/b + ArcSin[1 + I*d*x^2])/2]))/(Cos[
ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))/(32*b^3*x)
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^(11/2) + 2*b*d^(11/2))*x^10 - 2*(3*I*a*d^(9/2) + 7*I*b*d^(9/2))*
x^8 - (11*a*d^(7/2) + 36*b*d^(7/2))*x^6 - 2*(-I*a*d^(5/2) - 20*I*b*d^(5/2))
*x^4 - 4*(3*a*d^(3/2) - 4*b*d^(3/2))*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*I*a*
d^3 + 8*I*b*d^3)*x^5 - 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(-I*a*d - I*b*d)*x)*(d
*x^2 - 2*I)^(3/2) + (3*(a*d^(9/2) + 2*b*d^(9/2))*x^8 - 6*(2*I*a*d^(7/2) + 5
*I*b*d^(7/2))*x^6 - 2*(8*a*d^(5/2) + 25*b*d^(5/2))*x^4 - 10*(-I*a*d^(3/2) -
 3*I*b*d^(3/2))*x^2 + 4*a*sqrt(d) + 4*b*sqrt(d))*(d*x^2 - 2*I) + (b*d^(11/2
)*x^10 - 6*I*b*d^(9/2)*x^8 - 11*b*d^(7/2)*x^6 + 2*I*b*d^(5/2)*x^4 - 12*b*d^
(3/2)*x^2 + (b*d^4*x^7 - 3*I*b*d^3*x^5 - 4*b*d^2*x^3 + 4*I*b*d*x)*(d*x^2 -
2*I)^(3/2) + (3*b*d^(9/2)*x^8 - 12*I*b*d^(7/2)*x^6 - 16*b*d^(5/2)*x^4 + 10*
I*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2*I) + (3*b*d^5*x^9 - 15*I*b*d^4*x^
7 - 23*b*d^3*x^5 + 7*I*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2*I) + 8*I*b*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(5*I*a*d^4 + 12*I*b*d^4)*x^7 - (23*a*d^3 + 76*b*d^3)*x^5 - (-7*I*a*d^2
 - 64*I*b*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2*I) + 8*I*a*sqrt(d)
)/(a^2*b^2*d^(11/2)*x^9 - 6*I*a^2*b^2*d^(9/2)*x^7 - 12*a^2*b^2*d^(7/2)*x^5
+ 8*I*a^2*b^2*d^(5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*I*b^4*d^(9/2)*x^7 - 12*b^
4*d^(7/2)*x^5 + 8*I*b^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*I*b^4*d^3*x^4 - 3*b^
4*d^2*x^2 + I*b^4*d)*(d*x^2 - 2*I)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*I*b^4*d^(
7/2)*x^5 - 5*b^4*d^(5/2)*x^3 + 2*I*b^4*d^(3/2)*x)*(d*x^2 - 2*I) + 3*(b^4*d^
5*x^8 - 5*I*b^4*d^4*x^6 - 8*b^4*d^3*x^4 + 4*I*b^4*d^2*x^2)*sqrt(d*x^2 - 2*I
))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I)^2 + (a^2*b^2*d^4*x^6 - 3*I*
a^2*b^2*d^3*x^4 - 3*a^2*b^2*d^2*x^2 + I*a^2*b^2*d)*(d*x^2 - 2*I)^(3/2) + 3*
(a^2*b^2*d^(9/2)*x^7 - 4*I*a^2*b^2*d^(7/2)*x^5 - 5*a^2*b^2*d^(5/2)*x^3 + 2*
I*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2*I) + 2*(a*b^3*d^(11/2)*x^9 - 6*I*a*b^3*d^(9
/2)*x^7 - 12*a*b^3*d^(7/2)*x^5 + 8*I*a*b^3*d^(5/2)*x^3 + (a*b^3*d^4*x^6 - 3
*I*a*b^3*d^3*x^4 - 3*a*b^3*d^2*x^2 + I*a*b^3*d)*(d*x^2 - 2*I)^(3/2) + 3*(a*
b^3*d^(9/2)*x^7 - 4*I*a*b^3*d^(7/2)*x^5 - 5*a*b^3*d^(5/2)*x^3 + 2*I*a*b^3*d
^(3/2)*x)*(d*x^2 - 2*I) + 3*(a*b^3*d^5*x^8 - 5*I*a*b^3*d^4*x^6 - 8*a*b^3*d^
3*x^4 + 4*I*a*b^3*d^2*x^2)*sqrt(d*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)
*sqrt(d)*x - I) + 3*(a^2*b^2*d^5*x^8 - 5*I*a^2*b^2*d^4*x^6 - 8*a^2*b^2*d^3*
x^4 + 4*I*a^2*b^2*d^2*x^2)*sqrt(d*x^2 - 2*I)) + integrate(1/8*(d^6*x^12 - 8
*I*d^5*x^10 - 27*d^4*x^8 + 56*I*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*I*d^3*x
^6 - 3*d^2*x^4 - 8*I*d*x^2 + 4)*(d*x^2 - 2*I)^2 - 96*I*d*x^2 + 2*(2*d^(9/2)
*x^9 - 10*I*d^(7/2)*x^7 - 15*d^(5/2)*x^5 - I*d^(3/2)*x^3 - 11*sqrt(d)*x)*(d
*x^2 - 2*I)^(3/2) + 3*(2*d^5*x^10 - 12*I*d^4*x^8 - 26*d^3*x^6 + 24*I*d^2*x^
4 + 3*d*x^2 + 10*I)*(d*x^2 - 2*I) + 2*(2*d^(11/2)*x^11 - 14*I*d^(9/2)*x^9 -
 39*d^(7/2)*x^7 + 61*I*d^(5/2)*x^5 + 61*d^(3/2)*x^3 - 30*I*sqrt(d)*x)*sqrt(
d*x^2 - 2*I) - 48)/(a*b^2*d^6*x^12 - 8*I*a*b^2*d^5*x^10 - 24*a*b^2*d^4*x^8
+ 32*I*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b^2*d^4*x^8 - 4*I*a*b^2*d^3*x^
6 - 6*a*b^2*d^2*x^4 + 4*I*a*b^2*d*x^2 + a*b^2)*(d*x^2 - 2*I)^2 + 4*(a*b^2*d
```

```
^(9/2)*x^9 - 5*I*a*b^2*d^(7/2)*x^7 - 9*a*b^2*d^(5/2)*x^5 + 7*I*a*b^2*d^(3/2
)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 - 2*I)^(3/2) + 6*(a*b^2*d^5*x^10 - 6*I*a*
b^2*d^4*x^8 - 13*a*b^2*d^3*x^6 + 12*I*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2)*(d*x^2
 - 2*I) + (b^3*d^6*x^12 - 8*I*b^3*d^5*x^10 - 24*b^3*d^4*x^8 + 32*I*b^3*d^3*
x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 - 4*I*b^3*d^3*x^6 - 6*b^3*d^2*x^4 + 4*I
*b^3*d*x^2 + b^3)*(d*x^2 - 2*I)^2 + 4*(b^3*d^(9/2)*x^9 - 5*I*b^3*d^(7/2)*x^
7 - 9*b^3*d^(5/2)*x^5 + 7*I*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 - 2*I
)^(3/2) + 6*(b^3*d^5*x^10 - 6*I*b^3*d^4*x^8 - 13*b^3*d^3*x^6 + 12*I*b^3*d^2
*x^4 + 4*b^3*d*x^2)*(d*x^2 - 2*I) + 4*(b^3*d^(11/2)*x^11 - 7*I*b^3*d^(9/2)*
x^9 - 18*b^3*d^(7/2)*x^7 + 20*I*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d
*x^2 - 2*I))*log(d*x^2 + sqrt(d*x^2 - 2*I)*sqrt(d)*x - I) + 4*(a*b^2*d^(11/
2)*x^11 - 7*I*a*b^2*d^(9/2)*x^9 - 18*a*b^2*d^(7/2)*x^7 + 20*I*a*b^2*d^(5/2)
*x^5 + 8*a*b^2*d^(3/2)*x^3)*sqrt(d*x^2 - 2*I)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*x*log(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I) + a*d*x - 8*(b^4*d*log
(d*x^2 + sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^
2 - 2*I*d)*x - I) + a^2*b^2*d)*integral(1/8/(b^3*log(d*x^2 + sqrt(d^2*x^2 -
 2*I*d)*x - I) + a*b^2), x) + 2*sqrt(d^2*x^2 - 2*I*d)*b)/(b^4*d*log(d*x^2 +
 sqrt(d^2*x^2 - 2*I*d)*x - I)^2 + 2*a*b^3*d*log(d*x^2 + sqrt(d^2*x^2 - 2*I*
d)*x - I) + a^2*b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + b\,\mathrm{asinh}\,(d\,x^2 - \mathrm{i}))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^3,x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^3, x)
```

3.328 $\int \left(a+ib\textbf{ArcSin}(1-idx^2)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$15b^2x\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}-\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\text{ArcSin}(1-idx^2)\right)^{3/2}}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right.\right.$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((-I/b)^(1/2)*(a-I*b*ar
csin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/
(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(-I/b)^(1/2)-15*b
^3*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*co
sh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2
))+sin(1/2*arcsin(-1+I*d*x^2)))-5*b*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)*(2*I*d
*x^2+d^2*x^4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4895}

$$-\frac{15\sqrt{\pi}\sqrt{-\frac{i}{b}}\,b^3x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}+15b^2x\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}-\frac{5b\sqrt{d^2x^4+2idx^2}\left(a+ib\text{ArcSin}(1-idx^2)\right)^{3/2}}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] - (5*b*Sqrt[(2*I)*d*x^2 + d^2*x^
4]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x) + x*(a + I*b*ArcSin[1 - I*d*x
^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[
1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)/b]*(
Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])) - (15*Sqrt[(-I)/b
]*b^3*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/
Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] -
Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
```

```
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}\,dx = -\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}{dx}+x\left(a+ib\sin^{-1}\left(1-id\right.\right.$$

$$= 15b^2x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}-\frac{5b\sqrt{2idx^2+d^2x^4}\left(a+ib\sin^{-1}\left(1-i\right.\right.}{dx}$$

Mathematica [A]
time = 0.19, size = 337, normalized size = 0.97

$$-\frac{5b\sqrt{dx^2\left(2i+dx^2\right)}\left(a+ib\text{ArcSin}(1-idx^2)\right)^{3/2}}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{5/2}+\frac{15b^2x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{\pi}\,S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(5/2),x]
```

```
[Out] (-5*b*Sqrt[d*x^2*(2*I + d*x^2)]*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2))/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[(-I)/b]*Sqrt[a +
I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]
)/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + Sqrt[Pi]*FresnelS[(Sqrt[(-I
)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a
/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x
^2]/2]))
```

Maple [F]
time = 1.16, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(5/2),x)
[Out] int((a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

3.329 $\int \left(a+ib\text{ArcSin}(1-idx^2)\right)^{3/2}\,dx$

Optimal. Leaf size=312

$$-\frac{3b\sqrt{2idx^2+d^2x^4}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}+\frac{3\sqrt{ib}\,b\sqrt{\pi}\,x\text{FresnelC}\Big(\frac{\sqrt{}}{}}{\cos\left(\frac{1}{2}\text{ArcSin}\right.}$$

```
[Out] x*(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(-1+I*d*x^
2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(c
os(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)+3*b*x*F
resnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*
a/b)-sinh(1/2*a/b))*(I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))-3*b*(2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(-1+I*d*
x^2))^(1/2)/d/x
```

Rubi [A]
time = 0.08, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4903}

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{3b\sqrt{d^2x^4+2idx^2}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{dx}+\frac{3\sqrt{\pi}\,\sqrt{ib}\,bx\left(-\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}\,\sqrt{ib}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(2*I)*d*x^2 + d^2*x^4]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x)
+ x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*Sqrt[I*b]*b*Sqrt[Pi]*x*Fresnel
C[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)]
- Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
 - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*
Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
```

```
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}\,dx=-\frac{3b\sqrt{2idx^2+d^2x^4}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\big(a+ib\sin^{-1}\big(1-idx$$

$$=-\frac{3b\sqrt{2idx^2+d^2x^4}\,\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{dx}+x\big(a+ib\sin^{-1}\big(1-idx$$

Mathematica [A]
time = 0.15, size = 258, normalized size = 0.83

$$-\frac{3b\sqrt{dx^2\left(2i+dx^2\right)}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{dx}+x\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}+\frac{3b^2\sqrt{\pi}\,x\left(-S\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(2*I + d*x^2)]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/(d*x) +
x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2) + (3*b^2*Sqrt[Pi]*x*(-(FresnelS[Sqrt[
a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[
a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi]
)]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{arcsinh}\left(d\,x^2+i\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(I+d*x^2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(3/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

3.330 $\int \sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}\,dx$

Optimal. Leaf size=263

$$x\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}+\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-\frac{i}{b}}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\ldots$$

```
[Out] x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(-I/b)^(1/2)-b*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d
*x^2))^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I/b)^(1/2)*Pi^(1/2
)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))+x*(a-I*b*arcsin
(-1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4895}

$$-\frac{\sqrt{\pi}\sqrt{-\frac{i}{b}}\,bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}+x\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sq
rt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]
))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
 - (Sqrt[(-I)/b]*b*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1
- I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I
*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
```

```
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}\,dx = x\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)} + \frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{-\frac{i}{b}}\,\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)-\sin\left(\right.\right.}$$

Mathematica [A]
time = 0.04, size = 259, normalized size = 0.98

$$\frac{x\left(\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)+\sqrt{\pi}\,S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{-\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
[Out] (x*(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]) - Sqrt[Pi]*FresnelC[(Sqrt[(-I)/b]*Sqrt[a
+ I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) + S
qrt[Pi]*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]
*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[(-I)/b]*(Cos[ArcSin[1 - I*d*x^2]
/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\text{arcsinh}\left(d\,x^2+i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
[Out] int((a+b*arcsinh(I+d*x^2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

3.331 $\int \frac{1}{\sqrt{a+ib\mathbf{ArcSin}\left(1-idx^2\right)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a+ib\mathrm{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\mathrm{FresnelC}\left(\frac{\sqrt{a+ib\mathrm{ArcSin}\left(1-idx^2\right.}}{\sqrt{ib}\,\sqrt{\pi}}\right.}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)-\mathrm{si}\right.}$$

```
[Out] -x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsi
n(-1+I*d*x^2)))/(I*b)^(1/2)-x*FresnelC((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*
b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin
(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{4903\}$

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\mathrm{FresnelC}\left(\frac{\sqrt{a+ib\mathrm{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}\,\sqrt{ib}}\right)}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\mathrm{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\,\sqrt{\pi}}\right)}{\sqrt{ib}\,\left(\cos\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\mathrm{ArcSin}\left(1-idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcS
in[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(
Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ib\sin^{-1}(1-idx^2)}}\,dx = -\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a+ib\sin^{-1}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1-idx^2)\right)\right)} - \frac{\sqrt{\pi}}{\sqrt{}}$$

Mathematica [A]
time = 0.00, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}\,x\left(-S\left(\frac{\sqrt{a+ib\text{ArcSin}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\text{FresnelC}\left(\frac{\sqrt{a+ib\text{ArcSin}(1-idx^2)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]],x]
[Out] (Sqrt[Pi]*x*(-(FresnelS[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[P
i])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelC[Sqrt[a + I*b*ArcSin[1 -
I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I
*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{arcsinh}(d\,x^2+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 + I) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(1/2), x)
```

3.332 $\int \frac{1}{\left(a+ib\textbf{ArcSin}(1-idx^2)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,x\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)\right.}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}$$

```
[Out] -(-I/b)^(3/2)*x*FresnelC((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(
1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(-1+I*d*x^2))
+sin(1/2*arcsin(-1+I*d*x^2)))+(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*a
rcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-(2*I*d*x^2+d^2*x
^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{4906\}$

$$-\frac{\sqrt{d^2x^4+2idx^2}}{bdx\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\,\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
```

```
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}}-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xC\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\sin^{-1}}}{\sqrt{\pi}}\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1-idx^2\right)\right)}$$

Mathematica [A]
time = 0.25, size = 291, normalized size = 1.00

$$-\frac{\sqrt{2idx^2+d^2x^4}}{bdx\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,x\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}+\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xS\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])) -
(((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I
*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*
x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS
[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)]
+ I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2
])
```

Maple [F]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^{2}+i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1{}\mathrm{i}\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(3/2), x)
```

3.333 $\int \frac{1}{\left(a+ib\mathbf{ArcSin}\left(1-idx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\text{si}\right.}$$

```
[Out] -1/3*x*FresnelS((a-I*b*arcsin(-1+I*d*x^2))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cos
h(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1
/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(-1+I*d*x^2
))^(1/2)/(I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2
/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))/(I*b)^(1/2)-1/3*
(2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/3*x/b^2/
(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4903}

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}\sqrt{ib}}\right)}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)}{3\sqrt{ib}\,b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x}{3b^2\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\sqrt{d^2x^4+2idx^2}}{3bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)
) - x/(3*b^2*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt
[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh
[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I
*d*x^2]/2])) - (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt
[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[I*b]*b^2*(Cos[A
rcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} + \cdots$$

$$= -\frac{\sqrt{2idx^2+d^2x^4}}{3bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+ib\sin^{-1}\left(1-idx^2\right)}} - \cdots$$

Mathematica [A]
time = 0.55, size = 308, normalized size = 0.94

$$-\frac{\frac{b\sqrt{dx^2(2i+dx^2)}}{dx\left(a+ib\text{ArcSin}(1-idx^2)\right)^{3/2}}+\frac{x}{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}+\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)}+\frac{\sqrt{\pi}\,x\text{FresnelC}\left(\frac{\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{ib}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)\right)}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*((b*Sqrt[d*x^2*(2*I + d*x^2)])/(d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2
)) + x/Sqrt[a + I*b*ArcSin[1 - I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[Sqrt[a + I*
b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b
)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
+ (Sqrt[Pi]*x*FresnelC[Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]/(Sqrt[I*b]*Sqrt[Pi
])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I*b]*(Cos[ArcSin[1 - I*d*x^2]/
2] - Sin[ArcSin[1 - I*d*x^2]/2])))/b^2
```

Maple [F]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2+i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2),x)
[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(5/2), x)
```

3.334 $\int \frac{1}{\left(a+ib\mathbf{ArcSin}(1-idx^2)\right)^{7/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}}-\frac{\sqrt{2idx^2+d^2x^4}}{15b^3dx\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(-1+I*d*x^2))^(3/2)-1/15*(-I/b)^(3/2)*x*FresnelC((
-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sin
h(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d
*x^2)))+1/15*(-I/b)^(3/2)*x*FresnelS((-I/b)^(1/2)*(a-I*b*arcsin(-1+I*d*x^2)
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arc
sin(-1+I*d*x^2))+sin(1/2*arcsin(-1+I*d*x^2)))-1/5*(2*I*d*x^2+d^2*x^4)^(1/2)
/b/d/x/(a-I*b*arcsin(-1+I*d*x^2))^(5/2)-1/15*(2*I*d*x^2+d^2*x^4)^(1/2)/b^3/
d/x/(a-I*b*arcsin(-1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {4912, 4906}

$$-\frac{\sqrt{d^2x^4+2idx^2}}{15b^3dx\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}-\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}+\frac{\sqrt{\pi}\left(-\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}\left(1-idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1-idx^2\right)\right)\right)}-\frac{x}{15b^2\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{3/2}}-\frac{\sqrt{d^2x^4+2idx^2}}{5bdx\left(a+ib\text{ArcSin}\left(1-idx^2\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] -1/5*Sqrt[(2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a + I*b*ArcSin[1 - I*d*x^2])^(5/2)
) - x/(15*b^2*(a + I*b*ArcSin[1 - I*d*x^2])^(3/2)) - Sqrt[(2*I)*d*x^2 + d^2
*x^4]/(15*b^3*d*x*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]]) - (((-I)/b)^(3/2)*Sqrt
[Pi]*x*FresnelC[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*
(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Si
n[ArcSin[1 - I*d*x^2]/2])) + (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)
/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(15*b^2*(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
```

```
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}+$$

$$=-\frac{\sqrt{2idx^2+d^2x^4}}{5bdx\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+ib\sin^{-1}\left(1-idx^2\right)\right)^{3/2}}-$$

Mathematica [A]
time = 0.65, size = 365, normalized size = 0.94

$$\frac{\frac{-\frac{3b\sqrt{dx^2(2i+dx^2)}}{d}-x^2\left(a+ib\text{ArcSin}(1-idx^2)\right)+\frac{\sqrt{dx^2(2i+dx^2)}\left(-ia+b\text{ArcSin}(1-idx^2)\right)^2}{bd}}{x\left(a+ib\text{ArcSin}(1-idx^2)\right)^{5/2}}-\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,x\,\text{FresnelC}\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)}+\frac{\left(-\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,x\,S\left(\frac{\sqrt{-\frac{i}{b}}\sqrt{a+ib\text{ArcSin}(1-idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1-idx^2)\right)}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*b*ArcSin[1 - I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(2*I + d*x^2)])/d - x^2*(a + I*b*ArcSin[1 - I*d*x^2]) +
(Sqrt[d*x^2*(2*I + d*x^2)]*((-I)*a + b*ArcSin[1 - I*d*x^2])^2)/(b*d))/(x*(a
 + I*b*ArcSin[1 - I*d*x^2])^(5/2)) - (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(S
qrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 - I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I
*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2])
+ (((-I)/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[(-I)/b]*Sqrt[a + I*b*ArcSin[1 -
```

```
 I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 - I*
d*x^2]/2] - Sin[ArcSin[1 - I*d*x^2]/2]))/(15*b^2)
```

Maple [**F**]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d \,x^{2}+i\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
[Out] int(1/(a+b*arcsinh(I+d*x^2))^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 + I) + a)^(-7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(I+d*x**2))**(7/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsinh(I+d*x^2))^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2+1\mathrm{i}\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2),x)

[Out] int(1/(a + b*asinh(d*x^2 + 1i))^(7/2), x)

3.335 $\int \left(a-ib\mathbf{ArcSin}(1+idx^2)\right)^{5/2}\,dx$

Optimal. Leaf size=348

$$15b^2x\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}-\frac{5b\sqrt{-2idx^2+d^2x^4}\,\left(a-ib\text{ArcSin}(1+idx^2)\right)^{3/2}}{dx}+x\left(a-ib\text{ArcSin}\left(1+idx^2\right.\right.$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(5/2)+15*b^2*x*FresnelS((I/b)^(1/2)*(a-I*b*arcs
in(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)-15*b^2*x*F
resnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b
)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*
d*x^2)))/(I/b)^(1/2)-5*b*(a-I*b*arcsin(1+I*d*x^2))^(3/2)*(-2*I*d*x^2+d^2*x^
4)^(1/2)/d/x+15*b^2*x*(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4895}

$$-\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\,\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}+\frac{15\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\,\sqrt{a-ib\text{ArcSin}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}+15b^2x\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}-\frac{5b\sqrt{d^2x^4-2idx^2}\,\left(a-ib\text{ArcSin}(1+idx^2)\right)^{3/2}}{dx}+x\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
```

```
[Out] 15*b^2*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] - (5*b*Sqrt[(-2*I)*d*x^2 + d^2*x
^4]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x) + x*(a - I*b*ArcSin[1 + I*d*
x^2])^(5/2) + (15*b^2*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1
+ I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[A
rcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (15*b^2*Sqrt[Pi]*x*F
resnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*
b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin
[1 + I*d*x^2]/2]))
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
```

```
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}\,dx=-\frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}{dx}+x\left(a-ib\sin^{-1}\left(1+\right.\right.$$

$$=15b^2x\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}-\frac{5b\sqrt{-2idx^2+d^2x^4}\left(a-ib\sin^{-1}\left(1-\right.\right.}{dx}$$

Mathematica [A]
time = 0.19, size = 337, normalized size = 0.97

$$-\frac{5b\sqrt{dx^2(-2i+dx^2)}\left(a-ib\text{ArcSin}(1+idx^2)\right)^{3/2}}{dx}+x\left(a-ib\text{ArcSin}(1+idx^2)\right)^{5/2}+\frac{15b^2x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}(1+idx^2)}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)+\sqrt{\pi}\,S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(5/2),x]
[Out] (-5*b*Sqrt[d*x^2*(-2*I + d*x^2)]*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2))/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2) + (15*b^2*x*(Sqrt[I/b]*Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqr
t[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi]*FresnelC[(Sqrt[I/b]*Sqr
t[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])
))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Maple [F]
time = 1.13, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(5/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

3.336 $\int \left(a-ib\textbf{ArcSin}(1+idx^2)\right)^{3/2}\,dx$

Optimal. Leaf size=310

$$-\frac{3b\sqrt{-2idx^2+d^2x^4}\,\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{dx}+x\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}-\frac{3b^2\sqrt{\pi}\,xS\left(\frac{\sqrt{a-ib\text{ArcS}}}{\sqrt{-i}}\right.}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\,(1\right.\right.}$$

```
[Out] x*(a-I*b*arcsin(1+I*d*x^2))^(3/2)-3*b^2*x*FresnelS((a-I*b*arcsin(1+I*d*x^2)
)^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(co
s(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-3*b*x*Fre
snelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/
b)+sinh(1/2*a/b))*(-I*b)^(1/2)*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2
*arcsin(1+I*d*x^2)))-3*b*(-2*I*d*x^2+d^2*x^4)^(1/2)*(a-I*b*arcsin(1+I*d*x^2
))^(1/2)/d/x
```

Rubi [A]

time = 0.07, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4898, 4903}

$$-\frac{3\sqrt{\pi}\,b^2x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\text{ArcSin}\,(idx^2+1)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\,(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\,(1+idx^2)\right)\right)}-\frac{3b\sqrt{d^2x^4-2idx^2}\,\sqrt{a-ib\text{ArcSin}\,(1+idx^2)}}{dx}-\frac{3\sqrt{\pi}\,\sqrt{-ib}\,bx\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\,(1+idx^2)}}{\sqrt{\pi}\,\sqrt{-ib}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\,(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\,(1+idx^2)\right)}+x\left(a-ib\text{ArcSin}(1+idx^2)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[(-2*I)*d*x^2 + d^2*x^4]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x)
 + x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*b^2*Sqrt[Pi]*x*FresnelS[Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Si
nh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*
d*x^2]/2])) - (3*Sqrt[(-I)*b]*b*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 +
 I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Cos
[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4898

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcSin[c + d*x^2])^n, x] + (-Dist[4*b^2*n*(n - 1), Int[(a + b*ArcSin[
c + d*x^2])^(n - 2), x], x] + Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b
*ArcSin[c + d*x^2])^(n - 1)/(d*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^
2, 1] && GtQ[n, 1]
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
```

```
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}\,dx=-\frac{3b\sqrt{-2idx^2+d^2x^4}\,\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\big(a-ib\sin^{-1}\left(1+\right.$$

$$=-\frac{3b\sqrt{-2idx^2+d^2x^4}\,\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{dx}+x\big(a-ib\sin^{-1}\left(1+\right.$$

Mathematica [A]
time = 0.17, size = 255, normalized size = 0.82

$$-\frac{3b\sqrt{dx^2\left(-2i+dx^2\right)}\,\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{dx}+x\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}-\frac{3(-ib)^{3/2}\sqrt{\pi}\,x\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-S\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[d*x^2*(-2*I + d*x^2)]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/(d*x) +
 x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2) - (3*((-I)*b)^(3/2)*Sqrt[Pi]*x*(-(Fr
esnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(
2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqr
t[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(Cos[ArcSin[1 + I*
d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Maple [F]
time = 0.95, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(3/2),x)
[Out] int((a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

3.337 $\int \sqrt{a - ib\textbf{ArcSin}\left(1 + idx^2\right)}\, dx$

Optimal. Leaf size=262

$$x\sqrt{a - ib\text{ArcSin}\left(1 + idx^2\right)} + \frac{\sqrt{\pi}\, xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\text{ArcSin}\left(1 + idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right)\right)} - \frac{\sqrt{\pi}\, x\text{F}\cdots}{\cdots}$$

```
[Out] x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*
a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1
+I*d*x^2)))/(I/b)^(1/2)-x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1
/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*
d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(I/b)^(1/2)+x*(a-I*b*arcsin(1+I*d*x^2))
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4895}

$$-\frac{\sqrt{\pi}\, x\left(\cosh\left(\frac{a}{2b}\right) + i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\text{ArcSin}\left(1 + idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right)\right)} + \frac{\sqrt{\pi}\, x\left(\cosh\left(\frac{a}{2b}\right) - i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a - ib\text{ArcSin}\left(idx^2 + 1\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right) - \sin\left(\frac{1}{2}\text{ArcSin}\left(1 + idx^2\right)\right)\right)} + x\sqrt{a - ib\text{ArcSin}\left(1 + idx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[
a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/
(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sq
rt[Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(
Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] -
Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4895

```
Int[Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[x*Sq
rt[a + b*ArcSin[c + d*x^2]], x] + (-Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a
/(2*b)])*(FresnelC[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[c/b]
*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] + Simp[Sqrt[
Pi]*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*A
rcSin[c + d*x^2]]]/(Sqrt[c/b]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c +
```

```
d*x^2]/2]))), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}\,dx = x\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)} + \frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-\sin\left(\right.\right.}$$

Mathematica [A]
time = 0.04, size = 259, normalized size = 0.99

$$\frac{x\left(\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)+\sqrt{\pi}\,S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{\frac{i}{b}}\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] (x*(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]*(Cos[ArcSin[1 + I*d*x^2]/2]
 - Sin[ArcSin[1 + I*d*x^2]/2]) + Sqrt[Pi]*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*
ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]) - Sqrt[Pi
]*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/
(2*b)] + I*Sinh[a/(2*b)])))/(Sqrt[I/b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2]))
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh(-I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(d*x^2 - 1i))^(1/2),x)
```

```
[Out] int((a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

3.338 $\int \frac{1}{\sqrt{a-ib\mathbf{ArcSin}\left(1+idx^2\right)}}\,dx$

Optimal. Leaf size=231

$$-\frac{\sqrt{\pi}\,x\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+id\right.}}{\sqrt{-ib}\,\sqrt{\pi}}\right.}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right.\right.}$$

```
[Out] -x*FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/
2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin
(1+I*d*x^2)))/(-I*b)^(1/2)-x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b
)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(
1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4903}

$$-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}\,\sqrt{-ib}}\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\text{ArcSin}\left(idx^2+1\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
```

```
[Out] -((Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt
[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d
*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*
b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(
2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]
/2]))
```

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-
Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi
]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c
*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/
(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(
Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /;
FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-ib\sin^{-1}(1+idx^2)}}\,dx = -\frac{\sqrt{\pi}\,xC\left(\frac{\sqrt{a-ib\sin^{-1}(1+idx^2)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)-\sin\left(\frac{1}{2}\sin^{-1}(1+idx^2)\right)\right)} - \frac{\sqrt{\pi}\,x}{\sqrt{-}}$$

Mathematica [A]
time = 0.00, size = 180, normalized size = 0.78

$$\frac{\sqrt{\pi}\,x\left(-\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)-S\left(\frac{\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{-ib}\,\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\right)}{\sqrt{-ib}\,\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]],x]
[Out] (Sqrt[Pi]*x*(-(FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqr
t[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)])) - FresnelS[Sqrt[a - I*b*ArcSin[1
 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)])))/(
Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Maple [F]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(d*x^2 - I) + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(1/2), x)
```

3.339 $\int \frac{1}{\left(a-ib\textbf{ArcSin}\left(1+idx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}+\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right.}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}$$

```
[Out] (I/b)^(3/2)*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2)
)*(cosh(1/2*a/b)-I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2*arcsin(1+I*d*x^2))-sin(
1/2*arcsin(1+I*d*x^2)))-(I/b)^(3/2)*x*FresnelC((I/b)^(1/2)*(a-I*b*arcsin(1+
I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/(cos(1/2
*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-(-2*I*d*x^2+d^2*x^4)^(1/2)/
b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4906}

$$-\frac{\sqrt{d^2x^4-2idx^2}}{bdx\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}+\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
```

```
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
```

```
b, c, d}, x] && EqQ[c^2, 1]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}+\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\sin^{-1}\left(\right.}}{\sqrt{\pi}}\right.}{\cos\left(\frac{1}{2}\sin^{-1}\left(1+idx^2\right)\right)-}$$

Mathematica [A]
time = 0.25, size = 291, normalized size = 1.00

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{bdx\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}+\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,xS\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}-\frac{\left(\frac{i}{b}\right)^{3/2}\sqrt{\pi}\,x\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-3/2),x]
[Out] -(Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])) +
 ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^
2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] - I*Sinh[a/(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2]) - ((I/b)^(3/2)*Sqrt[Pi]*x*FresnelC[(Sqrt[I
/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cosh[a/(2*b)] + I*Sinh[a/
(2*b)]))/(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])
```

Maple [F]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(3/2), x)
```

3.340 $\int \frac{1}{\left(a-ib\text{ArcSin}(1+idx^2)\right)^{5/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\,xS\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\,\sqrt{\pi}}\right.}{3\sqrt{-ib}\,b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right.}$$

[Out] -1/3*x*FresnelS((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))/(-I*b)^(1/2)-1/3*x*FresnelC((a-I*b*arcsin(1+I*d*x^2))^(1/2)/(-I*b)^(1/2)/Pi^(1/2))*(I*cosh(1/2*a/b)+sinh(1/2*a/b))*(-I*b)^(1/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/3*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/3*x/b^2/(a-I*b*arcsin(1+I*d*x^2))^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4912, 4903}

$$-\frac{\sqrt{\pi}\sqrt{-ib}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}\sqrt{-ib}}\right)}{3b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\,x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{a-ib\text{ArcSin}\left(idx^2+1\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)}{3\sqrt{-ib}\,b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x}{3b^2\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}-\frac{\sqrt{d^2x^4-2idx^2}}{3bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]

[Out] -1/3*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - x/(3*b^2*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(3*Sqrt[(-I)*b]*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) - (Sqrt[(-I)*b]*Sqrt[Pi]*x*FresnelC[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))

Rule 4903

```
Int[1/Sqrt[(a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(-Sqrt[Pi])*x*(Cos[a/(2*b)] - c*Sin[a/(2*b)])*(FresnelC[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] - Simp[Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelS[(1/(Sqrt[b*c]*Sqrt[Pi]))*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Sqrt[b*c]*(Cos[ArcSin[c + d*x^2]/2] - c*Sin[ArcSin[c + d*x^2]/2]))), x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}\,dx=-\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}+\frac{\int \cdots}{\cdots}$$

$$=-\frac{\sqrt{-2idx^2+d^2x^4}}{3bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}-\frac{x}{3b^2\sqrt{a-ib\sin^{-1}\left(1+idx^2\right)}}-\frac{\sqrt{\pi}\cdots}{3\sqrt{\cdots}}$$

Mathematica [A]
time = 0.56, size = 308, normalized size = 0.94

$$-\frac{\frac{b\sqrt{dx^2\left(-2i+dx^2\right)}}{dx\left(a-ib\text{ArcSin}(1+idx^2)\right)^{3/2}}+\frac{x}{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}+\frac{\sqrt{\pi}\,x\,\text{FresnelC}\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)-i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}+\frac{\sqrt{\pi}\,x\,S\left(\frac{\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{-ib}\sqrt{\pi}}\right)\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)}{\sqrt{-ib}\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-5/2),x]
```

```
[Out] -1/3*((b*Sqrt[d*x^2*(-2*I + d*x^2)])/(d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(3/
2)) + x/Sqrt[a - I*b*ArcSin[1 + I*d*x^2]] + (Sqrt[Pi]*x*FresnelC[Sqrt[a - I
*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*b]*Sqrt[Pi])]*(Cosh[a/(2*b)] - I*Sinh[a/
(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2
]/2])) + (Sqrt[Pi]*x*FresnelS[Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]/(Sqrt[(-I)*
b]*Sqrt[Pi])]*(Cosh[a/(2*b)] + I*Sinh[a/(2*b)]))/(Sqrt[(-I)*b]*(Cos[ArcSin[
1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])))/b^2
```

Maple [F]
time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
```

```
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2),x)
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(5/2), x)
```

3.341 $\int \frac{1}{\left(a-ib\mathbf{ArcSin}(1+idx^2)\right)^{7/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}}-\frac{\sqrt{-2idx^2+d^2x^4}}{15b^3dx\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}$$

```
[Out] -1/15*x/b^2/(a-I*b*arcsin(1+I*d*x^2))^(3/2)-1/15*(I/b)^(3/2)*x*FresnelC((I/
b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(cosh(1/2*a/b)+I*sinh(1/
2*a/b))*Pi^(1/2)/b^2/(cos(1/2*arcsin(1+I*d*x^2))-sin(1/2*arcsin(1+I*d*x^2))
)+1/15*x*FresnelS((I/b)^(1/2)*(a-I*b*arcsin(1+I*d*x^2))^(1/2)/Pi^(1/2))*(I*
cosh(1/2*a/b)+sinh(1/2*a/b))*(I/b)^(1/2)*Pi^(1/2)/b^3/(cos(1/2*arcsin(1+I*d
*x^2))-sin(1/2*arcsin(1+I*d*x^2)))-1/5*(-2*I*d*x^2+d^2*x^4)^(1/2)/b/d/x/(a-
I*b*arcsin(1+I*d*x^2))^(5/2)-1/15*(-2*I*d*x^2+d^2*x^4)^(1/2)/b^3/d/x/(a-I*b
*arcsin(1+I*d*x^2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{4912, 4906\}$

$$-\frac{\sqrt{d^2x^4-2idx^2}}{15b^3dx\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}+\frac{\sqrt{\pi}\sqrt{\frac{i}{b}}\,x\left(\sinh\left(\frac{a}{2b}\right)+i\cosh\left(\frac{a}{2b}\right)\right)S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(idx^2+1\right)}}{\sqrt{\pi}}\right)}{15b^3\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{\sqrt{\pi}\left(\frac{i}{b}\right)^{3/2}x\left(\cosh\left(\frac{a}{2b}\right)+i\sinh\left(\frac{a}{2b}\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}\left(1+idx^2\right)}}{\sqrt{\pi}}\right)}{15b^2\left(\cos\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)-\sin\left(\frac{1}{2}\text{ArcSin}\left(1+idx^2\right)\right)\right)}-\frac{x}{15b^2\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{3/2}}-\frac{\sqrt{d^2x^4-2idx^2}}{5bdx\left(a-ib\text{ArcSin}\left(1+idx^2\right)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] -1/5*Sqrt[(-2*I)*d*x^2 + d^2*x^4]/(b*d*x*(a - I*b*ArcSin[1 + I*d*x^2])^(5/2
)) - x/(15*b^2*(a - I*b*ArcSin[1 + I*d*x^2])^(3/2)) - Sqrt[(-2*I)*d*x^2 + d
^2*x^4]/(15*b^3*d*x*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]) - ((I/b)^(3/2)*Sqrt[
Pi]*x*FresnelC[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(Cos
h[a/(2*b)] + I*Sinh[a/(2*b)]))/(15*b^2*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[Ar
cSin[1 + I*d*x^2]/2])) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a -
 I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15
*b^3*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2]))
```

Rule 4906

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[-
Sqrt[-2*c*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcSin[c + d*x^2]]), x] + (-Si
mp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] + c*Sin[a/(2*b)])*(FresnelC[Sqrt[c/
(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Cos[(1/2)*ArcSin[c + d*x^2]] - c*Si
```

```
n[ArcSin[c + d*x^2]/2])), x] + Simp[(c/b)^(3/2)*Sqrt[Pi]*x*(Cos[a/(2*b)] -
c*Sin[a/(2*b)])*(FresnelS[Sqrt[c/(Pi*b)]*Sqrt[a + b*ArcSin[c + d*x^2]]]/(Co
s[(1/2)*ArcSin[c + d*x^2]] - c*Sin[ArcSin[c + d*x^2]/2])), x]) /; FreeQ[{a,
 b, c, d}, x] && EqQ[c^2, 1]
```

Rule 4912

```
Int[((a_.) + ArcSin[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcSin[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Dist[1/(
4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcSin[c + d*x^2])^(n + 2), x], x] + Sim
p[Sqrt[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcSin[c + d*x^2])^(n + 1)/(2*b*d*(n +
 1)*x)), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ[
n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{7/2}}\,dx = -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}+$$

$$= -\frac{\sqrt{-2idx^2+d^2x^4}}{5bdx\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a-ib\sin^{-1}\left(1+idx^2\right)\right)^{3/2}}-$$

Mathematica [A]
time = 0.65, size = 370, normalized size = 0.95

$$\frac{\frac{-\frac{3b\sqrt{dx^2(-2i+dx^2)}}{d}-x^2\left(a-ib\text{ArcSin}(1+idx^2)\right)+\frac{\sqrt{dx^2(-2i+dx^2)}\left(ia+b\text{ArcSin}(1+idx^2)\right)^2}{bd}}{x\left(a-ib\text{ArcSin}(1+idx^2)\right)^{5/2}}+\frac{\sqrt{\frac{i}{b}}\sqrt{\pi}\,x\,\text{FresnelC}\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{\pi}}\right)\left(-i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{b\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}+\frac{\sqrt{\frac{i}{b}}\sqrt{\pi}\,x\,S\left(\frac{\sqrt{\frac{i}{b}}\sqrt{a-ib\text{ArcSin}(1+idx^2)}}{\sqrt{\pi}}\right)\left(i\cosh\left(\frac{a}{2b}\right)+\sinh\left(\frac{a}{2b}\right)\right)}{b\left(\cos\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)-\sin\left(\frac{1}{2}\text{ArcSin}(1+idx^2)\right)\right)}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*b*ArcSin[1 + I*d*x^2])^(-7/2),x]
```

```
[Out] (((-3*b*Sqrt[d*x^2*(-2*I + d*x^2)])/d - x^2*(a - I*b*ArcSin[1 + I*d*x^2]) +
 (Sqrt[d*x^2*(-2*I + d*x^2)]*(I*a + b*ArcSin[1 + I*d*x^2])^2)/(b*d))/(x*(a
- I*b*ArcSin[1 + I*d*x^2])^(5/2)) + (Sqrt[I/b]*Sqrt[Pi]*x*FresnelC[(Sqrt[I/
b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]])/Sqrt[Pi]]*((-I)*Cosh[a/(2*b)] + Sinh[
a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/2] - Sin[ArcSin[1 + I*d*x^2]/2])) +
(Sqrt[I/b]*Sqrt[Pi]*x*FresnelS[(Sqrt[I/b]*Sqrt[a - I*b*ArcSin[1 + I*d*x^2]]
```

```
)/Sqrt[Pi]]*(I*Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b*(Cos[ArcSin[1 + I*d*x^2]/
2] - Sin[ArcSin[1 + I*d*x^2]/2])))/(15*b^2)
```

Maple [F]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arcsinh}\left(d\,x^2-i\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
[Out] int(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(d*x^2 - I) + a)^(-7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*asinh(-I+d*x**2))**(7/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real -I
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsinh(-I+d*x^2))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(d\,x^2-\mathrm{i}\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2),x)
```

```
[Out] int(1/(a + b*asinh(d*x^2 - 1i))^(7/2), x)
```

3.342 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\operatorname{Int}\left(\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
[Out] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
[Out] -integrate((b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
[Out] -int((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.343 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{3b\left(a+b\sin\right.}{}$$

```
[Out] -1/4*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+3/2*b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*po
lylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/
2*b^2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+3/4*b^3*polylog(4,1/((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6813, 5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$\frac{3b^2\text{Li}_3\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c}+\frac{3b^3\text{Li}_4\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (3*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog
[2, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, E^(-2*ArcSinh[Sqrt[1 - c*x]/S
qrt[1 + c*x]])])/(2*c) + (3*b^3*PolyLog[4, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt
[1 + c*x]])])/(4*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
```

```
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\right.\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(\right.}{c}$$

Mathematica [A]

time = 0.04, size = 244, normalized size = 0.93

$$\frac{\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{b} - 4\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) - 6b\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) + 6b^2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(3, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right) - 3b^3\text{PolyLog}\left(4, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/b - 4*(a + b*ArcSinh[Sqrt[1
 - c*x]/Sqrt[1 + c*x]])^3*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]]
)] - 6*b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[3, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] - 3*b^
3*PolyLog[4, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(4*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(285) = 570$.
time = 4.03, size = 1175, normalized size = 4.50

method	result	size
default	Expression too large to display	1175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+1/4*b^3/c*arcsinh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^4-b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x
+1)+1)^(1/2))+6*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(-c*
x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-6*b^3/c*polylog(4,-(-c
*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-b^3/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))^3*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1
)+1)^(1/2))-3*b^3/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(-c*x
+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*
x+1)+1)^(1/2))-6*b^3/c*polylog(4,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*
x+1)+1)^(1/2))+a*b^2*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3/c-3*a*b^2/c*ar
csinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-
c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*po
lylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))+6*a*b^2/c
*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-3*a*b^
2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/
2)-((-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/
2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))+6*a*
b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))+3/
2*a^2*b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/c-3*a^2*b/c*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+
1)^(1/2))-3*a^2*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+
1)+1)^(1/2))-3*a^2*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-3*a^2*b/c*polylog(2,(-c*x+1
)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^3/c + integrate(1/8*((sqrt(2)*b^3
 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^3 - 6*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a
*b^2)*log(c*x + 1)^2 - 6*(4*sqrt(2)*a*b^2 - 2*(sqrt(2)*b^3 + sqrt(-c*x + 1)
*b^3)*log(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b
^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2 + 12*(sq
rt(2)*a^2*b + sqrt(-c*x + 1)*a^2*b)*log(c*x + 1) - 6*(4*sqrt(2)*a^2*b + 4*s
qrt(-c*x + 1)*a^2*b + (sqrt(2)*b^3 + sqrt(-c*x + 1)*b^3)*log(c*x + 1)^2 - 4
*(sqrt(2)*a*b^2 + sqrt(-c*x + 1)*a*b^2)*log(c*x + 1))*log(sqrt(2) + sqrt(-c
*x + 1)))/(sqrt(2)*c^2*x^2 + (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arcsinh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^3}{c^2x^2-1}\,dx-\int\frac{b^3\operatorname{asinh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3ab^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3a^2b\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*asinh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*asinh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.344 $\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{b\left(a+b\sinh\right.}{}$$

```
[Out] -1/3*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/
(c*x+1))^(1/2))^2)/c+b*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(
2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c+1/2*b^2*
polylog(3,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6813, 5775, 3797, 2221, 2611, 2320, 6724}

$$\frac{b\,\text{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c}+\frac{b^2\text{Li}_3\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -1/3*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1
 + c*x]])])/c + (b*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2,
E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, E^(-2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b\sinh^{-1}(x))^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\right.\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(\right.}{c}$$

Mathematica [A]
time = 0.04, size = 187, normalized size = 0.96

$$\frac{2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-3b\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-6b^2\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)+3b^3\text{PolyLog}\left(3,e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*(a + b*ArcSinh[Sqrt[1 - c
*x]/Sqrt[1 + c*x]] - 3*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])
]) - 6*b^2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, E^(2*Arc
Sinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + 3*b^3*PolyLog[3, E^(2*ArcSinh[Sqrt[1
- c*x]/Sqrt[1 + c*x]])])/(6*b*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(214) = 428.
time = 2.62, size = 649, normalized size = 3.35

method	result
default	$\frac{a^2\ln(cx+1)}{2c}-\frac{a^2\ln(cx-1)}{2c}+\frac{b^2\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c}-\frac{b^2\operatorname{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{-c}{cx}}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)+1/3*b^2*arcsinh((-c*x+1)^(1/2)/(c*x
+1)^(1/2))^3/c-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+(-c*x+1)^
(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1
)+1)^(1/2))+2*b^2/c*polylog(3,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+
1)+1)^(1/2))-b^2/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(-c*x+1)^(1
/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-2*b^2/c*arcsinh((-c*x+1)^(1/2
)/(c*x+1)^(1/2))*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1
)^(1/2))+2*b^2/c*polylog(3,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1
)^(1/2))+a*b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/c-2*a*b/c*arcsinh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+
1)+1)^(1/2))-2*a*b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x
+1)+1)^(1/2))-2*a*b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(-c*x+1)^(
1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-2*a*b/c*polylog(2,(-c*x+1)^(
1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1))^2/c + integrate(-1/4*((sqrt(2)*b^
2 + sqrt(-c*x + 1)*b^2)*log(c*x + 1)^2 - 4*(sqrt(2)*a*b + sqrt(-c*x + 1)*a*
b)*log(c*x + 1) + 2*(4*sqrt(2)*a*b - 2*(sqrt(2)*b^2 + sqrt(-c*x + 1)*b^2)*l
og(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-
c*x + 1))*sqrt(-c*x + 1))*log(sqrt(2) + sqrt(-c*x + 1)))/(sqrt(2)*c^2*x^2 +
 (c^2*x^2 - 1)*sqrt(-c*x + 1) - sqrt(2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arcsinh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2}{c^2x^2-1}\,dx-\int \frac{b^2\operatorname{asinh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{2ab\operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*asinh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*asinh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.345 $\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc}-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c}+\frac{b\text{PolyLog}\left(\right.}{}$$

```
[Out] -1/2*(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arcsinh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x+1)/(c
*x+1))^(1/2))^2)/c+1/2*b*polylog(2,1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+(1+(-c*x
+1)/(c*x+1))^(1/2))^2)/c
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {212, 6813, 5775, 3797, 2221, 2317, 2438}

$$-\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(1-e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\text{Li}_2\left(e^{-2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -1/2*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(b*c) - ((a + b*ArcSinh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 - E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/c + (b*PolyLog[2, E^(-2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(
2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(\frac{\sqrt{1}}{\sqrt{1}}\right.\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-\right.}{c}$$

$$= \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1-\right.}{c}$$

Mathematica [A]
time = 0.02, size = 127, normalized size = 0.95

$$\frac{\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)-2b\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)\right)-b^2\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] ((a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*(a + b*ArcSinh[Sqrt[1 - c*x]/
Sqrt[1 + c*x]] - 2*b*Log[1 - E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])]) -
 b^2*PolyLog[2, E^(2*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*b*c)
```

Maple [A]
time = 1.68, size = 263, normalized size = 1.98

method	result
default	$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}+\frac{b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c}-\frac{b\,\text{arcsinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{-cx+1}{cx+1}}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)+1/2*b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1
/2))^2/c-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(-c*x+1)^(1/2)/(c*x
+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))-b/c*polylog(2,-(-c*x+1)^(1/2)/(c*x+1)
^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-b/c*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)*ln(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)-((-c*x+1)/(c*x+1)+1)^(1/2))-b/c*polylog
(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)/(c*x+1)+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(2) + sqrt(-c*x + 1)))/c + 8*integrate(-1/4*(sqrt(2)*log(c*x
+ 1) - sqrt(2)*log(-c*x + 1))/(sqrt(2)*c*x + (c*x - 1)*sqrt(-c*x + 1) - sqr
t(2)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx - \int \frac{b \operatorname{asinh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*asinh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.346 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\operatorname{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arcsinh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{1}{a c^{2} x^{2} - a + b c^{2} x^{2} \operatorname{asinh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)} - b \operatorname{asinh}{\left(\frac{\sqrt{- c x + 1}}{\sqrt{c x + 1}} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*asinh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*asinh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.347 $\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\sinh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcSinh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arcsinh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] -4*(sqrt(2) + sqrt(-c*x + 1))/(2*sqrt(2)*a*b*c^2*x - 2*sqrt(2)*a*b*c - 4*sq
rt(-c*x + 1)*a*b*c - (sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x + 1)*
b^2*c)*log(c*x + 1) + 2*(sqrt(2)*b^2*c^2*x - sqrt(2)*b^2*c - 2*sqrt(-c*x +
1)*b^2*c)*log(sqrt(2) + sqrt(-c*x + 1))) - integrate((4*c*x + (sqrt(2)*c*x
- 3*sqrt(2))*sqrt(-c*x + 1) - 4)/(2*a*b*c^3*x^3 - 6*a*b*c^2*x^2 + 6*a*b*c*x
 - 4*(a*b*c*x - a*b)*(c*x - 1) - 2*a*b - (b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*b
^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)*b^2*c^2*x^2 - 2*sqr
t(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(c*x + 1) + 2*(b^2*c^3*x^3 -
 3*b^2*c^2*x^2 + 3*b^2*c*x - 2*(b^2*c*x - b^2)*(c*x - 1) - b^2 - 2*(sqrt(2)
*b^2*c^2*x^2 - 2*sqrt(2)*b^2*c*x + sqrt(2)*b^2)*sqrt(-c*x + 1))*log(sqrt(2)
 + sqrt(-c*x + 1)) - 4*(sqrt(2)*a*b*c^2*x^2 - 2*sqrt(2)*a*b*c*x + sqrt(2)*a
*b)*sqrt(-c*x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arcsinh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arcsinh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*asinh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arcsinh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arcsinh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*asinh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.348 $\int \sinh^{-1}\left(ce^{a+bx}\right)\,dx$

Optimal. Leaf size=76

$$-\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}+\frac{\text{PolyLog}\left(2,e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}$$

```
[Out] -1/2*arcsinh(c*exp(b*x+a))^2/b+arcsinh(c*exp(b*x+a))*ln(1-(c*exp(b*x+a)+(1+
c^2*exp(b*x+a)^2)^(1/2))^2)/b+1/2*polylog(2,(c*exp(b*x+a)+(1+c^2*exp(b*x+a)
^2)^(1/2))^2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2320, 5775, 3797, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -1/2*ArcSinh[c*E^(a + b*x)]^2/b + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(ce^{a+bx}\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int x\coth(x)\,dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} - \frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \sinh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1-\right.\right.}{} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\right.}{} \\
&= -\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{Li}_2\left(e^{2\sinh^{-1}\left(ce^{a+bx}\right.}\right.}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.40, size = 76, normalized size = 1.00

$$
-\frac{\sinh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\sinh^{-1}\left(ce^{a+bx}\right)\log\left(1-e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{PolyLog}\left(2, e^{2\sinh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c*E^(a + b*x)],x]
```

```
[Out] -1/2*ArcSinh[c*E^(a + b*x)]^2/b + (ArcSinh[c*E^(a + b*x)]*Log[1 - E^(2*ArcS
inh[c*E^(a + b*x)])])/b + PolyLog[2, E^(2*ArcSinh[c*E^(a + b*x)])]/(2*b)
```

Maple [A]
time = 2.75, size = 153, normalized size = 2.01

method	result
derivativedivides	$\frac{-\frac{\operatorname{arcsinh}\left(e^{bx+a}c\right)^{2}}{2}+\operatorname{arcsinh}\left(e^{bx+a}c\right)\ln\left(1+e^{bx+a}c+\sqrt{1+e^{2bx+2a}c^{2}}\right)+\operatorname{polylog}\left(2,-e^{bx+a}c-\sqrt{1+e^{2bx+2a}c^{2}}\right.}{b}$
default	$\frac{-\frac{\operatorname{arcsinh}\left(e^{bx+a}c\right)^{2}}{2}+\operatorname{arcsinh}\left(e^{bx+a}c\right)\ln\left(1+e^{bx+a}c+\sqrt{1+e^{2bx+2a}c^{2}}\right)+\operatorname{polylog}\left(2,-e^{bx+a}c-\sqrt{1+e^{2bx+2a}c^{2}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(exp(b*x+a)*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*arcsinh(exp(b*x+a)*c)^2+arcsinh(exp(b*x+a)*c)*ln(1+exp(b*x+a)*c+(
1+exp(b*x+a)^2*c^2)^(1/2))+polylog(2,-exp(b*x+a)*c-(1+exp(b*x+a)^2*c^2)^(1/
2))+arcsinh(exp(b*x+a)*c)*ln(1-exp(b*x+a)*c-(1+exp(b*x+a)^2*c^2)^(1/2))+pol
ylog(2,exp(b*x+a)*c+(1+exp(b*x+a)^2*c^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) + c*e^(b*x + a) + (c^2*e^
(2*b*x + 2*a) + 1)^(3/2)), x) + x*log(c*e^(b*x + a) + sqrt(c^2*e^(2*b*x + 2
*a) + 1)) - 1/4*(2*b*x*log(c^2*e^(2*b*x + 2*a) + 1) + dilog(-c^2*e^(2*b*x +
 2*a)))/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}\left(ce^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c*exp(b*x+a)),x)
[Out] Integral(asinh(c*exp(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c*exp(b*x+a)),x, algorithm="giac")
[Out] integrate(arcsinh(c*e^(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asinh}\left(c\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c*exp(a + b*x)),x)
[Out] int(asinh(c*exp(b*x)*exp(a)), x)
```

3.349 $\int e^{\sinh^{-1}(a+bx)} x^3 \, dx$

Optimal. Leaf size=165

$$\frac{e^{-3\sinh^{-1}(a+bx)}}{48b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}+\frac{a(3-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}$$

```
[Out] 1/48/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))^3+3/16*a/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2
))^2+1/8*(6*a^2-1)/b^4/(b*x+a+(1+(b*x+a)^2)^(1/2))+1/16*a*(-4*a^2+3)*(b*x+a
+(1+(b*x+a)^2)^(1/2))^2/b^4-1/24*(-6*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b
^4-3/32*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^4+1/80*(b*x+a+(1+(b*x+a)^2)^(1/2)
)^5/b^4+1/8*a*(-4*a^2+3)*arcsinh(b*x+a)/b^4
```

Rubi [A]
time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5873, 2320, 12, 1642}

$$\frac{(3-4a^2)\,ae^{2\sinh^{-1}(a+bx)}}{16b^4}+\frac{(3-4a^2)\,a\sinh^{-1}(a+bx)}{8b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}-\frac{(1-6a^2)\,e^{3\sinh^{-1}(a+bx)}}{24b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}-\frac{3ae^{4\sinh^{-1}(a+bx)}}{32b^4}+\frac{e^{-3\sinh^{-1}(a+bx)}}{48b^4}+\frac{e^{5\sinh^{-1}(a+bx)}}{80b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcSinh[a + b*x])) + (3*a)/(16*b^4*E^(2*ArcSinh[a + b*x])) -
 (1 - 6*a^2)/(8*b^4*E^ArcSinh[a + b*x]) + (a*(3 - 4*a^2)*E^(2*ArcSinh[a + b
*x]))/(16*b^4) - ((1 - 6*a^2)*E^(3*ArcSinh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcSinh[a + b*x]))/(32*b^4) + E^(5*ArcSinh[a + b*x])/(80*b^4) + (a*(3 - 4*a
^2)*ArcSinh[a + b*x])/(8*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x^3\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{16b^3x^4}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)^3}{x^4}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{16b^4}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^4}-\frac{6a}{x^3}-\frac{2(-1+6a^2)}{x^2}+\frac{2a(3-4a^2)}{x}+2a(3-4a^2)\,x+2(-1+6a^2)\,x^2-6ax\right.\right.}{16b^4}$$
$$= \frac{e^{-3\sinh^{-1}(a+bx)}}{48b^4}+\frac{3ae^{-2\sinh^{-1}(a+bx)}}{16b^4}-\frac{(1-6a^2)\,e^{-\sinh^{-1}(a+bx)}}{8b^4}+\frac{a(3-4a^2)\,e^{2\sinh^{-1}(a+}}{16b^4}$$

Mathematica [**A**]
time = 0.06, size = 119, normalized size = 0.72

$$\frac{30ab^4x^4+24b^5x^5-\sqrt{1+a^2+2abx+b^2x^2}\,(16-83a^2+6a^4+a(29-6a^2)\,bx+2(-4+3a^2)\,b^2x^2-6ab^3x^3-24b^4x^4)+15a(3-4a^2)\sinh^{-1}(a+bx)}{120b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(16 - 83*a^2
 + 6*a^4 + a*(29 - 6*a^2)*b*x + 2*(-4 + 3*a^2)*b^2*x^2 - 6*a*b^3*x^3 - 24*b
^4*x^4) + 15*a*(3 - 4*a^2)*ArcSinh[a + b*x])/(120*b^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(205) = 410$.
time = 0.22, size = 464, normalized size = 2.81

method	result

default	$\frac{x^2(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{5b^2} - \frac{7a\left(\frac{x(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} \cdots\right.}{\cdots}\right.}{4b \cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-7/5*a/b*(1/4*x*(b^2*x^2+2*a*b*x+a
^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(2*
b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2
)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)
))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1
/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/(b^2)^(1/2)))-2/5*(a^2+1)/b^2*(1/3*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4
*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/(b^2)^(1/2)))+1/5*b*x^5+1/4*a*x^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(205) = 410$.
time = 0.27, size = 491, normalized size = 2.98

$$\frac{1}{5}bx^5 + \frac{1}{4}ax^4 + \frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}x^2}{5b^2} - \frac{(a^2+1)a^3 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{5b^4} - \frac{7(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}ax}{20b^3} + \frac{\sqrt{b^2x^2+2abx+a^2+1}(a^2+1)ax}{5b^3} + \frac{(a^2+1)^2 a \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{5b^4} + \frac{7(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}a^2}{12b^4} + \frac{\sqrt{b^2x^2+2abx+a^2+1}(a^2+1)a^2}{5b^4} + \frac{7(5a^2b^2-(a^2+1)b^2)a^3 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{40b^6} - \frac{2(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}(a^2+1)}{15b^4} - \frac{7(5a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2abx+a^2+1}\,ax}{40b^5} - \frac{7(5a^2b^2-(a^2+1)b^2)(a^2+1)a \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{40b^6} - \frac{7(5a^2b^2-(a^2+1)b^2)\sqrt{b^2x^2+2abx+a^2+1}\,a^2}{40b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x^2/b^2 - 1
/5*(a^2 + 1)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)
)/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*a*x/b^3 + 1/5*(a^2 + 1)^2*a*arcsinh(2*(b^2*
x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^4 + 7/12*(b^2*x^2 + 2*a*b*x
```

```
+ a^2 + 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 + 1)*
a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*a^3*arcsinh(2*(b^2*x + a*b)/sqrt
(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3
/2)*(a^2 + 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*a*arcsinh(
2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^6 - 7/40*(5*a^2*b^2 -
 (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^6
```

Fricas [A]
time = 0.34, size = 138, normalized size = 0.84

$$\frac{24\,b^5x^5+30\,ab^4x^4+15\,(4\,a^3-3\,a)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+(24\,b^4x^4+6\,ab^3x^3-2\,(3\,a^2-4)b^2x^2-6\,a^4+(6\,a^3-29\,a)bx+83\,a^2-16)\sqrt{b^2x^2+2\,abx+a^2+1}}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + 15*(4*a^3 - 3*a)*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1)) + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 - 4)*b^2*
x^2 - 6*a^4 + (6*a^3 - 29*a)*b*x + 83*a^2 - 16)*sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))/b^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A]
time = 0.40, size = 173, normalized size = 1.05

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4+\frac{1}{120}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\left(4\,x+\frac{a}{b}\right)x-\frac{3\,a^2b^5-4\,b^5}{b^7}\right)x+\frac{6\,a^3b^4-29\,ab^4}{b^7}\right)x-\frac{6\,a^4b^3-83\,a^2b^3+16\,b^3}{b^7}\right)+\frac{(4\,a^3-3\,a)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{8\,b^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/120*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*(4*x
 + a/b)*x - (3*a^2*b^5 - 4*b^5)/b^7)*x + (6*a^3*b^4 - 29*a*b^4)/b^7)*x - (6
*a^4*b^3 - 83*a^2*b^3 + 16*b^3)/b^7) + 1/8*(4*a^3 - 3*a)*log(-a*b - (x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^3*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
[Out] int(x^3*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.350 $\int e^{\sinh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}-\frac{(1-4a^2)\sinh^{-1}}{8b^3}$$

```
[Out] -1/16/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2))^2-1/2*a/b^3/(b*x+a+(1+(b*x+a)^2)^(1/2
))-1/16*(-4*a^2+1)*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^3-1/6*a*(b*x+a+(1+(b*x+a
)^2)^(1/2))^3/b^3+1/32*(b*x+a+(1+(b*x+a)^2)^(1/2))^4/b^3-1/8*(-4*a^2+1)*arc
sinh(b*x+a)/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5873, 2320, 12, 1642}

$$-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{(1-4a^2)\sinh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}-\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\sinh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] -1/16*1/(b^3*E^(2*ArcSinh[a + b*x])) - a/(2*b^3*E^ArcSinh[a + b*x]) - ((1 -
 4*a^2)*E^(2*ArcSinh[a + b*x]))/(16*b^3) - (a*E^(3*ArcSinh[a + b*x]))/(6*b^
3) + E^(4*ArcSinh[a + b*x])/(32*b^3) - ((1 - 4*a^2)*ArcSinh[a + b*x])/(8*b^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\sinh^{-1}(a+bx)}x^2\,dx &= \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2\,dx,x,\sinh^{-1}(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int\frac{(1+2ax-x^2)^2(1+x^2)}{8b^2x^3}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{b}\\ &= \frac{\text{Subst}\left(\int\frac{(1+2ax-x^2)^2(1+x^2)}{x^3}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{8b^3}\\ &= \frac{\text{Subst}\left(\int\left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1+4a^2}{x}+(-1+4a^2)\,x-4ax^2+x^3\right)\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{8b^3}\\ &= -\frac{e^{-2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\sinh^{-1}(a+bx)}}{2b^3}-\frac{(1-4a^2)\,e^{2\sinh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\sinh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\sinh^{-1}}}{32}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 102, normalized size = 0.89

$$\frac{8ab^3x^3+6b^4x^4+\sqrt{1+a^2+2abx+b^2x^2}\,(2a^3+3bx-2a^2bx+6b^3x^3+a(-13+2b^2x^2))+3(-1+2a)(1+2a)\sinh^{-1}(a+bx)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2*a^3 + 3*b*x
 - 2*a^2*b*x + 6*b^3*x^3 + a*(-13 + 2*b^2*x^2)) + 3*(-1 + 2*a)*(1 + 2*a)*Ar
cSinh[a + b*x])/(24*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(143) = 286$.
time = 0.22, size = 288, normalized size = 2.50

method	result

default	$\frac{x(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4b^2} - \frac{5a\left(\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2} - \frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2} + \frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2 \cdots}{\sqrt{\cdots}}\right.}{\cdots}\right.}{b}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-5/4*a/b*(1/3*(b^2*x^2+2*a*b*x+a^2+1
)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*
(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/(b^2)^(1/2)))-1/4*(a^2+1)/b^2*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^
2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^
2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/4*b*x^4+1/3*a*x^3
```

Maxima [A]
time = 0.26, size = 273, normalized size = 2.37

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3 + \frac{(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}x}{4\,b^2} - \frac{5\,(b^2x^2 + 2\,abx + a^2 + 1)^{\frac{3}{2}}a}{12\,b^3} - \frac{(5\,a^2b^2 - (a^2 + 1)b^2)a^2\,\operatorname{arsinh}\left(\frac{2\,(b^2x + ab)}{\sqrt{-4\,a^2b^2 + 4\,(a^2 + 1)b^2}}\right)}{8\,b^5} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,x}{8\,b^4} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)(a^2 + 1)\,\operatorname{arsinh}\left(\frac{2\,(b^2x + ab)}{\sqrt{-4\,a^2b^2 + 4\,(a^2 + 1)b^2}}\right)}{8\,b^5} + \frac{(5\,a^2b^2 - (a^2 + 1)b^2)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,a}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*x/b^2 - 5/1
2*(b^2*x^2 + 2*a*b*x + a^2 + 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 + 1)*b^
2)*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/
8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^4 + 1/8
*(5*a^2*b^2 - (a^2 + 1)*b^2)*(a^2 + 1)*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*
b^2 + 4*(a^2 + 1)*b^2))/b^5 + 1/8*(5*a^2*b^2 - (a^2 + 1)*b^2)*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1)*a/b^5
```

Fricas [A]
time = 0.34, size = 117, normalized size = 1.02

$$\frac{6\,b^4x^4 + 8\,ab^3x^3 - 3\,(4\,a^2 - 1)\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + (6\,b^3x^3 + 2\,ab^2x^2 + 2\,a^3 - (2\,a^2 - 3)bx - 13\,a)\sqrt{b^2x^2 + 2\,abx + a^2 + 1}}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 - 3*(4*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1)) + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 - 3)*b*x -
 13*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A]
time = 0.40, size = 140, normalized size = 1.22

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{1}{24}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\left(3\,x+\frac{a}{b}\right)x-\frac{2\,a^2b^3-3\,b^3}{b^5}\right)x+\frac{2\,a^3b^2-13\,ab^2}{b^5}\right)-\frac{(4\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{8\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/24*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*(3*x + a
/b)*x - (2*a^2*b^3 - 3*b^3)/b^5)*x + (2*a^3*b^2 - 13*a*b^2)/b^5) - 1/8*(4*a
^2 - 1)*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(
b^2*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x^2*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.351 $\int e^{\sinh^{-1}(a+bx)}x\,dx$

Optimal. Leaf size=67

$$\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}$$

```
[Out] 1/4/b^2/(b*x+a+(1+(b*x+a)^2)^(1/2))-1/4*a*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b^2
+1/12*(b*x+a+(1+(b*x+a)^2)^(1/2))^3/b^2-1/2*a*arcsinh(b*x+a)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5873, 2320, 12, 1642}

$$-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}+\frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcSinh[a + b*x]) - (a*E^(2*ArcSinh[a + b*x]))/(4*b^2) + E^(3*Ar
cSinh[a + b*x])/(12*b^2) - (a*ArcSinh[a + b*x])/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
```

```
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)}x\,dx = \frac{\text{Subst}\left(\int e^x\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{4bx^2}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(-1-x^2)(1+2ax-x^2)}{x^2}\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{2a}{x}-2ax+x^2\right)\,dx,x,e^{\sinh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{e^{-\sinh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\sinh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\sinh^{-1}(a+bx)}}{12b^2}-\frac{a\sinh^{-1}(a+bx)}{2b^2}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 1.09

$$\frac{1}{6}\left(3ax^2+2bx^3+\frac{\sqrt{1+a^2+2abx+b^2x^2}\,(2-a^2+abx+2b^2x^2)}{b^2}-\frac{3a\sinh^{-1}(a+bx)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 - a^2 + a*b*x +
2*b^2*x^2))/b^2 - (3*a*ArcSinh[a + b*x])/b^2)/6
```

Maple [A]
time = 0.22, size = 145, normalized size = 2.16

method	result
default	$\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3b^2}-\frac{a\left(\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{(4b^2(a^2+1)-4a^2b^2)\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a}\right.}{8b^2\sqrt{b^2}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/b^2-a/b*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2
+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2
)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+1/3*b*x^3+1/2*a*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(83) = 166.
time = 0.26, size = 175, normalized size = 2.61

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2+\frac{a^3\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,ax}{2\,b}-\frac{(a^2+1)a\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2}{2\,b^2}+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4
*(a^2 + 1)*b^2))/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*x/b - 1/2*(a
^2 + 1)*a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^2 -
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x + a
^2 + 1)^(3/2)/b^2
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.39

$$\frac{2\,b^3x^3+3\,ab^2x^2+3\,a\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+(2\,b^2x^2+abx-a^2+2)\sqrt{b^2x^2+2\,abx+a^2+1}}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + 3*a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1)) + (2*b^2*x^2 + a*b*x - a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx+\sqrt{a^2+2abx+b^2x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))*x,x)
```

```
[Out] Integral(x*(a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1)), x)
```

Giac [A]
time = 0.41, size = 106, normalized size = 1.58

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2+\frac{1}{6}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(\left(2\,x+\frac{a}{b}\right)x-\frac{a^2b-2\,b}{b^3}\right)+\frac{a\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/6*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*((2*x + a/b)*
x - (a^2*b - 2*b)/b^3) + 1/2*a*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))*abs(b))/(b*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x),x)
```

```
[Out] int(x*(a + ((a + b*x)^2 + 1)^(1/2) + b*x), x)
```

3.352 $\int e^{\sinh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{e^{2\sinh^{-1}(a+bx)}}{4b}+\frac{\sinh^{-1}(a+bx)}{2b}$$

```
[Out] 1/4*(b*x+a+(1+(b*x+a)^2)^(1/2))^2/b+1/2*arcsinh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {5872, 2320, 12, 14}

$$\frac{\sinh^{-1}(a+bx)}{2b}+\frac{e^{2\sinh^{-1}(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x],x]
```

```
[Out] E^(2*ArcSinh[a + b*x])/(4*b) + ArcSinh[a + b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5872

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\sinh^{-1}(a+bx)}\,dx &= \frac{\text{Subst}\left(\int e^x\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{\sinh^{-1}(a+bx)}\right)}{2b}\\
&= \frac{e^{2\sinh^{-1}(a+bx)}}{4b} + \frac{\sinh^{-1}(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.48

$$\frac{(a+bx)\left(a+bx+\sqrt{1+a^2+2abx+b^2x^2}\right)+\sinh^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]) + ArcSinh[a + b*x]
)/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.
time = 0.35, size = 89, normalized size = 2.87

method	result	size
default	$ax+\frac{(2b^2x+2ab)\sqrt{b^2x^2+2abx+a^2+1}}{4b^2}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{2\sqrt{b^2}}+\frac{x^2b}{2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/2*ln((b^2*x+a*b
)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)+1/2*x^2*b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(35) = 70$.

time = 0.29, size = 141, normalized size = 4.55

$$\frac{1}{2}\,bx^2+ax-\frac{a^2\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b}+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\,x+\frac{(a^2+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x - 1/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2
+ 1)*b^2))/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x + 1/2*(a^2 + 1)*arcs
inh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b + 1/2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*a/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 0.33, size = 73, normalized size = 2.35

$$\frac{b^2x^2+2\,abx+\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx+a)-\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x + a) - log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+bx+\sqrt{(a+bx)^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt((a + b*x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.
time = 0.39, size = 80, normalized size = 2.58

$$\frac{1}{2}\,bx^2+ax+\frac{1}{2}\,\sqrt{b^2x^2+2\,abx+a^2+1}\left(x+\frac{a}{b}\right)-\frac{\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x + a/b) - 1/2*log
(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int a+\sqrt{(a+b\,x)^2+1}+b\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x,x)
[Out] int(a + ((a + b*x)^2 + 1)^(1/2) + b*x, x)
```

3.353 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x} dx$

Optimal. Leaf size=89

$$bx+\sqrt{1+a^2+2abx+b^2x^2}+a\sinh^{-1}(a+bx)-\sqrt{1+a^2}\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)+a\log$$

```
[Out] b*x+a*arcsinh(b*x+a)+a*ln(x)-arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2
*a*b*x+a^2+1)^(1/2))*(a^2+1)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5878, 14, 748, 857, 633, 221, 738, 212}

$$\sqrt{a^2+2abx+b^2x^2+1}-\sqrt{a^2+1}\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)+a\sinh^{-1}(a+bx)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] - Sqrt[1 + a^2
]*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2
])] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)}}{x} dx = \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x} dx$$
$$= \int \left(b+\frac{a}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}\right) dx$$
$$= bx + a\log(x) + \int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x} dx$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - \frac{1}{2}\int \frac{-2(1+a^2)-2abx}{x\sqrt{1+a^2+2abx+b^2x^2}} dx$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - (-1-a^2)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}} dx$$

$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\log(x) - \left(2\left(1+a^2\right)\right)\text{Subst}\left(\int \frac{1}{4(1+a^2)-x^2} dx, x$$
$$= bx + \sqrt{1+a^2+2abx+b^2x^2} + a\sinh^{-1}(a+bx) - \sqrt{1+a^2}\tanh^{-1}\left(\frac{1+a}{\sqrt{1+a^2}\sqrt{1+}}\right.$$

Mathematica [A]
time = 0.05, size = 99, normalized size = 1.11

$$bx+\sqrt{1+a^2+2abx+b^2x^2}+a\sinh^{-1}(a+bx)+\left(a+\sqrt{1+a^2}\right)\log(x)-\sqrt{1+a^2}\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + a*ArcSinh[a + b*x] + (a + Sqrt[1
+ a^2])*Log[x] - Sqrt[1 + a^2]*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
 a^2 + 2*a*b*x + b^2*x^2]]
```

Maple [A]
time = 0.30, size = 126, normalized size = 1.42

method	result
default	$\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{\sqrt{b^2}}-\sqrt{a^2+1}\ \ln\left(\frac{2a^2+2+2abx+2\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b
*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1
/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)+b*x+ln(x)*a
```

Maxima [A]
time = 0.26, size = 160, normalized size = 1.80

$$bx + a\,\mathrm{arsinh}\left(\frac{2\,(b^2x + ab)}{\sqrt{-4\,a^2b^2 + 4\,(a^2+1)b^2}}\right) + a\log(x) - \sqrt{a^2+1}\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2 + 4\,(a^2+1)b^2}\,|x|} + \frac{2\,a^2}{\sqrt{-4\,a^2b^2 + 4\,(a^2+1)b^2}\,|x|} + \frac{2}{\sqrt{-4\,a^2b^2 + 4\,(a^2+1)b^2}\,|x|}\right) + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] b*x + a*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)) + a*log
(x) - sqrt(a^2 + 1)*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs
(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b
^2 + 4*(a^2 + 1)*b^2)*abs(x))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
```

Fricas [A]
time = 0.36, size = 136, normalized size = 1.53

$$bx - a\log\left(-bx - a + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right) + a\log(x) + \sqrt{a^2+1}\,\log\left(-\frac{a^2bx + a^3 + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\left(a^2 - \sqrt{a^2+1}\,a + 1\right) - (abx + a^2 + 1)\sqrt{a^2+1} + a}{x}\right) + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] b*x - a*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + a*log(x) + sqrt
(a^2 + 1)*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 - sq
rt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) + sqrt(b^2*x^2
 + 2*a*b*x + a^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x, x)
```

Giac [A]
time = 0.43, size = 158, normalized size = 1.78

$$bx - \frac{ab\log\left(-ab - \left(x|b| - \sqrt{b^2x^2 + 2\,abx + a^2 + 1}\right)|b|\right)}{|b|} + a\log(|x|) + \sqrt{a^2+1}\,\log\left(\frac{\left|-2\,x|b| + 2\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1} - 2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b| + 2\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1} + 2\,\sqrt{a^2+1}\right|}\right) + \sqrt{b^2x^2 + 2\,abx + a^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*b*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))
/abs(b) + a*log(abs(x)) + sqrt(a^2 + 1)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 + 1))) + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)
```

Mupad [B]
time = 1.04, size = 180, normalized size = 2.02

$$\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+b\,x+a\,\ln\left(x\right)-\frac{\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}-\frac{a^2\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}+\frac{a\,b\,\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x,x)
```

```
[Out] (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + b*x + a*log(x) - log(a*b + (a^2 + 1)/
x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x)/(a^2 + 1)^(1/2
) - (a^2*log(a*b + (a^2 + 1)/x + ((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))/x))/(a^2 + 1)^(1/2) + (a*b*log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1
/2) + (a*b + b^2*x)/(b^2)^(1/2)))/(b^2)^(1/2)
```

3.354 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2} dx$

Optimal. Leaf size=99

$$-\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\sinh^{-1}(a+bx)-\frac{ab\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{\sqrt{1+a^2}}+b\log(x)$$

```
[Out] -a/x+b*arcsinh(b*x+a)+b*ln(x)-a*b*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*
x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(1/2)-(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/x
```

Rubi [A]
time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5878, 14, 746, 857, 633, 221, 738, 212}

$$-\frac{\sqrt{a^2+2abx+b^2x^2+1}}{x}-\frac{ab\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{\sqrt{a^2+1}}+b\sinh^{-1}(a+bx)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]/x + b*ArcSinh[a + b*x] - (a*b*Ar
cTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])])
/Sqrt[1 + a^2] + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)}}{x^2}\,dx = \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^2}\,dx$$
$$= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\right)dx$$
$$= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^2}\,dx$$
$$= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\int \frac{2ab+2b^2x}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx$$
$$= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,dx+b^2\int$$
$$= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\log(x)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4b^2}}}\,dx, x, 2ab+2b^2x\right)$$
$$= -\frac{a}{x}-\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x}+b\sinh^{-1}(a+bx)-\frac{ab\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2a}}\right.}{\sqrt{1+a^2}}$$

Mathematica [A]
time = 0.11, size = 110, normalized size = 1.11

$$b\sinh^{-1}(a+bx)-\frac{a+\sqrt{1+a^2+2abx+b^2x^2}+\left(-1-\frac{a}{\sqrt{1+a^2}}\right)bx\log(x)+\frac{abx\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\sqrt{1+a^2}}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^2,x]
```

```
[Out] b*ArcSinh[a + b*x] - (a + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 - a/Sqrt[
1 + a^2])*b*x*Log[x] + (a*b*x*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 +
a^2 + 2*a*b*x + b^2*x^2]])/Sqrt[1 + a^2])/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(91) = 182$.
time = 0.27, size = 284, normalized size = 2.87

method	result

default	$-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{3}{2}}}{\left(a^{2}+1\right) x}+\frac{a b \left(\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}+\frac{a b \ln\left(\frac{b^{2} x+a b}{\sqrt{b^{2}}}+\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}\right)}{\sqrt{b^{2}}}-\sqrt{a^{2}+1}\right.}{a^{2}+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b
^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*
x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+
(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2))+b*ln(x)-a/x
```

Maxima [A]
time = 0.27, size = 170, normalized size = 1.72

$$-\frac{ab\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{\sqrt{a^2+1}}+b\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)+b\log(x)-\frac{a}{x}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] -a*b*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(s
qrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)))/sqrt(a^2 + 1) + b*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 +
 4*(a^2 + 1)*b^2)) + b*log(x) - a/x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(91) = 182$.
time = 0.35, size = 183, normalized size = 1.85

$$\frac{\sqrt{a^2+1}\,abx\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\,\left(a^2-\sqrt{a^2+1}\,a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-(a^2+1)bx\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)+(a^2+1)bx\log(x)-a^3-(a^2+1)bx-\sqrt{b^2x^2+2\,abx+a^2+1}\,(a^2+1)-a}{(a^2+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + 1)*a*b*x*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/x) - (
a^2 + 1)*b*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) + (a^2 + 1)*
b*x*log(x) - a^3 - (a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(a^2 +
 1) - a)/((a^2 + 1)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+\sqrt{a^2+2abx+b^2x^2+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(91) = 182.
time = 0.45, size = 234, normalized size = 2.36

$$\frac{ab\log\left(\frac{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}-2\sqrt{a^2+1}\right|}{\left|-2x|b|+2\sqrt{b^2x^2+2abx+a^2+1}+2\sqrt{a^2+1}\right|}\right)}{\sqrt{a^2+1}}-\frac{b^2\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)|b|\right)}{|b|}+b\log\left(|x|\right)-\frac{a}{x}+\frac{2\left(\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)ab^5+a^2b^4|b|+b^4|b|\right)}{\left(\left(x|b|-\sqrt{b^2x^2+2abx+a^2+1}\right)^2-a^2-1\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] a*b*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(a^2
+ 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^2 +
1)))/sqrt(a^2 + 1) - b^2*log(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^
2 + 1))*abs(b))/abs(b) + b*log(abs(x)) - a/x + 2*((x*abs(b) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 + 1))*a*b^5 + a^2*b^4*abs(b) + b^4*abs(b))/(((x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 - 1)*b^4)
```

Mupad [B]
time = 1.26, size = 269, normalized size = 2.72

$$b\ln(x)-\frac{a}{x}+\ln\left(\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}+\frac{x\,b^2+a\,b}{\sqrt{b^2}}\right)\sqrt{b^2}-\frac{\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)}+\frac{a^3\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{{\left(a^2+1\right)}^{3/2}}-\frac{a^2\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x\,\left(a^2+1\right)}+\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,x\,a+1}{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}\right)}{{\left(a^2+1\right)}^{3/2}}-\frac{2\,a\,b\,\ln\left(a\,b+\frac{a^2+1}{x}+\frac{\sqrt{a^2+1}\,\sqrt{a^2+2\,a\,b\,x+b^2\,x^2+1}}{x}\right)}{\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x + log((a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2) + (a*b + b^2*x)/(b
^2)^(1/2))*(b^2)^(1/2) - (a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2)/(x*(a^2 + 1))
+ (a^3*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x
+ 1)^(1/2))))/(a^2 + 1)^(3/2) - (a^2*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/(
x*(a^2 + 1)) + (a*b*atanh((a^2 + a*b*x + 1)/((a^2 + 1)^(1/2)*(a^2 + b^2*x^2
 + 2*a*b*x + 1)^(1/2))))/(a^2 + 1)^(3/2) - (2*a*b*log(a*b + (a^2 + 1)/x + (
(a^2 + 1)^(1/2)*(a^2 + b^2*x^2 + 2*a*b*x + 1)^(1/2))/x))/(a^2 + 1)^(1/2)
```

3.355 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=116

$$-\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2}-\frac{b^2\tanh^{-1}\left(\frac{1+a^2+abx}{\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}}\right)}{2(1+a^2)^{3/2}}$$

```
[Out] -1/2*a/x^2-b/x-1/2*b^2*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x
+a^2+1)^(1/2))/(a^2+1)^(3/2)-1/2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2
)/(a^2+1)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5878, 14, 734, 738, 212}

$$-\frac{(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{2(a^2+1)x^2}-\frac{b^2\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2(a^2+1)^{3/2}}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] -1/2*a/x^2 - b/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2
*(1 + a^2)*x^2) - (b^2*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
```

```
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^3}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^3}\,dx \\
&= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\right)dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2}+\frac{b^2\int \frac{1}{x\sqrt{1+a^2+2abx+b^2x^2}}\,d\cdots}{2(1+a^2)} \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2}-\frac{b^2\text{Subst}\left(\int \frac{1}{4(1+a^2)-x^2}\,dx,x,\frac{\cdots}{\sqrt{1\cdots}}\right)}{1+a^2} \\
&= -\frac{a}{2x^2}-\frac{b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)x^2}-\frac{b^2\tanh^{-1}\left(\frac{1+a^2+ab\cdots}{\sqrt{1+a^2}\sqrt{1+a^2+\cdots}}\right)}{2(1+a^2)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 129, normalized size = 1.11

$$
\frac{1}{2}\left(-\frac{a}{x^2}-\frac{2b}{x}-\frac{(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{(1+a^2)x^2}+\frac{b^2\log(x)}{(1+a^2)^{3/2}}-\frac{b^2\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - ((1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])
/((1 + a^2)*x^2) + (b^2*Log[x])/(1 + a^2)^(3/2) - (b^2*Log[1 + a^2 + a*b*x
+ Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(3/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(102) = 204$.
time = 0.22, size = 459, normalized size = 3.96

method	result
default	$-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{3}{2}}}{2\left(a^{2}+1\right) x^{2}}-\frac{a b\left(-\frac{\left(b^{2} x^{2}+2 a b x+a^{2}+1\right)^{\frac{3}{2}}}{\left(a^{2}+1\right) x}+\frac{a b\left(\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}+\frac{a b \ln\left(\frac{b^{2} x+a b}{\sqrt{b^{2}}}+\sqrt{b^{2} x^{2}+2 a b x}\right.}{\sqrt{b^{2}}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/
x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+
a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(
a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/
2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*
a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^
(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(
1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2))/x))-1/2*a/x^2-b/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(102) = 204$.
time = 0.27, size = 313, normalized size = 2.70

$$\frac{a^{2} b^{2} \operatorname{arsinh}\left(\frac{2 a b x}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}+\frac{2 a^{2}}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}+\frac{2}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}\right)}{2(a^{2}+1)^{\frac{3}{2}}}-\frac{b^{2} \operatorname{arsinh}\left(\frac{2 a b x}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}+\frac{2 a^{2}}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}+\frac{2}{\sqrt{-4 a^{2} b^{2}+4(a^{2}+1) b^{2}}\,|x|}\right)}{2 \sqrt{a^{2}+1}}+\frac{\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}\, b^{2}}{2(a^{2}+1)}+\frac{\sqrt{b^{2} x^{2}+2 a b x+a^{2}+1}\, a b}{2(a^{2}+1) x}-\frac{b}{x}-\frac{a}{2 x^{2}}-\frac{(b^{2} x^{2}+2 a b x+a^{2}+1)^{\frac{3}{2}}}{2(a^{2}+1) x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3/2) - 1/2*b^2*arcsinh(2*a*b*x/(sqrt(-4*a
```

```
^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^
2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/sqrt(a^2 + 1) +
 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^2/(a^2 + 1) + 1/2*sqrt(b^2*x^2 + 2
*a*b*x + a^2 + 1)*a*b/((a^2 + 1)*x) - b/x - 1/2*a/x^2 - 1/2*(b^2*x^2 + 2*a*
b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^2)
```

Fricas [**A**]
time = 0.36, size = 181, normalized size = 1.56

$$\frac{\sqrt{a^2+1}\, b^2x^2 \log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\,\left(a^2-\sqrt{a^2+1}\,a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right) - a^5 - (a^3+a)b^2x^2 - 2\,a^3 - 2\,(a^4+2\,a^2+1)bx - (a^4+(a^3+a)bx+2\,a^2+1)\sqrt{b^2x^2+2\,abx+a^2+1} - a}{2\,(a^4+2\,a^2+1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + 1)*b^2*x^2*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt(a^2 + 1) + a)/
x) - a^5 - (a^3 + a)*b^2*x^2 - 2*a^3 - 2*(a^4 + 2*a^2 + 1)*b*x - (a^4 + (a^
3 + a)*b*x + 2*a^2 + 1)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - a)/((a^4 + 2*a^
2 + 1)*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**3,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(102) = 204.
time = 0.43, size = 384, normalized size = 3.31

$$\frac{b^2 \log\left(\frac{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}-2\,\sqrt{a^2+1}\right|}{\left|-2\,x|b|+2\,\sqrt{b^2x^2+2\,abx+a^2+1}+2\,\sqrt{a^2+1}\right|}\right)}{2\,(a^2+1)^{\frac{3}{2}}} - \frac{2\,bx+a}{2\,x^2} + \frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 a^2b^2 + 2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^4b^2 + 4\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2 a^3b|b| + \left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^3 b^2 + 3\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)a^2b^2 + 4\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2 ab|b| + \left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)b^2}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)^2-a^2-1\right)^2 (a^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sqrt(
a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt(a^
2 + 1)))/(a^2 + 1)^(3/2) - 1/2*(2*b*x + a)/x^2 + (2*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^3*a^2*b^2 + 2*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
a^2 + 1))*a^4*b^2 + 4*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*
```

```
b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*b^2 + 3*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^2 + 4*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^2*a*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 + 1))*b^2)/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2 - a^2 -
1)^2*(a^2 + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3,x)
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^3, x)
```

3.356 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4} \, dx$

Optimal. Leaf size=156

$$-\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2(1+a^2)^2x^2}-\frac{(1+a^2+2abx+b^2x^2)^{3/2}}{3(1+a^2)x^3}+\frac{ab^3\tanh^{-1}\Big(\frac{}{\sqrt{1+a^2}\sqrt{}}}{2(1+}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2-1/3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^3+1/2*a*b^
3*arctanh((a*b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1
)^(5/2)+1/2*a*b*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^2/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {5878, 14, 744, 734, 738, 212}

$$\frac{ab(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{2(a^2+1)^2x^2}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{3(a^2+1)x^3}+\frac{ab^3\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\sqrt{a^2+2abx+b^2x^2+1}}\right)}{2(a^2+1)^{5/2}}-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2) + (a*b*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^
2*x^2])/(2*(1 + a^2)^2*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2)/(3*(1 + a
^2)*x^3) + (a*b^3*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sqrt[1 + a^2 + 2
*a*b*x + b^2*x^2])])/(2*(1 + a^2)^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
```

```
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\sinh^{-1}(a+bx)}}{x^4}\,dx &= \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^4}\,dx \\
&= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\right)dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}-\frac{(ab)\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^3}\,dx}{1+a^2} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}- \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+ \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{2\left(1+a^2\right)^2x^2}-\frac{\left(1+a^2+2abx+b^2x^2\right)^{3/2}}{3\left(1+a^2\right)x^3}+
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 162, normalized size = 1.04

$$
\frac{1}{6}\left(-\frac{2a}{x^3}-\frac{3b}{x^2}-\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(2+2a^4+abx+a^3bx+2b^2x^2+a^2(4-b^2x^2)\right)}{\left(1+a^2\right)^2x^3}-\frac{3ab^3\log(x)}{\left(1+a^2\right)^{5/2}}+\frac{3ab^3\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{\left(1+a^2\right)^{5/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(2 + 2*a^4 + a
*b*x + a^3*b*x + 2*b^2*x^2 + a^2*(4 - b^2*x^2)))/((1 + a^2)^2*x^3) - (3*a*b
^3*Log[x])/(1 + a^2)^(5/2) + (3*a*b^3*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*S
qrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(5/2))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(136) = 272.
time = 0.23, size = 502, normalized size = 3.22

method	result

default	$-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{3(a^2+1)x^3}-\frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2}-\frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x}+\frac{ab\left(\sqrt{b^2x^2+2abx+a^2+1}+\frac{ab\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a^2+1)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-a*b/(a^2+1)*(-1/2/(a^2+1)/x^
2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*
b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a
*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln
((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/
(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(
a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1
/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b
^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1
/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x)))
-1/3*a/x^3-1/2*b/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(136) = 272$.
time = 0.26, size = 352, normalized size = 2.26

$$-\frac{a^3b^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{2\,(a^2+1)^{\frac{5}{2}}}+\frac{ab^3\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{2\,(a^2+1)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,ab^3}{2\,(a^2+1)^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2b^2}{2\,(a^2+1)^2x}+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{2\,(a^2+1)^2x^2}-\frac{b}{2\,x^2}-\frac{a}{3\,x^3}-\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{3\,(a^2+1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*a^3*b^3*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) +
2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*
(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(5/2) + 1/2*a*b^3*arcsinh(2*a*b*x/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)
```

```
*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(3
/2) - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/(a^2 + 1)^2 - 1/2*sqrt(b^
2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^2/((a^2 + 1)^2*x) + 1/2*(b^2*x^2 + 2*a*b*x
 + a^2 + 1)^(3/2)*a*b/((a^2 + 1)^2*x^2) - 1/2*b/x^2 - 1/3*a/x^3 - 1/3*(b^2*
x^2 + 2*a*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^3)
```

Fricas [A]
time = 0.46, size = 230, normalized size = 1.47

$$\frac{3\sqrt{a^2+1}\,ab^3x^3\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2+\sqrt{a^2+1}\,a+1\right)+(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-2\,a^7+(a^4-a^2-2)b^3x^3-6\,a^5-6\,a^3-3\,(a^6+3\,a^4+3\,a^2+1)bx-(2\,a^6-(a^4-a^2-2)b^2x^2+6\,a^4+(a^5+2\,a^3+a)bx+6\,a^2+2)\sqrt{b^2x^2+2\,abx+a^2+1}-2\,a}{6\,(a^6+3\,a^4+3\,a^2+1)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + 1)*a*b^3*x^3*log(-(a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x
 + a^2 + 1)*(a^2 + sqrt(a^2 + 1)*a + 1) + (a*b*x + a^2 + 1)*sqrt(a^2 + 1) +
 a)/x) - 2*a^7 + (a^4 - a^2 - 2)*b^3*x^3 - 6*a^5 - 6*a^3 - 3*(a^6 + 3*a^4 +
 3*a^2 + 1)*b*x - (2*a^6 - (a^4 - a^2 - 2)*b^2*x^2 + 6*a^4 + (a^5 + 2*a^3 +
 a)*b*x + 6*a^2 + 2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*a)/((a^6 + 3*a^4
 + 3*a^2 + 1)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**4,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(136) = 272.
time = 0.44, size = 715, normalized size = 4.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/2*a*b^3*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) - 2*sq
rt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1) + 2*sqrt
(a^2 + 1)))/((a^4 + 2*a^2 + 1)*sqrt(a^2 + 1)) - 1/6*(3*b*x + 2*a)/x^3 + 1/3
*(20*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^5*b^3 + 12*(x*abs(b
) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^7*b^3 + 6*(x*abs(b) - sqrt(b^2*x^2
```

```
 + 2*a*b*x + a^2 + 1))^4*a^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2*x^2 + 2*a
*b*x + a^2 + 1))^2*a^6*b^2*abs(b) + 2*a^8*b^2*abs(b) + 3*(x*abs(b) - sqrt(b
^2*x^2 + 2*a*b*x + a^2 + 1))^5*a*b^3 + 32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1))^3*a^3*b^3 + 33*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))
*a^5*b^3 + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^2*b^2*abs(
b) + 48*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^4*b^2*abs(b) + 8
*a^6*b^2*abs(b) + 12*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a*b^3
 + 30*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^3*b^3 + 6*(x*abs(b)
- sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*b^2*abs(b) + 24*(x*abs(b) - sqrt(b^2
*x^2 + 2*a*b*x + a^2 + 1))^2*a^2*b^2*abs(b) + 12*a^4*b^2*abs(b) + 9*(x*abs(
b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a*b^3 + 8*a^2*b^2*abs(b) + 2*b^2*ab
s(b))/((a^4 + 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2
- a^2 - 1)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\sqrt{(a+b\,x)^2+1}+b\,x}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4,x)
```

```
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^4, x)
```

3.357 $\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5}\,dx$

Optimal. Leaf size=207

$$-\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2}-\frac{(1+a^2+2abx+b^2x^2)^{3/2}}{4\,(1+a^2)\,x^4}+\frac{5ab(1+a^2+2\cdots}{12\,(1+\cdots}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3-1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4+5/12*a*b
*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)^2/x^3+1/8*(-4*a^2+1)*b^4*arctanh((a*
b*x+a^2+1)/(a^2+1)^(1/2)/(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(a^2+1)^(7/2)+1/8*(
-4*a^2+1)*b^2*(a*b*x+a^2+1)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)/(a^2+1)^3/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$,
Rules used = {5878, 14, 758, 820, 734, 738, 212}

$$\frac{(1-4a^2)\,b^2(a^2+abx+1)\sqrt{a^2+2abx+b^2x^2+1}}{8\,(a^2+1)^3\,x^2}-\frac{(a^2+2abx+b^2x^2+1)^{3/2}}{4\,(a^2+1)\,x^4}+\frac{5ab(a^2+2abx+b^2x^2+1)^{3/2}}{12\,(a^2+1)^2\,x^3}+\frac{(1-4a^2)\,b^4\tanh^{-1}\left(\frac{a^2+abx+1}{\sqrt{a^2+1}\,\sqrt{a^2+2abx+b^2x^2+1}}\right)}{8\,(a^2+1)^{7/2}}-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] -1/4*a/x^4 - b/(3*x^3) + ((1 - 4*a^2)*b^2*(1 + a^2 + a*b*x)*Sqrt[1 + a^2 +
2*a*b*x + b^2*x^2])/(8*(1 + a^2)^3*x^2) - (1 + a^2 + 2*a*b*x + b^2*x^2)^(3/
2)/(4*(1 + a^2)*x^4) + (5*a*b*(1 + a^2 + 2*a*b*x + b^2*x^2)^(3/2))/(12*(1 +
 a^2)^2*x^3) + ((1 - 4*a^2)*b^4*ArcTanh[(1 + a^2 + a*b*x)/(Sqrt[1 + a^2]*Sq
rt[1 + a^2 + 2*a*b*x + b^2*x^2])])/(8*(1 + a^2)^(7/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
```

```
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 5878

```
Int[E^(ArcSinh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[1 + u^
2])^n, x] /; RationalQ[m] && IntegerQ[n] && PolyQ[u, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)}}{x^5}\,dx = \int \frac{a+bx+\sqrt{1+(a+bx)^2}}{x^5}\,dx$$

$$= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\right)dx$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{1+a^2+2abx+b^2x^2}}{x^5}\,dx$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{(1+a^2+2abx+b^2x^2)^{3/2}}{4\,(1+a^2)\,x^4}-\frac{\int \frac{(5ab+b^2x)\sqrt{1+a^2+2abx+b^2x^2}}{x^4}\,dx}{4\,(1+a^2)}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{(1+a^2+2abx+b^2x^2)^{3/2}}{4\,(1+a^2)\,x^4}+\frac{5ab(1+a^2+2abx+b^2x^2)^{3/2}}{12\,(1+a^2)^2\,x^3}-\frac{((1-4a^2)}{}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2}-\frac{(1+a^2+2abx+}{4\,(1+a^2)\,x}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2}-\frac{(1+a^2+2abx+}{4\,(1+a^2)\,x}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\frac{(1-4a^2)\,b^2(1+a^2+abx)\sqrt{1+a^2+2abx+b^2x^2}}{8\,(1+a^2)^3\,x^2}-\frac{(1+a^2+2abx+}{4\,(1+a^2)\,x}$$

Mathematica [**A**]
time = 0.51, size = 192, normalized size = 0.93

$$\frac{1}{24}\left(-\frac{6a}{x^4}-\frac{8b}{x^3}-\frac{\sqrt{1+a^2+2abx+b^2x^2}\left(6+\frac{2abx}{1+a^2}-\frac{(-3+2a^2)b^2x^2}{(1+a^2)^2}+\frac{a(-13+2a^2)b^3x^3}{(1+a^2)^3}\right)}{x^4}+\frac{3(-1+2a)(1+2a)b^4\log(x)}{(1+a^2)^{7/2}}-\frac{3(-1+2a)(1+2a)b^4\log\left(1+a^2+abx+\sqrt{1+a^2}\sqrt{1+a^2+2abx+b^2x^2}\right)}{(1+a^2)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]*(6 + (2*a*b*x)
/(1 + a^2) - ((-3 + 2*a^2)*b^2*x^2)/(1 + a^2)^2 + (a*(-13 + 2*a^2)*b^3*x^3)
/(1 + a^2)^3))/x^4 + (3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[x])/(1 + a^2)^(7/2) -
(3*(-1 + 2*a)*(1 + 2*a)*b^4*Log[1 + a^2 + a*b*x + Sqrt[1 + a^2]*Sqrt[1 + a^
2 + 2*a*b*x + b^2*x^2]])/(1 + a^2)^(7/2))/24
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(183) = 366.
time = 0.23, size = 1003, normalized size = 4.85

method	result
default	$-\frac{(b^2x^2+2abx+a^2+1)^{\frac{3}{2}}}{4(a^2+1)x^4} - \frac{5ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{3\left(a^2+1\right)x^3} - \frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{2\left(a^2+1\right)x^2} - \frac{ab\left(-\frac{\left(b^2x^2+2abx+a^2+1\right)^{\frac{3}{2}}}{\left(a^2+1\right)x} + \frac{ab\left(\sqrt{b^2x^2 -}\right.}{}\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)/(a^2+1)/x^4-5/4*a*b/(a^2+1)*(-1/3/(a^2+1
)/x^3*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-a*b/(a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+
2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^
(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(
1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2
*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/
4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^
2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^
(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(
```

```
b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a
^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/x))))-1/4*b^2/(
a^2+1)*(-1/2/(a^2+1)/x^2*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)-1/2*a*b/(a^2+1)*(-1/
(a^2+1)/x*(b^2*x^2+2*a*b*x+a^2+1)^(3/2)+a*b/(a^2+1)*((b^2*x^2+2*a*b*x+a^2+1
)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)
^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a*b*x+a
^2+1)^(1/2))/x))+2*b^2/(a^2+1)*(1/4*(2*b^2*x+2*a*b)/b^2*(b^2*x^2+2*a*b*x+a^
2+1)^(1/2)+1/8*(4*b^2*(a^2+1)-4*a^2*b^2)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^
2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)))+1/2*b^2/(a^2+1)*((b^2*x^2+2*a*b*x
+a^2+1)^(1/2)+a*b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))
/(b^2)^(1/2)-(a^2+1)^(1/2)*ln((2*a^2+2+2*a*b*x+2*(a^2+1)^(1/2)*(b^2*x^2+2*a
*b*x+a^2+1)^(1/2))/x)))-1/3*b/x^3-1/4*a/x^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(183) = 366$.
time = 0.26, size = 594, normalized size = 2.87

$$\frac{5\,a^4b^4\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{8\,(a^2+1)^{\frac{7}{2}}}-\frac{3\,a^2b^4\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{4\,(a^2+1)^{\frac{5}{2}}}+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}\,a^2b^4}{8\,(a^2+1)^3}+\frac{b^4\,\mathrm{arsinh}\left(\frac{2\,abx}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2\,a^2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}+\frac{2}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}\,|x|}\right)}{8\,(a^2+1)^{\frac{3}{2}}}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,b^4}{8\,(a^2+1)^2}+\frac{5\,\sqrt{b^2x^2+2\,abx+a^2+1}\,a^3b^3}{8\,(a^2+1)^3x}-\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,ab^3}{8\,(a^2+1)^2x}-\frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}a^2b^2}{8\,(a^2+1)^3x^2}+\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}b^2}{8\,(a^2+1)^2x^2}+\frac{5\,(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}ab}{12\,(a^2+1)^2x^3}-\frac{b}{3\,x^3}-\frac{a}{4\,x^4}-\frac{(b^2x^2+2\,abx+a^2+1)^{\frac{3}{2}}}{4\,(a^2+1)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 5/8*a^4*b^4*arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2
*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(
a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(7/2) - 3/4*a^2*b^4*arcsinh(2*a*b*x/(sqrt(
-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1
)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)))/(a^2 + 1)^(
5/2) + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^2*b^4/(a^2 + 1)^3 + 1/8*b^4*
arcsinh(2*a*b*x/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2*a^2/(sqrt(-
4*a^2*b^2 + 4*(a^2 + 1)*b^2)*abs(x)) + 2/(sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2
)*abs(x)))/(a^2 + 1)^(3/2) - 1/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*b^4/(a^2
 + 1)^2 + 5/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a^3*b^3/((a^2 + 1)^3*x) - 1
/8*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*a*b^3/((a^2 + 1)^2*x) - 5/8*(b^2*x^2 +
 2*a*b*x + a^2 + 1)^(3/2)*a^2*b^2/((a^2 + 1)^3*x^2) + 1/8*(b^2*x^2 + 2*a*b*
x + a^2 + 1)^(3/2)*b^2/((a^2 + 1)^2*x^2) + 5/12*(b^2*x^2 + 2*a*b*x + a^2 +
1)^(3/2)*a*b/((a^2 + 1)^2*x^3) - 1/3*b/x^3 - 1/4*a/x^4 - 1/4*(b^2*x^2 + 2*a
*b*x + a^2 + 1)^(3/2)/((a^2 + 1)*x^4)
```

Fricas [A]
time = 0.36, size = 295, normalized size = 1.43

$$\frac{3\,(4\,a^2-1)\sqrt{a^2+1}\,b^4x^4\log\left(-\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2+1}\left(a^2-\sqrt{a^2+1}\,a+1\right)-(abx+a^2+1)\sqrt{a^2+1}+a}{x}\right)-6\,a^9-(2\,a^5-11\,a^3-13\,a)b^4x^4-24\,a^7-36\,a^5-24\,a^3-8\,(a^8+4\,a^6+6\,a^4+4\,a^2+1)bx-(6\,a^8+(2\,a^5-11\,a^3-13\,a)b^3x^3+24\,a^6-(2\,a^6+a^4-4\,a^2-3)b^2x^2+36\,a^4+2\,(a^7+3\,a^5+3\,a^3+a)bx+24\,a^2+6)\sqrt{b^2x^2+2\,abx+a^2+1}-6\,a}{24\,(a^8+4\,a^6+6\,a^4+4\,a^2+1)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 - 1)*sqrt(a^2 + 1)*b^4*x^4*log(-(a^2*b*x + a^3 + sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1)*(a^2 - sqrt(a^2 + 1)*a + 1) - (a*b*x + a^2 + 1)*sqrt
(a^2 + 1) + a)/x) - 6*a^9 - (2*a^5 - 11*a^3 - 13*a)*b^4*x^4 - 24*a^7 - 36*a
^5 - 24*a^3 - 8*(a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 - 1
1*a^3 - 13*a)*b^3*x^3 + 24*a^6 - (2*a^6 + a^4 - 4*a^2 - 3)*b^2*x^2 + 36*a^4
 + 2*(a^7 + 3*a^5 + 3*a^3 + a)*b*x + 24*a^2 + 6)*sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1) - 6*a)/((a^8 + 4*a^6 + 6*a^4 + 4*a^2 + 1)*x^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a^2 + 2abx + b^2x^2 + 1}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)**2)**(1/2))/x**5,x)
```

```
[Out] Integral((a + b*x + sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1))/x**5, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. 2(183) = 366.
time = 0.45, size = 1173, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(1+(b*x+a)^2)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2*b^4 - b^4)*log(abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1) - 2*sqrt(a^2 + 1))/abs(-2*x*abs(b) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 +
1) + 2*sqrt(a^2 + 1)))/((a^6 + 3*a^4 + 3*a^2 + 1)*sqrt(a^2 + 1)) - 1/12*(4*
b*x + 3*a)/x^4 + 1/12*(32*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*
a^6*b^4 + 256*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^8*b^4 + 96
*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^10*b^4 + 144*(x*abs(b) -
sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^7*b^3*abs(b) + 224*(x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^9*b^3*abs(b) + 16*a^11*b^3*abs(b) - 12*(x
*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^7*a^2*b^4 + 140*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^4*b^4 + 716*(x*abs(b) - sqrt(b^2*x^2 +
 2*a*b*x + a^2 + 1))^3*a^6*b^4 + 372*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a
^2 + 1))*a^8*b^4 + 432*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a^5
*b^3*abs(b) + 704*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^7*b^3*
abs(b) + 80*a^9*b^3*abs(b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1
))^7*b^4 + 129*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*a^2*b^4 + 6
85*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^3*a^4*b^4 + 543*(x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^6*b^4 + 432*(x*abs(b) - sqrt(b^2*x^
2 + 2*a*b*x + a^2 + 1))^4*a^3*b^3*abs(b) + 768*(x*abs(b) - sqrt(b^2*x^2 + 2
```

```
*a*b*x + a^2 + 1))^2*a^5*b^3*abs(b) + 160*a^7*b^3*abs(b) + 21*(x*abs(b) - s
qrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^5*b^4 + 246*(x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 + 1))^3*a^2*b^4 + 357*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2
+ 1))*a^4*b^4 + 144*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^4*a*b^3*
abs(b) + 320*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a^3*b^3*abs(b
) + 160*a^5*b^3*abs(b) + 21*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^
3*b^4 + 93*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*a^2*b^4 + 32*(x*a
bs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))^2*a*b^3*abs(b) + 80*a^3*b^3*abs(
b) + 3*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*b^4 + 16*a*b^3*abs(b)
)/((a^6 + 3*a^4 + 3*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)
)^2 - a^2 - 1)^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+\sqrt{{\left(a+b\,x\right)}^2+1}+b\,x}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5,x)
[Out] int((a + ((a + b*x)^2 + 1)^(1/2) + b*x)/x^5, x)
```

3.358 $\int e^{\sinh^{-1}(a+bx)^2} x^3 \, dx$

Optimal. Leaf size=359

$$-\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{16b^4e}+\frac{3a^2\sqrt{\pi}\,\mathrm{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^4e}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(2-\sinh^{-1}(a+bx)\right)}{32b^4e^4}-\frac{\sqrt{\pi}\,\mathrm{Er}}{}$$

```
[Out] -1/32*erfi(-2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)+1/16*erfi(-1+arcsinh(b*x+
a))*Pi^(1/2)/b^4/exp(1)-3/8*a^2*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1)
-1/16*erfi(1+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1)+3/8*a^2*erfi(1+arcsinh(b*x
+a))*Pi^(1/2)/b^4/exp(1)+1/32*erfi(2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(4)-3/
16*a*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+arcsi
nh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)
/b^4/exp(1/4)+3/16*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3
*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arcsinh(b*x
+a))*Pi^(1/2)/b^4/exp(9/4)
```

Rubi [A]
time = 0.55, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$-\frac{\sqrt{\pi}a^3\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}b^4}-\frac{\sqrt{\pi}a^3\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a^2\mathrm{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^4}+\frac{3\sqrt{\pi}a^2\mathrm{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^4}-\frac{3\sqrt{\pi}a\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-3\right)\right)}{16e^{9/4}b^4}+\frac{3\sqrt{\pi}a\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{16\sqrt[4]{e}b^4}+\frac{3\sqrt{\pi}a\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{16\sqrt[4]{e}b^4}-\frac{3\sqrt{\pi}a\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+3\right)\right)}{16e^{9/4}b^4}-\frac{\sqrt{\pi}\mathrm{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{16eb^4}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(2-\sinh^{-1}(a+bx)\right)}{32e^4b^4}-\frac{\sqrt{\pi}\mathrm{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{16eb^4}+\frac{\sqrt{\pi}\mathrm{Erfi}\left(\sinh^{-1}(a+bx)+2\right)}{32e^4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] -1/16*(Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[
1 - ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcSinh[a + b*x]])/(3
2*b^4*E^4) - (Sqrt[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt
[Pi]*Erfi[1 + ArcSinh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcSinh[a +
 b*x]])/(32*b^4*E^4) - (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2])/(16
*b^4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^
(1/4)) - (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) +
 (3*a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sq
rt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Er
fi[(3 + 2*ArcSinh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int e^{\sinh^{-1}(a+bx)^2}x^3\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^3\,dx,x,\sinh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))^3}{b^3}\,dx,x,\sinh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))^3\,dx,x,\sinh^{-1}(a+bx)\right)}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-a^3e^{x^2}\cosh(x)+3a^2e^{x^2}\cosh(x)\sinh(x)-3ae^{x^2}\cosh(x)\sinh^2(x)+e^{x^2}\text{c}\right.\right.}{b^4} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^3(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\text{Subst}\left(\int e^{x^2}\cosh(x)\text{sinh}\right.}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2}+\frac{1}{8}e^{-2x+x^2}-\frac{1}{8}e^{2x+x^2}+\frac{1}{16}e^{4x+x^2}\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^4} - \\
&= -\frac{\text{Subst}\left(\int e^{-4x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Subst}\left(\int e^{4x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{16b^4} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}\,dx,x,\sinh^{-1}(a+\text{b}\right.}{16b^4e^4} \\
&= -\frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{16b^4e} + \frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^4e} + \frac{\sqrt{\pi}\,\text{erfi}\left(2-\text{s}\right.}{32\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 198, normalized size = 0.55

$$
\frac{\sqrt{\pi}\left(2a(-3+4a^2)e^{15/4}\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+2(-1+6a^2)e^3\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)+6ae^{7/4}\text{Erfi}\left(\frac{3}{2}-\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(2-\sinh^{-1}(a+bx)\right)+6ae^{15/4}\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)-8a^3e^{15/4}\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)-2e^3\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)+12a^2e^3\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)-6ae^{7/4}\text{Erfi}\left(\frac{3}{2}+\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(2+\sinh^{-1}(a+bx)\right)\right)}{32b^4e^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(2*a*(-3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcSinh[a + b*x]] + 2*(-1 +
 6*a^2)*E^3*Erfi[1 - ArcSinh[a + b*x]] + 6*a*E^(7/4)*Erfi[3/2 - ArcSinh[a +
 b*x]] + Erfi[2 - ArcSinh[a + b*x]] + 6*a*E^(15/4)*Erfi[1/2 + ArcSinh[a + b
*x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcSinh[a + b*x]] - 2*E^3*Erfi[1 + ArcSinh
[a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcSinh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
 ArcSinh[a + b*x]] + Erfi[2 + ArcSinh[a + b*x]]))/(32*b^4*E^4)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$
\int e^{\operatorname{arcsinh}(bx+a)^2}x^3\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(asinh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arcsinh(b*x + a)^2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \, e^{\operatorname{asinh}(a+b\,x)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(asinh(a + b*x)^2),x)
[Out] int(x^3*exp(asinh(a + b*x)^2), x)
```

3.359 $\int e^{\sinh^{-1}(a+bx)^2} x^2 \, dx$

Optimal. Leaf size=251

$$-\frac{a\sqrt{\pi}\,\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4b^3e}-\frac{a\sqrt{\pi}\,\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)}{4b^3e}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(-3+2\sinh^{-1}(a+bx)\right)\right)}{16b^3e^{9/4}}-\sqrt{}$$

```
[Out] 1/4*a*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1)-1/4*a*erfi(1+arcsinh(b*x+
a))*Pi^(1/2)/b^3/exp(1)+1/16*erfi(-3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4
)-1/16*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/4*a^2*erfi(-1/2+ar
csinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)-1/16*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/
b^3/exp(1/4)+1/4*a^2*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*er
fi(3/2+arcsinh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)
```

Rubi [A]

time = 0.42, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$\frac{\sqrt{\pi}\,a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,a^2\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4eb^3}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{4eb^3}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-3\right)\right)}{16e^{9/4}b^3}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{16\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{16\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+3\right)\right)}{16e^{9/4}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -1/4*(a*Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(b^3*E) - (a*Sqrt[Pi]*Erfi[1 +
 ArcSinh[a + b*x]])/(4*b^3*E) + (Sqrt[Pi]*Erfi[(-3 + 2*ArcSinh[a + b*x])/2]
)/(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E
^(1/4)) + (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4))
- (Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[
Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 +
 2*ArcSinh[a + b*x])/2])/(16*b^3*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)^2\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(a-\sinh(x))^2}{b^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(a-\sinh(x))^2\,dx,x,\sinh^{-1}(a+bx)\right)}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(a^2e^{x^2}\cosh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh(x)\sinh^2(x)\right)\,dx,x,\sinh^{-1}\right.}{b^3}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh^2(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(\right.}{b^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}-\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^3} - \frac{(2a)\text{Subs}}{}$$

$$= \frac{\text{Subst}\left(\int e^{-3x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{8b^3} - \frac{\text{S}}{}$$

$$= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{8b^3e^{9/4}}$$

$$= -\frac{a\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{4b^3e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{4b^3e} + \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-3+\right.}{16}$$

Mathematica [A]
time = 0.12, size = 138, normalized size = 0.55

$$-\frac{\sqrt{\pi}\left((-1+4a^2)e^2\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+4ae^{5/4}\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(\frac{3}{2}-\sinh^{-1}(a+bx)\right)+e^2\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)-4a^2e^2\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)+4ae^{5/4}\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)-\text{Erfi}\left(\frac{3}{2}+\sinh^{-1}(a+bx)\right)\right)}{16b^3e^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x^2,x]
```

```
[Out] -1/16*(Sqrt[Pi]*((-1 + 4*a^2)*E^2*Erfi[1/2 - ArcSinh[a + b*x]] + 4*a*E^(5/4
)*Erfi[1 - ArcSinh[a + b*x]] + Erfi[3/2 - ArcSinh[a + b*x]] + E^2*Erfi[1/2
+ ArcSinh[a + b*x]] - 4*a^2*E^2*Erfi[1/2 + ArcSinh[a + b*x]] + 4*a*E^(5/4)*
Erfi[1 + ArcSinh[a + b*x]] - Erfi[3/2 + ArcSinh[a + b*x]]))/(b^3*E^(9/4))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arcsinh}(bx+a)^2}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x^2,x)
[Out] int(exp(arcsinh(b*x+a)^2)*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="maxima")
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="fricas")
[Out] integral(x^2*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\operatorname{asinh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x**2,x)
[Out] Integral(x**2*exp(asinh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x^2,x, algorithm="giac")
[Out] integrate(x^2*e^(arcsinh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{e}^{\mathrm{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(asinh(a + b*x)^2),x)
```

```
[Out] int(x^2*exp(asinh(a + b*x)^2), x)
```

3.360 $\int e^{\sinh^{-1}(a+bx)^2} x \, dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^2e}+\frac{\sqrt{\pi}\,\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)}{8b^2e}-\frac{a\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)}{4b^2\sqrt[4]{e}}-\frac{a\sqrt{}}{}$$

```
[Out] -1/8*erfi(-1+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1)+1/8*erfi(1+arcsinh(b*x+a))
*Pi^(1/2)/b^2/exp(1)-1/4*a*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)-
1/4*a*erfi(1/2+arcsinh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

Rubi [A]

time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5873, 6873, 12, 6874, 5624, 2266, 2235, 5625}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\sinh^{-1}(a+bx)+1\right)}{8eb^2}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcSinh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcSin
h[a + b*x]])/(8*b^2*E) - (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcSinh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5625

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5873

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-a/b + Sinh[x]/b)^m*f^(c*x^n)*Cosh[x], x], x, ArcSinh
[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\sinh^{-1}(a+bx)^2}x\,dx = \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\left(-\frac{a}{b}+\frac{\sinh(x)}{b}\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}\cosh(x)(-a+\sinh(x))}{b}\,dx,x,\sinh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)(-a+\sinh(x))\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\cosh(x)+e^{x^2}\cosh(x)\sinh(x)\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx,x,\sin\right.}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)\,dx,x,\sinh^{-1}(a+bx)\right)}{b^2} - \frac{a\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x}}{}\right.\right.}{}$$

$$= -\frac{\text{Subst}\left(\int e^{-2x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{4b^2} -$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx,x,\sinh^{-1}(a+bx)\right)}{4b^2e} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx,x,\sinh^{-1}(a+bx\right.}{4b^2e}$$

$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\sinh^{-1}(a+bx)\right)}{8b^2e} + \frac{\sqrt{\pi}\,\text{erfi}\left(1+\sinh^{-1}(a+bx)\right)}{8b^2e} - \frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-1+2\right.}{4b^2\sqrt[4]{}}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(2ae^{3/4}\text{Erfi}\left(\frac{1}{2}-\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(1-\sinh^{-1}(a+bx)\right)-2ae^{3/4}\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(1+\sinh^{-1}(a+bx)\right)\right)}{8b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(2*a*E^(3/4)*Erfi[1/2 - ArcSinh[a + b*x]] + Erfi[1 - ArcSinh[a +
b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[1 + ArcSinh[a + b*x
]]))/(8*b^2*E)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int e^{\text{arcsinh}(bx+a)^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)*x,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)*x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="maxima")
```

```
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="fricas")
```

```
[Out] integral(x*e^(arcsinh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\operatorname{asinh}^{2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)*x,x)
```

```
[Out] Integral(x*exp(asinh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)*x,x, algorithm="giac")
```

```
[Out] integrate(x*e^(arcsinh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{e}^{\mathrm{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(asinh(a + b*x)^2),x)
```

```
[Out] int(x*exp(asinh(a + b*x)^2), x)
```

3.361 $\int e^{\sinh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}$$

```
[Out] 1/4*erfi(-1/2+arcsinh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arcsinh(b*x+
a))*Pi^(1/2)/b/exp(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5872, 5624, 2266, 2235}

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{1}{2}\left(2\sinh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[(-1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erfi
[(1 + 2*ArcSinh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5624

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5872

```
Int[(f_)^(ArcSinh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Cosh[x], x], x, ArcSinh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\sinh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)\,dx, x, \sinh^{-1}(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \sinh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}\\
&= \frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(1+2\sinh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.68

$$\frac{\sqrt{\pi}\left(\text{Erfi}\left(\frac{1}{2}+\sinh^{-1}(a+bx)\right)+\text{Erfi}\left(\frac{1}{2}\left(-1+2\sinh^{-1}(a+bx)\right)\right)\right)}{4b\sqrt[4]{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcSinh[a + b*x]^2,x]
```

```
[Out] (Sqrt[Pi]*(Erfi[1/2 + ArcSinh[a + b*x]] + Erfi[(-1 + 2*ArcSinh[a + b*x])/2]
))/(4*b*E^(1/4))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arcsinh}(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2),x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{asinh}^{2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2),x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2),x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2),x)
```

```
[Out] int(exp(asinh(a + b*x)^2), x)
```

3.362 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x},x\right)$$

```
[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[E^ArcSinh[a + b*x]^2/x,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx = \int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x,x]
```

```
[Out] Integrate[E^ArcSinh[a + b*x]^2/x, x]
```

Maple [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsinh}(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asinh}^{2}{\left(a + bx \right)}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x,x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\operatorname{asinh}(a+b\,x)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2)/x,x)
```

```
[Out] int(exp(asinh(a + b*x)^2)/x, x)
```

3.363 $\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\sinh^{-1}(a+bx)^2}}{x^2}, x\right)$$

```
[Out] CannotIntegrate(exp(arcsinh(b*x+a)^2)/x^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Int[E^ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] Defer[Int][E^ArcSinh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx = \int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Mathematica [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{e^{\sinh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcSinh[a + b*x]^2/x^2,x]
```

```
[Out] Integrate[E^ArcSinh[a + b*x]^2/x^2, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{arcsinh}(bx+a)^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

```
[Out] int(exp(arcsinh(b*x+a)^2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(e^(arcsinh(b*x + a)^2)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{asinh}^{2}(a+bx)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asinh(b*x+a)**2)/x**2,x)
```

```
[Out] Integral(exp(asinh(a + b*x)**2)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsinh(b*x+a)^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(e^(arcsinh(b*x + a)^2)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{e^{\operatorname{asinh}(a+b\,x)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(asinh(a + b*x)^2)/x^2,x)
```

```
[Out] int(exp(asinh(a + b*x)^2)/x^2, x)
```

3.364 $\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx$

Optimal. Leaf size=60

$$-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}+\frac{\operatorname{PolyLog}\left(2,e^{2\sinh^{-1}(a+bx)}\right)}{2d}$$

```
[Out] -1/2*arcsinh(b*x+a)^2/d+arcsinh(b*x+a)*ln(1-(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/
d+1/2*polylog(2,(b*x+a+(1+(b*x+a)^2)^(1/2))^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5859, 12, 5775, 3797, 2221, 2317, 2438}

$$\frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)}{2d}-\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -1/2*ArcSinh[a + b*x]^2/d + (ArcSinh[a + b*x]*Log[1 - E^(2*ArcSinh[a + b*x]
)])/d + PolyLog[2, E^(2*ArcSinh[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5859

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx &= \frac{\text{Subst}\left(\int \frac{b\sinh^{-1}(x)}{dx}\,dx,x,a+bx\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh^{-1}(x)}{x}\,dx,x,a+bx\right)}{d}\\
&= \frac{\text{Subst}\left(\int x\coth(x)\,dx,x,\sinh^{-1}(a+bx)\right)}{d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d}-\frac{2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx,x,\sinh^{-1}(a+bx)\right)}{d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}-\frac{\text{Subst}\left(\int \log\left(1-e^{2x}\right)\right.}{}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}-\frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x\right.}{2d}\\
&= -\frac{\sinh^{-1}(a+bx)^2}{2d}+\frac{\sinh^{-1}(a+bx)\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)}{d}+\frac{\text{Li}_2\left(e^{2\sinh^{-1}(a+bx)}\right)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.87

$$\frac{-\sinh^{-1}(a+bx)\left(\sinh^{-1}(a+bx)-2\log\left(1-e^{2\sinh^{-1}(a+bx)}\right)\right)+\text{PolyLog}\left(2,e^{2\sinh^{-1}(a+bx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (-(ArcSinh[a + b*x]*(ArcSinh[a + b*x] - 2*Log[1 - E^(2*ArcSinh[a + b*x])]))
 + PolyLog[2, E^(2*ArcSinh[a + b*x])])/(2*d)
```

Maple [A]
time = 2.26, size = 134, normalized size = 2.23

method	result
derivativedivides	$\frac{-\frac{b\,\text{arcsinh}(bx+a)^2}{2d}+\frac{b\,\text{arcsinh}(bx+a)\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b\,\text{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+}{b}$
default	$\frac{-\frac{b\,\text{arcsinh}(bx+a)^2}{2d}+\frac{b\,\text{arcsinh}(bx+a)\ln\left(1+bx+a+\sqrt{1+(bx+a)^2}\right)}{d}+\frac{b\,\text{polylog}\left(2,-bx-a-\sqrt{1+(bx+a)^2}\right)}{d}+}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*b/d*arcsinh(b*x+a)^2+b/d*arcsinh(b*x+a)*ln(1+b*x+a+(1+(b*x+a)^2)^
(1/2))+b/d*polylog(2,-b*x-a-(1+(b*x+a)^2)^(1/2))+b/d*arcsinh(b*x+a)*ln(1-b*
x-a-(1+(b*x+a)^2)^(1/2))+b/d*polylog(2,b*x+a+(1+(b*x+a)^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arcsinh(b*x + a)/(b*d*x + a*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\operatorname{asinh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(asinh(a + b*x)/(a + b*x), x)/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(b*x + a)/(d*x + a*d/b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asinh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(asinh(a + b*x)/(d*x + (a*d)/b), x)
```

3.365 $\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\mathrm{Shi}\left(\sinh^{-1}(x)\right)$$

```
[Out] Shi(arcsinh(x))
```

Rubi [A]
time = 0.04, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5819, 3379}

$$\mathrm{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
```

```
[Out] SinhIntegral[ArcSinh[x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx &= \mathrm{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \sinh^{-1}(x)\right)\\ &= \mathrm{Shi}\left(\sinh^{-1}(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 3, normalized size = 1.00

$$\mathrm{Shi}\left(\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*ArcSinh[x]),x]
[Out] SinhIntegral[ArcSinh[x]]
```

Maple [A]
time = 2.08, size = 4, normalized size = 1.33

method	result	size
default	hyperbolicSineIntegral (arcsinh (x))	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(x)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] Shi(arcsinh(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2+1}\,\operatorname{asinh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(x)/(x**2+1)**(1/2),x)
[Out] Integral(x/(sqrt(x**2 + 1)*asinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(x)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^2 + 1)*arcsinh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.33

$$\int \frac{x}{\operatorname{asinh}(x)\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(asinh(x)*(x^2 + 1)^(1/2)),x)
[Out] int(x/(asinh(x)*(x^2 + 1)^(1/2)), x)
```

3.366 $\int x^3 \sinh^{-1}(a+bx^4)\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{1+(a+bx^4)^2}}{4b}+\frac{(a+bx^4)\sinh^{-1}(a+bx^4)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arcsinh(b*x^4+a)/b-1/4*(1+(b*x^4+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6847, 5858, 5772, 267}

$$\frac{(a+bx^4)\sinh^{-1}(a+bx^4)}{4b}-\frac{\sqrt{(a+bx^4)^2+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSinh[a + b*x^4],x]
```

```
[Out] -1/4*Sqrt[1 + (a + b*x^4)^2]/b + ((a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int x^3 \sinh^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \sinh^{-1}(a+bx)\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx, x, a+bx^4\right)}{4b}\\ &= \frac{(a+bx^4)\sinh^{-1}\left(a+bx^4\right)}{4b} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx, x, a+bx^4\right)}{4b}\\ &= -\frac{\sqrt{1+\left(a+bx^4\right)^2}}{4b} + \frac{(a+bx^4)\sinh^{-1}\left(a+bx^4\right)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.91

$$\frac{-\sqrt{1+\left(a+bx^4\right)^2}+(a+bx^4)\sinh^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSinh[a + b*x^4],x]
[Out] (-Sqrt[1 + (a + b*x^4)^2] + (a + b*x^4)*ArcSinh[a + b*x^4])/(4*b)
```

Maple [A]
time = 0.29, size = 38, normalized size = 0.84

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\,\mathrm{arcsinh}(b\,x^4+a)-\sqrt{1+\left(b\,x^4+a\right)^2}}{4b}$	38
default	$\frac{(b\,x^4+a)\,\mathrm{arcsinh}(b\,x^4+a)-\sqrt{1+\left(b\,x^4+a\right)^2}}{4b}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsinh(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*((b*x^4+a)*arcsinh(b*x^4+a)-(1+(b*x^4+a)^2)^(1/2))
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.82

$$\frac{(bx^4+a)\,\mathrm{arsinh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arcsinh(b*x^4 + a) - sqrt((b*x^4 + a)^2 + 1))/b
```

Fricas [A]
time = 0.35, size = 66, normalized size = 1.47

$$\frac{(bx^4+a)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2+1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))/b
```

Sympy [A]
time = 0.20, size = 61, normalized size = 1.36

$$\begin{cases} \frac{a\,\mathrm{asinh}\,(a+bx^4)}{4b}+\frac{x^4\,\mathrm{asinh}\,(a+bx^4)}{4}-\frac{\sqrt{a^2+2abx^4+b^2x^8+1}}{4b} & \text{for } b\neq 0\\ \frac{x^4\,\mathrm{asinh}\,(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asinh(b*x**4+a),x)
```

```
[Out] Piecewise(((a*asinh(a + b*x**4)/(4*b) + x**4*asinh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*asinh(a)/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(39) = 78.
time = 0.41, size = 105, normalized size = 2.33

$$\frac{1}{4}\,x^4\log\left(bx^4+a+\sqrt{(bx^4+a)^2+1}\right)-\frac{1}{4}\,b\left(\frac{a\log\left(-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,abx^4+a^2+1}}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsinh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 + 1)) - 1/4*b*(a*log(-a*b - (x^4
*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))*abs(b))/(b*abs(b)) + sqrt(b^
2*x^8 + 2*a*b*x^4 + a^2 + 1)/b^2)
```

Mupad [B]
time = 0.57, size = 88, normalized size = 1.96

$$\frac{x^4\,\mathrm{asinh}\left(b\,x^4+a\right)}{4}-\frac{\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}}{4\,b}+\frac{a\,\ln\left(\sqrt{a^2+2\,a\,b\,x^4+b^2\,x^8+1}+\frac{b^2\,x^4+a\,b}{\sqrt{b^2}}\right)}{4\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*asinh(a + b*x^4),x)
```

```
[Out] (x^4*asinh(a + b*x^4))/4 - (a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2)/(4*b) + (a
*log((a^2 + b^2*x^8 + 2*a*b*x^4 + 1)^(1/2) + (a*b + b^2*x^4)/(b^2)^(1/2)))/
(4*(b^2)^(1/2))
```

3.367 $\int x^{-1+n} \sinh^{-1}(a + bx^n) \, dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{1+(a+bx^n)^2}}{bn} + \frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arcsinh(a+b*x^n)/b/n-(1+(a+b*x^n)^2)^(1/2)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6847, 5858, 5772, 267}

$$\frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn} - \frac{\sqrt{(a+bx^n)^2+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcSinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[1 + (a + b*x^n)^2]/(b*n)) + ((a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5858

```
Int[((a_.) + ArcSinh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcSinh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+n}\sinh^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \sinh^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \sinh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{1+x^2}}\,dx,x,a+bx^n\right)}{bn}\\
&= -\frac{\sqrt{1+(a+bx^n)^2}}{bn}+\frac{(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.89

$$\frac{-\sqrt{1+(a+bx^n)^2}+(a+bx^n)\sinh^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcSinh[a + b*x^n],x]
[Out] (-Sqrt[1 + (a + b*x^n)^2] + (a + b*x^n)*ArcSinh[a + b*x^n])/(b*n)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int x^{-1+n}\operatorname{arcsinh}(a+b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arcsinh(a+b*x^n),x)
[Out] int(x^(-1+n)*arcsinh(a+b*x^n),x)
```

Maxima [A]
time = 0.26, size = 39, normalized size = 0.85

$$\frac{(bx^n+a)\operatorname{arsinh}(bx^n+a)-\sqrt{(bx^n+a)^2+1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arcsinh(b*x^n + a) - sqrt((b*x^n + a)^2 + 1))/(b*n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(44) = 88$.
time = 0.37, size = 152, normalized size = 3.30

$$\frac{\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a\right)\log\left(b\cosh\left(n\log\left(x\right)\right)+b\sinh\left(n\log\left(x\right)\right)+a+\sqrt{\frac{2\,ab+\left(a^{2}+b^{2}+1\right)\cosh\left(n\log\left(x\right)\right)-\left(a^{2}-b^{2}+1\right)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right)-\sinh\left(n\log\left(x\right)\right)}}\right)-\sqrt{\frac{2\,ab+\left(a^{2}+b^{2}+1\right)\cosh\left(n\log\left(x\right)\right)-\left(a^{2}-b^{2}+1\right)\sinh\left(n\log\left(x\right)\right)}{\cosh\left(n\log\left(x\right)\right)-\sinh\left(n\log\left(x\right)\right)}}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 + 1)*cosh(n*log(x)) - (a^2 - b^2 + 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 21.20, size = 76, normalized size = 1.65

$$\begin{cases} \log\left(x\right)\operatorname{asinh}\left(a\right) & \text{for}\: b = 0 \wedge n = 0 \\ \log\left(x\right)\operatorname{asinh}\left(a+b\right) & \text{for}\: n = 0 \\ \frac{x^{n}\operatorname{asinh}\left(a\right)}{n} & \text{for}\: b = 0 \\ \frac{a\operatorname{asinh}\left(a+bx^{n}\right)}{bn} + \frac{x^{n}\operatorname{asinh}\left(a+bx^{n}\right)}{n} - \frac{\sqrt{a^{2}+2abx^{n}+b^{2}x^{2n}+1}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*asinh(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*asinh(a), Eq(b, 0) & Eq(n, 0)), (log(x)*asinh(a + b), Eq(
n, 0)), (x**n*asinh(a)/n, Eq(b, 0)), (a*asinh(a + b*x**n)/(b*n) + x**n*asin
h(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) + 1)/(b*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.
time = 0.44, size = 113, normalized size = 2.46

$$-\frac{b\left(\frac{a\log\left(-ab-\left(x^{n}|b|-\sqrt{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}+1}\right)|b|\right)}{b|b|}+\frac{\sqrt{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}+1}}{b^{2}}\right)-x^{n}\log\left(bx^{n}+a+\sqrt{\left(bx^{n}+a\right)^{2}+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arcsinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1))*ab
s(b))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 + 1)/b^2) - x^n*log(b
*x^n + a + sqrt((b*x^n + a)^2 + 1)))/n
```

Mupad [B]
time = 0.36, size = 99, normalized size = 2.15

$$\frac{x^n \operatorname{asinh}(a + b\,x^n)}{n} - \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n + 1}}{b\,n} + \frac{a\,\ln\left(\frac{a\,b + b^2\,x^n}{\sqrt{b^2}} + \sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n + 1}\right)}{n\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*asinh(a + b*x^n),x)
```

```
[Out] (x^n*asinh(a + b*x^n))/n - (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^(1/2)/(b*n)
+ (a*log((a*b + b^2*x^n)/(b^2)^(1/2) + (a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1)^
(1/2)))/(n*(b^2)^(1/2))
```

3.368 $\int \sinh^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=49

$$\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\tanh^{-1}\left(\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}\right)}{b}$$

```
[Out] (b*x+a)*arccsch(a/c+b*x/c)/b+c*arctanh((1+1/(a/c+b*x/c)^2)^(1/2))/b
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5870, 6449, 379, 272, 65, 213}

$$\frac{c\tanh^{-1}\left(\sqrt{\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}+1}\right)}{b}+\frac{(a+bx)\text{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcCsch[a/c + (b*x)/c])/b + (c*ArcTanh[Sqrt[1 + (a/c + (b*x)/c)^
(-2)]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 379

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Dist[u^m/(Coeff
icient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{
a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Rule 5870

```
Int[ArcSinh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcCsch[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6449

```
Int[ArcCsch[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcCsch[c + d*
x]/d), x] + Int[1/((c + d*x)*Sqrt[1 + 1/(c + d*x)^2]), x] /; FreeQ[{c, d},
x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^{-1}\left(\frac{c}{a+bx}\right) dx &= \int \operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right) dx \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1+\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}} dx \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x} dx, x, \frac{a}{c}+\frac{bx}{c}\right)}{b} \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}\right)}{2b} \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b} \\
&= \frac{(a+bx)\operatorname{csch}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}+\frac{c\tanh^{-1}\left(\sqrt{1+\frac{c^2}{(a+bx)^2}}\right)}{b}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 330 vs. 2(49) = 98.

time = 0.67, size = 330, normalized size = 6.73

$$x\sinh^{-1}\left(\frac{c}{a+bx}\right)-\frac{(a+bx)\sqrt{\frac{a^2+c^2+2abx+b^2x^2}{(a+bx)^2}}\left(2a\left(b+\sqrt{b^2}\right)\tanh^{-1}\left(\frac{a+\sqrt{b^2}\,x-\sqrt{a^2+c^2+2abx+b^2x^2}}{c}\right)+2a\left(-b+\sqrt{b^2}\right)\tanh^{-1}\left(\frac{a-\sqrt{b^2}\,x+\sqrt{a^2+c^2+2abx+b^2x^2}}{c}\right)+c\left(\sqrt{b^2}\log\left(-a-\sqrt{b^2}\,x+\sqrt{a^2+c^2+2abx+b^2x^2}\right)+\left(-b+\sqrt{b^2}\right)\log\left(a-\sqrt{b^2}\,x+\sqrt{a^2+c^2+2abx+b^2x^2}\right)+b\log\left(ab^2+(b^2)^{3/2}x-b^2\sqrt{a^2+c^2+2abx+b^2x^2}\right)\right)\right)}{2b^2\sqrt{a^2+c^2+2abx+b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[c/(a + b*x)],x]
```

```
[Out] x*ArcSinh[c/(a + b*x)] - ((a + b*x)*Sqrt[(a^2 + c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*(2*a*(b + Sqrt[b^2])*ArcTanh[(a + Sqrt[b^2]*x - Sqrt[a^2 + c^2 +
 2*a*b*x + b^2*x^2])/c] + 2*a*(-b + Sqrt[b^2])*ArcTanh[(a - Sqrt[b^2]*x + S
qrt[a^2 + c^2 + 2*a*b*x + b^2*x^2])/c] + c*(Sqrt[b^2]*Log[-a - Sqrt[b^2]*x
+ Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]] + (-b + Sqrt[b^2])*Log[a - Sqrt[b^2]
*x + Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]] + b*Log[a*b^2 + (b^2)^(3/2)*x - b
^2*Sqrt[a^2 + c^2 + 2*a*b*x + b^2*x^2]])))/(2*b^2*Sqrt[a^2 + c^2 + 2*a*b*x
+ b^2*x^2])
```

Maple [A]
time = 2.14, size = 46, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\mathrm{arctanh}\left(\frac{1}{\sqrt{1+\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$	46
default	$-\frac{c\left(-\frac{(bx+a)\,\mathrm{arcsinh}\left(\frac{c}{bx+a}\right)}{c}-\mathrm{arctanh}\left(\frac{1}{\sqrt{1+\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*c*(-(b*x+a)/c*arcsinh(c/(b*x+a))-arctanh(1/(1+1/(b*x+a)^2*c^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*I*c*(log(I*(b^2*x + a*b)/(b*c) + 1) - log(-I*(b^2*x + a*b)/(b*c) + 1))
/b + 1/2*(2*b*x*log(c + sqrt(b^2*x^2 + 2*a*b*x + a^2 + c^2)) + a*log(b^2*x^
2 + 2*a*b*x + a^2 + c^2) - 2*(b*x + a)*log(b*x + a))/b + integrate((b^2*c*x
```

```
^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c + c^3 + (b^2*x^2 + 2*a*b*x + a
^2 + c^2)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(47) = 94$.
time = 0.39, size = 242, normalized size = 4.94

$$\frac{bx\log\left(\frac{(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)+a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a+c\right)-a\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a-c\right)-c\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2+c^2}{b^2x^2+2\,abx+a^2}}-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x*log(((b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x
 + a^2)) + c)/(b*x + a)) + a*log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x +
 a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a + c) - a*log(-b*x + (b*x + a)*sq
rt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a - c) - c*
log(-b*x + (b*x + a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 + c^2)/(b^2*x^2 + 2*a*b*
x + a^2)) - a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asinh}\left(\frac{c}{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(c/(b*x+a)),x)
```

```
[Out] Integral(asinh(c/(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(c/(b*x + a)), x)
```

Mupad [B]
time = 1.09, size = 41, normalized size = 0.84

$$\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{c^2}{{\left(a+b\,x\right)}^2}+1}\right)}{b}+\frac{\mathrm{asinh}\left(\frac{c}{a+b\,x}\right)\,\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(c/(a + b*x)),x)
[Out] (c*atanh((c^2/(a + b*x)^2 + 1)^(1/2)))/b + (asinh(c/(a + b*x))*(a + b*x))/b
```

3.369 $\int \frac{x}{\sinh^{-1}(\sinh(x))} dx$

Optimal. Leaf size=27

$$\sinh^{-1}(\sinh(x)) + \log\left(\sinh^{-1}(\sinh(x))\right)\left(-\sinh^{-1}(\sinh(x)) + x\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\right)$$

```
[Out] arcsinh(sinh(x))+ln(arcsinh(sinh(x)))*(-arcsinh(sinh(x))+x*sech(x)*(cosh(x)
^2)^(1/2))
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} dx$$

Verification is not applicable to the result.

```
[In] Int[x/ArcSinh[Sinh[x]],x]
[Out] Defer[Int][x/ArcSinh[Sinh[x]], x]
```

Rubi steps

$$\int \frac{x}{\sinh^{-1}(\sinh(x))} dx = \int \frac{x}{\sinh^{-1}(\sinh(x))} dx$$

Mathematica [A]
time = 0.36, size = 28, normalized size = 1.04

$$-\sinh^{-1}(\sinh(x))\left(-1 + \log\left(\sinh^{-1}(\sinh(x))\right)\right) + x\sqrt{\cosh^2(x)}\,\log\left(\sinh^{-1}(\sinh(x))\right)\mathrm{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcSinh[Sinh[x]],x]
[Out] -(ArcSinh[Sinh[x]]*(-1 + Log[ArcSinh[Sinh[x]]])) + x*Sqrt[Cosh[x]^2]*Log[Ar
cSinh[Sinh[x]]]*Sech[x]
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arcsinh}(\sinh(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arcsinh(sinh(x)),x)
```

```
[Out] int(x/arcsinh(sinh(x)),x)
```

Maxima [A]
time = 0.49, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.32, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{asinh}\left(\sinh\left(x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/asinh(sinh(x)),x)
```

```
[Out] Integral(x/asinh(sinh(x)), x)
```

Giac [A]
time = 0.39, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arcsinh(sinh(x)),x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.22, size = 1, normalized size = 0.04

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/asinh(sinh(x)),x)
```

```
[Out] x
```

3.370 $\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

```
[Out] arcsinh((b*x^2-1)^(1/2))^(1+n)*(b*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5871, 5783}

$$\frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{bx^2-1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
```

```
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5871

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^n}{\sqrt{-1+bx^2}}\,dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{\sinh^{-1}(x)^n}{\sqrt{1+x^2}}\,dx, x, \sqrt{-1+bx^2}\right)}{bx}\\
&= \frac{\sqrt{bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.00

$$\frac{\sqrt{bx^2}\,\sinh^{-1}\left(\sqrt{-1+bx^2}\right)^{1+n}}{b(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[Sqrt[-1 + b*x^2]]^n/Sqrt[-1 + b*x^2],x]
[Out] (Sqrt[b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]^(1 + n))/(b*(1 + n)*x)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcsinh}\left(\sqrt{x^2b-1}\right)^n}{\sqrt{x^2b-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
[Out] int(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(arcsinh(sqrt(b*x^2 - 1))^n/sqrt(b*x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(33) = 66$.
time = 0.36, size = 108, normalized size = 2.92

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\sinh\left(n\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)\right)}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="fricas")
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 - 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 - 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{2x}{\pi} & \text{for } b = 0 \wedge n = -1 \\ -ix\left(\frac{i\pi}{2}\right)^n & \text{for } b = 0 \\ \int \frac{1}{\sqrt{bx^2-1}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx & \text{for } n = -1 \\ \frac{\sqrt{bx^2}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)\operatorname{asinh}^n\left(\sqrt{bx^2-1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh((b*x**2-1)**(1/2))**n/(b*x**2-1)**(1/2),x)
```

```
[Out] Piecewise((-2*x/pi, Eq(b, 0) & Eq(n, -1)), (-I*x*(I*pi/2)**n, Eq(b, 0)), (I
ntegral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x), Eq(n, -1)), (sqrt
(b*x**2)*asinh(sqrt(b*x**2 - 1))*asinh(sqrt(b*x**2 - 1))**n/(b*n*x + b*x),
True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh((b*x^2-1)^(1/2))^n/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)^n}{\sqrt{b\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2),x)
```

```
[Out] int(asinh((b*x^2 - 1)^(1/2))^n/(b*x^2 - 1)^(1/2), x)
```

3.371 $\int \frac{1}{\sqrt{-1+bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{-1+bx^2}\right)\right)}{bx}$$

```
[Out] ln(arcsinh((b*x^2-1)^(1/2)))*(b*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5871, 5782}

$$\frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{bx^2-1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
```

```
[Out] (Sqrt[b*x^2]*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/(b*x)
```

Rule 5782

```
Int[1/(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sy
mbol] :> Simp[(1/(b*c))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*Log[a + b*A
rcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

Rule 5871

```
Int[ArcSinh[Sqrt[-1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[-1 + (b_.)*(x_)^2], x_Symbo
l] :> Dist[Sqrt[b*x^2]/(b*x), Subst[Int[ArcSinh[x]^n/Sqrt[1 + x^2], x], x,
Sqrt[-1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx^2}\sinh^{-1}\left(\sqrt{-1+bx^2}\right)}\,dx &= \frac{\sqrt{bx^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sinh^{-1}(x)}\,dx, x, \sqrt{-1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{bx^2}\log\left(\sinh^{-1}\left(\sqrt{-1+bx^2}\right)\right)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.83

$$\frac{x\log\left(\sinh^{-1}\left(\sqrt{-1+bx^2}\right)\right)}{\sqrt{bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x^2]*ArcSinh[Sqrt[-1 + b*x^2]]),x]
[Out] (x*Log[ArcSinh[Sqrt[-1 + b*x^2]]])/Sqrt[b*x^2]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arcsinh}\left(\sqrt{x^2b-1}\right)\sqrt{x^2b-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
[Out] int(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 - 1)*arcsinh(sqrt(b*x^2 - 1))), x)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.14

$$\frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2-1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 - 1) + sqrt(b*x^2)))/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2-1}\operatorname{asinh}\left(\sqrt{bx^2-1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/asinh((b*x**2-1)**(1/2))/(b*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 - 1)*asinh(sqrt(b*x**2 - 1))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arcsinh((b*x^2-1)^(1/2))/(b*x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 0.25, size = 23, normalized size = 0.79

$$\frac{\ln\left(\operatorname{asinh}\left(\sqrt{b\,x^2-1}\right)\right)\,\sqrt{x^2}}{\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(asinh((b*x^2 - 1)^(1/2))*(b*x^2 - 1)^(1/2)),x)
```

```
[Out] (log(asinh((b*x^2 - 1)^(1/2)))*(x^2)^(1/2))/(b^(1/2)*x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
            # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
     m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
     m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
     return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
           #is checked before calling the grading function that is passed.
           #but kept it here.
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
     print ("Enter grade_antiderivative for sagemath")
     print("Enter grade_antiderivative, result=",result)
     print("Enter grade_antiderivative, optimal=",optimal)
     print("type(anti)=",type(result))
     print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```